THE DRIFT VELOCITY OF ELECTRONS IN HYDROGEN AND NITROGEN

by

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SUMMARY

This thesis reports an investigation to determine the drift velocity of electrons in hydrogen and nitrogen. Results have been obtained at 293°K and 77.6°K over the range $0.001 \leqslant E/p \leqslant 20$ volt cm^{-1} mm Hg^{-1}, using the method of Bradbury and Nielsen. The overall accuracy is 1% at 293°K and 2% at 77.6°K.

To obtain results of this accuracy it has been necessary to analyse the errors which occur due to the effect of diffusion of electrons within the drift chamber. This analysis includes an account of the influence of back diffusion, the effect of the boundary condition imposed by the receiving electrode and the variation of the mean agitational speed of the electrons within an electron pulse. It is shown that effects due to diffusion can lead to spuriously high measurements of the drift velocity, introducing a relative error of the order of $3/h(W/D)$. (h being the distance between the shutters and W/D the ratio of the drift speed to the diffusion coefficient). Reasonable agreement has been obtained between the theoretical predictions of the pressure dependence of the observed value of the drift velocity with the results obtained experimentally. Variations of over 10% have been observed at some values of E/p. The theoretical analysis of the effect of diffusion has also been applied to the methods of Pack and Phelps, and Hornbeck.

A systemmatic investigation has been made of all likely sources of experimental error in drift velocity determinations using the method of Bradbury and Nielsen. This investigation includes an examination of the possible influence of gaseous impurities, which has been determined by measuring the change in the drift velocity when known amounts of impurity are added to the gas sample. Further experiments have been conducted to determine the influence of non-uniformity of the d.c. electric field, field interpenetration due to the a.c. voltages on the shutters, temperature gradients, space charge repulsion and phase differences in the voltages applied to the shutters.

A demonstration of the accuracy of the shutter method has been made by measuring the velocity of potassium ions in vacuum in a field free space, and comparing the results with the velocity predicted theoretically from the voltage used to accelerate the ions initially.

The final values of the drift velocity that are presented are corrected for the influence of diffusion and also for the variation of the behaviour of nitrogen from that of a perfect gas at 77.6°K. Measurements are also given for mixtures of nitrogen and hydrogen and comparisons are made with predictions of the theory of mixtures developed by Bailey. Several applications of drift velocity data are reexamined using the present results for low electron energies.

STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university. To the best of my knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

John J. Lowke.

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NOTATION

A	Collision Cross Section of a Gas Molecule (cm^2)
C	Speed of a Gas Molecule ($cm\ sec^{-1}$)
c	Speed of an Electron ($cm\ sec^{-1}$)
D	Coefficient of Diffusion
E	Electric Field Strength ($volt\ cm^{-1}$)
e	Electronic Charge (4.803×10^{-10} esu)
F	Defined by $k_T = F\ k_1$
f	Frequency with which Shutters Open.
f_n	Twice the Frequency of the a.c. signal corresponding to the mth current maximum of the current/frequency curve.
h	The Drift Distance.
I	Current
J	Particle Current
k_T	Townsends Energy Factor
k_1	Defined in equation (5)
ℓ	Mean Free Path (cm)
M	Mass of a gas Molecule ($H = 1.67 \times 10^{-24}$ gm)
m	Mass of an Electron (9.107×10^{-28} gm)
N	Molecular density of Gas
N_o	Avogadros Number ($6.025 \times 10^{23}\ gm\ mol^{-1}$)
n	Electron Density
p	Gas Pressure
Q	Energy of Electrons
Q_o	Average Energy of Gas Molecules

R_o	Universal Gas Constant (8.317 x 10^7 erg/mol/°K)
r	Relative Error
T	Temperature (Degrees Kelvin)
W	Drift Velocity (cm sec^{-1})
W_M	Measured Value of the Drift Velocity (cm sec^{-1})
η	Fractional Energy Loss of an Electron in a Collision
ν	Collision Frequency of Electrons with Gas Molecules.

SECTION I : INTRODUCTION

Chapter 1 : The Motion of Electrons in Gases

1.1 General historical review.

It is over fifty years since the classical experiments were performed which established the existence of the electron. Among the most important of these experiments were the first studies of the conduction of electricity through gases. For example an important experiment of Townsend showed that $N_o e$ (N_o is Avagadros number, e the charge) for particles which carried current through dry gases, appeared to be numerically equal to the value of $N_o e$ obtained in Faradays experiments, where, in this case, e is the charge of the ions which carry the electric current in the process of electrolysis. Townsend was also able to obtain one of the first estimates of the value of e using an experiment which applied Stokes Law to a falling cloud of newly condensed water droplets in air. The water droplets were assumed to have condensed around single charged particles. The determinations made by J.J. Thomson of e/m for the electrons of cathode rays involved more general studies of the passage of electrons through gases at various pressures. It is from initial experiments such as these that the later work on the properties of electrons in gases has developed.

Innumerable experiments on the motion of electrons and ions in gases have now been carried out. It has resulted, for example, in the discovery that the process of a collision of an electron with a gas molecule can be most complicated. In addition to elastic collisions the processes of ionization and attachment can occur. There is evidence that the molecules of slight traces of impurities, particularly of gaseous organic compounds, can result in the formation of 'clusters' by aggregating around electrons or ions. Although the collisions of electrons of low energy with monatomic gas molecules are usually perfectly elastic, collisions with diatomic gas molecules can result in inelastic collisions in which the energy of the electron excites the molecule into rotational or vibrational states. Furthermore gas molecules can be excited such that if they should later collide with a metal electrode through their normal motion through the gas by diffusion, an electron can be emitted. Such gas molecules are said to be excited into metastable states.

For electrons of very low energies, it is generally impossible to observe the motion of a single electron to determine knowledge of the processes that occur when electrons travel through the gas. Instead one can only make measurements of macroscopic quantities relating to large groups of electrons. The microscopic properties have to be inferred from the macroscopic phenomena.

Two of the most important macroscopic properties of any group of electrons which travel through a gas under the influence of an electric field are the drift velocity (W) and the coefficient of diffusion (D). From these two macroscopic quantities it is possible to obtain much information of the microscopic quantities which describe the collisions of an electron with gas molecules. Such microscopic quantities are, the effective collision cross section of the gas molecules (A); the mean free path of electrons between collisions (ℓ), which is related to A by $\ell = 1/NA$, N being the molecular density of the gas); the average fraction of the energy that an electron loses in a collision (η) and the average velocity of the individual electrons ($\bar{c}$). The quantities A, ℓ, and η are themselves functions of c and their evaluation from experimental values of W and D, which refer to groups of electrons having a wide energy distribution, is not simple.

It has been usual to make various assumptions, for example, as to the nature of the velocity distribution of the electrons, to enable the microscopic quantities to be estimated. Even though these estimates are not accurate, valuable information as to the nature of the collision processes can then be derived. For example, it is found that the values of η that are estimated from W and D measurements made in diatomic gases differ widely from $\frac{2m}{M}\left(1 - \frac{1}{k_T}\right)$ which would apply if collisions were perfectly elastic. (k_T is the ratio of the energy of the electrons to the energy of the gas molecules, m/M is the ratio of the mass of the electron to the gas molecule). Thus we know that inelastic collisions, such as occur in rotational excitation, are likely to be present. Furthermore a comparison of even the approximate derived values of A with the values of A that have been predicted theoretically, can provide evidence as to the validity of the assumptions of the theoretical evaluation of A, and thus elucidate the processes that occur.

Whenever microscopic quantities, such as A, are deduced from the macroscopic experimental quantities, such as W, it is desirable that the raw data be obtained to as high an accuracy as possible. There are many investigations of the drift velocity reported in the literature and a great variety of methods have been used. Nevertheless when comparisons are made of the results of different investigations, it is found that differences occur, which, for example, in hydrogen and nitrogen are well over 5%. Not only is it desirable to obtain data of greater precision than this, but the fact that the results of earlier investigations usually differ by more than the claimed experimental error makes it possible that factors are influencing the experimental results of which no account has been taken. Furthermore it is frequently not possible to compare accurately the results of different investigations because of the different temperatures at which the results have been taken and also because the graphical presentation of the results either obscures the degree of accuracy of the initial data or makes its recovery impossible.

1.2 The Present Investigation.

This thesis reports an investigation in which the method of Bradbury and Nielsen was used to measure the drift velocity of electrons in hydrogen and nitrogen. A systematic investigation was made of all likely sources of error. In the first place, the influence of diffusion, often neglected in time of flight methods, has been taken into account in assessing the results. Secondly, since gas purity is often cited as a possible explanation of the discrepancies between the results of various workers, the magnitude of the errors introduced by known amounts of likely impurities has been determined. In addition an examination of the possible influence of non uniformity of the electric field, field interpenetration due to voltages on the shutters, temperature gradients, space charge repulsion and phase differences in the voltages applied to the shutters have been made. A demonstration of the accuracy of the method has been made by measuring the drift velocity of positive ions in vacuum in a field free space and comparing the results with the velocity predicted theoretically from the voltage used to accelerate the ions initially.

When this project was commenced, no measurements were available for the drift velocity in hydrogen and nitrogen for values of E/p less than 0.05 or for gas temperatures other than room temperature. (E/p is the ratio of electric field strength to gas pressure, where the units of E/p are always given in volt cm^{-1} mm Hg^{-1}). To obtain measurements corresponding to very low electron energies it was desirable to obtain measurements at very low values of E/p at low gas temperatures. However, the measurements reported in this thesis that were made for a gas temperature of 77.6°K for values of E/p as low as 0.001 were preceded by the less accurate measurements of Pack and Phelps (1961).

As this thesis is particularly concerned with drift velocity measurements, the concept of the drift velocity, and the relationship of W to the microscopic parameters, will now be discussed in greater detail.

1.3 The Drift Velocity. (W).

If a group of electrons in a gas is under the influence of a uniform electric field E, each electron will gain momentum because of the force acting on it of E e (e is the electronic charge). However when the electrons collide with the gas molecules they lose momentum. A state of equilibrium is attained in which the average rate of increase in momentum of the group of electrons due to E equals the average rate of loss of momentum due to collisions with the gas molecules, and the group of electrons then travels with a uniform velocity of drift, called the drift velocity. This velocity of drift is the vector average of the individual random velocities of each electron. In general W is only a few per cent of c.

The drift velocity is expected on theoretical grounds to be a function of E/N and with few exceptions this has been verified by experimental results. A simple theoretical justification that W depends on E/N can be given by regarding all electron paths between collisions as straight lines. Then, if we consider any electron path of length s, the gain in energy of the electron from the electric field while it traverses this path will be given by

$$\tfrac{1}{2}m\,(v^2 - u^2) = \tfrac{1}{2}m\,2\,\underline{a}\,.\,\underline{s}$$

where $\underline{a}$ is the acceleration of the electron due to $\underline{E}$, $\underline{s}$ is the vector representing the electron path, and $\underline{u}$ and $\underline{v}$ are the initial and final velocities of the electron. If both E and N are doubled, $\underline{a}$ will be doubled but $\underline{s}$ will be halved so that the product $\underline{a}\,.\,\underline{s}$ remains constant. Thus the gain in energy of any electron between collisions remains constant for a given ratio of E/N. As W is the vector average of the individual velocities of the electrons, W should thus also be a simple function of E/N. It can be shown that the gain or loss of energy for the more general case of curved electron paths between collisions is also a function of $\underline{a}\,.\,\underline{s}$ and thus of E/N.

At 293°K, $N = 3.30\,.\,10^{16}\,p$, where p is the gas pressure in mm Hg. Thus the parameter E/N can be expressed as a constant times E/p, and results taken at a constant gas temperature can be given as a function of E/p.

The dependence of W on E/N enables an experimental check to be made on the consistency of an experimental result for any value of E/N by comparing results at different gas pressures (at constant temperature). It is because careful measurements of the drift velocity gave slightly varying results at different pressures for a given value of E/N that the investigation of Chapter 8 was undertaken.

The motion of a group of electrons travelling through a gas under the influence of a uniform electric field is determined not only by the electric field, but also by the energy of the gas molecules. For very low values of E/N the average energy of the electrons is determined largely by the energy of the gas molecules and becomes equal to that of the gas molecules in the limiting case where E/N tends to zero. As a consequence, the values of W at low E/N are dependent on the temperature of the gas, but at high E/N where the motion of the electrons is governed largely by E and is independent of the energy of the gas molecules, the values of W are independent of temperature. It is of interest to determine W at different gas temperatures; the results that have been obtained at 77.6°K and 293°K are compared graphically in Fig. 23 for hydrogen and Fig. 24 for nitrogen.

The relation of the drift velocity with various microscopic quantities mentioned previously, can be derived theoretically by making various simplifying assumptions. The derivations of Allis (1956) and Chapman and Cowling (1952) use the Boltzmann transport equation. Huxley and Crompton (1962), by making a reasonable assumption as to the form of the velocity distribution, have derived the same result much more simply, and a simplified version of this proof is given in Appendix I. The resulting expression for W is

$$W = \frac{E\,e}{3m}\;\overline{c^{-2}\,\frac{d}{dc}\,(\ell\,c^{2})} \qquad (1)$$

The chief assumptions on which the derivation is based are as follows:

(a) The proportion of the mean free path along which there is interaction of an electron with a gas molecule during a collision, is small.

(b) All collisions are elastic. For diatomic molecules, this assumption is not true because some collisions cause rotational and vibrational excitation. However the frequency of these collisions is small. Furthermore if the scattering of the electrons from the gas molecules is isotropic, the loss of momentum for electrons of velocity c when averaged over all directions of scattering is $m\,c$ ($m \ll M$), which is the same as the loss of momentum for inelastic collisions. Equation (1) is thus considered to still apply for diatomic molecules with an error of much less than 1%.

(c) Approximations are involved in the derivation of the formula. In the derivations using the Boltzmann equation only the first two terms of the series solution are retained and in the derivation of Huxley, approximations are also made (see Appendix I). Nevertheless these approximations are estimated to have negligible error and equation (1) has been used extensively in the literature.

Two other macroscopic quantities of great importance are the coefficient of diffusion and Townsends energy factor and these will now be discussed briefly.

1.4 The Coefficient of Diffusion. (D).

The coefficient of diffusion for electrons in a gas is defined in the usual way, i.e. the nett rate of transport due to diffusion of electrons across unit area is given by -D grad n, where n is the electron density.

D can also be expressed in terms of microscopic quantities, the resulting expression being (Appendix I)

$$D = \frac{1}{3} \overline{\ell c} \qquad (2)$$

1.5 Townsends Energy Factor. (k_T)

Townsends energy factor is the ratio of the average kinetic energy of an electron to the average kinetic energy of a gas molecule, i.e. $$k_T = \tfrac{1}{2}\, m\, \overline{c^2} / \tfrac{1}{2}\, M\, \overline{C^2} \qquad (3)$$

A most important relationship between the ratio W/D and k_T can be derived from equations (1) and (2), using in addition the relation

$$M\, \overline{C^2} / 2 = 3\, R\, T / 2\, N_o \qquad (4)$$

(T is the temperature of the gas in degrees K, R is the universal gas constant and N_o is Avogadros number). Equation (1) is divided by equation (2) to obtain

$$\frac{W}{D} = \frac{E\, e\, N_o}{R\, T\, k_T}\; \overline{c^{-2}\, \frac{d}{dc}\,(\ell\, c^2)}\;\; \overline{c^2} / \overline{\ell c} = \frac{E\, e\, N_o}{R\, T\, k_1} \qquad (5)$$

where $$k_T = k_1\; \overline{c^{-2}\, \frac{d}{dc}\,(\ell\, c^2)}\;\; \overline{c^2} / \overline{\ell c} = F\, k_1 \qquad (6)$$

The expression above, which was derived by Huxley and Crompton (1962) is a more accurate form of the expression derived originally by Townsend (Healey and Reed 1941). The expression of Townsend is obtained from (5) and (6) by putting F? B = 1. The difference between the two results arises from the earlier analysis of Townsend being based on the assumption that $\frac{\partial p}{\partial x}$ can be regarded as analagous to a force. The analogy is not perfect as a pressure gradient can never result in an increase in the energy of the electrons.

Chapter 2 : Methods of Measuring the Drift Velocity

The first reasonably accurate measurements of the drift velocity were made as early as 1898 by J. Zeleny. Since this time many investigations of the drift velocity have been made, using a great variety of methods, and a comprehensive history would be very long. It is proposed to describe in detail only the four methods which are used in investigations outlined in the current literature. These are the methods of Townsend, Bradbury and Nielsen, Hornbeck and Bortner.

2.1 The Method of Townsend.

In the method of measuring the drift velocity used by Townsend and Tizard (1913), a magnetic field is applied perpendicular to the applied electric field. The strength of the magnetic field is adjusted until the centre of the electron stream is deflected a known angle θ, this deflection being measured by means of divided electrodes. The relation between θ and W was derived to be

$$\tan \theta = V/W = B e W/E e \qquad (7)$$

where V is the transverse velocity produced by the magnetic field B. Townsend used this method in what was the first systemmatic investigation of the properties of electrons in gases. However, Huxley (1937) has shown that equation (7) is inaccurate because account must be taken of the velocity distribution of the electrons.

In the most recent form of the method (Huxley and Zaazou 1949) the electrons emerge from a very small hole and travel under the influence of a uniform electric field until they are collected by a divided electrode as shown in Fig. 1. A known magnetic field B is applied in a direction parallel to the direction of the slit in the collecting electrode, whose segments are connected together so that the ratio R_1 of the currents on the two halves of the collecting electrode can be measured. It has been shown by Huxley and Zaazou that R_1 is a function not only of B, W and k_1 but also of F' where F' is a dimensionless factor which depends both on the velocity distribution of the electrons and also on the variation of A with c. k_1 can be measured in the same apparatus by rearranging the connections to the segments of the collecting electrode to measure the ratio R_2 of the current falling on the central disc to the current falling on the outer annulus when there is no applied magnetic field. A further expression of Huxley (1940) relates R_2 with k_1.

Hall (1955a, 1955b) has used this method to determine W in hydrogen and deuterium by assuming that A is independent of c and that either a Maxwellian or Druyvesteyn electron velocity distribution exists.

The chief current interest in this method is not so much to use it to determine W, as to provide information regarding

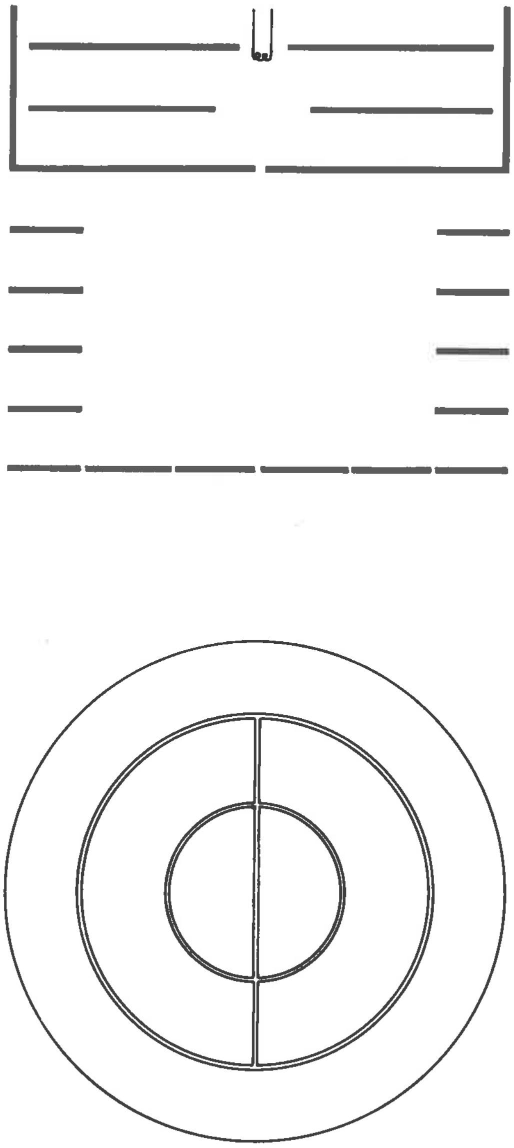


Fig 1

the velocity distribution of the electrons. If the value of W is known independently from direct time of flight measurements, comparison with magnetic deflection data enables F' to be evaluated. As F' is a known function of velocity averages, it should be possible to determine the extent of the departure of the velocity distribution from that of Maxwell for various values of k_T.

2.2.1 Method of Bradbury and Nielsen.

In 1936 Bradbury and Nielsen developed a direct time of flight method of measuring drift velocities which is extremely simple in principle, and which enables measurements to be made over a very wide range of E/p. Furthermore the systemmatic analysis of all sources of error that is made later in this thesis suggests that the method is the most accurate for drift velocity determinations. The only differences in the method of the present investigation with that of Bradbury and Nielsen is that in the present method a hot filament was used as the source of electrons and the results were taken using the later technique of Nielsen in which the frequency instead of the applied d.c. electric field is varied.

The experimental apparatus is illustrated in Fig. 2. Electrons emitted from a filament F drift through the gas to the collecting electrode B under the influence of a uniform electric field which is maintained between the electrodes A

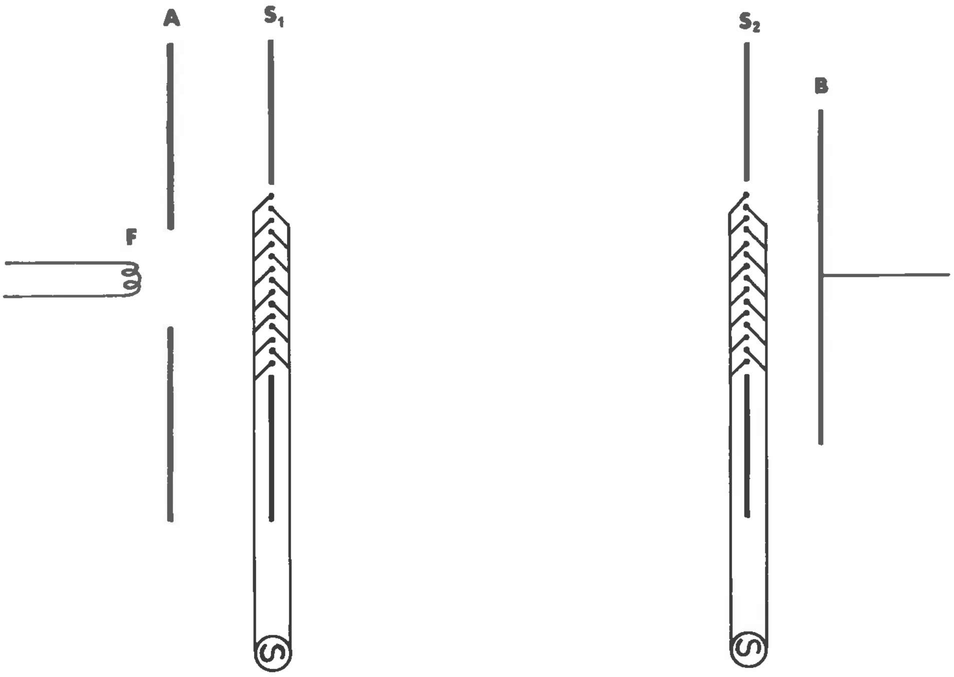


Fig 2

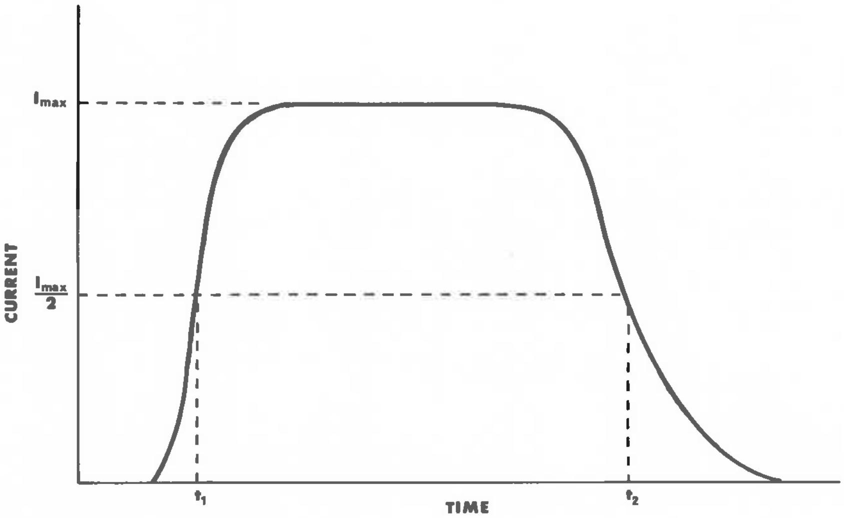


Fig 3

and B. Two shutters, S_1 and S_2, each consisting of a plane grid of parallel wires, of which alternate wires are connected together, are placed in the path of the electrons, the distance h between the two shutters being accurately known. An a.c. signal, whose frequency can be varied while maintaining a constant voltage, is applied to the two halves of S_1. The voltages applied to each half of the shutter are exactly 180 degrees out of phase. Similar voltages exactly in phase with those applied to S_1 are applied to S_2. All a.c. signals have superimposed on them the appropriate d.c. voltage required to maintain the uniform field between A and B. The frequency of the a.c. signal is then varied and the current collected at B is recorded. When a graph is drawn of current against frequency a series of maxima will be obtained as shown in Figs. 20 and 21.

The action of S_1 is to divide the stream of electrons into a series of pulses, as electrons will be admitted only when the a.c. signal on the shutter wires approaches zero. If $f/2$ is the frequency of the a.c. signal, f will be the frequency with which the shutters open. S_2 will have a similar influence in the electron current and the first maximum of the current/frequency curve will correspond to the frequency f_1 for which pulses of electrons produced by S_1 arrive at S_2 when the shutters are again about to open. Other current maxima will occur at integral multiples of f_1. The drift velocity is then determined from

$$W = h\,f_1 \quad \text{or} \quad W = h\,f_m/m \qquad (8)$$

where f_m is twice the frequency of the a.c. signal corresponding to the mth current maximum of the current/frequency curve.

Since the original experiments of Bradbury and Nielsen, there have been several developments which have an important bearing on this thesis. In 1957 R.A. Duncan published a paper analysing the influence of diffusion on the pulses of electrons formed in the apparatus of Bradbury and Nielsen. The case of a point source of electrons with a small collecting electrode was analysed and it was shown that the simple interpretation of the current/frequency curves was complicated in the following respects:

(a) The ultimate resolution of the method depends on the sharpness of the current peaks of the current/frequency curves. The degree of sharpness of the current peaks, which is generally increased by increasing the shutter voltage, is ultimately limited by the diffusion of the electrons. Duncan predicted that the effect of diffusion on the width of the pulses was so serious that, for example, at $E/p = 1$ $p = 1$ in hydrogen, only the first of the current peaks would be observed in the current/frequency curve.

(b) Furthermore, and more seriously, it was predicted that the values of f_m would be displaced due to diffusion so that spuriously high values of the drift velocity would be recorded. Thus the accuracy of the method was seriously limited under conditions of high electron diffusion. The work of Duncan has been considerably extended in Section III.

In 1957 Crompton, Hall and Macklin carried out an experimental investigation using the method of Bradbury and Nielsen in which the measurements of W were first reported to be pressure dependent at a given value of E/p. Results were taken in hydrogen at room temperatures for drift distances h of 3, 5.9 and 10 cm between the two shutters of the apparatus. The measured values of W

(1) for fixed h decreased to a limiting value as p was increased, and,

(2) at a given pressure decreased to a limiting value as h was increased.

These results were in qualitative agreement with the variations predicted from the theoretical work of Duncan, but the variation was very much larger than would be expected from theory.

In this thesis the variation of the results with gas pressure has been investigated experimentally in detail for hydrogen and nitrogen, and also in helium. This investigation includes a repetition of the measurements of Crompton, Hall and Macklin, using their original apparatus. A full discussion is given in Chapter 9. Reasonable agreement exists between the new experimental data and the extended theory developed in Section III, in which account is taken of the boundary conditions of the electrons at the shutters.

2.2.2 Modification of the Method of Bradbury and Nielsen made by Pack and Phelps.

Since the commencement of the work presented in this thesis an extensive experimental investigation has been reported by Pack and Phelps (1961) using a method based on the method of Bradbury and Nielsen. Drift velocities have been measured in many gases at temperatures extending to the temperature of liquid nitrogen. The results of the present investigation are in agreement with the results of Pack and Phelps, although their results were only obtained to an accuracy of the order of 6%. The analysis of errors due to diffusion in Section III has also been applied to the method of Pack and Phelps, and their method is now described.

The method differs from that of Bradbury and Nielsen in the following respects. Instead of the electron pulses being formed by a shutter, the pulses are produced by photo-electric emission from a cathode by means of a pulsed source of ultra-violet light. The electron pulses then travel to a shutter, but this shutter is not operated by the application of a.c. sine wave voltages as in the Bradbury and Nielsen method. Instead a constant d.c. bias is maintained between the two halves of the grid to close the shutter, and this bias is removed for a short time to open the shutter. The time interval between the flash of ultra-violet light and the opening of the shutter is varied and the current from the electron pulses transmitted by the receiving shutter recorded. From the time at which maximum current occurs, the transit time of an electron pulse from the cathode to the shutter is determined. A second shutter is included in the apparatus further from the cathode than the first shutter, and the transit time of an electron pulse to the second shutter can similarly be determined. The difference in these two transit times gives the time taken for an electron pulse to travel the distance between the two shutters, independent of any end effects occurring at the cathode. On knowing the distance between the two shutters, the drift velocity is then calculated.

2.3 The Method of Hornbeck - and other oscillographic methods.

The development of oscilloscopes having very high sweep frequencies and of amplifiers with fast rise times has enabled a whole group of new methods to be used in measuring the drift velocity. The most extensive investigation using these techniques is the recent work of Bowe (1960) who used the method of Hornbeck (1951). This method, which is closely related to the earlier method of Herreng (1943), has also been used in a modified form by Biondi and Chanin (1954) to measure the mobility of positive ions. The method of Hornbeck will now be described.

A flash of ultra-violet light produces an initial pulse of electrons at the cathode, which then travels under the influence of a uniform electric field to the anode. The cathode and anode are usually two large parallel circular plates. The resulting current pulse is then amplified and viewed oscillographically so that a photograph of the pulse can be obtained. Omitting influences due to diffusion the current I travelling in a Townsend gap is given by

$$I = N_1 e W/h \qquad (9a)$$

where N_1 is the total number of electrons in the gap (assuming that there are no positive ions due to ionization), e is the electronic charge, W the drift velocity and h the electrode separation. (See Appendix II). Thus, as the electrons are produced in the chamber, the current rises to a constant level which is maintained while the electrons are traversing the gap. Then, as electrons are absorbed at the anode, the current will decrease, becoming zero when the pulse is completely absorbed. The form of a typical current trace is given in Fig. 3.

The transit time of the pulse is assumed to be $t_2 - t_1$ where t_1 and t_2 are the times at which the current is half of its maximum value. This procedure in determining say t_2 is based on the assumption that when half of the electrons have been absorbed, the centre of the pulse is then at the collecting electrode. This assumption is not quite accurate, as will be shown in Chapter 10.

In many cases the oscillographic trace that is obtained is not as simple as is shown in Fig. 3. For example, Fig. 4 is a copy of a trace obtained by Hornbeck for positive ions in Argon. Although the initial ripple is of no significance, being due to the amplifying equipment, the existence of the two 'steps' in the trace indicates the presence of two different ions having different mobilities. Later these ions were identified as A_2^+ and A^+. The form of the trace given in Fig. 5, which has been taken from the results of Biondi and Chanin, is evidence of the existence of back diffusion. The initial spike is caused by the number of ions (N_1 of equation (9a)) in the chamber being diminished soon after the formation of the pulse, because ions are reabsorbed by the emitting electrode due to the influence of back diffusion.

A number of other investigations employing oscillographic methods have been used. The investigations of Colli and Facchini (1952), Klema and Allen (1950) and Nagy, Nagy and Dési (1960) differ in principle from that of Hornbeck only in that the resistance of the collecting electrode to earth is made very high. Consequently the oscillographic trace, which represents the voltage of the collecting electrode, is a measure not of the instantaneous current, but of the integral of the current. The trace in Fig. 6 is similar to that published by Nagy, Nagy and Dési, the transit time in this case is $t_2 - t_1$.

Kirschner and Toffollo (1952) have used a much less direct

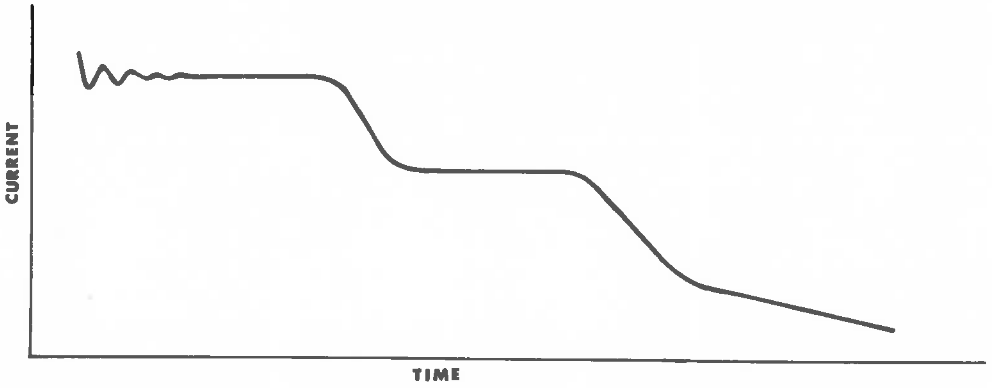


Fig 4

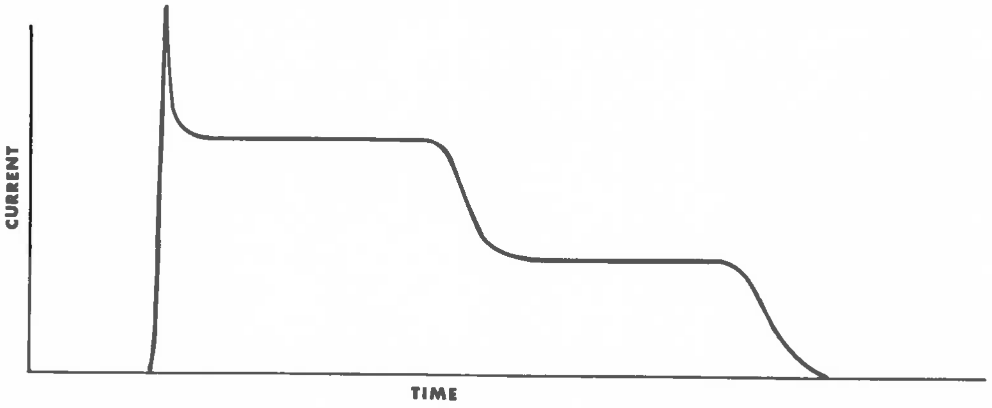


Fig 5

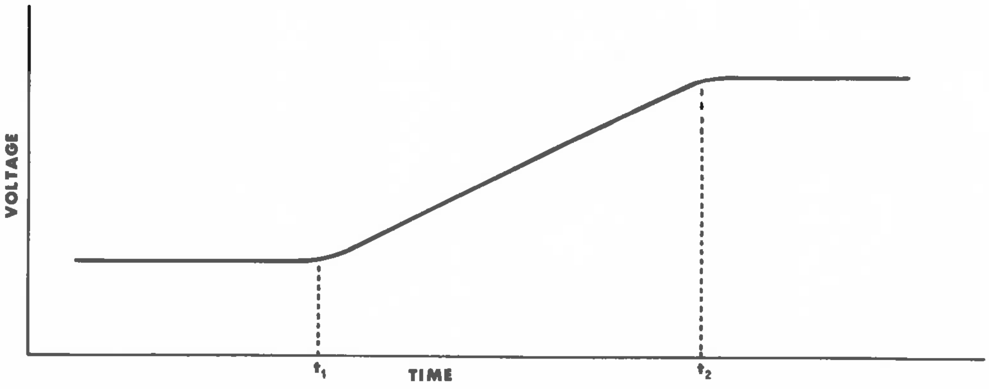


Fig 6

method, which was first developed by Hudson. The whole of the space in a Townsend gap is suddenly exposed to X-rays to form uniform ionisation. The form of the resultant rise in current has been analysed theoretically, and the transit time is estimated from the oscillographic trace representing the current.

The accuracy of all oscillographic methods is dependent on the accuracy of high speed sweep times and is limited by the accuracy with which time intervals can be read from the oscillographic trace. The possibility of errors being introduced due to distortion of the form of the pulse from the electronic amplifier equipment must also be carefully considered.

2.4 The Method of Bortner et al.

Bortner, Hurst and Stone (1956) have conducted direct time of flight measurements of drift velocities using Geiger Counters. A stream of α particles is used to initiate electron pulses by ionisation, the time of the formation of each pulse being recorded as each α particle activates a Counter. The time of arrival of the electron pulse, after it has travelled a known drift distance, is recorded by a second counter. By incorporating the use of a delay line before the first counter, the resultant pulses are brought to coincidence by varying the delay on the delay line, a procedure which it is claimed results in high accuracy.

