GEOID STUDIES OF SOUTH AUSTRALIA

by

John Robert Gilliland, B. Tech., M.Sc.(Oxon.)

A thesis submitted to the University of Adelaide in fulfilment of the requirements for the degree of Doctor of Philosophy.

December, 1982.

CONTENTS

Contents (contd.) Page No.

Contents (contd.) Page No.

Contents (Contd.) Page No.

FIGURES AND TABLES

Figures

Tables

SUMMARY

Techniques used for the determination of the deflections of the vertical and the geoid separation are reviewed. These may be basically described as astro-geodetic, gravimetric, dynamic and geometric satellite, and combination methods.

Correlation of free air anomalies with height is investigated and the results show that the correlation is dependent on the size of the area and the variability of the Bouguer anomaly within the area. Some methods of interpolation of mean gravity anomalies are reviewed in order to determine a suitable technique for interpolating gravity anomalies in local areas where the gravity information is reasonably dense.

Gravity data is used to determine the free air geoid separation and the deflections of the vertical at half degree geographical grid points in the region of South Australia. Mean gravity anomalies from the Rapp 5° equal area and 1° x 1° data sets are used to evaluate the outer zones in both Stokes' integral and Vening-Meinesz formulae and the inner zones are evaluated using mean anomalies derived from observed gravity station data. The contributions to the free air geoid solution are evaluated for each zone. The total contribution shows a geoid separation N increasing in a north easterly direction with numerical values

ranging from -17 metres in south west to +23 metres in the north east of the State. The north-south component of the deflection of the vertical ξ ranges in value from approximately -20" to +4" with both extreme values being located in the north west where the gravity field is rapidly changing. The east-west component of the deflection of the vertical η changes in value from -12" to +5" with the maximum rate of change occurring in the northern Flinders Ranges.

Estimations of the standard deviation of N, ξ, and η are ±1.25 metres, ±0".55 and ±0".50 respectively. The major contribution to the standard deviation of N comes from the Outer Zone ($\psi > 20°$) while the major contributions to the standard deviations of ξ and η are derived from the Inner and Innermost Zones ($\psi \leqslant 1.5$).

The free air geoid solution is compared with results obtained by the use of astro-geodetic techniques, Doppler methods, spherical harmonic coefficient, and previous gravimetric investigations. In order to facilitate these comparisons detailed gravimetric surveys were performed in the immediate vicinity of thirteen trigonometric stations and the free air geoid values of N, ξ, and η at each station are evaluated. The comparisons show no evidence of position dependent systematic errors.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any university, and to the best of my knowledge and belief, it contains no material previously published or written by another person, except where due reference is made in the text.

J.R. Gilliland.

ACKNOWLEDGEMENTS

I am indebted to Professor D.M. Boyd and Dr. P.I. Brooker for their advice and encouragement during the preparation of this dissertation.

Gravity and surveying data for the South Australian region were obtained from:

Department of Lands, South Australia
Department of Mines and Energy, South Australia
Bureau of Mineral Resources, Geology and Geophysics, Canberra
Division of National Mapping, Canberra.

Professor R.H. Rapp, Department of Geodetic Science, Ohio State University, made available the 5° equal area and 1° x 1° geographical block mean gravity anomalies, and Dr. C. Rizos, Deutsches Geodätisches Forschungsinstitut, provided the computer program for the analysis of the GEM-10B spherical harmonic coefficients.

Acknowledgements are given to those students of the School of Surveying, South Australian Institute of Technology, who accompanied me on field trips, to Miss A. Carey for the typing of the first draft of this thesis, and to Mr. B. Couzner for assistance in the preparation of diagrams. Thanks are also due to Professor G.J.F. Holden for his encouragement and assistance with the final preparation of the documentation.

I am also particularly grateful to my wife Jennifer for the layout and typing of the final draft of this thesis.

CHAPTER 1

INTRODUCTION

The determination of the geoid with respect to some mathematical model has been and is one of the major aims in the field of geodesy. This information is needed for the determination of the figure of the earth's surface. Since most classical measurements made on or near the earth's surface are dependent in some way on the gravity field, the results are related to a potential surface. These results are then usually referenced or reduced to the geoid which is not a simple mathematical model, and then to the reference model. This latter step requires a knowledge of the geoid undulation.

1.1 The Concept of the Geoid

The potential of gravity, W, at any point on the earth's surface is the sum of the potential of the centrifugal force due to the diurnal rotation of the earth and the potential of the gravitational force. The surfaces defined by W = constant, are known as geopotential surfaces or geops. For each value of W there is another geopotential surface, thus there are a whole family of geops. Because of the variation in the earth's density these geops will tend to have irregular curvature characteristics. The direction of gravity is normal to all equipotential surfaces.

One particular geop is called the geoid. The geoid, with slight idealization, is coincident with the mean sea level extended over the whole of the earth's surface (Heiskanen & Moritz, 1967, p. 49).

The concept of the geoid was first proposed by Gauss as the mathematical force of the earth, and it was later termed the geoid by the geodesist Listing (Rizos, 1980). When the scientific community talk of the earth's shape it is this geoidal surface that is being referenced and not the actual physical surface of the earth with its valleys and mountains.

Like all geops, the geoid is not a uniformly symmetrical figure, but it approximates to an ellipsoid of revolution, and it is with respect to an ellipsoid that the geoid is usually mapped. The geoid is not a difficult surface (with acceptable tolerances) to physically locate by sea surface studies and levelling techniques. The difficulty arises when attempting to determine the relationship between the geoid and the mathematical reference surface, i.e. the geoid-ellipsoid separation and the deflections of the vertical.

1.2 The Reference Ellipsoid

The ellipsoid of revolution that closely approximates the earth is one rotated about its minor axis. Its physical size is usually defined by the lengths of the semi-major and semi-minor axis or by the semi-major axis and the flattening. An ideal reference ellipsoid is one where the dimensions are selected so that the geoid separations are minimal. An ellipsoid which gives a good fit for one region on the earth's surface may not be suitable for another.

Along with the concept of a geometrical reference figure it is also possible to consider that this figure is dynamic. That is to say, the reference ellipsoid is given a mass and an angular velocity. Thus, the surface of the ellipsoid is an equipotential surface and is referred to as the normal potential. Assuming a rigid earth, the size, shape, and potential characteristics of the ellipsoid are usually defined by the following factors (Mather, 1973.b):

(a) the value of GM where G is the gravitational constant and M is the mass;

(b) the constant rate of rotation ω of the ellipsoid;

(c) the semi-major axis a of the ellipsoid;

(d) the flattening f of the ellipsoid or the equivalent dynamic factor J_2.

Other factors are required to define the position of the reference ellipsoid in space. These are usually the location of the zero meridian, the centre, and the minor axis of the ellipsoid.

With the change of any one of the above parameters or factors a different reference system will result. The local geoid shape and lack of intercontinental ties in the past has led to the adoption of many different reference ellipsoids by various mapping agencies throughout the world. The International Association of Geodesy (IAG) meeting in Moscow in 1971 approved and adopted a reference ellipsoid known as the Geodetic Reference System 1967 (GRS-67) with the following parameters:

equatorial radius, a = 6378 160 metres

geocentric gravitational constant, $GM = 398603 \times 10^9 m^3 s^{-2}$

dynamic form factor, J_2 = 0.0010827

angular velocity, ω = $7.2921151467 \times 10^{-5}$ rad/sec

From the above, other parameters can be determined. Examples of these accurate to the number of decimal places given, are:

semi-minor axis, b = 6356774.5161

flattening, f = 0.003352 923712 99

reciprocal flattening, f^{-1} = 298.247167427

Using these parameters the normal gravity γ on the surface of the ellipsoid at a latitude ϕ, may be calculated from

$$\gamma = 978.03185(1 + 0.005278895 \sin^2\phi + 0.000023462 \sin^4\phi) \text{ gal.}$$

With the ever increasing information, particularly from satellite observations, the "ideal" world reference system is in continuing need of redefinition if the system is to reflect the most recent, best fitting values of the parameters. Against this is the need for a reference system that can be used as a base or reference for data measurement over a reasonable time period, without the need to continuously update that information. In Canberra, the IAG in December 1979 adopted a new reference system called the Geodetic Reference System, 1980 (GRS-80). All parameters referred to above, were redefined; one major change was the redefinition of the semi-major axis as a = 6378137 metres. The definition of the other paramters and equations can be found in GRS-80 (1980).

1.3 The Aim

The aim of this study is to determine a free air geoid solution for South Australia using all available gravity information and to evaluate the effects of random and systematic errors on the results obtained. The random errors are determined from a consideration of the standard deviations or errors of representation of the individual mean gravity anomalies used in the solution. The existence of any systematic errors is evaluated by comparisons with results determined using:

(a) astro-geodetic techniques,

(b) spherical harmonic coefficients,

(c) Doppler derived values,

(d) previous gravimetric type solutions.

No precise details of computer programs are given in this thesis, but all computations have been carried out on a Cyber 173 computer located at the Levels Campus of the South Australian Institute of Technology. All programs other than the acknowledged use of a program originating from Rizos (1979) for the analysis of the geoid undulations from spherical harmonic coefficients, have been written by the author. All computer results have been sampled and checked by independent methods using either different computer techniques on the Cyber, or a "desk top" computer or calculator when storage requirements allowed.

Since this work commenced before the introduction of the GRS-80 system, the free air geoid solution is referenced to the GRS-67 system. This has the added advantage that the Australian Reference Ellipsoid and the reference ellipsoid defined by GRS-67 have the same value for the semi-major axis and for most practical purposes the same value for the flattening, the value of 1/298.25 being the flattening for the Australian Reference Ellipsoid.

It is interesting to note that on comparisons made in the South Australian region, the free air geoid solution reference to the GRS-67 and GRS-80 give identical values for the deflection component in the east-west direction, which is to be expected from theoretical considerations, almost identical solutions for the deflection component in the north-south direction and differences in the geoid separation value of approximately 0.1 metres with a small variation in the north-south direction.

1.4 The Study Region

South Australia lies between the meridians 129°E and 141°E and is bounded in the north by the parallel 26°S, and the Southern Ocean as its southern boundary. The total area of the State is 984377 square kilometres, which is approximately one eighth the area of the continent of Australia. The land is generally of low relief with approximately 50 percent of the total topography having heights less than 150 metres above mean sea level, and over 80 percent having less than 300 metres (S.A. Year Book, 1981).

The inland area is largely covered by featureless plains and deserts. There are two major regions where the terrain elevations readily exceed 300 metres. One of these regions is the Mount Lofty-Flinders Ranges which extends north from the Southern Ocean, approximately centred on the 138°.5 E meridian, for some 800 kilometres. The highest point in these ranges is 1166 metres, which is low on world standards. The other ranges, comprising the Everard-Musgrave chain, are situated in the far north west region of the State. The highest point in these ranges, and the State, is 1440 metres.

South Australia is fully covered with 1:250 000 topographic maps, but many of these are not contoured. Larger scale maps with contours are available for the southern settled areas of the State, but for the majority of the State, detailed terrain height information as obtained from topographical maps is not available.

1.5 Synopsis of Contents

Chapter 2 discusses the techniques that are used for the determination of the deflections of the vertical and the geoid separation. These may be basically described as astro-geodetic, gravimetric, dynamic and geometric satellite, and combination methods. An outline of the methods is given along with definition of the deflections of the vertical and the geoid separation. At the end of the Chapter, in section 2.4.4, general comments and comparisons are made.

Geoid studies in Australia are reviewed in Chapter 3. This includes an outline of the considerations and methods used to select the Australian Geodetic Datum. Methods used by Fischer and Slutsky (1967) to determine a preliminary astro-geodetic geoid are described, along with the early attempts of Mather (1968.a), to determine a gravimetric solution of the geoid for South Australia. This was later extended to embrace the Australian Mainland (1969). In section 3.2, the 1970 free air geoid solution of Australia, which is an improved solution on the earlier attempts, is outlined, as is the gravimetric geoid solution of Gruskinsky and Sazhina (1971) in section 3.3. The 1971 geoid was determined using all available astro-geodetic deflections of the vertical in a series of loop closures covering Australia, with the 1970 free air geoid information being used to interpolate the values of the geoid separation N and the deflections of the vertical ξ and η within the loops. The techniques employed in this 1971 determination are discussed in section 3.4, and in section 3.5 the more recent geoid investigations are reviewed.

The gravity data including both satellite and terrestrial derived information, available for the computation of a free air geoid solution in South Australia along with some methods of interpolation, are assessed in Chapter 4. In the same Chapter, formulae for the practical evaluation of Stokes' and Vening Meinesz methods of determining N, ξ and η are given, and the Zone locations, compartment or block sizes, and the mean free air anomalies are defined. Methods of assessing the standard deviations of these mean free air anomalies are also discussed.

In Chapter 5, the solution of the free air geoid at half degree geographical grid points is given, and estimations of the standard deviations of N, ξ and η are evaluated. The effects of systematic errors, and errors due to approximation and omissions in the computational formulae in determining these results are discussed.

The free air geoid solution is compared with results obtained by the use of astro-geodetic techniques, Doppler methods, spherical harmonic coefficients, and previous gravimetric investigations in Chapter 6. In order to facilitate these comparisons, detailed gravimetric surveys were performed in the immediate vicinity of thirteen trigonometric stations, and the free air geoid values of N, ξ and η at each station are evaluated. These details are also given in Chapter 6.

In Chapter 7, the results and conclusions drawn in the previous Chapters are summarised and the future developments and uses are briefly discussed.

CHAPTER 2

TECHNIQUES USED FOR THE DETERMINATION OF DEFLECTION OF THE VERTICAL AND THE UNDULATIONS OF THE GEOID

Geoidal undulations and deflections of the vertical may be determined by a variety of methods. For geodetic purposes these may be grouped into the following categories:

Astro-geodetic Methods

Gravitational Methods

Satellite Methods

Combined Methods.

Each of these methods are discussed in some detail in the following sections.

2.1 Astro-geodetic Methods.

Consider two points A and B on the earth's surface separated by a distance S, as shown in Figure 2.1. The deflections of the vertical at each of these points can be determined if the geodetic co-ordinates ϕ and λ and the astronomical co-ordinates Φ and Λ along with the orthometric heights are known. The astronomical co-ordinates are reduced to the geoid and the deflections of the vertical are obtained from the following relationships.

$$\xi = \Phi_{RED} - \phi \qquad \text{................ (2.1a)}$$

and

$$\eta = (\Lambda_{RED} - \lambda) \cos \phi \qquad \text{................ (2.1b)}$$

where ξ and η are the components of the deflections of the vertical in the meridian and the prime vertical respectively, and the subscript RED refers to the astronomical co-ordinates which have been reduced to the geoid. ξ is positive if the astronomical latitude is greater than the geodetic latitude and similarly, the η component is positive when the astronomical longitude is greater.

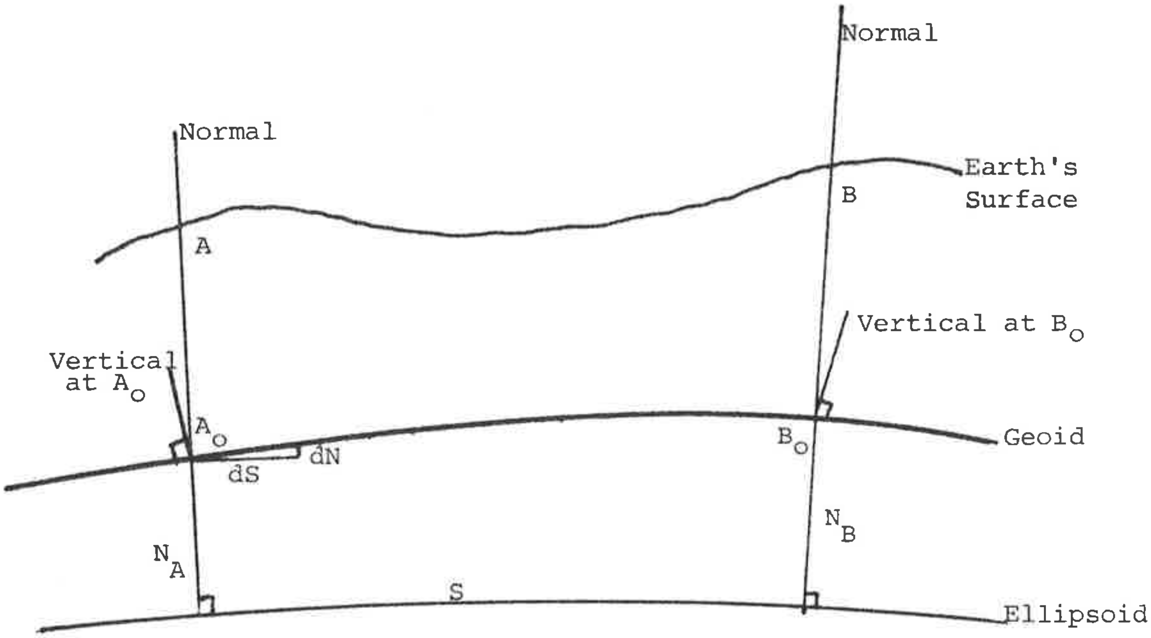


FIGURE 2.1

To reduce astronomical co-ordinates from the earth's surface to the geoid requires a detailed knowledge of the curvature of the plumb line at that point. Heiskanen and Moritz (1967, p. 193-195) show that:

$$\Phi_{RED} = \Phi + \delta\phi \qquad \text{................} \quad (2.2a)$$

and

$$\Lambda_{RED} = \Lambda + \delta\lambda \qquad \text{................} \quad (2.2b)$$

where

$$\delta\phi = -\int_0^H \frac{1}{g}\frac{\partial g}{\partial x}.dh$$

$$\delta\lambda \cos\phi = -\int_0^H \frac{1}{g}\frac{\partial g}{\partial y}.dh$$

H is the orthometric height of the astronomical station, and $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ are the horizontal gradients of gravity in the meridian and the prime vertical directions respectively. Without a detailed gravity survey around each astronomical station it is impossible to obtain satisfactory results from these formulae.

