HYDROJET

DUCTED PROPULSION SYSTEM

IMPELLER INDUCED VIBRATORY PRESSURES

and

PERFORMANCE CHARACTERISTICS

by

MALCOLM R. HALE B.E.

NOVEMBER 1966

Thesis for the degree of Doctor of Philosophy.
TABLE OF CONTENTS

ABSTRACT

STATEMENT BY THE AUTHOR

ACKNOWLEDGEMENTS

INTRODUCTION

1.0 HYDROJET PROPULSION

1.1 ROTODYNAMIC PROPULSION UNITS 1.1

1.2 PERFORMANCE OF A HYDROJET 1.3

1.2.1 Propulsive Efficiency 1.4

1.2.2 Estimation of Duct Losses 1.6

2.0 IMPELLER DESIGN AND MANUFACTURE

2.1 IMPELLER DESIGN 2.1

2.1.1 Introduction 2.1

2.1.2 Hydrodynamic Model of the Impeller 2.2

2.1.3 Blade Element Properties 2.5

2.1.4 Optimum Design 2.8

2.2 IMPELLER CHARACTERISTICS AND MODEL IMPELLER 2.10

2.3 MACHINING TECHNIQUE FOR THE IMPELLER 2.12

3.0 MODEL ARRANGEMENT AND TESTING PROCEDURE

3.1 MODEL ARRANGEMENT 3.1

3.2 DESIGN OF THE DYNAMOMETER 3.2

3.3 MODEL TESTING PROCEDURE 3.4
4.0 INSTRUMENTS AND APPLIED MEASURING TECHNIQUES.

4.1 SENSING ELEMENTS - TRANSDUCERS
   4.1.1 Torque and Thrust Transducers
   4.1.2 Pressure Transducer
   4.1.3 Event Marker - Transducer

4.2 SIGNAL AMPLIFIERS AND CONDITIONING UNITS

4.3 MATHEMATICAL ANALYSIS OFRecorded INFORMATION
   4.3.1 Pressure Field
   4.3.2 Vibratory Impeller Forces

4.4 SPHERICAL 5-HOLE PITOT AND ITS CALIBRATION
   4.4.1 Theory
   4.4.2 Calibration
   4.4.3 Construction Details

4.5 DYNAMIC CHARACTERISTICS OF THE DYNAMOMETER
   4.5.1 Dynamic Mechanical Coefficients
   4.5.2 Dynamic Hydro-Mechanical Coefficients

5.0 RESULTS OF MODEL EXPERIMENTS.

5.1 PRESSURE ON DUCT SURFACE
   5.1.1 Effect of Filtering the Signal and
       of its Periodicity
   5.1.2 Reynold's Number Effect
   5.1.3 Vibratory Pressures with Uniform Flow
   5.1.4 Vibratory Pressures with Non-Uniform Flow
   5.1.5 Possible Measurement Errors
5.2 IMPELLER CHARACTERISTICS
   5.2.1 Impeller Performance Characteristics 5.20
   5.2.2 Velocity in the Wake of the Impeller 5.21

6.0 CONCLUSIONS
   6.1 IMPELLER INDUCED VIBRATORY PRESSURES 6.1
   6.2 IMPELLER EFFICIENCY 6.2
   6.3 FEASIBILITY OF A HYDROJET 6.3

REFERENCES

FIGURES

APPENDICES
A1. THE DESIGN OF DUCTED IMPELLERS USING A VORTEX LINE ANALYSIS
    AND AN OPTIMIZING COMPUTER TECHNIQUE
A2. COMPUTATION OF RECTANGULAR MACHINING CO-ORDINATES FOR AN
    ARBITRARY IMPELLER DESIGN
A3. MACHINING PROCEDURE FOR THE MODEL IMPELLER
A4. PROPULSIVE EFFICIENCY OF DUCTED PROPULSION SYSTEM
A5. HYDROJET PROPULSION REDUCES VIBRATION
A6. THE ANALYSIS AND CALIBRATION OF THE FIVE-HOLE SPHERICAL PITOT
A7. MOVADAS -- DATA ACQUISITION SYSTEM
A8. COMPUTING PROCEDURE

---0000---
ABSTRACT.

Theoretical and experimental studies were conducted on a Hydrojet ducted impeller system with the aim of developing an efficient and vibration-free propulsion unit for ships. Initial estimates of the propulsive efficiency of the Hydrojet showed this to be a practical means of propulsion and further detailed investigations were undertaken.

A theoretical analysis of an impeller operating in an infinitely long duct was developed by assuming the impeller to be replaced by a simple bound vortex-line. The optimum geometry for an impeller was taken to be that design for which the induced and profile drags had minimum values, the maximum blade stress was equal to the design stress, and the blade sections operated free from cavitation at the design conditions.

Experimental and theoretical values of impeller efficiency showed that values in the order of 0.85 to 0.90 were possible.

Impeller-induced vibratory pressures on the inner duct surface were measured under various impeller loading conditions and different inflow velocity patterns to the impeller. An intense pressure field existed near the impeller plane but its magnitude rapidly attenuated with distance from the impeller. Pressure coefficients $\frac{P}{\rho n^2 D^2}$ in the order of 0.5 were measured at the impeller plane, but values of less than 0.01 existed at distances greater than 0.4 of the impeller diameter (D). The blade frequency harmonic content of the total pressure was extremely large, with the 2nd and 3rd harmonics being approximately 60% and 25% respectively of the 1st harmonic. The phase angle of all harmonics of the induced pressure were constant forward of the impeller and varied linearly with distance behind the impeller. The trailing vortices of the impeller blades appeared to originate near the mid-point
of the blade chord and had a tendency to move forward as the harmonic number increased. The helical vortex line at the duct surface had a constant pitch.

The pressure due to non-uniform flow were approximately dependent on the mean velocity taken over the blade length for a given angular position of the impeller.

In the course of the investigation the following studies were also conducted.

(a) An accurate method of machining impellers was developed. The impeller was machined with a spherical milling cutter which was controlled to move at all times tangential to the desired surface. The position of the cutter on the desired locus was limited to a rectangular grid pattern. A digital computer programme was developed to calculate the co-ordinates of points on the surface of an impeller blade of arbitrary shape.

(b) An analysis and associated technique was developed for the calibration of a five-hole spherical pitot in a flow whose direction is only approximately known.

(c) A complete analogue data-acquisition system was designed and developed capable of recording time-varying data with a flat frequency response from DC to 15 kc. The analogue data was recorded on to magnetic tape which was capable of being digitized at rates up to 12,000 samples per second.
STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of the author's knowledge and belief, this thesis contains no material previously published or written by another person, except where due reference is made in the text.

Malcolm R. Hale

Nov. 1966.
ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to the following individuals and organizations:

Professor H.H. Davis, Dr. J. Mannam and the staff of the Mechanical Engineering Department, University of Adelaide and Professor D.H. Norrie of the Department of Mechanical Engineering, Calgary (formerly of Adelaide) for their guidance, assistance, suggestions and encouragement during this project.

Messrs. R. Schumann, R. Garnham, G. Morgan and H. Bode, who helped with the design and construction of the data acquisition system used for this investigation.

Mr. J. Dunne and his staff of the Computing Electronics Division of Weapons Research Establishment (W.R.E.) Salisbury, South Australia, for their guidance and assistance during the development of the voltage to frequency converters.

Messrs. R. Fitton, R. Truesman and the staff of the Mechanical Engineering workshop, for their co-operation in constructing the models and mechanical components used in this research project.

Messrs. J.H. Fowler of Mechanical Engineering Department and D. Knight of C.S.I.R.O. computing centre for their assistance whilst developing the computer programmes. Miss P. Yates of Mathematical Services Division of W.R.E. for processing the recorded information by converting the analogue data to digital. The use is greatly appreciated of the computing facilities and services of the Commonwealth Scientific and Industrial Research

Commonwealth Government of Australia and the C.S.I.R.O. for providing research scholarships, which the author was fortunate enough to hold during this post graduate study.

U.S. Government Department of the Navy and the David Taylor Model Basin for the financial assistance which made this research possible.
INTRODUCTION

Naval architects and marine engineers are continually endeavouring to improve the propulsive efficiency and power of ships to obtain higher speeds. Unfortunately, the increase in power and speed is accompanied by an increase in the unsteady forces which are created by the propeller and which act on the hull of a ship. The vibrations produced by these forces may reach magnitudes which will have a detrimental effect on the machinery as well as producing considerable discomfort to passengers and crew. Thus designers are faced with the problem of reducing these unwanted forces to more acceptable levels.

Two types of forces are generated:--

(a) bearing forces
(b) surface forces

The former forces are transmitted to the hull through the propeller shaft and bearings. They arise from the propeller operating in a ship's wake which has a non-uniform velocity field. This field is primarily caused by the viscous boundary layer of the hull and to lesser extent by the potential flow field.

The latter forces are created through the interaction of the pressure field with the hull surface. The pressure field surrounding the propeller is dependant on the instantaneous loading on the propeller, on the blade geometry and, in particular, on the blade thickness. The resultant surface force on the hull is governed by the geometrical relationship between the propeller and the surface.

A considerable number of experimental investigations have been carried out to determine comfortable environmental levels of vibration, the levels existing in ships, the magnitude of the forces present, and
the effect the geometry of the afterbody has on these forces. (Ref. 0.1 and 0.2).

The results of these investigations lead to the conclusion that, with conventional hull form and propeller design, it is difficult to reduce the vibration to acceptable levels. It also suggests that it will be necessary to examine other forms of marine propulsion where the propeller-excited vibrations may be reduced and the propulsive efficiency increased. One such system is a ducted propulsion unit.

This thesis deals with a theoretical and experimental investigation into the feasibility of a ducted propulsion system, herein called a "Hydrojet". The Hydrojet consists of an impeller operating in a cylindrical duct of several impeller diameters in length. The following investigation was undertaken:

(a) An estimate of the propulsive efficiency of the Hydrojet.
(b) A theoretical analysis of the impeller.
(c) A model impeller based on the above theory was constructed to a high dimensional accuracy by a new machining technique.
(d) The pressure induced by the impeller operating in a duct were measured at the inner duct surface.
(e) Experimental values of the impeller characteristics were measured and compared with those obtained by theory.
1.1

**HYDROJET PROPULSION**

**ROTODYNAMIC PROPULSION UNITS.**

Hitherto the screw propeller, which is a roto-dynamic device, has been accepted as the conventional propulsion unit for marine vessels. One of the problems encountered with this type of propulsion device is the generation of large fluctuating propeller-induced forces. As already mentioned in the Introduction, extensive investigations have been undertaken to understand this problem and thus make it possible to reduce the vibrations to acceptable levels.

It is known that these vibratory forces can be eliminated if the following two conditions are satisfied.

(a) The magnitude and direction of the forces, acting on the blades, must be independant of time. This will be the case when a rigid propeller operates in a uniform flow field.

On the other hand, if the propeller operates in a non-uniform flow field, the blade section pitch will have to be controlled if the above conditions are to be satisfied.

(b) The net force due to the pressure field acting on the boundaries of the propulsion unit must be invariant with time. This can only be achieved with a rotor having finite number of blades and the boundaries near the rotor being both rigid and symmetrical about the rotor axis.

Based on these requirements, rotodynamic propulsion units can be broadly divided into two groups:

(a) An 'Open' configuration where the solid boundaries near the rotor are effectively outside the intense fluctuating pressure field which is generated by the rotor. Normal screw propulsion falls within this category.
(b) A 'Ducted' unit where the solid boundaries are within the intense pressure field of the rotor. Here the rotor operates in a rigid duct which is substantially symmetrical about the rotor axis near the plane of the rotor and the duct diameter approaches that of the rotor.

Many investigators have studied the open configuration with a view to increasing the efficiency and reducing the vibration levels due to the bearing and surface forces (Ref. 1.1). The factors governing the efficient operation of such a propeller are well known and can easily be adjusted when selecting a propeller for a given duty. The vibratory forces however are not easily controlled and in many instances it is difficult to obtain acceptable levels. Therefore it was decided to carry out a preliminary examination of various types of propulsive systems to see whether it would be possible to reduce the propeller-induced unsteady forces to even lower values.

For any new propulsion system to be suitable for large displacement vessels there are a number of requirements, in addition to those mentioned above (p 1.1), which must be satisfied, viz:--

(a) Propulsive efficiency must be comparable with that of a conventional screw propeller.

(b) Steering power must be adequate.

(c) The ability to maintain satisfactory performance in varying conditions of seaway must be maintained.

(d) The ability to withstand damage in exceptional conditions must be comparable with screw propellers.

The following configurations considered would have inherently low vibration characteristics:--
(a) A conventional screw propeller behind a hull with modified after-body.

(b) Various configurations with the propeller mounted at or forward of the bow.

(c) Various multi-hull configurations in which the propeller or propellers would operate in more uniform wakes than with the conventional screw system.

(d) A submersible hull consisting of a cigar-shaped main body completely submerged with a narrow stream-lined superstructure protruding above the water line.

(e) A hull with propulsive units in pods on outriggers.

(f) Various forms of internal duct system (hydraulic jet propulsion or Hydrojet).

Preliminary studies of the above configurations indicated that the type mentioned in (f) was most suited as a replacement for the screw propeller on large displacement vessels. For this reason it was considered profitable to carry out a more detailed analysis of this type of propulsion system.

1.2

PERFORMANCE OF A HYDROJET.

A Hydrojet or ducted impeller system is one in which the impeller operates in a duct or tunnel of dimensions comparable with the impeller diameter. The duct diameter ($D_d$) would be very nearly that of the impeller ($D$). The length would be of the order of several impeller diameters depending on its application.
In order to develop the desired thrust by increasing the axial momentum of the fluid, there is a contraction at the exit of the duct so that the exit or jet velocity \( V_j \) exceeds that of the intake \( V_I \).

### 1.2.1 Propulsive Efficiency

The fundamental nature of the energy losses in a Hydrojet is extremely complicated. To obtain an estimate of this, the hydrodynamic energy losses are divided into:

(a) Impeller losses.

(b) The losses due to the intake, duct and nozzle.

(c) Loss due to the interaction of hull and duct.

The losses associated with the impeller are accounted for in the impeller efficiency \( \eta_E \). The intake, duct and nozzle losses, termed the duct loss, may be expressed as a fraction of the intake kinetic energy.

\[
\text{Ductwork losses per unit mass flow} = \eta \frac{V_I^2}{2} \quad \ldots \quad 1.1
\]

where \( \eta \) = the loss factor  
\( V_I \) = intake velocity

The hull efficiency \( \eta_H \) takes into account the effect of operating the Hydrojet in the vicinity of a hull (Appendix A4). Hence the propulsive efficiency \( \eta_P \) can be expressed as follows:

(See Appendix A4 for the derivation).

\[
\eta_P = \eta_E \eta_H \frac{2\mu (1 - \mu)}{(1 - \mu^2)(1 - \frac{V_I}{V_J})} \quad \ldots \quad 1.2
\]

where \( \mu = \frac{V_I}{V_J} \) = velocity ratio

and \( V_J \) = velocity at exit of duct
A more useful relationship of $\eta_P$ can be given in terms of the thrust load coefficient ($C_{TL}$) based on the area ($A_o$) and the relative axial velocity ($V_a$) of the fluid at the impeller.

Thus

$$\eta_P = \eta_E \eta_H \frac{4(C_{TL} K_A)}{\left(\frac{T}{\frac{1}{2} \rho V_a^2 A_o}\right)^2 + 4(C_{TL} K_A) + 4 \delta}$$

...1.3

where $C_{TL} = \frac{T}{\frac{1}{2} \rho V_a^2 A_o}$

and $T =$ thrust of the Hydrojet

$K_A =$ ratio of area of impeller annulus to area of duct

Fig. 1.1 shows curves of propulsive efficiency ($\eta_P$) against thrust load coefficient ($C_{TL}$) for different duct loss factor ($\delta$) as expressed by equation 1.3. It will be observed that for a fixed value of $\delta$, the efficiency $\eta_P$ will increase with increasing $C_{TL}$ and reach a maximum; further increasing the $C_{TL}$ results in a decrease of propulsive efficiency ($\eta_P$).

The maximum impeller efficiency ($\eta_{Popt}$) of a ducted propulsion system for a given duct geometry and hence loss factor is:

$$\eta_{Popt} = \eta_E \eta_H \frac{1}{1 + \sqrt{\delta}}$$

$$= \eta_E \eta_H \frac{2}{(C_{TLopt} K_A) + 2}$$

...1.4

This maximum efficiency occurs when

$\mu_{opt} = \frac{1}{1 + \sqrt{\delta}}$ ...

and

$C_{TLopt} = \frac{2 \sqrt{\delta}}{K_A}$
Drawing a line passing through the maximum efficiency ($\gamma_{\text{Popt}}$) of each individual curve of $\gamma$ shows that the $\gamma_{\text{Popt}}$ will increase with decreasing thrust load coefficient and approach unity. Hence for high efficiency the duct loss factor and the thrust load coefficient must be kept as low as possible (Ref. 1.3 and 1.4).

It might be mentioned that below the maximum efficiency ($\gamma_{\text{Popt}}$) line the propulsive efficiency is greatly dependant on the thrust load coefficient ($C_{\text{Tl}}$) and hence the velocity ratio ($\mu$) for a given $\gamma$; above this line, however, $\gamma_P$ is not significantly affected by a change in $C_{\text{Tl}}$.

In order to apply the theoretical results shown in Fig. 1.1 it is necessary to estimate the duct loss factor (\(\frac{C}{\gamma}\)) and the impeller efficiency ($\gamma_E$).

1.2.2 **Estimation of Duct Losses.**

An initial estimate of the hydrodynamic loss can be made by first assuming the impeller does not effect the growth of the boundary layer in the duct. Therefore it is possible to replace the duct by a flat plate, having the same length and area as the inner surface of the duct. It is also assumed that the plate is in a uniform fluid stream having a velocity equal to the intake velocity of the propulsion unit. The loss factor (\(\frac{C}{\gamma}\)), which is the ratio of energy loss to the kinetic energy of fluid at the leading edge, is expressed by the following equation:--

\[
\frac{C}{\gamma} = \frac{\text{energy loss}}{\text{K.E. at inlet}} = \frac{1}{2} \left[ \frac{C_D \rho V_I^2 \left( \pi D_i L \right)}{\rho \left( \frac{\pi D_i^2}{4} \right) V_I^2} \right] V_I
\]

\[
= \frac{4 C_D L}{D_d}
\]

Where $C_D = \text{drag coefficient of the plate}$ and $D_d = \text{diameter of duct}$
and assuming \[ C_D = C_F + \Delta C_D \] \[ \ldots 1.7 \]

where \( C_F \) is given by the ATTC (1947) mean friction line as expressed in the relationship

\[ (C_F) - 0.5 = 4.132 \log_{10} (R_n C_F) \] \[ \ldots 1.8 \]

and \( R_n \) is Reynolds number of the plate based on its length.

Also \( \Delta C_D = 0.001 \) is an allowance for the surface and its coating.

The magnitude of \( \xi \) for varying duct geometry and operating conditions is given in Fig. 1.2. The values are based on the assumption that the density of fluid is 1.99 slugs/cub.ft.

The value of the duct loss factor \( (\xi) \) for a practical arrangement of ducting which has a length-to-diameter ratio of about 5 is, from Fig. 1.2, between 0.004 and 0.006. The corresponding value of the optimum performance ratio \( \frac{\eta_F}{\eta_H} \) from Fig. 1.1 is between 0.81 and 0.84.

The propulsive efficiency of the Hydrojet can be determined, if the impeller efficiency \( \eta_E \) and the hull efficiency \( \eta_H \) are known. An estimate for the impeller efficiency is of the order of 0.90, based on high specific-speed axial-pump data (Ref. 1.5). Assuming a value of 1.0 for the hull efficiency then the value of the propulsive efficiency for a practical Hydrojet system would be between 0.72 and 0.76.
2.1 IMPELLER DESIGN AND MANUFACTURE.

2.1.1 Introduction.

Propeller design procedures in use today, do not attempt to determine the optimum propeller geometry for a given set of operating conditions, apart from one recorded exception given in Ref. 2.1.

The methodical series of propellers are based on an optimum value for one of the major variables and the optimum values for all other design variables are not determined. For example, from the design data for the NSMB (or Van Manen) screw series, the optimum diameter can be chosen and hence the corresponding pitch and mean blade area ratio can be determined to avoid cavitation under the operating conditions. In this series, the blade outline, blade sections, and variation of maximum blade thickness with radius have been previously fixed.

In some of the more theoretical design procedures, it is possible to calculate the circulation distribution, so that the energy loss caused by the induced flow is minimised. It is not possible, however, to determine from these theories an optimum blade shape for strength and for the desired circulation distribution.

The theoretical knowledge of propeller operation has progressed to a stage where an attempt should be made to develop a design technique based on these theories which would determine the
best propeller geometry to suit a given set of operating conditions. A technique of this magnitude would require many mathematical statements and decisions. If such a design is to be economical in both time and cost, the resources of a high-speed digital computer and store are necessary. With the advent of more rigorous and complex theoretical approaches to the design of propellers, a type of optimum design procedure will become necessary in the future, if full advantage is to be taken of the acquired theoretical knowledge.

A programme described in Appendix A 1 was an initial attempt at an optimizing design procedure. Although the design method used in this programme was not the most rigorous, the solutions given by the programme show that optimization could be usefully employed for more complex design theories. The programme was developed to study the feasibility of the Hydrojet propulsion unit.

2.1.2 Hydrodynamic Model of the Impeller.

Since the present research project was directed towards estimating the capabilities of a ducted propulsion system, it was considered that a simplified vortex-line theory could be satisfactorily applied to the design of the impeller.

The following theory is applicable to the design of ducted propellers where the induced circulation around the duct can be neglected. It can also be applied to the design of axial flow pump units.
Consider an impeller operating in a long cylindrical duct in which the fluid can be considered as irrotational upstream of the impeller. It is assumed that the impeller has negligible tip clearance. Thus the impeller diameter equals the duct internal diameter. The finite size boss is assumed to have negligible effect on the induced flow.

It is assumed that the axial velocity profile is uniform across the duct upstream of the impeller.

It is also assumed that for a sufficient distance upstream and downstream of the impeller, the duct is parallel. The impeller design is selected to have no axial component of induced velocity. The velocity diagram is as shown in Fig. 2.1.

It is considered to be sufficiently accurate for the interference flow to be assumed constant circumferentially at any radius. This is equivalent to assuming the interference flow is generated by an infinite number of lifting-lines of variable strength in the radial direction. Using the Betz's criteria for minimum induced energy loss in the wake of an inviscous fluid, and the Kutta-Joukowski relationship, the ideal thrust and torque gradients at any section can be derived as follows:— (see Fig. 2.1)

\[
\frac{dT_i}{dr} = 4\pi \rho \omega^2 r^3 \eta_i (1 - \eta_i) \quad \ldots \quad 2.1
\]

\[
\frac{dQ_i}{dr} = 4\pi \rho \omega V_a r^3 (1 - \eta_i) \quad \ldots \quad 2.2
\]
If it is assumed that the circulation distribution for minimum energy loss is not greatly affected by the variation of profile drag with radius, then the actual thrust and torque gradient can be evaluated as follows:— (see Fig. 2.1).

\[
\frac{dT}{dx} = \frac{dT_i}{dx} \left(1 - \xi \tan \beta_i\right) \quad \ldots \quad 2.3
\]

\[
= K_1 \eta_i (1 - \eta_i) (1 - \frac{\xi}{\delta x \eta_i}) x^3 \quad \ldots \quad 2.4
\]

and

\[
\frac{dQ}{dx} = \frac{dQ_i}{dx} \left(1 + \frac{\xi}{\tan \beta_i}\right) \quad \ldots \quad 2.5
\]

\[
= K_1 \frac{V_a}{\omega} (1 - \eta_i) (1 + \xi \delta x \eta_i) x^3 \quad \ldots \quad 2.6
\]

where

\[
\delta = \frac{\omega E}{V_a} \quad \xi = \frac{C_D}{C_L} \quad \omega = \frac{r}{R} \quad K_1 = \frac{4\pi \rho \delta^4 V_a^4}{\omega^2}
\]

\[
\tan \beta_i = \frac{1}{\delta \eta_i x}
\]

The total thrust \(T\) and torque \(Q\) of the impeller can only be evaluated by summation over the blade length if the drag to lift ratio \(\xi\) is known at each section. Since this ratio depends on the blade profile which is in turn dependent on the strength, cavitation and hydrodynamic requirements, a simple expression for thrust and torque cannot be obtained.
An approximation to the impeller geometry can be obtained by assuming the drag to lift ratio constant with the radius. In this case integration gives the following equations for total thrust $T$ and overall efficiency $\eta_o$.

\[ T = A(1 - \eta_i)(\eta_i B - C) \quad ... \quad 2.7 \]

\[ \eta_o = \frac{\eta_i B - C}{B + \eta_i D} \quad ... \quad 2.8 \]

\[ T = A(1 - \eta_i)(B + \eta_i D) \quad ... \quad 2.9 \]

where

\[ A = \frac{\rho V a^4 \delta^3}{\omega^2} \quad C = \frac{k^3}{3} \xi (1 - K^3) \]

\[ B = \delta (1 - K^4) \quad D = \frac{k^3}{3} \xi \delta^2 (1 - K^5) \]

\[ K = \frac{\text{radius of boss}}{R} \]

2.1.3 Blade Element Properties.

1) Blade element characteristics.

The design lift coefficients of the section ($C_L$) can be expressed as,

\[ C_L = \frac{1}{2} \rho V R^2 \frac{c}{c} d \frac{d}{d} = \frac{8\pi \delta(1 - \eta_i)x^2 R}{2c \sqrt{(x \delta \eta_i)^2 + 1}} \quad ... \quad 2.10 \]

where $c = \text{chord of blade section.}$
The theoretical lift coefficient \( C_{li} \) required to develop the design lift is assumed to be greater than \( C_L \) by a factor \( \mu_m \), the viscosity factor. According to the potential theory of thin wing sections (see Chapter 5, Ref. 2.2), the theoretical lift coefficient of the section is a function of the camber to chord ratio \( \frac{m}{c} \) only if the section operates at shock-free entry conditions, hence

\[
C_{li} = l_m \frac{m}{c} \tag{2.11}
\]

The blade section chosen for this impeller was an NACA-16 thickness distribution with a mean line of \( a = 1.0 \) and the values for viscosity factor \( (\mu_m) \) and lift camber factor \( (l_m) \) at shock-free conditions are given in Appendix A 1, eqn. 18 to 20 and also in Table 5.6, p 175 in Ref. 2.2.

(2) **Blade strength.**

The stresses at a blade section were calculated by the simple theory for bending of a beam as suggested by Tingey (Ref. 2.3, also Ref. 2.2).

It is important to note that this theory can be applied only to designs where chordwise bending due to the pressure distribution over a section can be ignored. This implies that the blades should have relatively large thickness to chord ratio \( \frac{t}{c} \) and not excessively wide chords. The blade geometry chosen for the impeller is consistent with the assumptions of the stress calculation by Tingey.
(3) Cavitation.

Using the theory of thin wings, the cavitation parameter, the "pressure minima cavitation number" $\bar{c}_p$ is determined for the given blade section as follows: (Ref. 2.2 p 209)

$$
\bar{c}_p = \left(1 + 1.14 \frac{t_x}{c} + \frac{c_1}{4}\right)^2 - 1 \quad \ldots \quad 2.12
$$

The sectional cavitation number $\bar{c}_s$ is defined by

$$
\bar{c}_s = \frac{(P_r - e)}{\frac{1}{2} \rho V_R^2} \quad \ldots \quad 2.13
$$

where

$p_r = \text{pressure at blade section radius } r \text{ and at minimum immersion}$

$e = \text{saturated vapour pressure}$

When applying these equations to an actual impeller an overall factor $f_o$ which makes allowance for irregular and viscous flow is introduced to effectively increase $\bar{c}_p$.

Thus

$$
\bar{c}_s \geq f_o \bar{c}_p \quad \ldots \quad 2.14
$$

where $f_o = 1.2$ (Ref. 2.2 p. 209)
A similar method was used by Matthews and Strazzak to estimate the inception of cavitation in screw propeller designs. (Ref. 2.4).

2.1.4 **Optimum Design.**

The optimum combination of blade sections for a Hydro-jet impeller was chosen to be that which satisfies the following:--

(a) The hydrodynamic equations for minimum energy loss.
(b) The lowest possible profile drag providing that:--
(c) The blade section is strong enough to limit the sectional stresses to a value equal to or less than the maximum design stress.
(d) The blades must also operate free from cavitation.
(e) As a consequence of meeting the above conditions, the weight of the impeller will be a minimum for the chosen operating conditions.

Details of the design procedure are given in Appendix A1.

Although the impeller dimensions calculated by this design method lead to an optimum blade section arrangement for the given conditions, it is not necessarily the optimum design for a given duty, for example, for a given thrust (T).

The optimum design must be selected by studying closely, the results of a series of systematically varied impellers, all designed for optimum arrangement and satisfying the requirements of a given duty. Before deciding upon the final impeller geometry, certain other factors
affecting the operation of an impeller or rotor-dynamic propulsion unit must also be taken into account—e.g.

(a) Is the largest diameter impeller that can be installed in the vessel also the optimum diameter?

(b) Is the number of blades and rotational speed conducive to exciting critical modes of vibration when the propulsion system is operating?

(c) What is the economical range of rotational speeds of the prime mover?

The sectional lift coefficient is another important variable which must be studied before selecting the final design.

The design programme given in this report does not achieve the ideal objective. This could be stated as "the selection of an optimum impeller geometry to suit a particular duty by considering every possible arrangement which satisfies all known laws, principles and facts associated with its operation." All these decision points could be inserted into a programme for the logical selection of the ideal impeller, and would require extremely careful planning. Although the complete optimum design is not specified directly by the programme of this report, it is considered that the technique given for selecting an optimum blade geometry is a radical departure from the usual propeller design procedures.
2.2 IMPELLER CHARACTERISTICS AND MODEL IMPELLER.

The impeller design method outlined in the previous section determines the optimum blade dimensions which satisfy the design assumptions and the given set of operating conditions. Unfortunately it is not possible to represent the parameters which govern the impeller characteristics by dimensionless groups, as for each set of operating conditions there is only one arrangement of blade dimensions which satisfy all the design requirements.

A design study was made of a twin Hydrojet propulsion unit to replace an existing single screw propeller on a 19,000 tdw ore carrier. A drawing of the proposed layout is given in the Appendix A5.

The main design requirements of the impeller were

\[ \text{Thrust/Unit} = 44,800 \text{ lbf} \]
\[ \text{Velocity at impeller disc} = 19.3 \text{ fps} \]

To meet the above operating conditions, a number of impellers were designed according to the procedure given in Section 2.1 by varying the impeller radius, rotational speed and blade area ratio. The impeller efficiency \( \eta_E \) for these various designs are shown in Figs. 2.2 to 2.4. The blade section chord was assumed linear with radius, varying from CID at the boss to COD at the blade tip.

A study of these characteristics shows the radius at which the impeller efficiency is a maximum is not significantly affected by blade-area ratio or geometry changes (Fig. 2.2). Similarly, the optimum rotational speed is not greatly affected by the varying impeller geometry (Fig. 2.3) for the fixed impeller diameter and rotational speed. Fig. 2.4 shows the small change in impeller efficiency due to a change in the blade geometry.
The impeller chosen to meet the design conditions was as follows:

| Thrust/Unit | 44,800 lbf |
| Velocity at impeller disc | 19.3 fps |
| Diameter | 15 ft |
| Blades | 4 |
| Rotational Speed | 1.5 rps |
| Expanded Blade Area Ratio | 0.543 |

**Linear chord distribution**

| Tip chord | 5.0 ft |
| Boss chord | 3.0 ft |
| Chord section | NACA-16, a = 1.0 |

**Theoretical efficiency of**

| the impeller | 0.87 |
| Thrust load coefficient | 0.688 |

An 8 inch diameter model of this impeller is shown in Fig. 2.5 and Fig. 2.9 and a list of dimensions of the impeller is given in Table 4 of Appendix A1.

The value of the theoretical impeller efficiency compares favourable with that taken in Section 1.2.2 as an initial estimate.
2.3 MACHINING TECHNIQUE FOR THE IMPELLER.

Manufacturing technology, in general has progressed rapidly over the past decade, but it has not significantly affected the machining techniques used to construct model or prototype propellers.

Changes in manufacturing processes are necessary if accuracy is to be increased and production time decreased. Accuracy is important where measured values of a variable from model experiments are to be used to predict the prototype values or are to be compared with theoretical values.

The common method of machining an impeller or propeller is to use a cutter which moves (relative to the blade) on a cylindrical path with centre on the axis of the impeller. The cutter motion is controlled by a follower moving over a series of templates which may be either cylindrical or expanded-cylindrical sections depending on the mechanism used to convert movement of the follower to the cutter.

Another method, which is suited for the majority of milling machines, is the spot-machining of points on the blade surface in either a polar or a rectangular grid pattern. This method requires considerable computation especially when cartesian co-ordinates are used, because the object is naturally defined by polar co-ordinates.

In all the machining techniques commonly employed it is usual to use only a small number of sections to define the complete blade shape. Hand machining is thus necessary to "fair in" between the machined regions.
Disadvantages of the above methods are:

(1) The time taken to hand "fair in" between the accurately machined sections.

(2) The blade surface can only be as accurate as the templates.

(3) The accuracy is dependant on the size of the cutter since a correction should be made to the templates to define the locus of the cutter moving over a blade surface at the desired section. Because the template shape is usually that of a blade section, an appreciable error is introduced unless the cutter has a cutting edge radius small in comparison with the radius of curvature of the blade surface at the point being considered.

The ideal method of machining an impeller is by an automatically-controlled milling machine using a magnetic tape as an input medium for all machining instructions and control. The technique used to manufacture the model impeller associated with this research project, could be adapted to generate the above machining instructions.

Appendix A 2 gives a method of determining the co-ordinates of points on the locus-surface generated by the centre of a spherical cutter which would machine the surface of an arbitrarily defined impeller blade. The points are obtained in a rectangular grid pattern of a predetermined dimension.
The 8 inch diameter model impeller, Fig. 2.9, (scale ratio of 22.5) was machined in a Pantograph copying machine (Fig. 2.7 and 2.8) from a three-dimensional master template. This master template was four times larger than the model and was machined in a universal miller (Fig. 2.6) to dimensions calculated by the programme in Appendix A 2.

The master templates were used to increase the accuracy of the final impeller and to reduce the number of machining co-ordinates necessary to obtain the required accuracy.

The machined points were placed 0.25 of an inch in the radial direction and either 0.125 or 0.025 of an inch in a direction normal to it, depending on the rate of curvature of the surface at the point being machined. To define the blade surfaces 6550 points were used, i.e. 3275 points for each blade surface. The spherical cutter used to machine the master templates was 1.25 of an inch in diameter.

The accuracy of the finished model impeller was within $\pm$ 0.001 of an inch. The accuracy of computation can be selected and depends on the size of the model and master templates.

A brief description of the machining technique used is given in Appendix A 3.
Investigations were conducted to determine the impeller characteristics and surface pressures on the duct of a simple two-dimensional Hydrojet model as shown in Fig. 3.1. This model was tested in the 18 inch diameter Research Water-Tunnel of the Department of Mechanical Engineering.

The model consisted of several plane cylindrical sections of 8 inch internal diameter, to which was attached a contoured entrance section and a converging exit nozzle. The duct from intake to exit was 4.625 impeller diameters in length and the impeller was located in a plastic section 2.75 diameters from the intake. The plastic section of the duct surrounding the impeller had a blade tip clearance of 0.0028 impeller diameters.

The entrance section was a portion of an NACA 0015 basic thickness profile with an NACA 250 mean line. (Ref. 3.1, Nozzle 19 in Table 2). The exit nozzle had an area ratio \( \frac{A_J}{A} \) of 0.745 which corresponds to the design thrust load coefficient of the impeller as expressed in equation 1.5 of Section 1.2.1.
The pressure transducer (Section 4.1.2) could be mounted in any one of a series of locations placed on a plane parallel to the duct centre-line.

The other basic component of the model Hydrojet was the impeller dynamometer and drive shaft assembly. The drive shaft which housed the dynamometer was supported by bearings aft of the duct. Some difficulty was met with the design of these bearings because the impeller, dynamometer and drive shaft acted as a torsional pendulum and was excited by the slightest change in resistive torque of the bearing. However, the vibration of the dynamometer was reduced to workable levels by firstly inserting a carbon liner into the bearing and secondly by heavily damping the bearing supports. The damping was provided by lead weights which were attached to the supports and which were free to vibrate as dynamic absorbers.

### DESIGN OF THE DYNAMOMETER

The design requirements for the dynamometer were as follows,

(a) The new system had to fit into the existing drive shaft without modification.

(b) The system had to measure the mean values of torque and thrust.

(c) The fluctuating components of the torque and thrust had to be measured.

The arrangement finally selected for the dynamometer is shown in Fig. 3.2 and 3.3.

The bearing which carried the impeller was elastically connected to the drive shaft by a thin-walled cylindrical sensing element. This allowed relative motion between the shaft and impeller and at the same time effectively supported the impeller.

A tapered-land hydrostatic bearing (Ref. 3.2) was chosen because the change in eccentricity of the bearing and journal is very small for a
change in the bearing load. Thus the damping properties of the bearing were very nearly constant and the damping was viscous.

As the impeller was effectively supported by the bearing, the design of the sensing element was controlled by the maximum loading conditions of the impeller, providing the natural frequencies of the dynamometer were also acceptable. Also another design requirement of the sensing element was that its strain levels under normal operating conditions had to be large enough to be detected by the measuring instruments.

The characteristics chosen for the dynamometer were as follows:--

<table>
<thead>
<tr>
<th></th>
<th>THRUST</th>
<th>TORQUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal max. loading - (Limited at present by electronic instrumentation)</td>
<td>100 lbf</td>
<td>200 lbf ins</td>
</tr>
<tr>
<td>Max. Design Loading</td>
<td>400 lbf</td>
<td>350 lbf ins</td>
</tr>
<tr>
<td>Natural Frequency with impeller in air</td>
<td>2000 cps</td>
<td>370 cps</td>
</tr>
<tr>
<td>Sensitivity of dynamometer and amplifier (Sect. 4.2)</td>
<td>0.031 volts/lbf</td>
<td>0.043 volts/lbf.in</td>
</tr>
<tr>
<td>Sensing element dimensions</td>
<td>1.300 in. dia.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.300 in. length</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.030 in. wall thickness</td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>Cast Monel.</td>
<td></td>
</tr>
</tbody>
</table>

The forces acting on the dynamometer were detected by a fully active Wheatstone bridge circuit incorporating resistance wire strain gauges supplied with a DC voltage.

Because the sensing element initially was open to the surrounding water, the gauges were waterproofed by a thin coating (0.050 ins.) of a silicone rubber compound. This proved to be unsatisfactory because the moisture content gradually increased over a period of several weeks to a level that upset the balance of the D.C.
pre amplifiers in the recording instrumentation. (Appendix A7.). For this reason an alternative method was used, which consisted of forming an enclosed space above the strain gauges with a thin rubber diaphragm (Fig. 3.3). The space was then filled with silicone oil to exclude the entry of water into the space. This arrangement was an improvement on the previous method.

Unfortunately after completing the model tests discussed herein it was found that a small amount of water had entered the cavity and that the silicone oil had affected the cement used to bond the strain gauges to the sensing element. During the tests however, the dynamometer behaved satisfactorily.

3.3

MODEL TESTING PROCEDURE.

Tests were conducted to determine the following:-

(a) The fluctuating pressures acting on the inner surface of the duct near the plane of the impeller for,
   (i) Uniform flow at the impeller.
   (ii) Non-uniform flow at the impeller.

(b) Impeller Performance Characteristics.
   (i) Steady state values for thrust and torque.
   (ii) Velocity in the wake of the impeller.

The above characteristics were investigated at both 15 and 20 cps of the impeller, for various advance coefficients \( J_I \), where

\[
J_I = \frac{V_I}{nD}
\]

\( V_I \) = duct velocity at intake (fps)

\( n \) = impeller rotational speed (cps)

\( D \) = impeller diameter (ft)

When the model was tested under non-uniform inflow conditions, a series of wire-mesh patches, which formed a velocity wake inducer
(Ref. 3.3), were located forward of the duct intake. The complete wake inducer could be rotated during these tests about a longitudinal axis of the duct. This enabled the complete pressure distribution on the duct surface to be measured by only one transducer which could be moved axially along the duct.

Since it is known that the induced vibratory pressure of an impeller is periodic in relation to impeller rotation, and that it has a fundamental frequency equal to the blade passing frequency (i.e., rotational speed of the impeller times the number of blades), it was decided to express the pressure by a Fourier Series.

As facilities were available to process recorded analogue data into digital information, the Fourier components were determined analytically. The design and construction of a high-speed magnetic tape recording system with a Frequency Modulated (FM) recording mode was thus undertaken and is discussed in the following Section 4.0 and also in Appendix A7.
4.1

INSTRUMENTS AND APPLIED MEASURING TECHNIQUES.

The design and construction of much of the instrumentation for measurement and data recording was undertaken for this research project.

The recording mode selected for this project was F.M. (frequency modulated) to I.R.I.G. specifications.

The recorded F.M. analogue signal was converted to digital information before analysing the data on a digital computer.

The recording instruments can be divided into the following basic groups:

(1) Sensing elements - transducers
(2) Signal amplifiers and conditioning units
(3) Recording system - high-speed tape recorder

A simplified block diagram of the instrumentation is given in Fig. 4.1. Details of each unit and its calibration are given in the following sections and Appendix A7.

4.1

SENSING ELEMENTS - TRANSDUCERS.

4.1.1 Torque and Thrust Transducers.

Torque and thrust transducers were mounted in the dynamometer, details of which are given in Section 3.2.

The impeller which was supported on a hydrodynamic tapered bearing was elastically attached to the drive shaft via a thin-walled cylindrical tube, or strain shell.
The sensing elements, resistance wire strain gauges, were cemented to the strain shell in the directions of the principal stresses, and were connected to form a fully-active Wheatstone bridge circuit which was sensitive to either axial or torsional loading.

The strain gauges in the torque and thrust bridges consisted of four 600-Ω Phillips strain gauges, PR 9812 with a nominal gauge factor of $K = 2.0$.

A direct current (D.C.) strain-gauge system was adopted, and the signals were voltage amplified in D.C. pre-amplifiers attached to the flywheel of the impeller drive shaft.

Monel-metal slip-rings with silver carbon brushes were used to transmit the signals to MOVADAS, the recording system.

4.1.2 Pressure Transducer.

The pressure transducer used was a Statham PM 222 TC + 5-200 Bi-directional Differential Pressure Transducer which had a nominal output of 370 μV/psi/volt and a maximum excitation voltage of 3 volts. The diaphragm of the transducer which was 0.25 of an inch in diameter, was flush mounted as an external surface of the transducer. Because of its construction, the transducer had to be enclosed in a water-proof holder with only the sensing diaphragm and the signal leads open to the atmosphere. It was found necessary, during the tests, to apply an internal pressure to the transducer diaphragm in excess of the static pressure at that particular point to ensure no possible entry of water into
the holder.

Before and after each series of measurements, the transducer was statically calibrated. The following is a brief specification of the transducer.

<table>
<thead>
<tr>
<th>Table 4.1.</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE TRANSDUCER SPECIFICATION.</td>
</tr>
<tr>
<td>Model</td>
</tr>
<tr>
<td>Pressure Range</td>
</tr>
<tr>
<td>Transduction</td>
</tr>
<tr>
<td>Normal Bridge Resistance</td>
</tr>
<tr>
<td>Excitation</td>
</tr>
<tr>
<td>Nominal Output</td>
</tr>
</tbody>
</table>

4.1.3 Event Marker - Transducer.

This simple transducer, namely a photo-electric cell was arranged to give a voltage spike once per revolution at a known position of the impeller. This voltage signal, after amplification, triggered a monostable multivibrator to generate a square pulse which was recorded, in F.M. form, on a track of the tape-recorder.

In the subsequent mathematical analysis, the phase of the Fourier components of the wave were determined with respect to the Event Marker signal.
The Units comprising the Event Marker are discussed in Appendix A 7.

4.2

**SIGNAL AMPLIFIERS AND CONDITIONING UNITS.**

The signal amplifier and conditioning units form a data acquisition system which is called MOVADAS (Modulated Voltage Analogue Data Acquisition System).

MOVADAS consists of the following units (see Fig. 4.1.).

1. D.C. voltage pre-amplifiers and mean-reading instruments.
2. High-pass filter
3. Driver amplifier
4. Voltage to Frequency (V-F) Converter
5. Programming Switch

Details of these units are given in Appendix A 7.

In order to obtain a signal capable of driving the Voltage to Frequency (V-F) Converter, the voltage output of the transducers were firstly D.C. amplified using balanced push-pull stages to gain stability and a satisfactory common mode rejection ratio. High-pass filtering was necessary to remove tunnel noise before further amplifying the AC components and converting the signal to a modulated frequency in the V-F Converters.

Prior to recording the desired F.M. signal on the tape, the programming switch inserted binary coded pulses to identify the
type of data and the data set or number. Calibration signals were also fed into the V-F converter to give an instantaneous calibration of the converter. The identification and calibration signals were applied to all active data channels simultaneously so that the resultant digital information could be cross correlated between channels.

The F.M. signals in the range $54 \text{ Kc} \pm 40\%$ were recorded at 60 ips. The analogue tape was later converted to digital information at a rate of 4000 samples per second of actual recording time.

The major difficulties encountered while developing the measuring instrumentation are as follows:--

(1) **D.C. Pre-amplifiers.**

Because the voltage gains of the torque and thrust amplifiers were approximately 2500 and 4000 respectively, any small voltage change on the input side of the amplifier caused large signal changes at the output.

Originally, the rotating pre-amplifiers and strain gauges were supplied with power from external batteries via slip-rings. This arrangement was found to be unsatisfactory because:--

(i) Changes in brush to slip-ring resistance caused the strain gauge supply voltage to fluctuate.

(ii) Thermal emf's were generated on the slip-rings.

(iii) Power leads cutting stray magnetic fields generated small voltages,
The noise was cut to acceptable limits by attaching to the rotating amplifiers, the complete power-supply unit, including batteries and a voltage regulation circuit.

The output from the amplifiers were the only signals transmitted through the slip-rings.

(2) Mean Reading Instrument.

Large fluctuations in the thrust and torque signals were recorded after the D.C. pre-amplifiers. It was not possible to record consistent mean readings on a normal volt-meter.

