Spin Dependent Effects in Quantum Chromodynamics

by Steven David Bass

Thesis submitted for the degree of

Doctor of Philosophy

at the University of Adelaide

(Department of Physics and Mathematical Physics)

January 1992

Contents

Abstract

Polarisation data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self-protection

J. D. Bjorken, Proc. Adv. Research Workshop on QCD Hard Hadronic Processes, St. Croix, Virgin Islands (1987).

This thesis is about the theoretical interpretation of the EMC Spin Effect. EMC measured the polarised proton structure function $g_1(x)$ and found $\int_0^1 dx \ g_1(x) = 0.126 \pm 0.010 \pm 0.015$. In the naive parton model the flavour singlet contribution Δq_0 to this quantity determines the fraction of the proton's helicity which is carried by its quarks. EMC found $\Delta q_0 = 0.120 \pm 0.094 \pm 0.138$, consistent with zero!

We focus on the parton model and the role of the axial anomaly in polarised deep inelastic scattering. We argue that the factorisation of mass singularities is not sufficient to define the parton model in spin dependent QCD. (It is certainly a necessary condition.) We need an extra locality condition to properly define the parton distributions. We show that the spin dependent gluon distribution $\Delta g(x)$ is not relevant to the EMC data on $g_1(x)$. The EMC Spin Effect can be understood as a violation of Zweig's Rule, which is catalysed by the strong U(1) axial anomaly in QCD. The anomaly means that Δq_0 has nothing to do with quark spin after all. Finally, we consider the polarised photon structure function $g_1^\gamma(x)$ and derive an exact sum-rule for the first moment of $g_1^\gamma(x)$ for real photons. There is no rigorous sum-rule for $g_1(x)$ for hadronic targets.

Statement

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis. I consent to the thesis being made available for photocopying and loan if applicable, providing that due acknowledgement is given.

Acknowledgements

In the last three years I have had many interesting discussions with colleagues and friends alike about theoretical physics problems. Firstly, I would like to thank my supervisor (friend and teacher) Tony Thomas for his encouragement and guidance over the past six and a half years - and especially for being so liberal with my travel instincts. I have had a lot of fun working with Tony, and also with Boris Ioffe, Kolya Nikolaev and Tim Londergan. I would like to thank Kolya for a pleasant visit to Moscow and for teaching me a little of the art of effectiveness!

Thanks are also due to my experimental friends Roland Windmolders and Peter Schuler who introduced me to the EMC Spin Effect whilst on a visit to CERN in January 1989 and for discussions since. On the theoretical side, it is a pleasure to acknowledge spin discussions with Guido Altarelli, Stan Brodsky, Frank Close, Anatoly Efremov, John Ellis, Piet Mulders, Dick Roberts, Graham Ross, Andreas Schafer, John Taylor and Gabriele Veneziano. Especially, I would like to thank Rod Crewther, Vladimir Gribov and Angas Hurst for many theoretical discussions and teaching me what I know of field theory.

More locally (and this thesis is all about locality) I would like to thank my fellow students Alex Kalloniatis, Amanda Michels and Andreas Schreiber for many fruitful exchanges about theoretical problems. A special mention is due to the staff and fellow students of Kathleen Lumley College (my homely home for the last 5 and a half years) and especially the Master David Clements for his interest in my progress since I entered the University of Adelaide in 1984.

I would also like to thank the University of Adelaide for providing me with the George Fraser Scholarship for my Ph. D. and also a Don Stranks Memorial Travelling Fellowship. I have also enjoyed travel support from the Queen Elizabeth II Silver Jubilee Trust for Young Australians and the British Council.

Lastly (and the best is saved to last), I would like to thank my dear friend Julia just for being herself.

1. Introduction

1.1 The Deep Inelastic Scattering Experiment

Deep Inelastic Scattering (DIS) experiments provide perhaps our cleanest window on hadron structure at large momentum transfer squared Q^2. Indeed, the experiments at SLAC in the 1960's showed that hadrons behave as a collection of (almost) free fermionic *parton* constituents when they are probed at large Q^2. This remarkable observation was recently celebrated when the 1990 Nobel Prize in Physics was awarded to the experimenters Friedman, Kendall and Taylor [1]. It gave rise to the original parton ideas of Feynman [2,3] and Bjorken and Paschos [4,5].

DIS experiments involve a fixed hadronic target and a high energy (typically 100GeV) lepton beam. The beam consists of either electrons (at SLAC) or muons or, neutrinos (at CERN and Fermilab). The total inclusive cross section is measured. This is shown in Fig. 1. Here, q_μ denotes the momentum transfer to the target hadron, which has momentum p_μ.

The hadronic form factors which appear in the differential cross-section will (in general) be functions of the two scalar variables Q^2 and $\nu = p.q$. However, let us increase Q^2 and $p.q$ at the same rate so that the Bjorken variable $x = \frac{Q^2}{2p.q}$ remains fixed. Then, for $Q^2 > 2\text{GeV}^2$, the form factors behave as (structure) functions of the single variable x. (In reality, there is a slow variation of the structure functions with $\ln Q^2$.) This *scaling* property was anticipated by Bjorken [4]. It reveals a local interaction between the photon and charged elementary partons (quarks) inside the proton. It is the same effect as in Rutherford's α particle scattering experiments which revealed the nucleus inside the atom [6] – only at a much deeper level inside the nuclear "onion".

Further DIS experiments were done with (anti-)neutrino beams at BEBC (the Big European Bubble Chamber) at CERN. This time, parity odd structure functions could be measured and we were able to isolate the quark-antiquark sea inside the proton. For a review, see [7].

Our understanding of nuclear structure was then boosted by the discovery of Gross and Wilczek [8,9] (see also Georgi and Politzer [10])) that the approximate scaling observed in DIS is a prediction of any gauge theory based on the groups $SU(N)$. These theories exhibit the property of *asymptotic freedom*, whereby the quark - gluon (gauge boson) coupling vanishes logarithmically at large Q^2 ($\rightarrow \infty$). Hence, one sees the (almost) non-interacting partons of Feynman and Bjorken. Considerations of hadron spectroscopy soon lead one to a theory based on the *colour* gauge group SU(3). This theory is called Quantum Chromodynamics (QCD). For QCD to be correct it must also explain the absence of any coloured states in nature. This is the yet unsolved problem of confinement. I refer the reader to the work of Wilson [11] and Gribov [12] for a discussion of confinement (and also some optimism that the problem is solvable within QCD!).

More recently, the experimenters have gone beyond the initial DIS measurements with a nucleon target and unpolarised beams. In particular, the EMC (European Muon Collaboration), and its successors the NMC and SMC (N=New and S=Spin) at CERN have found some remarkable and unpredicted results. Firstly, they considered a nuclear (A$>$ 1) target and found that the structure functions per nucleon in a nucleus are modified compared to those obtained with a nucleon target. This result is known as the EMC Nuclear Effect and was announced in 1983 [13]. The theoretical interpretation of the EMC Nuclear Effect is still a subject of much debate and a plethora of models have been invented to fit the data (for a review see [14]). In 1987, EMC announced perhaps it most surprising result. Using a polarised muon beam and a polarised proton target, they measured the polarised structure function $g_1(x)$ [15]. In the old parton model of Feynman, the first moment of $g_1(x)$ determines the fraction of the proton's helicity which is carried by its quarks. Hence, there was great surprise in the high energy physics community when this "spin content" came out to be zero! This second EMC measurement is known as the EMC Spin Effect. It is the starting point for our story in this thesis.

1.2 Plan of the Thesis

However, before we can understand the EMC results it will be necessary to first review some of the kinematics and formalism of DIS. This is the subject of chapter 2, in which we describe the EMC $g_1(x)$ experiment. Chapter 3 reviews the QCD description of DIS in terms of the operator product expansion (OPE) and the renormalisation group. A more detailed treatment of this subject is to be found in the excellent reviews by Buras [16] and the textbooks of Muta [17] and Yndurain [18]. This formal description of DIS leads us to the QCD Improved Parton Model (IPM), which is based on the factorisation of mass singularities (chapter 4). The IPM has application to a whole host of large momentum hadronic interactions (see eg. the reviews [19, 20]).

Efremov and Teryaev [21] and Altarelli and Ross [22] have considered $g_1(x)$ in the context of the IPM. They proposed the idea that one can reconcile the EMC Spin Effect with the quark model expectation by invoking a large polarised gluon component $\Delta g \sim 5$ in the proton at the scale of the experiment. At $Q^2 \sim 10\text{GeV}^2$, 5 units of the proton's spin $\frac{1}{2}$ might reside in gluon helicity! Alternatively, Brodsky et al. [23] have speculated that the strange quark sea may carry a significant negative polarisation $\Delta s = -0.19$ in the spin $\frac{1}{2}$ proton. We examine both of these ideas in chapters 4 and 5. Chapter 4 concentrates on the spin dependent gluon distribution $\Delta g(x)$. We examine the gluon contribution to $g_1(x)$ in a variety of factorisation schemes [24] and find that it contributes to $g_1(x)$ only at very small x - essentially outside the measured data [25, 26]. (The calculations presented here strengthen the conclusions of ref. [27].)

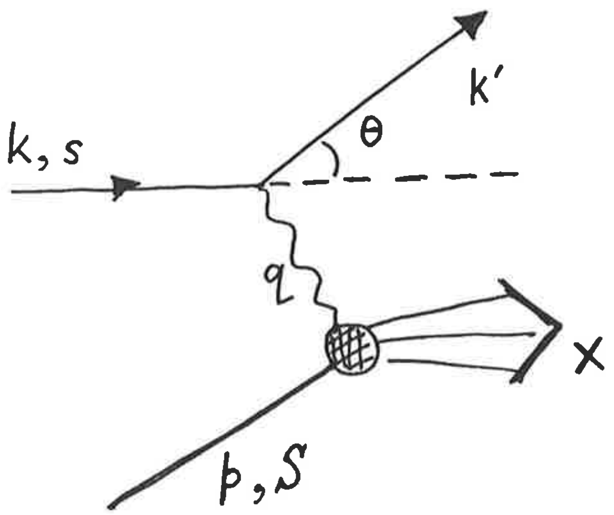
In Chapter 5 we discuss the axial anomaly in QCD and its consequences for polarised DIS. The anomaly means that $g_1(x)$ does not measure quark spin after all. Rather, it catalyses the violation of OZI (Zweig's Rule) that was measured by EMC [28, 29]. Furthermore, the anomaly implies that factorisation per se is not sufficient to define the IPM. We need an extra locality condition to completely define the gluon distributions consistent with gauge invariance [30].

Chapter 6 details the polarised photon structure function $g_1^\gamma(x)$. The polarised photon is an ideal physical system in which to discuss the axial anomaly in polarised DIS. For a real photon target we derive an exact sum-rule for $g_1^\gamma(x)$ [31].

In Chapter 7 we summarise our conclusions.

Let us now begin our story.

Figure 1. Deep Inelastic Scattering. Here, we choose to work in the LAB frame where the proton target (mass M) has momentum $p^\mu = (M; 0_T, 0)$ and polarisation S^μ. The incident muon (mass m) carries momentum $k^\mu = (E; \vec{k})$ and polarisation s^μ. It is scattered through an angle θ so that the scattered muon emerges with momentum $k'^\mu = (E'; \vec{k}')$. The exchanged photon carries momentum $q^\mu = (k - k')^\mu$. We measure the total inclusive cross section so that the final state hadrons X are not separated.



2. Deep Inelastic Scattering

2.1 Kinematics

In this section we consider DIS with a muon beam and proton target. The generalisation to a neutrino beam is straightforward; it is found in [32].

Muon nucleon scattering is described at leading order in electromagnetic interactions by the one photon exchange diagram of Figure 1. Here, we choose to work in the LAB frame where the proton target (mass M) has momentum $p^\mu = (M; 0_T, 0)$ and polarisation S^μ. The incident muon (mass m) carries momentum $k^\mu = (E; \vec{k})$ and polarisation s^μ. It is scattered through an angle θ so that the scattered muon emerges with momentum $k'^\mu = (E'; \vec{k}')$. The exchanged photon carries momentum $q^\mu = (k-k')^\mu$. The scattering is then characterised by the two invariants $Q^2 = -q^2$ and $\nu = p.q$ ($\nu = m(E - E')$ in the LAB frame). The Bjorken variable x is defined by the ratio $x = \frac{Q^2}{2\nu}$. We are interested in the inclusive hadronic cross section so that the final state hadrons X are not separated. (We sum over all accessible hadronic final states and scattered muon spins.)

The final state hadrons have invariant mass squared (centre of mass energy squared)

$$W^2 = (p+q)^2 = M^2 + Q^2(\frac{1-x}{x}) \geq M^2 \tag{1}$$

by energy momentum conservation. This clearly constrains $0 \leq x \leq 1$. (The case $x \leq 0$ corresponds to the process $p \to \gamma^* X$.) When $x = 1$ we have elastic scattering $W^2 = M^2$.

The differential cross section for the one photon exchange process (figure 1) is given by

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2}{q^4}\frac{E'}{E} L_{\mu\nu} W^{\mu\nu} \tag{2}$$

Here, α is the fine structure constant; $L_{\mu\nu}$ and $W_{\mu\nu}$ describe the muon and hadronic vertices respectively. Since the muon is pointlike we can write an exact expression for

$L_{\mu\nu}$, viz.

$$L_{\mu\nu} = \sum_{s'} [\overline{u}_a(k,s)(\gamma_\mu)_{ab} u_b(k',s')][\overline{u}_c(k',s')(\gamma_\nu)_{cd} u_d(k,s)]$$
$$= 2\left[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k.k' - m_\mu^2) + i\epsilon_{\mu\nu\rho\sigma} q^\rho s^\sigma\right] \qquad (3)$$

Here, $\epsilon_{0123} = +1$.

The hadronic tensor $W^{\mu\nu}$ is determined by symmetry arguments. Since the proton is not elementary we cannot write down an exact expression for $W^{\mu\nu}$, in contrast to the situation where we scatter from a lepton target. We choose to write $W_{\mu\nu}$ as a sum of symmetric and antisymmetric contributions; viz. $W_{\mu\nu} = W^S_{\mu\nu} + iW^A_{\mu\nu}$. Then the cross-section receives finite contributions from $L^{\mu\nu}_S W^S_{\mu\nu}$ and $L^{\mu\nu}_A W^A_{\mu\nu}$ where $L^S_{\mu\nu}$ and $L^A_{\mu\nu}$ are symmetric and antisymmetric contributions to $L_{\mu\nu}$ respectively. (The contraction of terms with mixed symmetry (S and A) vanishes). The structure of $W_{\mu\nu}$ is fixed by covariance, even parity and current conservation ($q^\mu W_{\mu\nu} = 0$). It reads

$$W^S_{\mu\nu} = \frac{1}{M}(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2})F_1(x,Q^2) + \frac{1}{M\nu}(p_\mu - \frac{p.q}{q^2}q_\mu)(p_\nu - \frac{p.q}{q^2}q_\nu)F_2(x,Q^2) \qquad (4)$$

and

$$W^A_{\mu\nu} = \frac{\epsilon_{\mu\nu\lambda\sigma} q^\lambda S^\sigma}{\nu} g_1(x,Q^2) + \frac{\epsilon_{\mu\nu\lambda\sigma} q^\lambda(\nu S^\sigma - q.Sp^\sigma)}{\nu^2} g_2(x,Q^2) \qquad (5)$$

Current conservation at the muon vertex ($q_\mu L^{\mu\nu} = 0$) implies that the terms in $W_{\mu\nu}$ which are proportional to q_μ or q_ν do not contribute to the cross section, equ.(2). Hence, we would be justified to ignore these terms.

The form factors contain all of the target dependent information. In unpolarised scattering, where we average over the target polarisation, the antisymmetric part of $W_{\mu\nu}$ vanishes and we measure the two form factors $F_1(x,Q^2)$ and $F_2(x,Q^2)$. The two form factors $g_1(x,Q^2)$ and $g_2(x,Q^2)$ contribute to the cross section in the product of $W^A_{\mu\nu}$ and the antisymmetric part of $L_{\mu\nu}$. They can only be measured with a polarised nucleon target and a polarised muon (or electron) beam. If the target were pointlike (eg. a muon) then the scattering is elastic and the structure functions become trivial. For an elementary fermion target we find $F_2(x) = x\delta(x-1)$ and $g_1(x) = \delta(x-1)$.

Deep inelastic scattering occurs when both $Q^2 \gg M^2$ and $W^2 \gg M^2$. That is, we are out of the resonance region where W^2 may coincide with the mass squared of one of the excited nucleon resonances. We are free to choose the z axis in our LAB frame so that $q^\mu = (q^0; 0_T, q^3)$. Since $-q^2 = Q^2$ we may write

$$q^\mu = (q^0; 0_T, q^0\sqrt{1 + \frac{Q^2}{q_0^2}}) \tag{6}$$

whereby $q_+ = \frac{1}{\sqrt{2}}(q_0 + q_3) =$ finite and $q_- = \frac{1}{\sqrt{2}}(q_0 - q_3) \to \infty$ in the deep inelastic (Bjorken) limit. The Bjorken variable becomes $q_+ = -p_+ x$.

In the kinematic region of DIS the structure functions behave as functions of x alone. The Q^2 dependence becomes almost trivial and is observed only as a slow logarithmic variation $\sim \ln Q^2$. This scaling property inspired Feynman to deduce the parton model of the nucleon.

Feynman supposed that the nucleon can be viewed as a box of long lived elementary partons. The partons are described by distributions so that $q^\uparrow(\xi)$ determines the probability of finding a parton (quark) with momentum fraction $p_+^q = \xi p_+$ which is polarised in the same direction as the proton polarisation. Similarly, $q^\downarrow(\xi)$ is the p_+ distribution of quarks polarised in the opposite direction to the proton; $\overline{q}^\uparrow(\xi)$ and $\overline{q}^\downarrow(\xi)$ denote the corresponding antiquark distributions. In the parton model, muons scatter elastically from the pointlike partons at large momentum transfer. Hence, $p_+^q = -q_+ = \xi p_+$ and we identify ξ as the Bjorken variable. The total hadronic cross section is the incoherent sum of elastic muon-parton scatterings.

Thus, one can write down parton model expressions for the structure functions

$$F_2(x) = 2xF_1(x) = x\sum_q \mathrm{e_q}^2(q+\overline{q})(x) \tag{7}$$

and

$$g_1(x) = \frac{1}{2}\sum_q \mathrm{e_q}^2 \Delta q(x) \tag{8}$$

Here, $q(x) = (q^{\uparrow} + q^{\downarrow})(x)$ and $\overline{q}(x) = (q^{\uparrow} + q^{\downarrow})(x)$ are the unpolarised quark and anti-quark distributions respectively;

$$\Delta q(x) = (q^{\uparrow} + \overline{q}^{\uparrow})(x) - (q^{\downarrow} + \overline{q}^{\downarrow})(x) \tag{9}$$

is the polarised quark distribution. Ignoring a small contribution from heavy charm c quarks in the proton we sum over the light quark flavours up u, down d and strange s; $\mathrm{e_q}$ denotes the quark charges. It follows that $g_1(x)$ measures the fraction of the proton's spin that is carried by its quarks in this simple parton model. The structure function $g_2(x)$ does not have a simple interpretation in the parton model [33, 34]. (It is sensitive to the transverse momentum of the partons [35, 36].) In DIS experiments with (anti-)neutrinos there is also a parity odd term in $W_{\mu\nu}$. The new structure functions $F_3(x)$ and $g_3(x)$ are described by the *valence* quark distributions $q_V(x) = (q - \overline{q})(x)$ and

$$\Delta q_V(x) = (q^{\uparrow} - \overline{q}^{\uparrow})(x) - (q^{\downarrow} - \overline{q}^{\downarrow})(x) \tag{10}$$

respectively. These valence distributions measure the effect of subtracting out the Dirac sea of quark and anti-quark excitations in the proton. We now turn the EMC measurement of $g_1(x)$.

2.2 The EMC Spin Effect

Let us consider a muon beam with definite polarisation and initial energy E, which scatters from a polarised proton target in a DIS experiment. The scattered muon will have final state energy $E^{'}$ and we choose the LAB frame ($p^{\mu} = (M; 0_T, 0)$). We let $\uparrow\downarrow$ denote the longitudinal beam polarisation and $\Uparrow\Downarrow$ denote (longitudinal) polarisation of the target proton. Then the differential cross sections are

$$\frac{d^2\sigma \uparrow\Uparrow}{d\Omega dE^{'}} + \frac{d^2\sigma \uparrow\Downarrow}{d\Omega dE^{'}} = \frac{8\alpha^2 (E^{'})^2}{mQ^4}\left[2\sin^2\frac{\theta}{2}\ F_1(x,Q^2) + \frac{M^2}{\nu}\cos^2\frac{\theta}{2}\ F_2(x,Q^2)\right] \tag{11}$$

and

$$\frac{d^2\sigma \uparrow\Uparrow}{d\Omega dE^{'}} - \frac{d^2\sigma \uparrow\Downarrow}{d\Omega dE^{'}} = \frac{4\alpha^2 E^{'}}{Q^2 E\nu}\left[(E + E^{'}\cos\theta)\ g_1(x,Q^2) - 2xM\ g_2(x,Q^2)\right] \tag{12}$$

Equ.(11) is twice the unpolarised cross section (which is defined by averaging over initial target polarisations). The ratio of the two cross sections defines the asymmetry

$$A(x,Q^2) = \frac{\frac{d^2\sigma\uparrow\Uparrow}{d\Omega dE^{'}} - \frac{d^2\sigma\uparrow\Downarrow}{d\Omega dE^{'}}}{\frac{d^2\sigma\uparrow\Uparrow}{d\Omega dE^{'}} + \frac{d^2\sigma\uparrow\Downarrow}{d\Omega dE^{'}}} \tag{13}$$

The EMC (European Muon Collaboration) measured $A(x,Q^2)$ for x between 0.01 and 0.7, and Q^2 between 1.5 GeV^2 and 70 GeV^2 [15, 37, 38, 39]. The mean Q^2 for the experiment was 10.7 GeV^2. The muon beam was a secondary beam with the primary protons taken from the CERN SpS. Muon beam energies of 100 GeV, 120 GeV and 200 GeV were used in the experiment. The target consisted of two sections polarised in opposite directions. Each was exposed simultaneously to the beam to reduce systematic errors. The beam polarisation was calculated as $(82 \pm 6)\%$ and the target polarisation was typically 75-80%. With a beam energy of 100 GeV the contribution of $g_2(x)$ to the asymmetry is suppressed with respect to $g_1(x)$ by the kinematic factor $\frac{M}{E} \sim 0.01$. Hence, it is lost among the errors and can be ignored. (The way to measure $g_2(x)$ is with a proton target polarised at right angles to the beam. In this case there is no large kinematic suppression factor with respect to $g_1(x)$ [33, 39].) When the data on $A(x, Q^2)$ was combined with measurements of the unpolarised structure functions EMC determined $g_1(x)$. The EMC data on $g_1(x)$ is shown in Fig. 2, along with earlier data from SLAC [40]. Here, the small x extrapolation was made (following Regge theory [41]) as $g_1(x) \sim x^{-0.12}$ as $x \to 0$. No significant deviation from scaling was observed in the data.

The parton model expression for $g_1(x)$ (equ.(8)) may be written as

$$g_1(x) = \frac{1}{12}\Delta q_3(x) + \frac{1}{36}\Delta q_8(x) + \frac{1}{9}\Delta q_0(x) \tag{14}$$

when we substitute in the values of the quark charges (viz. $\mathrm{e_u} = +\frac{2}{3}$, $\mathrm{e_d} = -\frac{1}{3}$ and $\mathrm{e_s} = -\frac{1}{3}$). Here, $\Delta q_3(x) = \Delta u(x) - \Delta d(x)$, $\Delta q_8(x) = \Delta u(x) + \Delta d(x) - 2\Delta s(x)$ and $\Delta q_0(x) = \Delta u(x) + \Delta d(x) + \Delta s(x)$. In the parton model $\Delta q = \int_0^1 dx\ \Delta q(x)$ measures the fraction of the proton's polarisation that is carried by quarks and antiquarks with flavour q. Hence, $\Delta q_0 = \Delta u + \Delta d + \Delta s$ is the quark spin contribution to the proton's spin.

As I explain in the next chapter, the first moments of $\Delta q_3(x)$ and $\Delta q_8(x)$ can be deduced from the low energy neutron and hyperon beta decays respectively. One finds $\Delta q_3 = \Delta u - \Delta d = 1.254 \pm 0.006$ and $\Delta q_8 = \Delta u + \Delta d - 2\Delta s = 0.688 \pm 0.035$ [42]. Hence, Δq_0 can be extracted from the $g_1(x)$ data.

Strange quarks are found to play a very small role in unpolarised DIS. It is tempting to think that the same result may hold in the polarised case too. This view was taken by Ellis and Jaffe [43] who supposed that $\Delta s = 0$ and predicted $\int_0^1 dx\ g_1(x) = 0.200 \pm 0.005$ corresponding to $\Delta q_0 = 0.69 \pm 0.03$. (The rest of the proton's spin would be carried by quark and gluon orbital angular momentum and gluonic S_z.) This prediction is known as the Ellis-Jaffe sum rule.

The surprise of the EMC experiment was that the sum rule is badly broken. After the small x extrapolation EMC found

$$\int_0^1 dx\ g_1(x) = 0.126 \pm 0.010(stat.) \pm 0.015(syst.) \tag{15}$$

which is two standard deviations from the Ellis-Jaffe prediction. It corresponds to

$$\Delta q_0 = 0.120 \pm 0.094(stat.) \pm 0.138(syst.) \tag{16}$$

ie. compatible with zero [44]! For each flavour, one finds $\Delta u = 0.78$, $\Delta d = -0.47$ and $\Delta s = -0.19$. Close and Roberts [45] have suggested that one should be careful about adopting the Regge extrapolation for x as large as ~ 0.02 and that (perhaps) this might explain some of the discrepancy. After all, the error bars on $g_1(x)$ are quite large at small x. The convergence of the first moment $\int_{x_m}^1 dx\ g_1(x)$ as a function of x_m is shown in Fig. 3. The large errors in the low x data points do not have a significant impact on the integral. (Remember that Fig. 2 is a logarithmic plot in x so that the errors are not as large as might appear at a casual glance. This point has been emphasised in ref. [46].) For this reason, we shall accept the EMC result as valid. The new SMC experiment at CERN [47] will measure the low x data points with greater precision and we look forward to seeing their results.

The EMC result (if it is true) poses problems for our understanding of proton structure. Indeed, the eightfold way patterns of Gell-Mann and Ne'eman which so beautifully described hadron spectroscopy assume that the (constituent) quarks' S_z add up to the proton's S_z. If the EMC spin data had been available in the 1960's we might have abandoned the quark model altogether! The EMC spin measurement has sometimes been called the proton "spin crisis" [40].

Before we can hope to understand Δq_0 in QCD we first need to familiarise ourselves with the formal theory of DIS: the operator product expansion and renormalisation group. This is the subject of the next chapter.

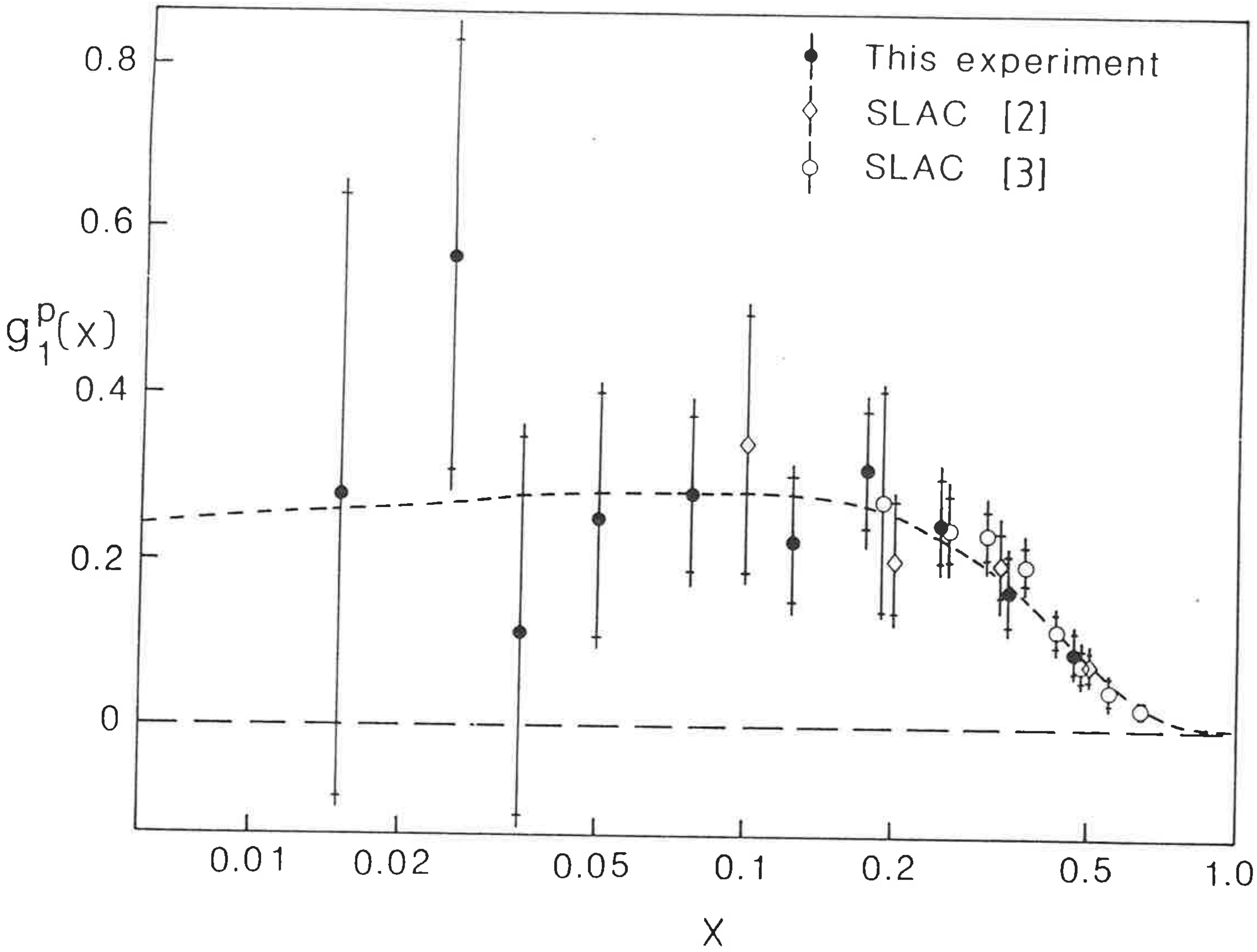


Figure 2. The EMC data on $g_1(x)$ [37].

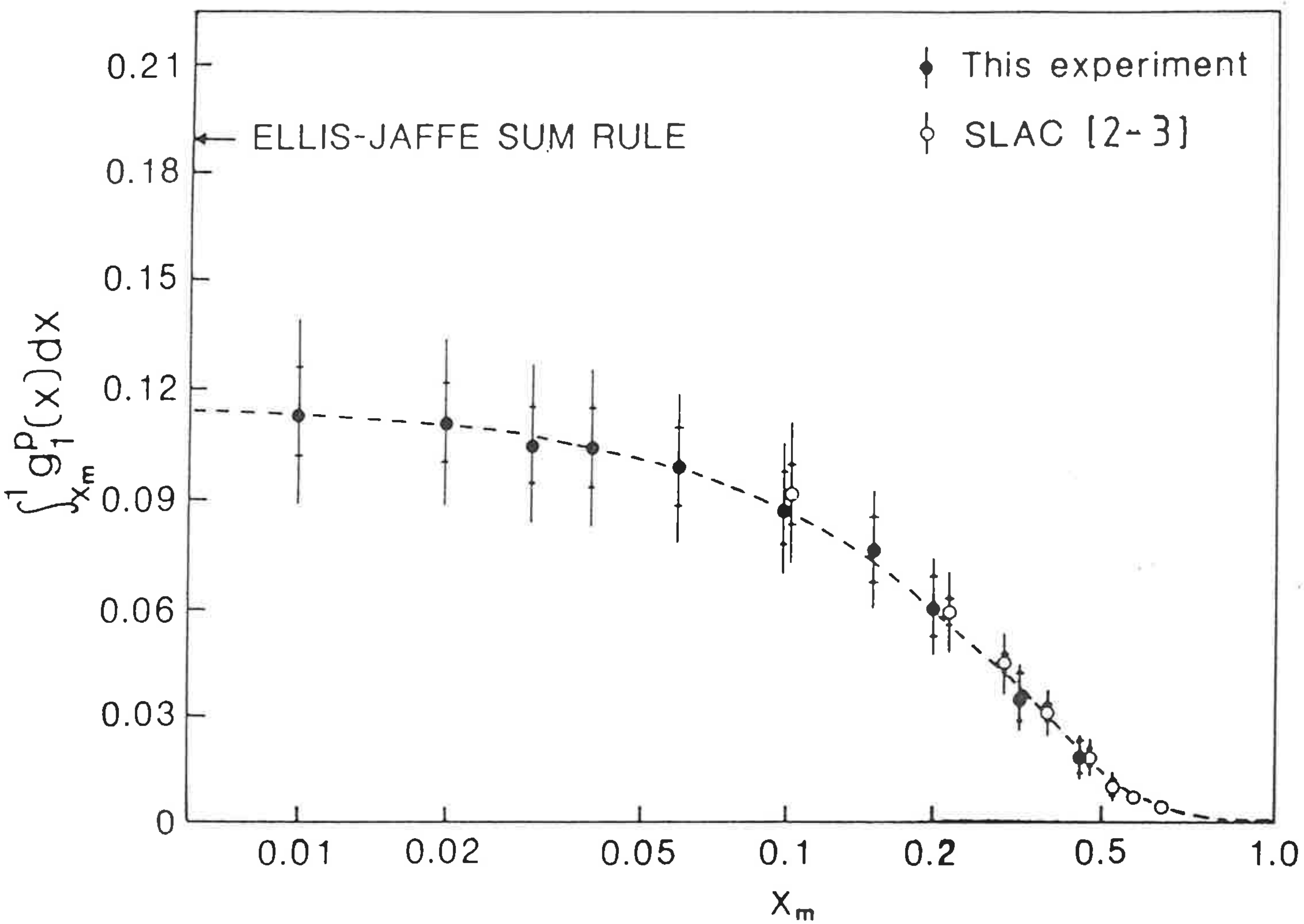


Figure 3. The convergence of the first moment $\int_{x_m}^1 dx\, g_1(x)$ as a function of x_m in the EMC data [37].

