THE UNIVERSITY OF ADELAIDE

Department of Mechanical Engineering

PROPAGATION AND REACTIVE ATTENUATION OF LOW FREQUENCY SOUND IN HARD-WALLED DUCTS WITH AND WITHOUT FLOW

by

C.R. Fuller, B.E.

Thesis for the Degree of Doctor of Philosophy

July, 1978
SUMMARY

Propagation and reflection of low frequency sound in single and coupled pairs of straight and curved ducts is investigated with and without flow of the propagating medium. The work is divided into three sections.

The first section deals with general propagation theory in straight and curved radial ducts of rectangular cross section with and without uniform flow. The effects of flow on the propagation of energy and the cut-off frequencies of higher modes are investigated experimentally and theoretically. A theoretical explanation in terms of wave number of why the cut-off frequency is independent of direction of propagation of sound relative to the flow is presented. The behaviour of phase velocity and group velocity near cut-on is also considered.

Sound propagation in radial bends of rectangular cross section is investigated using two methods, the first of which utilizes the traditional solution of the wave equation in cylindrical co-ordinates. An iterative method of solution of the characteristic equation is discussed and used to predict the acoustic pressure and particle velocity distributions of two propagating modes. Comparison is made with experimental results and the results of other workers. Good agreement is obtained. The theoretical investigation using cylindrical co-ordinates is limited to the case without flow.

An approximate method of analysis of low frequency sound in
radial bends is developed using conformal mapping techniques. As well as overcoming the need to evaluate complicated expressions of Bessel and Neumann functions, this approach allows theoretical consideration of the effects of flow. Simple equations are developed which predict the angular wave numbers of the (0,0) mode and higher evanescent modes as well as the cut-off frequencies of higher modes with and without flow. The results of the analysis agree with results obtained by other workers using cylindrical co-ordinates solution. The effects of flow on the pressure distribution and cut-off frequencies of higher modes are considered.

The second part of the thesis is concerned with sound generation and propagation in duct systems. Sound generated in a rectangular straight duct with rigid walls by a dipole piston source is theoretically investigated. The pistons are of equal area and fill the cross section of the duct. The characteristic impedance and radiation efficiency of the source is investigated for varying phase angle between pistons. The source is shown to be an extremely good radiator of sound when the pistons are in phase and to radiate no sound power at all below the cut-off frequency of the first cross mode of the duct when the pistons are \( \pi \) radians out of phase.

The effects of a curved axial partition on the impedance of a curved bend are investigated theoretically and experimentally. Whereas previous investigations have established that a curved bend provides negligible discontinuity to acoustic propagation, the presence of a central partition is found to drastically modify the propagational characteristics of the bend resulting in high reflection of sound at a number of discrete frequencies. By contrast, presence of a central partition in a straight duct would have no effect at all below the
cut-off frequency for the first cross mode of the duct.

The third part of the work deals with the development and testing of two reactive attenuators, based upon a principle mentioned by Rayleigh (Theory of Sound, Vol. II, p.63) and attributed to Herschel. Sound propagation in a single duct is caused to split into two parts which travel along separate parallel ducts and when recombined produce non-propagating modes. The sound is thus reflected and the device becomes an effective attenuator in prescribed frequencies. Such attenuators might be used for rigid walled ducts.

The first attenuator is designed to fit into a 90 degree bend in a rectangular duct system and relies on a center body to create an impedance mismatch at the attenuator exit. The center body is shaped to provide a low pressure drop across the device. The performance of the attenuator is theoretically analysed with and without flow. The analysis allows the redesign of the configuration of the attenuator to optimize its performance. An optimum attenuator is developed which provides a transmission loss of at least 10 dB over three quarters of an octave and losses of 30 to 50 dB at a number of discrete frequencies in the three quarters octave frequency range.

Flow is found to substantially reduce the high attenuation obtained at the discrete frequencies but a transmission loss of 10 dB is still obtained over three quarters of an octave for a flow rate of \( M = 0.08 \) in the upstream straight duct. The effects of flow on the design frequency and the pressure reflection coefficient are quantified.