A consistent average was obtained by counting the number of cycles on the output of the V-F converter over a period of 10 seconds. For this test, the high-pass filter was removed from the normal recording circuit.

It was thought that the large fluctuations in the signals were caused by non-uniform flow at the impeller.

(3) High-pass Filter.

The level of the background tunnel noise was approximately equal to the magnitude of the maximum pressure signals due to the impeller. A frequency analysis of the total pressure showed that most of its components were below 60 cps. The major component of the pressure noise occurred between 30 and 35 cps.
The noise was due to changing static pressure in the working section caused by:

(i) Average flow rate changing with time.
(ii) Vibrations transmitted to the working section and water tunnel structure from the drive motors, pumps and auxiliary units.

Since the frequency of the major component of the noise was below the blade passing frequency, filtering was adopted. After filtering out 50 cps and below, the pressure signal showed properties of a stationary periodic wave (refer Section 5.1).

The simple filter used did not have a sharp cut-off frequency, as it was only a single stage T filter. It was thus necessary to measure the complete transfer function of the filter and correct the measured signal. A typical plot of the transfer function components of one of the filters is shown in Fig. 4.2.

4.3

MATHEMATICAL ANALYSIS OF RECORDED INFORMATION.

After analogue to digital (A-D) conversion, the recorded digital information was identified and converted into words compatible to the word structure of the digital computer, CDC 6400 which was used in subsequent analyses (see Appendix A 7 and A 8 for details).
The list of digits which constituted a single experimental reading of one variable was termed a file. In each file there were a number of revolutions of the impeller, depending on the recording time. It was possible to choose from a complete file a list of numbers which corresponded to any one or more of the recorded revolutions of the impeller. This was possible because the identification pulses, from the programming switch, were recorded simultaneously on all active channels.

A Fourier analysis of the recorded signals was performed by considering the points as constituting a series of pseudo square waves whose Fourier coefficients can be expressed mathematically. The Fourier coefficients of the complex wave were determined by summing the coefficients of the pseudo square waves (see Ref. 4.1 and Appendix A 8).

The orientation of the reference axes used to describe the position of the impeller or a point on the duct surface is as follows:

Consider a right-handed set of axes OX, OY, and OZ with origin on the impeller axis and axes OY and OZ in the plane of the impeller (see Fig. 5.1). The plane YOZ was taken in these model studies to be the plane which contains the centroids of all the blade sections. (Section 2.3 of Appendix A 1 shows this to be a property of the impeller design).

The OX axis is in the same direction as the axial displacement of the impeller with respect to the fluid. The OY axis is
taken as the reference line for all angular measurements. The angular position of a given blade from the OY axis is $\Theta$ and the position of a point in space is given by the coordinates $X, r, \gamma$.

4.3.1 **Pressure Field.**

The Fourier components of the pressure fluctuations at a point on the duct surface $X, R_d, \gamma$ (see Fig. 5.1) were reduced to the following form, where $R_d$ is the duct radius,

$$ P = \sum_{i=1}^{\infty} A_i \cos (4\pi(\Theta - \gamma) + \varepsilon_i) $$

where $A_i =$ modulus of pressure component of the $i^{th}$ harmonic of blade frequency, i.e. single amplitude of vibratory pressure

$\Theta =$ blade angle with respect to axis, Fig. 5.1

$\varepsilon_i =$ phase-lead angle of $i^{th}$ harmonic

The magnitude of the pressures are also given in the form of non-dimensional coefficients $K_{Pi}$ and $K_{Pt}$ where

$$ K_{Pi} = \frac{A_i}{\rho n^2 D^2} \quad \text{and} \quad K_{Pt} = \frac{P_t}{\rho n^2 D^2} $$

where $P_t =$ single amplitude of total (all frequencies) vibratory pressure, peak to peak

$n =$ propeller revolutions per second

$D =$ propeller diameter

$\rho =$ density of fluid
4.3.2 Vibratory Impeller Forces.

The direct measurement of fluctuating forces acting on an impeller propulsion system is extremely difficult because the forces must be determined from a knowledge of the vibrations of the propulsion system. This requires the complete dynamic properties of the impeller, drive and supporting system to be known. A further complication arises, however, due to a fundamental property of an impeller. The various modes of vibration of a propulsion system are cross coupled by the impeller.

The dynamics of this type of system are investigated, in part, in Ref. 4.4. Here a theoretical analysis is undertaken of an elastically supported impeller being excited by a sinusoidal gust velocity. Experimental values of the dynamic coefficients mentioned in this report are given in Ref. 4.5.

Consider a simple, elastically supported impeller vibrating about its longitudinal axis (ie. axial and torsional). The complete transfer function of an impeller-mass-spring system is not only dependent on the mechanical properties of the mass-spring system, but also on the geometry of the impeller.

The equations of motion describing this coupled system can be expressed in the following:

\[
\begin{bmatrix}
\ddot{\xi}_Z M_Z + \dot{\xi}_Z C_Z + \xi_Z C_F \\
\ddot{\xi}_Z F_Z + \dot{\xi}_Z F_Z + \dot{\varphi}_Z F_Z + \ddot{\varphi}_Z F_Z + \varphi_Z C_F Z
\end{bmatrix} = F_Z
\]

...4.8
\[
\begin{align*}
\left[ \dddot{\varphi}_Z \frac{T_Z}{\varphi_Z} + \dddot{\varphi}_Z \frac{T_Z}{\varphi_Z} + \varphi_Z \frac{C_T}{Z} \right] & + \\
& \left[ \dddot{\varphi}_Z \frac{F_Z}{\varphi_Z} + \dddot{\varphi}_Z \frac{F_Z}{\varphi_Z} + \dddot{\varphi}_Z \frac{F_Z}{\varphi_Z} \right] = T_Z \\
& \left. \right\} \text{... 4.9}
\end{align*}
\]

The first group of terms in both equations refer to an elastically supported impeller vibrating in air. These coefficients are called the dynamic mechanical coefficients. The second group of variables refer to the hydrodynamic and cross coupling properties of the impeller system, the dynamic hydro-mechanical coefficients. The definitions of the above quantities are given below in Table 4.2.

Combining the second group of variables in equations 4.8 and 4.9 as:

\[
\begin{align*}
\left[ \dddot{\varphi}_Z \frac{T_Z}{\varphi_Z} + \dddot{\varphi}_Z \frac{T_Z}{\varphi_Z} + \varphi_Z \frac{C_T}{Z} \right] & + \\
& \left[ \dddot{\varphi}_Z \frac{F_Z}{\varphi_Z} + \dddot{\varphi}_Z \frac{F_Z}{\varphi_Z} + \dddot{\varphi}_Z \frac{F_Z}{\varphi_Z} \right] = \\
& \left. \right\} \text{... 4.10}
\end{align*}
\]

and assuming that differential equations 4.8 and 4.9 are linear, with constant coefficients, they can be expressed as:-
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_Z$</td>
<td>Longitudinal displacement</td>
<td>ft.</td>
</tr>
<tr>
<td>$\varphi_Z$</td>
<td>Angular displacement</td>
<td>rad.</td>
</tr>
<tr>
<td>$F_Z$</td>
<td>Axial load - thrust</td>
<td>lbf.</td>
</tr>
<tr>
<td>$T_Z$</td>
<td>Torsional load - torque</td>
<td>lbf.ft.</td>
</tr>
<tr>
<td>$M_Z$</td>
<td>Effective mass of vibratory system in air</td>
<td>slug</td>
</tr>
<tr>
<td>$C_Z$</td>
<td>Damping in longitudinal direction</td>
<td>lbf/sec/ft.</td>
</tr>
<tr>
<td>$C_F$</td>
<td>Stiffness in longitudinal direction</td>
<td>lbf/ft.</td>
</tr>
<tr>
<td>$I_Z$</td>
<td>Effective inertia of system in air</td>
<td>lbf.ft.sec$^2$</td>
</tr>
<tr>
<td>$G_Z$</td>
<td>Damping in torsional direction</td>
<td>lbf.ft.sec.</td>
</tr>
<tr>
<td>$C_Z^T$</td>
<td>Stiffness in &quot; &quot;</td>
<td>lbf.ft/radian</td>
</tr>
<tr>
<td>$C_Z^{FT}$</td>
<td>Cross coupling stiffness between $\varepsilon_Z$ and $\varphi_Z$ displacements</td>
<td>u</td>
</tr>
<tr>
<td>$C_Z^{TF}$</td>
<td></td>
<td>lbf/ft.</td>
</tr>
</tbody>
</table>
Table 4.2 continued.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{F_Z}{\dot{\psi}_Z} )</td>
<td>Velocity coupling</td>
<td>lbf·sec.</td>
</tr>
<tr>
<td>( \frac{F_Z}{\ddot{\psi}_Z} )</td>
<td>Acceleration coupling</td>
<td>lbf·sec^2</td>
</tr>
<tr>
<td>( \frac{F_Z}{\dddot{\psi}_Z} )</td>
<td>Axial damping</td>
<td>lbf·sec/ft</td>
</tr>
<tr>
<td>( \frac{T_Z}{\dot{\psi}_Z} )</td>
<td>Axial entrained mass</td>
<td>slugs</td>
</tr>
<tr>
<td>( \frac{T_Z}{\ddot{\psi}_Z} )</td>
<td>Torsional damping</td>
<td>lbf·ft·sec</td>
</tr>
<tr>
<td>( \frac{T_Z}{\dddot{\psi}_Z} )</td>
<td>Entrained moment of inertia</td>
<td>lbf·ft·sec^2</td>
</tr>
</tbody>
</table>

**Dynamic Hydro-mechanical Coefficients** (Ref. 4.4, Table 1.3)
\[
\left[ \frac{1}{H_E} + \frac{1}{H_H E} \right] \varepsilon_Z + H_4 \varphi_Z = F_Z \quad \ldots \quad 4.12
\]
\[
\left[ \frac{1}{H_\varphi} + \frac{1}{H_H \varphi} \right] \varphi_Z + H_3 \varepsilon_Z = T_Z \quad \ldots \quad 4.13
\]

Here \( H_i \) represent the transfer function of a particular section \( i \) of the vibratory system

where

\( i = \varepsilon \) simple longitudinal system in air

\( i = \varphi \) simple torsional system in air

\( i = H E \) hydrodynamic system for longitudinal displacement

\( i = H \varphi \) hydrodynamic system for torsional displacement

and \( i = 3, 4 \) coupling between longitudinal and torsional displacement

A further simplification can be made by substituting

\[
\frac{1}{H_1} = \frac{1}{H_E} + \frac{1}{H_H E} \quad \ldots \quad 4.14
\]

and

\[
\frac{1}{H_2} = \frac{1}{H_\varphi} + \frac{1}{H_H \varphi} \quad \ldots \quad 4.15
\]

into equation 4.12 and 4.13

\[
F_Z = \frac{\varepsilon_Z}{H_1} + H_4 \varphi_Z \quad \ldots \quad 4.16
\]

\[
T_Z = \frac{\varphi_Z}{H_2} + H_3 \varepsilon_Z \quad \ldots \quad 4.17
\]

This permits the impeller-mass spring system and associated hydrodynamic effects to be illustrated by a block diagram as follows:
The actual impeller loading can be calculated from measured displacements $E_z$, $\varphi_z$ by equations 4.16 and 4.17 once the respective transfer functions have been determined either mathematically or experimentally.

It was intended that the fluctuating forces acting on the impeller of the Hydrojet model would be measured in these studies, but this was not possible because of a faulty dynamometer (Section 3.2).

An outline of the method used to calibrate the dynamometer is given in Section 4.5.
Until now, the calibration of the 5-hole spherical pitot was conducted in a fluid in which the relative direction of flow was known accurately. This necessitated the use of a towing tank, in which the relative motion of carriage and water was precisely known, or the use of a very accurate wind or water tunnel.

A method was devised to accurately calibrate a spherical pitot in a flow whose direction is only approximately known. Calibration is thus possible in a small wind tunnel, water tunnel or duct.

The calibration procedure and theoretical analysis which is given in detail in Appendix A 6, is briefly discussed in the following section.

4.4.1 Theory.

The potential solution of the flow around a sphere gives the pressure at a point as:--

\[
\frac{p - p_0}{\frac{1}{2} \rho v^2} = 1 - \frac{9}{4} \sin^2 \beta
\]

\[\ldots 4.3\]

where
\begin{align*}
    p &= \text{pressure at point considered} \\
    p_0 &= \text{free stream pressure} \\
    V &= \text{magnitude of velocity vector } \vec{V} \text{ of free stream} \\
    \rho &= \text{fluid density} \\
    \beta &= \text{angular position of point from stagnation point}
\end{align*}
For three equispaced holes a, b, c on a great circle of a sphere Pien (Ref. 4.2) has shown that (see Fig. 4.3) --

\[
C_{ph} = \frac{p_a - p_b}{\frac{1}{2} \rho V_h^2} = \frac{9}{4} \sin 2\alpha \sin 2\beta_h \quad \ldots \quad 4.4
\]

\[
C_{Ph} = \frac{p_a - p_b}{2p_c - p_a - p_b} = \frac{\sin 2\alpha}{1 - \cos 2\alpha} \tan 2\beta_h \quad \ldots \quad 4.5
\]

where

- \( p_a, p_b, p_c \) = pressures at a, b, c.
- \( C_{ph}, C_{Ph} \) = pressures coefficients
- \( \alpha \) = angle between adjacent holes
- \( V_h \) = orthogonally projected component of the velocity \( \vec{V} \) onto the plane of the great circle
- \( \beta_h \) = angle between \( V_h \) and the stagnation point

From measurements of \( p_a, p_b, p_c \) the value of \( \beta_h \) can be calculated from equation 4.5, whence \( V_h \) can be obtained from equation 4.4. If on the great circle through c orthogonal to that through a, b, c, two equispaced holes a' and b' are placed as shown in Fig. 4.3, the component \( V_v \) of the velocity \( \vec{V} \) in the plane of this circle can be determined from measurement of \( p_a', p_c', p_b' \) by equations analogous to 4.4 and 4.5. The three components \( V_x', V_y', V_z' \) along the sphere axes 0X, 0Y and 0Z of the velocity \( \vec{V} \) are then given by:--
\[ V_x = V_h \cos \beta_h \]
\[ V_y = V_h \sin \beta_h \]
\[ V_z = V_v \sin \beta_v \]

In Ref. 4.2 and 4.3 in place of equation 4.4 the appropriate relations for

\[ \frac{p_c - p_a}{\frac{1}{2} \rho V_h^2} \quad , \quad \frac{p_c - p_b}{\frac{1}{2} \rho V_v^2} \]

are used. The first or second of these pressure coefficients is taken depending on whether \( V_h \) is on one side or the other side of the centre hole. This introduces an unnecessary complication. Use of equation 4.4 is also recommended because its pressure coefficient has a greater variation at any given value of \( \beta_h \) than these other two coefficients, thus giving a greater sensitivity.

4.4.2 Calibration.

4.4.2.1 Initial Angular Calibration.

The pitot is set up in a flow whose direction is approximately known, so that it can be rotated about two axes \( OY, OZ \) which are normal to each other and to the nominal flow direction. \( OY \) and \( OZ \) pass through the sphere centre and lie approximately in the plane of the holes. The axis \( OX \) also passes through the sphere centre and is orthogonal to \( OY \) and \( OZ \). The axes \( OX, OY, OZ \) are
are called the pitot axes.

Values of the expression for $C_{ph}$ and $C_{ph}$ versus $\beta_h$ and $C_{pv}$ and $C_{pv}$ versus $\beta_v$ are obtained by rotating the pitot axes OY and OZ respectively.

The error introduced by the flow velocity $\vec{V}$, not being in the calibration plane will be quite small since the velocity in the calibration plane will differ from $V$ only by $V(1 - \cos\phi)$ where $\phi$ is the angle of $\vec{V}$ to the calibration plane. Usually $\phi$ will not be more than a few degrees.

4.4.2.2 Orientation of Pitot Axes with reference to Datum Instrument Axes.

On the pitot base there will have been machined location faces. These can be used to define a set of orthogonal axes at the sphere centre, which will be called the datum instrument axes $OXX$, $OYY$ and $OZZ$. The problem is now to determine the orientation or position of the pitot axes with reference to these known instrument axes. As shown below, this orientation can be determined in a stream whose flow direction is only approximately known, by three inversions of the sphere position, providing the pitot is set up in the flow so that the sphere can be rotated about $OXX$, $OYY$ or $OZZ$ without the centre of the sphere moving in space.

(1) Rotation about $OXX$ Axis.

The sphere is initially positioned so that the flow vector $\vec{V}$ is at some estimated angle $\varphi$ to the centre hole (of between $5^\circ$ and $10^\circ$). The sphere is then rotated
successively by $90^\circ$ about the instrument axis $OXX$.

By solving equations 12 to 15 of Appendix A 6 the orientation of the pitot axis $OX$ with respect to the instrument axes can be determined.

(2) **Inversion about $OYY$ axis**

The sphere is positioned so that the velocity $\mathbf{V}$ will make an angle $\theta = 5^\circ$ with the $XXY$ plane as shown in Fig. 4.4. Since the flow direction is known approximately, an estimate of the value of $\theta$ can be made. The sphere is then rotated about $OYY$ so that $\mathbf{V}$ will make an angle of approximately $-\theta$ with the $XXY$ plane. Using values of the pressures at the initial and final positions, the orientation of pitot axis $OZ$ with respect to the instrument axes can be determined (by solving equations 24 and 25, Appendix A.6).

(3) **Inversion about $OZZ$ axis**

By repeating the procedure outlined in the section above, but in this case, for inversion about the $OZZ$ axis, the orientation of $OY$ axis can be determined.

4.4.2.3 **Calibration Assumptions.**

The initial calibration referred to in section 4.4.2.1 is based on an assumption, that the flow velocity is in the calibration plane. The accuracy of calibration can be increased by repeating the calibration procedure now that the fluid direction with respect to the instrument can be calculated.
Another assumption is that the pressure relationship between the holes is only dependent on the velocity component which can be considered as existing in the respective plane of calibration and that the pressure distribution is independent of the magnitude of the velocity component in the other calibration plane.

This is true for a perfect sphere having infinitely small holes which lie on a great circle through the centre hole.

The calibration of this instrument should include a check on its accuracy for fluid velocities whose directions are not in the calibration plane.

The velocity gradient of the free stream is also assumed to be small over the area of cross section of the sphere.

4.4.3 Construction Details.

The diameter of the spherical head was chosen to be 0.375 of an inch. An angular distance between the centre hole and the side holes of 20° and a hole diameter of 0.024 of an inch was selected.

The major difficulty in constructing an accurate spherical pitot is to accurately position the pressure tapping holes in the sphere. The following method of manufacture was used to partly alleviate this problem.

(1) The head was rough machined to a cylindrical form.

(2) This blank was mounted on a precision vertical drilling machine and pressure tapping holes were drilled parallel to the axes of the pitot head.
These holes were positioned so that a point on their centre line would be 20° to the axis of the spherical head when the head was machined spherical to 0.375 of an inch in diameter.

(3) Small bore stainless steel tubes were inserted into the holes and soldered in place.

(4) The head was accurately machined spherical.
DYNAMIC CHARACTERISTICS OF THE DYNAMOMETER.

Before a dynamic calibration could be undertaken, the linearity of the
dynamometer had to be proven for various loading combinations of thrust
and torque. This test was conducted, and the dynamometer and associated
recording instruments were shown to be linear with load.

The dynamic properties of an impeller-dynamometer system, as discussed
in Section 4.3.2, can be broadly divided into the determination of —
(a) The dynamic mechanical coefficients
(b) The dynamic hydro-mechanical coefficients

4.5.1 Dynamic Mechanical Coefficients.

A common method of determining the transfer function of a dynamic
system is to excite the system with a sinusoidal force and measure the
overall response of the system. This method can be applied in nearly all
cases, but certain restrictions and experimental procedures must be met,
depending on the physical properties of the system and the force exciter.

However, the only possible way of applying an exciting force to the
dynamometer, (Section 3.2) was by an external force exciter. If the
connection of this force exciter to the dynamometer, was to have a
negligible effect on the actual transfer function of the latter, then the
mass, stiffness and damping of the exciter had to be small in comparison
with that of the dynamometer.

This was true for the axial mode but in the torsional mode, which had
a low stiffness and natural frequency, this situation did not exist. In
this case, the dynamometer-plus-excitier system had to be considered as a
compound system, each component of which had its own transfer function.
Here the torsional transfer function of the dynamometer was obtained by a vectorial subtraction of the transfer function for the exciter from that of the exciter plus dynamometer system. This method of determining the vibration characteristics of a compound system is discussed in Ref. 4.6.

(a) Transfer Function of the Exciter.

The force exciter, which was used to determine the transfer functions, was an electro-magnetic loud-speaker coil assembly.

Two methods can be used to determine the transfer function of electro-magnetic exciters. The experimental method was adopted for these studies in preference to a theoretical derivation. This required connecting the exciter to a known vibratory system and measuring the complete transfer function of the combined system.

A simple cantilever beam was selected as the known vibratory system. Its natural frequency and stiffness was chosen to be approximately equal to that of the dynamometer.

A certain portion of the mass of the exciter appeared to behave as a point mass loading on the beam. The magnitude of this mass was determined from a knowledge of the natural frequency of the system. Knowing this and the physical properties of the beam, the transfer function for the beam and effective mass loading was calculated. Now the vibratory characteristics of the exciter can be obtained by vectorial subtraction of the transfer function of the beam from that of the combined system.

It was found that the exciter had only a small influence on the combined system and that it behaved as an extra mass loading.
(b) Transfer Function of the Dynamometer in Air.

Tests, similar to those above, were conducted to determine the dynamic mechanical coefficients of the dynamometer as expressed in equation 4.9. of Section 4.3.2.

The dynamometer-plus-excite system was considered as a compound system, as discussed previously.

The mass or inertia added to replace the impeller during these tests was equal to that of the impeller.

4.5.2 Dynamic Hydro-Mechanical Coefficients.

It was anticipated that the following tests would be undertaken, but as previously mentioned, a fault in an electrical component of the dynamometer terminated these.

The values of dynamic hydro-mechanical coefficients were to be computed theoretically (Ref. 4.4) and compared with those determined experimentally from a knowledge of the free vibrations of the impeller-dynamometer when immersed in water (Ref. 4.7).

It is worthwhile to mention that the theory of Wereldsma (Ref. 4.4) is based on an unsteady two-dimensional airfoil theory presented by Von Karman and Sears (Refs. 4.8 and 4.9). This theory may be better adapted to ducted impellers than open water propellers, because in a ducted impeller the radial velocities and the velocity gradients over the blade length are small.
5.1 RESULTS OF MODEL EXPERIMENTS.

5.1 PRESSURE ON DUCT SURFACE.

The reference axis used to describe orientation of the impeller and the position of a point \( P (X, r, \gamma) \) is shown in Fig. 5.1. The right handed set of axes \( OX, OY, OZ \) with origin on the impeller axis has the axes \( OY \) and \( OZ \) in the reference plane of the impeller.

The reference plane, in this case, is the plane which contains the centroids of all the blade sections.

The direction of the \( OX \) axis is opposite to the fluid motion. All angular measurements are taken from the \( OY \) axis. The angular position of the impeller is given by \( \Theta \) (Fig. 5.1). The coordinates of a point on the duct surface are \( X, R_d, \gamma \).

The induced fluctuating pressures \( p \) measured on the duct for the uniform in-flow conditions were taken at a fixed angular position of \( \gamma = 0 \) and various axial distances \( (X) \). The pressures at these points were expressed by the following equation:

\[
p = \sum_{i=1}^{\infty} A_i \cos \left[ \frac{\pi}{4} (\Theta - \gamma + \epsilon_i) \right] \tag{5.1}
\]

where \( A_i \) = magnitude of the \( i \)th harmonic of blade frequency pressure (i.e. single amplitude of the vibratory pressure).

\( \Theta \) = impeller blade angle with respect to the axes in Fig. 5.1.

\( \gamma \) = angular position of the point being considered (Fig. 5.1).

\( \epsilon_i \) = phase-lead angle of \( i \)th harmonic component.

For the uniform flow conditions, the relationship above applied to all values of \( \gamma \) on the same axial section at \( X \), because \( A_i \) and \( \epsilon_i \) were independent of \( \gamma \).

For the non-uniform in-flow to the impeller, however, the pressure
measurements were taken for various values of $\gamma$ and $X$. The pressures were again expressed by equation 5.1, but for this type of pressure distribution the values of $A_1$ and $E_1$ were dependent on $\gamma$.

The test results are presented in a non-dimensional form which has been justified by other investigators (Ref. 5.1 and 5.3).

$$K_{Pi} = \frac{A_1}{\rho n^2 D^2} \quad \text{and} \quad K_{Pt} = \frac{P_t}{\rho n^2 D^2} \quad \ldots 5.2$$

where $K_{Pi}$ = pressure coefficient of $i^{th}$ harmonic of blade frequency.

$K_{Pt}$ = Pressure coefficient of total pressure.

$P_t$ = half the peak to peak value of the total pressure (all frequencies) ($\text{lbf/ft}^2$).

$n$ = impeller rotational speed (rpm).

$D$ = impeller diameter (ft).

$\rho$ = density of fluid ($\text{slug/ft}^3$).

5.1.1 Effect of Filtering the Signal and of its Periodicity.

The pressure measured by the transducer had two components. The periodic component in Fig. 5.2 was due to the induced pressure field of the impeller and the lower frequency component, apparently non-periodic, was generated by various external mechanisms remote from the impeller-duct system. The latter component had major frequencies between 30 and 35 cps. This was due to the excitation of the working section and supporting structure of the water tunnel at its natural frequency. The vibration was excited by force transmission through the pipe work from the main pumps and drive motors.
Because the level of the unwanted pressure fluctuations above 50 cps approached the noise level of the recording amplifiers, it was decided to select the operating conditions of the impeller so that the fundamental frequency of the induced pressure exceeded 50 cps. The impeller rotational speeds were thus chosen to be 15 and 20 cps, which corresponded to a fundamental frequency for the fluctuating pressure of 60 and 80 cps respectively.

It was decided to filter out the unwanted pressure component electronically, because without the use of a filter, it was difficult to study the induced pressure at distances greater than \( \frac{X}{D} = 0.15 \) where the noise to signal ratio exceeded 2. With the unwanted signal component removed it was possible to monitor the impeller induced pressure signal prior to recording. The recording accuracy was also improved because the signal due to the induced pressure was capable of driving the recording instruments over their full operating range.

A Fourier analysis of the filtered pressure signal was performed on the digital information obtained from the recording instruments. Over a period of 8 revolutions of the impeller, various samples of the recorded information were analysed and compared with those obtained from an analysis over the 8 revolutions. Fig. 5.3 shows the results of the Fourier analysis where the pressure coefficient for various samples (◻, ○, △) are compared with the average pressure coefficient (▽) at the different harmonics of blade frequency. The pressure coefficients in Fig. 5.3 for the samples were computed from digital information between the NPI\(^{th}\) and the (NPI + NP)\(^{th}\) revolution of the impeller. The values of NPI and NP for the samples are given in Fig. 5.3.
The results given in Fig. 5.3 show remarkable agreement with each other for the various harmonics of the blade frequency. Thus the pressure signals detected were stationary. A frequency analysis of the measured pressure is given in Fig. 5.4 for an impeller rotational speed of 20 cps. It can be seen that the pressure is periodic with a fundamental frequency of 80 cps. As this corresponds to the blade passing frequency, the results of Figs. 5.3 and 5.4 show that the pressure fluctuations were stationary and periodic.

Although the computed Fourier components were not significantly affected by the sample length, the results discussed in the following sections were the average components taken over 8 revolutions of the impeller for the uniform flow condition, and the average over 4 revolutions in the non-uniform flow conditions.

5.1.2 Effect of Reynolds Number.

The Fourier components of the pressure, namely the pressure coefficient and the phase angle, for impeller rotational speeds of 15 and 20 cps are given in Figs. 5.8 and 5.12 for various values of advance coefficient ($J_r$).

The values for the measured pressure components are independent of the rotational speed. Thus it is assumed that the Reynolds number, at least over the small range applicable to model impellers, has no effect on the induced vibratory pressures of a ducted impeller.

It has also been shown (Ref. 5.1) that the free-space pressures of model open-water propellers are independent of Reynolds number.
5.1.3 Vibratory Pressures with Uniform Flow.

The pressure on the duct was measured for three advance coefficients \( J_I \) equal to 0.74, 0.80 and 0.85 where

\[
J_I = \text{intake advance coefficient} = \frac{V_I}{nD} \quad \text{where} \quad V_I = \text{duct velocity at intake (fps)}
\]

\[
n = \text{impeller rotational speed (cps)}
\]

\[
D = \text{impeller diameter (ft)}
\]

The test results are presented in Figs. 5.5 to 5.17 and are discussed under the following headings:

(a) Vibratory pressure of the blade frequency harmonics.
(b) Peak-to peak values of the total pressure.
(c) Phase angle of the blade frequency harmonics.
(d) Phase angle of the maximum pressure.

(a) Vibratory Pressure of the Blade Frequency Harmonics.

The magnitude of the Fourier components of the induced vibratory pressure acting on the duct surface is shown in Figs. 5.5 to 5.7 for the various advance coefficients.

The axial pressure distribution of the ducted impeller system shows similar characteristics to the free space pressures of an open-water propeller, (Ref. 5.2) but the absolute value of the pressure is far greater. The attenuation in magnitude with axial distance from the impeller is also greater than that of an open-water propeller.

The major pressure region of a Hydrojet extends over a relatively small distance between + 0.3 to - 0.1 of the impeller diameter.
This length is comparable with the projected length of the impeller blade in the axial direction of $\frac{X}{D} = +0.043$ to $-0.048$. The magnitude of the pressure outside this region is of the same order as the pressure which exists at much larger tip clearances ($\frac{r}{D} = 0.6$ to $0.7$) for an open-water propeller.

It is known (Ref. 5.2 and 5.3) that the pressure distribution near an open-water propeller in a uniform velocity field is dependent on,

(i) the blade loading

(ii) the blade thickness

The pressure distribution associated with blade loading is symmetrical about the propeller, while that attributed to the blade thickness effect is asymmetrical. The effect of this same asymmetrical pressure distribution was detected for the ducted impeller. (Figs. 5.5 to 5.8). For this reason, the pressure due to the blade thickness effects must have a significant influence on the resulting vibratory pressures of a ducted impeller.

The magnitude of the pressure coefficients ($K_{pi}$) for the higher harmonics are large in relation to the fundamental. Values of the $K_{pi}$ for the second and third harmonics were approximately 60% and 25% respectively of the fundamental value. This large harmonic content is not present in the near pressure field of an open-water propeller with the same number of blades. (Fig. 21 of Reference 5.1).

For a ducted impeller however the blade frequency harmonics are important and must be considered if the peak to peak amplitudes
are to be measured or calculated.

It can also be seen (Figs. 5.5 to 5.7) that for a given blade frequency harmonic, the axial distance at which the maximum pressure occurs moves forward as the harmonic number increases. The phase angle of the harmonics also reveal a forward shift of the harmonics with increasing harmonic number (Fig. 5.9 to 5.11). A discussion of this is left to a following section (c).

(b) Peak to Peak Values of the Total Pressure.

In Fig. 5.8 the magnitude of half the peak-to-peak pressure coefficient is given for the total pressure at various advance coefficients. The peak-to-peak values of the total pressure were determined by synthesizing the actual total pressure from its Fourier components.

As the blade frequency harmonics are very nearly in phase behind the impeller and are definitely in phase forward of the impeller plane, (see following section) the peak-to-peak values of the total pressure (Fig. 5.8) are approximately the sum of the pressures for the odd harmonic components.

(c) Phase Angle of the Blade Frequency Harmonics.

The blade frequency pressure harmonics lead the impeller blade position by an angle $\xi_{i}$, where $i$ is the harmonic number (eqn. 5.1). Therefore, the maximum pressure of the harmonic components on the duct surface are at an angle $\xi_{i}$ to the impeller blade position in the direction of the angular motion of the impeller.

The determination of the phase angle of any experimentally
derived signal is difficult, and this was the case with the measured pressure signals (Section 5.15). After carefully studying the results and relating them to the hydrodynamics of an impeller, the following interpretation of the results is given.

It can be seen from Figs. 5.9 to 5.17 that the pressure field consists of two separate regions, one forward and the other behind the impeller. Near the plane of the impeller there is a discontinuity in phase of the induced pressure.

(i) **Forward of the Impeller**, the phase angle of each blade frequency is very nearly independent of axial distance \( \frac{x}{D} \) and of advance coefficient of the impeller. This is clearly shown in Figs. 5.9 to 5.11 where the results for \( J_I = 0.74 \) are given. The pattern of variation of the phase angle for the higher harmonics and higher advance ratios is not so well defined but the results do appear to follow the same trend as the lower harmonics.

As previously mentioned on p.5.6, the induced pressure is dependant on the blade thickness and on the blade loading. For an open-water propeller, the effect of blade thickness has been estimated from a source-sink distribution over the blade surface (Ref. 5.2). Also the pressure due to blade loading is effectively the pressure distribution of an array of doublets whose axes are perpendicular to the helicoidal surface swept out by the advancing blade. Thus the phase angle of the blade thickness pressure component at any axial position is constant with respect to the blade position while
the phase angle due to the blade loading component is
dependant on the distance from the propeller.

For a ducted impeller, however, the blade thickness must
control the total pressure field ahead of the impeller because
the phase angle of the pressure harmonics are constant with
distance.

It has been shown by Breslin (Ref. 5.2, Figs. 2a to 3b)
that the blade thickness effect controls the magnitude of the
pressure field forward of a propeller. However, only limited
evidence has been put forward for the phase relationship of
the pressure in this region. Other investigations (Refs. 5.3
and 5.4) have indicated that there is a region forward of a
propeller where the phase is substantially independant of
distance from the impeller.

However, the phase angle of the harmonic components of
the pressure for the ducted impeller ceases to be constant as
the blade frequency harmonic number increases. The phase
angle for the 3rd blade frequency harmonic, shown in Fig. 5.11
decreases with axial distance from the impeller. The
pressure component due to blade thickness effect in this case
must have a reduced influence on the total pressure field.
(ii) In the Region of the Impeller Plane, the trailing vortices or blade loading effect suddenly affects the pressure field and the phase angle of the Fourier components show a discontinuity (Fig. 5.9). The point at which this discontinuity occurs is not constant but moves forward slightly with increasing blade frequency harmonic, (Figs. 5.9 to 5.11). As the maximum pressure peak also moves forward with increasing blade frequency harmonic, the effective location of the bound vortices emanating from the impeller blades would appear to move forward.
(iii) **Aft of the Impeller.** Although the phase angle \( \varepsilon_i \) of the harmonics components (Figs. 5.9 to 5.15) are for values between 0 and \( 2\pi \), they may also be expressed by the addition of multiples of \( 2\pi \), i.e. \( \varepsilon_i \pm 2\pi n \) where \( n = 1, 2, 3, \ldots \). This is possible because the fluctuating pressures are periodic over \( 2\pi \) radians. For this reason, the value of phase angles in the above figures are not discontinuous at 0 and \( 2\pi \), but the values do, in fact, represent a continuous function.

**Aft of the impeller,** the phase angles of the pressure harmonics decrease linearly with axial distance from the impeller plane (Figs. 5.9 to 5.15). The phase change is in the same direction as the helicoidal wake pattern generated by the impeller. This indicates that the pressure field is mainly governed by the pressure associated with the blade loading, i.e. the pressure due to the trailing vortices. The trailing vortices have approximately constant pitch equal to that of the impeller. It is expected that the pitch of the trailing vortices should be constant in this ducted impeller case, because previous investigators have shown theoretically and experimentally that they are constant for most open-water propellers.

It is difficult to determine from the results a relationship between the phase angle of various harmonic components after the impeller. The indication is that they are not in phase (Figs. 5.9 to 5.11). This may be due to --
The effective location of the bound vortex lines moving forward.

Inaccuracies in measurement and computation of the phase angle.

With increasing advance coefficient $J_I$, the phase angle of the pressure has a tendency to be continuous from forward of the impeller to well aft (Fig. 5.16). For this to occur, the pressure field due to the trailing vortices must be insignificant in comparison with that due to the blade thickness. This is possible because at $J_I = 0.85$ the impeller loading, which in fact controls the magnitude of the pressures associated with the trailing vortices, is extremely small compared with the loading value at $J_I = 0.74$ (Fig. 5.25).

**Phase Angle of Maximum Pressure.**

The phase angle of the maximum pressures shown in Figs. 5.12, 5.15, and 5.17 are very nearly equal to that of the first blade frequency harmonic, because there is little variation in phase angle between the harmonics.

**5.1.4 Vibratory Pressures with Non-uniform Flow.**

Pressure measurements were taken with the impeller operating in a non-uniform flow. The velocity distribution at the intake of the duct and forward of the impeller is shown in Figs. 5.18 and 5.19. From these figures it can be seen that the velocity distribution
is effectively unchanged from the intake to the impeller.

Measurement of the fluctuating pressure on the duct was taken at only one advance coefficient $J_{\text{avg}} = 0.74$. The operating conditions were selected to maintain the same mass flow through the duct as in the previous test at $J = 0.74$ for a uniform flow condition. It would have been better to maintain a constant thrust loading condition between these two tests, but this was not possible due to the malfunction of the dynamometer. The pressure was measured at various angular positions between $\gamma = 0$ and $\frac{\pi}{2}$ (Fig. 5.1) because the velocity pattern was basically symmetrical about two normal diametral axes.

A frequency analysis of the pressure revealed that only blade frequency harmonics were significant as was the case for the uniform flow condition. It was difficult to determine a relationship between the pressure and the wake velocity from the limited number of experimental readings. However it can be said that the pressure at a given angular position ($\gamma$) is dependant, to the first approximation, on the average velocity over the radial line to that position.

The pressure near the impeller plane between $\frac{x}{D} = 0.15$ and -0.10 is influenced by the velocity distribution (Fig. 5.20 to 5.22). Outside the region the pressure is substantially constant and equals that for the uniform flow condition. In contrast to this, the pressure field of an open-water propeller operating in a wake is greatly dependant on the wake pattern even at extremely large distances from the propeller (Ref. 5.4, Fig 4). It has been stated by Breslin (Ref. 5.4) that the "non-uniform inflow velocities persist to larger distances since the signal associated with the non-uniform conditions
decays more slowly than the corresponding signal associated with the uniform conditions."

This statement, however, was based on a limited number of calculations at a considerable distance from the propeller where \( \frac{r}{D} = 0.6 \). The assumptions of this theory may not apply to a region extremely close to a ducted impeller.

On the other hand the phase angle of the pressure on the duct at various axial distances was not dependant on the wake pattern at the impeller (Figs. 5.23 and 5.24).

5.1.5 Possible Measurement Errors.

Factors which affected the accuracy of the measured pressure components were --

(a) Recording instrumentation and analysis.
(b) A finite size pressure transducer.
(c) Static pressure drop across the model.

(a) Recording Instrumentation and Analysis.

Although the analogue pressure signal and the event signal were recorded simultaneously on different tracks of a magnetic tape (Appendix A7), the sampling of the analogue signal and subsequent conversion to digital information was executed for each track in turn. Thus the digital data from each track was within one sample of the true event. When comparing information from two tracks, the phase relationship between them may be in error by up to \( + 1 \) sample in the total number of samples per period of the event. As the analogue to digital conversion was 4000 samples per second
for all the tests, this corresponded to $\pm 1.8$ degrees of phase shift at 20 cps of the impeller. Unfortunately the phase error in the harmonics of the blade frequency was considerably greater and equal

$$\pm 1.8 Z i \text{ degrees,} \ldots 5.3$$

where $Z =$ number of blades (4)
and $i =$ blade frequency harmonic number.

This represents a significant phase error for the higher harmonics. It should be noted that the magnitudes of the pressure components are not affected by this inaccurate phase relationship.

The accuracy of the recording instrumentation and subsequent analogue to digital (A-D) conversion was extremely high. The voltage to frequency (V-F) converters were linear to 0.5% of the correct value. For an input voltage of $\pm 1.4$ volts to the V-F converters, i.e. the normal input voltage range, the digital information after A-D conversion had a range of approximately 700 units. Also, at 4000 samples per second, which was the digital sampling rate for these tests, there were at least 8 digital samples per cycle of the 6th blade frequency harmonic. Thus the computational errors due to aliasing were small even for the higher harmonics that were considered.

(b) **Pressure Transducer.**

The measured signal from the transducer was not a true 'point measurement' but was in fact a reading which was dependant on the pressure distribution on the diaphragm of the transducer. As
previously discussed, the pressure distribution in the region of
the impeller showed considerable change in both magnitude and phase
with axial distance and it is in this region the accuracy of the
measured signal was doubtful.

In order to estimate analytically the accuracy of the recorded
pressure, the transducer diaphragm is assumed to be a uniform
circular plate, rigidly supported at the edge. Also the measured
signal is assumed to be proportional to the deflection of the
centre of the plate.

Consider the deflection of the plate due to the following
pressure distributions acting on it.

(i) Linear pressure variation across the plate.

(ii) Fluctuating pressure with a linear change in its
magnitude and phase across the plate.

(i) **Linear Pressure Variation.**

Consider a linear pressure variation from $P_1$ to $P_2$
across the disc.

\[
\begin{array}{c}
\text{The deflection of a simply supported plate under the}
\text{above loading conditions is given in Ref. 5.5 (p.256 eqn(5)).}
\text{There the deflection of the plate at the centre is depen-
\text{dant on the mean pressure i.e. } \frac{1}{2} (P_1 + P_2) \text{ and not on the}
\end{array}
\]
distribution, providing it is symmetrical.

This solution will also apply to the centre of a plate with fixed edges as above.

Forward of the impeller, the pressure on the transducer was approximately linear with constant phase. Therefore the measured components of the pressure in this region are effectively point measurements. However, the pressure measurements behind the propeller where large variations in magnitude and phase were detected, are inaccurate as shown in the following.

(ii) Linear Change in Magnitude and Phase on the Plate.

Consider a slowly varying sinusoidal pressure acting on a plate with the same boundary conditions as for (i) above. The magnitude and phase of this pressure is assumed to vary linearly across the disc. The deflection at the centre of the disc \(w\) due to this pressure \(P\) acting on a small unit area \(dx\) by \(dy\) is given in Ref. 5.5, p.290 viz.

\[
w = \frac{P}{8\pi D} \left( b^2 \log \frac{b}{a} + \frac{a^2 - b^2}{2} \right)
\] ...5.4
Assuming the pressure is linear with distance $x$
i.e. between $A_1$ and $A_2$, the magnitude and phase are constant.
The resultant deflection ($w'$) of the centre of the plate
due to the pressure loading from $A_1$ to $A_2$ is;

$$w' = \frac{P}{8 \pi D} \int_{A_1}^{A_2} \left( \frac{b^2}{a} \log \frac{b}{a} + \frac{a^2 - b^2}{2} \right) dx dy \quad \ldots 5.5$$

where $D$ = flexural rigidity of the plate.
Substituting $b = B \sec \theta$ and $y = B \tan \theta$ into equation 5.5
and evaluating between $A_1$ and $A_2$

$$w' = \frac{PB^3}{4\pi D} \left[ \sqrt{\left( \frac{a^2 - B^2}{B^2} \right)} \left( \frac{2a^2}{9B^2} - \frac{8}{9} \right) + \frac{2}{3} \arccos \frac{B}{a} \right] dx \quad \ldots 5.6$$

Assuming a phase distribution similar to that measured
aft of the impeller, then the response of the transducer
can be determined by an integration in the $x$ direction.

From Fig. 5.10 the change in phase for a change in
$X/D$ of 0.031 (i.e. dimension of the transducer) is approxi-
mately 0.68 for the region aft of the impeller.

A numerical solution of equation 5.6 with the above
distribution of pressure was obtained. Under these condi-
tions the deflection of the centre of the plate was reduced
by 4.8% and the phase lead by 1.8 degrees compared to a
uniform pressure distribution equal to $\frac{1}{2}(P_1 + P_2)$ and no
phase change across the plate.

Thus the size of the transducer does affect the accuracy
of the pressure measurements in a region aft of the impeller.
Here the recorded pressures of the Hydrojet should be
increased by approximately 5%.

(c) **Static Pressure Drop Across the Model.**

A large pressure drop was experienced along the length of the model during the tests. This resulted from the model being situated in a bounded fluid, where the model duct diameter was large in comparison with the diameter of the working section of the water tunnel. However, it was assumed that the fluctuating pressure induced by the impeller was not affected by the change in static pressure along the model. Thus the vibratory pressure on this model can be assumed to equal that of a model which operates in an infinite fluid.
5.2

IMPELLER CHARACTERISTICS.

5.2.1 Impeller Performance Characteristics.

Unfortunately the values of thrust and torque coefficients given in Fig. 5.25 may be significantly in error. As mentioned in Section 4.1.1 the voltage signals from the dynamometer were amplified by the rotating D.C. pre-amplifiers before the voltage signals were measured. Due to the extremely large voltage gain of these amplifiers, there was considerable voltage drift during the measurement of the thrust and torque signals. The zero readings of the amplifiers taken before and after each experimental reading showed a voltage drift equivalent to approximately 10% and 2% of the measured values of thrust and torque respectively. In determining the values of torque and thrust, the drift was assumed linear with time. However, assuming the experimental values given in Fig. 5.25 are true values, then the experimental values are lower than the theoretical values at the design advance coefficient as shown below,

<table>
<thead>
<tr>
<th>Intake Advance Coefficient $J_I$</th>
<th>Theoretical</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{V_u}{nD}$</td>
<td>0.82</td>
<td>0.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Thrust Coefficient $K_T$</th>
<th>0.198</th>
<th>0.123</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{T}{\rho n^2 D^4}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Torque Coefficient $K_Q$</th>
<th>0.0311</th>
<th>0.0204</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{Q}{\rho n^2 D^5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{J_I K_T}{2 \pi K_Q}$</td>
<td>0.834</td>
<td>0.77</td>
</tr>
</tbody>
</table>

| Impeller Efficiency (at $J_I = 0.82$) | 0.834 | 0.77 |

| Maximum Experimental Impeller Efficiency | --    | 0.95 |

| Impeller Efficiency (at $J_I \approx 0.74$) | 0.95 |

Although there is a large variation between the theoretical and the experimental values for thrust and torque, the value of advance coefficient, at which the theoretical value of thrust is produced, is only 0.792. This represents a decrease of less than 4% in the design advance coefficient. Assuming the measured values are correct, the results indicate that the pitch of the impeller should be increased by approximately 1.04 to maintain the design conditions.