3. A Formal Description of DIS

3.1 Light Cone Dominance

The general expression for the hadronic tensor $W_{\mu\nu}$ (equs. (4) and (5)) is given by

$$W_{\mu\nu} = \frac{1}{2\pi}(2\pi)^4 \delta(p + q - p_X) \sum_X < p, S|j_\mu(0)|X >< X|j_\nu(0)|p, S > \qquad (17)$$

Here, $j_\mu(x)$ is the electromagnetic current operator and we sum over all accessible final hadronic states $|X >$. Translational invariance means that the choice of origin in $j_\mu(0)$ is arbitrary. The energy momentum conserving delta function can be absorbed into the proton matrix elements; we take out the complete set set of hadronic states and obtain

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{2\pi} \int d^4z \; e^{iq.z} < p, S|J_\mu(z)J_\nu(0)|p, S >_c \\ &= \frac{1}{2\pi} \int d^4z \; e^{iq.z} < p, S|[J_\mu(z), J_\nu(0)]|p, S >_c \end{aligned} \qquad (18)$$

where the second step follows from energy momentum conservation. The extra term in the commutator corresponds to an unphysical $\delta(p - q + p_X)$ term (or $W^2 \leq M^2$). Hence, it does not contribute. We are only interested in the connected matrix element where the photon interacts with the target (and not the vacuum). This is indicated by the subscript $_c$ on the proton matrix elements.

The hadronic tensor $W_{\mu\nu}$ is equal to the absorptive part of the forward Compton scattering amplitude $T_{\mu\nu}$ where

$$T_{\mu\nu}(q,p) = T_{\nu\mu}(-q,p) = i \int d^4z \; e^{iq.z} < p, S|T(j_\mu(z)j_\nu(0))|p, S > \qquad (19)$$

We have

$$\begin{aligned} W_{\mu\nu} &= \frac{1}{\pi}\text{Abs } T_{\mu\nu} \\ &= \frac{1}{\pi}\frac{1}{2i}[T_{\mu\nu}(q_0 + i\epsilon) - T_{\mu\nu}(q_0 - i\epsilon)] \qquad (\epsilon = 0^+) \end{aligned} \qquad (20)$$

The Riemann Lebesgue Theorem tells us that the integral in the the definition of $W_{\mu\nu}$ and $T_{\mu\nu}$ receives a finite contribution only when there is small oscillation in the exponential (viz. $q.z \sim 1$). We choose a frame where $q^\mu = (q^0; 0_T, q^3)$ so that $q_+ = \frac{1}{\sqrt{2}}(q_0 + q_3) =$ finite and $q_- = \frac{1}{\sqrt{2}}(q_0 - q_3) \to \infty$ in the Bjorken limit. The dominant (finite) contribution to $T_{\mu\nu}$ then comes from

$$q.z = q_+ z_- + q_- z_+ - z_T . q_T \sim 1 \tag{21}$$

or $z_- \sim \frac{1}{q_+}$ and $z_+ \sim \frac{1}{q_-} \to 0$ ($q_T = 0$ in our frame of reference). Since $q_+ = -p_+ x$ it follows that z_- need not be small, particularly at small x. A further constraint follows from the causality condition

$$[j_\mu(z), j_\nu(0)] = 0 \quad \text{with } z^2 < 0 \tag{22}$$

We get a non-zero contribution to the hadronic cross section only when

$$z^2 = 2z_+ z_- - z_T^2 \sim \frac{2}{q_- q_+} - z_T^2 \sim \frac{4}{q^2} - z_T^2 \geq 0 \tag{23}$$

or when $z_T^2 \to 0$. Thus, $z^2 \to 0$ in the Bjorken limit and deep inelastic scattering is said to probe QCD physics "on the light cone".

3.2 The Operator Product Expansion

The product of two currents $j_\mu(z)$ and $j_\nu(0)$ has to be treated carefully in a renormalised quantum field theory. Indeed, the operator product may become singular when either $z_\mu \to 0$ or $z^2 \to 0$. This singular behaviour is treated systematically when one uses the Wilson operator product expansions [48]. There is a well defined expansion of the current operator product in each of the short distance and light cone limits. The light cone expansion (henceforth denoted OPE) is relevant to deep inelastic scattering.

On the light cone it may be shown that the product of two electromagnetic currents is equal to an infinite sum over gauge invariant, local, composite renormalised operators - each of which is multiplied by a coefficient function of z^2, which may be singular is at $z^2 = 0$. The idea is that we probe the quark and gluonic structure of the proton at large Q^2 and receive contributions from the proton matrix elements of all the quark and gluonic operators with the right quantum numbers. The light cone expansion has the form

$$T[j_\mu(z)j_\nu(0)] = \tau^S_{\mu\nu}(z,0) + i\tau^A_{\mu\nu}(z,0) \tag{24}$$

where the symmetric part is given by

$$\begin{aligned}
&\tau^S_{\mu\nu}(z,0) = \\
&(-g_{\mu\nu}\partial^2 + \partial_\mu\partial_\nu)\frac{1}{z^2 - i\epsilon z_0} \sum_{n=2,even}^{\infty} \sum_i C^n_{i,1}(z^2 - i\epsilon z_0, g) O_i^{\alpha\mu_1\ldots\mu_n}(0) z_{\mu_1}\ldots z_{\mu_n} \\
&-(g_{\mu\lambda}g_{\nu\sigma}\partial^2 - g_{\mu\lambda}\partial_\nu\partial_\sigma - g_{\nu\sigma}\partial_\mu\partial_\lambda + g_{\mu\nu}\partial_\lambda\partial_\sigma) \\
&\sum_{n=0,even}^{\infty} \sum_i C^n_{i,2}(z^2 - i\epsilon z_0, g) O_i^{\alpha\lambda\sigma\mu_1\ldots\mu_n}(0) z_{\mu_1}\ldots z_{\mu_n}
\end{aligned} \tag{25}$$

and the anti-symmetric part is given by

$$\begin{aligned}
&\tau^A_{\mu\nu}(z,0) = \\
&-\epsilon_{\mu\nu\lambda\sigma}\partial^\lambda \frac{1}{z^2 - i\epsilon z_0} \sum_{n=0,even}^{\infty} \sum_i E^n_{i,1}(z^2 - i\epsilon z_0, g) R_{i,1}^{\alpha\sigma\mu_1\ldots\mu_n}(0) z_{\mu_1}\ldots z_{\mu_n} \\
&-(\epsilon_{\mu\rho\lambda\sigma}\partial_\nu\partial^\rho - \epsilon_{\nu\rho\lambda\sigma}\partial_\mu\partial^\rho - \epsilon_{\mu\nu\lambda\sigma}\partial^2) \\
&\sum_{n=1,odd}^{\infty} \sum_i E^n_{i,2}(z^2 - i\epsilon z_0, g) R_{i,2}^{\alpha\lambda\sigma\mu_1\ldots\mu_n}(0) z_{\mu_1}\ldots z_{\mu_n}
\end{aligned} \tag{26}$$

Here, $O_i^{\alpha\mu_1\ldots\mu_n}(0)$, $R_{i,1}^{\alpha\sigma\mu_1\ldots\mu_n}(0)$ and $R_{i,2}^{\alpha\lambda\sigma\mu_1\ldots\mu_n}(0)$ denote the local operators; the C_i^n and E_i^n are the singular Wilson coefficients. The sum over n is determined by the crossing symmetry of $T_{\mu\nu}$ (equ.(19)). Only even n operators contribute to $\tau^S_{\mu\nu}(z,0)$. The anti-symmetric term $\tau^A_{\mu\nu}(z,0)$ receives contributions only from operators with an odd number of indices. The sum over i runs over quark and gluonic operators, which we label as F and V respectively. It comes from the sum over quark charges in $j_\mu(z)j_\nu(0)$, viz.

$$\sum_{\mathrm{q}} \mathrm{e}_{\mathrm{q}}^2 = \frac{1}{6}\lambda^3 + \frac{1}{6\sqrt{3}}\lambda^8 + \frac{1}{3}\sqrt{\frac{2}{3}}\lambda^0 \tag{27}$$

where λ^a are the Gell-Mann matrices normalised so that Tr $\lambda^a\lambda^b = 2\delta^{ab}$. The quark operators may be flavour non-singlet (carrying a λ^3 or λ^8) or flavour singlet (carrying a λ^0). The gluonic operators are all flavour singlet.

The strength of the singularity in the coefficient functions is determined by power counting arguments. The renormalised current operators $j_\mu(z)$ have canonical dimension +3. We let $d_{Op}(n)$ denote the dimension of an operator $Op(n)$ on the right hand side of the operator product expansion which is accompanied by n factors of z_μ. Dimensional consistency requires that the corresponding Wilson coefficient behaves as $\sim (z^2)^{(-6+d_{Op}(n)-n)/2}$ for $z^2 \to 0$. The strongest singularity occurs when $\tau = d_{Op}(n) - n$ is a minimum. We call τ the *twist* of the operator and n is its *spin*. DIS is dominated by the operators with lowest twist (=2) in the OPE.

The lowest dimension operators that can appear in the symmetric part of the OPE $\tau^S_{\mu\nu}(z,0)$ are the quark operator ($S\ \overline{q}\gamma_\mu D_\nu q$) and the gluonic operator (S Tr $G_{\mu\alpha}G^\alpha{}_\nu$). Here, $G_{\mu\nu}$ is the gluon field tensor, "S" denotes symmetrisation over μ and ν, and $D^\mu = \partial^\mu + igA^\mu_a t^a$ is the gauge covariant derivative where t^a are the group generators of SU(3) colour. For the gluonic operator, we take the trace over SU(3) colour in order to preserve gauge invariance. These operators have canonical dimension four and are accompanied by two factors of z_μ in the OPE. They are twist two. The other twist two operators which contribute to $\tau^S_{\mu\nu}(z,0)$ are constructed by adding an arbitrary even number of gauge covariant derivatives. For each accompanying (dimension +1) gauge covariant derivative there is an extra factor of z_μ. We find (n = even) [9]

$$
\begin{aligned}
O_F^{a\mu_1...\mu_n}(0) &= i^{n-1} S\ \overline{q}\gamma^{\mu_1}D^{\mu_2}...D^{\mu_n}\frac{\lambda^a}{2}q \qquad (\mathrm{a}=3,8,0) \\
O_V^{0\mu_1...\mu_n}(0) &= i^{n-2} S\ \frac{1}{2}\mathrm{Tr}\ G^{\mu_1\alpha}D^{\mu_2}...D^{\mu_{n-1}}G_\alpha^{\ \mu_n}
\end{aligned}
\tag{28}
$$

Here, $\frac{\lambda^a}{2}$ are the generators of SU(N) flavour; S denotes symmetrisation over $\mu_1, ..., \mu_n$. These operators have the proton matrix elements

$$
< P, s|O_i^{a\mu_1...\mu_n}(0)|P, s >_c = a_i^{a,n} p^{\mu_1}...p^{\mu_n} \tag{29}
$$

The flavour singlet ($a = 0$) quark operator and the gluonic operator mix under renormalisation.

The lowest dimension operator which can contribute to $\tau^A_{\mu\nu}(z, 0)$ is the axial vector current ($\overline{q}\gamma_\mu\gamma_5 q$). It has dimension three and is accompanied by one factor of z_μ; it is also twist two. There is no spin-one, gauge invariant, local gluonic operator which could contribute to $\tau^A_{\mu\nu}(z, 0)$ with dimension less than five. This operator (Tr $G_{\alpha\beta}D_\mu\tilde{G}^{\alpha\beta}$) has twist four. The twist two operators which contribute to $\tau^A_{\mu\nu}(z, 0)$ are all of R_1 variety and odd spin. They are (n = even) [49]

$$
\begin{aligned}
R_{1,F}^{a\sigma\mu_1...\mu_n}(0) &= i^n S\ \overline{q}\gamma^\sigma\gamma_5 D^{\mu_1}...D^{\mu_n}\frac{\lambda^a}{2}q \qquad (\mathrm{a}=3,8,0) \\
R_{1,V}^{0\sigma\mu_1...\mu_n}(0) &= i^{n-1} S\ \frac{1}{2}\epsilon^{\sigma\alpha\beta\gamma}\mathrm{Tr}\ G_{\beta\gamma}D^{\mu_1}...D^{\mu_{n-1}}G_\alpha^{\ \mu_n}
\end{aligned}
\tag{30}
$$

These R_1 operators have proton matrix elements

$$
< p, S|R_{1,i}^{a\sigma\mu_1...\mu_n}(0)|p, S >_c = b_i^{a,n+1}[S^\sigma p^{\mu_1}...p^{\mu_n} + \frac{n}{n+1}(p^\sigma S^{\mu_1} - S^\sigma p^{\mu_1})p^{\mu_2}...p^{\mu_n}] \tag{31}
$$

In equs.(30) and (31) S denotes symmetrisation over $\sigma, \mu_1, ..., \mu_n$.

The R_2 operators start at twist three. They are (n = odd) [49]

$$R_{2,F}^{a\lambda\sigma\mu_1...\mu_n}(0) = i^{n+1}S^{'}\ \overline{q}\gamma^\lambda\gamma_5 D^\sigma D^{\mu_1}...D^{\mu_n}\frac{\lambda^a}{2}q \quad (\mathrm{a} = 0, ..., 8)$$
$$R_{2,V}^{0\lambda\sigma\mu_1...\mu_n}(0) = i^n S^{'}\ \frac{1}{2}\epsilon^{\sigma\alpha\beta\gamma}\mathrm{Tr}\ G_{\beta\gamma}D^{\mu_1}...D^{\mu_n}G_\alpha^{\ \lambda} \tag{32}$$

which have proton matrix elements

$$< p, S|R_{2,i}^{a\lambda\sigma\mu_1...\mu_n}(0)|p, S >_c = d_i^{a,n+2}\frac{1}{2}(p^\sigma S^\lambda - S^\sigma p^\lambda)p^{\mu_1}...p^{\mu_n} \tag{33}$$

In equ.(32) $S^{'}$ is used to denote symmetrisation over $\sigma, \mu_1, ..., \mu_n$ and antisymmetrisation over λ and σ.

The forward Compton amplitude $T_{\mu\nu}$ is given by the Fourier transform of the OPE (equs.(24-26)). In twist two approximation it reads

$$\begin{aligned}
T_{\mu\nu} = i \int d^4z\ e^{iq.z} < P, s|T[j_\mu(z)j_\nu(0)]|P, s >_c = \\
2i\Bigg[[g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}] \sum_{n=2,even}^{\infty} \sum_{i=F,V} a_i^{a,n}\tilde{C}_{i,1}^n(-q^2)\ \omega^n \\
- [g_{\mu\nu} + p_\mu p_\nu\frac{q^2}{(p.q)^2} - \frac{p_\mu q_\nu + p_\nu q_\mu}{p.q}] \sum_{n=2,even}^{\infty} \sum_{i=F,V} a_i^{a,n}\tilde{C}_{i,2}^n(-q^2)\ \omega^n \\
+ \frac{i\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma}{p.q} \sum_{n=1,odd}^{\infty} \sum_{i=F,V} b_i^{a,n}\tilde{E}_{i,1}^n(-q^2)\ \omega^n \\
+ \frac{i\epsilon_{\mu\nu\lambda\sigma}q^\lambda[s^\sigma p.q - p^\sigma q.s]}{(p.q)^2} \sum_{n=3,odd}^{\infty} \sum_{i=F,V} (\frac{1}{2}d_i^{a,n}\tilde{E}_{i,2}^n(-q^2) - b_i^{a,n}\frac{n-1}{n}\tilde{E}_{i,1}^n(-q^2))\ \omega^n \Bigg]
\end{aligned} \tag{34}$$

Here, $\omega = \frac{1}{x}$. The Wilson coefficients relevant to unpolarised DIS are defined in momentum space by the Fourier transform

$$iq_{\mu_1}...q_{\mu_n}(\frac{2}{-q^2})^{n+1}\tilde{C}_{i,1}^n(-q^2, g) = \int d^4z\ e^{iq.z}z_{\mu_1}...z_{\mu_n}\frac{1}{z^2 - i\epsilon z_0}C_{i,1}^n(z^2 - i\epsilon z_0, g)$$
$$2iq_{\mu_1}...q_{\mu_n}(\frac{2}{-q^2})^{n+2}\tilde{C}_{i,2}^{n+2}(-q^2, g) = \int d^4z\ e^{iq.z}z_{\mu_1}...z_{\mu_n}C_{i,2}^n(z^2 - i\epsilon z_0, g) \tag{35}$$

The coefficients relevant to polarised DIS Fourier transform as

$$-\frac{1}{2}iq_{\mu_1}...q_{\mu_n}(\frac{2}{-q^2})^{n+1}\tilde{E}^n_{i,1}(-q^2,g) = \int d^4z\ e^{iq.z}z_{\mu_1}...z_{\mu_n}\frac{1}{z^2-i\epsilon z_0}E^n_{i,1}(z^2-i\epsilon z_0,g)$$
$$-\frac{1}{2}iq_{\mu_1}...q_{\mu_n}(\frac{2}{-q^2})^{n+2}\tilde{E}^n_{i,2}(-q^2,g) = \int d^4z\ e^{iq.z}z_{\mu_1}...z_{\mu_n}E^n_{i,2}(z^2-i\epsilon z_0,g) \tag{36}$$

The contribution of the twist four operators to $T_{\mu\nu}$ is suppressed by $\frac{1}{Q^2}$ with respect to the twist two contribution. Hence, they can be ignored at large Q^2 where scaling is observed.

The hadronic tensor $W_{\mu\nu}$ can now be determined from $T_{\mu\nu}$. A comparison with the Lorentz structure of $W_{\mu\nu}$ in equs.(4) and (5) shows that the first, third and fourth series expansions in $T_{\mu\nu}$ correspond to the unpolarised structure function $F_1(x)$ and the polarised structure functions $g_1(x)$ and $g_2(x)$ respectively. The second term in $T_{\mu\nu}$ corresponds to a linear combination of $F_1(x)$ and $F_2(x)$. Equ.(33) is a power series in ω which converges only for $|\omega| < 1$. Energy momentum conservation fixes $0 \leq x \leq 1$ (or $\omega \geq 1$) so that $T_{\mu\nu}$ is unphysical exactly where it converges. The forward Compton amplitude $T_{\mu\nu}$ has branch cuts along the real ω axis for $\omega \leq -1$ and $\omega \geq 1$. Hence, we may calculate the hadronic tensor $W_{\mu\nu} = \frac{1}{\pi}\text{Abs}\ T_{\mu\nu}$ by performing a Cauchy integration around the cuts (for example using the contour C shown in Fig. 4). The moments of $W_{\mu\nu}$ are given by

$$2\int_0^1 dx\ x^{n-2}W_{\mu\nu} = \frac{2}{\pi}\int_1^\infty d\omega\frac{T_{\mu\nu}}{\omega^n} = \frac{1}{2\pi i}\oint_C d\omega\frac{T_{\mu\nu}}{\omega^n} \tag{37}$$

where use has been made of the crossing symmetry $T_{\mu\nu}(q,p) = T_{\nu\mu}(-q,p)$. The Cauchy integration makes use of the identity

$$\frac{1}{2\pi i}\oint_C d\omega\ \omega^{m-n} = \delta_{m,n-1} \tag{38}$$

It follows that each even moment of $W^S_{\mu\nu}$ and each odd moment of $W^A_{\mu\nu}$ projects out the proton matrix element of a different set of local operators from the OPE (multiplied by the relevant Wilson coefficents). The higher moments determine the behaviour of the structure functions near $x = 1$ whilst the first moment tells us about the small x region.

We mentioned earlier that there are new parity-odd structure functions which appear in DIS experiments with an (anti-)neutrino beam. The unpolarised (anti-)neutrino structure function $F_3(x)$ is described by the odd spin quark operators $O_F^{a\mu_1\cdots\mu_{2n+1}}(0)$. The spin dependent (anti-)neutrino structure function $g_3(x)$ is described the even spin quark axial tensor operators $R_{1,F}^{a\sigma\mu_1\cdots\mu_{2n+1}}(0)$ [49]. The corresponding gluonic operators have the opposite charge conjugation property and do not contribute to DIS [16].

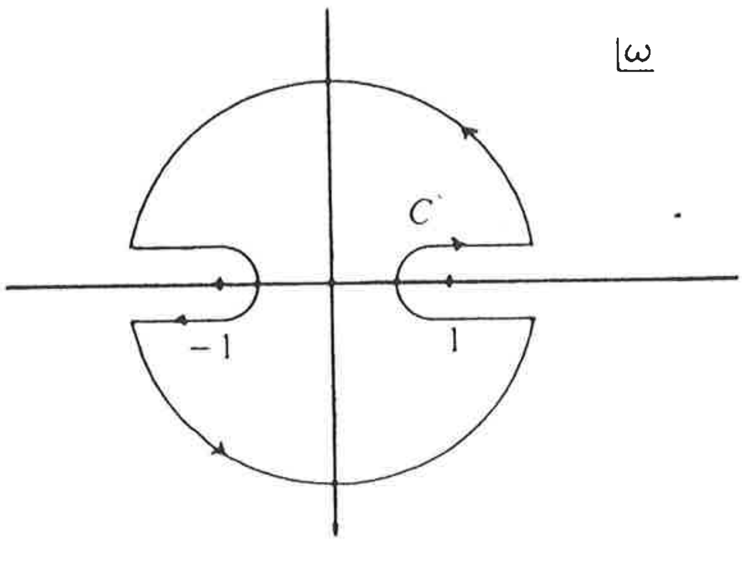


Figure 4. The forward Compton amplitude $T_{\mu\nu}$ has branch cuts along the real ω axis for $\omega \leq -1$ and $\omega \geq 1$. The hadronic tensor $W_{\mu\nu} = \frac{1}{\pi}\text{Abs } T_{\mu\nu}$ is calculated by performing a Cauchy integration along the contour C around the cuts.

3.3 The Coefficient Functions and the Renormalisation Group

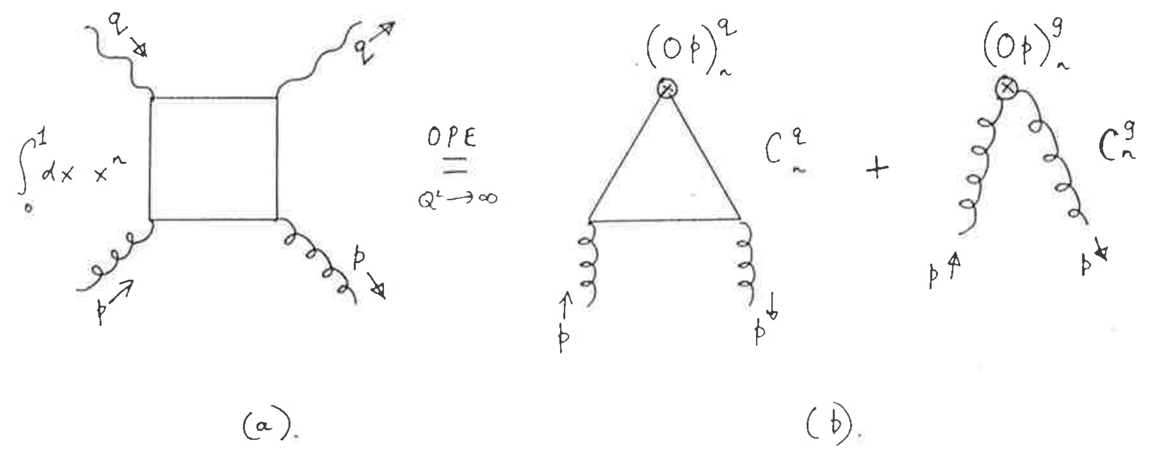
The OPE is an operator relation and does not depend upon the states that we choose to evaluate the operator matrix elements between. This means that the Wilson coefficients are target independent and may be evaluated using perturbative QCD. The calculation goes as follows. We consider DIS from an ideal free quark or gluon target and evaluate $T_{\mu\nu}$ for each target up to some fixed order in the QCD coupling constant α_s. The next step is to evaluate the quark and gluon matrix elements of the operators appearing in the right hand side of the OPE. The coefficients may be read off by comparing the two results at a given order in α_s.

The non-singlet quark operators do not mix with any gluonic operators under renormalisation. To an arbitrary order in α_s the non-singlet coefficients are just the normalisation necessary to scale the nth non-singlet quark operator matrix element to the nth moment of $T_{\mu\nu}$ via equ.(37). The singlet case is more complicated because the singlet quark and gluonic operators mix under renormalisation. Since gluons do not carry electromagnetic charge they can only contribute to DIS at $O(\alpha_s)$. The leading order $O(1)$ result is trivial: the quark structure functions are just Dirac delta functions at 1 and we deduce $C_q^n = 1+O(\alpha_s)$ for the quark coefficients. At $O(\alpha_s)$ we first consider $T^g_{\mu\nu}$ for a gluon target, which is given by the box graphs of Fig. 5a and the corresponding cross graphs. The OPE for nth moment of $T^g_{\mu\nu}$ has the form (Fig. 5b)

$$(T^g_{\mu\nu})_n =< g|(Op)^q_n|g > C^q_n + < g|(Op)^g_n|g > C^g_n \tag{39}$$

where $(Op)^q_n$ and $(Op)^g_n$ are the quark and gluon operators respectively. The matrix elements $< g|(Op)^q_n|g >\sim O(\alpha_s)$ and $< g|(Op)^g_n|g >= 1$. It follows that the gluon coefficient C^g_n at $O(\alpha_s)$ is just the difference between $(T^g_{\mu\nu})_n$ and $< g|(Op)^q_n|g >$.

Figure 5. The gluonic Wilson coefficients are calculated in perturbative QCD by applying the OPE to the box graph for photon gluon fusion.



The quark coefficients at $O(\alpha_s)$ are calculated by repeating this process for $(T^g_{\mu\nu})_n$ at one loop. Since $< q|(Op)^g_n|q >$ and C^g_n are both $\sim O(\alpha_s)$, the only contribution to the nth moment of $T^q_{\mu\nu}$ at $O(\alpha_s)$ is given by the product of $< q|(Op)^q_n|q >$ and C^q_n, each calculated to $O(\alpha_s)$. We can calculate the operator matrix element and read off the quark coefficient as a normalisation factor. The virtuality of the external quark or gluon in these calculations is always contained in the operator matrix elements because the Wilson coefficients are independent of the target.

The composite operators which appear in the OPE need to be renormalised. This is evident when one calculates the coefficients. There are divergences associated with the loop integrals which have to be subtracted out. In the case of the operator matrix elements these divergences cannot all be absorbed into the usual vertex and wavefunction renormalisation constants. They are intrinsic to the operator itself. The renormalisation procedure introduces a new momentum scale μ^2 into the calculation. This is true even in the case of a massless theory, where no momentum scale appears in the Lagrangian. The scale μ^2 has a different interpretation depending upon how one does the regularisation. For example, it may be an ultraviolet cut-off on divergent loop integrals or a Pauli-Villars regulator. One commonly used renormalisation prescription is minimal subtraction. Here, we start by continuing the dimension of a divergent loop integral from $d = 4$ to $d = 4 \pm \epsilon$ so that it becomes convergent in the new number of dimensions. The divergence at $d = 4$ appears as a pole in the dimensionality (with the singularity at $d = 4$), which is subtracted to complete the renormalisation. The scale μ^2 is introduced in this *dimensional regularisation* method to keep the coupling constant α_s dimensionless whatever the value of d. For example, when $d \rightarrow 4 + \epsilon$ we have $\alpha_s \rightarrow (\mu^2)^{-\epsilon}\alpha_s$. An example of minimal subtraction will be presented in section 5.3 when we discuss the axial anomaly.

Although we have to introduce a renormalisation scale in the calculation of a physical process like DIS, it is obvious that the physical cross section is independent of μ^2. Any observable is clearly independent of how a theorist might choose to calculate it! This is the idea of the renormalisation group equations.

Let us consider a renormalised operator Ω_R, which is constructed from a bare operator Ω. We shall initially be interested in the case where Ω_R does not mix with any other operators under renormalisation and where it is multiplicatively renormalised. This example applies to the flavour non-singlet operators in the OPE. We write $\Omega_R = Z_\Omega^{-1}\Omega$. Here, Z_Ω is the renormalisation constant for Ω. The divergences associated with the composite operator are isolated using some ultraviolet regularisation and subtracted at a renormalisation scale μ^2. The bare operator Ω is obviously independent of μ^2. This implies the differential equation

$$\frac{d}{d\ \ln\mu^2}\Omega = (\frac{d}{d\ \ln\mu^2}Z_\Omega)\Omega_R + Z_\Omega(\frac{d}{d\ \ln\mu^2}\Omega_R) = 0 \tag{40}$$

which we may rewrite as

$$(\frac{d}{d\ \ln\mu^2} + \gamma_\Omega)\Omega_R = 0 \tag{41}$$

Here

$$\gamma_\Omega = \frac{1}{Z_\Omega}\frac{d}{d\ \ln\mu^2}Z_\Omega \tag{42}$$

is called the anomalous dimension of the operator Ω.

The renormalisation group equation for the two electromagnetic current Greens function is

$$(\frac{d}{d\ \ln\mu^2} + 2\gamma_j) < p, S|j_\mu(z)j_\nu(0)|p, S >_c = 0 \tag{43}$$

where γ_j is the anomalous dimension of the photon currents. For conserved currents one finds $\gamma_j = 0$. The scale dependence of our non-singlet operator in the OPE is

$$(\frac{d}{d\ \ln\mu^2} + \gamma_\Omega) < p, S|\Omega_R|p, S >= 0 \tag{44}$$

We subtract and find

$$(\frac{d}{d\ln\mu^2} + 2\gamma_j - \gamma_\Omega)\ C_\Omega(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) = 0 \tag{45}$$

This equation may be written as

$$(\frac{d}{dg} + \frac{2\gamma_j - \gamma_\Omega}{\beta(g)})\ C_\Omega(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) = 0 \tag{46}$$

where

$$\beta(g) = \frac{\partial}{\partial\ln\mu^2}\ g \tag{47}$$

is the QCD beta function. We call equ.(46) the renormalisation group equation. It has the solution

$$C_\Omega(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) = \exp[\int_{g(\mu_0^2)}^{g(\mu^2)} dg\ \frac{2\gamma_j - \gamma_\Omega}{\beta(g)}]C_\Omega(\frac{Q^2}{\mu_0^2}, \alpha_s(\mu_0^2)) \tag{48}$$

For DIS, $\gamma_j = 0$ corresponding to a conserved electromagnetic current. We usually write equ.(48) with $\mu_0^2 = Q^2$ to eliminate any large $\ln\frac{Q^2}{\mu^2}$ terms. In QCD perturbation theory the beta function and anomalous dimensions have the form

$$\begin{aligned} \beta(g) &= -\beta_0 g^3 - \beta_1 g^5 + O(g^7) \\ \gamma_\Omega &= \gamma_0 g^2 + \gamma_1 g^4 + O(g^6) \\ C_\Omega(1, g) &= C_{\Omega,0} + C_{\Omega,1} g^2 + O(g^4) \end{aligned} \tag{49}$$

Substituting into equ.(48), we find that the coefficients evolve with Q^2 as

$$C_\Omega(\frac{Q^2}{\mu^2}, g(\mu^2)) = \left[\frac{g(Q^2)}{g(\mu^2)}\right]^{-\frac{\gamma_0}{\beta_0}} C_{\Omega,0} \tag{50}$$

at leading order.

The renormalisation group equation is slightly more complicated in the singlet case due to operator mixing. The anomalous dimensions for each moment γ_Ω now form a 2x2 matrix. Setting $\gamma_j = 0$ (for DIS) the singlet version of equ.(48) is

$$\begin{pmatrix} C^q \\ C^g \end{pmatrix}(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)) = T \exp \int_{g(Q^2)}^{g(\mu^2)} dg \begin{bmatrix} \gamma^F_{FF} & \gamma^F_{VV} \\ \gamma^V_{FF} & \gamma^V_{VV} \end{bmatrix}(g) \begin{pmatrix} C^q \\ C^g \end{pmatrix}(1, \alpha_s(Q^2)) \tag{51}$$

Since the anomalous dimensions are now a matrix, the terms in the expansion of the exponential will not commute. This is denoted by the T outside the exponential in equ.(51). The eigenstates of the renormalisation group equations are found diagonalising the anomalous dimension matrix. For example, the first moment of $F_2(x)$ contains the QCD energy momentum tensor, which is obviously conserved. It is an operator with quark and gluonic terms and has zero anomalous dimension. Hence, it scales with increasing Q^2.