In practice it is usual to substitute the normal gravity γ in place of g in the above formula and this results in

$$\delta\phi = -0''.17\ H \sin 2\phi$$
$$\delta\lambda = 0 \qquad \text{................} \quad (2.2c)$$

where H is in kilometres and the value of $\delta\phi$ is in seconds of arc. $\delta\lambda$ is equal to zero since the normal plumb line curvature is zero in the prime vertical direction because the reference ellipsoid is rotationally symmetrical with respect to the spin axis. The substitution of these values $\delta\phi$ and $\delta\lambda$ in equation (2.1) is an approximation and in some cases the irregularities of the actual plumb line curvature may be greater than those computed from the 'normal' part (Ibid, p.197). The corrections are height dependent and any discrepancies will have a proportionally larger effect in mountain regions than in low land regions.

Having obtained the astronomical deflections of the vertical at points A and B the difference in the geoidal undulation between these two points can now be evaluated. Consider a point p situated on the geoid in the normal plane AB at a distance ds from A, then the change in the geoid separation dN is given by:

$$dN = -\varepsilon \, ds \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.3)$$

where $\varepsilon = \xi_A \cos \alpha + \eta_A \sin \alpha$ and is the component of the deflection of the vertical at A in the azimuth α of the profile AB. On integration of equation (2.3) between points A and B in Figure 2.1

$$N_B = N_A - \int_A^B \varepsilon \, ds \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.4)$$

If A and B are separated by a relatively short distance and the value of ε varies uniformly along the profile AB, the equation (2.4) may be represented by

$$N_B - N_A = -\frac{1}{2} (\varepsilon_A + \varepsilon_B).S \qquad \ldots\ldots\ldots\ldots\ldots \quad (2.5)$$

where the subscripts A and B refer to stations A and B. This formula assumes that the section of the geoid between points A and B can be approximated by the arc of a circle. In practice the aim is to have the spacing S between stations as large as possible in order to reduce the number of time consuming astronomical observations but this depends on the terrain roughness along the profile. In moderately level areas a station distance of 25 km may be satisfactory but a spacing of 10 km may be unacceptable in high mountainous regions (Ibid, p. 201). The assumption that ε varies uniformly between stations can be assisted by avoiding the sighting of astronomical stations where local irregularities may have unwanted effects on the resulting deflections of the vertical. These regions may be indicated by rapid geological changes or non-symmetrical topographical effects (Mather et al, 1971).

The deflections of the vertical obtained from formulae (2.1a) and (2.1b) are dependent on the geodetic coordinate at the stations. These geodetic coordinates may be related to an earth centred ellipsoid but this is not usually the case. The geodetic coordinate systems used by most countries or regions are related to "local" reference ellipsoids whose parameters were selected from the available known data at the time to best suit that country's or region's mapping requirements, e.g. the Australian Geodetic Datum. Thus the deflections of the vertical, excluding observation and computational procedure errors, obtained by this method will not generally agree with those obtained by gravimetric or satellite methods. Comparisons can be made if the local reference ellipsoid is orientated to an earth centred ellipsoid or vice versa. This will be discussed in more detail later.

The difference in the geoid separation is referenced to the geodetic datum that is used to determine the astro-geodetic deflections of the vertical. What has been stated in the previous paragraph applies to the geoid undulation with the additional problem that formula (2.5) only gives the geoid undulation difference. At some point in the geodetic network, usually at the origin, a value for N is either assumed or computed by an alternative method. Having "obtained" this value of N at one station formula (2.5) is used to determine values at other stations along geodetic traverse lines. This method of obtaining the value of N at discrete points along traverse or profile lines is known as "astro-geodetic levelling" and as the name implies loops can be mathematically adjusted in a similar manner to levelling traverses.

The accuracy of this relative astro-geodetic geoid depends on:

(1) The spacing, location and density of astro-geodetic stations within the country or region. The spacing and location has been briefly discussed. The distance between stations can be increased using interpolation techniques such as astro-gravimetric levelling and other methods which are described in Heiskanen and Moritz (1967, p. 202-204). The interpolation errors are reduced proportionally with the increase in density of astro-geodetic levelling loops.

(2) The accuracy of the astronomical observations. These in turn depend on the precise determination of the refraction corrections, the UT1 instant of observation, the precision of the observed star coordinates and the observation techniques used. Lachapelle (1978) estimates the accuracy of the astrogeodetic deflections of the vertical in Canada to be "around 0".8" in the meridian component and to range between 0".5 to 1".5 in prime vertical component. This range in the latter value is mostly a function of the epoch of observations, the older ones being less accurate.

(3) Errors in the reduction of astronomical observations to the geoid. If the approximation formula (2.2c) is used, errors could result, particularly in high mountain areas.

(4) Errors in the geodetic network and hence in the resulting deflections of the vertical. These errors would generally have little effect since the geodetic coordinates are generally known to at least one order better than the equivalent astronomical coordinates, but the effect may be detectable on the peripheries of a large region.

2.2 Gravimetric Methods

At any point the difference between the actual potential of the earth, W, and the normal potential, U, is the disturbing or anomalous potential T, so that:

$$W = U + T \qquad \text{................} \quad (2.6)$$

At this stage the potential of the geoid is assumed to be equal to the normal potential on the reference or base ellipsoid i.e. $W_0 = U_0$. These two surfaces, although of equal potential, are generally separated in space by a distance N, measured along the normal to the ellipsoid. This distance is referred to as the geoidal undulation or geoid-ellipsoid separation.

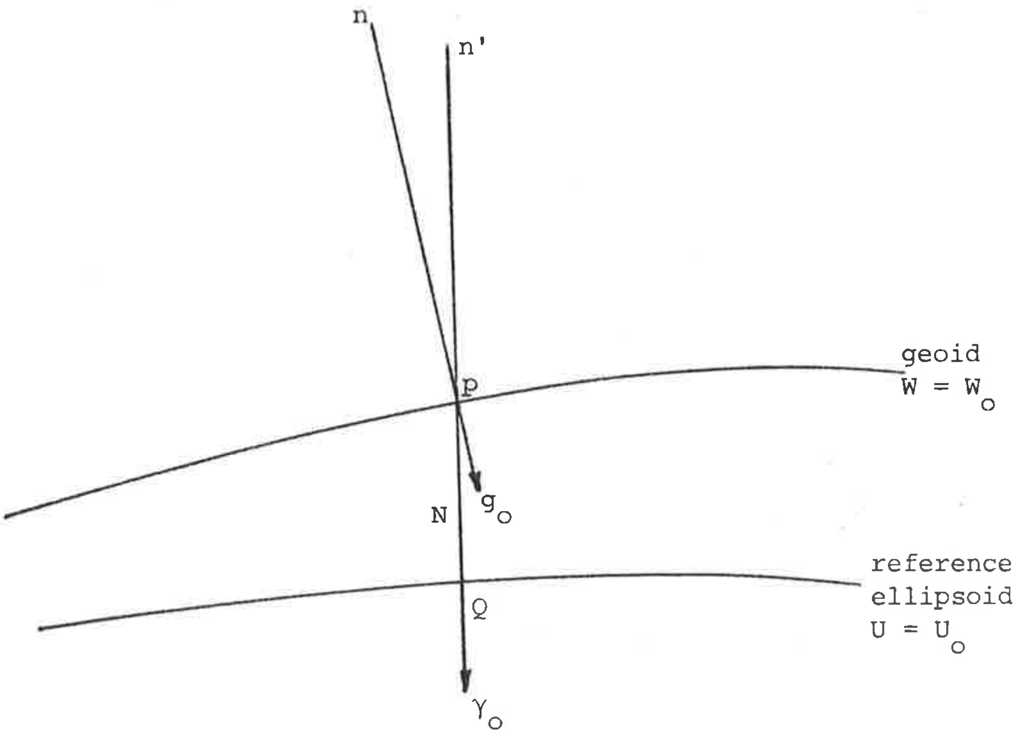


FIGURE 2.2

In Figure 2.2 the unit vectors in the direction of the actual gravity and normal gravity are denoted by $\underline{n}$ and $\underline{n}'$ respectively. The magnitude of the vectorial difference of

$$\underline{\Delta g}_0 = \underline{g}_0 - \underline{\gamma}_0$$

is the gravity anomaly, Δg, where

$$\Delta g = g_0 - \gamma_0 \qquad \text{................} \quad (2.7)$$

(the small difference in direction of $\underline{n}$ and $\underline{n}'$ is ignored in this expression).

The difference in direction between $\underline{n}$ and $\underline{n}'$ i.e. between the normal to the ellipsoid (called the normal) and the normal to the geoid (called the vertical) is the deflection of the vertical which is divided into two components, a north-south component, ξ, and an east-west component η. The direction of $\underline{n}$ and $\underline{n}'$ are respectively defined by the astronomical coordinates ϕ and Λ reduced to the geoid, and the geodetic coordinates ϕ and λ.

The normal potential on the geoid at point p (Figure 2.2) would be

$$U_p = U_0 + \frac{\partial U}{\partial n'} N = U_0 - \gamma N$$

and thus $$W_0 = U_p + T = U_0 - \gamma N + T$$

Hence $$T = \gamma N$$

or $$N = \frac{T}{\gamma} \qquad \text{................} \quad (2.8)$$

This is Bruns' formula (Heiskanen and Moritz, 1967, p. 85) which relates the disturbing potential to the geoid undulation.

Consider now the gravity anomaly Δg

$$\Delta g = g_0 - \gamma_0$$

$$= g_0 - \gamma_P + \frac{\partial \gamma}{\partial h} N$$

Again, ignoring the slight discrepancy in n and n' and reckoning the height, h, to be measured along the vertical, this expression may be rewritten as

$$\Delta g = \frac{\partial T}{\partial h} + \frac{\partial \gamma}{\partial h} N \qquad \text{................ (2.9)}$$

Thus, on substitution of Bruns formula

$$\frac{\partial T}{\partial h} - \frac{1}{\gamma}\frac{\partial \gamma}{\partial h} T + \Delta g = 0 \qquad \text{................ (2.10)}$$

This equation is called the fundamental equation of physical geodesy because it relates Δg to the unknown anomalous potential T (Heiskanen and Moritz, 1967, p. 86). Using the spherical approximation

$$\frac{1}{\gamma}\frac{\partial \gamma}{\partial h} = -\frac{2}{r}$$

where r is the distance from the earth's centre, equation (2.10) may be rewritten in the form:

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r} T \qquad \text{................ (2.11)}$$

If the masses external to the geoid are removed by computation then the anamalous potential T is harmonic and satisfies Laplace's equation.

$$\nabla^2 T = 0 \qquad \text{................} \quad (2.11a)$$

Thus, assuming Δg is known at every point on the grid, the solution of T in equation (2.11) is a third boundary value problem of potential theory.

Using equation (2.11) and the upward continuation integral of the gravity anomaly (Heiskanen and Moritz, 1967, p. 90) Stokes' Formula can be derived in the general form as:

$$T = \frac{R}{4\pi} \iint_{\sigma} \Delta g \; S(\psi) \; d\sigma \qquad \text{................} \quad (2.12)$$

or

$$N = \frac{R}{4\pi \bar{\gamma}} \iint_{\sigma} \Delta g \; S(\psi) \; d\sigma \qquad \text{................} \quad (2.12a)$$

where

$$S(\psi) = \frac{1}{\sin\left(\frac{\psi}{2}\right)} - 6 \sin\frac{\psi}{2} + 1 - 5\cos\psi -$$

$$3\cos\psi \ln\left(\sin\frac{\psi}{2} + \sin^2\frac{\psi}{2}\right) \qquad \text{........} \quad (2.13)$$

and is called Stokes' function.

$\bar{\gamma}$ is the mean value of gravity over the earth.

$d\sigma$ is an element of surface area on a unit sphere.

ψ is the angular distance between the point of computation & the elements of surface area $d\sigma$.

These formulae of Stokes have been derived with the following conditions:

(a) $U_0 = W_0$ i.e. the potential of the geoid is equal to the potential of the reference ellipsoid,

(b) the mass enclosed by the reference ellipsoid is equal to that enclosed by the geoid,

(c) the centre of gravity of the geoid and the ellipsoid coincide,

(d) no mass is external to the geoid,

(e) the rotational potential of the geoid and the ellipsoid are equal.

If an arbitrary reference ellipsoid is selected and conditions (a) and (b) are not met but the deviations of the geoid from ellipsoid are linear, then Stokes Integral may be written as:

$$N = \frac{G\delta M}{R\,\bar{\gamma}} - \frac{\delta W}{\bar{\gamma}} + \frac{R}{4\Pi\bar{\gamma}} \iint_{\sigma} \Delta g \; S(\psi) \; d\sigma \quad \ldots\ldots \; (2.14)$$

where the first and second term on the right hand side of the equation allow for the difference in mass δM, and the difference in potential δW between the geoid and the ellipsoid. This expression is often alternatively expressed as

$$N = N_0 + \frac{R}{4\Pi\bar{\gamma}} \iint_{\sigma} \Delta g \; S(\psi) \; d\sigma \quad \ldots\ldots\ldots\ldots \; (2.15)$$

Stokes' formula is derived using spherical approximations and this results in an error in the value of N of the order of flattening i.e. approximately (N/300). As will be seen later in this report the value of the separation does not exceed a numerical value of 25 metres in the South Australian region and hence the expected maximum error resulting from spherical approximation would not exceed 0.1 metres.

By differentiating equation (2.14) separately with respect to the distance components in the meridian and prime vertical, and after substitution, the Vening Meinesz formulae for the deflections of the vertical are obtained (Heiskanen & Moritz, 1967, p. 112). These may be written in the general form as:

$$\left.\begin{matrix}\xi \\ \eta\end{matrix}\right\} = \frac{1}{4\pi\gamma} \iint_{\sigma} \Delta g \, \frac{dS(\psi)}{d\,\psi} \begin{Bmatrix}\cos\,\alpha \\ \sin\,\alpha\end{Bmatrix} d\sigma \quad \ldots\ldots\ (2.16)$$

where $$\frac{dS(\psi)}{d\,\psi} = \frac{\cos(\frac{\psi}{2})}{2\sin^2(\frac{\psi}{2})} + 8\sin\psi - 6\cos\frac{\psi}{2} - 3\frac{(1-\sin(\frac{\psi}{2}))}{\sin\psi}$$

$$+ 3\sin\psi\,\ln\left(\sin(\frac{\psi}{2}) + \sin^2(\frac{\psi}{2})\right) \quad \ldots\ (2.17)$$

and is known as Vening Meinesz' function. α is the azimuth of the surface element $d\sigma$ from the computational point.

The Vening Meinesz formula (2.16) for the deflections of the vertical is valid for an arbitrary ellipsoid since the term N_0 in equation (2.15) disappears after differentiation.

The gravity anomaly Δg used in both the Stokes' and Vening Meinesz Formulae, is defined by equation (2.7) as being the difference in magnitude between the actual gravity on the geoid and the corresponding value of the normal gravity on the ellipsoid. Gravity observations are generally taken on the surface of the earth and then the value at the geoid is obtained by one of several reduction techniques which also attempt to comply with the boundary value condition that no masses exist outside the geoid. These reduction techniques are discussed in detail in Heiskanen and Moritz (1967, Chapter 3).