The method of Comunetti and Huber (1960) differs chiefly in the method of recording the time of arrival of the electron pulse. At the end of the drift distance, the electrons pass through a gauze into a region of high electric field which accelerates the electrons to the extent necessary to cause ionisation. The light produced from this ionization is used to excite a photomultiplier to record the arrival of the electron pulse.

SECTION II : THE PRESENT INVESTIGATION

EXPERIMENTAL

Chapter 3 : Description of Apparatus

3.1 The Electrode System.

The electrode system is shown in Fig. 7. The electrodes, which were made of brass are labelled 0, 1, 2 ... 15. All metal surfaces were coated with gold prior to assembly to reduce errors due to contact potential differences. Electrode 13 is in the form of a cylindrical cup to prevent electrons from diffusing sideways and travelling down the chamber without passing through the first shutter. The electrodes were positioned by accurately ground glass spacers to maintain the required electric field, which was uniform between electrodes 0 and 14. The guard rings 1 to 12 were 0.5 cm apart and the distance between the two shutters was 5.985 ± .005 cm. A high electric field, usually of 180 volt cm^{-1}, was applied between electrodes 14 and 15 to obtain maximum current from the filament, one side of which was connected externally to electrode 15. Provision was made so that this high voltage (called the "pull off" voltage) could be varied, thus enabling an additional means of regulating the magnitude of the current through the chamber to the normal method of controlling the filament temperature. The electric field between electrodes 13 and 14, which was the same as the uniform field between 0 and 13, ensured that the electrons reach the first shutter at the equilibrium energy and drift velocity appropriate to the particular value of the uniform electric field applied between the shutters. It was necessary to include a grid across the central aperture of electrode 14 to prevent the high "pull off" field from penetrating to the region of the shutters and thus influencing the results.

The collecting electrode (E in Fig. 8), was held in position by two glass rods, which were clamped to the back of the base electrode 0.

The electrode assembly was secured by three brass tie rods which passed through each guard ring and were screwed into electrode 13. These tie rods were insulated from the other electrodes by glass sheaths. The brass nuts at the ends of these tie rods were rounded to minimize the possibility of electric breakdown. It will be noted that the highest electric field in the apparatus was between these nuts and the base electrode, so that this precaution is most important.

3.2 The Experimental Tube.

It was required that it be possible to immerse the apparatus in liquid nitrogen to enable measurements to be made at very low gas temperatures. Thus the lead from the collecting electrode was brought from the experimental tube via the curved tube

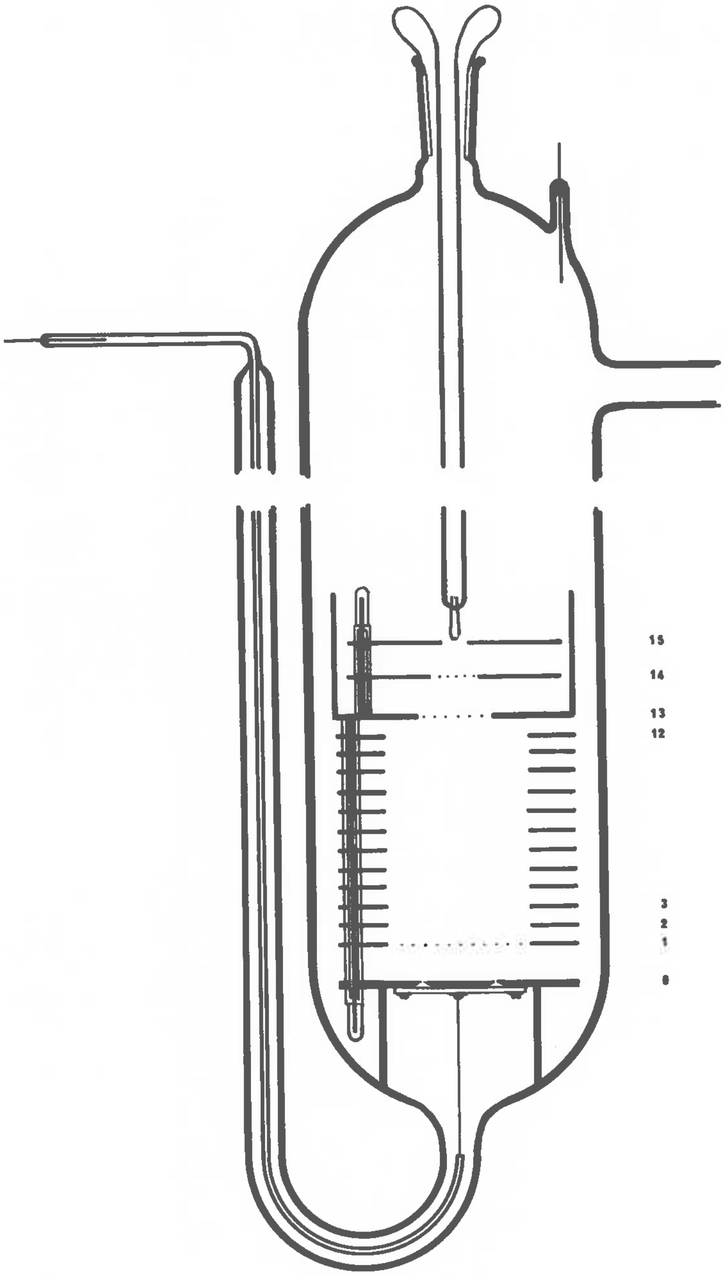


Fig 7

shown in Fig. 7. This tube was coated with an earthed layer of 'Aquadag' to provide electrical shielding. The wire to the electrometer was kept central in this tube by a smaller glass tube, to ensure that the capacity to earth of the electrometer lead was as low as possible.

The tungsten seals by which the leads to the electrodes were brought from the experimental tube were placed at the top of the glass envelope so that it was not necessary to immerse them in liquid nitrogen. The leads from these seals to the appropriate electrodes were enclosed in silica tubes to minimize the possibility of discharge between the wires.

The whole apparatus was placed in a 2½ inch thick foam polystyrene container constructed from 'Coolag'. The container was lined with a water tight copper can which could be filled with liquid nitrogen. The insulation properties of the 'Coolag' were quite adequate to prevent the liquid nitrogen from rapidly boiling away.

The filament was inserted into the apparatus by means of a glass filament holder which could be removed by means of the cone join at the top of the experimental tube.

3.3 The Electron Source.

The source of electrons was obtained by thermionic emission from a filament. Both platinum and tungsten filaments were used, but tungsten was found to be preferable when doing measurements in helium as higher filament temperatures were possible. Initially experiments in nitrogen were carried out using an oxide coated filament with a coating preparation supplied by Philips Electrical Industries. The advantage of this filament was that sufficient electron emission could be obtained at very low filament temperatures thus reducing errors due to the introduction of temperature gradients. However subsequent coated filaments used in hydrogen were found to emit large proportions of negative ions, possibly due to the presence of water vapor emitted from the coating preparation. All final results quoted in this thesis were obtained with an uncoated filament because it was desired to reduce the possibility of gas contamination and the presence of negative ions to a minimum.

3.4 The Shutters.

The construction of the shutters presented a technical problem of some difficulty. It was desirable to have the spacing between the grid wires as small as possible so that the voltage necessary to produce a given cross field between adjacent wires would be a minimum. Furthermore, a given voltage on the shutter wires produces less distortion to the uniform d.c. field as the wire spacing is reduced. The spacing that was achieved in the final shutters was 0.5 mm.

Two stringent requirements have to be satisfied in the construction of the shutters:

(a) All materials must have a very low vapor pressure as the experimental results are affected by impurities and gas pressure.

(b) The shutter must withstand cooling to the temperature of liquid nitrogen.

3.4.1 The First Shutters.

Initially the shutters were made of two pieces of mica, between which was an adhesive of cold setting 'Araldite' to support the shutter wires. The outgassing rate of the 'Araldite' had been measured and it was considered unlikely that the results in diatomic gases would be affected. It was necessary to make 3 mm holes in both upper and lower pieces of mica to prevent shearing between the mica and the 'Araldite' when the temperature of the shutter was reduced to 77.6°K.

The surfaces of the shutters were coated with gold by vacuum evaporation to provide a conducting layer to which the correct d.c. potential could be applied to maintain the uniform electric field.

When, however, these shutters were used to measure drift velocities it was found that the measured value of the drift velocity, for any given gas sample, slowly increased with time and a background current appeared in the current frequency curves. This increase in the value of the drift velocity was shown to be due to gas contamination from the 'Araldite', because later results using shutters free of 'Araldite' did not show this increase with time.

The increase in the value of the drift velocity with time is given in Table 1 for nitrogen at E/p = 0.2 at p = 20 mm. The time given, is the age of the gas sample after preparation from sodium azide.

Table 1

Time (hours)	0	2	3	Equilibrium
Drift Velocity (10^5 cm/sec)	3.90	3.92	3.97	4.06

Results taken without the use of liquid air traps indicate that the equilibrium value results from the condition where the rate of evolution of the vapor in the experimental tube equals the rate at which vapor is condensed in the liquid air traps.

As the outgassing rate of the apparatus in vacuum without the use of liquid air traps was only 0.1μ in 20 minutes, these increases in the drift velocity were, at the time, most unexpected. It was known that the drift velocity results using the inert gases are highly susceptible to impurities, but it was thought unlikely that results in the diatomic gases would be affected by impurities of 1 part in 10^4. However, investigations that are reported later in this thesis show that the drift velocity is significantly increased by traces of water vapor at this order of concentration, which is consistent with the belief that the increase of the results is due to the emission of a condensible vapor, possibly water, from the 'Araldite'. It was thus necessary to construct shutters that were free of 'Araldite'.

3.4.2 The Final Shutters.

The new shutters were made by threading fine nichrome wire of 0.003 inches diameter through holes pierced in mica. The techniques involved were only slight modifications of the method used by Crompton and Elford (1959) to make smaller shutters required for use with positive ions. For measurements with electrons it was desirable to have the bottom shutter of as wide a diameter as possible to avoid any unknown effects due to lateral diffusion. The dimensions of the apertures in the shutters that were made were 1.5 inches for the bottom shutter and 0.7 inches for the top shutter. The outside diameter was 2.5 inches in each case and the wire spacing 0.5 mm. The mica was held between two curved pieces of aluminium during the threading of the wire, the curvature enabling the wires to be brought to greater tension when the mica was later clamped between two flat face plates of brass. The shutter wires were insulated from the face plates by additional thin pieces of mica.

The diameter of the aperture of the additional pieces of mica and the mica former was made greater than the diameter of the aperture of the face plates so that no area of mica was exposed to the electron stream. Such areas of mica could accumulate electric charges and introduce errors into the results. The spurious results discussed in 6.4.2 indicate that it is most important that there be no possibility of electric charges developing near the shutters or on the shutter wires. Particular care was taken in the final shutters that were used to highly polish the surfaces of the face plates and to attempt to remove all traces of dirt that were visible when the wires were viewed with a binocular microscope. The shutters, including the shutter wires, were coated with gold by vacuum evaporation after final cleaning by means of electron bombardment.

Chapter 4 : Subsidiary Equipment

4.1 Electrometry.

The electron currents used in the drift chamber were from 3.10^{-13} to 10^{-12} amp and were measured by means of a Vibron electrometer in conjuction with a 10^{10} ohm resistance.

For the electrometer to detect rapid changes of current it was necessary that the time constant of the circuit, which is proportional to R C, be sufficiently low (R and C are equal to the resistance and capacity of the collecting electrode E to earth). C was made as low as possible by such precautions as:

(1) undercutting E to reduce the area in close proximity to the outer earthed ring,

(2) ensuring that the lead from E is central in the curved glass tube (Fig. 7), and

(3) the use of low capacity cable for the lead to the electrometer.

The lower limit of C is the input capacity of the electrometer, which is 20 pF.

The glass of the tungsten seal by which the lead from E is brought from the experimental tube was coated with a 2% solution of Silicone Fluid (MS200) in carbon tetrachloride and baked at 300°C for an hour. This procedure prevented electrometer instabilities caused by the formation of thin films of water on the surface of the glass at the seal, which otherwise would have to be removed by the use of an infra red lamp.

The lead from E to the electrometer was completely enclosed in an earthed shield to prevent interference from induced electric currents.

4.2 The d.c. Voltage Supply.

The voltages applied to the electrodes to maintain the uniform d.c. field were derived, by means of a voltage divider, from a Fluke type 301E power supply. This power supply provided voltages which could be set with an error of less than 0.1%. The voltage divider was constructed from a chain of 68K HS resistors. The voltage division was checked at various intervals during the period in which measurements were made and in all cases the voltage applied to each electrode was correct to well within 0.2%

The power supplies for the "pull off" voltage and the filament current were provided by 180 volt and 6 volt batteries. The detail of the wiring is shown in Fig. 9.

4.3 The a.c. Voltage Supply.

The a.c. voltages that are applied to the shutters should ideally have the following characteristics:

a. The phase difference between the two voltages applied to a shutter should be 180°.

b. There should be no phase difference between the corresponding voltages that are applied to each shutter.

c. The RMS voltages of the two signals applied to a shutter

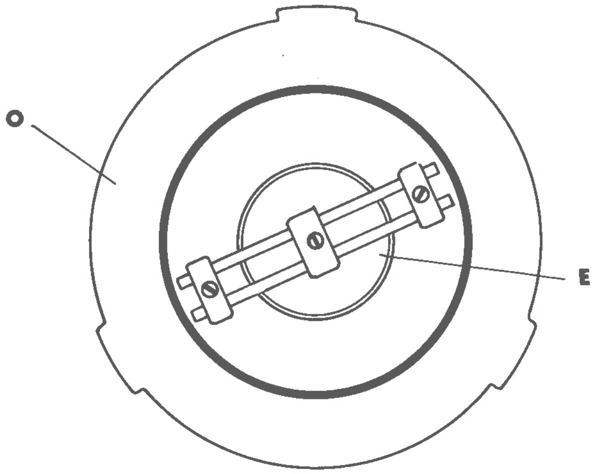

O
E

Fig 8

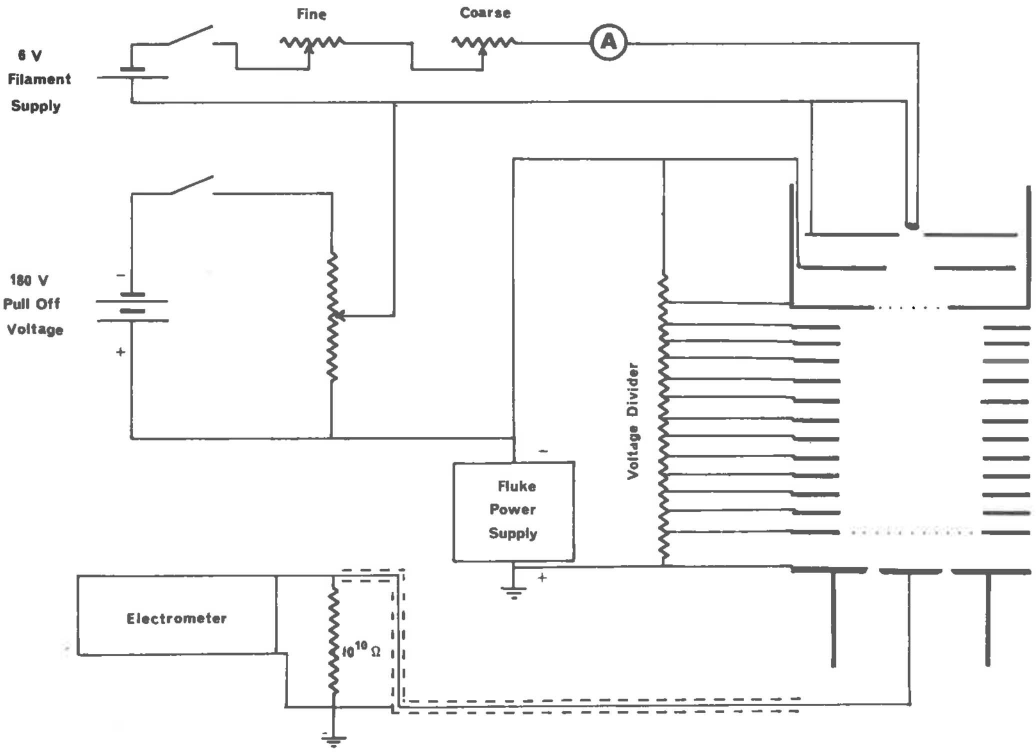

6 V
Filament
Supply
Fine
Coarse
A
180 V
Pull Off
Voltage
-
+
Voltage Divider
-
Fluke
Power
Supply
+
Electrometer
$10^{10}\ \Omega$

Fig 9

should be equal.

d. The magnitude of the RMS voltages of the signals at the shutters should remain constant as the frequency is varied.

e. The a.c. voltages for each shutter should be superimposed on the d.c. voltage necessary to maintain the required uniform d.c. field in the drift space.

f. The RMS value of the a.c. voltage must be sufficiently high so that when the voltage is at its peak value the shutter wires collect 80% or more of the electrons reaching the shutter.

The equipment to provide voltages satisfying the above requirements consisted of three components:

(1) An oscillator to provide the initial a.c. signal,

(2) An a.c. amplifier to provide two signals which differ in phase by 180°, and

(3) A resistance capacitance network to enable these signals to be superimposed on the appropriate d.c. voltages of each shutter.

The three components are now discussed.

4.3.1 The Oscillators.

An Airmec Model 201 Signal Generator, together with a Philips low frequency oscillator were used to provide a.c. voltages over the required frequency range of 1 kc/s to 1 Mc/s. Both oscillators satisfied the requirements that the output voltage was constant to $\frac{1}{3}$ d.b. with change of frequency over each range covering a frequency range of 3 : 1. The scale length was sufficient to enable frequencies to be read with an accuracy of $< 1\%$. The calibration of the oscillators was determined with a precision counter to an accuracy of better than $\frac{1}{2}\%$ at various intervals during the period over which measurements were taken.

4.3.2 The a.c. Amplifier.

4.3.2.1 Use of Transformer.

Initially an attempt was made to use a transformer as the amplifier. A special transformer was wound, using a "ferroxcube" core of the E type. The secondary winding had its centre connected to earth potential so that the output voltages were out of phase by 180°. However the frequency response of the transformer was unsatisfactory in that the amplification was not constant over the required frequency range.

As the electron current transmitted by the shutters is a function of the applied RMS voltage, it is necessary to maintain this voltage at a constant value during the determination of values of f_m. Thus, if the frequency response of the amplifier is not constant, adjustments of the output of the oscillator are required during the taking of measurements, to keep the voltage at the shutters constant. Because the emission of electrons from the filament can also vary, it is impossible to take accurate experimental readings if such adjustments are necessary.

An examination of principles of transformer design indicated that it would be impossible to construct a single transformer having a constant response over the required range of frequency, and that a series of transformers would be required.

The difficulty was overcome by designing an electronic amplifier with the required characteristics.

4.3.2.2 Electronic Amplifier.

The design of this amplifier was achieved under the guidance of Mr. G. Rose of the University of Adelaide.

The circuit diagram is given in Fig. 10. The initial amplifier is a 6V6 valve, tetrode connected, followed by a 6CD6 valve used as a phase splitter. Two further 6CD6 valves are used as cathode followers in the output stages.

It is difficult to satisfy the requirement that the amplification be constant with change of frequency because of the capacitive nature of the load. The capacity of each shutter is only of the order of 50 pF, but this results in an effective load of the order of 200 pF on each of the two outputs of the amplifier. At 1Mc/s the effective load is then as low as 1 K, so that an amplifier of low output impedance is required.

A further difficulty arises because the low value of the load at 1 Mc/s results in the slope of the a.c. load line becoming very great. Consequently, for most valves, the

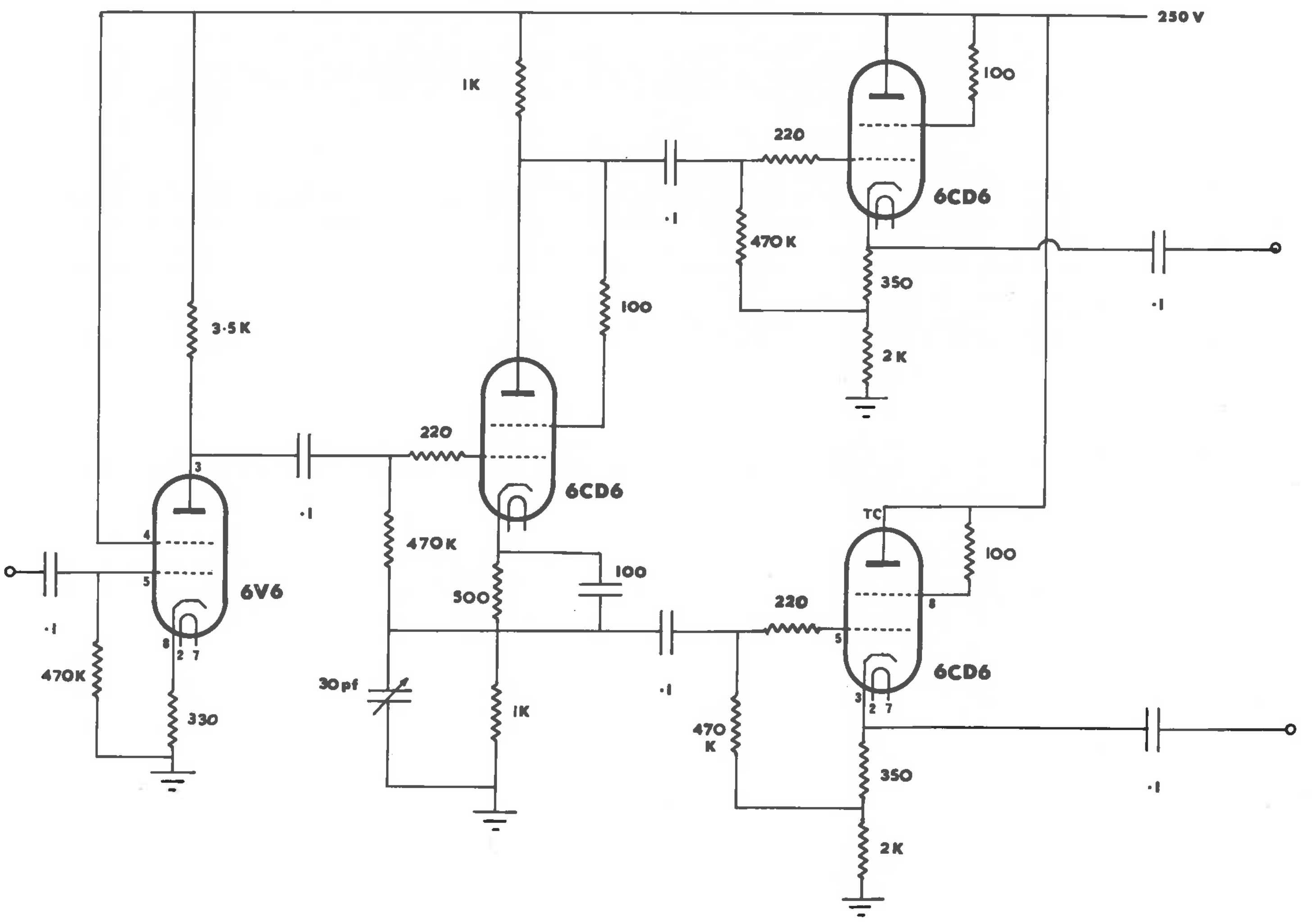

250 V
IK
100
220
6CD6
·I
470 K
350
100
2 K
3·5 K
220
·I
6CD6
470 K
TC
100
100
500
220
6V6
·I
470K
330
30 pf
IK
·I
6CD6
470 K
350
·I
2K
3
4
5
8
2
7
8
5
3
2
7

Fig 10

maximum output voltage that is obtainable for moderate plate voltages of the order of 250 volts is much lower than 20 volts RMS. However with the 6CD6 valves, which carry high current, this output is possible.

The 30 pF trimming condenser shown in Fig. 10 was used to compensate for the effect of stray leakage capacitances, which introduce slight variations from the ideal condition of the difference in phase of 180° between the two output signals. The optimum position of adjustment was ascertained by connecting the two signals direct to the plates of an oscilloscope and observing the resulting Lissajous figure.

The 250 volt 250 ma power supply used to drive the amplifier was constructed using the circuit of Fig. 11.

The frequency response of the electronic amplifier is compared in Fig. 12 with the response of the transformer used formerly.

4.3.3 The Resistance Capacitance Network.

This network, which enables the signals to be supplied to the shutters at the appropriate d.c. voltage, is illustrated in Fig. 13.

To reduce capacitive loading on the amplifier, the length of low capacity cable that is required to carry the a.c. voltages is reduced to a minimum by mounting the resistance capacitance network immediately above the experimental tube.

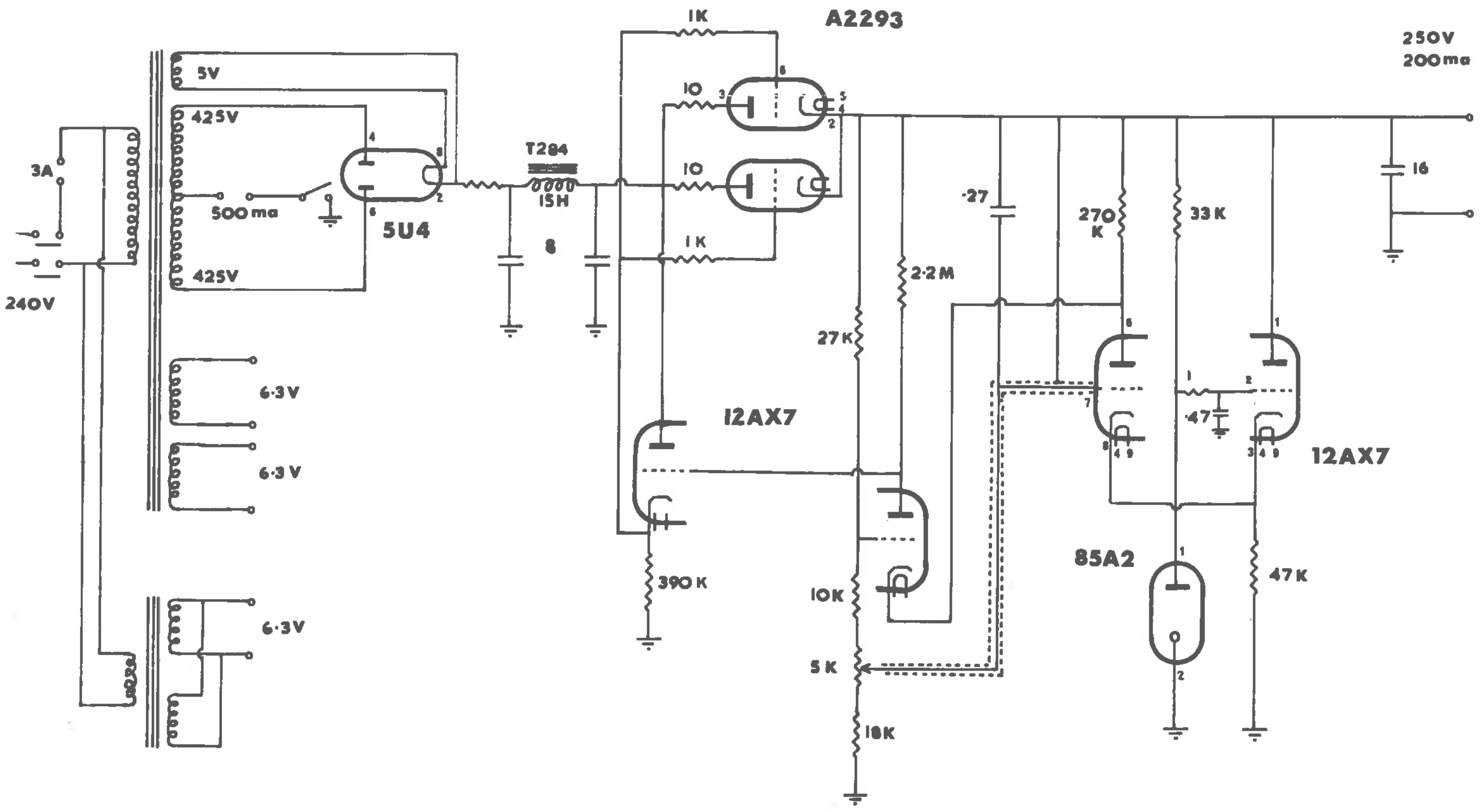

A2293
1K
10
10
1K
250V
200ma
5V
425V
425V
500ma
3A
240V
5U4
T284
15H
8
16
·27
270 K
33K
2·2M
27K
12AX7
390K
10K
5K
18K
6·3V
6·3V
6·3V
85A2
47K
·47
12AX7

Fig 11

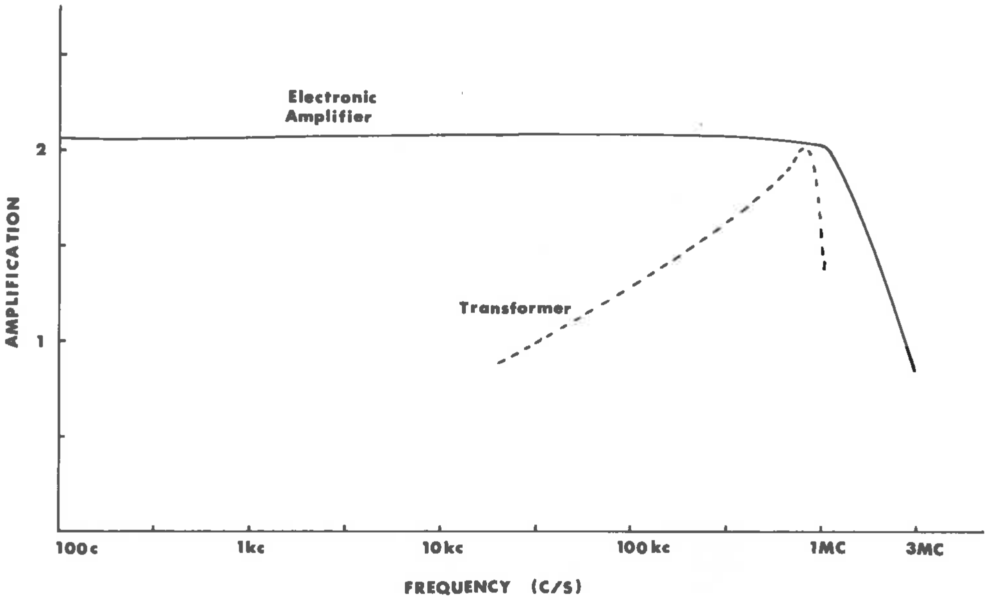

Electronic
Amplifier
Transformer
AMPLIFICATION
2
1
100c
1kc
10kc
100kc
1MC
3MC
FREQUENCY (C/S)

Fig 12

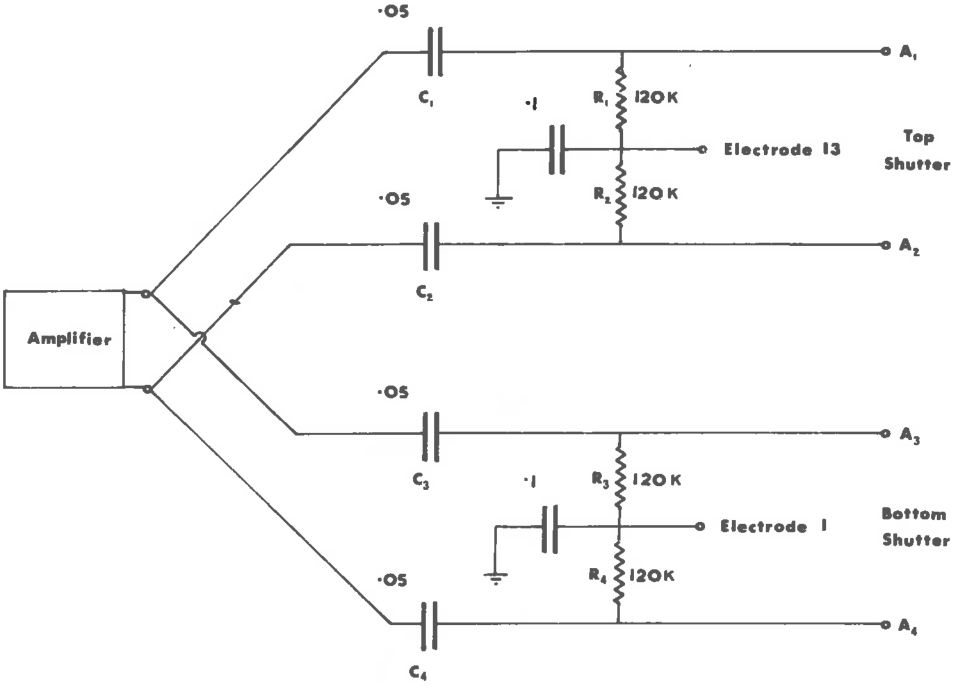

Amplifier
·05
C1
·05
C2
·05
C3
·05
C4
·1
·1
R1 120K
R2 120K
R3 120K
R4 120K
A1
A2
A3
A4
Electrode 13
Electrode 1
Top
Shutter
Bottom
Shutter

Fig 13

To satisfy the requirements, previously mentioned, of the phase of the signals at A_1 A_2 A_3 and A_4 (Fig. 13) it is essential that $C_1 = C_2 = C_3 = C_4$ and $R_1 = R_2 = R_3 = R_4$. The value of C_1 is chosen to be $0.05\,\mu$F, which is high enough for the condensers to have a reactance that is still only a few per cent of 120 K at the lowest frequencies that are required. Consequently the low frequency response is not adversely affected.

4.4 Measurement of the a.c. Signal.

It is necessary to ensure that the method of measurement of the a.c. signal on the shutters does not introduce errors because of the loading effect of the measuring instrument. Thus a probe was built for use with the vacuum tube volt meter, with which measurements were made, and the resultant loading on the shutters was only of the order of several pF. The value of the 4.7 M resistor, shown in the circuit of the probe which given in Fig. 14, is selected to obtain the correct scale reading on the VTVM. The probe was mounted together with the resistance capacitance network on top of the experimental tube so that it was as close to the shutters as possible.

4.5 The Vacuum System.

A schematic diagram of the vacuum system is given in Fig. 15. The positions of liquid air traps and taps are indicated. The traps immediately before the experimental tube and immediately after the gas production units were filled with indented copper

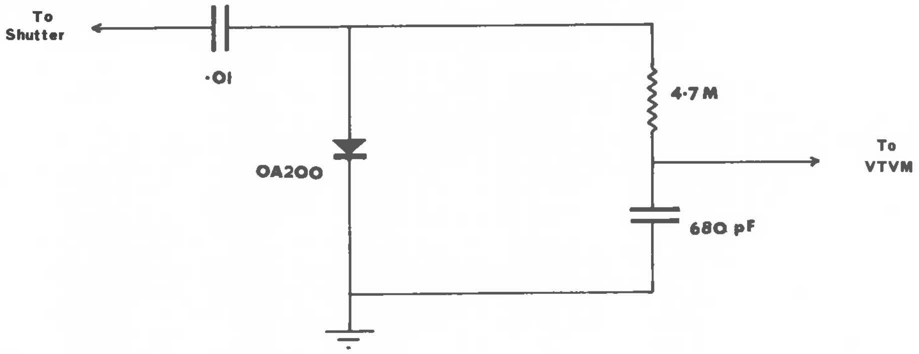

To
Shutter
·01
4·7 M
OA200
To
VTVM
680 pF

Fig 14

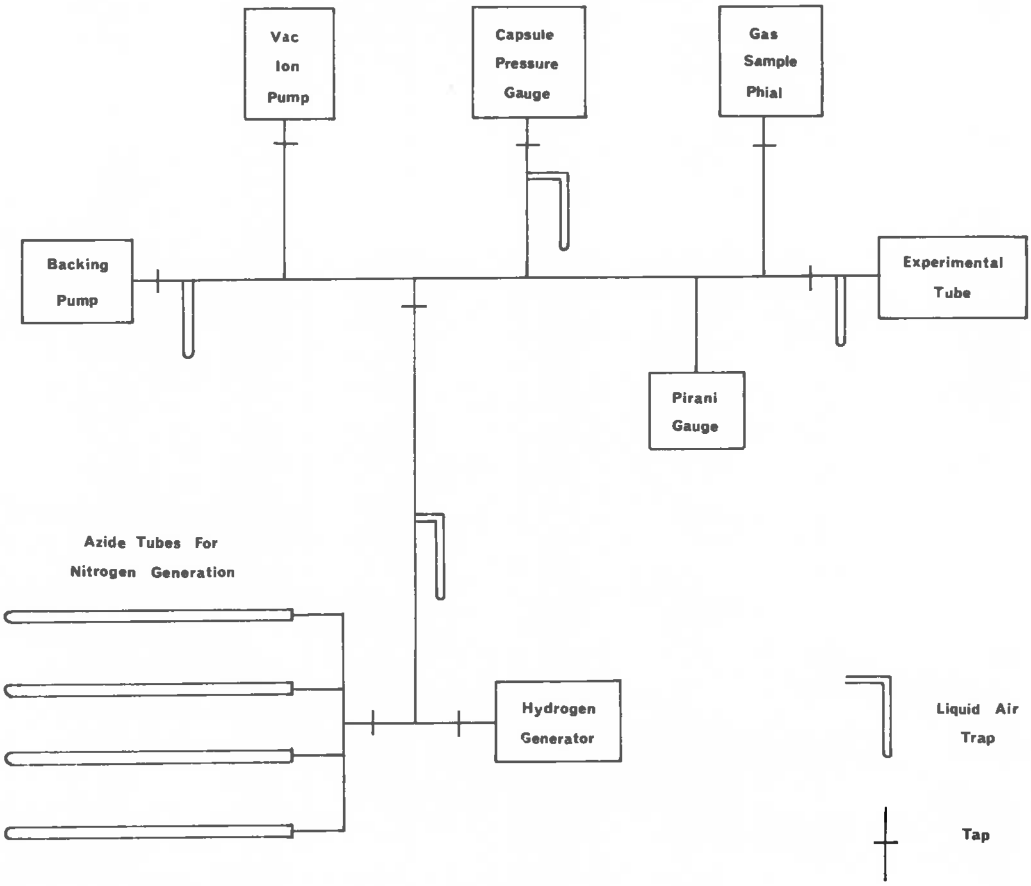

Vac
Ion
Pump
Capsule
Pressure
Gauge
Gas
Sample
Phial
Backing
Pump
Experimental
Tube
Pirani
Gauge
Azide Tubes For
Nitrogen Generation
Hydrogen
Generator
Liquid Air
Trap
Tap

Fig 15

foil to increase the efficiency of the cold trap by increasing the probability of the gas molecules making a large number of collisions with a cold surface as the gas passes through the trap.