Although the measured impeller efficiency, at the design advance coefficient, was considerable less than that calculated, the maximum experimental efficiency was considerably greater and was equal to 0.95. If the measurements were in error by 12% then the maximum efficiency would be approximately 0.79. This value, although much lower than the theoretical efficiency, is considerably greater than an equivalent open-water propeller. For example, an open-water efficiency of 0.64 can be expected for a propeller of the Troost B-55 Series, which has approximately the same dimensions and operating conditions as the impeller tested in these studies.

The results indicate that a favourable impeller efficiency is obtainable from a ducted impeller, but further tests must be conducted to accurately determine the correlation between the theoretical and experimental performance characteristics.

5.2.2 Velocity in the Wake of the Impeller.

By measuring the velocities in the wake behind the impeller, the calculated load distribution along the blade may be checked. The results of such a wake survey with a 5-hole spherical pitot (Section 4.4) are given in Fig. 5.26.
In Fig. 5.26 the axial velocity ($V_A$) and the tangential velocity ($V_T$) are given for various radial distances at two advance coefficients. These velocities are not as predicted in Section 2.1. The theoretical design assumed the axial velocity distribution was constant and the tangential velocity was linear with radial distance. This variation between theory and practice is to be expected because of the limitations of the simple theoretical analysis.

It has also been shown that large differences do exist between calculated and theoretical values of the wake velocities for open-water propellers, even when they are compared with the most rigorous propeller theories (Ref. 5.6).
6.0

CONCLUSIONS.

6.1

IMPELLER-INDUCED VIBRATORY PRESSURES.

Although the fluctuating pressures induced by a ducted impeller are similar in nature to that of an open-water propeller, several important differences do exist.

(1) The harmonic content of the 2nd and 3rd blade frequency harmonics of the impeller-induced pressure is approximately 60% and 25% respectively of the 1st harmonic pressure. Thus theoretical and experimental determination of pressures at points which are extremely close to an impeller, i.e., about 0.3% of the impeller diameter, must include the blade frequency harmonic components.

(2) Although the maximum pressure coefficient \( \frac{P_t}{\rho \frac{nD^2}{2}} \) is approximately 0.5, its magnitude rapidly attenuates to 10% of its maximum value at distances of \( \frac{X}{D} \) equal to + 0.2 and - 0.1 from the impeller plane.

(3) The blade thickness controls the pressure field forward of the impeller and results in a constant phase angle with respect to the blade position for all the harmonic components of the pressure.

(4) Aft of the impeller, both the blade thickness and the blade loading, influence the pressure field. When the blade loading is high, the pressures associated with the trailing vortices control the pressure, but as the thrust loading approaches zero, the blade thickness effects become significant.

(5) A marked discontinuity in the pressure distribution occurs near the plane which contains the mid-points of the blade sections of the impeller. Behind the impeller plane, the phase is constant
with respect to a uniform pitch helical surface which is, in fact, the trailing vortices of the blades. The position of the discontinuity in the pressure field, i.e. where the trailing vortices appear to emanate from the blade, has a tendency to move forward with an increase in the blade frequency harmonic component.

For this reason, the bound vortex lines, which represent the impeller in a theoretical model, must be situated at the mid-point of the blade sections.

(6) For a non-uniform inflow velocity distribution at the impeller, the resultant induced pressure is dependent, to a first approximation, on the mean velocity component existing over blade length.

(7) Care must be exercised when measuring pressures aft of an impeller, because the response of a finite size transducer is dependent on the phase relationship of the pressures across the diaphragm of the transducer. For example, the pressure measured behind the impeller with a transducer which has a diaphragm diameter of 0.03 of the impeller diameter, is approximately 4% lower than the true pressure. However, the phase of the pressure is only in error by about 2 degrees.

**6.2 Impeller Efficiency.**

There is substantial evidence that impeller efficiencies in the order of 0.90 are possible from a ducted impeller. The experimental value of the maximum impeller efficiency was 0.95, but this unfortunately may be an optimistic estimate because of instrumentation errors. A simple vortex-line analysis of the impeller predicted an efficiency
of only 0.83.

However, further experimental studies must be conducted to determine the actual performance characteristics of the ducted impeller.

6.3

FEASIBILITY OF A HYDROJET.

Only two characteristics of a Hydrojet propulsion unit were considered in these studies. They were the propulsive efficiency and the impeller-induced vibratory pressures on the duct surface.

The propulsive efficiency of a Hydrojet suitable for large displacement vessels is expected to exceed 0.75 providing the ratio of duct length can be reduced to twice its diameter, then the propulsive efficiency is significantly increased to approximately 0.80.

Before accurate predictions can be made for the propulsive efficiency of a Hydrojet propulsion unit, detailed investigations into the frictional resistance of a duct-impeller system and also into the characteristics of the impeller are necessary.

The other characteristic of the Hydrojet that was considered was the impeller induced pressures on the duct surface. Although the magnitudes of these pressures were extremely high, the extent of this intense fluctuating pressure field was very small. Before it is possible to say whether the resultant forces due to the pressure and hence the vibration levels of the ship are acceptable, detailed studies of the duct structure surrounding such an impeller must be undertaken. It is anticipated that such a structure can be designed to effectively cancel the cyclic forces due to the induced pressures of an impeller operating in a uniform velocity field. This would then leave only those forces due to the non-uniformity of the wake to force excite the hull.
REFERENCES.

0.1 NORRIE, D.H. Studies in Marine Propulsion Vibration.
Doctor of Philosophy Thesis, University of Adelaide, Mechanical

0.2 BROWN, N.A. Periodic Propeller Forces in Non-uniform Flow.
Massachusetts Institute of Technology, Dept. of Naval Architecture

1.1 BRESLIN, J.P. Review of Theoretical Predictions of Vibratory Pressures
and Forces Generated by Ship Propellers.
Stevens Institute of Technology, Davidson Laboratory, Note 717.
(Prepared for the Second International Ship Structures Congress,
Delft. July 1964.)


1.3 GASUINAS, A and LEWIS, W.P. Hydraulic Jet Propulsion - A Theoretical
and Experimental Investigation into the Propulsion of Seacraft
by Water Jets.
Presented to Ordinary Meeting of the Institution of Mechanical

1.4 GONGWER, C.A. The Influence of Duct Losses on Jet Propulsion Devices.
References Continued--

1.5 VAN MANEN, J.D. Fundamentals of Ship Resistance and Propulsion.

2.1 BRITISH PROPELLER DESIGN.
Marine Engineer and Naval Architects, Vol.88 No.1069, April, 1965

2.2 O'BRIEN, T.P. The Design of Marine Screw Propellers.

2.3 TINGEY, R.H. Marine Engineering, Vol.1 Chap. IX
(Editor - Seward, H.L.) Society of Naval Architects and Marine
Engineers, 1942.

2.4 MATTHEWS, S.T. and STRASZAK J.S.C. Optimum Length and Thickness of
Propeller Blade Sections for Cavitation and Strength Considerations.
M. B.-231 April, 1961.

3.1 VAN MANEN, J.D. and SUPERINA, A. The Design of Screw Propellers in
Nozzles.

3.2 MANNAM, J.; FOWLER, J.H. and CARPENTER, A.L.
Tapered Lands Hydrostatic Journal Bearings.
References continued --

3.3 McCARTHY, J.H. A Method of Wake Production in Water Tunnels.

4.1 CLARKE, A.P. Computation of the Coefficients of a Fourier Series
   Expansion of a Function defined by Sampled Data Points.
   Weapons Research Establishment, Tech. Memo. T.RD 71, ADDS.

4.2 PIEN P.C. The Five-Hole Spherical Pitot Tube.

4.3 SILOVIC, V. The Five-Hole Spherical Pitot Tube for Three Dimensional
   Wake Measurements.
   Hydro-0g Aerodynamisk Laboratorium, Lyngby, Denmark.

4.4 WERELDSMA, R. Dynamic Behaviour of Ship Propellers.

4.5 WERELDSMA, R. Experiments on Vibrating Propeller Models.
   Journal of Ship Research, 1965
   also Netherlands Ship Model Basin Publication No. 252.

4.6 PENDERED, J.W. and BISHOP, R.E.D. Extraction of Data for a Sub-system
   from Resonance Test Results.
References continued –

4.7  LEIBOWITZ, R.C. and KENNARD, E.H.  Theoretical and Experimental
determination of Damping Constants of One- to Three Dimensional
Vibrating Systems.

4.8  KARMAN, TH. VON and SEARS, W.R.  Airfoil Theory for Non-uniform Motion.
Journal of Aeronautical Sciences, 1938.

4.9  SEARS, W.R.  Some Aspects of Non-stationary Airfoil Theory and its
Practical Application.
Journal of Aeronautical Sciences, 1941.

5.1  TACHMINDJI, A.J. and DICKERSON, M.C.  The Measurement of Oscillating
Pressures in the Vicinity of Propellers.

5.2  BRESLIN, J.P.  Review and Extension of Theory for Near-Field Propeller
Induced Vibratory Effects.
Fourth Symposium on Naval Hydrodynamics, Vol.2 Washington DC
August, 1962.

5.3  BRESLIN, J.P. and KOWALSKI, T.  Experimental Studies of Propeller-
Induced Vibratory Pressures on Simple Surfaces and Correlation
with Theoretical Predictions.

5.4  TSAKONAS, S; BRESLIN, J.P. and JEN, N.  Pressure Field around a
Marine Propeller Operating in a Wake.
Steven Institute of Tech. Davidson Laboratory, R-857, May 1962.
References continued --

5.5 TIMOSHENKO, S and WOINOWSKY-KRIEGER, S. Theory of Plates and Shells. 

5.6 JOHNSSON, C.A. Comparison of Propeller Design Techniques. 
Fourth Symposium on Naval Hydrodynamics. 
Fig. 1.1 - Estimated Performance of Hydrojet Propulsion Unit. (Assuming $K_A = 1$)
Fig. 1.2 - Estimated Ducting Loss Factor $f_d$

Based on,

1. $\rho = 1.99$ slug/cub ft.
2. $C_m$ ATTC. (1947) Mean Friction Line.
3. Equivalent straight duct.
Fig. 2.1 - Velocities and Forces at Blade Section.
Fig. 2.2 - Optimum Efficiency for Varying Radius.
Fig. 2.3 - Optimum Efficiency at Varying Shaft Speed.

Fig. 2.4 - Optimum Efficiency with Varying Blade Geometry.
Fig. 2.6 - MACHINING MASTER TEMPLATES FOR IMPELLER.

Fig. 2.7 - MACHINING MODEL IMPELLER IN COPYING MACHINE.
**Fig. 2.8** - MACHINING MODEL IMPELLER.

**Fig. 2.9** - MODEL IMPELLER.
Fig. 3.1 - MODEL DUCT ARRANGEMENT.

Fig. 3.2 - VIEW OF DYNAMOMETER BEFORE ASSEMBLY.
FIG. 3.3 CROSS SECTION OF DYNAMOMETER
Fig. 4.0 – GENERAL VIEW OF THE INSTRUMENTATION.
Fig. 4.1 - Block Diagram of Instrumentation
Fig. 4.2 - Typical Transfer Function Characteristics of the High-pass Filter.
Fig. 4.3 - Orientation of Velocity components and pressure points to the Axes.
Fig. 4.4 - Inversion about OYY Axis.

Notation of Velocity Components with respect to the Instrument Axes.
Fig. 5.1 - Orientation of axes of impeller
Fig. 5.2 - TYPICAL PRESSURE SIGNALS FOR UNIFORM FLOW \( J_1 = 0.74 \), \( N = 1200 \text{ rpm} \), \( \frac{R_D}{D} = 0.5028 \)
UNIFORM FLOW
IMPELLER - 4 BLADES

Fig. 5.3 - Periodicity of Recorded Pressure.
UNIFORM FLOW
IMPELLER - 4 BLADES

\[
\frac{R_d}{D} = 0.5028
\]

\[
\frac{X}{D} = -0.05
\]

\[
J_I = 0.74
\]

Fig. 5.4 - Frequency Spectra of Pressure
Fig. 5.5 - Axial Variation of Blade Frequency Harmonic Pressure Coefficient. \( J_1 = 0.74 \).
Fig. 5.6 - Axial Variation of Blade Frequency Harmonic Pressure Coefficient. $J_1 = 0.80$. 

UNIFORM FLOW
IMPELLER - 4 BLADES

$\frac{N_d}{\omega} = 0.5028$

$J_1 = 0.80$
Fig. 5.7 - Axial Variation of Blade Frequency Harmonic Pressure Coefficient. $J_1 = 0.85$. 

UNIFORM FLOW IMPELLER - 4 BLADES

$R_{ld} = 0.5028$

$J_1 = 0.85$
Fig. 5.8 - Axial Variation of the Total Pressure Coefficient.
UNIFORM FLOW
IMPELLER-4 BLADES
1st BLADE HARMONIC

\[ \frac{R_d}{D} = 0.5028 \]
\[ J_1 = 0.74 \]
\[ = 900 \text{ rpm} \]
\[ ° = 1200 \text{ rpm} \]

Fig. 5.9 - Axial Variation of Phase Angle (Lead) For 1st Blade Frequency Harmonic. \( J_1 = 0.74 \)
Fig. 5.10 - Axial Variation of Phase Angle (Lead) for 2nd Blade Frequency Harmonic. $J_I = 0.74$. 
Fig. 5.11  -  Axial Variation of Phase Angle (Lead) for 3rd Blade Frequency Harmonic. $J_1 = 9.74$
Fig. 5.12 - Axial Variation of Phase Angle (Lead) for Total Pressure.
Fig. 5.13 - Axial Variation of Phase Angle (Lead) for 1st Blade Frequency Harmonic. $J_1 = 0.80$. 
UNIFORM FLOW
IMPELLER - 4 BLADES
2nd BLADE HARMONIC

\[ \frac{R_d}{D} = 0.5028 \]

\[ J_1 = 0.8 \]

\[ N = 900 \text{ rpm} \]

\[ \Theta = 1200 \text{ rpm} \]

---

Fig. 5.14 - Axial Variation of Phase Angle (Lead) for 2nd Blade Frequency Harmonic. \( J_1 = 0.80 \).
Fig. 5.15 - Axial Variation of Phase Angle (Lead) for Total Pressure. $J_1 = 0.80$. 

UNIFORM FLOW
IMPELLER—4 BLADES
TOTAL PRESSURE

$R_d = 0.5028$

$J_1 = 0.80$

$\square = 900 \text{ rpm}$

$\bigcirc = 1200 \text{ rpm}$
Fig. 5.16 - Axial Variation of Phase Angle (Lead) for 1st Blade Frequency Harmonic. $J_1 = 0.85$. 

UNIFORM FLOW
IMPELLER - 4 BLADES
1st BLADE HARMONIC

$\frac{R_g}{D} = 0.5028$

$J_1 = 0.85$

$\omega = 900 \text{ rpm}$

$\omega = 1200 \text{ rpm}$
Fig. 5.17- Axial Variation of Phase Angle (Lead) for Total Pressure. $\theta_I = 0.85$. 

UNIFORM FLOW
IMPELLER - 4 BLADES
TOTAL PRESSURE

$R_d = 0.5028$
$J_1 = 0.85$
$\square = 900 \text{ rpm}$
$\bigcirc = 1200 \text{ rpm}$
Fig. 5.18 - Velocity Profile in the Duct for Wake-2.
Fig. 5.19 - Velocity Profile at the Impeller for Wake - 2.
Fig. 5.20 - Pressure Coefficient for the 1st Blade Frequency Harmonic with Wake-2.
Fig. 5.21 - Pressure coefficient for the 2nd Frequency Harmonic with Wake-2.
Fig. 5.22 - Pressure Coefficient of the Total Pressure with Wake-2.
Fig. 5.23 - Phase Angle (Lead) for the 1st Blade Frequency Harmonic with Wake-2.
Fig. 5.24 - Phase Angle (Lead) for the 1st Blade Frequency Harmonic with Wake-2.
THRUST ($k_T$), TORQUE ($k_Q$) COEFFICIENT

Fig. 5.25 - Impeller Performance Characteristics
Fig. 5.26 - Induced Velocities behind the Impeller at $\frac{X}{D} = -0.49$. 
APPENDIX A2

THE DESIGN OF DUCTED IMPELLERS
USING A VORTEX LINE ANALYSIS

and

AN OPTIMIZING COMPUTER TECHNIQUE

by

M.R.Hale

UNIVERSITY OF ADELAIDE
DEPARTMENT OF MECHANICAL ENGINEERING
TABLE OF CONTENTS

ABSTRACT 1

1.0 INTRODUCTION 2

2.0 THE DESIGN OF THE IMPELLER 3

2.1 Hydrodynamic model of the impeller 3

2.2 Blade-element lift and drag characteristics 7

2.3 Blade strength 8

2.4 Cavitation 10

3.0 THE OPTIMUM DESIGN OF AN IMPELLER 12

3.1 The Optimizing Ducted-Impeller Design Technique 13

4.0 INPUT INSTRUCTIONS 18

REFERENCES 20

TABLES 22

FIGURES 38
INTRODUCTION

Propeller design procedures in use today (with one recorded exception - Ref. 1) do not attempt to determine the optimum propeller geometry for a given set of operating conditions. The methodical series of propellers are based on an optimum value for one of the major variables, such as diameter or blade area. The optimum values for all the other design variables are not determined. For example, from the design data for the NSMB (or Van Manen) screw series the optimum diameter can be chosen and hence the corresponding pitch and mean blade area ratio can be determined to avoid cavitation under the operating conditions. But in this series the blade outline, blade sections and variation of maximum blade thickness with radius has previously been fixed.

In some of the more theoretical design procedures, it is possible to calculate the circulation distribution so that the energy loss caused by the induced flow is minimized. It is not possible however to determine from these theories a blade shape which is an optimum from the point of view of strength and other requirements yet capable of producing the desired circulation distribution.

The theoretical knowledge of propeller operation has progressed to a stage where an attempt should be made to develop a design technique based on these theories which would determine the best propeller geometry to suit given operating conditions.

A technique of this magnitude would require many mathematical statements and decisions. If such a design is to be economical in both time and cost, the resources of a high-speed digital computer and store are necessary. With the advent of more rigorous and complex
theoretical approaches to the design of propellers, this type of optimum
design procedure will become necessary in the future, if full advantage
is to be taken of the acquired theoretical knowledge.

The programme described in this report was an initial attempt at
an optimizing design procedure. Although the design method used in
this programme was not the most rigorous, the solutions given by the
programme show that this type of optimization could be usefully employ-
ed for more complex design theories. The programme was developed as
part of a feasibility study of a hydrojet propulsion unit in which an
impeller operates in a cylindrical duct.

The design technique given in this report determines the optimum
geometry for an impeller having minimum induced drag energy loss and
minimum profile-drag energy loss. The blade thickness required to
maintain the maximum stress at any section constant and equal to the
design stress is determined. Blade surface pressure and cavitation
margin are also taken into account in selecting the blade sections.

The impeller design as given by the optimum solution will, within
the limitations of the theory used, have the maximum possible efficiency,
the maximum utilization of material and the minimum weight.

2.0 THE DESIGN OF THE IMPPELLER

2.1 Hydrodynamic model of the impeller

Since the present research project was directed towards estimating
the capabilities of a ducted propulsion system, it was considered that
a simplified vortex-line theory could be satisfactorily applied to the
design of the impeller.

Consider an impeller operating in a long cylindrical duct in which
the fluid can be considered as irrotational upstream of the impeller. It is assumed that the impeller has negligible tip clearance. Thus the impeller diameter equals the duct internal diameter. The finite size boss is assumed to have negligible effect on the induced flow.

It is assumed that the axial velocity profile is uniform across the duct upstream of the impeller. It is also assumed that for a sufficient distance upstream and downstream of the impeller the duct is parallel. The impeller design is selected to have no axial component of induced velocity. The velocity diagram is as shown in Fig. 1.

It is considered to be sufficiently accurate for the interference flow to be assumed constant circumferentially at any radius. This is equivalent to assuming the interference flow is generated by an infinite number of lifting-lines of variable strength in the radial direction. Using the Betz's criteria for minimum induced energy loss in the wake, and the Kutta-Joukowski relationship, the ideal thrust and torque gradients at any section can be derived as follows. (See Fig. 1).

Betz's theory can be stated as: "When the distribution of circulation along the blade is such that, for a given thrust, the energy lost per unit time is a minimum, then the flow far behind the screw is the same as if the screw surface formed by the trailing vortices was rigid, and moved backwards in the direction of its axis with a constant velocity, the flow being that of classical hydrodynamics in an inviscid fluid, continuous, irrotational and without circulation." (See Ref. 2).

The Betz minimum energy condition is expressed as (Ref. 3)

$$\frac{\tan \theta}{\tan \theta_1} = \eta_1 = \text{constant with radius} \quad \ldots \quad 1.$$
where $\beta$ = the advance angle

$\beta_i$ = the hydrodynamic pitch angle

$\eta_i$ = the ideal blade element efficiency

Substituting for $\tan \beta$ and $\tan \beta_i$ from Fig. 1 gives

$$\frac{\tan \beta}{\tan \beta_i} = 1 - \frac{U_{It}}{2r\omega}$$

... 2

where $U_{It}$ is the circumferential component of induced velocity at a sufficient distance downstream of the impeller.

The expression for thrust grading $\frac{dT_i}{dr}$ and torque grading $\frac{dQ_i}{dr}$ (see Ref. 3) become in the present case:-

$$\frac{dT_i}{dr} = \rho \Gamma Z(r\omega - \frac{U_{It}}{2})$$

... 3

$$= 4\pi \rho \omega^2 r^3 \eta_i(1 - \eta_i)$$

... 4

$$\frac{dQ_i}{dr} = \rho \Gamma Z Va r$$

... 5

$$= 4\pi \rho \omega Va r^3 (1 - \eta_i)$$

... 6

where $\Gamma = \frac{2\pi r U_{It}}{Z}$ is the circulation around a blade.

$Z$ = the number of blades ...

If it is assumed that the circulation distribution for minimum energy loss is not greatly affected by the variation of profile drag with radius, then the actual thrust and torque gradient can be evaluated as follows. (See Fig. 1).

$$\frac{dT}{dx} = \frac{dT_i}{dx} (1 - \xi \tan \beta_i)$$

... 8

$$= K_i \eta_i (1 - \eta_i)(1 - \frac{\xi}{6x \eta_i}) x^3$$

... 9
and \( \frac{dQ}{dx} = \frac{dQ_i}{dx} \left(1 + \frac{\xi_i}{\tan \beta_i}\right) \)

\[ \tan \beta_i = \frac{1}{\delta \eta_i^x} \]

where \( \delta = \frac{\omega R}{V_a} \quad \xi_i = \frac{C_D}{C_L} \quad x = \frac{r}{R} \quad K_i = \frac{4\pi \rho \delta^4 V_a^4}{\omega^2} \]

The total thrust \((T)\) and torque \((Q)\) of the impeller can only be evaluated by summation over the blade length if the drag to lift ratio \(\xi_i\) is known at each section. Since this ratio depends on the blade profile which is in turn dependent on the strength, cavitation and hydrodynamic requirements, a simple expression for thrust and torque cannot be obtained.

An approximation to the impeller geometry can be obtained by assuming the drag to lift ratio constant with the radius. In this case integration gives the following equations for total thrust \(T\) and overall efficiency \(\eta_o\).

\[ T = A(1 - \eta_i)(\eta_i B - C) \quad \ldots \quad 12 \]

\[ \eta_o = \frac{\eta_i B - C}{B + \eta_i D} \quad \frac{T}{A(1 - \eta_i)(B + \eta_i D)} \quad \ldots \quad 13 \]

where \( A = \frac{\pi \rho V_a^4 \delta^3}{\omega^2} \quad C = \frac{4}{3} \xi_i (1 - K^3) \quad B = \delta (1 - K^4) \quad D = \frac{4}{5} \xi_i \delta^2 (1 - K^5) \)
2.2 Blade-element lift and drag characteristics

The lift on a blade element is given by the Kutta-Joukowski relationship:

\[ dL = \rho \Gamma V_R \, dr \] \hspace{1cm} \ldots \hspace{1cm} 15

where

\[ V_R = \sqrt{(r \omega - \frac{U_R^2}{2}) + V_{a}^2} \]

Using equations 2, 7, 11, 15, the design lift coefficient of the section \((C_L)\) can be expressed as,

\[ C_L = \frac{dL}{\frac{1}{2} \rho V_R^2 c \, dr} = \frac{8\pi \delta (1 - \eta_i) x^2 R}{Zc \sqrt{(x \delta \eta_i)^2 + 1}} \] \hspace{1cm} \ldots \hspace{1cm} 16

where \(c = \) chord of blade section

The theoretical lift coefficient \((C_{1i})\) required to develop the design lift is assumed to be greater than \(C_L\) by a factor \(\mu_m\), the viscosity factor. According to the potential theory of thin wing sections (see Chapter 5, Ref. 4) the theoretical lift coefficient of the section is a function of the camber to chord ratio \(\frac{m_x}{c}\) only if the section operates at shock-free entry conditions, hence

\[ C_{1i} = \frac{m_x}{c} \] \hspace{1cm} \ldots \hspace{1cm} 17

The blade section chosen for this impeller was an NACA-16 thickness distribution with a mean line of \(a = 1.0\), and the following are the values for viscosity factor \((\mu_m)\) and lift camber factor \((l_m)\) at shock-free conditions. (Table 5.6, p.175 in Ref. 4).

\[ \mu_m = 0.74 \] \hspace{1cm} \ldots \hspace{1cm} 18

\[ l_m = 18.13 \] \hspace{1cm} \ldots \hspace{1cm} 19

hence \(\frac{m_x}{c} = \frac{C_{1i}}{l_m} = \frac{C_L}{\mu_m l_m} = 0.0745 C_L\) \hspace{1cm} \ldots \hspace{1cm} 20
The above method of calculating blade section geometry to obtain a desired lift coefficient has been experimentally proven valid by O'Brien of the National Physics Laboratory, (Ref. 5). Unfortunately there is no corresponding derivation at present available for the analysis of blade-section profile drag. The usual assumption that the profile drag is a function of the thickness to chord ratio $\frac{t}{c}$ and angle of incidence was therefore adopted. The values of drag coefficient $C_D$ used in the design were taken from the data of Hill (Ref. 6).

A 10th order polynomial of the form

$$C_D = \sum A_i \left( \frac{t}{c} \right)^i$$

where $i = 0, 1, 2, \ldots, 10$

was fitted to the data for aerofoil sections at zero angle of incidence. The coefficients $A_i$ are given in Table 1. These coefficients were calculated using IBM Programme Library No. 7.0.002 in which a set of simultaneous equations representing the condition for least square deviation is solved using a modified Gaussian elimination technique followed by a back substitution.

A tenth order polynomial was necessary to determine the value of $C_D$ to an accuracy of 0.2% over the range of $\frac{t}{c}$ from 0.0 to 0.3.

This $C_D$ relationship will apply to the NACA - 16, $a = 1.0$ section used in the design since the ideal angle of incidence is constant and equal to zero at shock-free entry conditions. (Ref. 4, Table 5.6).

2.3. **Blade Strength**

The stresses at a blade section were calculated by the simple theory for bending of a beam as suggested by Tingey (Ref. 7, also Ref. 4).

It is important to note that this theory can only be applied to designs where chordwise bending due to the pressure distribution over a section can be ignored. This implies that the blades should have
relatively large thickness to chord ratio \( \frac{t}{c} \) and not excessively wide chords.

The assumptions made by Tingey for the determination of stresses in a propeller with zero rake are:

(1) The section through which the blade would fracture if overstressed (which is a plane section approximately parallel to the axis of rotation) has the same geometric properties as a corresponding cylindrical blade section at the same radius from the axis of rotation.

(2) The centres of area of all the cylindrical blade sections (or the centres of gravity of all the cylindrical blade sectional elements) are on a straight line intersecting the axis of rotation and normal to this axis.

(3) The simple theory of the bending of beams can be applied in assessing the stresses due to the bending moments caused by hydrodynamic forces acting on the blades.

(4) The principal axes of inertia of a cylindrical blade sectional element coincide with two perpendicular axes in the plane of the expanded element. One axis is parallel to the chord line, and the other intersects it at the centre of the element.

(5) The blades do not deflect.

The stress \( \sigma \) at a given point on the section due to the hydrodynamic loading and the centrifugal force is then given by

\[
\sigma = y \frac{M_n}{I_n} + h \frac{M_p}{I_p} + \frac{F}{A_s} \quad \ldots \quad 22
\]

where \( M_n = M_T \cos \phi + M_Q \sin \phi \),
\[ M_p = M_T \sin \phi - M_Q \cos \phi \]

\[ \phi = \text{Blade element pitch angle} \]

\[ \phi = \beta \text{ in this design} \]

The relevant geometrical properties of the NACA - 16, \( a = 1.0 \) blade section is given in Table 2. Information for other sections can be found in Table 8.4 in Ref. 4.

2.4 Cavitation

Using the theory of thin wings, the cavitation parameter "pressure minima cavitation number" \( \xi_p \) can be determined for a particular geometry of blade section.

\[ \xi_p = \frac{(p - p_{l1}^\prime)}{\frac{1}{2} \rho V_R^2} \]

... 23

where \( p = \) free stream pressure

\[ p_{l1}^\prime = \text{minimum value of local pressure } p_{l1} \]
on surface of section.

For the blade section under discussion, (Ref. 4, p.209)

\[ \xi_p = (1 + 1.14 \frac{t_x}{c} + \frac{C_{l1}}{4})^2 - 1 \]

... 24

The sectional cavitation number \( \xi_s \) is defined by

\[ \xi_s = \frac{(p_r - e)}{\frac{1}{2} \rho V_R^2} \]

... 25
where $p_r =$ pressure at blade section radius $r$ and at minimum immersion

$e = $ saturated vapour pressure

When applying these equations to an actual impeller an overall factor $f_o$ which makes allowance for irregular and viscous flow is introduced to effectively increase $\delta_p$.

Thus $\delta_s \geq f_o \delta_p$ \hfill ... 26

where $f_o = 1.2$ (Ref. 4 p.209)

A similar method was used by Matthews and Straszak to estimate the inception of cavitation in screw propeller designs. (Ref. 8).

For simplicity in calculating the static pressure at a blade section it was assumed that there is negligible head loss in the intake up to the impeller, and that the operating conditions of the impeller-duct system are such that the fluid velocity at the intake equals the fluid velocity just forward of the intake. If the duct is uniform in diameter, the static pressure at the impeller plane equals the static pressure at the corresponding point at the intake.

For a non-uniform duct, and other operating conditions, the impeller static pressure would need to be estimated from parameters of the system. The influence of duct friction could also be allowed for if applicable.
Applicability of the Design Analysis.

It should be noted that the hydrodynamic design of the impeller used in this paper differs in a number of significant respects from that commonly used for axial-flow pumps, and to a lesser extent from methods used for axial flow fans and compressors.

The design analysis was developed for a feasibility study of a ducted impeller (i.e. axial-flow pump) operating at conditions far removed from the normal range for axial-flow pumps. The application was for propulsion of large ships, where the unit would have a large diameter, extremely high capacity, low head, and low static head at the impeller. The specific speed would thus be very large, a typical value of

\[
N_s = \frac{\frac{0.5}{NQ}}{H^{0.75}} \quad (N \text{ in rpm, } Q \text{ in gpm, } H \text{ in ft})
\]

... 27

being 40,000. An impeller designed for such conditions has very low values of induced tangential velocities, these being of the order of 2% and 10% of the axial velocities at hub and tip respectively.

Further differences from axial pump practice arise because the design uses an optimizing procedure. This also has the incidental effect of reducing the impeller weight since not only is the distribution of maximum stress along the blade uniform but also the profile drag (and hence blade sectional area) is minimized.
In axial-flow machines, the impeller can be designed for any
arbitrary variation of tangential velocity with radius, i.e. vortex
distribution. If there is to be no radial flow ("simple radial
equilibrium") it can be shown (Ref. 9, p.427) that the axial velo-
city must satisfy the relation.

\[
\left( \frac{V_a}{V_{a_0}} \right) = 1 - \left( \frac{r}{n} - 1 \right) \left( \frac{U_{It_0}}{V_{a_0}} \right)^2 \left( \frac{r}{r_0} \right) - 1 \]

... 28

where \( V_a \) = axial velocity at radius \( r \)
\( V_{a_0} \) = axial velocity at hub
\( U_{It_0} \) = induced tangential velocity at hub

and where \( n \) is the index in the vortex distribution, described
in the form

\[
U_{It} r^n = \text{constant} \]

... 29

For axial-flow pumps, free-vortex design (i.e. \( n = 1 \)) is
often used and 28 shows this leads to the design condition of
constant axial velocity. Such a design does not necessarily
result in the highest overall efficiency at the design condition
(Refs. 10, 11).

The aim of the present design is to maximise the overall
efficiency. This requires minimizing the energy losses i.e.
profile drag losses and induced energy losses. The former are
minimized in the hydrodynamic analysis of Section 2.1 by using the
Betz condition of minimum energy loss. This latter condition
expressed in equations 1 and 2 leads to the requirement $\frac{U_{1t}}{r^{-1}}$ = constant i.e. forced-vortex or solid-rotation design for which $n = -1$ in equation 29. Substitution of the representative design values of $V_{ao} = 25$ f.p.s. and $U_{1t_0} = 0.5$ f.p.s. into equation 28 shows that $V_a$ varies only 0.5% from hub to tip, and hence be assumed constant in this design. The design thus ensures radial equilibrium, for the application considered.

Solid-rotation gives a non-constant total-energy distribution with radius after the impeller. For axial-compressors, as noted in Ref. 12, this does not lead to a decreased efficiency as compared with the free-vortex design with its constant total-energy distribution, and there appears to be no reason to believe that the same will not be true for the present design. In any case the total-energy variation is small in this design. It is also noted in Ref. 12 that the solid-rotation design has for the same diameter and speed, the advantage of a greater work capacity than the free-vortex design.

It is interesting to note that several axial-flow compressor designs which are commonly used do violate the simple radial equilibrium condition (Ref. 9, pp.424-425; Ref. 13). The same is true for marine propellers. These two types of machine bracket as it were the axial-flow pump and ducted-impeller, and indicate that departure from simple-radial equilibrium may not necessarily introduce an efficiency penalty in pump design. Removal of the radial-equilibrium restriction would allow greater flexibility in design. Some of the design types discussed in Ref. 14 might lead to a higher overall efficiencies for pumps and ducted impellers.
(It is noted on p.435 of Ref. 9 that pump efficiencies are not as high as might perhaps be expected). Optimizing programs of the type described in this paper could be developed for the various kinds of design, and could facilitate comparison of their merits.

The present analysis assumes the interference flow to be constant circumferentially at any radius (i.e. assumes an infinite number of blades). The effect of a finite number of blades could be taken into account by incorporating into the analysis a "blade-number factor" for circumferential induced velocity similar to the Goldstein factor used in open-water screw-propeller design. Such a factor can be computed from equation 32 of Ref. 15. The analysis could then be used to optimize the number of blades.

It should be noted that the optimizing technique can be adapted to other impeller design methods, such as those commonly used for axial-flow pumps or compressors. It could also be adapted to turbines of various kinds.
3.0

THE OPTIMUM DESIGN OF AN IMPELLER

The optimum combination of blade sections for a hydrojet impeller must satisfy the conditions expressed in the hydrodynamic equations for minimum energy loss, have the lowest profile drag that is possible, but still be strong enough to limit the sectional stresses to a value equal to or less than the maximum design stress. The blades must also operate free from cavitation. For this design, the weight of the impeller will be a minimum for the chosen operating conditions of $T$, $n$ and $D$.

Although the impeller calculated by this design method has an optimum blade section arrangement for the given conditions it is not necessarily the optimum design for a given duty, i.e. given $T$. The optimum design must be selected by studying closely the results of a series of systematically varied impellers, all designed for optimum arrangement and satisfying the requirements of a given duty. Before deciding upon the final impeller geometry, certain other factors affecting the operation of an impeller or rotor-dynamic propulsion unit must also be taken into account - e.g. --

1. What is the largest diameter that can be installed in the vessel, and is this greater than the optimum diameter?

2. Is the number of blades and rotational speed conducive to exciting critical modes of vibration when the propulsion system is operating?

3. What is the economical range of rotational speeds of the prime mover?

The sectional lift coefficient is another important variable which must be studied before selecting the final design since the value of lift
coefficient selected in the programme is not limited. Under certain
circumstances, the sectional lift coefficient as computed may exceed
the stall value. Usually this only occurs when the operating
conditions differ greatly from the optimum operating conditions.

The design programme given in this report does not achieve the
ideal objective which could be stated as "the selection of an optimum
impeller geometry to suit a particular duty by considering every
possible arrangement which satisfies all known laws, principles and
facts associated with its operation." All these decision points
could be inserted into a programme for the logical selection of the
ideal impeller and would require extremely careful planning. Although
the complete optimum design is not specified directly by the programme
of this report, it is considered that the technique given for
selecting an optimum blade geometry is a radical departure from the
usual propeller design procedures. It is felt to be a worthwhile
step towards designing the most efficient unit possible using available design techniques and aids.

3.1. The Optimizing Ducted-Impeller Design Technique

An outline is given below of the major steps in the design
programme. The bold-type letters in the margin refer to sections
of the programme (see a listing of the programme in Table 3).
The flow diagram of the programme given in Fig. 2 shows the major
computing steps and the information flow paths.

The impeller design method is an iterative process in which the
blade sectional variables $\frac{dT}{dx}$, $\frac{dQ}{dx}$, $C_L$, $t_x$ etc. and the impeller
efficiency are computed using successive estimated values for the
blade element efficiency $\eta_i$ until the impeller efficiency calculated
is to within a small pre-determined percentage of its preceding value.
The design then has the maximum impeller efficiency, since it is assumed that curves of impeller efficiency versus the sectional variables are smooth single-maximum curves with no points of inflection.

D. **INITIALIZATION**

Before commencing the main calculation, an initial estimate of $\eta_1$ is computed by solving equation 11 in which the drag to lift ratio is assumed constant with radius.

It has been found from previous experience that a value for drag to lift ratio $\xi$ of 0.03 determines the initial value of $\eta_1$ to within a few per cent of its final value (for which profile drag is considered a variable).

E. Using the estimated value of $\eta_1$ and the assumed value of $\xi$, the blade sectional variables are computed from the hydrodynamic equations 1 - 11, 15 - 20 for a selected number of circumferential sections at radius fractions $x$.

F. **SELECTION OF BLADE SECTIONS**

The aim of this section of the routine is to determine the blade section geometry necessary to satisfy the hydrodynamic requirements. The calculation commences at the blade tip where some of the variables are known or can be selected, and then proceeds with the selection of blade sections properties towards the boss.

The tip and root chords widths are selected values which form part of the input to the programme. The remaining chord widths are then calculated using a given input chord distribution which for simplicity was assumed linear for the hydro-jet impeller design.
These chords widths may be adjusted later to prevent cavitation causing departure from the initially assumed linear chord distribution.

Because stress on the tip section is not a limiting factor, the tip thickness is selected for minimum profile drag \( t \frac{K}{c} = 0.045 \) from equation 21).

With the above selected blade sectional variables and those calculated from part E above, the sectional cavitation number \( \xi_s \) at the tip is determined and compared with the pressure minimum cavitation number \( \xi_p \) of the section. If the conditions at this section induce cavitation, the chord width must be increased to reduce the lift coefficient. This will result in a decrease in the pressure minimum cavitation number i.e. an increase in the minimum value of local pressure on the section at which the inception of cavitation occurs. The width is progressively adjusted until cavitation free operation of the section is obtained.

G. BLADE STRESS CALCULATION

This routine is progressively applied from tip to root to all the chosen radial stations or radius fractions, excluding the tip station.

The first estimate of blade thickness at the station being considered is based on the blade thickness at the preceding station (i.e. the next station radially outwards) and maintains the thickness to chord ratio constant. Since the maximum-sectional stress is zero at the tip and increases with decreasing radius, the thickness to chord ratio can be maintained constant over the outer portion of the blades until a section is reached where the maximum stress is equal to the design stress. These outer sections, therefore, operate with
minimum profile drag.

The stress at four points on the blade section is calculated and the maximum numerical value is compared with the allowable stress. These four points are - the leading and trailing edges and the maximum ordinate position on both the back and face. If the calculated maximum stress is equal to or less than the allowable design stress, the section must now be checked for cavitation. If cavitation exists, the chord at the section must be increased and the stress calculation for that radius fraction repeated. However, if the maximum stress at the section is greater than the design stress, the thickness is increased and the stress calculation repeated until the maximum sectional stress is equal to the design stress at which point the section is checked for cavitation. If cavitation exists, the chord is increased and the stress calculation repeated on the basis that thickness to chord ratio is the same as that of the preceding station. After successive calculations, a stage is reached at which the working stress equals the design stress, and the section is free of cavitation.

The method used to determine the required thickness is a combined iterative and convergence procedure called "Regula Falsi". The independent variable, thickness, is progressively increased by a known amount until the dependent variable, maximum sectional stress minus design stress, changes its numerical sign. When this state has been reached, the desired value of the independent variable lies between the last two consecutive values, providing there is not more than one root of the equation in this interval. As the equation for blade thickness has only one solution, a solution is always possible no matter what stepping interval is selected.
Having located the solution within a range of values a forced convergence is applied. The convergence method used computes the dependent variable for a value of the independent variable midway between the two values surrounding the solution. The numerical sign of the value of dependent variable obtained is investigated. Two values of the independent variable, thickness, for which the corresponding values of dependent variables have opposite numerical signs are then selected from the last three consecutive values. The forced convergence is repeated until the desired accuracy of the solution is reached. Since the convergence is based on the mid-point value, the accuracy of the computation is governed by the number of complete passes through the calculating routine, after the initial values spanning the solution have been determined.

The stress calculation, blade thickness determination and cavitation estimate are repeated for every station up to the boss, using the estimated value of the blade element efficiency. Having now determined the blade thickness distribution, the values of the following variables lift coefficient and drag-to-lift ratio at the selected radial stations can be evaluated.

A position has now been reached where a close estimate of the blade element efficiency is required.

Since the blade element efficiency cannot be expressed as a simple function of thrust and blade section properties, the "Regula Falsi" method of convergence must again be employed to determine the design blade element efficiency. The efficiency is progressively decreased from an assumed value of 99.9% until the computed thrust obtained by summing the values of thrust gradient at each station over the complete blade length equals the design thrust. The
thrust gradient at each station is calculated using the current value of blade element efficiency and the sectional characteristics determined in Section J. above.

L. The final value from section K above is now used to re-compute the torque gradients, lift coefficients, and maximum ordinate of the mean line.

If the final value of blade efficiency from Section K is not equal to the initial value chosen in Section D to the desired accuracy, further convergence is necessary and the calculation from Section F to Section K must be repeated using the newly determined value of the blade element efficiency from Section K as an input variable. This calculation yields a new output value of blade element efficiency which must be compared with the input value. If the two values are not equal to within the desired accuracy, the process must be repeated until this condition is satisfied whence the computation is complete.

4.0 INPUT INSTRUCTIONS

The input data is divided basically into two groups: one group for constant data, such as blade section properties, material densities etc., and the other group for variable impeller design conditions and geometry.

The variable input data - characters TT to NTEST (see Table 3) form a group. Any number of such groups may be placed after the constant data - characters DTOL to A10, if systematically varied impellers are to be investigated.

LSTEP is a control on the number of stations chosen to satisfactorily define the impeller. LSTEP must be odd and
for most calculations it is suggested that LSTEP be selected equal to 17.

NTEST controls the type of print-out from the programme;

NTEST - ve causes print-out of impeller characteristics and geometry after each estimate of $\eta_i$.

NTEST = 0 causes print-out of impeller characteristics and geometry after $\eta_i$ has been determined to the required accuracy.

NTEST + ve causes progressive print-out of all major decisions and relevant variables as well as the impeller characteristics and geometry.

The definitions of the characters defined in the input statements of Table 3 are given in the nomenclature list Table 6. The form of the input can be seen from the example given in Table 5.

The programme as written in FORTRAN for CDC 3200 occupies about 2,100 words of storage, not including the storage required for the total number of subscripted variables. The execution time for the CDC 3200 machine is approximately 3 minutes per single set of data.
REFERENCES

1. BRITISH PROPELLER DESIGN
   Marine Engineer and Naval Architect
   Vol. 88 No.1069, April 1965.

2. GOLDSTEIN, S. On the Vortex Theory of Screw Propellers
   Proceedings of the Royal Society of London,

3. van MANEN, J.D. Fundamentals of Ship Resistance and
   Propulsion. Part B.

4. O'BRIEN, T.P. The Design of Marine Screw Propellers

5. SILVERLEAF A. & O'BRIEN T.P. Some Effects of Blade Section
   Shape on Model Screw Performance.
   Transactions of the North-East Coast Institution of

6. HILL, J.G. The Design of Propellers.
   Trans. Society of Naval Architects & Marine Engineers,

7. TINGEY, R.H. Marine Engineering, Vol.1, Chap. IX,
   (Editor - Seward, H.L.) Society of Naval Architects
   and Marine Engineers, 1942.
References continued:

9. SHEPHERD D.C

Principles of Turbomachinery.
MacMillan 1956

10. BARN A P.S.

Equilibrium of Flow in Axial Flow Fans Designed for Constant
Lift-Drag Ratio
Proceedings of First Australasian Conference on Hydraulic
and Fluid Mechanics,

11. BARN A P.S.

Preliminary Aerodynamic Design Considerations of Axial
Flow Fans.
Auburn University, Engineering Experiment Station,
Bulletin 50, April 1965

12. DE K O V A TS A. & DESMUR, G.

Pump Fans & Compressors,
Blackie 1958, p.288

13. COHEN H. & ROGERS G.F.C.

Gas Turbine Theory,
Longmans Green 1954, pp. 144-145.
14. WATTENDORF F.L.