One particularly favoured reduction technique is the free air reduction resulting in the free air anomaly. If the intervening effect of the mass between the geoid and the earth's surface is ignored then the value of gravity at the geoid g_0 may be obtained from:

$$g_0 = g_s - \frac{\partial g}{\partial h} . H \qquad \text{............... (2.18)}$$

where g_s is the gravity at the earth's surface, H is the orthometric height, and the second term on the right hand side of the equation is the free air reduction. This may be rewritten as:

$$g_0 = g_s - \left(\frac{\partial \gamma}{\partial h} + \frac{\partial \Delta g}{\partial h}\right) H \qquad \text{............... (2.18a)}$$

and for most practical purposes the anomalous part of the vertical gradient may be ignored thus in effect substituting the normal gravity gradient in place of the actual gravity gradient. This results in an approximation of equation (2.18) and on substitution is given by:

$$g_0 = g_s - \frac{\partial\gamma}{\partial h} H$$

$$= g_s + F \qquad \text{................} \quad (2.19)$$

where F = 0.3086 H mGal and H is measured in metres. This simple but effective reduction technique is also a close approximation to Helmert's condensation reduction method. In this method the intervening mass is condensed on the geoid so that the mass of the earth is not affected. If the attraction of this condensed mass equals that of the actual mass above the geoid (which it nearly does) then the condensation reduction is equal to the free air reduction.

Any removal or shifting of masses implied in the reduction of gravity to the geoid, changes the geopotential and hence the shape of the geoid. This change is known as the "indirect effect" of the gravity reduction. Thus the surface computed by the use of Stokes' formula using gravity obtained by reducing surface gravity to the geoid is not the geoid but another surface referred to as the co-geoid.

The geoid undulation is then defined by:

$$N = N_0 + N^c + \delta N \qquad \text{................} \quad (2.20)$$

where N^c is the undulation defined by the co-geoid and δN is the separation between the geoid and co-geoid. The gravity anomalies used in Stokes' formula should refer to the co-geoid and hence requires a correction of $+0.3086\delta N$ mGal where δN is in metres.

The indirect effect on the deflections of the vertical is given by (Heikanen & Moritz, 1967, p. 143):

$$\delta\xi = -\frac{1}{R}\frac{\partial \delta N}{\partial \phi} \quad \ldots\ldots\ldots\ldots\ldots (2.21a)$$

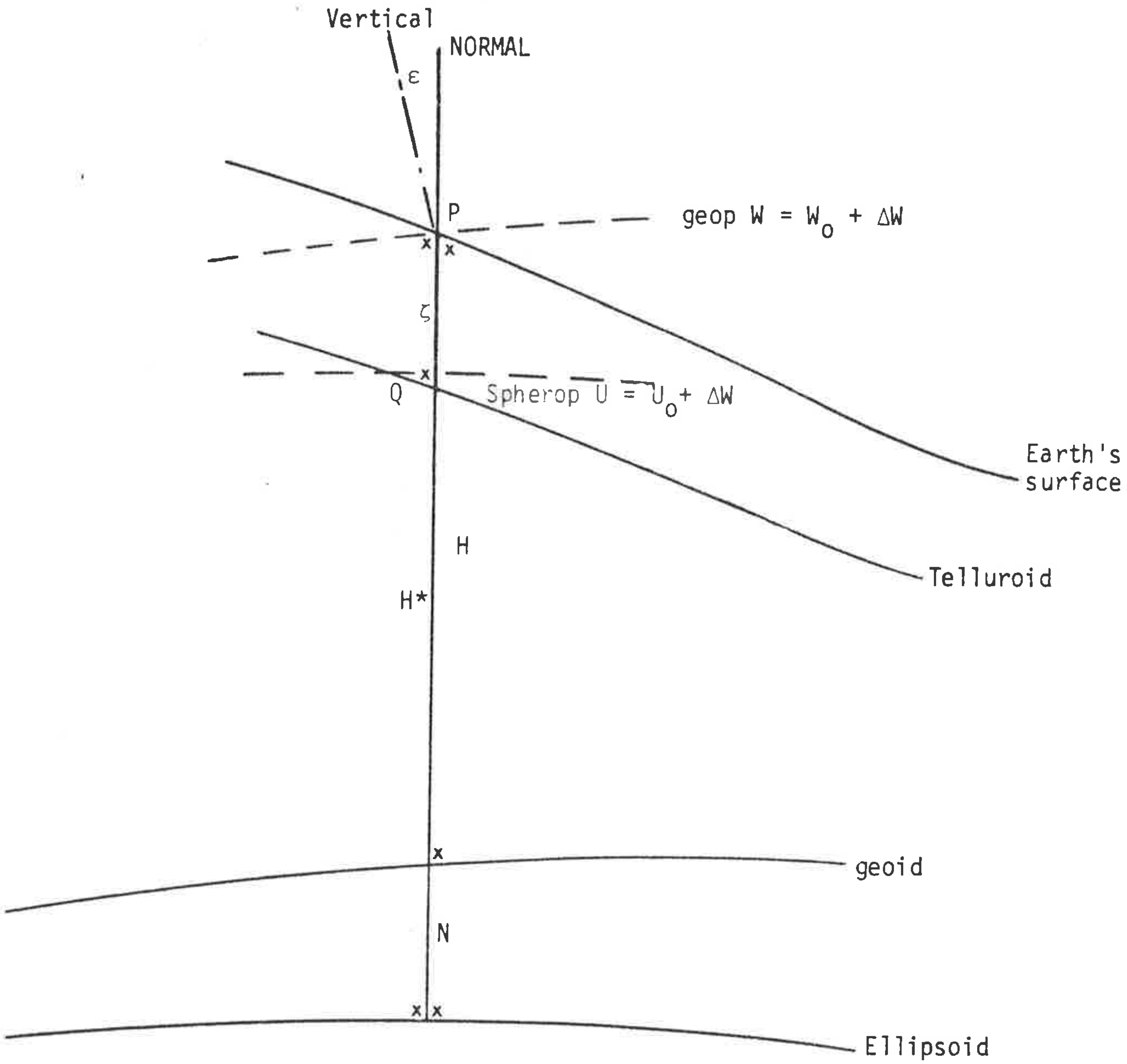
and

$$\delta\eta = -\frac{1}{R\cos\phi}\frac{\partial \delta N}{\partial \lambda} \quad \ldots\ldots\ldots\ldots\ldots (2.21b)$$

and as shown depend on the rate of change of δN in the meridian and prime vertical respectively.

The magnitude of the indirect effects δN, $\delta\xi$ and $\delta\eta$ depend on the reduction process used. As an example the indirect effect δN resulting from the use of a Bouguer reduction is larger than the value of N, whereas the indirect effect has a magnitude of only a few metres if the free air reduction technique is used. The co-geoid derived from the use of free-air anomalies is often called the free air geoid. The indirect effect and its relationship to the free air geoid with particular reference to the South Australian region, and the practical solution of both Stokes' and Vening Meinesz formulae will be discussed in later chapters.

To overcome the problems of the Stokesian solution of the boundary value problem Molodenskii proposed a different solution to the problem where the bounding surface is not the geoid but the



Telluroid and Geoid
Figure 2.3

earth's physical surface (Molodenskii et al, 1962). The gravity anomaly is referenced to the earth's surface and thus problems associated with the reduction of observed gravity to the geoid are avoided. The geometrical concept of this method and its relationship to the geoid are shown in Figure (2.3).

The ellipsoid height is given by the numerical sum of the orthometric height and the geoidal undulation, i.e.

$$h = H + N \qquad \ldots\ldots\ldots\ldots\ldots (2.22)$$

or alternatively it may be represented by the normal height H^* and the height anomaly ζ i.e.

$$h = H^* + \zeta \qquad \ldots\ldots\ldots\ldots\ldots (2.23)$$

where the normal height is the height of the spherop $U = U_0 + \Delta W$ above the ellipsoid and may be determined by geodetic levelling techniques using analytical expressions that are free of any terms requiring assumptions on the density of the earth's crust. ΔW is the difference between the geopotential at the surface point P and the geoid. If the spheropotential at P was the same as the geopotential at P then $H^* = h$, but this does not generally occur and hence the difference between the two heights is known as the height anomaly.

The telluroid is a "non-potential" surface obtained by plotting the normal heights above the ellipsoid and has been defined as the locus of points Q where the spheropotential equals that of the geopotential W at the surface of the earth (Heiskanen & Moritz, 1967, p. 292). If the potential of the ellipsoid and the geoid are not equal, i.e. $U_o \neq W_o$ then a more practical definition "is the locus of those points Q which have the same difference in potential ΔW in relation to the reference ellipsoid as the difference in geopotential between the surface point P and the geoid" (Rizos, 1980).

By equating equations (2.22) and (2.23) and substituting formulae for the orthometric and normal heights then the geoid undulation is related to the height anomaly by:

$$N = \zeta + \frac{(\bar{g} - \bar{\gamma}_o)}{\bar{\gamma}_o} H \qquad \text{................ (2.24)}$$

where $\bar{\gamma}_o$ is the mean normal gravity along the normal between the ellipsoid and the telluroid and $\bar{g}$ is the mean gravity along the plumb line between the geoid and the earth's surface. Thus the geoid undulation may be obtained indirectly from the height anomaly.

The solution to Molodenskii's problem for the height anomaly and the deflections of the vertical at the surface of the earth have been the subject of much work, e.g. (Mather, 1973.a),(Heiskanen & Moritz, 1967, chapter 8), and these solutions can be approximated to:

$$\begin{aligned} \zeta &= \zeta_s + \zeta_1 \\ \xi' &= \xi_s + \xi_1 \qquad \text{................ (2.25)} \\ \eta' &= \eta_s + \eta_1 \end{aligned}$$

The value of ζ_s is derived from Stokes' formulae, equation (2.12a) using surface free air anomalies in place of free air anomalies on the geoid, and the remaining term ζ_1 is referred to as the non-Stokesian contribution and is much smaller than ζ_s e.g. an approximate value of ζ_i for Mt Blanc was reported to be -0.2 metres (Heiskanen & Moritz, 1967, p. 329) while the value of $(\zeta - N)$ was 1.8 metres. In a similar way the deflections of the vertical at the surface of the earth ξ' and η' are composed of two components. The first components ξ_s and η_s are obtained using Vening Meinesz equation (2.16) with surface free air gravity anomalies and the second components approximate to the terrain correction effect. These components ξ_1 and η_1 may obtain a magnitude of several seconds (Ibid, 1967, p. 329) but Kearsley (1976) obtained a maximum value of $-0\overset{''}{.}64$ and a root mean value (RMS) of approximately $\pm 0\overset{''}{.}3$ for ξ_1 and smaller values for η_1 for seven stations situated in the north west of New South Wales in "rough" terrain. For a further 5 stations situated in relatively flat topography the effect was negligible. For regions of predominantly flat terrain, such as the South Australian region the values of ξ_1 and η_1 are, in general, small. Mather (1968b) shows that the solution of the free air geoid is a good approximation for both the geoid and the telluroid. In the following chapters details of the practical solution for the free air geoid using Stokes' and Vening Meinesz formulae are given.

2.3 Satellite Methods

Satellite methods used for the determination of geoid undulation and the deflections of the vertical may be divided into two basic categories. These are the dynamic methods where the solution is obtained from consideration of the earth's potential, and the geometrical methods where the geoid undulations are deduced from geometrical considerations. The geometrical methods generally do not lend themselves to solution for the deflections of the vertical.

2.3.1 Dynamic Methods

The geopotential W of the solid earth may be represented as:

$$W = W_G + W_R \qquad \text{................} \quad (2.26)$$

where W_G is the gravitational potential of the solid earth (including the oceans and atmosphere) and W_R is the rotational potential. As the gravitational potential is harmonic in space, equation (2.26) may be rewritten for some point P with spherical coordinates (ϕ, λ, r) as:

$$W_p = \frac{GM}{r_p} \sum_{n=0}^{\infty} \left(\frac{a}{r_p}\right)^n \sum_{m=0}^{n} (\bar{C}_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda) \bar{P}_{nm}(\sin \phi)$$

$$+ \frac{\omega^2 r^2}{2} \cos \phi \qquad \text{................} \quad (2.27)$$

where GM is the product of the gravitational constant and the mass of the earth including the atmosphere, a is the radius of some arbitary sphere to which the spherical harmonic co-efficients are referred, $\bar{P}_{nm}(\sin \phi)$ is the fully normalised Legendre polynomial of degree n and order m, and $\bar{C}_{nm}$ and $\bar{S}_{nm}$

are fully normalised spherical harmonic coefficients. The second term on the right hand side is the rotational potential expressed as a function of the angular velocity ω.

The disturbing potential T given by equation (2.6) is the difference between the geopotential and the normal potential, and since the rotational potential of the normal gravity field may be defined as equal to that of the geopotential, T is harmonic in space and for a point P is expressed as:

$$T_p = \frac{GM}{r_p} \sum_{n=0}^{\infty} \left(\frac{a}{r_p}\right)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}^*_{nm} \sin m\lambda) . \bar{P}_{nm}(\sin \phi) \qquad (2.28)$$

where $\bar{C}^*_{nm}$ and $\bar{S}^*$ are the differences between the actual potential coefficients and those implied by the adopted reference ellipsoid. If the reference ellipsoid is an ellipsoid of revolution then $\bar{S}^*_{nm}$ is the same $\bar{S}_{nm}$ since all equivalent coefficients implied by the reference ellipsoid are zero.

This disturbing potential T can be downward continued to the geoid (Rizos, 1980) and by application of Brun's equation (2.8) an expression for N in terms of spherical harmonics is obtained by:

$$N_p = \frac{GM}{r_p \gamma_p} \sum_{n=0}^{\infty} \left(\frac{a}{r_p}\right)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}^*_{nm} \sin m\lambda) . \bar{P}_{nm}(\sin \phi) \qquad (2.29)$$

The values of $\bar{C}_{nm}$ and $\bar{S}_{nm}$ may be obtained from an analysis of the orbital perturbations of artificial satellites and in practice the summation is not to degree ∞ but rather to n' where n' is the highest degree to which the coefficients $\bar{C}_{nm}$ and $\bar{S}_{nm}$ are known. If the mass of the earth including the atmosphere is equal to the mass of the ellipsoid and the centre of the reference ellipsoid is coincident with the geocentric, the first and second degree harmonics are zero.

Hence, equation (2.29) may be rewritten as:

$$N_p = \frac{GM}{r_p \gamma_p} \sum_{n=2}^{n'} \left(\frac{a}{r_p}\right)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}^* \sin m\lambda) .$$

$$\bar{P}_{nm}(\sin \phi) \qquad \ldots\ldots\ldots\ldots\ldots \quad (2.30)$$

Equation (2.30) requires a knowledge of the geoid height H since r_p is a geocentric distance and thus, for geoidal solutions requiring sub-metre precision, an iterative solution is required (Rizos, 1980). By approximating:

$$r_p = a \quad \text{and} \quad \frac{GM}{r_p \gamma_p} = R$$

where R is the mean radius of the earth, then equation (2.30) may be rewritten in the form:

$$N_p = R \sum_{n=2}^{n'} \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}^*_{nm} \sin m\lambda) \bar{P}_{nm}(\sin \phi)$$

$$\ldots\ldots\ldots\ldots\ldots \quad (2.31)$$

Formulae for the deflections of the vertical are obtained by differentiating equation (2.30) or (2.31) with respect to the distance components in the prime vertical and the meridian. From equation (2.31) this results in (Lachapelle, 1978):

$$\xi(\phi,\lambda) = -\sum_{n=2}^{n'}\sum_{m=0}^{n} (\overline{C}^*_{nm}\cos m\lambda + \overline{S}^*_{nm}\sin m\lambda) \cdot \frac{\partial\overline{P}_{nm}(\sin\phi)}{\partial\phi} \qquad \text{................ (2.32)}$$

and

$$\eta(\phi,\lambda) = -\frac{1}{\cos\phi}\sum_{n=2}^{n'}\sum_{m=0}^{n} (-\overline{C}^*_{nm}\, m\sin m\lambda + \overline{S}^*_{nm}\, m\cos m\lambda) \cdot \overline{P}_{nm}(\sin\phi) \qquad \text{................ (2.33)}$$

Another method to determine the geoid height N is to find a and r values which satisfy equation (2.27) for a point P situated on the geoid. The solution for r is found by iteration, knowing the geoid potential and other quantities. The value of N is then found by differencing r with the corresponding r value of the reference ellipsoid (Rapp & Rummel, 1975) and is given by:

$$N = r - \frac{a\sqrt{1 - e^2}}{\sqrt{1 - e^2\cos^2\phi}} \qquad \text{................ (2.34)}$$

where a, the semi major axis, and e the eccentricity, refer to the reference ellipsoid. This method is the basis of a combination method discussed later.

2.3.2 Geometric Methods

The geoid height N, the ellipsoid height h, and the orthometric height H are related by equation (2.22) viz.

$$h = H + N \quad \text{................} \quad (2.22)$$

Thus, if both the ellipsoidal height and the orthometric height are known the value of N can be obtained directly from equation (2.22). The orthometric height is deduced from geodetic levelling and the ellipsoidal height may be deduced from the geocentric coordinates derived from three dimensional satellite positioning fixes.

