ANALYSIS OF CONCEPTS OF WEAK CONFIRMATION

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SUMMARY

The study of confirmation is viewed in this dissertation as concerned with the evidential grounds for rationally believing a hypothesis to be true. A fundamental distinction underlying the study of confirmation generally is the distinction between two senses of 'to confirm', *viz.*, the senses of 'to make firm' and 'to make firmer'. The former sense is spoken of here as the strong sense of 'to confirm'; the latter sense is spoken of as a weak sense of 'to confirm' and is referred to as 'to confirm_F'. Several confirmation theorists have recently noted that 'to confirm' and its cognates and synonyms have been understood in more than one non-strong or weak sense. These weak senses of 'to confirm' form a cluster of related concepts which are referred to as concepts of weak confirmation. Attention is restricted to the qualitative form, rather than to the comparative form or the quantitative form, of these concepts.

After distinguishing four concepts of weak confirmation from one another, attention is drawn to the fact that the concept of confirmation_F includes an essential reference to prior evidence whereas the other concepts of weak confirmation explicitly avoid any reference to prior or background evidence. The significance of the distinction between prior-evidence-dependent and prior-evidence-independent concepts of confirmation is

demonstrated with reference to Hempel's influential study of confirmation. After reviewing his definition of confirmation and noting some interesting features of his study of confirmation in relation to Nicod's study of confirmation, the problem of identifying the concept which Hempel is concerned to explicate is considered. After the identifications suggested by Scheffler and Carnap are canvassed and rejected, Hempel's explicandum is identified as being a prior-evidence-independent concept.

It is argued that the concept of confirmation$_F$ is the concept of weak confirmation which is most central to and most important for the study of weak confirmation. The significance of the prior-evidence-dependent concepts is evaluated in the light of this contention, and a role which these concepts might play in relation to the study of confirmation$_F$ is suggested.

In the last two chapters the problem of explicating concepts of weak confirmation is discussed. Two approaches to the matter of explication are considered: one approach consists of assigning a formal probabilistic measure to a concept, thereby determining the characteristics of that concept; the other approach consists of imposing upon a concept various logical conditions of adequacy, based upon an intuitive understanding of that concept, and requiring any proposed explicatum to fulfill the conditions of adequacy. The latter approach is found to be rather indecisive, and is consequently unsatisfactory. The former approach is discussed in some detail in connection with the concept of confirmation$_F$. A certain probabilistic measure often construed as a necessary and sufficient condition of confirmation$_F$ is evaluated. In conclusion, suggestions are made for further study of the concept of confirmation$_F$.

STATEMENT

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university, nor, to the best of my knowledge and belief, any material previously published or written by another person, except when due reference is made in the text of the thesis.

Signed,

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CHAPTER ONE

INTRODUCING THE STUDY OF WEAK CONFIRMATION

1. Confirmation as a factor involved in the acceptance of hypotheses.

The study of confirmation is one which has gained prominence in the last three decades, although philosophers and logicians have been interested in the topic under other names for many centuries. The study of confirmation has been an important part of the study of inductive procedures in science and is an integral part of that discipline commonly known as "induction" or "inductive logic." Roughly speaking, confirmation deals with the relation between two items in which the first is taken to be evidence for the second.

It has become a commonplace of current philosophy of science that theories or hypotheses of a general character are not mechanically inferred from empirical data of a particular nature by some inductive rule, although many inductive logicians of earlier eras seem to have held such a view. It is now widely believed that the method by which a hypothesis or theory is invented or obtained is not a matter which is susceptible to rational analysis, for theories and hypotheses are often arrived at by anything but systematic and rational procedures. They are often invented by the exercise of creative imagination, as writers such as Albert Einstein, William Whewell, and, more recently, Carl Hempel, have emphasized. Once a hypothesis or theory is obtained, however, irrespective of how it is obtained, a rational procedure can be followed in order to determine whether or not that hypothesis or theory is worthy of being regarded as plausible or true. Hempel puts it this way: "Although no restrictions are imposed upon the invention of theories, scientific objectivity is safeguarded by making their acceptance dependent upon the outcome of careful tests."[1]

Hempel is one of a number of theorists who discuss the role of confirmation theory in connection with the methodological problem of elucidating the rules governing the testing of a scientific hypothesis and its subsequent acceptance or rejection. This approach has a number of difficulties attending it, as the following discussion will show, although it also has an element of truth. Hempel distinguishes three phases in the test of an empirical hypothesis culminating in its acceptance or rejection.[2] The first phase consists of carrying out suitable

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[1]Hempel, "Recent Problems of Induction," in Mind and Cosmos (Pittsburgh: University of Pittsburgh Press, 1966), ed. R. G. Colodny, p. 116. Emphasis in the original.

[2]Hempel, "Studies in the Logic of Confirmation," pp. 41 - 42, first published in Mind, vol. 54 (1945), pp. 1 - 26 and 97 - 121; reprinted with changes in Aspects of Scientific Explanation and Other Essays in the Philosophy of Science (New York: Free Press, 1965), ed. Hempel, pp. 3 - 46. I shall refer to this article as "Studies," and all references will be to the reprint. The reprint has the important "Postscript (1964) on Confirmation" appended, pp. 47 - 51.

tests of the empirical hypothesis after it has been proposed (irrespective of how it has been obtained), and expressing these test results as evidence reports. The second phase consists of confronting the hypothesis with the test results obtained, that is, ascertaining whether the reports constitute confirming, disconfirming, or neutral evidence. The third and final phase consists of accepting or rejecting the empirical hypothesis in the light of all the test results obtained, or, if sufficient evidence has not been obtained to warrant the acceptance or rejection of the hypothesis on the basis of the available evidence, of suspending judgment concerning the hypothesis. A theory of confirmation is relevant to the second phase, and is supposed to supply criteria which determine whether a given test report is confirming, disconfirming, or neutral with respect to the hypothesis. The third phase of testing is construed by Hempel as pragmatic, although he thinks that it is worthwhile to attempt to give a reconstruction in purely logical terms of this phase as well as of the second phase. Criteria for acceptance and rejection would include a reference to the kind and amount of confirming and disconfirming evidence, but possibly to other factors as well, such as the simplicity of the hypothesis, and the manner in which it fits into the system of previously accepted theories.

It is widely recognized among theorists in inductive logic that a number of factors are important in an analysis of acceptance. Henry E. Kyburg enumerates a number of factors which philosophers of science have considered important. Some of them are (i) the probability of the hypothesis h, relative to the total evidence, (ii) the information content of h (perhaps relativized to the total evidence also), (iii) the simplicity of h, and (iv) the fruitfulness of h.[3] One other factor which has affected acceptance, but is rightly regarded as not generally relevant by philosophers of science, is the political and moral utility of a given hypothesis. It is evident from these enumerated factors, as well as from factors mentioned by Hempel, that the problem of analyzing the concept of acceptance constitutes a complex problem which promises to be at least as difficult as the problem of analyzing the concept of confirmation.

Not only is the analysis of any interesting concept of acceptance bound to be complex, but there is the additional difficulty that the term 'acceptance' has been used in a variety of senses, all of which it might be impossible to enumerate. Y. Bar-Hillel discusses some of the ways in which this "high-prestige" term is used, among them being the following: to

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[3]Henry E. Kyburg, Jr. "The Rule of Detachment in Inductive Logic," in The Problem of Inductive Logic (Amsterdam: North-Holland Publishing Company, 1968), ed. I. Lakatos, pp. 101 - 102.

accept a theory may mean to regard the theory as true in some final sense, so that no empirical findings will be counted as evidence against it; to accept a theory may be to base research on the assumption that the theory is true; to accept a theory may be to select the theory as the one most worth superseding by a better one; and finally, to accept a theory may be to dedicate more time to the exposition of the theory in classroom teaching than to competing ones.[4] Bar-Hillel's attitude to an analysis of this concept is summed up in the remark that ". . . 'to accept' is not only a definitely pragmatic term in ordinary usage but also one which it is not worthwhile, or possible, adequately to explicate as a purely semantic term."[5] R. G. Swinburne similarly lists several senses of 'to accept', *viz*., (i) to believe to be true, (ii) to assume to be true for future theoretical investigation, and (iii) to assume to be true in determining a course of action in some field.[6] Some of the senses enumerated by Swinburne perhaps coincide with some of the uses

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[4] Y. Bar-Hillel, "On alleged rules of detachment in inductive logic," p. 124, and "The acceptance syndrome," p. 154, in *The Problem of Inductive Logic*. These constitute replies to Kyburg and to other authors involved in the discussion of Kyburg's paper, "The Rule of Detachment in Inductive Logic."

[5] Bar-Hillel, "The acceptance syndrome," pp. 154 - 155.

[6] R. G. Swinburne, "Choosing Between Confirmation Theories," *Philosophy of Science*, vol. 37 (1970), p. 604.

of 'to accept' identified by Bar-Hillel. It is important to draw these distinctions if we are to coherently discuss the study of confirmation theory in terms of acceptance. The complexities involved in analyzing the concept of acceptance and the ambiguities associated with the term 'to accept' suggest that a more perspicuous way of explaining the study of confirmation be sought.

2. Confirmation as related to elaborating standards of rational belief.

A second, less ambiguous way of elucidating the study of confirmation is by relating it to the epistemological problem of elaborating the standards of rational belief. Observational results and experimental findings which are brought to bear upon a hypothesis, the truth of which is uncertain, do not usually afford conclusive, deductive evidence for the truth of that hypothesis, but such findings do provide a basis for rationally believing that hypothesis to be true. To elaborate the standards of rational belief for a hypothesis is to provide a theory which specifies the observational and experimental findings that constitute the basis for rational belief in the truth of such a hypothesis. This theoretical problem of elaborating standards of rational belief is just the problem of characterizing the conditions under which a body of evidence can be said to confirm, to disconfirm, or to be irrelevant to, a hypothesis of empirical character. This second way of characterizing the study of confirmation suggests that the sense of 'to accept' in which the study of confirmation is a study of the experimental tests and observational findings which partly determine whether a hypothesis should be accepted or rejected is the sense of 'to accept' meaning the same as 'to rationally believe to be true'. This is the view of confirmation theory which I embrace. In keeping with this way of explaining the study of confirmation, I shall introduce in the next chapter the concept of rational credibility in terms of which all the concepts of confirmation which are discussed will be expressed. The concept of credibility is of course intimately connected with the concept of rational belief, although it would be unwise to uncritically assume that these concepts are identical or even materially equivalent. The term 'credible', which means the same thing as the phrase 'worthy of belief', appropriately captures the idea that confirmation theory involves specifying the grounds which make some hypothesis worthy of being believed.

In the foregoing paragraph I have described the study of confirmation as pertaining to the basis for rationally believing a hypothesis to be true, rather than simply the basis for rationally believing a hypothesis. The preferred formulation highlights the fact that the study of confirmation is concerned with the important question of whether or not a hypothesis is true in a way that the simpler formulation does not. The experimental tests and observational findings which confirm a hypothesis have an important bearing upon whether we are warranted in believing that hypothesis to be true. Such a view of confirmation theory effectively rules out an instrumentalist view of scientific theories according to which theories are not meaningful statements which are either true or false but rather are mere computational devices which enable one to derive sentences describing observable phenomena. The simpler, unpreferred formulation does not have a definite anti-instrumentalist bias and might be preferred by someone who is sympathetic to the instrumentalist view. The instrumentalist view, in my opinion, is objectionable for reasons outlined by a number of authors.[7] I shall assume here a realist philosophy of science.

Although there is a large contingent of theorists and scientists who maintain that there are grounds for believing hypotheses and theories of science to be true, there is at least one influential school of thought which denies that confirmation theory has anything to do with justifying, even

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[7]See, for example, the critique of instrumentalism by J. J. C. Smart, in _Between Science and Philosophy: An Introduction to the Philosophy of Science_ (New York: Random House, 1968), pp. 141 - 157.

partially, the truth of hypotheses or with specifying the grounds for believing a hypothesis to be true. The school of thought to which I refer is the Popperian school led by Karl Popper. One of the important factors in scientific activity, according to Popper, is trying to assess what tests or trials a hypothesis has withstood, i.e., trying to assess how far a hypothesis ". . . has been able to prove its fitness to survive by standing up to tests."[8] Popper sees his theory of confirmation, or "corroboration" as he prefers to call it, as concerned with exactly this matter. Popper emphasizes that the tests of a hypothesis *h* are ". . . reports of the outcome of sincere attempts to refute *h*, rather than of attempts to verify *h*."[9] We can tentatively 'accept' a theory or hypothesis which is capable of being tested and has in fact passed the severest tests we are able to devise, ". . . but only in the sense that we select it as worthy to be subjected to further criticism, and to the severest tests we can design."[10] Conjectures may pass the severest tests we are able to devise, but these conjectures can never be positively justified by such tests, ". . . they can neither be established as certainly true nor

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[8]Popper, *The Logic of Scientific Discovery* (London: Hutchinson, 1968), p. 251.

[9]*Ibid.*, p. 414.

[10]*Ibid.*, p. 419.

even as 'probable'."[11] Popper maintains that his metrical measure of corroboration, *viz.*, his degree of corroboration function, ". . . must not be interpreted, therefore, as a degree of the rationality of our belief in the *truth* of *h*,"[12] although it can be interpreted as ". . . [degree] of the rationality of our belief in *h*, in the light of tests."[13] In this context, however, as Popper explains elsewhere, ". . . the term 'belief' is taken to cover our critical acceptance of scientific theories — a *tentative* acceptance combined with an eagerness to revise the theory if we succeed in designing a test which it cannot pass."[14] Hence, the purpose which Popper's theory of corroboration serves in his philosophy of science is quite different from that which confirmation theory is taken as serving by those who hold that experimental tests and observational findings can be legitimately construed as providing grounds for believing hypotheses and theories of science to be true.

In spite of Popper's frequent protestations that his

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[11] Popper, *Conjectures and Refutations* (New York and Evanston: Harper & Row, 1965), p. vii.

[12] Popper, *The Logic of Scientific Discovery*, p. 415. Emphasis in the original.

[13] *Ibid.*, p. 414.

[14] Popper, *Conjectures and Refutations*, p. 51. Emphasis in the original.

theory of corroboration is not part of the inductivist enterprise, his theory has been interpreted as a theory which is a rival to inductivist theories and has been cast in the same role which inductivist theories play. An instructive example of such an interpretation of Popper's theory can be seen in a recent article by Wesley Salmon.[15] Salmon discusses the problem of showing how it is possible that we have knowledge of unobserved fact, and considers eight different approaches to this problem, including among them Popper's deductivist approach. Salmon correctly points out that Popper does not regard hypotheses as firmly established or established as true just because they are highly corroborated. Salmon maintains, however, that Popper's account of corroboration is a mode of nondemonstrative, ampliative inference and that Popper's theory of corroboration is not free of all inductivist elements, for the theory of corroboration is used in order to select among unfalsified hypotheses and thus ". . . is a way of providing for the acceptance of hypotheses even though the content of these hypotheses goes beyond the content of the basic statements [i.e., observation statements]."[16] Now Salmon's remark that Popper's theory of corroboration is

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[15] Salmon, "The Foundations of Scientific Inference," in <u>Mind and Cosmos</u>, ed. Colodny, pp. 135 - 275.

[16] <u>Ibid</u>., p. 160.

designed to specify the grounds for accepting a hypothesis is quite correct, but the sense which Popper attaches to 'accept' is quite different from the sense of 'accept' which is basic to most, if not all, of the inductivist program. Popper does not develop the theory of corroboration in order to show how we can have knowledge of unobserved fact, and Salmon's suggestion that Popper developed his theory for that purpose is wrong. Popper develops his theory in order to spell out the basis for selecting a theory as worthy of being subjected to further criticism and to the severest tests which can be designed. Popper's program is concerned with the growth of science rather than with establishing as true or even as probable the theories which comprise 'science' at any given time. And when Popper speaks of the growth of scientific knowledge (all knowledge in fact) he means ". . . the repeated overthrow of scientific theories and their replacement by better or more satisfactory ones."[17] The Popperians, in Bar-Hillel's words, ". . . [are] mostly interested in the diachronic growth of science."[18]

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[17] Popper, Conjectures and Refutations, p. 215.

[18] Bar-Hillel, "Inductive logic as 'the' guide of life," p. 69, in The Problem of Inductive Logic, ed. Lakatos. Bar-Hillel's paper constitutes a reply to Salmon, "The Justification of Inductive Rules of Inference," pp. 24 - 43, and to other authors involved in the discussion of Salmon's paper.

The view of confirmation which I take in this dissertation implies that Popper's theory of corroboration is, strictly speaking, irrelevant to the central purpose of confirmation theory. For the tests of a hypothesis, according to Popper, cannot justify, even partially, the truth of a hypothesis and thus do not provide good reasons for believing a hypothesis to be true. The tests of a hypothesis, i.e., the sincere attempts to falsify a hypothesis, only warrant one in regarding the hypothesis tested as worthy of being subjected to further criticism. In spite of the irrelevance of Popper's theory of corroboration to the central aims of confirmation theory, it has been and can be construed as specifying grounds for believing a hypothesis to be true. It should only be remembered that this is not how Popper intends his theory of corroboration to be interpreted and that he does not share the basic inductivist assumptions. I shall assume in this dissertation, along with many other theorists and scientists, that there are grounds for believing hypotheses and theories to be true and that confirmation theory is concerned with specifying just those grounds.

3. Confirmation and the problem of justifying induction.

One of the most notorious issues in inductive logic has been the problem of justifying induction. This problem has been so prominent in the study of induction that it is frequently referred to as _the_ problem of induction. The problem was raised and made famous by Hume, and since Hume's time it has been tackled from many angles in an effort to avoid his skeptical conclusions. I shall not examine the various ploys which have been tried in order to avoid skepticism, for the study of confirmation is concerned with a problem which is logically prior to the problem of justification. The study of confirmation is concerned with the prior problem of specifying and clarifying that which is to be justified. The classical problem expressed in the question: "What justification is there for rationally believing some hypothesis to be true on the basis of deductively incomplete evidence?" requires a specification of that which constitutes the rational basis for believing a hypothesis to be true. Once the descriptive problem has been answered, we will be in a better position to see how the problem of justification, if there still is any problem, might be solved.

Nelson Goodman has ingeniously argued that the problem of the justification of induction is not something over and above the problem of describing or defining valid induction or confirmation.[19] He draws a parallel with the problem of justifying deduction and the problem of defining valid deduction in order to

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[19] Goodman, _Fact, Fiction, and Forecast_, Second edition Indianapolis, New York, Kansas City: Bobbs-Merrill, 1965), pp. 62 - 66.

make his point plausible. In order to justify a particular deduction, we show that it conforms to the general rules of deductive inference. An argument that conforms to the general rules is justified or valid, and an argument which does not so conform is fallacious. Although we do not usually ask what justifies the rules, this question must eventually be posed. Goodman's answer is as follows: "Principles of deductive inference are justified by their conformity with accepted deductive practice. Their validity depends upon accordance with the particular deductive inferences we actually make and sanction. . . . .Justification of general rules thus derives from judgments rejecting or accepting particular deductive inferences."[20] If we happen to endorse a rule which, it turns out, yields an inference which we cannot accept, then the rule must be amended. The process of justification consists of making adjustments in both rules and in accepted inferences until they are in perfect agreement, and in this agreement lies the only justification we shall ever find. The same answer is applicable to induction, Goodman contends. A particular inductive inference is justified by conformity to general rules and these general rules are justified by conformity to accepted inductive inferences. Canons of induction are valid

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[20] *Ibid.*, pp. 63 - 64.

". . . if they accurately codify accepted inductive practice."[21] The problem of induction thus consists in the problem of defining the difference between a valid and an invalid inductive argument, i.e., the problem of specifying the circumstances under which those inductive judgments are made that are normally regarded as valid. This is just the problem of defining the relation of confirmation.

One of the outstanding dissimilarities between inductive logic and deductive logic is that we are in possession of many more definite judgments regarding rules of deductive inference than we are of rules of inductive inference. If someone should happen to endorse an argument which conforms to some invalid deductive rule, it seems to be relatively easy to convince him that he has appealed to an invalid rule by means of a counter-example, i.e., an argument with obviously true premises and obviously false conclusion which nevertheless conforms to the invalid rule. We are in possession of such definite and widely held judgments concerning valid and invalid rules of deductive inference that if someone should happen to appeal to an invalid rule of inference he is usually convinced of his error by relatively little effort. The situation within induction is somewhat different. There is no group of rules of inductive

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[21] _Ibid._, p. 64.

inference which commands assent by almost everyone. It is quite conceivable, and perhaps likely, that particular inductive inferences are endorsed which conform to general inductive rules which would turn out to be objectionable if a close scrutiny of such rules were made. A general inductive rule which some theorists seem to have regarded as highly as deductive rules of inference is the rule: An instance of a hypothesis always confirms the hypothesis. Closer scrutiny of this rule has revealed that it cannot be endorsed without qualification. Hence, although some theorists have accepted it almost without argument, others have rejected it or amended it so that it conforms to approved particular inferences. The discussion of this rule also illustrates the delicate problem of altering general inductive rules and particular inductive inferences in order to reach a state of agreement between rules and inferences.

One of the objections which might be raised to Goodman's argument which equates the problem of justification with the problem of description is that a rule of inference cannot be justified by an appeal to its conformity with accepted inferential practice, since the accepted inferential practice might include making invalid inferences. It might be argued, for example, that among particular deductive inferences which people make there are some in which the fallacy of affirming the consequent is committed. Thus accepted deductive inferential practice might include this invalid inference, and to justify deductive rules of inference by appealing to their conformity to accepted deductive practice would be to justify the rule of affirming the consequent. Goodman, however, is not denying that invalid inferences are sometimes made or even that invalid rules are sometimes endorsed. Thus it may well be the case, and in fact is the case, that the fallacy of affirming the consequent is committed in particular arguments. If this fallacy is committed, however, it is probably because of insufficient reflection upon the form of the argument in which the fallacy occurs or upon similar arguments. The fact is that anyone who does argue fallaciously will normally be made to see, with the help of examples similar to the argument in which the fallacy occurs, that he is subscribing to an invalid rule and will thus be led to reject what he at first endorsed, perhaps from ignorance or from haste. A person who did not so acquiese would just be hopelessly "deductively blind." The situation would presumably be the same in connection with inductive inferences, although it is an assumption at this stage that there are rules of inductive inference which could command the assent of the human community comparable to the assent which is given to rules of deductive inference such as modus ponens, modus tollens, and so on.

4. Outline of the dissertation.

The topic generally known as confirmation theory includes a number of distinct but related topics. The areas I have in mind have been quite extensively discussed by Carnap in *Logical Foundations of Probability*, and by many authors besides, especially since the book just named was originally published.[22] Carnap maintained in the first edition that three different concepts of confirmation had to be distinguished from one another, *viz.*, a metrical or quantitative concept, a comparative concept, and a classificatory or qualitative concept. He likened these three concepts to the concepts of temperature, being warmer than, and being warm, respectively, and drew helpful and at the same time misleading parallels between the concepts of confirmation and the concepts of physical warmth. In the second edition of *Logical Foundations of Probability* Carnap has included a very important preface which greatly clarified his position on confirmation. It includes an important section distinguishing two different classes of concepts of confirmation corresponding to two different senses of 'to confirm'. The informal concepts which he uses to distinguish the two classes of concepts are the concept of firmness and the concept of increase in firmness. It turns out that the metrical and comparative concepts which

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[22]Published in Chicago by The University of Chicago Press, first edition 1950, second edition 1962. I shall refer to the book hereinafter as '*LFP*'. All references will be to the second edition.

Carnap studies in his book are concepts of firmness, while the qualitative concept is a concept of increase in firmness. I shall have more to say about Carnap's important distinction in the next chapter, but let it suffice to say here that the concept of increase in firmness is the concept in which I am particularly interested in this dissertation. It is, moreover, the qualitative form of the concept of increase in firmness which I am concerned to analyze. My study here, therefore, is closely related to the study of the concept of confirmation carried out by authors such as Hempel, Carnap, Goodman, and Popper, as well as by a number of other writers who have contributed to the study of confirmation, e.g., R. H. Vincent, J. L. Mackie, Jean Nicod, Israel Scheffler, H. G. Alexander, David Stove, and Howard Smokler, to name a few.

One of the issues with which this dissertation is centrally concerned is different concepts of confirmation which are related to but distinguishable from the qualitative form of the concept of increase in firmness isolated by Carnap. I contend that 'to confirm', and related locutions, has been understood by different theorists not only in the senses of 'to make firm' and of 'to make firm<u>er</u>', but also in a number of other senses. The significant feature of these other senses of 'to confirm' is that they are in qualitative form and are similar to the sense of 'to confirm' meaning 'to make firmer' rather than the sense of 'to confirm' meaning 'to make firm'. In order to fruitfully compare these senses of 'to confirm', as well as the sense of 'to confirm' meaning 'to make firmer', I introduce the uninterpreted concept of rational credibility in terms of which all these concepts are then expressed.

In order to demonstrate the significance of distinguishing the different senses of 'to confirm', I turn in chapters three and four to an examination of Hempel's study of confirmation. His studies have proved to be of considerable interest and have generated a considerable amount of controversy. I summarize his theory of confirmation in chapter three and include a comparison of it with Nicod's theory of confirmation. In chapter four I consider two critiques of Hempel's theory and try to show how the distinctions drawn in chapter two concerning different senses of 'to confirm' are significant in evaluating Hempel's study of confirmation. The critiques which are singled out for examination are those of Scheffler and Carnap. They are of importance mainly because they pertain to Hempel's *explicandum*, rather than to his *explicatum*.

In the fifth chapter I return to a consideration of the concepts of confirmation which were distinguished in chapter two, and I argue that one among the different concepts which I have distinguished is of central importance to the study of confirmation generally. I argue that the qualitative form of the concept of increase in firmness is most important among all the senses of 'to confirm' distinguished, and I try to show how the examination of other concepts can be related to the one important sense of 'to confirm'. I suggest that some of the theorists who may have intended to concern themselves with the important sense of 'to confirm' have perhaps been misled into considering some concept of confirmation which is of lesser importance. I also try to show how their studies might nonetheless be construed as being of at least some value in relation to the central concept of confirmation.

In chapter six I turn to the matter of explication and consider the two main approaches which have been taken to this problem in recent literature on confirmation. I draw particular attention to the use of conditions of adequacy by means of which particular characteristics of a concept of confirmation are expressed. The first approach I consider to the problem of explication is one in which a concept of confirmation is required to meet some probabilistic requirement. I demonstrate which of the conditions of adequacy hold for and which are violated by each of the concepts of confirmation distinguished in chapter two, on the supposition that a plausible probabilistic requirement is imposed on each concept of confirmation. I then consider the second approach to the problem of explication, according to which a set of conditions of adequacy is assigned to each concept of confirmation on the basis of an intuitive understanding of the concepts. These conditions of adequacy thus assigned place restrictions on the nature of the *explicata* one might propose for a given concept of confirmation. I demonstrate that the task of specifying characteristics for a concept of confirmation is plagued by indeterminacy. The failure of the second approach to explication suggests that the first approach might be more fruitful, i.e., the approach in which a plausible probabilistic requirement is imposed upon a given concept of confirmation.

In the last chapter, then, I return to a consideration of the central concept of confirmation whose role was discussed in chapter five, and I assess the plausibility of the probabilistic requirement imposed upon this concept in chapter six. It turns out that the probabilistic requirement in question has been frequently proposed, and I consider the main arguments for and against it. I indicate my qualified sympathy with it and with the use of probabilistic requirements in general in connection with the task of explication. I conclude the dissertation with some general remarks concerning some of the problems which still require solution and indicate what I take to be the direction in which answers appear to be forthcoming.

5. Comments on several assumptions made in the dissertation.

I made reference in preceding pages to various senses of 'to confirm' which will be distinguished in the succeeding pages. A number of authors have drawn attention to the fact that 'to confirm' is sometimes used in a pragmatic, person-dependent sense. Scheffler points out, for example, that 'to confirm' is sometimes taken to mean 'to strengthen a hypothesis at a given time for a given person'.[23] Max Black contends that ". . . the formal and artificially constructed concept of confirmation" is appreciably different from certain familiar but ill-defined, pragmatic concepts in common use rendered by "empirical support" and "empirical evidence," of which the concept of confirmation is supposed to be a technical reconstruction.[24] Carnap also takes note of the fact that some uses of 'to confirm' are person-dependent and that he at one time construed 'to confirm' in such a pragmatic sense.[25] None of the concepts of confirmation which will be subsequently distinguished, however, are pragmatic, person or time-dependent concepts. They are all semantic concepts expressed as relations which hold between two or more sentences. One concept which will be discussed includes an

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[23]Israel Scheffler, The Anatomy of Inquiry (New York: Alfred A. Knopf, 1963), pp. 277 - 278.

[24]Max Black, Margins of Precision (Ithaca, New York: Cornell University Press, 1970), p. 192.

[25]Carnap, LFP, p. xviii, n. 3.

essential reference to temporally prior evidence and thus might appear to be time-dependent. The concept in question only appears to be time-dependent, however, for that concept is not so much time-dependent as prior-evidence-dependent.

Anyone with even a superficial acquaintance with the literature on confirmation will be aware of the fact that the argument-places of confirmation relations have been variously filled. Confirmation relations have been variously construed as holding between propositions, events, sentences, and so on. Some hold that the first argument-place in a relation of confirmation can be filled by objects, i.e., that objects confirm propositions, sentences, etc. I shall adopt a meta-linguistic view of relations of confirmation according to which the argument-places be filled by variable names of sentences. This is the approach taken, for example, by Carnap in Logical Foundations of Probability and by Hempel in his studies on confirmation. This approach permits the language in which relations of confirmation are expressed to be extensional and the underlying logic to be non-modal. Some of the other approaches do not yield these desirable results. The issues connected with this general problem are rather complicated ones and include the theory of meaning in a significant way. This matter does not substantially affect the topics which I discuss in this dissertation. Although I shall not address myself to this problem, I do draw attention to the fact that it is a controversial one.[26]

One of the obvious desiderata of any adequate theory of confirmation is that its results should be applicable to science as it is actually practiced. Sometimes theories of confirmation are offered in which hypothesis or evidence sentence of only one or two kinds are represented. The fact is, however, that in the course of scientific discussion sentences of every conceivable logical form are used as hypotheses and as evidence. Sometimes theorists offer a theory of confirmation in which they deliberately limit the evidence sentences and/or the hypotheses in order that at least a first approximation to an adequate theory might be obtained. Hempel, for example, offers a definition of confirmation in which the evidence sentences are non-general, but acknowledges the fact that ". . . it would seem desirable to free the definition of confirmation from the restricting condition."[27] I assume in this dissertation that sentences of every logical form are capable of being used as evidence sentences and as hypotheses.

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[26] See Howard Smokler, "The Equivalence Condition," *American Philosophical Quarterly*, vol. 4 (1967), pp. 300 - 307, for a discussion of this problem and for references to other authors whose studies are relevant.

[27] Hempel, "A Purely Syntactical Definition of Confirmation," *The Journal of Symbolic Logic*, vol. 8 (1943), p. 143. I shall refer to this article hereinafter as "Definition."

Goodman limits the sentences which can serve as hypotheses to lawlike sentences, for he claims: "Only a statement that is <u>lawlike</u> . . . is capable of receiving confirmation from an instance of it; accidental statements are not."[28] He gives the following example in order to illustrate his view: the fact that a given piece of copper conducts electricity increases the credibility of the lawlike sentence asserting that all copper conducts electricity and thus confirms the hypothesis that all copper conducts electricity; but the fact that a given man now in this room is a third son does not increase the credibility of the accidental sentence asserting that all men now in this room are third sons and thus does not confirm the hypothesis that all men now in this room are third sons.

The central problem with which Goodman is concerned is that of distinguishing between lawlike and accidental universal sentences. He attacks this problem by contending that an instance[29] of a lawlike sentence makes that sentence more

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[28] Goodman, <u>Fact, Fiction, and Forecast</u>, p. 73. Emphasis in the original.

[29] The concept of an instance is unclear, as readers familiar with recent literature on confirmation will know. We can harmlessly assume for the purposes of the argument here that the concept of an instance employed is what I shall call "the concept of a logical instance" (see 3.1 below), according to which <u>z</u> is a logical instance of <u>y</u> if <u>y</u> is a universal conditional sentence of the form '$(x)(Px \supset Qx)$' and <u>z</u> is a conjunction of the form '$Px \& Qx$'.

credible and thus confirms it whereas an instance of an accidental universal sentence does not make the latter sentence more credible and thus does not confirm it. Goodman of course is not defining "accidental universal sentence" as one which is not made more credible by its instances, but is asserting a generalization based on his judgment of characteristics of universal accidental sentences. "Is this judgment plausible?" we might well ask. The answer depends upon our assessment of the credibility relation, and this is inclined to be ambiguous. The assessment of Goodman's position given by Arthur Pap is of interest here — an assessment with which I am in agreement.[30] He maintains that the accidental universal sentence "all of the marbles in this urn are white" is confirmed, in terms of the intuitive notion of "confirmation," by the evidence that ten drawings of marbles from the urn in question are white, and thus denies the validity of Goodman's contention that accidental universals are not confirmed by their instances whereas lawlike sentences are. It seems, intuitively speaking, just incorrect to judge that instances of an accidental universal sentence do not make that sentence more credible than it was prior to the acquisition of those instances. In saying this I am not

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[30] Pap, "Review of *Fact, Fiction, and Forecast* by Nelson Goodman," *The Review of Metaphysics*, vol. 9 (1955-56), p. 298.

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[31] *Ibid.*, p. 298.

examples which could be quite easily changed if any insuperable objection were sustained. In other cases, principles concerning confirmation are at stake, and I have tried to justify my position in such instances. The rules of adequacy discussed in chapter six are especially affected by judgments concerning relations of credibility. The discussion there also shows that definite, intuitively plausible, judgments are often impossible to give. This problem will be discussed in more detail in the study to follow.

CHAPTER TWO

DISTINGUISHING CONCEPTS OF CONFIRMATION

1. The concepts of confirmation$_S$ and confirmation$_F$ distinguished by Carnap.

The central notions of confirmation theory are frequently expressed by such locutions as 'confirming evidence', 'supporting evidence', and 'corroborating evidence'. A significant number of writers consider these locutions to be synonymous with one another, and thus use them interchangeably. No doubt the verbs 'to confirm', 'to support', and 'to corroborate', as well as their corresponding adjectives and nouns, as used in ordinary, non-technical discourse, are highly similar, if not identical, to one another in meaning. One noteworthy difference between these terms as they are used in ordinary discourse, however, is that the phrase '*x* corroborates *y*' often suggests that both *x* and *y* are evidence sentences such that each strengthens or bolsters the support given by the other to some undisclosed hypothesis, whereas the phrases '*x* supports *y*' and '*x* confirms *y*' suggest that *x* is an evidence report and *y* is a hypothesis. The object of my study is epistemic or evidential relations between evidence reports and hypotheses. Insofar as 'to corroborate' has frequently been used as a synonym for 'to support' and 'to confirm', I shall construe 'to corroborate' as also expressing relations between evidence sentences and hypotheses, contrary to its connotation as sometimes used in ordinary discourse. Since I shall consider the verbs 'to confirm', 'to support', and 'to corroborate' to be synonymous, I shall henceforth use mostly the term 'to confirm' and its cognates. This choice is suitable also in view of its frequent occurrence in studies of confirmation. Although the object of my study is evidential relations between evidence sentences and hypotheses, this does not rule out the possibility of considering a given sentence at one time as an evidence sentence and at another time as a hypothesis. To consider a sentence as an evidence sentence or as a hypothesis is rather to specify epistemological roles which a sentence may play.

Several important studies of confirmation have been undertaken in recent years, and in some of these studies, a number of analyses of concepts of confirmation have been proposed. It is my conviction that authors have often been too hasty in proposing formal analyses of concepts in the field of confirmation theory, and that insufficient attention has been devoted to the preanalytical clarification of the concepts which are to be defined. Stephen Barker's remark that ". . . premature formalizations have little power to illuminate the philosophically interesting questions which cluster round the problem of the role of simplicity in scientific thinking"[1] seems to me to be apt not only

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[1]Stephen F. Barker, "On Simplicity in Empirical Hypotheses," <u>Philosophy of Science</u>, vol. 28 (1961), p. 162.

with respect to the study of the role of simplicity but also with respect to the study of concepts of confirmation. I shall begin the study of confirmation by considering various concepts of confirmation to which other theorists have drawn attention. The first author whose work I shall consider is Carnap.

It is a fact, well-known to confirmation theorists, that the key locution 'to confirm' has been used ambiguously. The best-known discussion of this ambiguity is probably that of Carnap's, contained in the Preface to the Second Edition of his initial monumental work on probability and confirmation.[2] Carnap observes that 'to confirm' has the connotation both of 'making firm' and of 'making firmer'. This ambiguity, noticed only relatively recently by philosophers, is evident from a careful perusal of several of the meanings assigned to the word 'confirm' by the _Oxford English Dictionary_. The first and seventh entries under 'confirm' are, respectively: "to make firm or more firm, to add strength to, to settle, establish firmly," and "to corroborate, or to add support to (a statement, etc.); to make certain, verify, put beyond doubt."[3] The verbs 'to corroborate' and 'to support' appear to suffer from the same ambiguity,

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[2] Carnap, _LFP_, pp. xv-xviii.

[3] _A New English Dictionary on Historical Principles_, commonly known as the _Oxford English Dictionary_ (Oxford: Clarendon Press, 1884), ed. James A. H. Murray.

although to a lesser extent perhaps. The nouns and adjectives related to 'to confirm', 'to support', and 'to corroborate' certainly display the same ambiguity, although to varying degrees. Some of the near-synonyms of 'to confirm' in the sense of 'making firm' are 'to make certain', 'to verify', 'to put beyond all reasonable doubt', and 'to establish'. This sense of 'to confirm' has been of interest to many confirmation theorists, but I shall not address myself to it. However, in order to distinguish this sense — the strong sense — of 'to confirm', from other, weaker senses of this term, I shall append the subscript 'S' in order to indicate that the strong sense is meant. Thus the statement 'Evidence _b_ confirms$_S$ hypothesis _h_' shall be understood to mean 'Evidence _b_ establishes, verifies, makes firm, makes certain, puts beyond all reasonable doubt, etc. hypothesis _h_'. Evidence _b_ is understood to be the total available evidence relevant to _h_.

It is helpful to distinguish qualitative, comparative, and metrical forms of the concept of confirming$_S$ evidence.[4] This sort of distinction is made in concepts of the physical sciences, and Carnap is one theorist who has also introduced it into his study

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[4]Carnap speaks in _LFP_ of qualitative, comparative, and metrical concepts of confirmation, but I shall follow R. G. Swinburne who speaks rather of the qualitative form of the concept of confirmation, the comparative form, etc. Cf. Swinburne, "Choosing Between Confirmation Theories," _Philosophy of Science_, vol. 37 (1970), pp. 602 - 603.

of confirmation and probability. The qualitative, or classificatory, form of the concept of confirmation_S is expressed in statements which state <u>whether or not</u> a given hypothesis is confirmed_S (that is, verified, established, etc.) by some specified evidence. The metrical or quantitative form of the concept presupposes that it is possible to metricize the concept of confirmation_S and that one can reasonably assign a numerical value to a pair of sentences consisting of an evidence sentence and a hypothesis representing the degree to which the hypothesis has been established or verified by the evidence sentence. The metrical form of the concept is expressed in such statements as 'The degree to which *h* has been confirmed_S by *b* is 3/4', and 'The degree of confirmation_S of *h* on *b* is 7/8'. The comparative form of the concept of confirming_S evidence is expressed in such statements as 'Hypothesis *h* is better established by evidence *b* than is hypothesis *k* by the same evidence', in which a comparison is made with respect to the amount hypotheses are made firm by evidence reports.

Although it is not my intention to discuss in detail the concept of confirming_S evidence, the foregoing remarks concerning this concept have been necessary in view of the fact that Carnap elucidates the concept of confirmation in the sense of 'making firmer' by reference to the concept of confirmation_S. The concept of confirmation which has the sense of 'making firmer' might be described as a weak concept of confirmation, for evidence which only makes a hypothesis firmer but does not make the hypothesis firm, is weak in comparison with evidence which makes the hypothesis firm. It appears that there is not just one non-strong (or weak) concept of confirmation, but a number of weak concepts of confirmation which are seldom recognized and hence infrequently distinguished from one another. I shall subsequently attempt to distinguish several weak concepts of confirmation, and in order to facilitate the discussion, I shall introduce a second subscript 'F' which shall identify confirming evidence in the sense of 'making firmer'. Thus the statement 'Evidence *e* confirms$_F$ hypothesis *h*' shall be understood as 'Evidence *c* makes hypothesis *h* firmer (although not necessarily firm)'. The concept of confirmation$_F$ is thus one of a number of weak concepts of confirmation. Other weak concepts of confirmation will be introduced in due course.

2. Carnap's definition of confirmation$_F$ using the concept of confirmation$_S$.

The central concept in Carnap's study of confirmation is the concept of confirming$_S$ evidence, i.e., the concept of confirmation which has the sense of 'making firm'. It is, moreover, the metrical form of the concept which Carnap is concerned to clarify and explicate. A satisfactory explication of this form of the concept would of course ensure a successful explication of the qualitative and comparative forms of the concept. The metrical form of the concept of confirming_S evidence is expressed by the phrase 'The degree of confirmation_S (firmness) of a hypothesis h on the total evidence b', which Carnap symbolizes by 'c(h,b)'. The expression 'The degree of confirmation_S of h on b' is regarded as synonymous with the expression 'The logical probability of h on b', and 'c(h,b)' is considered to satisfy the axioms and theorems of the Probability Calculus.

It is in terms of the measure 'c(h,b)' that the concept of confirming_F evidence is discussed and also explicated. Carnap distinguishes three forms of the concept of confirming_F evidence, just as three forms of the concept of confirming_S evidence were distinguished.[5] These are again the qualitative, comparative, and metrical forms of the concept. The simplest form of the concept of confirming_F evidence is the qualitative form, which is expressed in an evaluative statement which merely states whether or not a given evidence report makes a specified hypothesis firmer. The metrical or quantitative form of the concept is expressed in statements which state the degree to which a given evidence report makes a specified hypothesis

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[5]Carnap, LFP, pp. xv - xvi; also pp. 19 - 23.

firmer, e.g., 'The degree to which *h* is made firmer by *e* is ¼'. The comparative form of the concept of confirmation$_F$ is expressed in statements which compare the amounts to which hypotheses are made firmer by evidence reports, e.g., 'The amount by which *h* is made firmer by *e* is the same as (more than, or less than) the amount by which hypothesis *k* is made firmer by evidence *f*'. I shall confine my attention to the simplest form of the concept — the qualitative form — and consider Carnap's explication of it.

Carnap makes clear that the concept of confirming$_F$ evidence is to be understood in the following way: Evidence *e* makes a hypothesis *h* firmer insofar as *e* is a new evidence report (describing a new observational result, for example, or the result of a new experimental test of *h*) which, when added to the prior evidence *b*, contributes positively to the degree of firmness of *h*.[6] The prior evidence *b* consists of the total evidence that is available before the new evidence item *e* is acquired. Hypothesis *h* has a certain degree of firmness (confirmation$_S$) on the basis of *b*, and if the degree of firmness of *h* on the basis of *e* and *b* combined is greater than the degree of firmness of *h* on the prior evidence *b* alone, then *e* is correctly said to make *h* firmer. Carnap's

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[6] *Ibid.*, p. 463.

definition of confirming$_F$ evidence in terms of the function '$c(h,b)$' is:

'e confirms$_F$ h on the basis of b' = df '$c(h,e \& b) > c(h,b)$'.[7] (1)

In the case that no prior evidence exists, e.g., just after a hypothesis has been proposed and no tests of the hypothesis have been carried out, the prior evidence b is counted as null or tautological (symbolized by 't'), and the conjunction 'e & b' in (1) reduces to 'e', while 'b' is replaced by 't'. In this special case, e is said to be <u>initially confirming$_F$ evidence</u> for h and is defined as:[8]

'e initially confirms$_F$ h' = df '$c(h,e) > c(h,t)$'. (2)

It is easy to supply a correlative definition for the qualitative form of the concept of disconfirming evidence, that is, evidence which makes a hypothesis less firm than it was before. This sense of 'to disconfirm' shall be identified by also appending to it the subscript 'F', thereby showing that is a sense analogous to 'to confirm$_F$'. The relevant definition in terms of '$c(h,b)$' is:

'e disconfirms$_F$ h on the basis of b' = df '$c(h,e \& b) < c(h,b)$'. (3)

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[7] *Ibid.*, p. 463. Important definitions, criteria, statements, etc., to which further reference is likely, shall be numbered consecutively in the right-hand margin as shown.

[8] *Ibid.*, pp. 463 - 464.

Although Carnap does not explicitly define the concept of disconfirming$_F$ evidence, (3) is reasonable in view of the fact that it is derivable from (1), elementary arithmetical rules, and the following definition of disconfirming$_F$ evidence:[9]

'$\underline{e}$ disconfirms$_F$ $\underline{h}$ on the basis of $\underline{b}$' = df '$\underline{e}$ confirms$_F$ -$\underline{h}$ on the basis of $\underline{b}$'. (4)

The derivation of (3) from (1) and (4) proceeds as follows: '$\underline{e}$ disconfirms$_F$ $\underline{h}$ on the basis of $\underline{b}$' = '$\underline{e}$ confirms -$\underline{h}$ on the basis of $\underline{b}$' (from (4)) = '$\underline{c}(-\underline{h},\underline{e}$ & $\underline{b}) > \underline{c}(-\underline{h},\underline{b})$' (from (1)) = '$1 - \underline{c}(\underline{h},\underline{e}$ & $\underline{b}) > 1 - \underline{c}(\underline{h},\underline{b})$' (since $\underline{c}(\underline{x},\underline{y}) + \underline{c}(-\underline{x},\underline{y}) = 1$) = '$\underline{c}(\underline{h},\underline{b}) > \underline{c}(\underline{h},\underline{e}$ & $\underline{b})$' (by simple arithmetic).

The correlative definition of irrelevant$_F$ or neutral$_F$ evidence, that is, evidence which makes a hypothesis neither firmer nor less firm than it was before, is now easily obtainable from (1) and (3). From the definition:

'$\underline{e}$ is irrelevant$_F$ to $\underline{h}$ on the basis of $\underline{b}$' = df '$\underline{e}$ neither confirms$_F$ nor disconfirms$_F$ $\underline{h}$ on the basis of $\underline{b}$',

we obtain, in conjunction with (1) and (3), the following:

'$\underline{e}$ is irrelevant to $\underline{h}$ on the basis of $\underline{b}$' = df '$\underline{c}(\underline{h},\underline{e}$ & $\underline{b}) \leq \underline{c}(\underline{h},\underline{b})$ and $\underline{c}(\underline{h},\underline{e}$ & $\underline{b}) \geq \underline{c}(\underline{h},\underline{b})$',

from which it follows that:

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[9] This is a relativized version of the definition of disconfirming$_F$ evidence: '$\underline{e}$ disconfirms$_F$ $\underline{h}$' = df '$\underline{e}$ confirms$_F$ -$\underline{h}$', which is approvingly quoted by Carnap, LFP, p. 479.