Simplified Design Comparisons of Axial Compressors,
Journal of Aeronautical Sciences, Vol.18, 1951, p.447

15. TACHMINDJI, A.J.

Potential Problem of the Optimum Propeller with Finite
Number of Blades - Operating in a Cylindrical Duct
Table 1 - Coefficients of Polynomial for Drag Coefficient of an Aerofoil at Zero Incidence

<table>
<thead>
<tr>
<th>i</th>
<th>Ai</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.9299158 E-02</td>
</tr>
<tr>
<td>1</td>
<td>- .3499790 E-01</td>
</tr>
<tr>
<td>2</td>
<td>- .2191732 E+01</td>
</tr>
<tr>
<td>3</td>
<td>.7515442 E+02</td>
</tr>
<tr>
<td>4</td>
<td>- .8086659 E+03</td>
</tr>
<tr>
<td>5</td>
<td>.3367858 E+04</td>
</tr>
<tr>
<td>6</td>
<td>.6849617 E+04</td>
</tr>
<tr>
<td>7</td>
<td>- .1329265 E+06</td>
</tr>
<tr>
<td>8</td>
<td>.5739262 E+06</td>
</tr>
<tr>
<td>9</td>
<td>- .1119756 E+07</td>
</tr>
<tr>
<td>10</td>
<td>.8516336 E+06</td>
</tr>
</tbody>
</table>

\[ C_D = \sum Ai \left( \frac{t}{c} \right)^i \]

where \( i = 0, 1, 2, \ldots, 10 \)
### Table 2. Geometrical Properties of NACA-16, \( a = 1.0 \) Profile

(Ref. 4, Table 8.4)

<table>
<thead>
<tr>
<th>Mathematical Symbol</th>
<th>Area Factor</th>
<th>Co-ordinates of Centroids</th>
<th>Moment of Inertia Factors</th>
<th>Distance from Centroid to</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_s )</td>
<td>( h/c )</td>
<td>( y/x )</td>
<td>( t_x )</td>
</tr>
<tr>
<td>Programme Symbol</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AS</td>
<td>HBC</td>
<td>YETA</td>
<td>YBIB</td>
<td>PPIP</td>
</tr>
<tr>
<td>Numerical Value</td>
<td>0.736</td>
<td>0.516</td>
<td>0.5-0.182 ( m_x ) ( t_x )</td>
<td>0.0418</td>
</tr>
</tbody>
</table>

where

\[
A_s = a_s t_x c = AAS \\
i_n = i_n t_x c = PIN \\
i_p = i_p t_x c^3 = PIP
\]
Table 3 - Programme Listing.

1330 M R HALE/FOWLER MECH ENG DEPT U OF A TEL 461

010 FORMAT (1H1,1X,38-[DESIGN PARAMETERS HYDROJET IMPELLER/1H0,3X,18H
1DESIGN ASSUMPTIONS/1H 4X,25HBEZF MIN ENERGY CONDITION/1H 4X,23HC
2CONSTANT AXIAL VELOCITY/1H 4X,30HNAHCA PROFILE SHOCK FREE ENTRY/77]
020 FORMAT (1H,72H RADIUS VELOCITY ROTATION BLADES IDEAL EFF TOT
1AL EFF THRUST SHP;
030 FORMAT (F7.3,F6.4,F8.4,F8.4,E10.3,F7.2)
050 FORMAT (5E14.7/5E14.7/5E14.7)
060 FORMAT (F9.1,F7.2,F6.2,F9.1,F7.2,F6.5,F7.2,F6.5)
070 FORMAT (1H,66H TIP CHORD BOSS CHORD BOSS RATIO TIP IMMERSION AL
1LOWABLE STRESS)
080 FORMAT (1H,F7.3,F5.5,F7.3,F7.3,F7.3,F7.3,F7.3,F7.3)
090 FORMAT (1H,-69HRAD FRAC CHORD THICKNESS MEAN LINE ANGLE BI LIF
1T COEF DRAg/LIFT)
110 FORMAT (1H,-79HRAD FRAC ST F ST B ST E ST N CAVT S CAVT P
1 THRUST GRAD TORQUE GRAD)
120 FORMAT (1H,F6.2,F7.0,F8.3,F2X,E10.3,F4X,E10.3)
130 FORMAT (1H,6H EFF=1=F7.4,F6.4,F1T=1=F9.1)
140 FORMAT (1H,-50H SCAN EFF BOSS THICKNESS BOSS CHORD MAX STRESS)
150 FORMAT (1H,F9.4,F5X,F6.6,F10X,F6.3,F6X,F6.0)
160 FORMAT (1H,10H RAD FRAC=1=F6.2)
170 FORMAT (1H,-13,H4X11HMAX STRESS=1=F7.0,F4X,6HCHORD=1=F7.3,
14X,10HTHICKNESS=1=F7.4)
180 FORMAT (1H,11H CAVITATION,4X,7HCavit S=1=F8.3,F2X,7HCavit P=1=F8.3,F8.3
1)

DIMENSIONDL(17),T(17),Q(17),CX(17),LL(17),AM(17),ST(17),4,ANG(17),
1TX(17),CAVs(17),CAVP(17),X(17)
READ(60,030)DL,TL,TCOD,DLN,DLN,STAL,HTIP
READ(60,040)AS,PPIP,PPINA,PPINB,YBTA,YBTA,HBTC,YBCT,HBC,CLI,CAV1,
Table 3 - Continued.

1CAV2,PAT,VAP,DEVN1,DEVN2,DEVN3,STEP1
READ(60,045)A0,A1,A2,A3,A4,A5,A6,A7,A8,A9,A10
100 READ(60,046)TT,VEL,ROT,RAD,DX,Z,COD,CID,LSTEP,NTES
CALL=CCK(60,J)
GOTO(450,460),J
460 WRITE(61,010)
IF(NTES)102,102,101
101 WRITE(61,095)
C
102 PI=3.1415927
OMEG=2.0*PI*ROT
DEL=OMEG*RAD/VEL
RSTEP=LSTEP-1
RATIO=1.0-DX*RSTEP
CON1=4.0*PI*DENW*(DEL**4)/(VEL**4)/(OMEG*OMEG)
CON2=DELNW/OMEG**2/RAD**2
CON=8.0*PI*DEL**2/RAD
CON=PAT+HTP*32.2*DENW-VAP
A=(PI*DENW*(DEL*VEL)**3)*VEL)/(OMEG*OMEG)
C=(1.0-RATIO**3)*DTOL+1.3333333
D=(1.0-RATIO**5)*DTOL*DEL*DEL*0.8
ROOT=SQR(1.0-C/B)**2-(4.0*TT)/(A*B))
ETA1=1.0+C/B+ROOT)*0.3
D103K=1,LSTEP
XD=K-1
X(K)=1.0-XD*DX
DL(K)=DTOL
T(K)=CON1*(X(K)**3)*ETA1*(1.0-ETA1)*(1.0-DL(K)/(DEL*X(K)*ETA1))
Q(K)=CON1*VEL*(X(K)**3)*(1.0-ETA1)*(1.0+DL(K)*DEL*X(K)*ETA1)/OMEG
CX(K)=COD-(COD-CID)*(1.0-X(K))/(1.0-RATIO)
CL(K)=CON3*(X(K)**3)*(1.0-ETA1)/(SQR((X(K)**2*DEL*ETA1)**2+1.0)**
L1C(K))
103 AM(K)=CL*K*X(K)**CX(K)
TX(LSTEP)=0.0
STRES=0.0
F
104 ETA=ETA1


<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>105</td>
<td>WRITE(61,096)EFF1, TX(LSTEP), CX(LSTEP), STRES</td>
</tr>
<tr>
<td>106</td>
<td>IF(NTEST)106, 106, 105</td>
</tr>
<tr>
<td></td>
<td>EFF1=ETA</td>
</tr>
<tr>
<td></td>
<td>MSTEP=0</td>
</tr>
<tr>
<td></td>
<td>TX(1)=TCOD*COD</td>
</tr>
<tr>
<td></td>
<td>ANGB=ATANF(1.0/(DEL*EFF1))</td>
</tr>
<tr>
<td></td>
<td>ANG(1)=ANGB*180.0/PI</td>
</tr>
<tr>
<td></td>
<td>CAVS(1)=CON4+(1.0-X(1))<em>32.2</em>DENW*RAD)<em>2.0</em>(SINF(ANGB)*2)/</td>
</tr>
<tr>
<td></td>
<td>1(VEL<em>VEL</em>DENW)</td>
</tr>
<tr>
<td>107</td>
<td>CAVP(1)=(1.0+1.14<em>TX(1)/CX(1)+CL(1)/(0.74</em>CAV2))**2-1.0</td>
</tr>
<tr>
<td></td>
<td>IF(CAVS(1)&lt;CAVP(1))CAVP(1)=CAV1108, 109, 109</td>
</tr>
<tr>
<td>108</td>
<td>CLA=CL(1)</td>
</tr>
<tr>
<td></td>
<td>CL(1)=0.74<em>CAV2</em>(SRTF(CAVS(1)/CAV1+1.0)-1.0-1.14*TX(1)/CX(1))</td>
</tr>
<tr>
<td></td>
<td>CX(1)=CLA*CX(1)/CL(1)</td>
</tr>
<tr>
<td></td>
<td>TX(1)=TCOD*CX(1)</td>
</tr>
<tr>
<td></td>
<td>IF(NTEST)107, 107, 119</td>
</tr>
<tr>
<td>119</td>
<td>WRITE(61,099)CAVS(1), CAVP(1), CX(1)</td>
</tr>
<tr>
<td></td>
<td>GOTO107</td>
</tr>
<tr>
<td>109</td>
<td>IF(NTEST)111, 111, 110</td>
</tr>
<tr>
<td>110</td>
<td>WRITE(61,097)X(1)</td>
</tr>
<tr>
<td></td>
<td>WRITE(61,098)MSTEP, STRES, CX(1), TX(1)</td>
</tr>
<tr>
<td>111</td>
<td>DL(1)=A0+A1<em>TCOD+A2</em>(TCOD<strong>2)+A3*(TCOD</strong>3)+A4*(TCOD**4)+A5*(TCOD*</td>
</tr>
<tr>
<td></td>
<td>1.5)+A6*(TCOD<strong>6)+A7*(TCOD</strong>7)+A8*(TCOD<strong>8)+A9*(TCOD</strong>9)+A10*(TCOD*</td>
</tr>
<tr>
<td></td>
<td>2)**10))/CL(1)</td>
</tr>
<tr>
<td></td>
<td>ST(1,1)=0.0</td>
</tr>
<tr>
<td></td>
<td>ST(1,2)=0.0</td>
</tr>
<tr>
<td></td>
<td>ST(1,3)=0.0</td>
</tr>
<tr>
<td></td>
<td>ST(1,4)=0.0</td>
</tr>
<tr>
<td></td>
<td>DQ2801=2, LSTEP</td>
</tr>
<tr>
<td></td>
<td>CX(1)=COD-(COD-C1D)*(1.0-X(1))/(1.0-RATIO)</td>
</tr>
<tr>
<td></td>
<td>IF(NTEST)113, 113, 112</td>
</tr>
<tr>
<td>112</td>
<td>WRITE(61,097)X(1)</td>
</tr>
<tr>
<td>113</td>
<td>WRITE=C.C</td>
</tr>
<tr>
<td></td>
<td>STB=0.0</td>
</tr>
<tr>
<td></td>
<td>STM=0.0</td>
</tr>
</tbody>
</table>
Table 3 - Continued.

STAB=0.0
STRES=0.0
TX(I)=TX(I-1)*CX(I)/CX(I-1)
MSTEP=0
114 MSTEP=MSTEP+1
115 WRITE(61,098)MSTEP,STRES,CX(I),TX(I)
116 IF(I/2-(I-1)/2)450,130,120
120 NSTEP=I-2
BMT=T(I-1)*DX*DX*0.5*RAD/Z
BMQ=Q(I-1)*DX*DX*0.5/(Z*X(I-1))
FC=CON2*(X(I)*TX(I)*CX(I)+X(I-1)*TX(I-1))*CX(I-1))**0.5*DX
GOTO140
130 NSTEP=I-1
BMT=0.0
BMQ=0.0
FC=0.0
140 IF(I-2)450,170,150
150 DO160L=2,NSTEP,2
TM1=T(L-1)*(X(L-1)-X(I))
TM2=T(L)*(X(L)-X(I))
TM3=T(L+1)*(X(L+1)-X(I))
BMT=BMT+(TM1+4.0*TM2+TM3)*RAD*DX/(3.0*Z)
QM1=Q(L-1)*/(1.0-X(I)/X(L-1))
QM2=Q(L)*(1.0-X(I)/X(L))
QM3=Q(L+1)*(1.0-X(I)/X(L+1))
BMQ=BMQ+(QM1+4.0*QM2+QM3)*DX/(3.0*Z)
F1=X(L-1)*TX(L-1)*CX(L-1)
F2=X(L)*TX(L)*CX(L)
F3=X(L+1)*TX(L+1)*CX(L+1)
160 FC=FC+CON2*(F1+4.0*F2+F3)*DX/3.0
170 ANGB=ATANF(1.0/(DEL*EFF1*X(I))))
BMN=BMT*COSF(ANGB)+BMQ*SINF(ANGB)
BMQ=BMT*SINF(ANGB)-BMQ*COSF(ANGB)
ANG(I)=ANGB*180.0/PI
AAS=AS*TX(I)*CX(I)
Table 3 - Continued.

IF(CAVS(I)-CAVP(I)*CAV1)270,280,280
270 CLA=CL(I)
   CL(I)=0.74*CAV2*(SQRTF(CAVS(I)/CAV1+1.0)-1.0-1.14*TX(I)/CX(I))
   CX(I)=CLA*CX(I)/CL(I)
   IF(NTEST)113,113,275
275 WRITE(61,099)CAVS(I),CAVP(I),CX(I)
   GOTO113
280 CONTINUE
   CALTA=0.0
   CALTB=0.0
   EFFI=0.999
290 CALT=0.0
   DO300K=2,LSSTEP,2
   T(K-1)=EFFI*(X(K-1)**3)*(1.0-EFFI)*(1.0-DL(K-1)/(DEL*X(K-1)*EFFI))
   T(K)=EFFI*(X(K)**3)*(1.0-EFFI)*(1.0-DL(K)/(DEL*X(K)*EFFI))
   T(K+1)=EFFI*(X(K+1)**3)*(1.0-EFFI)*(1.0-DL(K+1)/(DEL*X(K+1)*EFFI))
300 CALT=CALT+(T(K-1)+4.0*T(K)+T(K+1))*DX/3.0
   IF(NTEST)306,306,305
305 WRITE(61,094)EFFI,CALT
306 IF(TT-CALT)330,360,310
310 CALTA=TT-CALT
   EFFA=EFFI
   IF(CALT>T)340,320,450
320 EFFI=EFFA-0.0010
   GOTO290
330 CALTB=TT-CALT
   EFB=EFFI
340 IF(EFFA-EBF-DEVN2)360,360,350
350 EFF1=(EFFA+EFB)/2.0
   GOTO290
360 ET8=EBF
   CALQ=0.0
   DO380K=2,LSSTEP,2
Table 3 - Continued.

\[ Q(K-1) = (1.0 + DL(K-1) \times X(K-1) \times ETB) \times CON1 \times VEL / OMEG \]
\[ Q(K) = (1.0 + DL(K) \times X(K) \times ETB) \times CON1 \times VEL / OMEG \]
\[ Q(K+1) = (1.0 + DL(K+1) \times X(K+1) \times ETB) \times CON1 \times VEL / OMEG \]

380 CALQ = CALQ + (Q(K-1) + (4.0 * Q(K) + Q(K+1)) * DX / 3.0)
385 IF (NTEST) 410, 385, 410
400 ETAI = ETB
   DO 390I = 1, LSTEP
      CX(K) = COD - (COD - CID) * (1.0 - X(K)) / (1.0 - RATIO)
      CL(K) = CON3 * X(K) * (1.0 - ETB) / (SQRDF((X(K) * DEL * ETB) ** 2 + 1.0) * CX(K))
   390 AM(K) = CL * CL(K) * CX(K)
   GOTO 104
410 ETAO = TT * VEL / (CALQ * OMEG)
   SHP = CALQ * OMEG / 550.0
   WRITE (61, 020)
   WRITE (61, 050) RAD, VEL, ROT, N, ETB, ETAO, TT, SHP
   WRITE (61, 055)
   ALST = STAL / 144.0
   WRITE (61, 056) CX(1), CX(LSTEP), RATIO, HTIP, ALST
   WRITE (61, 060)
   DO 420J = 1, LSTEP
   WRITE (61, 070) X(J), CX(J), TX(J), AM(J), ANG(J), CL(J), DL(J)
   WRITE (61, 080)
   DO 440J = 1, LSTEP
      CAVP(J) = CAVP(J) * CAV1
   DO 430I = 1, 4
   430 ST(J) = ST(J) / 144.0
   440 WRITE (61, 090) X(J), ST(J, 1), ST(J, 2), ST(J, 3), ST(J, 4), CAVS(J), CAVP(J),
      1T(J), Q(J)
   IF (NABS (ETA - ETB) - DEVN3) 100, 100, 400
450 STOP
END
Table 4 - Output for HYDRO - 1 Impeller.

**DESIGN PARAMETERS HYDROJET IMPELLER**

**DESIGN ASSUMPTIONS**
- BETZ MIN ENERGY CONDITION
- CONSTANT AXIAL VELOCITY
- NACA PROFILE
- SHOCK FREE ENTRY

<table>
<thead>
<tr>
<th>RADIUS</th>
<th>VELOCITY</th>
<th>ROTATION</th>
<th>BLADES</th>
<th>IDEAL EFF</th>
<th>TOTAL EFF</th>
<th>THRUST</th>
<th>SHP</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>19.30</td>
<td>1.50</td>
<td>4</td>
<td>0.9723</td>
<td>0.8699</td>
<td>44800</td>
<td>1807</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIP CHORD</th>
<th>BOSS CHORD</th>
<th>BOSS RATIO</th>
<th>TIP IMMERSION</th>
<th>ALLOWABLE STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.000</td>
<td>2.000</td>
<td>0.200</td>
<td>11.50</td>
<td>7250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RAD FRAC</th>
<th>CHORD</th>
<th>THICKNESS</th>
<th>MEAN LINE</th>
<th>ANGLE BI</th>
<th>LIFT COEF</th>
<th>DRAG/LIFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>5.000</td>
<td>0.2250</td>
<td>0.0926</td>
<td>15.67</td>
<td>0.249</td>
<td>0.030</td>
</tr>
<tr>
<td>0.95</td>
<td>4.875</td>
<td>0.2194</td>
<td>0.0876</td>
<td>16.45</td>
<td>0.241</td>
<td>0.031</td>
</tr>
<tr>
<td>0.90</td>
<td>4.750</td>
<td>0.2137</td>
<td>0.0826</td>
<td>17.31</td>
<td>0.233</td>
<td>0.032</td>
</tr>
<tr>
<td>0.85</td>
<td>4.625</td>
<td>0.2081</td>
<td>0.0776</td>
<td>18.26</td>
<td>0.225</td>
<td>0.033</td>
</tr>
<tr>
<td>0.80</td>
<td>4.500</td>
<td>0.2025</td>
<td>0.0726</td>
<td>19.32</td>
<td>0.217</td>
<td>0.034</td>
</tr>
<tr>
<td>0.75</td>
<td>4.375</td>
<td>0.1969</td>
<td>0.0675</td>
<td>20.51</td>
<td>0.207</td>
<td>0.036</td>
</tr>
<tr>
<td>0.70</td>
<td>4.250</td>
<td>0.1912</td>
<td>0.0625</td>
<td>21.84</td>
<td>0.197</td>
<td>0.038</td>
</tr>
<tr>
<td>0.65</td>
<td>4.125</td>
<td>0.2035</td>
<td>0.0574</td>
<td>23.34</td>
<td>0.187</td>
<td>0.040</td>
</tr>
<tr>
<td>0.60</td>
<td>4.000</td>
<td>0.2302</td>
<td>0.0523</td>
<td>25.06</td>
<td>0.175</td>
<td>0.044</td>
</tr>
<tr>
<td>0.55</td>
<td>3.875</td>
<td>0.2562</td>
<td>0.0471</td>
<td>27.02</td>
<td>0.163</td>
<td>0.049</td>
</tr>
<tr>
<td>0.50</td>
<td>3.750</td>
<td>0.2813</td>
<td>0.0419</td>
<td>29.29</td>
<td>0.150</td>
<td>0.056</td>
</tr>
<tr>
<td>0.45</td>
<td>3.625</td>
<td>0.3056</td>
<td>0.0367</td>
<td>31.94</td>
<td>0.136</td>
<td>0.066</td>
</tr>
<tr>
<td>0.40</td>
<td>3.500</td>
<td>0.3289</td>
<td>0.0315</td>
<td>35.04</td>
<td>0.121</td>
<td>0.080</td>
</tr>
<tr>
<td>0.35</td>
<td>3.375</td>
<td>0.3508</td>
<td>0.0263</td>
<td>38.71</td>
<td>0.104</td>
<td>0.100</td>
</tr>
<tr>
<td>0.30</td>
<td>3.250</td>
<td>0.3723</td>
<td>0.0211</td>
<td>43.08</td>
<td>0.087</td>
<td>0.130</td>
</tr>
<tr>
<td>0.25</td>
<td>3.125</td>
<td>0.3915</td>
<td>0.0160</td>
<td>48.29</td>
<td>0.069</td>
<td>0.178</td>
</tr>
<tr>
<td>0.20</td>
<td>3.000</td>
<td>0.4056</td>
<td>0.0112</td>
<td>54.51</td>
<td>0.050</td>
<td>0.264</td>
</tr>
</tbody>
</table>
### Table 4 - Continued.

<table>
<thead>
<tr>
<th>RAD FRAC</th>
<th>ST F</th>
<th>ST B</th>
<th>ST E</th>
<th>ST N</th>
<th>CAVT S</th>
<th>CAVT P</th>
<th>THRUST GRAD</th>
<th>TORQUE GRAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.555</td>
<td>0.347</td>
<td>0.181E 06</td>
<td>0.425E 05</td>
</tr>
<tr>
<td>0.95</td>
<td>140</td>
<td>-127</td>
<td>113</td>
<td>113</td>
<td>0.615</td>
<td>0.340</td>
<td>0.155E 06</td>
<td>0.363E 06</td>
</tr>
<tr>
<td>0.90</td>
<td>505</td>
<td>-555</td>
<td>385</td>
<td>386</td>
<td>0.685</td>
<td>0.333</td>
<td>0.132E 06</td>
<td>0.308E 06</td>
</tr>
<tr>
<td>0.85</td>
<td>1149</td>
<td>-1330</td>
<td>834</td>
<td>834</td>
<td>0.766</td>
<td>0.325</td>
<td>0.111E 06</td>
<td>0.259E 06</td>
</tr>
<tr>
<td>0.80</td>
<td>2089</td>
<td>-2476</td>
<td>1444</td>
<td>1441</td>
<td>0.861</td>
<td>0.317</td>
<td>0.092E 05</td>
<td>0.216E 06</td>
</tr>
<tr>
<td>0.75</td>
<td>3392</td>
<td>-4061</td>
<td>2229</td>
<td>2216</td>
<td>0.973</td>
<td>0.309</td>
<td>0.0759E 05</td>
<td>0.177E 06</td>
</tr>
<tr>
<td>0.70</td>
<td>5095</td>
<td>-6113</td>
<td>3168</td>
<td>3139</td>
<td>1.106</td>
<td>0.300</td>
<td>0.0616E 05</td>
<td>0.144E 06</td>
</tr>
<tr>
<td>0.65</td>
<td>6204</td>
<td>-7246</td>
<td>3300</td>
<td>3245</td>
<td>1.265</td>
<td>0.303</td>
<td>0.0492E 05</td>
<td>0.115E 06</td>
</tr>
<tr>
<td>0.60</td>
<td>6454</td>
<td>-7248</td>
<td>2758</td>
<td>2673</td>
<td>1.457</td>
<td>0.318</td>
<td>0.0386E 05</td>
<td>0.090E 05</td>
</tr>
<tr>
<td>0.55</td>
<td>6657</td>
<td>-7247</td>
<td>2330</td>
<td>2201</td>
<td>1.660</td>
<td>0.334</td>
<td>0.0296E 05</td>
<td>0.069E 05</td>
</tr>
<tr>
<td>0.50</td>
<td>6829</td>
<td>-7248</td>
<td>1989</td>
<td>1800</td>
<td>1.975</td>
<td>0.349</td>
<td>0.0221E 05</td>
<td>0.0528E 05</td>
</tr>
<tr>
<td>0.45</td>
<td>6976</td>
<td>-7249</td>
<td>1717</td>
<td>1446</td>
<td>2.327</td>
<td>0.365</td>
<td>0.0159E 05</td>
<td>0.0387E 05</td>
</tr>
<tr>
<td>0.40</td>
<td>7101</td>
<td>-7247</td>
<td>1503</td>
<td>1122</td>
<td>2.726</td>
<td>0.381</td>
<td>0.0110E 05</td>
<td>0.0274E 05</td>
</tr>
<tr>
<td>0.35</td>
<td>7211</td>
<td>-7247</td>
<td>1346</td>
<td>816</td>
<td>3.303</td>
<td>0.397</td>
<td>0.00719E 04</td>
<td>0.0185E 05</td>
</tr>
<tr>
<td>0.30</td>
<td>7247</td>
<td>-7184</td>
<td>1236</td>
<td>507</td>
<td>3.969</td>
<td>0.415</td>
<td>0.00433E 04</td>
<td>0.0118E 05</td>
</tr>
<tr>
<td>0.25</td>
<td>7247</td>
<td>-7099</td>
<td>1191</td>
<td>194</td>
<td>4.779</td>
<td>0.432</td>
<td>0.00228E 04</td>
<td>0.00695E 04</td>
</tr>
<tr>
<td>0.20</td>
<td>7251</td>
<td>-7030</td>
<td>1239</td>
<td>-135</td>
<td>5.727</td>
<td>0.446</td>
<td>0.00920E 03</td>
<td>0.00365E 04</td>
</tr>
</tbody>
</table>

### Table 5 - Input for HYDRO - 1 Impeller.

| 0.030 | 0.0450 | 1.9905 | 14.907 | 1.044E+06 | 11.50 |
| 0.736 | 0.0418 | 0.0445 | 0.029 | 0.5 | -0.182 | -0.016 | 0.818 | 0.516 | 0.0745 |
| 1.20  | 4.0000 | 2116.2 | 35.28 | 0.10E-03 | 0.10E-04 | 0.10E-03 | 0.10E-01 |
| 0.9299158E-02 | 0.349979E-01 | 0.2191732E+01 | 0.7515442E+02 | 0.8086659E+03 |
| 0.3567858E+04 | 0.6849617E+04 | 0.1329265E+06 | 0.5739262E+06 | 0.1119756E+07 |
| 0.8516336E+06 | 4.48000 | 19.30 | 1.50 | 7.50 | 0.50000000E-01 | 4.0 | 5.000 | 3.000 | 17.1 |
Table 6  List of Symbols

<table>
<thead>
<tr>
<th>PROGRAMME SYMBOL</th>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>MATHEMATICAL SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1, A10</td>
<td>Coefficients of polynomial $C_D = \Sigma A_i \left( \frac{r}{c} \right)^i$</td>
<td></td>
<td>$A_i$</td>
</tr>
<tr>
<td>AM(I)</td>
<td>Max. ordinate of mean line</td>
<td>ft</td>
<td>$m_x$</td>
</tr>
<tr>
<td>ANG(I)</td>
<td>Hydrodynamic pitch angle of blade element</td>
<td>degree</td>
<td>$\beta_i$</td>
</tr>
<tr>
<td>ANGB</td>
<td>&quot;</td>
<td>radian</td>
<td>$\beta_i$</td>
</tr>
<tr>
<td>AS</td>
<td>Blade area factor (Table 2)</td>
<td></td>
<td>$A_s$</td>
</tr>
<tr>
<td>AAS</td>
<td>Area of section</td>
<td>$r_t^2$</td>
<td>$A_s$</td>
</tr>
<tr>
<td>BMN</td>
<td>Section bending moment about axis normal to chord</td>
<td>lbf ft</td>
<td>$M_n$</td>
</tr>
<tr>
<td>BMNP</td>
<td>Section bending moment about axis parallel to chord</td>
<td></td>
<td>$M_p$</td>
</tr>
<tr>
<td>BMQ</td>
<td>Section bending moment due to Q</td>
<td></td>
<td>$M_Q$</td>
</tr>
<tr>
<td>BMT</td>
<td>&quot;</td>
<td></td>
<td>$M_T$</td>
</tr>
<tr>
<td>CALQ</td>
<td>Calculated torque of impeller</td>
<td></td>
<td>$Q$</td>
</tr>
<tr>
<td>CALT</td>
<td>Calculated thrust of impeller</td>
<td>lbf</td>
<td>$T$</td>
</tr>
<tr>
<td>CAV1</td>
<td>Overall factor for non potential flow</td>
<td>eqn 26</td>
<td>$f_o$</td>
</tr>
<tr>
<td>CAV2</td>
<td>Constant in 3rd term of eqn. 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAVP(I)</td>
<td>Pressure min. cavitation no. at a section</td>
<td></td>
<td>$\sigma_p$</td>
</tr>
<tr>
<td>CAVS(I)</td>
<td>Cavitation no. of a section</td>
<td></td>
<td>$\sigma_s$</td>
</tr>
<tr>
<td>CID</td>
<td>Section chord width at boss</td>
<td>ft</td>
<td>$\sigma_s$</td>
</tr>
<tr>
<td>CL(I)</td>
<td>Sectional lift coefficient</td>
<td></td>
<td>$C_L$</td>
</tr>
<tr>
<td>CLI</td>
<td>Constant in eqn. 20 (for NACA-16, =0.0745)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>COD</td>
<td>Section chord width at tip</td>
<td>ft</td>
<td>$K_1$</td>
</tr>
<tr>
<td>CON1</td>
<td>Force parameter (see Section 2.1)</td>
<td>lbf</td>
<td>$K_1$</td>
</tr>
</tbody>
</table>
Table 6 cont.  List of Symbols

<table>
<thead>
<tr>
<th>PROGRAMME SYMBOL</th>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>MATHEMATICAL SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CX(I)</td>
<td>Sectional chord width of section</td>
<td>ft</td>
<td>c</td>
</tr>
<tr>
<td>DEL</td>
<td>Advance coefficient</td>
<td></td>
<td>δ</td>
</tr>
<tr>
<td>DENM</td>
<td>Density of impeller material</td>
<td>slug/ft$^3$</td>
<td>$\rho_m$</td>
</tr>
<tr>
<td>DENW</td>
<td>Density of working fluid</td>
<td>&quot;</td>
<td>$\rho$</td>
</tr>
<tr>
<td>DEVN1</td>
<td>Calculation accuracy of TX</td>
<td>ft</td>
<td></td>
</tr>
<tr>
<td>DEVN2</td>
<td>&quot;</td>
<td></td>
<td>EFFI from thrust gradient</td>
</tr>
<tr>
<td>DEVN3</td>
<td>Calculation accuracy of $\eta_i$</td>
<td></td>
<td>= $\frac{C_D}{C_L}$</td>
</tr>
<tr>
<td>DL(I)</td>
<td>Drag to lift ratio of section</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DTOL</td>
<td>Drag to lift ratio for initial approximation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DX</td>
<td>Distance between sections as percentage of R</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EFFI</td>
<td>Ideal blade element efficiency</td>
<td></td>
<td>$\eta_i$</td>
</tr>
<tr>
<td>ETAI</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot; estimation at beginning of calculation</td>
</tr>
<tr>
<td>ETAO</td>
<td>Impeller efficiency</td>
<td></td>
<td>$\eta_o$</td>
</tr>
<tr>
<td>FC</td>
<td>Centrifugal force acting on a section</td>
<td>lbf</td>
<td>$F_c$</td>
</tr>
<tr>
<td>HB</td>
<td>Blade section property (Table 2)</td>
<td>ft</td>
<td>$h$</td>
</tr>
<tr>
<td>HBC</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;$F_c$</td>
</tr>
<tr>
<td>HBT</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;$F_t$</td>
</tr>
<tr>
<td>HBTC</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;$F_t$</td>
</tr>
<tr>
<td>HTIP</td>
<td>Depth of immersion of blade tip</td>
<td>ft</td>
<td>$H$</td>
</tr>
<tr>
<td>LSTEP</td>
<td>Number of stations (see section 4.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTEST</td>
<td>Control for progressive print out of intermediate calculated variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OMEG</td>
<td>Angular velocity of impeller</td>
<td>rad/s</td>
<td>$\omega$</td>
</tr>
</tbody>
</table>
Table 6 cont.  List of Symbols

<table>
<thead>
<tr>
<th>PROGRAMME SYMBOL</th>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>MATHEMATICAL SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAT</td>
<td>Atmospheric pressure</td>
<td>lb/ft(^2)</td>
<td>P(_A)</td>
</tr>
<tr>
<td>PIN</td>
<td>Section moment of inertia about an axis</td>
<td>in(^4)</td>
<td>I(_n)</td>
</tr>
<tr>
<td>PIP</td>
<td>Section moment of inertia about an axis</td>
<td>&quot;</td>
<td>I(_p)</td>
</tr>
<tr>
<td>PPINA</td>
<td>Blade section property (Table 2)</td>
<td>&quot;</td>
<td>i(_n)</td>
</tr>
<tr>
<td>PPINB</td>
<td>&quot; &quot; &quot; &quot; &quot;</td>
<td>&quot;</td>
<td>i(_p)</td>
</tr>
<tr>
<td>Q(I)</td>
<td>Torque gradient at section</td>
<td>lbf/ft</td>
<td>(\frac{dQ}{dx})</td>
</tr>
<tr>
<td>RAD</td>
<td>Radius of impeller</td>
<td>ft</td>
<td>R</td>
</tr>
<tr>
<td>RATIO</td>
<td>Hub radius to impeller radius</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ROT</td>
<td>Rotational speed</td>
<td>rps</td>
<td>n</td>
</tr>
<tr>
<td>SHP</td>
<td>Shaft horse power at impeller</td>
<td>hp</td>
<td></td>
</tr>
<tr>
<td>STAL</td>
<td>Allowable design stress</td>
<td>lbf/ft(^2)</td>
<td>(\sigma_{design})</td>
</tr>
<tr>
<td>STEP1</td>
<td>Increase in blade thickness in &quot;Regula Falsi&quot;</td>
<td>ft</td>
<td></td>
</tr>
<tr>
<td>STM</td>
<td>Maximum numerical stress at a section</td>
<td>lbf/ft(^2)</td>
<td>(</td>
</tr>
<tr>
<td>ST(I J)</td>
<td>Stress at extremities of a section</td>
<td>&quot;</td>
<td>(\frac{dT}{dx})</td>
</tr>
<tr>
<td>T(I)</td>
<td>Thrust gradient at a section</td>
<td>lbf</td>
<td></td>
</tr>
<tr>
<td>TCOD</td>
<td>Thickness to chord ratio for minimum profile drag</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TT</td>
<td>Design thrust</td>
<td>lbf</td>
<td>T</td>
</tr>
<tr>
<td>TX(I)</td>
<td>Maximum thickness of a section</td>
<td>ft</td>
<td>(t_x)</td>
</tr>
</tbody>
</table>
Table 6 cont. List of Symbols

<table>
<thead>
<tr>
<th>PROGRAMME SYMBOL</th>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>MATHEMATICAL SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAP</td>
<td>Vapour pressure of working fluid</td>
<td>lb/ft²</td>
<td>e</td>
</tr>
<tr>
<td>VEL</td>
<td>Axial velocity of fluid at impeller</td>
<td>ft/s</td>
<td>vₐ</td>
</tr>
<tr>
<td>X(I)</td>
<td>Radius fraction of section</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>YB</td>
<td>Blade section properties (Table 2)</td>
<td>ft</td>
<td>ỹ</td>
</tr>
<tr>
<td>YBC</td>
<td></td>
<td></td>
<td>ỹₛ</td>
</tr>
<tr>
<td>YBCCT</td>
<td></td>
<td></td>
<td>ỹₛ / t</td>
</tr>
<tr>
<td>YBTA</td>
<td></td>
<td></td>
<td>ỹₜ</td>
</tr>
<tr>
<td>YBTB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>No. of blades</td>
<td></td>
<td>Z</td>
</tr>
</tbody>
</table>
Table 6 cont. List of Symbols

<table>
<thead>
<tr>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>MATHEMATICAL SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sectional drag coefficient (2-dimensional flow)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Diameter of impeller</td>
<td>ft</td>
<td>D</td>
</tr>
<tr>
<td>Profile drag of blade element</td>
<td>lbf</td>
<td>dD</td>
</tr>
<tr>
<td>Actual lift</td>
<td></td>
<td>dL</td>
</tr>
<tr>
<td>Actual torque</td>
<td>lbf ft</td>
<td>dQ</td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>dQ&lt;sub&gt;i&lt;/sub&gt;</td>
</tr>
<tr>
<td>Actual thrust</td>
<td>lbf</td>
<td>dT</td>
</tr>
<tr>
<td>Ideal</td>
<td></td>
<td>dT&lt;sub&gt;i&lt;/sub&gt;</td>
</tr>
<tr>
<td>Distance of point on section from centroid parallel to chord line</td>
<td>ft</td>
<td>h</td>
</tr>
<tr>
<td>Lift camber factor, eqn. 20</td>
<td></td>
<td>l&lt;sub&gt;m&lt;/sub&gt;</td>
</tr>
<tr>
<td>Free stream static pressure</td>
<td>lbf/ft&lt;sup&gt;2&lt;/sup&gt;</td>
<td>P</td>
</tr>
<tr>
<td>Min. value of local pressure &lt;sub&gt;p_e&lt;/sub&gt; on surface of section</td>
<td>lbf/ft&lt;sup&gt;2&lt;/sup&gt;</td>
<td>p&lt;sub&gt;e&lt;/sub&gt;</td>
</tr>
<tr>
<td>Pressure at blade section radius &lt;sub&gt;r&lt;/sub&gt; at minimum immersion</td>
<td></td>
<td>p&lt;sub&gt;r&lt;/sub&gt;</td>
</tr>
<tr>
<td>Radius of blade element</td>
<td>ft</td>
<td>r</td>
</tr>
<tr>
<td>Circumferential component of induced velocity in fully developed wake</td>
<td>ft/s</td>
<td>U&lt;sub&gt;It&lt;/sub&gt;</td>
</tr>
<tr>
<td>Velocity of flow relative to blade including induced flow effects</td>
<td></td>
<td>V&lt;sub&gt;R&lt;/sub&gt;</td>
</tr>
<tr>
<td>Normal distance of point on section from a line through centroid parallel to chord line</td>
<td>ft</td>
<td>y</td>
</tr>
<tr>
<td>Advance angle of blade element</td>
<td>rad</td>
<td>θ</td>
</tr>
<tr>
<td>Circulation</td>
<td>ft&lt;sup&gt;2&lt;/sup&gt;/s</td>
<td>Γ</td>
</tr>
<tr>
<td>Viscosity factor eqn. 20</td>
<td></td>
<td>μ&lt;sub&gt;m&lt;/sub&gt;</td>
</tr>
<tr>
<td>Fibre stress at a point on a section</td>
<td>lbf/ft&lt;sup&gt;2&lt;/sup&gt;</td>
<td>S</td>
</tr>
<tr>
<td>Pitch angle of blade element equals θ&lt;sub&gt;i&lt;/sub&gt; in the example impeller</td>
<td>rad</td>
<td>φ</td>
</tr>
</tbody>
</table>
Fig. 1 - Velocities and Forces at Blade Section.
Fig. 2 - Programme Flow Diagram.
I

260

NO

WILL
SECTION
CAVITATE?

YES

INCREASE
CHORD
WIDTH

1

280

J

SELECT $\gamma_4 = 0.999$

K

CALCULATE THRUST
BY SUMMATION OF
THRUST GRADIENTS

NO

CONVERGENCE
ROUTINE

YES

IS
CALCULATED
THRUST
DESIGN?

DECREASE
$\gamma_4$

N

M

NOWITHIN DESIRED
ACCURACY OF
PREVIOUS VALUE?

NO

SET UP NEW
SECTION
VARIABLES

YES

OUTPUT

L

CALCULATE TORQUE
BY SUMMATION OF
TORQUE GRADIENTS

IS $\gamma_4$
EQUAL

40. Fig. 2
Fig. 2 - Continued. "Regula Falsi" Convergence Routine for Blade Thickness.
APPENDIX A2

COMPUTATION OF
RECTANGULAR MACHINING CO-ORDINATES
for
AN ARBITRARY IMPELLER DESIGN

by

M.R.Hale

UNIVERSITY OF ADELAIDE
DEPARTMENT OF MECHANICAL ENGINEERING
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>1.0 INTRODUCTION</td>
<td>3</td>
</tr>
<tr>
<td>2.0 THE DEFINITION OF BLADE SHAPE</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Restriction on blade shape</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Blade shape equations</td>
<td>5</td>
</tr>
<tr>
<td>2.3 The blade shape of the Hydrojet Impeller</td>
<td>9</td>
</tr>
<tr>
<td>3.0 THE METHOD OF COMPUTING THE MACHINING CO-ORDINATES</td>
<td>10</td>
</tr>
<tr>
<td>3.1 Computing Stages</td>
<td>10</td>
</tr>
<tr>
<td>3.2 Points, Reference Planes and Axes</td>
<td>10</td>
</tr>
<tr>
<td>3.3 Computation of Surface Points</td>
<td>15</td>
</tr>
<tr>
<td>3.4 Direction Cosines at a Point</td>
<td>18</td>
</tr>
<tr>
<td>4.0 STABILITY OF CALCULATION</td>
<td>21</td>
</tr>
<tr>
<td>4.1 Convergence in computing boundary points</td>
<td>21</td>
</tr>
<tr>
<td>4.2 Convergence in computing points on surface</td>
<td>22</td>
</tr>
<tr>
<td>5.0 INPUT/OUTPUT INSTRUCTIONS AND EXECUTION TIME</td>
<td>23</td>
</tr>
<tr>
<td>5.1 Input Instructions</td>
<td>23</td>
</tr>
<tr>
<td>5.2 Output Instructions</td>
<td>25</td>
</tr>
<tr>
<td>5.3 Execution Time</td>
<td>26</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
</tr>
<tr>
<td>TABLES</td>
<td></td>
</tr>
<tr>
<td>FIGURES</td>
<td></td>
</tr>
</tbody>
</table>
COMPUTATION OF
RECTANGULAR MACHINING CO-ORDINATES FOR AN ARBITRARY IMPELLER DESIGN

by M.R. Hale

ABSTRACT

A digital-computer programme in FORTRAN has been developed to calculate the co-ordinates of points on the surface of an impeller (or propeller) blade of arbitrary shape from a series of specifying equations. The programme then computes the co-ordinates of points on the locus-surface of the centre of a spherical cutter which would machine the prescribed surface. All the points calculated lie on a rectangular grid. Although the impeller shape can be arbitrary, it must be possible to define the blade shape by a series of equations of given form. Certain minor restrictions on impeller geometry must also be satisfied.

The blade-shape equations which are required are those which define the following:

(1) Radial chord distribution.
(2) Maximum-sectional-thickness distribution.
(3) Maximum-ordinate of mean line distribution.
(4) Blade angle distribution.
(5) Thickness form of the blade sections.
(6) Profile of the mean line.

The restrictions placed on the impeller geometry are the following:

(1) The blade profiles at all sections must have a similar basic form, superimposed on the mean line.
(2) The mean lines must be similar at all sections, but may have varying maximum ordinates.
(3) The centroids of all sections must lie on a straight radial line, i.e., the blades must have no rake or skew.

The number of computed surface points is limited by the storage capacity of the computer. With the IBM 709/7090 II computer, having a total storage capacity of 32,561 words (about 28,000 usable words) a matrix of points 53 by 120 can be calculated for each side of the blade.

The output of the programme is a typewritten list of machining coordinates and instructions in a form suitable for the operator of a hand-operated vertical miller.

The programme can be modified to cope with impeller or propeller designs which do not satisfy the above restrictions. It can also be extended to generate machining instructions for an automatic miller with magnetic tape control.
The most common method of machining an impeller or propeller is to use a cutter which moves (relative to the blade) on a cylindrical path with centre on the axis of the impeller. The cutter motion is controlled by a follower moving over a series of templates which may be either cylindrical or expanded-cylindrical sections depending on the mechanism used to convert movement of the follower to movement of the cutter.

Another method, which is suited for the majority of milling machines, is based on spot-machining of points on the blade surface using either a polar or a rectangular grid. This method requires considerable computation to transform an object naturally defined by polar co-ordinates into one defined by cartesian co-ordinates if the latter are used.

In all the machining techniques commonly employed it is usual to use only a small number of sections to define the complete blade shape. Hand machining is then used to "fair in" between the machined regions.

Disadvantages of the above methods are:-

(1) The time consumed in hand-fairing between the accurately machined sections.
(2) The blade surface can only be as accurate as the templates.
(3) The accuracy is dependent on the size of the cutter. It is not usual to attempt to define the locus of a cutter moving over the blade surface at the desired section. As the template shape is usually that of a blade section, an appreciable error is thus introduced unless the cutter has a cutting-edge radius small in comparison with the radius of curvature of the blade surface at the point being considered.
Perhaps the ideal method of machining an impeller would be to make use of an automatically-controlled milling machine using a magnetic tape as an input medium for all machining instructions and control. The technique described in the paper does not satisfy this ideal completely but is a step towards the final objective.

This paper outlines a method of determining the co-ordinates of points on the locus-surface of the centre of a spherical cutter, such that the surface being machined would be the surface of an arbitrarily defined impeller blade. The machined points are obtained in a rectangular grid pattern.

The use of the programme is illustrated by considering the design of impeller which is being used in a model study of a Hydrojet Propulsion Unit (Ref. 1). The 8 inch model impeller (scale ratio of 22.5) was machined in a Pantograph copying machine from a three-dimensional master template. This master template which was four times larger than the model, was machined in a universal miller to dimensions calculated by this programme. The intermediate stage of using a master template was introduced to reduce hand-fairing errors, and to reduce the number of machining co-ordinates necessary to obtain the required accuracy.

2.0

THE DEFINITION OF BLADE SHAPE

2.1 Restrictions on blade shape.

The programme as described in the following sections restricts the blade shape by requiring zero rake and skew, and requiring that the centroids of all sections lie on a straight radial line. This restriction could easily be removed by inserting extra equations defining these parameters in terms of the radius.

The most important limitation concerns the distribution of blade section
shapes. Corresponding ordinates of blade sections at all radii from tip to boss must be proportional to the maximum thickness of the sections. The mean line shape must also be similar at all sections i.e., corresponding ordinates of the mean line must be proportional to the maximum displacements of the mean lines. These restrictions simplify the problem of defining the whole impeller blade by a series of related equations.

2.2 Blade shape equations

The following shape parameters must be defined in terms of equations:

(1) Chord-width distribution with respect to radius.
(2) Maximum sectional thickness with respect to radius.
(3) Maximum ordinate of the mean line with respect to radius.
(4) Blade angle distribution with respect to radius.
(5) Thickness form of the blade section with respect to unit thickness and unit chord width.
(6) Profile of the mean line with respect to unit chord width.