At one loop $O(\alpha_s)$, the quark and gluon matrix elements of the operators in the OPE have a $\gamma_\Omega \ln \frac{\mu^2}{p^2}$ term after we carry out the renormalisation. This term comes from the integration region where the quark/gluon in the loop is collinear with the target quark/gluon. The renormalisation scale will appear both in the definition of the composite operators in the OPE and also in the Wilson coefficients (because the latter are defined by subtracting out the operator matrix elements as described above). If we generalise our treatment to include heavy quark contributions to DIS then the charm quark mass $m_c = 2$GeV may be comparable to the renormalisation scale μ^2 and should be included in the relevant Wilson coefficients [50].

Before returning to our discussion of DIS we just mention that in general the OPE quark and gluon operators will mix with gauge dependent operators under renormalisation (eg. those constructed out of Fadeev Popov ghost fields). The beautiful result which makes the OPE and RGE approach relevant to DIS is that the gauge dependent operators form a subgroup of the complete set of composite operators under renormalisation so far as mixing is concerned and can be treated as if the gauge dependent operators do not exist [9].

The anomalous dimensions relevant to unpolarised DIS were first presented in refs. [9, 10]. The corresponding Wilson coefficients have been calculated to $O(\alpha_s)$ in ref.[51]. Higher order calculations appear in ref.[52]. In the spin dependent theory, the anomalous dimensions and Wilson coefficients have been calculated to $O(\alpha_s)$. The anomalous dimensions appear in ref.[49] and the coefficients were first calculated in refs.[53-55].

3.4 Parton Distributions in QCD

Quark and gluon parton distributions can be defined in QCD. Formally, these quark (gluon) distributions are defined so that the nth moment of the distribution is equal to the target matrix element of the nth quark (gluon) operator in the OPE. (These operator matrix elements will be renormalisation scheme dependent at order α_s.) Thus, the set of all even spin $< p, S|O_i^{a\mu_1\ldots\mu_n}(0)|p, S >$ define the unpolarised parton distributions (measured in unpolarised DIS). The spin dependent parton distributions are determined from the set of all odd spin $< p, S|R_{i,1}^{a\sigma\mu_1\ldots\mu_n}(0)|p, S >$. The Mellin theorem [56] tells us that these distributions, which are defined with respect to the OPE, are unique (up to the choice of renormalisation prescription). The Mellin theorem reads as follows: Let $g(x)$ be a piecewise smooth function for $x > 0$, and $\int_0^\infty dx\ x^{\sigma-1}g(x)$ be absolutely convergent for $\alpha < \sigma < \beta$. Now let $s = \sigma + it$ and define

$$f(s) = \int_0^\infty dx\ x^{s-1}g(x) \tag{52}$$

then

$$g(x) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty} ds\ x^{-s}f(s) \tag{53}$$

The proof is found in the textbook by Courant and Hilbert [56].

One can apply the same trick to define a coefficient distribution. The nth moment of the quark (gluon) coefficient distribution is given by the nth quark (gluon) Wilson coefficient from the relevant series. We shall choose the renormalisation scale $\mu^2 = Q^2$. The convolution property then enables us to write structure functions as a convolution of the parton and coefficient distributions. For example, the singlet contribution to the unpolarised structure function $F_2(x)$ is given by

$$F_2(x)|_S = x \int_x^1 \frac{dy}{y} q(y, Q^2) C^q(\frac{x}{y}, \alpha_s(Q^2)) + x \int_x^1 \frac{dy}{y} g(y, Q^2) C^g(\frac{x}{y}, \alpha_s(Q^2)) \qquad (54)$$

Here $q(x, Q^2)$ and $g(x, Q^2)$ are the unpolarised quark and gluon distributions respectively; $C^q(x, \alpha_s(Q^2)) = \delta(x-1) + O(\alpha_s(Q^2))$ and $C^g(x, \alpha_s(Q^2)) \sim O(\alpha_s(Q^2))$ are the corresponding coefficient distributions.

The parton distributions are defined via light cone correlation functions [57, 58]. In the unpolarised case we find (for each flavour)

$$q(x) = \frac{1}{2\pi} \int dz_- \ e^{-ixz_- p_+} < p, S|[\overline{q}(z_-)\gamma_+ \mathrm{P} e^{i \int_0^{z_-} dy_- A_+} q(0)]|p, S >_c \qquad (55)$$

for the quark distribution and

$$\overline{q}(x) = -\frac{1}{2\pi} \int dz_- \ e^{+ixz_- p_+} < p, S|[\overline{q}(z_-)\gamma_+ \mathrm{P} e^{i \int_0^{z_-} dy_- A_+} q(0)]|p, S >_c \qquad (56)$$

for the antiquark distribution. The path ordered exponential in equs.(55) and (56) is taken along the line y_- [57]. It becomes trivial in light cone gauge $A_+ = 0$, where $D_+ = \partial_+$.

To check that these distributions have the correct moments we write x^n as a derivative (in z_-) acting on $e^{\pm ixz_- p_+}$ and then integrate by parts. The exponential integrates to give a Dirac delta function $\delta(z_-)$ and we project out the desired local operator matrix element. This construction holds for the bare (unrenormalised) operators. After renormalisation, the non-local operator in equs.(54) and (55) has a series expansion in terms of the local operators in the OPE, viz. [58]

$$
\begin{aligned}
&< p,S|[\overline{q}(z_-)\gamma_+ \mathrm{P} e^{i\int_0^{z_-} dy_- A_+} q(0)]|p,S>_c = \\
&\qquad \sum_n \frac{(-z_-)^n}{n!} < p,S|[\overline{q}(0)\gamma_+(D_+)^n q(0)]|p,S>_c
\end{aligned}
\tag{57}
$$

Similarly, one can define an unpolarised gluon distribution. It is

$$
\begin{aligned}
xg(x) = \frac{1}{2\pi p_+} \int dz_- \ (e^{-ixz_-p_+} + e^{ixz_-p_+}) \\
< p,S|\mathrm{Tr}[G_{+\nu}(z_-)\mathrm{P}e^{i\int_0^{z_-} dy_- A_+} G^\nu{}_+(0)]|p,S>_c
\end{aligned}
\tag{58}
$$

where

$$
\begin{aligned}
&< P,s|\mathrm{Tr}[G_{+\nu}(z_-)\mathrm{P}e^{i\int_0^{z_-} dy_- A_+} G^\nu{}_+(0)]|P,s>_c = \\
&\qquad \sum_n \frac{z_-^n}{n!} < P,s|\mathrm{Tr}[G_{+\nu}(0)(D_+)^n G^\nu{}_+(0)]|P,s>_c
\end{aligned}
\tag{59}
$$

in the renormalised theory. Each of the operators in the right hand sides of equs.(57) and (59) is renormalised independently within some choice of gauge invariant renormalisation scheme (eg. minimal subtraction). In obvious notation, these distributions have the moments

$$
\begin{aligned}
\int_0^1 dx \ x^{2n+1}(q+\overline{q})_a(x) = a_{F,a}^{2n+2} \\
\int_0^1 dx \ x^{2n} \ xg(x) = a_{V,0}^{2n+2}
\end{aligned}
\tag{60}
$$

The combination $q_V(x) = (q-\overline{q})(x)$ is the valence quark distribution, which is measured in unpolarised DIS with (anti-)neutrinos.

In light-cone gauge one often thinks of the light cone operator matrix element $< P,s|[\overline{q}(z_-)\gamma_+ q(0)]|P,s>_c$ in the following way. We take a quark out from (or insert an anti-quark into) the target proton at position 0 and re-insert the quark (take out the anti-quark) at a position z_- along the light cone. This intuitive approach is the idea behind recent attempts to calculate structure functions within low energy models of hadronic structure [59-64] (eg. the MIT Bag [65]). The Bag is a mean field theory so that vacuum fluctuations at very small distances are averaged out. This means that we might hope to calculate the twist two piece of the nucleon structure function reliably for correlation lengths $z_- > 0.1\text{fm}$. A further theoretical constraint follows from the fact that the Bag breaks translational invariance (it is fixed at a point in space-time). Model calculations will thus have limited accuracy beyond approximately $z > 0.8\text{fm}$ (for a bag radius $R = 1.0\text{fm}$). The model structure functions are calculated at the Bag scale $\mu^2 \sim 0.5\text{GeV}^2$ and evolved up to the scale of DIS experiments, where they may be compared with data. In practical calculations this evolution is carried out using perturbative QCD [59, 63]. Whilst there will be problems with applying perturbative QCD at the low end of the evolution, it is hoped that the errors are not so large in the valence sector where we exclude nucleon sea effects such as pions. The agreement between model structure functions and DIS data is quite good for the valence distributions $q_V(x)$ [59-64]. At the beginning of my Ph.D. I spent some time working on this program resulting in ref.[64], which is appended to this thesis.

The parton distributions relevant to polarised deep inelastic scattering are defined in the same way as the unpolarised distributions. However, one has to be careful about applying one's physical intuition (developed in the the unpolarised world) to the polarised world. Indeed, as we will show in the following chapters, one has to be very careful about what one calls a "quark parton" and what one calls a "gluon parton". This conceptual problem is intimately related to the Adler-Bell-Jackiw anomaly [66] which arises in renormalised gauge field theory.

We now define the spin dependent quark and antiquark distributions

$$(q^{\uparrow} - q^{\downarrow})(x) = \frac{1}{2\pi}\frac{p_+}{2Ms_+}\int dz_-\ e^{-ixz_-p_+} < p, S|[\overline{q}(z_-)\gamma_+\gamma_5 \mathrm{P}e^{i\int_0^{z_-} dy_- A_+} q(0)]|p, S >_c \tag{61}$$

and

$$(\overline{q}^{\uparrow} - \overline{q}^{\downarrow})(x) = \frac{1}{2\pi}\frac{p_+}{2Ms_+}\int dz_-\ e^{+ixz_-p_+} < p, S|[\overline{q}(z_-)\gamma_+\gamma_5 \mathrm{P}e^{i\int_0^{z_-} dy_- A_+} q(0)]|p, S >_c \tag{62}$$

A caveat is due here: the reader should not be too quick so as to use her or his intuition that these distributions may have something to do with polarised quarks in the proton! The reason for this will become apparent as we continue. (The factor $\frac{p_+}{s_+}$ in equs.(61) and (62) is missing in ref. [67].) In the renormalised theory

$$\begin{aligned} &< p, S|[\overline{q}(z_-)\gamma_+\gamma_5 \mathrm{P}e^{i\int_0^{z_-} dy_- A_+} q(0)]|p, S >_c = \\ &\qquad \sum_n \frac{(-z_-)^n}{n!} < P, s|[\overline{q}(0)\gamma_+\gamma_5(D_+)^n q(0)]|P, s >_c \end{aligned} \tag{63}$$

Similarly, we define a spin dependent gluon distribution

$$\begin{aligned} x\Delta g(x) = \frac{1}{2\pi}\frac{1}{2Ms_+}\int dz_-\ &(e^{-ixz_-p_+} - e^{ixz_-p_+}) \\ &< p, S|\mathrm{Tr}[G_{+\nu}(z_-)\mathrm{P}e^{i\int_0^{z_-} dy_- A_+}\tilde{G^{\nu}}_+(0)]|p, S >_c \end{aligned} \tag{64}$$

where

$$\begin{aligned} &< p, S|\mathrm{Tr}[G_{+\nu}(z_-)\mathrm{P}e^{i\int_0^{z_-} dy_- A_+}\tilde{G^{\nu}}_+(0)]|p, S >_c = \\ &\qquad \sum_n \frac{z_-^n}{n!} < p, S|\mathrm{Tr}[G_{+\nu}(0)(D_+)^n \tilde{G}^{\nu}_+(0)]|p, S >_c \end{aligned} \tag{65}$$

Not withstanding our earlier notation in chapter 2, we shall define the linear combinations

$$\Delta q(x) = (q^{\uparrow} - q^{\downarrow})(x) + (\overline{q}^{\uparrow} - \overline{q}^{\downarrow})(x) \tag{66}$$

and

$$\Delta q(x)_V = (q^{\uparrow} - q^{\downarrow})(x) - (\overline{q}^{\uparrow} - \overline{q}^{\downarrow})(x) \tag{67}$$

With these definitions, we may write the polarised proton structure function $g_1(x, Q^2)$

as a sum over flavour singlet and non-singlet pieces, viz.

$$g_1(x,Q^2) = g_1(x,Q^2)|_S + g_1(x,Q^2)|_{NS} \tag{68}$$

where

$$g_1(x,Q^2)|_{NS} = \frac{1}{6}\int_x^1 \frac{dz}{z}\Big(\Delta q_3(z,Q^2) + \frac{1}{\sqrt{3}}\Delta q_8(z,Q^2)\Big) C^q_{NS}(\frac{x}{z}, \alpha_s(Q^2)) \tag{69}$$

and

$$g_1(x,Q^2)|_S = \frac{1}{3}\sqrt{\frac{2}{3}}\int_x^1 \frac{dz}{z}\Delta q_0(z,Q^2) C^q_S(\frac{x}{z}, \alpha_s(Q^2)) + \frac{1}{9}\int_x^1 \frac{dz}{z}\Delta g(z,Q^2) C^g_S(\frac{x}{z}, \alpha_s(Q^2)) \tag{70}$$

for the non-singlet and singlet contributions respectively. Here, the $C(x, \alpha_s(Q^2))$ are the spin dependent coefficient distributions. The spin dependent valence distribution $\Delta q_V(x)$ could be measured in a DIS experiment with a polarised proton target and a polarised (anti-) neutrino beam. It is not observed in polarised DIS with a muon beam. We now re-consider the first moment of $g_1(x)$.

3.5 EMC Revisited

Using our new OPE language, the first moment of $g_1(x)$ is

$$\int_0^1 dx\ g_1(x,Q^2) = \frac{1}{6}\Big[(\Delta q_3 + \frac{1}{\sqrt{3}}\Delta q_8)(1 - \frac{\alpha_s(Q^2)}{\pi}) + 2\sqrt{\frac{2}{3}}\Delta q_0(Q^2)(1 - \frac{\alpha_s(Q^2)}{\pi}\frac{33-8N_f}{33-2N_f})\Big] \tag{71}$$

where the first moments of the parton densities are defined by the proton matrix elements of the axial currents, viz.

$$2Ms_\mu \Delta q_a = < p, S|\bar{q}\gamma_\mu\gamma_5 \frac{\lambda^a}{2} q|p, S >_c \tag{72}$$

The factor $(1 - \frac{\alpha_s(Q^2)}{\pi})$ is the non-singlet Wilson coefficient and $(1 - \frac{\alpha_s(Q^2)}{\pi}\frac{33-8N_f}{33-2N_f})$ is the singlet coefficient.

The non-singlet axial vector currents are soft operators; they are conserved in the limit of massless quarks. This means that they are not renormalised and have vanishing anomalous dimension. The singlet axial vector current is not conserved due to the axial anomaly [66]. When this composite operator is constructed in any gauge invariant renormalisation scheme it is multiplicatively renormalised and has an anomalous dimension which starts at two loops [55, 68]. In the perturbative domain where α_s is small the evolution of Δq_0 with Q^2 proceeds very slowly. However, it is reasonable to suppose that there may be some rapid variation in Δq_0 at low scales ($Q^2 < 1\text{GeV}^2$) [69, 70]. This will be explained in chapter five (on axial anomalies).

The main point to realise at this stage is that if the Ellis-Jaffe hypothesis were to hold at one scale it would not hold (exactly) at any other. The matrix elements of the non-singlet axial vector currents describe the neutron and hyperon beta decays. Thus, polarised DIS is seen to relate low-energy weak-interaction physics and the strong interaction at high energy. Indeed, the difference in $g_1(x)$ for the proton and neutron is given by Bjorken's sum-rule [4]

$$\int_0^1 dx \left(g_1^p(x) - g_1^n(x) \right) = \frac{1}{6}\Delta q_3 (1 - \frac{\alpha_s}{\pi}) \tag{73}$$

which is devoid of any axial anomaly. It is often regarded as a fundamental test of QCD. This relationship between polarised DIS and the weak interactions allows us to identify $\Delta q_3 = g_A = F + D$ and $\Delta q_8 = \frac{1}{\sqrt{3}}(3F - D)$ in the limit of good SU(3) flavour. Here, F and D are the antisymmetric and symmetric SU(3) couplings. The effect of SU(3) flavour violations has been investigated by Lipkin [71, 72] (and found to be small). Although we cannot choose $\Delta s = 0$ independent of scale one might suppose an Ellis-Jaffe hypothesis at the Q^2 of the EMC experiment. After taking into effect the small perturbative corrections in the Wilson coefficients one would predict

$\int_0^1 dx\ g_1(x) = 0.189 \pm 0.005$ at $Q^2 \sim 10\text{GeV}^2$, which is still two standard deviations from the data [39].

Earlier, we noted that there is no spin-one, gauge invariant, local gluonic operator which can appear in the OPE for $g_1(x)$. This is the reason that there is no $\Delta g = \int_0^1 dx\ \Delta g(x)$ term in the first moment of $g_1(x)$. One has

$$\Delta g \int_0^1 dx\ C_S^g(x, \alpha_s(Q^2)) = 0 \qquad (74)$$

for an arbitrary choice of target. (This result differs from the unpolarised structure function where all the (even) moments of $F_2(x)$ contain contributions from quark and gluonic terms in the OPE.) Although the first moment of $\Delta g(x)$ is not given by any single local gluonic operator there is no reason to suppose that it is identically zero - especially for an ideal free gluon target. One can divide the infinite series for $x\Delta g(x)$ in equs.(64) and (65) by x and write the first moment as an infinite series defined by all the higher moments of the function. Hence, the first moment of $\Delta g(x)$ is defined via an infinite series of gauge invariant, local composite gluonic operators. The Mellin Theorem tells us that Δg is unique (within any choice of renormalisation prescription) up to a Dirac delta function at the origin. Since Δg varies as a function of the target, it follows that the target independent coefficient in equ.(74) must vanish to all orders in perturbation theory. This result has been argued strongly by Bodwin and Qiu [73] for any choice of gauge invariant renormalisation scheme and verified by them in the MS and subsequently by Manohar in the $\overline{MS}$ [74].

4. Observing the Spin Dependent Gluon Distribution

4.1 The QCD Improved Parton Model

In chapter 3 we explained how deep inelastic scattering is described in QCD by the OPE and the renormalisation group. Ideally, we would like a QCD description of more general hadron interactions at high energy (eg. Drell Yan production and exclusive processes like $p\overline{p} \to c\overline{c}X$). This is the subject of the QCD Improved Parton Model [19, 75-77], henceforth denoted IPM. It is constructed following the OPE result for DIS. We factor the hadronic cross section into a convolution of the parton distributions of each parent hadron with a hard parton scattering cross section.

For example, consider the production of charm jets $c\overline{c}$ with large transverse momentum squared ($k_T^2 \geq \Lambda^2$) in polarised DIS. This is an exclusive measurement out of the total inclusive DIS cross section. It gives a contribution to $g_1(x)$ for the proton of

$$g_1^{c\overline{c}}(x) = \frac{5}{36}\int_x^1 \frac{dz}{z}\Delta g(z,\Lambda^2)\Delta\sigma_{\gamma g\to c\overline{c}}(\frac{x}{z},\frac{Q^2}{\Lambda^2},\alpha_s(\Lambda^2)) \tag{75}$$

Here, $\Delta g(x,\Lambda^2)$ represents the IPM spin dependent gluon distribution of the proton, defined at a factorisation scale Λ^2. It is to be understood as the probability for finding a gluon in the proton with momentum fraction $p_+^{gluon} = xp_+^{proton}$ and transverse momentum squared $p_T^2 \leq \Lambda^2$. The hard factor $\Delta\sigma_{\gamma g\to c\overline{c}}(x,\frac{Q^2}{\Lambda^2},\alpha_s(\Lambda^2))$ is the spin dependent cross section for producing $c\overline{c}$ with transverse momentum squared $k_T^2 \geq \Lambda^2$ from photon-gluon fusion (Fig. 6). We choose to calculate this partonic cross section in a frame where both the photon and gluon have zero transverse momentum. After all, the transverse momentum of parton constituents does not play any significant role in the naive parton model. There will be some part of the total phase space for $\gamma g \to c\overline{c}$ which describes $c\overline{c}$ with $k_T^2 \leq \Lambda^2$. This is understood to be factored into the spin dependent quark distribution $\Delta q(x, \Lambda^2)$, which describes the x distribution of quarks with transverse momentum squared $k_T^2 \leq \Lambda^2$ in the proton. The picture that we have just described is the simplest form of factorisation. It dates from the work of Gribov and Lipatov [78]. The k_T^2 cut-off parton model has been developed to all orders of perturbation theory [58, 76, 79] and verified in explicit calculations to $O(\alpha_s^2)$ [80]. In light cone gauge $A_+ = 0$ these IPM parton distributions can (apparently) be defined in a similar way to the light cone OPE definitions in chapter 3 [58]. (The word "apparently" anticipates a subtlety that will appear towards the end of this chapter.) We could also have considered a 2 jet definition with a cut-off on the quark virtuality $m^2 - k^2 \geq \Lambda^2$ in Fig. 6.

In general, the principal of factorisation goes as follows [58, 75, 76]. Let us consider the scattering of a hard photon (large Q^2) or soft parton (low virtuality $p_1^2 \leq \Lambda_{QCD}^2$) from another soft parton (virtuality $p_2^2 \leq \Lambda_{QCD}^2$) so that there is at least one large mass scale in the process. This may be the invariant mass squared of of the final state partons. We work at $O(\alpha_s)$. All such partonic cross sections will possess a mass singularity. If Q^2 is the large momentum scale in the problem, then we find a term $\sim \ln \frac{Q^2}{p_2^2}$, which is divergent when $p_2^2 \to 0$. The mass singularity comes from that part of phase space where a (massless) parton splits collinearly into two (massless) partons. At first glance, the mass singularity is an unpleasant result. Gauge invariance requires that we put the external partons on mass shell: $p^2 = 0$ for a gluon and $p^2 = m^2 \to 0$ for the light quarks. (The reason for this is the gauge parameter in the gluon propagator.) The trick (or escape route) is to write

$$\begin{aligned} 1 + \alpha_s \ln \frac{Q^2}{p^2} &= 1 + \alpha_s \ln \frac{Q^2}{\Lambda^2} + \alpha_s \ln \frac{\Lambda^2}{p^2} \\ &= (1 + \alpha_s \ln \frac{Q^2}{\Lambda^2})(1 + \alpha_s \ln \frac{\Lambda^2}{p^2}) + O(\alpha_s^2) \end{aligned} \tag{76}$$

where Λ^2 is the factorisation scale. It separates the cross section into a product of long and short distance contributions. The hard term $(1 + \alpha_s \ln \frac{Q^2}{\Lambda^2})$ is now free of any mass singularity; the long distance effects (including the mass singularity) are factored into the quark and gluon parton distributions of the proton wavefunction. Factorisation theorems have been proved which show that this factorisation procedure can be carried to all orders in perturbation theory up to power corrections $\sim O(\frac{p^2}{Q^2})$ [58, 76, 79].

There is no unique way to carry out the factorisation. One physically motivated approach is to apply the k_T^2 cut-off ($k_T^2 \geq \Lambda^2$) on the real quark and gluon jets that are produced. In this case, there will also be a renormalisation scale μ^2 associated with vertex and propagator renormalisation at higher orders. Here, the k_T^2 cut-off represents an infra-red regularisation whilst the renormalisation involves the subtraction of ultraviolet divergences. One chooses $\Lambda^2 = \mu^2$ to simplify the calculations. It is often a good idea to take $\Lambda^2 = Q^2$ too to eliminate large logarithm terms $\sim \ln \frac{Q^2}{\Lambda^2}$. Alternatively, one can set $p^2 = m^2 = 0$ and calculate the parton scattering cross section using dimensional regularisation (see eg. Pokorski [81]). The mass singularity is then manifest as a pole at d=4, which is factored out. While this latter procedure is equally valid it is does not have the intuitive appeal of the Gribov-Lipatov approach [78].

The factorisation procedure can be applied simultaneously to a host of different hadronic processes. This means that the IPM parton distributions which are measured in one process (eg. DIS) can be used to make testable predictions in other hadron interactions (eg. Drell Yan and exclusive jet production) so long as we use the same factorisation convention. There is excellent agreement with experiment. (For reviews see [19, 20, 82].

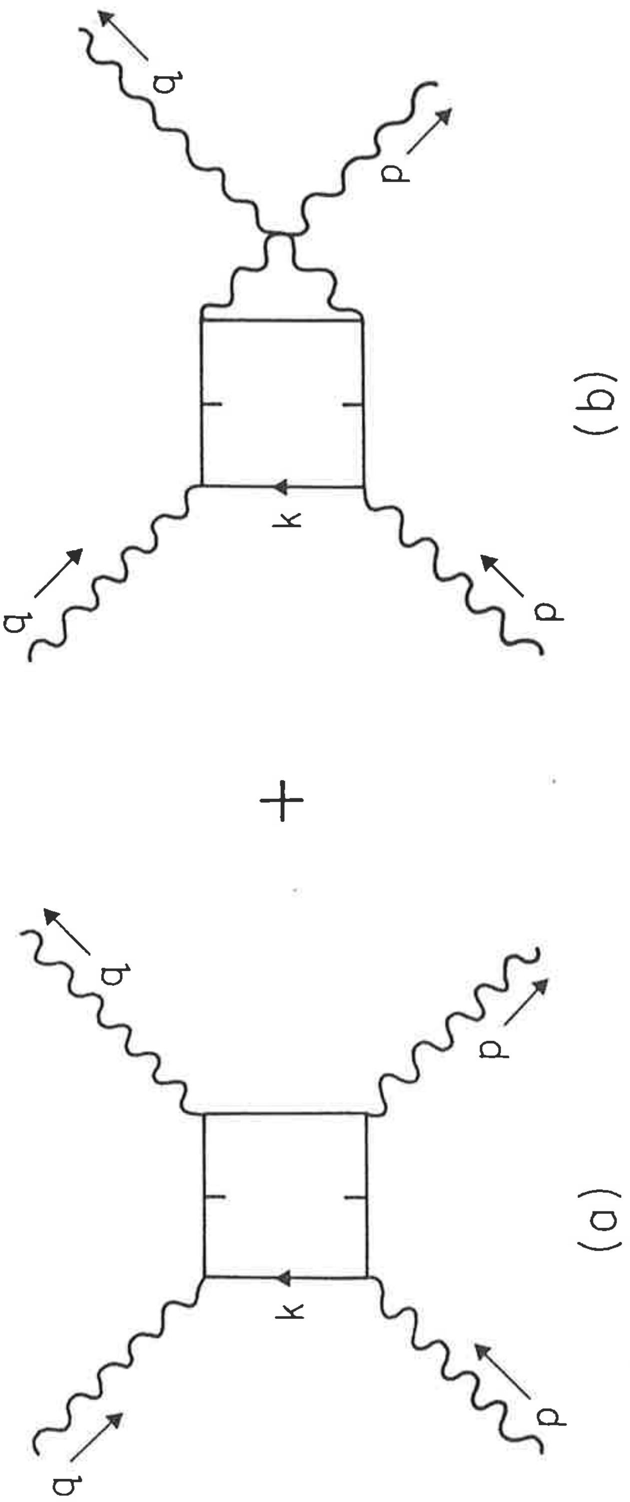
Finally, we remark that the coefficients $P_{ij}(x)$ which multiply the logarithms $P_{ij}(x) \ln \frac{Q^2}{p^2}$ in the parton cross sections are just the inverse Mellin transforms of the flavour singlet anomalous dimensions that we encountered in chapter 3 ($\int_0^1 dx\ P_{ij}(x) = \gamma^i_{jj}$). Hence, we can write a renormalisation group equation for the x dependence of the

IPM parton distributions. This is the Altarelli-Parisi formulation [83]. The P_{ij} are the Altarelli-Parisi splitting functions. They have the nice physical interpretation of being the probability of finding a parton i in a parton j.

Large k_T^2 two-quark-jet events represent an ideal probe of the IPM gluon distributions in deep inelastic scattering [75]. This point has been emphasised recently by Ross and Roberts [84] who use these events as a phenomenological definition of the gluon distribution. Alternatively, one could measure the structure functions at a range of Q^2 and separate the quark and gluon distributions by a fit to the data involving Altarelli-Parisi evolution.

We now explore these ideas through a detailed study of spin dependent photon-gluon fusion. This process has attracted considerable attention in the EMC Spin Effect literature following work by Efremov and Teryaev [21, 85], and Altarelli and Ross [22]. These authors realised that Altarelli-Parisi evolution implies that $\Delta g(Q^2) = \int_0^1 dx\ \Delta g(x, Q^2)$ evolves as $\frac{1}{\alpha_s}$ at large Q^2; the combination $(\alpha_s \Delta g)(Q^2)$ tends to a constant in the scaling limit. Furthermore, they found a higher order correction of $-\frac{\alpha_s}{2\pi}\Delta g$ to the first moment of $g_1(x)$ for each accessible quark flavour, which is induced by photon-gluon fusion. (Of course, they were using a particular choice of IPM or factorisation procedure.) The reader may be surprised at this result: we have already argued that there is no contribution to the first moment of $g_1(x)$ from the (OPE defined) $\Delta g(x)$. The Efremov-Teryaev, Altarelli-Ross (ETAR) term has been the source of much debate in the EMC Spin Effect literature. The rest of this thesis is devoted to an analysis of the anomalous ETAR result.

Figure 6. The lowest order perturbative QCD (box) diagrams for photon-gluon scattering.



4.2 Polarised Photon-Gluon Scattering

In this section we calculate $g_1(x)$ for a gluon target to one loop in perturbation theory. This box calculation is the difference between the two cross sections

$$\Delta\sigma = \sigma(\gamma \uparrow \ g \uparrow \rightarrow q\bar{q}) - \sigma(\gamma \uparrow \ g \downarrow \rightarrow q\bar{q}) \tag{77}$$

For deep inelastic scattering we shall be interested in the case where the photon is a long way from on mass shell (hard photon) and the gluon is just slightly virtual (soft gluon). The cross section for $\gamma\ g \rightarrow q\bar{q}$ is related to the forward (one loop) Compton scattering amplitude by the Optical Theorem.

We define q^μ and p^μ to be the photon and gluon momenta respectively. Then $Q^2 = -q^2$ and $P^2 = -p^2$ are the photon and gluon virtualities respectively. We use m to denote the quark mass. The box graphs (Fig. 6) are calculated in the $\gamma^* - g$ centre of mass frame, whereby $p^\mu = (p^0, 0_T, p^3)$ and $q^\mu = (q^0, 0_T, q^3)$ with $q^3 = -p^3$; k^μ denotes the quark loop momentum. We shall keep all terms in P^2 and m^2 until the end of the calculation and then take the limit $Q^2 \gg P^2, m^2$. The Bjorken variable is $x = \frac{Q^2}{2p.q}$.

The starting expression for $\Delta\sigma(x, Q^2, P^2, m^2)$ may be written down from the Feynman rules for the box graphs. We add in the contribution where the quarks circulate around the loop in the opposite direction and obtain

$$\begin{aligned}
&\Delta\sigma(x, Q^2, P^2, m^2) = \\
&-\frac{2N_f T g^2}{(2\pi)^4} \int d^4k \frac{(2\pi)\delta((p-k)^2 - m^2)\delta((q+k)^2 - m^2)}{(k^2 - m^2 + i\epsilon)^2} \\
&\frac{1}{4}\epsilon_{ij}\epsilon_{lm} \mathrm{Tr}\big[\gamma_j(\hat{k} + m)\gamma_m(\hat{q} + \hat{k} + m)\gamma_l(\hat{k} + m)\gamma_i(\hat{k} - \hat{p} + m) \\
&+ \gamma_j(\hat{k} + m)\gamma_m(\hat{q} + \hat{k} + m)\gamma_i(\hat{k} + \hat{q} - \hat{p} + m)\gamma_l(\hat{k} - \hat{p} + m)\big]
\end{aligned} \tag{78}$$

Here, ϵ_{ij} is antisymmetric in the two transverse directions (with $\epsilon_{12} = +1$); N_f is the number of quark flavours. We shall consider the contributions from light quark

$m \sim 0$ production ($N_f = 3$) and heavy ($m = 2$GeV) charm quark production ($N_f = 1$) separately.