'_e_ is irrelevant to _h_ on the basis of _b_' = df '$c(h,e \& b) = c(h,b)$'.

An intuitively appropriate and suggestive synonym for the term 'confirming$_F$ evidence' is 'strengthening evidence', for evidence which makes a hypothesis firmer than it was on the basis of prior evidence alone could be equally appropriately described as evidence which strengthens that hypothesis, or makes that hypothesis stronger than it was before on the then available evidence. An appropriate synonym for 'disconfirming$_F$ evidence' is likewise 'weakening evidence', for evidence which makes a hypothesis less firm than it was on the basis of prior evidence alone could aptly be described as weakening that hypothesis. The term 'to strengthen' is frequently used in studies of confirmation as a synonym for 'to confirm', and 'to weaken' is also taken as a synonym for 'to disconfirm'. L. Jonathon Cohen writes, for example, that ". . . 'to confirm' means, literally, to strengthen, i.e., to make something stronger than it was before."[10] Hempel also uses the terms 'to strengthen' and 'to weaken' as intuitively acceptable synonyms for the concepts of confirmation and disconfirmation that he studies.[11] Now the introduction of an intuitively suggestive synonym does

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[10]L. Jonathon Cohen, _The Implications of Induction_ (London: Methuen, 1970), p. 7, n. 2.

[11]Hempel, "Definition," p. 122.

not add much to the net clarification of a concept. In fact, different authors have sometimes used a rather imprecise but suggestive locution in order to partially elucidate quite different concepts. The terms 'to strengthen' and 'to weaken' seem, however, particularly appropriate synonyms for 'to confirm_F' and 'to disconfirm_F', respectively.

3. The concepts of confirmation_A, confirmation_B, and confirmation_C distinguished by Vincent and Rescher, and remarks on the use of the notion of credibility in defining the three mentioned concepts.

The distinction that Carnap has made between two senses of 'to confirm' has become firmly established within confirmation theory in general. There are other theorists, however, who wish to draw not only the distinction drawn by Carnap, but also other distinctions with respect to the way in which 'to confirm' and related expressions in non-strong or weak senses are sometimes used. One such theorist is R. H. Vincent.[12] Vincent distinguishes the locution '*e* confirms (that is, establishes, or puts beyond reasonable doubt) *h*' from the locution '*e* is confirmatory with respect to *h*', stressing the relative weakness of the latter expression. Vincent thus restricts the use of '*e* confirms *h*'

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[12] R. H. Vincent, "Corroboration and Probability," *Dialogue: Canadian Philosophical Review*, vol. 2 (1963-64), pp. 194 - 205.

to what I call the strong sense of confirmation, i.e., confirmation$_S$, whereas the locution '*e* is confirmatory with respect to *h*' is reserved for use with weak concepts of confirmation. Inasmuch as I find Vincent's preferred locution somewhat cumbersome and because other theorists have not followed his example, I shall use '*e* confirms *h*' for the strong and weak sense of confirmation, adding the appropriating subscripts in order to avoid confusion. Vincent describes '*e* is confirmatory with respect to *h*' as an evaluative locution used to make an "epistemic" or "evidential" appraisal of a body of information on a specified hypothesis. Vincent thinks that this locution has been used in three different ways and gives the following three definitions to represent the different ways in which philosophers have informally construed this locution:[13]

'*e* confirms$_A$ *h*'= df '*h* is more credible than -*h* relative to *e*'; (5)

'*e* confirms$_B$ *h*' = df '*h* is more credible relative to *e* than relative to logical truth alone'; (6)

'*e* confirms$_C$ *h*' = df '*h* is more credible than -*h* relative to *e*, and *h* is more credible relative to *e* than relative to logical truth alone'. (7)

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[13] *Ibid.*, p. 202. Symbols and subscripts have been altered in order to comply with conventions already laid down in this dissertation. Vincent's preferred locution 'is confirmatory with respect to' has also been replaced by 'confirms'.

Vincent adds the following clarificatory comment concerning the *definientia* of (5), (6), and (7): '*h* is more credible than -*h* relative to *e*' is short for 'the claim that *h* is true is more rationally credible than the claim that -*h* is true relative to the information *e*', and the expression '*h* is more credible relative to *e* than relative to logical truth alone' is short for 'the claim that *h* is true is more rationally credible relative to the information *e* than relative to logical information alone'.[14]

The key concept in each of the definitions is of course the concept of credibility expressed in comparative form. The three concepts of confirmation can be easily and readily compared with one another insofar as the one concept in the same form is used in each definition. The differences between the three concepts of confirmation can be readily appreciated even though the concept of credibility is not itself fully explicated. These definitions do not commit one to complete analyses of the concepts thus defined, although they would do so if the concept of credibility was fully (or sufficiently) explicated. In such a case, the above definitions would require careful justification. In the present situation, however, in which no explication of credibility is assumed, the definitions represent an attempt to give the sense of three concepts of confirmation by making use of

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[14] *Ibid.*, pp. 202 - 203.

an important epistemological concept.

Associated with the comparative form of the concept of credibility is the quantitative form of the concept. The three concepts of confirmation defined in (5), (6), and (7) could be defined in an alternative way as follows:

'*e* confirms$_A$ *h*' = df 'the (degree of) credibility of *h* on *e* is greater than the (degree of) credibility of -*h* on *e*'; (5')

'*e* confirms$_B$ *h*' = df 'the (degree of) credibility of *h* on *e* is greater than the (degree of) credibility of *h* on a logical truth alone'; (6')

'*e* confirms$_C$ *h*' = df 'the (degree of) credibility of *h* on *e* is greater than the (degree of) credibility of -*h* on *e* and the (degree of) credibility of *h* on a logical truth alone'. (7')

These definitions presuppose that the quantitative form of the concept of credibility exists. Some authors are reluctant to concede that the quantitative form of certain epistemic concepts are used in the course of epistemological discussion, while there is in general less reluctance to admit the existence of the comparative form of epistemic concepts. I shall adopt the comparative form of the concept of credibility for the purpose of defining various concepts of weak confirmation. Thus the definitions given in (5), (6), and (7) for the three concepts —

confirmation$_A$, confirmation$_B$, and confirmation$_C$ — will be considered the primary definitions for these concepts.

Nicholas Rescher distinguishes three concepts of evidence which have an obvious similarity to the concepts of confirmation distinguished by Vincent.[15] Rescher distinguishes three concepts which he refers to as the concepts of presumptive evidence, supporting evidence, and confirming evidence. Unlike myself, Rescher does not construe 'to support' and 'to confirm' as synonymous terms. His informal, explanatory comments concerning these concepts are rather unhelpful, but the formal interpretation indicates their similarity to the concepts defined by Vincent. Rescher uses the concept of the likelihood of *h* on *e*, symbolized as '$L(h,e)$', to define the three concepts of evidence.[16] The definitions are as follows:

'*e* is presumptive evidence for *h*' = df '$L(h,e) \geq L(-h,e)$';

'*e* is supporting evidence for *h*' = df '$L(h,e) \geq L(h)$';

'*e* is confirming evidence for *h*' = df '$L(h,e) \geq L(-h,e)$ & $L(h,e) \geq L(h)$'.

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[15] Nicholas Rescher, "A Theory of Evidence," *Philosophy of Science*, vol. 25 (1958), pp. 83 - 94.

[16] Rescher says that his likelihood measure *L* is identical with the measure Carnap introduced in *LFP* in connection with the definitions of degree of confirmation. This measure should not be confused with the likelihood measure introduced by R. A. Fisher in "On the Mathematical Foundations of Theoretical Statistics," *Philosophical Transactions of the Royal Society*, Series A, vol. 222 (1922), pp. 309 - 368.

The concept of presumptive evidence is defined by a comparison of the likelihood of _h_ on _e_ with the likelihood of -_h_ on _e_, similar to the concept of confirmation_A which is defined by a comparison of the credibility of _h_ on _e_ with the credibility of -_h_ on _e_. Rescher's concept of supporting evidence is defined by the comparison of the likelihood of _h_ on _e_ with the likelihood of _h_ on a logical truth, similar to the concept of confirmation_B which is defined by a comparison of the credibility of _h_ on _e_ with the credibility of _h_ on a logical truth. An analogous similarity exists between Rescher's concept of confirming evidence and the concept of confirmation_C.

There are several obvious dissimilarities between Vincent's and Rescher's definitions. In the first place Rescher uses in his _definientia_ the relation of being greater than or equal to, whereas Vincent uses the simpler relation of being greater than. It seems to me that Rescher's relation yields some counter-intuitive results which Vincent's relation does not. It is _prima facie_ counter-intuitive, for example, to allow _e_ to be supporting evidence for _h_ even if the likelihood of _h_ on _e_ is no greater than, but is equal to, the likelihood of _h_ on a logical truth. This strikes me as a clear case of irrelevance. There does not appear to be any obvious reason to use the relation of being greater than or equal to, rather than the simpler relation of being greater than. I suggest that the simpler relation yields results which accord better with intuitive judgments and is the more suitable relation of the two. The second dissimilarity in the two accounts is Vincent's use of the concept of credibility, in contrast to Rescher's use of the concept of likelihood. The concepts of credibility and likelihood are *prima facie* different concepts and the suggestion that Vincent and Rescher are offering definitions for the same three concepts deserves comment.

There is a family of terms, including 'probable', 'likely', and 'credible' and their cognates, which are frequently used in various discussions of confirmation theory. Locutions in which these terms occur are sometimes construed in a technical way, i.e., in accordance with a system of axioms and theorems, and at other times such locutions are understood non-technically, e.g., the locution 'the probability of *x* on *y*' is often construed in accordance with a set of axioms and theorems known as the Probability Calculus, but at other times it is understood non-technically. The terms of this family, when preanalytically understood, are closely related to one another in meaning it seems, although the relationships are not clear and are difficult to specify. Much more definite judgments are possible of course when the terms are used technically. Because these terms are closely related, theorists have often selected one term or another in their expositions of concepts of confirmation, sometimes specifying that the term is to be understood in accordance with certain formal requirements, and at other times construing the term in such a way that no formal requirements are imposed on its use. Thus Vincent and Rescher, for example, use closely related locutions in order to define three different concepts of confirmation. Rescher imposes several formal requirements on his use of 'likelihood', unlike Vincent who imposes no formal requirements on his use of 'credibility'. The number of different terms which could be used for the purpose of defining concepts of confirmation is large. For example, the term 'likelihood' can be construed, not only informally, but in accordance with one or more axioms of the Probability Calculus. Someone might choose to understand the locution 'the likelihood of x on y' in such a way that the only axiom which is specified as holding is the axiom which asserts that the likelihood of x on y plus the likelihood of -x on y equals 1. Another might choose to understand the term in such a way that some other axiom is specified as holding, or perhaps several axioms are specified as holding. The resulting number of interpretations of the term 'likelihood' could be quite large. The terms 'probability' and 'credibility', as well as others perhaps, could similarly be construed in a variety of ways. Someone might be tempted to suggest that all the different definitions that might be given for what is said to be a single concept of confirmation, where these definitions differ only insofar as different senses of 'likelihood', 'probability', 'credibility', etc. appear in them, are really definitions for different concepts of confirmation. Such a suggestion, in my opinion, would constitute multiplying concepts beyond necessity and reasonableness. Moreover, different definitions in which the key term may or may not have formal requirements are normally proposed by different authors in an effort to capture the sense in which some concept is preanalytically understood. Thus I consider Vincent's and Rescher's definitions to be two attempts to define the same three uses of 'to confirm'.

In order to define various concepts of weak confirmation and to effectively compare and contrast these concepts — in particular, the concepts to which I have already drawn attention — I shall make use of the locution '*x* is more credible than *y*'. I shall use this locution in a non-technical way and understand it in accordance with Vincent's clarificatory statement quoted above. The locutions '*x* is more likely than *y*' or '*x* is more probable than *y*', understood in a non-technical fashion, could perhaps be used as effectively to define the different concepts of confirmation. The locution '*x* is more credible than *y*' as understood here, however, brings out more clearly than do the other locutions that a theory of confirmation deals with the rational basis for believing a hypothesis to be true. I shall also use the related locution 'the degree of credibility of _x_ on _y_' ('the credibility of _x_ on _y_' for short) as in (5'), (6'), and (7') above, although '_x_ is more credible than _y_' shall be the locution primarily used for defining concepts of weak confirmation. Using one locution with which to define the various concepts of confirmation facilitates the comparing and contrasting of the concepts thus defined. Since the locution I use is understood in a non-technical sense, the concepts are freed from an interpretation which involves the explicit use of a formal probability function. I do not wish to be interpreted here as asserting that the concept of credibility cannot be explicated with reference to a formal probability function. I wish to leave the question open at this time.

4. Definition of confirmation$_F$ using the notion of credibility, and the distinction between prior-evidence-independence and prior-evidence-dependence.

I drew attention in section 2.2 to Carnap's discussion of confirmation and to the distinction he makes between the concept of confirming$_S$ evidence and the concept of confirming$_F$ evidence. The latter concept is of considerable interest here. I shall employ the concept of credibility in order to express the concept of confirming$_F$ evidence. The concept of confirmation$_F$ becomes more readily comparable with the concepts of confirmation defined in (5), (6), and (7), since the same concept is used to express the different senses of confirmation. Moreover, the concept of confirmation$_F$ is freed from an interpretation which involves the explicit use of a formal probabilistic function, i.e., a function which satisfies the axioms and theorems of the Probability Calculus. Carnap's function, '*c*(*h*,*b*)' which is used in (1) and (2) is just that kind of function. The concept of confirming$_F$ evidence which was defined in (1) above, can be alternatively defined in the following way:

'*e* confirms$_F$ *h* on the basis of *b*' = df '*h* is more credible relative to *e* and *b* than relative to *b* alone', (8)

and the concept of initially confirming$_F$ evidence which was defined in (2), can be alternatively defined as:

'*e* initially confirms$_F$ *h*' = df '*h* is more credible relative to *e* than relative to logical truth alone'. (9)

The *definiens* of (8) can be clarified, in analogy to Vincent's clarification of the *definientia* of (5), (6), and (7), as follows: '*h* is more credible relative to *e* and *b* than relative to *b* alone' is short for 'the claim that *h* is true is more rationally credible relative to the information *e* and *b* than relative to the information *b* alone'. The *definiens* of (9) is identical with the *definiens* of (6) which has already been clarified.

The concepts defined in (8) and (9) can also be defined using the metrical form of the concept of credibility as follows:

'$\underline{e}$ confirms$_F$ $\underline{h}$ on the basis of $\underline{b}$' = df 'the (degree of) credibility of $\underline{h}$ on $\underline{e}$ and $\underline{b}$ is greater than the (degree of) credibility of $\underline{h}$ on $\underline{b}$ alone'; (8')

'$\underline{e}$ initially confirms$_F$ $\underline{h}$' = df 'the (degree of) credibility of $\underline{h}$ on $\underline{e}$ is greater than the (degree of) credibility of $\underline{h}$ on a logical truth alone'. (9')

I shall make reference mainly to (8) and (9) rather than (8') and (9') in keeping with my decision to define concepts of confirmation _via_ the comparative form of the concept of credibility. Definitions (8') and (9') show a strong resemblance to definitions (1) and (2), for wherever the locution 'the degree of confirmation$_S$' occurs in (1) and (2), the locution 'the degree of credibility' occurs in (8') and (9'). The difference between (1) and (2), and (8') and (9'), respectively, is that the locution 'the degree of confirmation$_S$' in (1) and (2) is interpreted technically, i.e., in accordance with the Probability Calculus, whereas the locution 'the degree of credibility' in (8') and (9') is not thus interpreted. Although some authors, e.g., Carnap, assert that the credibility of $\underline{x}$ on $\underline{y}$ must be identified with the logical probability of $\underline{x}$ on $\underline{y}$, I shall leave the question open.

One of the noteworthy characteristics of the concept of confirmation$_F$ is its capacity to capture the changing background of temporally prior evidence for a hypothesis. This concept expresses the relation of a given evidence sentence to a hypothesis in view of the temporally prior evidence which has been accumulated up until the time the new evidence is acquired. Thus three items are related: a new evidence report e, a hypothesis h, and a sentence describing the temporally prior evidence b. The reference to prior evidence thus constitutes an integral part of the concept of confirming_F evidence. The effect of changing background of prior evidence on the relation of a given evidence sentence to a given hypothesis is not negligible either, for it is possible for there to be sentences b' and b'' such that a certain evidence sentence e' confirms_F a certain hypothesis h' given b' but e' does not confirm_F h' given b''. I shall illustrate below what I consider to be examples of just such a situation. Whenever the prior evidence for a hypothesis is null or absent, the concept of initially confirming_F evidence is expressed, in which the temporally prior evidence is equivalent to a logical truth 't'. The sort of situation in which the concept of initially confirming_F evidence is capable of being expressed is that in which a hypothesis h has just been proposed and no evidence for h has been acquired. The prior evidence is equivalent to a tautology and the concept of confirming_F evidence then expressed, *viz.*, the concept of initially confirming_F evidence, is expressed in a two-termed relation.

The concepts of confirmation which Vincent defines, in contrast to the concept of confirmation$_F$, include no reference to prior evidence. This characteristic of his definitions (likewise of Rescher's) is important and warrants examination. I wish to maintain that in some studies of confirmation the relation of confirmation is conceived in such a way that the context of prior evidence is completely ignored. In such studies a report which is put forward as possible evidence is completely divorced from the context of prior evidence. The context of prior or background evidence constitutes a normal part of empirical inquiry. In such studies of confirmation, a sentence *e*, which is to be assessed with respect to the hypothesis *h*, might be the report of the first test of *h*, or the fiftieth test, or the hundredth test of *h*, etc. Whether or not *e* confirms *h* depends only upon whether or not there is a certain relation, independent of prior evidence, between *e* and *h*. The prior evidence for or against *h* does not enter into the assessment of whether *e* is confirmatory with respect to *h* or not. I refer to such a relation of confirmation as a relation which is independent of prior evidence or a prior-evidence-independent relation. The concept of confirmation is so conceived that no effective mention of the background of changing prior evidence is possible.[17] This is in direct contrast, for example, to the

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[17] I shall argue in the next chapter that the concept of confirmation in Hempel's study of confirmation is most plausibly construed as such a concept.

concept of $confirmation_F$ which includes as an integral part reference to the prior evidence b. The concept of $confirmation_F$ can thus be appropriately called a prior-evidence-dependent concept of confirmation. The distinction which I am drawing here between prior-evidence-dependent and prior-evidence-independent concepts is quite close to a distinction drawn by Cohen. Cohen examines a concept called "the concept of inductive support" which is ". . . familiarly exemplified, as the concept of a certain timeless relation between propositions, in assessment of the support that experimental evidence gives to scientific hypotheses."[18] This logical relation is said to be often presupposed by some "historically or heuristically important relation" such as the relation of confirmation which has a definite temporal implication.[19] Insofar as the concept of $confirmation_F$ makes an essential reference to the temporally prior evidence it has an implicit temporal implication, unlike the prior-evidence-independent concepts of confirmation which express relations between e and h that are unaffected by prior evidence. The prior-evidence-independent concepts express relations which are akin to logical relations between e and h and might well be explicable by means of logical apparatus

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[18] Cohen, *The Implications of Induction*, p. 12.

[19] *Ibid.*, pp. 7 - 8.

involving only e and h. Prior-evidence-dependent concepts, such as confirmation_F, express relations involving three variables. Consequently, the relationship of a given evidence sentence e and a given hypothesis h will be vitally affected by the prior evidence b. Hence, the concept of confirmation_F will not be explicable by some logical apparatus involving only e and h but will also require the inclusion of b.

It is difficult to show conclusively that the concepts which Vincent distinguishes are of prior-evidence-independent relations. There is no reference to prior evidence in his definitions, in contrast to my definition for the concept of confirmation given in (8) which makes explicit reference to prior evidence. The possibility remains, however, that the concepts expressed in (5), (6), and (7) are considered by Vincent as special cases of general concepts which include a reference to prior evidence.[20] There is considerable reason to think, however, that the concept expressed in (5), i.e., the concept of confirmation_A, is of a prior-evidence-independent relation, since (5) does not appear to be the special case (that is, the case when the prior evidence is null) of a general concept which includes an essential reference to prior evidence. One might propose as

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[20] Vincent does not say that these concepts are special cases of more general concepts which include a reference to prior evidence, and that is some reason to think that he did not consider them to be special cases. On the other hand, arguments from silence are sometimes risky.

a formulation of the general concept of which (5) is to be the special case the following:

'h is more credible relative to e and b than -h relative to e and b', (10)

but the concept expressed in (10) does not have greater generality than the concept expressed in (5). The only difference between the concepts as expressed in (5) and (10) is that the conjunction 'e and b' appears in (10) where 'e' appeared in (5). This does not constitute a difference in generality however, for any sentence consisting of the conjunction 'e and b' is perfectly acceptable as a substitution instance of 'e' in (5). One might suggest as a second proposal for the general concept of which the concept expressed in (5) is to be the special case the following:

'h is more credible relative to e and b than -h relative to e alone', (11)

but (11) is unacceptable as well. The concept expressed in (5) concerns the comparative effect of a constant body of evidence on a hypothesis and the negation of that hypothesis. The concept expressed in (11), however, concerns the comparative effect of changing evidence (because of one reference to prior evidence) to a hypothesis and the negation of that hypothesis. Any proposal for the general concept, of which the concept expressed in (5) is to be the special case, which involves a single reference to prior evidence will be a concept which involves changing evidence. Such a proposal will always be out of harmony with the central feature of the concept expressed in (5), and for that reason will be unacceptable. A general concept, however, of which the concept expressed in (5) is a special case (in a different sense), is the concept expressed in the following statement:

'*h* is more credible than *h'* relative to *e*, where *h* and *h'* are logically incompatible'. (12)

The concept expressed in (5) is a special case of the concept expressed in (12), not in the sense that the concept expressed in (5) is the concept expressed when the prior evidence is null (for (12) makes no reference to prior evidence), but rather insofar as -*h* is a special case of *h'* which is logically incompatible with *h*.

The concept of confirmation_B expressed in (6) is a more likely candidate for the special case (when the prior evidence is null) of a prior-evidence-dependent relation of confirmation. The *definiens* of (9) is identical with the *definiens* of (6), and (9) is the special case (when the prior evidence is null) of (8). This is evidence in favor of the assertion that (6) is a special case of (8), a prior-evidence-dependent concept. Whether or not this is how Vincent understands the concept of confirmation_B it is difficult to tell. Vincent does not say very much about the concept of confirmation$_B$, except to say that the *definiens* of (6) ". . . clarifies to some extent one sense of the everyday expression *e is favourably relevant to h.*"[21] Carnap, on the other hand, says that the concept of initially confirming$_F$ evidence can be understood as *initially positively relevant evidence*, or evidence which is *initially positive.*[22] The important qualifier present in Carnap's exposition, but absent in Vincent's, is the word 'initially'. The presence of this qualifier in Carnap's exposition stresses the fact that the concept of initially confirming$_F$ evidence is the special case of confirming$_F$ evidence when the prior evidence is null. There is one other feature of Vincent's exposition which supports the suggestion that the concept of confirmation$_B$ expresses a relation that is independent of prior evidence. The concept of confirmation$_C$ is a combination of the concepts of confirmation$_A$ and confirmation$_B$. I have argued that the concept of confirmation$_A$, expressed in (5), is a prior-evidence-independent concept. If this is indeed the case, and if in addition the concept of confirmation$_B$ were a prior-evidence-dependent concept, the concept of confirmation$_C$ would be an unworkable combination of a prior-

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[21] Vincent, *op. cit.*, p. 203. I have changed symbols here again in order to conform with conventions already laid down in this thesis.

[22] Carnap, *LFP*, pp. 347, 356, and 462 - 464.

evidence-independent concept and a prior-evidence-dependent concept. It is highly unreasonable to suggest that Vincent would specify such a concept as important to and present in studies of confirmation undertaken by other theorists. In any case I shall assume that Vincent construes the concept of $confirmation_B$ as a prior-evidence-independent concept. I believe that there have been studies in which the prior-evidence-independent concept expressed in (6) has been discussed, and so I shall need some subscripted sense of 'to confirm' to stand for this sense. Thus (6) will serve my purposes very nicely. The concept of $confirmation_B$ is similar in some respects to the concept of initial $confirmation_F$, as I have indicated above. Insofar as both (6) and (9) have the same _definiens_, the concept of credibility, which has been used to elucidate various senses of confirmation, does not succeed in showing a difference between the concepts of $confirmation_B$ and initial $confirmation_F$, which I claim exists. I shall attempt to explain how the prior-evidence-independent concept expressed in (6) differs from the prior-evidence-dependent concept expressed in (9).

5. _The difference between the concepts of $confirmation_B$ and initial $confirmation_F$._

I have assumed that the concept of $confirmation_B$ is independent of prior evidence. This assumption has been made explicit just in case Vincent's intention was not to propose such a concept, although I have indicated several reasons which suggest that he too construes the concept of $confirmation_B$ to be independent of prior evidence. The evidence *e* which is to be assessed for $confirmation_B$-value can be legitimately drawn from any stage in the testing of hypothesis *h*. Evidence *e* need not be the first test of *h*, but can be any test of *h*, or the report of any observation made during the examination of *h*. Evidence *e* is thus divorced from the context of inquiry. If *h* is more credible on the basis of *e* than on the basis of a tautology *t*, then *e* $confirms_B$ *h*. If *h* is less credible on the basis of *e* than on the basis of *t*, on the other hand, then *e* is correctly said to $disconfirm_B$ *h*. If *h* is neither more nor less credible on the basis of *e* than on the basis of *t*, then *e* is $neutral_B$ or $irrelevant_B$ with respect to *h*.

The concept of initially $confirming_F$ evidence, by contrast, is expressible only in contexts in which no evidence for or against a hypothesis has been acquired. If the effect of such initial evidence, say *e'*, is greater on the credibility of a hypothesis *h* than the effect of a logical truth *t*, i.e., if *h* is more credible on *e'* than on *t*, then e' is initially $confirming_F$ evidence for *h*. Any evidence *e"*, which is acquired after *e'* is acquired, could not possibly qualify, in this situation, as initially $confirming_F$ evidence for *h*, although *e"* might have qualified as initially confirming$_F$ evidence had it been the first evidence for h acquired. Evidence e" might be confirming$_F$ evidence for h, that is, h might be more credible on the basis of e' and e" combined than on the basis of e' alone, but e" cannot be initially confirming$_F$ evidence for h. Given that e' is the first evidence report to be evaluated with respect to h, the *only* evidence sentence which can ever qualify as initially confirming$_F$ evidence for h in the whole history of the tests of h will be e'. The concept of confirmation$_B$, however, is such that e', e", and all other evidence sentences can be individually evaluated with respect to their being confirmatory$_B$ or not. The concept of confirmation$_B$ treats evidence sentences *as if* they were all reports of the first test of h.

The difference between the two concepts can be further brought out in the following illustrations. Let us suppose that the hypothesis:

h_1: $(x)(Px \supset Qx)$[23]

is proposed and that evidence for or against h_1 is sought. Let us assume that at some given time no evidence for or against h_1 exists and that the first test of h_1 yields the evidence

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[23] I shall dispense with quotation marks and underlines in sentences set out in separate lines in symbolic notation. I frequently dispense with underlines in sentences not set out on separate lines too.

sentence:

$\underline{e}_1$: Pa & Qa.

Evidence $\underline{e}_1$ is the sort of result which is widely thought to render $\underline{h}_1$ more credible than would a logical truth, and I shall assume that this is so. Thus $\underline{e}_1$ is confirming$_B$ evidence for $\underline{h}_1$, and in view of the fact that $\underline{e}_1$ is the report of the first test of $\underline{h}_1$, $\underline{e}_1$ is also initially confirming$_F$ evidence for $\underline{h}_1$. Let us suppose that the second test of $\underline{h}_1$ yields the evidence sentence:

$\underline{e}_2$: Pb & Qb.

Evidence $\underline{e}_2$ is the sort of result widely thought to render $\underline{h}_1$ more credible than $\underline{h}_1$ is on the basis of prior evidence which is in this case $\underline{e}_1$, i.e., $\underline{h}_1$ is widely thought to be more credible on the basis of $\underline{e}_1$ and $\underline{e}_2$ than it is on the basis of $\underline{e}_1$ alone, and, supposing this is true, $\underline{e}_2$ is confirming$_F$ evidence for $\underline{h}_1$ given $\underline{e}_1$. Evidence $\underline{e}_2$, of course, is not initially confirming$_F$ evidence for $\underline{h}_1$ since there already exists prior evidence, viz., $\underline{e}_1$. However, $\underline{e}_2$ is also confirming$_B$ evidence for $\underline{h}_1$ insofar as $\underline{e}_2$ can be considered on its own, without relating it to the prior evidence or to any other sort of evidence for or against $\underline{h}_1$. Since $\underline{h}_1$ is more credible on the basis of $\underline{e}_2$ than on a logical truth, $\underline{e}_2$ is confirming$_B$ evidence for $\underline{h}_1$. The context in which an evidence report is acquired is completely and deliberately ignored. Evidence $\underline{e}_2$

is treated as if it is the report of the first test of h_1 which is then assessed in order to determine whether or not it is initially confirming$_F$ evidence. The difference between initially confirming$_F$ evidence and confirming$_B$ evidence can be summed up in the following emendations of (6) and (9):

> 'e confirms$_B$ h' = df 'h is more credible relative to e than relative to logical truth alone, where e is any evidence sentence which is independent of the background of prior evidence, i.e., e is the report of any test of or observation relevant to h irrespective of prior evidence'; (6")
>
> 'e initially confirms$_F$ h' = df 'h is more credible relative to e than relative to logical truth alone, where e is the report of the first test of h, i.e., e is temporally prior to any other test of or observation relevant to h'. (9")

Although the concept of confirmation$_B$ differs from the concept of initial confirmation$_F$, a criterion which is designed to determine whether or not h is more credible relative to e than relative to a logical truth alone would be of value for both concepts. In order to determine that e initially confirms$_F$ h, however, we would have to determine in addition whether or not e is the report of the first test of h. If e is not the report of the first test of h it could not possibly qualify as initially confirming$_F$ evidence for $\underline{h}$. Criteria for the concept of confirmation$_B$ and the general concept of confirmation$_F$, on the other hand, would be markedly different. The main reason for this is that the concept of confirmation$_F$ involves an essential mention of prior evidence, and as the prior evidence varies, so does the relation between a given evidence sentence and a given hypothesis. There are several kinds of illustrations which show this dependence, and which also serve to bring out in a graphic way the difference between a prior-evidence-independent concept and one that is dependent on prior evidence. Hardly more than a few, modest, intuitively plausible assumptions concerning credibility comparisons is required in these illustrations.

The first illustration has to do with a hypothesis which has been falsified in the course of its testing. Let us imagine that the hypothesis $\underline{h}_1$ (used in illustration several pages back) continues to be tested and that the third experimental test of $\underline{h}_1$ yields the evidence sentence:

$\underline{e}_3$: Pc & -Qc.

Evidence $\underline{e}_3$ falsifies $\underline{h}_1$, or renders it infirm. Let us further suppose that a fourth experimental test of $\underline{h}_1$ yields the evidence sentence:

$\underline{e}_4$: Pd & Qd.

It is plausible to judge $\underline{e}_4$ to be confirmatory$_B$ with respect to $\underline{h}_1$, since $\underline{e}_4$ by itself makes $\underline{h}_1$ more credible than would a logical truth. It is implausible to maintain, however, that $\underline{e}_4$ is confirming$_F$ evidence for $\underline{h}_1$, for $\underline{h}_1$ is hardly firmer on $\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$, and $\underline{e}_4$ combined than it is on $\underline{e}_1$, $\underline{e}_2$, and $\underline{e}_3$ alone, since $\underline{e}_3$ which renders $\underline{h}_1$ completely infirm is contained in both conjunctions. To assess $\underline{e}_4$ as confirming$_F$ evidence for $\underline{h}_1$ would eventuate in a falsified hypothesis being "revived" by suitable evidence obtained after the falsifying evidence was obtained. This would of course be absurd. Thus, no evidence obtained after a falsifying report is obtained can be confirming$_F$ evidence for the falsified hypothesis.

A second illustration of the difference between the concepts of confirmation$_B$ and confirmation$_F$ involves the use of existential hypotheses which have been rendered certain. Suppose that the hypothesis:

$\underline{h}_2$: (Ex)Kx

has been rendered certain by the evidence sentence 'Ka'. Any experimental result or observational information obtained after the acquisition of 'Ka' cannot render $\underline{h}_2$ firmer than it is on the basis of 'Ka', since it is certain on 'Ka', and thus cannot be confirming$_F$ evidence for $\underline{h}_2$, e.g., the report 'Kb' which is obtained after the acquisition of 'Ka' cannot make $\underline{h}_2$ firmer than it is on 'Ka' since $\underline{h}_2$, _ex hypothesi_, is certain or firm (confirmed$_S$) on the basis of 'Ka'. A concept of confirmation which is independent of prior evidence, such as the concept of confirmation$_B$, however, is such that each evidence report is eligible to be assessed independently of the prior evidence background, and the evidence 'Kb' is confirming$_B$ evidence for $\underline{h}_2$ just as is 'Ka'.

The third example illustrating the difference between the concepts of confirmation$_B$ and confirmation$_F$ involves universal generalizations. There have been authors who have held that some universal hypotheses may be rendered firm or certain by evidence of a singular character and that the evidence acquired in addition to the confirming$_S$ evidence does not provide a further increase in firmness to such hypotheses. R. M. Eaton, for example, writes:[24]

> In a highly integrated science, like modern physics, a single observation or experiment — a shift in the spectral lines from a remote star — may establish an important result. The scientist does not seriously consider that sheer repetition of experiments, beyond two, three, or a dozen trials to eliminate possible errors, contributes much to the solidity of the theory tested by them.

John Stuart Mill expresses a similar idea in the question: "Why is a single instance, in some cases, sufficient for a complete induction, while, in others, myriads of concurring instances, without a single exception known or presumed go such a very

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[24]R. M. Eaton, General Logic (New York: Charles Scribner's, 1931), pp. 484 - 485.

little way toward establishing a universal proposition?"[25] A more recent writer, Stephen Toulmin, remarks that ". . . physicists seem to be satisfied with far fewer observations than logicians would expect them to make: one finds in practice none of that relentless accumulation of confirming instances which one would expect from reading books on logic," adding also that ". . . to establish the form of a regularity in physics only a few careful observations are needed."[26] On the supposition that these authors are right and that some universal hypotheses can be established by a finite number of evidence reports, then further evidence reports would not constitute confirming_F evidence for established hypotheses, that is, further evidence would not make a hypothesis already established any stronger, although such evidence may well be confirming_B evidence. Thus $\underline{h}_1$, for example, might finally be rendered certain by the evidence report:

$$\underline{e}_n\text{:} \quad Px_n \ \& \ Qx_n.$$

Then any subsequent evidence report, e.g.:

$$\underline{e}_{n+1}\text{:} \quad Px_{n+1} \ \& \ Qx_{n+1}$$

would not make $\underline{h}_1$ firmer, that is, $\underline{e}_{n+1}$ would not confirm_F $\underline{h}_1$,

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[25] J. S. Mill, A System of Logic, Eighth Edition (London: Longmans, Green, 1930), Book 3, Chapter III.

[26] Stephen Toulmin, The Philosophy of Science (London: Hutchinson, 1953), p. 99.

given $\underline{e}_1$, $\underline{e}_2$, . . . , $\underline{e}_n$, although $\underline{e}_{n+1}$ would be confirming$_B$ evidence for $\underline{h}_1$ insofar as $\underline{h}_1$ would be more credible on the basis of $\underline{e}_{n+1}$ than on the basis of logical truth alone.

Some theorists have also thought that some universal hypotheses, at least, are incapable of receiving support from a certain kind of evidence after much evidence of that kind has been accumulated, even though the evidence already accumulated fails to render the hypothesis certain. There is a suggestion of this in the passage from Mill quoted several pages back. Hypothesis $\underline{h}_1$, for example, might be confirmed$_F$, but not established, by evidence of the form 'Px & Qx' until the evidence report:

$\underline{e}_m$: Px_m & Qx_m

is reached, while the evidence acquired thereafter no longer confirms$_F$ $\underline{h}_1$. The concept of confirmation$_B$, however, is such that all evidence of the form 'Px & Qx', including the evidence:

$\underline{e}_n$: Px_n & Qx_n,

where $n > m$, confirms$_B$ $\underline{h}_1$.

The foregoing examples illustrate the need, I contend, to distinguish concepts which are dependent on prior evidence from those without such a dependence. Criteria for concepts which are prior-evidence-independent frequently specify only the logical form of evidence sentences, and this is sufficient since every sentence of a given logical form is either confirming, disconfirming, or neutral with respect to a given hypothesis. The concept of confirmation$_B$, for example, is such that every sentence of the form 'Px & Qx' confirms$_B$ a hypothesis of the form '(x)(Px ⊃ Qx)', assuming that sentences of the form 'Px & Qx' make '(x)(Px ⊃ Qx)' more credible than does a logical truth. This is not the case with the concept of confirmation$_F$, inasmuch as '(x)(Px ⊃ Qx)' is not always confirmed$_F$, given the prior evidence, by evidence reports of the form 'Px & Qx', e.g., in such cases in which '(x)(Px ⊃ Qx)' has already been falsified or been rendered firm. Concepts of confirmation which are independent of prior evidence thus take on the character of purely logical relations, because with respect to a specific hypothesis _h_, every sentence of a given logical form either confirms, disconfirms, or is neutral to _h_. In the following chapters I wish to demonstrate, with reference to an actual theory of confirmation, the value and importance of distinguishing concepts of confirmation which are prior-evidence-independent from those which are prior-evidence-dependent.

CHAPTER THREE

THE CRITERIA OF CONFIRMATION OFFERED BY NICOD AND BY HEMPEL

1. Nicod's definition of confirmation and its shortcomings.

The value and importance of distinguishing weak senses of 'to confirm' from one another can be graphically shown, in my estimation, with reference to Hempel's famous study of qualitative confirmation and to several critiques of his study. Hempel's study is interesting in its own right insofar as it represents one of the first careful probes into the crucial question: "What should count as constituting evidence for a hypothesis?" Hempel's study, moreover, utilizes a number of techniques which promise to aid the study of confirmation. A number of issues will be discussed while Hempel's study of confirmation is briefly sketched and discussed in this chapter. In the following chapter I shall consider several critiques of Hempel's study.

Hempel's study of confirmation is contained primarily in the two papers already mentioned, *viz.*, "A Purely Syntactical Definition of Confirmation," and "Studies in the Logic of Confirmation." "Studies" is the more informal paper of the two, while "Definition" consists of a careful, formal study. Hempel begins his examination of the concept of confirmation in "Studies" with a scrutiny of Nicod's criterion of confirmation.

Nicod held what I call "the instantial view of confirmation." Although the criterion of confirmation which Nicod presents is badly deficient, it embodies several ideas which play an important role in Hempel's theory of confirmation. Hence, an understanding of the instantial view of confirmation will facilitate an appreciation of Hempel's work.

The instantial view of confirmation is the view that a hypothesis *h* is confirmed or supported by its instances. Consider the hypothesis that everything which is *P* is also *Q*, that is:

h_1: $(x)(Px \supset Qx)$.

This is supported by the report of an object *a* which is *P* and *Q*, that is:

e_1: $Pa \mathbin{\&} Qa$.

Nicod expresses such a criterion of confirmation in the following passage:[1]

> Consider the formula or the law: *A entails B*. How can a particular proposition, or more briefly, a fact, affect its probability? If this fact consists of the presence of B in a case of A, it is favourable to the law "A *entails B*"; on the contrary, if it consists of the absence of B in a case of A, it is unfavourable to this law. It is conceivable that we have here the only two direct modes in which a fact can influence the probability of a law. Given the hypothesis, either a fact realizes the

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[1] Jean Nicod, *Foundations of Geometry and Induction* (London: Kegan Paul, Trench, Trubner, 1930), translated by P. P. Wiener, p. 219. Emphasis in the original.

> conclusion and lends support to the law, or else it does not realize the conclusion and refuses to support the law . . . Thus, the entire influence of particular truths or facts on the probability of universal propositions or laws would operate by means of these two elementary relations which we shall call confirmation and invalidation.

Thus the hypothesis which asserts that all objects having the property P have also the property Q is confirmed by the proposition, for example, that object a is both P and Q, and is invalidated by the proposition that b is P but not Q.

Nicod construes the arguments of the relation of confirmation to be propositions. However, since the discussion of Nicod's criterion is prefatory to the discussion of Hempel's definition, and Hempel's definition is given for a relation between sentences rather than between propositions, I shall construe Nicod's criterion as given for a relation which holds between sentences. Nicod's criterion of confirmation can be expressed as follows:

e confirms h if and only if (iff) e is an instance of h. (13)

An instance, in Nicod's sense, is a sentence which describes an object such that the antecedent and the consequent of a universal conditional sentence are true of it. I shall refer to an instance, in this sense, as being a logical instance of a hypothesis. A logical instance of the sentence C_1: '$(x)(Px \supset (P_1x \supset Qx))$', for example, is the sentence '$Pa \,\&\, (P_1a \supset Qa)$', for the latter sentence describes an object a such that the antecedent and the consequent of C_1 are true of a. A more precise expression of Nicod's criterion of confirmation, appropriate for symbolic languages, is as follows:

> e confirms h iff e is a conjunction consisting of the instantiated antecedent and the instantiated consequent of a universal conditional sentence h, where the same variable in the matrix of h is instantiated by the same individual constant throughout.[2] (14)

A universal conditional is invalidated, according to Nicod, by a sentence which gives a description of an object such that the antecedent is true of it but the consequent is false of it. Such sentences, sometimes called "negative instances," conclusively falsify a universal conditional sentence and are universally recognized by confirmation theorists as providing disconfirming evidence of the strongest kind. The sentence C_1, for example, is invalidated or falsified by the sentence '$Pb \,\&\, -(P_1b \supset Qb)$', for the antecedent of C_1 is true of b but the consequent of C_1 is false of b. Nicod's criterion for disconfirming evidence can be more precisely expressed for symbolic languages as follows:

> e disconfirms h iff e is a conjunction consisting of the

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[2] This criterion closely resembles Hempel's version of Nicod's criterion offered in "Definition," p. 129.

instantiated antecedent and the negation of the instantiated consequent of the universal conditional h, where the same variable in the matrix of h is replaced by the same individual constant throughout. (15)

This criterion of disconfirmation uses the same logical apparatus as the criterion of confirmation expressed in (14) above.

There are several noteworthy deficiencies in Nicod's criteria. First of all, Nicod's criteria for confirmation and disconfirmation do not allow the widely held definition of disconfirmation *via* confirmation:

'*e* disconfirms *h*' = df '*e* confirms -*h*', (16)

to hold, for if *h* is a universal conditional sentence, -*h* will be an existential sentence and existential sentences do not have logical instances. This defect in Nicod's criteria goes hand in hand with the first of two defects outlined by Hempel, *viz.*, that Nicod's criterion of confirmation is limited to hypotheses of universal conditional form, providing no standard of confirmation for sentences of any other form, such as existential or particular sentences.[3] The passage quoted above from Nicod makes this evident, for he makes reference only to laws or universal propositions. Hempel also contends that Nicod's criteria are applicable to universal conditional sentences

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[3]Hempel, "Studies," p. 11.

having only monadic predicates, because universal conditionals with dyadic predicates are capable of being disconfirmed and confirmed, on Nicod's criteria, by the same sentence. Hempel's example of this involves the hypothesis h': '(x)(y)((-Lxy & Lyx) ⊃ (Lxy & -Lyx))', which is confirmed according to Nicod's criterion by the same sentence e': '-(Lab & Lba) & (Lab & -Lba)', and is alleged to be disconfirmed on Nicod's criterion of disconfirmation by the logically equivalent sentence e": '-(Lba & Lab) & (-Lba & Lab)'.[4] Now '(Lba & Lab)' is the instantiated antecedent of h' but '(-Lba & Lab)' is not the negation of the instantiated consequent of h' which is '-(Lba & -Lab)'. Thus e" does not disconfirm h' according to the criterion of disconfirmation expressed in (15). This indicates that Hempel has interpreted Nicod's criterion of disconfirmation in a way different from that expressed in (15). Since '(-Lba & Lab)' does entail the negation of the instantiated consequent of h', '-(Lba & -Lab)', Hempel perhaps interprets Nicod's criterion of disconfirmation as follows:

> e disconfirms h iff e is a conjunction consisting of the instantiated antecedent and a sentence s entailing the negation of the instantiated consequent of the universal conditional h, where the same variable in the matrix of

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[4]Hempel, "Recent Problems of Induction," p. 125.

h is replaced by the same individual constant throughout. (15')

Suitable restrictions would probably have to be placed on s in order that (15') would continue to represent Nicod's notion of disconfirmation, e.g., s would likely have to be non-logically false. Hempel's criticism that Nicod's criteria are applicable to universal conditional sentences having only monadic predicates thus depends for its plausibility upon the acceptance of Hempel's interpretation of Nicod's notion of disconfirmation. The criterion of disconfirmation expressed in (15) strikes me as being more adequate to Nicod's conception of disconfirmation than the criterion of disconfirmation expressed in (15'). Hence, it is not obvious that Hempel has succeeded in uncovering a second defect in Nicod's criteria. Even if Hempel's interpretation of Nicod's conception of disconfirmation is rejected, Hempel has certainly pinpointed a serious deficiency in Nicod's criteria, namely, that they are restricted to universal conditional sentences.

The second defect in Nicod's criterion of confirmation, according to Hempel, is that it ". . . makes confirmation depend not only on the content of the hypothesis, but also on its formulation."[5] The hypothesis 'All swans are white', for example, can

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[5]Hempel, "Studies," p. 12.

be equivalently formulated as 'Whatever is not white is not a swan', but a sentence reporting the sighting of a white swan a confirms the swan-hypothesis only in its first formulation, not in its second, according to Nicod's criterion. Letting 'S' stand for 'is a swan' and 'W' for 'is white', the two sentences can be symbolized as follows:

$\underline{h}_2$: $(x)(Sx \supset Wx)$,

$\underline{h}_2'$: $(x)(-Wx \supset -Sx)$,

and the report of a white swan can be symbolized as:

$\underline{e}_2$: Sa & Wa.