The form of these equations expressing the blade shape parameters is not restricted in any way. If any of the equations differ from those used for the Hydrojet impeller, as expressed below, then these equations must replace the corresponding equations in the programme as written in Table 2. The majority of statements which refer to the shape parameters are found between statements numbered 600 and 660.

In the case of the Hydrojet impeller some of these relationships were expressed in the form of polynomial expansions. A previous programme (Ref. 1) had carried out the design of the impeller. From its output tabular values defining the above distributions (See Table 1) were taken. For some of the blade-shape distributions polynomials were fitted to the tabular values using the least-square-deviation technique. The
polynomial coefficients were calculated with the aid of IBM Library Programme No. 7.0.002. This uses a modified Gaussian elimination method to solve equations representing the condition for least-square deviation, the polynomial coefficients then being calculated by back substitution.

The equations representing the design data for the impeller under consideration are given below. Reference should be made to Table 5 for the nomenclature used.

(1) **Chord width:** The impeller design chosen has a linear chord distribution and this was represented by the following relationship between the chord at the tip COD, and the chord at the boss CID, for varying radii RX:

\[
CX = COD - (COD - CID) \times \frac{(RADM - RX)}{(RADM * (1.0 - RATIO))}
\]

... 1

This assumes a hub to diameter ratio of RATIO.

(2) **Maximum Section Thickness:** The thickness distribution with respect to radius was expressed by three different equations, each being applicable over a certain region.

Between the radius fraction 0.7 and 1.0, where the maximum thickness TX of the section was determined by minimum profile drag conditions, the relationship was

\[
TX = 0.045 \times CX
\]

... 2

Over the interval between radius fraction 0.65 and 0.7 where the working stress is made equal to the allowable stress, but where the change in design thickness is small, the thickness to be used in the programme was assumed constant.
For the region between radius fraction 0.2 and 0.65 where the thickness is determined by the design stress, a third-order-polynomial expression was used to specify the thickness to within an accuracy of 0.00012 ft. or 0.3%.

\[ T_x = \leq T_i (XR)^i \]  \hspace{1cm} \ldots \hspace{1cm} 3.

where \( i = 0, 1, 2, 3 \)

and \( XR = \) radius fraction

(3) **Maximum mean-line ordinate:** This distribution was defined by a fourth-order polynomial to an accuracy of 0.5%, the equation being

\[ AM = \leq A_i (XR)^i \]  \hspace{1cm} \ldots \hspace{1cm} 4.

where \( i = 0, 1, 2, 3, 4 \).

(4) **Blade angle:** The impeller was designed to have the chord line at all sections lying on a helical surface. The blade angle was defined as:

\[ \text{ANG} = \arctan \left( \frac{\text{PITCH}}{2\pi RX} \right) \]  \hspace{1cm} \ldots \hspace{1cm} 5.

(5) **Thickness form of the blade section:** The blade section used for the impeller was a NACA - 16 profile. (Ref. 2, p155). Because of the large change in curvature from the leading edge to the trailing edge, an equation, differing in form from a polynomial, had to be found to describe the shape of the section.

An equation was first chosen to give the correct curvature on the leading edge. The profile was assumed in this region to be given (See Ref. 3) by a parabola of the form

\[ y_o^2 = 2Kx \]

where \( y_o \) is the y ordinate at a fractional distance x along the chord.
Since radius of curvature \( \rho = \frac{\left[ 1 + \left( \frac{d\chi}{dy} \right)^2 \right]^{3/2}}{\frac{d^2\chi}{dy^2}} \) \( 6. \)

then at the L.E. where \( y_o = 0, \ x = 0 \) \( 7. \)

\[ \rho = K \]

For this profile \( \rho = K \) may be calculated at the L.E. as 0.4888% of chord for unit thickness

At the T.E. where \( x = 1.0 \) the profile equation must satisfy \( y_o = 0, \ x = 1.0 \)

To satisfy this condition the parabolic equation is modified to

\[ y_o = (1 - x) \sqrt{2Kx} \] \( 8. \)

The ordinates given by equation 8, if subtracted from the corresponding ordinates of the NACA - 16 profile give residual ordinates which can be approximated by a polynomial expansion. The final equation for the profile thus has the following form:

\[ Y_0 = (1 - XX)[\sqrt{0.98879XX}] + \varepsilon Y_i XX^i \] \( 9. \)

where \( i = 0, 1, ..., 9 \)

A tenth-order polynomial was necessary if the maximum percentage error was to be limited to 0.17%. The overall accuracy of this method for defining such an aerofoil section can be judged from the deviations shown in Fig. 1.

The ideal trailing edge thickness of the blade is zero. As this is impractical, and in any case is impossible to machine, the edge thickness of the model impeller was made equal to the smallest machinable dimension. Hence the blade section ordinates on the trailing edge half of the blade were increased above the calculated theoretical value
$Y_0^* = Y_0 + \text{ALTE} * \text{SCALM} * (XX - 0.5) / (\text{SCALE} * TX)$  

This equation applies to the model in question, which has a scale ratio of SCALM (1:22.5). The allowable edge thickness ALTE was chosen to be 0.020 of an inch. The factor SCALE in equation is the scale ratio of the master template to the prototype.

(6) **Profile of mean line:** The NACA $a = 1.0$ mean line was used in the impeller design. The equation representing the shape of the mean line is (Ref. 4, equation 4.26)

$$\frac{y_m}{c} = \frac{C_{li}}{4\pi} \left[ (1 - x) \log_e (1 - x) + x \log_e x \right]$$  

where $y_m = YM = y$ ordinate of mean line

$C_{li} = \text{Ideal lift coefficient for non-viscous fluid}$

$c = \text{Chord width}$

$x = XX = \text{Fractional distance along chord}$

but $\frac{f}{c} = 0.05515 C_{li}$

where $f = AM = \text{Max. ordinate of mean line}$

thus $\frac{YM}{AM} = \frac{1}{4\pi} (0.05515) \left[ (1 - XX) \log_e (1 - XX) + XX \log_e XX \right]$  

$2.3 \ \text{The Blade Shape of the Hydrojet Impeller}$

The dimensions of the Hydrojet impeller are given together with the estimated operating characteristics of the impeller in Table 1. From this data equations representing the blade shape parameters were determined in the form described in Section 2.2. These equations formed part of the input to the machining co-ordinate programme.
3.0 THE METHOD OF COMPUTING THE MACHINING CO-ORDINATES

3.1 Computing Stages

The programme can be divided into two major stages:

(1) From the blade shape equations previously discussed, the co-ordinates of the following surface points are calculated:

(i) Points on the leading and trailing edges of the blade surface.

(ii) Points on the face and back surfaces of the blade.

(2) The direction cosines of the normal to the surface at each of the above points are determined. By making the length of each normal equal to the radius of the spherical cutter, the extra-surface points on the required locus-surface described by the cutter centre are obtained. For convenience of the machinist the co-ordinates of the bottom of the cutter are specified in the programme output.

A flow diagram representing the important steps in the computation is given in Fig. 2.

The complete programme listing is given in Table 2.

3.2 Points, Reference Planes and Axes

To describe the position of any point in space four sets of axes were used. These can be divided into two groups - one groups being used in Stage (1) of Section 3.1 and the other in Stage (2) of the same Section.

3.2.1 Surface Points

For the calculation of surface points (Stage 1) the following pairs of reference axes are used (See Fig. 3):-
(1) A set of right-handed cartesian axes $X, Y, Z$, aligned so that $X$ lies along the line of centroids of the blade sections and $Z$ along the axis of the impeller.

(2) A set of right-handed cartesian axes $XB, YB, ZB$, aligned parallel to $X, Y, Z$, and with $YB$ co-linear with $Y$, as shown in Fig. 3. The distance between the $X, Z$ plane and $XB, ZB$ plane is denoted $YD$.

To simplify the computation, the points chosen to define the blade surface were those resulting from the intersection of a two-dimensional fundamental mesh (Ref. 6) with the blade surface. The planes forming the fundamental mesh were aligned respectively parallel to $X, Z$ axes and the $Y, Z$ axes.

Planes parallel to $X, Z$ axes are called column planes and planes parallel to $Y, Z$, axes are called row planes or stations. The distance between column planes is not equal (See below). The distance between row planes is equal, and has a value $DX$.

The row planes are identified as $I = 1, 2, 3, \ldots$ as shown in Fig. 5. It was necessary to select $I = 1$ outside the blade surface to simplify the computing routine. Thus the row plane passing through the tip of the impeller on the $X$ axes (see Fig. 5) was chosen to be $I = 5$ row plane.

The column planes associated with each row plane are named separately (Fig. 5) and by two different methods:

1. **Nomenclature of column planes - Method 1.**

   The plane containing the axes $X, Z$, is chosen as the datum plane $N = 0$ and planes on either side are named by $N = 1, 2, 3, \ldots$, an additional identification of $M = 1$ or 2 being required.
to indicate which side of the datum is being considered. This distance between planes is (up to a certain distance from \( N = 0 \)) equal, and of value \( \text{SPACX} \). At a predetermined distance from the \( N = 0 \) plane the spacing between the column planes is changed to \( \text{SPACN} \) (one fifth of \( \text{SPACX} \)) and the identification of the planes continued numerically without further interruption. The predetermined distance is chosen to be equal to or greater than a given fractional distance of the value of the \( Y \) ordinate of the point on the leading or trailing edge on the same row plane. This fractional value \( \text{PCX} \) is given as an input instruction to the programme. The reason for using planes with a smaller spacing (\( \text{SPACN} \)) over part of the blade is to define more accurately the blade surface in the region where the surface curvature is increasing rapidly near the blade edges.

The numerical value of \( N \) associated with the column planes where the spacing distance changes from \( \text{SPACX} \) to \( \text{SPACN} \) is recorded as the value of \( \text{MLE} (I) \) and of \( \text{MTB} (I) \) where these correspond to the leading and trailing edge respectively at a given row plane \( I \). The total number of \( N \) planes on either side of \( X, Z \) plane are recorded as \( \text{NLE} (I) \) and \( \text{NTB} (L) \) for the leading and trailing edge side at a given row plane \( I \) (Fig. 5).

It should be noted that column planes are not equally spaced over the entire blade surface, and also that points on the same plane parallel to the \( X, Z \) axes but in different row planes may not have the same numerical value of \( N \).

Associated with any point on the blade surface a curved plane \( \text{YR}, \text{ZR} \) (Figs. 3, 5) passing through the point can be defined as a portion of a cylindrical surface with its axis coincident with the impeller axis.
The above identification of column planes applies only to the computation in Stage (1). The following nomenclature was used in the programme to store values associated with these planes:

(2) **Nomenclature of column planes - Method 2.**

The column planes $N$ are identified in another way by being numbered consecutively from the trailing edge to the leading edge with the identifier (subscript) of $NNN$. The values of $NNN$ are chosen so that the numerical value of the $NNN$ plane containing the $X$, $Z$ axes (i.e. $N = 0$ plane) is made equal to a specified value $NN$. The value of $NN$ must be chosen so that the identifier $NNN$ for any column planes on the blade surface is positive and at least greater than $NNN = 5$. The chosen value of $NN$ must be specified as an input statement. For the Hydrojet impeller, the value of $NN$ was chosen to be 60.

It should be noted that although the column plane $NNN$ are numbered consecutively their spacing is not constant and that for different row planes the column planes which hold the same value of $NNN$ are not necessarily co-planar.

A point on the blade surface can be identified by the numerical values of the row and column planes which pass through it. The additional subscript $J = 1$ indicates back or face surface of the blade. Thus a point $(I, NNN, J) = (6, 13, 2)$ would be a point on the face surface of the blade at $I = 6$ whose identifier has a value of $NNN = 13$.

The co-ordinates of points associated with the axes $XB$, $YB$, $ZB$ are given by $XB(I, NNN, J)$, $YB(I, NNN, J)$ and $ZB(I, NNN, J)$. For example $ZB(I, NNN, J)$ is the distance from the point $(I, NNN, J)$ to the $XB$, $YB$ plane.
Before the direction cosines of the normal to the surface at a point can be calculated, a number of neighbouring points must be located from the complete array of points which was located in the store of the computer. To aid the identification of points in this section of the programme, each point is now renamed according to its distance from the \( N = 0 \) plane.

Consider a series of planes parallel to \( N = 0 \) plane, which have a spacing interval of \( \text{SPACN} \). On the trailing-edge side of the axes \( X \), \( Z \) a column plane is chosen as a datum plane \( NK = 0 \) and all the column planes spaced \( \text{SPACN} \) are identified consecutively from the datum as \( NK = 1, 2, 3, \ldots \) (Fig. 5). The numerical value of the \( NK \) plane containing the \( X \), \( Y \) axes (i.e. \( N = 0 \) plane) is represented by \( \text{NKKN} \) in the programme. The value of \( \text{NKKN} \) is controlled by an input statement and its value is further discussed in Section 5.1.

Thus in the centre of the blade surface where the spacing of the surface points is \( \text{SPACX} \), every fifth \( NK \) plane will be an \( N \) plane. Points which hold the same value of the identifier \( NK \) will be physically on the same column plane, and hence are easily distinguished from the complete array of points.

The 16 points which surround a given mesh point from which a normal is to be erected are located using the \( NK \) identification above. These points are then re-identified by a separate nomenclature \( \text{KA} \). The mesh point is chosen to be the point \( \text{KA} = 5 \). The points on the row plane passing through \( \text{KA} = 5 \) are identified as \( L = 1 \) and \( \text{KA} = 1 \) to 9 in the same sense as the positive direction of \( \text{NNN} \) (See Fig. 5). The points on a column plane through \( \text{KA} = 5 \) are denoted by \( L = 2 \) and \( \text{KA} = 1 \) to 9 in the same sense as the positive \( X \) direction. Each of these points \( \text{KA} = 1 \) to 9 must have the same spacing interval.
3.2.2 Extra - Surface Points

To identify points outside the blade surface in the section of the programme which computes the direction cosines of the normals, and the machining co-ordinates, (i.e. Stage (2) Section 3.1) the following pair of reference axes are used (Fig. 4).

(1) A set of right-handed cartesian axes XN, YN, ZN which are parallel to the X, Y, Z axes but with XN displaced a distance YD from X, and with YN displaced a distance ZD from Y (See Fig. 4). This set of axes is only used to identify the machining points associated with the face surface.

(2) A set of right-handed cartesian axes XM, YM, ZM obtained by rotating bodily the set of axes XN, YN, ZN about the X axis as shown in Fig. 4. This set of axes identifies machining points associated with the back surface.

The co-ordinates of points related to these axes are specified as XXX(J), YYY(J), ZZZ(J) where \( J = 1 \) or 2 refers to the back or face and hence identifies which set of axes is being considered.

3.3 Computation of Surface Points

The computation of the co-ordinates of points on the blade surface from the blade shape equations is divided into two sections:

(1) Computation of points on the boundary edge of the blade surface for each row plane.

(2) Computation of points on the blade surface at the intersection of each row and column plane.

3.3.1 Computation of Boundary Points

The co-ordinates of points on the leading and trailing edges on a
given row plane are calculated and stored so that the calculation of points in the following section 3.3.2 can be confined to only the blade surface (See Fig. 2/1).

The iterative convergence procedure called "Regular Falsi" is used to compute these boundary points (See Fig. 2/13). Here the independent variable, radius RX, is adjusted to make the dependent variable, length-of-the-arc CA equal to CB a known proportion of the chord at the radial distance RX. These distances are shown in Fig. 5.

Before entering the "Regular Falsi" routine the distance RX (See Fig 2) was approximated by

$$RX = RADM \times PERCR + X$$

where $$X = X$$ ordinate of the row plane

The value of PERCR must be selected to produce CA just greater than CB for all row planes of the impeller.

The Regular Falsi procedure is as follows.

The value of RX is progressively decreased until the difference between CA and CB changes its numerical sign. At this stage, the desired value of RX is between the two previous values. The calculation is then repeated for a value of RX midway between these two values. The values of RX which surround the desired value are again selected from the preceding two values of RX and the current value. The convergence procedure is continued until the required accuracy is obtained.

At each station, the leading and then trailing edge co-ordinates are computed and then stored as subscripted variable in I (the row plane number).

The computation in this section is shown diagramatically in the flow diagrams Fig. 2/1 and 2/13.
Before entering the computing routine for the surface points, the subscripted variable $YB(I,NNN)$ (i.e. the Y ordinate) for each point on the two-dimensional fundamental mesh which covers the blade surface is equated to zero. This is a necessary condition for the calculation stage to be described in Section 3.4.

The geometry of each blade section is specified by three variables $CX$, $TX$, $AM$ and by equations for the thickness form and mean line profile (See Fig. 6). From this data the Z ordinate (of the surface) at a given surface point is computed by an iterative procedure (See Figs. 2/2 to 2/6). By considering a cylindrical blade section passing through a point $(I, N)$ an initial approximation of the distance along the chord line to the desired surface point at $(I, N)$ is made by assuming this distance to be equal to $XC$ the distance between $E$ and $Q$ (See Fig. 6).

At this cylindrical distance $XC$ from the leading edge, the co-ordinates for the points on the blade surface are computed and compared with the desired values. The value of $XC$ is then progressively adjusted using the "Regular Falsi" iterative convergence technique until the co-ordinates are determined to within the desired accuracy.

The calculated co-ordinates of the chosen points on the surface are designated by the subscripted variable $(I, NNN)$. The subscripted index $NNN$ refers to the numerical order of the points in the positive $YB$ direction from the trailing edge to the leading edge, and is centred about a point on the $X, Z$ plane where $NNN = NN$. This method of subscripting is in contrast with the more usual method of naming points according to their distance for a reference axis. The former method was chosen because the available storage locations in the main store of
the IBM 709/7090 II computer were limited. The execution time for this programme would have been greatly increased if these variables had to be placed on a magnetic tape store. These factors necessitated keeping a numerical count of the number of points on each row plane (I) which had spacing intervals SPACX and SPACN. A knowledge of this numerical count was also necessary before the co-ordinates of points surrounding any given point could be selected from the stored variables specifying the given point.

3.4 Direction Cosine at a Point

Since the mesh of points set up to cover the blade surface is rectangular, some of the mesh points lie in a region outside the blade area and are therefore disregarded in the following computational routine. These points are easily distinguished because their YB ordinates have previously been equated to zero (Section 3.3.2) whereas all other points on the surface have values of YB(I,NNN, J) greater than zero (Fig. 2/7).

In turn, at each surface mesh points (I, NNN, J) the co-ordinates of sixteen points arranged about the given mesh points as shown in Fig. 5 are selected from the complete store of points (See Fig. 2/7 to 2/12). The spacing distance between points in the row plane and the column plane must be equal. These points are re-identified as KA = 1 to 9 on both the row plane and the column plane, in both cases being centred about the given mesh point (KA = 5). From these sixteen selected points, the most symmetrical array of eight points surrounding the given mesh point on the blade surface is selected (Fig. 2/12).
The intersection of the row plane through the mesh point, and the surface of the blade, determines a space curve through the point — the "row space curve". Similarly the intersection of the column plane through the mesh point, and the blade surface, determines the "column space curve". The gradient (i.e. differential coefficient) of the row space curve at the mesh point, can now be found by substituting the ordinates of the four selected points on the row plane plus the ordinate of the mesh point into the appropriate equation for a five-point Gregory-Newton differentiation. Similarly, the gradient of the column space curve at the mesh point can be found. Each gradient is of course, equal to the slope of the tangent to the curve.

By combining the gradients of the two space curves through the mesh point, the direction cosines of the normal to the surface is calculated as follows.

Equation of tangent to row space curve at mesh point $XB, YB, ZB$ is given by,

$$ z = a(y - YB) + ZB $$  

where $a = \text{GRAD}(1, J) = \text{Slope of tangent to surface in YB direction}$

Similarly the equation to column space curve at mesh point is given by,

$$ z = b(x - XB) + ZB $$  

where $b = \text{GRAD}(2, J) = \text{Slope of tangent to surface in XB direction}$

Hence equation of plane containing both tangents is given by,

$$ ax + by = z = aXB + bYB - ZB $$
The direction cosines COSA, COSB, COSC of the outward pointing normal to the back surface are

\[
- \frac{\text{GRAD}(2, J)}{\sqrt{\text{GRAD}^2(2, J) + \text{GRAD}^2(1, J) + 1}} \quad \ldots \quad 18
\]

Note that:

\[
\text{Direction cosines of normal to back surface} = - \text{Direction cosines of normal to face surface} \quad \ldots \quad 19
\]

The position of the centre of the milling cutter along the normal to the given surface at the mesh point is now calculated, and specified in terms of the machining axes (Fig. 4). The information required by the machinist is not the blade surface dimensions, but the motions of the three lead screws necessary to traverse the cutter from the known origin of the axes to the point in question. Hence it was necessary to convert each co-ordinate of the cutter centre into an equivalent number of complete turns and parts of a turn (i.e. thousandths of an inch) of the appropriate lead screw. Since the machinist can easily check the position of the bottom of the cutter, it is more useful for the position of this point to be specified than the position of the cutter centre. Thus the final print-out is in terms of the position of the bottom of the cutter.

Since the blade surface has so far only been determined accurately up to but not at the blade edge, further points are required to define the actual blade edge. These points are calculated by a similar differentiation technique to that used for the points on the surface. These edge points are then specified with reference to the same machining axes described above and illustrated in Fig. 4. The corresponding
positions of the cutter are also calculated. The blade shape is further
defined by specifying the radius of the leading edge (RLE(I)) at each
row plane.

4.0 STABILITY OF CALCULATIONS

When applying this computing technique to an impeller or propeller
design, care must be taken to ensure that the iterative routines in-
corporated in the solution do in fact converge. The rate and
accuracy of the convergence must also be checked.

The two iterative procedures in the programme are considered sep-
arrately in the following.

4.1 Convergence in computing boundary points. (Ref. Section 3.3.1)

This routine is convergent provided that the initial value assumed
for RX is greater than the actual value of RX at a given row plane.
This is governed by statement number (170 + 0002) Table 2.

\[ RX = X + \text{RADM} \times \text{PERCR} \]

\[ \cdots 20 \]

The value of PERCR can be adjusted for each impeller design and is
submitted to the programme by an input statement.

The rate of convergence is governed by statement

\[ 230 \quad RX = RX - \text{STEPL} \]

\[ \cdots 21 \]

The accuracy of the calculation of the boundary point is determined
by the test statement

\[ \text{IF}(\text{ABSF}(\text{CA} - \text{CB}) - \text{DEVN1}) 270, 270, 260 \]

\[ \cdots 22 \]

This terminates the calculation of CB when an accuracy of less
than + or - (DEVN1) of an inch is obtained.
(1) In Stage 1 of the computation the N plane on which the surface point is located is progressively moved closer to the trailing edge of the blade. The position of the surface point on an N plane near the trailing edge needs careful investigation. This surface point must be located on the actual blade surface, i.e. the face or back surface, and not on the square region of the edge (See detail A Fig. 6).

In statement (430 + 0001) Table 2, the Y ordinate of the N<sup>th</sup> plane is tested to determine whether the plane cuts the blade on the blade surface or not. The significant variable in this expression is YYTE. This variable represents the value of the distance, parallel to the X, Y plane, from the intersection of the chord line and the trailing edge to the intersection of the face surface of the blade and the trailing edge, at a given row plane. The value of YYTE is derived from the allowable edge thickness ALTE, and the actual blade thickness Y0, at XC = 1.0. This value Y0 is referred to in the programme as DEVN2. (See statement 348, Table 2). DEVN2 exists because of the inaccuracies in the equation defining the thickness form. The actual value of Y0 at XC = 1.0 for the NACA - 16 profile is zero. The extra factor of 1.0001 in statement 348 increases the distance YYTE to allow for accumulated errors in the computation.

If another thickness form is used in place of the NACA - 16, the value of DEVN2 submitted as input to this programme should be a zero or positive value although its actual value may be
(2) By choosing an initial value of XC as given in statement (654 + 0004) (Table 2) a solution is always possible provided --

\[ 0 \leq XX \leq 1.0 \text{ where } XX = XC/CX \]

... 23.

i.e. XX must be within the blade section.

If during the iteration the value of XX (See statements 790, 810) falls outside the above range, it is forced to hold one of the limiting values.

As with the first convergence routine, the rate of convergence and accuracy are governed by statements 790, 810, and 820, (Table 2) and values of STEP2 and DEVN3.

5.0 INPUT/OUTPUT INSTRUCTIONS AND EXECUTION TIME

5.1 Input instructions

The programme as written (Table 2) will accept data for any impeller or propeller satisfying the stated conditions and whose blade shape is capable of being expressed by equations of the form given in Section 2.0.

An example of the input data, taken from the HYDRO - 1 impeller, is shown in Table 3. The values of most the variable have dimensions in feet and apply to the prototype impeller (See Table 5).

The following items in the input data require further discussion.

The values of the constants YD and ZD must be chosen to maintain all values of YN(1,NNN), PTY(J) and PTZ(J) positive. This places the origin of the axes XE, YE, ZE and XN, YN, ZN, and XM, YM, ZM outside the fundamental mesh which covers the blade surface.

The value of NN must be at least four (4) more than the maximum
number of points to be calculated in any one row plane on the trailing
edge side of the \( N = 0 \) plane section. The value of \( NKKN \) must be
divisible by five (5) and equal to or greater than the maximum value of

\[
\text{NTE}(I) + 4 \times \text{NTE}(I) + 4 
\]

... 24.

The maximum value of this expression occurs where the projected
blade area on the \( X, Y \) plane has the maximum \( Y \) dimension.

The dimension of \( YB(I,NNN) \) in the \( X \) direction, i.e. \( I \), must be
at least 8 more than the total number of row planes required. The
other subscripted variables referring to row planes \( RXLE(I), NLE(I) \)
etc. must have the \( I \)th subscript greater than the total number of row
planes, by at least four (4).

The dimension of \( YB(I,NNN) \) and \( ZB(I,NNN,J) \) in the \( YB \) direction
i.e. \( NNN \), must be at least twice the value of \( NN \).

The machining dimensions can be calculated for points between any
two row planes. The input variables controlling this are the values
of \( IP \) and \( IPAT \). The value of \( IP \) selects the beginning point for the
calculation and is the \( (IP + 4) \)th row plane. The value of \( IPAT \)
determines the end row plane with \( I = (IPAT + 4) \) where the calculation
ceases. If the \( (IP + 4) \) and \( (IPAT + 4) \) row planes lie within the
blade surface area and are not on the edge of this surface then the
variables associated with the first two and the last two row planes
have small errors due to the differentiation in Section 3.4.

Care must be taken when choosing the value of \( DEVN2 \), and reference
should be made to Section 4.2 for a discussion on its value.

As previously mentioned in Section 4.0 the initial value of
\( RX \) and also the rate and accuracy of convergence in the two "Regular
Falsi" routines, must be carefully selected. If the leading and trailing edges are to be adequately defined then the value of PCX must be less than 0.90. This variable PCX determines the point where the spacing of column planes change from SPACX to the smaller spacing of SPACN.

If a progressive print-out of all major calculations and decisions in the programme is needed NTEST should be set to a positive number, otherwise it should be zero or negative.

The programme as written occupied approximately 25,300 words in the store of an IBM 709/7090 II computer with main storage capacity of 32,561 words.

5.2 Output Instructions

The machining technique governs the form of the output instructions. The output for the Hydrojet impeller was chosen to be punched cards, which were later listed. These output instructions are in a form suitable for the machinist of a hand-operated vertical milling machine.

The machining instructions for the back and face surfaces of the blade could not be separated in the computer without increasing storage and running time. Each alternate card of output thus refers to the same blade surface. The deck of output cards may be later processed by an IBM Collator, to separate the alternate cards.

In the output listing the row planes are called stations and are numbered from one (1) at the blade tip. Thus each station is actually the $(I + 4)^{th}$ row plane.

Associated with each point on the blade surface and its machining co-ordinates there is a reference number. This reference number is actually the number of the $NK^{th}$ column plane which passes through the point. All surface points with the same reference number $NK$ are
physically on the same column plane.

The example of the machining instructions given in Table 4 is for the HYDRO-1 impeller at a station on the back surface. Fig. 7 shows a plot of computed points on the back and face surface of the impeller at selected sections.

5.3 Execution Time

The average time taken by the IBM 709/7090 to calculate the machining co-ordinates for 100 mesh points (i.e., 100 points on both the back and face surface) was approximately 21 seconds. Thus the execution time required for the 3,666 mesh points of the impeller HYDRO-1 was 765 seconds.
REFERENCES

1. HALE, M.R: The Design of Ducted Impellers using a Vortex line Analysis and an Optimizing Computer Technique.
University of Adelaide, Dept. of Mech. Eng.

2. O'BRIEN, T.P: The Design of Marine Screw Propellers.


4. ABBOTT, I.H. and VON DOENHOFF, A.E: Theory of Wing Sections - including a Summary of Aerofoil Data.

5. NEWELL H.B: "Vector Analysis"
### Table 1. Impeller Dimensions and Characteristics

<table>
<thead>
<tr>
<th>DESIGN PARAMETERS</th>
<th>HYDROJET IMPELLER</th>
</tr>
</thead>
<tbody>
<tr>
<td>DESIGN ASSUMPTIONS</td>
<td></td>
</tr>
<tr>
<td>BETZ MIN ENERGY CONDITION</td>
<td></td>
</tr>
<tr>
<td>CONSTANT AXIAL VELOCITY</td>
<td></td>
</tr>
<tr>
<td>NACA PROFILE</td>
<td>SHOCK FREE ENTRY</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RADIUS</th>
<th>VELOCITY</th>
<th>ROTATION</th>
<th>BLADES</th>
<th>IDEAL EFF</th>
<th>TOTAL EFF</th>
<th>THRUST</th>
<th>SHP</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.50</td>
<td>19.30</td>
<td>1.50</td>
<td>4</td>
<td>0.9783</td>
<td>0.8599</td>
<td>44800</td>
<td>1207</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TIP CHORD</th>
<th>BOSS CHORD</th>
<th>BOSS RATIO</th>
<th>TIP IMMERSION</th>
<th>ALLOWABLE STRESS</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.000</td>
<td>3.000</td>
<td>0.200</td>
<td>11.50</td>
<td>7250</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>RAD FRAC</th>
<th>CHORD</th>
<th>THICKNESS</th>
<th>MEAN LINE</th>
<th>ANGLE BI</th>
<th>LIFT COEF</th>
<th>DRAG/LIFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>5.000</td>
<td>0.2250</td>
<td>0.0926</td>
<td>15.67</td>
<td>0.249</td>
<td>0.030</td>
</tr>
<tr>
<td>0.95</td>
<td>4.875</td>
<td>0.2194</td>
<td>0.0976</td>
<td>16.45</td>
<td>0.241</td>
<td>0.031</td>
</tr>
<tr>
<td>0.90</td>
<td>4.750</td>
<td>0.2137</td>
<td>0.0926</td>
<td>17.31</td>
<td>0.223</td>
<td>0.032</td>
</tr>
<tr>
<td>0.85</td>
<td>4.625</td>
<td>0.2081</td>
<td>0.0976</td>
<td>18.26</td>
<td>0.225</td>
<td>0.033</td>
</tr>
<tr>
<td>0.80</td>
<td>4.500</td>
<td>0.2025</td>
<td>0.0926</td>
<td>19.32</td>
<td>0.217</td>
<td>0.034</td>
</tr>
<tr>
<td>0.75</td>
<td>4.375</td>
<td>0.1969</td>
<td>0.0975</td>
<td>20.17</td>
<td>0.207</td>
<td>0.026</td>
</tr>
<tr>
<td>0.70</td>
<td>4.250</td>
<td>0.1912</td>
<td>0.0975</td>
<td>21.84</td>
<td>0.197</td>
<td>0.028</td>
</tr>
<tr>
<td>0.65</td>
<td>4.125</td>
<td>0.1962</td>
<td>0.0974</td>
<td>22.93</td>
<td>0.187</td>
<td>0.040</td>
</tr>
<tr>
<td>0.60</td>
<td>4.000</td>
<td>0.2039</td>
<td>0.0923</td>
<td>24.06</td>
<td>0.175</td>
<td>0.044</td>
</tr>
<tr>
<td>0.55</td>
<td>3.875</td>
<td>0.2052</td>
<td>0.0971</td>
<td>25.62</td>
<td>0.163</td>
<td>0.047</td>
</tr>
<tr>
<td>0.50</td>
<td>3.750</td>
<td>0.2319</td>
<td>0.0919</td>
<td>27.39</td>
<td>0.150</td>
<td>0.056</td>
</tr>
<tr>
<td>0.45</td>
<td>3.625</td>
<td>0.3056</td>
<td>0.0367</td>
<td>31.94</td>
<td>0.136</td>
<td>0.056</td>
</tr>
<tr>
<td>0.40</td>
<td>3.500</td>
<td>0.3289</td>
<td>0.0315</td>
<td>35.04</td>
<td>0.121</td>
<td>0.080</td>
</tr>
<tr>
<td>0.35</td>
<td>3.375</td>
<td>0.3508</td>
<td>0.0263</td>
<td>38.71</td>
<td>0.104</td>
<td>0.100</td>
</tr>
<tr>
<td>0.30</td>
<td>3.250</td>
<td>0.3723</td>
<td>0.0211</td>
<td>43.03</td>
<td>0.087</td>
<td>0.130</td>
</tr>
<tr>
<td>0.25</td>
<td>3.125</td>
<td>0.3915</td>
<td>0.0160</td>
<td>48.39</td>
<td>0.069</td>
<td>0.178</td>
</tr>
<tr>
<td>0.20</td>
<td>3.000</td>
<td>0.4056</td>
<td>0.0112</td>
<td>54.51</td>
<td>0.050</td>
<td>0.264</td>
</tr>
<tr>
<td>RAD FRAC</td>
<td>ST F</td>
<td>ST B</td>
<td>ST E</td>
<td>ST N</td>
<td>CAUT S</td>
<td>CAUT P</td>
</tr>
<tr>
<td>----------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>1.00</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
<td>0.555</td>
<td>0.347</td>
</tr>
<tr>
<td>0.95</td>
<td>140°</td>
<td>-127°</td>
<td>113°</td>
<td>113°</td>
<td>0.615</td>
<td>0.340</td>
</tr>
<tr>
<td>0.90</td>
<td>505°</td>
<td>-550°</td>
<td>385°</td>
<td>386°</td>
<td>0.685</td>
<td>0.333</td>
</tr>
<tr>
<td>0.85</td>
<td>1149°</td>
<td>-1330°</td>
<td>834°</td>
<td>824°</td>
<td>0.766</td>
<td>0.325</td>
</tr>
<tr>
<td>0.80</td>
<td>2089°</td>
<td>-2475°</td>
<td>1444°</td>
<td>1462°</td>
<td>0.861</td>
<td>0.317</td>
</tr>
<tr>
<td>0.75</td>
<td>3392°</td>
<td>-4061°</td>
<td>2229°</td>
<td>2216°</td>
<td>0.973</td>
<td>0.309</td>
</tr>
<tr>
<td>0.70</td>
<td>5095°</td>
<td>-6113°</td>
<td>3162°</td>
<td>3139°</td>
<td>1.075</td>
<td>0.300</td>
</tr>
<tr>
<td>0.65</td>
<td>6204°</td>
<td>-7246°</td>
<td>3300°</td>
<td>3245°</td>
<td>1.255</td>
<td>0.303</td>
</tr>
<tr>
<td>0.60</td>
<td>6454°</td>
<td>-7248°</td>
<td>2758°</td>
<td>2672°</td>
<td>1.457</td>
<td>0.318</td>
</tr>
<tr>
<td>0.55</td>
<td>6657°</td>
<td>-7247°</td>
<td>2530°</td>
<td>2201°</td>
<td>1.690</td>
<td>0.334</td>
</tr>
<tr>
<td>0.50</td>
<td>6829°</td>
<td>-7248°</td>
<td>1589°</td>
<td>1800°</td>
<td>1.975</td>
<td>0.349</td>
</tr>
<tr>
<td>0.45</td>
<td>6976°</td>
<td>-7249°</td>
<td>1717°</td>
<td>1446°</td>
<td>2.037</td>
<td>0.265</td>
</tr>
<tr>
<td>0.40</td>
<td>7102°</td>
<td>-7249°</td>
<td>1503°</td>
<td>1122°</td>
<td>2.762</td>
<td>0.318</td>
</tr>
<tr>
<td>0.35</td>
<td>7211°</td>
<td>-7247°</td>
<td>1345°</td>
<td>816°</td>
<td>3.003</td>
<td>0.397</td>
</tr>
<tr>
<td>0.30</td>
<td>7247°</td>
<td>-7184°</td>
<td>1236°</td>
<td>507°</td>
<td>3.969</td>
<td>0.415</td>
</tr>
<tr>
<td>0.25</td>
<td>7247°</td>
<td>-7099°</td>
<td>1191°</td>
<td>194°</td>
<td>4.779</td>
<td>0.432</td>
</tr>
<tr>
<td>0.20</td>
<td>7251°</td>
<td>-7030°</td>
<td>1230°</td>
<td>-135°</td>
<td>5.727</td>
<td>0.446</td>
</tr>
<tr>
<td>Line</td>
<td>Description</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td><strong>1330 M R HALE/FOWLER MECH ENG DEPT U OF A TEL 461</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td><strong>RECT COORDINATES MACHINING DIMENSIONS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td><strong>PROGRAM NO 0022/7090</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>010</td>
<td>FORMAT(1H4,42H PATTERN COORDINATES OF HYDROJET IMPELLER)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>015</td>
<td>FORMAT(1H6,47H MACHINING COORDINATES UNIFORM RECTANGULAR GRID)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>020</td>
<td>FORMAT(1H6,15H MODEL SCALE =F7.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>021</td>
<td>FORMAT(1H6,15H MODEL RADIUS =F7.3,3HINS)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>025</td>
<td>FORMAT(1H6,20H REFERENCE DISTANCES)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>030</td>
<td>FORMAT(1H6,41H REF PT TO LINE OF CENTROID IN Y DIFCN =F7.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>035</td>
<td>FORMAT(1H6,41H REF PT TO LINE OF CENTROID IN Z DIFCN =F7.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>040</td>
<td>FORMAT(1H6,13I,11H STATION NO.9X,16HBACK COORDINATES)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>041</td>
<td>FORMAT(1H6,13I,11H STATION NO.9X,16HFACE COORDINATES)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>045</td>
<td>FORMAT(1H6,30H RADIAL DISTANCE TO STATION =F8.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>048</td>
<td>FORMAT(1H6,11X,22H MACHINING COORDINATES,16X,16HBOTTOM OF CUTTER)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>049</td>
<td>FORMAT(1H6,6X,17H POINTS ON SURFACE)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>050</td>
<td>FORMAT(1H6,9X,1HX,12X,1HY,12X,1HZ,12X,1HX,8X,1HY,8X,1HZ)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>051</td>
<td>FORMAT(1H6,5X,1HX,8X,1HY,8X,1HZ)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>055</td>
<td>FORMAT(1H6,39H NO REV THOU REV THOU)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>060</td>
<td>FORMAT(1H6,14H,14I,15I,18I,15I,18I,15I,4X,3F9.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>062</td>
<td>FORMAT(1H6,4X,F9.3,4X,F9.3,4X,F9.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>065</td>
<td>FORMAT(1H6,18H ST CHCRD WIDTH =F8.3,7X,11HLE RADIUS =F6.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>066</td>
<td>FORMAT(1H6,3F9.3,16)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>067</td>
<td>FORMAT(1H6,3X,3F9.3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>068</td>
<td>FORMAT(1H6,14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>070</td>
<td>FORMAT(4E14.7,5I7,5E14.7,3E14.7,5E14.7,4E14.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>080</td>
<td>FORMAT(3E14.7,4E14.7,5E14.7,3E14.7,5E14.7,5E14.7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>090</td>
<td>FORMAT(5F10.5,F8.3,15)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>091</td>
<td>FORMAT(4F10.3,18)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DIMENSION RXLE(57),RTE(57),YLE(57),YTE(57),RLE(57),NLE(57),MLE(57)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1,NTE(57),MTE(57),YB(61,120),Z(2),ZB(57,120,2),YC(2,9),PI(2,9,2),</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ZGRAD(2,2),PTX(2),PTY(2),PTZ(2),VYV(2),ZZZ(2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>COMMONB</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>READ INPUT TAPE 2.070, RADIUS, SCALE, SCALM, IF, IPAT, NN, NKKN, NTEST,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1DX, SPACX, SPACN, PCX, PERCS, YD, 2D, ALTE, STEP1, STEP2, DEVN1, DEVN2, DEVN3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2. - Continued.

2CUTR, XMIIR, YMIIR, ZMIIR
READ INPUT TAPE 2, 080, COD, CID, CX7, T0, T1, T2, T3, A0, A1, A2, A3, A4, AMO,
1AMI, ANG0, ANGI, PD, HBC, YBCT, RADL, E, Y0, Y1, Y2, Y3, Y4, Y5, Y6, Y7, Y8, Y9
UNIT=12.0/SCALE
RADM=RAD*UNIT
PUNCH010
PUNCH010
PUNCH015
PUNCH015
PUNCH020, SCALE
PUNCH020, SCALE
PUNCH021, RADM
PUNCH021, RADM
PUNCH025
PUNCH025
PUNCH030, YD
PUNCH030, YD
PUNCH035, ZD
PUNCH035, ZD

**COMPUTING END POINTS AT SPECIFIED RADIAL DISTANCES**
PI=3.1415927
PITCH=PD*2.0/RADM
COD=COD*UNIT
CID=CID*UNIT
AMO=AMO*UNIT
AMI=AMI*UNIT

YR=(1.0-HBC)*COD*COSF(ANG0)+YBCT*AMO*SINF(ANG0)
XOL=RADM*COSF(YR/RADM)
YR=(1.0-HBC)*CID*COSF(ANGI)+YBCT*AMI*SINF(ANGI)
X1L=RATIO*RADM*COSF(YR/(RATIO*RADM))
YR=-HBC*COD*COSF(ANG0)+YBCT*AMO*SINF(ANG0)
X0L=RADM*COSF(YR/RADM)
YR=-HBC*CID*COSF(ANGI)+YBCT*AMI*SINF(ANGI)
X1L=RATIO*RADM*COSF(YR/(RATIO*RADM))
IPP=IP+4
Il=IPAT+4
Table 2. - Continued.

D0330I*IP,P,I
A=I-5
X=RA DM-DX*A
D0310M=1,2
IF(M-1)1000,110,140
110 IF(X=XOL)130,130,120
120 RXLE(I)=RADM
RL E(I)=0.0
YLE(I)=SQR TF(RADM*RADM-X*X)
GOT0310
130 IF(X=XIL)1000,170,170
140 IF(X=XOT)160,160,150
150 RXTE(I)=RADM
YTE(I)=-SQR TF(RADM*RADM-X*X)
GOT0310
160 IF(X=XIT)1000,170,170
170 RB=0.0
RA=0.0
RX=X+RA DM*PERCR
180 CA=RX*ATANF(SQR TF(RX*RX-X*X)/X)
IF(M-1)1000,190,200
190 K=1
GOT0600
195 CB=(1.0-HB C)*CX*COSF(ANG)+YB CT*AM*SINF(ANG)
GOT0210
200 K=2
GOT0600
205 CB=HB C*CX*COSF(ANG)-YB CT*AM*SINF(ANG)
210 IF(CA-CB)240,270,220
220 RA=RX
IF(RB)1000,230,250
230 RX=RX-STEP1
GOT0100
240 RB=RX
250 IF(ABS F(CA-CB)-DE VN1)270,270,260
260 RX=0.5*(RA+RB)
Table 2. - Continued.

GOTO180
270 IF(M-1)1000,280,300
280 RXLE(I)=0.5*(RA+RB)
    YLE(I)=SQRF(RXLE(I)*RXLE(I)-X*X)
    RX=RXLE(I)
    K=3
GOTO600
290 RLE(I)=RADLE*TX*TX/CX
GOTO310
300 RXTE(I)=0.5*(RA+RB)
    YTE(I)=SQRF(RXTE(I)*RXTE(I)-X*X)
310 CONTINUE
    IF(NTEST)330,330,320
320 WRITE OUTPUT TAPE 3,090,X,RXLE(I),RXTE(I),YLE(I),YTE(I),RLE(I),I
330 CONTINUE

**CALCULATING COORDINATES OF POINTS ON SURFACE**

**UNIFORM GRID SPACING**

III=IPAT+8
NN2=DN+2
DO340I=1,III
DO340N=1,NN2
340 YB(I,N)=0.0
DO550I=IPP,II
A=I-5
X=RA+DX*A
    IF(I-5)1000,344,346
344 NLE(I)=0
MLE(I)=0
NTE(I)=0
MTE(I)=0
GOTO515
346 NLE(I)=10*NN
    NTE(I)=10*NN
    RX=RXTE(I)
    K=8
GOTO600
Table 2. - Continued.

348 YYTE=(ALTE*SCALM/2.0+DEVN2*TX)*1.0001*SIN(ANG)/SCALE
DO510M=1,2
Y=0.0
DO510N=1,NN
IF(M-1)1000,350,400
350 IF(N-NLE(I)-1)360,510,510
360 IF(Y-PCX*YLE(I))370,380,380
370 Y=Y+SPACX
MLE(I)=N
GOTO450
380 Y=Y+SPACN
IF(Y-YLE(I))450,390,390
390 NLE(I)=N-1
GOTO510
400 IF(N-NTE(I)-1)410,510,510
410 IF(Y-PCX*YTE(I))430,430,420
420 Y=Y-SPACX
MTE(I)=N
GOTO450
430 Y=Y-SPACN
IF(Y-(YTE(I)+YYTE))440,440,450
440 NTE(I)=N-1
GOTO510
450 RX=SQRTF(X*X+Y*Y)
IF(RX-RATIO*RADM)460,470,470
460 Z(1)=0.0
Z(2)=0.0
YA=-YD
GOTO480
470 K=4
YA=Y
IF(M-1)1000,471,472
471 YR=+RX*ATANF(SQRTF(RX*RX-X*X)/X)
GOTO600
472 YR=-RX*ATANF(SQRTF(RX*RX-X*X)/X)
GOTO600
Table 2. - Continued.