Dirac Algebra for Box 1

We now evaluate the Dirac algebra, beginning with the uncrossed box (box 1). With a few "tricks" the traces are not as forbidding as it might first appear (with eight γ^μ matrices). This will become apparent as we continue. Box 1 gives the trace

$$\frac{1}{4}\epsilon_{ij}\epsilon_{lm}\mathrm{Tr}[\gamma_j(\hat{k}+m)\gamma_m(\hat{k}+\hat{q}+m)\gamma_l(\hat{k}+m)\gamma_i(\hat{k}-\hat{p}+m)] \tag{79}$$

To simplify the calculation, we write $s^\mu = (k+q)^\mu$ and $t^\mu = (k-p)^\mu$ and consider

$$\frac{1}{4}\epsilon_{ij}\epsilon_{lm}\mathrm{Tr}[\gamma_j(\hat{k}+m)\gamma_m(\hat{s}+m)\gamma_l(\hat{k}+m)\gamma_i(\hat{t}+m)] \tag{80}$$

The trace is readily simplified via the identity

$$\epsilon_{ij}\gamma_i(\hat{p}+m)\gamma_j = \{\gamma_2\gamma_1, \hat{p}\}_+ - 2\gamma_2\gamma_1 m \tag{81}$$

We write $\hat{p} = \hat{p}_L + \hat{p}_T$ so that $\hat{p}_L = p^0\gamma_0 + p^3\gamma_3$ and $\hat{p}_T = p^1\gamma_1 + p^2\gamma_2$. Then

$$\{\gamma_2\gamma_1, \hat{p}_T\}_+ = 0 \tag{82}$$

and

$$\{\gamma_2\gamma_1, \hat{p}_L\}_+ = 2\gamma_2\gamma_1\hat{p}_L \tag{83}$$

Thus, we obtain

$$\epsilon_{ij}\gamma_i(\hat{p}+m)\gamma_j = 2\gamma_2\gamma_1(\hat{p}_L - m) \tag{84}$$

When this result is applied to $(\hat{s}+m)$ and $(\hat{t}+m)$, equ.(80) simplifies to

$$\begin{aligned}&\mathrm{Tr}\Big[\ (m-\hat{t}_L)\gamma_1\gamma_2(\hat{k}+m)\gamma_2\gamma_1(m-\hat{s}_L)(\hat{k}+m)\ \Big]\\ =&\mathrm{Tr}\Big[\ (m-\hat{t}_L)(\hat{k}_L-\hat{k}_T+m)(m-\hat{s}_L)(\hat{k}+m)\ \Big]\end{aligned} \tag{85}$$

Note that we now have to consider a trace of at most four γ^μ matrices. Equ.(85) is evaluated using the trace theorems listed in the Appendix. The terms in m to an odd power vanish and we find

$$4\Big[m^4+m^2\Big(t_L.s_L+k.(k_L-k_T-2t_L-2s_L)\Big)+\Big(2k.s_Lk.t_L-t_L.s_L(k_L^2+k_T^2)\Big)\Big] \tag{86}$$

The expressions for s and t are now substituted back. Since the quarks with momentum s and t are both on shell (via the Optical Theorem) it follows that

$$s^2=(k+q)^2=m^2 \tag{87}$$

and

$$t^2=(k-p)^2=m^2 \tag{88}$$

Hence,

$$2q.k=(m^2-k^2)+Q^2 \tag{89}$$

and

$$2p.k=(k^2-m^2)-P^2 \tag{90}$$

respectively. With this information, the trace of the first (uncrossed) box equ.(79) simplifies to

$$4\big[\frac{1}{2}(k^2-m^2)^2+(k^2-m^2)(q.p-k_T^2)+\frac{1}{2}Q^2P^2+2k_T^2q.p\big] \tag{91}$$

We now turn to the crossed box.

Dirac Algebra for Box 2

For box 2 we have to evaluate

$$\frac{1}{4}\epsilon_{ij}\epsilon_{lm}\mathrm{Tr}[\gamma_j(\hat{k}+m)\gamma_m(\hat{k}+\hat{q}+m)\gamma_i(\hat{k}+\hat{q}-\hat{p}+m)\gamma_l(\hat{k}-\hat{p}+m)] \tag{92}$$

Again, we define $s^\mu = (k+q)^\mu$, $t^\mu = (k-p)^\mu$ and also $u^\mu = (k+q-p)^\mu$. Thus, we consider

$$\frac{1}{4}\epsilon_{ij}\epsilon_{lm}\mathrm{Tr}[\gamma_j(\hat{k}+m)\gamma_m(\hat{s}+m)\gamma_i(\hat{u}+m)\gamma_l(\hat{t}+m)] \tag{93}$$

It is helpful to expand out equ.(93) as

$$\begin{aligned}\frac{1}{4}\mathrm{Tr}\Big[&\gamma_2(\hat{k}+m)\gamma_2(\hat{s}+m)\gamma_1(\hat{u}+m)\gamma_1(\hat{t}+m)\\ &-\gamma_2(\hat{k}+m)\gamma_1(\hat{s}+m)\gamma_1(\hat{u}+m)\gamma_2(\hat{t}+m)\\ &-\gamma_1(\hat{k}+m)\gamma_2(\hat{s}+m)\gamma_2(\hat{u}+m)\gamma_1(\hat{t}+m)\\ &+\gamma_1(\hat{k}+m)\gamma_1(\hat{s}+m)\gamma_2(\hat{u}+m)\gamma_2(\hat{t}+m)\Big]\end{aligned} \tag{94}$$

and treat each term separately. The trick is to reduce this expression to a trace over at most four γ^μ matrices. This may be accomplished via the identities

$$\gamma_2(\hat{k}+m)\gamma_2 = -2k^2\gamma_2 + (\hat{k}-m) \tag{95}$$

and

$$\gamma_1(\hat{k}+m)\gamma_1 = -2k^1\gamma_1 + (\hat{k}-m) \tag{96}$$

We define the four vectors $k'^\mu = (k^0; -k^1, k^2, k^3)$ and $k''^\mu = (k^0; k^1, -k^2, k^3)$ so that equ.(94) becomes

$$\begin{aligned}\frac{1}{4}\mathrm{Tr}[&(\hat{k}'' - m)(\hat{s}+m)(\hat{u}' - m)(\hat{t}+m)\\ &+(\hat{k}' - m)(\hat{s}+m)(\hat{u}'' - m)(\hat{t}+m)\\ &-(\hat{k}+m)(\hat{s}' - m)(\hat{u}+m)(\hat{t}'' - m)\\ &-(\hat{k}+m)(\hat{s}'' - m)(\hat{u}+m)(\hat{t}' - m)]\end{aligned} \tag{97}$$

Now we can apply the trace theorems with little difficulty. As with box 1, there are no terms in an odd power of m. For the crossed graph (box 2) the m^4 term also drops out. The coefficient of m^2 is

$$t.[2s - (u' + u'') - (k' + k'')] - s.(u' + u'' + k' + k'') + u.(-2k + t' + t'' + s' + s'')$$
$$+ k.(s' + s'' + t' + t'') + k'.u'' + k''.u' - t''.s' - t'.s'' \tag{98}$$

Most of this term vanishes by inspection with the identity $(k' + k'')^\mu = 2k_L^\mu$; the m^2 term becomes

$$2t.s' - 2u.k + k'.u'' + k''.u' - t''.s' - t'.s'' \tag{99}$$

We now substitute the expressions for s, t and u. After a little algebra the whole m^2 term simplifies to zero. The coefficient of m^0 is

$$\begin{aligned}&(k''.s\ u'.t + k'.s\ u''.t) - (k.s'\ t''.u + k.s''\ t'.u)\\ &+ (k''.t\ u'.s + k'.t\ u''.s) - (u.s'\ t''.k + u.s''\ t'.k)\\ &+ 2(k.u\ t''.s' - s.t\ k'.u'')\end{aligned} \tag{100}$$

We expand out the first and second lines of equ.(100) and collect all terms in $k_L.s_L$, k^1s^1 and k^2s^2, and $k_L.t_L$, k^1t^1 and k^2t^2 respectively. Each vanishes. The third line of equ.(99) does give a finite contribution. We substitute in the expressions for for s, t and u and find

$$4k_T^2 q.p \tag{101}$$

for the second box. (Note that $k_T^2 \geq 0$).

Kinematics and Phase Space

We calculate the box graphs in the centre of mass frame. The photon and gluon momenta are $q^\mu = (q^0; 0_T, -p^3)$ and $p^\mu = (p^0; 0_T, p^3)$ respectively. Hence, we can write

q^0 and p^0 in terms of Q^2 and P^2 as $p^0 = \sqrt{(p^3)^2 - P^2}$ and $q^0 = \sqrt{(p^3)^2 - Q^2}$. The momenta of the produced quark antiquark pair are $s^\mu = (s^0; -\vec{s})$ and $t^\mu = (s^0; \vec{s})$. We now derive a few kinematic relations, which are useful in evaluating $\Delta\sigma(x, Q^2, P^2, m^2)$.

The s-channel invariant mass squared (or centre of mass energy squared) is given by

$$W^2 = (p+q)^2 = Q^2 \frac{(1-x)}{x} - P^2 \tag{102}$$

In the centre of mass frame W^2 becomes

$$W^2 = (p^0 + q^0)^2 = (s+t)^2 = 4(s^0)^2 \tag{103}$$

We let $W = +\sqrt{W^2} = 2s^0$. Since the produced quark and antiquark are treated as on shell via the optical theorem (viz. $\vec{s}^2 = (s^0)^2 - m^2$), it follows that the quark three momentum has magnitude $s = |\vec{s}| = \frac{1}{2} W \sqrt{1 - \frac{4m^2}{W^2}}$. The photon (and gluon) three momentum is obtained by squaring

$$\begin{gathered} W - p^0 = q^0 \\ W - \sqrt{(p^3)^2 - P^2} = \sqrt{(p^3)^2 - Q^2} \end{gathered} \tag{104}$$

to give

$$2Wp^0 = W^2 - P^2 + Q^2 \tag{105}$$

We eliminate p^0 in terms of p^3 and find

$$4W^2 (p^3)^2 = 4\left[(p.q)^2 - P^2 Q^2\right] \tag{106}$$

or

$$\frac{p.q}{(p^3)^2} = \frac{2\left[(1-x) - \frac{P^2 x}{Q^2}\right]}{1 - \frac{4x^2 P^2}{Q^2}} \tag{107}$$

To evaluate the loop integral and phase space in equ.(78) we must first parametrise

k^μ. We let $t^\mu = (s^0; (s\sin\theta)_T, s^3 = s\cos\theta)$ (ignoring ϕ angles), whence

$$k^\mu = (p-t)^\mu = (p^0 - s^0; -(s\sin\theta)_T, p^3 - s\cos\theta) \tag{108}$$

The denominator in the quark propagator may then be written

$$\begin{aligned} m^2 - k^2 &= P^2 + 2(p^0 s^0 - p^3 s^3) \\ &= P^2 + 2(p^0 s^0 - p^3 s\cos\theta) \\ &= 2p^3 s(1 - \cos\theta + \delta) \end{aligned} \tag{109}$$

where

$$\delta = \frac{2p^0 s^0 - 2p^3 s + P^2}{2p^3 s} \tag{110}$$

Similarly, the quark propagator in the right leg of the crossed diagram (box 2) has denominator

$$m^2 - u^2 = 2p^3 s(1 + \cos\theta + \delta) \tag{111}$$

The quark transverse momentum squared ($k_T^2 = u_T^2$) is

$$k_T^2 = s^2 \sin^2\theta \tag{112}$$

Finally, we need the following result

$$2p^3 s(1+\delta) = 2p^0 s^0 + P^2 = p^0 W + P^2 = q.p \tag{113}$$

which follows from equ.(105). It corresponds to

$$\delta = \frac{1}{\sqrt{1 - \frac{4m^2}{W^2}}\sqrt{1 - \frac{4x^2P^2}{Q^2}}} - 1 \tag{114}$$

The two body phase space in equ.(78) is

$$\int d^4k \; \delta(s^2 - m^2) \; \delta(t^2 - m^2) \tag{115}$$

It is evaluated by inserting the unity factor (energy momentum conserving delta function) $1 = \int ds^0 \delta(s^0 + t^0 - W)$ inside the integral $\int d^4k$. Since $t^\mu = (k-p)^\mu$

we can transform the integration variable from $\int d^4k$ to $\int d^4t$ to obtain

$$\int d^4t \int ds^0 \delta(s^0 + t^0 - W)\delta(s^2 - m^2)\, \delta(t^2 - m^2) \tag{116}$$

We now do the integrals $\int dt^0$ and $\int ds^0$, which leaves a trivial integral in Euclidean 3-space. The phase space integrals reduce to

$$\int d(\cos\theta) \int ds s^2 2\pi \frac{1}{W^2}\delta(2\sqrt{s^2+m^2} - W) = \frac{\pi}{4}\sqrt{1 - \frac{4m^2}{W^2}} \int d(\cos\theta) \tag{117}$$

(Integration over the ϕ angle gives 2π.) We now multiply by the constant term $-\frac{2TN_f g^2}{(2\pi)^3}$ from equ.(78) (where $T = \frac{1}{2}$ in QCD) to obtain a phase space factor

$$-\frac{\alpha_s}{2\pi} N_f \sqrt{1 - \frac{4m^2}{W^2}} \frac{1}{4} \tag{118}$$

The phase space factor $\sqrt{1 - \frac{4m^2}{W^2}}$ determines the kinematic cut-off

$$x_{max} = \frac{Q^2}{Q^2 + P^2 + 4m^2} \tag{119}$$

so that $\Delta\sigma(x, Q^2, P^2, m^2) = 0\ \forall x \geq x_{max}$.

Evaluating the Boxes

At this point, we define $\eta = 1 - \cos\theta + \delta$ so that the quark virtualities are

$$m^2 - k^2 = 2p^3 s\eta \tag{120}$$

and

$$m^2 - u^2 = 2p^3 s[2(1+\delta) - \eta] \tag{121}$$

The quark transverse momentum squared has an expression

$$\begin{aligned} k_T^2 &= s^2 \sin^2\theta \\ &= -s^2\left[\eta^2 - 2\eta(1+\delta) + \delta(2+\delta)\right] \end{aligned} \tag{122}$$

in terms of η. For the integration variable,

$$\int_{-1}^{+1} d(\cos\theta) = \int_{\delta}^{2+\delta} d\eta \tag{123}$$

Thus, we can write a (yet to be integrated) expression for box 1

$$\frac{s}{2p^3}\eta + \frac{1}{2} + \left[\frac{q.p(1+\delta)}{(p^3)^2} + \frac{s^2\delta(2+\delta)}{2p^3 s}\right]\frac{1}{\eta} + \frac{2q.p}{4(p^3)^2 s^2}\left[s^2\delta(2+\delta) - \frac{1}{2}xP^2\right]\frac{1}{\eta^2} \tag{124}$$

The denominator in box 2 is the product of $m^2 - k^2$ and $m^2 - u^2$. Therefore, we need to consider

$$\frac{1}{\eta[2(1+\delta)-\eta]} = \frac{1}{2(1+\delta)}\left[\frac{1}{\eta} + \frac{1}{2(1+\delta)-\eta}\right] \tag{125}$$

Since $(1+\delta+\xi) = 2(1+\delta) - (1+\delta-\xi)$, the two terms in the square bracket on the right hand side of this equation each give an equal contribution to box 2. Hence, the (yet to be integrated) contribution from box 2 to $\Delta\sigma$ is

$$-\frac{q.p}{4(p^3)^2}\frac{1}{(1+\delta)}\frac{\eta^2 - 2\eta(1+\delta) + \delta(2+\delta)}{\eta} \tag{126}$$

Equs.(124) and (126) are simplified with the kinematic relations (equs. (102-114)). We add the contributions from boxes 1 and 2 (equs.(124) and (126)) and multiply by the phase space factor equ.(118) to obtain

$$\begin{aligned}
&\Delta\sigma(x, Q^2, P^2, m^2) = \\
&-\frac{\alpha_s}{2\pi}N_f \int d\eta \left[\frac{(2x-1)(1-\frac{2xP^2}{Q^2})}{(1-\frac{4x^2P^2}{Q^2})}\left(\frac{1}{2}\sqrt{1-\frac{4m^2}{W^2}} - \frac{1}{\eta\sqrt{1-\frac{4x^2P^2}{Q^2}}}\right)\right. \\
&\left.-\frac{2(1-x-\frac{xP^2}{Q^2})}{(1-\frac{4x^2P^2}{Q^2})^2}\frac{(2m^2(1-\frac{4x^2P^2}{Q^2}) - P^2x(2x-1)(1-\frac{2xP^2}{Q^2}))}{\eta^2 W^2\sqrt{1-\frac{4m^2}{W^2}}}\right]
\end{aligned} \tag{127}$$

where the η integral ranges between δ and $2+\delta$ for the full phase space. To evaluate

$\Delta\sigma$ we have only to do the trivial integrals

$$\int_{\delta}^{2+\delta} d\eta \; 1 = 2$$
$$\int_{\delta}^{2+\delta} d\eta \; \frac{1}{\eta} = \ln(\frac{2+\delta}{\delta}) \tag{128}$$
$$\int_{\delta}^{2+\delta} d\eta \; \frac{1}{\eta^2} = \frac{2}{\delta(2+\delta)}$$

Putting things together, the exact expression for $\Delta\sigma$ is

$$\begin{aligned}
&\Delta\sigma(x, Q^2, P^2, m^2) = \\
&-\frac{\alpha_s}{2\pi} N_f \frac{\sqrt{1-\frac{4m^2}{W^2}}}{1-\frac{4x^2P^2}{Q^2}} \Bigg[(2x-1)(1-\frac{2xP^2}{Q^2}) \\
&\left(1 - \frac{1}{\sqrt{1-\frac{4m^2}{W^2}}\sqrt{1-\frac{4x^2P^2}{Q^2}}} \ln(\frac{1+\sqrt{1-\frac{4x^2P^2}{Q^2}}\sqrt{1-\frac{4m^2}{W^2}}}{1-\sqrt{1-\frac{4x^2P^2}{Q^2}}\sqrt{1-\frac{4m^2}{W^2}}})\right) \\
&+(x-1+\frac{xP^2}{Q^2}) \frac{\left(2m^2(1-\frac{4x^2P^2}{Q^2}) - P^2x(2x-1)(1-\frac{2xP^2}{Q^2})\right)}{m^2(1-\frac{4x^2P^2}{Q^2}) - P^2x(x-1+\frac{xP^2}{Q^2})} \Bigg]
\end{aligned} \tag{129}$$

In the Bjorken limit $Q^2 \gg P^2, m^2$ $\Delta\sigma(x, Q^2, P^2, m^2)$ becomes

$$\begin{aligned}
\Delta\sigma(x, Q^2, P^2, m^2) = \frac{\alpha_s}{2\pi} N_f \Bigg[&(2x-1)\left(\ln\frac{Q^2}{\mu^2} + \ln\frac{1-x}{x} - 1\right) \\
&+ (2x-1)\ln\frac{\mu^2}{x(1-x)P^2+m^2} + (1-x)\frac{2m^2 - P^2x(2x-1)}{x(1-x)P^2+m^2} \Bigg]
\end{aligned} \tag{130}$$

Here, we have inserted a scale μ^2 to separate out the $\ln Q^2$ term from the x dependent scaling terms. The term $(2x-1)$ is the Altarelli-Parisi splitting function for polarised $g \rightarrow q\bar{q}$.

Equ. (130) is not well defined in the limit that both m^2 and P^2 go to zero. There is a *mass singularity* in the Q^2 independent logarithm. This comes from the $\frac{1}{\eta}$ term in our box calculation. The last term in equ.(130) depends on the ratio of m^2 to P^2. It comes from the $\frac{1}{\eta^2}$ term and is not well defined when both m^2 and P^2 are identically zero.

In both cases, the problem comes from the small η part of the total phase space. This translates to low k_T^2 or, equivalently, low $m^2 - k^2$. Due to the gauge parameter in the general gluon propagator, the box diagram with gluon external legs is gauge dependent when we take $P^2 \neq 0$ (unless we restrict ourselves to gauge transformations with a fixed momenta P^2). Hence, a gauge invariant calculation of the photon-gluon fusion goes to light quarks process has a singularity in the collinear region of phase space where the two quarks are produced with zero transverse momentum. However, we know that the perturbative quark propagator is unrealistic in this dynamical region due to confinement [86]. If quarks (or gluons) were permitted to reach their mass shell then we would see coloured states in nature, which does not occur. The perturbative gluon propagator is similarly unphysical when the gluon approaches on-mass shell; quark and gluon virtualities in the proton are typically $P^2 \geq O(\Lambda_{QCD}^2)$. The solution to these problems is to factor out this soft (long distance) part of the box's phase space into the quark distribution [19, 58, 75-81], as we describe below.

The final state quark-antiquark pair are placed on mass shell in our formalism when we apply the optical theorem and take the absorptive part of the forward Compton scattering amplitude. We can physically justify this since the struck quark is given a substantial amount of energy by the incident photon. This energy is sufficient to remove it from the target hadron without any significant final state interaction with the remaining hadronic debris. It then undergoes hadronisation into a jet of fast moving (colourless) hadrons in the same direction as the final state quark jet in our perturbative calculation. The on-shellness of the final state quarks is a mathematical statement of the negligible final state interaction with the residual target debris. The reason that we keep both P^2 and $m^2 - k^2$ away from the mass shell is related to QCD dynamics within the target proton.

The first moment of $\Delta\sigma(x, Q^2, P^2, m^2)$ in the scaling limit $Q^2 \to \infty$ is [25]

$$G_1 = -\frac{\alpha_s}{2\pi} N_f \left[1 + \frac{2m^2}{P^2} \frac{1}{\sqrt{1+\frac{4m^2}{P^2}}} \ln\left(\frac{\sqrt{1+\frac{4m^2}{P^2}} - 1}{\sqrt{1+\frac{4m^2}{P^2}} + 1} \right) \right] \tag{131}$$

This expression simplifies [87] to $-\frac{\alpha_s}{2\pi} N_f$ when $P^2 \gg m^2$. It is zero when $m^2 \gg P^2$ (for example, in $c\,\bar{c}$ production). Clearly, there is an ambiguity in G_1 when both m^2 and P^2 go to zero. In this case, G_1 depends upon the ratio in which m^2 and P^2 vanish, and not that they do vanish. The result $G_1 = -\frac{\alpha_s}{2\pi} N_f$ was obtained by Altarelli and Ross [22] for real gluons ($P^2 = 0$). Unfortunately, the $\frac{1}{\eta^2}$ term was lost in their calculation of the box.

The non-local $\frac{m^2}{P^2}$ term in equ.(131) is a general characteristic of photon-gluon scattering. Ioffe and collaborators [88, 89, 24] have examined the box calculation for the scattering of a hard photon from a longitudinal gluon (or photon). They found that the cross-section vanishes (as expected) in the limit that the longitudinal gluon goes on shell ($P^2 = 0$) with a non-zero fermion mass in the loop. However, if we start with an identically massless theory then the cross section for scattering from a longitudinal gluon (or photon) is finite - even in the limit $P^2 \to 0$! They speculate that this result implies that fermions must have a finite mass in any gauge theory (see also Gribov [90]).

4.3 Photon Gluon Fusion and the IPM

As we noted in our discussion of factorisation in section 4.1, the correct procedure is to factorise the low k_T^2 or $m^2 - k^2$ part of the box phase space into the quark distribution of the hadronic wavefunction. We now investigate the effect of this factorisation by calculating the box diagram with a cut-off on the k_T^2, and then on the quark virtuality in the loop [24]. Since

$$m^2 - k^2 = P^2 x + \frac{m^2 + k_T^2}{(1-x)} \tag{132}$$

it is easy to translate between the two results. Note that a cut-off on the k_T^2 cut-off is covariant [91]; it is defined via

$$k_T^\mu = R^{\mu\nu} k_\nu \tag{133}$$

where

$$R^{\mu\nu} = -g^{\mu\nu} + \frac{q^2 p^\mu p^\nu + p^2 q^\mu q^\nu - q.p(q^\mu p^\nu + q^\nu p^\mu)}{(q.p)^2 - q^2 p^2} \tag{134}$$

That is, k_T^μ is a Lorentz four vector defined to be orthogonal to the plane in Lorentz space spanned by q^μ and p^μ.

The phase space corresponding to low k_T^2 is separated out by evaluating the loop integrals with the constraint $\theta(k_T^2 - \Lambda^2)$. Since $k_T^2 = u_T^2$ in box 2 this restriction applies equally to the cross graph; we have no problems with gauge invariance. The cut-off on $k_T^2 = s^2 \sin^2\theta$ is equivalent to a cut-off on $\cos^2\theta \leq (1 - \frac{\Lambda^2}{s^2})$; that is we do the η integral between new limits of integration, viz.

$$1 + \delta - \sqrt{1 - \frac{\Lambda^2}{s^2}} \leq \eta \leq 1 + \delta + \sqrt{1 - \frac{\Lambda^2}{s^2}} \tag{135}$$

In this cut-off field theory the complete expression for $\Delta\sigma$ is

$$\begin{aligned}
&\Delta\sigma(x, Q^2, P^2, m^2) = \\
&-\frac{\alpha_s}{2\pi} N_f \frac{\sqrt{1 - \frac{4(m^2+\Lambda^2)}{W^2}}}{1 - \frac{4x^2P^2}{Q^2}} \Bigg[(2x-1)(1 - \frac{2xP^2}{Q^2}) \\
&\left(1 - \frac{1}{\sqrt{1 - \frac{4(m^2+\Lambda^2)}{W^2}}\sqrt{1 - \frac{4x^2P^2}{Q^2}}} \ln\left(\frac{1 + \sqrt{1 - \frac{4x^2P^2}{Q^2}}\sqrt{1 - \frac{4(m^2+\Lambda^2)}{W^2}}}{1 - \sqrt{1 - \frac{4x^2P^2}{Q^2}}\sqrt{1 - \frac{4(m^2+\Lambda^2)}{W^2}}}\right)\right) \\
&+ (x - 1 + \frac{xP^2}{Q^2}) \frac{\left(2m^2(1 - \frac{4x^2P^2}{Q^2}) - P^2 x(2x-1)(1 - \frac{2xP^2}{Q^2})\right)}{(m^2 + \Lambda^2)(1 - \frac{4x^2P^2}{Q^2}) - P^2 x(x - 1 + \frac{xP^2}{Q^2})} \Bigg]
\end{aligned} \tag{136}$$

Note that this equation reproduces equ.(129) when $\Lambda^2 = 0$. The effect of the k_T^2 cut-off is just to replace m^2 by $m^2 + \Lambda^2$ everywhere except in the numerator of the $\frac{1}{\eta^2}$ term.

For $Q^2 \gg \Lambda^2$ (and m^2 and P^2) we find

$$\Delta\sigma(x, Q^2, P^2, m^2) = \frac{\alpha_s}{2\pi} N_f \Bigg[(2x-1)\left(\ln \frac{Q^2}{\Lambda^2} + \ln \frac{(1-x)}{x} - 1 \right) + (2x-1) \ln \frac{\Lambda^2}{x(1-x)P^2 + (m^2+\Lambda^2)} + (1-x) \frac{2m^2 - P^2 x(2x-1)}{x(1-x)P^2 + m^2 + \Lambda^2} \Bigg] \tag{137}$$

Here, we have written all of the m^2 and P^2 dependence in the second line of the cross section. We may now safely set $P^2 = 0$ (to ensure gauge invariance) and also $m^2 = 0$ for the light quarks. There is no need to worry about the mass singularity, which occurs when we integrate over the total phase space. The second line is relevant only for heavy quark production (eg. $c\bar{c}$).

In this cut-off field theory there is a further restriction on the allowed values of x. We have the new kinematic constraint

$$x_{max} = \frac{Q^2}{Q^2 + P^2 + 4(\Lambda^2 + m^2)} \tag{138}$$

which means that $x < 1$ for finite Λ^2 and Q^2. We never hit the logarithmic singularity at $x = 1$. The two quark jet contribution to $g_1(x)$ for the proton (equ.(75)) is pushed to lower x in the data as we increase the minimum k_T^2 acceptance threshold.

It is now a trivial exercise to evaluate the moments of $\Delta\sigma$; viz.

$$G_n = \int_0^1 dx x^{n-1} \Delta\sigma(x, Q^2, P^2, m^2) \tag{139}$$

For $n \geq 2$ G_n behaves as

$$G_n \sim \frac{\alpha_s}{2\pi} N_f (\Delta\gamma_{VV}^F)_n \ln \frac{Q^2}{\Lambda^2} \tag{140}$$

as we expect from the Altarelli Parisi formalism. Here, $(\Delta\gamma_{VV}^F)_n = \int_0^1 x^{n-1}(2x-1)$.

This result holds independently of the choice of cut-off. The leading contribution to G_n ($n \geq 2$) comes from integration over $m^2 - k^2$ or k_T^2 between $\sim [\Lambda^2, Q^2]$. It depends on the value of the finite cut-off Λ^2.

The first moment ($n = 1$) is particularly interesting. It has no $\ln Q^2$ dependence. We evaluate G_1 as a function of the k_T^2 cut-off Λ^2 and find ($\Lambda^2 =$ constant)

$$G_1(\Lambda^2) = -\frac{\alpha_s}{2\pi} N_f \left[1 + \frac{2m^2}{P^2} \frac{1}{\sqrt{1 + \frac{4(m^2+\Lambda^2)}{P^2}}} \ln \left(\frac{\sqrt{1 + \frac{4(m^2+\Lambda^2)}{P^2}} - 1}{\sqrt{1 + \frac{4(m^2+\Lambda^2)}{P^2}} + 1} \right) \right] \tag{141}$$

Once again, this result reduces to equ.(131) when $\Lambda^2 = 0$. For $P^2 >> m^2$ G_1 reduces to $-\frac{\alpha_s}{2\pi} N_f$, regardless of the value we choose for Λ^2. For the case $m^2 >> P^2$ [92] equ.(141) becomes $G_1 = -\frac{\alpha_s}{2\pi} N_f \frac{\Lambda^2}{\Lambda^2 + m^2}$, which is zero when Λ^2 vanishes and $-\frac{\alpha_s}{2\pi} N_f$ when $\Lambda^2 >> m^2$ (ie. when we can neglect both the m^2 and P^2 scales). The latter case is relevant to factorisation with light quarks: we set $m^2 = P^2 = 0$ and find a finite result for G_1 that is independent of the finite cut-off Λ^2. Thus, there is an unambiguous contribution to the first moment of $\Delta\sigma$, which comes from a purely pointlike part of the box (at very large k_T^2). It means that there is an effective photon-gluon contact interaction which provides a local measurement of the target (perturbative) gluon's helicity [87]. Furthermore, the first moment of $g_1(x)$ receives a contribution from polarised glue in the IPM. For each flavour q, one should replace

$$\Delta q(Q^2) \to \Delta q(Q^2) - \frac{\alpha_s}{2\pi} \Delta g(Q^2) \tag{142}$$

in the large k_T^2 version of the IPM expression for $\int_0^1 dx \; g_1(x)$ [24, 92]. This is the Efremov-Teryaev, Altarelli-Ross result [21, 22, 85, 87]. These authors suggested that one could retain the Ellis-Jaffe hypothesis that $\Delta s = 0$ if there were a large polarised gluon component in the proton $\Delta g \sim 5$ at $Q^2 \sim 10\text{GeV}^2$.

There are a few surprising things about these results. Firstly, it appears that gluons can behave just like quarks in polarised DIS. Photons scatter locally from quarks because the quarks carry electric charge. However, gluons are electrically neutral so one would expect photon-gluon scattering to be predominantly non-local. Secondly and more formally, we seem to have a contradiction between the OPE and the QCD Improved Parton Model. The OPE told us that the first moment of the gluon coefficient distribution relevant to $g_1(x)$ is exactly zero in any gauge invariant renormalisation scheme. Clearly, the IPM and OPE results are different. We shall return to this problem in the next chapter when we discuss the axial anomaly in QCD.

Before we do this however, it is useful to check the factorisation scheme dependence of our results. We shall investigate this by carrying out the factorisation with another choice of infrared cut-off (on the virtuality of the quark in the loop). This corresponds to subtracting out a different slice of the total phase space - and hence to a different two-quark-jet definition [25]. The cut-off on the virtuality is x dependent with respect to the k_T^2 cut-off, equ.(132). The small x part of the nucleon structure functions is described by Regge theory whilst the large x region is described by resonance production at realistic Q^2. Therefore, an x dependent cut-off might be convenient in a full non-perturbative analysis of the structure functions. We shall also look at a few related gluon fusion processes to check whether the local photon-gluon interaction could be observable in different experiments.

We now consider a cut-off on the quark virtuality [24, 25] and evaluate the box graph loop integrals with the constraint $\theta(m^2 - k^2 - \Gamma^2)$. One must be careful to apply the same restriction to $m^2 - u^2$ in the crossed box (box 2). From equ.(111) we know that $m^2 - k^2 \geq \frac{Q^2}{x} - \Gamma^2$ whenever $m^2 - u^2 \leq \Gamma^2$, and vice versa. For consistency, we should constrain $m^2 - k^2 \leq \frac{Q^2}{x} - \Gamma^2$ in both boxes. This extra constraint has little effect when $Q^2 \gg \Gamma^2$. The virtuality cut-off is consistent with positive k_T^2 whenever $\Gamma^2 \geq P^2$ (see equ.(132)). We set $Q^2 \gg P^2, m^2, \Gamma^2$. The cut-off $(m^2 - k^2) \geq \Gamma^2$ is equivalent to evaluating the η integrals between $2x\frac{\Gamma^2}{Q^2} \leq \eta \leq 2$. We find

$$\Delta\sigma(x, Q^2, P^2, m^2, \Gamma^2) = \frac{\alpha_s}{2\pi} N_f \left[(2x-1)\left(\ln\frac{Q^2}{\Gamma^2} - \ln x - 1\right) + \frac{2m^2 - P^2 x(2x-1)}{\Gamma^2} \right] \tag{143}$$

which has the first moment

$$G_1(\Gamma^2) = -\frac{\alpha_s}{2\pi} N_f \left(\frac{1}{2} + \frac{1}{6}\zeta + \frac{2m^2}{\Gamma^2} \right) \tag{144}$$

where $\zeta = \frac{P^2}{\Gamma^2}$. (If $P^2 \leq \Gamma^2$ we have to be careful not to include any negative k_T^2. For $m^2 \ll \Gamma^2$ we then find $G_1(\Gamma^2) = -\frac{\alpha_s}{2\pi} N_f \left(1 - \frac{1}{3}\zeta^2 + \zeta(1-\zeta)\ln\zeta\right)$ [24].) The factorisation procedure is completed by setting $m^2 = P^2 = 0$ whence $G_1(\Gamma^2) = -\frac{\alpha_s}{2\pi} N_f \frac{1}{2}$. The virtuality cut-off yields a contribution at infinity which is one half of what we found with the k_T^2 cut-off [24, 25]. The contribution to G_1 "at infinity" depends upon the x dependence of the infra-red cut-off. There is no unique way to define "infinity". The factorisation scheme dependence of G_1 was first pointed out by Kolya Nikolaev, Tony Thomas and myself in ref. [25]. It was later taken up by Manohar in a series of papers [67, 93].