The second attenuator is designed for use in straight ducts of circular cross section and relies on an acoustic delay line to generate a series of evanescent modes and a resultant impedance mismatch at the device exit. The attenuator is investigated
experimentally with and without flow for speeds up to $M = 0.37$. The device is shown to provide higher levels of attenuation and a 10 dB rejection band over an octave for the case without flow. The disturbance to the fluid flow in the main duct is negligible. A theoretical analysis of the delay line attenuator over its operating frequency range is not attempted. However, it may be theoretically described for the very low frequency portion of its range using a lumped circuit analysis. The theory predicts reasonably well the positions of the amplification of sound, measured in the very low frequency range.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statement of Originality</td>
<td>1</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>ii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>iii</td>
</tr>
<tr>
<td>List of Plates</td>
<td>ix</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>List of Symbols</td>
<td>xi</td>
</tr>
<tr>
<td>GENERAL INTRODUCTION</td>
<td>1</td>
</tr>
</tbody>
</table>

**PART I - PROPAGATION THEORY**

**Chapter 1**  
SOUND PROPAGATION IN STRAIGHT DUCTS OF RECTANGULAR CROSS SECTION  
3

1.1 Introduction  
3

1.2 Straight Rectangular Ducts without Flow  
6

1.3 Straight Rectangular Ducts with Uniform Flow  
18

1.3.1 Cut-off Frequencies with Flow  
22

1.3.2 Energy Transmission  
34

1.4 Experimental Investigation - Equipment, Procedure and Results  
39

1.5 Discussion of Results  
52

1.6 Summary  
56

**Chapter 2**  
SOUND PROPAGATION IN RADIAL BENDS OF RECTANGULAR CROSS SECTION  
58

2.1 Introduction  
58

2.2 Cylindrical Co-ordinates Solution without Flow  
63

2.3 Analysis of Sound Propagation in Radial Bends by Conformal Mapping  
76
2.3.1. Derivation of Mapping Equations 81
2.3.2 Sound Propagation in Radial Bends without Mean Flow 81
2.3.3 Sound Propagation in Radial Bends with Mean Flow 95
2.3.4 Range of Applicability of the Conformal Mapping Method 99
2.4 Experimental Investigation - Equipment and Procedure. 100
2.5 Discussion of Results 108
2.5.1. The Cylindrical Solution of the Wave Equation 108
2.5.2 The Conformal Mapping Solution without Mean Flow 112
2.5.3 The Conformal Mapping Solution with Mean Flow 117
2.6 Summary 118

PART II - DUCT SOURCES AND SYSTEMS

Chapter 3 SOUND RADIATION FROM A DIPOLE SOURCE IN A SEMI-INFINITE RECTANGULAR DUCT 120
3.1 Introduction 120
3.2 Analysis 123
3.3 Discussion 148
3.4 Summary 152

Chapter 4 PROPAGATION OF SOUND IN A CURVED BEND CONTAINING A CURVED AXIAL PARTITION 153
4.1 Introduction 153
4.2 Analysis 155
4.3 Experimental Apparatus and Method 159
4.4 Theoretical Predictions 167
4.5 Discussion

4.5.1 Power Reflection Coefficient

4.5.2 Characteristic Impedance

4.5.3 Transmission Loss

4.6 Summary

PART III - REACTIVE ATTENUATORS

Chapter 5 A REACTIVE ACOUSTIC ATTENUATOR

5.1 Introduction

5.2 Design

5.3 Analysis

5.4 Experimental Apparatus and Method

5.5 Theoretical Predictions

5.6 Discussion of Results

5.6.1 Power Transmission Coefficient

5.6.2 Transmission Loss

5.6.3 Curved Duct Radial Pressure Distribution

5.7 General Design Guide of Attenuators

5.8 Summary

Chapter 6 THE EFFECT OF FLOW ON THE PERFORMANCE OF A REACTIVE ACOUSTIC ATTENUATOR

6.1 Introduction

6.2 Analysis

6.3 Experimental Apparatus and Method

6.4 Theoretical Predictions

6.5 Discussion of Results

6.5.1 Pressure Reflection Coefficient

6.5.2 Power Reflection and Transmission Coefficients
6.5.3 Transmission Loss

6.6 Summary

Chapter 7 A DELAY LINE ATTENUATOR FOR USE IN FLOW DUCTS OF CIRCULAR CROSS SECTION.