These satellite positioning fixes may be obtained using passive or active satellites. In both cases if the geocentric coordinates of a station are required a derived knowledge of the satellite's position at the time of observation is needed. Passive satellite techniques include laser ranging to artificial satellites and the moon. Active satellite techniques include satellite altimetry and Doppler frequency shift measurements.

For Doppler measurements an active satellite with a transmitter on board transmits a signal at a constant frequency. The observer receives this signal and using Doppler techniques the range difference can be computed (Torge, 1980, section 4.4.6). The observations are corrected for systematic errors of the Doppler counter, the time system and the effects of refraction. The ionospheric refraction is determined by comparing the distances obtained from two different frequencies transmitted from the satellite. Improved reduction techniques are available if more than one receiver is operating within a medium sized region at the same time and "short arc techniques" are used. Brown (1976) demonstrates techniques which have an absolute estimated RMS accuracy in position of 1-2 metres and a relative RMS accuracy of 0.2-0.4 metres using 4 receivers simultaneously.

The method of satellite altimetry is based on a satellite-bourne altimeter which transmits radar pulses in the vertical direction to the ocean surface and these are in turn reflected back to the satellite. From the time of propagation of the signal the height of the satellite above the sea surface is deduced. If the position of the satellite is known the height of the sea surface above the reference ellipsoid can be deduced. This sea surface approximates to the geoid, the difference being the stationary sea surface topography which has a magnitude of 1-2 metres (Rizos, 1980). Hence a map of geoidal undulations can be obtained from satellite altimetry to the order of 1-2 metres. It should be noted that the altimeter does not sense a point of the sea surface but due to the divergent radar beam the result is a mean value of an area 15 km in diameter. Satellite altimetry is only successfully used over oceans and in this context it will not be discussed further in this work.

2.4 Combination Techniques

2.4.1 Satellite Determined Gravity Anomalies

To evaluate Stokes' and Vening Meinesz formulae the procedure is based on system quadratures which are derived from equations (2.12a) and (2.16) using the mean value theorem, and may be written as:

$$N = \frac{R}{4\pi\overline{\gamma}} \sum_i \sum_j S(\psi)_{ij} \, \Delta g_{ij} \Delta\sigma_{ij} \qquad \text{..........} \quad (2.35)$$

$$\left.\begin{matrix}\xi \\ \eta\end{matrix}\right\} = \frac{1}{4\pi\overline{\gamma}} \sum_i \sum_j \frac{dS(\psi)}{d\psi}_{ij} \begin{Bmatrix}\cos\alpha_{ij} \\ \sin\alpha_{ij}\end{Bmatrix} \Delta g_{ij} \Delta\sigma_{ij} \quad \text{..} \quad (2.36)$$

where $\Delta\sigma_{ij}$ is the ij surface area element measured in steradians and all other terms have been previously defined with ij referring to the mean value for the surface area element.

In order to evaluate equations (2.12a) and (2.16) a knowledge of gravity is required at all points on the earth's surface whereas the evaluation of equations (2.35) and (2.36) require a knowledge of all the mean gravity anomalies Δg_{ij} for all surface area elements $\Delta\sigma_{ij}$. For ease of computation these surface area elements may vary in size, increasing in area as the distance from the computational point increases. This will be discussed further in later chapters.

The earth's gravity field is not completely defined by surface gravity anomalies and hence the complete evaluation of formulae (2.35) and (2.36) is not possible without supplementary data. This data may be obtained by techniques using either a spherical harmonic model of the earth or geoid undulations obtained in practice from satellite altimeter data (e.g. Rapp, 1977).

Using a spherical harmonic model of the earth's potential field (e.g. equation (2.27)), the gravity anomaly Δg_p at a point P with spherical coordinates (ϕ,λ,r) using the notation previously defined is given by (Rizos, 1980):

$$\Delta g_p = \frac{GM}{r_p^2} \sum_{n=2}^{n'} (n-1) \left(\frac{a}{r_p}\right)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}^*_{nm} \sin m\lambda) .$$

$$\bar{P}_{nm}(\sin \phi) \qquad \ldots\ldots\ldots\ldots\ldots \quad (2.37)$$

Procedures used for the recovery of gravity anomalies from altimeter derived undulations, have been reviewed by Rapp (1977). Brief summaries of the methods are:

<u>Stokes' Equation Solution</u> - the essence of this method is that given sufficient values of N then equation (2.12a) may be used to solve for Δg values using a least square adjustment process. There are problems achieving the required results since an infinite quantity of gravity anomalies are required to obtain a value of N.

<u>Inverse Stokes' Equation</u> - a direct determination of point anomalies from geoid undulations can be computed from

(Ibid, 1977)

$$\Delta g_p = -\frac{G}{R} N_p - \frac{G}{16\pi R} \int_\sigma \frac{N_q - N_p}{\sin^3(\frac{\psi}{2})} d\sigma_q \quad \ldots\ldots \; (2.38)$$

The kernel in equation (2.38) is difficult to evaluate and there is difficulty in finding a reliable estimate for the mean anomalies.

<u>Fourier Transform Solution</u> - another technique for predicting gravity anomalies is by means of a transformation to the spectral domain using:

$$\Delta\hat{g}_k = \Delta\bar{g}_k + \Delta t \sum_{j=0}^{m+1} h_j (x_{k-j} + X_{k+j}) \quad \ldots\ldots \; (2.39)$$

where the notation is the same as that given in (Ibid, 1977). $\Delta\hat{g}_k$ is the reference gravity anomaly with respect to some earth model (e.g. GEM9), Δt is the data spacing, h_j are the weights, and X is the difference between the altimeter derived value and the value derived from the earth model. The detailed use of this technique is described in (Sjoberg, 1977). This technique has the advantage in processing many arcs simultaneously without significant increase in computer time (Ibid, 1977).

<u>Least Squares Collocation</u> - adopting the notation used in (Ibid, 1977) the predicted mean anomaly Δg derived from point altimeter measurements using least squares collocation may be obtained from:

$$\Delta g = \underline{C}_{gh} (\underline{C}_{hh} + D)^{-1} \underline{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.40)$$

and the standard deviation m_g of the predicted gravity anomaly is given by:

$$m_g^2 = \underline{C}_{gg} - \underline{C}_{gh} (\underline{C}_{hh} + D)^{-1} \underline{C}_{hg} \quad \ldots\ldots\ldots \quad (2.41)$$

where $\underline{h}$ is the error of altimeter data, $\underline{C}_{gh}$ is the co-variance matrix between the mean anomaly to be predicted and the point altimeter derived undulation data, $\underline{C}_{hh}$ is the co-variance matrix between the measured undulation data, $\underline{C}_{gg}$ is the variance matrix of an anomaly block of the size being predicted, and D is the error covariance matrix of the alti-meter observations. Using this technique, anomalies can be predicted with respect to a reference ellipsoid or to a higher degree surface defined by a set of potential co-efficients. This method has been used by Rapp, (1978.a) to determine 12144 1° x 1° mean gravity anomalies and 377 5° equal area mean gravity anomalies. Some of these values are used later in the work to define the gravity field in the Middle and Outer Zones, in the determination of the free air geoid undulations and the deflections of the vertical in the South Australian Region.

2.4.2 Combination of potential coefficients and terrestrial anomaly data

A commonly used procedure for determining the geoid undulation is to combine terrestrial anomaly data with potential coefficients and subdivide the free air geoid undulation N into three components (Rapp and Rummel, 1975), such that the sum yields the undulation.

$$N = N_1 + N_2 + N_3 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.42)$$

There are basically two different methods of obtaining these three components.

<u>In the first method</u> the value of N_1 is the geoid undulation implied from a given set of potential coefficients. The value of N_1 may be obtained using equation (2.30), (2.31) or (2.34). The N_2 component is computed from

$$N_2 = \frac{R}{4\pi\bar{\gamma}} \iint_{\sigma_c} (\overline{\Delta g}^{o} - \overline{\Delta g}_s)\, S(\psi)\, d\sigma \qquad \ldots\ldots \quad (2.43)$$

where $\overline{\Delta g}_s$ is the mean anomaly implied by the potential coefficients used in computing N_1, $\overline{\Delta g}^{o}$ is the mean free air anomaly which may be corrected for the effect of the atmosphere, σ_c is a limited cap about the computation point and all other terms have been previously defined. The component N_2 is the contribution from Stokes' integral for a cap size σ_c after the contribution for this region from N_1 obtained from the potential coefficients, has been removed.

The component N_3 is given by:

$$N_3 = \frac{R}{4\pi\bar{\gamma}} \iint_{\sigma-\sigma_c} (\overline{\Delta g}^{o} - \overline{\Delta g}_s)\, S(\psi)\, d \qquad \ldots\ldots. \quad (2.44)$$

where $(\sigma - \sigma_c)$ represents the remaining global cap not included in σ_c and hence not obtained from N_2. If the σ_c is chosen correctly the value of N_3 can be neglected (Ibid, 1975).

In the second method the values of the components of the geoid undulations are obtained by splitting Stokes' integral into two parts, i.e.

$$N = \frac{R}{4\pi\bar{\gamma}} \int_{\psi=0}^{\psi_o} \int_{\alpha=0}^{2\pi} \Delta g \; S(\psi) \; \sin\psi \; d\psi \; d\alpha$$

$$+ \int_{\psi=\psi_o}^{2\pi} \int_{\alpha=0}^{2\pi} \Delta g \; S(\psi) \; \sin\psi \; d\psi \; d\alpha \qquad \text{...... (2.45)}$$

where the first term on the right hand side is N_2 and is the geoid undulation component obtained directly from Stokes' integral for a cap size of radius ψ_o. The second term on the right hand side can be rewritten as (Heiskanen & Moritz, 1967, p. 260):

$$\delta N = \frac{R}{2\bar{\gamma}} \sum_{n=2}^{\infty} Q_n \; \Delta g_n \qquad \text{................ (2.46)}$$

where Q_n is Mododenskii's truncation function and is given by:

$$Q_n = \int_{\psi_o}^{\pi} S(\psi) \; P_n(\cos\psi) \; \sin\psi \; d\psi \qquad \text{....... (2.47)}$$

and the value of Δg_n is the n^{th} degree component of the gravity anomaly implied by a set of potential coefficients and is given by:

$$\Delta g_n = \frac{GM}{r^2} (n-1) (\frac{a}{r})^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}^*_{nm} \sin m\lambda) . \bar{P}_{nm}(\sin \phi) \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.48)$$

Since in practice the summation in equation (2.46) takes place from n=2 to n=n' and not infinity as previously explained, the value of δN may be represented as:

$$\delta N = \frac{R}{2\gamma} \sum_{n=2}^{n'} Q_n \Delta g_n + \frac{R}{2\gamma} \sum_{n=n'+1}^{\infty} Q_n \Delta g_n \quad \ldots (2.49)$$

$$= N_1 + N_3$$

where the last term N_3 is neglected.

The deflection of the vertical may be obtained by a combination of potential coefficients and terrestrial gravity anomalies using similar methods as those described above to obtain the geoid undulations. Using the method described by Sjoberg (1977), the deflections of the vertical may be considered to consist of three components such that:

$$\xi = \xi_o + \xi_1 + \xi_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.50a)$$

$$\eta = \eta_o + \eta_1 + \eta_2$$

where the values of ξ_o and η_o are the deflections of the vertical component from a set of potential coefficients and are obtained from equations (2.32) and (2.33) or from the more generalised form given in (Ibid, 1977) as:

$$\xi_o = -\sum_{n=2}^{n'} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (\overline{C}^*_{nm} \cos m\lambda + \overline{S}^*_{nm} \sin m\lambda) . \frac{\partial \overline{P}_{nm}(\sin \phi)}{\partial \phi} \qquad \ldots\ldots\ldots\ldots\ldots \quad (2.51)$$

$$\eta_o = -\frac{1}{\cos \phi} \sum_{n=2}^{n'} \left(\frac{a}{r}\right)^{n+1} \sum_{m=0}^{n} (-\overline{C}^*_{nm} m \sin m\lambda + \overline{S}^*_{nm} m \cos m\lambda) . \overline{P}_{nm}(\sin \phi) \qquad \ldots\ldots\ldots\ldots\ldots \quad (2.52)$$

The values of ξ_2 and η_2 are obtained by applying Vening Meinesz integral to gravity anomalies that are formed as in equation (2.43) by subtracting the contribution Δg_s implied by the potential coefficients in computing ξ_2 and η_2 from the terrestrial mean free air anomaly, i.e.:

$$\left.\begin{matrix}\xi_2 \\ \eta_2\end{matrix}\right\} = \frac{1}{4\pi\gamma} \iint_{\sigma_2} (\overline{\Delta g}^o - \overline{\Delta g}_s) \frac{ds(\psi)}{d\psi} \left\{\begin{matrix}\cos \alpha \\ \sin \alpha\end{matrix}\right\} d\sigma \qquad (2.53)$$

where σ_2 represents the zone from ψ_1 to ψ_2.

The values ξ_1 and η_2 may be obtained using equation (2.53) but with the integration for the inner most zone defined by $\psi = 0$ to $\psi = \psi_1$. Thus the deflections of the vertical regional effects are defined by ξ_o and η_o and the detailed localised variations are obtained from ξ_1, η_1, ξ_2 and η_2.

In equation (2.50a) and (2.50b) no allowance has been made for terrain corrections and there is no term equivalent to N_3 in the equations since this term is normally neglected in practice.

2.4.3 Combination methods using Collocation Techniques

The covariance function of the disturbing potential k(A,B) following the notation used in (Lachapelle, 1975) may be written as:

$$k(A,B) = Cov(T_A,T_B)$$

$$= \sum_{n=2}^{\infty} k_n \left(\frac{R_b^2}{r_A r_B}\right)^{n+1} P_n (\cos \psi) \qquad .... (2.54)$$

where $Cov(T_A,T_B)$ is the covariance of the disturbing potential between points A and B, k_n are the degree variances of the anomalous potential, R_b is the radius of the Bjerhammar sphere, and r_A and r_B are the geocentric radii of points A and B separated by a spherical distance ψ.

The covariance between two quantities derived by applying certain operations on T, can be obtained by applying the same operations on k(A,B). This is referred to as the "law of propagation of covariances" (Tscherning and Rapp, 1974). Thus covariances of, and between T, N, ξ, η and Δg may be derived from equation (2.54). The relevant formulae are given in various publications e.g. (Ibid 1974, Lachapelle, 1975).

These covariances can be used to determine the geoid height and deflections of the vertical by combining heterogeneous data such as gravity anomaly data and astrogeodetic deflections of the vertical (Lachapelle, 1975). Collocation techniques may also be used to determine a component of the geoid height i.e. N_2 as defined in equation (2.43), or a component of the deflections of the vertical as defined by ξ_1 and η_1 in equations (2.50a) and (2.50b). This is given by (Lachapelle, 1976):

$$\begin{bmatrix} N_2 \\ \xi_2 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \underline{c}_{Nx} \\ \underline{c}_{\xi x} \\ \underline{c}_{\eta x} \end{bmatrix} \underline{c}_{xx}^{-1} \, \underline{x}'(\sigma_1) \quad \ldots\ldots\ldots\ldots\ldots \quad (2.55)$$

where $\underline{x}'(\sigma_1) = \underline{x}(\sigma_1) - \underline{x}_1(\sigma_1)$

$\underline{x}(\sigma_1)$ is the vector containing the measurements (free air gravity anomalies and/or geocentric astro-geodetic components of the deflections of the vertical) from the inner zone σ_1 and referring to the reference ellipsoid, and $\underline{x}_1(\sigma_1)$ is the corresponding vector derived from the potential coefficients using equations (2.51), (2.52) and (2.57).

Thus the vector $\underline{x}'(\sigma_1)$ contains the measurements referred to the surface implied by the set of potential coefficients. $\underline{c}_{Nx}$, $\underline{c}_{\xi x}$, and $\underline{c}_{\eta x}$ are the signal cross-covariance vectors between N and $\underline{x}'(\sigma_1)$, ξ and $\underline{x}'(\sigma_1)$, and η and $\underline{x}'(\sigma_1)$ respectively. $\underline{c}_{xx}$ is the covariance matrix of $x'(\sigma_1)$.

2.4.4 Comments on methods used to determine the geoid height and the deflection of the vertical

Astro-geodetic differences in geoidal height have historically been determined from the deflections of the vertical along traverse loops and these loops have been adjusted. At some point within the region a value of N is selected using methods to obtain a "best fitting" reference ellipsoid for the region or country (see Heiskanen & Moritz, 1967, p. 215), or possibly by using gravimetric or satellite techniques if the reference ellipsoid is required to be earth centred. The practical problem that arises is that values of N, ξ and η are known at discrete points along the traverse loops. Mather et al (1971) in Australia used a gravimetric solution of the geoid on a half degree geographical grid to obtain interpolated astro-geodetic values on a half degree grid. Within each adjusted loop the gravimetric solution was aligned to the astro-geodetic known values around the traverse loop (see Chapter 3 for more detail).

