Newtonian and Post-Newtonian Cosmology

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Abstract

Newtonian cosmology is commonly used in astrophysical problems, because of its obvious simplicity when compared with general relativity. However it has inherent difficulties, the most obvious of which is the non-existence of a well-posed initial value problem. Also, Newtonian cosmology is not actually a specialization of the linearization of general relativity as it is usually assumed to be. Both of these problems seem to be due to a loss of information when taking this approximation. This leads to the derivation of a new higher order theory, the post-Newtonian approximation, in which these difficulties are overcome.

The post-Newtonian theory is compared with the fully general relativistic theory in the context of the $k = 0$ Friedmann-Robertson-Walker cosmologies. It is found that the post-Newtonian theory reproduces the results of its general relativistic counterpart, whilst the Newtonian theory does not. Essentially, in Newtonian cosmology the pressure does not enter the dynamics and thus it is impossible to reproduce the complete set of solutions available

in general relativity. In fact, since the solutions to the Newtonian theory do not vary with the input barotropic equation of state, it follows that only the matter dominated case can be described.

It is also explored how far the post-Newtonian theory goes in overcoming the difficulties associated with anisotropic homogeneous cosmologies in the Newtonian approximation. In Newtonian cosmology it is not possible to reproduce the complete set of Bianchi types corresponding to the nine unknowns related to the expansion, the shear and the rotation. Rather, each of the nine Bianchi types must be treated individually by imposing ad-hoc and therefore unjustifiable restrictions on a subset of unknowns. Secondly, anisotropy leads to solutions to which there are no general relativistic counterparts. On the other hand, the cosmological equations of the post-Newtonian approximation are much more in the spirit of general relativity.

On the basis of the aforementioned issues the post-Newtonian approximation seems to be a better approximation of the fully general relativistic theory than the standard Newtonian theory.

Summary

- Standard Newtonian theory is recovered from general relativity as a specialization of the linearized approximation and is studied in detail.
- Newtonian cosmology is Newtonian theory supplemented with the hydrodynamic equations (Bianchi identities).
- It is shown that the Bianchi identities are not obtainable from the field equations.
- Newtonian theory has no well-posed initial value problem when it is applied to cosmology since there are no boundary conditions. This is due to the non-existence of an evolution equation for the Newtonian potential ϕ, and so it is not possible to solve uniquely for ϕ.
- Information seems to be lost in taking the Newtonian approximation. This motivates the need for a higher order approximation.

The results of these aspects of Newtonian theory feature in chapter 4 and are published in Szekeres and Rainsford (1999), [99].

- By keeping higher order terms the Newtonian theory can be reformulated as a theory in which the Bianchi identities are consistent with the field equations. This new theory of order c^{-4} should replace the former standard Newtonian theory since it is the lowest order approximation to be complete.

- However, although consistent, this modified Newtonian theory still does not have a well-posed initial value problem. Thus, it is necessary to go to the post-Newtonian level in order to achieve a physically viable cosmological theory. This new post-Newtonian approximation is based on the reformulation of the field equations as wavelike equations.

  The reformulation of Newtonian theory and the derivation of the post-Newtonian approximation are summarised in chapter 5 and are published in Szekeres and Rainsford (1999), [99].

- The Newtonian theory and the post-Newtonian approximation are then applied to homogeneous cosmologies and are compared against the fully general relativistic case.

- The Newtonian cosmological theory is only able to reproduce the Friedmann-Robertson-Walker $k = 0$ models for the special case of dust because pressure in no way enters the dynamics. Solutions of the Newtonian theory do not vary with the input barotropic equation of state which relates the density and pressure. It follows that Newtonian theory can only describe the matter-dominated case.

- The post-Newtonian approximation, on the other hand, overcomes this difficulty by including the pressure into the dynamics through an extra function of time, $A(t)$. It is then possible to vary the equation of state and have correspondingly varying solutions as in the fully general relativistic case.

The application of the Newtonian theory and the post-Newtonian approximation to the FRW ($k = 0$) models is covered in chapter 6 and published in Rainsford (2000), [89].

- In anisotropic homogeneous Newtonian cosmology it is not possible to reproduce the complete set of Bianchi types that correspond to the nine unknowns - the expansion, shear and rotation. Each of the nine Bianchi types must be treated individually.

- Anisotropic homogeneous Newtonian cosmology predicts solutions to which there are no general relativistic counterparts: Theorems by Ellis state that shear-free dust must be either rotation-free or expansion-free. The rotation-free case is equivalent to the FRW models which already have been explored. The expansion-free case, however, produces models which are singularity free. Theorems of Hawking state that all solutions of general relativity must have a singularity.

- The cosmological equations of the post-Newtonian approximation are more in the spirit of general relativity.

  The results of this work is to be found in chapter 7 and in Rainsford (2001), [90].

- Thus the post-Newtonian approximation seems to be a favourable approximation of the fully general relativistic theory.

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