Evidence $\underline{e}_2$ is a logical instance of $\underline{h}_2$ but not of $\underline{h}_2'$ (although '-Wb & -Sb' would be), and so $\underline{e}_2$ confirms $\underline{h}_2'$ according to Nicod's criterion. Hypotheses $\underline{h}_2$ and $\underline{h}_2'$ are logically equivalent sentences, and Hempel considers any criterion of confirmation, including Nicod's, which fails to assess an evidence sentence uniformly with respect to logically equivalent sentences to be defective. Hempel's criticism is of course based on his acceptance of the logical condition of adequacy that if e confirms h, then e confirms every sentence logically equivalent to h. A number of conditions of adequacy have been discussed in the relevant literature on confirmation, including the above condition known as the equivalence condition for hypotheses. This condition and other important conditions as well will be the subject of later discussion.

2. <u>Hempel's definition of confirmation</u>.

Whereas Nicod's criterion of confirmation is based upon the antecedent and the consequent of only universal conditional sentences being true of an object, Hempel's criterion is based upon <u>sentences</u> of any logical form being true of an object (or a sequence of objects). Hempel's definition of confirmation is based upon the notion of <u>satisfaction</u> — evidence <u>e</u> confirms the hypothesis <u>h</u> if the hypothesis is satisfied in the class of objects mentioned in the evidence report. Put another way: <u>e</u> confirms <u>h</u> if one can infer from the information contained in <u>e</u> that <u>h</u> holds true in the finite class of objects which are mentioned in <u>e</u>. The criterion which is expressed in Hempel's definition is thus dubbed "the satisfaction criterion of confirmation."[6]

Since Hempel's definition is not restricted to hypotheses of universal conditional form, but embraces hypotheses of any logical form whatsoever, the possible hypothesis-sentences include existential sentences, universal sentences, particular sentences, and "mixed generalizations," i.e., sentences with at least one existential quantifier and at least one universal quantifier when symbolized in the appropriate symbolic language. The evidence-sentences in Hempel's study are required to be of

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[6] <u>Ibid</u>., p. 37.

molecular form. A molecular sentence, or a molecule, is a sentence which is either atomic or consists of atomic sentences and sentence connectives. An atomic sentence is a sentence consisting of a predicate followed by as many individual constants as the degree of the predicate requires, e.g., 'Pa' and 'Rab' are atomic, whereas 'Pa & -Rab' is molecular. Permitting only molecular sentences to be evidence sentences is a limitation, Hempel admits, but his study is designed to represent ". . . only a first attempt to arrive at a systematic logical theory of confirmation."[7] This feature of evidence sentences, moreover, corresponds to the idea that the data obtained in the test of hypotheses are usually particular in character.

Hempel takes the relation of confirmation to hold between two sentences of an idealized language L which has the logical structure of a quantification theory without identity. In the account to follow, which is based on Hempel's study, '<u>h</u>' is a variable ranging over hypotheses or hypothesis-sentences, and '<u>e</u>' is a variable ranging over molecular sentences providing evidence; the first four letters of the English alphabet, with or without subscripts, are used as individual constants, and 'x', 'y', and 'z', with or without subscripts, are used as individual variables; and predicate symbols consist of the

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[7]Hempel, "Definition," p. 143.

letters 'P', 'Q', and 'R', with or without subscripts, the first two being used to symbolize predicates of the first degree. Language L contains the usual logical connectives, quantifier signs, auxiliary signs of symbolic languages, etc.[8] The sentences of L are formed according to the usual rules for well-formedness of quantification theory. The individual constants of L are thought of as names of particular things which form the object of some inquiry, and the predicates of L are true or false of those objects.

Hempel illustrates the basic idea of his definition of confirmation with reference to the following hypothesis S and the evidence sentence M:[9]

S: $(x)(Px \supset Qx)$,

M: Pa & Qa & Pb & Qb & -Qc & -Pc.

M is said to be confirming evidence for S in the following sense:[10]

> S asserts that the extension of P is included in the extension of Q (and, consequently, the extension of -Q in that of -P); and for the objects referred to in M, namely, a, b, and c, it is indeed the case that all

[8] Because of typographical limitations, '&' is used throughout for the conjunction symbol instead of the raised dot, '-' is used for negation, and the sign for existential quantification is '(E)' instead of the more customary inverted 'E'. For complete details of Hempel's account see "Definition."

[9] Hempel, "Definition," p. 130.

[10] *Ibid.*, p. 130.

> those reported in *M* as belonging to the extension of *P* are also reported as belonging to the extension of *Q*, and those reported as belonging to the extension of *-Q* are also reported as belonging to the extension of *-P*.

This idea is generalized to include hypotheses which cannot be interpreted as asserting a relation of inclusion, e.g., existential hypotheses such as '(Ex)(y)(Px v Rxy)', so that the concept of confirming evidence is expanded as follows:[11]

> The above molecule *M* determines a certain class of individuals /‾*a*,*b*,*c*_/; and we may say that *M* confirms *S* because from the information contained in *M* it can be inferred that in /‾*a*,*b*,*c*_/ *S* is completely satisfied; or, to use a metaphor: in a world containing exclusively the individuals *a*, *b*, and *c*, the sentence *S* would be true, according to the information contained in M.

In such a finite world, i.e., in a world containing exclusively the individuals *a*, *b*, and *c*, the content of *S* could be expressed as the molecular sentence:

I: $(Pa \supset Qa) \& (Pb \supset Qb) \& (Pc \supset Qc)$,

and the assertion that *S* would be true in that world, according to the information contained in *M*, can be more precisely expressed by saying that in that world *I* is a logical consequence of *M*. *I* is called "the development of *S* for the individual constants contained in *M*," or "the *IM*-development of *S*." The idea of *IM*-development is generalized by means of a recursive

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[11]*Ibid.*, p. 130. Because of typographical limitations, diagonal brackets are used to symbolize classes.

definition.[12] The central idea of Hempel's definition of confirming evidence can be expressed for the sentences S and M used in illustration as follows: "A sentence S is confirmed by a molecule M if M entails the IM-development of S."[13]

Refinements are made upon this central idea in order to obtain a satisfactory definition of 'e confirms h'. First, the two-termed relation of direct confirmation (Cfd) is introduced which captures the basic and central idea. The definition of Cfd(e,h), that is, 'e is directly confirming evidence for h', is put in the following form:[14]

> Cfd(e,h) iff (a) e is a molecule, (b) the Ie-development of h is not analytic or h is analytic, and (c) e entails the Ie-development of h.

A few comments about this definition are in order. Condition (a) restricts the evidence to molecules — a restriction I have commented on above; condition (b) is added by Hempel to avoid some untoward consequences of a minor nature, on which I shall not comment; and (c) of course expresses the central idea of the definition of confirming evidence. From further discussion of the definition for Cfd in "Studies," it is evident that Hempel would revise condition (c) to something like:

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[12] For details see "Definition," p. 131f.

[13] Ibid., p. 131.

[14] Ibid., p. 142.

(c') *e* entails the *Ie*-development of *h* for individual constants occurring essentially (in non-analytic components) in *e*.

The reason for altering condition (c) is that according to the definition for *Cfd* given above (with condition (c)) the evidence report:

e_3: Pa & (Qb v - Qb)

does not entail the Ie_3-development of:

h_3: (x)(Px),

which is the sentence:

Ie_3: Pa & Pb.

That is, e_3 does not entail Ie_3, which is the development of h_3 for the individual constants mentioned in e_3, *viz.*, 'a' and 'b'. Hence e_3 does not directly confirm h_3. This fact, says Hempel, ". . . is obviously due to the circumstance that e_3 contains the individual constant 'b', without asserting anything about *b*: The object *b* is mentioned only in an analytic component of e_3."[15] This deficiency in the definition of *Cfd* is easily remedied by replacing condition (c) with (c').

The second refinement to which I shall draw attention is the introduction of the relation *Cf*(*e*,*h*) (*e* is confirming evidence for *h*) which is a broader relation than *Cfd* and which is based upon the definition for *Cfd*. One of the shortcomings of

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[15] Hempel, "Studies," p. 38, n. 46.

the definition of Cfd is that it violates the general consequence condition.[16] This condition of adequacy is that evidence which confirms a hypothesis also confirms all consequences of that hypothesis. Thus, for example, Hempel regards 'Pa' as being confirming evidence not only for '(x)(Px)', but also for all consequences of '(x)(Px)', such as 'Pb'. The definition for Cfd which he gives does not imply that 'Pa' is confirming evidence for 'Pb' and so has only a limited application. The relation of Cf(e,h) overcomes this drawback, among others, and is consequently defined as follows:[17]

> Cf(e,h) iff (a) e is a molecule; and (b) there is a set of hypotheses K which entails h and every member of the set K either is entailed by e or is directly confirmed (i.e., stands in the relation Cfd) by e.

Thus 'Pa', to revert back to the example, confirms 'Pb', (although not directly) for '(x)(Px)' entails 'Pb', and 'Pa' directly confirms '(x)(Px)'. In this example the set K required by the definition of Cf consists of the sentence '(x)(Px)'.

The most important standard against which the adequacy of a definition for confirmation is measured is the set of logical adequacy conditions, one of which is the general consequence condition. These conditions express principles of confirming evidence which, according to Hempel, are essential to the concept

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[16] Hempel, "Definition," p. 136.

[17] *Ibid.*, p. 142.

of confirmation. He lists a total of eleven conditions which deal with features of evidence sentences and hypotheses such as consistency, relations of entailment, etc. insofar as these features pertain to the relation of confirmation. Thus the definitions of _Cfd_ and _Cf_ offered by Hempel not only have to meet the rather vague requirement of yielding results which are in close agreement with intuitive demands, but they also have to meet the more specific requirement of satisfying various logical conditions of adequacy which are thought to embody the most crucial features of the concept of confirmation.

3. _Consequences of Hempel's definition of direct confirmation, and the paradoxes of confirmation._

One of the consequences of Hempel's study of confirmation is his discovery of the paradoxes of confirmation. I shall not deal with solutions to the paradoxes of confirmation, although in the subsequent discussion I shall occasionally refer to them. The paradoxes are most popularly generated from two apparently highly plausible assumptions, _viz._:

A. The equivalence condition for hypotheses: If _e_ confirms _h_ then _e_ confirms every sentence logically equivalent to _h_; and

B. A logical instance of a universal conditional hypothesis confirms that hypothesis.

Assumption B expresses Nicod's criterion of confirmation as a sufficient condition for confirmation, and Hempel indeed construes Nicod's criterion as such. Assumption A is endorsed by Hempel, as I have previously mentioned. According to assumption B the hypothesis 'Whatever is not a swan is not white' is confirmed by a evidence sentence reporting an observation of a non-white non-swan $\underline{b}$ (which is perhaps a blue vase), i.e., the evidence sentence:

$\underline{e}_4$: $-Wb \,\&\, -Sb$

confirms the hypothesis:

$\underline{h}_2'$: $(x)(-Wx \supset -Sx)$.

The hypothesis asserting that all swans are white, i.e.,

$\underline{h}_2$: $(x)(Sx \supset Wx)$,

is logically equivalent to $\underline{h}_2'$ and is thus confirmed by $\underline{e}_4$ according to assumption A. Thus the report of a non-white non-swan confirms the hypothesis that all swans are white. This is one of the so-called "paradoxes of confirmation," since it is paradoxical to hold, according to some theorists, that the report of a non-white non-swan can confirm the hypothesis that all swans are white. The report of a white swan would presumably paradoxically confirm the hypothesis that whatever is not white is not a swan. The paradox consists in the fact that on the basis of some intuitive assessments certain evidence reports are neutral, whereas on the basis of intuitively plausible assumptions such as A and B those evidence reports must be judged as confirming.

Hempel endorses assumptions A and B, and with that accepts the paradoxically confirmatory evidence sentences as confirmatory. His definition also implies that $\underline{e}_4$ confirms $\underline{h}_2$. In fact, $\underline{e}_4$ directly confirms $\underline{h}_2$. A total of five different evidence sentences have been regarded as paradoxically confirmatory with respect to universal conditional sentences. I shall consider the sentences which have been considered paradoxically confirmatory with respect to the hypothesis:

$\underline{h}_1$: $(x)(Px \supset Qx)$.

Instead of merely enumerating these sentences, I shall consider in more generality the possible evidence sentences which one might have concerning the object a using the predicates 'P', '-P', 'Q', and '-Q'. The number of logically distinct molecular sentences which can be formed is sixteen, and are as follows:

(i) -Pa & -Qa,
(ii) -Pa & Qa,
(iii) -Pa,
(iv) -Pa v Qa,
(v) Qa,
(vi) Pa ≡ Qa,
(vii) Pa & Qa,
(viii) Pa & -Pa,
(ix) (Pa v -Pa) & (Qa v -Qa),
(x) Pa ≡ -Qa,
(xi) -Pa v -Qa,
(xii) Pa v -Qa,
(xiii) Pa v Qa,
(xiv) Pa,
(xv) -Qa,
(xvi) Pa & -Qa.

According to Hempel's definition, sentences (i) - (viii) are confirming evidence for $\underline{h}_1$ (and to every sentence logically equivalent to $\underline{h}_1$), sentences (ix) - (xv) are neutral or irrelevant, and sentence (xvi) is disconfirming. Sentences (i) - (v) are paradoxically confirmatory with respect to $\underline{h}_1$. Thus sentence (iii), for example, which might describe an object $\underline{a}$ which is $-\underline{P}$, is confirmatory with respect to the hypothesis that everything which is $\underline{P}$ is also $\underline{Q}$.

It should be noticed that an evidence sentence is paradoxically confirmatory with respect to a hypothesis only if that hypothesis is expressed in one form and not another. Thus (i) is paradoxically confirmatory with respect to $\underline{h}_1$ but not paradoxically confirmatory with respect to the logically equivalent sentence:

$$\underline{h}_1': \quad (x)(-Qx \supset -Px).$$

So the sentences (i) - (v) are paradoxically confirmatory with respect to $\underline{h}_1$, but not necessarily to every other sentence logically equivalent to $\underline{h}_1$. Thus it is possible that there are sentences logically equivalent to $\underline{h}_1$ with respect to which some or all of sentences (vi) - (viii) are also paradoxically confirmatory. Whereas the form of a sentence determines whether or not a given evidence sentence is <u>paradoxically</u> confirmatory with respect to it, the form of a sentence does not determine whether or not a given evidence sentence is <u>confirmatory</u> with respect to it, and thus sentences (i) - (viii) will be assessed according to Hempel's definition as confirming evidence for $\underline{h}_1$ and to every sentence logically equivalent to $\underline{h}_1$. Some evidence sentences, presumably, will be paradoxically confirmatory with respect to some of these equivalent formulations. This fact, in itself, lends strong support to the contention that the paradoxes of confirmation arise from the vagaries of intuition.

The sixteen possible evidence sentences listed above include the logically true sentence (ix) and the logically false sentence (viii). Since (ix) is irrelevant anyway, an objection to it is not likely to arise, but Hempel's inclusion of (viii) as confirmatory might well raise a question in someone's mind. The fact remains, however, that according to the information contained in a logically false evidence sentence, one can indeed infer that a hypothesis $\underline{h}$ holds true in the finite class of objects which are mentioned in the evidence sentence, since a logically false sentence entails any sentence whatsoever. Thus the admission of logically false evidence sentences is in harmony with the basic notion underlying Hempel's definition of confirmation. The objection to such evidence sentences might be based on the fact that in actual scientific practice logically false evidence sentences are of no interest and seldom occur, and if they do occur they are probably ignored. If any insuperable difficulty should arise in one's theory of confirmation from the admission of such evidence sentences, one could harmlessly stipulate that they be dropped as possible evidence sentences. Their innocuous inclusion, on the other hand, gives a sense of completeness to a study of confirmation.

4. Hempel's definition of direct confirmation and its relation to Nicod's definition of confirmation.

The comparison of Nicod's criterion of confirmation with Hempel's definition not only yields an interesting result in itself but also is of value in connection with Scheffler's critique of Hempel which I shall consider in the next chapter. I shall make extensive reference to the hypothesis h_1 and the sixteen possible evidence sentences which have been listed in the previous section. Nicod countenances only sentence (vii) as confirming evidence for h_1, while (xvi) is correctly assessed by Nicod as disconfirming evidence for h_1. Hempel, on the other hand, assesses a total of eight of the sixteen possible evidence sentences as confirming evidence for h_1, *viz.*, sentences (i) - (viii), while (xvi) is also assessed as disconfirming. There is an interesting relationship between Nicod's criterion and Hempel's definition for direct confirmation when attention is restricted to just the class of sentences considered by Nicod, namely, universal conditional sentences with monadic predicates. In particular, it can be shown that the eight evidence sentences which Hempel assesses as confirmatory with respect to h_1 are also logical instances of universal conditional sentences logically equivalent to $\underline{h}_1$ and thus confirmatory with respect to $\underline{h}_1$ on Nicod's criterion. Moreover, the eight remaining sentences, *viz.*, sentences (ix) - (xvi), which do not constitute confirming evidence for $\underline{h}_1$ on Hempel's definition, neither are logical instances of any sentences logically equivalent to $\underline{h}_1$. The establishing of this relationship between Nicod's and Hempel's criteria for the class of sentences mentioned requires as an assumption the equivalence condition for hypotheses and also the equivalence condition for evidence sentences. The latter condition asserts that if $\underline{e}$ confirms $\underline{h}$ then any sentence logically equivalent to $\underline{e}$ also confirms $\underline{h}$. These conditions are endorsed by many authors, including Hempel. Thus Nicod's criterion of confirmation is a necessary and sufficient condition for Hempel's criterion for direct confirmation of universal conditionals with monadic predicates.[18]

The following table gives the eight evidence sentences which stand in the relation *Cfd* to the sentence $\underline{h}_1$, and a corresponding sentence which is logically equivalent to $\underline{h}_1$ of which the specified evidence sentence is a logical instance. Some of the evidence sentences are listed in a simplified form, indicating

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[18] Hempel himself comments on the fact that Nicod's criterion is a sufficient condition for the confirmation of universal conditionals with monadic predicates, "Studies," p. 14.

that the equivalence condition for evidence sentences is assumed. In some of the cases there are a number of hypothesis-sentences which could serve in the place of the Related Hypothesis-Sentence, e.g., '-Pa' is a logical instance of $\underline{h}_{iii}$ below as well as of '(x)(-(Px & Qx) ⊃ -Px)'.

TABLE A

Evidence Sentence	Related Hypothesis-Sentence
(i) -Pa & -Qa	($\underline{h}_1$) (x)(-Qx ⊃ -Px)
(ii) -Pa & Qa	($\underline{h}_{ii}$) (x)(((Px v Qx) & (-Px v -Qx)) ⊃ (Px ⊃ Qx))
(iii) -Pa	($\underline{h}_{iii}$) (x)((-(Px & Qx) ⊃ (Px ⊃ Qx))
(iv) -Pa v Qa	($\underline{h}_{iv}$) (x)((-Px v Px) ⊃ (Px ⊃ Qx))
(v) Qa	($\underline{h}_v$) (x)((Px v Qx) ⊃ (Px ⊃ Qx))
(vi) Pa ≡ Qa	($\underline{h}_{vi}$) (x)((Qx ⊃ Px) ⊃ (Px ⊃ Qx))
(vii) Pa & Qa	($\underline{h}_{vii}$) (x)(Px ⊃ Qx)
(viii) Pa & -Pa	($\underline{h}_{viii}$) (x)((Px & -Qx) ⊃ (Px & -Px))

Although some of the hypotheses in Table A are easily obtainable by trial and error, a general method enabling one to obtain these and other suitable hypotheses exists.[19] I shall illustrate the method using the evidence sentence '-Pa'.

Let $\underline{h}'$ be some universal conditional sentence logically equivalent to $\underline{h}_1$ and of which '-Pa' is a logical instance. Since

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[19] I am indebted to Dr. A. L. Reeves for helpful discussion on the method of proof used here.

_h_' is in the form of a universal conditional, it can be symbolized as '(x)(Cx ⊃ Dx)', where 'Cx' and 'Dx' are complex expressions formed using only 'Px', 'Qx', and logical symbols, but in a combination not yet known. Since $\underline{h}_1$ is logically equivalent to _h_', the material equivalence of these sentences also holds, i.e., '(x)(Px ⊃ Qx) ≡ (x)(Cx ⊃ Dx)', from which it follows that 'Pa ⊃ Qa ≡ Ca ⊃ Da'. Hence 'Pa ⊃ Qa' and 'Ca ⊃ Da' will have the same truth tables. Now 'Ca & Da' is a logical instance of _h_', and since, _ex hypothesi_, '-Pa' is a logical instance of _h_', 'Ca & Da' will have the same truth table as '-Pa'. The following truth tables for 'Pa ⊃ Qa' and '-Pa', and hence for 'Ca ⊃ Da' and 'Ca & Da' respectively, can be constructed:

TABLE B

Pa	Qa	Pa ⊃ Qa	-Pa	Ca ⊃ Da	Ca & Da
T	T	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

From the truth tables for 'Ca ⊃ Da' and 'Ca & Da' in Table B, a partial truth table for 'Ca' and 'Da' can be constructed as follows:

TABLE C

Ca	Da
F	?
T	F
T	T
T	T

The truth values for the second, third, and fourth rows of Table C are obvious from the values for 'Ca ⊃ Da' and 'Ca & Da'. The first row must have the value F for 'Ca', but the value for 'Da' can be either T or F. The function of 'Pa', 'Qa', and logical symbols which has the values for 'Ca' shown in Table C is '-(Pa & Qa)', i.e., 'Ca' = '-(Pa & Qa)'. The two functions of 'Pa', 'Qa', and logical symbols which has the values for 'Da' shown in Table C are '-Pa' (when the value F is assigned to row one), and 'Pa ⊃ Qa' (when the value T is assigned to row one), i.e., 'Da' = '-Pa' or 'Da' = 'Pa ⊃ Qa'. Thus 'Ca ⊃ Da' is expressible as '-(Pa & Qa) ⊃ -Pa' or as '-(Pa & Qa) ⊃ (Pa ⊃ Qa)'. Since 'Pa ⊃ Qa ≡ Ca ⊃ Da', we obtain '(Pa ⊃ Qa) ≡ (-(Pa & Qa) ⊃ -Pa)' and '(Pa ⊃ Qa) ≡ (-(Pa & Qa) ⊃ (Pa ⊃ Qa))' from which we obtain, respectively, '(x)(Px ⊃ Qx) ≡ (x)(-(Px & Qx) ⊃ -Px)' and '(x)(Px ⊃ Qx) ≡ (x)(-(Px & Qx) ⊃ (Px ⊃ Qx))'. Thus *h*' can be suitably construed as '(x)(-(Px & Qx) ⊃ -Px)' or as '(x)(-(Px & Qx) ⊃ (Px ⊃ Qx))', and an examination of both quickly reveals that '-Pa' is indeed an instance of either. The same method can be used to obtain universal conditional hypotheses which are logically equivalent to '(x)(Px ⊃ Qx)', each of which has one of the sentences (i) - (viii) as a logical instance.

In order to show that the sentences (ix) - (xvi) are not logical instances of any universal conditional sentences logically equivalent to h_1, it is sufficient to proceed using the *reductio* *ad absurdum* method. I shall first illustrate this for the sentence 'Pa v Qa' (xiii), that is, I shall show that there is no universal conditional sentence *h*", which is logically equivalent to h_1, of which 'Pa v Qa' is a logical instance.

Let us suppose that 'Pa v Qa' is a logical instance of a universal conditional *h*" which is logically equivalent to h_1 and has the form: '(x)(Cx ⊃ Dx)', where 'Cx' and 'Dx' are formed as before. Again 'Pa ⊃ Qa' will have the same truth table as 'Ca ⊃ Da', and 'Pa v Qa' will have the same truth table as 'Ca & Da'. The following table of values is obtained:

TABLE D

Pa	Qa	Pa ⊃ Qa	Pa v Qa	Ca ⊃ Da	Ca & Da
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	F

Now consider the possibility of obtaining truth values for 'Ca' and 'Da' from the values for 'Ca ⊃ Da' and 'Ca & Da'. Only the second row of the values for 'Ca ⊃ Da' and 'Ca & Da' needs to be considered. Since 'Ca ⊃ Da' has the value F, 'Da' must have the value F. Since 'Ca & Da' has the value T, 'Da' must have the value T. Contradiction! Hence 'Pa v Qa' is not a logical instance of any universal conditional sentence *h*" logically equivalent to h_1.

The same method establishes that none of the sentences

(ix) - (xvi) are logical instances of universal conditionals logically equivalent to $\underline{h}_1$. This can be easily seen by considering the following table in which values are assigned to the sentences (ix) - (xvi):

<u>TABLE</u> <u>E</u>

Pa	Qa	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)	(xv)	(xvi)
T	T	T	F	F	T	T	T	F	F
T	F	T	T	T	T	T	T	T	T
F	T	T	T	T	F	T	F	F	F
F	F	T	F	T	T	F	F	T	F

Again only the second row needs to be examined. Since 'Ca ⊃ Da', which has the same values as 'Pa ⊃ Qa', is F in this row, 'Da' must have the value F. Since 'Ca & Da' is T for each of (ix) - (xvi) in the second row, 'Da' must have the value T. Thus 'Da' must be both T and F. From this contradiction one can infer that none of (ix) - (xvi) are logical instances of universal conditional sentences logically equivalent to $\underline{h}_1$.

The foregoing proofs demonstrate that Nicod's criterion, in conjunction with the two equivalence conditions specified earlier, yields the same results as Hempel's definition of direct confirmation when attention is restricted to probably the most discussed and most interesting class of sentences, namely, universal conditionals with monadic predicates. This close relationship further illustrates, perhaps, the influence of Nicod's studies in confirmation on those undertaken by Hempel. Other authors have suggested that the concept of confirmation that Hempel is concerned to study is even more closely linked with the concept of a logical instance central to Nicod's exposition. This question shall be examined in the following chapter in which I consider several important critiques of Hempel's study.

CHAPTER FOUR

IDENTIFYING HEMPEL'S *EXPLICANDUM*

1. *Scheffler's analysis of Hempel's explicandum*.

In this chapter I wish to try to determine which concept of confirmation, among the concepts distinguished in chapter two, Hempel attempts to explicate. That is, I shall try to identify Hempel's *explicandum*. One of the problems associated with explication is the imprecision with which the *explicandum* is necessarily expressed, for any concept which is precisely expressed is *ipso facto* not in need of explication. This problem thus surrounds to some extent the examination of Hempel's *explicandum* undertaken here. I shall consider two critiques of Hempel's study in this chapter and then offer my own interpretation of his study. Before doing so, however, I wish to draw brief attention to a feature of Hempel's study as presented in the foregoing chapter which has a bearing on the assessment of Hempel's *explicandum*.

Hempel draws a distinction between two relations of confirmation — direct confirmation (*Cfd*) and confirmation in general (*Cf*). The relation *Cf* is more inclusive than the relation *Cfd*, as his definitions show. This distinction is analogous to the distinction which Ernest Nagel draws between two kinds of evidence, namely, direct evidence and indirect evidence, although his criteria are somewhat different from the criteria expressed in Hempel's definitions.[1] A third theorist, J. L. Mackie, similarly distinguishes several kinds of non-deductive support, namely, probabilification, direct confirmation, and indirect support, maintaining that one of the mistakes that has been made in studies of confirmation has been to assume that there is just one kind of non-deductive support.[2] The criteria which Mackie offers are different again from the criteria offered by Hempel and Nagel. Thus Hempel is not alone in maintaining a distinction between direct confirmation and confirmation in general, for analogous distinctions have been drawn by other authors.

There are a number of explanations which might be offered to account for the distinction Hempel draws. It might be suggested that the relations Cfd and Cf express different concepts of confirmation and that Hempel's explicandum is not just one concept but two. Now this suggestion seems implausible in connection with the distinctions Hempel has drawn, for Hempel never suggests that the definitions of the relations Cfd and Cf are of two different concepts. Moreover, it is clear from the technical article "Definition" that the relation Cf is introduced and

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[1]Nagel, *The Structure of Science* (London: Routledge & Kegan Paul, 1961), pp. 64 - 65.

[2]Mackie, "The Relevance Criterion of Confirmation," *The British Journal for the Philosophy of Science* ("*BJPS*" hereinafter), vol. 20 (1969), p. 37 f.

separately defined because the definition of the relation <u>Cfd</u> fails to satisfy all the required conditions of adequacy.[3]

A more plausible — and more charitable — explanation then for Hempel's introduction of the two relations <u>Cfd</u> and <u>Cf</u> is that the definition for the relation first introduced, namely <u>Cfd</u>, did not satisfy all the desiderata Hempel thought important. A second relation <u>Cf</u> was thus introduced to make up for the deficiencies in the definition of <u>Cfd</u>. Although only one concept of confirmation is being defined, the definition takes a relatively complex form involving the defining of two relations. I shall eschew, then, the suggestion that Hempel's study of confirmation is a confused study of two concepts of confirmation, and explicitly assume that Hempel's study is a coherent analysis of one concept of confirmation. I shall now consider the provocative position adopted by Scheffler with respect to Hempel's <u>explicandum</u>.

Israel Scheffler devotes a considerable portion of his analysis of confirmation in <u>The Anatomy of Inquiry</u> to a scrutiny of Hempel's study of confirmation. Of particular interest here is an interpretation of Hempel's <u>explicandum</u>. Scheffler understands Hempel's key expression '<u>e</u> confirms <u>h</u>' as '<u>e</u> represents a positive instance of <u>h</u>', as the following passage clearly

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[3]Hempel, "Definition," p. 136.

indicates:[4]

> He [Hempel] wants . . . to define the conditions under which *e* accords with *h*, or represents a positive instance of *h*. . . What he wants is a definition which may be applied to a *given pair* of statements *e* and *h*, in such a way as to enable us to decide whether or not *e* accords with *h*, or, as he puts it, *e* confirms *h*.

In dealing with Scheffler's interpretation of Hempel's *explicandum*, I shall not consider the locution '*e* accords with *h*'. Scheffler evidently considers this locution to be a synonym (or near-synonym) of '*e* represents a positive instance of *h*'. Hempel himself uses 'accords with' as a synonym (or near-synonym) of 'confirms' and so Scheffler's remark in this respect is of no moment.[5] Certain expressions, for example, 'accords with', are so vague and uncertain in meaning that they can be used, and have been used, as synonyms for quite different concepts of confirmation with the result that no net clarification is achieved. The locution 'represents a positive instance of', or, more simply, 'is a positive instance of', although not completely determinate in meaning, has been partially understood, so that Scheffler's interpretation of Hempel's *explicandum* constitutes a substantial one.

Scheffler is very concerned to avoid begging the question

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[4]Scheffler, *The Anatomy of Inquiry*, p. 237. Emphasis in the original.

[5]Hempel, "Studies," p. 4.

as to whether or not ". . . every instance that *accords with* a hypothesis *h* in fact *confirms* it."[6] Scheffler here understands '*e* confirms *h*' to mean '*e* accords with *h* but not also with the contrary of *h*', or '*e* is a positive instance of *h* but not of the contrary of *h*', i.e., '*e* confirms (in Hempel's sense) *h* but not the contrary of *h*'.[7] Hempel, he contends, tries to explicate the underlying notion of an instance of a hypothesis. Now I shall not comment extensively on Scheffler's conception of confirmation — called "selective confirmation" — except to say that it seems to be a special case of the concept of confirmation expressed in (12) above, which is as follows:

'*h* is more credible than *h*' relative to *e*, where *h* and *h*' are logically incompatible'.[8] (12)

Scheffler's *explicandum* seems to be the special case of (12) where *h*' is the contrary of *h* and all subject terms are taken to be denoting. Swinburne says that Scheffler means by 'to selectively confirm a hypothesis',". . . to add to the likelihood of its truth more than to that of a rival,"[9] but this strikes me

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[6]Scheffler, *op. cit*., p. 237. Emphasis in the original.

[7]*Ibid*., p. 291f.

[8]See section 2.4 above where (12) was introduced.

[9]Swinburne, "The Paradoxes of Confirmation — A Survey," *American Philosophical Quarterly*, vol. 8 (1971), p. 320.

as being incorrect. If we render this notion uniform with the other concepts of confirmation discussed in this dissertation by substituting the concept of credibility for the concept of likelihood, Swinburne can be interpreted as saying that '*e* selectively confirms *h*' means '*e* increases the (degree of) credibility of *h* more than it increases the (degree of) credibility of *h*', the rival of *h*'. The concept expressed here, however, is in comparative form, not in qualitative form, since it *compares* the increase in credibility of *h* on the basis of *e* with the increase in credibility of *h*' on the basis of *e*. A qualitative concept, when defined using the notion of increase in credibility (that is, increase in firmness), does not compare two increases in credibility with each other, but merely states that there is an increase in credibility of a hypothesis. The four concepts of confirmation (all in qualitative form) discussed in chapter two above were secondarily defined in (5'), (6'), (7'), and (8') using the notion of (degree of) credibility. Each definition asserts only that there is an increase in credibility, and does not *compare* two increases in credibility. However, the comparative form of these concepts, if defined using the notion of (degree of) credibility, would require the comparison of two increases in credibility. Thus Swinburne, in effect, is asserting that Scheffler is investigating a concept in comparative form. It would be absurd, of course, for Scheffler to carry out a critique of Hempel's study of qualitative confirmation by comparing and contrasting Hempel's explicandum, which is in qualitative form, with a concept in comparative form. I do not think that Scheffler is misdirected on that score, and suggest that Swinburne's assessment of Scheffler's concept of selective confirmation is incorrect.

In his discussion of Hempel's concept of confirmation, Scheffler appears to have the Aristotelian conception of a contrary in mind, for he uses the definite article in connection with it, e.g., he says that the contrary of the sentence '(x)(-Bx ⊃ -Rx)' is '(x)(-Bx ⊃ Rx)'.[10] On the Aristotelian account of contrariety, S is the contrary of T iff S and T are universal conditionals which have the same subject term but whose predicate terms are contradictory. According to Aristotle's account, contraries cannot be true together,[11] and since '(x)(-Bx ⊃ -Rx)' and '(x)(-Bx ⊃ Rx)' are contraries on Aristotle's scheme, the implicit presupposition is that the subject term is a denoting term. Most logicians hold the view that S is a contrary of T iff S and T cannot be true together. Hence '(x)(-Bx ⊃ -Rx)' and '(x)(-Bx ⊃ Rx)', for example, are not contraries since both are true if nothing is -B. On the latter view of

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[10]Scheffler, op. cit., p. 288.

[11]Aristotle, De Interpretatione, Chapter 7, in The Basic Works of Aristotle (New York: Random House, 1941), ed. Richard McKeon, p. 44.

contrariety, a given sentence has many contraries, not just one.

Scheffler's interpretation, then, amounts to construing Hempel's *explicandum* as the concept of an instance of a hypothesis. Now Hempel indeed asserts that the concept of confirmation he is concerned to explicate is closely connected with the concept of an instance of a hypothesis:[12]

> A closely related concept is that of instance of a hypothesis. The so-called method of inductive inference is usually presented as proceeding from specific cases to a general hypothesis of which each of the special cases is an "instance" in the sense that it conforms to the general hypothesis in question, and thus constitutes confirming evidence for it.

Hempel is not claiming here, however, that his *explicandum* is the concept of an instance of a hypothesis. He only asserts that the concept of an instance of a hypothesis is closely related to his *explicandum*. He also suggests, in the passage quoted, that a sufficient condition of a sentence's confirming a given hypothesis is that that sentence is an instance of the hypothesis. Hempel goes on to say that the concept of an instance of a hypothesis is in need of precise analysis, thereby implying that there is a sense of 'is an instance of' which is in need of explication. Scheffler is thus partly justified in identifying this concept with Hempel's *explicandum*, for both concepts are in need of explication.

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[12]Hempel, "Studies," p. 5.

2. <u>The inadequacy of Scheffler's analysis, and the concept of an instance of a hypothesis</u>.

The locution '*x* is an instance of *y*' has of course been understood in the definite sense which I have called the concept of a logical instance, and is defined as follows:

> '*x* is a logical instance of *y*' = df '*y* is a sentence of universal conditional form, and *x* is a conjunction having as its conjuncts the instantiated antecedent of the matrix of *y* and the instantiated consequent of the matrix of *y*, where the same variable in the matrix of *y* is instantiated with the same individual constant throughout'. (17)

The concept of a logical instance has been discussed in connection with Nicod's criterion of confirmation, for his criterion is based upon this concept. This concept is frequently used in the literature on confirmation in accordance with the definition I have given. The concept of a logical instance is not in need of explication, and Scheffler could not be reasonably interpreted as saying that Hempel's *explicandum* is the concept of a logical instance. There presumably is some other sense of 'is an instance of' which Hempel thinks requires analysis and which Scheffler identifies with Hempel's *explicandum*. I shall call this as yet unclarified sense of the locution '*x* is an instance of *y*' the concept of an *inductive instance*.

Hempel has correctly pointed out that the concept of an inductive instance of a hypothesis has been associated with a method of inductive inference according to which a general hypothesis is inductively "inferred" from an instance of that hypothesis. Contemporary inductive theorists are disinclined to speak of "inferring" general hypotheses from instances of those hypotheses, since such kind of talk suggests that hypotheses can be routinely derived from data of a particular character — a suggestion which completely misrepresents the actual procedure of science and critical thinking. Partly as a result of the association of the concept of an inductive instance with this method, however, the concept of an inductive instance has been so construed that sentences which can be spoken of as having instances are _general_ sentences, and in particular, universal sentences.[13] Thus an important characteristic of the concept of an inductive instance is that the sentences of which _x_ is an inductive instance are general sentences. The explication which Hempel advances for the concept of confirming evidence clearly reveals that the sentences which are allowed to serve as hypotheses are sentences of any logical form whatsoever, and not just general sentences. There is nothing about Hempel's _explicandum_, moreover, which prevents sentences of molecular form

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[13]A general sentence is any sentence which contains a non-vacuous quantifier when adequately symbolized in an appropriate symbolic language.

from being considered as hypotheses. There is therefore, *prima facie*, a considerable difference between Hempel's *explicandum* and the concept of an inductive instance which counts against the plausibility of Scheffler's identification of these two concepts. A related reason for rejecting Scheffler's identification requires a reference to the logical conditions of adequacy which Hempel consider in his study of confirmation.[14] These conditions represent important characteristics of his *explicandum* — characteristics which would generally be sought in an adequate explication. One important condition which Hempel requires any adequate explication of the concept of confirmation to fulfill is the entailment condition: If *e* entails *h* then *e* confirms *h*. If Scheffler is correct in his interpretation of Hempel, then Hempel must be construed as specifying in the logical conditions of adequacy important characteristics of the concept of an inductive instance; in particular, Hempel must be construed as requiring that the following condition be fulfilled: If *e* entails *h* then *e* is an inductive instance of *h*. It is very implausible to suggest, it seems to me, that an important characteristic of the concept of an inductive instance is represented in such a condition, e.g., it is highly implausible to suggest that '(x)(Px)' is an inductive instance of 'Pa'! Some of the

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[14]Cf. Hempel, "Definition," pp. 127 - 128, and "Studies," pp. 30 - 35.

conditions of adequacy listed by Hempel do not yield the same absurdity that the entailment condition does, but this example, in my estimation, shows the implausibility of Scheffler's interpretation of Hempel's *explicandum*.

The reply that Scheffler might want to make to these objections is that Hempel is offering a rational reconstruction (or an explication) of the concept of an inductive instance in which certain intuitive judgments will have to be sacrificed for the sake of generality and comprehensiveness. Scheffler would of course be correct in saying that Hempel offers a reconstruction. An adequate explication, however, must meet the material condition of adequacy, as Hempel has himself maintained, i.e., a proposed *explicatum* must be in sufficiently close agreement with the customary meaning of the *explicandum*. Given Scheffler's interpretation of Hempel's *explicandum*, the following implausible results, for example, would also be obtained. First, 'Pa' would have to be construed as an inductive instance of 'Pb', since 'Pa' confirms 'Pb' on Hempel's definition; second, 'Pa & Qb' would have to be construed as an inductive instance of 'Pa', since the latter is confirmed by the former on Hempel's definition. Many more examples could be given. Such examples, in my estimation, constitute exactly the kinds of counterintuitive results which count decisively against Scheffler's identification and must be construed as such if the material condition of adequacy is to be taken seriously. Hence, I conclude that Scheffler's interpretation of Hempel's *explicandum* as the concept of an inductive instance is implausible and worthy of rejection.

I do not think that the concept of an inductive instance is of very great importance to confirmation theory. It does not warrant as much attention as other concepts within confirmation theory do warrant. There are, however, several *explicata* which readily suggest themselves which are of passing interest. The following definition:[15]

'*x* is an inductive instance of *y*' = df '*x* is a logical instance of *y* or of any sentence logically equivalent to *y*' (18)

is suggested by Hempel in one of his recent discussions of confirmation.[16] In connection with the hypothesis:

h_5: $(x)(Rx \supset Bx)$

he considers molecular evidence sentences of three types:

Type I: Ra & Ba,

Type II: -Ba & -Ra,

Type III: -Ra v Ba.

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[15] I suspect that the concept of a logical instance has always been predominant in the use of the locution '*x* is an instance of *y*', and that some authors have used this locution only in the sense of the concept of a logical instance.

[16] Hempel, "Recent Problems of Induction," p. 119f.

Each type of evidence sentence is said to be a positive instance of $\underline{h}_5$ by virtue of being a logical instance of $\underline{h}_5$ or of some sentence logically equivalent to $\underline{h}_5$ — 'Ra & Ba' is a logical instance of $\underline{h}_5$, '-Ba & -Ra' is a logical instance of '(x)(-Bx ⊃ -Rx)' which is logically equivalent to $\underline{h}_5$, and '-Ra v Ba' is a logical instance of '(x)((-Rx v Rx) ⊃ (-Rx v Bx))' which is logically equivalent to $\underline{h}_5$. Although Hempel does not mention this fact, the types of evidence sentence could be increased to a maximum of eight, each one of which is a logical instance of some sentence logically equivalent to $\underline{h}_5$.[17] The concept of a logical instance, in comparison, is narrower, for 'Pa' is not a logical instance of '(x)(Px)', since '(x)(Px)' is not a universal conditional, but 'Pa' is an inductive instance of '(x)(Px)', since 'Pa' is a logical instance of the sentence '(x)((Px ⊃ Px) ⊃ Px)' which is logically equivalent to '(x)(Px)'. An interesting consequence of (18) is that every universal instantiation of a universal generalization is an inductive instance of that universal generalization. This is so by virtue of the fact that any universal generalization C: '(x)(. . . Sx . . .)' can be cast in the logically equivalent form C': '(x)(((. . . Sx . . .) ⊃ (. . . Sx . . .)) ⊃ (. . . Sx . . .))'. A logical instance of C' has the form: '(. . . Sa . . .)',

[17] This was proved in section 3.4 above, in connection with Nicod's criterion of confirmation.

the concept of an inductive instance. If universal generalizations alone are construed as capable of having inductive instances, the second *explicatum* is likely to be rejected in favor of (18); if all general sentences are construed as capable of having inductive instances, (18) is likely to be rejected in favor of the second *explicatum*. In any case, the matter will tend to turn on the importance with which the concept of an inductive instance is regarded. I shall not consider the issue in any more detail, but will now turn to the second evaluation of Hempel's study of confirmation.

3. *Carnap's analysis of Hempel's explicandum*.

A second assessment of Hempel's study of confirmation to which I shall devote attention is Carnap's. It is necessary first of all to briefly recall the distinction Carnap draws between confirming_F evidence and initially confirming_F evidence. The general form of the concept of confirming_F evidence is relativized to prior evidence *b*, while the special case of the concept — initially confirming_F evidence — is relativized to null or tautological prior evidence. Thus in the special case of the concept of confirming_F evidence there is no reference to prior evidence at all. Carnap makes the following significant comment concerning Hempel's *explicandum*:[19]

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[19]Carnap, *LFP*, p. 468. Emphasis in the original.

> His explicandum is as follows: a sentence . . . represents confirming$_F$ (corroborating, favorable) evidence or constitutes a confirming$_F$ instance for a given hypothesis. In his general discussion and in the examples, no reference is made to any prior evidence. Thus Hempel's explicandum corresponds to our dyadic relation Co(h,e) ('h is confirmed$_F$ by e') rather than to the triadic relation C(h,e,b) ('h is confirmed$_F$ by e on the basis of the prior evidence b').

Thus Carnap considers Hempel's explicandum to be the concept of initially confirming$_F$ evidence, represented by Carnap's dyadic relation Co(h,e), rather than the general concept of confirming$_F$ evidence (C(h,e,b)). Carnap is correct in his observation that Hempel, in his general discussion and in his examples, makes no reference to any prior evidence, but the conclusion that Carnap draws from this observation, I shall argue, is incorrect. Thus I shall argue that Hempel's explicandum is not the concept of initially confirming$_F$ evidence.

The following example helps to illustrate the concept of confirmation which Carnap interprets as Hempel's explicandum. Let us suppose that the hypothesis:

h_1: $(x)(Px \supset Qx)$

is proposed and that evidence for or against it is sought, e.g., specific experiments are devised in order to test h_1. Suppose also that at some given time no evidence for or against h_1 exists and that the first test of h_1 yields the following evidence sentence:

e_1: Pa & Qa.

$\underline{e}_1$ is confirming evidence for $\underline{h}_1$ according to Hempel's criterion — a result presumably in agreement with numerous people's intuitions on the matter, including Carnap's.[20] Since there is no evidence for or against $\underline{h}_1$ prior to the acquisition of $\underline{e}_1$, $\underline{e}_1$ constitutes initially confirming$_F$ evidence for $\underline{h}_1$. Let us also suppose that a report of the second test of $\underline{h}_1$ is acquired, namely:

$\underline{e}_2$: Pb & Qb.

Now the acquisition of $\underline{e}_2$ in this example involves a background of factual prior evidence, and $\underline{e}_2$ could not qualify as initially confirming$_F$ evidence for $\underline{h}_1$. However, $\underline{e}_2$ might well qualify as confirming$_F$ evidence for $\underline{h}_1$, i.e., $\underline{h}_1$ might be confirmed$_F$ by $\underline{e}_2$ given $\underline{e}_1$, but $\underline{e}_2$ could not qualify as initially confirming$_F$ evidence for $\underline{h}_1$ since prior evidence for $\underline{h}_1$ has already been acquired, viz., $\underline{e}_1$. The concept which Hempel is concerned to explicate, according to Carnap, obtains between an evidence sentence and a hypothesis only when there is no factual prior evidence for or against $\underline{h}_1$. Since $\underline{e}_2$, in the above example, is obtained in the context in which there is prior evidence, it is not and cannot

[20] In spite of the fact that in Carnap's system the degree of confirmation$_S$ of a universal hypothesis on finite evidence is zero, Carnap speaks of confirming$_F$ evidence for such hypotheses. See LFP, p. 223; p. 466, example 1; and p. 469. The confirming$_F$ evidence might be measured in such cases by taking into account the relevant measures of instance confirmation$_S$. See LFP, p. 571f for a discussion of instance confirmation$_S$.

qualify as <u>initially</u> confirming$_F$ evidence for $\underline{h}_1$, although it may well be confirming$_F$ evidence for $\underline{h}_1$ given $\underline{e}_1$. According to Carnap's interpretation of Hempel, therefore, $\underline{e}_2$ could not be construed as confirming $\underline{h}_1$.