Least squares collocation techniques can be used to obtain the astro-geodetic heights from deflections of the vertical. In the countries surrounding the North Sea this technique was used to overcome such problems as linear interpolation, profile selection, and neglection of vertical deflection stations with only one observed component (Wenzel, 1979). The resulting geoid was estimated to have an accuracy of ±0.5 metres relative to the level of that of satellite doppler derived geoid heights.

When using orbital satellites to obtain a geopotential model of the earth the value of $(\frac{a}{r})^n$ in equation (2.29) becomes increasingly less than one as the value of the degree n increases. Thus there is a limiting maximum value of the degree n' which can be determined. Odd numbered Goddard Earth Models (GEM 1, 3, 5, 7, 9) are examples of geopotential models determined from satellite orbital data.

The GEM 9 model has been determined to a degree and order of 20. Thus features of the geopotential and hence the geoid height are smoothed to a wavelength greater than 2 x 10^3km (Rizos, 1980). These geopotential models are improved by incorporating additional information such as terrestrial gravity data. The even numbered GEM models (GEM 2, 4, 6, 8, 10) are examples of this.

In Canada values of N, ξ, and η were obtained using solutions of equations (2.31), (2.32) and (2.33). Potential coefficients from GEM 8, 9 and 10 and GRIM 2 (a European Geopotential model) were used separately to obtain values of N at 237 stations where Doppler derived values of N were available (Lachapelle, 1978). After scaling the NWL 9D Doppler system by -0.4 ppm the results were compared using:

$$\sigma(\Delta N) = \left[\frac{\Sigma(N_{coeff} - N_D)^2}{n - 1} \right]^{\frac{1}{2}} \quad \text{............. (2.56)}$$

where n is the number of stations, N_{coeff} is the geopotential derived value of N, and N_D is the scaled Doppler derived value of N. The values of $\sigma(\Delta N)$ for all GEM models ranged between ±2.0 metres for GEM 10A to ±2.6 metres for GEM 9. The values from the GRIM 2 model resulted in a $\sigma(\Delta N)$ of ±4.8 metres. The use of the "GEM type" solutions to derive the geoid height for reducing distances to the reference ellipsoid is adequate for national geodetic control surveys (Ibid, 1978). Solution for ξ and η using GEM 8 potential coefficients when compared to astro-geodetic values after orientation to the geocentre were found to have an "overall fit" of 4".34 in ξ and 4".85 in η which is relatively poor. This reflects the fact that deflections of the vertical are strongly affected by the local gravity field whereas geoid heights are much less so.

Kearsley (1976) gives an example where more than half the total signal of a component of the deflection of vertical is obtained from the innermost cap about the computation point.

Because of the insensitivity of geopotential type solutions to short wave effects, it is common to use combination techniques (as described in Section 2.4.2 or 2.4.3, e.g. Marsh & Changs, 1976; Rapp, 1978a; Vincent & Marsy, 1973). The two methods described in Section 2.4.2 for the determination of the value of N by combining potential coefficient data and terrestrial gravity data yield essentially the same result but the error analysis of the second method which uses truncation techniques is simpler (Rapp & Rummel, 1975).

Using these techniques the atmospheric correction for the mass of the atmosphere external to the geoid cannot be ignored when determining the value of N since the correction for a cap size of 20 degrees is of the order of 2 metres (Ibid, 1975). The atmospheric correction has no practical effect on the deflections of the vertical (Sjobert, 1977).

The advantage of the use of least squares collocation to determine the inner zone contribution for N, ξ and η is that it permits both terrestrial gravity data and astro-geodetic data to be used. In Canada the root mean square difference between astro-geodetic and predicted components of the deflections of the vertical using least squares collocation techniques for ξ_1 and η_1 in equations (2.50a) and (2.50b) was $\pm 1\overset{''}{.}26$ and $\pm 1\overset{''}{.}48$ respectively (Lachapelle, 1976). In this study an inner zone of radius 0.75 degrees was used with an outer zone of 8 degrees. In a more recent study by Sjoberg (1977) it was shown that a significant gain in accuracy is achieved if the outer boundary of the outer zone is increased to at least 30 degrees and preferably 40 degrees for the determination of the deflection of the vertical.

In the same study (Ibid, 1977) it was shown that the total root mean square error of the deflection of the vertical θ given by:

$$\theta = \sqrt{\xi^2 + \eta^2} \qquad \ldots\ldots\ldots\ldots\ldots \quad (2.57)$$

was ±2".2 for an inner zone radius of 1 degree and an outer zone boundary of 40 degrees using collocation techniques. If an intensive determination of the inner zone using Vening Meinesz formula was used the total root mean square error in θ was reduced to ±1".32. The collocation method uses more computer time than the use of Stokes & Vening Meinesz formulae, and this time difference rapidly increases with an increase in data points. For regions where there are several hundred or more gravity points within a radius of one degree of the computation point (as is the case in the South Australian Region) the use of the Stokes & Vening Meinsz formulae is preferable.

The use of combination methods was basically initiated because of the lack of suitable detailed gravity data in many parts of the world, thus avoiding the problems of interpolation in data sparse regions encountered by many researchers (e.g. Mather, 1968a, 1970). However in recent years the world coverage of mean gravity anomalies has greatly improved and this has radically altered the situation. Recently a gravimetric geoid of England and Ireland has been produced by Olliver (1979) using Stokes Integral and mean gravity anomalies obtained from terrestrial observations and satellite techniques. This method has the advantage that although the computation time may be longer than some combination methods the detailed analysis of random errors is simple. This is the method that is employed later in this work to produce the free air geoid for the South Australian region.

Altimeter data gives the height of the instantaneous sea surface above a reference surface. This enables the mapping of the geoid at sea with sufficient precision to permit the mapping of deviations of the actual sea surface from the geoid, and hence aids in the unification of regional levelling datums (Rizos, 1980). Another use of altimeter data which is of importance to this present study is the determination of mean gravity anomalies in ocean regions. This has proved to be a valuable source of gravity data. Using altimeter data, Rapp (1978a) produced 377 five degree equal area mean anomalies and 12144 one degree by one degree (1° x 1°) geographical block mean anomalies. This data is used in this work to assist in the definition of the gravity field in the Middle and Outer Zones.

Doppler derived values of the geoid height are determined after reduction of the data, obtained by observations at the stations which are extended over a period of several days. For this reason Doppler values are usually available at relatively few points in a county or region as is the present case for Australia. Thus Doppler values of the geoid height are not used in general to determine a detailed geoid (at present) but rather to orientate an astro-geodetic derived geoid to an earth centred ellipsoid (e.g. Wenzel, 1979) or to determine the zero order effect in a gravimetric or potential solution of the geoid (e.g. Olliver, 1980).

CHAPTER 3

A REVIEW OF GEOID STUDIES IN AUSTRALIA

3.1 Geoid Studies Prior to 1970

The first geoid studies in Australia were done in the Woomera Region of South Australia (Bomford, 1963) using astro- geodetic techniques to determine the separation of the Clarke 1858 Spheroid from the geoid at 60 local stations. The purpose of this study was to aid in the precise determination of rocket flight paths using camera techniques.

Prior to 1963 most geodetic observations in Australia were referenced to the Clarke 1858 Spheroid with its origin located in Sydney. After investigations by the Division of National Mapping a spheroid of equatorial radius a, and a flattening f, given by:

$$a = 6{,}378{,}165 \text{ metres}; \quad f = 1/298.3$$

with an origin at Maurice trigonometrical station in South Australia was used by National Mapping for an initial geodetic network adjustment. The spatial orientation of this ellipsoid with respect to the earth was determined by obtaining the deflections of the vertical at 150 Laplace stations spread over the continent, and applying these values as corrections to the geodetic coordinates at the Maurice Origin (Mather and Fryer, 1970.a). The consequence of this was a revision of all geodetic values in the network. A new origin was then defined in terms of the Grundy trigonometric station located in the centre of Australia. √

In 1965 the reference ellipsoid was changed to that adopted by the International Astronomical Union with parameters

$$a = 6{,}378{,}160 \text{ metres}; \quad f = 1/298.25$$

and this is called the Australian National Spheroid (ANS). The ANS has similar dimensions to the Reference Ellipsoid 1967 which was adopted by the International Association of Geodesy at the 1967 General Assembly. After studying the deflections of the vertical at 275 Laplace stations the Division of National Mapping decided not to change the coordinates of the existing centrally located origin, but rather to redefine the origin in terms of the Johnston Geodetic Station. This origin is now known as the Johnston Origin and was assigned a zero geoid ellipsoid separation value because of lack of information to the contrary (Mather and Fryer, 1970.a). The Australian Geodetic Datum (AGD) is defined by the ANS and a set of geodetic coordinates adopted for the Johnston Origin.

In 1967 the U.S. Army Map Service (Fischer & Slutsky, 1967) produced an astro-geodetic geoid for Australia referred to the AGD. About 550 widely distributed astro-geodetic stations were used to determine the geoid ellipsoid separation and then the results from each were combined by adjustment. One procedure was to use large loops of closely spaced astro-geodetic stations and compute the geoidal increments ΔH between stations using an adaptation of equation (2.5).

$$\Delta H = \frac{1}{412530} (\xi''.S_m + \eta''.S_p) \qquad \ldots\ldots\ldots\ldots\ldots \; 3.1$$

where ξ'' and η'' are the deflections of the vertical determined astronomically and S_m and S_p are the distances between the two stations along a meridian and the mean parallel respectively. The large loops used were selected because they had closures of one metre or less. The geoidal separations along the paths of these loops were determined to conform with N = 0 at the Johnston Origin. The second procedure was to interpolate values of ξ and η along each meridian and parallel respectively at full degrees of longitude and latitude at 30 minute intervals from charts prepared showing ξ and η isolines at one second intervals. Using these values, geoid profiles along the meridians and parallels were calculated using the projection method formulae (Fischer & Slutsky, 1967) and the geoid separation at each one degree latitude or longitude point. At these one degree points of intersection the values of the separation calculated along the meridian and the parallels did not usually agree. These values were then adjusted to agree taking into account the surrounding four adjacent one degree sections, two on the meridians and two on the parallels with the further constraint imposed by the fixed framework of the geoidal loops computed by the first procedure. The resulting geoidal contour lines have a random character with higher values in the east and the west than in the centre or the north (Fischer & Slutsky, 1967). The value of N from this astro-geodetic computation on the AGD ranges from -5 metres to 18 metres.

The first gravimetric geoidal solution in Australia was prepared by Mather (1968a) in 1967 for the South Australian Region. Free air anomalies were used in Stokes & Vening Meinesz formulae to compute the values of N, ξ and η. Two solutions were computed, one being referenced to the International Spheroid, 1930 using the corresponding International gravity formulae and the other solution being referenced to an ellipsoid with gravity formulae acceptable to the International Astronomical Union. This latter reference ellipsoid and parameters was later to be known as the Geodetic Reference System, 1967 (GRS-67). This gravimetric solution was a "composite solution" in the sense that all gravity data within 20° of the computation point was based on surface gravity observations whereas for regions beyond this inner zone, a set of 5° x 5° area mean free air anomalies derived from satellite data and surface gravity was used in the formulae. In the inner 20° zone mean free air anomalies were computed for various size compartments (see Table 3.1) from observed, interpolated and extrapolated gravity values.

This determination of the free air geoid was extended by Mather in 1968 for the entire mainland of Australia using the same techniques as those used in the South Australian study but with one additional outer Data Set thus giving two solutions. Both outer data sets were derived from a combination of satellite and surface data but were prepared independently by Kaula and

Range of ψ	Date Type	Compartment Size
$\psi < 0.°1$	Surface Gravity	Individual Readings
$0.°1 < \psi < 1.°5$	Surface Gravity	0.°1 x 0.°1
$1.°5 < \psi < 5°$	Surface Gravity	0.°5 x 0.°5
$5° < \psi < 20°$	Surface Gravity	1° x 1°
$\psi > 20°$	Combined Satellite and surface data	5° x 5°

Table 3.1

Gravity Data and Compartment Sizes for corresponding ranges of ψ.

Rapp using different techniques (Mather 1969) and were found to have a global comparison error of ±12.5 mGals. The use of each of these data sets gave similar solutions for the free air geoid of Australia with a systematic difference of approximately 4 metres in the value of N.

Using the Rapp data for the outer zone the value of N ranges from -14 metres to +80 metres with value generally increasing in the north east direction. Both components of the deflection of the vertical had predominantly negative values with a range of -14" to +2" and -12" to +6" for ξ and η respectively. These values for the deflections of the vertical do not include the contribution from the inner four 0.°1 x 0.°1 blocks about the computation point because these contributions were found to be as large as 1" under not uncommon circumstances. Mather (1969) states "... the magnitude of the deflection of the vertical can be significantly affected by the approximate technique used for the evaluation of the inner zone ...". It is worth noting that at the time of these studies the existing gravity coverage in South Australia was poor in comparison with most other regions of the Australian Mainland and in fact, for twelve 1° x 1° blocks there was no observed gravity data available.

The gravimetric geoid solutions were compared to the astro-geodetic solution obtained by Fischer & Slutsky by converting the AGD to the datum of the free air geoid. The conversion was made by using three correction parameters to the coordinates at the Johnston Origin. These were ΔN_0 to the geoid-spheroid separation, $\Delta\xi_0$ to the meridian component, and $\Delta\eta_0$ to the prime vertical component and they were derived using a least square adjustment between the free air geoid and the astro-geodetic solution at approximately 700 points. Several solutions were obtained by using separately the Kaula & Rapp data and various geographical constraints. The results obtained for $\Delta\xi_0$ and $\Delta\eta_0$ were approximately -4".6 and -4".2 respectively. Depending on the data and the geographical constraints used the value of ΔN_0 varied from 6.0 metres to 20.4 metres with an average value of 14.9 metres. The range in the correction values of ΔN_0 was due to the weakness of the gravity field in the north west of South Australia (Mather, 1969). A comparison between the corrected astro-geodetic values of the separation N_a and the values of the separation N_g determined using Rapp data for the outer zone, show the value of $(N_a - N_g)$ to have a high value of +10 metres in the north west region of South Australia and low values of -16 metres and -14 metres on the west and north east cost of Australia respectively.

In a subsequent study (Mather & Fryer, 1970.a) using the free air geoid computed with Rapp data representing the outer zone, the Fischer-Slutsky astro-geodetic geoid, 506 Laplace stations and a least square solution, orientation corrections to AGD were again computed. The indicated values for these corrections were:

$$\Delta\xi_0 = -4''.7$$

$$\Delta\eta_0 = -4''.4$$

$$\Delta N_0 = 14.0 \text{ metres}$$

The orientation parameters were used to represent the astro-geodetic geoid on the Reference Ellipsoid, 1967 and this "Corrected astro-geodetic geoid" was said to be "the best representation of the geoid at present ...". (Mather & Fryer, 1970.b). The "corrected" geoid had a minimum separation value of -26 metres in the south west and a maximum value of +68 metres in the north east of Australia.

3.2 1970 Free Air Geoid

In 1970 Mather recomputed the free air geoid for Australia using the Rapp data set to represent the Outer Zone and a more comprehensive and consistent data set for the gravity field in the Australian region. In the comparison between the 1968 free air geoid and the Fischer and Slutsky astro-geodetic geoid, after translation into a common system the value of ΔN:

$$\Delta N = N_a - N_g$$

was found to have a root mean square value of ±5.3 metres over the continent about a mean not significantly different from zero. The values of ΔN were observed to be position dependent. Significant inconsistancies existed in the gravity anomaly field used in the near zones and this was thought to be partly the cause of the differences between the two geoid solutions. In the 1968 solution the data sets were independent of one another and this led to discrepancies between different data sets representing the same region in partially surveyed or represented areas. No inconsistencies occurred in fully surveyed or totally unsurveyed areas (Mather, 1970).

The 1970 free air geoid solution used the same sub-divisional units defining the data sets and geographical blocks as the 1968 solution. The field gravity data was enhanced by the addition of helicopter and marine gravity surveys carried out by the Bureau of Mineral Resources, Canberra and information from other sources. Compatible mean free air anomalies were computed for 0.°1 x 0.°1, 0.°5 x 0.°5, 1° x 1° and 5° x 5° blocks using consistent common data based on free air anomalies on a 0.°1 grid and using a two dimensional trigonometric function to interpolate missing values. The area means were used for 5° x 5° blocks if the total number of surface gravity values was in excess of 1000, but if this was not the case Rapp data was substituted.

The resulting 1970 free air geoid on comparison with the Fischer and Slutsky astro-geodetic determined geoid after translation to a common system, showed that major inconsistencies had been eliminated by the techniques briefly described above. The root mean square value of ΔN was reduced to ±2.5 metres. Some systematic differences still occurred between the two differently derived geoids but these existed only over limited geographical extents and generally on the peripheries with the one exception being the geoid low over the Officer Basin in South Australia where ΔN reached a value of 7 metres. Although the available gravity data used in this solution was more comprehensive than that used in the 1968 solution there was still a paucity of data in some regions and in South Australia there were ten 1° x 1° blocks having no observation data and these were situated in the vicinity of Officer Basin.