480 IF \( M=1 \) 1000, 490, 495
490 NNN = NNN + N
GOTO 496
495 NNN = NNN - N
496 ZB(I, NNN, 1) = Z(1)
ZB(I, NNN, 2) = Z(2)
YB(I, NNN) = YA + YD
IF (NTEST) 510, 510, 500
500 WRITE OUTPUT TAPE 3, 091, ZB(I, NNN, 1), YB(I, NNN), ZB(I, NNN, 2), X, NNN
510 CONTINUE
515 RX = X
YR = 0.0
K = 5
IF (RX = RATIO * RADM) 520, 600, 600
520 Z(1) = 0.0
Z(2) = 0.0
YR = -YD
530 ZB(I, NNN, 1) = Z(1)
ZB(I, NNN, 2) = Z(2)
YB(I, NNN) = YR + YD
IF (NTEST) 550, 550, 540
540 WRITE OUTPUT TAPE 3, 091, ZB(I, NNN, 1), YB(I, NNN), ZB(I, NNN, 2), X, NNN
550 CONTINUE

C **DETERMINATION OF DIRECTION COSINES AT A POINT**
C **HENCE CORRECTING COORDINATES FOR THOSE AT BOTTOM OF CUTTER**
DO 30000 I = 1, PP, 11
A = I - 5
IA = I - 4
X = RADM - DX * A
PUNCH040, IA
PUNCH040, IA
PUNCH045, X
PUNCH045, X
PUNCH048
PUNCH048
PUNCH049
**TRAILING EDGE COORDINATES**

\[
SPX = 12.0*DX
\]

\[
IF(X+2.0*DX-XOT)1140,1180,1110
\]

\[
1110 IF(X+DX-XOT)1130,1130,1115
\]

\[
1115 IF(X-XOT)1120,1120,1400
\]

\[
1120 GDT = (3.0*YTE(I+4)-16.0*YTE(I+1)+36.0*YTE(I+2)-48.0*YTE(I+1)+125.0*YTE(I))/SPX
\]

\[
GOTO1200
\]

\[
1130 GDT = (-YTE(I+3)+6.0*YTE(I+2)-18.0*YTE(I+1)+10.0*YTE(I)+3.0*YTE(I-1))/SPX
\]

\[
GOTO1200
\]

\[
1140 IF(X-2.0*DX-XIT)1150,1180,1180
\]

\[
1150 IF(X=DX-XIT)1170,1160,1160
\]

\[
1160 GDT = (-3.0*YTE(I+1)-10.0*YTE(I)+18.0*YTE(I-1)-6.0*YTE(I-2)+YTE(I-3))/SPX
\]

\[
GOTO1200
\]

\[
1170 GDT = (-25.0*YTE(I)+48.0*YTE(I-1)-36.0*YTE(I-2)+16.0*YTE(I-3)-13.0*YTE(I-4))/SPX
\]

\[
GOTO1200
\]

\[
1180 GDT = (YTE(I+2)-8.0*YTE(I+1)+8.0*YTE(I-1)-YTE(I-2))/SPX
\]

\[
1200 RX = RXTE(I)
\]

\[
K = 6
\]

\[
GOTO600
\]

\[
1210 ANGLE = ATANF(GDT)
\]

\[
PZ = HBC*CX*SINF(ANG) - YBCT*AM*COSF(ANG)
\]

\[
BZ = ZD - PZ + CUTR
\]

\[
FZ = ZD + PZ + CUTR
\]

\[
FY = YD + YTE(I) - CUTR*COSF(ANGLE)
\]

\[
BY = YD - YTE(I) + CUTR*COSF(ANGLE)
\]
Table 2. - Continued.

```
PX*+CUTR*SIN(ANGLE)
PUNCHO67, PX, BY, B2
PUNCHO67, PX, FY, FZ

C **POINTS IN BLADE AREA--RENUMBERING POINTS ABOUT LINE OF CENTROID
C ** AS N=NN HENCE SELECTING 4 PTS SURROUNDING THE PT ON EACH SIDE

1400 NA=NN-NTE(I)
    NB=NN-MTE(I)
    NC=NN+MLE(I)
    ND=NN+NLE(I)
    DO2600N=NA, ND
    IF(YB(I,N))1000,2600,1410
1410 IF(N-NB)1430,1420,1420
1420 IF(N-NC)1550,1550,1650
1430 NK=NKNN-4*MTE(I)-NN+N
    SPAC=SPACN
    DO1540KA=1,9
    NKK=NK-5+KA
    NNN=NKKN-NKKN+4*MTE(I)+NN
    IF(NNN-NB)1480,1480,1470
1470 YC(I,K)=0.0
    GOT01490
1480 YC(I,K)=YB(I,NNN)
    PT(I*KA+1)=ZB(I,NNN,1)
    PT(I*KA+2)=ZB(I,NNN,2)
1490 III=I+5-KA
    NNN=NK-NKKN+4*MTE(III)+NN
    IF(NNN-(NN-MTE(III))1530,1530,1500
1500 IF((NK+4)/5-NK/5)1000,1520,1510
1510 YC(2,K)=0.0
    GOT01540
1520 NNN=(NK-NKKN)/5+NN
1530 IF(NNN)1535,1535,1534
1534 IF(NNN-NN2)1536,1536,1535
1535 YC(2,K)=0.0
    GOT01540
1536 YC(2,K)=YB(III,NNN)
```
Table 2. - Continued.

\[ PT(2, KA, 1) = ZB(III, NNN, 1) \]
\[ PT(2, KA, 2) = ZB(III, NNN, 2) \]

1540 CONTINUE
GOTO2000

1550 NK = NKKN - 5 * (NN - N)
SPAC = SPACX
DO 1640 KA = 1, 9
NKK = NK - 25 + 5 * KA
NNN = (NKK - NKKN) / 5 + NN
IF (NNN - NB) 1560, 1560, 1570

1560 NNN = NKK - NKKN + 4 * MTE(I) + NN
IF (NNN) 1585, 1585, 1590

1570 IF (NNN - NC) 1590, 1580, 1580

1580 NNN = NKK - NKKN - 4 * MLE(I) + NN
IF (NNN - NN2) 1590, 1585, 1585

1585 YC(1, KA) = 0.0
GOTO1595

1590 YC(1, KA) = YB(I, NNN)

1595 PT(1, KA, 1) = ZB(I, NNN, 1)
PT(1, KA, 2) = ZB(I, NNN, 2)
III = I + 5 - KA
NNN = (NK - NKKN) / 5 + NN
IF (NNN - (NN - MTE(III))) 1600, 1600, 1610

1600 NNN = NK - NKKN + 4 * MTE(III) + NN
GOTO1630

1610 IF (NNN - (NN + MLE(III))) 1630, 1620, 1620

1620 NNN = NK - NKKN - 4 * MLE(III) + NN

1630 IF (NNN) 1635, 1635, 1634

1634 IF (NNN - NN2) 1636, 1636, 1635

1635 YC(2, KA) = 0.0
GOTO1640

1636 YC(2, KA) = YB(III, NNN)
PT(2, KA, 1) = ZB(III, NNN, 1)
PT(2, KA, 2) = ZB(III, NNN, 2)

1640 CONTINUE
GOTO2000
****Table 2. - Continued.**

1650 NK=NKKN+4*MLE(I)-NN+N
SPAC=SPACN
DO1730KA=1,9
NKK=NK-5+KA
NNN=NKK-NKKN-4*MLE(I)+NN
IF(NNN-NC)1660,1670,1670
1660 YC(1,KA)=0.0
GOTO1680
1670 YC(1,KA)=YB(I,NNN)
P(T(1,KA,2)=ZB(I,NNN,2)
1680 III=I+5-KA
NNN=NK-NKKN-4*MLE(III)+NN
IF(NNN-((NN+MLE(III)))1690,1720,1720
1690 IF((NK+4)/5-NK/5)1000,1710,1700
1700 YC(2,KA)=0.0
GOTO1730
1710 NNN=(NK-NKKN)/5+NN
1720 IF(NNN)1725,1725,1724
1724 IF(NNN-NN2)1726,1726,1725
1725 YC(2,KA)=0.0
GOTO1730
1726 YC(2,KA)=YB(III,NNN)
P(T(2,KA,1)=ZB(III,NNN,1)
P(T(2,KA,2)=ZB(III,NNN,2)
1730 CONTINUE
C **ROUTINE FOR DETERMINING COORDINATES OF CUTTER AT EACH XYZ PTS**
2000 DO2570J=1,2
NNN=10
DO2480L=1,2
IF(L-1)1000,2010,2020
2010 SP=12.0*SPAC
GOTO2030
2020 SP=12.0*DX
2030 IF(NNN-1)1000,2480,2040
2040 IF(YC(L,3))1000,2350,2380
Table 2. - Continued.

2350 IF(YC(L,4))1000,2360,2370
2360 IF(YC(L,9))1000,2420,2430
2370 IF(YC(L,6))1000,2420,2440
2380 IF(YC(L,7))1000,2390,2450
2390 IF(YC(L,6))1000,2400,2410
2400 IF(YC(L,1))1000,2420,2470
2410 IF(YC(L,2))1000,2420,2460
2420 NNN=1
2430 GOTO2480
2435 IF(YC(L,6)*VC(L,7)*YC(L,3))1000,2420,2435
2440 GOTO2480
2445 GRAD(L,J)=-25.0*PT(L+5,J)+48.0*PT(L+6,J)-36.0*PT(L+7,J)+16.0*PT(L+8,J)-3.0*PT(L+9,J)/SP
2450 GOTO2480
2455 IF(YC(L,6)*YC(L,1))1000,2420,2435
2465 GRAD(L,J)=PT(L+3,J)-8.0*PT(L+4,J)+8.0*PT(L+5,J)-PT(L+7,J)/SP
2470 GOTO2480
2475 IF(YC(L,4))1000,2420,2465
2485 GRAD(L,J)=-PT(L+2,J)+6.0*PT(L+3,J)-18.0*PT(L+4,J)+10.0*PT(L+5,J)+13.0*PT(L+6,J)/SP
2490 CONTINUE
2495 IF(J=1)1000,2485,2490
2495 YY(J)=2.0*YD-YBI(I,N)
2495 ZZ(J)=-ZD-ZBI(I,N,J)
2495 GOTO2494
2495 YY(J)=YBI(I,N)
2495 ZZ(J)=-ZD-ZBI(I,N,J)
2495 IF(NNN=1)1000,2495,2500
2495 PUNCH68,NK
<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
<td>[ \text{GOTO2570} ]</td>
</tr>
<tr>
<td>2500</td>
<td>[ \text{DEN} = \text{SORTF}(1+\text{GRAD}(1,J) \times \text{GRAD}(1,J) + \text{GRAD}(2,J) \times \text{GRAD}(2,J)) ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{IF } (J = 1,1000) \rightarrow \text{GOTO } 2520 ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{COSA} = -\text{GRAD}(2,J) / \text{DEN} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{COSB} = -\text{GRAD}(1,J) / \text{DEN} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{COSC} = 1.0 / \text{DEN} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{CPX} = \text{X} \times \text{CUTR} \times \text{COSA} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{CPY} = \text{Y} \times \text{CUTR} \times \text{COSB} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{CPZ} = \text{Z} \times \text{CUTR} \times \text{COSC} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{PTX}(J) = \text{CPX} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{PTY}(J) = 2.0 \times \text{YD} - \text{CPY} ]</td>
</tr>
<tr>
<td>2510</td>
<td>[ \text{PTZ}(J) = \text{ZD} - \text{CPZ} \times \text{CUTR} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{GOTO2560} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{COSA} = \text{GRAD}(2,J) / \text{DEN} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{COSB} = \text{GRAD}(1,J) / \text{DEN} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{COSC} = 1.0 / \text{DEN} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{CPX} = \text{X} \times \text{CUTR} \times \text{COSA} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{CPY} = \text{Y} \times \text{CUTR} \times \text{COSB} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{CPZ} = \text{Z} \times \text{CUTR} \times \text{COSC} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{PTX}(J) = \text{CPX} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{PTY}(J) = \text{CPY} ]</td>
</tr>
<tr>
<td>2520</td>
<td>[ \text{PTZ}(J) = \text{ZD} + \text{CPZ} \times \text{CUTR} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{B} = \text{PTX}(J) / \text{XMINLR} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{KX} = \text{B} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{BX} = \text{KX} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{CX} = \text{XMINLR} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{KXX} = 1000.0 \times (\text{PTX}(J) - \text{C}) + 0.5 ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{B} = \text{PTY}(J) / \text{YMINLR} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{KY} = \text{B} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{BY} = \text{KY} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{C} = \text{B} \times \text{YMINLR} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{KYY} = 1000.0 \times (\text{PTY}(J) - \text{C}) + 0.5 ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{B} = \text{PTZ}(J) / \text{ZMINLR} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{KZ} = \text{B} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{B} = \text{KZ} ]</td>
</tr>
<tr>
<td>2560</td>
<td>[ \text{C} = \text{B} \times \text{ZMINLR} ]</td>
</tr>
<tr>
<td>Table 2. - Continued</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>K2Z = 1000.0 \cdot (PTZ(J) - C) + 0.5</td>
<td></td>
</tr>
<tr>
<td>PUNCH060, NK, XX, KXX, KY, KYY, KZ, KZZ, PTX(J), PTY(J), PTZ(J)</td>
<td></td>
</tr>
</tbody>
</table>

2570 CONTINUE
PUNCH066, X, YYY(1), ZZZ(1), NK
PUNCH066, X, YYY(2), ZZZ(2), NK

2600 CONTINUE

C **LEADING EDGE COORDINATES**

IF(X+2.0*DX-XOL) GOTO 2640
2610 IF(X+DX-XOL) GOTO 2630
2615 IF(X-XOL) GOTO 2620

2620 GDT = (3.0*YLE(I+4) - 16.0*YLE(I+3) + 36.0*YLE(I+2) - 48.0*YLE(I+1) + 25.0*YLE(I+0) |
        \cdot  \times SPX
GOTO 2700

2630 GDT = (-YLE(I+3) + 6.0*YLE(I+2) - 18.0*YLE(I+1) + 10.0*YLE(I) + 3.0*YLE(I-1) |
        \div SPX
GOTO 2700

2640 IF(X-2.0*DX-XIL) GOTO 2650
2650 IF(X-2.0*DX-XIL) GOTO 2660

2660 GDT = (-3.0*YLE(I+1) - 10.0*YLE(I) + 18.0*YLE(I-1) - 6.0*YLE(I-2) + YLE(I-3) |
        \times SPX
GOTO 2700

2670 GDT = (-25.0*YLE(I) + 48.0*YLE(I-1) - 36.0*YLE(I-2) + 16.0*YLE(I-3) - 3.0*YLE(I-4) |
        \div SPX
GOTO 2700

2680 GDT = (YLE(I+2) - 8.0*YLE(I+1) + 8.0*YLE(I-1) - YLE(I-2)) / SPX

2700 RX = RXLE(I)
K = 7
GOTO 2600

2710 ANGLE = ATANF(GDT)
PX = X - CUTF*SINF(ANGLE)
FY = YD + YLE(I) + CUTF*COSF(ANGLE)
BY = YD - YLE(I) - CUTF*COSF(ANGLE)
PZ = (1.0 - HBC) \times CUTF*SINF(ANGLE) - YBCT\times AM\times COSF(ANGLE)
BZ = 2D - PZ + CUTF
FZ = 2D + PZ + CUTF
PUNCH067, PX, BY, BZ
**Routine for calculating properties of blade sections**

**Also Z ordinate for a given X,Y coordinate**

600 CX=COD-(COD-CID)*(RADM-RX)/(RADM*(1.0-RATIO))

XR=RX/RADM
IF(RX=0.7*RADM) 620, 610, 610

610 TX=0.045*CX
GOTO650

620 IF(RX=0.65*RADM) 640, 630, 630

630 TX=0.045*CX7*UNIT
GOTO650

640 TX=(T0+T1*XR+T2*(XR**2)+T3*(XR**3))*UNIT

650 ANG=ATANF(PITCH/(2.0*PI*RX))

AM=IA0+A1*XR+A2*(XR**2)+A3*(XR**3)+A4*(XR**4))*UNIT
GOTO(195, 205, 290, 654, 654, 1210, 2710, 348)*K

654 DO840 J=1, 2
MTEST=0
XCA=0.0
XCB=0.0
XC=(1.0-HBC)*CX-YR/COSF(ANG)

655 XX=XC/CX
660 MTEST=MTEST+1
IF(MTEST=30) 669, 670, 670

670 Z[1]=0, 0
Z[2]=0, 0
YA=-YD
GOTO840

669 IF(XX) 661, 661, 662

661 XX=0.0
GOTO664

662 IF(XX-1.0) 665, 663, 663
Table 2 - Continued.

663 XX=1.0
664 YO=0.0
665 YM=0.0
GOTO666
666 YO=(1.0-XX)*0.98879*SQRIF(XX)+Y0+Y1*XX+Y2*(XX**2)+Y3*(XX**3)
666 YO=Y0+Y4*(XX**4)+Y5*(XX**5)+Y6*(XX**6)+Y7*(XX**7)+Y8*(XX**8)
666 YO=Y0+Y9*(XX**9)
666 YM=-(1.0-XX)*LOGF(1.0-XX)+XX*LOGF(XX)/0.693147
667 IF(XX-0.5)668,669,667
668 XC=XX*CX
   IF(J=1)400,490,700
   690 YDC=(YM-YBC)*AM+YO*TX
   GOTO710
   700 YDC=(YM-YBC)*AM-YO*TX
   710 GL=(1.0-HBC)*CX-XC
   GP=SQRIF(YDC*YDC+GL*GL)
   IF(GL)720,730,760
   720 GAMMA=ATANF(YDC/GL)+PI
   GOTO770
   730 IF(YDC)740,1000,750
   740 GAMMA=-PI/2+0
   GOTO770
   750 GAMMA=PI/2+0
   GOTO770
   760 GAMMA=ATANF(YDC/GL)
   770 DELTA=GAMMA+ANG
   Z(J)=GP*SINF(DELTA)
   YJR=GP*COSF(DELTA)
   IF(YR-YJR)800,840,780
   780 XCA=XC
   IF(XCB)1000,790,820
   790 XX=XX+STEP2
   GOTO660
   800 XCB=XC
   IF(XCA)1000,810,820
Table 2. - Continued.

810 XX=XX+STEP2
     GOTO660
820 IF(XCA-XCB-DEVN3)840,840,830
830 XC=(XCA+XCB)*0.5
     GOTO655
840 CONTINUE
     GOTO(1000,1000,1000,400,530,1000,1000,1000,1000,1000)
1000 CALL EXIT
     END
### Table 4. - Output Machining Instructions.

**49 STATION NO**  | **BACK COORDINATES**
---|---
**RADIAL DISTANCE TO STATION = 4.000**

<table>
<thead>
<tr>
<th>NO</th>
<th>REV THOU</th>
<th>REV THOU</th>
<th>REV THOU</th>
<th>MACHINING COORDINATES</th>
<th>BOTTOM OF CUTTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>405</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>406</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>407</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>408</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>409</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>410</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NO</th>
<th>REV THOU</th>
<th>REV THOU</th>
<th>REV THOU</th>
<th>MACHINING COORDINATES</th>
<th>BOTTOM OF CUTTER</th>
</tr>
</thead>
<tbody>
<tr>
<td>415</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>420</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>425</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>430</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>435</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>440</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>445</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>450</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>455</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>460</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>465</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>470</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>475</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>480</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>485</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>490</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>495</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>500</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>505</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
<tr>
<td>510</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>X: 49.118 Y: 48.60 Z: 48.60</td>
<td>X: 3.7564 Y: 8.995 Z: 7.114</td>
</tr>
</tbody>
</table>
Table 4. -- Continued.

<table>
<thead>
<tr>
<th>515</th>
<th>16</th>
<th>86</th>
<th>48</th>
<th>59</th>
<th>25</th>
<th>52</th>
<th>4.086</th>
<th>6.059</th>
<th>3.177</th>
</tr>
</thead>
<tbody>
<tr>
<td>520</td>
<td>16</td>
<td>98</td>
<td>47</td>
<td>52</td>
<td>24</td>
<td>51</td>
<td>4.098</td>
<td>5.927</td>
<td>3.051</td>
</tr>
<tr>
<td>525</td>
<td>16</td>
<td>109</td>
<td>46</td>
<td>44</td>
<td>23</td>
<td>55</td>
<td>4.109</td>
<td>5.794</td>
<td>2.928</td>
</tr>
<tr>
<td>530</td>
<td>16</td>
<td>120</td>
<td>45</td>
<td>35</td>
<td>22</td>
<td>58</td>
<td>4.120</td>
<td>5.660</td>
<td>2.808</td>
</tr>
<tr>
<td>535</td>
<td>16</td>
<td>130</td>
<td>44</td>
<td>26</td>
<td>21</td>
<td>67</td>
<td>4.130</td>
<td>5.526</td>
<td>2.692</td>
</tr>
<tr>
<td>540</td>
<td>16</td>
<td>140</td>
<td>43</td>
<td>17</td>
<td>20</td>
<td>80</td>
<td>4.140</td>
<td>5.392</td>
<td>2.580</td>
</tr>
<tr>
<td>545</td>
<td>16</td>
<td>149</td>
<td>42</td>
<td>6</td>
<td>19</td>
<td>96</td>
<td>4.149</td>
<td>5.256</td>
<td>2.471</td>
</tr>
<tr>
<td>550</td>
<td>16</td>
<td>157</td>
<td>40</td>
<td>119</td>
<td>18</td>
<td>116</td>
<td>4.157</td>
<td>5.119</td>
<td>2.366</td>
</tr>
<tr>
<td>555</td>
<td>16</td>
<td>164</td>
<td>39</td>
<td>106</td>
<td>18</td>
<td>14</td>
<td>4.164</td>
<td>4.981</td>
<td>2.264</td>
</tr>
<tr>
<td>560</td>
<td>16</td>
<td>170</td>
<td>38</td>
<td>91</td>
<td>17</td>
<td>42</td>
<td>4.170</td>
<td>4.841</td>
<td>2.167</td>
</tr>
<tr>
<td>565</td>
<td>16</td>
<td>174</td>
<td>37</td>
<td>74</td>
<td>16</td>
<td>75</td>
<td>4.174</td>
<td>4.699</td>
<td>2.075</td>
</tr>
<tr>
<td>570</td>
<td>16</td>
<td>177</td>
<td>36</td>
<td>53</td>
<td>15</td>
<td>111</td>
<td>4.177</td>
<td>4.553</td>
<td>1.986</td>
</tr>
<tr>
<td>575</td>
<td>16</td>
<td>178</td>
<td>35</td>
<td>27</td>
<td>15</td>
<td>27</td>
<td>4.178</td>
<td>4.402</td>
<td>1.902</td>
</tr>
<tr>
<td>580</td>
<td>16</td>
<td>174</td>
<td>33</td>
<td>116</td>
<td>14</td>
<td>74</td>
<td>4.174</td>
<td>4.241</td>
<td>1.824</td>
</tr>
</tbody>
</table>

ST CHORD WIDTH = 4.717  LE RADIUS = 0.046

Table 3. -- Input Data for HYDRO - 1 Impeller.

<table>
<thead>
<tr>
<th>1</th>
<th>53</th>
<th>60</th>
<th>500</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7500000E+01</td>
<td>2000000E+00</td>
<td>5.625000E+01</td>
<td>2.250000E+02</td>
<td></td>
</tr>
<tr>
<td>0.2500000E+00</td>
<td>1.250000E+00</td>
<td>2.500000E+01</td>
<td>9.300000E+00</td>
<td>6.500000E-01</td>
</tr>
<tr>
<td>0.6000000E+01</td>
<td>4.000000E+01</td>
<td>2.000000E+00</td>
<td>1.9986E-04</td>
<td>5.000000E-03</td>
</tr>
<tr>
<td>0.1000000E+00</td>
<td>1.000000E+00</td>
<td>1.000000E+00</td>
<td>3.230000E-04</td>
<td>5.000000E-03</td>
</tr>
<tr>
<td>0.6250000E+00</td>
<td>2.500000E+00</td>
<td>1.280000E+00</td>
<td>1.250000E+00</td>
<td></td>
</tr>
<tr>
<td>0.5000000E+01</td>
<td>3.000000E+01</td>
<td>4.250000E+00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.
<table>
<thead>
<tr>
<th>POINTS ON SURFACE</th>
<th>POINTS ON SURFACE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>X</strong></td>
<td><strong>Y</strong></td>
</tr>
<tr>
<td>4.000</td>
<td>8.375</td>
</tr>
<tr>
<td>4.000</td>
<td>8.350</td>
</tr>
<tr>
<td>4.000</td>
<td>8.325</td>
</tr>
<tr>
<td>4.000</td>
<td>8.300</td>
</tr>
<tr>
<td>4.000</td>
<td>8.275</td>
</tr>
<tr>
<td>4.000</td>
<td>8.250</td>
</tr>
<tr>
<td>4.000</td>
<td>8.125</td>
</tr>
<tr>
<td>4.000</td>
<td>8.000</td>
</tr>
<tr>
<td>4.000</td>
<td>7.875</td>
</tr>
<tr>
<td>4.000</td>
<td>7.750</td>
</tr>
<tr>
<td>4.000</td>
<td>7.625</td>
</tr>
<tr>
<td>4.000</td>
<td>7.500</td>
</tr>
<tr>
<td>4.000</td>
<td>7.375</td>
</tr>
<tr>
<td>4.000</td>
<td>7.250</td>
</tr>
<tr>
<td>4.000</td>
<td>7.125</td>
</tr>
<tr>
<td>4.000</td>
<td>7.000</td>
</tr>
<tr>
<td>4.000</td>
<td>6.875</td>
</tr>
<tr>
<td>4.000</td>
<td>6.750</td>
</tr>
<tr>
<td>4.000</td>
<td>6.625</td>
</tr>
<tr>
<td>4.000</td>
<td>6.500</td>
</tr>
<tr>
<td>4.000</td>
<td>6.375</td>
</tr>
<tr>
<td>4.000</td>
<td>6.250</td>
</tr>
</tbody>
</table>
Table 5. Nomenclature of Programme Symbols.

<table>
<thead>
<tr>
<th>ITEM</th>
<th>DESCRIPTION</th>
<th>UNIT</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0, ..., A4</td>
<td>Coeff. of polynomial - Max. ord. of mean line</td>
<td>inches</td>
<td>a_d</td>
</tr>
<tr>
<td>ALTE</td>
<td>Min. allowable T.E. thickness of model</td>
<td>feet</td>
<td>a_m</td>
</tr>
<tr>
<td>AM</td>
<td>Max. mean line ordinate</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>AMI</td>
<td>&quot; &quot; &quot;    at hub</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>AMO</td>
<td>&quot; &quot; &quot;    at tip</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>ANG</td>
<td>Blade pitch angle</td>
<td>radians</td>
<td>β</td>
</tr>
<tr>
<td>ANGI</td>
<td>&quot; &quot; &quot; at hub</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>ANGO</td>
<td>&quot; &quot; &quot; at tip</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>CID</td>
<td>Chord width at hub</td>
<td>feet</td>
<td>&quot;</td>
</tr>
<tr>
<td>COD</td>
<td>&quot; &quot; &quot; tip</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>COSA</td>
<td>Direction cosines - normal to surface</td>
<td>&quot;</td>
<td>x</td>
</tr>
<tr>
<td>COSB</td>
<td>&quot; &quot; &quot; &quot; &quot;</td>
<td>&quot;</td>
<td>β</td>
</tr>
<tr>
<td>COSC</td>
<td>&quot; &quot; &quot; &quot; &quot;</td>
<td>&quot;</td>
<td>γ</td>
</tr>
<tr>
<td>CUTTR</td>
<td>Rad. of spherical milling cutter</td>
<td>inches</td>
<td>&quot;</td>
</tr>
<tr>
<td>CX7</td>
<td>Chord width at R0.7</td>
<td>feet</td>
<td>&quot;</td>
</tr>
<tr>
<td>CX</td>
<td>&quot; &quot; at rad. fraction x</td>
<td>inches</td>
<td>c_x</td>
</tr>
<tr>
<td>DELTA</td>
<td>See Fig. 5</td>
<td>radians</td>
<td>δ = β + γ</td>
</tr>
<tr>
<td>DEVN1</td>
<td>Computing accuracy of CB - Sec. 4.1</td>
<td>inches</td>
<td>&quot;</td>
</tr>
<tr>
<td>DEVN2</td>
<td>Edge allowance on T.E. - Sec. 5.1</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>DEVN3</td>
<td>Computing accuracy of XC - Sec. 4.2</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>DX</td>
<td>Dist. between row planes - model</td>
<td>inches</td>
<td>&quot;</td>
</tr>
<tr>
<td>GAMMA</td>
<td>See Fig. 5</td>
<td>radians</td>
<td>&quot;</td>
</tr>
<tr>
<td>GDT</td>
<td>Grad. of tangent to blade edge in X, Y plane</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>GL, GP</td>
<td>See Fig. 5</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>GRAD(L,J)</td>
<td>Gradient of surface J in direction L</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>HBC</td>
<td>Blade section characteristic (Ref. 3 p 293)</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>IP</td>
<td>No. of row plane where calculations begin</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>IPAT</td>
<td>&quot; &quot; &quot; end</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>KX, KXX</td>
<td>No. of rotn. &amp; parts of lead screw X direc.</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>KY, KYY</td>
<td>&quot; &quot; &quot; Y</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>KZ, KZZ</td>
<td>&quot; &quot; &quot; Z</td>
<td>&quot;</td>
<td>&quot;</td>
</tr>
<tr>
<td>ITEM</td>
<td>DESCRIPTION</td>
<td>UNIT</td>
<td>SYMBOL</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
<td>--------</td>
</tr>
<tr>
<td>MLE(I)</td>
<td>No. of points, (Y=0) to (Y=PCX) of L.E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTE(I)</td>
<td>&quot; &quot; &quot; &quot; &quot; &quot; &quot; of T.E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NKN</td>
<td>No. of point on the (N=0) plane sec. 5.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NLE(I)</td>
<td>No. of points from (Y=0) to L.E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NN</td>
<td>No. of the NNN plane on line of centroids</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTE(I)</td>
<td>&quot; &quot; of points from (Y=0) to T.E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTEST</td>
<td>O Punch (0/P, +ve) punch (0/P) &amp; intermediate calculation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PCX</td>
<td>Fraction of CX where spacing is SPACX</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>Pitch to diameter ratio</td>
<td></td>
<td>P/D</td>
</tr>
<tr>
<td>FERCR</td>
<td>Initial estimate of RX - Sec. 4.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PT(L,KJ)</td>
<td>ZB ord. of points surrounding pt. I, NNN</td>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>PTX(J)</td>
<td>ordinates of bottom of cutter (X) direction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PTY(J)</td>
<td>&quot; &quot; &quot; &quot; &quot; &quot; (Y) &quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PTZ(J)</td>
<td>&quot; &quot; &quot; &quot; &quot; &quot; (Z)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RAD</td>
<td>Rad. of impeller - prototype</td>
<td>feet</td>
<td>R</td>
</tr>
<tr>
<td>RADLE</td>
<td>Rad. of L.E. factor (RLE(I) = \frac{t^2}{c})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RADM</td>
<td>Rad. of blade tip of master template</td>
<td>inches</td>
<td>(R_m)</td>
</tr>
<tr>
<td>RATIO</td>
<td>Ratio of boss to tip radius</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RLE(I)</td>
<td>Rad. of L.E. at section</td>
<td>inches</td>
<td>(r_x)</td>
</tr>
<tr>
<td>RX</td>
<td>Radial distance to point</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RXLE(I)</td>
<td>&quot; &quot; &quot; &quot; L.E. at row plane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RXTE(I)</td>
<td>&quot; &quot; &quot; &quot; T.E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SCALE</td>
<td>Scale of prototype to master template</td>
<td>R/R_M</td>
<td></td>
</tr>
<tr>
<td>SCALM</td>
<td>Scale of prototype to model</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPACN</td>
<td>Minimum spacing between column planes</td>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>SPACX</td>
<td>Maximum &quot; &quot; &quot; &quot; &quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STEPI</td>
<td>Forced convergence interval - Sec. 4.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>STEP2</td>
<td>&quot; &quot; &quot; &quot; Sec. 4.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T0,...,T3</td>
<td>Coeff. of polynomial for max. thickness</td>
<td></td>
<td>(t_i)</td>
</tr>
<tr>
<td>TX</td>
<td>Blade section maximum thickness</td>
<td></td>
<td>(t_x)</td>
</tr>
<tr>
<td>X</td>
<td>X ordinate</td>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>ITEM</td>
<td>DESCRIPTION</td>
<td>UNIT</td>
<td>SYMBOL</td>
</tr>
<tr>
<td>----------</td>
<td>-------------------------------------------------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>XTL</td>
<td>X ordinate of L,E. at hub</td>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>XIT</td>
<td>&quot; &quot; &quot; T,E. &quot; &quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XMLR</td>
<td>Pitch of lead screw of miller - x direc.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XOL</td>
<td>X ordinate of L,E. at tip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XOT</td>
<td>&quot; &quot; &quot; T,E. &quot; &quot;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XR</td>
<td>Radius fraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XX</td>
<td>Fraction of chord</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Y_0,...,Y_9</td>
<td>Coeff. of polynomial - NACA-16 profile</td>
<td>inches</td>
<td>y_i</td>
</tr>
<tr>
<td>Y</td>
<td>Y ordinate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YΒ(I,NNN)</td>
<td>Y ordinate w.r. to axes XB, YB, ZB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YΒCT</td>
<td>Blade section characteristics (Ref. 3 p293)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YС(L,KA)</td>
<td>Y ordinate of PT(L,KA,J) w.r. to YB</td>
<td>inches</td>
<td></td>
</tr>
<tr>
<td>YD</td>
<td>Distance from X, Z plane to XB, ZB plane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YDC</td>
<td>See Fig. 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YЛE(I)</td>
<td>Y ordinate of L,E. at section</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YM</td>
<td>See Fig. 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YМILR</td>
<td>Pitch of lead screw of miller - Y direction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YO</td>
<td>Ordinate of NACA - 16 profile</td>
<td></td>
<td></td>
</tr>
<tr>
<td>XR</td>
<td>Circumferencial distance from N = 0 plane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YТE(I)</td>
<td>Y ordinate to T,E. at section</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YΤТЕ</td>
<td>Difference in Y of back &amp; face at T,E.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>YΥΥ(J)</td>
<td>Y ordinate of point w.r. to machining axes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ζ(J)</td>
<td>Z &quot; &quot; &quot; X,Y,Z axes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΖΒ(I,ΝΝΝ,J)</td>
<td>&quot; &quot; &quot; XB,YB,ZB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZD</td>
<td>Distance from X,Y plane XN, YN .plane</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZМILR</td>
<td>Pitch of lead screw of miller - z direction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΖΖΖ(J)</td>
<td>Z ordinate of point w,r. to machining axes</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ DV = Y_{\text{NACA}} - Y_0 \]

where \( Y_{\text{NACA}} \) = Ordinate Value given by NACA - 16 Profile.

\[ Y_0 = (1 - XX) \left[ \frac{1}{\sqrt{0.98879XX} + Y_i XX^i} \right] \]

where \( i = 0, 1, \ldots, 9 \)

Fig. 1 - Deviation from NACA - 16 Profile of Unit Thickness.
Fig. 2/1

Fig. 2/1 - Computing Boundary Points.

NOTE:
SEE DETAILED
FIGURE 2/13
DO 550
  I = IPP, II
CALC X
STOP
I - 5
NLE(I) = 0
MLE(I) = 0
NTE(I) = 0
MTE(I) = 0
NLE(I), NTE(I)
RX, XR
K = 8
600
348
YYTE
DO 510
M = 1, 2
Y = 0, 0
DO 510
N = 1, NN
STOP
M - 1
N-NLE(I)-1
Y-PCX*YLE(I)
Y = Y + SPACX
MLE(I) = N
450
Y = Y + SPACN
NLE(I) = N-1
450
Y-YLE(I)
Y-YTE(I)
Y-YYTE
Y = Y - SPACN
MTE(I) = N
Y = Y - SPACX
NTE(I) = N-1
YYTE
DO N
DO M
Fig. 2/2 - Calculating Surface Points.
Fig. 2/3 - Calculation of Blade Surface Points.
Fig. 2/4 - Trailing Edge Machining Co-ordinates.
Fig. 2/5 - Routine for Calculating Blade Section Variables.
Fig. 2/6 - Routine for Calculating Blade Section Variables and Surface Point Co-ordinates.
Fig. 2/7 - Determining Region of Surface Points.
Fig. 2/8 - Locating Neighbouring Surface Points - Region I
Fig. 2/9 - Locating Neighbouring Surface Points - Region II
Fig. 2/10 - Calculating Machining Co-ordinates for Surface Points.
Fig. 2/11 - Output for Machining Instructions.
Fig. 2/12 - Calculating Gradient of Surface at a Point.
Fig. 2/13 - Convergence Routine for Boundary Points.
Fig. 3 - Calculation Axes

Fig. 4 - Machining Axes.
Fig. 5 - Blade Surface Variables.
Fig. 6 - Blade Section Variables.
Fig. 7 - Computed Blade Surfaces at Selected Stations HYDRO-1 Impeller.
APPENDIX A3

MACHINING PROCEDURE FOR THE MODEL IMPELLER.
MACHINING PROCEDURE FOR THE MODEL IMPELLER.

1. The master templates were cast with free cutting alluminium alloy (6% Si, 0.2% Cu, 0% Fe).

2. After rough machining the base of the templates and heat treating them, the surfaces were accurately machined and hand-scraped.

3. To accurately position the templates on the table of a milling machine, it was necessary to machine two surfaces normal to each other and normal to the base of the templates. It was arranged that these surfaces were parallel to the axes which were used to describe the geometry of the impeller blade.

4. A datum jig (Fig. 2.6) was constructed and fixed relative to the above three surfaces to locate the origin from which the machining coordinates were computed. (details in Appendix A2)

5. After having positioned the milling cutter directly over the datum jig, the milling machine positioning dials were pre-set to zero.

6. The surface, as defined by the points, was then machined (Fig. 2.6).

7. A small amount of hand filing and scraping was necessary to remove the excess material from between the accurately machined points.

8. The inner and outer cylindrical surfaces of the templates were machined and a dummy hub fitted to the inner surface.

9. The model impeller material was rough machined, heat treated, and bored to final size.
10. A precision indexing table was mounted on the pantograph copying machine to support the model impeller (Fig. 2.7). Care was taken to align the master templates and the model material in the correct position relative to the follower and cutter of the copying machine.

11. The model impeller was then roughly machined all over to within 0.040 of an inch (Fig. 2.8).

12. One surface of all the blades was then finished.

13. Before machining the second side of the blades, the blades were supported from behind with plastic putty to increase the stiffness of the material and reduce deflections when machining (Fig. 2.7).

14. Finally the accuracy and balance of the model was checked (Fig. 2.9).
PROPULSIVE EFFICIENCY OF DUCTED PROPULSION SYSTEM.
APPENDIX A 4

PROPULSIVE EFFICIENCY OF A DUCTED PROPULSION SYSTEM.

Assuming all the losses in the ducting can be expressed as a fraction \( \frac{f}{j} \) of the total kinetic energy of the fluid at the intake to the duct.

\[
\text{duct loss /unit mass flow} = \frac{f}{j} \frac{V_i^2}{2} \quad \ldots \text{A4.1}
\]

Then the total head across the impeller is, referring to figure below.

\[
H_i = H_4 - H_1 + h \quad \ldots \text{A4.2}
\]

\[
= \frac{V_j^2}{2g} - \frac{V_i^2}{2g} + \frac{f}{j} \frac{V_i^2}{2g} \quad \ldots \text{A4.3}
\]

\[
= \frac{1}{2g} \left[ V_j^2 - V_i^2 \left( 1 - \frac{j}{f} \right) \right] \quad \ldots \text{A4.4}
\]

The power supplied to an impeller which has an efficiency \( \eta_E \)

\[
P = \frac{\frac{1}{\eta_E} g M H_i}{\text{A4.5}}
\]

where \( M \) = mass of fluid flowing per second.
Assuming the intake and jet velocities are uniform, then the thrust developed by unit is equal to rate of change of momentum.

\[ T = M (V_J - V_I) \] ... A4.6

To estimate the expended horse power delivered by the impeller a hull efficiency factor must be included because the resistance of the ship does not usually equal the thrust of the propulsion unit and the ship's velocity differs from the intake velocity to the propulsion system.

Defining the hull efficiency \( \gamma_H \) in the normal way as

\[ \gamma_H = \frac{(1 - t)}{(1 - w)} \] ... A4.7

where \( t = \) thrust deduction factor

and \( w = \) wake factor

Hence the expended horse power is

\[ \text{E.H.P.} = \gamma_H (V_J - V_I) M V_I \] ... A4.8

and the propulsive efficiency \( \gamma_P \) of the unit is

\[ \gamma_P = \frac{\text{E.H.P.}}{\text{Power input}} = \gamma_E \gamma_H \frac{2 \frac{V_I}{V_J} (V_J - V_I)}{\left[ \frac{V_J^2}{V_I^2} - (1 - \xi) \right]} \] ... A4.9

\[ = \gamma_E \gamma_H \frac{2 \mu (1 - \mu)}{\left[ 1 - \mu^2 (1 - \xi) \right]} \] ... A4.10

where \( \mu = \frac{V_I}{V_J} \)
The maximum efficiency for a given $\xi$ is

$$\gamma_p \text{ opt} = \gamma_E \gamma_H \frac{1}{[1 + \sqrt{\xi}]^2}$$

...A4.11

and occurs where $\mu \text{ opt} = \frac{1}{1 + \sqrt{\xi}}$

The propulsive efficiency can also be expressed in terms of the thrust load coefficient ($C_{TL}$) of the impeller.

$$\gamma_p \text{ opt} = \gamma_E \gamma_H \frac{4 (C_{TL} K_A)}{[ (C_{TL} K_A)^2 + 4 (C_{TL} K_A) + 4 \xi ]}$$

...A4.12

where $C_{TL} = \frac{T}{\frac{1}{2} \rho V_a^2 A_o}$

$$= 2 K_A [ \frac{1}{\mu} - 1 ]$$

where $V_a$ = axial velocity at the impeller plane

$A_o$ = area of impeller annulus

$K_A$ = ratio of impeller annulus area to area of duct at intake.

The optimum propulsive efficiency for a given $\xi$ corresponds to a unique value of thrust load coefficient where,

$$\gamma_p \text{ opt} = 2 \gamma_E \gamma_H \frac{1}{[C_{TL \text{ opt}} K_A + 2]}$$

...A4.13

where $C_{TL \text{ opt}} = \frac{2 \sqrt{\frac{\xi}{K_A}}}{K_A}$
APPENDIX A5

HYDRO-JET PROPULSION REDUCES VIBRATION

by

M.R.Hale and D.H.Norrie

Engineering, 24 July 1964

NOTE:

This publication is included in the print copy of the thesis held in the University of Adelaide Library.
APPENDIX A6

THE ANALYSIS AND CALIBRATION OF THE FIVE-HOLE SPHERICAL PITOT

by

M.R.Hale and D.H.Norrie

UNIVERSITY OF ADELAIDE

DEPARTMENT OF MECHANICAL ENGINEERING

Report Mech. Eng. E66/1

March, 1966.
THE ANALYSIS & CALIBRATION OF THE FIVE-HOLE SPHERICAL PITOT

by M.R. Hale & D. H. Norrie

SUMMARY

The theory of the five-hole spherical pitot is summarized and the reasons for the actual characteristics deviating from the theoretical discussed. An analysis and associated technique for the calibration of the pitot, even in a flow whose direction is only approximately known, is given. The results of the calibration can be put in a form suitable for the reduction of test data by a digital computer. The technique described allows a calibration of high accuracy to be obtained with a simple flow duct and should enable a much wider use to be made of this simple instrument for measuring simultaneously the magnitude and direction of fluid velocity.
INTRODUCTION.

A variety of instruments are available for determining separately the magnitude and direction of the velocity at a point in a fluid stream. (Ref. 1). There are few, however, which determine simultaneously both magnitude and direction. Of those with this capability the simplest depend on the measurement of pressure at points on the surface of a sphere. Spherical pitots using five holes were used by Taylor (Ref. 2) in 1915, Meyer and Borren (Ref. 3) in 1928 and Gutsche (Ref. 4) in 1931. A 13-hole instrument was early developed by the David Taylor Model Basin (Ref. 2), Krisam (Ref. 5), Jegerow (Ref. 6) & Eckert (Ref. 7) also reported on these pitots. A notable advance was made by Pien (Ref. 8) in 1958, who utilized the fact that three pressure measurements on a great circle of a sphere determine uniquely the velocity component in that plane. By reducing the angular distance between the side and centre holes to 20° from the previously used 40-50° Pien also increased the accuracy and range of the instrument. The Pien pitot has subsequently been reported on favourably by Silovic (Ref. 9).

Up to the present, accurate calibration of the instrument has required a flow whose direction is known accurately. This has necessitated the use of a towing tank, in which the relative motion of carriage and water is precisely known, or a very precise wind or water tunnel. It is the purpose of this paper to show that an accurate calibration can be carried out in a flow whose direction is only approximately known. Calibration is thus
possible in a small wind tunnel, water tunnel or duct. Although this paper is concerned with the application of the probe in incompressible flows, it can also be used in compressible flows (Ref. 1, 10) and the technique described below can be adapted to this case.

2.0 **THEORY.**

The potential solution of the flow around a sphere gives the pressure at a point as:

\[
\frac{p - p_o}{\frac{1}{2} \rho V^2} = 1 - \frac{9}{4} \sin^2 \beta \tag{1}
\]

where

- \( p \) = pressure at point considered
- \( p_o \) = free stream pressure
- \( V \) = magnitude of velocity vector \( \vec{V} \) of free stream
- \( \rho \) = fluid density
- \( \beta \) = angular position of point from stagnation point.

For three equispaced holes a, b, c on a great circle of a sphere (see Fig. 1) Pien (Ref. 8) has shown that:

\[
C_{ph} = \frac{p_a - p_b}{\frac{1}{2} \rho V^2 h} = \frac{2}{9} \sin 2\alpha \sin 2(\beta_h) \tag{2}
\]

\[
C_{Ph} = \frac{p_a - p_b}{2p_c - p_a - p_b} = \frac{\sin 2\alpha}{1 - \cos 2\alpha} \tan 2(\beta_h) \tag{3}
\]

where \( p_a, p_b, p_c \) = pressures at a, b, c

\( C_{ph}, C_{Ph} \) = pressure coefficients

\( \alpha \) = angle between adjacent holes
\[ V_h = \text{Orthogonally projected component of the velocity} \]
\[ \vec{V} \text{ onto the plane of the great circle.} \]
\[ \beta_h = \text{angle between } V_h \text{ and the stagnation point.} \]

From measurements of \( p_a', p_b', p_c' \), the value of \( \beta_h \) can be calculated from equation 3, whence \( V_h \) can be obtained from equation 2. If on the great circle through \( c \) orthogonal to that through \( a, b, c \), two equispaced holes \( a' \) and \( b' \) are placed as shown in Fig. 1., the component \( V_\mathbf{v} \) of the velocity \( \vec{V} \) in the plane of this circle can be determined from measurement of \( p_a', p_c', p_b' \), by equations analogous to 2 and 3. The three components \( V_x, V_y, V_z \) along the sphere axes \( X, Y, Z \) of the velocity \( \vec{V} \) are then given by:

\[ V_x = V_h \cos \beta_h, \quad V_y = V_h \sin \beta_h, \quad V_z = V_\mathbf{v} \sin \beta_\mathbf{v} \]

In Refs. 8 and 9 in place of equation 2, the appropriate relations for

\[ \frac{p_c - p_a}{2 \rho V_h^2}, \quad \frac{p_c - p_b}{2 \rho V_h^2} \]

are used. The first or second of these pressure coefficients is taken depending on whether \( V_h \) is on one side or the other side of the centre hole. This introduces an unnecessary complication.