7.1 Introduction
7.2 Design of Attenuator
7.3 Construction of Attenuator
7.4 Experimental Apparatus and Procedure
    7.4.1 The Standing Wave Apparatus
    7.4.2 The "Suck Down" Flow Rig
    7.4.3 Experimental Procedure
7.5 Analysis
7.6 Discussion of Results
    7.6.1 No Flow of the Propagating Medium
    7.6.2 With Flow of the Propagating Medium
7.7 Summary

Chapter 8 GENERAL CONCLUSIONS

APPENDICES

Appendix 1 The Wave Equation
Appendix 2 Continuity Equations of the Analyses of Chapters 4, 5 and 6.
    2.A Continuity Equations of Interface B of the Analysis of Chapter 4
    2.B Continuity Equations at Interfaces B, C and D of the Analysis of Chapter 5
    2.C Continuity Equations at Interfaces B, C and D of the Analysis of Chapter 6
Appendix 3  Matrices of the analyses of Chapters 4, 5 and 6  

3.A The Matrices of the Analysis of Chapter 4  
3.B The Matrices of the Analysis of Chapter 5  
3.C The Matrices of the Analysis of Chapter 6  

Appendix 4  Reduction of Harmonic Noise Generated by a Centrifugal Fan  

4.A Introduction  
4.B The Skewed Cut-off  
4.C Discussion of Skewed Cut-off Experimental Results  
4.D Impedance Loading of the Fan.  

Appendix 5  Publications  

5.A "A Reactive Acoustic Attenuator".  

5.B "Propagation of Sound in a Curved Bend Containing a Curved Axial Partition".  

5.C "A Low Pressure Drop, Compact, Reactive Attenuator".  

List of References
GENERAL INTRODUCTION

As the demand for faster flow in duct systems increases so does the noise generated by the fan and fluid-duct interaction. Part of the noise generated by the fan and flow is of long wavelength relative to the dimensions of the duct, and the level of this low frequency noise is often unacceptable in such applications as libraries or sound studios. In the usual method of low frequency noise control a system of resonators or lined expansion chambers may be employed, but these devices have the disadvantage of excessive size or large pressure drop and usually operate over a narrow frequency range. Alternatively lining the duct with absorbent material is relatively ineffective at low frequencies and unsuitable for low frequency noise control.

The principal aim of this research work is thus to develop an alternative attenuator suitable for control of low frequency sound in rigid walled ducts. It is hoped to eliminate the stated problems of excessive size and pressure drop as well as narrow operating band. The operation of the attenuator is based on a principle described by Rayleigh and attributed to Hershel. It depends upon the generation of impedance mismatches at planes across the duct surface which then cause reflection of sound. The degree of impedance mismatch is controlled by the degree to which series of evanescent modes are generated at these surfaces. The performance of the attenuator is independent of the downstream terminating impedance, a characteristic which distinguishes it from the usual reactive devices such as an expansion chamber.
Two configurations of the attenuator are investigated; one suitable for use in bends in ducts of rectangular cross section and the other designed for use in straight ducts of circular cross section. Investigation of the effects of flow on the performance of these devices is included in the study.

Necessary to the theoretical understanding of the performance of the reactive attenuator is a detailed understanding of propagation theory in hard-walled ducts with and without flow. Such factors as the effect of flow on the cut-off frequencies of higher modes, sound propagation in curved bends with and without flow and the impedance of a surface whose pressure distribution is approximated by a Fourier sum of evanescent modes are closely related to the performance of the attenuator and are carefully considered in the investigation. A large part of the thesis is therefore dedicated to a theoretical and experimental study of propagation of low frequency sound in hard-walled ducts with and without flow. The remainder is concerned with investigation of the proposed attenuator.