That $\underline{e}_2$ cannot be considered as a candidate for <u>initially</u> confirming$_F$ evidence for $\underline{h}_1$ in the situation described above is incontrovertible. That Hempel so conceived the concept of confirmation he was concerned to explicate, however, so that it was applicable only in the situations in which there was no prior factual evidence, seems to me to be incorrect. Although there is no explicit statement to this effect in Hempel's work (for Hempel does not make reference to prior factual evidence, as Carnap correctly points out), there are several passages which strongly suggest that Carnap's interpretation of Hempel's <u>explicandum</u> is incorrect. Moreover, I shall argue that there is a prior-evidence-independent concept of confirmation with which Hempel's <u>explicandum</u> is much more suitably identified. I shall argue, in particular, that Hempel's <u>explicandum</u> is most suitably identified with the concept of confirmation expressed in (6") above, which I shall again state for easy reference:

> '$\underline{e}$ confirms$_B$ $\underline{h}$' = df '$\underline{h}$ is more credible relative to $\underline{e}$ than relative to logical truth alone, where $\underline{e}$ is an evidence sentence which is independent of the background of prior evidence, i.e., $\underline{e}$ is the report of any test of

or observation relevant to *h* irrespective of prior evidence'. (6")

The concept of confirmation$_B$ is similar to the concept of initially confirming$_F$ evidence:

'*e* initially confirms$_F$ *h*' = df '*h* is more credible relative to *e* than relative to logical truth alone, where *e* is the report of the first test of *h*, i.e., *e* is temporally prior to any other test of or observation relevant to *h*' (9")

and a criterion which determines for any *h* and *e* whether or not *h* is more credible relative to *e* than relative to logical truth alone will naturally be of value for both concepts. The two concepts are distinguishable from one another, however, insofar as the concept of confirmation$_B$ is prior-evidence-independent whereas the concept of initial confirmation$_F$ is prior-evidence-dependent.[21]

A number of important passages in Hempel's studies strongly suggest that he was not concerned to explicate the concept of initial confirmation$_F$, contrary to Carnap's allegations. I shall try to establish this negative point first. In one passage Hempel discusses his conception of confirmation with respect to the hypothesis 'Every *P* is a *Q*'. This hypothesis,

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[21] See section 2.5 above for a detailed comparison of these two concepts.

he says:[22]

> ". . . forbids the occurrence of any objects having the property _P_ but lacking the property _Q_, i.e. it restricts all objects whatsoever to the class of those which either lack the property _P_ or also have the property _Q_. Now, every object either belongs to this class or falls outside it, and thus, every object — and not only the _P_'s — either conforms to the hypothesis or violates it; there is no object which is not implicitly referred to by a hypothesis of this type."

This passage shows that Hempel does not see only the first object tested for the properties _P_ and _Q_ as either confirming or violating the hypothesis, but _all objects whatsoever_ either conform to the hypothesis and thus confirm the hypothesis or violate it. Thus objects which are examined after the initial object is examined can presumably be properly assessed as either confirming or disconfirming evidence for the hypothesis in question. Hempel similarly says in another passage that ". . . the hypotheses "All ravens are black" and "All ravens are green" are both confirmed by any one of the (presumably infinitely many) objects that are not ravens."[23] It is evident here again that in allowing an infinite number of objects to confirm a hypothesis, he is allowing more than just the first object examined in the test of

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[22] Hempel, "Studies," p. 18. At this stage in his discussion, Hempel informally took objects rather than sentences to be the _confirmantia_, i.e., the things mentioned as confirming a hypothesis, cf. David Stove, "Hempel's Paradox," _Dialogue: Canadian Philosophical Review_, vol. 4 (1965-66), p. 444.

[23] Hempel, "Empirical Statements and Falsifiability," _Philosophy_, vol. 33 (1958), p. 345.

a hypothesis to confirm that hypothesis. If the concept he was explicating was the concept of initially confirming_F evidence, he would be interested in considering only the first object (or the report of it).

There is a third passage which gives strong support to the position I am trying to establish here, viz., that Hempel's explicandum is not the concept of initially confirming_F evidence. Hempel considers the hypothesis that all ravens are black, which is symbolized as:

h_5: $(x)(Rx \supset Bx)$

in relation to three types of sentences having the following forms:

Type I: $Rx \& Bx$,

Type II: $-Bx \& -Rx$,

Type III: $-Rx \vee Bx$.[24]

Hempel then reports on an imaginary investigation which I shall quote in detail.[25]

> Suppose we are told that in the next room there is an object *i* which is a raven. Our hypothesis h_5 then tells us about *i* that it is black, and if we find that this is indeed the case, so that we have *Ri* & *Bi*, then this must surely count as bearing out, or confirming, the hypothesis.
> Next, suppose that we are told that in the adjoining room there is an object *j* that is not black. Again, our

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[24] Hempel, "Recent Problems of Induction," p. 119f.

[25] Ibid., p. 121.

> hypothesis tells us something more about it, namely, that it is not a raven. And if we find this is indeed so — i.e., that -Bj & -Rj, then this bears out, and thus supports, the hypothesis.
>
> Finally, even if we are told only that in the next room there is an object *k*, the hypothesis still tells us something about it, namely, that either it is no raven or it is black — i.e., that -Rk v Bk; and if this is found to be the case, it again bears out the hypothesis.

This imaginary investigation shows clearly that after the first test of the hypothesis was carried out, a second test of the hypothesis was performed in which the outcome was indeed confirming evidence for the hypothesis h_5. If Hempel was explicating the concept of initially confirming$_F$ evidence, he could not countenance second tests of a hypothesis as confirming evidence, for only the first test report could possibly qualify. Hempel's main intention in the passage just quoted is to show how evidence sentences of Types II and III can possibly confirm h_5. In order to make this even more plausible, he adduces a further, admittedly imprecise example in which *k* is the hypothesis 'All marbles in this bag are red'. If we suppose that there are twenty marbles in the bag, ". . . then the generalization *k* has twenty instances of Type I, each being provided by one of the marbles."[26] If each object in the bag which is a marble is checked then the hypothesis is exhaustively tested, and roughly speaking we could say that if each marble examined

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[26] *Ibid.*, p. 122.

is found to be red, then we have shown that one twentieth of the total content of the hypothesis is true. Hempel argues that in a similar way the hypothesis that any object that is not red is not a marble in this bag will have an infinite number of non-red objects as its instances and to examine each one and to determine that it is not a marble in the bag ". . . is therefore to check, and corroborate, only a tiny portion of all that the hypothesis affirms. Hence, a positive finding of Type II would indeed support our generalization, but only to a very small extent."[27] Hempel points out that the rough example sketched here involves a theory of degrees of confirmation. What this theory amounts to in the case of the hypothesis _k_ is that after the first marble has been found to be red, one twentieth of the content of _k_ has been shown to be true; after the second marble has been found to be red, one twentieth more of the content of _k_ has been found to be true so that a total of one tenth of the content of _k_ has been found to be true; after ten marbles have been found to be red a total of one-half of the content of _k_ has been found to be true; and finally, when all twenty marbles have been found to be red the total content of _k_ has been found to be true. Now it is obvious that Hempel does not regard _only_ the first test of _k_ capable of being

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[27] _Ibid._, p. 122.

confirming evidence for *k*, but the second, third, fourth, etc. tests of *k* all constitute confirming evidence for *k*. Thus Hempel's *explicandum* is not the concept of initially confirming_F evidence.

4. *Remarks on Hempel's methodological requirement in relation to Carnap's analysis of Hempel's explicandum, and my analysis of Hempel's explicandum.*

There is a feature of Hempel's study of confirmation which certain authors have used as a basis for asserting that Hempel construes his *explicandum* to be the concept of initially confirming_F evidence. This feature of Hempel's study is a methodological requirement to the effect that in assessing an evidence report with respect to a hypothesis, additional or extraneous information must not be included in the evidence report.[28] Hempel tries to show that some of the paradoxes of confirmation arise from the tacit introduction of additional information when judging the confirmation-value of a given evidence report. Hempel illustrates this using the hypothesis that all sodium salts burn yellow. Now the report of piece of pure ice being held in a colorless flame and failing to turn the flame yellow confirms the assertion that whatever does not burn yellow is no sodium salt, and hence, by the equivalence condition for hypotheses, also the original

[28] Hempel, "Studies," p. 19f.

hypothesis that all sodium salts burn yellow. This might strike one as paradoxical, Hempel remarks, and the reason becomes clear if this situation is compared with the situation in which an object whose chemical composition is not known is held in a colorless flame and fails to turn the flame yellow and is found later to contain no sodium salt. The latter outcome also confirms the hypothesis that all sodium salts burn yellow but not paradoxically. The problem arises, says Hempel, over the fact that in the seemingly paradoxical cases of confirmation we are tacitly introducing additional information into our evidence claim. In the above example, there has been a tacit introduction of the information that the substance used is ice and that ice contains no sodium salt. This information is tacitly added to the evidence report that the substance tested neither burns yellow nor contains sodium salt. In tacitly introducing additional information ". . . we fail to observe the methodological fiction, characteristic of every case of confirmation, that we have no relevant evidence for _h_ other than that included in _e_."[29]

Hempel stresses in another paper that the introduction of additional information must be avoided when the confirmatory relevance of a given evidence is being judged.[30] He replies to

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[29] _Ibid._, p. 19.

[30] Hempel, "The White Shoe: No Red Herring," _BJPS_, vol. 18 (1967), pp. 239 - 240.

I. J. Good who contends that not all (logical) instances of a universal conditional hypothesis confirm such a hypothesis.[31] Good argues that the following evidence sentence:

> S: W_1 or W_2 is the world we are in. W_1 contains 100 crows, all of them black, and one million other birds; W_2 contains 1001 crows, of which 1000 are black and one white, plus one million other birds. One bird has been selected equiprobably at random from all the birds in our world. It turns out to be a black crow.

gives strong support to the hypothesis that we are in world W_2, where not all crows are black, and concludes that the observation of a black crow in the circumstances described undermines the hypothesis that all crows are black. Hence a (logical) instance of the hypothesis that all crows are black undermines the hypothesis.

Hempel says in reply that in order to show that a positive instance of a hypothesis does not confirm the hypothesis, e.g., to show that all crows are black is not confirmed by the evidence '*c* is a crow and is black', one would have to consider the evidence sentence by itself and without reference to any other information. Good's example, however, concerns the confirmatory role of *S* which is vastly stronger than the evidence report that

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[31] I. J. Good, "The White Shoe is a Red Herring," *BJPS*, vol. 17 (1966), p. 322.

c is a crow and is black. Thus the introduction of additional information might even conceal the confirmatory role of an evidence sentence completely. This is simply illustrated by the evidence sentence 'c is a crow and is black and d is a crow and is not black' which disconfirms the hypothesis that all crows are black by virtue of its inclusion of the report of a non-black crow. Because of this disconfirmatory component, the confirmatory role of the report of the black crow c is concealed. If the confirmatory role of the evidence 'c is a crow and is black' alone is to be assessed, extraneous or additional information must not be included in the evidence sentence. Thus, the presence of additional information can make an evidence report paradoxically confirmatory or even conceal the confirmatory character of an evidence report altogether.

Some of the comments that have been made concerning Hempel's methodological requirement appear to interpret him as requiring that temporally prior evidence must be excluded. Such an interpretation of the methodological requirement would certainly lend support to Carnap's contention that Hempel's *explicandum* is the concept of initial confirmation$_F$. H. G. Alexander writes that ". . . Hempel's concept of confirmation seems outrageous because he is considering the ideal situation in which our minds though well-stocked with concepts are *tabulae*

<u>rasae</u> as far as facts are concerned."[32] Alexander says in another place that ". . . the paradoxes of confirmation are discussed by Hempel in those cases in which we consider the bearing of evidence E on hypothesis H <u>in the absence of all prior knowledge</u>."[33] A second author, J. L. Mackie, wonders whether we can plausibly speak of confirmation in a setting in which no reference is made to additional knowledge, or, ". . . whether any definition of confirmation that gives positive results in this setting can be carried over to ordinary contexts or will even throw light on confirmation in ordinary contexts."[34] Hempel makes abundantly clear, however, that the additional information which must be excluded is not, as Alexander and Mackie (perhaps) suggest, temporally prior knowledge or evidence. It is additional information which would mask or conceal completely the otherwise confirmatory role of some specified evidence with respect to a given hypothesis. Hempel's methodological requirement can be expressed as a maxim, as follows: When evaluating an evidence report in order to determine whether or not it is confirming evidence for

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[32] H. G. Alexander, "The Paradoxes of Confirmation," <u>BJPS</u>, vol. 9 (1958), p. 232.

[33] Alexander, "The Paradoxes of Confirmation — a Reply to Dr. Agassi," <u>BJPS</u>, vol. 10 (1959), p. 231. Emphasis in the original.

[34] Mackie, "The Paradox of Confirmation," <u>BJPS</u>, vol. 13 (1962), p. 266.

a given hypothesis, do not include, either implicitly or explicitly, any information in the evidence sentence besides the report whose confirmation-status you wish to determine. Thus the methodological requirement imposed by Hempel does not constitute a requirement that the temporally prior evidence must be excluded, and hence does not give support to Carnap's interpretation of Hempel's explicandum. I conclude that Hempel's explicandum is not the concept of initially confirming_F evidence.

I believe that the foregoing discussion not only shows that Hempel's explicandum cannot be the concept of initially confirming_F evidence, but also strongly suggests that Hempel's explicandum is a prior-evidence-independent concept of confirmation. The typical approach to the evaluation of a given evidence sentence with reference to a hypothesis consists of isolating the evidence sentence from the prior factual evidence (and even from additional or extraneous information), and then considering it on its own with reference to the hypothesis. The question which naturally arises is: Which of the prior-evidence-independent concepts of confirmation, namely, confirmation_A, confirmation_B, and confirmation_C, does Hempel's explicandum most resemble? I have indicated earlier that my answer to this question is that Hempel's explicandum most resembles the concept of confirmation_B, and I shall try to substantiate this answer or at least make it plausible.

The most compelling reason for selecting the concept of confirmation$_B$ as the concept closest to Hempel's explicandum comes from a suggestion from Hempel himself. Hempel has recently evaluated the prospect of formulating adequate criteria of qualitative confirmation.[35] He is inclined to think that the best approach to this problem has been suggested by Carnap which involves the use of the concept of degree of confirmation$_S$ (that is, the concept of logical probability). The general principle which Hempel is inclined to adopt is: ". . . on some suitable definition of logical probability, the probability of *h* on *e* should exceed the *a priori* probability of *h* whenever *e* qualitatively confirms *h*."[36] The *a priori* probability of *h* is the probability of *h* on a logical truth, and so Hempel's inclination is to accept a principle which requires that the probability of *h* on *e* be greater than the probability of *h* on a logical truth *t* whenever *e* confirms *h*. The concept of confirmation$_B$ expresses the comparison of the credibility of *h* on *e* with the credibility of *h* on a logical truth *t*, and so there is a close structural similarity between the concept of confirmation$_B$ and the require-

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[35] Hempel, "Postscript (1964) on Confirmation," in *Aspects of Scientific Explanation and Other Essays in the Philosophy of Science*, pp. 47 - 51.

[36] *Ibid.*, p. 50. Symbols have been altered slightly to comply with my usage.

ment that Hempel enunciates in the principle above.[37] If Hempel had suggested that whenever e confirms h then the probability of h on e exceeds the probability of -h on e, then this would support the view that Hempel's explicandum should be identified with the concept of confirmation_A. The principle which he has suggested, however, fits in best with the view that Hempel's explicandum is closest to the concept of confirmation_B.

Construing Hempel's explicandum as the concept of confirmation_B, moreover, helps to explain the paradoxes of confirmation. The paradoxically confirmatory evidence sentences have to be considered in the light of the meaning of the concept of confirmation_B. It may appear that the report of any black object, for example, does not confirm the hypothesis that all crows are black, but if that report is compared with the effect of a logical truth on the credibility of the hypothesis in question, it is reasonable to suggest that the evidence in question renders the given hypothesis more credible than would a logical truth alone. Thus the paradoxically confirmatory sentences have to be construed as truly confirmatory_B.

Lastly, construing Hempel's explicandum as the concept of

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[37] Some theorists, e.g., Carnap, moreover, consider the expression 'the degree of confirmation (or logical probability) of h on e' to be synonymous with the expression 'the degree of credibility of h on e'. If this was indeed so, the identification of the concept of confirmation_B with Hempel's explicandum would have even greater warrant.

confirmation$_B$ provides a plausible basis for explaining Scheffler's and Carnap's misinterpretations of Hempel's explicandum. The concept of an inductive instance is directly dependent on there being a logical relation between the hypothesis *h* and the sentence *i* which is an inductive instance of *h*. The concept of confirmation$_B$, as well as the concepts of confirmation$_A$ and confirmation$_C$ for that matter, is a prior-evidence-independent concept and has the character of a purely logical relation. Insofar as the concept of confirmation$_B$ has this character, it bears a similarity to the concept of an inductive instance, although no more similarity than that which is borne by the other prior-evidence-independent concepts to the concept of an inductive instance.[38] It may have been this common characteristic which misled Scheffler. Carnap, on the other hand, correctly perceives that the concept which Hempel attempts to explicate is not relativized to prior evidence but pertains to a comparison of the effect of a factual evidence claim with that of a tautology on a hypothesis. The reason that no reference is made to prior evidence, however, is not because the explicandum is the concept of initially confirming$_F$ evidence, but rather because the explicandum is a prior-evidence-independent concept. And Carnap is only partly correct in his interpretation of Hempel's explicandum.

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[38]See the end of section 2.5 for a comment concerning this feature of prior-evidence-independent concepts of confirmation.

CHAPTER FIVE

THE PRIMACY OF THE CONCEPT OF CONFIRMATION$_F$

1. An influential view of the place of confirming$_F$ evidence in relation to the study of confirmation$_S$.

I drew attention in chapter two to a number of weak concepts of confirmation and divided them up into two classes: the class of prior-evidence-independent concepts of confirmation which includes the concepts of confirmation$_A$, confirmation$_B$, and confirmation$_C$, and the class of prior-evidence-dependent concepts of confirmation which includes the concepts of confirmation$_F$ and initial confirmation$_F$.[1] I argued that in order to prevent further confusion and to promote clarity in the study of confirmation, it is desirable that these classes of confirmation be distinguished. I argued in the previous chapter that the foregoing distinction is helpful in considering Hempel's <u>explicandum</u> and in evaluating various critiques of Hempel's study of confirmation. I wish to return to a consideration of the concepts which were distinguished in chapter two and argue that among the weak concepts of confirmation which I have distinguished, the concept of confirmation$_F$ is most important for and central to the study of weak confirma-

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[1] I speak of the concept of initial confirmation$_F$ as if it was a concept separate from the concept of confirmation$_F$. I have treated the special case of the concept of confirmation$_F$ in this way because of the need to distinguish it from the concept of confirmation$_B$.

tion. I shall argue that this is so largely because of its being a prior-evidence-dependent concept of confirmation. I wish to consider the prior-evidence-independent concepts in the light of this contention, and try to assess the role they have played or can play in relation to the concept of confirmation_F.

I have urged above that the study of confirmation consists of the study of the grounds for rationally believing a hypothesis to be true.[2] For this reason the concept of rational credibility has been selected for use in the definitions for the various concepts of confirmation which have been discussed above. The very purpose of obtaining observational and experimental results in connection with some hypothesis is to supply grounds for rationally believing that hypothesis to be true. Theorists do not agree, of course, concerning the observational and experimental results which actually comprise the basis for rational belief in the truth of some conjecture, although there have been a number of popular suggestions which have had supporters for several centuries now. One of the objectives of confirmation theory has been to resolve this disagreement.

There are some theorists for whom the interest in weak confirmation is directly related and subservient to their interest in strong confirmation, that is, the problem of determining which

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[1]See section 1.2 above.

experimental and observational findings constitute confirming and disconfirming evidence (in a weak sense) has arisen for some theorists only in connection with the greater problem of determining the degree of firmness (strong confirmation or $confirmation_S$) a hypothesis might have on the total available evidence. The greater interest which theorists have shown in the study of $confirmation_S$ is readily understandable, for the concept of $confirmation_S$ pertains to the degree of certainty or firmness which it is rational to assign to a given hypothesis on the basis of the total available evidence. The study of weak confirmation, on the other hand, is concerned with the relatively less important problem of determining the comparative effect of the same evidence on different hypotheses or of changing evidence on the same hypothesis. Ascertaining the basis for attributing certainty or a degree of firmness to universal conjectures or hypotheses has proved to be extremely difficult, although attempts continue to be made. The rewards for successfully completing this task probably far outweigh the rewards for successfully analyzing the concepts (or the central concept, perhaps) of weak confirmation. The fact that the more important problem of adequately analyzing the concept of $confirmation_S$ has proved to be so troublesome makes the less important problem of analyzing the concepts of weak confirmation more important than it would otherwise be. Obtaining a solution to one problem, even if it is the less important problem, is better than having two problems for which we have no solutions.

One theorist who explicitly relates the significance of weak confirmation to the more important problem of determining the degree of confirmation$_S$ of a given hypothesis on the total evidence is Carnap. He holds that for the calculation of the degree of confirmation$_S$ of a hypothesis h on total evidence b there are several important logical factors which must be taken into account. The factors which he considers to be of prime importance include the following:[3] (i) the extension of the confirming observational material, (ii) the variety of the confirming material, (iii) the precision of the confirming material, (iv) the extension (and likewise the other factors just listed) of the disconfirming material. Carnap cites Hempel's study of confirmation as one investigation into the difficulties connected with explicating the concept of a confirming case, and so it is evident that when Carnap makes reference to confirming material he is making reference to a weak concept of confirmation. The degree of confirmation$_S$ of h on b thus depends, among other things, upon the number of confirming and disconfirming (in a weak sense) evidence reports which constitute b. This does not mean that Carnap obtains the

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[3] Carnap, LFP, p. 223.

numerical value representing the degree of confirmation$_S$ of h on b by counting the number of confirming and disconfirming evidence reports (which have been appropriately weighted, perhaps, depending on the variety and precision of the reports), although confirmation theorists in an earlier time may have been inclined to proceed in such a way. Carnap's explicatum for the degree of confirmation$_S$ of h on b does not make explicit use of the concept of weak confirmation, although the degree of confirmation$_S$ of h on b does in fact depend upon the (weakly) confirming and disconfirming evidence reports. The effect of the confirmatory and disconfirmatory data is implicitly taken into account in the calculation of the degree of confirmation$_S$.

In Carnap's exposition the (weakly) confirming and disconfirming evidence, upon which the degree of confirmation$_S$ depends, is presumably the confirming$_F$ and disconfirming$_F$ evidence, since this is the one weak sense of 'to confirm' which Carnap explicitly recognizes. The confirming$_F$ evidence reports for h consist of the sentences which provide an increase in the degree of confirmation$_S$ of h on the total prior evidence b, and the disconfirming$_F$ evidence reports for h consists of the sentences which provide a decrease in the degree of confirmation$_S$ of h on b. The prior evidence b continually changes as new, additional observational evidence is acquired. The following imaginary example serves to illustrate the place of the confirming$_F$ and disconfirming$_F$ evidence, upon which the degree of confirmation$_S$ partially depends. Suppose that the hypothesis h_1 is undergoing testing, but prior to any tests or observational findings have been obtained, h_1 has a certain degree of confirmation$_S$ on tautological evidence t. It is worth remarking that on Carnap's scheme the degree of confirmation$_S$ of a hypothesis on a tautological basis (the null confirmation$_S$) need not be 0 and can even be quite high.[4] Suppose that the first evidence report, e_1, is such that $c(h_1,e_1) > c(h_1,t)$. Evidence e_1 is then confirming$_F$ evidence for h_1 given t. In fact, e_1 is initially confirming$_F$ evidence for h_1. Suppose e_2 is such that $c(h_1,e_1 \& e_2) < c(h_1,e_1)$. Evidence e_2 is then disconfirming$_F$ evidence for h_1 given e_1. Suppose e_3 is such that $c(h_1,e_1 \& e_2 \& e_3) = c(h_1, e_1 \& e_2)$. Then e_3 is neutral$_F$ to h_1 given e_1 & e_2. Lastly, suppose e_4 is such that $c(h_1,e_1 \& e_2 \& e_3 \& e_4) > c(h_1,e_1 \& e_2 \& e_3)$. Then e_4 is confirming$_F$ evidence for h_1 given e_1 & e_2 & e_3. There are thus two confirming$_F$ reports, one disconfirming$_F$ report, and one irrelevant$_F$ report with respect to the hypothesis h_1, given the

[4] This point is clearly brought out by Yehoshua Bar-Hillel, "Further Comments on Probability and Confirmation: A Rejoinder to Professor Popper," BJPS, vol. 7 (1956), p. 246. In Carnap's system a sentence stating that the null confirmation$_S$ of h is a certain numerical value states a logical property of h without saying anything about facts. This value depends only upon the meaning of h or the semantic rules of the language to which h belongs. It does not depend upon facts and does not have to be 0. Cf. Carnap, LFP, p. 307f.

appropriate prior evidence in each of the four cases. There are a total of three confirming$_F$ and disconfirming$_F$ evidence reports, given the appropriate prior evidence in each case, which contribute to the degree of confirmation$_S$ of $\underline{h}_1$ on $\underline{e}_1$ - $\underline{e}_4$. An adequate theory of confirmation$_F$ would be required to contain an explication for the relation '$\underline{c}(\underline{h}, \underline{e}\ \&\ \underline{b}) > \underline{c}(\underline{h},\underline{b})$'. In Carnap's system this requirement is met once the concept of degree of confirmation$_S$ of $\underline{h}$ on $\underline{b}$, '$\underline{c}(\underline{h},\underline{b})$', is adequately explicated. It is of course possible to adequately explicate the relation '$\underline{c}(\underline{h}\ \&\ \underline{e}\ \&\ \underline{b}) > \underline{c}(\underline{h},\underline{b})$' without thereby providing a satisfactory explication for the concept of degree of confirmation$_S$.

The picture which Carnap presents concerning confirming$_F$ and disconfirming$_F$ evidence and its relation to the degree of confirmation$_S$ (firmness) of $\underline{h}$ on total evidence $\underline{b}$ is one in which the firmness of a hypothesis $\underline{h}$ is based on many individual evidence reports each one of which either contributes positively to the firmness of $\underline{h}$ (i.e., is confirming$_F$ evidence) or contributes negatively to the firmness of $\underline{h}$ (i.e., is disconfirming$_F$ evidence, although not necessarily conclusively disconfirming$_F$ evidence). The degree of firmness which it is rational to assign a hypothesis $\underline{h}$ depends upon the total observational and experimental results obtained in connection with tests of $\underline{h}$, with each observational and experimental result making a contribution toward the total degree of firmness of $\underline{h}$.

The Carnapian picture of the relationship between strong and weak confirmation, adumbrated above, is also endorsed by Hempel, although he, unlike Carnap, evidently does not construe weak confirmation to be confirmation_F. I have argued in the previous chapter that the weak concept of confirmation which Hempel analyzes is the concept of confirmation_B, rather than the concept of initial confirmation_F. When the article "Studies in the Logic of Confirmation" was originally written, Hempel thought that any attempt to define either the metrical form or the comparative form of the concept of confirmation_S would be likely to ". . . require a precise definition of the concepts of confirming and disconfirming instance of a hypothesis."[5] Hempel's remarks in the section of the paper from which this quotation is taken indicate that he thought the degree of confirmation_S (firmness) of a hypothesis would not only depend upon, but also be defined in terms of, the confirming and disconfirming evidence (in a weak sense) for and against that hypothesis. Carnap, on the other hand, takes the position that the definition of degree of confirmation_S need not be given in terms of (weakly) confirming and disconfirming evidence, although the effect of the (weakly) confirming and disconfirming evidence will be

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[5]Hempel, "Studies," p. 8. In 1964 Hempel notes that Carnap's definition of degree of confirmation_S does not require a definition of a confirming instance, ibid., p. 7, n. 7.

accounted for in the definition of degree of confirmation$_S$. A natural extension of Carnap's and Hempel's view concerning the relationship of weak confirmation to confirmation$_S$ is the view that (weakly) confirming and disconfirming evidence is also important in connection with the qualitative form of the concept of confirmation$_S$, i.e., that whether or not a hypothesis _h_ is _firm_ or _certain_ depends upon the (weakly) confirming and disconfirming evidence acquired in the course of testing _h_. This view is reflected in popular statements to the effect that knowledge of unobserved fact is dependent upon the total number of (weakly) confirming evidence reports.

2. _The primacy of confirming$_F$ evidence insofar as it is evidence which contributes to the total firmness of a hypothesis._

It might be suggested that any one of the prior-evidence-independent concepts of confirmation could be considered in the role which confirming$_F$ and disconfirming$_F$ evidence plays in Carnap's exposition of the relationship of weak confirmation to strong confirmation (confirmation$_S$). It might be suggested, for example, that the degree of firmness of _h_ on total evidence _b_ could be construed as depending upon confirming$_B$ and disconfirming$_B$ evidence,[6] i.e., that the degree of firmness of _h_ on

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[6]Disconfirming$_B$ evidence with respect to _h_ shall be construed as evidence which makes _h_ less credible than does a logical truth.

b depends upon the observational and experimental results which make h either more or less credible than does a logical truth. Someone might adduce as an example Hempel's study of confirming evidence, which I have argued is a study of $confirming_B$ evidence. Similar suggestions might be made in connection with the two other prior-evidence-independent concepts of confirmation, viz., $confirmation_A$ and $confirmation_C$. I submit, however, that the only evidence which can genuinely contribute to the degree of firmness of a hypothesis on total evidence is evidence which actually increases or decreases the firmness of the hypothesis, given the appropriate prior evidence — in short, $confirming_F$ and $disconfirming_F$ evidence. I will grant, however, that an approximation to such evidence might be found in $confirming_A$ evidence and $disconfirming_A$ evidence, or $confirming_B$ evidence, and $disconfirming_B$ evidence, or $confirming_C$ evidence and $disconfirming_C$ evidence, but such evidence will only be an approximation. I shall argue for this position in more detail a little later.

I illustrated in the previous section how some new evidence item e, which is $confirming_F$ evidence for h given b, increases the degree of firmness ($confirmation_S$) of h from that which it had on the total prior evidence b to that which it has on the total evidence upon acquiring e, viz., e & b. I also illustrated how a new evidence item e which is $disconfirming_F$ evidence with respect to _h_ given _b_ decreases the degree of firmness (confirmation$_S$) of _h_ from that which it had on the total prior evidence _b_ to that which it has on the total evidence upon acquiring _e_, _viz._, _e_ & _b_. This indicates how the concepts of confirming$_F$ and disconfirming$_F$ evidence in Carnap's system are in fact defined. This illustration shows clearly that confirming$_F$ and disconfirming$_F$ evidence genuinely contributes to the degree of firmness that a hypothesis has on the total evidence, with each item of evidence in the total evidence contributing some, although possibly small, amount to the degree of firmness of that hypothesis. I wish to argue, moreover, that confirming and disconfirming evidence, in any of the prior-evidence-independent senses of "confirming" and "disconfirming," cannot be cast in the role played by confirming$_F$ and disconfirming$_F$ evidence in the degree of firmness of _h_ on total evidence _b_, and that such evidence cannot genuinely contribute to the degree of firmness of _h_ on _b_. This divergence is brought about by the fact that the prior-evidence-independent concepts of confirmation make no reference to prior evidence, so that a given confirming evidence sentence _e_, in a prior-evidence-independent sense of "confirming," does not necessarily increase the degree of firmness of a hypothesis _from that which it was on the prior evidence_ (i.e., prior to the acquisition of _e_). The following illustration will serve to bring out this point with more force and clarity.

In the illustration I make reference to confirming$_B$ evidence, but the same illustration could be presented *mutatis mutandis* using either the concept of confirming$_A$ evidence or the concept of confirming$_C$ evidence.

Suppose that the hypothesis h_n is examined and that various experimental tests of h_n are carried out. Suppose that a report e_n is acquired which is found to be confirming$_B$ evidence for h_n, i.e., h_n is more credible relative to e_n than relative to logical truth alone. Characteristic of the prior-evidence-independent concepts of confirmation, e_n is assessed completely on its own, divorced from the context of prior evidence which normally surrounds the empirical investigation of hypotheses. Let us suppose that the circumstances surrounding the testing of h_n are normal, however, and that the prior evidence up until the time that report e_n is acquired is b_n. The fact that e_n is confirming$_B$ evidence for h_n is of course completely independent of b_n and would be independent of any other prior evidence sentence. Now the degree of firmness of h_n on e_n & b_n might in fact be greater than the degree of firmness of h_n on b_n alone, in which case e_n would not only be confirming$_B$ evidence for h_n but also confirming$_F$ evidence for h_n given b_n, but it need not be just as surely. From the fact that e_n confirms$_B$ h_n nothing whatever follows concerning whether or not e_n confirms$_F$ h_n given b_n. That is, e_n need not contribute positively to the firmness of $\underline{h}_n$ on $\underline{b}_n$ & $\underline{e}_n$ despite the fact that it is confirming$_B$ evidence for $\underline{h}_n$. If $\underline{h}_n$, for example, has been rendered certain by $\underline{b}_n$ (hence apart from $\underline{e}_n$), $\underline{e}_n$ cannot contribute positively to the firmness of $\underline{h}_n$, for $\underline{h}_n$ is already rendered certain or firm by other evidence. Or if $\underline{h}_n$ has been falsified by $\underline{b}_n$ (hence apart from $\underline{e}_n$), $\underline{e}_n$ cannot contribute positively to the firmness of $\underline{h}_n$, for $\underline{h}_n$ is once and for all infirm and is incapable of being "revived." Thirdly, if $\underline{h}_n$ has already been made as firm as it can possibly be by evidence having the same logical form as $\underline{e}_n$ (supposing that this in fact can occur), then $\underline{e}_n$ will not contribute to the firmness of $\underline{h}_n$. Thus the contribution of $\underline{e}_n$ toward the firmness of $\underline{h}_n$ might well be nil, despite the fact that $\underline{h}_n$ is more credible on $\underline{e}_n$ than on a logical truth alone.

An adequate theory of confirmation$_F$ will naturally take into account the background of prior evidence when assessing the confirmatory role of a given evidence report, but this is something a theory of confirmation$_B$ cannot do. The reason for this is that prior evidence is not taken into account and evidence reports might well be assessed as confirmatory$_B$ although those reports do not contribute to the degree of firmness a hypothesis enjoys. The confirming$_F$ and disconfirming$_F$ evidence for a hypothesis $\underline{h}$ is able to genuinely contribute to the firmness of $\underline{h}$. If the hypothesis $\underline{h}_n$ has been rendered certain by the prior evidence $\underline{b}_n$, an adequate theory of confirmation$_F$ will assess $\underline{e}_n$

as being neither confirming$_F$ evidence for $\underline{h}_1$ given $\underline{b}_n$ nor disconfirming$_F$ evidence for $\underline{h}_n$ given $\underline{b}_n$ (i.e., irrelevant$_F$ with respect to $\underline{h}$, given $\underline{b}_n$), and the degree of firmness of $\underline{h}_1$ on total evidence will not depend upon $\underline{e}_n$ in any way. Or if $\underline{h}_n$ has been falsified by some part (or all) of the prior evidence $\underline{b}_n$, the new evidence $\underline{e}_n$ will be correctly assessed as being irrelevant$_F$ with respect to $\underline{h}_n$ given $\underline{b}_n$, and again the degree of firmness of $\underline{h}_n$ on the total evidence $\underline{b}_n$ & $\underline{e}_n$ will not depend upon any contribution attributable to $\underline{e}_n$. The confirming$_F$ and disconfirming$_F$ evidence reports will thus be reports that genuinely contribute to the degree of firmness that $\underline{h}_n$ has on the total evidence. Confirming$_B$ and disconfirming$_B$ evidence, on the other hand, cannot be counted on as making a genuine contribution to the firmness of $\underline{h}_n$.

It is instructive, in this context, to make reference to Nicod's study of confirmation in order to further illustrate that a genuine contribution to the firmness of a hypothesis is provided only by reports which increase the firmness of that hypothesis. Nicod's criterion of confirmation was discussed earlier, and so I shall not repeat it here.[7] Nicod makes the following significant remark: "A favourable case increases more or less the probability of a law, whereas a contrary case

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[7]See Section 3.1 above.

annihilates it entirely."[8] This indicates that Nicod construes a favorable or confirming case, for which he offers a criterion, as increasing the probability (or the firmness) of a hypothesis from that which it was prior to the acquisition of the new confirming case. His criterion of confirmation being what it is, this means that any logical instance of a universal conditional sentence is construed as increasing the firmness of a hypothesis. This I submit, is incorrect. It cannot be assumed without qualification that all sentences of the form: 'Px & Qx' increase the firmness of the universal conditional sentence $\underline{h}_1$: '(x)(Px ⊃ Qx)'. Let us imagine that evidence sentences, both confirming and invalidating, are numbered in the order in which they are acquired. On Nicod's criterion there is only one kind of evidence sentence which confirms $\underline{h}_1$, namely, sentences of the form 'Px & Qx', so we shall not have to bother with sentences of any other form. Let us suppose that after $\underline{e}_6$: 'Px_6 & Qx_6' is acquired we obtain the first invalidating evidence report $\underline{e}_7$: 'Px_7 & $-Qx_7$'. Any evidence reports of the form 'Px & Qx' which are acquired after $\underline{e}_7$ is acquired cannot and will not contribute to the firmness of $\underline{h}_1$, for $\underline{h}_1$ has once and for all been invalidated by the evidence sentence $\underline{e}_7$. According to Nicod's criterion of confirmation, however, the evidence report $\underline{e}_8$:

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[8] Nicod, _Foundations of Geometry and Induction_, p. 220.

'Px_8 & Qx_8' is assessed as confirming $\underline{h}_1$. But the report $\underline{e}_8$ cannot increase the firmness of $\underline{h}_1$ from that which it has on evidence sentences $\underline{e}_1$ - $\underline{e}_7$, since $\underline{h}_1$ has been rendered infirm by $\underline{e}_7$ and no additional evidence report can "revive" $\underline{h}_1$. In order for Nicod to achieve the desired aim of providing a criterion of evidence which increases the firmness of a hypothesis, it is necessary to take into consideration the nature of the prior evidence.

There are several other possible contexts which illustrate the inadequacy of Nicod's criterion of confirmation. We can concede, perhaps, that a contribution to the firmness of $\underline{h}_1$ is afforded by each of the evidence sentences, $\underline{e}_1$, $\underline{e}_2$, . . . , $\underline{e}_n$, where each of these sentences is of the form 'Px & Qx' and where $\underline{n}$ is quite large. We might reach the stage, however, where $\underline{h}_1$ no longer receives an increase in probability or firmness from some sentence $\underline{e}_{n+1}$, either because $\underline{h}_1$ has been rendered firm or certain by evidence of the form 'Px & Qx', or, what is more likely, because $\underline{h}_1$ has received as much firmness from evidence of the form 'Px & Qx' as it possibly can, even though such evidence does not render $\underline{h}_1$ firm or certain. This is not an idle supposition, for many theorists have suggested that evidence of a given form, after much has been acquired, finally loses its significance. Such an opinion has been expressed especially in connection with laws. Now Nicod's criterion of confirmation will judge $\underline{e}_{n+1}$ to be favorable or confirming evidence for $\underline{h}_1$, since it is of the form 'Px & Qx', even though $\underline{e}_{n+1}$ does not actually contribute to the firmness of $\underline{h}_1$. In order to adequately capture the notion of an increase in firmness it is necessary to conceive of the relation of confirmation as involving an essential reference to temporally prior evidence so that the effect of such prior evidence will be accounted for in assessing a given evidence report. This feature is effectively captured by the concept of confirmation$_F$.

3. Construing prior-evidence-independent concepts of confirmation as approximating to confirmation$_F$.

Although it is the confirming$_F$ and disconfirming$_F$ evidence reports which genuinely contribute to the firmness of a hypothesis, it would be possible to construe confirming and disconfirming evidence reports, where "confirming" and "disconfirming" express prior-evidence-independent concepts, as reports which approximate to confirming$_F$ and disconfirming$_F$ evidence, provided that certain assumptions were made concerning not only prior evidence but also the relationship between the concept of confirmation$_F$ and prior-evidence-independent concepts of confirmation. For example, it would be possible and perhaps even plausible to construe reports of confirming$_B$ and disconfirming$_B$ evidence with respect to $\underline{h}$ as reports which approximate to confirming$_F$ and disconfirming$_F$ evidence with respect to h (given the appropriate prior evidence), provided certain assumptions were made about the prior evidence and provided there is some connection between the concept of confirmation$_F$ and the prior-evidence-independent concept of confirmation$_B$. If, for example, evidence which makes h more credible than does a logical truth (that is, confirming$_B$ evidence) was considered to be evidence which makes h more credible than h was on the basis of prior evidence alone, (that is, confirming$_F$ evidence) and if in addition the prior evidence is suitably restricted, then the confirming$_B$ evidence for h might well serve as an approximation to confirming$_F$ evidence for h (given the appropriate prior evidence). In order to indicate more clearly how confirming evidence, in a prior-evidence-independent sense of "confirming," might be used to approximate to confirming$_F$ evidence, and to indicate what kind of assumptions might have to be made concerning the prior evidence, I shall adduce an imaginary illustration in which the hypothesis h_1: '$(x)(Px \supset Qx)$' is considered. The prior-evidence-independent concept of confirmation I shall use is the concept of confirmation$_B$. I shall not indicate how disconfirming evidence, in a prior-evidence-independent sense of "disconfirming," might be used to approximate to disconfirming$_F$ evidence, for the illustration would be analogous to the case for confirming evidence.

Suppose that a number of experimental tests of and observational findings relevant to $\underline{h}_1$ have been acquired which comprise the prior evidence $\underline{b}_1$. Suppose also that some report $\underline{e}_n$: '$\underline{Pa}$ & $\underline{Qa}$' is then acquired, and that it is known that the prior evidence $\underline{b}_1$ does not contain any findings about $\underline{a}$. Suppose also that $\underline{e}_n$ is judged to be confirming$_B$ evidence for $\underline{h}_1$, i.e., $\underline{h}_1$ is judged to be more rationally credible on the basis of $\underline{e}_n$ than on the basis of a logical truth alone — a fairly reasonable supposition to which many theorists would probably be inclined to assent. Now the report $\underline{e}_n$ might be construed as an approximation to confirming$_F$ evidence for $\underline{h}_1$ given $\underline{b}_1$ on the basis of the following considerations. An evidence report which makes $\underline{h}_1$ more credible than a logical truth does might be assessed as evidence which increases the firmness of $\underline{h}$, on prior evidence $\underline{b}_1$, provided certain assumptions were made concerning $\underline{b}_1$ and its effect on $\underline{h}_1$. If it were known, for example, that $\underline{h}_1$ had not been rendered firm by some part (or all) of the evidence $\underline{b}_1$, then $\underline{h}_1$ would quite possibly be capable of receiving an increase in firmness from the new report $\underline{e}_n$. Secondly, if it were also known that $\underline{h}_1$ had not been rendered infirm (that is, falsified) by some part (or all) of the evidence $\underline{b}_1$, then $\underline{h}_1$ would again be quite possibly capable of receiving an increase in firmness from the new evidence report. In the third place, if it were also known that $\underline{b}_1$ did not contain so much evidence of the form 'Px & Qx' that further evidence of that form would no longer afford an increase in firmness of $\underline{h}_1$, then $\underline{h}_1$ would again stand to receive an increase in firmness from $\underline{e}_n$. All these restrictions on $\underline{b}_1$, and perhaps more restrictions still, would have to obtain in order for $\underline{h}_1$ to be in a position to receive a genuine increase in firmness from $\underline{e}_n$. If the appropriate restrictions on $\underline{b}_1$ are in effect and if, in addition, confirming$_B$ evidence for $\underline{h}_1$ is also confirming$_F$ evidence for $\underline{h}_1$ given $\underline{b}_1$, then the report of confirming$_B$ evidence $\underline{e}_n$ might well be used as an approximation to confirming$_F$ evidence for $\underline{h}_1$, given $\underline{b}_1$ or similarly appropriate prior evidence.

In a similar fashion, it is possible that confirming$_A$ or confirming$_C$ evidence might be construed as evidence which approximates to confirming$_F$ evidence, given not only the assumption that confirming$_A$ or confirming$_C$ evidence for a hypothesis $\underline{h}$ is evidence which confirms$_F$ $\underline{h}$ given the appropriate prior evidence $\underline{b}$, but also the additional assumption that the prior evidence $\underline{b}$ is of an appropriate sort. It is quite conceivable that different views might be held concerning the kind of confirming evidence, in a prior-evidence-independent sense of "confirming," which affords the most suitable approximation to confirming$_F$ evidence. One might maintain that an evidence report which makes a hypothesis more credible than the negation of that hypothesis (confirming$_A$ evidence) is most suitably regarded as approximating to evidence which increases the firmness of a hypothesis;

another might maintain that an evidence report which makes a hypothesis more credible than a logical truth does (confirming_B evidence) is most suitably regarded as approximating to evidence which increases the firmness of a hypothesis; and yet a third might maintain that an evidence report which makes a hypothesis more credible than a logical truth does and also makes that hypothesis more credible than the negation of that hypothesis (confirming_C evidence) is most suitably regarded as approximating to evidence which increases the firmness of a hypothesis. It seems to me that confirming_B evidence provides a better approximation to confirming_F evidence than either confirming_A evidence or confirming_C evidence does. For the judgment 'If *h* is more credible on *e* than on *t* (*e* confirms_B *h*) then *h* is more credible on *e* & *b* than on *b* alone (*e* confirms_F *h* given *b*)' strikes me as more plausible than either the judgment 'If *h* is more credible than -*h* on *e* (*e* confirms_A *h*) then *h* is more credible on *e* & *b* than on *b* alone (*e* confirms_F *h* given *b*)' or the judgment 'If *h* is more credible than -*h* on *e* and *h* is more credible on *e* than on *t* (*e* confirms_C *h*) then *h* is more credible on *e* & *b* than on *b* alone (*e* confirms_F *h* given *b*)'.