Use of equation 2 is recommended also because its pressure coefficient has a greater variation at any given value of \( \beta_h \) than these other two coefficients, thus giving a greater sensitivity.
Equations 2 and 3 are only true if the fluid has no viscosity, the sphere is perfect, the holes are exactly positioned, and the hole size approaches zero. Since none of these conditions is true in practice, calibration of the instrument is necessary to obtain curves of $C_{ph}$ and $C_{ph}$ versus $\beta_h$, which are used in place of equations 2 and 3. Similar calibration is necessary for $C_{pv}$ and $C_{pv}$.

Ideally, calibration for all combinations of values of $C_{ph}$, $C_{ph}$, $C_{pv}$, $C_{pv}$, $\beta_h$, $\beta_v$ is required at all Reynolds numbers. This is not practical, or in reality necessary. Adequate accuracy can be obtained by using the following approximations.

1. That the position of an orthogonal set of axes OX, OY, OZ (the 'pitot axes') passing through the sphere centre and fixed to the pitot can be found experimentally such that if $a$, $a'$, $c$, $b'$ are assumed to be on the orthogonal planes X0Y, X0Z respectively, the maximum error in the computed velocity vector $\vec{V}$ will be minimized within the variation of magnitude and direction of $\vec{V}$ it is wished to consider. These axes can be regarded as the 'best-fit' axes for the holes $a$, $a'$, $c$, $b$, $b'$. The actual hole positions will clearly be very close to the planes X0Y, X0Z for a well-made instrument.
(2) That the relationship between $C_{ph}$, $C_{Ph}$, and $\beta_h$ is independent of $V_v$ and $\beta_v$ to a high degree of accuracy. In theory, as 2 and 3 indicate, there is complete independence. In practice, there can be a slight dependence. Similarly that the relationship between $C_{pv}$, $C_{Pv}$, and $\beta_v$ is independent of $V_h$ and $\beta_h$.

For a perfect instrument and fluid, the velocity $\vec{V}$ will be along the X pitot axis when:

(a) the pressures at a, b, a', b' are equal.

(b) the pressures remain equal at a and b for rotation about OY, and at a' and b' for rotation about OZ.

For the actual case, if a stream of precisely known direction is available, the pitot can be orientated in this stream to obtain the best overall compromise between condition (a) and condition (b) for angular rotations within the range required, thus determining the 'pitot axes' OX, OY, OZ.

If such a stream is not available, the following method can be used.

3.1 Initial Angular Calibration

The pitot is set up in a flow whose direction is approximately known, so that it can be rotated about two axes OY', OZ' which are normal to each other and to the nominal flow direction. OY' and OZ' pass through the sphere centre which remains fixed in space. The axis OX' also passes through the sphere centre and is orthogonal to OY' and OZ'.
The axes OX', OY', and OZ' remain fixed in space and in practice represent the axes of a mounting device or gymbal. The sphere is then orientated about its centre until the best compromise is obtained to condition (b) above. Figs. 2a and 2b indicate the effects of the "lateral tilt" and "rotational tilt" on the pressure difference of a hole pair, and the way in which the shape of the pressure difference curve can be used to indicate the corrective orientation necessary. If OY' and OZ' were truly normal to the flow vector \( \vec{V} \) their position relative to the sphere would now closely coincide with the "pitot axes" OY, OZ defined earlier. In practice \( \vec{V} \) would only be a few degrees from the direction of X'O so that as a first approximation it can be assumed to be in the direction of X'O. Calibration graphs of \( C_{ph} \) and \( C_{ph} \) versus \( \beta_h \), and \( C_{pv} \) and \( C_{pv} \) versus \( \beta_v \) can now be determined by rotation about OZ', OY'. These pressure coefficients can be assumed to be a first approximation of those which would be obtained by rotation about the pitot axes OZ and OY with \( \vec{V} \) being initially exactly along XO. Note that the accuracy of the approximation decreases with increase in the angle of rotation about OZ' and OY', so that the initial orientation and calibration should only be for small values of \( \beta_h \) and \( \beta_v \) (eg. up to \( \pm 7^\circ \)). The magnitude of \( \vec{V} \) can be measured to to sufficient accuracy by a standard bull-nose pitot-tube, which is not very sensitive to a few degrees yaw in the flow.
The error introduced by the flow velocity \( \vec{V} \) not being in the calibration plane will be quite small since the velocity in the Calibration plane will differ from \( \vec{V} \) only by \( V(1 - \cos \phi) \) where \( \phi \) is the angle of \( \vec{V} \) to the calibration plane. Usually \( \phi \) will not be more than a few degrees.

3.2 Orientation of Pitot Axes with reference to Datum Instrument Axes.

On the pitot base there will have been machined location faces. These can be used to define a set of orthogonal axes at the sphere centre, which will be called the datum instrument axes \( OXX, OYY, OZZ \). The problem is to now determine the orientation or position of the pitot axes with reference to these known instrument axes. As shown below, this orientation can be determined in a stream whose flow direction is only approximately known, by three inversions of the sphere position, providing the pitot is set up in the flow so that the sphere can be rotated about \( OXX, OYY \) or \( OZZ \) without the centre of the sphere moving in space.

The direction cosines of the instrument axes \( OXX, OYY, OZZ \) with respect to the (unknown) pitot axes \( OX, OY, OZ \) will be denoted by:

<table>
<thead>
<tr>
<th></th>
<th>( OX )</th>
<th>( OY )</th>
<th>( OZ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( OXX )</td>
<td>( 11 )</td>
<td>( 12 )</td>
<td>( 13 )</td>
</tr>
<tr>
<td>( OYY )</td>
<td>( 21 )</td>
<td>( 22 )</td>
<td>( 23 )</td>
</tr>
<tr>
<td>( OZZ )</td>
<td>( 31 )</td>
<td>( 32 )</td>
<td>( 33 )</td>
</tr>
</tbody>
</table>

The direction cosines of the velocity \( \vec{V} \) with respect to the instrument axes \( OXX, OYY, OZZ \) will be denoted by \( l, m, n \).
3.2.1 Rotation about OXX Axis.

The sphere is initially positioned so that the flow vector $\vec{V}$ is at some estimated angle $\varphi$ to the centre hole (of between $5^\circ$ and $10^\circ$). The sphere is then rotated successively by $90^\circ$ about the instrument axis OXX, the initial position being that shown in Fig. 1 and denoted by the subscript 1, and the successive positions by subscripts 2, 3, 4. The direction of rotation is with the OZZ axis moving towards the position of the OYY axis. The velocity $\vec{V}$ becomes with respect to the axes, successively, $\vec{V}_1$, $\vec{V}_2$, $\vec{V}_3$, $\vec{V}_4$ with direction cosines relative to the instrument axes OXX, OYY, OZZ as shown below.

<table>
<thead>
<tr>
<th></th>
<th>OXX</th>
<th>OYY</th>
<th>OZZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{V}_1$</td>
<td>1</td>
<td>m</td>
<td>n</td>
</tr>
<tr>
<td>$\vec{V}_2$</td>
<td>1</td>
<td>n</td>
<td>$-m$</td>
</tr>
<tr>
<td>$\vec{V}_3$</td>
<td>1</td>
<td>$-m$</td>
<td>$-n$</td>
</tr>
<tr>
<td>$\vec{V}_4$</td>
<td>1</td>
<td>$-n$</td>
<td>m</td>
</tr>
</tbody>
</table>

The direction cosines of $\vec{V}_1$, $\vec{V}_2$, $\vec{V}_3$, $\vec{V}_4$ with respect to the pitot axes OX, OY, OZ will be

$$\frac{V_{x1}}{V_1}, \frac{V_{y1}}{V_1}, \frac{V_{z1}}{V_1}, \frac{V_{x2}}{V_2} \text{ etc.}$$

These can be denoted generally by

$$\frac{V_{ij}}{V_j} \quad \text{where } i = 1, 2, 3$$

according to whether $V_{ij}$ is the $x, y, z$ component, and $j$ takes the values 1, 2, 3, 4 according to whether $V_j$ is the magnitude of $\vec{V}_1$, $\vec{V}_2$, $\vec{V}_3$, $\vec{V}_4$. The ratio $\frac{V_{ij}}{V_i}$ will also be denoted by $a_{ij}$.
By the transformation of axes these direction cosines are given by:

$$a_{11} = l_{11}^1 + l_{21}^m + l_{31}^n \quad \text{with } i = 1, 2, 3 \quad \ldots \quad 6$$

$$a_{12} = l_{11}^1 + l_{21}^n - l_{31}^m \quad \text{with } i = 1, 2, 3 \quad \ldots \quad 7$$

$$a_{13} = l_{11}^1 - l_{21}^m - l_{31}^n \quad \text{with } i = 1, 2, 3 \quad \ldots \quad 8$$

$$a_{14} = l_{11}^1 - l_{21}^n + l_{31}^m \quad \text{with } i = 1, 2, 3 \quad \ldots \quad 9$$

Since the $a_{ij}$ of equations 6 to 9 can be computed from the pressure readings, the initial angular calibrations, and equation 4, we have a set of 12 equations in the 12 unknowns $l_{pq}$ $(p, q = 1, 2, 3)$ and $l, m, n$.

There are also 6 identities between the direction cosines $l_{pq}$ of the type

$$l_{11}^1 + l_{21}^2 + l_{31}^2 = 1 \quad \ldots \quad 10,$$

and a similar relation for the direction cosines $l, m, n$. There are thus 19 equations available in the 12 unknowns. On solving these equations it is found that they are not all independent and that the following relations exist between the direction cosines $a_{ij}$:

$$a_{11} + a_{13} = a_{12} + a_{14} \quad \text{with } i = 1, 2, 3 \quad \ldots \quad 11.$$  

The only explicit solutions for the direction cosines $l_{pq}$ and $l, m, n$ are those for $l_{11}, l_{12}, l_{13}$, and $l$. Addition of equations 6 to 9 in respective pairs gives

$$a_{11} + a_{13} = 2l_{11}^1, \quad i = 1, 2, 3 \quad \ldots \quad 12$$

$$a_{12} + a_{14} = 2l_{11}^1, \quad i = 1, 2, 3 \quad \ldots \quad 13.$$
Substituting equations 12 and 13 in 10 gives:

\[ l = \frac{1}{2} \left[ (a_{11} + a_{13})^2 + (a_{21} + a_{23})^2 + (a_{31} + a_{33})^2 \right]^{\frac{1}{2}} \]

\[ ... \]

\[ l = \frac{1}{2} \left[ (a_{12} + a_{14})^2 + (a_{22} + a_{24})^2 + (a_{32} + a_{34})^2 \right]^{\frac{1}{2}} \]

\[ ... \]

Since the velocities \((V_x', V_y', V_z', V)\)\(_{1,2,3,4}\) are determined experimentally, the experimental values for \(a_{ij}\) may not exactly satisfy equation 11. Equations 14 and 15 will yield two experimentally derived values of \(l\), and substitution respectively into 12 and 13 gives two sets of values for \(l_{11}', l_{12}', l_{13}'\). Since the initial angular calibration graphs used to calculate \((V_x', V_y', V_z')\)\(_{1,2,3,4}\) are a first approximation, these values of \(l, l_{11}', l_{12}', l_{13}'\) must also be regarded as first approximations.

### 3.2.2 Inversion about OYY Axis.

The sphere is rotated to the initial position 1 of Section 3.2.1. The velocity \(\bar{V}\) will make an angle \(\Theta\) with the XXYY plane as shown in Fig. 3. Since the flow direction is known approximately, an estimate of the value of \(\Theta\) can be made. The sphere is then rotated about OYY so that \(\bar{V}\) will make an angle of approximately \(-\Theta\) with the XXYY plane. The initial and final configurations will be denoted by subscripts 1 and 5.

The components of \(\bar{V}_1\) with respect to OXX, OYY, OZZ will be (Fig. 3).

\[ V_1 \cos \beta_1 \cos \Theta_1; \quad V_1 \sin \beta_1; \quad V_1 \cos \beta_1 \sin \Theta_1 \]

\[ ... \]

\[ 16. \]
and the corresponding direction cosines of $\vec{V}_1$ will thus be

$$\cos \beta_1 \cos \theta_1; \sin \beta_1; \cos \beta_1 \sin \theta_1 \quad \ldots \quad 17.$$  

The direction cosines $l_1, m_1, n_1$ of $\vec{V}_1$ with respect to $OX, OY, OZ$ are from equations 17 and 5 using the transformation-of-axes theorem.

$$l_1 = l_{11} \cos \beta_1 \cos \theta_1 + l_{21} \sin \beta_1 + l_{31} \cos \beta_1 \sin \theta_1$$

$$m_1 = l_{12} \cos \beta_1 \cos \theta_1 + l_{22} \sin \beta_1 + l_{32} \cos \beta_1 \sin \theta_1$$

$$n_1 = l_{13} \cos \beta_1 \cos \theta_1 + l_{23} \sin \beta_1 + l_{33} \cos \beta_1 \sin \theta_1 \quad \ldots \quad 18.$$  

A similar set of equations 19 to that of 18 can be derived for $\vec{V}_5$.

The components of the vector $\vec{V}_6 = \vec{V}_1 - \vec{V}_5$ with respect to $OX, OY, OZ$ are given by

$$V_{x6} = V_{11} - V_{51}$$

$$V_{y6} = V_{12} - V_{52}$$

$$V_{z6} = V_{13} - V_{53} \quad \ldots \quad 20$$

and substituting into equation 20 from 18 and 19 and putting $V_1 = V_5 = V$, and $\beta_1 = \beta_5 = \beta$ to simplify we obtain:

$$\frac{V_{x6}}{V} = \left( l_{11} \cos \beta \cos \theta_1 + l_{31} \cos \beta \sin \theta_1 \right) - \left( l_{11} \cos \beta \cos \theta_5 + l_{31} \cos \beta \sin \theta_5 \right)$$

$$\frac{V_{y6}}{V} = \left( l_{12} \cos \beta \cos \theta_1 + l_{32} \cos \beta \sin \theta_1 \right) - \left( l_{12} \cos \beta \cos \theta_5 + l_{32} \cos \beta \sin \theta_5 \right)$$
\[ \frac{V_{z6}}{V} = (l_{13} \cos \beta \cos \theta_1 + l_{33} \cos \beta \sin \theta_1) - (l_{13} \cos \beta \cos \theta_5 + l_{33} \cos \beta \sin \theta_5) \]

The components of \( \vec{V}_6 \) are also given by

\[ V_{x6} = V_{x1} - V_{x5} \]
\[ V_{y6} = V_{y1} - V_{y5} \]
\[ V_{z6} = V_{z1} - V_{z5} \]

Substituting equation 22 into 21 gives:

\[ \frac{V_{x1} - V_{x5}}{V \cos \beta} = l_{11} (\cos \theta_1 - \cos \theta_5) + l_{31} (\sin \theta_1 - \sin \theta_5) \]
\[ \frac{V_{y1} - V_{y5}}{V \cos \beta} = l_{12} (\cos \theta_1 - \cos \theta_5) + l_{32} (\sin \theta_1 - \sin \theta_5) \]
\[ \frac{V_{z1} - V_{z5}}{V \cos \beta} = l_{13} (\cos \theta_1 - \cos \theta_5) + l_{33} (\sin \theta_1 - \sin \theta_5) \]

Since \( \theta_5 \approx \theta_1 \), the first term of equation 23 is approximately zero and can be neglected.

Hence equation 23 reduces to:

\[ \frac{V_{x1} - V_{x5}}{V} = l_{31} A \]
\[ \frac{V_{y1} - V_{y5}}{V} = l_{32} A \]
\[ \frac{V_{z1} - V_{z5}}{V} = l_{33} A \]
where \( A = \cos \beta (\sin \theta_1 - \sin \theta_5) \)

From the pressure measurements at configurations 1 & 5 and using the initial angular calibrations \((V_x', V_y', V_z', V)\) 1 & 5 can be estimated, and substitution into equation 24 together with the condition that

\[
1_{31}^2 + 1_{32}^2 + 1_{33}^2 = 1 \quad \cdots \quad 25
\]

enables \(1_{31}', 1_{32}', 1_{33}'\) to be calculated.

By setting the pitot back to configuration 4 of Section 3.2.1 and inverting to configuration 6, in a similar manner, another set of equations similar to 24 can be obtained from which another set of values of \(1_{31}', 1_{32}', 1_{33}'\) can be calculated.

3.2.3. Inversion about OZZ Axis.

By repeating the procedure outlined in Section 3.2.2, but in this case for inversion about the OZZ axis from configuration 1 to configuration 7, values can be obtained for \(1_{21}', 1_{22}', 1_{23}'\), the relevant equations being

\[
\begin{align*}
\frac{V_{x1} - V_{x7}}{V} &= 1_{21} B \\
\frac{V_{y1} - V_{y7}}{V} &= 1_{22} B \\
\frac{V_{z1} - V_{z7}}{V} &= 1_{23} B 
\end{align*} \quad \cdots \quad 26
\]
and
\[ l_{21}^2 + l_{22}^2 + l_{23}^2 = 1 \]

A similar inversion about OZZ from configuration 2 to configuration 8 allows a second set of values of \( l_{21}, l_{22}, l_{23} \) to be computed.

3.2.4 **Iterative Procedure to Increase Accuracy**.

By the method outlined in Sections 3.1 and 3.2.1 to 3.2.3, first approximations have been obtained for the angular calibrations with respect to pitot axes, and for the orientation of the pitot axes with respect to datum instrument axes. This orientation being specified by the direction cosines \( l_{pq} \) (\( p,q = 1,2,3 \)). Also first approximations have been obtained to the flow direction.

The pitot can now be reset so that the calculated pitot axis 0X is in the calculated flow direction, and the procedure repeated as necessary to obtain both the angular calibrations and the direction cosines to the desired accuracy. For the final angular calibrations it is desirable to obtain graphs of \( C_{ph} \) and \( C_{Ph} \) versus \( \beta_h \) for several fixed values of \( \beta_v \), and to average these to obtain the final calibration graphs of \( C_{ph} \) and \( C_{Ph} \) versus \( \beta_h \). Similarly for the \( \beta_v \) calibration graphs. On physical grounds it can be seen that the iterative procedure described above will be convergent. In many cases, the first set of results will be sufficiently accurate.
4.0

**EFFECT OF REYNOLDS NUMBER.**

It has been noted (Ref. 11) that below a Reynolds Number \( R_n \) of the order of \( 10^4 \) there can be "some trouble with calibration," and "laminar separation and a change of the pressure distribution in the neighbourhood of the pressure holes" is suggested as the cause. Calibrations carried out by the authors in a wind tunnel with a 3/8 inch diameter Spherical pitot (see Fig. 4) show some variation below a \( R_n \) of about \( 1.4 \times 10^4 \). Above this value the effect of \( R_n \) would appear to be small.

5.0

**THE UTILIZATION OF CALIBRATION DATA.**

The calibrations at \( R_n = 1.6 \times 10^4 \) obtained by the authors for the 3/8" pitot are shown in Fig. 5. In this form, the calibration data is not suited to the analysis of test results by digital computer. The calibration data can be expressed for computer analysis by determining the difference between theoretical and experimental results in a polynomial form as follows:

Thus

\[
\left( \frac{p_a - p_b}{\frac{1}{2} \rho v^2} \right)_{\text{expt'al}} = \frac{\alpha}{4} (\sin 2\alpha \sin 2\beta_n) - \sum a_i \beta_n^i
\]

... 28
\[ (\tan 2\theta_h)_{\text{expt'al}} = \frac{p_a - p_b}{(2p_c - p_a - p_b)\sin 2\alpha} \sum_{i} b_i \left[ \frac{(p_a - p_b)}{2p_c - p_a - p_b} \right]_i \]

where the last right-hand-side terms are best-fit polynomials.

This numerical procedure allows the calibrations to be incorporated into a data reduction programme for test results.
REFERENCES

1. DEAN, R.C. Aerodynamic Measurements.
   M.I.T Press. 1953

2. JANES, C.E. Instruments & Methods for Measuring the Flow
   of Water around Ships & Ship Models.
   David Taylor Model Basin Report 487, March 1948

3. MEYER, H Onderzoek betreffend de meetmethode met de
   pitot-buis van Dr. Ing. J. J. Borren
   (Investigation on the Method of Measurement
   with Pitot Tube, Designed by Dr. J.J. Borren)
   De Ingenieur p.173, 7 July 1928.

4. GUTSCHE, F Das Zylinderstaurohr (The Cylindrical Pitot
   Tube). Schiffbau, Schiffart and Hafenbau
   pp.13-19, v. 32, n.1, January, 1931

5. KRISAM, F Speed & Pressure Recording in Three-Dimension-
   al Flow. NACA-TM 688, October, 1932

6. JEGEROW, G. Measurement of Direction, Velocity & Pressure
   in a Three-Dimensional Current.
   R.T.P Translation No.2498, Durand Reprinting
   Reprinting Comm., California Institute of Technology.

7. ECKERT, B. Experiences with Flow-Direction Instruments
   NACA-TM 969, March, 1941.

   David Taylor Model Basin Report 1229 May 1958
References.

9. SILOVIC, V
   The Five Hole Spherical Pitot Tube for Three
   Dimensional Wake Measurement.
   Hydro-0g Aerodynamisk Laboratorium.

10. ROBERT, B.G.
    Static Response of Hemispherical Head, Differential
    Pressure Incidence Meter from March
    No 1.6 to 2.6 Weapons Research Establishment Technical Notes HSA 46
    Nov. 1959.
    Extension of the Calibration to 30° Incidence
    and Mach. No 2.75. WRE Tech. Note HSA
    72 January 1961.

11. van MANEN, J.D.
    Unsteady Propeller Forces,
    Proceedings of 10th International Towing
Fig. 1. - Orientation of Velocity components and pressure points to the Axes.
Fig. 2a - Effect of "lateral Tilt" on Pressure Difference (Pa - Pb)

Fig. 2b - Effect of "rotational tilt" on Pressure Difference (Pa - Pb)
Fig. 3 - Inversion about OYY Axis.
Notation of Velocity Components with respect to the Instrument Axes.
Fig. 4 - Effect of Reynolds Number on Pressure Coefficient.
Fig. 5.1 - Typical Calibration Curve for 3/8" Pitot.
Fig. 5.2 - Typical Calibration Curve for 3/8" Pitot.
APPENDIX A7

MOVADAS - DATA ACQUISITION SYSTEM.
MOVADAS - DATA ACQUISITION SYSTEM

MOVADAS - (Modulated Voltage Analogue Data Acquisition System) is a Frequency Modulated Recording System based on the Standard Bandwidth IRIG Specifications as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tape speed</td>
<td>60</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>54</td>
</tr>
<tr>
<td>Modulation range</td>
<td>±40</td>
</tr>
</tbody>
</table>

MOVADAS consists of the following basic units:

1. Signal Amplifiers and conditioning units.
   (i) D.C. Voltage pre-amplifiers.
   (ii) Driver Amplifier and High-pass Filter.
   (iii) Voltage-frequency (V-F) Converters.
   (iv) Programming switch.
   (v) Event Marker.
   (vi) Reference frequency.

2. High speed tape recorder.

A simplified layout of MOVADAS is given in Fig. A7.1.
(1) **D.C. Voltage Pre-amplifiers**

The circuit for the D.C. Pre-amplifier was based on the conventional type of balanced differential transistor amplifier.

The circuit as shown in Fig. A7.9.1 was designed using the experience gained within the Mechanical Engineering Department at Adelaide over the past years. By careful selection of the components, it was possible to reduce the voltage drift and the amplifier noise level, due to changes in either ambient temperature or the supply voltage, to quite small values.

A brief specification of the D.C. Pre-amplifiers is as follows:

- **Input impedance**: 73 K\(\Omega\)
- **Output**: 230 \(\Omega\)
- **Voltage gains up to**: 7000
- **Input noise level**: 4 \(\mu\) V
- **Common mode rejection ratio**: Excellent
- **Offset voltage**: 0 V
- **Linear O/P voltage swing**: +3 volts
- **Frequency response (flat)**: DC-17KC (at least)
- **Supply voltage**: 18v DC

The first stage transistors (2S501 or 2S502, low noise transistors) were selected using the following technique,
(i) Each transistor (T₁) was compared with a fixed or control transistor (T₂), and the value of the load resistor RV₁ in the figure below was measured after RV₁ was adjusted to give voltage $e_o$ equal to 0 volts.

(ii) The transistors were paired according to equal load resistor RV₁.

(iii) The pairs with maximum gains were selected for the pre-amplifiers.

All the transistors in the amplifiers were symmetrically mounted in a circular aluminium block the function of which was to act as a heat sink for uniform temperature. Metallic-oxide film, high-stability resistors were used throughout to maintain stability and to reduce the noise level.

The amplifier power supply regulation circuit and strain gauge voltage and balance circuit are given in Figs. A7.9.2 and A7.9.3.

(2) **Driver Amplifier and High Pass Filter**

This provided either DC or AC coupled inputs through two Nuvistor triodes in push-pull cathode follower configuration.
The triodes gave a high input impedance and a low output impedance to connect directly to two push-pull transistor amplifier stages (Ref. A7.1). The gain control was obtained by feedback. The output voltage from an emitter follower to the V-F converter was limited by the forward conduction of silicon diodes.

The circuit diagram is shown in Fig. A7.2.1 and A7.2.2.

(3) Voltage-Frequency Converter

The IRIG specification as follows was adopted.

Carrier frequency \(54\ \text{Kc/s at } 60^\circ\text{s}\)

Modulation range \(\pm 40\%\) of carrier frequency

Input voltage \(\pm 1.4\ \text{volts for f.s.d.}\)

Each V-F converter consisted of:

(i) Input stage

(ii) Modulator

(iii) Schmitt Trigger

(iv) Binary divider/flip-flop plus output stage.

The circuit diagrams are given in Figs. A7.3.1 to A7.3.6.

(i) **Input Stage**

Provided approximately 10,000\(\Omega\) input impedance through an OC 140 emitter follower. This was temperature compensated by another OC 140 in the emitter circuit.

(ii) **Modulator**

This was a free-running voltage controlled R-L multivibrator adjusted to 54 Kc/s at zero input. The allowable linear deviation from this centre frequency was \(\pm 40\%\). The input signal was fed in through two 10mH
inductances, each one coupled to an emitter follower. A further emitter follower supplied a low output impedance to the Schmitt trigger.

(iii) **Schmitt Trigger**

This unit converted the modulator output voltage to a "square" wave form.

(iv) **Binary divider/Flip-flop**

At a recording speed of 30 ips both inputs A & B at this stage were connected to output B of the Schmitt Trigger to provide a binary division prior to the flip-flop.

At 60 ips both outputs A & B of the Schmitt were connected to inputs A & B respectively of the Flip-flop which acted as a signal clean-up stage.

A low impedance output stage followed to match that of the Record/Reproduce tape head.

(4) **Programming Switch**

This unit controlled the recording sequence and inserted pulses of known amplitudes to identify the recorded information, i.e. run and channel identification. An instantaneous calibration of the V-F converter was also provided, immediately prior to the recording of real data.

Format details of the recorded information can be found in a following Section A7.4.

A block diagram of the programming switch is given in Fig. A7.6.1 and the circuit of the basic monostable multivibrator used is given in Fig. A7.6.2.
The programming switch inserted the identification marks simultaneously on all active channels, except that channel which recorded the reference frequency [see (6) following].

(5) **Event Marker**

An FM signal changing from a static level of 54 Kc/s to a transient value of 70 Kc/s upon occurrence of a cyclic "Event" was recorded on a separate data channel during each recording sequence.

The event was signified by the interruption of a light beam impinging on a photo-cell.

An OC 71 transistor, which had the paint removed from its cover, was used as the photocell.

The voltage signal was fed into an AC coupled, two-stage pulse amplifier which provided a heavy positive pulse to the monostable multivibrator.

A variable pulse length was provided by the switching of a capacitor C3. (Refer Fig. A7.5.2).

The stages following were identical to those of the V-F converter.

The circuit diagrams are shown in Figs. A7.5.1 and A7.5.2.

(6) **Reference Frequency**

This was required for the analogue-to-digital conversion and was a 50 Kc/s, F-M signal recorded on a separate track of the magnetic tape.

The unit was controlled by a Verner Crystal Oscillator Type TS 25.
The basic layout for this unit is given in Fig. A7.4.

A7.2

**HIGH SPEED TAPE RECORDER**

An Epsylon Multitrack high speed tape recorder was modified to take 0.500 of an inch tape width at speeds of 60 and 30 ips. The magnetic heads were replaced with DRICO Series 50, Record/Reproduce heads to IRIG 7 track specifications.

A7.3

**ANALOGUE - DIGITAL CONVERSION**

The recorded F.M. information was converted into binary information at the rate of 4000 samples per second at an analogue tape speed of 60 ips. The recording mode of the digital information was compatible with IBM 7090 digital computer i.e.

- Packing density 200 bits per inch
- MOVADAS Data word 12 reading bits plus 2 parity bits
- Computer word 36 bits or three MOVADAS data words
- Record length 324 words

Because the A-D Converter used for the conversion was programmed to accept multiplexed telemetry data, the resulting structure of each computer record was as follows:

Each recorded track was converted separately and the digital information was recorded end-for-end on the digital tape or "transmittal tape."

The list of numbers which represented a complete single recorded run was termed a file. After each file an end of file
mark was recorded on the digital tape.

In the first record of a file, between the first (1) and the twenty-fourth (24) MOVADAS word, a time word of three (3) MOVADAS words in length was automatically inserted. The time word had the following structure, in octal notation:

\[4000_8, \ XXXX_8, \ YYY_8\]

where \[XXXX_8\] was greater than \[40_8\]

\[Yyyy_8\] was greater than \[4000_8\]

This information was a real time count and was redundant in the subsequent analysis.

After every twenty-fourth MOVADAS word the time word was repeated until the end of file.

A7.4

FORMAT OF RECORDED INFORMATION, ANALOGUE AND DIGITAL

The form of the input voltage to the V-F converter in MOVADAS, on automatic record mode, is sketched in Fig. A7.10.

The corresponding F.M. signal and digital output is also given in Fig. A7.10.

It was found necessary to commence each recording by a period of constant voltage, i.e. constant frequency on the F.M. signal, because the A-D Converter had to be manually operated.

The identification pulses prior to the recorded data represented the run number and channel (or track) number in binary form.
The run number consisted of five bits of information giving run numbers from 0 to 31 and the channel identification consisted of three bits. After this information a series of calibration voltages followed, enabling an instantaneous calibration to be made of the V-F converter.

The identification pulses and calibration voltages were applied to all active channels simultaneously. This was necessary because the recorded data from several tracks had to be phase compared in subsequent analysis.

The information recorded on the digital tape is as sketched in Fig. A7.10. The list of digits which constitutes a single experimental reading was termed a file and was terminated with an end of file mark. It should be noted that due to the programming switch in MOVADAS, the actual digital pulses were not ideally 'square cornered.' The initial discontinuity and leading edge was as desired, but oscillations did exist on the pulse top and trailing edge. These irregularities presented difficulties, when developing a technique for detecting the identification pulses during computer analysis of the data.

The bit structure of the recorded digital data is quoted in Section A7.3 and details of the analysis of the data are given in Appendix A8.

REFERENCE

A7.1 GRIGSON, C.W.B. Some Precision Direct-Coupled Transistor Amplifier and Approximate Design.

Electronic Engineering, July-August 1964 Vol.36 Nos.437, 438
Fig. A7.1

Layout of Movadas
**Fig. 4.2.1 - Driver Amplifier**

Filter and High Input Impedance Stage
Fig. 2.2 - Driver Amplifier
Amplification Stage.
Fig. 47.2.2 - Driver Amplifier
Amplification Stage.
Fig. 3.1 - V-F Converter

Block Diagram of Wiring between Basic Cards.

Tape Speed = 60
  30
  1.0v
Cal. Volts 0.3v
Zero

Resistors:
R4 2.7K to 3.3K

Resistor Variable:
RV2 5K sensitivity
RV3 100 zero set
Fig. 7.3.2 - V - F Converter
Input Amplifier

TRANISTORS
T1  0C140

ZENER DIODE
Z1  124.7V5
Z2  125.6V5

RESISTORS
R1  33
R2  100K  Electro sil
R3  10K
R4  2.7K to 3.3K
R5  0-330
R6  4.7K
R7  33K
R8  100K

RESISTORS VARIABLE
RV1  25K
RV2  5K
RV3  100
Fig. A/3.3 - V-F Converter

Astable Multivibrator - Voltage Controlled Modulator.
Fig. 3.4 - V-F Converter

Schmitt Trigger.
Fig. A7.3.5 - V-F Converter

Binary Divide/Astable Multivibrator
Fig. A7.3.6 - V-F Converter
Calibration Voltage Supply.
Fig. 7.4 - Reference Frequency
Block Diagram of Wiring between Basic Cards.
AMP & MONGSTABLE DEVICE

PHOTO ELECTRIC DEVICE

INPUT AMP MODULATOR BINARY STAGE

AMP T.P.

CAPACITORS
C1 64μF
C2 0.47μF
C3 1μF

RESISTORS
R4 2.7K to 3.3K
RV2 5K sensitivity
RV3 100 zero set

RESISTORS VARIABLE

OFF
ZERO
TEST
10M/S
20M/S PULSE LENGTH

FIG. A7.5.1 Event Marker
Block Diagram of Wiring between Basic Cards.
<table>
<thead>
<tr>
<th>RESISTORS</th>
<th>CAPACITORS</th>
<th>TRANSISTORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 10K</td>
<td>C1 0.047μf</td>
<td>T1 OC71</td>
</tr>
<tr>
<td>R2 56K</td>
<td>C2 100μf</td>
<td>T2 OC74N</td>
</tr>
<tr>
<td>R3 10K</td>
<td>C3 2.5μf</td>
<td>T3 AF14N</td>
</tr>
<tr>
<td>R4 82K</td>
<td>C4 0.01μf</td>
<td>T4 2N393</td>
</tr>
<tr>
<td>R5 1K</td>
<td>C5 4700pf</td>
<td>T5 OC71</td>
</tr>
<tr>
<td>R6 5.6K</td>
<td>C6 Variable time delay</td>
<td>Paint removed to make transistor light sensitive</td>
</tr>
<tr>
<td>R7 1K</td>
<td>C7 0.01μf</td>
<td></td>
</tr>
<tr>
<td>R8 10K</td>
<td>C8 1.0μf</td>
<td></td>
</tr>
<tr>
<td>R9 1meg</td>
<td>C9 25μf</td>
<td></td>
</tr>
<tr>
<td>R10 33K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R11 5.6K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R12 2.2K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R13 33K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R14 2.2K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R15 560</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R16 5.6K</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5.2 - Event Marker
Amplifier and Monostable Multivibrator.
Fig. A7.5.2 - Event Marker Amplifier and Monostable Multivibrator.
TRANISTORS
T4  0C71  C1  1 μf  RELAYS  SIEMENS
T2  AD140  C2  1 μf  RL 1  6500/416
T5  0C71  C3  1000 μf  RL 2  "
T4  AC128  C4  .68 μf  RL 3  "
T5  2N3638

ZENER DIODE
Z1  1Z30

RESISTORS
R1  68 Ω  RL 4  "
R2  1 K  RL 5  6500/418
R3  10 K  RL 6  "
R4  15 K
R5  10 K

CAPACITORS
C1  1 μf
C2  1 μf
C3  1000 μf
C4  .68 μf

MONOSTABLE DEVICE
F/F1
F/F2
F/F3
F/F4

RELAYS
RL 1  Connect Record/Reproduce Heads to V-F Converters
RL 2  Controls Tape Recorder Operation
RL 3  Connects Driver Amplifier and V-F Converters
RL 4  Controls Drive Coil of Uni-selector
RL 5  RL 6

Fig. 6.1 - Programme Switch
Block Diagram of Basic Cards.
Fig. 4.7.6.1 - Programme Switch

Block Diagram of Basic Cards
Fig.A7.6.2 - Programme Switch

Monostable Multivibrator.
Fig.A78.1 - D.C. Power Supply + 12 volts.
VALVES

V1  6X4
V2  6X4
V3  6u8 two in one valve
V4  6u8 "   "   "
V5  5651

RESISTORS

R1  100K.
R2  4.7K.
R3  100K.
R4  4.7K.
R5  220K.
R6  220K.
R7  1K.
R8  3.3K.
R9  3.3K.
R10 1K.
R11 1m.
R12 1m.
R13 39K.
R14 33K.
R15 6.8K.

RESISTORS VARIABLE

RV1  10K.
RV2  10K.

CAPACITORS

C1  70μF
C2  0.02μF
C3  2.0μF
C4  0.1μF
C5  40μF

Fig. B.2 - D.C. Power Supply

High Tension ± 100v
Fig. A7.8.2 - D.C. Power Supply. High Tension 100v
Fig. A78.3 - DC Power Supply
Heaters of Driver Amplifier.
<table>
<thead>
<tr>
<th>RESISTORS</th>
<th>TRANSISTORS</th>
<th>VARIABLE RESISTORS</th>
<th>ROTATING PRE AMP ONLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1 50K high stability</td>
<td>T1 28501</td>
<td>RV1 5KΩ w/w Amp Balance</td>
<td>RV1 1KΩ</td>
</tr>
<tr>
<td>R2 100K</td>
<td>T2 28703</td>
<td>RV2 100Ω Slide Wire</td>
<td>RV3 1KΩ</td>
</tr>
<tr>
<td>R3 100K</td>
<td>T3 2N3566</td>
<td>RV3 5KΩ w/w Amp Balance</td>
<td></td>
</tr>
<tr>
<td>R4 50K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R5 25K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R6 40K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R7 50K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R8 1K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R9 100K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R10 2.7K</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R11</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R13 gain setting</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R14 feed back</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R15 selected</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9.1 - D.C. Pre-Amplifiers
Basic Circuit Diagram.
Fig. 9.1 - D. C. Pre-Amplifiers
Basic Circuit Diagram.
Fig. A7.9.2 - D.C. Pre-Amplifiers

Amplifier Regulated Voltage Supply.
Fig. 47, 9.3 - D.C. Pre Amplifiers
Strain Guage Regulated Power Supply and Balance Circuit.
Fig. A7.10  Characteristics of a Single File from MOVADAS
APPENDIX A8.

COMPUTING PROCEDURE.

COMPUTING SEQUENCE.

The information recorded on magnetic tape by MOVADAS (Appendix A7) was a Frequency Modulated (FM) wave. Before this could be processed by a digital computer, the information had to be converted to an array of numbers in Binary Coded Decimal (BCD) format which represented the recorded signal sampled at equal intervals of time. The FM Analogue to Digital (A-D) Conversion was carried out at the Mathematical Services Division of Weapons Research Establishment, South Australia. Subsequent analysis of the data was conducted on the digital computers at the University.

The analysis of the data was divided into the following stages.

(1) **PROGRAM CONVERT** (Table A8.1).

The digital information, in the form discussed in Sections A7.3 and A7.4 of Appendix A7 was firstly decoded into a format compatible with the word structure of the computers, and then data files were recognized from the identification pulses. This conversion was necessary because the output from the A-D Converter was to IBM 7090 format which was incompatible with that of the CDC Computers at the University.

As mentioned in Section A7.4 of Appendix A7, the analogue information was simultaneously applied to all active channels of MOVADAS and recorded on separate tracks of the magnetic tape. Included in this analogue information were the identification pulses, i.e. run and channel numbers and instantaneous V-F
Converter calibrations. The analogue information from each track was converted separately to digital information at the same sampling frequency of 4000 samples per second. After A-D conversion, digital files were then written consecutively, 'end for end' on the digital tape. The occurrence of the last calibration voltage pulse was taken as the time zero origin, and was thus accurate to 1/4000 of a second. Hence a digital sample from one channel counted from the time origin had a corresponding sample at the same instant of time on all other channels.

The digital samples after time zero origin were selected and recorded, together with various identification numbers, on to another magnetic tape for subsequent analysis by program CONTROL.

(2) PROGRAM CONTROL (Table A8.2).

Using the recorded data from program CONVERT and input instructions in the form of punched cards, the analysis was as follows:

Subroutine SYNC

This routine selected from the digital file of an experimental data channel an array of numbers which corresponded to a given number of recorded cycles.

As previously mentioned, the digital information recorded from program CONVERT contained all samples from the time zero origin. From the digital event file, the particular locations of the event pulses from the time origin were determined. The corresponding points on the data files were also
located. Thus, it was possible to select from the complete
digital file an array of numbers between any two event pulses.

Subroutine CONV and REDFA.

The data was converted into real values of the measured
variable (pressure) because the sensitivities of each unit in
the recording system were known. The values of the transducers
and amplifiers sensitivities were supplied to the program
from card input information. The V-F converter sensitivity
was obtained from the known instantaneous calibration levels
which preceded the recorded data. (Section A7.4, Appendix A7.)

Subroutine FASQW

A Fourier analysis (Ref. A8.1) was performed on the data as
selected in subroutine SYNC above.

The phase of the Fourier component was with respect to the
event pulse. The Fourier coefficients were corrected in magni-
tude and phase because of the non-linear frequency response of
the high-pass filter used in the recording instrumentation
(Section 4.2).

The program printed the following results for each blade
frequency harmonic --
(a) The magnitude of the actual pressure.
(b) The pressure coefficient.
(c) Phase with respect to the position of the impeller
blades.

Subroutine CALMAX

From the calculated Fourier components the actual pressure
signal was synthesized. From this synthesized wave,
the maximum and minimum pressures and their phase angles were
determined. The wave form was also plotted using the follow-
ing subroutine MEKPLT.

Subroutine MEKPLT

This is an automatic plotting routine developed for a line
printer to plot the values stored in a double subscripted
array. This routine provides automatic scaling and choice of
origin for up to 10 separate functions plotted on the one sheet.
The coordinate axes are anotated and the user may specify the
titles for each axis. The input instructions are given at
the beginning of subroutine MEKPLT in Table A8.2.

A8.2 PROGRAM INPUT INSTRUCTIONS.

The programs are listed in Table A8.1 and A8.2.

The definitions of the input variables are as follows:--

(1) PROGRAM CONVERT

ITAPTRN Transmittal or digital tape number
ITAPCON The output tape number written by CONVERT
NNSER Series run number
LIST Control on type of output required
  - ve lists the data points
  0 lists the data points and writes on to
    magnetic tape
  + ve writes on magnetic tape only
LLEST (N M) Array of numbers to specify the number of files to
  be listed, and skipped and listed..... etc.
(2) PROGRAM CONTROL

ITAPA  No. of the F.M. analogue tape
ITAPT  No. of the transmittal or digital tape
ITAPCON No. of the tape recorded by program CONVERT
ITAPSYN Extra file identification if required
NPI    Number of the cycle of the event at which the
       analysis is to begin
NP     Number of cycles over which the analysis is to
       be conducted
LIST   - ve or 0 lists the data points over one cycle
       and also the Fourier components of the
       total number of cycles that were
       investigated. These Fourier components
       are not corrected for the effect of the
       high-pass filter.

+ ve    does not list the above
ICHN    No. of the data channel to be investigated
NTYPE   Identification for the type of data
KKD     No. of Fourier coefficients required per revolu-
       tion of the impeller

VOLTC(I),I=1.5 Value of the 5 voltage signals which were supplied
       automatically to the F.M. Converters on recorded
       mode (Appendix, A7.)

GI      Sensitivity of the transducer
REV     Rotational speed of the impeller (rpm)
SETANG Angle between the position where the event occurred
          and the position of the transducer
WAKEANG  Angle, at which the wake pattern is set with respect to the transducer

FMOD(I),I=1,7  Magnification ratio of the high-pass filter at the frequency of the blade harmonics

FPHASE(I),I=1,7  Phase angle for the filter at the frequency of the blade harmonics

AAPH(A(I),I=1,8  Comments which will form the title to the output information

NSER  Series run number

IRUN  Run number for the experimental reading and also the digital file number

NOCHN(I),I=1,7  The numbers associated with the channels of information to be investigated. The event channel number must precede the data channels

IG2, IG3, IG4  Gain switch positions on the recording instruments. The gain figures are given in the program as GG2(I) GG3(I), GG4(I).

THE FOURIER ANALYSIS ROUTINE FASCW

A mathematical derivation for this method of Fourier Analysis is given in Ref. A8.1. Briefly it is as follows:--

The Fourier Series expansion of a continuous function \( f(t) \) in the range \( 0 \leq t \leq 2\pi \) is,

\[
f(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi t}{c} + b_n \sin \frac{n\pi t}{c} \right)
\]  . A8.1
where \( a_0 = \frac{1}{2c} \int_{0}^{2c} f(t) \, dt \)

\[ a_n = \frac{1}{c} \int_{0}^{2c} f(t) \cos \frac{n \pi t}{c} \, dt \]

and \( b_n = \frac{1}{c} \int_{0}^{2c} f(t) \sin \frac{n \pi t}{c} \, dt \)

An estimate \( a'_n \) of \( a_n \) for a series of \( N \) equally spaced points \( f(h, k) \) where \( h \) is the sampling interval and \( k = 0, 1, \ldots, N-1 \) is:

\[ a'_n = \frac{2}{N-1} \sum_{k=0}^{N-1} W_k f(h,k) \cos k \theta \] \( \ldots A8.2 \)

where \( W_k \) = weighting coefficients

and \( \theta = \frac{2 \pi n}{N-1} \)

Consider now a series defined by

\[ a''_n = \frac{2}{N-1} \sum_{k=0}^{N-1} W_k f(h,k) \frac{\cos k \theta}{|\cos k \theta|} \] \( \ldots A8.3 \)

and since \( \frac{\cos k \theta}{|\cos k \theta|} \) is mathematically represented by

\[ \frac{4}{\pi} \sum_{j=1}^{\infty} \frac{(-1)^{j-1}}{2j-1} \cos (2j-1)k \theta \] \( \ldots A8.4 \)

Then equation A8.3 can be expressed by

\[ a''_n = \frac{4}{\pi} \sum_{j=1}^{\infty} \frac{(-1)^{j-1}}{2j-1} a'_n (2j-1)n \] \( \ldots A8.5 \)
A similar solution is given for $b_n''$ and $b_n'$ (Ref. A8.1). The ratio $\cos k \theta / |\cos k \theta|$ for a fixed $\theta$ and varying $k$ is obtained by sampling the squarewave of frequency $\theta$ at intervals of $k$ in time.