All joints and taps of the vacuum system were greased with Apiezon N grease.

The pumping out procedure was first to use a backing pump and then a getter ion pump. The apparatus could be evacuated so that the pressure as recorded by the getter ion pump was less than 10^{-6} mm Hg. When the experimental tube was isolated from the pumping system, the outgassing rate, as determined by the Pirani gauge and without the liquid air traps normally used, was found to be 0.1μ in 3 hours.

No attempt was made to bake out the apparatus at high temperature because of the likelihood of the shutters being damaged. However, the apparatus was baked out at low temperature (100°C) for 14 hours; but on admitting fresh gas, the drift velocity measurements were the same as the values obtained prior to baking the apparatus.

4.6 Pressure Measurement.

Gas pressures were recorded on two precision type capsule gauges (Crompton and Elford 1957) which together covered the required pressure range of 2 mm to 500 mm Hg. The low pressure gauge, which recorded pressures to 20 mm Hg, was calibrated with a dead weight pressure standard and the high pressure gauge with a standard mercury manometer as described in the original paper. The overall error in the measurement of pressure was of the order of 0.5% at the lowest scale readings and less for higher scale readings.

4.7 Gas Production.

4.7.1 Hydrogen.

Hydrogen was admitted to the apparatus by means of a palladium osmosis tube heated electrically to 200°C by means of a heater coil made of nichrome wire. The general procedure, together with the precautions necessary to prevent the palladium changing its phase during the heating process, is given by Crompton and Elford (1962).

4.7.2 Nitrogen.

Nitrogen was prepared by heating sodium azide using techniques that were developed by Dr. R.W. Crompton.

To reduce the possibility of introducing impurities to the gas due to outgassing of the glass tubes containing the azide, or the azide itself, the tubes were initially strongly flamed. Furthermore, while the tubes containing the azide were being evacuated, they were heated to a temperature just below the temperature for the breakdown of the azide.

To prevent possible damage due to the rapid evolution of gas during the preparation of nitrogen, care was taken to heat only a small amount of the azide at a time, the heating being carefully controlled electrically. This was achieved by spreading the azide thinly (never more than 0.2 inches deep) along the length of four $\frac{3}{4}$ inch diameter glass tubes, which were 24 inches long. The heating coil of nichrome wire could be slipped over the tubes, and not more than $\frac{1}{2}$ an inch of the length of a tube was heated at one time.

4.7.3 Helium.

The helium that was used, was stated to be spectroscopically pure, and was obtained from the British Oxygen Company.

4.8 Temperature Measurement.

As the drift velocity depends on both N and T it is necessary to determine accurately the temperature of the gas when measurements are being made. Temperatures were recorded by means of two copper constantan thermocouples. One thermocouple was connected to the guard ring just below the first shutter and enabled an assessment to be made of temperature variations caused by the heating action of the filament. The temperature rise as recorded by this thermocouple could be greatly reduced by immersing the apparatus in a water bath, which acted as a heat sink. The second thermocouple was placed in this bath.

When measurements of the drift velocity were made at 77.6°K, the bath surrounding the apparatus consisted of liquid nitrogen having a purity close to 100%. At these temperatures, the thermocouple that was attached to the electrode system could not be used as errors were introduced due to the presence of temperature gradients over the tungsten seals by which the thermocouple leads are brought out from the experimental tube. However an experiment conducted prior to the assembly of the apparatus, showed that the temperature inside of the experimental tube below the level of the liquid nitrogen bath was the same as the temperature of the bath, to within 0.5°K.

This experiment was carried out by introducing the thermocouple through the cone at the top of the tube in which the filament support is normally placed. The lower half of the tube was immersed in liquid air and the temperature of the air inside of the tube was measured at various depths from the top of the tube. The results are shown graphically in Fig.16.

It is seen that the largest temperature gradient occurs in the region between the top of the dewar, which contains the liquid air, and the surface of the liquid air. The temperature at the top of the tube is slightly below room temperature, so that a temperature gradient exists across the tungsten seals. Below the surface of the liquid air, the temperature is constant and is equal to the temperature of the surrounding bath.

The thermocouples were calibrated at 8 temperatures in the temperature range 77°K to 373°K following the procedure of Scott (1941). The deviation is found of the e m fs measured at the calibration temperatures from the e m fs given for the corresponding temperatures of a reference calibration that is given by Scott. A plot of these deviations against the measured value of the e m f gives a 'deviation curve'. When the calibrated thermocouple is used to measure a temperature, this deviation curve enables the temperature to be determined using the reference calibration.

The reference calibration, given by Scott, has been extended in Appendix III to cover the required temperature range, using the cubic equation:

$$E_s = -38.474\ T - 0.046046\ T^2 + .00003948\ T^3.$$

The coefficients of this expression were derived from values given in the reference calibration.

The calibration points, from which the deviation curve was obtained, are given in Table 2. E_L is the measured value of the e m f, E_s is the e m f obtained from the reference calibration and ΔE the deviation.

Table 2

Point	Temperature ^{o}K	E_L (μ v)	E_s (μ v)	Δ E $E_L - E_s$
Boiling Water	373	-4170	-4268	98
Hot Water	321.7	-1920	-1978	58
Hot Water	305.1	-1235	-1281	46
Freezing Mercury	234.1	1380	1426	-46
Cold Alcohol Bath	203.2	2405	2470	-65
CO_2 Sublimation Point	194.5	2650	2717	-67
Boiling Oxygen	90.1	5180	5255	-75
Boiling Nitrogen	77.2	5395	5472	-77

The reference junction of the thermocouple was in each case embedded in a thermos flask of ice made from distilled water. The deviation curve is given in Fig.17. The following are the main precautions that were taken in the determination of the e m fs at the calibration temperatures.

(1) Boiling Water. Glass beads were included in the beaker of distilled water to prevent super heating. Account was taken of the effect of deviations of the atmospheric pressure from 760 mm Hg.

(2) Hot Water. To obtain consistent results, the mouth of the vacuum flask containing the water was packed with cotton wool and the water was agitated by bubbles of nitrogen.

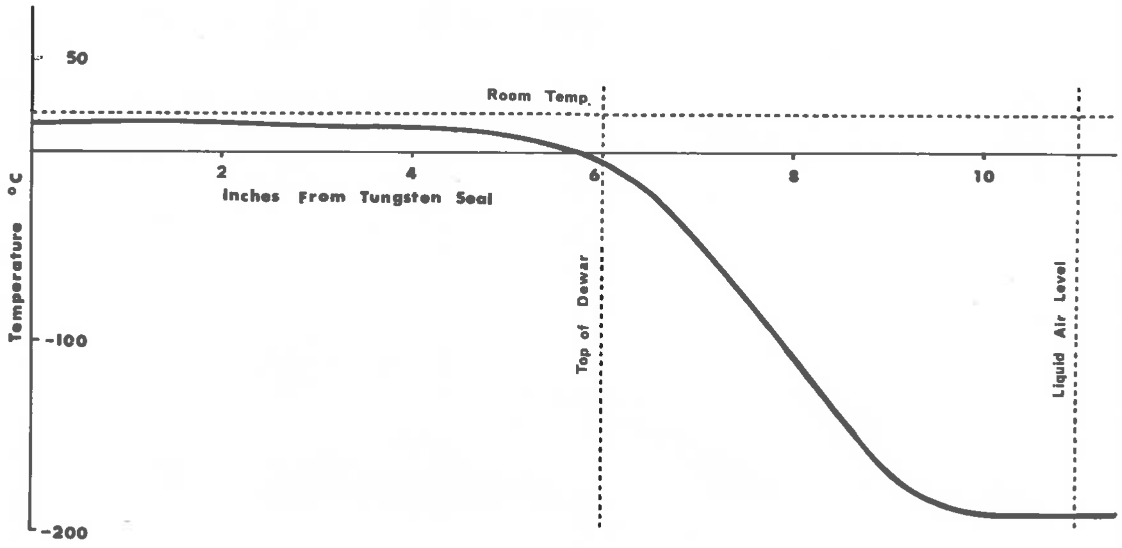

50
Room Temp.
2
4
6
8
10
Inches From Tungsten Seal
Temperature °C
-100
-200
Top of Dewar
Liquid Air Level

Fig 16

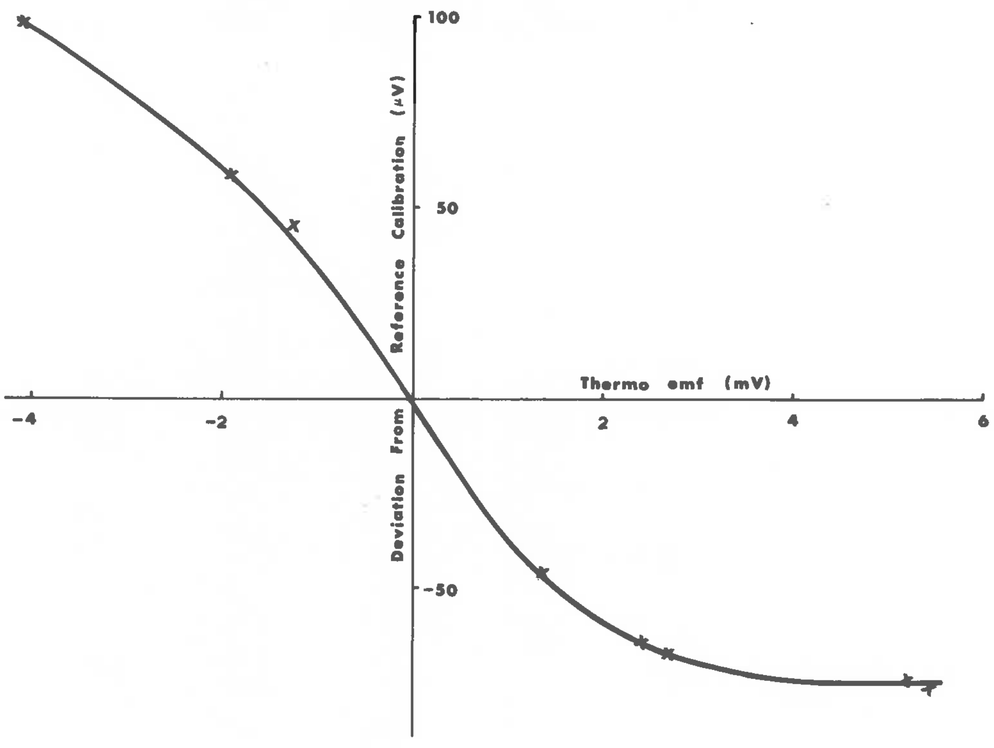

100
50
-50
Deviation From Reference Calibration (μV)
Thermo emf (mV)
-4
-2
2
4
6

Fig 17

(3) Freezing Mercury. The graph of the variation of e m f with time of a thermocouple initially embedded in frozen mercury is given in Fig. 18. It is seen that the e m f corresponding to the melting point of mercury can be determined accurately.

(4) Cold Alcohol Bath. Precautions similar to those of (2) were taken to reduce temperature gradients in the cold bath of alcohol.

(5) CO_2 Sublimation Point. It was necessary to ensure that the dry ice was in an atmosphere of CO_2 by enclosing the dry ice in a vacuum flask whose mouth was packed with cotton wool. The air initially present in the flask, which can suppress the sublimation temperature by $4^\circ K$, is expelled from the flask by the constant flow of CO_2 from the dry ice.

(6) Boiling Oxygen and Nitrogen. These calibration points are critical for the accuracy of drift velocity results for gas temperatures of the order of $77^\circ K$. The calibration points were obtained from the boiling point of samples of liquid oxygen and nitrogen which were stated to have a purity of greater than 99%. Values of the e m f obtained with three different samples of oxygen gave identical results.

To determine the dependence of the boiling point on the concentration of oxygen in a mixture of oxygen and

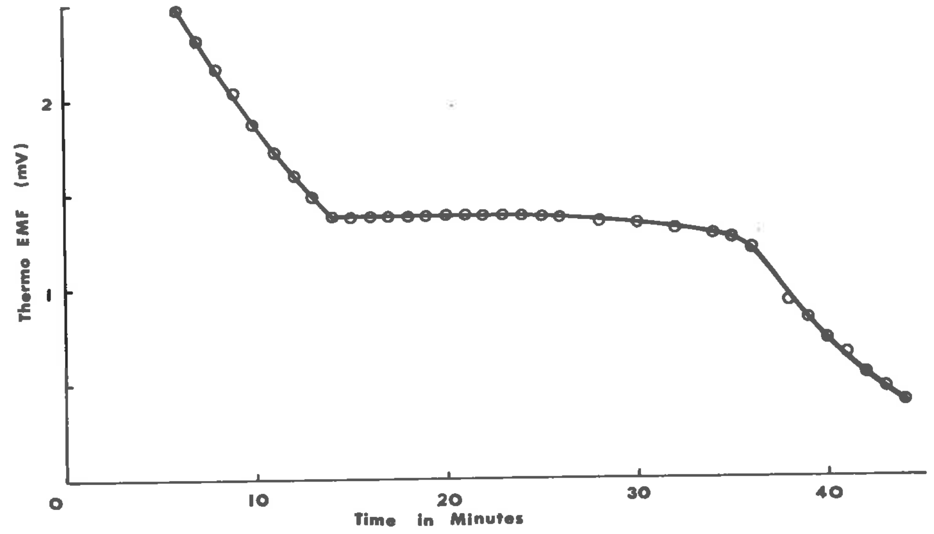

Thermo EMF (mV)
2
1
0
10
20
30
40
Time in Minutes

Fig 18

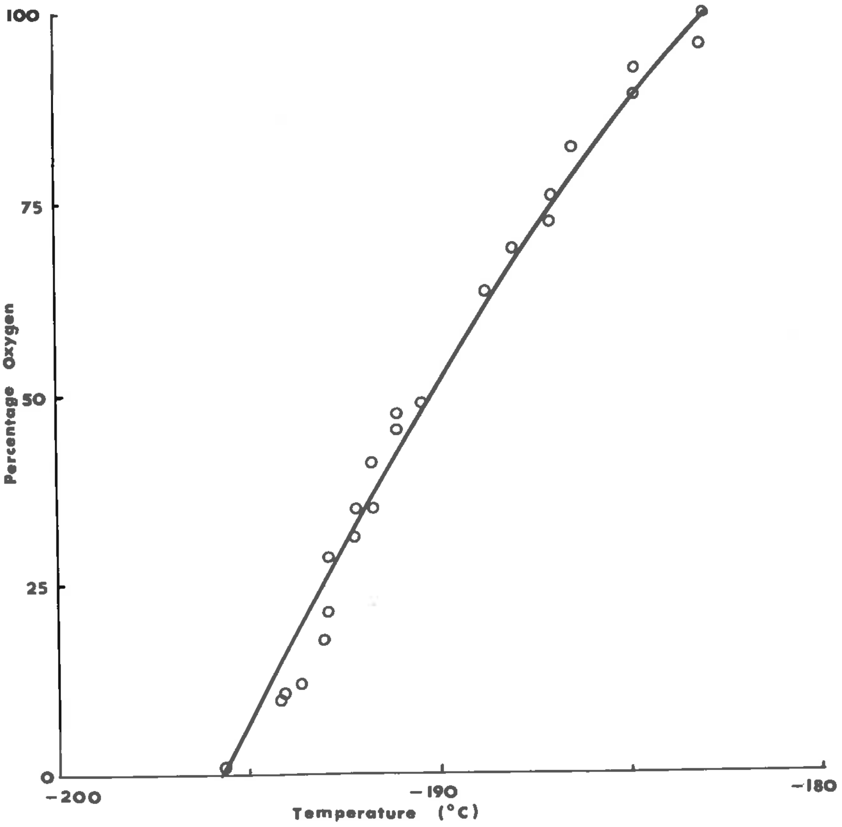

Percentage Oxygen
100
75
50
25
0
-200
-190
-180
Temperature (°C)

Fig 19

nitrogen the experimental curve of Fig. 19 was obtained. Because the curve is approximately linear, 1% impurity in the liquids would result in an error in the e m f from that of the pure liquid of less than 0.5°K. Nevertheless it is largely because of the uncertainty in determinations of the temperature that the drift velocity results taken at 77.6°K have a possible error of the order of 2%.

Chapter 5 : Experimental Results

5.1 Results in Pure Gases - Hydrogen and Nitrogen.

The drift velocity was measured by determining the frequency of the maximum of the first or second current peak of the current/frequency curve. Figs. 20 and 21 give two extreme examples of current/frequency curves and it is seen that the resolving power possible from any peak varies considerably under different conditions. The effect of diffusion on the resolving power is discussed in Chapters 8 and 9.

In all instances where current/frequency curves were prepared, as for example in Fig. 20 which exhibits 20 maxima, the values of f_m/m corresponding to each maximum were found to agree within 1%. This agreement is strong evidence that no errors are present due to:

(1) displacement of the current maximum due to a variation in the value of the a.c. voltage on the shutters with change of frequency, (discussed in 4.3.2.1).

(2) incorrect frequency calibration, and

(3) phase differences between a.c. voltages applied to the first and second shutter (discussed in 6.3.1).

At each determinationof f_m the temperature and pressure of the gas was recorded to enable the value of E/N to be calculated accurately. These values of E/N generally differed

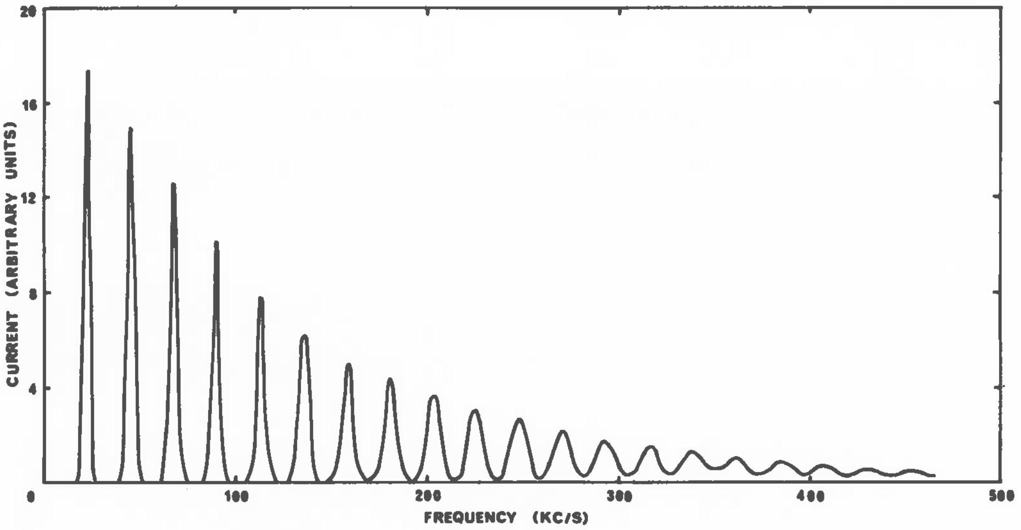


Fig 20

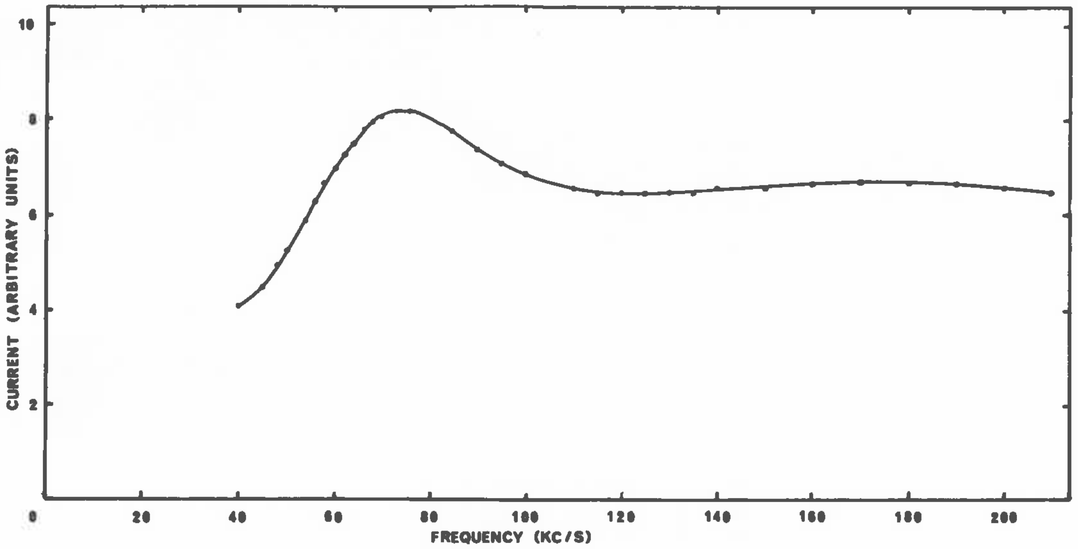


Fig 21

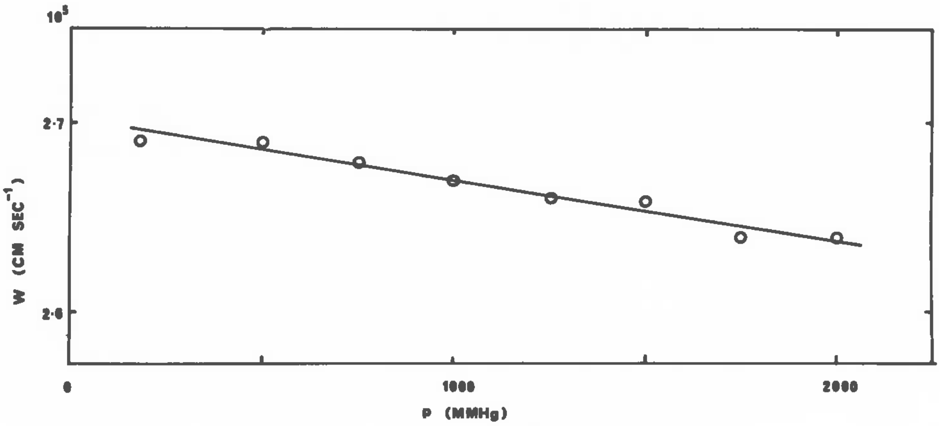


Fig 22

slightly from the standard values of E/N required, due to slight variations in the gas pressure and gas temperature caused, for example, by the variation of the levels in the liquid air traps and the heating effect of the filament. A correction was then made to the measured values of W, using the tables of Appendix IV, to obtain the equivalent measurement at the required value of E/N. The tables give the percentage change in W for a 1% change in E/N and were calculated by determining the gradient of the curve of W against E/N. As the experimental values of E/N were generally within 1% of the values required, it was unnecessary to know the gradients accurately.

At each of the standard values of N, two experimental sets of measurements were made using different gas samples. Agreement between the measurements was always to within 1% at high values of E. At low values of E experimental errors became larger because of the greater influence of spurious contact potential differences and also because the greater degree of diffusion of the electrons results in a loss of resolving power. Where differences of more than 1% existed between the measurements a third gas sample was used to take a further set of results, and the average of all three values was then determined.

Table 3. Hydrogen 293°K

Values of W x 10^{-5}cm sec^{-1}. E/p = 3.30.10^{16}E/N. N = 3.30.10^{16}p.

E/p \ p	2	5	10	20	50	100	Best Estimate
18	63.2						62.8
15	51.7						51.3
12	42.3						41.9
10	37.2						36.8
9	34.5						34.2
8	32.1						31.9
7	29.7	29.6					29.5
6	27.1	26.9					26.7
5	24.3	24.1					24.0
4	21.5	21.2	21.1				21.0
3	18.3	18.0	18.0				17.9
2.5	16.6	16.3	16.2				16.1
2	14.7	14.4	14.4	14.3			14.3
1.8	13.9	13.6	13.6	13.5			13.5
1.5	12.7	12.4	12.4	12.3			12.3
1.2	11.4	11.1	11.0	11.0			11.0
1.0	10.4	10.2	10.1	10.1			10.1
.9		9.72	9.63	9.59			9.56
.8		9.23	9.15	9.11	9.08		9.08
.7		8.73	8.64	8.62	8.58		8.57
.6		8.19	8.12	8.09	8.05		8.04
.5		7.59	7.53	7.50	7.46		7.45
.4		6.91	6.86	6.83	6.80	6.81	6.81
.3			6.02	5.98	5.97	5.97	5.97
.25			5.50	5.46	5.45	5.45	5.45

Table 3. Hydrogen 293^{o}K (continued)

Values of W x 10^{-5} cm sec^{-1}. E/p = 3.30.10^{16}E/N. N = 3.30.10^{16}p.

E/p \ p	10	20	50	100	200	400	500	Best Estimate
.2	4.89	4.85	4.84	4.84	4.85			4.85
.18		4.57	4.56	4.56	4.56			4.56
.15		4.09	4.08	4.08	4.09			4.09
.12		3.57	3.55	3.55	3.56			3.56
.1		3.16	3.14	3.15	3.15	3.15		3.15
.09			2.92	2.94	2.94	2.94		2.94
.08			2.68	2.70	2.70	2.70	2.70	2.70
.07			2.44	2.45	2.45	2.45	2.45	2.45
.06			2.18	2.18	2.19	2.18	2.18	2.18
.05			1.89	1.90	1.90	1.90	1.89	1.89
.04			1.58	1.58	1.59	1.58	1.59	1.59
.03				1.25	1.26	1.25	1.26	1.26
.025				1.06	1.07	1.07	1.07	1.07
.02				.876	.878	.879	.879	.878
.018					.799	.800	.798	.797
.015					.675	.675	.677	.676
.012					.547	.549	.550	.549
.01					.461	.462	.464	.463
.009						.419	.420	.419
.008						.374	.375	.374
.007						.330	.330	.329
.006						.284	.284	.283
.005						.236	.236	.235
.004							.189	.188

Table 4. Nitrogen 293°K.

Values of $W \times 10^{-5}$ cm sec^{-1}. $E/p = 3.30.10^{16} E/N$. $N = 3.30.10^{16} p$.

E/p \ p	2	3	4	5	10	20	50	100	Best Estimate
20	71.2								70.9
18	65.4								65.1
15	57.2	57.1							56.8
12	48.4	48.3	48.3						48.2
10	42.4	42.2	42.1	42.1					42.0
9	39.2		39.0	39.1					39.0
8	36.1		35.8	35.9					35.7
7	32.5			32.4					32.3
6	29.0			28.9					28.8
5	25.6			25.3					25.0
4	21.9			21.5	21.2				21.1
3	18.0			17.4	17.2				17.1
2.5	15.9			15.3	15.0				14.9
2	13.7			12.9	12.8	12.8			12.7
1.8	12.9			12.1	11.8	11.8			11.7
1.5	11.4			10.6	10.4	10.3			10.2
1.2	10.2			9.04	8.87	8.79			8.73
1.0	9.26			8.04	7.84	7.77			7.72
.9				7.52	7.33	7.25			7.19
.8				6.97	6.82	6.72	6.68		6.66
.7				6.49	6.31	6.22	6.20		6.18
.6				5.97	5.80	5.72	5.70		5.68
.5				5.49	5.31	5.23	5.21		5.19
.4				5.03	4.85	4.78	4.77	4.77	4.76
.3					4.39	4.32	4.28	4.29	4.28
.25					4.13	4.05	4.03	4.03	4.02

Table 4. Nitrogen 293°K (continued)

Values of $W \times 10^{-5}$ cm sec^{-1}. $E/p = 3.30.10^{16} E/N$. $N = 3.30.10^{16} p$.

E/p \ p	10	20	50	100	200	400	500	Best Estimate.
.2	3.84	3.77	3.76	3.75	3.76			3.76
.18		3.66	3.63	3.63	3.63			3.63
.15		3.46	3.43	3.43	3.43			3.43
.12		3.26	3.23	3.23	3.22			3.22
.1		3.12	3.09	3.10	3.09	3.09		3.09
.09			3.03	3.04	3.04	3.03		3.03
.08			2.97	2.97	2.97	2.96	2.96	2.96
.07			2.91	2.91	2.90	2.89	2.90	2.90
.06			2.83	2.83	2.83	2.82	2.81	2.81
.05			2.72	2.72	2.72	2.71	2.71	2.71
.04			2.56	2.56	2.55	2.55	2.55	2.55
.03				2.29	2.29	2.28	2.28	2.28
.025				2.07	2.08	2.08	2.08	2.08
.02				1.82	1.82	1.82	1.82	1.82
.018					1.71	1.70	1.71	1.71
.015					1.49	1.49	1.49	1.49
.012					1.25	1.26	1.25	1.25
.01					1.07	1.07	1.07	1.07
.009						.977	.975	.974
.008						.879	.878	.877
.007						.778	.776	.774
.006						.674	.673	.672
.005						.567	.565	.564
.004							.456	.455

Table 5. Hydrogen 77.6°K.

Values of W x 10^{-5} cm sec^{-1}. E/p = 3.30.10^{16} E/N. N = 3.30.10^{16} p.

E/p \ p	10	20	50	80	200	500	1000	2000	Best Estimate
3	18.3								18.1
2.5	16.5								16.3
2	14.7	14.5							14.4
1.8	13.9	13.7							13.6
1.5	12.7	12.6							12.5
1.2	11.4	11.2							11.1
1.0	10.5	10.4							10.3
.9	9.99	9.90							9.85
.8	9.55	9.46	9.42						9.40
.7	9.06	8.98	8.95						8.93
.6	8.55	8.49	8.44						8.42
.5	8.03	7.94	7.89	7.89					7.87
.4	7.39	7.31	7.29	7.30					7.29
.3	6.59	6.55	6.51	6.49					6.48
.25	6.10	6.03	6.01	6.02					6.01
.2	5.51	5.45	5.41	5.41	5.37				5.36
.18		5.17	5.13	5.13	5.12				5.11
.15		4.68	4.64	4.63	4.63				4.63
.12		4.12	4.09	4.10	4.08				4.08
.1		3.71	3.68	3.68	3.66				3.66
.09			3.44	3.44	3.41				3.41
.08			3.20	3.19	3.16	3.15			3.15
.07			2.94	2.93	2.91	2.90			2.90
.06			2.65	2.65	2.63	2.62			2.62
.05			2.34	2.34	2.33	2.32			2.32
.04			2.02	2.01	2.00	1.99	1.97		1.97
.03				1.63	1.62	1.63	1.62		1.62

Table 5. Hydrogen 77.6°K. (continued)

Values of $W \times 10^{-5}$ cm sec^{-1}. $E/p = 3.30.10^{16} E/N$. $N = 3.30.10^{16} p$.

E/p \ p	10	20	50	80	200	500	1000	2000	Best Estimate
.025				1.44	1.43	1.43	1.43		1.43
.02					1.23	1.24	1.24	1.23	1.23
.018					1.14	1.14	1.14	1.14	1.14
.015					1.01	1.01	1.01	1.01	1.01
.012					.868	.868	.871	.869	.869
.01					.764	.764	.768	.767	.767
.009						.709	.712	.715	.715
.008						.651	.656	.655	.655
.007						.590	.598	.594	.594
.006						.525	.533	.530	.530
.005						.457	.459	.458	.458
.004						.382	.381	.383	.383
.003							.300	.297	.297
.0025							.255	.254	.254
.002							.208	.207	.207
.0018								.188	.188
.0015								.158	.158
.0012								.129	.129
.001								.108	.108

Table 6. Nitrogen 77.6°K.

Values of W x 10^{-5} cm sec^{-1}. E/p = 3.30.10^{16} E/N. N = 3.30.10^{16} p.

E/p \ p	20	40	50	80	200	500	1000	2000	Best Estimate
2	12.9								12.8
1.8	11.9								11.8
1.5	10.4								10.3
1.2	8.93								8.81
1	7.89	7.82							7.77
.9	7.38	7.31							7.26
.8	6.82	6.80	6.77						6.73
.7	6.33		6.27						6.24
.6	5.80		5.76						5.73
.5	5.35		5.29	5.28					5.24
.4	4.89		4.84	4.82					4.80
.3	4.42		4.38	4.36					4.33
.25	4.20		4.14	4.11					4.09
.2	3.91		3.88	3.87	3.85				3.85
.18	3.80		3.76	3.75	3.74				3.74
.15	3.63		3.59	3.58	3.58				3.58
.12	3.49		3.44	3.44	3.43				3.43
.1	3.43		3.40	3.40	3.39				3.39
.09			3.40	3.40	3.39				3.39
.08			3.41	3.42	3.41	3.42			3.42
.07			3.45	3.46	3.46	3.47			3.46
.06			3.52	3.53	3.53	3.53			3.53
.05			3.60	3.61	3.60	3.60			3.60
.04			3.66	3.66	3.66	3.67	3.67		3.67
.03				3.67	3.68	3.67	3.68		3.68
.025				3.63	3.62	3.63	3.63		3.63
.02					3.49	3.49	3.49	3.48	3.48
.018					3.41	3.41	3.41	3.38	

Table 6. Nitrogen 77.6°K. (continued)

Values of W x 10^{-5} cm sec^{-1}. $E/p = 3.30.10^{16}$ E/N. $N = 3.30.10^{16}$ p.

E/p \ p	20	40	50	80	200	500	1000	2000	Best Estimate
.015					3.22	3.23	3.21	3.19	
.012					2.94	2.94	2.93	2.89	
.01					2.69	2.68	2.67	2.64	
.009						2.53	2.52	2.48	
.008						2.35	2.34	2.29	
.007						2.16	2.14	2.10	
.006						1.93	1.92	1.88	
.005						1.68	1.66	1.62	
.004						1.41	1.38	1.36	
.003							1.08	1.05	
.0025							.918	.894	
.002							.744	.724	
.0018								.656	
.0015								.548	
.0012								.443	
.001								.372	

The results at 293°K are presented in Tables 3 and 4 as a function of the ratio E/p for various values of p. As has been mentioned in 1.3 it is E/N and not E/p which is the basic parameter. Since $N = 3.30 \times 10^{16}$ p cm^{-3} at 293°K the parameter E/p is equivalent to 3.30×10^{16} E/N at this temperature.

The results at 77.6°K, which are presented in Tables 5 and 6 are also expressed as a function of 3.30×10^{16} E/N so that the results for any value of E/N can easily be compared at the two temperatures. At 77.6°K the values 3.30×10^{16} E/N can be regarded as being equal to E/p where p in this case has the particular meaning of being the pressure of the gas at 293°K that has the same concentration N as the gas at 77.6°K. In Tables 5 and 6 p is used in this way.

When the values of E/N are deduced from gas pressures measured at 77.6°K, account must be taken of the deviation of the behaviour of nitrogen from that of a perfect gas. The second virial coefficient B, has been measured in this temperature region by Keesom and Van Lammeren (1932) and also by Van Itterbeek, Lambert and Forrez (1956) using the indirect method involving the measurement of the velocity of sound. Both investigations give $B = -11.3.10^{-3}$ at 77.6°K where B is defined such that $pV = RT\ (1 + B/V)$ where p is the pressure of the gas in atmospheres, T is the temperature in degrees K and R is a constant such that when T = 273°K, p = V = 1. At the highest gas pressures used, the correction to E/N amounts to 2.5%.

For hydrogen at 77.6°K, the corresponding correction is much less than 1%.

From an inspection of the tables, it can be seen that the results for a given value of E/N are in general not entirely independent of N. It will be shown in 8.6 that the influence of electron diffusion leads to an increase in the measured value of the drift velocity from the true velocity, the measured value W_M being given approximately by

$$W_M = \left[1 + 3/h\ (W/D)\right]\ W \qquad (9)$$

where W is the true velocity, h is the drift distance and D is the coefficient of diffusion of the electrons. Equation (9) is not an exact relation because the increase in the measured value of the drift velocity from the true velocity is dependent on the fraction of the time that the shutters are effectively 'open' and the results are thus to some extent dependent on the amplitude of the a.c. signal. Nevertheless, as will be shown in 9.1 the variation of the measured values of W with N can usually be explained as being due to the influence of diffusion and is closely represented by equation (9).