Reference can again be made to Nicod's study of confirmation in order to further illustrate how a criterion of confirmation might be construed as yielding results which approximate to a criterion of confirmation_F. As long as there is no falsifying evidence of the form 'Px & -Qx', which would conclusively falsify $\underline{h}_1$, and as long as the evidence reports which his criterion assesses as confirmatory (namely, reports of the form 'Px & Qx') neither render $\underline{h}_1$ firm nor lose their efficacy due to prolificity, and perhaps more restrictions besides these mentioned also hold, then Nicod's criterion of confirmation can be regarded as approximating to a criterion of confirming$_F$ evidence. All of the above restrictions would have to be in force, however, in order for Nicod's criterion of confirmation to be construed as approximating to a criterion of confirmation$_F$. Since the concept of confirmation$_F$ makes an essential reference to the prior evidence background, it accounts for all these restrictions automatically. If some evidence sentence $\underline{e}_n$ conclusively disconfirms$_F$ $\underline{h}_1$ given $\underline{b}_n$, then an adequate theory of confirmation$_F$ will assess any evidence sentence $\underline{e}_{n+1}$ as not confirming$_F$ $\underline{h}_1$ given $\underline{b}_n$ & $\underline{e}_n$. If $\underline{h}_1$ is incapable of receiving any additional increase in firmness from $\underline{e}_{n+1}$, either because $\underline{h}_1$ is already firm or has received as much firmness as it can possibly receive from evidence having the same form as $\underline{e}_{n+1}$, then an adequate theory of confirmation$_F$ will assess $\underline{e}_{n+1}$ as not confirming$_F$ $\underline{h}_1$ given the prior evidence available up until $\underline{e}_{n+1}$ was acquired.

Thus, although an approximation to confirming$_F$ and disconfirming$_F$ evidence might be provided by confirming and disconfirming evidence, in a prior-evidence-independent sense of "confirming" and "disconfirming," such evidence will only be an approximation depending upon a number of restrictions all of which it might be impossible to specify. This demonstrates the primacy of the concept of confirmation$_F$ among the weak concepts of confirmation which I have distinguished, when weakly confirming and disconfirming evidence is regarded as providing the basis upon which a hypothesis is assessed as being firm or as having a certain degree of firmness.

I argued in the previous chapter that Hempel's <u>explicandum</u> is most plausibly identified as the concept of confirmation$_B$ among all the concepts of confirmation which I have distinguished. I argued for this identification on the basis of several of Hempel's statements and also by eliminating other concepts, e.g., the concept of initial confirmation$_F$, as plausible candidates. The argument in this chapter has been to the effect that the one weak concept of confirmation which is central to the aim and purpose of confirmation theory in general is the concept of confirmation$_F$. It follows that Hempel's study of confirmation is not as valuable and significant as it might appear to be, insofar as the concept which he explicates is not of central importance to confirmation theory. Inasmuch as Hempel explicates the concept of confirmation$_B$, he does not succeed in adequately specifying the weakly confirming and disconfirming evidence reports which genuinely contribute to the firmness of a hypothesis, for his definition assesses as confirmatory reports which do not contribute to the firmness of a hypothesis. For example, the report 'Pa' is assessed as confirming the hypothesis '(Ex)Px', but 'Pa' should not be assessed as contributing to the firmness of '(Ex)Px' if the latter is falsified or already made firm by prior evidence. Only the concept of confirmation_F, which relates an evidence report, a hypothesis, and the prior evidence background, can adequately assess which evidence reports contribute to the firmness of a hypothesis. Hempel's study is of some limited value, however, insofar as his definition can be construed as specifying the evidence reports which approximate to be confirming_F evidence. In order to construe it as an approximation, however, it is necessary to assume not only that the hypothesis being examined has not been falsified, nor rendered firm, etc. by other evidence but also that if e confirms_B h then e confirms_F h given b. Hempel is not the only theorist who has been preoccupied with a two-termed prior-evidence-independent relation of confirmation while a three-termed prior-evidence-dependent relation of confirmation should have been studied. Theorists who have developed more traditional theories of induction have made the same mistake. I shall briefly examine the traditional theories of induction in order to point out their deficiencies.

4. Remarks concerning traditional theories of weak confirmation and the view of confirmation underlying them.

The traditional answers to the question: What experimental results and observational findings contribute to the firmness of a hypothesis, i.e., render a hypothesis firm or confer a certain degree of firmness on a hypothesis? have been (1) enumerative instances of the hypothesis, (2) reports which eliminate alternatives to the hypothesis, or (3) predictions deduced from the hypothesis which are borne out by observation. Theories which are based on these three stock answers have become widely known as (1) the theory of enumerative induction, (2) the theory of eliminative induction, and (3) the method of hypothesis, respectively.[9] Theorists who have endorsed the theory of enumerative induction have thus taken the position that observational and experimental findings which contribute to the firmness of a hypothesis *h* are instances of *h*. The degree of firmness of *h* has been regarded as varying largely with the number of instances of *h*. The notion of an instance is uncertain, as I have mentioned above.[10] Assuming that this problem can be solved, the theory of

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[9]See Stephen Barker, Induction and Hypothesis (Ithaca, New York: Cornell University Press, 1957), chapters 3,4, and 8, for a lucid and concise discussion of these traditional positions.

[10]This issue has been examined in connection with Nicod and Scheffler in sections 3.1 and 4.2 above.

enumerative induction simply states that the instances of a hypothesis render that hypothesis firm or confer a degree of firmness upon that hypothesis. The theory of elimination induction states that reports which eliminate hypotheses which compete with *h* (that is, hypotheses with which *h* is logically incompatible) render *h* firm or confer a degree of firmness upon *h*. The chief difficulty with this theory has been that the number of hypotheses which compete with some hypothesis *h* is infinite, so the elimination of a finite number of these competing hypotheses contributes nothing to the firmness of *h*. According to the method of hypothesis, the deductive consequences of a hypothesis *h* which are borne out by observation render *h* firm, or confer a degree of firmness upon *h*. This theory, in a somewhat more elaborate form perhaps, has a large number of supporters.

I do not intend to discuss the strengths and weaknesses of these traditional theories in any more detail. I wish, rather, to draw attention to one important feature of all these theories, namely, that these theories consider the firmness of a hypothesis to depend upon evidence reports all of which are such that the prior evidence for or against a hypothesis is not taken into account.

According to the enumerative theory, the reports which contribute to the firmness of a hypothesis are its instances.

For example, all individual reports of the form 'Px & Qx' are considered as weakly confirming the hypothesis '(x)(Px ⊃ Qx)'.[11] There is no explicit consideration of the prior evidence for or against a hypothesis when an evidence item is considered by the theory. The theory, moreover, specifies which reports contribute to the firmness of a hypothesis by determining whether or not a given logical relation holds between the evidence sentence e examined and the hypothesis h, ignoring the prior evidence b which might already have been acquired.

According to the eliminative theory the reports which contribute to the firmness of a hypothesis are those reports which eliminate competing hypotheses. For example, the hypothesis h_1: '(x)(Px ⊃ Qx)' is weakly confirmed by the evidence report 'Pa & Qa', for the latter sentence eliminates or falsifies the hypothesis '((x)(Px ⊃ -Qx) & (Ex)(Px))' which is logically incompatible with h_1. In this theory there again is no explicit consideration of the prior evidence for or against a hypothesis h when an evidence sentence e is considered, and again the weakly confirming evidence sentences for a hypothesis are determined by a logical relation holding between h and e,

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[11] I ignore complexities connected with the concept of an instance here and assume that which I have called logical instances of a universal conditional hypothesis are instances in the sense required by the enumerative theory. The important question, of course, is whether logical instances exhaust the sense of 'instance' required by the enumerative theory.

ignoring the prior evidence *b* which might already have been acquired.

The method of hypothesis is similar to the theory of enumeration and the theory of elimination insofar as according to it evidence reports which contribute to the firmness of a hypothesis are considered in isolation from the prior evidence which might already have been acquired. Hempel formulates a criterion which captures the central notions of the method of hypothesis as follows:[12] evidence *e* confirms *h* if *e* can be divided into two mutually exclusive sentences *e*' and *e*" such that *e*" can be deduced from *h* & *e*' but not from *e*' alone. Thus, for example, 'Pa & Qa' confirms h_1 on this criterion since 'Qa' can be deduced from '(x)(Px ⊃ Qx) & Pa' but not from 'Pa' alone. Again the weakly confirming evidence sentences for a hypothesis are determined by means of a logical relation between a hypothesis *h* and an evidence sentence *e*, ignoring the prior evidence which might already have been acquired.

The concepts of weak confirmation which underlie the traditional theories have been two-termed and prior-evidence-independent rather than three-termed and prior-evidence-dependent. The theories have been put forward in order to account for the firmness enjoyed by a hypothesis, however. Because the

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[12] Hempel, "Studies," pp. 26 - 27.

significance of prior evidence has been overlooked, the traditional theories cannot adequately account for the firmness which is afforded to hypotheses by evidence reports. These traditional theories might also be viewed as providing results which approximate to confirming$_F$ evidence which genuinely contributes to the firmness of a hypothesis. Because prior evidence has been overlooked, the relation of confirmation which has been examined has been two-termed and prior-evidence-independent. A notable and commendable exception to this widespread tendency has been Carnap's examination of the three-termed, prior-evidence-dependent concept of confirmation$_F$.

It might be suggested that since confirming and disconfirming evidence, in a prior-evidence-independent sense of "confirming" and "disconfirming," can serve as an approximation to confirming$_F$ and disconfirming$_F$ evidence when suitable restrictions are imposed upon the prior evidence, we need not bother with investigating the concept of confirmation$_F$ but can limit our attention to one or more of the simpler prior-evidence-independent concepts. There are several things which I would say in reply to such a suggestion. In the first place, the concept of confirmation$_F$ is an interesting concept in its own right, and warrants investigation because of its primacy among the weak concepts of confirmation. Secondly, there may be more difficulty in specifying the restrictions which have to be imposed upon prior evidence so that a prior-evidence-independent concept can be used as an approximation to confirming_F evidence. The three restrictions which I have frequently used in illustration are that the hypothesis being examined is not already falsified, nor already rendered firm, nor already in possession of as much firmness as it can possibly receive from evidence of a certain kind. There could quite possibly be additional restrictions which one would have to impose on the prior evidence so that some prior-evidence-independent concept of confirmation could serve as an approximation to the concept of confirmation_F. In the third place, a successful explication of the qualitative form of the concept of confirmation_F is the first step in the direction of successfully explicating the quantitative form of that concept. It is not only of interest to know whether or not *e* confirms_F *h* given *b*, but it is also of interest to know the degree to which *e* increases the firmness of *h* from that which it had on the total prior evidence *b*. A qualitative theory of confirmation_F might assess 'Pa' as confirming_F evidence for '(x)Px' given *b*' and 'Pb' as confirming_F evidence for '(x)(Px)' given *b*" (where *b*' ≠ *b*"), but a quantitative theory of confirmation_F would determine the amount by which 'Pa' and 'Pb' each increase the firmness of '(x)(Px)'. Hence the concept of confirmation_F remains a central concept among the concepts of confirmation which have been distinguished in this dissertation.

5. The primacy of the concept of confirmation_F even when not related to the study of confirmation_S, and remarks on the value of the prior-evidence-independent concepts of confirmation.

The discussion in the preceding sections of this chapter has been based upon the view that the study of weak confirmation is important mainly in relation to determining the degree of firmness (confirmation_S) which a hypothesis has on the total available evidence. In Carnap's exposition the weakly confirming and disconfirming evidence upon which the degree of firmness of a hypothesis depends is the confirming_F and disconfirming_F evidence. For this reason the concept of confirmation_F is the most important concept among the concepts of weak confirmation which have been distinguished and discussed in this dissertation, for only confirming_F and disconfirming_F evidence sentences can genuinely contribute to the degree of firmness possessed by a hypothesis.

It is not necessary, of course, to explicitly relate the study of weak confirmation to the study of strong confirmation (confirmation_S), for the study of weak confirmation is of interest in itself apart from any significance it might have in connection with the study of confirmation_S. Even if the study of weak confirmation were not related to the study of confirmation_S, the concept of confirmation_F would be of considerable importance, for the concept of confirmation_F expresses the epistemologically significant notion of making a hypothesis firmer than it was on prior evidence, or of increasing the firmness of a hypothesis from that which it was on prior evidence.

It is not inconceivable that the concept of confirmation_F might someday be judged as being of greater epistemological importance than the concept of confirmation_S, although the common opinion among most inductive theorists today is that the latter is of greater importance than the former. There are a number of theorists who oppose attempts to explicate the metrical form of the concept of confirmation_S. Nagel, for example, claims: "It does not seem possible to assign a quantitative value to the degree of confirmation of a theory."[13] One reason often given is that the metrical form of the concept is not used in science as science is commonly practiced, i.e., it is not customary to assign numerical values to a hypothesis representing its degree of confirmation_S on the total available evidence. There is more tolerance among critics of attempts to explicate the metrical form of the confirmation_S of the comparative form of the concept of confirmation_S which is expressed in such statements as 'Hypothesis _h_ is firmer on total evidence _b_ than hypothesis _k_ is on total evidence _f_', although writers have pointed out that comparisons are not normally made, and cannot be reasonably made,

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[13] Nagel, _Principles of the Theory of Probability_ (Chicago: Chicago University Press, 1939), p. 68.

between any two hypotheses drawn from any fields of inquiry whatsoever. Misgivings have even been expressed concerning the qualitative form of the concept of confirmation_S which is expressed in simple statements such as 'Hypothesis h is firm or certain on total evidence b', for it has been pointed out that unless h is a tautology or unless b entails h, there really are very few, if any, empirical contexts expressed by b which warrant one in maintaining that h is certain or beyond any doubt whatsoever. Among recent writings which have contributed to the undermining of the belief in epistemological certainty are those of W. V. O. Quine who has made the by-now-famous claim that no statement in the totality of so-called knowledge (and this includes logical laws) is immune to revision.[14] It is conceivable, therefore, that the situation might someday arise in which less importance is given to the concept of firmness or confirmation_S (in any of its forms) than there presently is, and that theorists would be chiefly concerned with judgments involving the concept of confirmation_F. In such an event, of course, the concept of confirmation_F would have to be explicable without reference to the concept of confirmation_S. Although this situation is conceivable, it is not likely to arise in the near future, given the widespread belief that one can possess certain

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[14] Quine, From a Logical Point of View, Second edition (New York: Harper & Row, 1963), p. 43.

knowledge.

What importance can be attached to the prior-evidence-independent concepts of confirmation, given the primacy of the prior-evidence-dependent concept of confirmation$_F$? In the first place, they can perhaps be used, given the proper restrictions explained above, as an approximation to the concept of confirmation$_F$, although if the latter concept were adequately explicated there would be no value in using concepts which provide only an approximation to the concept of confirmation$_F$. In the second place, the prior-evidence-independent concepts are of interest in their own right, even though they are not as epistemologically central as the concept of confirmation$_F$ is. Thirdly, one or more of these concepts may prove to be significant in connection with some feature of scientific inquiry. For example, the concept of confirmation$_B$ might be used in delineating the field of observation in connection with testing a hypothesis in order to obtain confirming$_F$ evidence. We might look for confirming$_F$ evidence for a hypothesis _h_ among those evidence reports which are found, prior to the testing of _h_, to confirm$_B$ _h_. It cannot be expected that all evidence reports which confirm$_B$ _h_ also confirm$_F$ _h_ given the appropriate prior evidence. I have argued this at some length above in section 2.5. But it is not unreasonable to expect that an evidence report which confirms$_F$ _h_, given the appropriate prior evidence, also confirms$_B$ _h_. That is, it is reasonable to expect that if h is more credible relative to e and b than relative to b alone then h is more credible relative to e than relative to logical truth alone, where the evidence sentence e is construed in the second instance as evidence which is independent of the background of prior evidence. Confirmation$_B$ is definitely not a sufficient condition of confirmation$_F$ but it is a necessary condition. Thus the concept of confirmation$_B$ might be useful in delineating the field of testing prior to obtaining the actual confirming$_F$ evidence.

I wish to devote the remainder of this dissertation to the matter of explication. In view of the fact that all of the weak concepts of confirmation are of at least limited interest, I shall consider the matter of explication with respect to all of the concepts of weak confirmation which have been distinguished. Since there is one concept which is of special interest, I devote more attention to its explication in a separate chapter — chapter seven.

CHAPTER SIX

EXPLICATION AND THE USE OF LOGICAL CONDITIONS OF ADEQUACY

1. Logical conditions of adequacy discussed in current literature on confirmation.

An important feature of confirmation theory has been and continues to be the discussion of the characteristics of concepts of confirmation. This matter has an important bearing upon the task of explicating concepts of confirmation, although theorists are not agreed concerning the relation of the discussion of characteristics of concepts of confirmation to the explication of such concepts. Some theorists believe that prior to an adequate explication of a given concept it is necessary to clearly state everything that is known about the *explicandum*, i.e., it is necessary to enunciate all intuitively obvious characteristics of the *explicandum*. These characteristics then serve as a standard for explication. Kemeny and Oppenheim, for example, decry the tiral and error method of explication as "dangerous" and declare that ". . . we must first put down clearly all that our intuition tells us about the *explicandum*, and then find the precise definitions that satisfy our intuitive requirements."[1] This approach to the task of providing an adequate explication

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[1] John G. Kemeny and Paul Oppenheim, "Degree of Factual Support," *Philosophy of Science*, vol. 19 (1952), p. 308.

(or rational reconstruction) of concepts of confirmation was adopted by Hempel, whose study I have already partially examined. The characteristics of the explicandum were stated in statements which Hempel called "logical conditions of adequacy." This expression has been in frequent use ever since, and I shall use it too. One of the unsatisfactory features of this approach to explication is the fact that different theorists have different intuitions concerning the characteristics of concepts of confirmation, with the result that sometimes the set of characteristics drawn up by one theorist is incompatible with the set of characteristics drawn up by another. The problem is choosing between conflicting sets of logical conditions of adequacy is formidable. It is of course presupposed that a set of characteristics, serving as logical conditions of adequacy, is consistent.

A second approach has been taken to explication according to which a given concept of confirmation is required to meet a certain formal condition from which logical conditions of adequacy for a given concept can be deduced. Carnap, for example, requires that an adequate explicatum for the qualitative form of the concept of confirmation$_F$ must be in accord with at least one adequate explicatum for the metrical form of the concept of confirmation$_S$ (firmness), that is, there must be at least one function c that is a suitable explicatum for the concept of degree of confirmation$_S$ such that whenever *e* confirms$_F$ *h* given *b*, then the degree of confirmation$_S$ of *h* on *e* & *b* is greater than the degree of confirmation$_S$ of *h* on *b* alone ($c(h,e \,\&\, b) > c(h,b)$).[2] Once this formal requirement concerning the qualitative form of the concept of confirmation$_F$ is adopted, certain logical conditions of adequacy which describe various characteristics of the concept of confirmation$_F$ follow logically. A very similar approach to explication is taken by Nicholas Rescher who defines three concepts of confirmation in terms of a probabilistic function *L*.[3] From these definitions certain logical conditions of adequacy — "rules of evidence" in Rescher's terminology — follow. These conditions of adequacy then serve as a partial description of the concept that has been defined. The problem of choosing between two incompatible sets of adequacy conditions is circumvented by this second approach to explication, but what requires defense is the initial choice of definitions.

What then are some of the logical conditions of adequacy which authors have thought to be intuitively plausible? I shall begin to answer this question by making reference to Hempel's list

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[2] Carnap, *LFP*, p. 472.

[3] Nicholas Rescher, "A Theory of Evidence," p. 83f. See section 2.3 above for further discussion of the concepts defined by Rescher.

of logical conditions of adequacy; in particular, the three classes of conditions he considers. Hempel groups the conditions of adequacy he thought to be correct into three, each group headed by a condition entailing all the other conditions in that group. The three main conditions are:[4]

C1: Entailment Condition: If *e* entails *h*, then *e* confirms *h*.

C2: Consequence Condition: If *e* confirms every sentence of a class *K*, then *e* also confirms every logical consequence of *K*.

C3: Consistency Condition: Every consistent evidence sentence is logically compatible with all the sentences which it confirms.

Hempel lists some eight or nine additional conditions of adequacy, each of which is derivable from one of the above general conditions. A number of authors, besides Hempel, have enunciated conditions of adequacy which they have deemed important. The following list of conditions includes all of the important conditions of adequacy which have been discussed or mentioned in the literature on confirmation.[5] A number of conditions have

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[4]Hempel, "Definition," p. 127 and "Studies," pp. 31 - 33.

[5]See Hempel, "Definition," p. 127f, "Studies," p. 30f; Carnap, *LFP*, section 87; Rescher, *op. cit.*, p. 88f.; Howard Smokler, "Conflicting Conceptions of Confirmation," (continued)

become the subject of debate and have acquired names. I include these names for easy reference. The conditions have been numbered in order to indicate relations of entailment where they occur, e.g., conditions which are derivable from C2 are numbered C2.1, C2.2, etc. In the statement of some of the conditions I have included a somewhat simpler condition, e.g., C2.3, C2.5, C5.1, etc. When I come to examine the conditions, I shall for the most part consider only the simpler condition. Here then is the list of logical conditions of adequacy:

C1.1: If *h* is logically true, then *e* confirms *h*.

C1.2: If *e* is logically false, then *e* confirms *h*.

C1.3: Evidence *e* confirms *e*.

C2.1: Special Consequence Condition: If *e* confirms *h*, then *e* confirms every logical consequence of *h*, i.e., if *e* confirms *h*, and *h* entails *k*, then *e* confirms *k*.

C2.2: Equivalence Condition for Hypotheses: If *e* confirms *h*, then *e* confirms every sentence logically equivalent to *h*, i.e., if *e* confirms *h*, and *k* is logically equivalent to *h*, then *e* confirms *k*.

C2.3: Conjunction Condition: If *e* confirms every one of a finite number of sentences, then *e* confirms also their

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(continued from previous page) *The Journal of Philosophy*, vol. 65 (1968), pp. 300 - 312; and Hesse, "Theories and the Transitivity of Confirmation," *Philosophy of Science*, vol. 37 (1970), p. 50f, for various conditions and names of conditions.

conjunction, i.e., if *e* confirms *h*, and *e* confirms *k*, then *e* confirms *h* & *k*.

C2.4: If *e* confirms one of a finite number of sentences, then *e* confirms also their disjunction, i.e., if *e* confirms one of *h* and *k*, then *e* confirms *h* v *k*.

C2.5: If *e* confirms a conjunction, then *e* confirms each conjunct: If *e* confirms *h* & *k*, then *e* confirms *h* and *e* confirms *k*.

C3.1: Special Consistency Condition: The class K of all sentences confirmed by a consistent evidence sentence *e* is consistent.

C3.2: Compatibility Condition: If *e* and *h* are logically incompatible, then *e* does not confirm *h*.

C3.3: It is not the case that *e* confirms *h* and that *e* confirms -*h*.

C3.4: If *h* is inconsistent, then it is not the case that *e* confirms *h*.

C4: Converse Consequence Condition: If *e* confirms *h*, then *e* confirms every sentence of which *h* is a logical consequence, i.e., if *e* confirms *h*, and *h* is a logical consequence of *k*, then *e* confirms *k*.

C5: Converse Entailment Condition: If *h* entails *e*, then *e* confirms *h*.

C5.1: One term of a conjunction confirms the whole conjunction, i.e., *e* confirms *e* & *h*.

C6: If *e* confirms *h*, then *h* confirms *e*.

C7: If *e* confirms *h*, then -*h* confirms -*e*.

C8: Transitivity Condition: If *e* confirms *h*, and *h* confirms *k*, then *e* confirms *k*.

C9: If *e* confirms *h*, then any sentence which entails *e* confirms *h*, i.e., if *e* confirms *h*, and *k* entails *e*, then *k* confirms *h*.

C9.1: Equivalence Condition for Evidence Sentences: If *e* confirms *h*, then any sentence logically equivalent to *e* confirms *h*, i.e., if *e* confirms *h* and *k* is logically equivalent to *e*, then *k* confirms *h*.

In the foregoing list of conditions of adequacy, the relation of confirmation has been construed as a two-termed relation. There are of course three-termed relations of confirmation, e.g., the concept of confirming_F evidence expressed in the locution '*e* confirms_F *h* given *b*'. It is necessary to amend the foregoing conditions of adequacy so that they are capable of expressing characteristics of a three-termed relation of confirmation, e.g., the Transitivity Condition could be amended as follows:

CC8: Transitivity Condition: If *e* confirms *h* given *b*, and *h* confirms *k* given *b*, then *e* confirms *k* given *b*.

Condition CC8 is capable of expressing a characteristic of the concept of confirmation_F, whereas C8 as it stands is not so capable. The numbering of the amended conditions differs from the numbering of foregoing list of conditions only insofar as an additional 'C' is inserted in front in order to indicate that the amended condition is the correlative condition for three-termed concepts of confirmation. When I come to consider the various conditions of adequacy with respect to the concept of confirmation$_F$ a little later in this chapter I shall comment further on correlative conditions of adequacy.

2. Approaching explication by interpreting the concepts of confirmation probabilistically, and the consequence of this approach for assessing conditions of adequacy.

Some authors, as I have mentioned, attempt to state everything about an explicandum in conditions of adequacy, and then attempt to discover a suitable explicatum which yields results in agreement with the requirements expressed in the conditions of adequacy. Other authors require that a given concept must meet certain formal requirements, such as probabilistic requirements, and on the basis of such requirements they determine the characteristics of a given concept, i.e., they determine which conditions of adequacy hold for that concept and which conditions are violated. I shall begin to examine the relationship between concepts of confirmation and conditions of adequacy by considering the second approach to explication, i.e., I shall propose certain formal requirements for the concepts of confirmation distinguished in chapter two and determine which conditions of adequacy hold for and which conditions of adequacy are violated by the different concepts of confirmation thus determined.

In chapter two I distinguished several senses of 'to confirm' and drew particular attention to Carnap's concept of confirmation_F and Vincent's three concepts of confirmation — confirmation_A, confirmation_B, and confirmation_C. The formal requirements which readily suggest themselves and which have indeed been advanced by theorists are the following probabilistic ones:

$$\underline{e} \text{ confirms}_A\ \underline{h} \text{ iff } P(\underline{h},\underline{e}) > P(-\underline{h},\underline{e}) \qquad (19)$$

$$\underline{e} \text{ confirms}_B\ \underline{h} \text{ iff } P(\underline{h},\underline{e}) > P(\underline{h}) \qquad (20)$$

$$\underline{e} \text{ confirms}_C\ \underline{h} \text{ iff } P(\underline{h},\underline{e}) > P(-\underline{h},\underline{e}) \text{ and } P(\underline{h},\underline{e}) > P(\underline{h}) \qquad (21)$$

$$\underline{e} \text{ confirms}_F\ \underline{h} \text{ given } \underline{b} \text{ iff } P(\underline{h},\underline{e}\ \&\ \underline{b}) > P(\underline{h},\underline{b}) \qquad (22)$$

$$\underline{e} \text{ initially confirms}_F\ \underline{h} \text{ iff } P(\underline{h},\underline{e}) > P(\underline{h}) \qquad (23)$$

The probability function P used here satisfies the standard axioms and theorems of the Probability Calculus.[6] For any theory of credibility in which the metrical form of this concept is required to satisfy the axioms of the Probability Calculus,

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[6]These axioms and important theorems can be found in any standard work on probability, in which the arguments of the probability function are sentences, rather than events or properties. See, for example, Carnap, LFP, section 59, for axioms and theorems of importance; see section 62 for his remarks on axioms systems of various authors including Keynes, Jeffreys, and others.

the foregoing requirements on the five concepts of confirmation will be fulfilled. From the definition for conditional probability:

$$P(\underline{h},\underline{e}) = P(\underline{h} \,\&\, \underline{e})/P(\underline{e})$$ [7]

the probabilistic requirements set forth in (19) - (23) are equivalent to:

$\underline{e}$ confirms$_A$ $\underline{h}$ iff $P(\underline{h} \,\&\, \underline{e}) > P(-\underline{h} \,\&\, \underline{e})$ (19')

$\underline{e}$ confirms$_B$ $\underline{h}$ iff $P(\underline{h} \,\&\, \underline{e}) > P(h)\, P(\underline{e})$ (20')

$\underline{e}$ confirms$_C$ $\underline{h}$ iff $P(\underline{h} \,\&\, \underline{e}) > P(-\underline{h} \,\&\, \underline{e})$ and $P(\underline{h} \,\&\, \underline{e}) > P(\underline{h})\, P(\underline{e})$ (21')

$\underline{e}$ confirms$_F$ $\underline{h}$ given $\underline{b}$ iff $P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{b})\, P(\underline{b}) > P(\underline{e} \,\&\, \underline{b}) \times P(\underline{h} \,\&\, \underline{b})$ (22')

$\underline{e}$ initially confirms$_F$ $\underline{h}$ iff $P(\underline{h} \,\&\, \underline{e}) > P(\underline{h})\, P(\underline{e})$ (23')

Since we have at our disposal a considerable amount of information concerning probability statements, we are able to determine, with the help of this information, which logical conditions of adequacy hold for and which conditions of adequacy are violated by the five concepts of confirmation, given that these concepts meet the requirements expressed in (19) - (23). In order to prove that a given condition of adequacy holds for a given concept of confirmation, it is sufficient to show that the condition of adequacy necessarily holds when the concept of con-

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[7] The assumption is made here that the denominator is not zero.

firmation occurring in the condition of adequacy is uniformly interpreted in accordance with the appropriate probabilistic requirement, and each clause specifying whether some sentence is logically true or logically false, or enunciating some relation of entailment or logical compatibility is also uniformly interpreted as a statement of probability. The condition of adequacy C1, for example, can be shown to hold for the concept of confirmation$_A$ if '$\underline{e}$ entails $\underline{h}$' is interpreted probabilistically and '$\underline{e}$ confirms $\underline{h}$' is interpreted as '$P(\underline{h} \& \underline{e}) > P(-\underline{h} \& \underline{e})$' and the resulting conditional is shown to be true. In order to prove that a given condition of adequacy is violated for a certain concept of confirmation it is sufficient to show that there is a counter-instance of that condition of adequacy. In order to show that there is a counter-instance to a condition of adequacy it is necessary first of all to uniformly interpret probabilistically the concept of confirmation occurring in the condition of adequacy as well as the clauses which enunciate relations of entailment or logical compatibility and the clauses which specify sentences as being logically true or logically false. The condition of adequacy C5, for example, can be shown to be violated by the concept of confirmation$_A$ if '$\underline{h}$ entails $\underline{e}$' is interpreted probabilistically and '$\underline{e}$ confirms $\underline{h}$' is interpreted as '$P(\underline{h} \& \underline{e}) > P(-\underline{h} \& \underline{e})$' and the resulting conditional is shown to be false by a counterexample.

Evaluating conditions of adequacy with reference to the concepts of confirmation makes use of a number of important characteristics of probability functions. First of all, absolute probability statements are used rather than conditional probability statements. Secondly, use is made of the fact that the sum of the values of probability statements which canvass all possibilities of a given sentence (or sentences) is one, e.g., $P(\underline{x}) + P(-\underline{x}) = 1$, $P(\underline{x} \,\&\, \underline{y}) + P(\underline{x} \,\&\, -\underline{y}) + P(-\underline{x} \,\&\, \underline{y}) + P(-\underline{x} \,\&\, -\underline{y}) = 1$, etc. Thirdly, use is made of the fact that if a sentence is logically true, then the probability of that sentence is one; if a sentence is logically false, then the probability of that sentence is zero. Fourthly, use is made of the fact that relations of entailment and logical compatibility can be expressed using probability statements, e.g., $\underline{e}$ entails $\underline{h}$ iff $P(-\underline{h} \,\&\, \underline{e}) = 0$, and $\underline{e}$ is compatible with $\underline{h}$ iff $P(\underline{h} \,\&\, \underline{e}) \neq 0$. These are a number of the more important features of probability statements upon which the following discussion and proofs will depend.

I shall begin the examination of conditions of adequacy with reference to the concepts of confirmation_A, confirmation_B, confirmation_C, and initial confirmation_F. These concepts are all two-termed relations and so they will be examined with reference to the conditions of adequacy C1 - C9.1. Since the probabilistic requirement for the concept of confirmation_B is identical with the probabilistic requirement for the concept of initial confirmation$_F$, and this requirement determines which conditions of adequacy are violated by and which conditions hold for a given concept, it will not be necessary to separately examine the conditions of adequacy with reference to both concepts. The results which I obtain for the concept of confirmation$_B$ shall be interpreted as holding for the concept of initial confirmation$_F$ as well. In the proofs pertaining to these four concepts of confirmation I adopt the restriction that the evidence $\underline{e}$ is not logically false; hence $P(\underline{e}) \neq 0$.

The method of proof proceeds as follows.[8] First it is necessary to determine the number of sentence variables in the condition of adequacy which is to be examined, e.g., C1 has two sentence variables '$\underline{e}$' and '$\underline{h}$', C2.2 has three sentence variables '$\underline{e}$', '$\underline{h}$', and '$\underline{k}$'. Next it is necessary to assign values to the probability statements which canvass all possible combinations of the sentence variables involved in the condition of adequacy which is to be examined, e.g., when a condition of adequacy has the two sentence variables '$\underline{e}$' and '$\underline{h}$' it is necessary to assign values to the probability statements: '$P(\underline{h}\ \&\ \underline{e})$', '$P(\underline{h}\ \&\ -\underline{e})$', '$P(-\underline{h}\ \&\ \underline{e})$', and '$P(-\underline{h}\ \&\ -\underline{e})$'. I shall assign the values 'w', 'x', 'y', and 'z', respectively to

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[8]The method of proof used here is obtained from Rescher, "A Theory of Evidence."

the four probability statements just listed. This assignment of values in conjunction with the fact that the four probability statements can be used to express other probability statements involving '$\underline{e}$' and '$\underline{h}$' enables us to determine values for these other probability statements, e.g., $P(\underline{h}) = P(\underline{h} \ \& \ (\underline{e} \vee -\underline{e})) = P(\underline{h} \ \& \ \underline{e}) + P(\underline{h} \ \& \ -\underline{e}) = w + x$; $P(\underline{h} \vee \underline{e}) = P(-(-\underline{h} \ \& \ -\underline{e})) = 1 - P(-\underline{h} \ \& \ -\underline{e}) = 1 - z = w + x + y + z - z = w + x + y$. By the use of such simple techniques in conjunction with a suitable assignment of values to probability statements, the values of the probability statements in (19'), (20'), and (21') can also be obtained, e.g., $\underline{e}$ confirms$_A$ $\underline{h}$ iff $w > y$; $\underline{e}$ confirms$_B$ $\underline{h}$ iff $w > (w + x)(w + y)$. These values are used in the condition of adequacy whenever a relation of confirmation occurs in the condition being examined. I shall demonstrate, by way of example, that the entailment condition (C1) holds for the concept of confirmation$_A$. C1 asserts that if $\underline{e}$ entails $\underline{h}$, then $\underline{e}$ confirms $\underline{h}$. I shall demonstrate, in particular, that if $\underline{e}$ entails $\underline{h}$, then $\underline{e}$ confirms$_A$ $\underline{h}$. Now $\underline{e}$ entails $\underline{h}$ iff $P(-\underline{h} \ \& \ \underline{e}) = 0$ iff $y = 0$. And $\underline{e}$ confirms$_A$ $\underline{h}$ iff $P(\underline{h} \ \& \ \underline{e}) > P(-\underline{h} \ \& \ \underline{e})$ iff $w > y$. I have made the blanket assumption above that $\underline{e}$ is not logically false; hence $P(\underline{e}) = w + y \neq 0$. Given that $w + y \neq 0$, it follows that if $y = 0$ then $w > y$. Thus C1 holds for the concept of confirmation$_A$.

Using the same technique, it is easy to show, for example, that C_5 does not hold for the concept of confirmation$_A$. Condition

C5 asserts that if _h_ entails _e_, then _e_ confirms _h_. I shall demonstrate that the adequacy condition 'If _h_ entails _e_ then _e_ confirms$_A$ _h_' does not hold. Now _h_ entails _e_ iff P(_h_ & -_e_) = x = 0. Evidence _e_ confirms$_A$ _h_ iff w > y. From the fact that x = 0 it is impossible to determine anything concerning the relative values of w and y, e.g., it is possible for w = .2, x = 0, y = .5, and z = .3. Such an assignment allows P(_e_) = w + y = .7 ≠ 0 and so the restriction concerning _e_ is met. Such an assignment allows y > w and hence it is not the case that _e_ confirms$_A$ _h_. Thus it does not follow from the fact that _h_ entails _e_ that _e_ confirms$_A$ _h_. The condition of adequacy, C5, if it is slightly modified, does hold for the concept of confirmation$_B$, however. I determined above that _h_ entails _e_ iff x = 0. Now _e_ confirms$_B$ _h_ iff P(_h_ & _e_) > P(_h_) P(_e_) iff w > (w + x) (w + y). Given that x = 0 it is necessary to determine whether or not w > (w + x) (w + y), or, more simply, whether or not w > w (w + y), since x = 0. Now w > w (w + y) provided that w ≠ 0, w + y ≠ 0, and w + y ≠ 1. Since P(_e_) = w + y ≠ 0, the possibility that w + y = 0 is eliminated. Since w ≠ 0 iff w + x ≠ 0 (since x = 0) iff _h_ is logically false, and since w + y ≠ 0 iff _e_ is not logically false, the following modified condition holds for the concept of confirmation$_B$: If _h_ entails _e_, _h_ is not logically false, and _e_ is not logically true, then _e_ confirms _h_.

Certain conditions of adequacy are of considerable interest even though certain provisos are included in them. The restrictions which I have added to C5 above in order that it holds for the concept of confirmation$_B$ do not constitute a significant change in that condition of adequacy, for such restrictions pertain only to limiting cases of evidence sentences and hypotheses. When I consider modified conditions of adequacy, the modifications contained in such conditions of adequacy are only of this relatively insignificant kind. Logically false or true evidence sentences and hypotheses are of relatively little interest in scientific inquiry. I shall consider only the following four conditions of adequacy in a slightly modified form: C1, C1.3, C5, and C5.1. I shall indicate that a condition of adequacy has been modified by prefacing the number of a condition with an 'M'. The modified conditions are as follows:

MC1: If e entails h, and h is not logically true, then e confirms h.

MC1.3: If e is not logically true, then e confirms e.

MC5: If h entails e, h is not logically false, and e is not logically true, then e confirms h.

MC5.1: One term of a conjunction, provided it is not logically true, confirms the conjunction, provided the latter is not logically false: e confirms e & h, provided e is not logically true nor e & h logically false.

Using the method of proof illustrated above, each of the conditions of adequacy previously listed can be examined with respect to the four two-termed concepts of confirmation. It is obvious from the probabilistic requirement for Vincent's three concepts of confirmation that any condition of adequacy which is violated by either the concept of confirmation_A or the concept of confirmation_B will also be violated by the concept of confirmation_C, and that any condition of adequacy which holds for both of the former concepts of confirmation will hold also for the latter concept of confirmation. As I mentioned before, the conditions of adequacy which hold for and are violated by the concepts of confirmation_B and initial confirmation_F are identical.

The conditions of adequacy which hold for and are violated by the four two-termed concepts of confirmation are given in the following table.[9] Only condition of adequacy C1.2 is not considered, and reference to it is omitted here because it requires that the evidence sentence _e_ be false, in violation of the restriction which I have placed upon the evidence sentence _e_. I include C1.2 in the foregoing list because the correlative condition of adequacy for three-termed concepts of confirmation is not similarly eliminated.

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[9] See Appendix 1 for the proofs upon which Table F is based.

TABLE F

CONCEPT	VIOLATED BY	HOLDS FOR
$Confirmation_A$	C2, C2.3, C3, C3.1, C4, C5, C5.1, MC5, MC5.1, C6, C7, C8, C9,	C1, C1.1, MC1, C1.3, MC1.3, C2.1, C2.2, C2.4, C2.5, C3.2, C3.3, C3.4, C9.1,
$Confirmation_B$ and Initial $Confirmation_F$	C1.1, C1, C1.3, C2, C2.1, C2.3, C2.4, C2.5, C3, C3.1, C4, C5, C5.1, C8, C9,	MC1, C2.2, MC1.3, C3.2, C3.3, C3.4, MC5, MC5.1, C6, C7, C9.1,
$Confirmation_C$	C1, C1.1, C2, C2.1, C1.3, C2.3, C2.4, C2.5, C3, C3.1, C4, C5, C5.1, MC5, MC5.1, C6, C7, C8, C9,	MC1, MC1.3, C2.2, C3.2, C3.3, C3.4, C9.1.

The foregoing table illustrates the value of considering as conditions of adquacy not only very general conditions such as C2 and C3 but also a number of weaker conditions of adequacy derivable from such conditions. The general condition of adequacy may be violated by a given concept of confirmation whereas various weaker conditions, derivable from the general condition, hold for that concept of confirmation, e.g., C2 is violated by the concept of confirmation$_A$ but C2.1, C2.2, C2.4, and C2.5 hold for the concept of confirmation$_A$.

In order to consider the concept of confirmation$_F$ in relation to the conditions of adequacy, it is necessary, as I have previously pointed out, to amend the conditions of adequacy so that the concept of confirmation contained therein is a three-termed relation. Since it would be rather tedious and unnecessary to list the conditions of adequacy with this simple emendation, I shall not do so. All that is necessary in many instances is to read '*e* confirms *h*' as '*e* confirms *h* given *b*'. It is not quite that simple for all of the conditions of adequacy previously listed, however, since more variations can occur in the antecedent conditions of a condition of adequacy which is amended for three-termed concepts of confirmation. There are two correlative entailment conditions, for example, which must be considered:

CC1: If *e* entails *h*, then *e* confirms *h* given *b*.

CC1': If *e* & *b* entails *h*, then *e* confirms *h* given *b*.

The total number of conditions of adequacy which must be considered with respect to the concept of confirmation$_F$ is greater than the number of conditions which were considered with respect to two-termed concepts of confirmation. Other important correlative conditions of adequacy, besides CC1 and CC1', which require consideration include the following:

CC1.1: If h is logically true, then e confirms h given b.

CC1.1': If h & b is logically true, then e confirms h given b.

CC1.2: If e is logically false, then e confirms h given b.

CC1.2': If e & b is logically false, then e confirms h given b.

CC3.2: If e and h are logically incompatible, then e does not confirm h given b.

CC3.2': If e, h, and b are logically incompatible, then e does not confirm h given b.

CC3.4: If h is inconsistent, then e does not confirm h given b.

CC3.4': If h & b is inconsistent, then e does not confirm h given b.

CC5: If h entails e, then e confirms h given b.

CC5': If h & b entails e, then e confirms h given b.

As before, certain of the complete set of adequacy conditions, which require examination with respect to the concept of confirmation$_F$, hold only if they are suitably modified. The following modified conditions of adequacy hold for the concept of confirmation$_F$:

MCC1: If e entails h, and b entails neither h nor -e, then e confirms h given b.

MCC1': If *e* & *b* entails *h*, and *b* does not entail *h*, then *e* confirms *h* given *b*.

MCC1.3: If *b* does not entail *e*, *e* confirms *e* given *b*.

MCC5: If *h* entails *e*, and *b* entails neither *e* nor -*h*, then *e* confirms *h* given *b*.

MCC5': If *h* & *b* entails *e*, and *b* entails neither *e* nor -*h*, then *e* confirms *h* given *b*.

MCC5.1: Evidence *e* confirms *e* & *h* given *b*, provided *b* does not entail *e* & -*h*.

One general restriction which I shall impose upon the consideration of the correlative conditions of adequacy is that *e* & *b* is not logically false. This restriction eliminates CC1.2' as a possible condition of adequacy. The following table provides a concise summary of the conditions of adequacy which hold for and are violated by the concept of confirmation$_F$:[10]

TABLE G

VIOLATED BY	HOLDS FOR
CC1, CC1', CC1.1, CC1.1',	MCC1, MCC1', CC2.2, CC3.2,
CC1.2, CC2, CC2.1, CC1.3,	MCC1.3, CC3.2', CC3.3, CC3.4,
CC2.3, CC2.4, CC2.5, CC3,	CC3.4', MCC5, MCC5', MCC5.1,
CC3.1, CC4, CC5, CC5',	CC6, CC7, CC9.1.
CC5.1, CC8, CC9,	

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[10]See Appendix 1 for the proofs upon which Table G is based. Carnap considers several conditions of adequacy with respect to the concept of confirmation$_F$, *LFP*, section 87.

Since the general concept of confirmation$_F$ has a special case, i.e., the concept of initial confirmation$_F$, it should be the case that all conditions of adequacy which are violated by the concept of initial confirmation$_F$ are also violated by the general concept of confirmation$_F$, and also that all conditions which hold for the general concept hold also for the special case. A consultation of Tables F and G shows that this is so.

The argument in this section has been that once one adopts a probabilistic requirement for a concept of confirmation, the conditions of adequacy which hold for and are violated by that concept are easily determined. The conditions of adequacy which hold for a given concept of confirmation serve to characterize the concept of confirmation. The second approach to explication, in which one first attempts to state everything one can about the *explicandum* in conditions of adequacy, is much more problematic. I turn now to a consideration of this approach.

3. *A second approach to explication according to which intuitive assessments of conditions of adequacy are made for the different concepts of confirmation.*

I have dealt with one approach thus far to the problem of explicating concepts of confirmation, namely, interpreting a concept of confirmation in accordance with some formal probabilistic requirement and then determining which characteristics of a concept of confirmation are expressed in statements which I have called "logical conditions of adequacy." If the logical conditions of adequacy, which are determined by the formal requirement which is imposed upon a concept of confirmation, tally with one's intuitive understanding of that concept, that is some reason to consider the formal requirement which has been imposed as a plausible requirement. On the other hand, if any logical condition of adequacy, deducible from a formal requirement which is imposed upon a concept of confirmation, conflicts with one's intuitive understanding of that concept, that is some reason to reject as implausible the formal requirement which has been imposed.

Theorists have assessed the relative worth of formal requirements for a concept of confirmation and logical conditions of adequacy which characterize a concept of confirmation in varying ways. Carnap and Rescher, for example, attach a great deal of importance to the formal requirement for a concept of confirmation, the result of which is that all the conditions of adequacy for a given concept are thereby determined.[11] They apparently think that our intuitive judgments about conditions of adequacy are more unreliable than our judgments about formal requirements. Hempel, on the other hand (at least in 1945), laid

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[11] Rescher, "A Theory of Evidence;" Carnap, LFP, section 87.

great store upon conditions of adequacy which he thought were indispensible for his *explicandum*, but he later adopted a probabilistic requirement which yielded results which were incompatible with the conditions of adequacy he originally endorsed.[12] Hempel thus appears to have changed his mind about the relative worth of formal requirements and conditions of adequacy. Some authors, e.g., Mackie[13], have remarked that just as we can throw doubt on a condition of adequacy by accepting a formal requirement with which that condition of adequacy is incompatible, so we can throw doubt on a formal requirement by accepting a condition of adequacy with which that formal requirement is incompatible. The arbitrariness which afflicts this whole issue is lamentable.

