The University of Adelaide
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A Numerical Study of Bluff Body Flow

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Summary

A numerical scheme, based on discret-vortex and surface-vorticity boundary-integral methods, has been developed for simulating time-dependent, two-dimensional, viscous flow over arbitrary arrays of solid bodies of arbitrary cross-section. Flow-induced fluctuating force on bluff bodies immersed in the wakes of upstream bodies are major sources of diopole sound and the motivation was to make possible the calculation of such forces and hence of the aerodynamic noise which they generate. A further objective was to identify the various possible flow regimes which can occur on an array and their dependence on array geometry and to uncover the detailed flow mechanisms of wake-body interaction associated with each of them.

In the analysis, the flow is considered to be started impulsively from rest and its development with time is calculated. The numerical scheme models the natural processes of generation of vorticity at solid boundaries and its diffusion into the flow domain, and the evolution of the vorticity field in the flow domain by convection and diffusion. The time-histories of the distributions of elemental vortices and fluid-dynamic pressures on the bodies are the prime results of the calculations, from which streamline patterns, isovorticity contours, pressure distributions in the general flow field, and Strouhal numbers of vortex shedding are determined.

The underlying principles and development of the numerical scheme, and the implementation procedures for calculation of inviscid-flow over single bodies, viscous flow over single bodies, and finally viscous flow over arrays are presented. In the development of the method, number of refinements to accepted procedures have been made. Amongst these are the introduction of an innovative numerical procedure for calculation of the pressure distribution in the general flow field from the evolution of vorticity field; the use of a modified cell-to-cell algorithm to improve computational efficiency of the discrete vortex method; and the implementation of what has been termed the zero circulation correction, to improve accuracy of the surface-vorticity boundary-integral method.

For validation, the numerical scheme has been applied to the flow over circular cylinders at Reynolds numbers in the range $100 \leq Re_d \leq 10^4$, and thick rectangular plates with chord-to-thickness ratios $C = 1$, $C = 2$ and $C = 4$, at $100 \leq Re_h \leq 10^5$. Extensive comparisons with published experimental data show that the procedure gives a true representation of real flow.
patterns and regimes, and, in general, gives quantitatively accurate predictions of flow parameters. In many cases, the calculations additionally provide fine details of the flow not revealed by and therefore usefully complementing experimental findings. They also draw attention to the effects of three-dimensionality in real flows and to the Reynolds-number limitations that these impose on two-dimensional numerical calculations.

In light of the success of the simulation of the flow over circular cylinders and thick rectangular plates, the procedures have been applied with confidence to array flows, a class of bluff-body flows which has been very much less extensively studied, either experimentally or numerically, than the flow over individual bluff bodies. Simulations of flow over tandem arrays of two rectangular plates with \((C_1 = 1, C_2 = 1)\), \((C_1 = 1, C_2 = 4)\) and \((C_1 = 4, C_2 = 4)\), at a Reynolds number of \(Re_h = 500\), have been made. The broad flow regimes which have been identified, based on the position of impingement on the array of the shear layers separated from the leading corners of the upstream plate, are in accord with the limited available experimental results. More significantly, the fine detail provided by the simulations reveals a progressively changing flow in the gap between the plates as the gap is varied from negligible to very large values. In this progression, a number of characteristic sub-regimes can be identified: periodically-reversing transverse flow through the gap for very small gaps \(G < 0.5\); trapped-vortex flow in the gap for \(0.5 < G < 1\); periodic vortex-formation in the gap, without or with very little vortex convection, for \(1 < G < 2\); and fully-established vortex-street flow within the gap for large gaps \(G > 2\). The nature of the interaction between the downstream plate and vortices shed from the upstream plate varies from progressive loss of vorticity from the gap vortices to the external flow, for \(G < 1\), to vortex-street impingement on the downstream plate for large gaps, \(G > 2\). In the case of gaps large enough to accommodate a fully-established vortex street, the simulations show up the relation between vortices impinging on the downstream plate and vortex formation from the trailing edge of the downstream plate. In particular, they identify the condition under which phase-locking of vortex shedding from the upstream and downstream plates occurs, and, when it does, accurately predict phase differences. Flow parameters, such as Strouhal number of vortex shedding, pressure coefficients and drag coefficients, vary with \(G\), and in most cases, a change in the form of variation indicates a change from one flow regime to another. Overall, the results of the simulations provide detailed insights into the mechanisms of flow over tandem arrays of rectangular plates which it would be difficult or impossible to obtain experimentally.