DIFFERENTIAL GAMES WITH NO INFORMATION

by

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Thesis submitted for the degree of Doctor of Philosophy in the University of Adelaide, September, 1971
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SUMMARY

This thesis contains the account of an extensive examination of differential games in which the players are granted no information concerning the evolution of the game. Mostly, games considered herein are two person zero-sum deterministic differential games of fixed duration, but many of the methods developed for dealing with these games appear to be applicable to a much wider class of differential games.

Chapter 1 contains a general introduction, a brief subjective account of the historical development of the subject, and a list of notations and conventions used throughout the thesis.

The main theme is taken up in Chapter 2, with the definition of the games to be considered. The nature of, and the relation between, the pure strategies of differential games with perfect information, and those of imperfect information is then discussed, and the difficulties of defining strategies for games of perfect information is illustrated. The chapter concludes by showing that if a differential game with no information has a saddle point in pure strategies, then these strategies will constitute a saddle point of any differential game with the same dynamics, whether of perfect or imperfect information.

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Games with open-loop saddle points are discussed in Chapters 3 and 4. In Chapter 3 is given a fairly general necessary condition for a pair of open-loop strategies to constitute a saddle point. Certain conditions, first stated (but not proved) by Fichesfet, for the existence of these saddle points are proved in this chapter, and some other existence conditions of Fichesfet are given much simpler proofs. The chapter concludes with the description of a procedure for deciding whether a differential game of a certain class has an open-loop saddle point.

By treating the payoffs as quadratic functions on a Hilbert space, Chapter 4 gives some necessary and sufficient conditions for an open loop strategy pair to be a saddle point of a linear–quadratic differential game. In the light of these results, and those of previous chapters, the traditional treatment of quadratic games with noise corrupted measurements is criticised.

The question of mixed strategies is taken up in Chapter 5. It is shown that any conceivable mixed strategy for a differential game with no information can be represented in an especially simple way. The existence of a value for a wide class of differential games, and of mixed strategy saddle points for a more restricted class is then established, and a general necessary condition for a mixed strategy pair to constitute a saddle point is given.

The thesis concludes with a discussion of the limitations and possible extensions of the results which it contains.