The cut-off on the transverse momentum squared opens up the maximum amount of phase space with which to define the two-quark-jet events [84]. If we are at large k_T^2 then we are also at large $m^2 - k^2$. This result follows from equ.(132) whereby $m^2 - k^2 \geq k_T^2$. The converse does not hold. (We can be at a very large virtuality but small k_T^2 when $x \to 1$.) Furthermore, the dynamics of the problem lie in the plane in Lorentz space defined by p^μ and q^μ. Since k_T^2 is orthogonal to this plane, any cut-off on the transverse momentum squared will not violate any symmetries of the interaction. It is not clear that any other form of cut-off (eg. which encroaches into the p^μ q^μ plane) will have this nice property. (I would like thank V. Gribov for discussions on this point [94].) These two points suggest that the k_T^2 cut-off is the preferred method by which to define the factorisation.

4.4 Comparison with Other Processes

In the preceding sections we have spent much effort analysing spin dependent photon-gluon fusion in the IPM. However, there is nothing in the box calculation that is not already in QED. Transfering from the soft gluon to a soft photon just multiplies the group factor T by two. This means that the total x integrated spin dependent asymmetry equ.(77) for (hard γ)+(soft γ) $\rightarrow e^+e^-$ has a finite contribution from a local photon photon interaction.

We now look at two other physical processes to see whether the local photon gluon interaction might be observable in different experiments. The first process is unpolarised DIS and the box graphs relevant to $F_2(x)$. Next we consider the spin dependent asymmetry for gg $\rightarrow$ c$\overline{\text{c}}$, which is the leading parton process which contributes to the spin dependent asymmetry for p$\overline{\text{p}}$ $\rightarrow$ c$\overline{\text{c}}$X.

Unpolarised Photon-Gluon Fusion

The cross section for unpolarised hard photon (large Q^2) soft gluon (soft P^2) scattering is denoted by $\sigma(x, Q^2, P^2, m^2)$. At one loop $\sigma(x, Q^2, P^2, m^2)$ is calculated from the optical theorem as

$$\begin{aligned}\sigma(x, Q^2, P^2, m^2) = &-\frac{2N_f T g^2}{(2\pi)^4} \int d^4k \frac{(2\pi)\delta((p-k)^2 - m^2)\delta((q+k)^2 - m^2)}{(k^2 - m^2 + i\epsilon)^2} \\ &\frac{1}{4} g^T_{ij} g^T_{lm} \mathrm{Tr}\big[\gamma_j(\hat{k}+m)\gamma_m(\hat{q}+\hat{k}+m)\gamma_l(\hat{k}+m)\gamma_i(\hat{k}-\hat{p}+m) \\ &+ \gamma_j(\hat{k}+m)\gamma_m(\hat{q}+\hat{k}+m)\gamma_i(\hat{k}+\hat{q}-\hat{p}+m)\gamma_l(\hat{k}-\hat{p}+m)\big]\end{aligned} \tag{145}$$

Here, $g^T_{\mu\nu} = \mathrm{diag}[0; -1, -1, 0]$. The phase space and kinematics are the same as for $\Delta\sigma$, and the tricks for simplifying the Dirac algebra apply here too. For the numerator in box 1, the Dirac algebra yields

$$\frac{1}{4}\left[\frac{1}{2}(k^2 - m^2)^2 + (k^2 - m^2)(q.p + k_T^2) + k_T^2(k_T^2 + Q^2 + P^2) + \frac{1}{2}Q^2P^2\right] \tag{146}$$

For the crossed box (box 2) we find

$$\frac{1}{4}k_T^2\Big[2k_T^2 - q.p + Q^2 + P^2\Big] \tag{147}$$

The next step is write $k^2 - m^2$ and k_T^2 in terms of our variable η. After a lengthy (but straightforward) calculation following sections 4.2 and 4.3 we evaluate the η integrals corresponding to $k_T^2 \geq \Lambda^2$. With this restriction on the total phase space we obtain

$$\begin{aligned}
&\sigma(x, Q^2, P^2, m^2, \Lambda^2) = \\
&\frac{\alpha_s}{2\pi}N_f\Bigg[(2x^2 - 2x + 1)\Bigg(\ln(\frac{Q^2}{\Lambda^2}) + \ln(\frac{1-x}{x}) - 1\Bigg) \\
&+ (2x^2 - 2x + 1)\ln(\frac{\Lambda^2}{x(1-x)P^2 + m^2 + \Lambda^2}) + x(1-x)\frac{(2m^2 - P^2(2x^2 - 2x + 1))}{x(1-x)P^2 + m^2 + \Lambda^2}\Bigg]
\end{aligned} \tag{148}$$

Here, we have set $Q^2 \gg \Lambda^2$. The moments of σ are given by

$$\begin{aligned}
F_n &= \int_0^1 dx \; x^{n-1}\sigma(x, Q^2, P^2, m^2) \\
F_n &\sim \frac{\alpha_s}{2\pi}N_f(\gamma_{VV}^F)_n \ln\frac{Q^2}{\Lambda^2}
\end{aligned} \tag{149}$$

where $(\gamma_{VV}^F)_n = \int_0^1 dx \; x^{n-1}(2x^2 - 2x + 1)$. Hence, F_n is dominated by integration over k_T^2 in $\sim [\Lambda^2, Q^2]$ for each n (including $n = 1$). This is the same situation as occurs with the higher ($n \geq 2$) moments of $\Delta\sigma$ in the polarised case. Any pointlike piece will be swamped by the large logarithm.

Spin Dependent Gluon-Gluon Fusion

The spin dependent asymmetry for gg $\rightarrow$ c$\bar{\text{c}}$ receives a contribution from each of the two graphs in Fig. 7. It contributes to the asymmetry for p$\bar{\text{p}}$ $\rightarrow$ c$\bar{\text{c}}$X via the convolution

$$A^{c\bar{c}} = \int_0^1 \frac{dx_a}{x_a}\int_0^1 \frac{dx_b}{x_b}\Delta g(x_a, \Lambda^2)\Delta g(x_b, \Lambda^2)\Delta\sigma_{gg\rightarrow c\bar{c}}(x_a, x_b, \frac{W^2}{\Lambda^2}, \alpha_s(\Lambda^2)) \tag{150}$$

Here, $\Delta g(x, \Lambda^2)$ represents the IPM spin dependent gluon distribution of the proton (a) and anti-proton (b); W^2 is the invariant mass squared of the c$\bar{\text{c}}$ jets. The hard factor $\Delta\sigma_{gg\to c\bar{c}}(x, \frac{W^2}{\Lambda^2}, \alpha_s(\Lambda^2))$ is the spin dependent asymmetry for producing $c\bar{c}$ with transverse momentum squared $k_T^2 \geq \Lambda^2$. It receives contributions from the two Feynman graphs shown in Fig 7 (plus the cross terms). Here, we shall be interested only in the box contribution which mimics the QED process: two soft photons with large centre of mass energy decay to $\mu^+\mu^-$. This box contribution may be deduced from our previous calculation (equ.(136)) by moving to a different kinematic region. We work in the centre of mass frame. One gluon originates from the proton (momentum P_a^μ) with momentum $p^\mu = x_a P_a^\mu + p_T$. The other gluon originates from the antiproton (momentum P_b^μ) with momentum $q^\mu = x_b P_b^\mu + q_T$. We consider high energy $\mathrm{p\bar{p}}$ so that $E_a, E_b \gg m$ (even within the gg centre of mass frame). The invariant mass squared for the $\mathrm{gg} \to \mathrm{c\bar{c}}$ process is

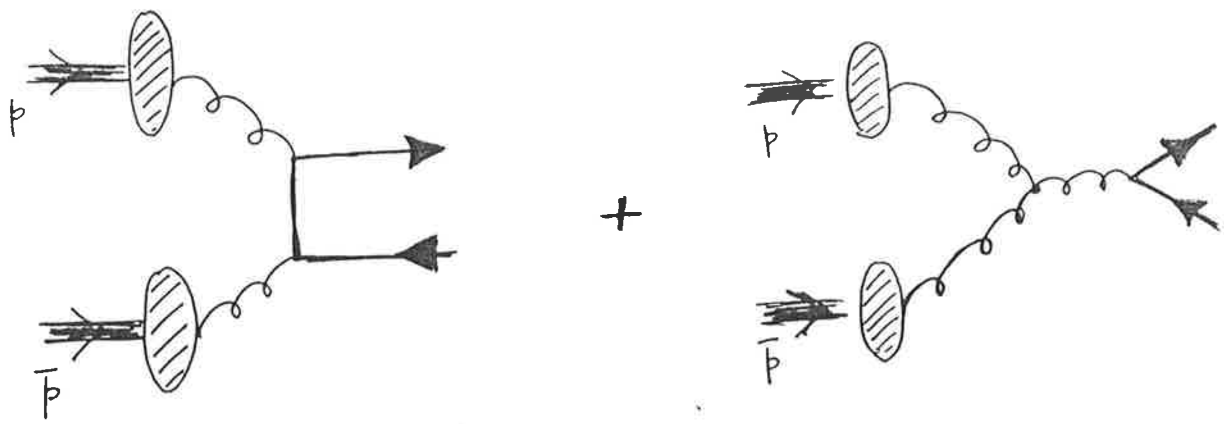
$$W^2 = (p+q)^2 = 4x_a x_b E_a E_b = x_a x_b W_{\mathrm{p\bar{p}}}^2 \tag{151}$$

Here, $W_{\mathrm{p\bar{p}}}^2$ is the invariant mass squared of the $\mathrm{p\bar{p}}$ system. Since $W^2 = Q^2 \frac{1-x}{x} - P^2$ in terms of the Bjorken variable, it follows that the gluon fusion process corresponds to the $x \to 0$ limit of equ.(136). We choose to look for $\mathrm{c\bar{c}}$ jets with $k_T^2 \geq \Lambda^2$ where the acceptance threshold is constrained by the kinematics to be $\Lambda^2 \leq \frac{1}{4}W^2 - m^2$. In the limit $W^2 \gg \Lambda^2$ we complete the factorisation and find

$$\begin{aligned} \Delta\sigma_{gg\to c\bar{c}}(x, \frac{W^2}{\Lambda^2}, \alpha_s(\Lambda^2)) &= \frac{\alpha_s}{2\pi}\left[1 - \ln\frac{W^2}{\Lambda^2} + \frac{2m^2}{m^2+\Lambda^2}\right] \\ &= \frac{\alpha_s}{2\pi}\left[1 - \ln\frac{W_{\mathrm{p\bar{p}}}^2}{\Lambda^2} - \ln x_a x_b + \frac{2m^2}{m^2+\Lambda^2}\right] \end{aligned} \tag{152}$$

which is dominated by the large (Λ^2 dependent) logarithm. Here (and in the corresponding QED process $\gamma\gamma \to \mu^+\mu^-$) any local term would be lost among the experimental errors.

Figure 7. The spin dependent asymmetry for gg $\rightarrow c\bar{c}$ receives a contribution from each of the two graphs in Fig. 7.



4.5 Gluonic Effects and the $g_1(x)$ Data : the Convolution

As we mentioned in section 4.1 the exclusive two-quark-jet contribution to $g_1(x)$ is given by the convolution

$$g_1^{q\bar{q}}(x) = \frac{1}{9}\int_x^1 \frac{dz}{z}\Delta g(z,\Lambda^2)\Delta\sigma_{\gamma g\to q\bar{q}}(\frac{x}{z},\frac{Q^2}{\Lambda^2},\alpha_s(\Lambda^2)) \tag{153}$$

where $\Delta g(x)$ is the spin dependent gluon distribution of the proton. In the absence of any independent measurement of $\Delta g(x)$ (eg. in polarised $\mathrm{p\bar{p}} \to \mathrm{c\bar{c}X}$ or prompt photon production [95]) we need to deduce a phenomenological parametrisation for $\Delta g(x)$. In the parton model $\Delta g(x,\Lambda^2)$ is interpreted in $A_+ = 0$ gauge as the difference between the x distributions of gluons with $p_T^2 \leq \Lambda^2$ which are polarised in the same direction as the proton's polarisation and where the gluons are polarised in the opposite direction to the proton's polarisation. Thus, we expect

$$|\Delta g(x) = (g^\uparrow - g^\downarrow)(x)| \leq g(x) = (g^\uparrow + g^\downarrow)(x) \tag{154}$$

One experimental parametrisation for $g(x,Q^2)$ at $Q^2 = 4\mathrm{GeV}^2$ is [96]

$$xg(x,Q^2 = 4GeV^2) = 0.88(1+9x)(1-x)^4 \tag{155}$$

We shall consider a form

$$\Delta g(x) = x^\alpha g(x) \tag{156}$$

This is the hardest possible $\Delta g(x)$ that we can write down consistent with the above inequality [25, 26]. The EMC low x data was measured at $< Q^2 >= 4\mathrm{GeV}^2$. Hence, our choice for $\Delta g(x)$ allows us to directly estimate the gluon component in this part of the data. In Fig. 8 we show the two-light-quark-jet contribution to the EMC data with $\Lambda^2 = Q^2 = 4\mathrm{GeV}^2$. We plot the combination $(g_1^{\mathrm{EMCdata}} + g_1^{q\bar{q}})(x)$, where $g_1^{q\bar{q}}(x)$

is evaluated via equ.(153). This plot allows us to see the gluonic contribution to the EMC data on $g_1(x)$ at a glance. The hard parton scattering cross section is taken from equ.(136). For $\alpha = 0.9, 0.8, 0.7$ and 0.2 we find that $\Delta g = \int_0^1 dx\ \Delta g(x) = 0.5, 0.6, 0.8$ and 4.0 respectively. The contribution to $g_1(x)$ is concentrated at small x and lies well within the experimental errors where data is available.

The earliest calculations of the gluonic contribution to $g_1(x)$ are due to Altarelli and Stirling [97] who choose to replace the hard photon-gluon scattering cross section by a delta function at one. As correctly pointed out by Ellis et al. [27], this leads to a gluon contribution (from $\Delta g(x)$) that is much too inflated at large x, where the data is recorded. The authors of ref.[27] chose to calculate the two-quark-jet piece without any cut-off on the k_T^2 or on the virtuality, and set $P^2 \ll m^2 \to 0$ when they evaluated the box for light quarks. Hence, their calculation is sensitive to the $\ln(1-x)$ singularity. This leads to a two-quark-jet cross section that is further inflated at large x than the large k_T^2 results presented here. The soft acceptance cut-off on the jets' k_T^2 implies the kinematic constraint on x in the box, equ.(138). This has the effect of shielding the convolution from the $\ln(1-x)$ singularity. Indeed, we find $\Delta\sigma(x_{max}) = 0$ in equ.(136) for finite Λ^2 and Q^2. The k_T^2 cut-off softens the x dependence of the gluon induced two-quark-jet contribution to $g_1(x)$.

It has been suggested [27] that even at the relatively low Q^2 of the EMC experiment charm quark production could give a significant contribution to $g_1(x)$. Unfortunately that paper contains a mistake in the sign of the charm contribution (viz. equ.(75)). The calculation of $g_1^{c\bar{c}}(x)$ is shown in Fig. 9. Here, we use the same parametrisation for $\Delta g(x)$ and the cross section for hard photon-gluon fusion is evaluated with $m = 2$GeV. Again, we find a negligible contribution to $g_1(x)$.

Brodsky and Schmidt have suggested the parametrisation

$$\Delta g(x)|_{BS} = \frac{1}{x}\left[5(1-x)^4 - 4(1-x)^5 - (1-x)^6\right] \tag{157}$$

using arguments from light-cone dynamics [98]. This corresponds to $\Delta g = 1.2$ and is supposed to hold at the nucleon scale $\Lambda^2 \sim 1\text{GeV}^2$. In Fig. 10 we show the large $k_T^2 \geq \Lambda^2$ two-quark-jet contributions to $g_1(x)$ at $Q^2 = 4\text{GeV}^2$ from both light and charm quarks with the Brodsky and Schmidt $\Delta g(x)$. Once again, the IPM two-quark-jet contribution is at small x - outside the range of the current data.

We have emphasised the difference between the IPM and OPE results for the hard photon-gluon scattering factor in the convolution, equ.(153). We found that $C^g(x) \neq \Delta\sigma(x)$ and that these two quantities have different first moments. This observation does not affect our conclusion that $\Delta g(x)$ is not relevant to the experimentally determined sum-rule between $x_{min} = 0.01$ and $x = 1$, as we show in Fig. 11. To illustrate this we have calculated the combination $(g_1^{EMC} + g_1^{q\bar{q}})(x)$ with the Brodsky and Schmidt gluon distribution $\Delta g(x)_{BS}$ and the Wilson coefficient distribution $C^g(x, \alpha_s)$, which has been evaluated in the $\overline{MS}$ renormalisation scheme as [73, 74]

$$C^g(x, \frac{Q^2}{\Lambda^2}, \alpha_s(\Lambda^2) = \frac{\alpha_s}{2\pi} N_f \left[(2x-1)\left(\ln\frac{Q^2}{\Lambda^2} + \ln\frac{1-x}{x} - 1\right) + 2(1-x)\right] \tag{158}$$

Fig. 11 shows both the light and charm quark contributions to $(g_1^{EMC} + g_1^{q\bar{q}})(x)$.

In summary, the gluonic contribution to $g_1(x)$ is essentially outside the present data; it is at too low x. This means that the large Δg hypothesis cannot be used to reconcile the EMC data with the Ellis-Jaffe sum-rule. Similar calculations have also appeared in refs. [84, 99, 100].

Figure 8. Here we show the two light-quark jet contribution to the EMC data with $\Lambda^2 = Q^2 = 4\mathrm{GeV}^2$. We plot the combination $(g_1^{\mathrm{EMCdata}} + g_1^{q\bar{q}})(x)$, where $g_1^{q\bar{q}}(x)$ is evaluated via equ.(153). We use the form $\Delta g(x) = x^{\alpha} g(x)$ where $g(x)$ is given by equ.(155) [96]. Results are shown for $\alpha = 0.9, 0.8, 0.7$ and 0.2 corresponding to $\Delta g = 0.5, 0.6, 0.8$ and 4.0 respectively. This gluonic contribution to $g_1(x)$ is concentrated at small x - essentially outside the range of the EMC data.

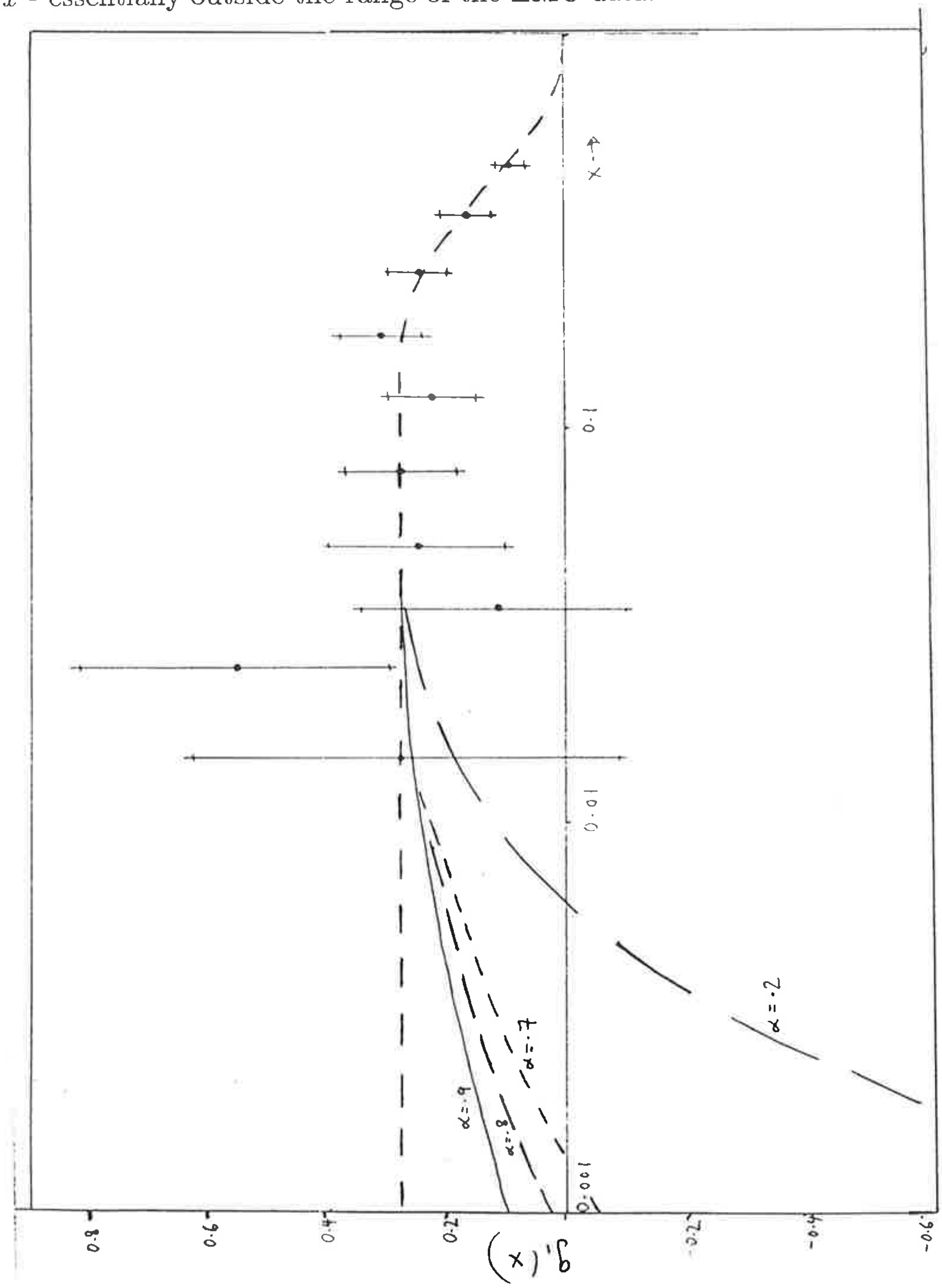


Figure 9. The two charm-quark jet contribution to the EMC data with $\Lambda^2 = Q^2 = 4\text{GeV}^2$. We plot the combination $(g_1^{\text{EMCdata}} + g_1^{c\bar{c}})(x)$, where $g_1^{c\bar{c}}(x)$ is evaluated via equ.(153). We use the form $\Delta g(x) = x^\alpha g(x)$ where $g(x)$ is given by equ.(155) [96]. Results are shown for $\alpha = 0.8$ and 0.2 corresponding to $\Delta g = 0.6$ and 4.0 respectively. The charm quark contribution to $g_1(x)$ is negligible at the Q^2 of the EMC experiment; it is concentrated at small x - essentially outside the range of the EMC data.

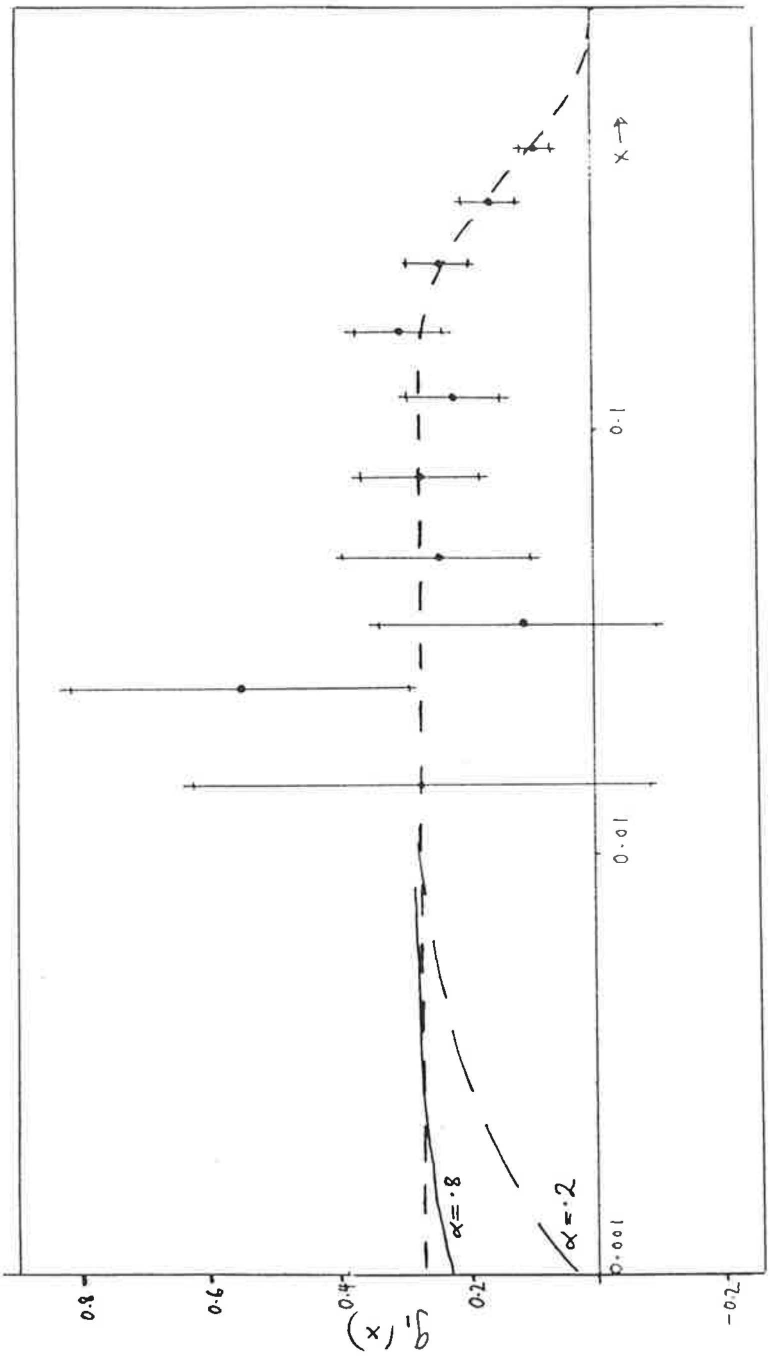


Figure 10. Here we show the large $k_T^2 \geq \Lambda^2$ two-quark-jet contributions to $g_1(x)$ at $Q^2 = 4\mathrm{GeV}^2$ from both light and charm quarks with the Brodsky and Schmidt $\Delta g(x)$, which is defined at a scale $\Lambda^2 = 1\mathrm{GeV}^2$ [98]. We plot the combination $(g_1^{\mathrm{EMCdata}} + g_1^{q\bar{q}})(x)$. Once again, the IPM two-quark-jet contribution is at small x - outside the range of the current data.

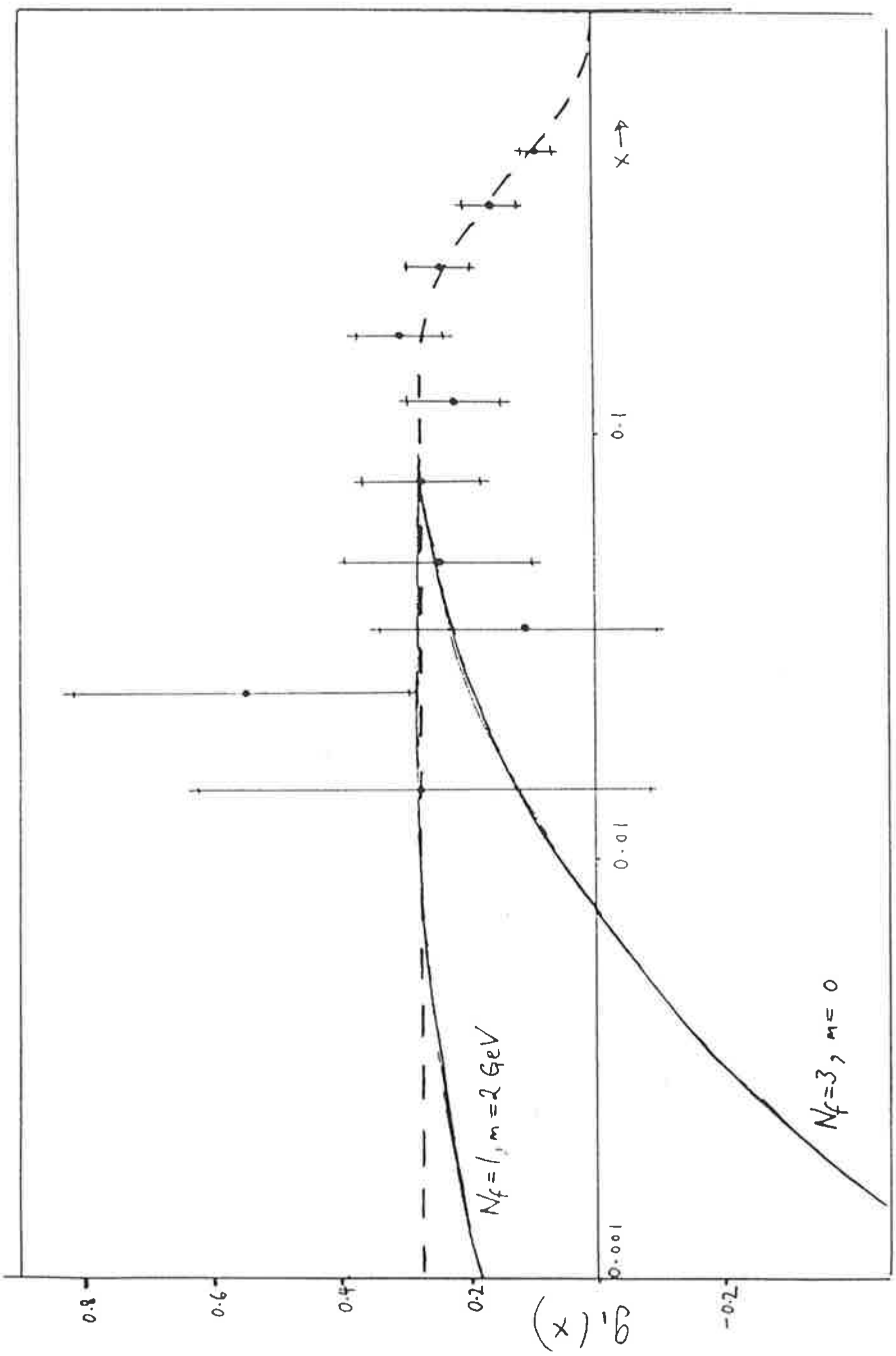
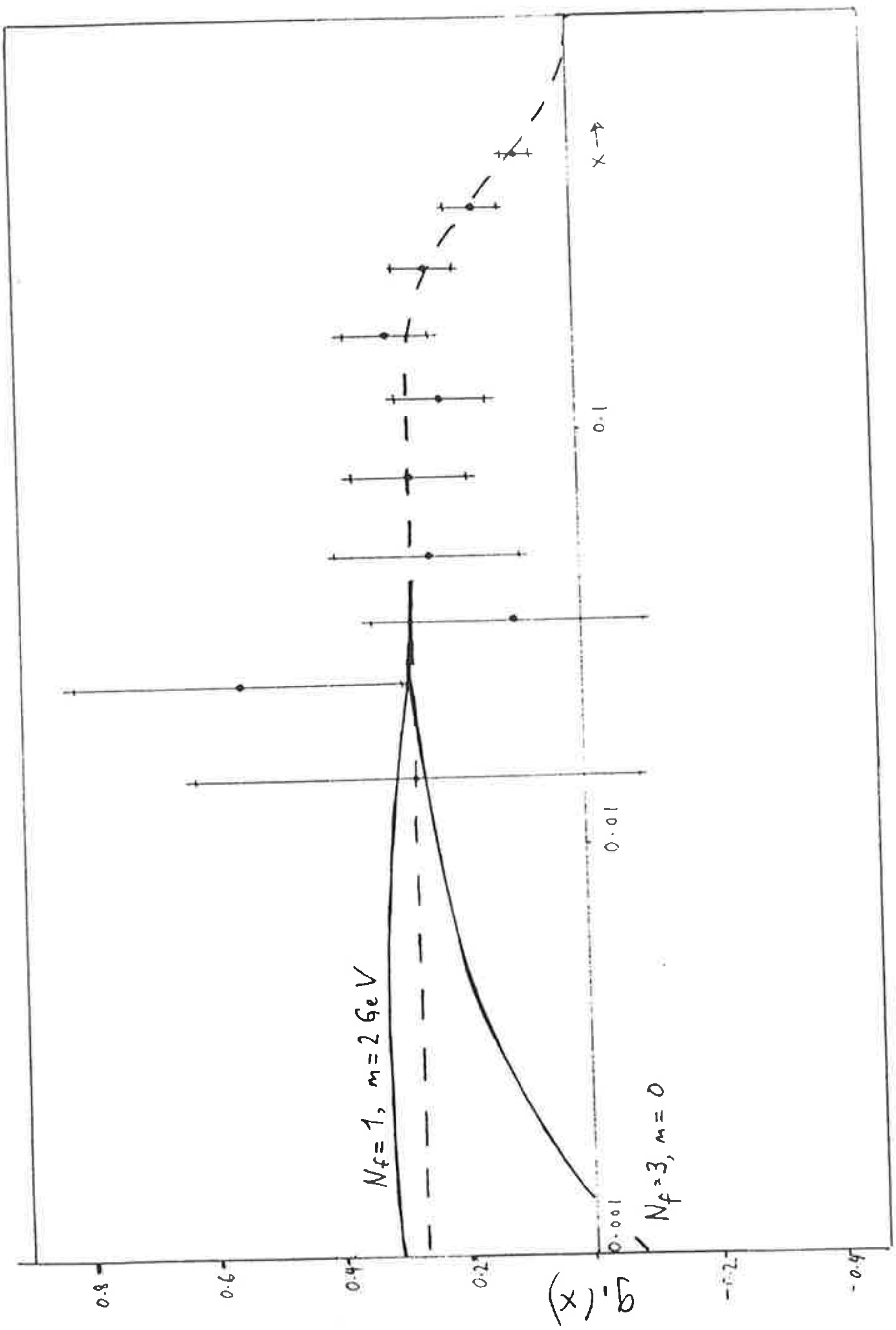


Figure 11. Here we plot $(g_1^{EMC} + g_1^{q\bar{q}})(x)$ with the Brodsky and Schmidt gluon distribution $\Delta g(x)_{BS}$ [98] and the Wilson coefficient distribution $C^g(x, \alpha_s)$, which has been evaluated in the $\overline{MS}$ renormalisation scheme [73, 74]. We show both the light and charm quark contributions to $(g_1^{EMC} + g_1^{q\bar{q}})(x)$. This OPE gluonic contribution to $g_1(x)$ is at small x. The observation that $C^g(x, \alpha_s)$ and $\Delta\sigma(x)$ have different first moments does not affect our conclusion that $\Delta g(x)$ is not relevant to the EMC data.