The variations of W with N at values of E/p less than 0.02 in nitrogen at 77.6°K cannot be explained by the influence of diffusion. For example at E/p = 0.005 the measured value of the drift velocity at p = 2000 is 3.1% lower than the result at p = 500. Equation (9) predicts a variation of W of less than 0.2% over this range of p, (W/D is obtained from results of Warren and Parker (1962)). As no variations with p at low values of E/p have been observed for the corresponding results in hydrogen, it is unlikely that the variation of the results in nitrogen is caused by an experimental error. The observed dependence of the results on N is illustrated in greater detail in Fig. 22 in which additional experimental points are given to those of Table 6. It is seen that the dependence of the results on N is such that the variations of W are closely proportional to N. No explanation has been found for this variation.

It is of interest to note that, if no correction had been made taking account of the high value of the second virial coefficient, the variation of the results at $E/p = 0.005$ would have been 4.8% instead of 3.1%.

The correct value of the drift velocity free of any falsification due to the influence of diffusion, is the asymptotic value that would be measured when very high gas pressures are used. The figures used in the tables under the column 'Best Estimate' are an estimate of these asymptotic values of the drift velocity, this estimate being based on the trend of the measured values given in the tables. Supplementary measurements were made in nitrogen for $p = 3$ and 4 at $293^{\circ}K$ and $p = 40$ at $77.6^{\circ}K$ to enable the asymptotic value to be assessed more accurately. Because of the observed real dependence of

W on N for E/p   0.02 in nitrogen at 77.6°K it is impossible to give a 'Best Estimate' of W at these values of E/p.

The pronounced effect of temperature on the values of the drift velocity in hydrogen and nitrogen at low E/N is illustrated in Figs. 23 and 24. From the graphs it is seen, as was mentioned in 1.3, that at high values of E/N W becomes more and more independent of the temperature of the gas.

5.2 Experiments with Gas Mixtures.

Although experiments are reported in this section for mixtures of hydrogen and nitrogen in which the percentage of nitrogen is varied from 0 to 100, the experiments were primarily designed to determine the influence of traces of gaseous impurities on the drift velocity values obtained in pure gases. A mass spectrographic analysis carried out using gas samples taken from the gas after an experimental set of results had been completed confirmed that the only impurities likely to be present were hydrogen in nitrogen, nitrogen in hydrogen and water vapor.

5.2.1 Mixtures of Hydrogen and Nitrogen.

5.2.1.1 Experimental Procedure.

In all experiments with mixtures of hydrogen and nitrogen, hydrogen was admitted first, by means of the palladium thimble, and nitrogen was added later by heating sodium azide. The relative pressures were measured with the capsule gauges.

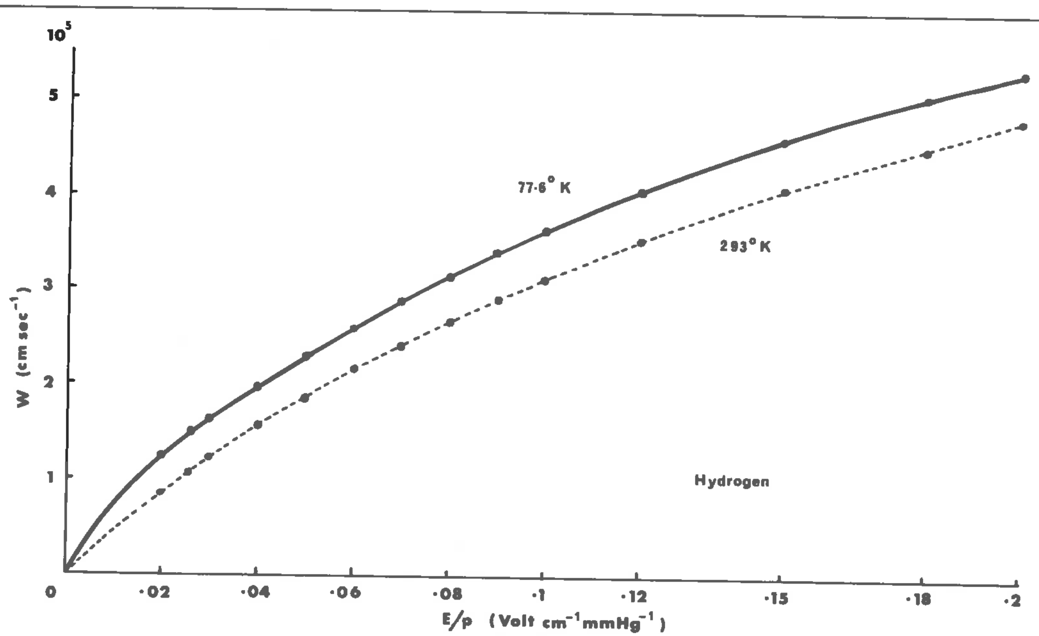

10^5
5
4
3
2
1
0
W (cm sec^-1)
77·6° K
293° K
Hydrogen
·02
·04
·06
·08
·1
·12
·15
·18
·2
E/p (Volt cm^-1 mmHg^-1)

Fig 23

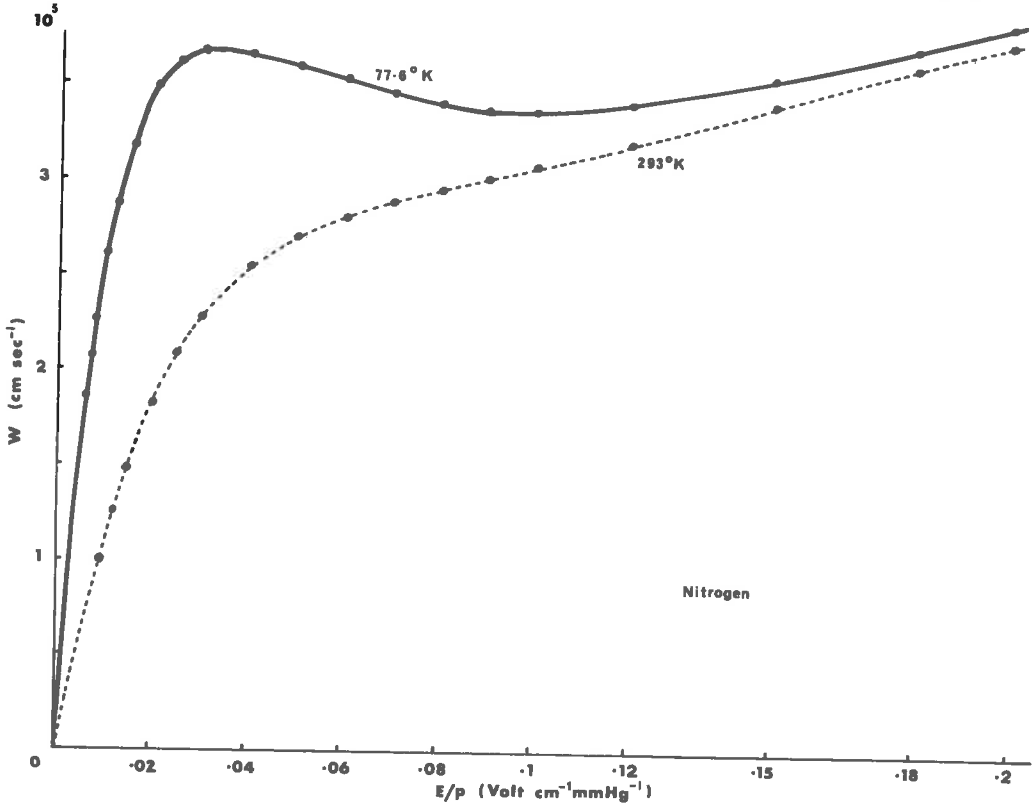

10^5
3
2
1
0
W (cm sec^-1)
77·6° K
293°K
Nitrogen
·02
·04
·06
·08
·1
·12
·15
·18
·2
E/p (Volt cm^-1 mmHg^-1)

Fig 24

It is difficult to produce accurately a gas mixture with a given ratio of two gases because it is necessary that the levels of liquid air in the traps be equal when making the two pressure measurements that define the ratio. However, the drift velocity was found to be not strongly dependent on this ratio and the errors possible in the final results presented are still considered to be of the order of 2%.

In experiments with mixtures of hydrogen and nitrogen it was found that the experimental results slowly varied with time until the gas mixture became uniform. A period of up to 60 hours was allowed for the gases to mix together. If a further period of 12 hours produced no change in the results, the time allowed for the gases to mix together was considered to be adequate.

During the taking of measurements the reduction of the pressure of a gas mixture by pumping was avoided, because of the possibility of changing the proportions of the gases in the mixture due to the action of the pump.

5.2.1.2 Impurities of Hydrogen in Nitrogen.

To determine the influence of hydrogen impurities on the drift velocity measurements in nitrogen, small quantities of hydrogen were added to nitrogen and the resulting change in the drift velocity noted. It was found that at least 0.5% of hydrogen had to be added to change the drift velocity by 1%, and the change in W was approximately proportional to the percentage of hydrogen added. Table 7 gives the percentage changes in W at E/p = 0.5 that were observed when various percentages were added.

Table 7

Percentage Hydrogen	½	1	2.5	5	10
Percentage Increase in W	1	2	6	10	16

Table 8 gives the percentage increase in the drift velocity that was observed at various values of E/p for a mixture of 2.5% hydrogen in nitrogen at 293°K.

Table 8

E/p	20	5	2	1	0.5	.2	.1	.05	.03	.018	.005
Experimental Increase			1	3	6	4	5	2	0	-3	
Theoretical Increase	0	0	1	4	6	5	5	2	-1	-2	-2

The table also gives the changes that would be expected in the drift velocity calculated using the theory of mixtures as developed by Bailey and Healey (1935). It is seen that good agreement is obtained between the theoretical predictions and the experimentally observed increases in the drift velocity. It is assumed that over the whole range of E/p for which the present results are taken, impurities of 0.5% or more of hydrogen are required to change the drift velocity results in nitrogen by 1%. The mass spectrographic analysis revealed that the percentage impurity of hydrogen was less than 0.01%. It is evident that the resultant percentage change in the values of the drift velocity is below 0.1%.

5.2.1.3 Impurities of Nitrogen in Hydrogen.

Similar experiments were conducted to determine the influence of small quantities of nitrogen on the drift velocity of electrons in hydrogen. In general it was found that a given percentage of nitrogen in hydrogen had much less effect in changing drift velocity values than the same percentage of hydrogen in nitrogen. Table 9 gives the percentage change in the drift velocity for a 10% mixture of nitrogen in hydrogen at 293°K. A comparison with the values as predicted by the theory of mixtures again gives good agreement.

Table 9

E/p	10	2	.5	.1	.08	.05	.01	.005
Experimental Increase			-1	3	4	5	6	
Theoretical Increase	1	-1	0	3	4	4	6	7

It is again evident from the results given in Table 9 that the drift velocity results could not be affected by the presence of impurities of nitrogen because a mass spectrographic analysis showed that the percentage of nitrogen present in samples of hydrogen was less than 0.01%.

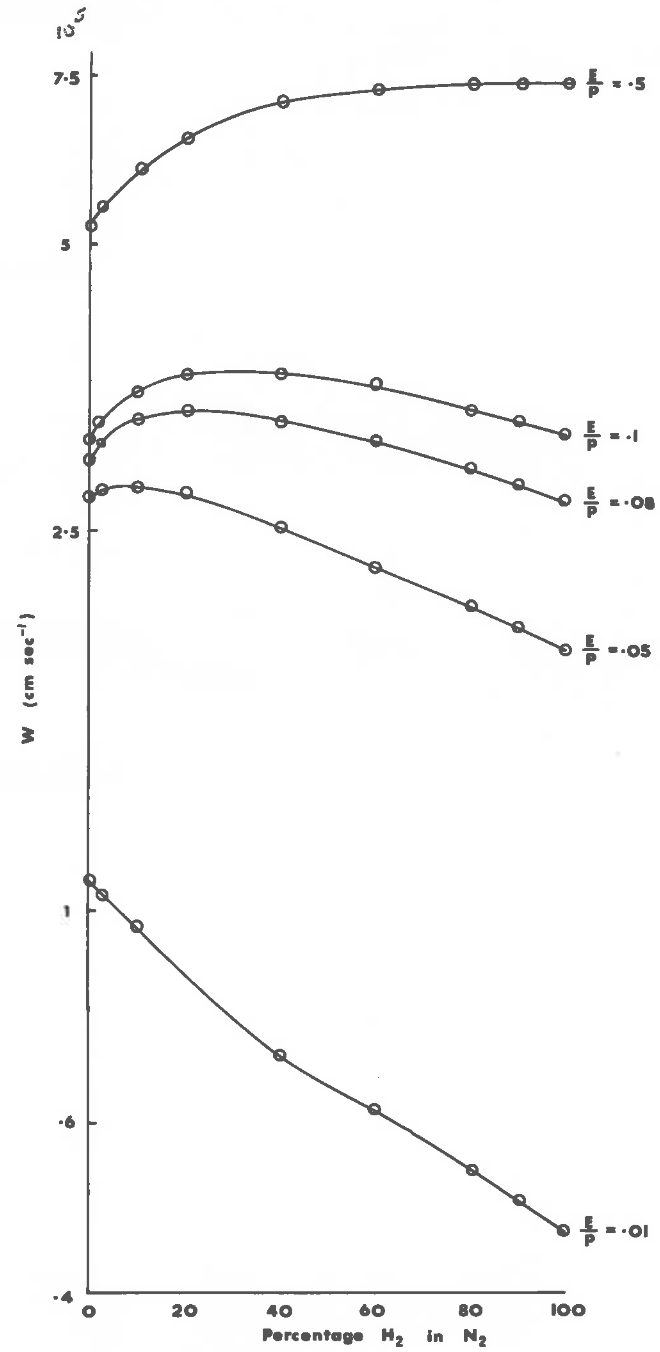
5.2.1.4 More General Results.

Additional results have been taken for percentages of hydrogen in nitrogen to cover the whole range 0 to 100%. These results are given in Table 10. The gas temperature was 293°K.

Table 10

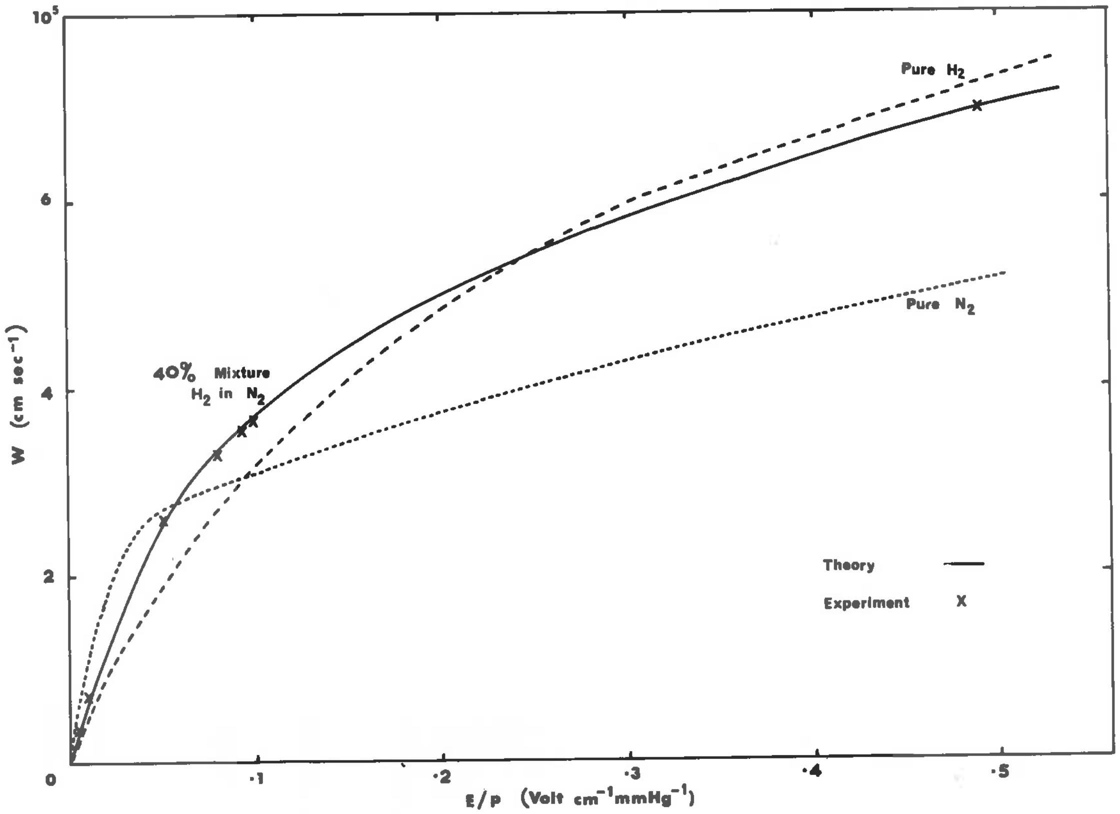
%H_2 E/p	0	2½	10	20	40	60	80	90	100
.5	5.19	5.51	6.00	6.50	7.05	7.32	7.42	7.45	7.45
.1	3.09	3.25	3.50	3.64	3.66	3.56	3.36	3.29	3.15
.08	2.96	3.10	3.28	3.35	3.29	3.13	2.92	2.82	2.70
.05	2.71	2.77	2.78	2.74	2.52	2.30	2.10	2.00	1.89
.01	1.07	1.04	.967	.866	.706	.622	.535	.499	.463

The results of Table 10 are presented graphically in Fig. 25. In Fig. 26 the results for 40% hydrogen in nitrogen are compared with the calculated values obtained using the theory of mixtures. It is seen that the agreement is good. In all theoretical calculations involving the theory of mixtures, use is made of the values of k_1 obtained by Crompton and Sutton (1952) and Crompton and Jory (1962).



10^5
7·5
5
2·5
1
·6
·4
W (cm sec^{-1})
0
20
40
60
80
100
Percentage H_2 in N_2
$\frac{E}{P}$ = ·5
$\frac{E}{P}$ = ·1
$\frac{E}{P}$ = ·08
$\frac{E}{P}$ = ·05
$\frac{E}{P}$ = ·01

Fig 25



10^5
6
4
2
0
W (cm sec^{-1})
·1
·2
·3
·4
·5
E/p (Volt cm^{-1}mmHg^{-1})
Pure H_2
Pure N_2
40% Mixture
H_2 in N_2
Theory
Experiment X

Fig 26

5.2.2 Impurities of Water Vapor in Nitrogen.

Experiments have also been carried out to ascertain the influence of water vapor on the values of the drift velocity of electrons in nitrogen. The results indicate that for a given percentage of water vapor impurity the percentage increase in W is over a hundred times greater than for the same percentage of hydrogen impurity.

Water vapor was introduced into the evacuated apparatus by connecting to the apparatus a cold trap containing a small quantity of ice. The amount of water vapor that is thus introduced is assumed to be the vapor pressure of ice corresponding to the temperature of the ice in the cold trap. This temperature was controlled by means of a cold bath of alcohol and measured by means of a thermocouple embedded in the ice. Although it is possible that the estimate of the concentration of water vapor may be in error by as much as 20%, the results quoted in Table 11 show the marked effect of traces of water vapor on the value of the drift velocity.

Table 11

Amount of Water Vapor Added.	Percentage of Water Vapor	Increase in Measured Value of W.
40	0.02%	6%
5	0.002%	0.7%

The results of Table 11 are for nitrogen at 293°K, $p = 250$ and $E/p = 0.16$.

The addition of water vapor also caused a change in the form of the current/frequency curves. The curves for the two experimental conditions of Table 11 are illustrated in Figs. 27 and 28.

It is seen that the normal series of current peaks is superimposed on a background current which increases with frequency, the background current consisting of negative ions formed by attachment. At high frequencies more ions are transmitted by the shutter because the field between the shutter wires changes too rapidly for the ions to be collected.

The existence of a steadily increasing background current will slightly displace the positions of the maxima of the current/frequency curves. By subtracting the background current from the total current it may be shown that the presence of this current accounts for only 0.1% of the 6% increase in W due to the presence of water vapor impurity.

The existence of a background current of negative ions is a valuable criterion for ascertaining whether water vapor is present in the gas. As is seen from Fig. 28, the amount of water vapor required to increase the value of the drift velocity by only 0.7% is still easily detectable by the presence of a background current. In all measurements that were taken in pure gas no background current could be detected and this is

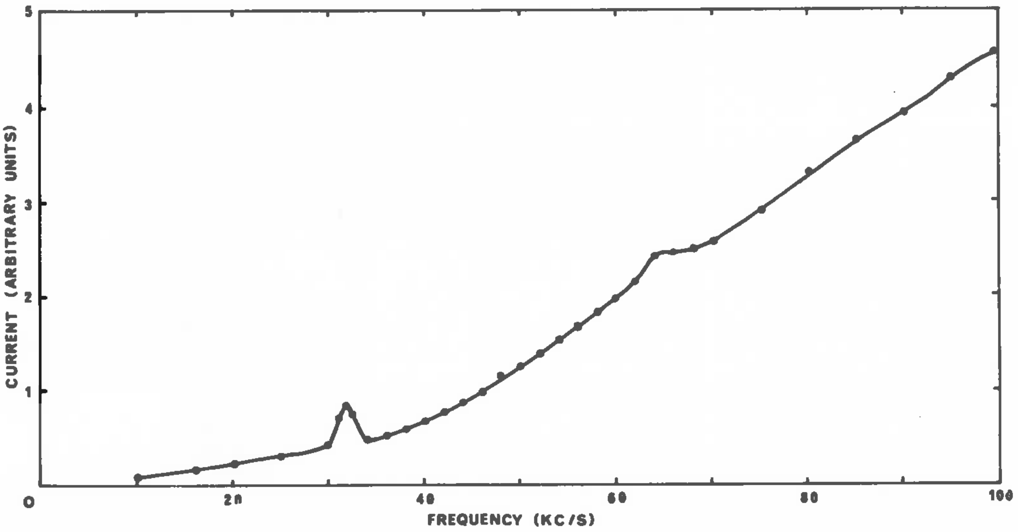

CURRENT (ARBITRARY UNITS)
5
4
3
2
1
0
20
40
60
80
100
FREQUENCY (KC/S)

Fig 27

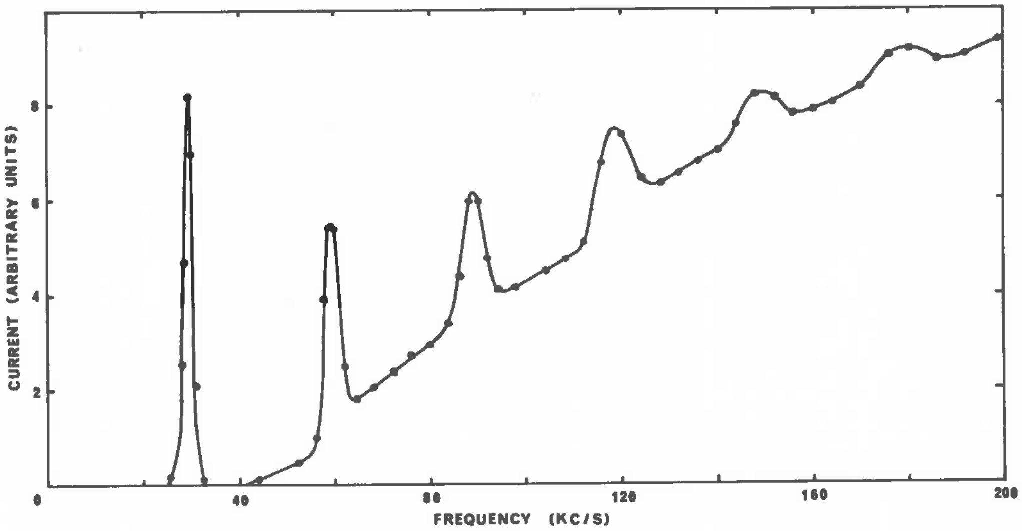

CURRENT (ARBITRARY UNITS)
8
6
4
2
0
40
80
120
160
200
FREQUENCY (KC/S)

Fig 28

regarded as being the best indication that the present results are not in error by more than 0.5% due to the presence of water vapor. Furthermore the measured value of the drift velocity for any one gas sample remained constant for 12 hours so that it is unlikely that errors were introduced by the outgassing of the experimental tube.

Chapter 6 : Analysis of Errors

In the previous chapter the possibility of the introduction of errors due to the presence of impurities in the gas has been discussed. The present chapter is concerned with the remaining experimental errors that are possible due to temperature gradients across the drift chamber, distortion of the electric field, phase errors of the a.c. voltages, contact potential differences and surface charges, and space charge repulsion. The magnitude of the errors that are present due to diffusion is also discussed on the basis of the theoretical work of Chapter 8.

A discussion of the effect of a broken lead to an electrode or a shutter is also included in this section.

6.1 Temperature Gradients.

The drift velocity is a function of both E/N and temperature. Consequently if the temperature of the gas deviates slightly from say 293°K, deviations of the measured value of the drift velocity from the value at 293°K are introduced due both to the change of the gas density with temperature, for any given value of E/p, and also due to the absolute dependence of the drift velocity on the temperature.

In taking measurements at 293°K the temperature as measured by the thermocouple within the drift tube slowly increased due to the heating action of the filament. To minimise the temperature gradient across the drift space the filament was turned off at all times other than during the actual determination of f_m. Under most conditions the temperature as measured by this thermocouple rose by approximately $1.5^{\circ}K$, the maximum rise recorded being $2.5^{\circ}K$. Changes in the temperature of the thermocouple in the bath surrounding the experimental tube were negligible. The best estimate of the temperature of the gas was taken to be the average of the two thermocouple readings, and this was the temperature on which corrections were based for variations in gas density from that corresponding to a temperature of $293^{\circ}K$.

A temperature change of $3^{\circ}K$ for gas at constant pressure is required to alter E/N by 1% at $293^{\circ}K$ and the change in W for a 1% change in E/N is generally less than 1% (as is seen from the tables of Appendix IV). Consequently, with the corrections that have been applied, it is considered that the error in the determination of E/N should be considerably less than 1%.

The variation of W with the temperature at constant E/N is greatest at the lowest values of E/N. As no correction was made for this effect, particular care was taken to ensure that the readings of W at the very lowest values of E/N were made when the gas temperature was within 1 degree of $293^{\circ}K$.

The measurements of W at $77.6^{\circ}K$ are more susceptible to errors in the determination of the temperature as it requires an error in the temperature determination of only 0.8°K to produce an error of 1% in the estimated value of E/N. Furthermore the temperature as recorded by the thermocouple cannot be determined with as great an accuracy as at 293°K, errors of 0.5°K being possible. Consequently the overall error in the drift velocity measurements at 77.6°K is expected to be of the order of 2%. It is seen that at E/p = 2 the values of the drift velocity at 77.6°K in both hydrogen and nitrogen are 1% higher than the value at 293°K at the same value of E/p. It is not known whether this difference is a result of inaccuracies in the thermocouple calibration, an effect due to the heating action of the filament or a genuine difference in the two sets of measurements due to the difference in the temperature. A likely explanation is that heat conduction from the leads to the electrodes, even though these leads were made of fine nichrome wire, is sufficient to raise the temperature of the gas above the temperature of the bath surrounding the experimental tube. It is also possible that the difference arises from too low an estimate of the effect of diffusion on the results at 77.6°K.

6.2 Distortion of the Electric Field.

6.2.1 Non Uniformity in the Applied d.c. Field.

The voltages that were applied to the electrodes to produce a uniform electric field were supplied by means of a carefully constructed potential divider. When these voltages were measured at intervals during the period over which measurements were taken they were always found to be within 0.2% of their correct value. Thus errors arising from field distortion from the application of incorrect potentials to the guard electrodes may be discounted. The possibility of more serious disturbances to the uniformity of the field can arise from:

(a) Slight buckling of the brass guard rings which resulted in there being deviations from 0.5 cm in the distance between corresponding points of the edges of adjacent guard rings of up to 0.010 cm.

(b) The influence of the tie rods. Measurements with an electrolytic tank indicated that this distortion was negligible within the drift space.

To test the effect on the results of any non uniformity of the electric field, the electric field was deliberately grossly distorted by changing the potentials of guard rings 4, 5 and 6 from their correct values to the potential of guard ring 3. At $E/p = 0.2$ for $p = 150$ using hydrogen at 293°K, the measured value of the drift velocity decreased by 2% from the value obtained when the guard electrodes were connected normally.

The remarkably small influence of pronounced field distortion was verified by finding the field distribution within the drift tube by means of an electrolytic tank and performing a rough calculation to find the time of flight of an electron pulse drifting through the region of non-uniform field between the shutters. The field distribution is given in Fig. 29. The total time of flight was calculated by adding the transit times for a pulse to travel between each of the intercepts of the equipotential lines with the axis of the tube, assuming that the electric field between each equipotential line is uniform. This calculation, which agreed with the experimental value of the drift velocity to 1%, showed that the effect of field distortion produces only a second order error in the measured value of the drift velocity, provided the correct potential difference is applied between the shutters.

Since the field distortion required to reduce the values of the drift velocity by 1% is so much greater than the distortion likely to be present experimentally, errors due to d.c. field distortion were considered to be negligible.

6.2.2 Distortion produced by the a.c. Voltages.

Errors are also possible from the a.c. voltages that are applied to the shutter wires. The field distortion produced by the a.c. voltages diminishes very rapidly with distance

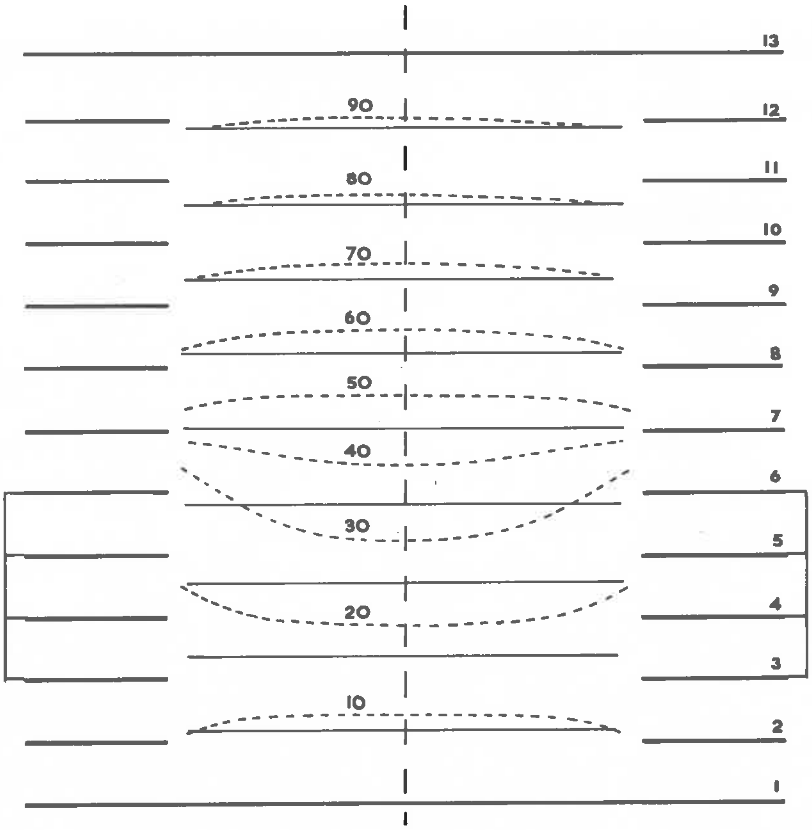


Fig 29

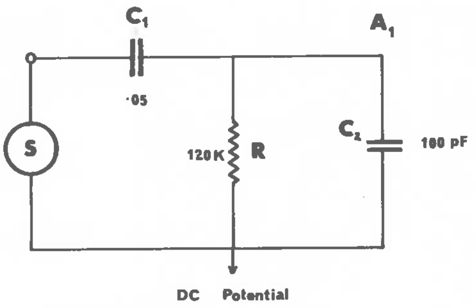


Fig 30

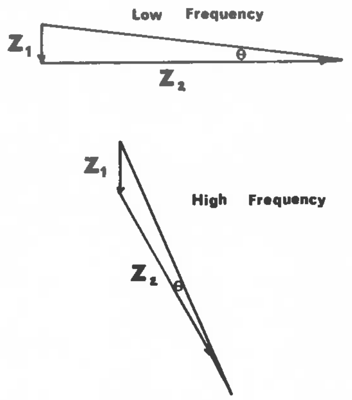


Fig 31

from the plane of the shutters because of the cancelling effect of the voltages on alternate shutter wires. In addition, when the frequency of the a.c. signal corresponds to the maximum of the first current peak (f_1), the field distortion is a maximum when the electron pulse is midway between the two shutters and thus furthest from the regions of field distortion. Furthermore, one of the advantages of the method of Bradbury and Nielsen is that any effects of field distortion that do occur are largely self cancelling due to the use of alternating voltages. The effect of field distortion on any electron pulse is compensated by the effect on the next pulse because the a.c. voltages have then changed their phase by 180^0. Consequently there are only second order effects on the measured value of the drift velocity. Thus, for example, errors due to inequalities of the amplitudes of the two a.c. voltages applied to a shutter, do not markedly affect drift velocity measurements. However the resolving power appropriate to the current peaks will be reduced because of the differing drift velocities of alternate electron pulses.

Experimental confirmation of the lack of influence of the a.c. voltages on the drift velocity values was obtained at $E/p = 0.005$ by varying the RMS voltage from 0.2 to 4 volts. The drift velocity at each RMS value remained constant to 1%, even though the d.c. field E was only 2.5 volt cm^{-1}. However, when the experimental conditions are such that W/D is small, the drift velocity has been observed to be a function of the a.c. voltage. This variation, can however, be shown to arise from the diffusion of the electron pulses, as will be shown in Section III.

6.3 Phase Errors of the a.c. Voltages.

6.3.1 A critical requirement for accurate determinations of the drift velocity is that the phase of the a.c. signals applied to the first shutter be equal to the phase of the signals applied to the second shutter.

An estimate was made of the phase differences that occur between the output voltage of the amplifier and the voltage on the shutters. The effect of a shutter, which is known to have a capacity of approximately 50 pF, was assumed to be equivalent to two 100 pF capacities in series, with the common point at a fixed d.c. potential. The effective load on one of the outputs of the amplifier produced by a shutter and the corresponding components of the resistance capacitance network of Fig. 13 is illustrated in the circuit of Fig. 30. This load consists of the sum of the impedance of C_1, (Z_1), and the impedance of R and C_2, (Z_2), where

$$Z_1 = \frac{-j}{C_1 \omega} \quad \text{and} \quad Z_2 = \frac{R\,(1 - j\,C_2 \omega R)}{1 + C_2^{\,2}\, \omega^2 R^2}$$

The phase difference between the voltage at A_1 and the voltage at the amplifier is given by θ in the vector diagrams of Fig. 31. It is seen that

$$\tan \theta < \frac{|Z_1|}{|Z_2|} = \frac{(1 + C_2^{\,2}\,\omega^2\,R^2)^{\frac{1}{2}}}{C_1\,\omega\,R}$$

$$\rightarrow \frac{1}{C_1\,\omega\,R} \quad \text{at low frequencies} \quad (2^\circ \text{ at } 1\ \text{kc/s})$$

$$\rightarrow \frac{C_2}{C_1} \quad \text{at high frequencies} \ (2^\circ \text{ at } 1\ \text{Mc/s}).$$

Similar phase shifts will occur at the second shutter and thus the difference in phase of the voltages at the two shutters should be much less than 1°. Thus errors in the drift velocity should be below 0.5%.

To verify that such phase differences were not present, a direct experimental check was made by observing the Lissajous figures that were formed when signals from the two shutters were connected direct to the plates of an oscilloscope. No phase differences were detected.

If phase differences were present, the differences would be expected to vary with frequency and thus to have given rise to disagreement in the values of f_m/m of the current frequency curves. As has been mentioned previously (5.1), no such disagreement was observed.

6.3.2 It is also possible that the phase difference between the two a.c. signals applied to the two halves of a shutter differs slightly from 180°. The variation of the voltage on the two halves of a shutter for the case of such a phase error is illustrated in Fig. 32. It is seen that at the times t_1, t_2, t_3 etc. when the shutter is open, the resulting d.c. potential of the shutter is displaced from its correct value to maintain the uniform d.c. field, by a voltage equal to ΔV. This displacement is alternately positive and negative for consecutive occasions that the shutter is open. As a consequence the resultant effects on successive electron pulses are compensating and the measured value of the drift velocity is unchanged. A slight broadening will occur, however, in the peaks of the current/frequency curves. A direct experimental comparison of the phases of the voltages applied to the shutter failed to detect any error.