As well as this general solution of the free air geoid of Australia, Mather selected 38 astro-geodetic stations in the AGD and by field observations intensified the observed gravity field in the four 0.°1 x 0.°1 blocks surrounding these stations. The values of N, ξ, and η computed for these 38 stations along with the values obtained from the general solution on a one degree geographical grid across Australia were then compared with the astro-geodetic determined geoid, and orientation parameters for the AGD were computed by various combinations of the available results (see Mather, 1970 for more detail). The results indicated that the inner-most zone contribution had marginal effects on the orientation parameters. The following values for the orientation parameters were obtained:

$$\Delta\xi_0 = -4''.2 \pm 0''.2$$

$$\Delta\eta_0 = -4''.5 \pm 0''.2$$

$$\Delta N_0 = +7.2 \pm 0.2 \text{ metres}$$

The value of ΔN_0 (not to be confused with $\Delta N = N_a - N_g$) includes a zero order term of -2.8 metres resulting from the mean of the free air anomalies used in the study. This mean for the whole of the earth surface was +0.4 mGal and not zero. No allowance has been made for the non-zero order correction for the separation of the free air geoid (co-geoid) and the geoid estimated to be an average of about 6.8 metres with less than one metre variation over the Australian region (Fryer, 1970). The errors quoted are based on detectable errors and do not indicate any systematic effects that may have existed in the low degree harmonics of the earth's gravitational field.

3.3 The Geoidal Studies of Australia by Grushinsky and Sazhina

Grushinsky and Sazhina (1971) produced a gravimetric geoid for the Australian region using slightly different techniques to those employed by Mather (1968, 1970). The basic data for the Australian region was obtained from the Bureau of Mineral Resources preliminary gravimetric maps on the scale of 1:2,500,000 and map sheets on the scale of 1:500,000 and 1:250,000 as were available in 1966 and this was supplemented with observed gravity data for the surrounding regions from a multitude of sources (see ibid, 1971, for details). The contribution to the geoid ellipsoid separation from the outer zone (ψ > 1000 km) was computed using spherical harmonics of the gravitational field obtained from Smithsonian Astrophysical Observatory. Mean values of the free air anomaly for 1° x 1° blocks were used to represent the inner area (ψ < 1000 km). For each of the four 1° x 1° blocks directly adjoining the computation point Stokes' function was integrated over the entire block and the resulting mean value used in the summation form of Stokes' formula. In the band of the twelve 1° x 1° blocks immediately surrounding the four innermost blocks, the mean value of Stokes' function computed to centres of the four 0.°5 x 0.°5 blocks within the larger block was used, and for the remaining 1° x 1° blocks Stokes' function was computed to the geometric centre of each block.

The geoidal undulations resulting from these studies had a minimum value of -40 metres in the south west and a maximum of +70 metres in the north east of Australia. When compared to the 1968 free air geoid the systematic differences between the two solutions is -2.2 metres and the root mean square of these differences over the entire continent is ±5 metres. On graphical comparison with the Mather 1970 free air geoid which has a total separation range of -24 metres to +64 metres the results are similar for most of the Australian continent except in the south west region of Australia where the Grushinsky and Sazhina geoid has a greater slope and the separation reaches a minimum value of -40m compared to the value of -24 metres in the 1970 free air geoid.

3.4 The Geoid in Australia - 1971

The 1971 Geoid map was based on a combination of gravimetric and astro-geodetic methods. In effect it was an astro-geodetic geoid with appropriately corrected gravimetric values of N being used to define the areas within the astro-geodetic levelling loops.

The available astro-geodetic levelling over Australia was divided into 49 loops with 76 junction points and comprised a total of 49,407 km of levelling. The levelling routes were carefully selected, taking into account any regions where local irregularities may have had unwanted effects on the deflections of the vertical. These regions were indicated by either rapid geological changes, poor siting of trigonometric stations e.g. on non symmetrical hills, or by a poor localised agreement with results from the 1970 free air geoid. The mean misclosure was ±2.0 metres in the individual loops and their average length was 1656 km. The value of N at Johnston Origin was held fixed at zero metres and the 76 junction points and the 49 loops were simultaneously adjusted resulting in astro-geodetic values of N for 1133 stations (Mather et al, 1971).

The gravimetric values of N were transformed to the AGD using the orientation parameters determined from the comparison of the Fischer and Slutsky astro-geodetic geoid with the 1970 free air geoid to determine ΔN in the relationship (Mather, 1969, p. 27) $Ng_a = N_g + \Delta N$; where ΔN is the correction to be applied to the gravimetric determined value of the geoid separation N_g referred to the 1967 Reference System, to obtain the equivalent value Ng_a referred to the AGD.

The residuals σ_N of the difference between the astro-geodetic value of the separation N_a and the corresponding value Ng were determined at each astro-geodetic station. The residuals σ_N ranged in value from -4 to +4 metres, but for approximately 75% of the Australian continent the absolute value was less than 2 metres. The larger values of σ_N occurred mainly in the coastal regions, the exceptions being three local areas. The available astro-geodetic density in these three areas was inadequate and in one area the gravity field was largely predicted (Mather, 1972). The gradients of σ_N along the controlling astro-geodetic loops were small when the station density was high.

The residuals $\bar{\sigma}_N$ at gravimetric stations where no astro-geodetic information existed were interpolated from the known residuals σ_N using (Mather et al, 1971):

$$\bar{\sigma}_N = \frac{\sum_i w_i \sigma_{N_i}}{\sum_i w_i} \qquad \text{................ } 3.2$$

The weight coefficients w_i were determined from

$$w_i = \ell_i^{-2}$$

where ℓ_i represented the distance between the ith station on the perimeter of the loop and the gravimetric station at which the interpolated value of $\bar{\sigma}_N$ was required. Using the gravimetric determined value of the geoid separation, N_g, the computed correction ΔN, and the estimated value of the residual $\overline{\sigma}_N$, an estimate of the astro-geodetic value of the geoid separation N_a was obtained at each gravity station from the relationship.

$$N_a = N_g + \Delta N + \overline{\sigma}_N \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.3$$

This method localised any effect of the residuals and permitted the gravimetric determined values of N to be used to define the geoid within the perimeters of the loops.

Figs 2

The result is an astro-geodetic geoid referenced to the AGD with values of N determined from the 1970 free air geoid being used as interpolation factors within the geodetic levelling loops. The dominant features of this geoid are a low of -4 metres centred near the Officer Basin in South Australia, geoidal rises associated with all mountain ranges, and a maximum geoidal high of +18 metres in the south west of Western Australia. The 1971 geoid map published in (Mather et al, 1971) and (Fryer, 1971) shows a geoid low of -10 metres in the Officer Basin and a high of +12 metres in Western Australia. This is due to a linear change in the value of N of -6 metres caused by adopting N = -6 metres at the Johnston Origin instead of N = 0.

Charts for both deflections of the vertical were produced on a similar basis to that used for the geoid separation. Using the orientation parameters the free air derived values were connected to the AGD and used for interpolation of results within the geodetic levelling loops. These converted values of the deflections of the vertical when compared to the astro-geodetic determined value at 1084 stations, had a root mean square residual of ±2". Caution must be exercised in the use of these contour maps of the deflections since the average data point spacing was 50 km (Fryer, 1971) and there was an inadequate sampling of gravity in the innermost zone of many stations (Mather et al, 1971). Deflections of the vertical may change rapidly in regions of relatively flat topography. In one region of South Australia a change of 25" occurred within a distance of 90 km.

The values of ξ ranged from -20" to +16" with rapid gradients occurring in a large region situated in Central Australia where the minimum value of -20" occurs, and also in the south east of the continent where the maximum value of +16" occurs. The value of η ranges from -16" to +16" with rapid gradients occurring in several regions, the largest of which are situated near Canberra, Perth, north of Esperance in W.A., and the Officer Basin.

A revised set of orientation parameters were computed from a comparison of the 1971 geoid and the 1970 free air geoid and the results were:

$$\Delta\xi = -4''.0$$

$$\Delta\eta = -4''.1$$

$$\Delta_N = +8.3 \text{ metres}$$

The value of ΔN does not include a zero order term. The root mean square of the residuals σ_N from 1133 station comparisons was ±1.6 metres as compared to ±2.6 metres obtained from the comparison between the 1971 free air geoid and the Fischer and Slutsky astro-geodetic field.

3.5 More Recent Geoid Investigations on the Australian Continent

3.5.1 Kearsley (1976) investigated gravimetric methods of evaluating the deflections of the vertical using 12 stations in the north west of New South Wales. These stations were selected because the astro-geodetic deflections had been previously determined at each of these points and they were situated in a variety of terrain types. Vening Meinesz formula was used to compute the deflections and then the corrections for the terrain effect were evaluated using two different methods. One method used the Molodenskii terrain correction as modified by Pellinen (1964) and the other used a modification of Green's Identity Approach (Kearsley, 1976).

The contributions to the Vening Meinesz deflections of the vertical of the inner zones, i.e. $\psi < 1.5°$ were evaluated using a unique computer technique based on Rice Rings and modified to suit the density of available data. Using the same type of subdivision and a height data bank the two types of terrain correction were determined. For the outer region effects i.e. $\psi > 1.5°$ the results from the 1970 free air geoid solution were used with additional effects for the merging between the Rice Ring type system and the grid system used for the 1970 free air geoid. No attempt was made to compute the effects of the terrain corrections for these outer regions since they only have possible significant effect near the computation point. The gravity coverage used in the 1970 free air geoid solution was supplemented with an additional 200 gravity stations situated in the immediate vicinity of the 12 stations.

The results including the terrain corrections were compared to the astro-geodetic values after application of the orien-parameters (Mather, 1970). The mean differences in ξ and η were $+0\overset{''}{.}06$ and $+1\overset{''}{.}47$ respectively. The large value of $+1\overset{''}{.}47$ was thought to indicate the existance of systematic errors in the gravimetric value of η (Clarke, 1978) and was possibly due to an inadequate knowledge of the gravity field off the east coast of Australia. After the removal of these systematic effects the precision of the gravimetric values of the deflections was $\pm 0\overset{''}{.}5$ as compared to ±2" obtained in the 1970 free air geoid solution.

The evaluation of the terrain correction obtained from the application of Green's Third Identity to the Earth's surface proved quite unstable due mainly to the sensitivity of the expression to errors in the determination of ground slopes. The Pellinen modified terrain correction effects were computed at only 7 stations, since the remaining stations were situated in relatively flat topography and the effect would have been negligible. The maximum correction was $-0''.64$ to ξ at one station and the mean corrections to ξ and η without respect to sign were $0''.3$ and $0''.15$ respectively.

3.5.2 A first order traverse loop was used by Clarke (1978) to study the effect of incorporating gravimetrically determined quantities in a three dimensional cartesian adjustment of geodetic data. This traverse loop consisted of 78 stations, had a perimeter of 2200 km, and was located in New South Wales and to a much lesser extent in Victoria.

The geoid ellipsoid separation at each station was computed using the same basic zone configuration and compartment size as Mather (1970) with the innermost zone ($\psi < 0.°1$) being subdivided into 0.°01 blocks. The deflections of the vertical were computed using the computer techniques and the methods developed by Kearsley (1976) but no terrain corrections were applied. The gravity data used was the same as that previously compiled by Mather (1970) and later used by Kearsley (1976) but supplemented with additional observed gravity in the immediate vicinity of each traverse station. Gravity points were observed at each traverse station and at each of the four cardinal points at 400, 1600, and 6,400 metres, making a total of 13 observed gravity points within the innermost zone.

Within the 200,000 square kilometer study region there were 47 stations at which astronomical observations were available.

Using orientation parameters determined by Mather (1970) the astro-geodetic and gravimetric determination values of N, ξ and η were compared. The mean of the residuals of the differences showed non-zero means of -3.69 metres, $-0\overset{''}{.}44$, and $1\overset{''}{.}07$ respectively. These results when considered along with those of Mather (1970) and Kearsley (1976) suggested that systematic errors in the gravimetric determinations of N, ξ and η existed in the Australian region. These systematic errors were position dependent and may have values as large as, or even larger than the random errors associated with the same quantities (Clarke, 1978).

The results of the three dimensional adjustment showed that the astronomical determined values of position and direction appeared to have a lower precision than that generally accepted, with standard deviation for latitude, longitude and azimuth of $\pm 0\overset{''}{.}5$, $\pm 1\overset{''}{.}2$, and $\pm 1\overset{''}{.}7$ respectively. Gravimetric determined values of N, ξ and η were shown to be at least as reliable to those determined by classical astro-geodetic methods if they were corrected for the systematic error effects.

In the same study it was demonstrated that the predominant features of the systematic errors could be modelled by computing pseudo corrections to the mean gravity anomalies for four compartments arbitrarily selected with a symmetrical distribution at a distance of approximately 20 degrees from the Johnston Origin. This investigation was later extended by Clarke (1981) to cover the whole of the AGD using a set of 83 control stations comprising the stations used by Mather (1970) and those used in the earlier study by Clarke (1978). The ξ and η residuals between the astro-geodetic values and the orientated gravimetric values were used to compute pseudo-corrections at selected locations suitable for use over the whole of the Australian continent.

CHAPTER 4

PRACTICAL EVALUATION OF STOKES' AND VENING MEINESZ FORMULAE

4.1 Stokes' and Vening Meinesz Formulae

For the practical evaluation of Stokes' integral (2.12a) and Vening Meinesz formulae (2.16) the surface integrals are replaced by finite summations resulting in:

$$N = \frac{R}{4\pi\gamma} \sum_i S(\psi)_i \, \Delta g_i \, \Delta\sigma_i \quad \ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

and

$$\left.\begin{matrix}\xi \\ \eta\end{matrix}\right\} = \frac{1}{4\pi\gamma} \sum_i \frac{dS(\psi)_i}{d\psi} \, \Delta g_i \begin{Bmatrix}\cos \alpha_i \\ \sin \alpha_i\end{Bmatrix} \Delta\sigma_i \quad \ldots\ldots \quad (4.2)$$

where Δg_i is the mean gravity anomaly of the surface element $\Delta\sigma_i$, and $S(\psi)_i$ and $\frac{dS(\psi)_i}{d\psi}$ are the mean values of the Stokes' and Vening Meinesz functions for the compartment or surface area $\Delta\sigma_i$. If $\Delta\sigma_i$ is bounded by meridians and parallels n and m degrees apart respectively, then:

$$\Delta\sigma_i = \frac{\pi^2}{180^2} \, n \, m \cos \phi_i \quad \ldots\ldots\ldots\ldots\ldots \quad (4.3)$$

where ϕ_i is the mid latitude of the compartment. It is customary to compute the values of both Stokes' and Vening Meinesz functions using the mid geographical coordinates ϕ_i and λ_i of the compartment $\Delta\sigma_i$ to represent the mean values of these functions within the compartment, that is:

$$S(\psi)_i = S(\psi)_{\phi_i,\lambda_i} \qquad \ldots\ldots\ldots\ldots\ldots (4.4)$$

$$\frac{dS(\psi)_i}{d\psi} = \frac{dS(\psi)_{\phi_i,\lambda_i}}{d\psi} \qquad \ldots\ldots\ldots\ldots\ldots (4.5)$$

This is an approximation and an error is introduced into the computational procedure. Paul and Nagy (1973) suggest for the evaluation of Stokes' function the introduction of secondary terms to the right hand side of equation (4.4) and a similar treatment to equation (4.5) could be used to reduce the error incurred in using these approximate formulae. The more usual approach is to divide the earth's surface into several zones and within these zones the compartments $\Delta\sigma_i$ are constant or near constant in area. Both functions are dependant on the value of ψ and rapidly increase as the value of $\psi \to 0$ with the reverse effects as $\psi \to 180°$. Thus, with careful selection of the zones and the compartment sizes, equations (4.4) and (4.5) may be used in the practical evaluation of equations (4.1) and (4.2).

The mean gravity anomaly Δg_i selected for use in equations (4.1) and (4.2) is the mean free air anomaly for reasons discussed in Chapter 2.2 and is given by:

$$\Delta g_i = \frac{1}{\Delta\sigma_i} \iint_{\Delta\sigma_i} \Delta g \, d\sigma \qquad \ldots\ldots\ldots\ldots\ldots (4.6)$$

This formula assumes that the gravity anomaly is known at every point within the compartment $\Delta\sigma_i$. In practice this is not the case and the actual determination of $\Delta\sigma_i$ depends on the gravity data available in that region.

4.2 Gravity Data Available for Geoidal Determinations in the South Australian Region.

4.2.1 B.M.R. Surface Gravity Data

The majority of the observed gravity data used in this work was supplied by the Bureau of Mineral Resources (B.M.R.) from their gravity data bank. This data bank contains information on more than 500,000 gravity stations observed by B.M.R. personnel, their contractors, State Mines Departments, private exploration companies and academic institutions.