5. Axial Anomalies and $g_1(x)$

5.1 Introduction

In chapter 4 we argued that the EMC data on $g_1(x)$ receives a negligible contribution from $\Delta g(x)$ after we carry out the convolution. This still leaves the problem of how to explain the violation of the Ellis-Jaffe sum-rule (viz. $\Delta s = 0$) observed by EMC. The failure of the Ellis-Jaffe hypothesis represents a violation of the OZI (or Zweig's) Rule [101]. If the Ellis-Jaffe sum-rule were an exact symmetry of nature we would have $\Delta q_0 = \Delta q_8$ independent of scale. Instead, the EMC data reveals $\Delta q_0 = 0.12 \pm 0.23$ in contrast with $\Delta q_8 = 0.68 \pm 0.03$, which is extracted from hyperon β decay measurements. The flavour singlet axial charge of the proton $\Delta q_0(Q^2)$ is about two standard deviations below the quark model expectation at $Q^2 \sim 10\text{GeV}^2$.

There is another, quite famous, anomalous effect associated with the flavour singlet axial-vector current: the U(1) problem of QCD [102-105]. The current A^0_μ acts as an interpolating field for the η' meson

$$< \text{vac}|A^0_\mu(x)|\eta'(q) > = iq_\mu f_{\eta'} e^{-iq.x} \tag{159}$$

Here $f_{\eta'}$ is the η' decay constant. Similarly, the non-singlet axial currents can be used to define interpolating operators for the 0^- meson octet - the would-be Goldstone bosons of chiral $SU(2)_L \otimes SU(2)_R$. However, there is no analogy of PCAC in the flavour singlet channel. The η' is too massive to be a would-be Goldstone boson [106] ($m_{\eta'} = 958\text{MeV}$). It remains massive even in the chiral limit where we set the quark masses to zero. Clearly, one has to be very careful when making phenomenological predictions with A^0_μ.

We also have to resolve the apparent discrepancy between the OPE and IPM results for the first moment of the hard part of the spin dependent asymmetry for photon-gluon fusion. As we saw in chapter 3, the first moment of the OPE coefficient distribution relevant to $g_1(x)$ is $\int_0^1 dx\ C^g(x, \alpha_s) = 0$. This contrasts with the IPM result where we found a finite contribution $\int_0^1 dx\ \Delta\sigma(x) = -\frac{\alpha_s}{2\pi}$ from very large $k_T^2 \sim Q^2$ in the Bjorken limit.

Both of these effects have a natural explanation in terms of the axial anomaly in QCD [66], which we now discuss.

5.2 The Axial Anomaly

The axial anomaly is a renormalisation effect associated with the axial vector current. There are no twist two and spin one, gauge invariant, local gluonic operators for A^0_μ to mix with under renormalisation. Nevertheless, the renormalisation of the axial vector current is non-trivial in any interacting gauge theory. It is well known that one cannot renormalise $A_\mu = \overline{q}\gamma_\mu\gamma_5 q$ in a gauge invariant manner and still satisfy the canonical divergence equation $\partial^\mu A^S_\mu = D_L = 2m\overline{q}i\gamma_5 q$ This is one statement of the axial anomaly [66].

At first glance the axial vector current looks like a gauge invariant operator. However, the renormalisation of A_μ involves gauge dependent counterterms - in particular the Chern Simons current k_μ. In QCD

$$k_\mu^{QCD} = \frac{\alpha_s}{2\pi}\epsilon_{\mu\alpha\beta\gamma}B^\alpha_a\left[\partial^\beta B^\gamma_a - \frac{1}{3}g f_{abc}B^\beta_b B^\gamma_c\right] \tag{160}$$

Here B^μ_a is the gluon field operator, g is the gluon coupling constant and f_{abc} are the structure constants of the gauge group SU(3). In QED

$$k_\mu^{QED} = \frac{\alpha}{2\pi}\epsilon_{\mu\lambda\alpha\beta}B^\alpha\partial^\lambda B^\beta \tag{161}$$

where B^μ is now the photon field operator. (Henceforth we use B_μ to denote the gluon or photon fields and A_μ to denote the axial-vector current.) In general, two different renormalisation prescriptions R and R' may yield renormalised axial vector currents that differ by a multiple of k_μ, viz.

$$A_\mu|_{R'} = A_\mu|_R + ck_\mu|_{R'} \tag{162}$$

If we define the symmetry current A^S_μ by

$$\partial^\mu A^S_\mu = D_L = 2m\bar{q}i\gamma_5 q \tag{163}$$

then one can show [66] that the gauge invariantly renormalised axial vector current is

$$A^{GI}_\mu = A^S_\mu + k_\mu \tag{164}$$

The divergence of the Chern Simons term defines the gauge invariant topological charge density [103]

$$\partial^\mu k_\mu = \frac{\alpha_s}{2\pi}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu} \tag{165}$$

Here $G_{\mu\nu}$ is the gluon (photon) field tensor in QCD (QED) and $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$ is the corresponding dual tensor. Since k_μ is gauge dependent it follows from equ.(163) that the symmetry current A^S_μ is also gauge dependent. The renormalisation mismatch between satisfying gauge invariance and the canonical equation of motion equ.(164) defines the axial anomaly. The anomaly was discovered in QED perturbation theory by Adler and by Bell and Jackiw [66] who showed that it correctly predicts the (finite) cross section for $\pi^0 \to 2\gamma$. This decay was previously unexplained and is forbidden without the anomaly. Since D_L is a soft operator (with dimension 3) the symmetry current is scale independent; it is not renormalised. The gauge invariant axial current A^{GI}_μ is multiplicatively renormalised, viz.

$$A^{GI}_\mu(\mu^2) = Z_5(\mu_0^2 \to \mu^2)A^{GI}_\mu(\mu_0^2) \tag{166}$$

It has an anomalous dimension which starts at two loops in perturbation theory. All of the scale dependence is in the renormalisation of k_μ [68].

We now consider the consequences of the QCD axial anomaly for the first moment of $g_1(x)$. The gauge invariant axial current appears in the OPE. Since k_μ^{QCD} is flavour independent there will be no anomaly contribution to the non-singlet axial vector currents A_μ^3 and A_μ^8. (These currents are not renormalised.) The QCD axial anomaly is important when we consider the flavour singlet axial vector current A_μ^0. For this reason it is often called the strong U(1) anomaly.

The matrix element of A_μ^0 between proton states with momentum p_μ and p'_μ has the form ($q_\mu = (p - p')_\mu$)

$$< p, S|A_\mu^0|p', S >_c = 2mS_\mu G_A^0(q^2) + q_\mu q.SG_P^0(q^2) \tag{167}$$

If we define the form factor $\kappa(q^2)$ by the divergence

$$q.S\kappa(q^2) =< p, S|D_L^0 + \frac{1}{2}\sqrt{\frac{2}{3}}N_f\frac{\alpha_s}{2\pi}\mathrm{Tr}G_{\mu\nu}\tilde{G}^{\mu\nu}|p'S >_c \tag{168}$$

then the anomalous divergence equation for A_μ^0 reads [28, 107]

$$2mG_A^0(q^2) + q^2G_P^0(q^2) = \kappa(q^2) \tag{169}$$

The quantity measured by EMC is $2m\Delta q_0 = G_A^0(q^2 = 0)$. Since A_μ^0 does not couple to any massless bosons (even in the chiral limit) there are no zero mass poles in $G_P^0(q^2)$. We find

$$2m\Delta q_0 = \kappa(0) \tag{170}$$

The singlet axial charge is a measure of the gluonic topological charge (or instanton) density in the proton [108, 109]. This result differs from the naive parton model which has no explicit gauge degrees of freedom and, hence, no anomaly. The strong U(1) anomaly must catalyse the apparent violation of OZI seen by EMC [28, 29]. 't Hooft has argued [102] that the anomaly successfully explains the U(1) problem in QCD. However, this subject is still a matter of controversy [103-105].

The anomaly also means that Δq_0 does not measure quark spin. In the naive parton model one can write down a Lagrangian and derive the Noether spin current corresponding to conservation of angular momentum. In the parton's rest frame the Pauli Lubanski spin operator defines the intrinsic spin operators [110]

$$S_k = \int d^3x \ (\bar{q}\gamma_k\gamma_5 q) \qquad k = 1, 2, 3 \tag{171}$$

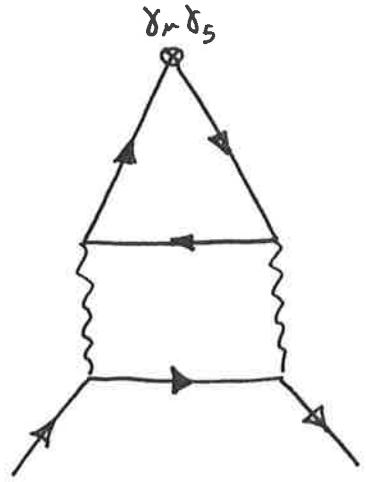
The S_k are time independent for massless quarks; they satisfy the algebra of SU(2) spin.

The angular momentum tensor in QCD receives contributions from both quark and gluonic operators [110]. After we carry out the renormalisation it no longer makes sense to call any operator purely fermionic or bosonic. Even the scale independent A^S_μ is sensitive to the gluon fields through its gauge dependence. Nevertheless, suppose that we try to construct an intrinsic spin operator for quarks in the renormalised theory following the free fermion result. Only the operators $S^S_k = \int d^3x \ A^S_k$ will satisfy exact SU(2). These operators are gauge dependent and have gauge dependent eigenstates. The spin $\frac{1}{2}$ "quark" defined by S^S_z is a gauge artifact! It is not observable. One might also consider the gauge invariant $S^{GI}_k(\mu^2) = \int d^3x \ A^{GI}_k(\mu^2)$. At any given scale μ_0^2 we are free to choose the overall normalisation so that the S^{GI}_k satisfy the commutation relations of SU(2). However, the scale dependence of $A^{GI}_\mu(\mu^2)$ means that at any other scale $\mu^2 \neq \mu_0^2$ we lose SU(2), viz.

$$[S^{GI}_i, S^{GI}_j](\mu^2) = i\epsilon_{ijk} Z_5(\mu_0^2 \to \mu^2) S^{GI}_k(\mu^2) \tag{172}$$

The renormalisation group factor $Z_5(\mu_0^2 \to \mu^2)$ starts at two loops in perturbation theory (Fig. 12) and has the form $Z_5 = (1 + O(\alpha^2) \ln \frac{\mu^2}{\mu_0^2})$. It is essentially trivial in QED and in electroweak theory due to the small coupling and large M_W^2, M_Z^2 respectively.

Figure 12. The flavour singlet axial current has a two loop anomalous dimension in perturbation theory.



In QCD there are reasons for believing that Z_5 may be significantly different from one. One might choose to set $\mu_0^2 \sim 0.2\text{GeV}^2$ where we expect quark models to be a good semiclassical approximation to QCD. This assumption has been made by Gluck and Reya [111] who chose an Ellis-Jaffe hypothesis at $\mu_0^2 \sim 0.2\text{GeV}^2$. Explicit calculations [112] show that one cannot reconcile this quark model expectation with the EMC data by purely perturbative evolution alone. The large OZI violation observed by EMC at $Q^2 \sim 10\text{GeV}^2$ is suggestive of a large Z_5 factor between the starting scale and where we trust perturbative QCD ($\mu^2 \geq 1\text{GeV}^2$). Such an effect has been discussed in refs. [69] and [70]. How might it be observed in data ? Resonances are seen at large x (> 0.8). Thus, any rapid scale dependence of $g_1^P(x, Q^2)$ here would be lost in the resonance "noise". At small x one could expect a clear signal. Here, "small x" is defined as away from the resonance region in x that we are familiar with from unpolarised DIS. Does scaling set in smoothly (like in unpolarised DIS) or is there any evidence for a phase transition at low x ? At present there is no data which could answer this question. If instantons are responsible for confinement it is reasonable to expect a phase transition in Δq_0 between hadronic degrees of freedom $\mu^2 \leq 0.5\text{GeV}^2$ and quark-gluon degrees of freedom $\mu^2 \geq 2\text{GeV}^2$. An experiment to look at the onset of scaling would certainly be worthy of consideration and might provide a test of instanton models of confinement.

One extra comment is in order here. The constituent quarks in our low energy models are not QCD quarks in that they do not couple to the full non-abelian gauge field. Confinement is usually built into the models by assuming an antisymmetric wavefunction in colour space, which follows from the Pauli principle. Thus, the constituent quarks are really only sensitive to electromagnetic gauge degrees of freedom through their electric charges. In making the Gluck and Reya hypothesis [111] we are marrying together two different pictures, which we hope are equivalent at the low energy scale of the model. The quark models are mean field theories so that any short distance correlations are averaged out to zero. This includes the anomaly as a short distance renormalisation effect. For this reason, gluonic exchange currents [113] and pionic corrections [114] are unable to accomodate the large OZI violation in Δq_0 in even the most sophisticated of bag models. One needs to generalise these models to include the anomaly - perhaps via the U(1) effective action of QCD [115, 116]. Some progress has been made in this area [117, 118].

One might also consider Δq_0 in the large N_c limit of QCD [119, 120]. Non abelian gauge theories like QCD simplify tremendously in the gedanken limit of a large number of colours (large N_c). Large combinatorial factors arise in the loop graphs of perturbation theory and one can calculate interactions to a given power in the number of colours. The coupling constant is normalised in the new theory by $g = g_0 N_c^{-\frac{1}{2}}$ with g_0 fixed. This corresponds to the gluon self energy ~ 1 as we increase N_c, viz. a stable gauge vacuum. Quark loops behave as $\frac{1}{N_c}$ and are suppressed. Thus, Zweig's rule and the Ellis-Jaffe sum-rule are exact in the large N_c limit of QCD.

Witten has pointed out the similarities between this large N_c suppression of the sea and our valence quark picture of the nucleon [120]. In many interactions one can treat 3 as a good approximation to ∞! The failure of the Ellis-Jaffe hypothesis and the derivation by Witten that $m^2_{\eta'} \sim N_c^{-1}$ [121] indicate that the large N_c approximation fails in the presence of the anomaly.

Brodsky et al. [23] have investigated the proton spin problem within a particular version of the Skyrme model of the proton. The singlet axial vector current A^0_μ decouples from the proton at leading order in N_c in this model. They claim this result as a success of the Skyrme model over more conventional quark models like the bag. In view of the restoration of OZI at large N_c this calculation should be treated with suspicion. It is well known that the Skyrme model prediction for g_A falls short by a factor of two. Thus the Skyrme model is unable to reproduce even the Bjorken sum-rule, equ.(73).

We now discuss the axial anomaly in perturbation theory to help clarrify the ideas presented above. This discussion will also help to resolve the apparent discrepancy of $-\frac{\alpha_s}{2\pi}\Delta g$ between the OPE and IPM results for the first moment of $g_1(x)$.

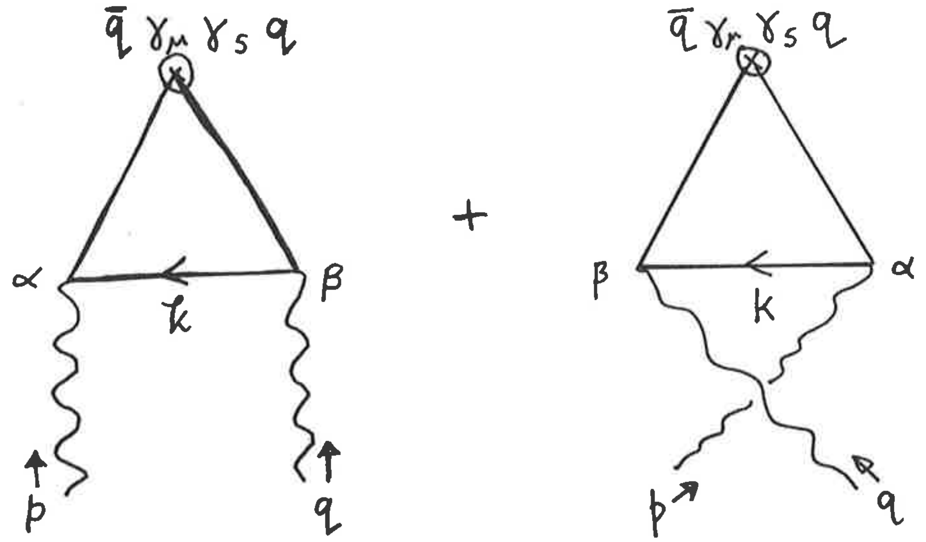


Figure 13. The axial anomaly is calculated in perturbation theory via the Adler-Bell-Jackiw (vector, vector, axial-vector) triangle diagram. The external vector legs may be either photons or gluons. They carry momentum p_μ and q_μ and have transverse polarisation $\epsilon_\mu = \frac{1}{\sqrt{2}}(0; 1, \pm i, 0)$.

5.3 The Axial Anomaly in Perturbation Theory

The axial anomaly was discovered by Adler and by Bell and Jackiw [66] within QED perturbation theory. There are many excellent reviews of this subject [122, 123]. Here, we briefly review the Adler-Bell-Jackiw (ABJ) results and explain how they relate to the IPM.

We start by considering the amplitude $\Delta_{\mu\alpha\beta}(p,q)$ for the vector, vector, axial-vector (VVA) triangle graph, which is shown in Fig. 13. The external vector legs may be either photons or gluons. They carry momentum p_μ and q_μ and have transverse polarisation $\epsilon_\mu = \frac{1}{\sqrt{2}}(0;1,\pm i,0)$. In QED perturbation theory $\Delta_{\mu\alpha\beta}(p,q)$ is calculated from the Feynman rules as

$$\Delta_{\mu\alpha\beta}(p,q) = -ig^2 \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\frac{1}{\hat{k}+\hat{p}-m}\gamma_\alpha \frac{1}{\hat{k}-m}\gamma_\beta \frac{1}{\hat{k}-\hat{q}-m}\gamma_\mu\gamma_5\right] + (\alpha \leftrightarrow \beta, p \leftrightarrow q) \tag{173}$$

The VVA triangle amplitude $\Delta_{\mu\alpha\beta}(p,q)$ has a superficial linear divergence which induces a surface term when we shift the loop momentum

$$k_\mu \to k_\mu + a_\mu \tag{174}$$

The shift variable a_μ has the general form

$$a_\mu = (a+b)p_\mu + bq_\mu \tag{175}$$

in Fig. 13a. (We swap $p_\mu \leftrightarrow q_\mu$ in the cross graph Fig. 13b to satisfy Bose symmetry.) If $\Delta_{\mu\alpha\beta}(p,q|a)$ denotes the shifted triangle amplitude then the surface term is

$$\Delta_{\mu\alpha\beta}(p,q|a) - \Delta_{\mu\alpha\beta}(p,q|0) = -\frac{\alpha}{2\pi}\epsilon_{\mu\lambda\alpha\beta}a(p-q)^\lambda \tag{176}$$

The triangle amplitude is defined up to a polynomial in the external momenta. The surface term is the two photon matrix element of the gauge dependent Chern-Simons term k_μ^{QED} in equ.(161). The general VVA amplitude does not satisfy current conservation at the vector legs. One finds

$$p^{\alpha}\Delta_{\mu\alpha\beta}(p,q|a) \neq 0 \neq q^{\beta}\Delta_{\mu\alpha\beta}(p,q|a) \tag{177}$$

in perturbation theory. Gauge invariance holds only for a particular choice of a_{μ}, which defines the physical VVA amplitude. The surface term has divergence

$$-(p+q)^{\mu}(-)\frac{\alpha}{2\pi}\epsilon_{\mu\lambda\alpha\beta}(p-q)^{\lambda} = \frac{\alpha}{\pi}\epsilon_{\mu\lambda\alpha\beta}p^{\lambda}q^{\mu} \tag{178}$$

which is the two photon matrix element of the topological charge $\frac{\alpha}{2\pi}F.\tilde{F}$. The same calculation holds in QCD perturbation theory where we replace the external photon legs by gluons. (There is an extra group factor $T = \frac{1}{2}$ in the QCD expression for $\Delta_{\mu\alpha\beta}(p,q)$.) In QCD the surface term equ.(176) is the two gluon matrix element of k_{μ}^{QCD}. The three gluon part of k_{μ}^{QCD} does not contribute to the ABJ triangle amplitude.

The restoration of gauge invariance via the surface term is only one procedure for defining the gauge invariant triangle amplitude. The anomaly k_{μ} contribution arises in any gauge invariant renormalisation scheme (eg. minimal subtraction [124] or the Pauli-Villars technique [66]). In the Pauli-Villars procedure the $M \to \infty$ regulator leaves behind the finite anomaly term after the divergence is subtracted out.

The forward limit $p = -q$ of the VVA triangle amplitude $\Delta_{\mu\alpha\beta}(p,q)$ is relevant to $g_1(x)$. The OPE tells us that the first moment of the one-loop spin dependent asymmetry for photon-gluon fusion ($\Delta\sigma(x,Q^2,P^2,m^2)$ in chapter 4) is given by the forward matrix element of the axial-vector current in the scaling limit $Q^2 \to \infty$. The cross section $\Delta\sigma(x,Q^2,P^2,m^2)$ is equal to the sum of the $O(\alpha_s)$ gluonic Wilson coefficient distribution and the spin dependent quark distribution of the gluon $\Delta q^{gluon}(x)$. This quark distribution is defined by the absorptive part of the forward VVA amplitudes, which are obtained by replacing A_{μ} by the general spin-even axial-tensor operator

$\overline{q}\gamma_+\gamma_5(D_+)^{2n}q$ in equ. (173). In $B_+ = 0$ gauge, Δq^{gluon} is given by [73, 87, 125]

$$2p_+\Delta q^{gluon}(x) = -ig^2 2N_f T$$
$$\int \frac{d^4k}{(2\pi)^4}\delta(x - \frac{k_+}{p_+})\delta((k-p)^2 - m^2)\frac{\mathrm{Tr}[(\hat{k}+m)\hat{\epsilon}^*(\hat{k}-\hat{p}+m)\hat{\epsilon}(\hat{k}+m)\gamma_+\gamma_5]}{(k^2-m^2)^2} \tag{179}$$

We may evaluate this amplitude via minimal subtraction. In this case the anomaly depends on how we continue the γ_5 into the regulator dimensions.

The correct procedure was established by 't Hooft and Veltman [124]. Gauge invariant regularisation is equivalent to the continuation

$$\begin{aligned} \{\gamma_\mu, \gamma_5\}_+ &= 0 \qquad \mu = 0,1,2,3 \\ [\gamma_\mu, \gamma_5]_- &= 0 \qquad \mu = \text{regulator dimensions} \end{aligned} \tag{180}$$

where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. (The choice $\{\gamma_\mu, \gamma_5\}_+ = 0\ \forall\mu$ defines the symmetry current A^S_μ, viz. no anomaly.) Clearly, the axial anomaly is in the definition of γ_5. There will be gauge dependent counterterms associated with each of the higher spin axial tensor operators $\overline{q}\gamma_+\gamma_5(D_+)^n q$. These gauge dependent counterterms can be isolated in QED by calculating the surface terms corresponding to the momentum shift $k_\mu \rightarrow k_\mu + a_\mu$ in the general VVA amplitude. There will be a contribution to the surface terms from the gauge invariant operators $F_{+\alpha}(\partial_+)^{2n-1}\tilde{F}^\alpha_+$ due to operator mixing under renormalisation. These terms are subtracted out to isolate the gauge dependent counterterm.

Returning to the expression equ.(179) for $\Delta q^{gluon}(x)$, the delta functions are evaluated via the k_+ and k_- integrals. We evaluate the Dirac trace in $4+\epsilon$ regulator dimensions and find

$$\Delta q^{gluon}(x) =$$
$$\pm\frac{\alpha_s}{2\pi^2}N_f\mu^{-2\epsilon}\int d^{2+\epsilon}k_T\left[\frac{(k_T^2+m^2)(1-2x) - 2m^2(1-x) - 2\frac{\epsilon}{2+\epsilon}k_T^2(1-x)}{(k_T^2+m^2+P^2x(1-x))^2}\right] \tag{181}$$

Here μ^2 is the renormalisation scale. The overall positive (negative) sign indicates a left (right) handed gluon polarisation. The $\frac{\epsilon}{2+\epsilon}k_T^2(1-x)$ term comes from the continuation of γ_5 into the regulator dimensions. If no ultraviolet cut-off is imposed the k_T^2 integral develops a $\frac{1}{\epsilon}$ pole. This pole cancels with $\frac{\epsilon}{2+\epsilon}k_T^2(1-x)$to give a finite contribution in the limit $\epsilon \to 0$. This is the axial anomaly. The dimensionally regulated expression for $\Delta q^{gluon}(x)$ is

$$\Delta q^{gluon}(x) = \pm\frac{\alpha_s}{2\pi}N_f\left[(\frac{2}{\epsilon} - \gamma_E + \ln 4\pi)(1-2x)\right. \\ \left. + (2x-1)\ln\frac{m^2 + P^2x(1-x)}{\mu^2} + 1 - \frac{m^2}{m^2+P^2x(1-x)}\right] \tag{182}$$

Here γ_E is the Euler constant. The first term proportional to $(\frac{2}{\epsilon} - \gamma_E + \ln 4\pi)$ contains the ultraviolet divergence. It is subtracted out into the operator renormalisation constant leaving the finite quark distribution. We evaluate

$$\int_0^1 dx\ \Delta q^{gluon}(x) == \pm\frac{\alpha_s}{2\pi}N_f\left[1 + \frac{2m^2}{P^2}\frac{1}{\sqrt{1+\frac{4m^2}{P^2}}}\ln\left(\frac{\sqrt{1+\frac{4m^2}{P^2}}-1}{\sqrt{1+\frac{4m^2}{P^2}}+1}\right)\right] \tag{183}$$

which is exactly the first moment of $\Delta\sigma(x, Q^2, P^2, m^2)$ without any infra-red cut-off (cf. equ.(131) in chapter 4). The unity factor in equ.(183) is the anomaly and the non-local $\frac{m^2}{P^2}$ term is the one-loop forward matrix element of the pseudoscalar density $2m\bar{q}i\gamma_5 q$. Since the anomaly is an ultraviolet (large $k_T^2 \to \infty$) effect it contributes to the triangle amplitudes if we impose an infra-red cut-off on the k_T^2 or, equivalently, on the quark virtuality $m^2 - k^2$. (Recall that these two cut-offs are related by equ.(132)). Of course, one should be careful to use the same choice of infra-red cut-off for both $\Delta\sigma(x)$ and $\Delta q^{gluon}(x)$.

5.4 Locality and the Parton Model

The new physics content is that there is a large $k_T^2 \to \infty$ contribution to the spin dependent quark distribution in the scaling limit, which was not anticipated in the original parton papers of Gribov and Lipatov [78]. The IPM was developed with the OPE results for DIS in mind. Factorisation of mass singularities is similar to the calculation of the Wilson coefficients in the OPE. We would like our parton model to reproduce the rigorous OPE result that the spin dependent gluon distribution does not contribute to the first moment of $g_1(x)$. When we applied the OPE to the spin dependent photon gluon fusion process we found that the large $k_T^2 \sim Q^2$ term *is* included in the quark distribution. It corresponds to the two gluon matrix element of the anomalous Chern-Simons term associated with the axial-vector current. (At $O(\alpha_s^2)$ there will be a contribution to spin dependent photon-gluon fusion associated with the non-abelian part of k_μ^{QCD}, which should also be included in the quark distribution. It could only be isolated after detailed renormalisation calculation.) The IPM and OPE approaches to DIS can be reconciled if we supplement the IPM with a new locality condition. The gluon distributions are to be defined as generating a non-local interaction between the gluon and the hard photon in the DIS process. The quark distribution contains all partons which make a local interaction with the photon. In this picture, the local photon gluon interaction is a renormalisation effect associated with the gluonic dressing of the quark partons.

Thus, the anomaly contributes to $g_1(x)$ through $\Delta q(x, Q^2)$ for any target. It is convoluted with the quark coefficient distribution $C_S^q(x) = \delta(x-1) + O(\alpha_s)$ in the OPE expression for $g_1(x)$ (equ.(70)). In principle the anomaly can be manifest over a complete range of x - even in the "valence region" $x \geq 0.2$. Some authors have noticed that k_μ^{QCD} and the Noether spin current for gluons coincide in light-one gauge $B_+ = 0$ [21, 22, 87] (see also [74, 107]). With this is mind, they suggested that one might use the forward proton matrix element of k_μ^{QCD} to define the first moment of $\Delta g(x)$ in the factorisation only IPM. The higher moments of $\Delta q_0(x)$ and $\Delta g(x)$ would be defined by the gauge invariant R_1 operators in section 3.2. In this picture, the IPM parton distributions would differ from their OPE counterparts only in their first moment, or equivalently, by a gauge dependent term proportional to $\delta(x)$, which has no effect on practical calculations.

Carlitz et al. [87] have suggested that the axial anomaly should be characterised by two-quark-jet events with large $k_T^2 \sim Q^2$. If this is correct then we should expect to see these events over all x in both polarised and unpolarised DIS. (Recall that $g_1(x)$ is measured in the difference of two spin dependent deep inelastic muon-proton cross sections - the sum of which defines the unpolarised structure function. Cross sections are positive definite by definition. Any effect seen in the difference of two positive quantities is seen in their sum.) A significant two-quark-jet cross section at large x in unpolarised DIS has no place in our understanding of the unpolarised gluon distribution. Furthermore, as we discuss in chapter 6 infra-red effects in QCD mean that one cannot isolate the axial anomaly from the axial-vector current. For this reason, I expect that the cross section for two quark jet events with $k_T^2 \sim Q^2$ will be found to be negligible if any experiment is made to measure it. This does not necessarily imply a small anomaly contribution to Δq_0. We mentioned in section 4.5 that Altarelli and Stirling [97] had investigated the possibility of reconciling the EMC data with the Ellis-Jaffe hypothesis by invoking a large Δg in the proton. Following previous work [21, 22], they tried to relate Δg to the anomaly in an IPM defined purely by factorisation. They convoluted $\Delta g(x)$ with $\delta(x-1)$ giving a gluon contribution that was much too inflated at large x. However, the preceding argument shows that they were on the right track - only that the axial anomaly has nothing to do with $\Delta g(x)$.

We mentioned above that there will be anomalous (gauge dependent) counterterms associated with each of the spin-odd axial-tensor operators, which define $\Delta q(x)$. The spin dependent valence distribution $\Delta q_V(x) = (q^\uparrow - \overline{q}^\uparrow)(x) - (q^\downarrow - \overline{q}^\downarrow)(x)$ could be measured in polarised DIS with a polarised proton target and (anti-)neutrino beam in the parity odd structure function $g_3(x)$. It is formally defined by the forward proton matrix elements of the spin-even axial tensor operators $\overline{q}(0)\gamma_+\gamma_5(D_+)^{2n-1}q(0)$ via the quark light-cone correlation functions in equs.(61-63) in section 3.4. The spin-even axial-tensor operators have odd charge conjugation. The spin-even gluonic operators $G_{+\alpha}(0)(D_+)^{2n-2}G^{\alpha}{}_{+}(0)$ are even under charge conjugation and do not contribute to DIS. There is no operator mixing between quark and gluonic operators with $\Delta q_V(x)$. Similarly, we cannot construct a generalised spin-even, gluonic Chern-Simons term with odd charge conjugation. This means that the valence distribution is anomaly free. The polarised analogy to the Gross-Llewellyn Smith sum-rule [126], viz.

$$\int_0^1 dx \; g_3^{\nu+\overline{\nu}}(x) = (\Delta q_V)_0 \tag{184}$$

(above the charm production threshold) does measure a valence quark spin component in the proton.