6.4 Contact Potential Differences and Surface Charges.

No results have been taken using values of E less than 2 volt cm^{-1} because of the possibility of stray contact potential differences and the development of surface charges on the grid wires introducing errors. The practice of always basing the 'Best Estimate' of the drift velocity for a given value of E/p on the measurement made at the highest possible gas pressure means that the value of E for these measurements is always greater than 5 volt cm^{-1} for all values

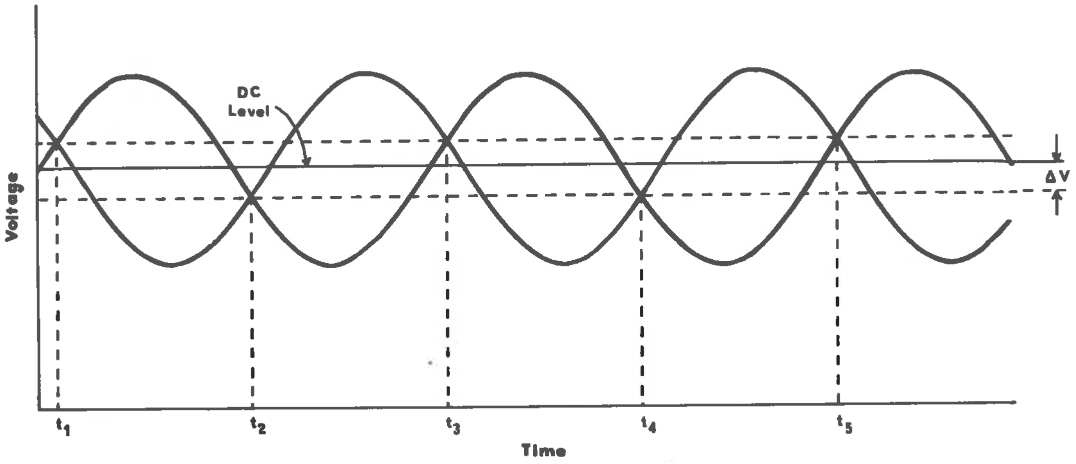


Fig 32

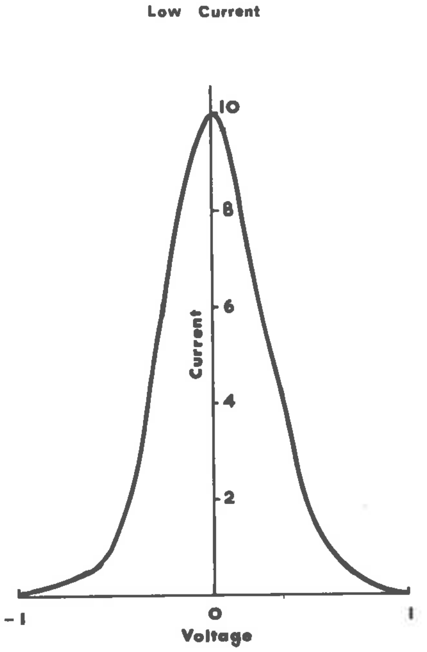


Fig 33

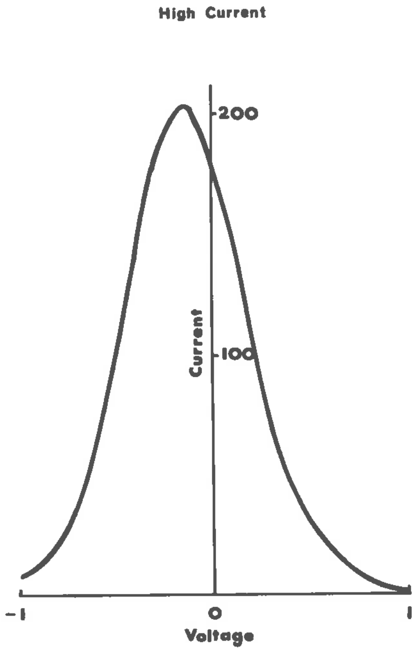


Fig 34

of E/p except $E/p \leqslant 0.01$. Thus if any effects due to contact potential differences or surface charges are in existence, the results most likely to be in error are the values for $E/p < 0.01$. However no evidence of such errors was found in the final results that were taken.

6.4.1 Contact Potential Differences.

Experiments performed by Dr. R.W. Crompton using the Kelvin method of determining contact potential differences have shown that contact potential differences of 200 mV can exist between gold surfaces if the gold is deposited by different methods. Although all metal surfaces were gilded it was considered possible that contact potential differences could introduce appreciable errors at values of $E < 2$ volt cm^{-1}.

6.4.2 Surface Charges.

On one occasion, when measurements were being made with the present apparatus when $E = 2$ volt cm^{-1}, the measured value of the drift velocity slowly increased by 6% when very high currents were used. It seems most likely that this effect is explained by the accumulation of a spurious charge at the surface of the grid wires of the top shutter. The evidence leading to this conclusion will now be discussed.

(1) With the filament running continuously and set to emit very high values of current, readings of f_1 increased with time for low values of E. The results of Table 12

give the variation of f_1 with time for nitrogen at 293°K with p = 400 and E = 2 volt cm^{-1}.

Table 12

time (in mins.)	0	1	4	8
f_1 (kc/s)	5.11	5.23	5.47	5.48

At values of E > 5 volt cm^{-1} the increase of f_1 with time was less than 1%.

(2) After the use of high emission current in the tube, readings of f_1 were made using low emission current and with the filament turned on for as short a time as possible. The results for the same conditions of E, p, and T as Table 12 are given in Table 13.

Table 13.

time (in hours)	0	½	1½	4	19	40
f_1 (kc/s)	5.48	5.33	5.31	5.27	5.21	5.16

(3) The filament was then run for ten minutes at high temperatures with all d.c. potentials reversed so that positive ions instead of electrons are emitted. The value of f_1 for electrons was then reduced as shown in Table 14.

Table 14 (Values of f_1 in kc/s)

Initial Value with Low Current.	After High Electron Current.	After Positive Ion Current.
5.11	5.46	5.01

This evidence suggests that the variation in the readings of f_1 is caused by accumulation of charge at the top shutter wires which alters the effective d.c. potential difference between the two shutters, thus varying the drift velocity results for electrons at low values of E.

(4) Further evidence that the charge occurs on the shutter wires was obtained from the experimental "cut off" curves. "Cut off" curves for a given shutter were obtained by plotting the current transmitted by the shutter as a function of the d.c. potential applied between the two sets of wires. The mean value of the potential of the shutter wires was kept at the potential required to maintain the uniform d.c. field. Checks were made during the measurements of the transmitted current to ensure that the current when $V_s = 0$ (V_s is the potential difference between the shutter wires) did not vary due to variations in the emission of the filament. The shutter wires of the other shutter were connected to the appropriate guard ring so as not to interfere with the electron current.

A typical "cut off" curve is shown in Fig. 33 for the top shutter in the case of nitrogen at 293°K, p = 400, E = 2 volt cm^{-1} and E/p = 0.004. The same "cut off" curve is given in Fig. 34 after the filament had been turned on at high emission current for ten minutes with

$V_s = -1$. The curve is no longer symmetrical about $V_s = 0$ suggesting that the set of wires to which the majority of electrons were attracted had acquired a surface charge thus varying the effective potential of the wires from the d.c. potential that was applied.

The apparatus was then dismantled, but close inspection of the shutter wires with a binocular stereo microscope did not reveal any possible cause for the effects that were observed. The wires were carefully cleaned as thoroughly as possible, the face plates of the shutter were repolished and both shutters were recoated with gold by vacuum evaporation. On reassembling the apparatus, the spurious effect of an increase in the value of f_1 with time at high currents was not observed.

It is of interest to note that F.W. Aston (1927) encountered spurious effects with his second mass spectrograph which he explained by a similar charging up of pure gold surfaces to potentials other than the d.c. potential which was applied.

6.5 Space Charge Repulsion.

As the measured value of the drift velocity in hydrogen at $E/p = 0.005$ at $293^{\circ}K$ was constant for values of the electron current which could be varied from 3.10^{-13} amps to 10^{-11} amps, it was considered unlikely that errors due to the influence of space charge repulsion were present.

6.6 Errors due to Diffusion.

It is shown in Chapter 8 that the relative error in any measurement due to diffusion is of the form $c_1/h(W/D)$ where c_1 is a constant. Thus for any value of E/N the errors due to diffusion are least for the results that are taken at the highest value of N. As these results differ from the 'Best Estimate' values at most by 1.3% it is seen that the magnitude of the correction is small and, as a consequence, it is considered that the values of the 'Best Estimate' are correct to 1%. The results most likely to be in error are for $E/p > 7$ where results could not be taken over a large range of N to enable an accurate assessment of the asymptotic limit to be made.

If equation (9) is regarded as giving the upper limit of the error due to diffusion, the results taken at the highest gas pressure for any value of E/p are in error due to diffusion by less than 0.5% for $0.006 \leqslant E/p \leqslant 0.8$ in hydrogen at 293°K, $0.006 \leqslant E/p \leqslant 0.4$ in nitrogen at 293°K, $E/p \leqslant 0.8$ in hydrogen at 77.6°K and $E/p \leqslant 0.2$ in nitrogen at 77.6°K.

6.7 Effect of a Broken Lead.

It is of assistance in searching for possible electrical faults to know the effect on measurements of the presence within the experimental tube of a broken lead to an electrode or set of shutter wires.

6.7.1 A Broken Lead to an Electrode.

The potential of the disconnected electrode usually differs from its correct potential and the resulting field distortion always reduces the measured value of the drift velocity.

The effect of distorting the potential of guard ring 3 from its correct potential of -8 volts is illustrated by the experimental values of f_1 given in Table 15 which apply for nitrogen at p = 400 with a potential difference of 48 volts between the shutters.

Table 15

Voltage of Electrode 3.	0	-1	-3	-5	-8	-10	-20	-30
f_1 (kc/s)	9.05	9.15	9.22	9.32	9.32	9.32	8.92	8.32

6.7.2 A Broken Lead to the Shutter Wires.

When this effect was simulated by disconnecting one of the leads to the shutters, it was found that in general current peaks were still obtained in the current/frequency curve, indicating that the disconnected shutter wires remained at a constant d.c. potential.

(1) If the d.c. voltage of the disconnected half of the shutter differs only slightly from its correct value,

the form of the variation of the voltages of the two halves of the shutter with time is then given in Fig. 35. The shutter is now open at $t_1 + \Delta t$, $t_2 - \Delta t$, $t_3 + \Delta t$, $t_4 - \Delta t$ etc., instead of at t_1, t_2, t_3 and t_4. As a result, for each current peak normally observed in the current/frequency curve, a double peak occurs; the frequency of separation of the two maxima being dependent on the amplitude of V_G.

(2) In the particular case where the d.c. voltage of the disconnected shutter wires coincides with the peak voltage of V_G as shown in Fig. 36, the current peaks occur at $f_1/2$, $3f_1/2$, $5f_1/2$ etc. (f_1 is the frequency of the first current maximum with the shutters connected normally).

(3) If the d.c. voltage of the disconnected shutter wires is as shown in Fig. 37 the shutter is never open, and no current peaks are observed.

The results for these three possibilities were verified experimentally by applying a distorting d.c. potential instead of the a.c. signal to one of the leads to a shutter.

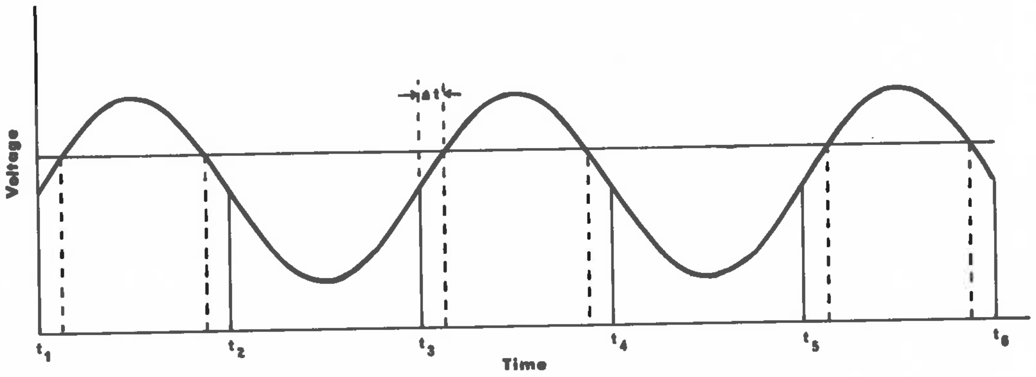


Fig 35

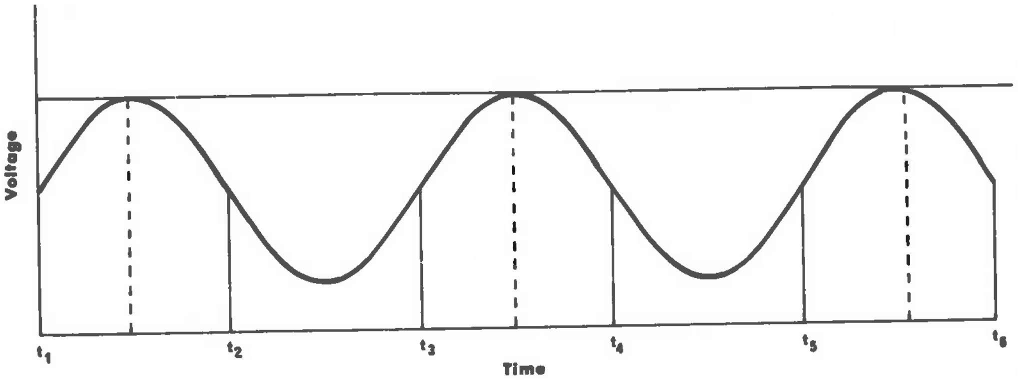


Fig 36

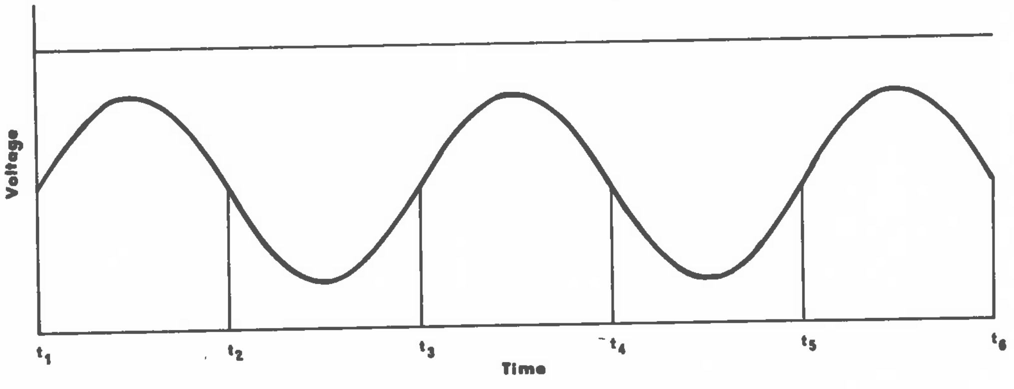


Fig 37

Chapter 7 : Further Test of Experimental Accuracy

An additional experiment that has been conducted gives further evidence that the method of Bradbury and Nielsen is not prone to errors from factors such as the existence of spurious phase differences or field distortion from the shutter voltages.

A similar shutter apparatus was used to measure the velocity of a collimated monoenergetic beam of positive potassium ions travelling in vacuum. The ions were initially accelerated by an accurately measured potential difference V before reaching the first shutter, and then travelled with constant velocity v in a region of field free space between the shutters. The velocity could then be calculated using the relation

$$\tfrac{1}{2} mv^2 = e V$$

(m is the mass of the positive ion and e the charge of the ion).

The potassium ion source was a coated tungsten filament of the type used by Blewitt and Jones (1936). Collimation was achieved by selecting ions from a very small solid angle by means of a 3 mm hole in a solid brass cylinder. The distance between the two shutters was 5.35 cm. Positive ions were used for this experiment since it was desirable to measure drift velocities in vacuum of the same order of magnitude as the drift velocities of electrons in hydrogen and nitrogen. If electrons had been used, drift velocities of this magnitude could only have been obtained by using very low accelerating voltages and the results would then have been prone to error due to the influence of stray contact potential differences.

The agreement which was obtained between theory and experiment, as is shown by Table 16, was within 1%.

Table 16

Accelerating Voltage	30	40	50	60	70	80	90	100	150	200
Theoretical Velocity $\times 10^{-5}$ cm/s	12.1	14.0	15.7	17.2	18.5	19.8	21.0	22.2	27.2	31.3
Experimental Velocity $\times 10^{-5}$ cm/s	12.0	14.0	15.6	17.1	18.4	19.7	20.9	22.1	27.0	31.0

A typical current/frequency curve is given in Fig. 38.

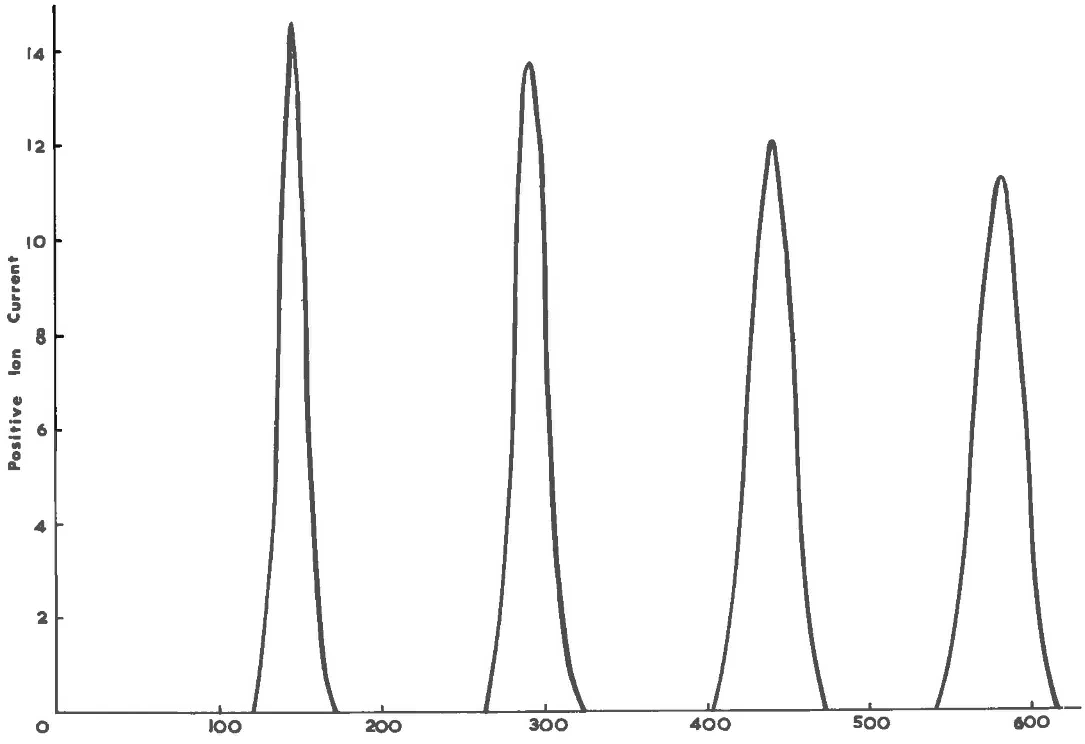

Positive Ion Current
14
12
10
8
6
4
2
0
100
200
300
400
500
600
Frequency in kc/s

Fig 38

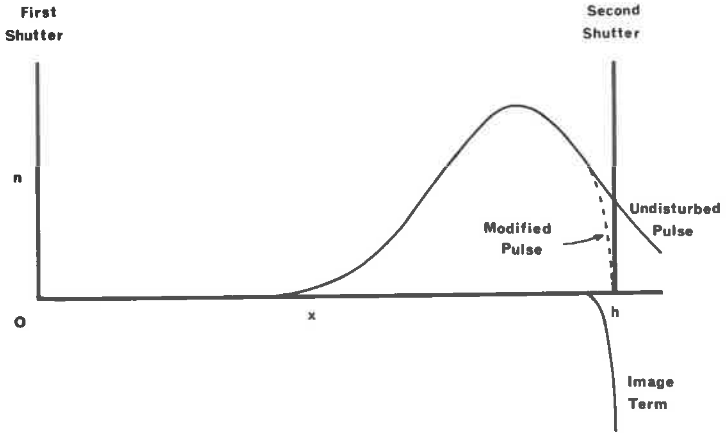

First Shutter
Second Shutter
n
Modified Pulse
Undisturbed Pulse
o
x
h
Image Term

Fig 39

SECTION III : THE EFFECT OF DIFFUSION ON MEASUREMENTS

In the work described in this section the primary concern has been to analyse the errors in measurements of the drift velocity that occur due to the diffusion of the electrons. The analysis, which is applied primarily to the method of Bradbury and Nielsen, predicts that the experimental results should be pressure dependent, and reasonable agreement is obtained with the pressure dependence of the experimental results given in Chapter 5.

The analysis has also been applied to the methods of Pack and Phelps, and Hornbeck, and an expression has been obtained for the resolving power appropriate to each current peak of the current/frequency curves obtained in the method of Bradbury and Nielsen.

Chapter 8 : Theoretical Analysis of Effect of Diffusion for Method of Bradbury and Nielsen.

8.1 Introduction.

In 2.2.1 it was stated that in the method of Bradbury and Nielsen, the relation $W = h f_1$ is used to calculate the drift velocity, where f_1 is twice the frequency of the a.c. signal corresponding to the maximum of the first current peak of the current/frequency curve. Use of this formula assumes, however, that the time required for the centre of an undisturbed pulse of electrons to travel the distance h is $1/f_1$. This assumption is not exactly true for the following reasons.

(1) Some electrons from each pulse will be absorbed owing to back diffusion by the first shutter after it is shut. This slightly alters the position of the maximum of the electron density for the pulse from that corresponding to the ideal case of an undisturbed pulse.

(2) The pulse is disturbed by the absorption of electrons by the second shutter before it opens.

(3) The time of arrival of an undisturbed electron pulse at the second shutter, which corresponds to the time that the electron density at the second shutter is a maximum, is not equal to the time that the instantaneous particle current is a maximum because of diffusion current.

(4) The number of electrons in each pulse varies as the frequency of the a.c. signal is varied. This variation results from the number of electrons in each pulse being determined by the effective time that the first shutter is 'open'.

(5) Current/frequency curves will not be symmetrical about each current maximum because there is a continuous decay of the maximum electron density of a pulse while it is passing through the second shutter.

(6) The shutters transmit electrons of high speed more readily than electrons of low speed. Thus errors will be introduced due to any variation of $\bar{c}$ along the length of the pulse. ($\bar{c}$ being the mean agitational speed of the electrons)

The influence of factors (3), (4) and (5) was analysed by Duncan (1957) for the case of a point source with a small collecting electrode.

Throughout Section III the terms "relative error" represented by r, and error, are used to imply variations in the measured results due to any of the above factors. Accidental experimental errors are referred to as 'accidental errors'.

8.2 Diffusion Errors other than the Error due to Back Diffusion.

This sub-section discusses the combined effect of factors (2), (3), (4) and (5) mentioned previously. First an expression is obtained for the order of the relative error which applies only to the limiting case of the second shutter being open for a very small percentage of the time. Later the effect of the second shutter being open for a finite time interval is considered.

8.2.1 Limiting Case when Shutter is Open for a Small Fraction of the Time.

The differential equation governing the motion of the electrons is

$$\nabla^2 n = \frac{W}{D}\frac{\partial n}{\partial x} + \frac{1}{D}\frac{\partial n}{\partial t} \qquad (10)$$

where x is the distance from the first shutter, t is the time and n = n(x, t) is the electron density. (Appendix V).

As the mean agitational speed of the electrons $\bar{c}$ is generally of the order of 100 times the drift speed W, it is justifiable to assume that the coefficient of diffusion parallel to the applied electric field is equal to the coefficient of diffusion perpendicular to the field.

It is assumed that the shutters can be considered as being either open or shut. This approximation of the experimental conditions is made to simplify the analysis. It is also assumed that the first shutter produces an infinitely thin plane source of electrons at $x = 0$ when $t = 0$ which diffuses as it passes down the chamber as in Fig. 39.

Then

$$n = \left[N_o/(4\pi Dt)^{\frac{1}{2}}\right] \exp\left[-(x - Wt)^2/4Dt\right] \qquad (11)$$

(where N_o is the number of electrons in the pulse per unit area of yz plane) is a solution of the differential equation (10), satisfying the initial conditions.

At any time t, the maximum value of n is $N_o/(4\pi Dt)^{\frac{1}{2}}$ which decays with time. The decay leads to the error described by factor (5) above.

When the second shutter is shut, it acts as a metal plane absorbing electrons in its vicinity, imposing a boundary condition $n = 0$ at $x = h$ on the solution to the differential equation. This boundary condition can be satisfied for all t by introducing a negative image term, of appropriate strength, which represents a pulse starting at $x = 2h$ and also travelling with velocity W. The modified solution of the differential equation is then:

$$n = \frac{N_o}{(4\pi Dt)^{\frac{1}{2}}}\left\{\exp\left[-\frac{(x-Wt)^2}{4Dt}\right] - \exp\left[\frac{hW}{D} - \frac{(x-2h-Wt)^2}{4Dt}\right]\right\} \qquad (12)$$

The instantaneous current density J is given by

$$\begin{aligned} J &= neW - De\frac{\partial n}{\partial x} \\ &= -De\left(\frac{\partial n}{\partial x}\right)_{x=h} \quad \text{at the second shutter} \qquad (13) \\ &= \frac{N_o e}{(4\pi Dt)^{\frac{1}{2}}}\,\frac{h}{t}\,\exp\left[-\frac{(h-Wt)^2}{4Dt}\right] \end{aligned}$$

To determine J as a function of frequency the following modifications to equation (13) are made.

(i) Substitute $t = 1/f$ to change the variable from time to frequency.

(ii) Substitute $N_o = N_1/f$ because the number of electrons admitted by the first shutter is proportional to the time that it is open (factor 4). This substitution is valid only in case of high diffusion where the undisturbed diffused pulse at $x = h$ can be closely represented by an exponential function of the form given in equation (11). In the case of low diffusion the electron density is more closely represented by a step function and the following analysis is not applicable.

On making these two substitutions the expression for J is

$$J = \frac{N_1 e h f^{\frac{1}{2}}}{(4\pi D)^{\frac{1}{2}}} \exp\left[- \frac{(h - W/f)^2 f}{4D} \right] \qquad (14)$$

As the time constant of the measuring circuit is large, the current measured will be independent of displacement currents, and will depend only on the number of electrons collected by the receiving electrode. (See Appendix II). The number of electrons received might be expected to be doubled when the frequency is doubled because of the increased number of times that the second shutter is open, but this is cancelled by the fact that, on each occasion, the interval of time that the second shutter is open is then halved. Thus I, the current measured, is simply given by $I = C_1 J$, where C_1 is a constant of proportionality. It is assumed that the second shutter is open for such a small interval of time that the effect of the removal of the boundary condition $n = 0$ at $x = h$ is negligible.

It is now required to find an expression to determine the approximate value of f for which the current I is a maximum. We change the variable in the expression for I from f to r where $r = (f - f_o)/f_o$, f_o being given by $W = hf_o$. Then on substituting $f = f_o(1 + r)$ in equation (14) we obtain for I

$$I = \frac{C_1 N_1 e h f_o^{\frac{1}{2}} (1 + r)^{\frac{1}{2}}}{(4\pi D)^{\frac{1}{2}}} \quad \exp\left[- \frac{h^2 f_o r^2}{4D(1 + r)} \right] \tag{15}$$

Maximum current occurs when $\partial I/\partial f = 0$, i.e. when $\partial I/\partial r = 0$, i.e. when

$$\tfrac{1}{2} = h^2 f_o r/2D - h^2 f_o r^2/4D(1 + r) \tag{16}$$

Thus r is the relative error in assuming that f_o is the frequency for which maximum current occurs. As r will be small, we can neglect the term in r^2 and obtain

$$r = 1/h(W/D). \tag{17}$$

From this expression we can calculate the percentage error to be expected in the limiting case when the shutters are open for a very small time interval, since the values of W/D are known from independent diffusion measurements. For example, at $E/p = 0.4$ V cm^{-1} mm Hg^{-1} at $p = 5$ mm of nitrogen, the measured value of W/D is 6.35 (from unpublished measurements made by Dr. R.W. Crompton using Townsend's method of determining W/D). For a drift velocity apparatus where the distance between the shutters is 6 cm, formula (17) predicts the measured value of the drift velocity to be 2.6% high. For the case of the inert gases, where the coefficient of diffusion is higher, the resultant errors will be greater.

8.2.2. Effect of the Shutters being Open for a Finite Time.

Equations are now derived which enable the relative error to be calculated when account is also taken of the finite time that the second shutter is open. The resulting equations are thus more general than the results of 8.2.1, but are generally not as useful because their application to particular experimental conditions involves numerical integration. Furthermore the fraction of the time that the second shutter is effectively open is not generally known in given experimental conditions. The relative error for varying fractions of the time that the second shutter is open has been obtained from the equations for the experimental conditions quoted at the end of 8.2.1.

Let T be the time that the bottom shutter opens. Then for $t < T$ the expression for the electron density will be given by equation (12),

$$n = \frac{N_o}{(4\pi Dt)^{\frac{1}{2}}} \left\{ \exp\left[-\frac{(x-Wt)^2}{4Dt}\right] - \exp\left[\frac{hW}{D} - \frac{(x-2h-Wt)^2}{4Dt}\right] \right\}$$

which satisfies the boundary condition $n = 0$ at $x = h$. When $t = T$ the shutter opens and the boundary condition is removed. For all $t > T$ we consider the electron density function to be the sum of an infinite number of elementary pulses at $x = X$ where X goes from $-\infty$ to h. The strength of each pulse is given by the value of n at $x = X$ and $t = T$. Then for $t > T$ the expression for n is given by

$$n = \int_{-\infty}^{h} \frac{N_o}{(4\pi DT)^{\frac{1}{2}}} \left\{ \exp\left[- \frac{(X - WT)^2}{4DT} \right] - \exp\left[- \frac{(X - 2h - WT)^2}{4DT} + \frac{hW}{D} \right] \right\}$$

$$\times \frac{1}{\left[4\pi D(t - T)\right]^{\frac{1}{2}}} \exp\left[- \frac{(x - X - W[t - T])^2}{4D(t - T)} \right] dX \qquad (18)$$

On rearranging the terms involving the exponentials we obtain

$$n = \int_{-\infty}^{h} \frac{N_o}{4\pi DT^{\frac{1}{2}}(t - T)^{\frac{1}{2}}} \left\{ \exp\left[- \frac{t(X - xT/t)^2}{4DT(t - T)} - \frac{(x - Wt)^2}{4Dt} \right] \right.$$

$$\left. - \exp\left[\frac{hW}{D} - \frac{t\{X - xT/t - 2h(t - T)/t\}^2}{4DT(t - T)} - \frac{(x - 2h - Wt)^2}{4Dt} \right] \right\} dX \qquad (19)$$

We now substitute $a = t/4DT(t - T)$, $b = xT/t$, and $c = xT/t + 2h - 2hT/t$. Then on further simplification n becomes

$$n = \frac{N_o}{2\pi D^{\frac{1}{2}} t^{\frac{1}{2}}} \left[\left\{ \frac{\pi^{\frac{1}{2}}}{2} + \text{erf}\left[(h - b)a^{\frac{1}{2}} \right] \right\} \exp\left[- \frac{(x - Wt)^2}{4Dt} \right] \right.$$

$$\left. - \left\{ \frac{\pi^{\frac{1}{2}}}{2} + \text{erf}\left[(h - c)a^{\frac{1}{2}} \right] \right\} \exp\left[\frac{hW}{D} - \frac{(x - 2h - Wt)^2}{4Dt} \right] \right], \qquad (20)$$

where $\text{erf}\, x = \int_{o}^{x} \exp\left[- u^2 \right] du$.

The electron density at x = h is given by

$$(n)_{x=h} = \frac{N_o}{2\pi D^{\frac{1}{2}}t^{\frac{1}{2}}} 2 \operatorname{erf}\left[(h - b)a^{\frac{1}{2}} \right] \exp\left[-\frac{(h - Wt)^2}{4Dt} \right] .$$

On differentiating n to find $\partial n/\partial x$ and substituting x = h, we obtain

$$\left(\frac{\partial n}{\partial x}\right)_{x=h} = \frac{N_o}{2\pi D^{\frac{1}{2}}t^{\frac{1}{2}}} \left\{ \frac{W}{D} \operatorname{erf}\left[(h - b)a^{\frac{1}{2}}\right] - \frac{h\,\pi^{\frac{1}{2}}}{2Dt} \right\} \exp\left[-\frac{(h - Wt)^2}{4Dt} \right] . \quad (21)$$

The instantaneous electron flux at the second shutter is given by J where

$$J = neW - De \frac{\partial n}{\partial x}, \qquad \text{at } x = h$$

$$= \frac{N_o e}{2\pi D^{\frac{1}{2}}t^{\frac{1}{2}}} \left\{ W \operatorname{erf}\left[\frac{h}{2} \left(\frac{t - T}{DTt} \right)^{\frac{1}{2}} \right] + \frac{h}{t} \frac{\pi^{\frac{1}{2}}}{2} \right\} \exp\left[-\frac{(h - Wt)^2}{4Dt} \right] . \quad (22)$$

As in equation (14) we now express J in terms of frequency by changing the variable from t to f and substituting $N_o = N_1/f$. If we assume that for any given frequency f, the shutter is open for 1/10th of the time for one cycle, we choose T = 19/20f as the opening time of the second shutter and t will equal (19 + a)/20f, where a will vary from 0 to 2 while the shutter is open. Then from reasoning similar to that presented in 8.2.1 the current I measured will be proportional to J, and I will be given by

$$I = \int_{0}^{2} \frac{AS^{\frac{1}{2}}}{k} \left\{ \operatorname{erf} \left[\frac{h(af_{o}S)^{\frac{1}{2}}}{2(19D)^{\frac{1}{2}}} \right] + \frac{S\pi^{\frac{1}{2}}}{2} \right\} \exp \left[-\frac{hW}{4D}(1 - 1/S)^{2}S \right] da, \qquad (23)$$

where A is a constant, $k = f/f_{o}$, and $S = \frac{20k}{19 + a}$.

This integration can be done numerically for each value of f, and by plotting I as a function of f, the frequency for which maximum current occurs can be found. In a similar way the frequency for the maximum current can be determined when the shutters are open for 0.05 and 0.01 of the time. Such calculations have been made for the experimental conditions quoted at the end of 8.2.1 and the relative error when the second shutter is open for 0.01, 0.05 and 0.1 of the time is given below.

Table 17

Fraction Open	0 +	0.01	0.05	0.1
Predicted Error	2.6%	2.1%	1.6%	1.5%

8.3 Error Due to Back Diffusion.

This subsection discussed factor (1). In cases of high diffusion, after a pulse is formed by the first shutter, some of the electrons which are diffusing backwards from the pulse are swept up by the closed shutter, which again is considered to act as a conducting plane. The centre of the pulse is thereby displaced slightly forward, and in the following analysis we obtain an expression for this displacement. In this case the solution to the differential equation has the boundary condition $n = 0$ at $x = 0$. Owing to the difficulty of representing mathematically the initial conditions, the following analysis has been developed to approximate the experimental conditions.

We introduce the electron pulse as an infinitely thin plane at $x = h/m$ from the first shutter at time $t = t_o$, where m is a number which may be made very large and t_o is the time required for the pulse to travel the distance h/m at velocity W, i.e. $t_o = h/mW$. It is found that the expression for the error approaches a limiting value as m is made very large. Again the boundary condition is satisfied by introducing an image term travelling in the same direction as the electron pulse, where in this case it starts at $x = -h/m$ at $t = t_o$. The strength of the image term is chosen such that $n = 0$ at $x = 0$ for all $t > t_o$ and the solution of the differential equation (10) is then

$$n = \frac{N_o}{\left[4\pi D(t - h/mW)\right]^{\frac{1}{2}}} \left\{ \exp\left[- \frac{(x - Wt)^2}{4D(t - h/mW)} \right] - \exp\left[- \frac{hW}{mD} - \frac{(x + 2h/m - Wt)^2}{4D(t - h/mW)} \right] \right\}. \qquad (24)$$

The value of n when the pulse is at the second shutter is given by substituting $t = h/W$; then

$$n = \frac{N_o W^{\frac{1}{2}}}{\left[4\pi Dh(1 - 1/m)\right]^{\frac{1}{2}}} \left\{ \exp \left[- \frac{(x - h)^2 W}{4Dh(1 - 1/m)} \right] - \exp \left[- \frac{hW}{mD} - \frac{(x + 2h/m - h)^2 W}{4Dh(1 - 1/m)} \right] \right\}. \qquad (25)$$

The maximum of n will occur for a value of x slightly different from h owing to the presence of the image term. This value of x will satisfy $\partial n/\partial x = 0$, i.e. x satisfies

$$x - h = (x + 2h/m - h) \exp \left[- \frac{hW}{mD} - \frac{W(h^2/m^2 + hx/m - h^2/m)}{Dh(1 - 1/m)} \right].$$

The variable is again changed to r, where $r = (x - h)/h$ is the relative error in taking the maximum of n to be at $x = h$. Omitting all terms of $1/m^2$ and higher order and all terms in r^2 and higher order, the above equation becomes

$$r = (r + 2/m) \exp \left[- \frac{hW(1 + r)}{Dm} \right],$$

i.e.

$$r = \frac{2}{m\left[\exp (hW/mD) - 1\right]} \;. \qquad (26)$$

If m is so large that hW/mD is small, r is given by

$$r \simeq 2/h(W/D). \qquad (27)$$

For the experimental conditions quoted earlier, equation (27) predicts an error of 5.2% as a consequence of back diffusion. This prediction applies for the limiting case of the first shutter being open for a very small interval of time.

