GROUP-THEORETICAL APPLICATIONS
IN A
TRI-LOCAL MODEL FOR BARYONS

by

A. J. Bracken B.Sc.

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Department of Mathematical Physics,
University of Adelaide,
South Australia.
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SUMMARY

This thesis deals with problems arising in an attempt to develop a tri-local model for baryons.

In this model it is postulated that each free baryon can be described in terms of wave functions having three Dirac bi-spinor indices, and depending on three sets of Lorentz four-vector coordinates.

The basic aim of the model is to exhibit the decomposition of a particular Hilbert space of such functions into $U(3)$ multiplets of unitary irreducible representations of the inhomogeneous Lorentz group, corresponding to multiplets of massive half-odd integer spin particles, with spin constant within each multiplet, but mass varying from one isospin sub-multiplet to another.

$U(3)$ assumes here a "geometrical" significance, rather than appearing as a group acting within an abstract "internal symmetry" space. In particular, transformations generated by the baryon number operator are associated with modified Born reciprocal transformations, and those
generated by the third component of the isospin operator, with cyclic permutation of the three coordinate four-vectors and also of three sets of Dirac matrices.

No attempt is made to describe a system of sub-particles bound together by mediative fields, the view being adopted that the extended space-time structure provides real internal degrees of freedom for the baryon. The associated "internal dynamics" are envisaged as essentially of harmonic oscillator type, and various multiplets as associated with different excited levels.

The attempt to formulate the model is made in what is essentially a "Lorentz basis". An analysis is given of the representations of \( \text{SO}(3,1) \) involved, incorporating a somewhat heuristic treatment of the reduction of unitary representations from the direct-product of infinite- and finite-dimensional non-unitary ones.

A discussion is given of the eigenvalue problem for a "relativistic harmonic oscillator operator" in various spaces associated with different types of representation of \( \text{SO}(3,1) \).
A technique is developed for the resolution of an arbitrary four-vector operator into shift operators for the invariants of the associated representations of $SO(3,1)$. Applications in the theory of the relativistic harmonic oscillator, and in the exhibition of "angular reciprocal transformations", are indicated. A further application to the energy-momentum four-vector leads to the presentation of a "generalized Dirac operator" which provides a reducible representation of certain well-known Gel'fand-Yaglom matrices.

The analogous resolutions of two-, three- and five-vector operators under $SO(2)$, $SO(3)$ and $SO(5)$, respectively, are also given.

A discussion is given of some possible descriptions of massive systems with spin $\frac{1}{2}$ or $\frac{3}{2}$, in terms of wave functions having three bi-spinor indices. This involves the reformulation of the Dirac-Fierz-Pauli (Rarita-Schwinger, Gupta) and Joos-Weinberg descriptions of spin $\frac{3}{2}$ systems in terms of wave equations involving three sets of
Dirac matrices. Some new insight is gained into the relationship between these, the Bargmann-Wigner and the \L{}ubański-Bhabha theories.

A partially successful attempt is made to complete the formulation of the model. Unacceptable features of this attempt are indicated, and the possible nature of further work necessary successfully to complete the formulation is indicated.