We now further investigate the role of the axial anomaly in polarised DIS through a polarised photon target. The spin dependent structure function $g_1(x)$ of the polarised photon is accessible to experiment.

6. Polarised Deep Inelastic Scattering from Photon Targets and Axial Anomalies

6.1 Introduction

Deep inelastic scattering from photon targets reveals many novel effects in QCD. The unpolarised photon structure function has been well studied both theoretically and experimentally [127, 128]. In this chapter we consider the hadronic part of the polarised photon structure function $g_1^\gamma(x)$ [129]. We start by considering $g_1^\gamma(x)$ in a free quark model (FQM). This is just the box calculation in chapter 4. We investigate how the FQM compares with a QCD analysis, first carried out by Witten [130] (Witten analysis), using the operator product expansion and the renormalisation group. The Witten analysis introduces the contact interaction, which occurs with deep inelastic scattering from photon targets. We then concentrate on the first moment of $g_1^\gamma(x)$. This quantity allows for a particularly clean discussion of the role of the axial anomaly in polarised deep inelastic scattering. The anomaly makes a contact measurement with the target photon. We derive an exact sum-rule for $G_1 = \int_0^1 dx g_1^\gamma(x)$ for real photons [31]. In ref. [131] a sum rule was suggested for the first moment of $g_1^\gamma(x)$ on the basis of the perturbative triangle diagram in QED. The result of ref. [131] is equivalent to applying the operator product expansion to the FQM, where the role of the strong U(1) anomaly is ignored. Finally, we contrast the roles played by the QED and QCD axial anomalies in polarised DIS.

The photon structure functions are observed experimentally in $e^+ e^- \to$ hadrons [127, 132] where for example a hard photon (large Q^2) probes the quark structure of a soft photon ($P^2 \sim 0$). This is shown in Fig. 14. For any P^2 of the target photon the measured structure functions receive a contribution both from contact photon fusion and also a hadronic piece, which is commonly associated with vector meson dominance

(VMD) of the soft target photon. The hadronic piece scales with Q^2 whilst the contact piece behaves as $\ln Q^2$ as we let Q^2 tend to ∞. This was discussed by Witten [130] in the unpolarised case. The $\ln Q^2$ scaling behaviour mimics the free quark model prediction, but the coefficient of the logarithm receives a finite renormalisation in QCD.

We now begin our detailed discussion of the polarised photon structure function $g_1^\gamma(x)$. Experimentally, $g_1^\gamma(x)$ may be measured with the promised polarised $e^+ e^-$ beams at SLAC.

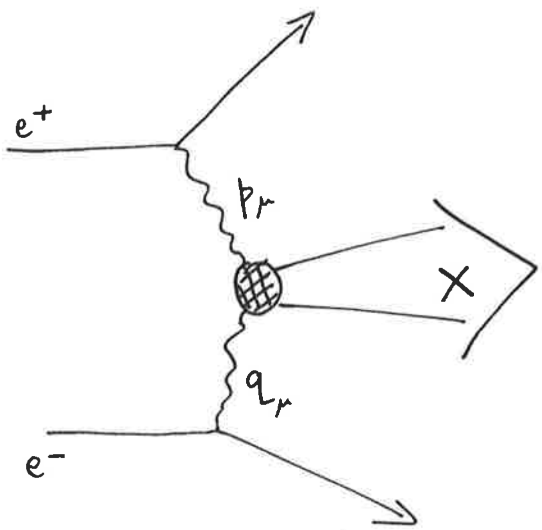


Figure 14. The photon structure functions are observed experimentally in $e^+ e^- \to$ hadrons where for example a hard photon (large Q^2) probes the quark structure of a soft photon ($P^2 \sim 0$).

6.2 $g_1^\gamma(x)$ in the Free Quark Model

In the free quark model the polarised photon structure function $g_1^\gamma(x)$ is calculated via the box diagrams of Fig. 6 on page 49. It is the same calculation as $\Delta\sigma(x, Q^2, P^2, m^2)$ in chapter 4 - without the infrared cut-off. Although $g_1^\gamma(x)$ will contain the mass singularity for real photons, there is no need to worry about continuing the quark propagator into the infra-red region $k_T^2 \sim 0$ in the free quark model.

We let q and p denote the probe and target photon momentum respectively. In the usual way we define $Q^2 = -q^2$, $P^2 = -p^2$ and $x = Q^2/2p.q$ is the Bjorken variable. All the photons have transverse helicity and we antisymmetrise over left and right handed polarisations. Finally, we let m be the quark mass and k is the free quark momentum in the loop (Fig. 6). In the deep inelastic limit ($Q^2 >> P^2, m^2$) we obtain

$$\begin{aligned} g_1^\gamma(x)_{FQM} = \frac{\alpha}{\pi} N_c \sum_q e_q^4 \Bigg[(2x-1)\left(\ln \frac{Q^2}{\mu^2} + \ln \frac{\mu^2(1-x)}{x^2(1-x)P^2 + xm^2} - 1 \right) \\ + (1-x)\frac{(2m^2 - P^2 x(2x-1))}{m^2 + P^2 x(1-x)} \Bigg] \end{aligned} \tag{185}$$

Here, we have inserted a scale μ^2 to separate out the $\ln Q^2$ term from the x dependent scaling terms. For three flavours (u, d and s) $N_c \sum_q e_q^4 = \frac{2}{3}$. As we found in chapter 4, there is an unambiguous contribution to the first moment of $g_1^\gamma(x)$ (in the FQM) which comes from a purely pointlike part of the box at very large $k_T^2 \sim Q^2$. The FQM contains a contact photon-photon interaction, which provides a local measurement of the target photon's helicity [87]. We shall return to this point later when we discuss a sum rule for gg. The higher ($n \geq 2$) moments of $g_1^\gamma(x)$ are dominated by the logarithm term in the FQM, viz.

$$G_n = \int_0^1 dx x^{n-1} g_1^\gamma(x) \sim \frac{\alpha}{\pi}\frac{2}{3}(\gamma_{VV}^F)_n \ln \frac{Q^2}{\mu^2} \tag{186}$$

In the next section we investigate what happens in QCD when we carry out a leading order Witten analysis of the structure function of the polarised photon. We consider a real photon target. Clearly things will not be so simple as in the FQM. For example, we expect a soft contribution to $g_1^\gamma(x)$ associated with vector meson dominance. The simplest way to view this in the framework of the FQM box calculation is to modify the soft photon vertices by a form factor $(\frac{m^2}{m^2-k^2})^\beta$ where m^2 is now some effective (constituent) quark mass squared and $\beta > 0$. For all positive β our modified (soft) $g_1^\gamma(x)$ will scale with Q^2 at large Q^2 [133].

6.3 $g_1^\gamma(x)$ in QCD : The Leading Order Witten Analysis

Formally, the total inclusive cross section for $\gamma\,\gamma \to$ hadrons may be written as the absorptive part of the forward $\gamma\,\gamma$ scattering amplitude [134]

$$T_{\mu\nu\alpha\beta} = \frac{1}{2}\int dx\; dy\; dz e^{iq.x} e^{ip.(y-z)} < \mathrm{vac}|T[j_\mu(x) j_\nu(0) j_\alpha(y) j_\beta(z)]|\mathrm{vac} > \qquad (187)$$

where $j_\alpha(y)$ and $j_\beta(z)$ are understood to couple to the soft photons. The forward matrix element in equ.(187) is a function of six distances x, y, z, $x-y$, $x-z$, $y-z$. It is only when all of these distances are small simultaneously that we can apply perturbative QCD (which includes the lowest order box calculation). For a real photon target, we contract equ.(187) with the target photon's polarisation vector $\epsilon^\alpha(\lambda)$ and consider

$$W_{\mu\nu} = \frac{1}{2\pi}\mathrm{Abs}\int dx\; e^{iq.x} < \gamma(p,\lambda)|T[j_\mu(x) j_\nu(0)]|\gamma(p,\lambda) > \qquad (188)$$

Since $Q^2 \to \infty$ in deep inelastic scattering we can use the phase oscillation argument in section 3.1 for the exponential to show that $W_{\mu\nu}$ receives a contribution only when $x^2 \to 0$. We can apply the operator product expansion to $j_\mu(x)$ and $j_\nu(0)$. In all, there are eight independent structure functions for the spin-one photon [135]. The antisymmetric part of $W_{\mu\nu}$ is described (up to a Lorentz factor) by the polarised deep inelastic structure function $g_1^\gamma(x)$, where the nth moment of $g_1^\gamma(x)$ is defined by [129]

$$iS\ \epsilon^{\mu\lambda\alpha\beta}\epsilon_\alpha\epsilon^*_\beta p_\lambda p^{\mu_1}...p^{\mu_{n-1}}\int\limits_0^1 dx\ x^{n-1}g_1^\gamma(x) = \frac{1}{2}\sum_\theta C_{\theta,n}(Q^2) < \gamma(p)|O_{\theta,n}^{\mu\mu_1..\mu_{n-1}}|\gamma(p) > \tag{189}$$

Here S denotes symmetrisation over μ, μ_1,..., μ_{n-1} and $\epsilon_{0123} = +1$. The relevant twist two operators $O_{\theta,n}$ are the hadronic operators $O_{H,n}^{\mu\mu_1..\mu_{n-1}}$, viz.

$$\begin{aligned} O_{q,n}^{a\mu\mu_1..\mu_{n-1}} &= i^{n-1}S\ \overline{q}\gamma^\mu\gamma_5 D^{\mu_1}...D^{\mu_{n-1}}\frac{\lambda^a}{2}q, \quad \mathrm{a} = 0,1,...,8 \\ O_{g,n}^{\mu\mu_1..\mu_{n-1}} &= \frac{1}{2}i^{n-2}S\ \mathrm{Tr_c}[G^{\mu\alpha}D^{\mu_1}...D^{\mu_{n-2}}\tilde{G}_\alpha^{\mu_{n-1}}] \end{aligned} \tag{190}$$

and also the photon operators

$$O_{\gamma,n}^{\mu\mu_1..\mu_{n-1}} = \frac{1}{2}i^{n-2}S\ F^{\mu\alpha}\partial^{\mu_1}...\partial^{\mu_{n-2}}\tilde{F}_\alpha^{\mu_{n-1}} \tag{191}$$

In equ.(190) D_μ is the full gauge covariant derivative involving both both gluon and photon fields and $\mathrm{Tr_c}$ denotes the trace over colour indices. The $O_{H,n}$ are the same operators which appear in deep inelastic scattering from hadronic targets; $G_{\mu\nu}$ and $F_{\mu\nu}$ $G_{\mu\nu}$ and $F_{\mu\nu}$ are the gluon and photon field tensors respectively with $\tilde{G}_{\mu\nu}$ and $\tilde{F}_{\mu\nu}$ the corresponding dual tensors (ie. $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$).

As it was first pointed out by Witten [130], the coefficient functions of the photonic and singlet hadronic operators will mix under the renormalisation group. The hadronic matrix elements are of leading order α_{QED} (henceforth written α) whilst the photon operator matrix elements are $O(1)$. Since the hadronic coefficient functions are $O(1)$ and the photon coefficient functions start at $O(\alpha)$ equ.(189) receives leading order (in α) contributions both from the hadronic and photonic channels.

At leading order in α the renormalisation group equation reads (cf. section 3.3)

$$(\frac{d}{d\ln\mu^2} - \gamma(g))C(\frac{Q^2}{\mu^2}, g, \alpha) = (\frac{\partial}{\partial\ln\mu^2} + \beta\frac{\partial}{\partial g} - \gamma)C(\frac{Q^2}{\mu^2}, g, \alpha) = 0 \tag{192}$$

Here, $C(\frac{Q^2}{\mu^2}, g, \alpha)$ is a column vector which runs over both the singlet and non-singlet hadronic coefficients, as well as the coefficients for the new photonic operators. (There is no $\beta_{QED}\frac{\partial}{\partial e}$ term written here in the total scale derivative; it does not occur at leading order in α.) The renormalisation group equation has the solution

$$\begin{aligned} \begin{pmatrix} C_H \\ C_\gamma \end{pmatrix}(\frac{Q^2}{\mu^2}, g, \alpha) &= T \exp \int_{\overline{g}}^{g} \frac{dt}{\beta(t)} \begin{bmatrix} \gamma_H & 0 \\ k & 1 \end{bmatrix}(t) \begin{pmatrix} C_H \\ C_\gamma \end{pmatrix}(1, \overline{g}(\frac{Q^2}{\mu^2}), \alpha) \\ &= \begin{bmatrix} M & 0 \\ X & 1 \end{bmatrix}(\frac{Q^2}{\mu^2}, g, \alpha) \begin{pmatrix} C_H \\ C_\gamma \end{pmatrix}(1, \overline{g}(\frac{Q^2}{\mu^2}), \alpha) \end{aligned} \tag{193}$$

Here, γ_H and k are the purely hadronic and photon-hadron mixing anomalous dimensions respectively. The hadronic part γ_H forms a 4x4 block diagonal matrix comprising the singlet anomalous dimensions in the upper left 2x2 entries and the non-singlet anomalous dimensions diagonally in the lower right 2x2 entries. The mixing term k is a 1x4 row vector and is proportional to γ_{VV}^F, viz. $k = -e^2 \frac{N_f}{T(R)} N_c \gamma_{VV}^F [\frac{2}{9}, 0, \frac{1}{6}, \frac{1}{18}] + O(e^2 g^2)$. It is the same calculation as for γ_{VV}^F - only with the group factors of QED instead of QCD. The constants N_f and N_c are number of quark flavours (=3) and colours (=3), and $T(R) = \frac{1}{2} N_f$. The numbers in the row vector in k are the coefficients of the SU(3) flavour decomposition of $\sum e_q^2$, see equ. (27).

In the solution to the renormalisation group equation M is the familiar hadronic evolution coefficient

$$M(\frac{Q^2}{\mu^2}, g, \alpha) = T \exp \int_{\overline{g}}^{g} dt \frac{\gamma_H(t)}{\beta(t)} \tag{194}$$

The mixing term is

$$X(\frac{Q^2}{\mu^2}, g, \alpha) = \int_{\overline{g}}^{g} dt \frac{k}{\beta(t)} T \exp \int_{\overline{g}}^{t} ds \frac{\gamma_H(s)}{\beta(s)} \tag{195}$$

These coefficients are readily evaluated by expanding γ_H in terms of its eigenvalues λ_k and the corresponding projection matrices P_k (viz. $\gamma_H = \sum \lambda_k P_k$) [9], [130]. Since

$P_iP_j = \delta_{ij}P_j$ this trick simplifies the matrix multiplication considerably. We find

$$X(\frac{Q^2}{\mu^2}, g, \alpha) = \frac{k}{2\beta_0 \overline{g}^2} \sum_k \frac{P_k}{(1+\frac{\lambda_k}{2\beta_0})} \quad + \text{ non} - \text{leading terms} \tag{196}$$

The mixing term X has been calculated by Witten [130] to leading order in α_s (and to next to leading order by Bardeen and Buras [136]) for the unpolarised case. The anomalous dimensions for polarised deep inelastic scattering are known only to leading order in α_s. After including the effect of (leading order) renormalisation group, the moments of $g_1^\gamma(x)$ are given by

$$\begin{aligned} &2iS\ \epsilon^{\mu\lambda\alpha\beta}\epsilon_\alpha\epsilon^*_\beta p_\lambda p^{\mu_1}..p^{\mu_{n-1}} \int_0^1 dx\ x^{n-1} g_1^\gamma(x,Q^2) \\ &= \sum_{O_H} M(\frac{Q^2}{\mu^2}, g, \alpha) C_{H,n}(1, \overline{g}(\frac{Q^2}{\mu^2}), \alpha) < \gamma | O_{H,n}^{\mu\mu_1..\mu_{n-1}}(\mu^2)|\gamma > \\ &+ \sum_{O_\gamma} (X(\frac{Q^2}{\mu^2}, g, \alpha) C_{H,n}(1, \overline{g}(\frac{Q^2}{\mu^2}), \alpha) + C_{\gamma,n}(1, \overline{g}(\frac{Q^2}{\mu^2}), \alpha)) < \gamma | O_{\gamma,n}^{\mu\mu_1..\mu_{n-1}}(\mu^2)|\gamma > \end{aligned} \tag{197}$$

Here, $< \gamma|O_{\gamma,n}|\gamma >= 1$ to leading order in α. After diagonalising γ_H, we construct the row vector X and contract it with the hadronic coefficients $C_{H,n}$. For the spin dependent case (with three flavours) we obtain

$$\begin{aligned} &X(\frac{Q^2}{\mu^2}, g, \alpha) C_{H,n}(1, \overline{g}, \alpha) = \\ &\frac{\alpha}{\pi}\frac{2}{3}(\gamma^F_{VV})_n \ln\frac{Q^2}{\mu^2}\left[\frac{2}{3}(1+\frac{(\gamma^V_{VV})_n}{2\beta_0})\frac{1}{\delta_n} + \frac{5}{12}\frac{1}{1+\frac{(\gamma^F_{FF})_n}{2\beta_0}}\right] \\ &+ \text{non} - \text{leading terms} \end{aligned} \tag{198}$$

The anomalous dimensions $(\gamma^F_{FF})_n$, $(\gamma^F_{VV})_n$, $(\gamma^V_{FF})_n$ and $(\gamma^V_{VV})_n$ are normalised so that $(\gamma)_n(\text{this paper}) = -\frac{\alpha_s}{2\pi}(\gamma)_n(\text{Ahmed and Ross [49]})$ and $\beta_0 = 11 - \frac{2}{3}N_f$. The coefficient

$$\delta_n = 1 + \frac{(\gamma^F_{FF})_n + (\gamma^V_{VV})_n}{2\beta_0} + \frac{(\gamma^F_{FF})_n(\gamma^V_{VV})_n - (\gamma^F_{VV})_n(\gamma^V_{FF})_n}{4\beta_0^2} \tag{199}$$

For $n \geq 2$ the $\ln Q^2$ factor, which we observed in the FQM, reappears in the mixing term. However, by comparison with equ.(186), we notice a finite QCD renormalisation of the coefficient of the logarithm. This logarithm term dominates the other (scaling) terms in the expression for $g_1^\gamma(x)$. The $C_\gamma < \gamma|O_\gamma|\gamma >$ $\left(C_H < \gamma|O_H|\gamma >\right)$ terms scale up to the purely photonic (hadronic) evolution respectively, which is due to the QED (QCD) anomalous dimensions.

Now, let us consider the first moment. The only twist two operators which contribute to G_1 are the non-singlet hadronic axial vector current operators $A^3_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\lambda_3}{2}q$, $A^8_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\lambda_8}{2}q$, and the singlet $A^0_\mu = \bar{q}\gamma_\mu\gamma_5\frac{\lambda_0}{2}q$. There are no twist two, spin one, gauge invariant photon or gluon operators. The operator product expansion gives us

$$\begin{aligned} G_1(Q^2) &= \int_0^1 dx g_1^\gamma(x) \\ &= \frac{1}{6}\left[\left(a_3 + \frac{1}{\sqrt{3}}a_8\right)\left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) + 2\sqrt{\frac{2}{3}}a_0(Q^2)\left(1 - \frac{\alpha_s(Q^2)}{\pi}\frac{33-8N_f}{33-2N_f}\right)\right] \end{aligned} \tag{200}$$

where $ia_k\epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha(\lambda)\epsilon^{*\beta}(\lambda) =< \gamma(p,\lambda)|A^k_\mu|\gamma(p,\lambda) >$.

The general form for the matrix element of the axial vector current A^i_μ between on-shell photon states (see Fig. 15) is given by covariance and parity. Allowing for Bose crossing symmetry we have

$$< \gamma(l,\lambda_2)|A^k_\mu|\gamma(p,\lambda_1) >= R^k_{\mu\alpha\beta}(p,-l)\epsilon^\alpha(\lambda_1)\epsilon^{*\beta}(\lambda_2) \tag{201}$$

where [66]

$$\begin{aligned} R^k_{\mu\alpha\beta}(p,-l) = \Big[&\epsilon_{\mu\nu\alpha\beta}(p^\nu + l^\nu)A^k(s^2) + \epsilon_{\mu\alpha\nu\delta}p^\nu l^\delta\left(B^k_1(s^2)p_\beta + B^k_2(s^2)l_\beta\right) \\ &+\epsilon_{\mu\beta\nu\delta}p^\nu l^\delta\left(B^k_1(s^2)l_\alpha + B^k_2(s^2)p_\alpha\right)\Big] \end{aligned} \tag{202}$$

and $s^\mu = (p-l)^\mu$. The A^k, B^k_1 and B^k_2 are scalar form factors of the variable s^2 (for real photons: $p^2 = l^2 = 0$). The forward matrix element, which is relevant to deep inelastic scattering, is given by

$$ia_k = 2A^k(s^2 = 0) \tag{203}$$

with $\lambda_1 = \lambda_2 = \lambda$. Here, we have tacitly assumed that there are no massless bosons coupled to A^k_μ. This is certainly true in nature. The would-be Goldstone bosons π^0 and η, which couple to A^3_μ and A^8_μ respectively, have finite mass due to the chiral symmetry breaking quark mass term in the QCD Lagrangian. After $\eta - \eta'$ mixing is allowed for, the physical η' also couples to A^8_μ. However, the η' is not a would-be Goldstone boson and remains massive even in the chiral limit. Both the η and η' couple to the singlet axial current A^0_μ. (There is also a small contribution to the axial current matrix elements from $\pi^0 - \eta$ and $\pi^0 - \eta'$ mixing.)

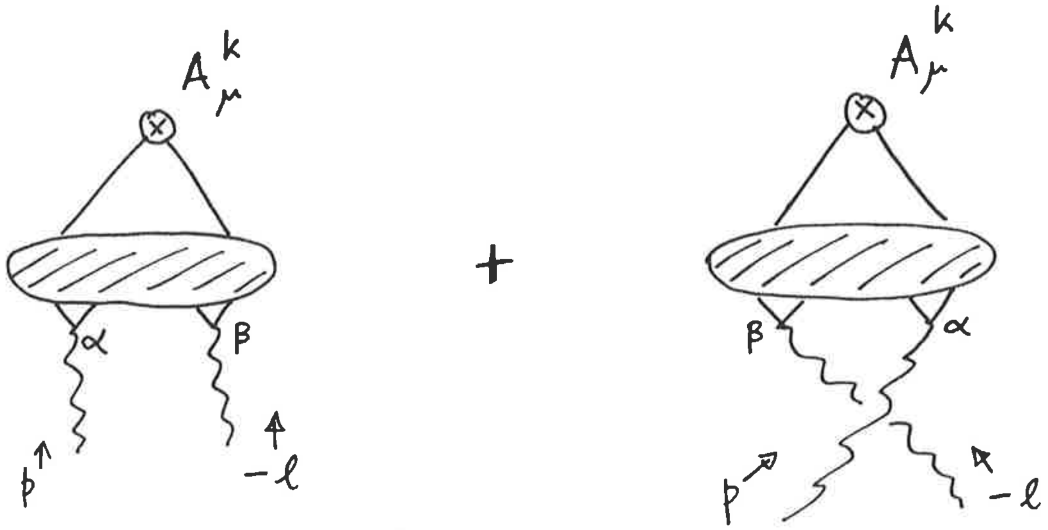


Figure 15. G_1 in QCD receives contributions from the photon matrix elements of the axial-vector currents A^k_μ.

Electromagnetic gauge invariance (current conservation) at the photon vertices, viz.

$$p^\alpha R^k_{\mu\alpha\beta} = l^\beta R^k_{\mu\alpha\beta} = 0, \tag{204}$$

leads to the identities

$$\begin{aligned} A^k(s^2) &= \frac{s^2}{2} B^k_1(s^2) + p^2 B^k_2(s^2) \\ A^k(s^2) &= \frac{s^2}{2} B^k_1(s^2) + l^2 B^k_2(s^2) \end{aligned} \tag{205}$$

respectively. Since $R^k_{\mu\alpha\beta}$ contains no massless poles we can drop the B^k_2 term in equs. (205). Clearly, $A^k(s^2 = 0) = 0$ for each of $k = 3, 8, 0$. Hence, electromagnetic current conservation and the absence of any massless bosons coupled to A^k_μ imply the exact (scale independent) sum-rule

$$G_1 = 0 \tag{206}$$

for the real photon. We now show that this zero comes from the cancellation of two matrix elements: one, which measures a purely pointlike interaction, and a hadronic term.

Current conservation at the photon vertices also means that the form factor $A^k(s^2)$ contains the Adler-Bell-Jackiw axial anomaly [66]. The on-shell photon matrix element of the anomalous divergence equation

$$\partial^\mu A^k_\mu = N_c \mathrm{Tr}[\frac{\lambda_\mathrm{k}}{2} \mathrm{e}^2_\mathrm{q}] \ \frac{\alpha}{2\pi} F.\tilde{F} + (2m_q \bar{q} i \gamma_5 \frac{\lambda_k}{2} q) + \delta_{k0} \frac{1}{2} \sqrt{\frac{2}{3}} \ N_f \frac{\alpha_s}{2\pi} \mathrm{Tr_c} G.\tilde{G} \tag{207}$$

is given by

$$-is^\mu R^k_{\mu\alpha\beta} = -2iA^k(s^2) \epsilon_{\mu\nu\alpha\beta} p^\mu l^\nu \tag{208}$$

In equ. (207) $\mathrm{Tr_c}$ denotes the trace over colour indices and $\mathrm{e_q}$ is the quark charge matrix. There is an s^2 independent contribution $-\frac{2\alpha}{\pi} N_c \mathrm{Tr}[\frac{\lambda^k}{2} \mathrm{e}^2_\mathrm{q}]$ to $-2iA^k(s^2)$, and hence to a_k, from the electromagnetic topological charge density. The QED axial anomaly is manifest in $g_1^\gamma(x)$ as the contact (local) photon matrix element of the photonic operator $\frac{\alpha}{2\pi}F\tilde{F}$. The Chern-Simons current in QED has gauge invariant forward VVA matrix elements. Clearly

$$< \gamma(p,\lambda)|k_\mu^{QED}|\gamma(p,\lambda) >= \frac{\alpha}{\pi}\epsilon_{\mu\nu\alpha\beta}p^\nu\epsilon^\alpha\epsilon^{*\beta} \tag{209}$$

vanishes if we replace one of the photon polarisation vectors by the photon momentum p^α. For this reason, one can associate the contact anomaly contribution to G_1 directly with k_μ^{QED}. This QED anomaly contribution corresponds to the local measurement of the target photon that we observed in the free quark model or box calculation. Since it is gauge invariant and not sensitive to any infra-red effects in QCD, we conclude that the QED anomaly contribution to $g_1^\gamma(x)$ is characterised by large k_T^2 two-quark-jet events. Following the discussion in chapter 5, there will be a (gauge-invariant) QED anomalous contribution to all of the higher moments of $g_1^\gamma(x)$. Thus, there are two spin dependent photon distributions relevant to $g_1^\gamma(x)$: one associated with the anomaly and the other associated with the photonic operators $F^{\mu\alpha}\partial^{\mu_1}...\partial^{\mu_{n-2}}\tilde{F}_\alpha^{\mu_{n-1}}$. Beyond leading order in α, these distributions will evolve differently under the renormalisation group.

The contact anomalous interaction contributes a total of $-\frac{2}{3}\frac{\alpha}{\pi}$ to G_1. For the real photon, this contribution is identically cancelled by the hadronic term associated with the operators $2m_q\bar{q}i\gamma_5\frac{\lambda_k}{2}q$ and $\frac{\alpha_s}{2\pi}\text{Tr}_c G.\tilde{G}$ in equ.(207). If we define

$$\begin{aligned}\chi^q(s^2)\epsilon_{\mu\nu\alpha\beta}p^\mu l^\nu\epsilon^\alpha(\lambda_1)\epsilon^{*\beta}(\lambda_2) &=< \gamma(l,\lambda_2)|2m_q\bar{q}i\gamma_5 q|\gamma(p,\lambda_1) > \\ N_f\chi^g(s^2)\epsilon_{\mu\nu\alpha\beta}p^\mu l^\nu\epsilon^\alpha(\lambda_1)\epsilon^{*\beta}(\lambda_2) &=< \gamma(l,\lambda_2)|N_f\frac{\alpha_s}{2\pi}\text{Tr}G.\tilde{G}|\gamma(p,\lambda_1) >\end{aligned} \tag{210}$$

with $p^2 = l^2 = 0$, then $\chi^q(0)$ and $\chi^g(0)$ satisfy the sum-rules

$$\begin{aligned}(\chi^u - \chi^d)(0) &= \frac{2\alpha}{\pi}(e_u^2 - e_d^2)N_c \\ (\chi^u + \chi^d - 2\chi^s)(0) &= \frac{2\alpha}{\pi}(e_u^2 + e_d^2 - 2e_s^2)N_c \\ (\chi^u + \chi^d + \chi^s + N_f\chi^g)(0) &= \frac{2\alpha}{\pi}(e_u^2 + e_d^2 + e_s^2)N_c\end{aligned} \tag{211}$$

This result (without the singlet gluon term) was first noted by Adler [122] before the advent of QCD.

6.4 Isolating the Strong U(1) Anomaly Contribution to Δq_0

Early in the EMC Spin debate it was pointed out that the Chern-Simons term and the Noether gluonic spin operator coincide in the axial gauge $B_+ = 0$ [22, 87, 107], which is closest to the parton model (section 3.5). It was then suggested by Jaffe and Manohar [107] that the forward matrix matrix elements of the QCD Chern-Simons current are gauge dependent. They argued that the contribution to a given matrix element from the non-abelian three gluon term in k_μ^{QCD} might change under large gauge transformations, which are associated with topological structure, eg. instantons [103]. Cheng and Li [137] then proposed that the analogous quantities to $\chi^q(0)$ and $\chi^g(0)$ in the proton might be used to define the proton's quark and gluon "spin content"; that is, one might choose to define the quark and gluon parton "spin content" of the spin one photon as $\Delta q^\gamma_{CL} = \frac{\pi}{2\alpha}\chi^q(0)$ and $\frac{\alpha_s}{2\pi}\Delta g^\gamma_{CL} = \frac{\pi}{2\alpha}\chi^g(0)$ respectively.

We have stressed before (in chapter 5) that the axial anomaly, which means both k_μ^{QCD} and $\chi^g(0)$, has nothing to with the spin dependent gluon distribution $\Delta g(x, Q^2)$. The anomaly is a property of the quark distribution $\Delta q(x, Q^2)$, which is defined by the the target matrix elements of the local axial tensor operators $\overline{q}(0)\gamma_+\gamma_5(D_+)^n q(0)$. Nevertheless, it is interesting to see whether the anomaly contribution to Δq_0 can be isolated in a different experiment or whether $\chi^q(0)$ and $\chi^g(0)$ are each sensitive to soft infra-red effects in QCD [107] (for example, the ratio of quark masses [28]). This problem has been addressed by a number of authors [28, 29, 138, 139, 140] for the proton target. Here we shall repeat these calculations and show how they generalise to the polarised photon target. We now calculate $\chi^q(0)$ and $\chi^g(0)$ via the QCD effective action [115, 116].

In a recent paper [28] (see also ref. [138]), Veneziano has discussed the EMC Spin Effect from the viewpoint of the effective chiral action for QCD [115, 116]

$$\Gamma = \frac{F_\pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{F_\pi^2}{4}\mathrm{Tr}M(U+U^\dagger) + \frac{i}{2}Q\mathrm{Tr}(\ln U - \ln U^\dagger) + \frac{1}{2\kappa(s^2)}Q^2 + \Gamma_{\mathrm{int}}(U,Q,B,\gamma) \tag{212}$$

Here $U = \exp(2i\phi_a \frac{\lambda^a}{2}/F_\pi)$ where ϕ_a is the nonet of would-be Goldstone bosons in QCD with perfect OZI (OZIQCD). In massless QCD the left and right handed quarks transform independently under rotations in flavour space: Chiral $SU(3)_L \otimes SU(3)_R$ is an exact global symmetry of the massless QCD Lagrangian [115]. The fields in U transform as

$$U \to LUR^\dagger \tag{213}$$

under a general independent left L and right R transformation in flavour space. The chiral symmetry is manifest in Nambu-Goldstone mode; it is spontaneously broken and the 0^- meson octet are the corresponding massless Goldstone bosons. The chiral $SU(3)_L \otimes SU(3)_R$ symmetry is explicitly broken by the finite quark masses in the QCD Lagrangian. This leads to the finite masses of the 0^- meson octet, which we call would-be Goldstone bosons. (They would be Goldstone bosons in the limit of zero quark mass.) This explicit breaking of chiral symmetry is represented in the effective action by the boson mass matrix M. It is proportional to the quark mass matrix in the QCD Lagrangian. The mass term breaks chiral symmetry and induces mixing among the ϕ_a to produce the physical bosons (π^0, η &tc).