It is of interest to note the effect of different values of $1/m$ on the value of r as calculated from equation (26).

Table 18

Values of r in relation to $1/m$ in equation (26)

$1/m$	0 +	0.01	0.05	0.1
Predicted error	5.2%	4.3%	1.8%	0.5%

In the case of the first shutter being open for a finite time, the initial electron density can be approximated by a step function of width h/M. We consider this step function to be a series of infinitely thin elemental pulses which start at distance h/m from the first shutter where $M < m < \infty$. As can be seen from Table 18, the closer to the first shutter that an elemental pulse starts, the greater is the error due to back diffusion. Thus the total error for a pulse originally a step function of width say $h/20$ would generally be expected to be greater than that given in Table 18 when $1/m = 1/20$.

8.4 Error due to Variation of $\bar{c}$ within the Pulse.

This subsection discusses factor (6). Townsend (1930) has shown that there will be a variation in the mean agitational speed of electrons within a pulse at any instant after its production in a gas, when it travels under the influence of a uniform electric field. The electrons in the leading section of a pulse will have gained a greater amount of energy from the field than electrons in the trailing edge of the pulse because they have travelled a greater distance along the field. Now the electrical shutters admit electrons of high agitational speed more readily than electrons of low agitational speed because the faster moving electrons are less likely to be swept aside by the electric field produced between the shutter wires. Thus the number of electrons admitted by the second shutter for any section of an electron pulse will be dependent not only on the value of n and $\partial n / \partial x$ but also on the value of $\bar{c}$ for that section of the pulse. Consequently, the effect of the variation of $\bar{c}$ within the pulse will be such as to increase the measured value of the drift velocity of a pulse above its true value. It is possible to obtain an estimate of the error due to this effect if it is assumed that the transmission of electrons through the shutters is proportional to some power q of the electron energy.

The equations of Townsend (1930), which were derived assuming that A is independent of c, are as follows

$$\frac{y}{y^2 + 1}\left[y \log \frac{y\,Q_o - Q_o}{y\,Q_o - Q} - \frac{1}{y} \log \frac{Q_o + yQ}{Q_o + yQ_o}\right] = \frac{s}{S} \qquad (28)$$

$$\text{where } y - 1/y = Sz/as, \qquad (29)$$

$y > 1$, Q is the average energy of electrons at distance z from the centre of the electron pulse, Q_o is the average energy of all electrons in the pulse, s is the average number of collisions undergone by the electrons, $a = Q_o/E\,e$ and $S = \bar{c}\,a/W\ell$ is the average number of collisions of electrons in travelling distance a. Equation (28) can be simplified to become

$$\frac{y - 1}{y - Q/Q_o} = \text{Exp}\left[(1 + 1/y^2)s/S + A_o\right] \qquad (30)$$

$$\text{where } A_o = \frac{1}{y^2} \log\left[\frac{1 + y\,Q/Q_o}{1 + y}\right] > -\log 2 \quad \text{for } Q/Q_o > \tfrac{1}{2}.$$

It will now be shown that when the pulse is at the second shutter, s/S, and thus the right hand side of equation (30), is very large. Thus the solution of this equation can be closely represented by $y = Q/Q_o$ and using equation (29) it is shown that the variation of energy within the pulse is represented by $Q/Q_o = 1 + z/2h$.

When the pulse is at the second shutter

$s/S \simeq s_1/S$ where s_1 is the average number of collisions of electrons to reach the second shutter.

$= h/a$ as the number of collisions is proportional to the distance travelled by the pulse .

$$
\begin{aligned}
s/S &= hEe/Q_o \\
&\simeq 27hE/k_1 \\
&> 40 \qquad \text{for all experimental conditions of this investigation.}
\end{aligned}
$$

Thus the right hand side of equation (30) is greater than exp (40 - log 2).

As a consequence the solution of equation (30) is closely represented by $y = Q/Q_o$.

From equation (29) it follows that

$$
\begin{aligned}
y &= Sz/2as \pm (1 + S^2z^2/4a^2s^2)^{\frac{1}{2}} \\
&= z/2h \pm (1 + z^2/4h^2)^{\frac{1}{2}} \qquad \text{as } S/s \simeq a/h \\
&= 1 + z/2h \qquad \text{to a first order.}
\end{aligned}
$$

$$\text{Thus } Q/Q_o = 1 + z/2h. \tag{31}$$

To determine the effect of the variation of Q within a pulse on the observed position of maximum current, we again assume, as in 8.2.1, that an electron pulse can be represented by equation (11). When the pulse is at the second shutter $Wt = h$ and the equation becomes

$$n = c_1 \exp\left[- Wz^2/4Dh\right] \tag{32}$$

where c_1 is a constant. As the shutter transmits electrons of high energy more readily than electrons of low energy, the variation in the energy of electrons along the length of a pulse, given by equation (31), results in the effective position of the maximum of the pulse being at a value of z greater than 0. To account for the energy variation, a weighting factor of $(1 + z/2h)^q \simeq 1 + qz/2h$ is introduced into the expression for current. Thus if diffusion current and the effects of boundary conditions are omitted, the current I is given by

$$I = c_2(1 + qz/2h) \quad \exp\left[-Wz^2/4Dh\right]$$

where c_2 is a constant. When $\partial I/\partial z = 0$, $z = q/(W/D)$, to a first order. The relative error due to the variation of $\bar{c}$ within the pulse is then given by

$$r = (z - 0)/h = q/h(W/D) \tag{33}$$

The value of q is unknown and its experimental determination is difficult because when Q is varied experimentally, W, which will also influence the electron transmission, also changes. However, as it is most likely that $0 < q \leq 1$, the result that has been obtained indicates the order of the error due to factor (6). It also indicates that the error due to factor (6), as for all other factors, is proportional (to a first approximation) to $1/h(W/D)$.

8.5 Effect of having a Point Source of Electrons.

The analysis of 8.2, 8.3 and 8.4 was based on the assumption of an infinite plane source of electrons. It is of interest to determine any change in the expressions for r on assuming that the electron pulse originates as a point source. The limiting values of r would then be known for any experimental conditions, which in general correspond neither to a point nor a plane source.

The expression for the electron density of a pulse of electrons from a point source satisfying the previous initial conditions is

$$n = \frac{N_o}{(4\pi Dt)^{3/2}} \exp\left[-\frac{(x - Wt)^2 + y^2 + z^2}{4Dt}\right]. \qquad (34)$$

Equation (34) again satisfies the differential equation (10), and is such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n\,dx\,dy\,dz = N_o \quad \text{for all } t > 0.$$

We consider first the case where the collecting electrode is infinite. The analysis of 8.2.1 can then be repeated with the expression for n given by (34), again modified to include an image term. The value of the instantaneous current at the second shutter will then be

$$\iint J\,dS = \frac{N_o e}{(4\pi Dt)^{3/2}}\,\frac{h}{t}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{(h - Wt)^2 + y^2 + z^2}{4Dt}\right] dy\,dz$$

$$= \frac{N_o e}{(4\pi Dt)^{\frac{1}{2}}}\,\frac{h}{t}\,\exp\left[-\frac{(h - Wt)^2}{4Dt}\right], \qquad (35)$$

as before in (13). Consequently the relative error is still $r = 1/h(W/D)$. In a similar way the analysis for back diffusion in the case of a point source again gives $r = 2/h(W/D)$.

However, the case of a point source with a collecting electrode which is not infinite does lead to a difference in the value of r from that obtained for a plane source. We consider the extreme case of a very small collecting electrode on the X axis. It is no longer necessary to integrate over the collecting plane, for the instantaneous current at the second shutter may be regarded simply as the value of J at the point $x = h$, $y = 0$, $z = 0$. Thus from (35)

$$J = \frac{N_o e}{(4\pi Dt)^{3/2}} \frac{h}{t} \exp\left[-\frac{(h - Wt)^2}{4Dt}\right]. \tag{36}$$

On making the substitutions of 8.2.1, the current I is given by

$$I = C_2(1 + r)^{3/2} \exp\left[-\frac{hWr^2}{4D(1 + r)}\right] \quad (C_2 \text{ is a constant}).$$

When $\partial I/\partial r = 0$,

$$r = 3/h(W/D). \tag{37}$$

The analysis of 8.3 for factor (1), appropriate to the case of a point source, is not altered by having a small electrode instead of an infinite one insofar as it still gives a value of r for back diffusion of $2/h(W/D)$.

8.6 Summary of Theoretical Predictions of Errors due to Diffusion.

In 8.2.1 a derivation for r was made of the combined effect of the four factors numbered (2), (3), (4) and (5) listed in 8.1. The result has also been calculated for the following combinations of factors: (3) + (4) + (5), (4) + (5), (5), (1) + (3) + (4) + (5), (1) + (4) + (5) and (1) + (4). In all cases the results so obtained are consistent with the hypothesis that the individual errors from each factor are simply additive. The individual contributions of each factor are given below for a plane source of electrons in the limiting case of the shutters being open for a very small interval of time.

(1) The relative error due to back diffusion as determined in 8.3 is given by $2/h(W/D)$.

(2) The influence of the boundary condition imposed by the second shutter gives a contribution to the relative error $1/h(W/D)$.

(3) The inclusion of the term $-D\,\partial n/\partial x$ to account for transport due to diffusion introduces a relative error of $1/h(W/D)$.

(4) The varying number of electrons in each pulse with change of frequency (represented by the substitution $N_o = N_1/f$) changes the relative error by $-2/h(W/D)$.

(5) The coefficient $1/(4\pi Dt)^{\frac{1}{2}}$ describing the decay with time

of each pulse introduces a relative error of $1/h(W/D)$.

(6) The varying value of $\bar{c}$ within the pulse introduces a relative error of $q/h(W/D)$ where the transmission of the shutter is assumed to be proportional to the qth power of the electron energy.

The effect of having a point source with a small collecting electrode increases the relative error from that of a plane source by $2/h(W/D)$.

The total relative error for all six factors in the case of the shutters being open for a very small time interval is thus

$$r = \frac{3+q}{h(W/D)} \quad \text{for a plane source of electrons and} \tag{38}$$

$$r = \frac{5+q}{h(W/D)} \quad \text{for a point source of electrons and a small collecting electrode.} \tag{39}$$

It is most likely that $0 < q \leqslant 1$. The error is such that the measured values of the drift velocity W_M, are higher than the true value of the drift velocity W.

Comparison with the experimental results has shown that a useful formula relating W_M with W for general experimental conditions is

$$W_M = W\left[1 + 3/h(W/D)\right] \tag{40}$$

If this formula were to represent the exact variation of the results with pressure, the theoretical interpretation from the previous analysis would be that the decrease in the error due to the finite time that the shutters are open is equal to

(1) the error due to factor (6) plus

(2) the error introduced because the electron source differs from that of a plane source.

8.7 Resolving Power.

The resolving power appropriate to each current peak of the current frequency curve is defined as the ratio of the frequency at which maximum current occurs divided by the frequency spread at half of the maximum current. If the value of the resolving power is high the current peak is very sharp and the drift velocity can be determined with high accuracy. The resolving power can generally be increased by increasing the amplitude of the a.c. voltages that are applied to the shutters, but there is a limit to the resolving power that is possible, which is determined by the coefficient of diffusion of the electrons.

For high a.c. voltages on the shutters, the electron pulses can be regarded as originating as infinitely thin plane sources as described by equation (11). Using this equation it is possible to obtain a theoretical expression for the limiting value of the resolving power in terms of D, W and h. From equation (11)

$$n = \frac{N_o}{(4\pi Dt)^{\frac{1}{2}}} \quad \exp\left[-\frac{(x - Wt)^2}{4Dt}\right].$$

The current measured is proportional to the instantaneous rate of flow of particles, J, where

$$J = (neW)_{x=h} = \frac{N_o eW}{(4\pi Dt)^{\frac{1}{2}}} \quad \exp\left[-\frac{(h - Wt)^2}{4Dt}\right].$$

The diffusion current has been neglected as only a first approximation of the resolving power is required. For the mth current peak $t = m/f$. Also, as in (14), $N_o = N_1/f$. Thus

$$J = \frac{N_1 eW}{f(4\pi Dm/f)^{\frac{1}{2}}} \quad \exp\left[-\frac{(h - Wm/f)^2}{4Dm/f}\right]. \tag{41}$$

It is assumed that the resolving power appropriate to the peak represented by equation (41) is approximately equal to the resolving power of the peak represented by

$$J = (J_o/m) \quad \exp\left[-(h - Wm/f)^2/(4Dm/f)\right] \tag{42}$$

where J_o is a constant.

The maximum value of J is J_o/m and occurs when $f = mW/h$. The frequency spread at half of the maximum current is $f'' - f'$ where f'' and f' are the roots of

$$(J_o/m) \quad \exp\left[-(h - Wm/f)^2/(4Dm/f)\right] = J_o/2m \tag{43}$$

This equation simplifies to

$$h^2f^2 - (2hWm + 4Dm \log_e 2)f + W^2m^2 = 0.$$

It can be shown that f" - f' is given by

$$f'' - f' = \left[(2hWm + 4Dm \log_e 2)^2 - 4W^2m^2h^2\right]^{\frac{1}{2}}/h^2$$

$$= \frac{4Dm \log_e 2}{h^2} \left(1 + \frac{hW}{D \log_e 2}\right)^{\frac{1}{2}}$$

$$\simeq \frac{4Dm \log_e 2}{h^2} \left(\frac{hW}{D \log_e 2}\right)^{\frac{1}{2}}$$

for all experimental conditions of this investigation.

Thus the resolving power is

$$\text{Resolving Power} = \frac{mW}{h(f'' - f')}$$

$$= \frac{1}{4}\left(\frac{hW}{D \log_e 2}\right)^{\frac{1}{2}} \qquad (44)$$

The expression for the resolving power, has important consequences in attempting to obtain optimum accuracy when experimental results are being taken.

(1) The expression is independent of m, the number of the current peak. In general the first peak was used to take measurements of W as it was desirable to use as little electron current as possible to minimise the

formation of temperature gradients across the drift space due to the hot filament.

(2) The resolving power is proportional to $h^{\frac{1}{2}}$. Thus long chamber lengths are desirable to obtain the greatest accuracy.

(3) As the resolving power is proportional to $(W/D)^{\frac{1}{2}}$ it is desirable to use high gas pressures to make the coefficient of diffusion as low as possible.

Chapter 9 : Comparison of Theoretical Predictions and Experimental Results for Method of Bradbury and Nielsen.

9.1 Errors Due to Diffusion.

From the theoretical predictions summarised in 8.6 it may be concluded that the measured values of the drift velocity should increase due to the influence of diffusion such that the relative error in measurements is of the order of $3/h(W/D)$, (equation 40). As a consequence, the measured value of the drift velocity at a given value of E/p is predicted to be pressure dependent, the error in the measured value tending to zero for very high gas pressures. Section 9.1 is concerned with comparing the pressure dependence that is observed experimentally with the pressure dependence that would be expected from equation (40).

9.1.1 Results of Crompton, Hall and Macklin.

The investigation of the pressure dependence of results in hydrogen made by Crompton, Hall and Macklin (1957) gave asymptotic values of W at high pressure which are in agreement with the 'Best Estimate' values of the present investigation, within the limits of the experimental error. However, the measured values of the drift velocity at low pressures were in some cases over 10% higher than the corresponding results of the present investigation.

To investigate this descrepancy, the results of Crompton, Hall and Macklin have been retaken, using their original experimental tube with h = 3 cm. The new measurements show that the variation of the results with gas pressure is much less than is indicated by the results of Crompton, Hall and Macklin. Typical results are given in Table 19 for $E/p = 1$.

Table 19. (Values of $W \times 10^{-5}$ cm/s)

Pressure	5	10	20	30
Results of C.H.M.	12.9	12.0	10.3	10.2
New results, using original apparatus.	10.9	10.2	10.1	10.1

Although the results that have been retaken are not directly comparable with the results of the apparatus of this investigation, which has a drift distance of 6 instead of 3 cm, the reduced variation of the results from the asymptotic limit suggests that the results of the present investigation give the true magnitude of the variation of W with pressure.

The discrepancy between the two sets of results taken with the apparatus of Crompton, Hall and Macklin has been attributed largely to the original electronic feeder network which supplied the a.c. voltages to the shutters and which gave a falling voltage with increase of frequency. In the case of high diffusion when low pressures are used, the current/frequency peaks are very flat so that even a slight decrease in the voltage on the shutters with increasing frequency can significantly increase the frequency at which maximum current occurs.

9.1.2 Results in Hydrogen and Nitrogen.

Calculations have been made of the variation of the results with pressure that is predicted by the formula, $W_M = \left[1 + 3/h(W/D)\right] W$. W_M is evaluated using the independent measurements of W/D that have been made at room temperature by Crompton and Sutton (1952) and Crompton and Jory (1962), and the unpublished results for 77°K made by Warren and Parker (1962).

The agreement between the predicted values of W_M and those obtained by experiment will now be discussed for various values of E/p over the range for which measurements were made. The values of E/p that were chosen were 0.05, 0.4, 4.0 and 7.0.

(1) Low Values of E/p.

For E/p = 0.05 the variation of W_M over the range of p of 50 to 500 is predicted to be $\leqslant$ 1% for hydrogen and nitrogen at both 293°K and 77.6°K. The variation in the corresponding four sets of experimental measurements is less than 1% in each case. Results are shown graphically in Fig. 40.

The decrease with N of the results in Table 4 for nitrogen at 77.6°K for $E/p < 0.02$ cannot be explained by the influence of diffusion as the decrease of the results is directly proportional to N (see Fig. 22). As has been mentioned in 5.1, the coefficient of diffusion for these

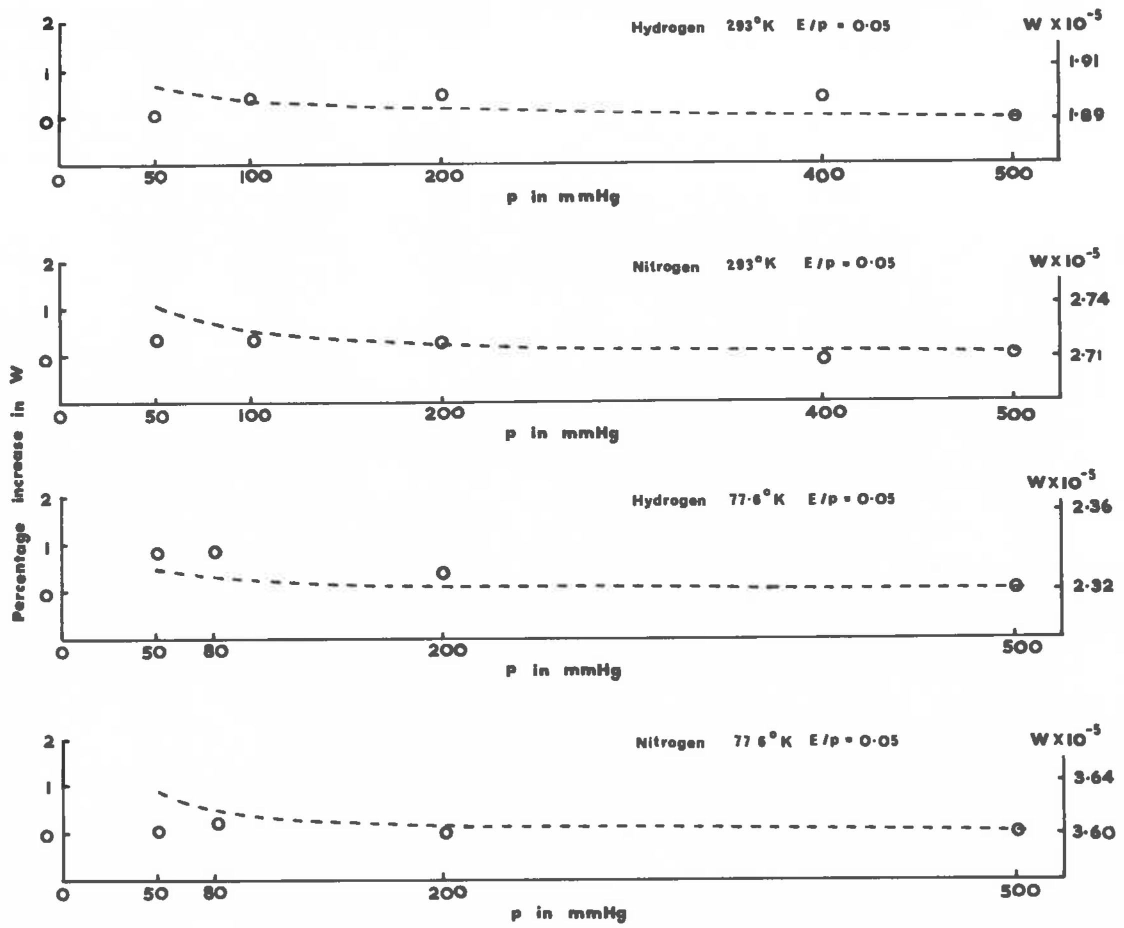

Hydrogen 293°K E/p = 0·05
W x 10⁻⁵
1·91
1·89
2
1
0
0
50
100
200
400
500
p in mmHg
Nitrogen 293°K E/p = 0·05
W x 10⁻⁵
2·74
2·71
Percentage increase in W
p in mmHg
Hydrogen 77·6°K E/p = 0·05
W x 10⁻⁵
2·36
2·32
80
p in mmHg
Nitrogen 77·6°K E/p = 0·05
W x 10⁻⁵
3·64
3·60
p in mmHg

Fig 40

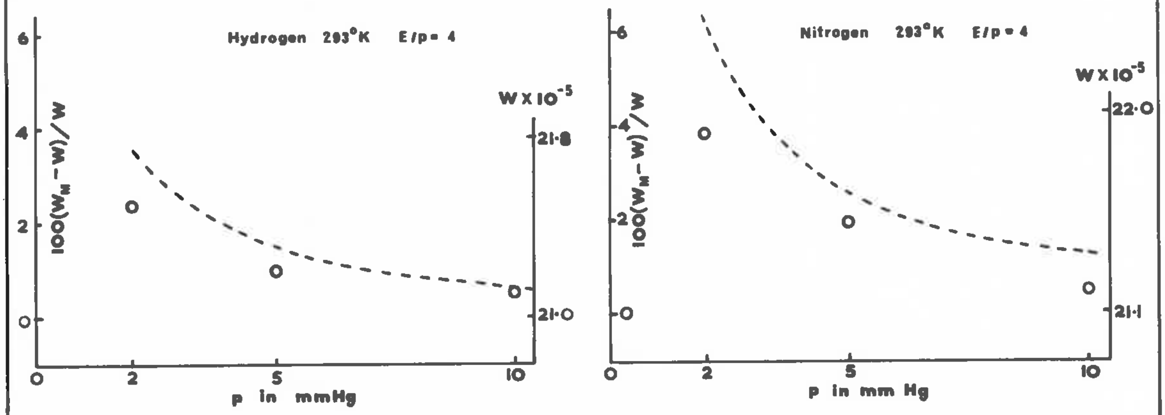

Hydrogen 293°K E/p = 4
W x 10⁻⁵
21·8
21·0
6
4
2
0
100(W_N − W)/W
0
2
5
10
p in mmHg
Nitrogen 293°K E/p = 4
W x 10⁻⁵
22·0
21·1
100(W_N − W)/W
p in mm Hg

Fig 42

measurements is so low that the variation with N, predicted by equation (40), is less than 0.3% over the range of N for which measurements were taken.

(2) Intermediate Values of E/p.

In the four graphs of Fig. 41 the variation with N of the experimental results taken at $E/p = 0.4$ is compared with the theoretical predictions of W_M. The experimental points are indicated by circles and the theoretical values of W_M by the broken curve. It will be noticed that, in each case, the relative error of the results from the asymptotic value of W obtained at high pressure is slightly less than $3/h(W/D)$.

The results at $E/p = 4.0$ are similarly compared with theoretical values of W_M in Fig. 42, and again it is seen that the observed value of the relative error is less than $3/h(W/D)$.

(3) High Values of E/p.

The range of pressures over which it was possible to take measurements at high values of E/p was limited because of electrical breakdown within the tube. Consequently the variation of measured values of W with p could not be investigated fully. Nevertheless it is apparent that the observed variation of the results with pressure is very much less than is predicted from equation (40). For example at $E/p = 7.0$ the predicted variation of W_M between

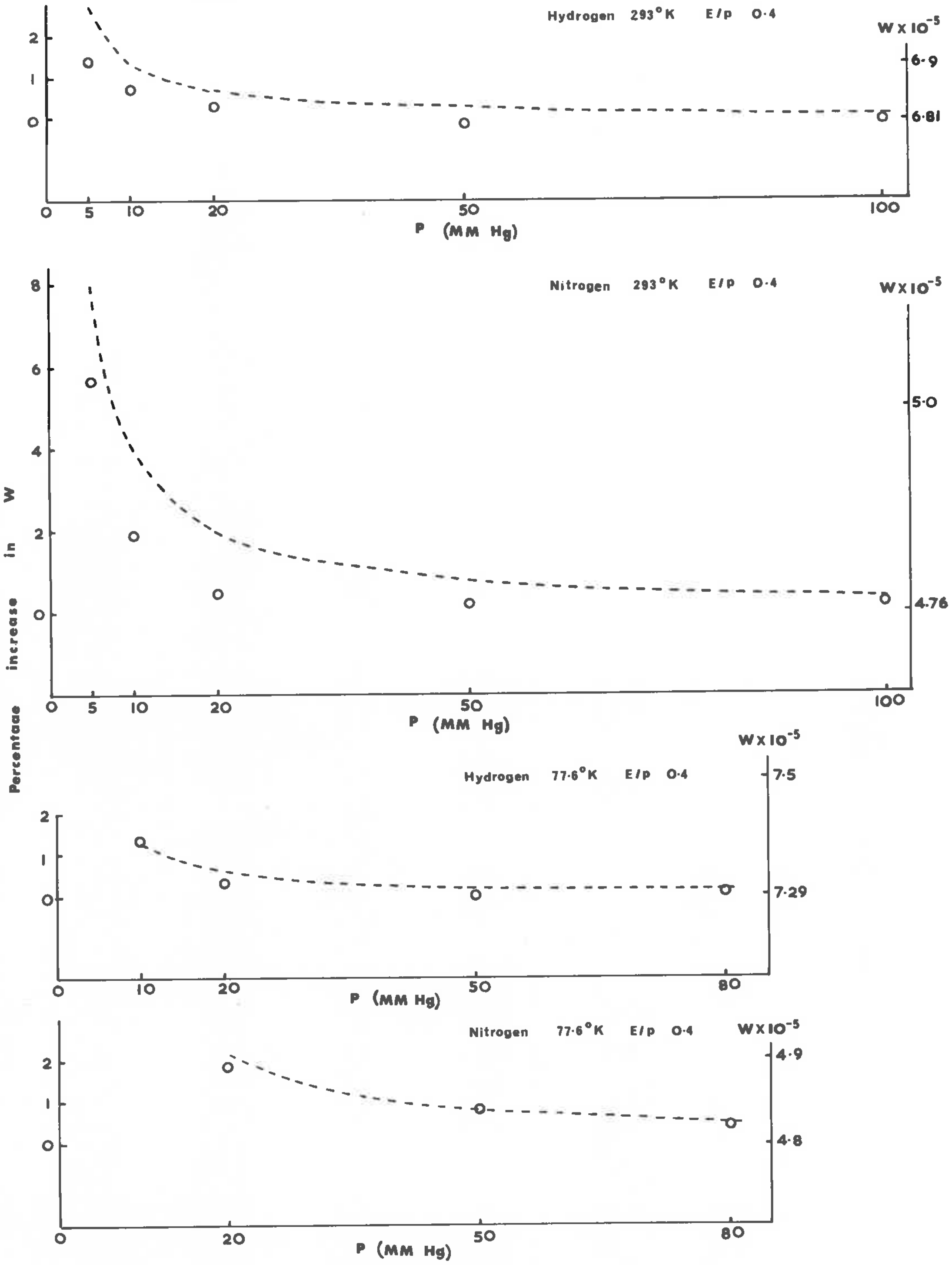

Hydrogen 293°K E/p 0·4
W x 10⁻⁵
6·9
6·81
2
1
0
0
5
10
20
50
100
P (MM Hg)
Nitrogen 293°K E/p 0·4
W x 10⁻⁵
5·0
4·76
8
6
4
2
0
Percentage increase in W
0
5
10
20
50
100
P (MM Hg)
Hydrogen 77·6°K E/p 0·4
W x 10⁻⁵
7·5
7·29
2
1
0
0
10
20
50
80
P (MM Hg)
Nitrogen 77·6°K E/p 0·4
W x 10⁻⁵
4·9
4·8
2
1
0
0
20
50
80
P (MM Hg)

Fig 41

$p = 5$ and 2 is 1.8% for hydrogen and 2.5% for nitrogen. The observed experimental variations are 0.3% in each case.

This small variation is attributed to the fact that very high a.c. voltages were not used for these measurements to avoid the possibility of electrical breakdown within the shutters. Such high a.c. voltages are necessary to obtain boundary conditions of the electron density at the shutters comparable with the boundary conditions that exist at low E/p when the electron energies are much lower. The effect of low a.c. voltages is to reduce the effective time that the shutters are open, and thus the error due to diffusion is decreased.

9.1.3 Results in Helium.

Results have also been taken in helium to test the validity of equation (40) for the case of a monatomic gas in which it is known that the coefficient of diffusion is high. The helium that was used was stated to be spectroscopically pure. It is known that drift velocities in monatomic gases in particular are subject to errors due to small traces of impurity. However, it is felt that the pressure dependent measurements which were taken with one gas sample are still valid even though the asymptotic limit of the measurements may be slightly in error due to traces of impurities. The measurements were repeated several times with different gas samples and in all cases the results were in agreement, within the experimental error.

Experimental results, which were obtained only for $E/p = 0.4$, are indicated in Fig. 43 by circles. The broken curve is obtained from equation (40). The full curve, which represents W_M if the relative error from the asymptotic limit is $5/h(W/D)$, has also been drawn for comparison with the experimental variation.

These results were taken before the procedure mentioned in 6.4.2 was carried out in which the shutters were recoated with gold. By comparison with similar results taken at this time in nitrogen, it is considered likely that the true increase of the results at $p = 5$ from the value of W at the asymptotic limit is 12% instead of 15%. In any case it is seen that the variation with p is, to a first order, predicted by equation (40).

9.1.4 Variation of Results with V_G.

The increase in the measured value of the drift velocity is predicted in 8.2.2 and 8.3 to be reduced if the effective time that the shutters are open is increased. The effective time that the shutters are open is increased for very low values of the RMS value of the a.c. voltage V_G. Thus under conditions of high diffusion, the measured value of the drift velocity would be expected to depend slightly on the amplitude of the a.c. signal that is applied to the shutters. Such a dependence was observed in nitrogen at $p = 7$, $T = 293^{\circ}K$ at $E/p = 0.4$, the results of which are given in Table 20.

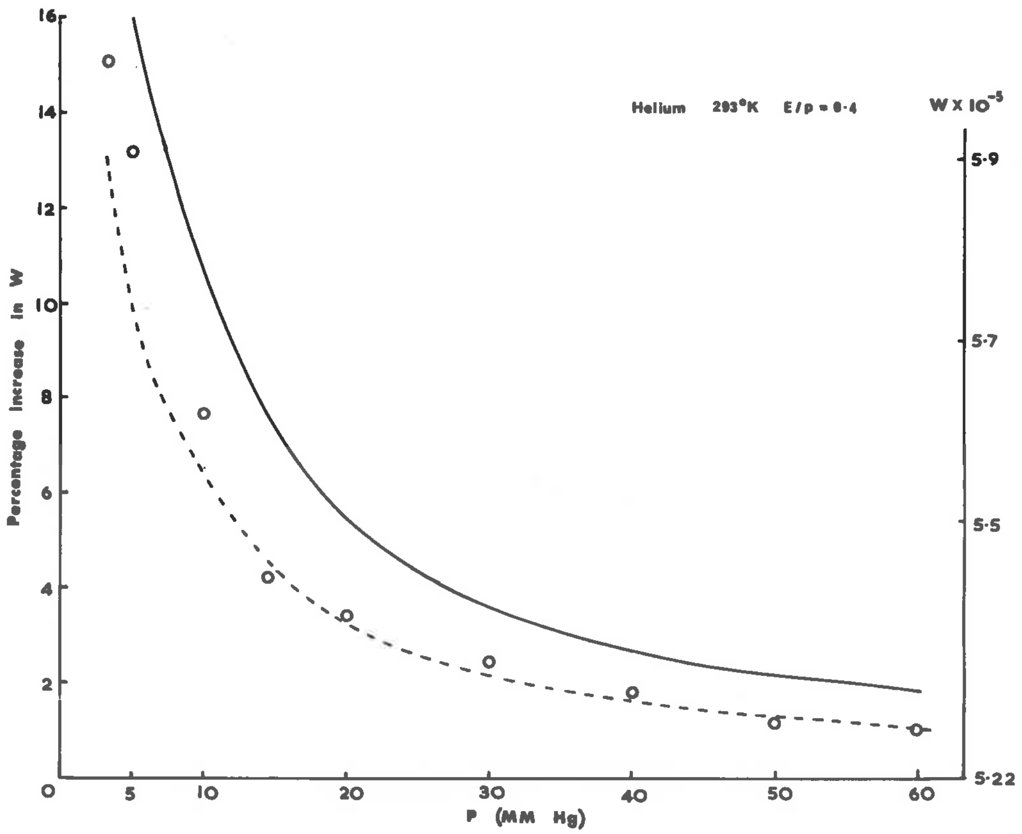

Helium 293°K E/p = 0·4
W x 10⁻⁵
Percentage Increase in W
16
14
12
10
8
6
4
2
0
5
10
20
30
40
50
60
P (MM Hg)
5·9
5·7
5·5
5·22

Fig 43

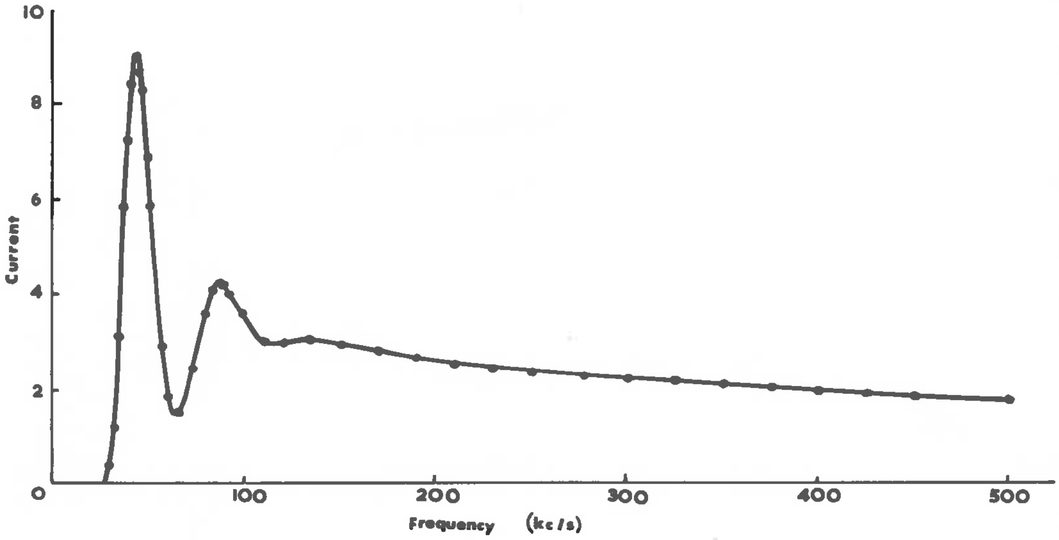

Current
10
8
6
4
2
0
100
200
300
400
500
Frequency (kc/s)

Fig 44

Table 20.

V_G (volts)	1.5	3.2	5.0	10	15
W ($\times 10^{-5}$ cm/s)	4.79	4.84	4.89	4.95	4.95

At $p = 100$ and $E/p = 0.4$, where the error due to diffusion is less than 1%, no dependence of the measured value of the drift velocity on V_G could be detected.

9.1.5 Conclusions.

The experimental results that have been obtained of the dependence of the measured value of W on N are strong evidence that diffusion effects of the type discussed theoretically can, under some conditions, introduce considerable errors into drift velocity measurements. If accurate results are to be obtained it is essential that an examination be made to determine if such errors due to diffusion are significant.