The largest individual contribution to this data bank was made by the B.M.R. and their private contractors using helicopter transport which gave an average station spacing of 11 km except in South Australia and Tasmania where the average is 7 km. In South Australia this was due to close co-operation between S.A. Mines Department and the B.M.R. The height differences between stations were measured using microbarometers and these were subsequently tied to the Australian Height Datum. The resulting gravity station heights are estimated to have an R.M.S. error of better than ±5 metres (Dooley and Barlow, 1976) within each survey. The gravity intervals between stations were measured with La Costa and Romberg or Worden gravity meters and the estimated precision with respect to the Australian National Gravity Network (ANGN) as a whole is ±0.5 mGal. (Mather et al, 1976). The remaining data which was supplied by the other agencies has been recomputed and adjusted to the same datum and scale as the B.M.R. observed values. The precision of the data is at least equal to, if not better than that of the B.M.R. reconnaissance surveys.

The ANGN originated with 59 Cambridge pendulum stations, one of which comprised the then National Gravity Base Station (NGBS) at Melbourne, which was included in the First Order World Gravity Net. Between the years 1964-1967 a series of east-west gravity traverses were run across Australia following approximately an "isogal" of gravity values to minimise the observed gravity differences between stations. These stations were situated at or near airfields and were approximately 150-250 km apart thus allowing brief travelling times by chartered aircraft between stations and hence reducing uncertainties in drift in the meter readings at each station (a minimum of three different gravity meters were used on each survey). Using the results from these "Isogal Surveys", 57 of the original 59 pendulum stations that were re-occupied, and a "mean Australian milligal", (Dooley and Barlow, 1976) the ANGN was redefined using May 1965 Isogal Values. All gravity values in the B.M.R. data bank are referenced in scale and datum to the ANGN and the May 1965 Isogal values.

In 1973 a new datum and scale were selected (Boulanger et al, 1973). To comply with the change brought about by the International Union of Geodesy and Geophysics adopting a new international gravity reference system in 1971 called the International Gravity Standardization Net 1971 (IGSN 71), the IGSN 71 station at Sydney was adopted as the new National Gravity Base Station for Australia in place of the Melbourne station. The scale change to the "mean Australian milligal" was determined by an accurate gravity survey by an Australian-Soviet team along the east coast of Australia using eight Soviet Gag-2 gravity meters (Mather et al, 1976). This survey showed the values of IGSN 71 stations, other than the Sydney station to be in error by 15 parts in 10^5. This was confirmed within experimental error in 1974 when Soviet OVM Pendulums were used to resurvey the line. As all Australian gravity data used in this present work is based on the up to date 1973 datum, all B.M.R. gravity values were adjusted by a linear transformation using:

$$g_{1973} = 979671.86 + 1.0005118\ (g_{1965} - 979685.74)$$

This results in an estimated standard error of less than 0.2 mGal throughout Australia (Dooley and Barlow, 1976).

For each gravity station the following information was obtained from the B.M.R. data bank.

(i) the geographical position.

(ii) the observed gravity value adjusted and relative to the May 1965 Isogal system.

(iii) the terrain and station height adjusted to the Australian Height Datum (AHD).

The geodetic latitude and longitude referred to the Australian Geodetic Datum (AGD) is used to locate the position of each field gravity stations. This geodetic latitude is not a geocentric latitude but nevertheless is used in the formula to compute the normal gravity. Since the discrepancy between the two is small, the resultant effect is not expected to be greater than ±5 μGal (Mather et al, 1976). The estimated error of 0.1 minutes of latitude (Anfiloff et al, 1976) in position location would result in a larger error than this.

4.2.2 Data Obtained from Field Observations

To facilitate tests for the interpolation of gravity data approximately 300 gravity stations were observed in the Flinders Ranges of South Australia. This area was selected for the ruggedness of terrain which, although not large on world standards, changes from 190 metres to 1000 metres elevation over a geographical area of approximately $\frac{1}{2}° \times \frac{1}{2}°$. The topographical surface is mountainous with terrain slopes varying rapidly in all directions and would represent the rugged extreme in the South Australian Region.

The gravity readings were made using a Worden gravity meter, geodesist model and reduced to the May 1965 Isogal Values. To do this the gravity survey was done in a series of loops and tied to two separate Isogal Stations 200 km apart. The gravity survey had an internal precision of better than ±0.2 mGals and the separate ties to the two Isogal stations agreed within 0.1 mGal.

The elevations of the stations were obtained by two methods. More than 25% of the gravity readings were taken on bench marks (B.M.'s) where the height had been previously established by third order levelling techniques and tied to the Australian Height Datum (AHD). The estimated precision of the elevations are ±2.0 metres in the AHD system. The position of other stations were located on aerial photographs using stereographic models and, later in the laboratory, the heights were obtained using a Zeiss Stereometrograph and diapositives with plotting control supplied by the South Australian Department of Lands. The heights obtained this way are estimated to have a precision of ±3 metres. As in each case the geographical position was obtained from the detailed 1:50,000 map sheets covering the area, the height of the station was also interpolated from the 10 metre contours on these maps to act as a check for any gross blunders.

In addition to the observation programme in the Flinders Ranges, gravity stations were observed in the immediate vicinity of selected trigonometric stations where astronomical and/or Doppler data were available from the Lands Department of South Australia or the Division of National Mapping of the Department of National Development.

The aim of these surveys was to establish with more detail than the existing data permitted, the gravity field of the Innermost Zone for detailed comparisons between gravimetric and non-gravimetric determination of the geoid. Gravity values were obtained at each trigonometric station and at eight symmetrically spaced points within the vicinity of 1-3 km of the station. It was not always possible to observe all eight gravity points because of the actual terrain and vegetation encountered.

These local gravity surveys were tied to the ANGN using either local identifiable gravity stations supplied by the Mines Department of South Australia or Isogal Stations. Trigonometric surveying methods were used to obtain the geographical location and the height of the gravity stations with the exception of the survey around the trigonometric station Lock. In this gravity survey both the geographical and height coordinates, due to weather conditions at the time, were scaled from a 1:50,000 topographical map and more details of these surveys, their location and use is given in Chapter 6.

4.2.3 Combined Satellite and Terrestrial 1° x 1° Mean Gravity Anomalies Set

This data set consists of 50650 mean free air anomalies referenced to the GRS-67 system. The data is derived from a combination of terrestrial and satellite data, and was obtained from R.H. Rapp of the Ohio State University in 1979.

The terrestrial data originated with a data set, known as the August '76 set, which was primarily based on a revised version of the United States Defence Mapping Agency Aerospace Centre. The set has subsequently been updated using information obtained from various sources around the world including new data from Australia supplied by R.S. Mather (Rapp, 1978.b).

The altimeter gravity data is derived from the use of the Geos-3 altimeter. Assuming the sea surface to be static and to coincide with the geoid, then, knowing the height of the satellite Geos-3, above a reference ellipsoid at any one point in time and using the altimeter to determine the distance between satellite and sea surface (assumed to be the geoid), the geoid ellipsoid separation can be derived. (Ch. 2) This is a model and in reality, it is not as simple as this, (Rummel & Rapp, 1977) but using these principles over a time period, a mean free air gravity anomaly can be computed from these derived undulations of the geoid.

The two data sets were merged to form the combined terrestrial altimeter 1° x 1° mean free air anomaly set. The altimeter data was given priority in the ocean areas and only supplemented to a very limited extent with terrestrial ocean data, and then combined with terrestrial data from land areas.

The standard deviation of each known 1° x 1° anomaly is given and the average of these is ±15 mGals for the terrestrial, and ±8 mGals for the altimeter derived data. Where the separate data sets overlap a comparison was made between all data showing the R.M.S. difference between the two to be ±15 mGals. If the comparison was limited to original data with standard deviations no greater than 10 mGals, the resulting R.M.S. difference is ±12 mGals (Rapp, 1978.b).

The result is a data set consisting of the mean free air anomaly, and its standard deviation and geographical location referenced to the GRS-67 for 50650 1° x 1° geographical blocks (hereafter referred to as the Rapp 1° data set).

4.2.4 Combined Satellite and Terrestrial 5° Equal Area Anomalies Set

The information in this data set is derived from the 5060 1° x 1° mean anomalies described in (4.2.3) and is referenced to the GRS-67 system. This data obtained from Prof. R.H. Rapp consists of 1654 5° equal area anomalies. The actual location of these 5° equal area blocks is the same as that used by Kaula (1966).

The anomaly values for each 5° equal area compartment were obtained using least square techniques and the existing 1° x 1° mean anomalies within that compartment. In the 157 compartments where no 1° x 1° mean anomaly data existed the value was predicted from the surrounding ten nearest 5° equal area values already determined (Rapp, 1977).

The resulting data set consists of 1654 5° mean area gravity anomalies giving complete cover of the earth's surface. With each mean anomaly in this set the latitude and longitude values of the boundaries and of the geographic centre, the spherical area on a unit sphere, and the standard deviation of the anomaly are given.

4.3 Zone Locations and Compartment Sizes

Originally both Stokes' and Vening Meinesz formulae were evaluated using a system of rings or zones centred around the computation point. Between each two successive rings the compartment size $\Delta\sigma_i$ was constant but increased in size, as did the spherical distance between the rings, as the value of ψ increased. The mean gravity anomalies were usually interpolated from maps using graphical methods. Thus, the gravity data obtained could not be used for a similar computation at even a neighbouring point on the earth's surface because the circular compartments would not correspond.

Since the advent of the electronic computer most solutions of N, ξ and η have used some form of grid or "square" system based on latitudes and longitudes. This type of method permits the simple storage of gravity data and the relevant position and compartment description information, and this data bank usually requires no change when the computations are repeated for a neighbouring point. One disadvantage of this method occurs when the computation point is not coincident with a grid point. In this case the required values of N, ξ and η are either interpolated from the neighbouring grid points or the grid may be further subdivided to accommodate the location of the point. Kearsley (1976) used a computer combination of both the rings and grid system to successfully compute deflections of the vertical at 12 trigonometric stations in north western New South Wales.

Since the geoidal solutions of South Australia are computed at half degree latitude and longitude grid points the grid system is used. (See Chapter 6 for non-grid points.)

Zone boundaries and compartment sizes are selected for the evaluation of Stokes' and Vening Meinesz formulae so that equations (4.4) and (4.5) respectively are practically satisfied. Since the function $S(\psi)$ and $\frac{dS(\psi)}{d\psi}$ vary differently with respect to ψ, it is feasible to select non-corresponding zone boundaries and compartment sizes for the solution of Stokes' & Vening Meinesz formulae, but this would greatly increase the data processing required. Thus zones and compartment sizes are selected for combined solutions so that both equations (4.4) and (4.5) are practically satisfied. The zones and compartment sizes within the zones selected for this study are given in Table 4.1.

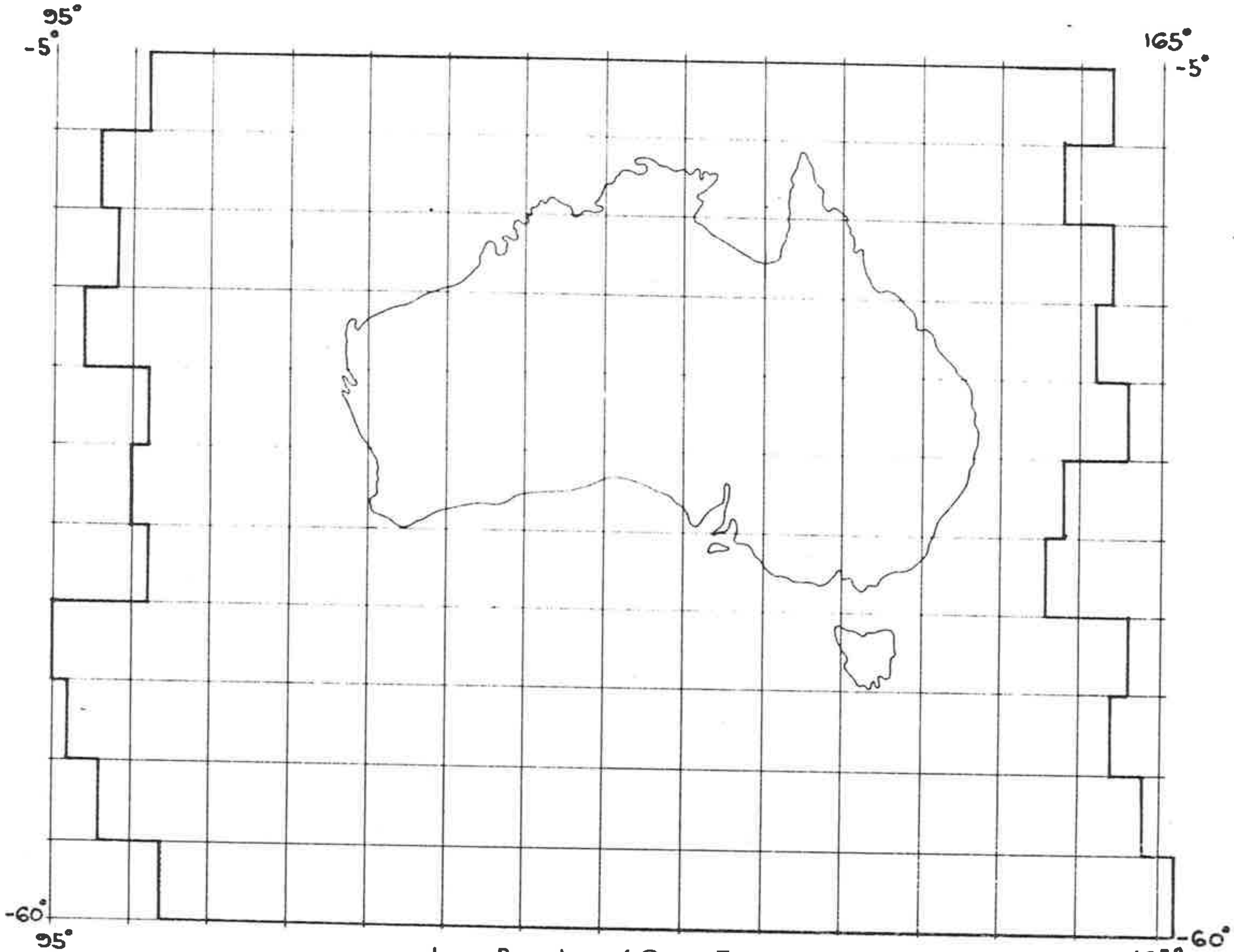
ZONE	RANGE OF	COMPARTMENT SIZE WITHIN THE ZONE
Outer	ψ > 20° approx	5° equal area blocks
Middle	5° approx < ψ < 20° approx	1° x 1° geographical blocks
Near	1.5° approx < ψ < 5° "	0.5° x 0.5° geographical blocks
Inner	0.2° approx < ψ < 1.5° "	0.1° x 0.1° geographical blocks
Innermost	ψ < 0.2° approx	0.1° x 0.1° geographical blocks and individual readings

TABLE 4.1

The Outer, Middle and Near Zone boundaries and compartment sizes are similar to those used by other authors studying the geoid in Australia (Kearsley and Van Gysen, 1979; Mather, 1970). For ease of computation the inner boundary of the Outer Zone is held constant for all computation grid points. The geographical location of this inner boundary, shown in Figure 4.1, was selected using two criteria:

(1) the zone edge is located at a minimum of 20° from any grid point,

and (2) the boundary is identical to that of a 5° equal area block, to which a gravity anomaly is allocated.



Inner Boundary of Outer Zone

(Inner Boundary Shown ———)

Figure 4.1

The inner boundary of the Middle Zone is a rectangular geographical block with the boundaries centred around the grid point so that $\psi \geqslant 5°$. This results in a boundary size of 10° x 10°, 11° x 11°, or 10° x 11°, depending on the actual location of the grid point. The outer boundary is coincident with the inner border of the Outer Zone. The inner boundary of the Near Zone is a 3° x 3° geographical block with the computational grid point as its centre.

During the computation procedures to determine the values N, ξ and η using equations (4.1) and (4.2) the contributions from the Middle and Near Zones were recomputed for the grid point 26°S and 129° E using values for $S(\psi)$ and $\frac{dS(\psi)}{d\psi}$ obtained from the mean values of the respective functions at 100 uniformly distributed points within each compartment.

The results when compared to values obtained using equations (4.4) and (4.5) showed no difference in the contributions to the deflections of the vertical and less than 0.08 metres to the geoid separation.

The outer boundary of the Innermost Zone was selected after testing the effects of the assumptions implied in equations (4.4) and (4.5) at the grid point 26°S and 129°E. This grid point was used because the surrounding mean gravity anomalies of the 0.1° x 0.1° blocks have rapidly changing values and this represents an extreme for the region being studied. The contribution for both the Inner and Innermost Zones were computed by several varying methods.