The second approach to the problem of explication uses as a standard of explication for a concept of confirmation a set of logical conditions of adequacy which is obtained by reflecting on the concept. This set is usually taken to be indispensible for the *explicandum*, and every proposed *explicatum* must satisfy the set of adequacy conditions adopted. One of the difficulties connected with discussing this approach to explication, particu-

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[12] See Hempel, "Postscript (1964) on Confirmation," p. 50, for a discussion of this point.

[13] Mackie, "The Relevance Criterion of Confirmation," *BJPS*, vol. 20 (1969), p. 36.

larly in relation to the literature on the topic, is the failure of authors to adequately discuss the concept of confirmation of which conditions of adequacy are taken to express characteristics. Very few theorists have distinguished different weak senses of 'to confirm', and fewer still have discussed conditions of adequacy in any detail. Several theorists, notably Carnap, and, more recently, Marsha Hanen, have recognized that 'to confirm' can be used in the sense of 'to confirm_S' and also in the sense of 'to confirm_F' and have pointed out that conditions of adequacy for these two senses of 'to confirm' might well differ.[14] It is of course absolutely necessary also to recognize the possibility that a condition of adequacy might be intuitively plausible for one weak concept of confirmation but not for another. According to the second approach to explication, the logical conditions of adequacy which one adopts as indispensible for some concept of confirmation can eliminate a probabilistic requirement which one might want to impose on the concept, e.g., if one adopts as a characteristic of the concept of confirmation_B that if *e* entails *h* then *e* confirms_B *h*, then the probabilistic requirement (20) imposed upon the concept of confirmation_B will have to be

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[14] See Carnap, LFP, section 87, for his discussion of Hempel's conditions of adequacy in which he suggests that Hempel might have in mind the qualitative form of the concept of confirmation_S when discussing conditions of adequacy; and Hanen, "Confirmation and Adequacy Conditions," Philosophy of Science, vol. 38 (1971), pp. 361 - 368.

rejected. In order for this approach to explication to be successful it is essential for us to have grounds upon which one can defend a certain condition of adequacy as expressing a characteristic of a concept of confirmation.

In order to ensure that our intuitive judgments regarding the characteristics of a given concept of confirmation are not colored by the ambiguity and uncertainty of the locution 'to confirm' and its synonyms and cognates, I shall express concepts in the least ambiguous way possible here, namely, by expressing concepts of confirmation using the credibility notion which was introduced in chapter two. Important points can be quite overlooked if the terminology employed is fraught with ambiguity. In the following discussion of various conditions of adequacy with respect to concepts of confirmation I shall thus express concepts of confirmation in a way which will maximally facilitate their discussion and will also promote clarity. It would be erroneous to think that all difficulties involved in judging whether or not a concept has a particular characteristic are eliminated by expressing the concepts of confirmation using the credibility notion, for they are not. A further problem, for which no ready solution is in sight, results from the fact that our intuitive judgments of credibility relations are themselves frequently uncertain. There may be some clear-cut judgments with which almost everyone concurs, but there surely are many more with which few would agree. Thus, for example, one person might judge that if h is more credible on e than on t, and h entails k, then k is more credible on e than on t (that is, that C2.1 holds for $confirmation_B$), while another person might not agree at all. This very fact minimizes the value of the second approach to explication. Moreover, it is one thing to consider a condition of adequacy as plausible for some concept of confirmation, but it is quite another thing to demonstrate in a reasonably conclusive fashion that that condition of adequacy is indispensible for the concept in question.

I remarked earlier that a set of logical conditions of adequacy which express characteristics of a concept of confirmation must be consistent. This is such an obvious requirement that it hardly needs stating. It is well known that certain of the conditions of adequacy which were listed earlier are incompatible, e.g., the special consequence condition and the converse consequence condition are incompatible.[15] The two forementioned conditions yield the untoward consequence that any sentence confirms every other sentence — a consequence which is objectionable for every sense of 'confirms'. That this is so can be seen by reflecting on each of the following statements, all of which are highly implausible: (a) for every e and every h, h is

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[15]See Hempel, "Studies," p. 32, for a discussion of this point.

more credible than -h on e, (b) for every e and every h, h is more credible on e than on t, (c) for every e and every h, h is more credible than -h on e and h is more credible on e than on t, and (d) for every e, every h, and every b, h is more credible on b and e than on b alone. Howard Smokler proposes a condition to express our intuitions about the untoward character of the consequence that any sentence confirms every other sentence. This condition is called the "nonuniversalizability condition," which states that for every e there is some h such that e does not confirm h.[16] On intuitive grounds it is apparent that this condition holds for every concept of confirmation. If two conditions of adequacy have as a consequence the denial of the nonuniversalizability condition ("NC" for short), we shall be forced to reject at least one of those two conditions. Although this requirement does not figure in the examination of conditions of adequacy singly, it is important when examining two or more conditions of adequacy.

I shall not assess the plausibility of each adequacy condition for all the concepts of confirmation, for such a task would not only be very lengthy and thus inappropriate here but would also prove to be fruitless in many instances because of the lack of clear-cut judgments concerning credibility relations.

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[16]Smokler, "Conflicting Conceptions of Confirmation," pp. 301 - 302.

I propose, instead, to examine a number of conditions of adequacy which have figured rather prominently in recent literature on confirmation. I shall be particularly concerned to distinguish between various senses of 'to confirm' and note the consequences for conditions of adequacy which result. Since I am paying attention to the credibility relations which are expressed by the different concepts of confirmation and not with any other feature of the concepts, I shall not make separate reference in the sequel to the concept of initial confirmation$_F$ but shall consider it to be effectively covered along with the concept of confirmation$_B$.

4. Consideration of the intuitive plausibility of several important conditions of adequacy, namely, the entailment condition, the special consequence condition, and the converse consequence condition.

One of the conditions of adequacy which has been widely endorsed is the entailment condition. Hempel, for example, treats entailment as a special case of confirmation, for if *e* entails *h* then *e* conclusively confirms or verifies *h*, and thus *e* should surely be counted as weakly confirming *h*. Although this condition has appeared obvious to many theorists, one recent author, Marsha Hanen, argues that there are obvious counterexamples for one sense of 'to confirm'. She points out that if *e* entails *h* where *h* is a tautology and *e* is any observation sentence, then ". . . *h* already has maximal degree of confirmation$_S$ regardless of *e*, so if '*e* confirms *h*' is construed as . . . '*e* increases the degree of confirmation$_S$ of *h* over what it was previously' then clearly such confirmation has not taken place, and we have a counterexample to the entailment condition."[17] Hanen is pointing out here that if *h* is already firm, a further observation sentence *e* will not make *h* any firmer, since *h* cannot be made any firmer. The hypothesis *h* is *exhausted*, says Hanen, and is thus incapable of receiving a further increase in firmness. It is evident that Hanen is discussing the concept which I have identified as the concept of confirmation$_F$ and is saying that the entailment condition does not always hold for this concept.

A second counterexample to the entailment condition, which Hanen adduces just in case someone objects to speaking of confirming tautologies or to the sort of entailment notion involved in the first counterexample, occurs when *h* is a synthetic sentence such as '(Ex)(Fx)' which is entailed by prior evidence *b* such as 'Fa'. If we suppose that *e* is some statement which also entails '(Ex)(Fx)', such as 'Fb' (where $a \neq b$), then we have

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[17] Hanen, *op. cit.*, p. 366. Symbols have been changed in order to comply with conventions already adopted in this dissertation. I have also inserted the subscript 'S' in order to indicate the sense of confirmation quite obviously meant.

another case where ". . . *e* entails *h* without . . . increasing its degree of confirmation$_S$ over what it was on prior evidence."[18] That is, '(Ex)(Fx)' is exhausted by 'Fa', and new evidence 'Fb' cannot make '(Ex)(Fx)' any firmer than it is on 'Fa', since '(Ex)(Fx)' has been exhausted by 'Fa'. Hence 'Fb' cannot confirm$_F$ '(Ex)(Fx)' given 'Fa', even though 'Fb' entails '(Ex)(Fx)'.

Hanen's discussion of the entailment condition with respect to the concept of confirmation$_F$ highlights the fact that unqualified approval cannot be given to the entailment condition in connection with the concept of confirmation$_F$. Her counterexamples do not invalidate the entailment condition as a plausible characteristic of the concept of confirmation$_F$, for it quite obviously is a plausible characteristic of that concept. Her counterexamples indicate the qualifications which have to be added to the entailment condition in order for it to express a plausible characteristic of the concept of confirmation$_F$.

It is significant that when the probabilistic requirement which is stated in (22) is imposed upon the concept of confirmation$_F$, the entailment condition (CC1) does not hold for the concept of confirmation$_F$ unless it is suitably modified to exclude the cases when *h* is already exhausted on the basis of prior evidence *b* (MCC1), i.e., the cases when *h* is incapable of

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[18]*Ibid.*, pp. 366 - 367. Symbols have again been suitably altered and the subscript 'S' has again been added.

receiving a further increase in firmness from some new evidence item e. In this case the probabilistic requirement which was considered in section 6.2 above results in a modification to the entailment condition which tallies with the modification which is demanded by our critical assessment of the entailment condition for the concept of confirmation_F.

The entailment condition (C1), or some slight modification thereof, also seems plausible for the two-termed relations of confirmation. It is plausible to judge that if e entails h then h is more credible than -h on e, i.e., it is intuitively plausible to judge that C1 holds for the concept of confirmation_A. It is also plausible to judge that if e entails h then h is more credible on e than on t, i.e., C1 is plausible for the concept of confirmation_B, although the one proviso which one might want to insist upon in this case is that h is not logically true, for if h is logically true, even e, which entails h, will not be able to improve upon the credibility given to h by a logical truth. The latter judgment is based upon the intuitive appraisal that a logical truth gives maximal credibility to another logical truth. The foregoing appraisals of the entailment condition with respect to the concepts of confirmation_A and confirmation_B suggest that C1 also holds for the concept of confirmation_C, again provided h is not logically true. The assessments which I have given here of the entailment condition with respect to the two-termed concepts of confirmation follow exactly the consequences of interpreting these three concepts probabilistically, as I did in section 6.2 above. For C1 holds for the concept of confirmation_A, and MC1 holds for the concepts of confirmation_B and confirmation_C, although C1 does not without modification.[19] The modification in MC1 is that <u>h</u> is not logically true.

A second condition of adequacy which has been frequently discussed is the special consequence condition. It has frequently been discussed in connection with the converse consequence condition, and it is profitable for us to do so here as well. The entailment condition has been widely endorsed, as I mentioned earlier. The significant thing about these three conditions is that they are incompatible, i.e., they yield a result which violates NC. Anyone who endorses the entailment condition is required to choose between the two consequence conditions. One can, of course, like Carnap, reject both of the consequence conditions.[20] The following proof makes clear the incompatibility of the three mentioned conditions:[21]

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[19]See Table F in section 6.2 above.

[20]Carnap, <u>LFP</u>, section 87.

[21]See Hempel, "Studies," p. 32, where this incompatibility is first mentioned.

(i) e entails e — Repetition,

(ii) e confirms e — by C1 on (i),

(iii) h & e entails e — Conjunction Elimination,

(iv) e confirms h & e — by C4 on (ii) and (iii),

(v) h & e entails h — Conjunction Elimination,

hence, (vi) e confirms h — by C2.1 on (iii) and (iv).

Since h and e are any sentences whatsoever, line (vi) violates NC. Several comments are necessary concerning the foregoing proof. In the first place, a two-termed relation of confirmation is used throughout. The same result, however, is obtainable using a three-termed relation of confirmation along with the correlative conditions of adequacy CC1, CC4, and CC2.1. In the second place, the foregoing proof is of interest in connection with the concepts of confirmation_B, confirmation_C, and confirmation_F, even though C1 does not hold for the concepts of confirmation_B and confirmation_C if it is unqualified and CC1 does not hold for the concept of confirmation_F if it is unqualified. In connection with the two-termed concepts we must assume that e is not logically true, for if e is not logically true then e confirms_B e and e confirms_C e, and line (ii) can be allowed to stand. In connection with the concept of confirmation_F we must make the assumption that b does not entail e, for if b does not entail e then e confirms_F e given b, and line (ii) can again be allowed to stand.

Both consequence conditions represent beliefs which are widely held among theorists. Some have argued that an observational consequence of a theory T which confirms T also confirms a more general theory from which T is derived. Ernest Nagel, for example, claims that the direct evidence for the Keplerian laws, the law for the period of the pendulum, and the law for freely falling bodies, is evidence for the Newtonian laws of motion from which all of the foregoing laws are deducible (taken together with various assumptions).[22] Others have claimed that an observational finding which confirms a theory T also confirms those theories deduced from T, e.g., the orbits of the planets have been taken to confirm gravitational theory and as a result the orbits of the comets. The latter laws are derivable from gravitational theory taken together with certain assumptions. Thus the question: "Which, if any, of the consequence conditions is a plausible condition warranting endorsement?" is an interesting one.

There is a second significant relationship involving C4 which is a strong argument against its plausibility, namely, its incompatibility with the entailment condition C1. This is shown in the following proof:

(i) e entails e v h - Disjunction Introduction,

(ii) e confirms e v h - by C1 on (i),

(iii) h entails e v h - Disjunction Introduction,

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[22]Nagel, The Structure of Science, p. 65.

hence, (iv) *e* confirms *h* - by C4 on (ii) and (iii). Since *h* and *e* are any sentences whatsoever, line (iv) violates NC. Again this proof could be carried out with a three-termed relation of confirmation and the correlative conditions of adequacy CC1 and CC4. Moreover, this result is significant for the concepts of confirmation_B and confirmation_C only if the assumption is made that *e* v *h* is not logically true, for only under this assumption does line (ii) hold for these concepts. In addition, this result is significant for the concept of confirmation_F only if the assumption is made that *b* does not entail *e* v *h*, for only under this assumption does line (ii) hold for the concept of confirmation_F. In view of the strong general accord given to the entailment condition, or a slightly modified form thereof, there is a strong reason to reject the converse consequence condition (C4 and CC4). This conclusion is in close agreement with consequences drawn from interpreting the concepts of confirmation probabilistically as I have done in section 6.2 above, for the converse consequence condition is violated by all concepts of confirmation and the entailment condition, or a slightly modified entailment condition, holds for each concept.[23] The question now requiring examination is whether the special consequence condition is a plausible condition of adequacy for any concept,

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[23] See Table F in section 6.2 above.

or concepts, of confirmation.

Hempel argues in favor of the special consequence condition by contending that any logical consequence of a group of hypotheses ". . . is but an assertion of all or part of the combined content of the original hypotheses and has therefore to be regarded as confirmed by any evidence which confirms all of the latter."[24] Mackie endorses C2.1 with the remark: "This condition is highly plausible, for in general the point of confirming hypotheses is to be able to draw predictions and other inferences from them, and we want some reliance on these inferences to be justified by whatever evidence has confirmed the hypothesis."[25] Thus C2.1 has had several able defenders. In order to examine the plausibility of C2.1 for the concepts of confirmation I am discussing, I shall write C2.1 in such a way that the concept of confirmation occurring in it is variously expressed using the notion of credibility developed earlier in this dissertation. The interpretations of C2.1 which need to be considered are as follows:

C2.1A: If _h_ is more credible than -_h_ on _e_, and _h_ entails _k_, then _k_ is more credible than -_k_ on _e_.

C2.1B: If _h_ is more credible on _e_ than on _t_, and _h_

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[24]Hempel, "Studies," p. 31.

[25]Mackie, "The Relevance Criterion of Confirmation," pp. 35 - 36.

entails *k*, then *k* is more credible on *e* than on *t*.

C2.1C: If *h* is more credible than -*h* on *e* and *h* is more credible on *e* than on *t*, and *h* entails *k*, then *k* is more credible than -*k* on *e* and *k* is more credible on *e* than on *t*.

CC2.1F: If *h* is more credible on *e* & *b* than on *b* alone, and *h* entails *k*, then *k* is more credible on *e* and *b* than on *b* alone.

The letters which follow the number of the condition of adequacy indicate the sense in which "confirms" is interpreted in that condition of adequacy.

There are several considerations which cast implausibility upon C2.1B in my estimation. These consist of examples which depend upon intuitive credibility assessments, and although they are not definitive, they are of some value in discrediting C2.1B. It is plausible to judge that 'Bozo is a dalmatian and Newton liked poetry' (e_1 & h_1) is more credible on the information 'Bozo is a dalmatian' (e_1) than on a logical truth alone, for e_1 here conclusively establishes half of e_1 & h_1 and thus seems to me to render e_1 & h_1 more credible than no information at all.[26] A

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[26] I have drawn this example from Vincent, "Corroboration and Probability," pp. 200 - 201. He maintains, however, that the information that Bozo is a dalmatian does not confirm the hypothesis 'Bozo is a dalmatian and Newton liked poetry'! He might be using a sense of 'confirm' other than 'confirm_B' though. If not, this shows how intuitions can differ widely.

logical consequence of $\underline{e}_1$ & $\underline{h}_1$ is of course $\underline{h}_1$, and according to C2.1B $\underline{h}_1$ should be more credible on $\underline{e}_1$ than on a logical truth. I submit, however, that 'Newton liked poetry' ($\underline{h}_1$) is not more credible on 'Bozo is a dalmatian' ($\underline{e}_1$) than it is on a logical truth alone (although it might be as credible on the factual information as it is on the logical information), and suggest that C2.1B is therefore false. A second example suggesting the implausibility of C2.1B is as follows. Let there be some h and some e such that h is more credible on e than on t. Since h entails t, it follows by C2.1B that t is more credible on e than on t. It is implausible to maintain, it seems to me, that a logical truth might be more credible on a factual truth than on another logical truth (or on itself), although it is not implausible to maintain, in my estimation, that a logical truth is as credible on a factual statment as on a logical truth. This example also suggests that C2.1B is false.

The two examples which I have used to discredit C2.1B can be used to discredit CC2.1F. This is so by virtue of the fact that the credibility relation which appears in C2.1B is a special case of the credibility relation which appears in CC2.1F, i.e., 'h is more credible on e than on t' is a special case of 'h is more credible on e & b than on b alone'. We can select an appropriate sentence representing the prior evidence b which will not affect the examples used against C2.1B above, and thus construct examples which suggest the falsity of CC2.1F. The assignment b = t will do, and is quite legitimate. Again, these examples depend for their efficacy upon the credibility assessments appearing in them, and thus are not as definitive as one might wish them to be. They do suggest, nevertheless, that CC2.1F is implausible.

Someone might be tempted to think that the falsity of C2.1B entails the falsity of C2.1C, since the credibility relations 'h is more credible on e than on t' and 'k is more credible on e than on t', which appear in C2.1B, appear also in C2.1C. This is not so, however, and can be easily shown if we first symbolize C2.1B and C2.1C in the following way. Let 'H' stand for 'h is more credible on e than on t', 'E' for 'h entails k', 'K' for 'k is more credible on e than on t', 'M' for 'h is more credible than -h on e', and 'N' for 'k is more credible than -k on e'. Now C2.1B and C2.1C can be symbolically expressed as follows:

C2.1B: $((H \& E) \supset K)$,

C2.1C: $((M \& H \& E) \supset (N \& K))$.

That the falsity of C2.1B does not entail the falsity of C2.1C is shown by the fact that we can assign the value T to 'H', 'E', and 'N', and the value F to 'K' and 'M', so that '$((H \& E) \supset K)$' has the value F and '$((M \& H \& E) \supset (N \& K))$' has the value T.

It is difficult to say anything definite about the plausibility of C2.1A and C2.1C. The first example which I used to discredit C2.1B above does not yield any definite result against C2.1A. The second example definitely does not discredit C2.1A. Let there be some h and some e such that h is more credible than -h on e. Since h entails t, it follows by C2.1A that t is more credible than -t on e. The latter judgment seems very plausible to me, for factual information e, which entails a logical truth, certainly makes that logical truth more credible than a logical falsehood (-t). Hence, the second example does not obviously discredit C2.1A.

The general problem encountered here of being unable to make a definite judgment about the validity or invalidity of conditions of adequacy for concepts of confirmation results from the general lack of clear intuitive assessments of various credibility statements. Unless we discover an example which can serve to discredit a condition of adequacy for a given concept of confirmation, our assessment of a condition of adequacy is likely to be uncertain. The value of the second approach to the matter of explication is greatly diminished by this difficulty. Before reconsidering the first approach to the problem of explication in connection with the concept of confirmation$_F$, I shall try to intuitively evaluate one more condition of adequacy.

5. Consideration of the intuitive plausibility of the equivalence condition for hypotheses.

Another condition of adequacy which has been frequently discussed is the equivalence condition for hypotheses (C2.2). It sometimes has been discussed together with the equivalence condition for evidence sentences (C9.1) because of the obvious similarity between these conditions. I shall devote my attention to C2.2, however, since it has figured more prominently in discussions than C9.1.

The idea behind C2.2 is that logically equivalent sentences can be substituted for one another in the argument place for hypotheses in any confirmation relation. One argument which has been advanced in favor of C2.2 is that it makes a confirmation relation independent of the way in which a hypothesis is formulated. It has seemed absurd to various theorists to think that a confirmation relation could be dependent upon the form in which a hypothesis is rendered.[27] Such theorists have contended that it is the content, not the form, of a hypothesis which is significant, and this can be expressed in many ways. Thus if the evidence report 'Pa & Qa' confirms '(x)(Px ⊃ Qx)', for example, then that evidence report should also be construed as confirming '(x)(-Qx ⊃ -Px)'. A second argument, which is more important

[27]Hempel adduces this argument in "Studies," p. 13.

than resort to a feeling of absurdity, is that a definition of confirmation would have to do justice to the way in which hypotheses are treated in scientific contexts such as explanation. In many contexts of explanation a hypothesis serves as a premise in an argument from which a sentence which describes an event to be explained is deduced, and in such contexts ". . . a scientist will feel free, in any theoretical reasoning involving certain hypotheses, to use the latter in whichever of their equivalent formulations are most convenient for the development of his conclusions."[28] These considerations have been endorsed by theorists other than Hempel, but a number of objections to C2.2 have also been made.[29]

The most common objection to C2.2 is that it, in conjunction with Nicod's criterion of confirmation, generates the paradoxes of confirmation.[30] I demonstrated in section 3.3 above that the paradoxes of confirmation are generated by C2.2 and Nicod's criterion. The problem for confirmation theory presented by the paradoxes of confirmation is a very complex one

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[28] _Ibid._, p. 13.

[29] The following writers all endorse C2.2 for the reasons given, or similar reasons: R. G. Swinburne, "The Paradoxes of Confirmation — A Survey," p. 321; Carnap, _LFP_, p. 474; and John R. Wallace, "Goodman, Logic, Induction," _The Journal of Philosophy_, vol. 63 (1966), pp. 311 - 312.

[30] See, for example, Smokler, "The Equivalence Condition," pp. 302 - 303.

concerning which there has been much discussion. I have not addressed myself to it in this dissertation because I have been concerned to grapple with the more basic problem of distinguishing different senses of 'to confirm' and the problem of approaches to explication. The subject of the paradoxes is probably large enough to constitute a dissertation topic in itself, so I shall not attempt to deal with it here. A number of competent theorists, beginning with Hempel, have argued that the so-called paradoxes of confirmation only appear on first sight to be irrelevant, and that when a closer scrutiny is made of those evidence reports which the criterion assesses as confirmatory but which are intuitively assessed as irrelevant it becomes quite obvious that the said reports are in fact confirmatory.[31] Thus there might not be, in fact, any problem for the equivalence condition by virtue of its entailing, along with Nicod's criterion, the paradoxes of confirmation.

A more discriminating position toward C2.2 is taken by Nelson Goodman who says that C2.2 is plausible only for some senses of confirmation but not for all of them. He says that there is a primary sense of confirmation for which C2.2 holds, but there is also a secondary sense of confirmation for which

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[31] Hempel's argument was first presented in "Studies," pp. 14 - 20. For a good survey of similar views, and a comprehensive survey (including bibliography) of the topic of the paradoxes, see Swinburne, "The Paradoxes of Confirmation — A Survey."

C2.2 does not hold.[32] It is not clear what sense of confirmation Goodman means by the term "primary sense of confirmation," although it could be a reference to Hempel's study of confirmation, but the secondary sense of confirmation to which he refers is Scheffler's concept of "selective confirmation," which I briefly commented on in section 4.1 above. I shall demonstrate that C2.2 does not hold for the concept of selective confirmation. I shall repeat Scheffler's criterion of selective confirmation, which is as follows:[33]

> $\underline{e}$ selectively confirms $\underline{h}$ iff $\underline{e}$ confirms (in Hempel's sense) but not the contrary of $\underline{h}$.

The hypothesis:

$\underline{h}_1$: $(x)(Rx \supset Bx)$

has as its contrary (according to Scheffler's usage) the hypothesis:

$\underline{h}_1'$: $(x)(Rx \supset -Bx)$.

The evidence sentence:

$\underline{e}_1$: Pa & Qa

confirms (in Hempel's sense, i.e., according to his definition)

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[32] Nelson Goodman, "Comments," The Journal of Philosophy, vol. 63 (1966), p. 331.

[33] Scheffler, The Anatomy of Inquiry, p. 293. Scheffler adopts the Aristotelian view of contrariety according to which a universal conditional has only one contrary. See section 4.1 above for a discussion of this point.

$\underline{h}_1$ but not $\underline{h}_1$'. In fact, $\underline{e}_1$ disconfirms (in Hempel's sense) $\underline{h}_1$'. Thus $\underline{e}_1$ selectively confirms $\underline{h}_1$. Now $\underline{h}_1$ is logically equivalent to the hypothesis:

$\underline{h}_2$: $(x)(-Bx \supset -Rx)$

which has as its contrary (according to Scheffler's usage) the hypothesis:

$\underline{h}_2$': $(x)(-Bx \supset Rx)$.

Since $\underline{e}_1$ confirms (in Hempel's sense) $\underline{h}_2$ and also $\underline{h}_2$', $\underline{e}_1$ does not selectively confirm $\underline{h}_2$. Thus we have the situation in which $\underline{e}_1$ selectively confirms $\underline{h}_1$ and $\underline{h}_1$ is logically equivalent to $\underline{h}_2$ but $\underline{h}_2$ does not selectively confirm $\underline{h}_2$.

The foregoing result for the concept of selective confirmation demonstrates that C2.2 is violated for at least one concept of confirmation. I suggested in section 4.1 above that Scheffler's concept of selective confirmation might be a special case of the concept which is expressed in (12):

$\underline{h}$ is more credible than $\underline{h}$' relative to $\underline{e}$, where $\underline{h}$ and $\underline{h}$' are logically incompatible. (12)

Moreover, it is noteworthy, in my estimation, that the concept expressed in (12) violates condition of adequacy C2.2 when a plausible probabilistic requirement is imposed upon (12). The probabilistic requirement which I have in mind is highly analogous to the probabilistic requirements imposed in section 6.2 above upon the concepts of confirmation$_A$, confirmation$_B$, confirmation$_C$, confirmation$_F$, and initial confirmation$_F$, and is as follows:

(12) iff $P(\underline{h},\underline{e}) > P(\underline{h}',\underline{e})$ and $\underline{h}$ and $\underline{h}'$ are logically incompatible. (24)

From the definition for conditional probability and the fact that $\underline{h}$ and $\underline{h}'$ are logically incompatible iff $P(\underline{h}\ \&\ \underline{h}') = 0$, (24) is equivalent to the following statement:

(12) iff $P(\underline{h}\ \&\ \underline{e}) > P(\underline{h}'\ \&\ \underline{e})$ and $P(\underline{h}\ \&\ \underline{h}') = 0$. (24')

The concept expressed in (12), when it is interpreted according to the requirement of (24'), violates condition of adequacy C2.2.[34] Scheffler's conclusion concerning his concept of selective confirmation is not surprising in view of the failure of (12), when given a plausible probabilistic interpretation, to hold for the condition of adequacy C2.2.

A question which is germane to my study which requires further comment is: "Is the equivalence condition for hypotheses a plausible characteristic of any of the concepts of confirmation I have discussed in this dissertation?" If we again write C2.2 in such a way that the concept of confirmation occurring in it is variously expressed using the credibility notion developed earlier, we are in a quite favorable position to assess the validity of C2.2, intuitively speaking, with respect to the concepts of confirmation being discussed. The relevant inter-

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[34] See Appendix 2 for the proof of this.

pretations of C2.2 are as follows:

C2.2A: If h is more credible than -h on e, and h is logically equivalent to k, then k is more credible than -k on e.

C2.2B: If h is more credible relative to e than to t, and h is logically equivalent to k, then k is more credible relative to e than to t.

C2.2C: If h is more credible than -h on e and h is more credible on e than on t, and h is logically equivalent to k, then k is more credible than -k on e and k is more credible on e than on t.

CC2.2F: If h is more credible on e & b than on b alone, and h is logically equivalent to k, then k is more credible on e & b than on b alone.

Each of the foregoing statements strikes me as being very plausible. Consider C2.2B, as one example. It in effect states that if e provides stronger grounds than does a tautology for rationally believing h to be true, and h is logically equivalent to k, then e provides stronger grounds than does a tautology for rationally believing k to be true. The latter judgment, in my estimation, is the quintessence of rationality and plausibility. It is in fact difficult to conceive of an example which would even threaten to discredit any one of the four interpretations of C2.2 given above. Even the so-called paradoxes of confirmation look relatively harmless! One of the paradoxical consequences of Nicod's criterion and the equivalence condition for hypotheses, for example, is that the report '-Pa & -Qa' confirms '(x)(Px ⊃ Qx)'. This comes about because '-Pa & -Qa' confirms '(x)(-Qx ⊃ -Px)' on Nicod's criterion, and '(x)(-Qx ⊃ -Px)' is logically equivalent to '(x)(Px ⊃ Qx)'. The intuitive judgment which has been thought to make this result paradoxical has of course been that -Pa & -Qa' is irrelevant or neutral to '(x)(Px ⊃ Qx)'. This example is no real problem for the four interpretations of C2.2, however, and I shall consider it with respect to C2.2A to illustrate my point. Supposing that it is plausible to judge that '(x)(-Qx ⊃ -Px)' (h) is more credible than '(Ex)(Px & -Qx)' (-h) on the report '-Pa & -Qa', and given that '(x)(-Qx ⊃ -Px)'(h) is logically equivalent to '(x)(Px ⊃ Qx)' (k), it follows by C2.2A that '(x)(Px ⊃ Qx)' (k) is more credible than '(Ex)(Px & -Qx)' (-k) on '-Pa & -Qa'. The credibility statement comprising the last clause,''(x)(Px ⊃ Qx)' is more credible than '(Ex)(Px & -Qx)' on '-Pa & -Qa'', does not strike me as implausible at all, and so there are no grounds for conceding that '-Pa & -Qa' paradoxically confirms_A '(x)(Px ⊃ Qx)'. Carrying through the same example with respect to the other interpretations of the equivalence condition would reveal, I believe, that the paradoxes of confirmation do not present an insuperable problem for the equivalence condition for hypotheses. I am thus inclined to agree with Hempel that the reports which are thought to be paradoxically confirmatory only appear on first sight to be irrelevant, and that when a closer scrutiny is made of the said evidence reports it becomes quite evident that the reports are in fact confirmatory. I do not presume, however, to have even begun to consider the problem of the paradoxes in the brief comments I have made here.

In considering the second approach to the problem of explication, I have considered only a select number of conditions of adequacy, and have attempted to determine on intuitive grounds whether or not the given conditions of adequacy are plausible for the various concepts of confirmation. One of the reasons for limiting myself to only four conditions is that the four which have been examined comprise those which have figured most frequently in the literature on conditions of adequacy. A second reason for not having considered all of the conditions of adequacy in relation to the concepts of confirmation which have been considered in this dissertation is the sheer number of statements which would require examination and intuitive assessment — a total of 111.[35] Moreover, examining all these statements would not only be an onerous task, but also an unpromising one in view of the inconclusiveness of many conditions of adequacy when an

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[35] Twenty-six conditions of adequacy would require examination with respect to the three two-termed concepts of confirmation, and thirty-three correlative conditions of adequacy would require examination with respect to the one three-termed concept of confirmation.

attempt is made to give an intuitive assessment.

The uncertainty and indecisiveness of the second approach to the matter of explication, due largely to the paucity of clear and defensible credibility assessments, is a general problem for this approach for which no solution is in immediate sight. The credibility assessments which I have made in the foregoing three sections have seemed plausible to me, but they are of course open to debate. I have indicated in several places that other authors do not share my convictions. For the second approach to explicating concepts of confirmation to be successful, it is necessary to investigate more thoroughly the concept of credibility. The first approach to the matter of explication has the decided advantage of yielding decisive results. Of course assigning a probability expression to a concept of confirmation as a means of interpreting that concept of confirmation requires considerable defense. In the chapter to follow I wish to consider the explication of one concept of confirmation by the demonstrably more effective and decisive method.

CHAPTER SEVEN

EXPLICATING THE CONCEPT OF CONFIRMATION$_F$

1. Comments on the characteristics of the concept of confirmation$_F$ when required to meet the probabilistic interpretation considered earlier.

In the previous chapter I have discussed two approaches to the problem of explication. This has revealed the following important results. The second approach to the problem of explication in which intuitively plausible conditions of adequacy are suggested as the standard against which any explication is measured, has proved to be rather indecisive insofar as judgments pertaining to conditions of adequacy are frequently uncertain. The first approach to the problem of explication, on the other hand, in which a formal probabilistic requirement is imposed upon a concept of confirmation, enables one to obtain definite judgments concerning the validity of conditions of adequacy for different concepts of confirmation. This approach to explication, however, requires a defence for the formal requirement selected for a given concept of confirmation.

In chapter five I have argued that the one weak concept among the weak concepts which I have distinguished which is most important to the theory of confirmation generally is the concept of confirmation$_F$, i.e., I have argued that the concept which it is of greatest importance to explicate and toward which most attention should be directed is the concept of confirmation$_F$. In this concluding chapter I shall consider the plausibility of approaching the problem of the explication of the concept of confirmation$_F$ using the formal probabilistic requirement offered in the previous chapter. I shall also consider various specific arguments pertaining to the probabilistic requirement and try to informally assess the plausibility of explicating the concept of confirmation$_F$ by using the probabilistic measure expressed in (22).

The probabilistic requirement which was imposed upon the concept of confirmation$_F$ in the previous chapter was as follows:

'$\underline{e}$ confirms$_F$ $\underline{h}$ given $\underline{b}$ iff $P(\underline{h},\underline{e}\ \&\ \underline{b}) > P(\underline{h},\underline{b})$'. (22)

This requirement on the concept of confirmation$_F$ determines which conditions of adequacy discussed in the previous chapter hold for the concept of confirmation$_F$ and which conditions of adequacy are violated by it. These results were summarized in Table G in section 6.2. On the basis of (22) a number of conditions of adequacy which have figured quite prominently in discussions of confirmation are violated. The special consequence condition (CC2.1) and the converse consequence condition (CC4) are both violated. The consistency condition (CC3) and the special consistency condition (CC3.1) are violated, as well as a number of other conditions which are not quite as well known as the ones I have mentioned. On the other hand, the equivalence conditions (CC2.2 and CC9.1) hold, as might be expected, as well as two slightly modified entailment conditions (MCC1 and MCC1'). An interesting condition which does hold is CC6 according to which if e confirms h given b then h confirms e given b. This condition holds promise for use in extending the application of an explication which might be initially proposed for a relation between non-general evidence sentences and general hypotheses.

Several other interesting conditions which hold for the concept of confirmation$_F$ on the basis of (22) are two slightly modified converse entailment conditions (MCC5 and MCC5'). Condition MCC5 asserts that if h entails e, and b entails neither e nor -h, then e confirms h given b. Criterion MCC5' asserts that if h & b entails e, and b entails neither e nor -h, then e confirms h given b. Condition MCC5' expresses a characteristic of the concept of confirmation$_F$ which corresponds closely to a principle which is frequently thought to be a sufficient condition of confirmation in some sense or other (usually unspecified but perhaps with the concept of confirmation$_F$ in mind). Condition MCC5' expresses what is sometimes called the prediction criterion of confirmation. The basic idea behind it is that if a predicted event, which is explained by some hypothesis h in conjunction with background or prior information b but not by the background or prior information b by itself, is found to occur, then that event confirms the hypothesis *h* relative to the background or prior information *b*. The theory of explanation which is often endorsed in this context is one based upon the strict covering-law model of explanation according to which an event is explained if it is deduced from lawlike statements (laws) in conjunction with a statement of initial conditions. The condition of adequacy MCC5' states that if an evidence sentence *e*, which describes some observable event, is deducible from a hypothesis *h* in conjunction with a statement of background or prior information *b*, but not from *b* alone, then *e* confirms_F *h* given *b*. An added proviso in MCC5' is that -*h* is not entailed by *b*, for if *b* does entail -*h*, then -*h* has already been made firm by *b* (and hence *h* has already been made infirm by *b*) so *e* cannot affect the status of *h*. Condition of adequacy MCC5 expresses a sufficient condition of confirmation_F very similar so that expressed by MCC5', except that according to MCC5 if *h* by itself entails *h*, where *b* entails neither *e* nor -*h*, then *e* confirms *h* given *b*.

Conditions MCC5 and MCC5' express only two of a large number of sufficient conditions for confirmation_F where these sufficient conditions consist of various relations of entailment between *h*, *e*, and *b*, similar to the relations of entailment found in MCC5 and MCC5'. Other sufficient conditions of confirmation_F include MCC1, MCC1', and the following examples:

If _e_ & _b_ entails _h_, and _b_ entails neither -_e_ nor _h_, then _e_ confirms$_F$ _h_ given _b_.

If _b_ entails _e_ & _h_ and _b_ entails neither -_e_ nor _h_, then _e_ confirms$_F$ _h_ given _b_.

In fact, the number of different sufficient conditions of confirmation$_F$, based on simple relations of entailment between _h_, _e_, and _b_, which can be easily constructed is fifteen thousand.[1] The conditions MCC1, MCC1', MCC5, and MCC5' are thus only four of a large number of sufficient conditions of confirmation$_F$ based on relations of entailment. They are important nevertheless and have been practically the only sufficient conditions of confirmation$_F$ which have received attention from confirmation theorists.

It might be suggested that it is not necessary to explicate the concept of confirmation$_F$ by making use of a probabilistic measure, such as the measure found in (22) perhaps, and that an adequate _explicatum_ could be constructed based on simple logical relationships such as entailment between _h_, _e_, and _b_. It must be acknowledged that among the _conceivable_ criteria for the concept of confirmation$_F$ are criteria which depend only upon simple logical relationships between _h_, _e_, and _b_. Carnap, for example, discusses the possibility of considering the following

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[1]See Appendix 3 for an explanation of how this figure is obtained.

proposal as an explication of the concept of $confirmation_F$:

a) *h* & *e* & *b* is not logically false,

b) *b* & -*e* & -*h* is not logically false, and

c) either *h* & *b* entails *e* or *e* & *b* entails *h* or both.[2]

Carnap points out that whenever the three foregoing conditions are met then *e* $confirms_F$ *h* given *b*. The important question, however, is whether or not the three foregoing conditions constitute a necessary condition of $confirmation_F$. It is easy to show that these conditions do not, for if *e* is the sentence 'Pa & Qa', *b* is the tautology '*t*', and *h* is the simple conditional law '(x)(Px ⊃ Qx)', it is not the case either that *h* entails *e* or that *e* entails *h*. Yet *e* would normally be regarded as being initially $confirming_F$ evidence for *h*.[3]

The general problem with suggested criteria which are based upon simple logical relationships between *h*, *e*, and *b* is that such criteria might well be sufficient conditions of $confirmation_F$, but they are not likely to be necessary conditions of $confirmation_F$. Such proposed criteria are just not sensitive enough to take account of all the different kinds of reports generally taken to be $confirming_F$ evidence for a hypothesis given the appropriate prior evidence. The inadequacy of this

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[2]Carnap, LFP, p. 465.

[3]Carnap, LFP, p. 466, for one, thinks so.

simple approach forces one to consider criteria which are capable of being used in connection with many different kinds of putative evidence. The following kind of example shows the inadequacy of criteria which are based upon simple logical relationships between _h_, _e_, and _b_. Suppose we are testing the law '(x)(Px ⊃ Qx)' and are interested in determining which of a variety of evidence reports are confirmatory$_F$. Assume that we are at the earliest stage in the testing of the law and that _b_ is a tautology. Now if we obtain the evidence report 'Pa & Qa' which we might intuitively judge to be confirming$_F$ evidence for the law given the null prior evidence, we can easily enough construct a criterion based on simple logical relationships between _h_, _e_, and _b_ which yields the result that 'Pa & Qa' confirms$_F$ '(x)(Px ⊃ Qx)' given no previous evidence. However, a criterion which was based on entailment relationships between _h_, _e_, and _b_ would not be capable of determining whether or not the evidence report 'Pa & Sa' was confirming$_F$ evidence for the law in question, where the prior evidence consists of the information that 90% of all things which are S are also Q. Such a proposal for confirming$_F$ evidence cannot be ignored, it seems, and a criterion other than one which is based on entailment relationships between _h_, _e_, and _b_ is needed. Similar examples could be constructed, perhaps involving complex hypotheses and complex evidence sentences, which would also indicate that a definition which is to express a necessary and sufficient criterion of confirmation$_F$ must be sought in some approach other than that which relies on simple entailment relations between h, e, and b. The most obvious direction in which to turn is that of the Probability Calculus, which is capable of handling more complex evidence. The criterion of confirmation expressed in (22) thus seems more likely to be adequate than some proposal based on entailment relations.

2. The relevance criterion and comments on its endorsement by Popper and Carnap.

The criterion of confirmation$_F$ expressed in (22) is called by some writers, e.g., Mackie, C. A. Hooker and D. Stove, 'the relevance criterion of confirmation'.[4] Hooker and Stove trace this criterion back to J. M. Keynes who spoke of the 'favorable relevance' of a hypothesis to an observation sentence. Writers sometimes make use of the fact that '$P(h,e \,\&\, b) > P(h,b)$' iff '$P(e,h \,\&\, b) > P(e,b)$' and Hooker and Stove, as well as Mackie, express the relevance criterion as follows:

'e confirms h in relation to b iff $P(e,h \,\&\, b) > P(e,b)$'.[5] (25)

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[4] C. A. Hooker and D. Stove, "Relevance and the Ravens," BJPS, vol. 18 (1967), pp. 305 - 315; J. L. Mackie, "The Relevance Criterion of Confirmation," BJPS, vol. 20 (1969), pp. 27 - 40.

[5] Hooker and Stove, op. cit., p. 305; and Mackie, (continued)

Both (22) and (25) are suitably spoken of as expressing the relevance criterion of confirmation$_F$ insofar as the probabilistic expressions occurring in them are equivalent. A non-relativized form is acknowledged by Hooker and Stove according to which e confirms h iff $P(e,h) > P(e)$.[6] The concept of confirmation supposedly meant, in my terminology, is initial confirmation$_F$.

The attitude of Hooker and Stove toward the relevance criterion is rather provocative. They do not deny that the relevance criterion is a plausible thesis, but they contend that it is not any more plausible than many other theses which philosophers treat with deep distrust and for which arguments are demanded. They also find it remarkable that ". . . without anyone ever having been obliged to advance arguments for it, the Relevance Criterion has acquired so unique an authority that it is treated as a settled fact, even by two great philosophers who are in other opinions opposed, and even when it is inconsistent with the other opinions of each."[7] The theorists to whom Hooker and Stove refer in the last sentence are Popper and Carnap. Although Popper and Carnap disagree about many issues

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(continued from previous page) op. cit., p. 27. Symbols have been changed in order to comply with the conventions laid down in this dissertation. The concept of confirmation apparently meant is confirmation$_F$ since a reference to Carnap's examination is made.

[6]Hooker and Stove, op. cit., p. 305, n. 3.

[7]Hooker and Stove, op. cit., p. 310.

in the theory of confirmation ("corroboration," in Popper's more recently adopted terminology), both are said to endorse the relevance criterion. This is certainly so in the case of Carnap, and very likely in the case of Popper, although there is considerable doubt in my mind that the concept for which Popper offers the relevance criterion is the concept of confirmation_F. I shall briefly discuss the positions taken by Popper and Carnap with respect to the relevance criterion.

There are a number of passages appearing in Popper's writings which partially support the view that he endorses the relevance criterion. He says in one place that a criterion of that fact that ". . . the evidence _e_ supports or corroborates or confirms a statement _h_ . . . is: 'that _e_ _increases_ the probability of _h_'."[8] This criterion is then formulated as:

'$\underline{Co}(\underline{h},\underline{e})$ iff $P(\underline{h},\underline{e}) > P(\underline{h})$', (26)

where '$\underline{Co}(\underline{h},\underline{e})$' is the symbol for '_h_ is supported or corroborated or confirmed by _e_'. The criterion expressed in (26) is identical with the relevance criterion in its non-relativized form. Popper claims that it has one defect, however, for if _h_ is a universal sentence and _e_ is an empirical sentence then $P(\underline{h}) = P(\underline{h},\underline{e}) = 0$. Thus the criterion expressed in (26) cannot measure the

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[8] Popper, _The Logic of Scientific Discovery_, p. 388. Emphasis in the original. Symbols have been altered to comply with the conventions adopted in this dissertation.

corroboration of a universal theory. This defect is overcome by making use of the fact that $P(\underline{h},\underline{e}) > P(\underline{h})$ iff $P(\underline{e},\underline{h}) > P(\underline{e})$ whenever $P(\underline{h}) \neq 0 \neq P(\underline{e})$. The revised criterion of corroboration is:[9]

'$\underline{Co}(\underline{h},\underline{e})$ iff $P(\underline{h},\underline{e}) > P(\underline{h})$ or $P(\underline{e},\underline{h}) > P(\underline{e})$'. (27)

The criterion given here by Popper is sufficiently similar to the relevance criterion in its non-relativized form. It must be borne in mind, however, that it is unlikely that Popper sees the concept of corroboration for which he provides a criterion as identical with the concept of confirmation$_F$ as this is understood by Carnap and in this dissertation as well. I made brief reference in 1.2 above to the aims of Popper's theory of corroboration and to the fact that he does not see corroborating evidence as making the claim that a certain hypothesis is true more rationally credible than it was on prior evidence. He sees corroborating evidence rather as rendering a hypothesis worthier to be subjected to further criticism and to the severest test we can devise. Still, Popper's criterion of corroboration might be capable of being construed as a criterion of confirmation$_F$, even if, as seems likely, Popper does not concern himself with the concept of confirmation$_F$.