It is impossible to make an exact comparison of the theoretical predictions of errors with the experimental results because of two unknown factors. Firstly, the shutters are fed with sine wave voltages which do not conform to the theoretical model where the shutters open and close instantaneously. The theoretical predictions are dependent on the time interval that the shutters are open, and it is difficult to determine what the equivalent time interval for the experimental conditions will be. Secondly, the error due to the variation of $\bar{c}$ within the pulse depends on the unknown dependence of the transmission of the shutters with the energy of the electrons. A further error, which is usually unknown, is the extent to which the experimental source of electron pulses, the first shutter, conforms to the theoretical model of either a point or a plane source. In this case, however, the limits of the error that would be predicted theoretically are known.

Despite these unknown factors, the theory does explain the direction and order of magnitude of the errors that are observed. It is therefore reasonable to assume that the observed pressure dependence of the results does not indicate any fault in experimental technique. It should be noted that the error predicted from $r = 3/h(W/D)$ will on the one hand overestimate the true error because of the finite time that the shutters are open, and on the other hand underestimate the true error because of the variation of $\bar{c}$ within the pulse. The experimental results would seem to indicate that the first of these effects is greater than the second and that $3/h(W/D)$ is generally slightly larger than the total relative error that is observed experimentally.

One of the most important applications of the result that the relative error due to diffusion is of the order $3/h(W/D)$ is that it enables the optimum experimental conditions to be chosen. By making either the chamber length or the gas pressure sufficiently large, errors due to diffusion can be made insignificantly small.

9.2 Current/Frequency Curves.

The marked effect of diffusion on current/frequency curves is illustrated in Figs. 44 and 45 which were each taken for nitrogen at $293^{\circ}K$ with $E/p = 0.4$ The curve shown in Fig. 44 corresponds to $p = 5$ mm and thus high electron diffusion relative to the curve of Fig. 45 where the gas pressure was 100 mm.

It is impossible to develop theory to predict the experimental current/frequency curves exactly, because of:

(1) The difficulty of taking account of the boundary conditions that exist at the two shutters.

(2) The fact that the substitution $N_o = N_1/f$ (in equation 14) is invalid at conditions of low electron diffusion when the electron density at $x = h$ is more closely represented by a step function than an exponential function.

An approximate theoretical expression for the current/frequency curve can be obtained as follows, using a procedure similar to that of Duncan (1957). We assume, as in equation (11), that the electron pulse can be expressed by:

$$n = \left[N_o/(4\pi Dt)^{\frac{1}{2}}\right] \exp \left[-(x - Wt)^2/4Dt\right].$$

The influence of the boundary conditions is neglected. Then J at $x = h$ is given by:

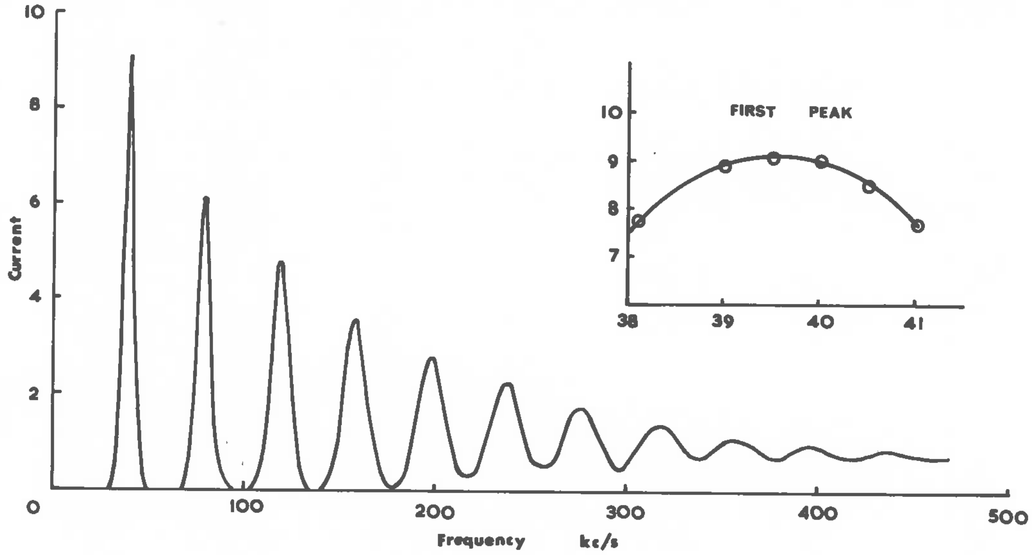

10
8
6
4
2
0
Current
100
200
300
400
500
Frequency kc/s
10
9
8
7
FIRST PEAK
38
39
40
41

Fig 45

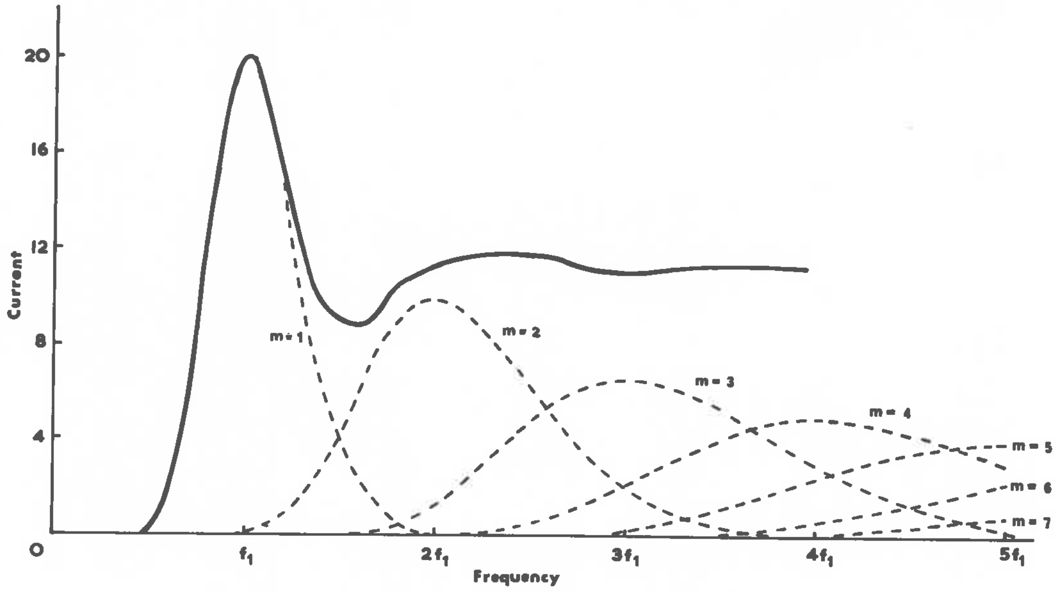

20
16
12
8
4
0
Current
m = 1
m = 2
m = 3
m = 4
m = 5
m = 6
m = 7
f_1
$2f_1$
$3f_1$
$4f_1$
$5f_1$
Frequency

Fig 46

$$J = nW - D\,\partial n/\partial x \qquad \text{at } x = h$$

$$= \frac{N_o}{(4\pi Dt)^{\frac{1}{2}}}\left[\frac{W}{2} + \frac{h}{2t}\right] \exp\left[-(h - Wt)^2/4Dt\right].$$

For the mth current pulse, $t = m/f$ and we assume $N_o = N_1/f$. The particle current at $x = h$ for all pulses is then:

$$J = \sum_m \frac{c_3}{(mf)^{\frac{1}{2}}} \left[1 + f/mf_o\right] \exp\left[-\frac{hW}{4D}\frac{f}{mf_o}\left(1 - \frac{mf_o}{f}\right)^2\right] \qquad (45)$$

where c_3 is a constant.

The theoretical curves for m = 1 to 7, together with the sum of the contributions for each pulse, are shown in Fig. 46 for the conditions of Fig. 44. Although the form of the current pulses will be considerably altered if the boundary conditions are taken into account, fair agreement is still obtained with the experimental curve of Fig. 44.

In a similar way, theoretical curves could be drawn for other experimental conditions. If equation (45) were used to plot a curve for the experimental conditions of Fig. 45, where electron diffusion is very low, the theoretical expression would not accurately represent the experimental data because of the substitution $N_o = N_1/f$. When this substitution is made, the value of the maximum current of the mth peak is predicted to be proportional to 1/m. This prediction can be seen from equation (45) when it is remembered that at the maximum of the mth peak, $f = mf_o$. However, the fall in the height of the current peaks of the experimental curve of Fig. 45 is not proportional to 1/m. For the case of $D = 0$, the pulses could be represented by step functions and the height of all current peaks of the current/frequency curve would be equal

9.3 Resolving Power.

No systemmatic investigation has been made to compare the resolving power of current peaks of experimental current/frequency curves with the values of the resolving power that are predicted from the theoretical expression for the resolving power given in equation (44),

i.e. $$\text{R.P.} = \frac{1}{4}(hW/D \log_e 2)^{\frac{1}{2}}$$

Resolving powers for the peaks of the experimental current/frequency curve of Fig. 20 for hydrogen at $E/p = 0.08$, $p = 500$ are given in Table 21.

Table 21

Peak	1	2	3	4	5	6	7	8	9	10	11	12	13
R.P.	5.7	9	14	18	23	23	23	24	24	24	24	24	24

The resolving power for these conditions predicted from equation (44) is 23. The low resolving power for the first peaks can be explained because at low frequencies the effective time that the shutters are open is greater than at high frequencies and the increase in the width of the pulse can result in a lower resolving power. It is seen that the theoretical prediction that the resolving power is independent of the number of the current peaks is largely verified.

Although there is good agreement between the theoretical and experimental values of the resolving power in the example above, other experimental values of the resolving power are up to 50% higher than the theoretical upper limit predicted from equation (44). For example, the results for the curve of Fig. 45 for nitrogen at E/p = 0.4 and p = 100 are given in Table 22.

<u>Table 22</u>

Peak	1	2	3	4	5
R.P.	7.9	11.3	10.8	11.3	11.3

The theoretical value from equation (44) is 8.2.

The value of the resolving power for the single peak of Fig. 46 is 2.6 compared with the theoretical value of 1.9.

The only explanation that can be given is that the effect of the boundary conditions at the shutters, which is not taken into account in the derivation of equation (44), can reduce the half width of the electron pulse by up to a third.

Even though there may be differences of 50% between experimental and theoretical values of the resolving power, equation (44) is still of value in indicating the order of magnitude of the resolving power. An important application is given in 11.3.1.2.

Chapter 10 : Methods of Pack and Phelps, and Hornbeck

An analysis similar to that of Chapter 8 will now be carried out to determine the influence of diffusion in introducing errors to drift velocity measurement made using the methods of Pack and Phelps, and Hornbeck.

10.1 Method of Pack and Phelps.

This method is similar to the method of Bradbury and Nielsen but with the following modifications. Instead of the electron pulses being formed by a shutter, the pulses are produced by photoelectric emission from a cathode by means of a pulsed source of ultraviolet light. The electron pulses then travel to a shutter, but this shutter is not operated by the application of a.c. sine wave voltages as in the Bradbury and Nielsen method. Instead, a constant d.c. bias is maintained between the two halves of the grid to close the shutter, and this bias is removed for a short time to open the shutter. The time interval between the flash of ultraviolet light and the opening of the shutter is varied, and the current from the electron pulses transmitted by the receiving shutter recorded. From the time at which maximum current occurs, the transit time of an electron pulse from the cathode to the shutter is determined. A second shutter is included in the apparatus further from the cathode than the first shutter, and the transit time of an electron pulse to the second shutter can similarly be determined. The difference in these two transit times should give the time taken for an electron pulse to travel the distance between the two shutters, independent of any end effects occurring at the cathode. On knowing the distance between the two shutters, the drift velocity is then calculated.

The calculation of diffusion errors for any transit time differs from that for the method of Bradbury and Nielsen. The number of electrons in each pulse is presumably proportional to the duration of each flash of ultraviolet light. This time is not varied when the time interval between the flash of ultraviolet light and the opening of the shutter is varied. Consequently, in the analysis appropriate to the method of Pack and Phelps, there is no substitution analogous to $N_o = N_1/f$ made in 8.2.1 and an increase in the value of r results. On the other hand, in the method of Bradbury and Nielsen, as the frequency of the signal applied to the shutters is increased, the first shutter is open for a shorter time and the number of electrons in each pulse is reduced. Consequently, there is a slight reduction in the frequency for which maximum current occurs which partially cancels other errors due to diffusion.

The expression for the instantaneous flux of particles at the receiving shutter is again given by J in equation (13). The variable may be changed from t to r where $r = (t - t_o)/t_o$ and t_o is given by $W = h/t_o$. After substituting $t = t_o(1 + r)$

in (13), J becomes

$$J = C(1 + r)^{-3/2} \exp\left[-\frac{hWr^2}{4D(1 + r)}\right] \text{ (C is a constant).} \qquad (46)$$

Maximum current occurs when $\partial J/\partial r = 0$, i.e. when $r = -3/h(W/D)$ (neglecting powers in r^2), where r is the relative error in assuming that t_o is the time at which maximum current occurs. If errors due to back diffusion also exist, the total relative error in any transit time measurement will be given by

$$r = -\ 5/h(W/D). \qquad (47)$$

This expression again applies only in the limiting case of a very narrow width of the initial pulse and for the receiving shutter being open for a very small interval of time.

The error predicted by formula (47) is quite large for some of the experimental conditions used by Pack and Phelps. For example, consider the conditions for an apparatus with drift distances of 2.54 cm and 5.08 cm for neon at a pressure of 48 mm Hg at 300°K. For $E/p = 0.5$ V cm^{-1} mm Hg^{-1}, W/D is approximately 7.1. (This figure is calculated using a value of Townsend's energy factor of 136 obtained by interpolation from the results of Bailey (1924).) The predicted errors in the two transit times are 28% for a drift distance of 2.54 cm and 14% for a drift distance of 5.08 cm. Differences should therefore exist in the drift velocities calculated from the two drift distances. Pack and Phelps have not reported any such differences so that it is possible that the errors due to diffusion have been masked by the finite width of the initial pulse and the finite time interval that the receiving shutters are open. These effects have not been considered in the above predictions.

The formulae for r that have been derived give the relative error as being inversely proportioned to h and so the absolute errors due to diffusion will be independent of h, to a first approximation. Thus when Pack and Phelps calculate drift velocities from the differences in the transit times for the two drift distances the absolute errors should largely cancel (see Pack and Phelps 1961, footnote 4). The errors, however, would still be expected to be greater than a second order effect, complete cancellation being unlikely as r is proportional to 1/h only in the limiting case of both the initial pulse width and the time interval that the shutters are open tending to zero.

10.2 Method of Hornbeck.

This method was used by Hornbeck (1951) to measure the drift velocity of electrons in helium, and has since been used by Bowe (1960) in an extensive and more accurate investigation. The method has also been used in a modified form by Biondi and Chanin to measure the mobility of positive ions. In the case of electrons, a flash of ultraviolet light produces an initial pulse of photoelectrons at the cathode, which then travels under the influence of a uniform electric field to the anode. The cathode and the anode are usually two large parallel circular plates. The resulting current pulse is then amplified and viewed oscillographically so that a photograph of the pulse can be obtained. Omitting influences due to diffusion the current I travelling in a Townsend gap is given by $I = NeW/h$, where N is the total number of electrons in the gap (assuming that there are no positive ions due to ionization), e is the electronic charge, W the drift velocity, and h the electrode separation. Thus, as the electrons are produced in the chamber the current rises to a constant level which is maintained while the electrons are traversing the gap. Then, as electrons are absorbed at the anode, the current will decrease, becoming zero when the pulse is completely absorbed. The form of a typical current trace is given in Fig. 47.

The transit time of the pulse is assumed to be $t_2 - t_1$, where t_1 and t_2 are the times at which the current is half of its maximum value. This procedure in determining say t_2 is based on the assumption that when half of the electrons have been absorbed, the centre of the pulse is then at the collecting electrode. This assumption is not quite accurate when account is taken of the fact that the collecting electrode acts as a conducting plane reducing the electron concentration at its surface to zero, thus distorting the shape of the pulse. The time at which half of the electrons have been absorbed will occur slightly earlier than the time at which the centre of the undisturbed pulse would have reached the collecting electrode. As a consequence, values of the drift velocity which are slightly high will be recorded.

The analysis made in this section applies only to the evaluation of t_2. The position of t_1 is determined by the rising characteristic of the current pulse which is dependent on the variation of the intensity of the light source with time and also on electron absorption due to back diffusion. The degree of back diffusion is further dependent on the energy with which the photoelectrons are emitted (see Theobold 1953).

The analysis of the position of t_2 follows closely that of 8.2.1. The first shutter in Fig. 39 is replaced by a surface from which pulses of photoelectrons are emitted. Shutter 2 is replaced by the receiving electrode. As in 8.2.1 the expression which satisfies the boundary condition $n = 0$ at $x = h$ is given by equation (12)

$$n = \frac{N_o}{(4\pi Dt)^{\frac{1}{2}}}\left\{\exp\left[-\frac{(x - Wt)^2}{4Dt}\right] - \exp\left[\frac{hW}{D} - \frac{(x - 2h - Wt)^2}{4Dt}\right]\right\}.$$

From elementary theory (as in Appendix II) the current in a Townsend gap is given by I_2, where

$$I_2 = NeW/h - (n_2 - n_1)De/h \qquad (48)$$

where N is the total number of electrons in the gap,

W the drift velocity,

h the length of the gap,

e the electronic charge,

n_2 the electron density at the collecting electrode,

n_1 the electron density at the electron emitting surface.

When the pulse is collected by the anode, the electron density at the cathode is assumed to be zero, so $n_1 = n_2 = 0$. Equation (48) then becomes

$$I_2 = \frac{N_o eW}{h} \int_{-\infty}^{h} \frac{1}{(4\pi Dt)^{\frac{1}{2}}} \exp\left[-\frac{(x - Wt)^2}{4Dt}\right] dx$$

$$- \frac{N_o eW}{h} \int_{-\infty}^{h} \frac{1}{(4\pi Dt)^{\frac{1}{2}}} \exp\left[\frac{hW}{D} - \frac{(x - 2h - Wt)^2}{4Dt}\right] dx$$

$$= \frac{N_o eW}{h\pi^{\frac{1}{2}}} \left[\frac{\pi^{\frac{1}{2}}}{2} + \operatorname{erf} \frac{h - Wt}{(4Dt)^{\frac{1}{2}}}\right] - \frac{N_o eW}{h\pi^{\frac{1}{2}}} \left[\frac{\pi^{\frac{1}{2}}}{2} - \operatorname{erf} \frac{h + Wt}{(4Dt)^{\frac{1}{2}}}\right] \exp \frac{hW}{D}, \qquad (49)$$

where $\operatorname{erf} x = \int_0^x \exp\left[-u^2\right] du$. If we ignore the effect of the collecting electrode, the image term in the expression for n is omitted and the expression for current as a function of time is given by I_1, where

$$I_1 = \frac{N_o eW}{h\pi^{\frac{1}{2}}} \left[\frac{\pi^{\frac{1}{2}}}{2} + \operatorname{erf} \frac{h - Wt}{(4Dt)^{\frac{1}{2}}}\right]. \qquad (50)$$

The shapes of the falling characteristics of I_1 and I_2 are shown in Fig. 48 in which the difference between I_1 and I_2 is shown to be much larger than actually occurs under usual experimental conditions.

When the pulse is at the collecting electrode, $t = h/W$ and I_1 becomes $I_1 = \frac{1}{2}N_o eW/h = \frac{1}{2}I_o$, where I_o is the maximum current. However, because of the boundary condition imposed by the collecting electrode, the oscillographic trace gives I_2 and not I_1. Thus, when t_2 is estimated from the half current maximum of I_2 there will be an error introduced equal to Δt (see Fig. 48). The relative error r, in the determination of t_2, will be given by

$$r = \frac{\Delta t}{t_2} \simeq \frac{\Delta t^1}{t_2} \simeq \frac{\Delta I}{t_2} \Bigg/ \left(\frac{dI_1}{dt}\right)_{t = t_2}, \qquad (51)$$

where

$$\left(\frac{dI_1}{dt}\right)_{t = t_2} = - \frac{N_o eW^2}{h(4\pi D t_2)^{\frac{1}{2}}},$$

and

$$\Delta I = \frac{N_o eW}{h\pi^{\frac{1}{2}}}\left[\frac{\pi^{\frac{1}{2}}}{2} - \text{erf}\left(\frac{hW}{D}\right)^{\frac{1}{2}}\right] \exp \frac{hW}{D}.$$

From a formula given in Comrie (1948) for large x,

$$\frac{\pi^{\frac{1}{2}}}{2} - \text{erf } x = \frac{e^{-x^2}}{2x}\left[1 - \frac{1}{2x^2} + \frac{3}{(2x^2)^2} - \dots\right]$$
$$\simeq e^{-x^2}/2x \quad \text{if } x > 2.$$

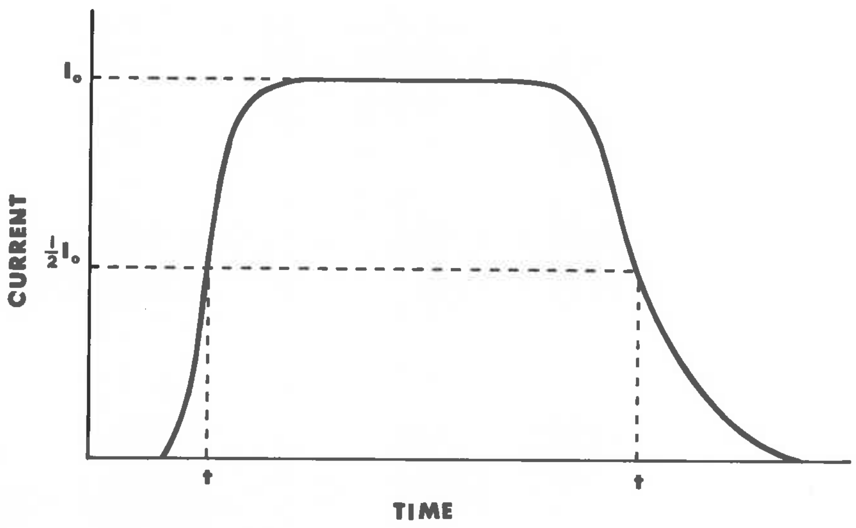


Fig 47

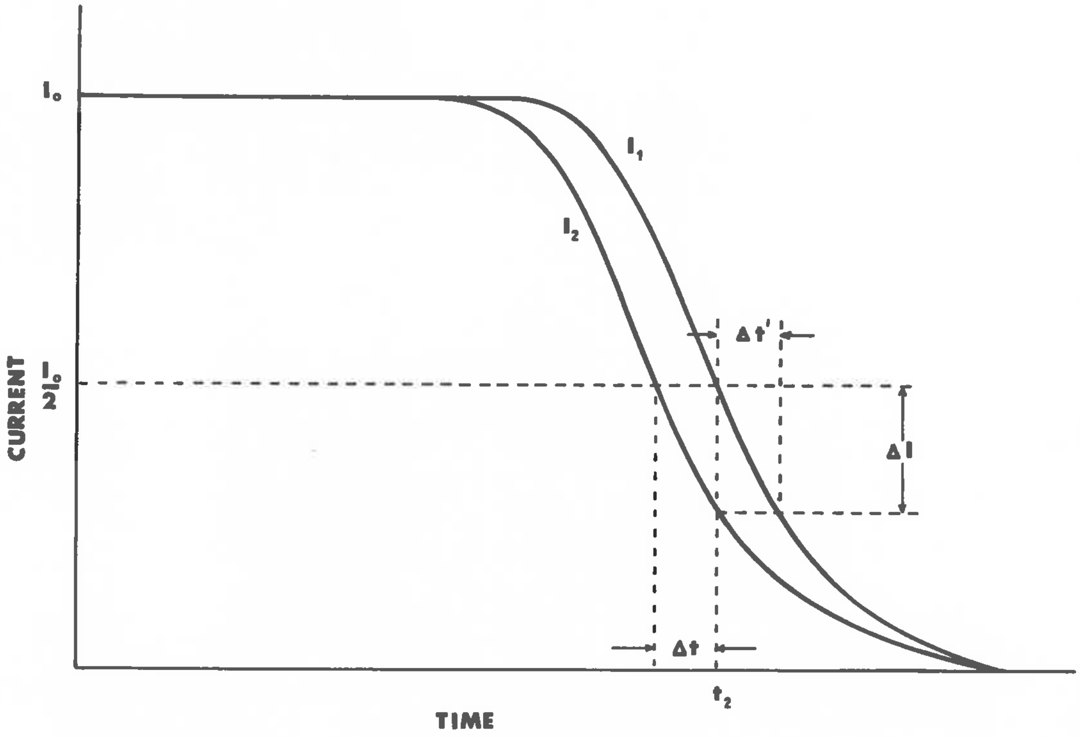


Fig 48

As experimental values of W/D and h are generally such that $(hW/D)^{\frac{1}{2}} > 2$, we can use this formula to simplify ΔI, and obtain

$$\Delta I \simeq \frac{N_o e W D^{\frac{1}{2}}}{2h\pi^{\frac{1}{2}}(hW)^{\frac{3}{2}}}. \qquad (52)$$

On substituting for ΔI and $(dI_1/dt)_{t = t_2}$, equation (51) becomes $r = -1/(hW/D)$. The error is such that the measured value of the drift velocity is too high.

Hornbeck (1951) has reported the appearance of spikes at the commencement of the oscillographic traces which indicate the loss of electrons due to back diffusion to the cathode. Biondi and Chanin (1954) have reported a similar effect in the case of positive ions. In the case under discussion the loss of some of the electrons from the trailing edge of the pulse to the cathode would lead to a shifting forward of the maximum electron density in the pulse. If the pulses are initially very narrow and are formed close to the surface, the analysis of 8.3 is directly applicable giving an additional relative error of $2/h(W/D)$. This error may be cancelled or enhanced by further errors which may exist in the determination of t_1. However, the formulae for r that have been derived should indicate the order of the diffusion errors that are likely for any given experimental conditions.

For the measurements of Bowe the errors due to diffusion predicted from $r = 3/h(W/D)$ are in almost every case within the experimental error of 5%.

SECTION IV

Chapter 11 : Discussion of Results

11.1 Comparison of Results with those of other Investigations.

The 'Best Estimate' values of the results of Tables 3, 4, 5 and 6 are now compared with the results of other investigations. When account is taken of the limits of the experimental accuracy of each investigation, the present results in nitrogen at room temperature agree with the results of Pack and Phelps (1961), Nielsen (1936), Commetti and Huber (1960) and Nagy, Nagy and Dési (1960). The results of Bowe (1960) and Bortner, Hurst and Stone (1951) are up to 6% lower than the present results, while the results of Klema and Allen (1950) and Colli and Facchini (1952) are generally higher than the present results. The recent results of Commetti and Huber were obtained using the method of Bortner et alia and the recent results of Nagy et alia were obtained using the method of Colli and Facchini. Thus agreement of drift velocity results in nitrogen has been obtained between three quite different methods, over the common range of E/p, within the experimental error. No explanation can be given for the low results of Bowe, which were obtained using the method of Hornbeck.

The results in hydrogen at room temperature agree with the previous measurements of Bradbury and Nielsen (1936) and Pack and Phelps (1961) to within the experimental error. The only previous measurements of the drift velocity at liquid air temperatures is the investigation of Pack and Phelps, with which agreement is obtained.

The most recent of the investigations is that of Pack and Phelps (1961) which gives results at four different gas temperatures and over a very wide range of E/p. However, as the graph on which the experimental results were plotted has a logarithmic scale, the width of the line passing through the experimental points corresponds to 7% of the drift velocity. On their graph, the experimental points are indicated by circles and show a scatter which is comparable with the width of the line.

The values of the drift velocity that have previously been most generally accepted are those of Nielsen (1936) and Bradbury and Nielsen (1936) who give values of the drift velocity at 293°K over a range of E/p from 0.05 to 20. The scatter of the experimental points indicates that the accuracy of their investigation was of the order of 4%.

A comparison of the results of various investigations is made graphically in Fig. 49 for nitrogen at 293°K over the range $0.2 < E/p < 2.2$. The shaded area corresponds to the line on the graph of the results of Pack and Phelps. The experimental points of Nielsen and Bortner et alia are given, together with every third point of the investigation of Bowe.

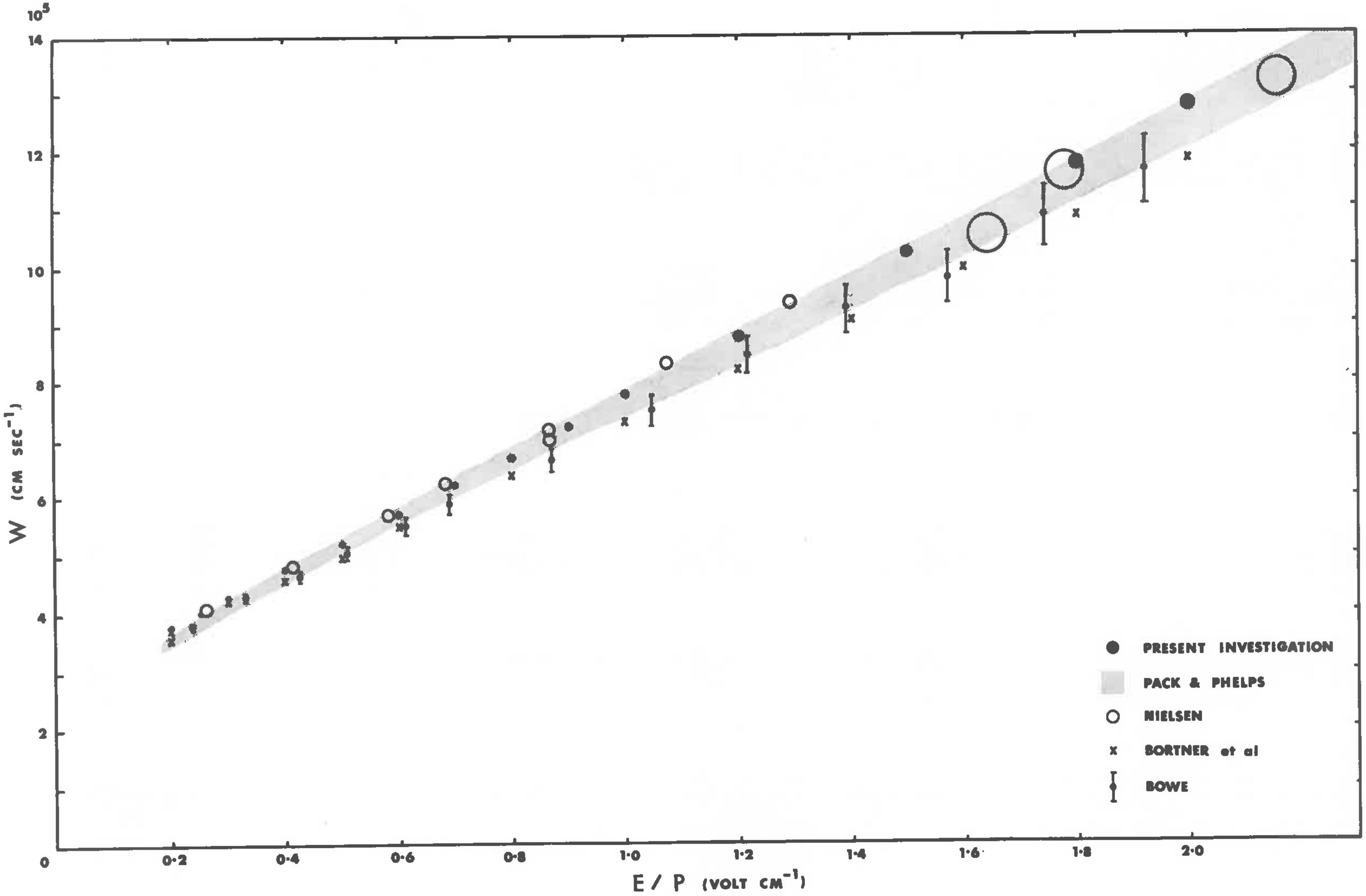

10^5
14
12
10
8
6
4
2
0
W (CM SEC^{-1})
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8
2·0
E / P (VOLT CM^{-1})
● PRESENT INVESTIGATION
▨ PACK & PHELPS
○ NIELSEN
x BORTNER et al
BOWE

FIG 49

The solid circles represent the experimental points of the present investigation, the size of the circles indicating the range ± 1% of the measured value. All results have been corrected to apply for a gas density corresponding to a temperature of 293°K.

11.2 Accuracy of Results.

The method of Bradbury and Nielsen has the advantage that the transit time of an electron pulse between two shutters is measured simply by determining the frequency of a sine wave a.c. signal, which can be done accurately with a frequency counter. The corresponding measurement in the methods of Pack and Phelps, and Bortner et alia, requires the use of a delay line. In the oscillographic methods the length of an oscillographic trace is required to be measured accurately.

With the method of the present investigation, it is possible to determine whether errors are present due to field distortion or phase errors from the a.c. voltages by checking whether measurements are independent of V_G (at conditions of low diffusion) or on the number of the peak that is chosen of the current/frequency curve.

From the general consistency with pressure of the final results and also from the analysis of errors that has been made, the errors that are possible in the 'Best Estimate' values of the results are considered to be of the order of 1% at 293°K and 2% at 77.6°K.

11.3 Applications of Results.

11.3.1 Theory of Effects of Diffusion.

11.3.1.1 Measurements at High E/p and Measurements in Air at Low E/p.

The theoretical expressions that have been derived in the analysis of the effect of diffusion are not only of value, for example, in providing an explanation for the pressure dependence of the observed values of W, but also have applications in the consideration of the possibility of further experiments.

For example, if it is desired to obtain values of W at very high values of E/p, it is desirable to have low applied voltages to prevent electrical breakdown within the tube. At a given value of E/p this requirement implies a low value of hp. However, for the error due to diffusion to be a minimum and the resolving power to be a maximum, it is desirable that the value of hW/D be as high as possible. Thus, at a given value of E/p, it is required to have hp as high as possible which is the opposite requirement to avoid breakdown within the tube.

In a similar way an assessment can be made of the difficulties of taking measurements of W in air at low values of E/p. Such measurements would be invaluable to provide a cross check on measurements that have been made involving the cross modulation of radio waves in the ionosphere. To avoid the loss of electrons due to attachment, in experiments to find W in air, it is desirable that hp be as low as possible.

For the effect of diffusion to be a minimum it is again desirable that hp be a maximum and thus similar difficulties are predicted in the obtaining of results as in the first example.

11.3.1.2 Application to Measurements in Water Vapor.

An important application has been made of the theoretical expression that has been derived for the resolving power (equation 44). In experiments to measure the drift velocity of electrons in water vapor, two sets of current peaks were obtained, the fundamental frequency f_1 of one set being approximately one hundred times the value of f_1 for the other set. It was naturally suspected that the particles with the higher velocity were electrons.

However, the fact that for these particles, the graph of W against E/p up to E/p = 6 was a straight line passing through the origin, together with other evidence, suggested that the energy of these particles was close to the thermal energy of the gas molecules. An additional independent estimate of the energy of the particles at E/p = 9 was made possible by the expression for the resolving power (44). The limiting value of the resolving power was determined by taking current/frequency curves at various RMS voltages applied to the shutters. The expression for the resolving power, equation (44), enabled an estimate of W/D to be made and use of equation (5) then led to a value of $k_1 = 1.5$. However, the value of k_1

for electrons in water vapor at $E/p = 9$ was known from independent measurements to be approximately 20. Thus the expression for the resolving power provided valuable evidence that the particles being measured were not electrons but negative ions of very high mobility.

11.3.1.3 Form of Current Frequency Curves.

A comparison of Figs. 44 and 45 indicates that the form of the current/frequency curves can vary very widely, it being possible under some conditions that even the first of the current peaks may not be clearly defined. The expression $3/h(W/D)$, from the theoretical analysis, provides a useful criterion as to whether peaks will be obtained. The conditions of Fig. 44 are such that $3/h(W/D) = 0.08$ and thus wherever $3/h(W/D) \geqslant 0.08$ we would not expect to obtain more than one peak of the current/frequency curve.

11.3.2 Measurements of the Drift Velocity.

Although it is not the primary concern of this thesis to make a detailed analysis and application of the results that have been obtained, two applications of the results are discussed in which use is made of the new data at very low electron energies.

11.3.2.1 Collision Cross Sections.

By making certain assumptions which simplify equation (2) ($D = \frac{1}{3}\overline{\ell c}$), an estimate of the effective collision cross section of the gas molecules can be made from the experimental values of W and k_1. Using a method originated by Townsend in which it is assumed that A is independent of c and the velocity distribution is Maxwellian, A can be calculated for various values of k_1. The values of A that are so obtained are not independent of c and Druyvesteyn (1934) has shown that the velocity distribution of the electrons is not Maxwellian for $k_1 > 1$. Nevertheless the values of A are a good first approximation of the correct values.

If A is independent of c equation (2) becomes $D = \bar{c}/3NA$. In the particular case where the velocity distribution of the electrons is Maxwellian it can be shown (Appendix VI) that

$$\bar{c} = 2\left(\frac{2\,R_o\,T}{N_o m \pi}\right)^{\frac{1}{2}} k_1^{\frac{1}{2}} \qquad (53)$$

where T is the temperature of the gas in degrees K, R_o is the universal gas constant in ergs, N_o is Avagadros number and m is the mass of the electron.

