

09PH
S824



ON-SHELL APPROACH TO THREE-BODY
SCATTERING

A Thesis Presented

by

Andris Talis Stelbovics B.Sc.(Hons.)
for the degree of Doctor of Philosophy.

Department of Mathematical Physics,
University of Adelaide.

February, 1975.

CONTENTS

| | | |
|----|--|----|
| 1. | INTRODUCTION | 1 |
| 2. | THREE-BODY AMPLITUDES IN THE SEPARABLE MODEL | 5 |
| | 2.1 Amado-Lovelace Equations | 5 |
| | 2.2 Equations for Identical Particles | 8 |
| | a) System of identical spinless bosons | 8 |
| | b) Elastic neutron-deuteron scattering | 9 |
| | c) Amado model | 10 |
| | 2.3 Diagrammatic Representation | 11 |
| 3. | CONSTRUCTION OF THE ON-SHELL AMPLITUDES | 13 |
| | 3.1 Solution of the AL equation | 15 |
| | 3.2 Singularity Structure of a General Perturbation Series Term | 20 |
| | 3.3 The Fredholm Solution of the Three-Body Scattering Amplitudes for Physical Energies | 26 |
| | a) Elastic amplitude | 27 |
| | b) Breakup amplitude | 29 |
| | c) Free particle amplitude | 35 |
| | 3.4 Rotation of Contours Method | 36 |
| 4. | ANALYTICITY IN ENERGY OF THE ON-SHELL ELASTIC AMPLITUDE | 38 |
| | 4.1 Analytic Continuation of On-Shell Elastic Amplitudes | 39 |
| | a) System of identical spinless bosons | 39 |
| | b) Neutron-deuteron elastic amplitudes | 47 |
| | 4.2 Position of Singularities | 49 |
| | 4.3 Equations with Zero Range Forces | 55 |
| | 4.4 Discussion | 57 |
| 5. | ANALYTICITY OF BREAKUP AND FREE SCATTERING AMPLITUDES | 60 |
| | 5.1 Breakup Amplitude | 61 |
| | a) Analyticity of $X_R(W; \underline{k}_F, \underline{k}'_B)$ | 63 |
| | b) Analyticity of $T_1(W; \underline{k}_F, \underline{k}_B)$ | 68 |

| | | |
|------------|--|-----|
| 5.2 | Free Particle Amplitude | 71 |
| | a) Analyticity of $X_{RF}(W; \underline{k}_F', \underline{k}_F)$ | 72 |
| | b) Analyticity of $T_2(W; \underline{k}_F', \underline{k}_F)$ | 73 |
| | c) Analyticity of $T_1(W; \underline{k}_F', \underline{k}_F)$ and $Z(W; \underline{k}_F', \underline{k}_F)$ | 74 |
| 5.3 | Position of Singularities | 77 |
| | a) Breakup Amplitude | 77 |
| | b) Free particle Amplitude | 81 |
| 5.4 | Concluding remarks | 82 |
| 6. | A STUDY OF THE ON-SHELL N/D METHOD OF SOLUTION FOR S-WAVE ELASTIC n-d SCATTERING IN THE CONTEXT OF THE AMADO MODEL | 85 |
| 6.1 | Dispersion Relations for Elastic n-d Scattering and Their N/D Representation | 88 |
| 6.2 | Restrictions on the Asymptotic Inelasticity | 92 |
| 6.3 | Driving Terms | 94 |
| 6.4 | Some Notes on Numerical Methods | 96 |
| | a) Solution of the Off-Shell Equations | 96 |
| | b) Calculation of Inputs | 98 |
| | c) Solution of the N/D equations | 100 |
| 6.5 | Results | 101 |
| 6.6 | Discussion and Conclusions | 113 |
| 7. | EIGENVALUE SPECTRA OF THE n-d KERNELS | 120 |
| 7.1 | General Properties | 120 |
| 7.2 | Numerical Calculation of Eigenvalues | 124 |
| 7.3 | Low Energy Behaviour of the Eigenvalues | 131 |
| 7.4 | Application to Low Energy S-wave n-d Scattering | 133 |
| 7.5 | Discussion | 137 |
| APPENDICES | | |
| A | The Landau Rules | 139 |
| B | Notes on Some Denominators | 139 |
| C | Properties of a Mapping Encountered in Chapter 5 | 142 |

D A Note on the Numerical Integration of

$$\int_0^{\infty} f(x) dx$$

143

REFERENCES

144

ABSTRACT

Although calculations of low energy elastic neutron-deuteron scattering based on on-shell dispersion relations have met with some success, it is difficult to assess their reliability because of the number of theoretical approximations and experimental uncertainties. In this thesis a study is made of the on-shell approach in the context of the Amado model of neutron-deuteron scattering where comparison with accurate numerical solutions of the off-shell Faddeev equations is possible.

The motivation for the work is discussed in Chapter 1 and the off-shell equations for the models are introduced in Chapter 2. In Chapter 3 the on-shell amplitudes for elastic, breakup and free scattering are constructed as Fredholm series solutions, whose convergence in the limit of physical energies is determined in each case by investigating their analytic structure. As a result we are able to rigorously justify the rotation of contours method used to solve the off-shell equations.

In Chapter 4 the elastic neutron-deuteron scattering amplitudes are continued to complex energies. It is shown that they are analytic on the entire physical sheet with the exception of the real axis along which the singularities are divided into two classes; the left hand potential cuts and right hand unitarity cuts. The special class of pinch singularities between energy denominators is studied and it is found that only a single pinch singularity is possible. Contained in the first iterated term of the multiparticle scattering series it contributes only to the left hand cut.

The methods developed for the elastic amplitudes are extended in Chapter 5 to obtain the analytic structure of the breakup and free scattering amplitudes.

In Chapter 6 a rigorous dispersion relation formulated for the elastic neutron-deuteron amplitude is solved in the s-wave quartet and doublet channels using various driving terms generated by the multiple scattering solution of the Amado model equations. To take account of the breakup channel ~~exact~~ model inelasticities computed from the off-shell equations are used. The numerical results indicate that the quartet channel scattering is well described with the simplest driving terms, but in the doublet channel the N/D solutions approach the exact model values only slowly as the order of the inputs is increased. The effect on the solutions of various approximations to the exact inelasticities is also tested. The results show that it is important to have a good description of inelastic effects in both the quartet and doublet channels. From the calculations it appears that although on-shell techniques are capable of reproducing the main features of the exact amplitudes they cannot compete in accuracy with the complete solutions of the off-shell Faddeev equations.

Chapter 7 is devoted to a numerical study of the eigenvalue spectra of the kernels of the off-shell equations and a description of the low energy behaviour of the phase shift in the s-wave doublet in terms of the eigenvalues is presented.