Despite the fact that the bare QCD Lagrangian is invariant under chiral $U(1)_L \otimes U(1)_R$, there is no would-be U(1) Goldstone boson in the hadron spectrum. The physical η' is too heavy [106] with a mass of 938 MeV; it remains massive in the chiral limit where all the quark masses go to zero. (Note, however, that $m^2_{\eta'} \sim \frac{1}{N_c}$ in the large N_c limit of QCD where Zweig's rule becomes exact.) The η' is generated in our effective theory by the pseudoscalar ghost field Q [28] which is associated with the QCD topological current K_μ. It mixes with the would-be Goldstone boson ϕ_0 to generate the large mass of physical η', viz. $m^2_{\eta'} = 2N_f F_\pi^{-2}\kappa(m^2_{\eta'})+$ quark mass terms. The interaction term $\Gamma_{\rm int}(U, Q, B, \gamma)$ contains the one particle irreducible couplings of ϕ_a to the baryon octet B and also the Wess Zumino term [141], which describes the axial anomaly induced decays $\phi_a \to 2\gamma$ ($a = 3, 8, 0$).

The effective action is not renormalisable [115] but it does not need to be. It describes only the chiral symmetry properties of QCD at low momenta, where the two derivative kinetic energy term is a sufficient approximation to nature - i.e. where we can neglect higher order terms in perturbation theory which need to be renormalised.

The quantities $\chi^q(0)$ and $\chi^g(0)$ are calculated as follows. We first write down the SU(3) flavour composition of the unmixed, would-be Goldstone bosons ϕ_a with mass squared μ_a^2; that is, we define the pure u, d and s flavour bosons (ϕ_u, ϕ_d and ϕ_s respectively) so that $\phi_3 = \frac{1}{2}(\phi_u - \phi_d)$, $\phi_8 = \frac{1}{2\sqrt{3}}(\phi_u + \phi_d - 2\phi_s)$ and $\phi_0 = \frac{1}{2}\sqrt{\frac{2}{3}}(\phi_u + \phi_d + \phi_s)$. The operators $\bar{q}i\gamma_5 q$, which define $\chi^q(0)$, couple directly to ϕ_q ($q = u, d, s$). The topological charge density $\frac{\alpha_s}{2\pi}{\rm Tr}_c G.\tilde{G}$, which defines $\chi^g(0)$, couples to the ghost Q.

From the quadratic part of Γ we can write down the bosonic propagator matrix, which contains off-diagonal entries due to the $Q - \phi_q$ coupling in the $\frac{i}{2}Q{\rm Tr}(\ln U - \ln U^\dagger)$ term in equ.(212). We need to invert the 4x4 matrix

$$K^{-1} = \begin{pmatrix} q^2 - \mu_u^2 & 0 & 0 & \sqrt{2}F_\pi^{-1} \\ 0 & q^2 - \mu_d^2 & 0 & \sqrt{2}F_\pi^{-1} \\ 0 & 0 & q^2 - \mu_s^2 & \sqrt{2}F_\pi^{-1} \\ \sqrt{2}F_\pi^{-1} & \sqrt{2}F_\pi^{-1} & \sqrt{2}F_\pi^{-1} & \kappa^{-1} \end{pmatrix}^{-1} \tag{214}$$

The propagator matrix simplifies considerably in the limit that $\mu_u^2, \mu_d^2 \ll \mu_s^2, F_\pi^{-2}\kappa$, or equivalently, $m_\pi^2 << m_\eta^2, m_{\eta'}^2$. One finds that the diagonal $Q - Q$ propagator vanishes at $s^2 = 0$ whilst the off-diagonal $Q - \phi_q$ propagators remain finite. The matrix elements $\chi^q(0)$ and $\chi^g(0)$ are calculated for a general DIS target T by inserting the propagator matrix between the initial amplitude for the operator-boson coupling and the amplitude for the would-be Goldstone boson to couple to the target. We let $g(\pi_q, T)$ denote the general target $T - \pi_q$ coupling constant. For the general target, we find [28]

$$\begin{aligned} N_f\chi^g(0) =& N_f\frac{F_\pi}{\sqrt{2}}\,\frac{g(\pi_u,T)m_d + g(\pi_d,T)m_u}{m_u+m_d} \\ \chi^u(0) =& \frac{F_\pi}{\sqrt{2}}\,(g(\pi_u,T) - g(\pi_d,T))\frac{m_u}{m_u+m_d} \\ \chi^d(0) =& -\frac{F_\pi}{\sqrt{2}}\,(g(\pi_u,T) - g(\pi_d,T))\frac{m_d}{m_u+m_d} \\ \chi^s(0) =& \frac{F_\pi}{\sqrt{2}}\left[g(\pi_s,T) - \frac{g(\pi_u,T)m_d + g(\pi_d,T)m_u}{m_u+m_d}\right] \end{aligned} \tag{215}$$

in the physical limit $m_\pi^2 << m_\eta^2, m_{\eta'}^2$. Here, we have used the result that μ_a^2 is proportional to the quark mass m_a [142].

For $g_1^\gamma(x)$ we are interested in the amplitude for the two photon decays of the would-be Goldstone bosons ($\pi_q \to 2\gamma$). This latter amplitude is determined from the Wess-Zumino term [141]. Hence, the QCD topological charge density contributes to a_0 for the photon via the decays $\phi_q \to 2\gamma$. The sum-rules in equ. (211) then describe the decay rates for the unmixed would-be Goldstone bosons ϕ_3, ϕ_8 and ϕ_0 respectively to two photons via the axial anomaly. For the photon target

$$g(\pi_a, \gamma) = N_c\frac{\sqrt{2}}{F_\pi}e_a^2 \tag{216}$$

We substitute these would-be Goldstone boson couplings to two photons into equ. () and find (in units of $\frac{2\alpha}{\pi}$) :

$$
\begin{aligned}
N_f \chi^g(0) =& N_c N_f \frac{e_u^2 m_d + e_d^2 m_u}{m_u + m_d} \\
\chi^u(0) =& N_c (e_u^2 - e_d^2) \frac{m_u}{m_u + m_d} \\
\chi^d(0) =& N_c (e_d^2 - e_u^2) \frac{m_d}{m_u + m_d} \\
\chi^s(0) =& N_c (e_s^2 - \frac{e_u^2 m_d + e_d^2 m_u}{m_u + m_d})
\end{aligned}
\tag{217}
$$

The linear combinations of $\chi^q(0)$ and $\chi^g(0)$ which couple to the axial currents A_μ^k (viz. equ. (211)) are clearly independent of the quark masses. However, the individual quantities $\chi^q(0)$ and $\chi^g(0)$ possess large isospin violations for all targets. They are not separately observable. This effect is well known for the the pion [143] and the proton [28, 138].

We take $m_u = \frac{1}{2} m_d$ and $N_c = N_f = 3$. In units of $\frac{2\alpha}{\pi}$ the polarised photon results become

$$
\chi^g(0) = 1, \; \chi^u(0) = \frac{1}{3}, \; \chi^d(0) = -\frac{2}{3}, \; \chi^s(0) = -\frac{2}{3} \tag{218}
$$

In other words, (with this choice for m_u/m_d) the gluon "spin content" of the polarised spin one photon is $\Delta g^\gamma = \frac{2\pi}{\alpha_s}$, or about 30 at realistic experimental Q^2! Clearly, this result is nonsense and illustrates the perils of defining "spin" in terms of these isospin dependent quantities.

We note that each of $\chi^q(0)$ and $\chi^g(0)$ are scale independent. For $\chi^q(0)$ this follows because $m\bar{q}i\gamma_5 q$ is a dimension three operator and therefore is not renormalised. The gluonic term $\chi^g(0)$ is relevant only to A_μ^0. At leading order in α, any scale dependence of a_0 is carried completely by $\chi^g(0)$ and satisfies the relation $\frac{\partial}{\partial \mu^2} a_0 = \frac{1}{2}\sqrt{\frac{2}{3}} N_f \frac{\partial}{\partial \mu^2} \chi^g(0)$. Electromagnetic gauge invariance means that $a_0 = 0$ independent of scale. Thus, $\chi^g(0)$ is QCD scale independent.

The electromagnetic axial anomaly contribution to the first moment of $g_1^\gamma(x)$ can be isolated exactly because the anomaly is manifest as a photon operator acting between external photon states. This compares with the strong U(1) anomaly contribution to the first moment of $g_1(x)$ for an arbitrary (hadronic or photon) target, which is sensitive to infra-red physics via the quark mass dependence. It is this property that makes the polarised photon a particularly clean system in which to study the role of the axial anomaly in polarised deep inelastic scattering. Furthermore, the polarised photon target is accessible to experiment.

The extension of our analysis to DIS from gluonic currents in the proton is non-trivial (at least beyond the one loop box calculation [87]). For example, we know that gluons in the proton are off-shell ($P^2 \neq 0$). This means that we cannot apply the operator product expansion between external gluon states in the usual way, following equs. (187) and (188). One must replace the target photon currents by gluon currents, which carry an explicit colour label, and also insert a path ordered exponential between the two gluon currents for gauge invariance. Applying the operator product expansion in the presence of the path ordered exponential is not straight-forward and requires an analysis that goes beyond the scope of this thesis. This technical point does not apply when we consider a real ($P^2 = 0$) gluon target. In this case, we can apply the operator product expansion between asymptotic gluon states. However, real gluons are not physical objects and are irrelevant to experiment.

7. Conclusions and Outlook

This thesis has been about the theoretical interpretation of the EMC Spin Effect. We have focussed on the parton model and the role of the axial anomaly in polarised deep inelastic scattering. This work was motivated by the pioneering papers of Efremov and Teryaev [21] and Altarelli and Ross [22] (ETAR). Our conclusions are fourfold.

1. Locality and the Parton Model in QCD

We have investigated $g_1(x)$ in several versions of the QCD Improved Parton Model to check the factorisation scheme dependence of the ETAR results.

In the OPE approach to DIS the first moment of $g_1(x)$ is given entirely by the first moment of the spin dependent quark distribution $\Delta q = \int_0^1 dx \Delta q(x, Q^2)$. This quark distribution is defined formally by the forward proton matrix elements of the spin-odd, local axial-tensor operators $\overline{q}(0)\gamma_+\gamma_5(D_+)^{2n}q(0)$. The spin dependent gluon distribution does not contribute to the first moment of $g_1(x)$.

In the IPM, which is based entirely on the factorisation of mass singularities, one finds that the IPM gluon distribution does contribute to the first moment of $g_1(x)$ - in apparent contradiction with the OPE. One finds that (for each flavour q)

$$\Delta q_{OPE} \rightarrow (\Delta q - \xi \frac{\alpha_s}{2\pi} \Delta g)_{IPM} \tag{219}$$

in the expression for the first moment of $g_1(x)$. The parameter ξ is factorisation scheme dependent. We may choose to define the quark and gluon distributions at a scale $\Lambda^2 \ll Q^2$ as describing all quarks and gluons with transverse momentum squared less than Λ^2 respectively. In this case, the gluon distribution is observed experimentally via large $k_T^2 \geq \Lambda^2$ two quark jet events. Factorisation via the k_T^2 cut-off offers some intuitive appeal; it leads to a value $\xi = 1$ (which agrees with ETAR). Alternatively, one could define the parton distributions by a cut-off on the quark virtuality $m^2 - k^2 \leq \Lambda^2 \ll Q^2$.

This corresponds to a different two quark jet definition and a different value for ξ, viz. $\xi = \frac{1}{2}$.

In both versions of the parton model the gluon term in the first moment is induced by a local photon gluon interaction, which appears to generate two quark jet events with k_T^2 or $m^2 - k^2$ of order Q^2. This is a new effect in the parton model. In unpolarised DIS we find that the analogous quantity to ξ receives contributions from two quark jet events with a range of k_T^2 between Λ^2 and Q^2. This is also true for the higher moments of $g_1(x)$. With the first moment of $g_1(x)$ the parameter ξ is determined only by the maximum possible $k_T^2 \sim Q^2$.

The IPM was developed with the OPE results for DIS in mind. Factorisation of mass singularities is similar to the calculation of the Wilson coefficients in the OPE. We would like our parton model to reproduce the rigorous OPE result that the spin dependent gluon distribution does not contribute to the first moment of $g_1(x)$. When we applied the OPE to the spin dependent photon gluon fusion process we found that the large $k_T^2 \sim Q^2$ term *is* included in the quark distribution. It corresponds to the two gluon matrix element of the anomalous Chern-Simons term associated with the axial-vector current. The IPM and OPE approaches to DIS can be reconciled if we supplement the IPM with a new locality condition. The gluon distributions are to be defined as generating a non-local interaction between the gluon and the hard photon in the DIS process. The quark distribution contains all partons which make a local interaction with the photon. In this picture, the local photon gluon interaction is a renormalisation effect associated with the gluonic dressing of the quark partons.

2. Effects of $\Delta g(x)$ in the EMC data

In section 4.5 we investigated the x dependence of the two quark jet contribution to $g_1(x)$, which is induced by $\Delta g(x)$. This gluon contribution was found to be at small x - essentially outside the range of the EMC data. Hence, one cannot use arguments about $\Delta g(x)$ to resolve the discrepancy between the data and the Ellis-Jaffe hypothesis. As we showed in Fig. 11, this conclusion holds independent of our arguments about locality and the definition of $\Delta g(x)$.

3. $g_1(x)$ measures a violation of OZI. It has nothing to do with "spin"

In chapter 5 we discussed the axial anomaly in polarised DIS. The flavour singlet contribution to the first moment of $g_1(x)$ is given by the proton matrix element of the gauge invariant, flavour singlet axial-vector current $A^0_\mu(\mu^2)$. This composite operator contains the strong U(1) axial anomaly, which catalyses the apparent violation of OZI seen in the EMC data. Thus, the EMC Spin Effect is seen to be related to the longstanding U(1) problem of QCD.

In section 5.2 we looked at the generators of SU(2) intrinsic spin for the free fermion. These are calculated in the fermion's rest frame via

$$S_k = \int d^3x \; \overline{q}(x)\gamma_k\gamma_5 q(x) \tag{220}$$

These operators are scale dependent in gauge-invariantly renormalised QCD. They do not satisfy the algebra of SU(2) spin. The gauge invariant axial-vector current, and hence $g_1(x)$, has nothing to do with an intrinsic quark spin component in the proton.

4. The polarised photon structure function $g_1^\gamma(x)$

The polarised photon is an ideal physical system in which to discuss the axial anomaly in polarised deep inelastic scattering. Unlike the case with hadronic targets, the anomaly makes a contact interaction with the photon target. The polarised photon target is unique in this respect: there are no free gluon targets in nature.

We have presented a detailed analysis of the polarised real photon structure function $g_1^\gamma(x)$ and shown that there is an exact sum-rule for the first moment of $g_1^\gamma(x)$; viz. $G_1 = 0$. This zero is the sum of two well defined finite quantities. One is purely pointlike in origin (contact interaction) and comes from the Adler-Bell-Jackiw axial anomaly. The other is a hadronic contribution associated with the decays of the unmixed would-be Goldstone bosons ϕ_3, ϕ_8, and ϕ_0 to two photons. The electromagnetic axial anomaly contribution to the first moment of $g_1^\gamma(x)$ can be isolated exactly because the anomaly is manifest as a photon operator acting between external photon states. This compares with the strong U(1) anomaly contribution to the first moment of $g_1(x)$ for an arbitrary (hadronic or photon) target, which is sensitive to infra-red physics via the quark mass dependence. It cannot be isolated in different experiments. The strong U(1) axial anomaly means that there is no rigorous sum-rule for $g_1(x)$.

Outlook

In this thesis we have clarrified the current theoretical understanding of the EMC Spin Effect. However, the dynamical mechanism responsible for the violations of OZI remains a mystery. Forte and Shuryak [109] have recently tried to generalise 't Hooft's treatment of the U(1) problem to the Spin Effect, although the U(1) problem itself is still a matter of some theoretical controversy [102-105]. The final resolution of the "spin crisis" may take some time and is sure to involve interesting new ideas, perhaps involving the confinement mechanism itself.

There are also many experimental challenges ahead with polarised DIS. In the short term, we await the forthcoming SMC results on $g_1(x)$. The SMC [39, 47] hope to halve the statistical errors in the EMC data. They will also measure $g_1(x)$ for the neutron - allowing a test of the Bjorken sum-rule to within 10%. A test of the Bjorken sum-rule is also planned at SLAC experiment E-142 [144]. The HERMES collaboration at HERA [145] have also planned to measure the spin dependent structure functions using the HERA electron beam and several internal gas targets. They hope to increase the accuracy of the EMC results on $g_1(x)$ by almost an order of magnitude.

Appendix A : Trace Theorems

$$
\begin{aligned}
\mathrm{Tr}\ \gamma_5 &= 0 \\
\mathrm{Tr}\ 1 &= 4 \\
\mathrm{Tr}\ \gamma^\mu \gamma^\nu &= 4g^{\mu\nu} \\
\mathrm{Tr}\ \gamma^\mu \gamma^\nu \gamma_5 &= 0 \\
\mathrm{Tr}\ \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \\
\mathrm{Tr}\ \gamma_5 \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma &= 4i\epsilon^{\mu\nu\rho\sigma} \\
\mathrm{Tr}\ [\text{odd number } \gamma^\mu \text{ matrices}] &= 0
\end{aligned}
\tag{221}
$$

Appendix B

List of Publications

This thesis work has resulted in the following publications.

"Gluons and The Proton Spin", S. D. Bass, N. N. Nikolaev, and A. W. Thomas,
Univ. Adelaide report ADP-90-133/T80 (1990).

"Improved Parton Distributions from the Quark Model",
S. D. Bass, A. Schreiber, A. W. Thomas and J. T. Londergan,
Aust. J. Phys. 44 (1991) 363.

"Modelling QCD", S. D. Bass and A. W. Thomas,
Proceedings of PANIC XII, Nucl. Phys. A527 (1991) 519c.

"On the Infrared Contribution to the Photon-Gluon Scattering and the Proton Spin Content",
S. D. Bass, B. L. Ioffe, N. N. Nikolaev and A. W. Thomas,
J. Moscow Phys. Soc. 1 (1991) 317.

"Polarised Deep Inelastic Scattering from Photon Targets and Axial Anomalies",
S. D. Bass, Univ. Adelaide preprint ADP-91-150/T91 (1991),
accepted for publication in Mod. Phys. Letts. A.

References

1. CERN Courier, Volume 30, number 9 (December 1990) 1.
2. R. P. Feynman, Phys. Rev. Lett. 23 (1969) 1415.
3. R. P. Feynman, "Photon-Hadron Interactions", Benjamin Cummings (1972).
4. J. D. Bjorken, Phys. Rev. 179 (1969) 1547.
5. J. D. Bjorken and E. A. Paschos, Phys. Rev. 185 (1969) 1975.
6. E. Rutherford, Phil. Mag. 21 (1911) 669.
7. E. Leader and E. Predazzi, "An Introduction to Gauge Theories and the New Physics", Cambridge UP (1983).
8. D. J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633.
9. D. J. Gross and F. Wilczek, Phys. Rev. D9 (1974) 980.
10. H. Georgi and H. D. Politzer, Phys. Rev. D9 (1974) 416.
11. K. G. Wilson, Phys. Rev. 179 (1974) 2445.
12. V. N. Gribov, Physica Scripta T15 (1987) 164.
13. EMC, J. J. Aubert et al., Phys. Lett. B123 (1983) 275.
14. R. P. Bickerstaff and A. W. Thomas, J. Phys. G. 15 (1989) 1523.
15. EMC, J. Ashman et al., Phys. Lett. B206 (1988) 364.
16. A. J. Buras, Rev. Mod. Phys. 52 (1980) 199.
17. T. Muta, "Foundations of Quantum Chromodynamics", World-Scientific (1987).
18. F. J. Yndurain, "Quantum Chromodynamics", Springer-Verlag (1983).
19. G. Altarelli, Physics Reports 81 (1982) 1.
20. P. D. B. Collins and A. D. Martin, "Hadron Interactions", Adam Hilger (1984).

21. A. V. Efremov and O. V. Teryaev, Dubna Preprint E2-88-287 (1988).

22. G. Altarelli and G. G. Ross, Phys. Lett. B212 (1988) 391.

23. S. J. Brodsky, J. Ellis and M. Karliner, Phys. Lett. B206 (1988) 309.

24. S. D. Bass, B. L. Ioffe, N. N. Nikolaev and A. W. Thomas, J. Moscow Phys. Soc. 1 (1991) 317.

25. S. D. Bass, N. N. Nikolaev and A. W. Thomas, Univ. Adelaide Report ADP-90-133/T80 (1990).

26. S. D. Bass and A. W. Thomas, Proc. PANIC XII, Nucl. Phys. A527 (1991) 519c.

27. J. Ellis, M. Karliner and C. Sachrajda, Phys. Lett. B231 (1989) 497.

28. G. Veneziano, Mod. Phys. Lett. A4 (1989) 1605.

29. G. Veneziano, CERN preprint TH-5840/90, published in "From Symmetries to Strings: 40 Years of Rochester Conferences", ed. A. Das, World-Scientific (1990)
.

30. S. D. Bass and A. W. Thomas, in preparation.

31. S. D. Bass, Univ. Adelaide preprint ADP-91-150/T91 (1991), to be published in Mod. Phys. Lett. A.

32. F. E. Close, "An Introduction to Quarks and Partons", Academic Press (1979).

33. R. L. Jaffe, Comments Nucl. Part. Phys. 19 (1990) 239.

34. L. Mankiewicz and A. Schafer, Univ. Heidelberg preprint (1991).

35. R. L. Jaffe and X. Ji, Phys. Rev. D43 (1991) 724.

36. L. Mankiewicz and Z. Ryzak, Phys. Rev. D43 (1991) 733.

37. EMC, J. Ashman et al., Nucl. Phys. B328 (1990) 1.

38. V. Hughes et al., Phys. Lett. B212 (1988) 511.

39. R. Windmolders, SMC-91-5 (1991), to be published in Int. J. Mod. Phys.

40. G. Baum et al., Phys. Rev. Lett. 51 (1983) 1135.

41. R. L. Heimann, Nucl. Phys. B64 (1973) 429.

42. M. Bourquin et al., Z. Phys. C21 (1983) 27.

43. J. Ellis and R. Jaffe, Phys. Rev. D9 (1974) 1444.

44. F. E. Close and R. G. Roberts, Phys. Rev. Letts. 60 (1988) 1471.

45. E. Leader and M. Anselmino, Z. Phys. C41 (1988) 239.

46. J. Ellis and M. Karliner, Phys. Lett. B213 (1988) 73.

47. SMC Proposal, CERN report CERN/SPSC 88-47 (1988).

48. K. G. Wilson, Phys. Rev. 179 (1969) 1499.

49. M. Ahmed and G. G. Ross, Nucl. Phys. B111 (1976) 441.

50. E. Witten, Nucl. Phys. B104 (1976) 445.

51. W. A. Bardeen et al., Phys. Rev. D18 (1978) 3998.

52. E. G. Floratos et al., Nucl. Phys. B129 (1977) 66; B152 (1979) 493.

53. J. Kodaira et al., Phys. Rev. D20 (1979) 627.

54. J. Kodaira et al., Nucl. Phys. B159 (1979) 99.

55. J. Kodaira, Nucl. Phys. B165 (1980) 129.

56. R. Courant and D. Hilbert, "Methods of Mathematical Physics" volume 1, Interscience (1953) 103.

57. J. C. Collins and D. E. Soper, Nucl. Phys. B194 (1982) 445.

58. C. H. Llewellyn Smith, Oxford preprint OX-89/88 (1988).

59. A. W. Thomas, Cargese lectures, in "Hadrons and Hadronic Matter", eds. D. Vautherin et al., NATO ASI Series B, Vol. 228 (Plenum N.Y.) pp. 263-292.

60. A. Signal and A. W. Thomas, Phys. Lett. B211 (1988) 481, Phys. Rev. D40 (1989) 2832; A. I. Signal, Ph. D. thesis, Univ. Adelaide (1988).

61. F. E. Close and A. W. Thomas, Phys. Lett. B212 (1988) 227.

62. A. W. Thomas et al., Phys. Lett. B233 (1989) 43.

63. A. W. Schreiber, Ph. D. thesis, Univ. Adelaide (1990).

64. S. D. Bass et al., Aust. J. Phys. 44 (1991) 363.

65. for a review, see eg. A. W. Thomas, Adv. Nucl. Phys. 13 (1983) 1.

66. S. L. Adler, Phys. Rev. 177 (1969) 2426; J. S. Bell and R. Jackiw, Nuovo Cimento 51A (1969) 47; S. L. Adler and W. A. Bardeen, Phys. Rev. 182 (1969) 1517.

67. A. V. Manohar, Phys. Rev. Letts. 65 (1990) 2511.

68. R. J. Crewther et al., Phys. Rev. D8 (1973) 3396.

69. R. L. Jaffe, Phys. Lett. B193 (1987) 101.

70. R. Ball, Phys. Lett. B266 (1991) 473.

71. H. Lipkin, Phys. Lett. B214 (1988) 429.

72. H. Lipkin, Phys. Lett. B230 (1989) 135.

73. G. T. Bodwin and J. Qiu, Phys. Rev. D41 (1990) 2755.

74. A. V. Manohar, Phys. Rev. Letts. 66 (1991) 289.

75. C. H. Llewellyn Smith, Acta Physica Austriaca Suppl. XIX (1978) 331.

76. J. Collins et al, in "Perturbative QCD", ed. A. Mueller, World Scientific (1989).

77. G. Altarelli, R. K. Ellis and G. Martinelli, Nucl. Phys. B143 (1978) 521.

78. V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 439. and 15 (1972) 675; L. N. Lipatov, Sov. J. Nucl. Phys. 20 (1974) 181.

79. D. Amati et al., Nucl. Phys. B140 (1978) 54, Nucl. Phys. B146 (1978) 29.

80. A. Carter and C. H. Llewellyn Smith, Nucl. Phys. B162 (1980) 397.

81. S. Pokorski, "Gauge Field Theories", Cambridge UP (1987).

82. R. K. Ellis and W. J. Stirling, CERN-JINR lectures, FERMILAB-Conf-90/164-T.

83. G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.

84. G. G. Ross and R. G. Roberts, Rutherford preprint RAL-90-062 (1990).

85. A. V. Efremov, J. Soffer and O. V. Teryaev, Nucl. Phys. B246 (1990) 97.

86. see eg. G. Krein et al., Argonne preprint PHY-6746-TH-90.

87. R. D. Carlitz, J. C. Collins and A. H. Mueller, Phys. Lett. B214 (1988) 229.

88. A. S. Gorski and B. L. Ioffe and A. Yu Khodjamirian, Phys. Lett. B227 (1989) 474.

89. A. S. Gorski and B. L. Ioffe, Particle World 1 (1990) 114.

90. V. N. Gribov, Nucl. Phys. B206 (1982) 103 and private communication.

91. L. Mankiewicz, Phys. Rev. D43 (1991) 64.

92. L. Mankiewicz and A. Schafer, Phys. Lett. B242 (1989) 455.

93. A. V. Manohar, Phys. Lett. B255 (1991) 579.

94. V. N. Gribov, private discussions

95. E. L. Berger and J. Qiu, Phys. Rev. D40 (1989) 778.

96. D. H. Saxon, Rutherford preprint RAL-89-078 (1989).

97. G. Altarelli and W. J. Stirling, Particle World 1 (1989) 40.

98. S. J. Brodsky and I. Schmidt, Phys. Lett. B234 (1990) 144.

99. A. Schafer, J. Phys. G16 (1990) L121.

100. M. Gluck, E. Reya and W. Vogelsang, Nucl. Phys. B351 (1991) 579.

101. S. Okubo, Phys. Lett. 5 (1963) 165; G. Zweig, CERN report No. 8419/TH412 (1964); J. Iizuka, Prof. Theor. Phys. Suppl. 37-38 (1966) 21.

102. G. 't Hooft, Phys. Rev. D14 (1976) 3432, Physics Reports 142 (1986) 357.

103. R. J. Crewther, Acta Physica Austriaca Suppl. XIX (1978) 47.

104. G. A. Christos, Physics Reports 116 (1984) 251.

105. R. J. Crewther and C. H. Llewellyn Smith, in preparation.

106. S. Weinberg, Phys. Rev. D11 (1975) 3583.

107. R. L. Jaffe and A. Manohar, Nucl. Phys. B337 (1990) 509.

108. S. Forte, Phys. Lett. B224 (1989) 189; Nucl. Phys. B331 (1990) 1.

109. S. Forte and E. V. Shuryak, Nucl. Phys. B357 (1991) 153.

110. see eg. J. D. Bjorken and S. Drell, "Relativistic Quantum Fields", McGraw Hill (1965).

111. M. Gluck and E. Reya, Z. Phys. C43 (1989) 679.

112. A. W. Schreiber, A. W. Thomas and J. T. Londergan, Phys. Lett. B237 (1990) 120; Phys. Rev. D42 (1990) 2226.

113. F. Myhrer and A. W. Thomas, Phys. Rev. D38 (1988) 1633; H. Hogaasen and F. Myhrer, Z. Phys. C48 (1990) 295.

114. A. Schreiber and A. W. Thomas, Phys. Lett. B215 (1988) 141.

115. see eg. H. Georgi, "Weak Interactions and Modern Particle Theory", Benjamin Cummings (1984).

116. C. Rosenzweig et al., Phys. Rev. D21 (1980) 3388; P. Di Vecchia and G. Veneziano, Nucl. Phys. B171 (1980) 253; E. Witten, Ann. of Phys. 128 (1981) 1789; P. Nath and R. Arnowitt, Phys. Rev. D23 (1981) 473.

117. M. C. Birse, Phys. Lett. B249 (1990) 291.

118. J. Schechter et al., Phys. Rev. Letts. 65 (1990) 2955.

119. G. 't Hooft, Nucl. Phys. B72 (1974) 461; B75 (1974) 461.

120. E. Witten, Nucl. Phys. B160 (1979) 57.

121. E. Witten, Nucl. Phys. B156 (1979) 269.

122. S. L. Adler, *in* Lectures on Elementary Particles and Quantum Field Theory, ed. S. Deser, M. Grisaru and H. Pendleton (MIT Press 1970).

123. R. Jackiw, *in* "Current Algebra and Anomalies", ed. S. B. Treiman, R. Jackiw, B. Zumino and E. Witten, World (1985).

124. G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189.

125. A. H. Mueller, Phys. Rev. D18 (1978) 3705; S. Gupta and A. H. Mueller, Phys. Rev. D20 (1979) 118; A. H. Mueller, Phys. Reports 73 (1981) 237.

126. D. Gross and C. H. Llewellyn Smith, Nucl. Phys. B14 (1969) 337.

127. Ch. Berger and W. Wagner, Physics Reports 146 (1987) 1.

128. J. D. Bjorken, Cargese Lectures 1989, SLAC - PUB - 5103 (1989).

129. A. V. Manohar, Phys. Lett. B219 (1989) 357.

130. E. Witten, Nucl. Phys. B120 (1977) 189.

131. A. V. Efremov and O. V. Teryaev, Phys. Lett. B240 (1990) 200.

132. S. J. Brodsky et al., Phys. Rev. Lett. 27 (1971) 280; T. Walsh, Phys. Lett. B36 (1971) 121.

133. R. L. Kingsley, Nucl. Phys. B60 (1973) 45.

134. M. A. Ahmed and G. G. Ross, Phys. Lett. B59 (1975) 369.

135. V. M. Budnev et al., Physics Reports 15 (1975) 181.

136. W. A. Bardeen and A. J. Buras, Phys. Rev. D20 (1979) 166.

137. T. P. Cheng and L. F. Li, Phys. Rev. Lett. 62 (1989) 1441.

138. G. M. Shore and G. Veneziano, Phys. Lett. B224 (1990) 75.

139. T. Hatsuda, Nucl. Phys. B329 (1990) 376; H. Fritzsch, Phys. Lett. B242 (1990) 451; X. Ji, Phys. Rev. Letts. 65 (1990) 408.

140. A. V. Efremov, J. Soffer and N. A. Tornqvist, Phys. Rev. Lett. 64 (1990) 1495; Phys. Rev. D44 (1991) 1369.

141. J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95.

142. J. Gasser and H. Leutwyler, Phys. Reports 87 (1982) 77.

143. B. L. Ioffe, Sov. J. Nucl. Phys. 29 (1979) 827; D. J. Gross, S. B. Treiman and F. Wilczek, Phys. Rev. D19 (1979) 2188.

144. R. Arnold et al., SLAC Proposal E-142 (Oct. 1989); E. Hughes, "SLAC Experiment E 142", SLAC-Pub-5373 (1990).

145. HERMES Proposal, K. Coulter et al., DESY/PRC 09-1 (1990).

Bass, S.D., Schreiber, A., Thomas, A.W. & Londergan, J.T. (1991) Improved parton distributions from the quark model.
Australian Journal of Physics, v. 44(4), pp. 363-368

NOTE:
This publication is included on pages 125-130 in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

http://doi.org/10.1071/PH910363

Bass, S.D. & Thomas, A.W. (1991) Modelling QCD: Probes of Hadron structure.
Nuclear Physics A, v. A527, pp. 519-524

NOTE:
This publication is included on pages 131-136 in the print copy of the thesis held in the University of Adelaide Library.

It is also available online to authorised users at:

http://doi.org/10.1016/0375-9474(91)90148-Y

Bass, S.D., Ioffe, B.L., Nikolaev, N.N. & Thomas, A.W. (1991) On the infrared contribution to the Photon-cluon scattering and the Proton spin content.
Journal of the Moscow Physical Society, v. 1 (3), pp. 317-333