There is a second feature of Popper's study which gives

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[9] Ibid., p. 389.

partial support to the view that he endorses the relevance criterion, this time in the relativized form as expressed in (25). Popper offers two measures of the degree of corroboration of *h* by *e* in the presence of *b*, which are as follows:[10]

$$C_1(\underline{h},\underline{e},\underline{b}) = \frac{P(\underline{e},\underline{h} \,\&\, \underline{b}) - P(\underline{e},\underline{b})}{P(\underline{e},\underline{h} \,\&\, \underline{b}) + P(\underline{e},\underline{b})} \;(1 + P(\underline{h},\underline{b})\; P(\underline{h},\underline{e} \,\&\, \underline{b}))$$

$$C_2(\underline{h},\underline{e},\underline{b}) = \frac{P(\underline{e},\underline{h} \,\&\, \underline{b}) - P(\underline{e},\underline{b})}{P(\underline{e},\underline{h} \,\&\, \underline{b}) - P(\underline{h} \,\&\, \underline{e},\underline{b}) + P(\underline{e},\underline{b})}$$

There has been a great deal of controversy over Popper's concept of degree of corroboration. This has arisen largely in connection with Carnap's study of confirmation_S and Popper's critiques of Carnap. There are a number of writers who have judged that Popper's concept of degree of corroboration is akin to Carnap's concept of degree of relevance or degree of confirmation_F.[11] Popper, on the other hand, appears to think that his concept of degree of corroboration is the same as Carnap's concept of degree of confirmation_S,[12] and that while he and Carnap have the

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[10] *Ibid.*, p. 400, n. *2; p. 401.

[11] See for example, John G. Kemeny, "Review of "Degree of Confirmation by Karl R. Popper"," *The Journal of Symbolic Logic*, vol. 20 (1955), p. 304; Y. Bar-Hillel, "Comments on 'Degree of Confirmation' by Professor K. R. Popper," *BJPS*, vol. 6 (1955), p. 155; and H. E. Kyburg, Jr., "Recent Work in Inductive Logic," *American Philosophical Quarterly*, vol. 1 (1964), p. 257, for a table in which the identification is implicitly made.

[12] Popper, *The Logic of Scientific Discovery*, p. 393.

same _explicandum_, they have different _explicata_. Carnap, however, wonders whether Popper doesn't have an _explicandum_ different from either degree of confirmation$_F$ (degree of increase in firmness) or degree of confirmation$_S$.[13] The question whether Popper's concept of degree of corroboration is akin to Carnap's concept of degree of confirmation$_S$ or Carnap's concept of degree of confirmation$_F$ or to neither is hard to answer. "The difficulty," writes Vincent," is that both writers are guilty of failure to give detailed-enough explanations of their chief explicanda. The difficulty is made worse because both writers provide a plurality of apparently non-equivalent explanations and yet do not make clear which explanation is supposed to be the chief one."[14]

If the controversy concerning Popper's concept were to be resolved and the measures $C_1(\underline{h},\underline{e},\underline{b})$ and $C_2(\underline{h},\underline{e},\underline{b})$ were indeed measures of degree of confirmation$_F$, (degree of increase in firmness) then Popper's measures would yield results which tally with the relevance criterion in its relativized form. For then $\underline{e}$ would confirm$_F$ $\underline{h}$ given $\underline{b}$ iff $C_1(\underline{h},\underline{e},\underline{b}) > 0$, or alternatively, $\underline{e}$ would confirm$_F$ $\underline{h}$ given $\underline{b}$ iff $C_2(\underline{h},\underline{e},\underline{b}) > 0$. Now $C_1(\underline{h},\underline{e},\underline{b}) > 0$ iff '$P(\underline{e},\underline{h} \& \underline{b}) - P(\underline{e},\underline{b})$' and '$1 + P(\underline{h},\underline{b})P(\underline{h},\underline{e} \& \underline{b})$' are both

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[13] Carnap, _LFP_, p. xviii, n. 3.

[14] Vincent, "Discussion: Concerning an Alleged Contradiction," _Philosophy of Science_, vol. 30 (1963), p. 190, n. 5.

positive or both negative. Since '1 + P(*h*,*b*) P(*h*,*e* & b)' cannot be negative but must be positive, 'P(*e*,*h*, & *b*) - P(*e*,*b*)' must also be positive, i.e., P(*e*,*h* & *b*) > P(*e*,*b*). Moreover, C_2(*h*,*e*,*b*) > 0 iff P(*e*,*h* & *b*) - P(*e*,*b*) > 0 iff P(*e*,*h* & *b*) > P(*e*,*b*). On the uncertain assumption that Popper's concept of corroboration is identical with the concept of confirmation$_F$ we can judge that Popper endorses the relevance criterion in both its relativized and non-relativized forms. Even if Popper's concept of corroboration is not identical with the concept of confirmation$_F$, Popper does endorse the relevance criterion for his concept of corroboration.

There is no doubt, of course, concerning Carnap's endorsement of the relevance criterion. Attention has been drawn to this fact already in chapter two above. An interesting remark which Hooker and Stove make concerning Carnap's study of confirmation$_F$ is that his acceptance of the relevance criterion ". . . seems to be so implicit as to indicate that, for him, favourable relevance is not so much a criterion of confirmation as confirmation itself."[15] By 'favourable relevance' Hooker and Stove mean the measure 'P(*h*,*e* & *b*) > P(*h*,*b*)'. This remark, when it is interpreted in the light of the distinctions I have drawn between various concepts of confirmation, means that for

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[15]Hooker and Stove, *op. cit.*, p. 310.

Carnap, the concept of an increase in the firmness of a hypothesis is equivalent to the concept of an increase in the (logical) probability of a hypothesis, or, put another way, that increase in credibility is identical with increase in (logical) probability. Hooker and Stove's judgment is probably quite close to the truth, for Carnap tends to treat the concept of (logical) probability and the concept of rational credibility as practically equivalent. This is clearly reflected in one of Carnap's papers in which he distinguishes the concept of objective (statistical) probability from the concept of subjective (personal) probability.[16] Carnap further distinguishes two versions of subjective probability, *viz*., one representing the *actual* degree of belief and the other representing the *rational* degree of belief. For the purposes of inductive logic, Carnap is interested, along with other authors such as F. P. Ramsey, Harold Jeffreys, J. M. Keynes, and I. J. Good, chiefly in the concept of probability which represents the rational degree of belief or rational credibility.[17] This is a key epistemological thesis for all of the foregoing authors listed and many more besides.

There have been many distinguished theorists, including

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[16] Carnap, "The Aim of Inductive Logic," in *Logic, Methodology and Philosophy of Science* (Stanford: Stanford University Press, 1962), ed. Nagel, *et. al*., p. 303f.

[17] *Ibid*., p. 307.

mathematicians, logicians, and philosophers, who have identified degree of rational credibility or degree of confirmation$_S$ with the degree of probability in the sense of the Probability Calculus. This thesis has been challenged by a number of theorists, however, which suggests that it is not as irreproachable as some may have thought. One of the strongest critics of the thesis that degree of rational credibility may be identified with the degree of probability is Popper who asserts and argues that 'degree of rational belief' is not one of the very large number of different interpretations of the calculus of probability.[18] A number of other authors have focussed on the general multiplication principle in their attack upon this thesis. The general multiplication principle of the Probability Calculus is:

$$P(\underline{h} \ \& \ \underline{k}, \underline{e}) = P(\underline{h}, \underline{k} \ \& \ \underline{e}) \ P(\underline{k}, \underline{e}).$$ [19]

John G. Kemeny, Hempel and Oppenheim, and R. H. Vincent all contend that the general multiplication principle is not satisfied by the concept of degree of confirmation$_S$ or degree of rational credibility,[20] and therefore maintain that degree of rational

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[18] Popper, The Logic of Scientific Discovery, p. 415.

[19] This principle or axiom commonly occurs in treatments of the probability calculus. See Carnap, LFP, p. 285, for example.

[20] John G. Kemeny, "Carnap on Probability," The Review of Metaphysics, vol. 5 (1951 - 52), p. 150f; Hempel and Oppenheim, "A Definition of "Degree of Confirmation"," (continued)

credibility is not one of the interpretations of the probability calculus. Fortunately, the foregoing very large and controversial issue does not have to be settled here in order to assess the relevance criterion. It is just that if the concept of degree of rational credibility were one of the interpretations of the Probability Calculus, the relevance criterion would hold for the concept of $confirmation_F$ as it has been elucidated using the concept of rational credibility.

I shall return now to the challenge which Hooker and Stove have issued in connection with the relevance criterion and consider Mackie's response to that challenge.

3. Mackie's qualified support of the relevance criterion and criticisms of his objections to it.

One of Mackie's defences of the relevance criterion involves a reference to what he calls the paradigm case of confirmation, *viz.*, the confirmation of a scientific hypothesis by an experimental test.[21] According to one simple version, an experiment is set up in such a way that, on the assumption that

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(continued from previous page) *Philosophy of Science*, vol. 12 (1945), pp. 111 - 112; and Vincent, "A Note on Some Quantitative Theories of Confirmation," *Philosophical Studies*, vol. 12 (1961), pp. 91 - 92, and "On My Cognitive Sensibility," *Philosophical Studies*, vol. 14 (1963), pp. 77 - 79.

[21]Mackie, "The Relevance Criterion of Confirmation," p. 29f.

certain background information *b* is true, it is expected that if the hypothesis *h* being tested is true then a certain observational result *e* will be obtained. The foregoing statement can be expressed in the following formal way: the conjunction of the background information *b* with the hypothesis *h* entails that the observational result *e* will obtain. Since *h* & *b* entails *e*, P(*e*,*h* & *b*) = 1. This version of a confirmational paradigm also includes the essential fact that *e* is not to be expected on the basis of *b* alone if *e* is actually to confirm *h*, i.e., *b* does not entail *e*; hence P(*e*,*b*) ≠ 1. Thus in the context just described P(*e*,*h* & *b*) > P(*e*,*b*). If the observational result *e* were entailed by *b*, then P(*e*,*b*) = 1 and confirmation would not occur.

In the foregoing example, if *e* were not observed then *h* would be falsified, assuming that *b* is true. Mackie goes on to maintain, however, that a hypothesis can also be confirmed by the observation of a result which that hypothesis would help to explain, even if ". . . failure to observe this result would not have been fatal for the hypothesis even in relation to *b* — that is, even where we could have explained away an unfavourable result without abandoning any of the background of accepted beliefs."[22] That is, Mackie allows that confirmation may occur

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[22] *Ibid*., p. 30.

in contexts in which _h_ explains _e_, where _e_ is otherwise in need of explanation. He remarks, however, that the explanation need not be thorough, i.e., it need not conform to the strict 'covering-law' model; ". . . the hypothesis need not make the occurrence of what has been observed inevitable in what we take the circumstances to be."[23] For example, the hypothesis that a ship of the Spanish Armada was wrecked in a particular bay would be confirmed if a search of the area produced some cannon and coins of the right period. But this hypothesis would not be falsified if such a search was fruitless (although it might well be disconfirmed). The desired discovery is not entailed by the hypothesis and the background information, although it is explained by the hypothesis. Mackie remarks in summary that ". . . the relevance criterion connects confirmation with explanation. Its basic form says that an observation confirms an hypothesis if and only if both (i) the hypothesis would explain what is observed and (ii) this was otherwise in need of explanation."[24] Mackie later provides a further clarification of his position claiming that it is not his intention to use the proposition that _h_ actually explains _e_ as part of the criterion of confirmation, ". . . but only to use the fact that _h_ has the role of an explanatory

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[23]Ibid., p. 33.

[24]_Ibid._, p. 33.

hypothesis in relation to _e_ to delimit the field within which relevance is a criterion of confirmation."[25] In the intervening portion of his article he offers certain considerations which suggest to him that the relevance criterion of confirmation is a criterion of confirmation _only_ in explanatory contexts, i.e., only in connection with explanatory hypotheses.

Mackie distinguishes three different varieties of "support," only one variety of which has as a criterion the relevance criterion.[26] Mackie argues that the relevance criterion is a sound criterion of _direct confirmation_, where direct confirmation is a variety of support given by scientific evidence to a hypothesis which helps to explain that evidence. A second variety of support is _indirect support_, which is the support given indirectly by directly confirming support to further inferences from the explanatory hypothesis. The third variety of support is _probabilification_, which is support given by the premises of a proportional syllogism to its conclusion, e.g., the information that _a_ is an A and that most things which are A are also B supports the conclusion that _a_ is a B. Mackie's central argument is to the effect that the relevance criterion is a sound criterion of direct confirmation, but it is not a sound criterion of support in general, i.e., the field within

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[25] _Ibid_., p. 39.

[26] _Ibid_., pp. 37 - 38.

which the relevance criterion is a criterion of confirmation is limited to that in which h has the role of an explanatory hypothesis in relation to some observational evidence e. He argues that there are ostensibly clear cases of indirect support which fail to obey the relevance criterion. He does not say much in regard to probabilification and its failure to obey the relevance criterion, but his arguments pertain mainly to the failure of indirect support to obey the relevance criterion. Mackie quite convincingly shows, in my estimation, that within the context of explanation the relevance criterion is a sound criterion of confirmation. He does not convincingly show, however, that there are genuine cases of confirmation_F which are not measured by the relevance criterion. I do not think that the cases of indirect support which he puts forward in which the relevance criterion fails are genuine cases of confirmation_F.

Mackie does not distinguish different concepts of confirmation as I have done, and so it is not completely clear what his contention that there are clear-cut cases of support which do not obey the relevance criterion amounts to. It is not clear, in particular, that his contention amounts to saying that there are clear-cut cases of confirmation_F which do not obey the relevance criterion. Since I am concerned to assess the adequacy of the relevance criterion for the concept of confirmation_F, I shall interpret Mackie's arguments against the adequacy of the relevance criterion for all kinds of support as arguments against the adequacy of the relevance criterion for all cases of confirmation$_F$, even if these arguments were not intended as such. I shall defend the relevance criterion against these arguments.

Mackie argues that the deductive consequences of a hypothesis, which is confirmed by *e*, are also confirmed by *e*. That is, Mackie endorses the special consequence condition CC2.1, which states that if evidence *e* confirms a hypothesis *h* given prior evidence *b*, and *h* entails sentence *k*, then *e* confirms *k* given *b*. This condition is incompatible with the relevance criterion, and, unlike Carnap who consequently rejects CC2.1, Mackie maintains that CC2.1 can be used to cast doubt on the relevance criterion.[27] Here Mackie is just pitting CC2.1 against the relevance criterion and opting for the former. The question remains: "Who, if any, is correct, Mackie or Carnap?" It seems to me that Mackie is incorrect in endorsing CC2.1 as plausible for the concept of confirmation$_F$. I discussed this issue in section 6.4 above and argued that CC2.1 is not a plausible characteristic of the concept of confirmation$_F$ from an intuitive standpoint. Hence, Mackie's argument here against the adequacy of the relevance criterion is unsuccessful.

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[27] *Ibid.*, p. 36. This remark, incidentally, strongly suggests that Mackie construes the relevance criterion as inadequate for the concept of confirmation$_F$.

Another important objection to the relevance criterion is developed from the ideal evidence paradox. One of the earliest formulations of the ideal evidence paradox is put forward by Popper who formulates it in connection with tossing a two-sided coin.[28] Mackie formulates the paradox in connection with tossing a six-sided die, as follows.[29] Hypothesis $\underline{h}_1$ is the statement that at the next throw of this die 6 will not come up, and $\underline{e}_1$ is the statement that in the last 600,000 throws 6 has come up only 100,000 times. The paradox consists in the fact that although, intuitively speaking, $\underline{e}_1$ is ideal evidence in favor of $\underline{h}_1$, $\underline{e}_1$ does not alter the probability of $\underline{h}_1$ and so, technically speaking, and according to the relevance criterion, $\underline{e}_1$ is irrelevant evidence with respect to $\underline{h}_1$. Mackie discusses this problem for the relevance criterion in connection with different kinds of background evidence. There is really only one case in which Mackie definitely asserts that $\underline{e}_1$ confirms $\underline{h}_1$ given a certain kind of background evidence, so I shall restrict my attention to that case.

Consider the case in which the background evidence $\underline{b}_1$ consists of the knowledge that the die $\underline{d}$ has the usual six sides so that there are six logically possible results of a throw, and

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[28] Popper, The Logic of Scientific Discovery, pp. 414 - 415.

[29] Mackie, "The Relevance Criterion of Confirmation," pp. 34 - 35.

we have no grounds for expecting the six sides to come up with equal frequency. Mackie says that the probability of $\underline{h}_1$ relative to $\underline{b}_1$ is 5/6, but this probability rests wholly on the set of logically possible alternatives and the principle of indifference. Now the probability of $\underline{h}_1$ on $\underline{e}_1$ and $\underline{b}_1$ is still 5/6, he maintains, so according to the relevance criterion $\underline{e}_1$ does not confirm $\underline{h}_1$ given $\underline{b}_1$. Mackie intuitively assesses $\underline{e}_1$ as confirming $\underline{h}_1$ given $\underline{b}_1$, however, so the relevance criterion is considered to be deficient. Concerning the confirmation of $\underline{h}_1$ (given $\underline{b}_1$) by $\underline{e}_1$ Mackie makes the following interesting comment:[30]

> It [i.e., $\underline{e}_1$] changes the type of evidence to which this probability is relative, and the way in which that evidence gives a fairly high probability to $\underline{h}_1$. Without $\underline{e}_1$, we rely heavily on the principle of indifference; once we have $\underline{e}_1$, we rely on a well-confirmed law about the behaviour of this die. Here too we can say that $\underline{e}_1$ confirms $\underline{h}_1$, but not in accordance with the relevance criterion, and in a different sense of 'confirms' from that which does obey this criterion.

It certainly appears to be true that $P(\underline{h}_1,\underline{b}_1) = P(\underline{h}_1, \underline{b}_1 \& \underline{e}_1)$ so that according to the relevance criterion $\underline{e}_1$ does not confirm$_F$ $\underline{h}_1$ given $\underline{b}_1$ but is irrelevant$_F$ to $\underline{h}_1$ given $\underline{b}_1$. Mackie claims, however, that $\underline{e}_1$ nevertheless does confirm, in some sense of 'confirm', $\underline{h}_1$ (given $\underline{b}_1$). What sense of 'confirm' is meant here? If some sense of 'confirm' other than 'confirm$_F$' is meant then there is no problem for the relevance criterion here, for

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[30] Ibid., p. 35. Symbols have been altered to comply with conventions adopted in this dissertation.

I am examining the relevance criterion as a criterion of confirmation$_F$ and not as a criterion of any other concept of confirmation. Mackie's remark that $\underline{e}_1$ confirms $\underline{h}_1$ in a sense of 'confirm' other than that which obeys the relevance criterion could be construed as meaning that $\underline{e}_1$ confirms $\underline{h}_1$ in a sense of 'confirms' other than 'confirms$_F$'. He perhaps means that $\underline{e}_1$ indirectly supports $\underline{h}_1$ given $\underline{b}_1$ rather than that $\underline{e}_1$ directly confirms $\underline{h}_1$ given $\underline{b}_1$, where the relevance criterion is a sound criterion only of direct confirmation and not of indirect support. In any case Mackie's remark is uncertain, and need not cause us any concern. What would be of significance though would be the confirmation$_F$ of $\underline{h}_1$ by $\underline{e}_1$ given $\underline{b}_1$ while the probability of $\underline{h}_1$ on $\underline{e}_1$ and $\underline{b}_1$ remained the same as the probability of $\underline{h}_1$ on $\underline{b}_1$ alone, for then the relevance criterion would not be a sound criterion of confirmation$_F$. I don't think Mackie is asserting exactly this, but suppose Mackie's statement is interpreted in such a way. Is it more plausible to maintain that $\underline{e}_1$ confirms$_F$ $\underline{h}_1$ given $\underline{b}_1$ rather than that $\underline{e}_1$ is irrelevant$_F$ or neutral$_F$ to $\underline{h}_1$ given $\underline{b}_1$? That is, is it more plausible to maintain that $\underline{h}_1$ is more credible on $\underline{e}_1$ and $\underline{b}_1$ than on $\underline{b}_1$ alone rather than that $\underline{h}_1$ is as credible on $\underline{e}_1$ and $\underline{b}_1$ as on $\underline{b}_1$ alone? Vincent advances several compelling arguments against the ideal evidence paradox which can be suitably altered to show that $\underline{e}_1$ is irrelevant$_F$

not confirming$_F$ evidence for $\underline{h}_1$ given $\underline{b}_1$.[31]

In the first place, it is not obvious that $\underline{e}_1$ confirms$_F$ $\underline{h}_1$ given $\underline{b}_1$. If after throwing $\underline{d}$ 600,000 times 6 has come up only 6,000 times (that is, 1/100 of the time), then this information would seem to make $\underline{h}_1$ more credible than $\underline{h}_1$ is on $\underline{b}_1$ alone; and if 6 has come up 194,000 times (that is, 97/300 or nearly 1/3 of the time) in the 600,000 throws then on this information the hypothesis $\underline{h}_1$ that 6 would not come up on the 600,001th throw would seem to be less credible than $\underline{h}_1$ is on $\underline{b}_1$ alone, i.e., this information would seem to disconfirm$_F$ $\underline{h}_1$ given $\underline{b}_1$. But when after throwing $\underline{d}$ 600,000 times we find that 6 comes up exactly 100,000 times (that is, 1/6 of the time), then this information prima facie neither confirms$_F$ nor disconfirms$_F$ $\underline{h}_1$ given $\underline{b}_1$, and is thus irrelevant$_F$ or neutral$_F$ with respect to $\underline{h}_1$ given $\underline{b}_1$.

In the second place the view which seems to be behind the judgment that $\underline{e}_1$ confirms$_F$ $\underline{h}_1$ given $\underline{b}_1$ is that $\underline{h}_1$ is more credible on $\underline{e}_1$ and $\underline{b}_1$ than on $\underline{b}_1$ alone because $\underline{e}_1$ and $\underline{b}_1$ provides more information about $\underline{d}$ than does $\underline{b}_1$ alone. Vincent contends that the principle of which this view is an instance is untenable and gives the following example to illustrate its untenability. Consider a patient who finds out from his doctor that he has

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[31] Vincent, "The Paradox of Ideal Evidence," The Philosophical Review, vol. 71 (1962), pp. 501 - 502.

disease D and that seven-eighths of all persons with disease D die within one year of its onset. Later, just as the doctor finds out that the patient has variety V of disease D, investigations show that seven-eighths of all patients with variety V of disease D die within one year of its onset. Here we have a single hypothesis and two bodies of information. The hypothesis that the patient will die within a year is no more credible given the weightier information than on the less substantial information. In the same way $\underline{h}_1$ is not made more credible by the addition of $\underline{e}_1$ to $\underline{b}_1$, even though more information concerning $\underline{d}$ is contained in $\underline{e}_1$ and $\underline{b}_1$ than in $\underline{b}_1$ alone, but $\underline{e}_1$ is neutral$_F$ with respect to $\underline{h}_1$ given $\underline{b}_1$. Hence, the ideal evidence paradox does not constitute a definitive objection to the relevance criterion.

4. Suggestions for further study in connection with the concept of confirmation$_F$, and concluding summary of the dissertation.

The foregoing discussion in this chapter has shown that the relevance criterion is a plausible criterion of confirmation$_F$, and that the objections which have been raised against it are susceptible to effective replies. Although the relevance criterion has much to commend it as a plausible criterion of confirmation$_F$, there is scope, nevertheless, for further work, and in this concluding section of this dissertation I shall mention several problems which require further attention before summarizing my main conclusions.

An important limitation on the use of the relevance criterion is the difficulty connected with determining whether or not an evidence report has increased, decreased, or left unchanged, the probability of a hypothesis on total prior evidence. That is, there is a difficulty connected with using the probabilistic expression '$P(h,e \;\&\; b) > P(h,b)$' itself, insofar as it is often difficult to determine, for any *h*, *e* and *b*, whether $P(h,e \;\&\; b) > P(h,b)$, $P(h,e \;\&\; b) < P(h,b)$, or $P(h, e \;\&\; b) = P(h,b)$. Effective use of the relevance criterion is thus somewhat limited at this stage. There are, of course, cases in which a change in the probability of a hypothesis is obvious, but there are many more in which this is not so. There also are cases where it is known that $P(h,e \;\&\; b)$ and/or $P(h,b)$ equals either 0 or 1, and in such cases one can usually determine whether or not there is a change in the probability of a hypothesis. This limitation on the use of the relevance criterion would be effectively eliminated if one could discover a way to assign values to '$P(x,y)$' for any *x* and *y*, for then it would be obvious whether $P(h,e \;\&\; b)$ was greater, less than, or equal to $P(h,b)$ for any given *h*, *e*, and *b*. The problem of assigning values to probability statements has been extensively discussed by various authors, but no successful solution has been obtained as yet.

A second important problem warranting further consideration is whether or not the credibility of a hypothesis is adequately measured by the probability of that hypothesis on total evidence, i.e., the problem of whether or not the concept of rational credibility is one interpretation of the Probability Calculus. This problem was briefly mentioned in section 7.2 above. If the concept of credibility were an interpretation of the Probability Calculus then the notion of an increase in credibility of *h* (i.e., the concept of confirmation_F) would be appropriately measured by an increase in the probability of *h*, i.e., then the relevance criterion is sound. It is possible that the notion of an increase in credibility of *h* might be adequately measured by an increase in the probability of *h* even if the concept of credibility were not an interpretation of the Probability Calculus, but it is not very likely. For example, if the degree of credibility of *h* on total evidence *b* were adequately measured by the function of probabilities '$P(h,b) / P(h)$', rather than simply by '$P(h,b)$', the increase in the credibility of *h* would still obey the relevance criterion. In this example the degree of credibility of *h* on *e* & *b* would equal $P(h,e \,\&\, b) / P(h)$, and *h* would be more credible on *e* & *b* than on *b* alone iff $(P(h,e \,\&\, b) / P(h)) > (P(h,b) / P(h))$ iff $P(h,e \,\&\, b) > P(h,b)$, provided $P(h) \neq 0$. It is not very likely, however, that the probabilistic measure of the concept of degree of credibility would be such that an increase in the credibility of _h_ would obey the relevance criterion. There is even a possibility that the concept of degree of credibility is not measured by any function of probabilities, as Cohen has recently argued.[32] In such a case it is rather unlikely that an increase in the credibility of _h_ would be measured by an increase in the logical probability of _h_. Hence, a general problem requiring further attention is the analysis of the concept of credibility. An adequate analysis of this concept could give a "second life" to the second approach to explication which depends so crucially upon the ability to make decisive credibility assessments.

In the study of confirmation which I have undertaken, I have argued a number of important theses. Among these theses has been the thesis that the concept of weak confirmation, as it appears in various studies of confirmation, has been conceived of as a prior-evidence-independent relation at some times, and at other times as a prior-evidence-dependent relation. I drew particular attention to the studies of confirmation undertaken by Vincent, Rescher, and Carnap in the discussion of the distinction between prior-evidence-independence and prior-evidence-dependence. A distinction which is logically prior to this distinction, of course, is the well-known distinction

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[32] Cohen, _The Implications of Induction_, section 3.

between strong and weak concepts of confirmation emphasized by Carnap. I have introduced the uninterpreted concept of credibility in order to facilitate the discussion of both strong and weak concepts of confirmation.

I have made use of, and illustrated the importance of, the distinction between prior-evidence-independent and prior-evidence-dependent concepts of confirmation in discussing Hempel's study of qualitative confirmation and various important critiques of his study. Anyone who is familiar with the literature written in the last two-and-a-half decades cannot help but be aware of the impact of Hempel's work upon the study of confirmation. I have not touched upon the question of the adequacy of his *explicatum*, because I have been interested in this dissertation in the more basic issue of the various *explicanda* appearing in the study of confirmation. I have argued that Hempel's study has been concerned with a prior-evidence-independent concept of confirmation.

Another important thesis which I have tried to establish has been the thesis that the three-termed, prior-evidence-dependent concept of confirmation_F is the primary concept of weak confirmation. I have emphasized the need to take into consideration the prior evidence background in order to accurately assess the ability of an evidence report to make an unqualified contribution to the firmness of a hypothesis. Because of the primacy of the prior-evidence-dependent concept of confirmation$_F$, a study of any prior-evidence-independent concept of confirmation can never assume as much importance as does the study of the former kind of concept. I have indicated, however, how studies of prior-evidence-independent concepts, especially the concept of confirmation$_B$, might be used in order to yield results which approximate to the results obtained from an adequate explication of the concept of confirmation$_F$.

The concept which requires and deserves special interest, in view of the arguments of the first five chapters, is of course the concept of confirmation$_F$. Before considering in some detail the explication of this concept, however, I have devoted my attention to the general problem of explicating concepts of confirmation. Two main approaches to this problem have been canvassed. I have considered the approach in which a concept is interpreted using a formal, probabilistic measure, and I have considered a second approach in which an attempt is made to assign intuitively plausible characteristics to a concept which then serve as a standard for any proposed explication. I have exposed the weaknesses of the second approach, which are mainly the result of the difficulty attached to making definite, defensible, credibility assessments. In the final chapter of this dissertation I have tried to assess the success of using the first approach to explication for the concept of confirmation$_F$. I have argued that the probabilistic measure 'P(h,e & b) > P(h,b)' is a plausible criterion of confirmation$_F$, and have defended this thesis against several objections. In closing I have indicated the direction which further studies might profitably take in order to more adequately assess the plausibility of the relevance criterion of confirmation$_F$.

APPENDIX 1

Most of the proofs concerning the conditions of adequacy which hold for and which are violated by the various concepts of confirmation parallel the examples given in the text. In order to show that a certain condition of adequacy is violated by a given concept of confirmation, it is sufficient to show that even if the antecedent of the condition of adequacy holds, the consequent of the condition of adequacy need not hold, given that the concept of confirmation occurring in the condition of adequacy is uniformly interpreted in accordance with the appropriate probabilistic requirement. In order to show that a certain condition of adequacy holds for a given concept of confirmation, it is sufficient to show that if the antecedent of the condition holds, then the consequent of that condition also holds, given that the concept of confirmation occurring in the condition of adequacy is uniformly interpreted in accordance with the appropriate probabilistic requirement. In order to reduce the required number of proofs as much as possible, I draw attention to the following relationships between certain of the concepts of confirmation. Separate proofs with respect to the concept of confirmation$_C$ are not necessary, for whenever a condition of adequacy holds for both the concepts of confirmation$_A$ and confirmation$_B$ that condition of adequacy will automatically hold for the concept of $confirmation_C$. Moreover, whenever a condition of adequacy is violated by either the concept of $confirmation_A$ or the concept of $confirmation_B$ (or both), that condition of adequacy is violated by the concept of $confirmation_C$. The conditions of adequacy which hold for and are violated by the concepts of $confirmation_B$ and initial $confirmation_F$ are identical, and in the proofs I shall simply refer to the concept of $confirmation_B$. Since the concept of initial $confirmation_F$ is a special case of the general concept of $confirmation_F$ (hence, the former is derivable from the latter), whenever a condition of adequacy is violated by the concept of initial $confirmation_F$, the correlative condition of adequacy will be violated by the concept of $confirmation_F$. Proofs for the concept of initial $confirmation_F$ will be given first, and these results will determine which correlative conditions of adequacy are violated by the concept of $confirmation_F$.

Most of the proofs to follow require an assignment of numerical values to the probability statements, the arguments of which involve all possible combinations of sentence variables included in any given condition of adequacy. The following three tables of assignments will be consulted in the proofs of adequacy conditions with respect to the concepts of $confirmation_A$ and $confirmation_B$:

TABLE I

$P(\underline{h} \,\&\, \underline{e}) = w$	$P(-\underline{h} \,\&\, \underline{e}) = y$
$P(\underline{h} \,\&\, -\underline{e}) = x$	$P(-\underline{h} \,\&\, -\underline{e}) = z$

TABLE II

$P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{k}) = x_1$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, \underline{k}) = x_5$
$P(\underline{h} \,\&\, \underline{e} \,\&\, -\underline{k}) = x_2$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, -\underline{k}) = x_6$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, \underline{k}) = x_3$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, \underline{k}) = x_7$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{k}) = x_4$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{k}) = x_8$

TABLE III

$P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{l} \,\&\, \underline{k}) = y_1$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, \underline{l} \,\&\, \underline{k}) = y_9$
$P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{l} \,\&\, -\underline{k}) = y_2$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, \underline{l} \,\&\, -\underline{k}) = y_{10}$
$P(\underline{h} \,\&\, \underline{e} \,\&\, -\underline{l} \,\&\, \underline{k}) = y_3$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, -\underline{l} \,\&\, \underline{k}) = y_{11}$
$P(\underline{h} \,\&\, \underline{e} \,\&\, -\underline{l} \,\&\, -\underline{k}) = y_4$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, -\underline{l} \,\&\, -\underline{k}) = y_{12}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, \underline{l} \,\&\, \underline{k}) = y_5$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, \underline{l} \,\&\, \underline{k}) = y_{13}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, \underline{l} \,\&\, -\underline{k}) = y_6$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, \underline{l} \,\&\, -\underline{k}) = y_{14}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{l} \,\&\, \underline{k}) = y_7$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{l} \,\&\, \underline{k}) = y_{15}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{l} \,\&\, -\underline{k}) = y_8$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{l} \,\&\, -\underline{k}) = y_{16}$

When conditions of adequacy having only two sentence variables are examined, it is sufficient to assign values to probability statements according to Table I. Table II is used to assign values to probability statements when conditions of adequacy with three sentence variables are examined, and Table III is used in connection with adequacy conditions with four sentence variables. The sum of the values in each table equals one, e.g., $x_1 + x_2 + \ldots + x_8 = 1$. The general proviso I have made is that e is not logically false; hence, $P(\underline{e}) \neq 0$, and so $w + y \neq 0$ (Table I), $x_1 + x_2 + x_5 + x_6 \neq 0$ (Table II), and $y_1 + \ldots + y_4 + y_9 + \ldots + y_{12} \neq 0$ (Table III).

I shall first consider the conditions of adequacy listed in the text with respect to the concepts of confirmation$_A$ and confirmation$_B$, and hence also in relation to the concepts of confirmation$_C$ and initial confirmation$_F$ by virtue of the relationships spelled out several pages back. The probabilistic interpretations of the concepts of confirmation$_A$ and confirmation$_B$ are as follows:

> e confirms$_A$ h iff $P(\underline{h},\underline{e}) > P(-\underline{h},\underline{e})$ iff $P(\underline{h}\ \&\ \underline{e}) > P(-\underline{h}\ \&\ \underline{e})$ iff (a) $w > y$ (according to Table I), (b) $x_1 + x_2 > x_5 + x_6$ (according to Table II), or (c) $y_1 + y_2 + y_3 + y_4 > y_9 + y_{10} + y_{11} + y_{12}$ (according to Table III);
>
> e confirms$_B$ h iff $P(\underline{h},\underline{e}) > P(\underline{h})$ iff $P(\underline{h}\ \&\ \underline{e}) > P(\underline{h})\ P(\underline{e})$ iff (a) $w > (w + x)(w + y)$ (according to Table I), (b) $x_1 + x_2 > (x_1 + x_2 + x_3 + x_4)(x_1 + x_2 + x_5 + x_6)$ (according to Table II), or (c) $y_1 + y_2 + y_3 + y_4 > (y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 + y_8)(y_1 + y_2 + y_3 + y_4 + y_9 + y_{10} + y_{11} + y_{12})$ (according to Table III).

The proofs for confirmation$_A$ and confirmation$_B$ now follow.

C1: If *e* entails *h*, then *e* confirms *h*.

e entails *h* iff P(-*h* & *e*) = 0 iff y = 0. Since y = 0 and w + y ≠ 0 by the general requirement, w > y.

e confirms_A *h* iff w > y. Hence C1 holds for confirmation_A.

e confirms_B *h* iff w > (w + x)(w + y). Since y = 0, w > (w + x)(w + y) provided w + x ≠ 1, i.e., provided *h* is not logically true. Thus the following modified condition holds for confirmation_B:

MC1: If *e* entails *h*, and *h* is not logically true, then *e* confirms *h*.

MC1 holds of course for confirmation_A, and so MC1 holds for confirmation_C.

C1.1: If *h* is logically true, then *e* confirms *h*.

h is logically true iff w + x = 1. Since w + x = 1, y + z = 0, and so y = 0. By the general requirement w + y ≠ 0 and so w > 0. Thus w > y.

e confirms_A *h* iff w > y. Hence C1.1 holds for confirmation_A.

e confirms_B *h* iff w > (w + x)(w + y). Since w + x = 1 and y = 0, w $\not>$ (w + x)(w + y), and C1.1 is violated by confirmation_B. Thus confirmation_C is also violated by C1.1.

C1.2 is eliminated from consideration in view of the restriction that *e* is not logically false.

C1.3: Evidence $\underline{e}$ confirms $\underline{e}$.

$\underline{e}$ confirms$_A$ $\underline{e}$ iff $P(\underline{e} \,\&\, \underline{e}) > P(-\underline{e} \,\&\, \underline{e})$ iff $P(\underline{e}) > 0$. This assumption is made in the general requirement, so C1.3 holds for confirmation$_A$.

$\underline{e}$ confirms$_B$ $\underline{e}$ iff $P(\underline{e} \,\&\, \underline{e}) > P(\underline{e})\, P(\underline{e})$ iff $P(\underline{e}) > P(\underline{e})\, P(\underline{e})$ iff $1 > P(\underline{e})$ iff $\underline{e}$ is not logically true. Thus the following modified condition holds for confirmation$_B$:

MC1.3: If $\underline{e}$ is not logically true, then $\underline{e}$ confirms $\underline{e}$.

MC1.3 holds for confirmation$_A$, and so MC1.3 holds for confirmation$_C$. Since C1.3 does not hold for confirmation$_B$, C1.3 does not hold for confirmation$_C$ either.

C2: If $\underline{e}$ confirms every sentence of a class $\underline{K}$, then $\underline{e}$ also confirms every logical consequence of $\underline{K}$.

In examining this condition I shall show that it is violated by both confirmation$_A$ and confirmation$_B$ when $\underline{K}$ has two members, $\underline{h}$ and $\underline{l}$. C2 is thus violated when $\underline{K}$ has more than two members also. I shall examine the condition: If $\underline{e}$ confirms $\underline{h}$, $\underline{e}$ confirms $\underline{l}$, and $\underline{h}$ & $\underline{l}$ entails $\underline{k}$, then $\underline{e}$ confirms $\underline{k}$.

$\underline{h}$ & $\underline{l}$ entails $\underline{k}$ iff $P(\underline{h} \,\&\, \underline{l} \,\&\, -\underline{k}) = 0$ iff $y_2 + y_6 = 0$ iff y_2 and $y_6 = 0$.

$\underline{e}$ confirms$_A$ $\underline{h}$ iff $y_1 + y_2 + y_3 + y_4 > y_9 + y_{10} + y_{11} + y_{12}$.

$\underline{e}$ confirms$_A$ $\underline{l}$ iff $P(\underline{l} \,\&\, \underline{e}) > P(-\underline{l} \,\&\, \underline{e})$ iff $y_1 + y_2 + y_9 + y_{10} > y_3 + y_4 + y_{11} + y_{12}$.

$\underline{e}$ confirms$_A$ $\underline{k}$ iff P($\underline{k}$ & $\underline{e}$) > P(-$\underline{k}$ & $\underline{e}$) iff $y_1 + y_3 + y_9 + y_{11} > y_2 + y_4 + y_{10} + y_{12}$.

The following assignment of values shows that the antecedent conditions can be true while the consequent is false:

$y_1 = .03$	$y_5 = .01$	$y_9 = .2$	$y_{13} = .02$
$y_2 = 0$	$y_6 = 0$	$y_{10} = .2$	$y_{14} = .04$
$y_3 = .1$	$y_7 = .02$	$y_{11} = .01$	$y_{15} = .02$
$y_4 = .3$	$y_8 = .02$	$y_{12} = .01$	$y_{16} = .02$

Thus C2 is violated by confirmation$_A$.

$\underline{e}$ confirms$_B$ $\underline{h}$ iff $y_1 + y_2 + y_3 + y_4 > (y_1 + \ldots + y_8) \times (y_1 + \ldots + y_4 + y_9 + \ldots + y_{12})$.

$\underline{e}$ confirms$_B$ $\underline{l}$ iff P($\underline{l}$ & $\underline{e}$) > P($\underline{l}$) P($\underline{e}$) iff $y_1 + y_2 + y_9 + y_{10} > (y_1 + y_2 + y_5 + y_6 + y_9 + y_{10} + y_{13} + y_{14})(y_1 + \ldots + y_4 + y_9 + \ldots + y_{12})$.

$\underline{e}$ confirms$_B$ $\underline{k}$ iff P($\underline{k}$ & $\underline{e}$) > P($\underline{k}$) P($\underline{e}$) iff $y_1 + y_3 + y_9 + y_{11} > (y_1 + y_3 + y_5 + y_7 + y_9 + y_{11} + y_{13} + y_{15})(y_1 + \ldots + y_4 + y_9 + \ldots + y_{12})$.

The following assignment of values shows that the antecedent conditions can be true while the consequent is false:

$y_1 = .01$	$y_5 = .01$	$y_9 = .01$	$y_{13} = .01$
$y_2 = 0$	$y_6 = 0$	$y_{10} = .01$	$y_{14} = .01$
$y_3 = .01$	$y_7 = .01$	$y_{11} = .01$	$y_{15} = .6$

$y_4 = .01 \qquad y_8 = .01 \qquad y_{12} = .2 \qquad y_{16} = .09$

Thus C2 is violated by confirmation$_B$ and consequently by confirmation$_C$ as well.

C2.1: If *e* confirms *h* and *h* entails *k*, then *e* confirms *k*.

h entails *k* iff $P(h \,\&\, -k) = 0$ iff $x_2 + x_4 = 0$.

e confirms$_A$ *h* iff $x_1 + x_2 > x_5 + x_6$.

Examine the consequent that *e* confirms$_A$ *k*:

e confirms$_A$ *k* iff $P(k \,\&\, e) > P(-k \,\&\, e)$ iff $x_1 + x_5 > x_2 + x_6$.

Since $x_2 = 0$, we obtain that *e* confirms$_A$ *k* iff $x_1 + x_5 > x_6$. From the antecedent conditions we obtain that $x_1 > x_5 + x_6$, and from this the consequent of C2.1 follows. Hence C2.1 holds for confirmation$_A$.

e confirms$_B$ *h* iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent of C2.1:

e confirms$_B$ *k* iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

The following assignment shows that the antecedent conditions can hold while the consequent is false:

$x_1 = .01 \qquad x_3 = .01 \qquad x_5 = .01 \qquad x_7 = .4$

$x_2 = 0 \qquad x_4 = 0 \qquad x_6 = .4 \qquad x_8 = .17$

Thus C2.1 is violated by confirmation$_B$ and hence also by confirmation$_C$.

C2.2: If e confirms h and h is logically equivalent to k, then e confirms k.

h is logically equivalent to k iff h entails k and k entails h iff P(h & -k) = 0 and P(-h & k) = 0 iff $x_2 + x_4 = 0$ and $x_5 + x_7 = 0$ iff x_2, x_4, x_5, and $x_7 = 0$.

e confirms$_A$ h iff $x_1 + x_2 > x_5 + x_6$ iff $x_1 > x_6$ since x_2 and $x_5 = 0$.

Examine the consequent of C2.2:

e confirms$_A$ k iff $x_1 + x_5 > x_2 + x_6$ iff $x_1 > x_6$ since x_2 and $x_5 = 0$. Thus C2.2 holds for confirmation$_A$.

e confirms$_B$ h iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$ iff $x_1 > (x_1 + x_3)(x_1 + x_6)$ since x_2, x_4, and $x_5 = 0$.

Examine the consequent of C2.2:

e confirms$_B$ k iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$ iff $x_1 > (x_1 + x_3)(x_1 + x_6)$ since x_2, x_5, and $x_7 = 0$. Thus C2.2 holds for confirmation$_B$ and also for confirmation$_C$, since C2.2 also holds for confirmation$_A$.

C2.3: If e confirms h and e confirms k then e confirms h & k.

e confirms$_A$ h iff $x_1 + x_2 > x_5 + x_6$.

e confirms$_A$ k iff $x_1 + x_5 > x_2 + x_6$.

Examine the consequent that e confirms$_A$ h & k.

e confirms$_A$ h & k iff P(h & k & e) > P(-(h & k) & e) iff $x_1 > x_2 + x_5 + x_6$.

The following assignment shows that the antecedent conditions can be true while the consequent is false:

$x_1 = .2$	$x_3 = .02$	$x_5 = .3$	$x_7 = .03$
$x_2 = .3$	$x_4 = .02$	$x_6 = .1$	$x_8 = .03$

Thus C2.3 is violated by confirmation_A.

$\underline{e}$ confirms_B $\underline{h}$ iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

$\underline{e}$ confirms_B $\underline{k}$ iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent that $\underline{e}$ confirms_B $\underline{h}$ & $\underline{k}$:

$\underline{e}$ confirms_B $\underline{h}$ & $\underline{k}$ iff $P(\underline{h} \& \underline{k} \& \underline{e}) > P(\underline{h} \& \underline{k})\ P(\underline{e})$ iff $x_1 > (x_1 + x_3)(x_1 + x_2 + x_5 + x_6)$.

The following assignment shows that the antecedent conditions can be true while the consequent is false:

$x_1 = .01$	$x_3 = .1$	$x_5 = .2$	$x_7 = .1$
$x_2 = .2$	$x_4 = .1$	$x_6 = .1$	$x_8 = .19$

Thus C2.3 does not hold for confirmation_B and hence neither for confirmation_C.

C2.4: If $\underline{e}$ confirms one of $\underline{h}$ and $\underline{k}$, then $\underline{e}$ confirms $\underline{h}$ v $\underline{k}$.

Let $\underline{e}$ confirm_A $\underline{h}$. Hence $x_1 + x_2 > x_5 + x_6$.

Examine the consequent that $\underline{e}$ confirms_A $\underline{h}$ v $\underline{k}$:

$\underline{e}$ confirms_A $\underline{h}$ v $\underline{k}$ iff $P((\underline{h} \vee \underline{k}) \& \underline{e}) > P(-\underline{h} \& -\underline{k} \& \underline{e})$ iff $x_1 + x_2 + x_5 > x_6$.

From $x_1 + x_2 > x_5 + x_6$ it follows that $x_1 + x_2 + x_5 > x_6$.

Thus C2.4 holds for confirmation_A.

Let e confirm_B h. Thus $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent that e confirms_B h v k:

e confirms_B h v k iff $P((h \vee k) \,\&\, e) > P(h \vee k)\, P(e)$ iff $x_1 + x_2 + x_5 > (x_1 + \ldots + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .1$	$x_3 = .01$	$x_5 = .01$	$x_7 = .84$
$x_2 = .01$	$x_4 = .01$	$x_6 = .01$	$x_8 = .01$

Thus C2.4 does not hold for confirmation_B and thus neither for confirmation_C.

