OFF-SHELL EFFECTS IN PION-NUCLEUS INTERACTIONS

By

M. N. SINHA ROY.

A thesis submitted in accordance with the requirements of the degree of

DOCTOR OF PHILOSOPHY

Department of Mathematical Physics,
The University of Adelaide.

OCTOBER 1981
ABSTRACT

One of the fundamental questions in few and many-body physics is to what extent are differences in the off-shell form of two particle amplitudes observable in the properties of composite systems. This question is particularly relevant in the field of pion-nucleus interactions where short range properties of the pion-nucleon interaction are poorly determined.

In this thesis we have studied two different problems to examine the dependence of physically observable quantities of the pion-nucleus many-body system on the off-shell behaviour of the pion-nucleon amplitudes. In the first part of the thesis we investigate the possible sources of off-shell dependence in the energy shift of the pion-deuteron system. In the second part of the thesis we calculate the differential scattering cross-section of the pion-carbon system using the second-order optical potential.

A consistent off-shell behaviour of the pion-nucleon scattering amplitude has been obtained from the Yamaguchi type separable potential for the pion-nucleon interaction. This particular type of two-body interaction also facilitates numerical calculations.

In Chapter II we provide different separable models for the pion-nucleon interaction. The parameters of the interaction have been determined from the experimental data. For the resonance (P33) and absorption (P11) channels a simple Yamaguchi type interaction with energy dependent strength has been used to fit appropriate experimental phase shifts (up to ~ 300 MeV), scattering lengths, the position of the resonance and the position of the pole at the nucleon mass. The models for the
other pion-nucleon channels also fit the experimental phase shifts and scattering lengths correctly.

By treating the strong interaction contribution as a perturbation to the Coulomb interaction, the energy shift of the 1S level of the pion-deuteron system has been calculated with the aid of the Paddeev theory from the knowledge of the two-body pion-nucleon scattering amplitudes. Having derived our exact expression for the energy shift in Chapter III, we have investigated critically the approximate nature of Deser et al's formula, relating the energy shift to the pion-nucleus scattering length. Our calculation of the energy shift of the pion-deuteron system demonstrates the importance of off-shell contributions due to

1. the momentum distribution of the pionic wave function

and

2. the momentum variation of the pion-nucleon amplitude at negative energies.

Deser et al assumed that these off-shell contributions were negligible in their original calculation. Our results suggest that if the experimental value of the energy shift is refined further, as seems likely in the near future, extraction of the scattering length from the energy shift will require a more careful treatment, taking account of the corrections indicated by the present calculation.

In Chapter IV the second-order optical potential for the pion-carbon system has been constructed in terms of the two-body pion-nucleon scattering amplitude and the nucleon-nucleon correlation function in order to investigate the effect of the nucleon-nucleon correlation in pion-carbon scattering at intermediate energies. The main part of the
nucleon-nucleon correlation is generated by the strong and repulsive nature of the short-range part of the nucleon-nucleon forces. The dependence of the differential scattering cross-section on the range parameters of the pion-nucleon interaction and on the nucleon-nucleon correlation length has been examined by using different sets of the pion-nucleon interaction potentials and varying the correlation length.

We find that different correlation lengths alter the numerical results only for the large angle scattering. The best fit is obtained for unrealistic values of the correlation length. The implication of this result is discussed in Section 4.8.

The differential cross-sections have been calculated from the second-order optical potential using two sets of pion-nucleon interaction potentials to check the off-shell dependence. We find that the scattering cross-section is sensitive (\( \sim 10 - 15\% \)) to the off-shell dependence of the pion-nucleon scattering amplitudes and to the range parameters of the pion-nucleon interaction for the different channels. However, it is difficult to state to what extent the scattering cross-section depends on the individual range parameters of the model.

Although the second-order optical potential gives a more complete description of the microscopic processes, in our calculations the addition of the second-order optical potential does not improve the agreement of the theoretical results obtained from the first-order optical potential with the experimental data. The basic assumptions of the model and possible results for this discrepancy are discussed in the final section.
ABSTRACT

STATEMENT

ACKNOWLEDGEMENTS

CHAPTER 1. Introduction 1

CHAPTER 2. Separable Models for Pion-Nucleon Interactions

2.1 Introduction 19

2.2 Determination of the pion-nucleon form factors from the experimental data 26

CHAPTER 3. Three-Body Perturbative Estimates of the Energy Shift in the Pion-Deuteron Atom

3.1 Introduction 38

3.2 Theory 44

3.3 Evaluation of the first-order energy shift 51

3.4 Angular momentum reduction of $A_{\beta, \alpha}(p', p, E)$ 56

3.5 The specific form of the driving term for a free exchange of a pion and the multiple scattering contributions 63
CONTENTS

3.6 Contribution from N-N re-scattering 65
3.7 Conclusion 66

CHAPTER 4. Second-Order Optical Potential for the Pion-Carbon System

4.1 Introduction 78
4.2 Formal theory of the optical potential 84
4.3 Explicit expressions for the first- and second-order optical potentials 90
4.4 Transformation of the pion-nucleon scattering amplitude from pion-nucleon co-ordinates to pion-nucleus co-ordinates 94
4.5 P33 and P11 channel interactions 96
4.6 The partial wave decomposition for the first-order optical potential 99
4.7 The partial wave decomposition for the second-order optical potential 101
4.8 Results and Discussion 109