From equation (2)

$$A = \bar{c}/3ND = (\bar{c}/3NW)W/D$$

$$= \frac{E}{N}\,\frac{2\,e}{3Wk_1^{\frac{1}{2}}}\left(\frac{2N_o}{R_oT\,m\pi}\right)^{\frac{1}{2}} \qquad \text{(using equations (53) and (5).)}$$

$$= \frac{E}{p}\,\frac{2\,e}{3Wk_1^{\frac{1}{2}}}\left(\frac{2\,R_oT}{N_o m\pi}\right)^{\frac{1}{2}}\frac{1}{300 \times 13.6 \times 98} \qquad \left(\text{using } p = \frac{1}{3}\overline{NM6^2}\right.$$

$$\left.\text{and } \tfrac{1}{2}\overline{M6^2} = \frac{3}{2}\,\frac{R_oT}{N_o}\right)$$

where p is in mm Hg, E in volts cm^{-1}.

Thus $A = 4.26 \times 10^{-9}\ (E/p)(1/Wk_1^{\frac{1}{2}})\ \text{cm}^2$ at 293°K

$= 8.27 \times 10^{-9}\ (E/p)(1/Wk_1^{\frac{1}{2}})\ \text{cm}^2$ at 77.6°K

where p is defined as in 5.1.

If μ is the energy of the electrons ~~in electron volts,~~

$$\mu = \tfrac{1}{2} m \overline{c^2} = \frac{3 R_o T}{2 N_o} k_1 \quad \text{ergs}$$

$$= 0.0374\ k_1 \ \text{ev} \quad \text{at } 293^\circ\text{K}$$

$$= 0.00990\ k_1 \ \text{ev} \quad \text{at } 77.6^\circ\text{K}.$$

For both hydrogen and nitrogen we have two independent sets of experimental values of W and k_1, one set being for 293°K and the other set for 77.6°K (the values of k_1 are taken from the investigations mentioned in 9.1.2). It is of interest to compare the agreement of values of A derived from the two sets of experimental data. These values of A for both hydrogen and nitrogen are shown graphically in Fig. 50. The agreement is good, considering that the assumptions that have been made are only approximately correct and the errors that are possible in the values of k_1 for 77.6°K are of the order of 7%. To obtain a better estimate of A as a function of c we could repeat this calculation again, using the dependence of A upon c of Fig. 50, instead of assuming A to be constant. The accuracy of the values of k_1 that are at present available, does not, at present, warrant this procedure.

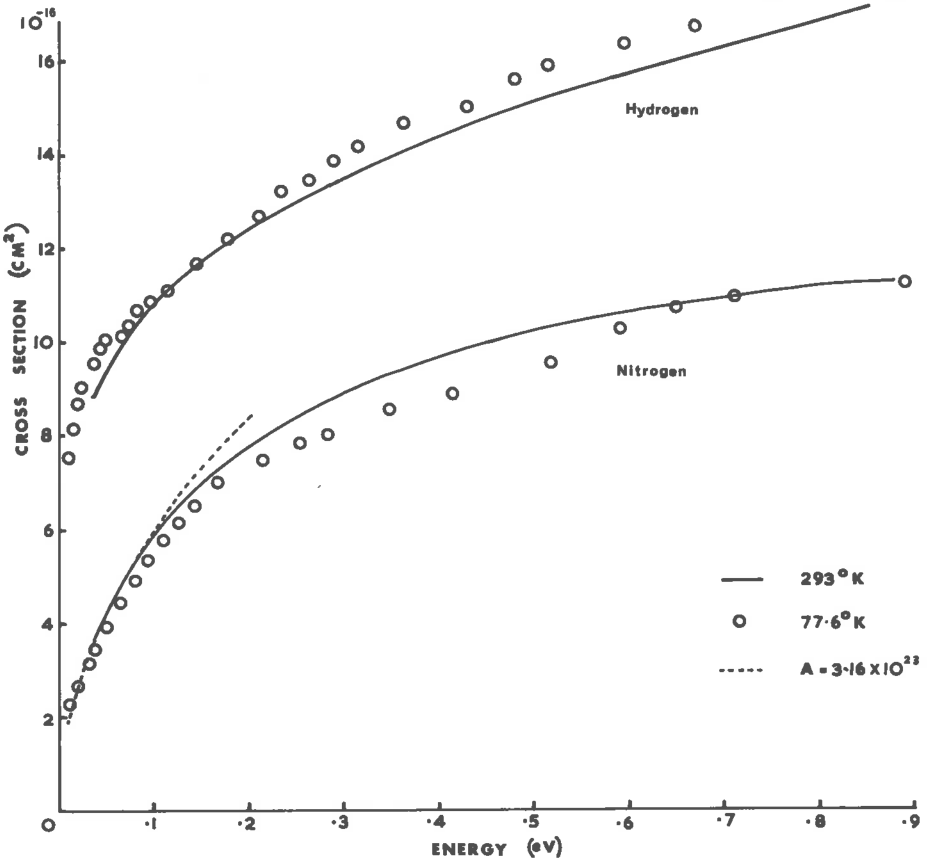

10⁻¹⁶
16
14
12
10
8
6
4
2
CROSS SECTION (CM²)
Hydrogen
Nitrogen
293°K
77·6°K
A = 3·16 X 10²³
0
·1
·2
·3
·4
·5
·6
·7
·8
·9
ENERGY (eV)

Fig 50

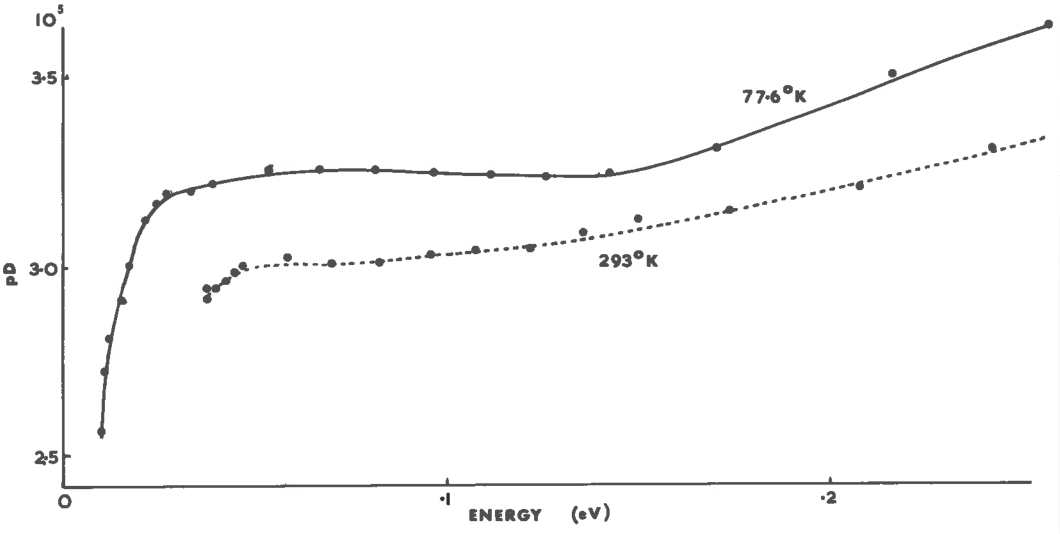

10⁵
3·5
3·0
2·5
pD
77·6°K
293°K
0
·1
·2
ENERGY (eV)

Fig 51

The consistency of the values of A derived from the results at 293°K with those derived from results at 77.6°K is strong evidence of the validity of the theoretical equations for W and D (equations (1) and (2).)

A recent investigation (Huxley 1959) derived a relation for the dependence of A upon c for nitrogen at low electron energies from the observation that the results at room temperature give pD = constant at very low values of E/p. If pD is constant for very low electron energies, $\overline{(c/A)}$ is constant (from equation (2)) and thus A is directly proportional to c, this deduction being independent of the form of the velocity distribution for the electrons. However the new results at 77.6°K indicate that at lower values of the electron energy than it is possible to obtain at 293°K, the value of pD is no longer constant. The values of pD deduced from the results at both 293°K and 77.6°K are given in Fig. 51. However, if we assume $pD = 3.2 \times 10^5$, it follows that

$$A = 3.16 \times 10^{-23} \; c \qquad (54)$$

It is seen from Fig. 50 that this is a good approximation for the collision cross section at low energies.

Considerable progress has been made by Frost and Phelps (1962) in the solution of the problem of deriving the exact dependence of A upon c from values of W and k_1, if these experimental quantities are known accurately. Their analysis, which employs the Boltzmann equation, is the first to take account of the modification to the velocity distribution which is caused by the inelastic collisions which occur during rotational and vibrational excitation.

11.3.2.2 Studies of the Ionosphere and Rotational Excitation.

The results of laboratory measurements of W and k_1 in nitrogen have an application in studies of radiowave interaction in the ionosphere. The theory that has been developed to explain Ionospheric Cross-modulation (Huxley 1952, and references contained therein) assumes that the mean energy ΔQ lost by an electron in a collision with a gas molecule is proportional to the excess of the mean energy Q of the electron above that of a gas molecule Q_o,

$$\text{i.e.} \qquad \Delta Q = G\,(Q - Q_o) \qquad (55)$$

where G is the constant of proportionality.
As $\Delta Q = \eta Q$ (where η is the mean fraction of the energy of an electron lost in a collision and $Q/Q_o = k_1$) and if the electrons are assumed to have a Maxwellian velocity distribution, equation (55) can be expressed as

$$\eta k_1 = G\,(k_1 - 1) \qquad (56)$$

From experiments investigating radiowave interaction it is possible to measure $G\nu$ for the region of the ionosphere in which the interaction takes place (ν is the collision frequency of the electrons with the gas molecules) and also the height of this region above the surface of the earth. Other ionospheric studies have determined ν at various heights and thus a value of G is obtained.

By using equation (56) it is also possible to evaluate G from laboratory measurements of W and k_1. Values of η can be obtained from the measurements of W and k_1 and the gradient of the graph of ηk_1 against k_1 at $k_1 = 1$ gives the value of G. Unfortunately the value of G for air cannot be evaluated accurately in this way as measurements have not as yet been made at very low values of E/p because of the difficulties mentioned in 11.3.1. However, Huxley (1959) has shown that results obtained from nitrogen should give values of G in close agreement with the values that would be obtained for air.

To draw the graph of ηk_1 against k_1 it is necessary to derive an expression for η in terms of W and k_1. A similar derivation to that which follows is given in Huxley and Crompton (1962). At equilibrium, the average rate of loss of the energy of electrons due to collisions with gas molecules $\left[\eta\, Q\, \overline{(c/\ell)}\right]$ is equal to the average rate of gain of energy from the electric field $\left[E\, e\, W\right]$.

$$\text{Thus} \quad \eta = \frac{E\, e\, W}{Q\, \overline{(c/\ell)}} = \frac{E\, e\, W}{F\, k_1\, Q_0 \overline{(c/\ell)}}$$

where $F = \overline{c^{-2} \frac{d}{dc}(\ell c^2)} \quad \overline{c^2}/\overline{\ell c}$ (See equation (6))

Using equation (1) and $Q_o = 3 R_o T/2 N_o$, η becomes

$$\eta = \frac{2\,\overline{c^2}}{3F^2\,\overline{(c/\ell)}\,\overline{(\ell c)}} \cdot \frac{N_o m}{R_o T} \cdot \frac{W^2}{k_1} \qquad (57)$$

In the particular case where ℓ is assumed to be independent of c and the velocity distribution is Maxwellian, the terms involving velocity averages can be evaluated using the data of Appendix VI. Then η becomes

$$\eta = \frac{\pi N_o m}{4 R_o T} \frac{W^2}{k_1}$$

$$= 1.769 \times 10^{-14} W^2/k_1 \quad \text{at } 293^\circ K$$

$$= 6.679 \times 10^{-14} W^2/k_1 \quad \text{at } 77.6^\circ K$$

The graph of ηk_1 against k_1 for nitrogen at $293^\circ K$ has been drawn using the above expression for η, where η has been determined using values of W from this investigation and unpublished values of k_1 obtained by Dr. R.W. Crompton. This graph, which is shown in Fig. 52, gives a value of G of 2.2×10^{-3}.

It is of interest to note that when results were available only at high E/p it was considered possible that the gradient of the graph at $k_1 = 1$ might be zero (Crompton, Huxley and

Sutton 1953). If this were to be true, equation (56) would be invalid and the theory of radiowave interaction would have to be reformulated.

Before comparison is made of the laboratory measurements of G with the value of G obtained from measuments in the ionosphere, account must be taken of the effect of temperature on the value of G, for the temperature of the region of interaction in the ionosphere is estimated to be 196°K. From an analysis of the effect of rotational excitation of diatomic gas molecules (Huxley 1956), Huxley has derived a formula for the dependence of B on temperature, where, by definition $BN = G\varsigma$ (Huxley 1959). Thus the temperature dependence of G can also be derived as follows:

$$G = BN/\varsigma \quad = \quad BN/(\overline{c/\ell}) \quad = \quad B/(\overline{cA}).$$

We now use the relation obtained by Huxley (1959) for A in nitrogen for electrons of low energy.

$$A \simeq 3.29 \times 10^{-23} \quad c$$

(Equation (54) is a more accurate expression for A in terms of c but its use would necessitate a re-evaluation of α and β of equation (58), which is not justified by the accuracy of the data for k_1 at present available). Also, as the electrons are close to the thermal energy of the gas molecules Q_o, $\frac{1}{2} m\overline{c^2} = Q_o$. It follows that $G = 1.385 \times 10^{-5}\ B/Q_o$.

Using the formula for B derived by Huxley (1959) the resulting expression for G is

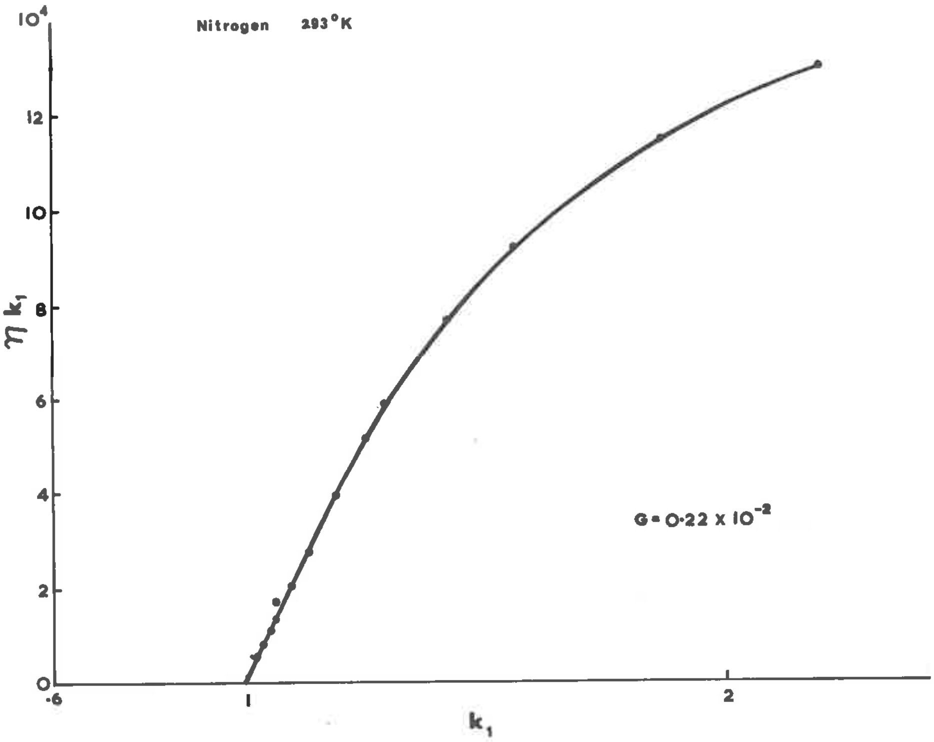
$$G = 1.385 \times 10^{-5} \left[Q_o^{-5/2} \alpha\beta \exp\left(-\beta/Q_o\right) + 3.78 \right]. \tag{58}$$

The values of α and β that were obtained by Huxley from results of W and k_1 in nitrogen at room temperature were $\alpha = 7.29 \times 10^{-18}$ and $\beta = 4.41 \times 10^{-14}$. G is temperature dependent through Q_o. Equation (58) predicts the following values of G at the temperatures given in Table 23.

Table 23

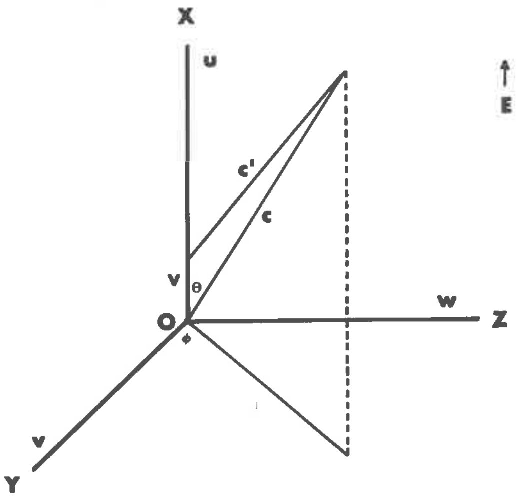
T	77.6	196	288
G	8.8×10^{-3}	4.6×10^{-3}	2.5×10^{-3}

The value of G at 293°K as obtained from Fig. 52 is 2.2×10^{-3}. (The close agreement with the value in Table 23 is to be expected as α and β are determined from data for room temperature.) A similar determination of G from the results at 77.6°K gave 7.6×10^{-3}. Experiments investigating ionospheric cross modulation give the approximate value of $G\nu$ as 2×10^{3} and ν is known to be approximately 5×10^{5} for this region, (Phelps and Pack 1959). Thus the resulting value of G from ionospheric measurements is 4×10^{-3}, which is known to apply for a temperature of approximately 196°K.



Nitrogen 293°K
10^4
12
10
8
6
4
2
0
ηk_1
G = 0·22 x 10^{-2}
·6
1
2
k_1

Fig 52



X
u
E
c'
c
v
θ
O
φ
w
Z
v
Y

Fig 53

The agreement with the values in Table 23 is good considering that:

(1) α and β have been evaluated using measurements of $k_1 - 1$ which are known to be approximate.

(2) A is only approximately proportional to c for low electron energies, as is shown in 11.3.2.1.

The agreement of the experimental values of G with theory is evidence supporting the theoretical analysis of rotational excitation that has been made by Huxley (1956). The results also appear to be consistent with the independent measurements in the ionosphere.

APPENDIX I

Formulae for W and D.

The derivations that are given in this Appendix are those of Huxley and Crompton (1962) for the simple case of isotropic scattering. In this section vectors such as the momentum of an electron of mass m and speed c are written as m $\underline{c}$. $|m\,\underline{c}|$ is written as m c.

Definitions.

The velocity distribution of electrons of uniform density n_o in space, travelling with drift velocity W due to an electric field E is represented by F (c, u). (u is the component of $\underline{c}$ in the direction of the electric field, v and w are the other components of $\underline{c}$). F is defined such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F\ du\ dv\ dw = 1 \qquad (59)$$

By definition, W is the vector average of $\underline{c}$

$$\text{i.e.} \qquad W = \bar{u} = \iiint u\ F\ du\ dv\ dw \qquad (60)$$

Assumption of Form of Velocity Distribution.

If E = 0, the surfaces in velocity space over which the density of points is uniform would be surfaces of a series of concentric spheres whose centres are the origin. In the presence of an electric field it is assumed that the surfaces remain as spheres but that the centres of the shperes are displaced in the direction of E by a velocity V where $V = V(c^1)$, c^1 being the radius of the sphere.

$$\text{Then} \quad \underline{c} = \underline{c}^1 + \underline{V}(c^1) \quad \text{and } F(c,u) = f(c^1)$$

As W is only a few per cent of $\bar{c}$, V is assumed to be generally very small compared with c. Referring to Fig. 53, c^1 is then approximately given by

$$c^1 = c - V(c^1) \cos\theta$$

$$\text{Thus} \quad F(c,u) \simeq f(c - V(c^1)\cos\theta) \simeq f(c) - V(c^1)\cos\theta \frac{\partial f}{\partial c} \qquad (61)$$

$V(c^1)$ can similarly be expressed in terms of c as follows:

$$V(c^1) = V(c - V(c^1)\cos\theta) \simeq V(c) - V(c^1)\cos\theta \frac{\partial V}{\partial c}$$

On substituting in (61) and ignoring the second order term we obtain

$$F(c,u) = f(c) - V(c)\cos\theta \frac{df}{dc} \qquad (62)$$

This expression for F can now be substituted in equations (59) and (60) and the integrations carried out using polar coordinates as indicated in Fig. 53.

$$\text{Equation (59) becomes} \qquad \int_0^\infty 4\pi c^2 f(c)\, d c = 1 \qquad (63)$$

Equation (60) becomes

$$W = - \int_0^\infty \frac{4\pi}{3} c^3 V(c) \frac{df}{dc} dc$$

$$= \int_0^\infty \frac{4\pi}{3} f(c) \frac{d}{dc} \left(c^3 V(c) \right) dc \qquad (64)$$

Expression for V(c).

The function V(c) is obtained by equating the average rate of loss of momentum of the electrons by collisions with the average rate of increase of momentum due to the action of the electric field.

The average rate of increase of momentum for an electron is E e. (65)

In evaluating the average rate of loss of momentum for an electron, averages have to be taken for all possible types of collision. We consider initially an electron of mass m and velocity c colliding with a gas molecule of mass M and velocity C and obtain the required average through the following steps.

a. All directions of C are equally probable, so that the average effect is obtained by putting C = 0.

b. Thus the average for all magnitudes C, is obtained by putting C = 0.

c. Scattering for an electron is assumed to be isotropic. The average for all directions of scattering when $m << M$ can be shown to be the average of the change of momentum for an electron scattered without change of direction (which is 0), with the change of momentum when the electron is scattered such that its velocity is reversed (which is $- 2m\underline{c}$). The average loss of momentum for an electron of velocity $\underline{c}$ for all directions of scattering is thus $m\underline{c}$.

As $c \gg C$ the speed of an electron relative to the gas molecules is very close to c and so c/ℓ is the average number of collisions in unit time. Thus the average rate of loss of momentum for an electron of velocity $\underline{c}$ is $m\underline{c}\, c/\ell$. The average rate of loss of momentum in the direction of E is $mc^2 \cos\theta/\ell$.

d. and e. Finally the average is obtained for all directions of $\underline{c}$ and for all magnitudes c. This average is performed by integration over all velocity space. The average rate of loss of momentum in the direction E is thus

$$\int_0^\infty \int_0^\pi \int_0^{2\pi} \frac{mc^2 \cos\theta}{\ell} \; F \; \cancel{dc} \; c \sin\theta \; d\varphi \;\; c\, d\theta\, dc$$

On substituting F, as given in equation (62), the average becomes

$$\int_0^\infty \frac{4\pi m}{3} \; f \; \frac{d}{dc} \left(c^4 V/\ell\right) \; dc \qquad (66)$$

Equating (65) and (66) we obtain

$$\int_0^\infty \frac{4\pi m}{3} \; f \; \frac{d}{dc} \left(c^4 V/\ell\right) \; dc \;=\; E\, e \qquad (67)$$

A solution of (67) is $V = E e \ell / mc$ as can be verified by substitution, using equation (63).

Formula for W.

W can now be evaluated by substituting the above expression for V in equation (64). W becomes

$$W = \int_0^\infty \frac{4\pi}{3} f \frac{d}{dc} (c^3 Ee\ell/mc) \, dc$$

$$= \frac{Ee}{3m} \overline{c^{-2} \frac{d}{dc} (\ell c^2)}$$

Formula for D.

We consider the rate of change of momentum of electrons in unit volume in the direction of x, at x, where the density of the electrons is a function of position, i.e. $n = n(x)$. This rate of change of momentum must be zero for $E = 0$ as there is then no external force acting on the particles.

The rate of loss of momentum per unit volume at x of particles of velocity c due to collisions is

$$n_c m \bar{c} c/\ell = n_c m w_c c/\ell \tag{68}$$

where n_c is the density of particles of velocity c and w_c is the average velocity in the x direction of all particles of speed c.

This rate of loss of momentum must be restored by the increase of momentum by the nett flow of particles to the unit volume.

The rate of transport of momentum in the direction OX at x by electrons of speed c into unit volume is

$$\tfrac{1}{2}\, n_c m\, \overline{u^2}$$

$$= \frac{\tfrac{1}{2}\, n_c\, m \int_0^{\frac{\pi}{2}} \int_0^{2\pi} (c\cos\theta)^2 \; c\, d\varphi \;\; c \sin\theta \;\; d\theta}{\int_0^{\frac{\pi}{2}} \int_0^{2\pi} c\, d\varphi \;\; c \sin\theta \;\; d\theta}$$

$$= n_c\, m\, c^2/6.$$

Momentum is also being transported out of unit volume through the unit area at x by electrons of speed c travelling in the direction - OX. The total rate of increase of momentum of particles in unit volume at x due to transport of particles through unit area at x, for particles of speed c is thus $n_c\, m\, c^2/3$.

Similarly at x + dx the total rate of loss of momentum to the unit volume is

$$(n_c + \frac{\partial n_c}{\partial x} dx)\;\; m\, c^2/3.$$

The total rate of gain of momentum per unit volume at x due to transport is thus

$$- \frac{1}{3}\; m\, c^2\, \frac{\partial n_c}{\partial x} \qquad (69)$$

Equating with equation (68) we obtain

$$n_c\, m\, w_c\; c/\ell \;=\; -\frac{1}{3}\, m\, c^2\, \frac{\partial n_c}{\partial x}$$

$$\therefore\; \frac{n_c\, w_c}{-\partial n_c/\partial x} \;=\; \frac{1}{3}\, \ell\, c$$

But D_c is defined such that $n_c w_c = -D_c\, \partial n_c/\partial x$.

Thus D_c is given by

$$D_c = \frac{1}{3}\, \ell\, c$$

and the formula for D is

$$D = \frac{1}{3}\, \overline{\ell\, c}\, .$$

APPENDIX II

Expressions for Current

1. Hornbeck's Method.

The following derivation is given by Cobine (1941) for the current in the external circuit when a point charge q moves between two large parallel plate electrodes separated by distance h, the potential difference between the plates being V.

If the charge moves a distance dx in time dt the work done is $Vq\,dx/h$. This energy is supplied by the external power supply and equals $V\,i\,dt$ where i is the current in the external circuit. Thus the current i is given by

$$i = \frac{q\,dx/dt}{h} \tag{70}$$

The case is now considered of a distribution of electrons within the gap, each electron having charge e, where $n = n(x)$ gives the density of electrons throughout the gap. The current due to the element of charge within a volume element $dx\,dS$ is given (from equation (70)) by di where

$$di = \frac{n\,e\,dx\,dS\,dx/dt}{h}$$

When diffusion occurs, the flux of particles at x, $n\,dx/dt$, is given by

$$n\,dx/dt = n\,W - D\,dn/dx.$$

Thus the total current I is given by

$$I = \iiint \frac{(n\,W - D\,dn/dx)\,e\,dx\,dS}{h}$$

$$= N\,eW/h - D\,e\,(n_2 - n_1)/h \qquad (71)$$

where N is the total number of electrons within the gap

n_2 is the electron density at the collecting electrode

n_1 is the electron density at the emitting surface.

Equation (71) is the relation used in 10.2

2. Method of Bradbury and Nielsen.

The most general expression for current at an electrode is

$$I = \iint (dD_1/dt + J)\,dS \qquad (72)$$

where D_1 is the electric displacement and J is the particle current.

The surface of integration is that of the collecting electrode.

Equation (72) follows from Maxwell's equation

$$\text{curl } H = \int (dD_1/dt + J).$$

It can be shown that for a point charge q moving within a gap that

$$\frac{q\,dx/dt}{h} = \iint \frac{d\,D_1}{dt}\,dS$$

In the method of Bradbury and Nielsen, the time constant of the electrometer is large and as a result displacement currents produced by the electron pulses are not recorded. As a result the term dD/dt of equation (72) can be ignored and the current is determined by J where

$$J = n e W - De\partial n/\partial x.$$

The average rate of receipt of charge by the collecting electrode is proportional to the value of J when the second shutter is open. Provided that V_G is kept constant as the frequency is varied, the total time that the shutter is open is independent of the frequency of the signal applied to the shutters. Thus the procedure in the theoretical analysis of 8.2.1 is justified.

APPENDIX III

Reference Thermocouple Calibration

The following table is given by Scott (1941) over the temperature range -192 to $0^{\circ}C$. The table has been extended to cover the wider range -200 to $100^{\circ}C$ by using the relation $E = -38.474\,T - 0.046046\,T^2 + 0.00003948\,T^3$. The coefficients of this expression were calculated from values taken from the table given by Scott.

$T\ ^{\circ}C$	E	$-\frac{dE}{dT}$
− 200	5537.2	15.5
− 198	5506.2	15.8
− 196	5474.7	16.0
− 194	5442.7	16.2
− 192	5410.2	16.6
− 190	5377.1	16.8
−188	5343.4	17.1
− 186	5309.2	17.4
− 184	5274.4	17.6
− 182	5239.1	17.9
− 180	5203.3	18.2
− 178	5166.9	18.5
− 176	5129.9	18.7
− 174	5092.5	19.0
− 172	5054.5	19.3
− 170	5015.9	19.5
− 168	4976.9	19.8
− 166	4937.3	20.1
− 164	4897.2	20.3
− 162	4856.6	20.6
− 160	4815.4	20.8
− 158	4773.7	21.1
− 156	4731.4	21.3
− 154	4688.8	21.6
− 152	4645.6	21.9
− 150	4601.9	22.1
− 148	4557.6	22.4
− 146	4512.9	22.6
− 144	4467.6	22.9
− 142	4421.8	23.1
− 140	4375.6	23.4
− 138	4328.8	23.6
− 136	4281.5	23.9
− 134	4233.8	24.1
− 132	4185.5	24.4
− 130	4136.7	24.6
− 128	4087.5	24.9
− 126	4037.8	25.1

T °C	E	$-\frac{dE}{dT}$
- 124	3987.5	25.4
- 122	3936.8	25.6
- 120	3885.6	25.8
- 118	3834.0	26.1
- 116	3781.8	26.3
- 114	3729.2	26.6
- 112	3676.1	26.8
- 110	3622.5	27.0
- 108	3568.4	27.3
- 106	3513.9	27.5
- 104	3458.9	27.7
- 102	3403.4	28.0
- 100	3347.5	28.2
- 98	3291.1	28.4
- 96	3234.3	28.7
- 94	3176.9	28.9
- 92	3119.2	29.1
- 90	3061.0	29.3
- 88	3002.3	29.6
- 86	2943.1	29.8
- 84	2883.6	30.0
- 82	2823.5	30.2
- 80	2763.1	30.5
- 78	2702.1	30.7
- 76	2640.8	30.9
- 74	2579.0	31.1
- 72	2516.7	31.3
- 70	2454.1	31.6
- 68	2391.0	31.8
- 66	2327.4	32.0
- 64	2263.4	32.2
- 62	2199.0	32.4
- 60	2134.2	32.6
- 58	2068.9	32.8
- 56	2003.3	33.0
- 54	1937.2	33.3
- 52	1870.6	33.5
- 50	1803.7	33.7
- 48	1736.3	33.9
- 46	1668.4	34.1
- 44	1600.4	34.3
- 42	1531.8	34.5
- 40	1462.8	34.7
- 38	1393.4	34.9
- 36	1323.6	35.1
- 34	1253.4	35.3
- 32	1182.8	35.5
- 30	1111.8	35.7
- 28	1040.3	35.9
- 26	968.5	36.1
- 24	896.3	36.3
- 22	823.8	36.5
- 20	750.8	36.7
- 18	677.4	36.9
- 16	603.7	37.1
- 14	529.7	37.3
- 12	455.0	37.4
- 10	380.1	37.6
- 8	304.8	37.8
- 6	229.2	38.0
- 4	153.2	38.2
- 2	76.8	38.4
0	0	38.6
2	- 77.1	38.8

T °C	E	$-\frac{dE}{dT}$
4	- 154.6	39.0
6	- 232.5	39.1
8	- 310.7	39.3
10	- 389.3	39.5
12	- 468.2	39.7
14	- 547.5	39.9
16	- 627.2	40.0
18	- 707.2	40.2
20	- 787.6	40.4
22	- 868.3	40.5
24	- 949.3	40.7
26	-1030.7	40.9
28	-1112.5	41.0
30	-1194.5	41.2
32	-1276.9	41.4
34	-1359.6	41.6
36	-1442.8	41.7
38	-1526.2	41.9
40	-1610.0	
100	-4268.0	

APPENDIX IV

Percentage Change in W for 1% Change in E/p

	293°K	
$\frac{E}{p}$	N_2	H_2
20	.7	1.0
18	.7	1.0
15	.7	1.0
12	.7	.9
10	.7	.7
9	.7	.7
8	.7	.6
7	.7	.6
6	.7	.6
5	.7	.6
4	.7	.6
3	.7	.6
2.5	.7	.6
2	.7	.5
1.8	.7	.5
1.5	.7	.5
1.2	.7	.5
1.0	.7	.5
.9	.6	.5
.8	.6	.5
.7	.6	.4
.6	.5	.4
.5	.5	.4
.4	.4	.4
.3	.3	.5
	293°K	
$\frac{E}{p}$	N_2	H_2
.25	.3	.5
.2	.3	.5
.18	.3	.6
.15	.3	.6
.12	.2	.6
.1	.2	.7
.09	.2	.7
.08	.2	.7
.07	.2	.7
.06	.2	.7
.05	.2	.8
.04	.3	.8
.03	.4	.8
.025	.5	.9
.02	.6	.9
.018	.6	.9
.015	.7	.9
.012	.7	.9
.01	.8	.9
.009	.9	.9
.008	.9	.9
.007	.9	1.0
.006	.9	1.0
.005	.9	1.0
.004	.9	1.0

77.6°K

$\frac{E}{p}$	N_2	H_2
2.5		.56
2		.52
1.8	.74	.51
1.5	.66	.51
1.2	.66	.47
1.0	.67	.42
.9	.70	.40
.8	.65	.39
.7	.56	.37
.6	.52	.36
.5	.42	.38
.4	.36	.38
.3	.30	.42
.25	.43	.46
.2	.42	.45
.18	.27	.53
.15	.21	.59
.12	.13	.59
.1	.04	.62
.09	- .04	.65
.08	- .08	.63
.07	- .11	.62
.06	- .13	.66

77.6°K

$\frac{E}{p}$	N_2	H_2
.05	- .09	.69
.04	- .03	.70
.03	.04	.67
.025	.13	.67
.02	.20	.70
.018	.29	.71
.015	.39	.68
.012	.47	.68
.01	.56	.70
.009	.62	.72
.008	.66	.73
.007	.69	.73
.006	.76	.77
.005	.81	.78
.004	.84	.82
.003	.89	.93
.0025	.94	.94
.002	.97	.95
.0018	.97	.96
.0015	.98	.98
.0012	.98	1.00
.001	1.00	1.00

APPENDIX V

Derivation of the Differential Equation (10)

We consider electrons travelling under the influence of a uniform electric field which causes them to acquire a drift velocity of W. From the definition of the coefficient of diffusion D, the rate of flow of particles into a volume V enclosed by a surface S is

$$- \iint (n\,W - D\nabla n) \cdot dS \qquad \text{(dS is directed outwards from V)}$$

where $n = n(x, y, z, t)$ is the electron density.

The equation of continuity therefore becomes

$$\iiint_V \frac{\partial n}{\partial t}\, dV = - \iint_S (n\,W - D\nabla n) . dS$$

$$= - \iiint_V \nabla \cdot (n\,W - D\nabla n)\, dV \qquad \text{(Gauss Theorem).}$$

As the above equation applies for any volume enclosure we may write

$$\frac{\partial n}{\partial t} = D\nabla^2 n - W \cdot \nabla n$$

If the x direction is chosen to be the direction of the electric field, the differential equation governing the motion of the electrons is then

$$\nabla^2 n = \frac{W}{D}\frac{\partial n}{\partial x} + \frac{1}{D}\frac{\partial n}{\partial t}$$

This equation is equation (10) of 8.2.1.

APPENDIX VI

Velocity Distributions

The following formulae enable the evaluation of velocity averages for either Maxwellian or Druyvesteyn velocity distributions.

Maxwellian distribution $$\frac{dN}{N} = \frac{4\,c^2}{a^3\sqrt{\pi}} \quad \exp\left(-\,c^2/a^2\right)\,dc$$

Druyvesteyn distribution $$\frac{dN}{N} = \frac{4\,c^2}{1.23\,a^3} \quad \exp\left(-\,c^4/a^4\right)\,dc$$

$$\int_0^a x^m \exp\left(-\,x^n\right)\,dx = \frac{1}{n}\,\Gamma\left(\frac{m+1}{n}\right)$$

$$\Gamma(n+1) = n\,\Gamma(n), \quad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}\,, \quad \Gamma(\tfrac{1}{4}) = 3.622, \quad \Gamma(\tfrac{3}{4}) = 1.225.$$

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