After summation of the series in equation A8.3 the linear system equation A8.5 is solved by back substitution.

This method of solving for the Fourier components is faster than the normal methods.

**MAJOR SYMBOLS USED IN THE PROGRAM.**

- **Y** is the one dimensional data list
- **N** is the number of data points in the data list
- **K** is the number of estimates of $a_k$ and $b_k$ required
- **YS** is the two dimensional array for $(a_k'', b_k'')$
- **YC** is the two dimensional array for $(a_k', b_k')$

**REFERENCE**

A8.1 CLARKE, A.P Computation of the Coefficients of a Fourier Series Expansion of a Function defined by Sampled Data Points.


T.M TRP 71 ADDS.
TABLE A8.1

PROGRAM CONVERT
CONVERTS MAG TAPE FROM A-D CONVERTER INFORMATION RECORDED
INITIALLY ON MOVADAS
CONVERTS THE LIST OF NUMBERS IN EACH FILE NAMED FROM IBM 7090
FORMAT TO CDC 3200 OR 6400 FORMAT
THE TIME WORDS ARE REMOVED, FILES THAT ARE NOT DATA FILES ARE
REJECTED, IDENTIFICATION AND CALIBRATION PULSES ARE LOCATED AND
DECODED, THE TRUE DATA IS PLACED ON MAG TAPE WITH THE NECESSARY
IDENTIFICATION
DATA OUTPUT IN BLOCKS OF 664 NUMBERS WITH DUMMY EOF AFTER EACH
BLOCK (-1,100,-10)
AFTER THE LAST BLOCK A DUMMY EOF IS (-2,-200,-20)
THIS IS FOLLOWED BY A TRUE END OF FILE

INTEGER SHIFT
INTEGER VAL
DIMENSION INPT(486), VAL(1836), FILE(8), ICAL(5)
DIMENSION LLEST(12,2)
DIMENSION IV(3), IY(3)

5 FORMAT(55X,10HEOF SENSED)
6 FORMAT(10X,22HEOF WRITTEN ON OUTTAPE)
10 FORMAT(10X,33HPARITY ERROR DETECTED AND IGNORED)
15 FORMAT(20X,19HCONVERTED RECORD NO,15)
20 FORMAT(1X,20HTOTAL NUMBER RECORDS,16)
25 FORMAT(1H-,11H FILE NUMBER,16)
30 FORMAT(1X,2015)
35 FORMAT(2413)
40 FORMAT(50X,22H DATA FILE NOT DETECTED)
45 FORMAT(5X,9H RECORD NO,15)
50 FORMAT(415)
55 FORMAT(20H TRANSMITTAL TAPE NO,16,3X,15H CONVERT TAPE NO,16)
60 FORMAT(1X,915)
65 FORMAT(50X,25HEOF DETECTED BEFORE IDENT)
70 FORMAT(7X,14H GOOD DATA FILE)
75 FORMAT(15X,18H LOOKING FOR IDENT)
80 FORMAT(10X,10H FILE IDENT,15,3H =,12,4X,11H NO OF STEPS,13)
85 FORMAT(3X,12H SKIP FILE NO,15)
90 FORMAT(50X,29H CALIBRATION SIGNALS INCORRECT)
95 FORMAT(915)
96 FORMAT(2015)
97 FORMAT(315)

IV(1) =-1
IV(2) =-100
IV(3) =-10
IW(1) =-2
IW(2) =-200
IW(3) =-20

READ TRANSMITTAL AND CONVERT TAPE NUMS, SERIAL NO OF DATA
LIST +VE IF MAG TAPE ONLY IS REQUIRED
LIST ZERO FOR MAG TAPE AND LIST OF POINTS
LIST -VE FOR LIST OF POINTS ONLY
READ(60,50) ITAPRNR, ITAPCON, NNSER, LIST
WRITE(61,55) ITAPRNR, ITAPCON
IF (LIST) 103,102,102
102 REWIND11
CALL DENS(10,1,REJ)
CR
READ NO OF FILES TO BE LISTED, SKIPPED, LISTED, SKIPPED,...ETC
READ(60,35)((LLEST(N,M),M=1,2),N=1,12)
NOFILE=0
DO106N=1,12
IF(LLEST(N,1),EQ,0,AND,LLEST(N,2),EQ,0)1000,104
104 IF(LLEST(N,1))1000,105,109
105 IF(LLEST(N,2))1000,1000,107
106 CONTINUE
107 LL=LLEST(N,2)
DO108LET=1,LL
NOFILE=NOFILE+1
WRITE(61,85)NOFILE
108 CALL SKIP(10)
GOTO106
109 NL=LLEST(N,1)
DO990LET=1,NL
NOREC=0
NODIS=0
IDENT=0
IP=0
JS=0
KFILE=1
JTEST=1
JDTES=6
IYD=35
IIYD=50
NOFILE=NOFILE+1
WRITE(61,25)NOFILE
BUFFERIN(10,1)(INPT(1),INPT(486))
100 CALL UNITS(T,10,K)
GOTO(100,136,110,120),K
110 IF(IP)1000,985,950
120 WRITE(61,10)
136 NOREC=NOREC+1
WRITE(61,45)NOREC
C
BY MASKING BREAKS THE WORD INTO TWO WORDS
DO140I=1,486
J=2*I+1+JS
VAL(J)=SHIFT(INPT(I),-12)
140 VAL(J+1)=AND(INPT(I),7777B)
BUFFERIN(10,1)(INPT(1),INPT(486))
NUM=NUM+1
C
CHECKING FOR TIME WORDS AND REMOVING THEM
IF(AND(VAJs+71, 4000B),EQ,4000B)AND,AND(VAJs+972, 408),EQ,40B
1,AND,AND(VAJs+1, 4000B),EQ,4000B)150,170
150 II=1+JS
III=969+JS
DO160J=II,III
160 VAL(I)=VAL(I+1)
NUM=NUM-3
GOTO200
170 IF(AND(VAJs+72, 4000B),EQ,4000B,AND,AND(VAJs+1, 408),EQ,40B,A
1ND,AND(VAJs+2, 4000B),EQ,4000B)180,200

TABLE A8.1

180  II=1+JS
   III=969+JS
   DO190 I=II,III
190  VAL(I)=VAL(I+2)
   NUM=NUM+3
   DO230 J=I,NUM
200  IF(AND(VAL(I),4000B),EQ,4000B,AND,AND(VAL(I+1),40B),EQ,40B,AND,AND
   1(VAL(I+2),4000B),EQ,4000B)220,235
220  NUM=NUM+3
   DO230 J=I,NUM
230  VAL(J)=VAL(J+3)
235  I=I+1
   IF(I-NUM+2)210,210,237
237  WRITE(61,15)NOREC
   IF(JS)1000,236,250
236  JS=864
   GOTO100
250  GOTO(400,455,700),KFILE
400  IF(NOREC=3)405,405,450
CHECKING FOR A DATA FILE, IE, INITIAL POINTS ARE APPROX. CONSTANT
405  DO420=36,864,18
   IF(IABS(VAL(I)=VAL(I-18))=IYD)420,420,410
410  JTEST=JTEST+1
420  CONTINUE
   GOTO(1000,900,430),NOREC
430  IF(JTEST-JDTEST)445,440,440
440  WRITE(61,40)
   CALL SKIP(10)
   GOTO990
445  WRITE(61,70)
450  KFILE=2
455  WRITE(61,75)
500  L=L+1
   IF(L-903)520,520,510
510  L=39
520  L=39
IDENTIFYING THE IDENTIFICATION PULSES
520  IF(IABS(VAL(L)=VAL(L-1))=IYD)500,500,525
525  IF(IABS(VAL(L)=VAL(L-10))=IYD)500,500,530
530  IF(IABS(VAL(L)=VAL(L-20))=IYD)500,500,535
535  IF(IABS(VAL(L-1)=VAL(L+1))=IYD)540,540,545
540  IF(IABS(VAL(L-1)=VAL(L+2))=IYD)500,500,545
545  LTEST=1
CHECKING THAT IT IS A PULSE:
D0555M=20,40
M=LN=M
   IF(IABS(VAL(MN)=VAL(MN-1))=IYD)555,555,550
550  LTEST=2
555  CONTINUE
   GOTO(575,560,1000),LTEST
ANOTHER CHECK FOR THE PULSE
560  D0570M=40,60
   M=LN=M
IF(IABS(VA(MN)-VA(MN-1))=IYD)570,570,565
565 CONTINUE
570 GOTO(1000,575,500),LTEST
575 KEEPS RUNNING COUNT OF THE NO OF DISCONTINUITIES AND SQUARE PULSES
580 IF(NODIS-16)=590,640
590 NODIS=NODIS+1
595 IF(NODIS/2-(NODIS+1)/2)=600,630,1000
600 IDENT=IDENT+1
605 IF(VAL(L)-VAL(L-1))=620,1000,610
610 IF(IMAGE(0))=0
620 GOTO630
630 L=L+100
640 IF(IMAGE(0))=0
650 ISUM=0
660 IDENT=IDENT+1
670 DETERMINING THE CALIBRATION LEVELS
680 DO670I=20,80
690 IAL=I*L
700 ISUM=ISUM+VAL(IAL)
710 SUM=ISUM
720 ICAL(0)=SUM/61.0
730 IF(IMAGE(13))=675,680,1000
740 L=L+100
750 GOTO650
760 DECODES RUN AND CHANNEL NOS
770 IIRUN=IMAGE(0)*16+IMAGE(2)*8+IMAGE(3)*4+IMAGE(4)*2+IMAGE(5)
780 ICHN=IMAGE(6)*4+IMAGE(7)*2+IMAGE(8)
790 WRITE(61,60)NSER,IIRUN,ICHN,(ICAL(/I),I=1,5),ITAPCON
800 WRITE(61,97)(IV(I),I=1,3)
810 CHECK ON VALUES FOR CALIBRATION
820 IF(ICAL(0).GE.ICAL(3).AND.ICAL(5).LT.ICAL(3))=684,984
830 VARIOUS OUTPUT FORMATS DEPENDING ON THE VALUE OF LIST
840 IF(LIST)=690,685,685
850 WRITE(11,99)NSER,IIRUN,ICHN,(ICAL(/I),I=1,5),ITAPCON
860 WRITE(11,97)(IV(I),I=1,3)
870 KFILE=3
880 LLL=L+250
890 IP=0
900 DO905J=LLL,1728
910 IP=IP+1
920 VAL(IP)=VAL(J)
930 IF(864-IP)=720,100,100.
940 DO710J=865,1728
950 IP=IP+1
960 VAL(IP)=VAL(J)
970 IF(LIST)=730,735,740
980 WRITE(61,30)(VAL(/I),I=1,864)
990 WRITE(61,97)(IV(/I),I=1,3)
1000 GOTO750
1010 WRITE(61,30)(VAL(/I),I=1,864)
1020 WRITE(61,97)(IV(/I),I=1,3)
740 WRITE(11,96)(VAL(I),I=1,864)
750 IP=IP-864
760 DO760 J=1,IP
770 VAL(J)=VAL(J+864)
780 GOTO100
790 DO910 I=1,864
800 VAL(I)=VAL(I+864)
810 GOTO100
820 IF(IDENT-13)985,955,1000
830 IF(LIST)960,965,970
840 WRITE(61,30)(VAL(I),I=1,IP)
850 WRITE(61,97)(IW(I),I=1,3)
860 GOTO980
870 WRITE(61,30)(VAL(I),I=1,IP)
880 WRITE(61,97)(IW(I),I=1,3)
890 LLL=IP+1
900 DO975 I=LLL,864
910 VAL(I)=0
920 WRITE(11,96)(VAL(I),I=1,864)
930 WRITE(11,97)(IW(I),I=1,3)
940 WRITE(61,20)NOREC
950 WRITE(61,5)
960 GOTO990
970 WRITE(61,90)
980 CALL SKIP(10)
990 GOTO990
1000 WRITE(61,65)
1001 WRITE(61,20)NOREC
1002 CONTINUE
1003 GOTO105
1004 CALL DENS(10,2,REJ)
1005 IF(LIST)1002,1001,1001
1006 END FILE 11
1007 WRITE(61,6)
1008 STOP
1009 END
PROGRAM CONTROL (INPUT, TAPE60 = INPUT, OUTPUT, TAPE61 = OUTPUT, TAPE02)
COMPLETE FOURIER ANALYSIS OF DATA FROM MOVADAS CONVERTED
TO CDC 6400 WORD STRUCTURE
COMMON Y (4321), ITAPCON, NSER, IRUN, NOCHN (8), ICAL (5)
COMMON VOLTC (5), NPI, NP, NPD, NPTS, LIST, ISTOP
COMMON NTEST, CAL (5), NPERD, M1, MF
COMMON FMOD (7), FPHASE (7), REV, SETANG, WAKEANG
COMMON G1, G2, G3, G4
COMMON AAPHA (8), KKD
DIMENSION GG2 (5), GG3 (2), GG4 (10)

010 FORMAT (9(1))
11 FORMAT (1H0, 12I5)
095 FORMAT (4(1))
096 FORMAT (1H1, 15HCONVERT TAPE NO, 15)
097 FORMAT (1H1, 13HANOLOGUE TAPE, I3, 4X, 16HTRANSMITTAL TAPE, I6)
098 FORMAT (3I3/313/5F10.3)
099 FORMAT (E10.3)
100 FORMAT (8A10)
101 FORMAT (1H, 8A10)
110 FORMAT (3I3)
120 FORMAT (F6.0, 2F6.1)
130 FORMAT (7F6.3)
140 FORMAT (7F6.1)
180 FORMAT (1H0, 7H SERIAL, 5X, 3HRUN, 5X, 7HCHANNEL, 5X, 4HTYPE, 3X, 10HNO PERIODS, 3X, 10HBLADE COEF, 3X, 9HNO POINTS, 3X, 21HINITIAL - MAX PERIODS)
190 FORMAT (1H0, 15, 4(5X, 15), 2(8X, 15), 19, 18)

CALIBRATION VALUES FOR PREAMP 2 FM 1
GG2 (1) = 107.1
GG2 (2) = 379.0
GG2 (3) = 771.0
GG2 (4) = 1170.0
GG2 (5) = 2440.0
GG3 (1) = 0.1008
GG3 (2) = 1.0
GG4 (1) = 10.21
GG4 (2) = 14.46
GG4 (3) = 20.60
GG4 (4) = 31.13
GG4 (5) = 41.07
GG4 (6) = 57.73
GG4 (7) = 82.44
GG4 (8) = 103.1

READ (60, 95) ITAPA, ITAPT, ITAPCON, ITAPSYY
READ (60, 98) NPI, NP, LIST, ICHN, NTYPE, KKD, (VOLTC (I), I = 1, 5)
READ (60, 99) G1
READ (60, 120) REV, SETANG, WAKEANG
READ (60, 130) (FMOD (I), I = 1, 7)
READ (60, 140) (FPHASE (I), I = 1, 7)
205 READ (60, 100) (AAPHA (I), I = 1, 8)
IF (ENDFILE 60) 1000, 200
200 READ (60, 101) NSER, IRUN, (NOCHN (I), I = 1, 7)
READ (60, 110) I3, I4, I5, I6
G2=GG2(G2)
G3=GG3(G3)
G4=GG4(G4+1)
ISTOP=1
WRITE(61,096)ITAPCON
WRITE(61,097)ITAPA,ITAPT
WRITE(61,101)(AAPHA(I),I=1,8)
WRITE(61,011)NSER,IRUN,(NOCHN(I),I=1,7),NPI,NP,LIST
NTEST=1
CALL SYNC
GOTO(220,1000),ISTOP
220 GOTO(221,205),NTEST
221 CALL CONV
GOTO(230,205),NTEST
230 CALL REDFA
WRITE(61,096)ITAPCON
WRITE(61,100)(AAPHA(I),I=1,8)
WRITE(61,097)ITAPA,ITAPT
WRITE(61,180)
WRITE(61,190)NSER,IRUN,ICHN,NTYPE,NPD,KKD,NPTS,NPI,NPERD
CALL FASOW
GOTO205
1000 STOP
END

SUBROUTINE SYNC
C SYNCHRONIZE DATA CHANNEL WITH EVENT MARKER SELECTS FILES, PERIODS,
C AND DATA POINTS
DIMENSION IX(3)
DIMENSION NPT(50)
COMMON(I4321),Y(I4321),ITAPCON,NSER,IRUN,NOCHN(8),ICAL(5)
COMMON VOLT(5),NPI,NP,NPD,NPTS,LIST,ISTOP
COMMON NTEST,CAL(5),NPERD,MI,HA
COMMON FMOD(7),FPHASE(7),REV,SETANG,WAKEANG
COMMON G1,G2,G3,G4
COMMON AAPHA(8),KKD
6 FORMAT(10X,44HIDENTIFICATION BETWEEN DATA BLOCKS INCORRECT)
10 FORMAT(1H,35HCONVERT TAPE IS NOT THE TAPE CALLED/10H TAPE READ,15
1/12H TAPE CALLED,15)
020 FORMAT(1H,27HDATA POINT APPEARS IN ERROR,18,3X,5HPONT/15H ORIGIN
1AL POINT,16,3X,12H CORRECTED TO,16/23H SURROUNDING POINTS ARE,518)
035 FORMAT(1H0.8X,7HPERIODS,15X,16HNUMBER OF POINTS,12X,6HPERIOD)
036 FORMAT(67H SELECT INITIAL MAX SELECT INITIAL FINAL TOTAL
1 VARIATION)
040 FORMAT(1H,15,18,17,110,218,17,18,16)
050 FORMAT(1H0.10HERROR TYPE,3(2X,15))
060 FORMAT(1H0.7HCHAN NO,16)
070 FORMAT(1H0,10I5)
080 FORMAT(1X,20I5)
95 FORMAT(9I5)
96 FORMAT(20I5)
97 FORMAT(3I5)
NOCHN(8)=0
TABLE A8.2

110 NT=5
111 IYD=5
112 IYD=50
113 NPTDD=5
114 INTPOL=100
115 IY(N)=0

C READS MAG TAPE AND SELCTS FILE REQUIRED FOR EVENT MARKER.
118 READ(02,95)NNSER,IRUN,IICHN,(ICAL(I),I=1,5),ITAPCO
119 IF(ENDIFILE 02)100,117
120 IF(ITAPCON(ITAPCO))60,119,600
121 IF(IRUN(IRUN))150,120,150
122 IF(NOCHN(I)-IICHN))150,160,150
123 CALL SKIP(02)
124 GOTO118
125 160 NPF=NPF+NPI
126 NPERD=0
127 NTOTE=0
128 KTEST=1
129 DO1051=1,50
130 105 NPT(I)=0
131 M=-863
132 MM=0

C READS DATA POINTS FROM FILE OF EVENT MARKER.
133 M=M+864
134 MM=MM+864
135 READ(02,97)(IX(I),I=1,3)
136 IF(IX(I).EQ.-1.AND.IX(2).EQ.-100.AND.IX(3).EQ.-10)162,161
137 IF(IX(I).EQ.-2.AND.IX(2).EQ.-200.AND.IX(3).EQ.-20)174,900
138 READ(02,96)(IY(I),I=M,MM)
139 IF(ENDIFILE 02)174,171
140 171 IF(HM=3456)170,170,172
141 CALL SKIP(02)
142 174 N=9
143 175 N=N+1
144 IF(N-MM+30)178,245,245
145 178 IF(IY(N)=1000)240,240,180

C DETERMINING THE EVENT PULSES,
146 IF(IABS(IY(N)=IY(N-1))=IYD)175,175,181
147 IF(IABS(IY(N)=IY(N-4))=IYD)175,175,182
148 IF(IABS(IY(N)=IY(N-9))=IYD)175,175,183
149 IF(IABS(IY(N-1)=IY(N+1))=IYD)173,173,189
150 IF(IABS(IY(N-1)=IY(N+2))=IYD)175,175,189
151 LTEST=1

C CHECKING TO MAKE SURE IT IS AN EVENT PULSE
152 DO185M=4,14
153 MN=M+N
154 IF(IABS(IY(MN)=IY(MN-1))=IYD)185,185,184
155 184 LTEST=2
156 155 CONTINUE
157 GOTO(200,186,1000),LTEST
158 DO188M=15,25
159 MN=M+N
160
TABLE A8.2

187 LTEST=3
188 CONTINUE
   GOTO(1000,200,240),LTEST
200 IF(IY(N)-IY(N-1)+IYD)220,220,205
205 IF(IY(N)-IY(N-1)-IYD)175,175,210

C NOTES NO. OF PERIOD AND NO. OF POINT WHERE IT OCCURED.
210 NPRED=NPRED+1
220 N=N+15
230 NTOTE=NTOTE
240 CONTINUE
245 REWIND02

C CHECK THAT PERIODS DID OCCUR.
250 IF(NPRED)250,250,255
   IERR=003
   NTEST=2
   WRITE(61,050)IERR,NPRED,NTOTE
   GOTO335

C CHECK THAT VARIATION IN NO. OF POINTS PER PERIOD IS ACCEPTABLE
255 NMIN=NPT(2)-NPT(1)
   NMAX=NMIN
   NN=NPRED-1
   DO290=2,NN
   NPTD=NPT(N+1)-NPT(N)
   IF(NMAX-NPTD)260,270,270

260 NMAX=NPTD
   GOTO290
270 IF(NMIN-NPTD)290,290,280
280 NMIN=NPTD
290 CONTINUE
   IF(NMAX-NMIN-NPTDD)305,305,300
300 IERR=004
   WRITE(61,050)IERR,NMIN,NMAX
   NTEST=2
   GOTO335

C CHECK THAT DESIRED NO. OF PERIODS IS OBTAINABLE
305 IF(NPRED-NPF)310,320,320
310 IERR=005
   WRITE(61,050)IERR,NPRED,NPF
   NPP=NPRED
   GOTO330
320 NPP=NPF
330 NPD=NPP-NPI
   IF(NPD)331,331,332
331 NTEST=2
   GOTO335

C HENCE DETERMINE NO. OF INITIAL AND FINAL POINTS, (TOTAL NO.
C OF POINTS)
332 MI=NPT(NPI)
   MF=NPT(NPP)
   NPTS=MF-MI+1
   WRITE(61,035)
   WRITE(61,036)
TABLE A8.2

```
WRITE(61,040)NPD,NPI,NPERD,NPTS,MI,MF,NTOTE,NMIN,NMAX
335 K=1
340 K=K+1
   IF(NOCHN(K))370,1001,380
370 IERR=006
   WRITE(61,050)IERR,K,K
   GOTO1000
380 WRITE(61,060)NOCHN(K)
   REWIND02
   D0360 N=1,4321
360 IY(N)=0

C: SELECT DATA FILE DESIRED FROM MAG,TAPE
390 READ(02,95)NSER,IIRUN,IICHN,(ICAL(I),I=1,5),ITAPCO
   IF(ENDFIL 02)1000,391
391 IF(IIRUN-IIRUN)430,400,430
400 IF(NOCHN(K)-IICHN)430,435,430
430 CALL SKIP(02)
   GOTO390
435 KTEST=1
   M=-863
   MM=0

C: READ DATA FROM MAG, TAPE
440 M=M+864
   MM=MM+864
   READ(02,97)(IX(I),I=1,3)
   IF(IX(I),EQ,-1,AND,IX(2),EQ,-100,AND,IX(3),EQ,-10)442,441
441 IF(IX(I),EQ,-2,AND,IX(2),EQ,-200,AND,IX(3),EQ,-20)500,900
442 READ(02,96)(IY(I),I=M,MM)
   IF(ENDFIL 02)500,450
450 IF(MM=3456)440,440,460
460 CALL SKIP(02)
500 GOTO(510,340),NTEST
510 WRITE(61,070)NSER,IIRUN,IICHN,(ICAL(I),I=1,5),NPTS,NPD

C: CHECK FOR ERRORS IN DATA AND INTERPOLATES FOR THE POINT.
   DO512I=MI,1,MF
      IF(ABS(2*IY(I-1)-IY(I-2)-IY(I))-INTPOL)512,511,511
511 IYY=IY(I)
   IYY(I)=(IY(I-1)+IY(I+1))/2
   WRITE(61,020)I,IIYY,IY(I),IY(I-2),IY(I-1),IY(I),IY(I+1),IY(I+2)
512 CONTINUE
   IF(LIST)515,515,340
515 IF(MF-MI=79)520,530,530
520 WRITE(61,080)IY(M),M=MI,MF
   GOTO340
530 MM=MI+79
   WRITE(61,080)IY(M),M=MI,MM
   MM=MF-79
   WRITE(61,080)IY(M),M=MM,MF
   GOTO340
600 WRITE(61,10)ITAPCO,ITAPCON
   GOTO1000
900 WRITE(61,6)
1000 ISTOP=2
1001 REWIND02
```
RETURN
END

SUBROUTINE CONV
CONVERT DATA POINTS INTO FLOATING POINT NUMBERS,
COMMON Y(4321), X(4321), ITACON, NSER, IRUN, NOCHN(8), ICAL(5)
COMMON VOLTC(5), NPI, NP, NPD, NPTS, LIST, ISTOP
COMMON NTEST, CAL(5), NPERD, MI, MF
COMMON FMOD(7), FPHASE(7), REV, SETANG, WAKEANG
COMMON G1, G2, G3, G4
COMMON AAPHA(8), KKD
II=0
DO100 I=MI, MF
II=II+1
100 Y(II)=IY(I)
DO300 I=1, 5
300 CAL(I)=ICAL(I)
RETURN
END

SUBROUTINE REDFA
FROM CALIBRATION VALUES DETERMINES BEST FIT STRAIGHT LINES AND
CONVERTS DIGITS TO REAL VALUES OF THE MEASURED VARIABLE,
COMMON Y(4321), X(4321), ITACON, NSER, IRUN, NOCHN(8), ICAL(5)
COMMON VOLTC(5), NPI, NP, NPD, NPTS, LIST, ISTOP
COMMON NTEST, CAL(5), NPERD, MI, MF
COMMON FMOD(7), FPHASE(7), REV, SETANG, WAKEANG
COMMON G1, G2, G3, G4
COMMON AAPHA(8), KKD
PI=3.1415927
XP=CAL(1)+CAL(2)+CAL(3)
XN=CAL(3)+CAL(4)+CAL(5)
YP=VOLTC(1)+VOLTC(2)+VOLTC(3)
YN=VOLTC(3)+VOLTC(4)+VOLTC(5)
ZP=CAL(1)**2+CAL(2)**2+CAL(3)**2
ZN=CAL(3)**2+CAL(4)**2+CAL(5)**2
WP=CAL(1)*VOLTC(1)+CAL(2)*VOLTC(2)+CAL(3)*VOLTC(3)
WN=CAL(3)*VOLTC(3)+CAL(4)*VOLTC(4)+CAL(5)*VOLTC(5)
AP=(ZP*YP-WP*XP)/(3.0*ZP-XP*XP)
AN=(ZN*YN-XN*XN)/(3.0*ZN-XN*XN)
BP=(3.0*WP*YP)/(3.0*ZP-XP*XP)
BN=(3.0*WN*YN)/(3.0*ZN-XN*XN)
SEN=1.0/(G1*G2*G3*G4)
CALINT=(AP-AN)/(BN-BP)
DO290 I=1, NPTS
IF(Y(I)-CALINT)270, 280, 280
270 Y(I)=(Y(I)+BN+AN)*SEN
GOTO290
280 Y(I)=(Y(I)+BP+AP)*SEN
290 CONTINUE
RETURN
END
Table A8.2

ROUTINE FASQW

DETERMINES FOURIER COEFFICIENTS OF THE VARIABLE.

DIMENSION PES (7), PANG (7)
DIMENSION YS (250, 2), YC (250, 2)
DIMENSION PMOD (250), PPHI (250)
COMMON IV (4321), Y (4321), IXAPCON, NSER, IRUN, NOCHN (8), ICAL (5)
COMMON VOLT (5), NPI, NP, NPD, NPTS, LIST, ISTOP
COMMON NTEST, CAL (5), NPERD, MI, MF
COMMON FMOD (7), FPHASE (7), REV, SETANG, WAKEANG
COMMON G1, G2, G3, G4
COMMON AAPH (8), KKD

120 FORMAT (1H1, 16X, 47HF, FOURIER COEFFICIENTS OF COMPLETE DATA ANALYSED)
125 FORMAT (1H0, 10X, 38HF, FOURIER COEFFICIENTS AT BLADE FREQUENCY)
130 FORMAT (1H0, 9X, 5HINDEX, 8X, 6HCO SINE, 13X, 4HSINE, 12X, 7HMODULUS, 9X, 5HPH
135 FORMAT (1H0, 82H BLADE HARMONIC PRESSURE COEFFICIENT)
136 FORMAT (1H0, 21X, 5H(PXI), 28X, 13H(LEAD DEGREE))
140 FORMAT (11X, 14, 3(4X, E14.7), 4X, F6.1)
145 FORMAT (1H0, 6X, 12, 10X, F8.5, 9X, F8.5, 13X, F7.2, 12X, F6.1)
150 FORMAT (1H0, 27H COUNTING ERROR EXIT CALLED)

KK=NPD*KKD
K=NPTS
PI=3.1415927
THETA=2.0*PI/FLOAT(N-1)
Y (1) =0.5* Y (1)
D50K=1, KK
ALPHA=PI/(FLOAT(K)*THETA)
BETA=1.0
YS(K, 1)=0.0
YS(K, 2)=0.0
D50J=1, 2
L=1
BETA=BETA+ALPHA/2.

IF (FLOAT(I) - GAMMA) 20*20, 11
11 DELTA2=FLOAT (I) - GAMMA
GAMMA=GAMMA+ALPHA
IF (L-1) 96, 12, 13
12 L=2
YS(K, J) =YS(K, J) + 5*Y (I-1) - DELTA2*(Y (I) + Y (I-1)) - 0.5*Y (1)
IF (I-N) 50, 14, 96
13 L=1
YS(K, J) =YS(K, J) - 5*Y (I-1) + DELTA2*(Y (I) + Y (I-1)) + 0.5*Y (1)
IF (I-N) 50, 14, 96
14 A=YS(K, J) - 0.5*Y (I)
YS(K, J) =1.57079632*A/FLOAT(N-1)
GOT050
20 M=L
IF (M-1) 30, 30, 45
30 YS(K, J) =YS(K, J) + Y (I)

560 PHI=90.0
WRITE (61, 80)
PI=3.1415927
D0100I=1, IBH
END
IF(I-N)50,39,50
39 A=YS(K,J)-0.5*Y(I)
   YS(K,J)=1.57079632*A/FLOAT(N-1)
   GOTO50
45 YS(K,J)=YS(K,J)-Y(I)
   IF(I-N)50,49,50
49 A=YS(K,J)+0.5*Y(I)
   YS(K,J)=1.57079632*A/FLOAT(N-1)
50 CONTINUE
   JJ=(2*KK)/3-1
   YC(KK,1)=YS(KK,1)
   YC(KK,2)=YS(KK,2)
   J=KK-1
   JX=J
   DO95LL=1,JX
   YC(J,1)=YS(J,1)
   YC(J,2)=YS(J,2)
   IF(L-JJ)95,70,70
70 DO91LL=1,L
   M=J+LL
   KA=M/J
   MM=M
   71 IF(MM-J)91,80,72
   72 MM=MM-J
   GOTO71
80 KKA=KA
73 IF(KKA-2)90,91,74
74 KKA=KKA-2
   GOTO73
90 YC(J,1)=YC(J,1)-YC(M,1)/(FLOAT(KA*(-1)%%((KA/2)+2)))
   YC(J,2)=YC(J,2)-YC(M,2)/(FLOAT(KA))
91 CONTINUE
95 J=J+1
   DO97LL=1,KK
   C=SQRFTF(YC(K,1)**2+YC(K,2)**2)
   PHI=ATANF(YC(K,2)/YC(K,1))*180.0/PI
   PHI=ABSF(PHI)
   IF(YC(K,2))470,510,540
470 IF(YC(K,1))480,490,500
480 PHI=180.0+PHI
   GOTO98
490 PHI=270.0
   GOTO98
500 PHI=360.0-PHI
   GOTO98
510 IF(YC(K,1))520,530,530
520 PHI=180.0
   GOTO98
530 PHI=0.0
   GOTO98
540 IF(YC(K,1))550,560,98
550 PHI=180.0-PHI
   GOTO98
560 PHI=90.0
TABLE A8.2

98 PMOD(K)=C
97 CONTINUE
   DEN=1.940/1728.0
   RAV=REV/60.0
   DIA=8.0
   CONS=(DEN*(RAV*DIA)**2)/12.0
   WRITE(61,125)
   WRITE(61,135)
   WRITE(61,136)
   M=0
   J=4*NPD
   DO570K=J,KK,J
   M=M+1
   A=M+4
   PPHASE=PMOD(K)/FMOD(M)
   PPHASE=PPHI(K)+FPHASE(M)+A*SETANG
   GOTO566
564 PPHASE=PPHASE-360.0
566 IF (PPHASE.LT.360.0)569,564
569 PRESK=PRESS/CONS
   PRESC(M)=PRESS
   PERCENT=PRESC(M)*100.0/Presc(1)
   WRITE(61,145)M,PRESC(M),PRESS,PPHASE,PERCENT
570 PANG(M)=PPHASE
   IBH=KK/(NPD*4)
   CALL CALMAX(PRESC,PANG,CONS,IBH,AAPHA)
   IF (LIST)580,580,99
580 WRITE(61,120)
580 WRITE(61,130)
580 DO590K=1,IBH
590 WRITE(61,140)K,YC(K,1),YC(K,2),PMOD(K),PPHI(K)
   GOTO99
96 WRITE(61,150)
97 NTEST=2
99 RETURN
END

SUBROUTINE CALMAX(Y,ANG,CONS,IBH,ALPHA)
DETERMINES PEAK TO PEAK VALES, AND PHASE OF SYNTHESIZED WAVE,
DIMENSION PP(100)
DIMENSION Y(7),ANG(7)
DIMENSION ALPHA(8)
50 FORMAT(1HO)
60 FORMAT(1HO,1OF10,5)
70 FORMAT(1HO,22X,24HVALUES OF TOTAL PRESSURE)
80 FORMAT(1HO,70HMAX PRESSURE - ANGLE MIN PRESSURE - ANGLE PRESSURE COEFFICIENT)
90 FORMAT(1HO,2(F10,5,F10,1,5X),4X,F10,5)
WRITE(61,70)
WRITE(61,80)
PI=3.1415927
DO100I=1,IBH
A = ANG(I) * PI
100 ANG(I) = A / 180, 0
    DANG = 0.0
    PMAX = 0.0
    PMIN = 0.0
150 P = 0.0
    DO 200 I = 1, 100
    A = 4 * I
200 P = P + Y(I) * COSF(ANG(I) * DANG * A)
    IF (P, GT, PMAX) 300, 400
300 PMAX = P
    ANGMAX = DANG * 180, 0 / PI
    GOTO 600
400 IF (P, LT, PMIN) 500, 600
500 PMIN = P
    ANGMIN = DANG * 180, 0 / PI
600 IF (DANG, GE, (PI / 2, 0)) 800, 700
700 DANG = DANG + 0.0005
    GOTO 150
800 PDIF = PMAX - PMIN
    PK = PDIF / (2.0 * CONS)
    WRITE (61, 90) PMAX, ANGMAX, PMIN, ANGMIN, PK
    DA = PI / 180.0
    DO 900 N = 1, 100
    B = N - 1
    DANG = B * DA
    PP(N) = 0.0
    DO 900 I = 1, 100
    A = 4 * I
900 PP(N) = PP(N) * Y(I) * COSF(ANG(I) * DANG * A)
    WRITE (61, 50)
    WRITE (61, 50)
    WRITE (61, 50)
    WRITE (61, 60) (PP(N), N = 1, 100)
    CALL MEKPLT (PP, 100, 1, 1, 1, ALPHA, 10H DEGREES, 10HPRESS)
1000 RETURN
END

C ROUTINE TO SKIP A FILE.
97 FORMAT (3I5)
100 READ (02, 97) (IX(I), I = 1, 3)
    IF (ENDFILE(O2) 1000, 200
200 IF (IX(1), EQ, 1, AND, IX(2), EQ, 100, AND, IX(3), EQ, -10) 400, 300
300 IF (IX(1), EQ, 2, AND, IX(2), EQ, -200, AND, IX(3), EQ, -20) 900, 1000
400 DO 500 I = 1, 44
500 READ (02, 97) (IX(I), I = 1, 3)
    GOTO 100
1000 STOP 02
900 RETURN
END
SUBROUTINE MEKPLT(YPT, IMAX, JMIN, JMAX, GTITLE, XTITLE, YTITLE,
1 FACTOR, GP0, GP1, GP2, GP3, GP4, GP5, GP6, GP7, GP8, GP9)
C GENERAL PURPOSE PLOTTING ROUTINE, AUTOMATIC SCALING, NO. OF PLOT
C COMBINED UP TO 10,
C YPT = NAME OF ARRAY TO PLOT WITH DIMENSIONS (IMAX,JMAX)
C PLOTS POINTS FROM THE JMIN VALUE OF THE ARRAY YPT TO JMAX VALUE
C PLOTS ONLY THE JMIN TO JMAX ARRAY OF POINTS IN YPT.
C GTITLE = NAME OF GRAPH
C XTITLE = NAME OF X AXIS
C YTITLE = NAME OF Y AXIS
C FACTOR = SCALE FACTOR TO CHANGE THE VALUES OF THE SUBSCRIPT I OF
C YPT(I,J) TO A GIVEN QUANTITY
C GP0, GP9 = CHARACTER TO DESIGNATE THE PARTICULAR GRAPH
C J OF YPT(I,J)
C DIMENSION GTITLE(8)
C DIMENSION YPT(IMAX,10), APT(100,10), ALPHA(100), BETA(10), IBETA(6)
10 FORMAT(1HI,30X,8A10)
20 FORMAT(1H,20X,2HI,10A10)
30 FORMAT(1H,20X,2HI,10A10,1HI)
40 FORMAT(1H,118,4H=I,100A1,3HI= )
50 FORMAT(1H,20X,2HI,100A1,1HI)
60 FORMAT(1H,1A10,10X,2HI,100A1,1HI)
61 FORMAT(1H,1H,4X,1I,1I,12X,2HI,100A1,1HI)
62 FORMAT(1H,2X,4H=I,10,14X,2HI,100A1,1HI)
70 FORMAT(1H,16X,1I,1I,16X,1I,1I,16X,1I,1I,16X,1I,1I,16X,1I)
71 FORMAT(1H,1H,73X,1I)
72 FORMAT(1H,71X,4H=10)
80 FORMAT(1H,65X,A10)
90 FORMAT(1H)
100 FORMAT(1H,5HYDIV=E16.9,5X,5HXDIV=E16.9)
C CALCULATE RANGE OF VALUES OF VARIABLE Y
AMAX=YPT(IMIN,1)
AMIN=YPT(IMIN,1)
D094=JMIN,JMAX
D094=IMIN,JMAX
IF(YPT(I,K),GT,AMAX)91,92
91 AMAX=YPT(I,K)
GOTO94
92 IF(YPT(I,K),LT,AMIN)93,94
93 AMIN=YPT(I,K)
94 CONTINUE
RY=AMAX-AMIN
C CALCULATE FACTOR TO EXPAND RANGE TO .GT., 50
IF(RY,GT,50)95,96
95 RF=1.0
IRF=1
GOTO97
96 RF=50/RY
IRF=ALOG10(RF)*1.0
RF=10**IRF
97 AMAX=AMAX*RF
AMIN=AMIN*RF
TABLE A8.2

DO981=I.MIN, I.MAX
DO98K=J.MIN, J.MAX
98 YPT(I.,K)=YPT(I.,K)*RF
PRINT 10, (GTITLE(I.), I=1,8)
PRINT90
C. CALCULATE RANGE OF VALUES OF VARIABLE X
IRX=I.MAX-I.MIN+1
IF(IRX.LT.100)110, 160
110 IRX5=(IRX+4)/5
120 IF(IRX5.EQ.(5*(IRX5/5)))140, 130
130 IRX5=IRX5+1
GOTO120
140 IRX=5*IRX5
IC=100/IRX
RC=IC
RC=1/RC
IXMIN=IRX5*(I.MIN/IRX5)
DO170K=J.MIN, J.MAX
DO170I=1, 100, IC
IXGRAD=IXMIN+(I-1)/IC
IF(IXGRAD.LT.I.MIN)170, 150
150 IF(IXGRAD.GT.I.MAX)170, 160
C. DISTRIBUTE POINTS OVER THE FIELD OF 100 SPACES.
160 APT(I.,K)=YPT(IXGRAD,K)
170 CONTINUE
GOTO250
C. FOR .GT. 100 POINTS, CALCULATE 'BEST FIT' X-RANGE AND NO. POINTS/S.
180 IRX5=(IRX+4)/5
190 IF(IRX5.EQ.(20*(IRX5/20)))210, 200
200 IRX5=IRX5+1
GOTO190
210 IRX=5*IRX5
IC=IRX5/20
RC=IC
IXMIN=IRX5*(I.MIN/IRX5)
DO240K=J.MIN, J.MAX
DO240I=1, 100
IXGRAD=IXMIN+(I-1)*IC
IF(IXGRAD.LT.I.MIN)240, 220
220 IF(IXGRAD.GT.I.MAX)240, 230
C.DISTRIBUTE POINTS OVER THE FIELD OF 100 SPACES
230 APT(I.,K)=YPT(IXGRAD,K)
240 CONTINUE
250 RY=AMAX-A.MIN
C. CALCULATE 'BEST FIT' Y-RANGE
IRY=RY
260 IF(IRY.EQ.(50*(IRY/50)))280, 270
270 IRY=IRY+1
GOTO260
280 C=IRY/50
C. CHOOSE MIN Y VALUE
IYMIN=A.MIN
IRY5=IRY/5
TABLE A8.2

284 IF(IYMIN.EQ.(IRY5*(IYMIN/IRY5)))285,286
286 IYMIN=IYMIN-1
GOTO284
285 IYMIN=IYMIN
YMAX=YMIN+50.0*C
IF(YMAX,LT.AMAX)270,287
C
PRINT GRADUATION MARKS ACROSS TOP
287 DO290K=1,10
290 BETA(K)=10H
I
PRINT20,(BETA(K),K=1,10)
C
PRINT LINE ACROSS TOP
DO300K=1,10
300 BETA(K)=10H
PRINT30,(BETA(K),K=1,10)
DO410J=1,51
DO310K=1,100
C
SET WHOLE FIELD BLANK
310 ALPHA(K)=1H
YGRAD2=YMAX-(J-1)*C
YGRAD2=YGRAD-C
IF(IRX,LT.100)320,330
320 IXDIV=100/IRX
GOTO340
330 IXDIV=1
C
IF Y-VALUE LIES BETWEEN VALUES OF 2 LINES, SET A POINT ON LOWEST L
340 DO365K=JMIN,JMAX
365 I=1,100,IXDIV
IX=I*K
IF(I,LT,IMIN)365,341
341 IF(I,GT,IMAX)365,342
342 IF(APT(I,K),LE,YGRAD)345,365
345 IF(APT(I,K),GT,YGRAD2)350,365
350 GOTO(500,501,502,503,504,505,506,507,508,509)K
500 ALPHA(I)=GP0
GOTO365
501 ALPHA(I)=GP1
GOTO365
502 ALPHA(I)=GP2
GOTO365
503 ALPHA(I)=GP3
GOTO365
504 ALPHA(I)=GP4
GOTO365
505 ALPHA(I)=GP5
GOTO365
506 ALPHA(I)=GP6
GOTO365
507 ALPHA(I)=GP7
GOTO365
508 ALPHA(I)=GP8
GOTO365
509 ALPHA(I)=GP9
GOTO365
365 CONTINUE
TABLE A8.2

PY10=ALOG10(YMAX)
IF(IRF.GT.0)366,367
367 IPY10=ALOG10(YMAX)
YGRAD=YGRAD/(10**IPY10)
IPY10=IPY10-2
IF(YGRAD.LT.0.0)371,372
371 IYGRAD=100*YGRAD-1.OE-01
GOTO368
372 IYGRAD=100*YGRAD+1.OE-01
GOTO368
366 IPY10=-IRF
IF(YGRAD.LT.0.0)373,374
373 IYGRAD=YGRAD+1.OE-01
GOTO368
374 IYGRAD=YGRAD+1.OE-01
368 IF(J.EQ.25)370,380
370 PRINT60,YTITLE,(ALPHA(I),I=1,100)
GOTO410
380 IF(J.EQ.26)381,382
381 PRINT61,IPY10,(ALPHA(I),I=1,100)
GOTO410
382 IF(J.EQ.27)383,385
383 PRINT62,(ALPHA(I),I=1,100)
GOTO410
385 IF((J-10*(J/10)-1),E0.0)390,400
390 PRINT40,IYGRAD,(ALPHA(I),I=1,100)
GOTO410
400 PRINT50,(ALPHA(I),I=1,100)
410 CONTINUE
C PRINT LINE ACROSS BOTTOM
PRINT30,(BETA(K),K=1,10)
C PRINT GRADUATION MARKS ACROSS BOTTOM
D0430K=1,10
430 BETA(K)=10H
PRINT20,(BETA(K),K=1,10)
C PRINT X-GRADUATIONS
XMAX=1MAX
IPX10=ALOG10(XMAX)
IX10=IPX10
PFAC=ALOG10(FACTOR)
IF(PFAC,LT.0.0)600,650
600 IPFAC=1.0-PFAC+1.0E-01
FAKTOR=FACTOR*(10**IPFAC)
IPFAC=-IPFAC
GOTO700
650 IPFAC=PFAC+1.0E-01
FAKTOR=FACTOR/(10**IPFAC)
700 D0440I=1,6
BETA(I)=(IXMIN*(I-1)*IRX5)*FAKTOR
BETA(I)=BETA(I)/(10**IX10)
440 IBETA(I)=100*BETA(I)+1.OE-01
IPX10=IPX10+2+IPFAC
PRINT70,(IBETA(I),I=1,6)
PRINT71,IPX10
PRINT72
PRINT90
PRINT80,XTITLE
PRINT VALUES OF LINE AND SPACE
XDIV=IRX5/20*FACTOR
YDIV=C/RF
PRINT100,YDIV,XDIV
RETURN
END