C2.5: If e confirms h & k then e confirms h and e confirms k.

e confirms_A h & k iff $x_1 > x_2 + x_5 + x_6$.

Examine the consequent for confirmation_A:

e confirms_A h iff $x_1 + x_2 > x_5 + x_6$, and e confirms_A k iff $x_1 + x_5 > x_2 + x_6$.

Given that $x_1 > x_2 + x_5 + x_6$, it follows that $x_1 + x_2 > x_2 + x_5 + x_6 + x_2 > x_5 + x_6$ and that $x_1 + x_5 > x_2 + x_5 + x_6 + x_5 > x_2 + x_6$.

Thus C2.5 holds for confirmation_A.

e confirms_B h & k iff $x_1 > (x_1 + x_3)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent for confirmation_B:

e confirms_B h iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 +$

$x_5 + x_6$), and e confirms$_B$ k iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .1$	$x_3 = .01$	$x_5 = .01$	$x_7 = .3$
$x_2 = .01$	$x_4 = .2$	$x_6 = .3$	$x_8 = .07$

Thus C2.5 is violated by confirmation$_B$ and hence also by confirmation$_C$.

C3: Every consistent evidence sentence is logically compatible with all the sentences which it confirms.

In order to examine this condition with respect to the concept of confirmation$_A$, I shall consider the case when e confirms three sentences. I shall show that the following condition is false: If e confirms$_A$ h, e confirms$_A$ k, and e confirms$_A$ l, then e, h, k, and l are consistent. Similar conditions in which more than three sentences are confirmed by e will also be violated by confirmation$_A$.

e confirms$_A$ h iff $y_1 + \ldots + y_4 > y_9 + \ldots + y_{12}$.

e confirms$_A$ k iff $y_1 + y_3 + y_9 + y_{11} > y_2 + y_4 + y_{10} + y_{12}$.

e confirms$_A$ l iff $y_1 + y_2 + y_9 + y_{10} > y_3 + y_4 + y_{11} + y_{12}$.

Examine the consequent that e, h, k, and l are consistent.

e, h, k, and l are consistent iff $P(h \& e \& l \& k) \neq 0$ iff $y_1 \neq 0$.

The following assignment shows that y_1 can be 0 while the antecedent conditions hold:

$y_1 = 0$	$y_5 = 0$	$y_9 = .2$	$y_{13} = .01$
$y_2 = .2$	$y_6 = .01$	$y_{10} = .1$	$y_{14} = .01$
$y_3 = .2$	$y_7 = .01$	$y_{11} = .1$	$y_{15} = .02$
$y_4 = .1$	$y_8 = .01$	$y_{12} = .01$	$y_{16} = .02$

Thus C3 does not hold for confirmation$_A$.

In order to examine C3 with respect to confirmation$_B$ it is sufficient to show that the following condition is false: If e confirms$_B$ h and e confirms$_B$ k, then e, h, and k are consistent. Every similar condition in which more than two sentences are confirmed by e will also be violated by confirmation$_B$.

e confirms$_B$ h iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

e confirms$_B$ k iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent that e, h, and k are consistent: e, h, and k are consistent iff $P(h \& e \& k) \neq 0$ iff $x_1 \neq 0$.

The following assignment shows that x_1 can be 0 while the antecedent conditions hold:

$x_1 = 0$	$x_3 = 0$	$x_5 = .4$	$x_7 = .01$
$x_2 = .4$	$x_4 = .01$	$x_6 = .01$	$x_8 = .17$

Thus C3 does not hold for confirmation$_B$ and hence neither for confirmation$_C$.

C3.1: The class K of all sentences confirmed by a consistent sentence *e* is consistent.

In order to show that C3.1 does not hold for confirmation_A it is sufficient to show that the following condition is false: If *e* confirms_A *h*, *e* confirms_A *k*, and *e* confirms_A *l*, then *h*, *k*, and *l* are consistent. Similar conditions in which more than three sentences are confirmed by *e* will also be violated by confirmation_A.

> The proof required here closely parallels the proof given under C3 above. In fact only the consequents are different. The consequent which requires examination here is that *h*, *k*, and *l* are consistent, which occurs iff $P(h \,\&\, k \,\&\, l) \neq 0$ iff $y_1 + y_5 \neq 0$. The assignment given under C3 shows that the antecedent conditions can hold while this consequent is false. Thus C3.1 is violated by confirmation_A.

In order to show that C3.1 does not hold for confirmation_B it is sufficient to show that the following condition is false: If *e* confirms_B *h* and *e* confirms_B *k* then *h* and *k* are consistent. Similar conditions in which more than two sentences are confirmed by *e* will also be violated by confirmation_B.

> The proof required here again closely parallels the proof given under C3 above for confirmation_B. Again only the consequents differ. The consequent which requires examination here is that *h* and *k* are consistent, which occurs iff

$P(\underline{h} \,\&\, \underline{k}) \neq 0$ iff $x_1 + x_3 \neq 0$. The assignment given under C3 shows that the antecedent conditions can hold while this consequent is false. Hence, C3.1 is violated by confirmation$_B$, and thus also by confirmation$_C$.

C3.2: If e and h are logically incompatible then e does not confirm h.

e and h are logically incompatible iff e & h is logically false iff $P(\underline{h} \,\&\, \underline{e}) = 0$ iff $w = 0$.

e does not confirm$_A$ h iff $w \leq y$. Since $w = 0$, $0 \leq y$.

Thus C3.2 holds for confirmation$_A$.

e does not confirm$_B$ h iff $w \leq (w + x)(w + y)$. Since $w = 0$, $0 = w \leq xy$ and C3.2 holds for confirmation$_B$. C3.2 holds for confirmation$_C$ as well.

C3.3: It is not the case that e confirms h and e confirms -h.

Assume the contradictory of C3.3 for confirmation$_A$.

e confirms$_A$ h iff $w > y$.

e confirms$_A$ -h iff $P(-\underline{h} \,\&\, \underline{e}) > P(\underline{h} \,\&\, \underline{e})$ iff $y > w$.

From these two statements it follows that $w > y$ and $y > w$ — a contradiction. Therefore the assumption is false.

Hence, C3.3 holds for confirmation$_A$.

Assume the contradictory of C3.3 for confirmation$_B$.

e confirms$_B$ h iff $w > (w + x)(w + y)$.

e confirms$_B$ -h iff $P(-\underline{h} \,\&\, \underline{e}) > P(-\underline{h})\, P(\underline{e})$ iff $y > (y + z)(w + y)$.

From the two foregoing statements it follows that $w + y > (w + x)(w + y) + (y + z)(w + y)$; hence, that $w + y > (w + y)(w + x + y + z)$ and thus that $w + y > w + y$ — a contradiction. Therefore the assumption is false. Hence C3.3 holds for confirmation$_B$ and, since C3.3 holds for confirmation$_A$, also for confirmation$_C$.

C3.4: If _h_ is inconsistent then _e_ does not confirm _h_.

_h_ is inconsistent iff $P(h) = 0$ iff $w + x = 0$ iff w and $x = 0$.

_e_ does not confirm$_A$ _h_ iff $w \leq y$. Since $w = 0$, $w \leq y$ and C3.4 holds for confirmation$_A$.

_e_ does not confirm$_B$ _h_ iff $w \leq (w + x)(w + y)$. Since $w = 0$ and $w + x = 0$, $0 \leq 0$ and C3.4 holds for confirmation$_B$. C3.4 also holds for confirmation$_C$.

C4: If _e_ confirms _h_, and _h_ is a logical consequence of _k_, then _e_ confirms _k_.

_h_ is a logical consequence of _k_ iff $x_5 + x_7 = 0$ iff x_5 and x_7 equal 0.

_e_ confirms$_A$ _h_ iff $x_1 + x_2 > x_5 + x_6$.

Examine the consequent that _e_ confirms$_A$ _k_:

_e_ confirms$_A$ _k_ iff $x_1 + x_5 > x_2 + x_6$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .1$ $x_3 = .2$ $x_5 = 0$ $x_7 = 0$

$x_2 = .1$ $x_4 = .2$ $x_6 = .1$ $x_8 = .3$

Thus C4 does not hold for confirmation$_A$.

e confirms$_B$ h iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent of C4 for confirmation$_B$:

e confirms$_B$ k iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .01$ $x_3 = .1$ $x_5 = 0$ $x_7 = 0$

$x_2 = .5$ $x_4 = .1$ $x_6 = .2$ $x_8 = .09$

Thus C4 does not hold for confirmation$_B$; hence, neither for confirmation$_C$.

C5: If h entails e then e confirms h.

h entails e iff $P(h \& -e) = 0$ iff $x = 0$.

Consider the consequent that e confirms$_A$ h:

e confirms$_A$ h iff $w > y$.

The following assignment shows that the antecedent can be true while the consequent is false:

$w = .1$ $x = 0$ $y = .5$ $z = .4$

Thus C5 is violated by confirmation$_A$.

Consider the consequent that e confirms$_B$ h:

e confirms$_B$ h iff $w > (w + x)(w + y)$ iff $w(w + y)$, given that $x = 0$. Now $w > w(w + y)$ provided $w + y \neq 1$, i.e., provided e is not logically true, and provided $w \neq 0$, thus provided $w + x \neq 0$ (since $x = 0$), i.e., provided h is not logically false. Thus the following modified condition holds for confirmation_B:

MC5: If h entails e, h is not logically false, and e is not logically true, then e confirms h.

Neither C5 nor MC5 hold for confirmation_C, since neither hold for confirmation_A.

C5.1: Evidence e confirms e & h.

e confirms_A e & h iff $P(e \& h \& e) > P(-(e \& h) \& e)$ iff $P(h \& e) > P((-e \vee -h) \& e)$ iff $P(h \& e) > P(-h \& e)$ iff $w > y$. This does not hold in general and so C5.1 does not hold for confirmation_A.

e confirms_B e & h iff $P(e \& h \& e) > P(h \& e)\, P(e)$ iff $P(h \& e) > P(h \& e)\, P(e)$. This will hold unless $P(h \& e) = 0$ or $P(e) = 1$. $P(h \& e) = 0$ iff h & e is logically false, and $P(e) = 1$ iff e is logically true. Thus the following modified condition holds for confirmation_B:

MC5.1: Evidence e confirms e & h, provided e is not logically true nor e & h logically false.

Neither C5.1 nor MC5.1 hold for confirmation_C, since neither hold for confirmation_A.

C6: If e confirms h then h confirms e.

e confirms_A h iff $w > y$.

$\underline{h}$ confirms$_A$ $\underline{e}$ iff $P(\underline{h} \& \underline{e}) > P(\underline{h} \& -\underline{e})$ iff $w > x$.

It does not follow from the fact that $w > y$ that $w > x$, so C6 does not hold for confirmation$_A$. Hence C6 does not hold for confirmation$_C$.

$\underline{e}$ confirms$_B$ $\underline{h}$ iff $P(\underline{h} \& \underline{e}) > P(\underline{h})\ P(\underline{e})$.

$\underline{h}$ confirms$_B$ $\underline{e}$ iff $P(\underline{h} \& \underline{e}) > P(\underline{h})\ P(\underline{e})$.

The consequent obviously follows from the antecedent, so C6 holds for confirmation$_B$.

C7: If $\underline{e}$ confirms $\underline{h}$ then $-\underline{h}$ confirms $-\underline{e}$.

$\underline{e}$ confirms$_A$ $\underline{h}$ iff $w > y$.

$-\underline{h}$ confirms$_A$ $-\underline{e}$ iff $P(-\underline{h} \& -\underline{e}) > P(-\underline{h} \& e)$ iff $z > y$.

It does not follow from the fact that $w > y$ that $z > y$, so C7 does not hold for confirmation$_A$. Hence C7 does not hold for confirmation$_C$ either.

$\underline{e}$ confirms$_B$ $\underline{h}$ iff $w > (w + x)(w + y)$.

$-\underline{h}$ confirms$_B$ $-\underline{e}$ iff $P(-\underline{h} \& -\underline{e}) > P(-\underline{h})\ P(-\underline{e})$ iff $z > (y + z)(x + z)$.

Since $1 = w + x + y + z$, $z = 1 - w - x - y$. Thus $-\underline{h}$ confirms$_B$ $-\underline{e}$ iff $1 - w - x - y > (y + 1 - w - x - y)(x + 1 - w - x - y)$ iff $w > (w + x)(w + y)$. The consequent obviously follows from the antecedent, and so C7 holds for confirmation$_B$.

C8: If $\underline{e}$ confirms $\underline{h}$ and $\underline{h}$ confirms $\underline{k}$, then $\underline{e}$ confirms $\underline{k}$.

$\underline{e}$ confirms$_A$ $\underline{h}$ iff $x_1 + x_2 > x_5 + x_6$.

$\underline{h}$ confirms$_A$ $\underline{k}$ iff $x_1 + x_3 > x_2 + x_4$.

Examine the consequent of C8 for confirmation$_A$:

$\underline{e}$ confirms$_A$ $\underline{k}$ iff $x_1 + x_5 > x_2 + x_6$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .1$	$x_3 = .2$	$x_5 = .01$	$x_7 = .37$
$x_2 = .2$	$x_4 = .01$	$x_6 = .01$	$x_8 = .1$

Thus C8 does not hold for confirmation$_A$.

$\underline{e}$ confirms$_B$ $\underline{h}$ iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

$\underline{h}$ confirms$_B$ $\underline{k}$ iff $x_1 + x_3 > (x_1 + \ldots + x_4)(x_1 + x_3 + x_5 + x_7)$.

Examine the consequent of C8 for confirmation$_B$:

$\underline{e}$ confirms$_B$ $\underline{k}$ iff $x_1 + x_5 > (x_1 + x_3 + x_5 + x_7)(x_1 + x_2 + x_5 + x_6)$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .02$	$x_3 = .2$	$x_5 = .02$	$x_7 = .2$
$x_2 = .2$	$x_4 = .04$	$x_6 = .2$	$x_8 = .12$

Thus C8 does not hold for confirmation$_B$; hence, neither for confirmation$_C$.

C9: If $\underline{e}$ confirms $\underline{h}$ and $\underline{k}$ entails $\underline{e}$, then $\underline{k}$ confirms $\underline{h}$.

$\underline{k}$ entails $\underline{e}$ iff $x_3 + x_7 = 0$ iff x_3 and x_7 equal 0.

$\underline{e}$ confirms$_A$ $\underline{h}$ iff $x_1 + x_2 > x_5 + x_6$.

Examine the consequent that *k* confirms$_A$ *h*:

k confirms$_A$ *h* iff $x_1 + x_3 > x_5 + x_7$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .1$	$x_3 = 0$	$x_5 = .3$	$x_7 = 0$
$x_2 = .4$	$x_4 = .05$	$x_6 = .1$	$x_8 = .05$

Thus C9 does not hold for confirmation$_A$.

e confirms$_B$ *h* iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$.

Examine the consequent that *k* confirms$_B$ *h*:

k confirms$_B$ *h* iff $x_1 + x_3 > (x_1 + \ldots + x_4)(x_1 + x_3 + x_5 + x_7)$.

The following assignment shows that the antecedent can be true while the consequent is false:

$x_1 = .01$	$x_3 = 0$	$x_5 = .1$	$x_7 = 0$
$x_2 = .5$	$x_4 = .1$	$x_6 = .1$	$x_8 = .19$

Thus C9 does not hold for confirmation$_B$; hence, neither for confirmation$_C$.

C9.1: If *e* confirms *h*, and *k* is logically equivalent to *e*, then *k* confirms *h*.

k is logically equivalent to *e* iff *k* entails *e* and *e* entails *k* iff P(*k* & -*e*) = 0 and P(-*k* & *e*) = 0 iff $x_3 + x_7 = 0$ and $x_2 + x_6 = 0$ iff x_2, x_3, x_6, and x_7 equal 0.

e confirms$_A$ *h* iff $x_1 + x_2 > x_5 + x_6$ iff $x_1 > x_5$ given that x_2 and x_6 equal 0.

Examine the consequent that k confirms$_A$ h:

k confirms$_A$ h iff $x_1 + x_3 > x_5 + x_7$ iff $x_1 > x_5$ given that x_3 and x_7 equal 0.

The consequent follows from the two antecedent conditions and so C9.1 holds for confirmation$_A$.

e confirms$_B$ h iff $x_1 + x_2 > (x_1 + \ldots + x_4)(x_1 + x_2 + x_5 + x_6)$ iff $x_1 > (x_1 + x_4)(x_1 + x_5)$ given that x_2, x_3, and x_6 equal 0.

Examine the consequent that k confirms$_B$ h:

k confirms$_B$ h iff $x_1 + x_3 > (x_1 + \ldots + x_4)(x_1 + x_3 + x_5 + x_7)$ iff $x_1 > (x_1 + x_4)(x_1 + x_5)$ given that x_2, x_3, and x_7 equal 0.

The consequent follows from the two antecedent conditions and so C9.1 holds for confirmation$_B$. C9.1 also holds for confirmation$_C$.

This completes the proofs for the concepts of confirmation$_A$, confirmation$_B$, confirmation$_C$, and initial confirmation$_F$ with respect to the conditions of adequacy listed in the text. I now turn to consider the concept of confirmation$_F$ with respect to the correlative conditions of adequacy. In order to reduce the number of proofs, I make use of the fact that if a condition of adequacy is violated by the concept of initial confirmation$_F$, then the correlative condition of adequacy will be violated by the concept of confirmation_F.

The following tables will be consulted in connection with the concept of confirmation_F:

TABLE IV

$P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{b}) = x_1$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, \underline{b}) = x_5$
$P(\underline{h} \,\&\, \underline{e} \,\&\, -\underline{b}) = x_2$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, -\underline{b}) = x_6$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, \underline{b}) = x_3$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, \underline{b}) = x_7$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{b}) = x_4$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{b}) = x_8$

TABLE V

$P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{b} \,\&\, \underline{k}) = y_1$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, \underline{b} \,\&\, \underline{k}) = y_9$
$P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{b} \,\&\, -\underline{k}) = y_2$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, \underline{b} \,\&\, -\underline{k}) = y_{10}$
$P(\underline{h} \,\&\, \underline{e} \,\&\, -\underline{b} \,\&\, \underline{k}) = y_3$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, -\underline{b} \,\&\, \underline{k}) = y_{11}$
$P(\underline{h} \,\&\, \underline{e} \,\&\, -\underline{b} \,\&\, -\underline{k}) = y_4$	$P(-\underline{h} \,\&\, \underline{e} \,\&\, -\underline{b} \,\&\, -\underline{k}) = y_{12}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, \underline{b} \,\&\, \underline{k}) = y_5$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, \underline{b} \,\&\, \underline{k}) = y_{13}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, \underline{b} \,\&\, -\underline{k}) = y_6$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, \underline{b} \,\&\, -\underline{k}) = y_{14}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{b} \,\&\, \underline{k}) = y_7$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{b} \,\&\, \underline{k}) = y_{15}$
$P(\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{b} \,\&\, -\underline{k}) = y_8$	$P(-\underline{h} \,\&\, -\underline{e} \,\&\, -\underline{b} \,\&\, -\underline{k}) = y_{16}$

The probabilistic interpretation of the concept of confirmation_F yields the following assignment of values:

$\underline{e}$ confirms_F $\underline{h}$ given $\underline{b}$ iff $P(\underline{h}, \underline{e} \,\&\, \underline{b}) > P(\underline{h}, \underline{b})$ iff $P(\underline{h} \,\&\, \underline{e} \,\&\, \underline{b})\ P(\underline{b}) > P(\underline{h} \,\&\, \underline{b})\ P(\underline{e} \,\&\, \underline{b})$ iff (a) $x_1(x_1 + x_3 + x_5 + x_7) > (x_1 + x_3)(x_1 + x_5)$ iff $x_1x_7 > x_3x_5$ (according to Table IV), or (b) $(y_1 + y_2)(y_1 + y_2 + y_5 +$

$y_6 + y_9 + y_{10} + y_{13} + y_{14}) > (y_1 + y_2 + y_5 + y_6)(y_1 + y_2 + y_9 + y_{10})$ iff $(y_1 + y_2)(y_{13} + y_{14}) > (y_5 + y_6)(y_9 + y_{10})$ (according to Table V).

The restriction that e & b is not logically false is adopted. Thus P(e & b) $\neq 0$ and so $x_1 + x_5 \neq 0$ (according to Table IV), and $y_1 + y_2 + y_9 + y_{10} \neq 0$ (according to Table V). The proofs for confirmation$_F$ now follow.

CC1: If e entails h then e confirms h given b.

e entails h iff $x_5 + x_6 = 0$ iff x_5 and x_6 equal 0.

e confirms$_F$ h given b iff $x_1x_7 > x_3x_5$. Since $x_5 = 0$, $x_1x_7 > x_3x_5$ in general provided $x_1 \neq 0$ and $x_7 \neq 0$ — thus provided $x_1 + x_5 \neq 0$ and $x_5 + x_7 \neq 0$, i.e., provided b does not entail -e and provided b does not entail h. Thus the following modified condition holds for confirmation$_F$:

MCC1: If e entails h, and b entails neither h nor -e, then e confirms h given b.

CC1': If e & b entails h then e confirms h given b.

e & b entails h iff P(-h & e & b) = 0 iff $x_5 = 0$.

e confirms$_F$ h given b iff $x_1x_7 > x_3x_5$. Since $x_5 = 0$, $x_1x_7 > x_3x_5$ in general provided $x_1 \neq 0$ and $x_7 \neq 0$. These provisos are identical with the provisos required in the preceeding condition CC1, and lead to the following modified condition holding for confirmation$_F$:

MCC1': If e & b entails h, b entails neither h nor -e, then e confirms h given b.

CC1.1: If h is logically true then e confirms h given b.

C1.1 is violated by initial confirmation$_F$, so CC1.1 is violated by confirmation$_F$.

CC1.1': If h & b is logically true then e confirms h given b.

h & b is logically true iff $x_1 + x_3 = 1$ iff $x_2 + x_4 + \ldots + x_8 = 0$ iff $x_2, x_4, \ldots, x_8 = 0$.

e confirms$_F$ h given b iff $x_1x_7 > x_3x_5$.

Since $x_5 = x_7 = 0$, $x_1x_7 \not> x_3x_5$ and thus CC1.1' does not hold for confirmation$_F$.

CC1.2 is eliminated from consideration because it violates the restriction that e & b is not logically false. CC1.2 requires that e be logically false, from which it follows that e & b is logically false.

CC1.2' is eliminated from consideration because it violates the restriction that e & b is not logically false.

CC1.3: Evidence e confirms e given b.

e confirms$_F$ e given b iff P(e & e & b) P(b) > P(e & b) x P(e & b) iff P(e & b) P(b) > P(e & b) P(e & b) iff P(b) > P(e & b) iff $x_1 + x_3 + x_5 + x_7 > x_1 + x_5$ iff $x_3 + x_7 > 0$ iff b does not entail e. Thus the following modified condition holds for confirmation$_F$:

MCC1.3: If b does not entail e then e confirms e given b.

CC2: If e confirms, given b, every sentence of a class K, then e also confirms, given b, every logical consequence of K.

C2 is violated by initial confirmation$_F$, so CC2 will be violated by confirmation$_F$.

CC2.1: If e confirms h given b, and h entails k, then e confirms k given b.

C2.1 is violated by initial confirmation$_F$, so CC2.1 will be violated by confirmation$_F$.

CC2.2: If e confirms h given b and k is logically equivalent to h, then e confirms k given b.

e confirms$_F$ h given b iff $(y_1 + y_2)(y_{13} + y_{14}) > (y_5 + y_6)(y_9 + y_{10})$.

k is logically equivalent to h iff k entails h and h entails k iff P(k & -h) = 0 and P(h & -k) = 0 iff $y_9 + y_{11} + y_{13} + y_{15} = 0$ and $y_2 + y_4 + y_6 + y_8 = 0$ iff $y_2, y_4, y_6, y_8, y_9, y_{11}, y_{13}, y_{15} = 0$.

Thus e confirms$_F$ h given b iff $y_1y_{14} > y_5y_{10}$, given that $y_2, y_6, y_9, y_{13} = 0$.

Examine the consequent of CC2.2:

e confirms$_F$ k given b iff $(y_1 + y_9)(y_6 + y_{14}) > (y_5 + y_{13})(y_2 + y_{10})$.

Given that $y_2, y_6, y_9, y_{13} = 0$, e confirms$_F$ k given b iff $y_1y_{14} > y_5y_{10}$, and so CC2.2 holds for confirmation$_F$.

CC2.3: If *e* confirms *h* given *b* and *e* confirms *k* given *b* then *e* confirms *h* & *k* given *b*.

C2.3 is violated by initial confirmation_F and so CC2.3 is violated by confirmation_F.

CC2.4: If *e* confirms, given *b*, one of *h* and *k*, then *e* confirms *h* v *k* given *b*.

C2.4 is violated by initial confirmation_F and so CC2.4 is violated by confirmation_F.

CC2.5: If *e* confirms *h* & *k* given *b*, then *e* confirms *h* given *b* and *e* confirms *k* given *b*.

C2.5 is violated by initial confirmation_F and so CC2.5 is violated by confirmation_F.

CC3: Every consistent sentence is logically compatible with all the sentences it confirms.

C3 is violated by initial confirmation_F and so CC3 is violated by confirmation_F.

CC3.1: If *e* confirms *h* given *b* and *e* confirms *k* given *b*, then *h* and *k* are consistent.

C3.1 is violated by initial confirmation_F and so CC3.1 is violated by confirmation_F.

CC3.2: If *e* and *h* are logically incompatible, then *e* does not confirm h given *b*.

e and *h* are logically incompatible iff P(*h* & *e*) = 0 iff $x_1 + x_2 = 0$ iff x_1 and x_2 equal 0.

e does not confirm$_F$ h given b iff $x_1x_7 \leq x_3x_5$.

Since $x_1 = 0$, $0 = x_1x_7 \leq x_3x_5$ and CC3.2 holds for confirmation$_F$.

CC3.2': If e, h, and b are logically incompatible, then e does not confirm h given b.

e, h, and b are logically incompatible iff P(h & e & b) = 0 iff $x_1 = 0$.

e does not confirm$_F$ h given b iff $x_1x_7 \leq x_3x_5$.

Since $x_1 = 0$, $0 = x_1x_7 \leq x_3x_5$ and CC3.2' holds for confirmation$_F$.

CC3.3: It is not the case that e confirms h given b and that e confirms -h given b.

Assume the contradictory of CC3.3.

e confirms$_F$ h given b iff $x_1x_7 > x_3x_5$.

e confirms$_F$ -h given b iff P(-h & e & b) P(b) > P(-h & b) P(e & b) iff $x_5(x_1 + x_3 + x_5 + x_7) > (x_5 + x_7)(x_1 + x_5)$ iff $x_3x_5 > x_1x_7$. Contradiction! Therefore the assumption is false and CC3.3 holds for confirmation$_F$.

CC3.4: If h is inconsistent then e does not confirm h given b.

h is inconsistent iff $x_1 + \ldots + x_4 = 0$ iff $x_1, \ldots, x_4 = 0$.

e does not confirm$_F$ h given b iff $x_1x_7 \leq x_3x_5$. Since $x_1 = 0$, $0 = x_1x_7 \leq x_3x_5$ and CC3.4 holds for confirmation$_F$.

CC3.4': If h & b is inconsistent, then e does not confirm h given *b*.

h & *b* is inconsistent iff $x_1 + x_3 = 0$ iff x_1 and $x_3 = 0$.

e does not confirm$_F$ *h* given *b* iff $x_1x_7 \leq x_3x_5$. Since $x_1 = 0$, $0 = x_1x_7 \leq x_3x_5$ and CC3.4' holds for confirmation$_F$.

CC4: If *e* confirms *h* given *b* and *k* entails *h* then *e* confirms *k* given *b*.

C4 does not hold for initial confirmation$_F$, and so CC4 will not hold for confirmation$_F$.

CC5: If *h* entails *e* then *e* confirms *h* given *b*.

h entails *e* iff $x_3 + x_4 = 0$ iff x_3 and x_4 equal 0.

e confirms$_F$ *h* given *b* iff $x_1x_7 > x_3x_5$. Since $x_3 = 0$, $x_1x_7 > x_3x_5 = 0$ in general provided $x_1 \neq 0$ and $x_7 \neq 0$ — thus provided $x_1 + x_3 \neq 0$ and provided $x_3 + x_7 \neq 0$, i.e., provided *b* does not entail -*h* and provided *b* does not entail *e*. Thus the following modified condition holds for confirmation$_F$:

MCC5: If *h* entails *e*, and *b* entails neither -*h* nor *e*, then *e* confirms *h* given *b*.

CC5': If *h* & *b* entails *e* then *e* confirms *h* given *b*.

h & *b* entails *e* iff P(*h* & *b* & -*e*) = 0 iff $x_3 = 0$.

e confirms$_F$ *h* given *b* iff $x_1x_7 > x_3x_5$. Since $x_3 = 0$, $x_1x_7 > x_3x_5 = 0$ in general provided $x_1 \neq 0$ and $x_7 \neq 0$. These provisos are the same as those considered under CC5 above and lead to the following modified condition holding for confirmation_F:

MCC5': If *h* & *b* entails *e*, and *b* entails neither -*h* nor *e*, then *e* confirms *h* given *b*.

CC5.1: Evidence *e* confirms *e* & *h* given *b*.

e confirms_F *e* & *h* given *b* iff P(*e* & *h* & *e* & *b*) P(*b*) > P(*e* & *h* & *b*) P(*e* & *b*) iff P(*e* & *h* & *b*) P(*b*) > P(*e* & *h* & *b*) P(*e* & *b*) iff P(*b*) > P(*e* & *b*) (provided P(*e* & *h* & *b*) ≠ 0, i.e., provided $x_1 \neq 0$) iff $x_1 + x_3 + x_5 + x_7 > x_1 + x_5$ iff $x_3 + x_7 \neq 0$. The provisos require that $x_1 + x_3 + x_7 \neq 0$, which obtains iff P((*h* v -*e*) & *b*) ≠ 0 iff *b* does not entail *e* & -*h*. Thus the following modified condition holds for confirmation_F:

MCC5.1: Evidence *e* confirms *e* & *h* given *b* provided *b* does not entail *e* & -*h*.

CC6: If *e* confirms *h* given *b* then *h* confirms *e* given *b*.

e confirms_F *h* given *b* iff P(*h* & *e* & *b*) P(*b*) > P(*h* & *b*) x P(*e* & *b*).

h confirms_F *e* given *b* iff P(*h* & *e* & *b*) P(*b*) > P(*h* & *b*) x P(*e* & *b*).

The antecedent is identical with the consequent, so CC6 obviously holds for confirmation_F.

CC7: If *e* confirms *h* given *b* then -*h* confirms -*e* given *b*.

e confirms_F *h* given *b* iff $x_1x_7 > x_3x_5$.

-*h* confirms_F -*e* given *b* iff P(-*h* & -*e* & *b*) P(*b*) > P(-*h* & *b*)

x P(-e & b) iff $x_7(x_1 + x_3 + x_5 + x_7) > (x_5 + x_7)(x_3 + x_7)$ iff $x_1x_7 > x_3x_5$. Thus CC7 holds for confirmation$_F$.

CC8: If e confirms h given b and h confirms k given b then e confirms k given b.

C8 is violated by initial confirmation$_F$, so CC8 is violated by confirmation$_F$.

CC9: If e confirms h given b and k entails e then k confirms h given b.

C9 is violated by initial confirmation$_F$, so CC9 is violated by confirmation$_F$.

CC9.1: If e confirms h given b, and k is logically equivalent to e, then k confirms h given b.

e confirms$_F$ h given b iff $(y_1 + y_2)(y_{13} + y_{14}) > (y_5 + y_6)(y_9 + y_{10})$.

k is logically equivalent to e iff k entails e and e entails k iff P(k & -e) = 0 and P(-k & e) = 0 iff $y_5 + y_7 + y_{13} + y_{15} = 0$ and $y_2 + y_4 + y_{10} + y_{12} = 0$ iff $y_2, y_4, y_5, y_7, y_{10}, y_{12}, y_{13}, y_{15} = 0$.

Thus e confirms$_F$ h given b iff $y_1y_{14} > y_6y_9$, given that $y_2, y_5, y_{10}, y_{13} = 0$.

Examine the consequent that k confirms$_F$ h given b:

k confirms$_F$ h given b iff P(h & k & b) P(b) > P(h & b) x P(k & b) iff $(y_1 + y_5)(y_1 + y_2 + y_5 + y_6 + y_9 + y_{10} + y_{13} + y_{14}) > (y_1 + y_2 + y_5 + y_6)(y_1 + y_5 + y_9 + y_{13})$ iff

$(y_1 + y_5)(y_{10} + y_{14}) > (y_2 + y_6)(y_9 + y_{13})$.

Given that y_2, y_5, y_{10}, $y_{13} = 0$, k confirms$_F$ h given b iff $y_1 y_{14} > y_6 y_9$, and so CC9.1 holds for confirmation$_F$. This completes the proofs for the concept of confirmation$_F$ with respect to the correlative conditions of adequacy.

APPENDIX 2

The proof that the concept expressed in (12), when required to meet the probabilistic requirement set out in (24), violates condition of adequacy C2.2 is analogous to the many proofs given in Appendix 1. This proof does differ from the foregoing proofs, however, insofar as we are required to consult the following table of probability values.

TABLE VI

$P(h \& e \& l \& k \& j) = z_1$	$P(-h \& e \& l \& k \& j) = z_{17}$
$P(h \& e \& l \& k \& -j) = z_2$	$P(-h \& e \& l \& k \& -j) = z_{18}$
$P(h \& e \& l \& -k \& j) = z_3$	$P(-h \& e \& l \& -k \& j) = z_{19}$
$P(h \& e \& l \& -k \& -j) = z_4$	$P(-h \& e \& l \& -k \& -j) = z_{20}$
$P(h \& e \& -l \& k \& j) = z_5$	$P(-h \& e \& -l \& k \& j) = z_{21}$
$P(h \& e \& -l \& k \& -j) = z_6$	$P(-h \& e \& -l \& k \& -j) = z_{22}$
$P(h \& e \& -l \& -k \& j) = z_7$	$P(-h \& e \& -l \& -k \& j) = z_{23}$
$P(h \& e \& -l \& -k \& -j) = z_8$	$P(-h \& e \& -l \& -k \& -j) = z_{24}$
$P(h \& -e \& l \& k \& j) = z_9$	$P(-h \& -e \& l \& k \& j) = z_{25}$
$P(h \& -e \& l \& k \& -j) = z_{10}$	$P(-h \& -e \& l \& k \& -j) = z_{26}$
$P(h \& -e \& l \& -k \& j) = z_{11}$	$P(-h \& -e \& l \& -k \& j) = z_{27}$
$P(h \& -e \& l \& -k \& -j) = z_{12}$	$P(-h \& -e \& l \& -k \& -j) = z_{28}$
$P(h \& -e \& -l \& k \& j) = z_{13}$	$P(-h \& -e \& -l \& k \& j) = z_{29}$
$P(h \& -e \& -l \& k \& -j) = z_{14}$	$P(-h \& -e \& -l \& k \& -j) = z_{30}$
$P(h \& -e \& -l \& -k \& j) = z_{15}$	$P(-h \& -e \& -l \& -k \& j) = z_{31}$
$P(h \& -e \& -l \& -k \& -j) = z_{16}$	$P(-h \& -e \& -l \& -k \& -j) = z_{32}$

The sum of all the values in Table VI equals 1. As before, <u>e</u> is not logically false, and so $z_1 + \ldots + z_8 + z_{17} + \ldots + z_{24} \neq 0$. The probabilistic requirement of (24) yields the following result for the concept expressed in (12): *h* is more credible than *l* relative to *e*, where *h* and *l* are logically incompatible, iff $P(h,e) > P(l,e)$ and *h* & *l* is logically false iff $P(h \& e) > P(l \& e)$ and $P(h \& l) = 0$.

The statement requiring examination is as follows: If *h* is more credible than *l* relative to *e*, where *h* & *l* is logically false, and *h* is logically equivalent to *k*, then *k* is more credible than *j* relative to *e*, where *k* & *j* is logically false. This statement is just C2.2 with "*e* confirms *h*" and "*e* confirms *k*" substituted by the concept expressed by (12). That this statement is not logically true is shown by finding values such that the antecedent conditions hold while the consequent conditions do not hold. The proof proceeds as follows.

h is more credible than *l* relative to *e* iff $P(h \& e) > P(l \& e)$ iff $(z_1 + \ldots + z_8) > (z_1 + \ldots + z_4 + z_{17} + \ldots + z_{20})$ iff $z_5 + \ldots + z_8 > z_{17} + \ldots + z_{20}$.

h & *l* is logically false iff $P(h \& l) = 0$ iff $z_1 + \ldots + z_4 = 0$ iff z_1, z_2, z_3, and $z_4 = 0$.

h is logically equivalent to *k* iff $P(h \& -k) = 0$ and $P(-h \& k) = 0$ iff $z_3 + z_4 + z_7 + z_8 + z_{11} + z_{12} + z_{15} + z_{16} = 0$ and $z_{17} + z_{18} + z_{21} + z_{22} + z_{25} + z_{26} + z_{29} + z_{30} = 0$ iff $z_3, z_4, z_7, z_8, z_{11}, z_{12}, z_{15}, \ldots, z_{18}, z_{21}, z_{22}, z_{25}, z_{26}, z_{29}, z_{30} = 0$.

The three foregoing conditions taken together yield the following probabilistic statement which represents the antecedent: $z_5 + z_6 > z_{19} + z_{20}$.

Examine the consequent of the above condition.

k is more credible than j relative to e iff P(k & e) > P(j & e) iff $z_1 + z_2 + z_5 + z_6 + z_{17} + z_{18} + z_{21} + z_{22} > z_1 + z_3 + z_5 + z_7 + z_{17} + z_{19} + z_{21} + z_{23}$ iff $z_2 + z_6 + z_{18} + z_{22} > z_3 + z_7 + z_{19} + z_{23}$.

k & j is logically false iff $z_1 + z_5 + z_{17} + z_{21} = 0$ iff z_1, z_5, z_{17}, and $z_{21} = 0$.

If we take into consideration the probability values which are determined to be 0 by the antecedent conditions, then the consequent reduces to the following: $z_6 > z_{19} + z_{23}$.

It is now obvious that values can be assigned which will make the antecedent conditions true and the consequent false — $z_5 = 0$, $z_6 = .3$, $z_{19} = .1$, $z_{20} = .1$, and $z_{23} = .3$ will do. A quick check shows that this assignment of values allows P(e) > 0, since $z_6 > 0$, so the general requirement concerning e has been met. Hence condition of adequacy C2.2 is violated by the concept expressed in (12) when that concept is interpreted according to the probabilistic requirement expressed in (24).

APPENDIX 3

In order to show which relations of entailment between *h*, *e*, and *b* constitute sufficient conditions of confirmation_F, and in order to determine the number of such sufficient conditions, I consider the probabilistic statement '$P(h,e \& b) > P(h,b)$' which is a necessary and sufficient condition of confirmation_F. Now $P(h,e \& b) > P(h,b)$ iff $P(h \& e \& b)P(b) > P(h \& b) \times P(e \& b)$ iff $x_1x_7 > x_3x_5$ when Table IV in Appendix 1 above is consulted. Table IV is the appropriate table of probability values to consult in this case since we are considering the three statements *h*, *e*, and *b*.

The general conditions which have been adopted concerning $x_1, x_2, \ldots, x_8$ are only that (a) $x_1 + x_2 + \ldots + x_8 = 1$, (b) each one of $x_1, x_2, \ldots, x_8$ is non-negative, and (c) $x_1 + x_5 \neq 0$, representing that *e* & *b* is not logically false. Under these general conditions there are an infinite number of value-assignments which can be given to x_1, x_3, x_5, and x_7 such that the product x_1x_7 is greater than the product x_3x_5. There are, however, a finite number of value-assignments which can be given to x_1, x_3, x_5, and x_7 such that neither x_1 nor x_7 equal 0 but at least one of x_3 and x_5 equal 0. In such cases, of course, the left-hand side of the inequality '$x_1x_7 > x_3x_5$' is positive and the right-hand side is equal to 0. The value-assignments in such cases can be expressed by using the entailment relation, that is, either affirming or denying that there is a relation of entailment between two or more of h, e, and b. Consider the following example. Condition MCC5 asserts that a sufficient condition of confirmation_F is that h entails e and that b entails neither e nor -h. These entailment conditions ensure that x_1x_7 is positive and that x_3x_5 equals 0. For h entails e iff P(h & -e) equals 0 iff $x_3 + x_4 = 0$ iff x_3 and x_4 equal 0, and b entails neither e nor -h iff P(b & -e) $\neq$ 0 and P(b & h) $\neq$ 0 iff $x_3 + x_7 \neq 0$ and $x_1 + x_3 \neq 0$. Hence, since $x_3 = 0$, the right-hand side of the inequality '$x_1x_7 > x_3x_5$' equals 0, and since $x_3 + x_7 \neq 0$ and $x_1 + x_3 \neq 0$ (whereas $x_3 = 0$), it follows that both x_1 and x_7 are positive and the left-hand side of the inequality in question is positive. All of the different cases in which at least one of x_3 and x_5 equals 0 while both x_1 and x_7 are positive constitute sufficient conditions of confirmation_F. I calculate their number to be 15,000 as follows.

If $x_3 = 0$ then the product $x_3x_5 = 0$. In order for the left-hand side (LHS) of the inequality '$x_1x_7 > x_3x_5$' ("the inequality" hereinafter) to be positive, given that $x_3 = 0$, it is necessary for either $x_1 \neq 0$ and $x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_7 \neq 0$, or $x_1 \neq 0$ and $x_3 + x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_3 + x_7 \neq 0$. Corresponding to each of the foregoing value-assignments — $x_3 = 0$, $x_1 \neq 0$, $x_3 + x_7 \neq 0$, etc. — there is some entailment relation between two or more of h, e, and b, e.g., $x_3 = 0$ iff h & b entails e, $x_1 \neq 0$ iff h & e does not entail -b, and $x_1 + x_3 \neq 0$ iff h does not entail -b. There are therefore four different sufficient conditions of confirmation$_F$ corresponding to the assignments $x_3 = 0$ and either $x_1 \neq 0$ and $x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_7 \neq 0$, or $x_1 \neq 0$ and $x_3 + x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_3 + x_7 \neq 0$.

If $x_5 = 0$ then the product $x_3x_5 = 0$, and in order for the LHS of the inequality to be positive it is necessary that either $x_1 \neq 0$ and $x_7 \neq 0$, or $x_1 + x_5 \neq 0$ and $x_7 \neq 0$, or $x_1 \neq 0$ and $x_5 + x_7 \neq 0$, or $x_1 + x_5 \neq 0$ and $x_5 + x_7 \neq 0$. Entailment relations again correspond to each of these assignments, with the result that there are another four sufficient conditions of confirmation$_F$.

There are nine conditions involving two values, including either x_3 or x_5 (or both), in which x_3 and/or x_5 equal 0, viz., $x_2 + x_3 = 0$, $x_3 + x_4 = 0$, $x_3 + x_5 = 0$, $x_3 + x_6 = 0$, $x_3 + x_8 = 0$, $x_2 + x_5 = 0$, $x_4 + x_5 = 0$, $x_5 + x_6 = 0$, and $x_5 + x_8 = 0$. It must be remembered that $x_1 + x_5 \neq 0$ by the general requirement that e & b is not logically false. For each of these nine conditions, under which at least one of x_3 and x_5 equals 0, there is a relation of entailment between two or more of h, e, and b, e.g., $x_5 + x_6 = 0$ iff -h entails -e. Under each of the nine foregoing conditions the right-hand side (RHS) of the inequality equals 0. For each one of the nine conditions there are sixteen different conditions under which the LHS of the inequality is positive. For example, if $x_3 = 0$ by virtue of the assignment $x_2 + x_3 = 0$, then in order for the LHS of the inequality to be positive it is necessary for $x_1 \neq 0$ and $x_7 \neq 0$, or $x_1 + x_2 \neq 0$ and $x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_7 \neq 0$, or $x_1 + x_2 + x_3 \neq 0$ and $x_7 \neq 0$, or $x_1 \neq 0$ and $x_2 + x_7 \neq 0$, or $x_1 \neq 0$ and $x_3 + x_7 \neq 0$, or $x_1 \neq 0$ and $x_2 + x_3 + x_7 \neq 0$, or $x_1 + x_2 \neq 0$ and $x_2 + x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_3 + x_7 \neq 0$, or $x_1 + x_2 \neq 0$ and $x_3 + x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_2 + x_7 \neq 0$, or $x_1 + x_2 + x_3 \neq 0$ and $x_2 + x_7 \neq 0$, or $x_1 + x_2 + x_3 \neq 0$ and $x_3 + x_7 \neq 0$, or $x_1 + x_2 \neq 0$ and $x_2 + x_3 + x_7 \neq 0$, or $x_1 + x_3 \neq 0$ and $x_2 + x_3 + x_7 \neq 0$, or $x_1 + x_2 + x_3 \neq 0$ and $x_2 + x_3 + x_7 \neq 0$. An entailment relation of course corresponds to each of the sixteen foregoing statements. Hence, there are 144 sufficient conditions of confirmation_F in this group.

There are 16 conditions involving three values, including either x_3 or x_5 (or both), under which x_3 and/or x_5 equal 0, e.g., $x_2 + x_3 + x_5 = 0$, and $x_5 + x_6 + x_8 = 0$. For each of these 16 conditions there are 64 $((2^3)^2)$ conditions under which x_1 and x_7 do not equal 0. Hence, there are 1024 sufficient conditions of confirmation_F in this group.

There are 14 conditions involving four values, including either x_3 or x_5 (or both), under which x_3 and/or x_5 equal 0, e.g., $x_3 + x_4 + x_5 + x_8 = 0$, and $x_2 + x_3 + x_6 + x_8 = 0$. For each of these 14 conditions there are 256 $((2^4)^2)$ conditions under which x_1 and x_7 do not equal 0. There are therefore 3584 sufficient conditions of confirmation$_F$ in this group.

There are six conditions involving five values, including either x_3 or x_5 (or both), under which x_3 and/or x_5 equal 0, e.g., $x_2 + x_3 + x_5 + x_6 + x_8 = 0$. For each of these six conditions there are 1024 ($(2^5)^2$) conditions under which x_1 and x_7 do not equal 0. There are therefore 6144 sufficient conditions of confirmation$_F$ in this group.

There is one condition involving six values, including both x_3 and x_5, under which both x_3 and x_5 equal 0, *viz.*, $x_2 + x_3 + x_4 + x_5 + x_6 + x_8 = 0$. For this one condition there are 4096 ($(2^6)^2$) conditions under which x_1 and x_7 do not equal 0. There are therefore 4096 sufficient conditions of confirmation$_F$ in this group.

When all the sufficient conditions of confirmation$_F$ are added together these total exactly 15,000.

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