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GENERALIZED QUANTIZATION AND COLOUR ALGEBRAS

by

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ABSTRACT

An important solution of the paracommutation relations is the so-called Green ansatz. Recently it was observed that this may be constructed from an algebra which shall be termed in this thesis, a colour algebra. Colour algebras are natural generalizations of the better known superalgebras. Their generality suggests they may be the key to exploring further forms of quantization.

In Chapter 2 colour algebras are studied in their own right. It is observed that a colour algebra can be described by an abelian grading group and a complex valued commutation factor defined on this group. It is further observed that these two objects are, in general, not fixed for a particular colour algebra and in fact, a unique canonical pair may be found.

Another aspect of the classification problem for colour algebras is considered in section 3, where it is shown that there is an abstract algebraic map between colour algebras and "canonical" superalgebras. In section 4 it is shown how this abstract map may be implemented by a Klein transformation and how this allows one to show that a representation of a colour algebra can be obtained in a simple manner from a representation of its "canonical" superalgebra.

In Chapter 3 another method of quantization called modular quantization is examined. This is shown also to have a colour algebra ansatz solution— the relevant colour algebra being different to that for paraquantization. The uniqueness of this solution for Fock representations is examined and an algebraic vacuum condition (being a generalization of a similar paraquantization condition) is found which implies the solution. It is further shown that the only ansatz type solution is the one given. Relativistic complications are also examined.

In section 3 the question of suitable observables is discussed. A condition known as strong locality is imposed and a set of observables is demonstrated to satisfy the condition. Moreover these observables are shown to satisfy commutation relations that are a generalization of the paracommutation relations. Restrictions on the algebraic order of strongly local observables are then discussed.

Section 4 contains a comparison of a modular field theory and a normal field theory with a hidden $U(m)$ global gauge symmetry. This comparison is made possible by the Klein transfomation.

Finally in Chapter 4, a generalization of the modular quantization is examined.