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ON SOME MATHEMATICAL ASPECTS OF DEFORMATIONS OF INHOMOGENEOUS ELASTIC MATERIALS

by

JEFFRY KUSUMA

Drs. (Matematika), Universitas Hasanuddin

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Department of Applied Mathematics

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CONTENTS

SUMMARY i

SIGNED STATEMENT ii

ACKNOWLEDGEMENTS iii

CHAPTER 1 INTRODUCTION

CHAPTER 2 ONE-DIMENSIONAL ELASTIC DEFORMATIONS

2.1 Introduction 5

2.2 Basic equations 6

2.3 Waves in inhomogeneous rods 7

2.4 Perturbation method 12

2.5 Floquet waves in inhomogeneous periodic materials 16

CHAPTER 3 AXIALLY SYMMETRIC DEFORMATIONS

3.1 Introduction 28

3.2 Basic equations 28

3.3 General solution using the separation of variables method 30

3.4 General solution using the Laplace transformation 32

3.5 Double walled thick cylinder 36

3.6 Numerical solutions 41

CHAPTER 4 SPHERICALLY SYMMETRIC DEFORMATIONS

4.1 Introduction 53

4.2 Basic equations 54

4.3 Some simple solutions for homogeneous materials 54

4.3.1 Homogeneous spheres 54

4.3.2 Spherical shells 55

4.3.3 Spheres with two layered materials 56

4.3.4 Two layered spherical shells 57

4.4 Simple solutions for inhomogeneous materials 58

4.4.1 Bessel differential equations 58

4.4.2 Some numerical results 60

4.5 Analytic solution for the time dependent problem 65

CHAPTER 5 ANTI-PLANE DEFORMATIONS FOR ISOTROPIC MATERIALS

5.1 Introduction 67

5.2 Basic equations 68

5.3 Analytical solution by the method of separation of variables 69

5.4 Boundary element method for static case 71

5.4.1 General solution in term of an arbitrary harmonic function 71

5.4.2 Boundary integral equation 74

5.4.3 Integral equation for interior points 76

5.4.4 Numerical examples 77

5.5 Boundary element method for dynamic case 83

5.5.1 Boundary integral equation 85

5.5.2 Numerical technique 86

5.5.3 Another shear moduli 87

5.5.4 Numerical examples 88

5.6 Boundary element method and perturbation technique 93

5.6.1 Perturbation technique 93

5.6.2 Boundary element method 94

5.6.3 Numerical results 95

5.7 Further technique for the dynamic case 99

5.7.1 General solution 99

5.7.2 Boundary integral equation 100

5.7.3 Numerical results 101

CHAPTER 6 ANTI-PLANE DEFORMATIONS FOR ANISOTROPIC MATERIALS

6.1 Introduction 105

6.2 Basic equations 105

6.3 Boundary element method for static case 106

6.3.1 General analytical solution 106

6.3.2 Boundary integral equation 109

6.3.3 Numerical approximations 111

6.3.4 Numerical example 114

6.4 Boundary element method for dynamic case 120

6.4.1 General analytical solution 120

6.4.2 Integral equation 121

6.4.3 Numerical example 123

6.4.4 Two inhomogeneous anisotropic materials 124

6.4.5 Numerical example 129

CHAPTER 7 PLANE DEFORMATIONS

7.1 Introduction 133

7.2 Governing differential equation and the fundamental singular solution 134

7.3 General second order elliptic system for anisotropic media and the fundamental singular solution 136

7.4 Boundary integral equation 137

7.4.1 Perturbation technique 137

7.4.2 Two homogeneous materials 142

7.5 Numerical methods 145

7.5.1 Perturbation technique 145

7.5.2 Two homogeneous materials 147

7.6 Numerical results 149

CHAPTER 8 SURFACE EFFECTS DUE TO INCIDENT PLANE *SH* WAVES

8.1 Introduction 158

8.2 Ground motion on an alluvial inhomogeneous anisotropic valley . . . 159

8.2.1 Problem formulation 159

8.2.2 Integral equation 162

8.2.3 Numerical results 165

8.3 Ground motion effect due to subterranean alluvial deposits 170

8.3.1 Problem formulation and integral equation 170

8.3.2 Numerical results 171

APPENDIX A 175

APPENDIX B 177

BIBLIOGRAPHY 179

SUMMARY

Two types of inhomogeneous elastic materials are considered in this thesis. The first type of material is made up of different regions with the elastic coefficients constant in each region. For the second type of material the elastic coefficients vary continuously with the spatial coordinates.

The thesis may be thought of as being composed of three parts. In the first part, the one-dimensional propagation of waves through an inhomogeneous elastic material is considered. Numerical solutions for certain materials are obtained by using the finite difference method. In the second part of the thesis, axially symmetric and spherically symmetric deformations problems of inhomogeneous materials are considered by employing analytical techniques. In the final and major part of the thesis, we consider some antiplane and plane deformation problems for isotropic and anisotropic inhomogeneous elastic materials. In the antiplane and plane problems emphasis is placed on the development of the boundary element method for the numerical solution of particular boundary value problems. The kernel of the integral equation for some specific materials is derived so that the standard boundary element method may be employed.