(a) Using equations (4.1), (4.2), (4.4) and (4.5) and the mean gravity anomalies for each 0.1° x 0.1° block.

(b) Using equations (4.1) and (4.2) and the mean gravity anomalies for each 0.1° x 0.1°, but using Stokes' and Vening Meinesz functions obtained from the mean values of the function at 100 uniformly distributed points within each 0.1° x 0.1° block.

(c) Using method (a) for all 0.1° x 0.1° blocks except for the Innermost Zone. The contributions for these compartments were computed using method (b).

(d) The processes (a), (b) and (c) were repeated but the four Innermost 0.1° x 0.1° blocks were excluded from the calculations.

The boundary between the Inner and Innermost Zones was selected as shown in Table 4.1 because the discrepancies between the use of method (a) and (b) for the Inner Zone was less than 0.01 metres and 0".03 for the contributions to N and both ξ and η. On comparison of the results from method (d) where the four Innermost 0.1° x 0.1° blocks were excluded, the contributions varied little in the value of the contribution to N, but discrepancies of greater than 0".1 occurred in the contribution to the deflections of the vertical. Furthermore, the large differences of the order of several seconds for the deflections of the vertical between the results obtained using method (d) and the corresponding methods (a),

(b) and (c), indicated the sensitivity of the deflections of the vertical to the contribution from the four Innermost 0.1° x 0.1° blocks. These results indicate that a more detailed survey is required within this region as suggested by Mather et al (1971) to enable the satisfactory determination of mean gravity anomalies for smaller geographical blocks.

4.4 Gravity Prediction Methods

In order to obtain the values of the mean gravity anomaly Δg_i it is sometimes necessary to extend the existing known gravity field or to estimate a gravity anomaly at a point. Many different techniques have been used for this purpose but only three of the more popular methods will be briefly reviewed.

4.4.1 Using Height Correlation

The free air anomaly of a mountain station usually has a greater numerical value than that of a nearby valley station and hence there appears to be a correlation between the free air anomaly and elevation. The correlation is approximately expressed by:

$$\Delta g = a + b.h \qquad \text{................ (4.7)}$$

where Δg is the free air anomaly, a and b are variables, and h is the elevation.

Uotila (1960) suggests the a value changes from area to area, but its variability is much less than the free air anomaly, and b is almost constant and approximates to the constant in the Bouguer reduction. Thus, the a value approximates the Bouguer anomaly.

It is commonly stated that equation (4.7) is approximate, but for a limited area the variables a and b may be regarded as constants. Using this assumption, equation (4.7) is often used for the following purposes:

- Prediction of point values of free air anomalies (e.g. Moritz, 1963).
- Computation of the mean free air anomaly Δg_i using

  $$\Delta g_i = a + b\bar{h} \qquad \text{................} \quad (4.8)$$

  where $\bar{h}$ is the mean height of the area (e.g. Uotila, 1960).
- For theoretical mathematical derivations (e.g. Moritz, 1966 and 1968).

The definition of this "limited area" is often not given. Mather (1975) reports a lack of correlation in excess of 50 km whereas other authors infer correlation exists over greater areas. In order to verify the area over which heights and free air-anomalies are correlated some tests have been made using 1° x 1°, 0.5° x 0.5° and 0.25° x 0.25° geographical blocks and the surface gravity data described in sections 4.21 and 4.22 (Gilliland, 1978).

In general, the test area was subdivided into equi-angular areas with sides of C degrees. The free air anomaly Δg, was then computed for each gravity station within the area using:

$$\Delta g \;=\; g_{1973} - \gamma_0 + 0.3086\ h \quad \ldots\ldots\ldots\ldots \quad (4.9)$$

where

h	is the orthometric height in metres.
γ_0	is the normal gravity computed using the 1967 Geodetic Reference system GRS-67.
and g_{1973}	is the observed gravity adjusted to the 1973 Australian datum (Dooley and Barlow, 1976).

Assuming equation (4.7) to be correct over a limited block area of C° x C°, the values of a and b were obtained by two methods:

(i) least square adjustment of equation (4.7) having two unknowns a and b, and n gravity stations; and

(ii) using the usual value of 0.1119 for the Bouguer reduction in place of b, and then computing the value of a, as the mean of n values obtained from equation (4.7).

If the number of stations n, was less than k (the allowable minimum per area), or greater than an upper limit governed by computer limitations (approximately 10^3), that particular area was deleted from the computation procedure. The values of C, k and the total number of blocks tested are shown in

Table 4.2.

C°	k	Number of blocks
1	60	137
0.5	30	583
0.25	5	2229

TABLE 4.2: Block Size and Distribution

Having obtained the values of a and b from both the above-mentioned methods, the standard deviations $\sigma_{\Delta g}$ of an individual free air anomaly about the line of regression a + bh, were computed using:

$$\sigma^2_{\Delta g} = \frac{\Sigma(\Delta g - (a + bh))^2}{(n - m)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.10)$$

where m has a value of 2 in method (i) and a value of 1 in method (ii).

The frequency of gravity stations within the geographical blocks used in this study are shown in Table 4.3

Area	1° x 1°	0°.5 x 0°.5	0°.25 x 0°.25
Maximum no. of Gravity Stations	1000	980	400
Minimum no. of Gravity Stations	60	30	5
Average no. of Gravity Stations	300	75	18

TABLE 4.3: Frequency of Gravity Stations

Table 4.4 shows the average value of the standard deviation $\sigma_{\Delta g}$ about the regression line for each block size and the frequency distribution of the values obtained for each individual block.

Block Size	1° x 1°		0°.5 x 0°.5		0°.25 x 0°.25	
Method	(i)	(ii)	(i)	(ii)	(i)	(ii)
Average $\sigma_{\Delta g}$	±11.3 mGal	±12.9 mGal	±7.9 mGal	±9.1 mGal	±5.1 mGal	±6.0 mGal
$\sigma_{\Delta g}$ mGal	FREQUENCY OF OCCURRENCE					
0 - 2	0	0	4	3	259	164
2 - 4	3	1	94	68	856	716
4 - 6	15	8	168	155	510	587
6 - 8	32	35	119	106	288	307
8 - 10	26	25	70	87	127	172
10 - 12	19	19	52	67	79	97
12 - 14	11	10	14	22	41	55
14 - 16	9	8	19	11	24	42
16 - 18	3	6	11	16	21	23
18 - 20	2	2	8	7	6	14
20 - 22	4	5	7	6	6	15
22 - 24	2	3	4	7	5	16
24 - 26	5	3	4	4	2	6
26 - 28	2	3	1	6	0	4
28 - 30	2	1	2	3	1	6
30 - 32	0	2	1	3	1	1
32 - 34	1	1	1	2	1	1
34 - 36	1	2	4	3	0	1
36 - 38	0	1	0	3	0	1
38 - 40	0	0	0	1	0	0
40 - 42	0	1	0	1	0	0

TABLE 4.4: Standard Deviations & Frequency Distributions

The use of Method (i) improves the value of the average standard deviation $\sigma_{\Delta g}$ over that for Method (ii) thus:

$(12.9^2 - 11.3^2) = \pm 6.2$ mGal for 1°0 square block.

$(9.1^2 - 7.9^2) = \pm 4.5$ mGal for 0°5 square block.

$(6.0^2 - 5.1^2) = \pm 3.1$ mGal for 0°25 square block.

This effect gives an indication of the validity of the Bouguer gradient when compared to the general linear regression within each particular block.

On the basis of the data studied there is no apparent relationship between the number of stations in a given test block and the standard deviation obtained. In the half degree area where 230 additional stations were observed, the values of a, b and $\sigma_{\Delta g}$ computed were virtually the same as those obtained using only the 58 BMR stations.

From a study of the individual 0°25 square blocks the height variation within the block does not effect the value of $\sigma_{\Delta g}$ but the geographical position in relation to the Bouguer anomaly gradient does. On comparing the location of the blocks with large standard deviations with the 1:5000.000 Gravity Map of Australia published by BMR, the great majority are situated in areas where the Bouguer Anomaly is rapidly changing, e.g. the north-west of South Australia.

There appears to be a tendency for the standard deviation to increase as the mean elevation of the block increases but this is not conclusive as the majority of the 0°.25 square blocks have a mean elevation of less than 200 metres. For the 1958 blocks with a mean elevation less than 500 metres the average standard deviation is ±4.7 mGal, whereas that for the 271 blocks with a mean elevation greater than 500 metres is ±7.5 mGal.

The value of the standard deviation $\sigma_{\Delta g}$ of an individual gravity anomaly about the line of regression gives a definite indication of the error obtained if equation (4.7) is used for prediction of additional free air anomalies within the same area. Hence it would be most useful in an error analysis of any computed results that have used this form of prediction to generate basic data. This standard deviation is also similar to the error of representation used by some authors (e.g. Mather et al, 1976) to indicate the variation of known and predicted values of the free air anomaly within an area, but, in this case the variation due to height has been removed.

From the results it can be seen that two basic factors govern the magnitude $\sigma_{\Delta g}$. These are the size of the area and the consistency of the Bouguer anomaly within the area. If b has the approximate value of 0.1119 then the a-value is in effect the Bouguer anomaly and it follows that the smaller the change in the Bouguer anomaly the larger the area over which equation (4.8) will be satisfied for any given $\sigma_{\Delta g}$.

For the three block sizes tested, the mean value of $\sigma_{\Delta g}$ increased with block size from ±0.5 mGal to ±11.3 mGal when computed by Method (i) and ±6.0 mGal to ±12.9 mGal when computed by Method (ii). The difference in mean values of $\sigma_{\Delta g}$ obtained by each method was approximately one mGal but shows a definite tendency to increase with the block size.

4.4.2 Least Squares Collocation Techniques for gravity anomaly interpolation in local areas

The simple use of least squares collocation for the interpolation of gravity anomalies is represented by:

$$\Delta g_p = [C_{p_1} C_{p_2} \ldots C_{p_n}] \begin{bmatrix} C_{11} C_{12} \ldots C_{1n} \\ C_{21} C_{22} \\ \cdot \\ \cdot \\ \cdot \\ C_{n1} \ldots\ldots C_{nn} \end{bmatrix}^{-1} \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \cdot \\ \cdot \\ \Delta g_3 \\ \Delta g_n \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots (4.11)$$

where $\Delta g_1 \ldots \Delta g_n$ are the gravity anomaly values at points 1, 2, 3 ... n and Δg_p is the resultant interpolated value of the gravity anomaly at point p. The C_{ij}'s, where $i = 1 \rightarrow n$ and $j = 1 \rightarrow n$ and C_{pi}'s where $i = 1 \rightarrow n$ are covariance functions.

and $C_{ij} = C(S_{ij})$

$C_{pi} = C(S_{pi})$

where S_{ij} is the distance between gravity anomaly points i and j and S_{pi} is the distance between points p and i. The covariance functions C(S) are obtained from the expression

$$C(S) = \frac{\sum_{i=1}^{n} (\Delta g \Delta g')_i}{n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.12)$$

where Δg and $\Delta g'$ are separated by a distance S and n is the total number of combinations of gravity points at a distance S apart. The use of equation (4.11) infers that the gravity field is both isotropic and homogeneous. Thus, the covariance function is dependent on distance and not direction, and is independent of the location of the field.

The assumption of a homogeneous gravity field has been queried by many authors. Rapp (1964) obtained significantly different covariance functions for different areas in the United States of America, and again Tscherning and Rapp (1974) allude to the variations when describing a method to obtain the "local" covariance function. Kearsley (1976) warns of the dangers of using a covariance function determined for one local area in an adjacent region. Using a test area in the U.S.A. Kearsley (1977) shows the presence of an isotropy in the covariance function determined from gravity data and he refers to other authors (e.g. Vyskocil, 1970), who question the isotropic properties of a local gravity field.

4.4.3 Weighted Means

This method of weighted means as applied to gravity anomaly prediction at a point, takes a weighted mean of the nearest observations surrounding the point. The weights are assigned to the observations inversely proportional to distance ℓ of the known points from the prediction point, raised to some power x. The mathematical model is given by:

$$E_{\Delta g} = \frac{\sum_{i=1}^{n} w_i \, \Delta g_i}{\sum_{i=1}^{n} w_i} \qquad \text{................ (4.13)}$$

where $E_{\Delta g}$ is the predicted value of the gravity anomaly, n is the total number of known gravity anomalies Δ_{gi} surrounding the point and $w = \frac{1}{\ell_i^{x}}$. The value of x is usually taken as two, i.e. the weight is inversely proportional to the square of the distance, but other values have been used. Mather, et al (1976), when determining an Australian gravity data bank for sea surface topography determinations (AUSGAD 76) used a variation of the form x = 1 for interpolating one tenth degree grid values from existing values within a range of 50 km. In this case the weight w_i ranged from 5 to 1 as the distance from the point of prediction varied from 10 km to 50 km.

In a randomly selected area of 1.1 degrees by 2.4 degrees in the north western region of South Australia the values of nine unknown 0.1° x 0.1° mean gravity anomalies were estimated using equation (4.13) and w_i defined by:

(a) $w_i = 6 - 10\ell_i$

and (b) $w_i = \frac{1}{\ell_i^{2}}$

TABLE 4.5: Randomly selected area showing 0.°1 x 0.°1 mean gravity anomalies

-61.61	-57.02	-50.64	-52.75	-48.45	-48.53	-52.33	-54.01	C	-44.42	-64.07
-55.07	-57.64	-55.62	-47.34	-48.41	-49.57	-47.03	-47.07	-46.26	-51.11	-62.63
-56.68	-56.68	-54.54	-49.23	-41.56	-45.18	-43.02	-34.67	-26.31	-39.83	-41.70
-56.37	-58.72	-50.02	-48.07	-41.16	-38.86	-34.53	-30.70	-25.53	-15.48	-19.28
-51.40	-46.20	-46.65	-48.70	-42.64	-32.24	-33.53	-29.33	-28.78	-26.76	-25.54
-44.03	-46.59	-43.77	-37.57	-30.14	-26.87	-29.78	-19.56	-19.19	-18.23	-20.93
-36.56	-36.46	-29.97	-29.44	-25.41	-24.31	-18.29	-16.21	-17.81	-20.74	-21.59
-26.86	-29.59	-24.74	A	-16.43	-13.88	-10.23	-13.14	D	-21.82	-21.66
-19.44	-20.06	-20.33	-15.83	-12.95	-13.44	-4.34	-8.99	-13.65	-18.75	-16.71
-20.61	-21.47	-15.62	-14.29	-12.21	-8.59	-8.65	-3.93	-10.58	-12.71	-20.72
-22.34	-23.83	-22.94	-16.35	-13.86	-12.12	-7.63	-3.09	-5.75	-9.72	-11.79
-19.73	-24.49	-22.57	-25.60	-27.28	-22.81	-18.71	-15.33	-22.20	-22.22	-19.79
-20.99	-18.83	-28.50	-30.49	-27.79	-28.03	-33.71	-28.83	E	-30.79	-35.26
-20.62	-28.05	-32.04	-34.89	-34.45	-38.44	-37.28	-41.14	-44.29	-46.97	-40.33
-32.76	-38.29	-45.99	-46.79	-48.71	-54.26	-55.45	-57.24	-60.40	-55.02	-50.96
-39.24	-35.76	-44.95	-46.56	-53.12	-62.50	-61.25	-59.45	-61.52	-60.04	H
-57.30	-58.33	-59.33	-63.00	-62.76	-73.88	-70.25	-77.82	-77.78	-74.11	-74.36
-43.59	-58.94	-63.52	-70.86	-76.75	-73.06	-71.38	-74.43	-70.60	G	I
-58.73	-62.66	-65.25	B	-68.56	-65.52	-64.25	-67.94	F	-64.47	-60.91
-48.72	-44.99	-59.07	-62.88	-66.04	-55.62	-52.27	-63.15	-58.83	-59.75	-60.25
-41.62	-36.73	-50.86	-51.01	-47.29	-47.90	-46.35	-14.59	-46.24	-49.10	-50.55
-28.58	-16.69	-35.98	-33.02	-36.77	-31.77	-30.64	-25.72	-27.16	-31.83	-43.53
- 1.01	-11.20	-9.81	-5.91	5.92	15.17	7.24	-.58	-2.90	-10.03	-5.29
28.197	13.72	1.89	2.97	9.53	13.31	33.28	28.50	37.54	33.04	24.17

where ℓ_i is the spherical distance in degrees and has a maximum value of 0.5°. Method (a) is similar to that used for the establishment of AUSGAD 76. The known mean gravity values are shown in Table 4.5 and the letters A to I represent the location of the unknown values. Table 4.6 shows the predicted values of these unknowns using the two methods of defining the weights. Both methods give similar results but it does appear that the method of weighting using reciprocal of the square of the distance gives slightly better results.

Point	Method (a) $(w_i = 6 - 10\ell_i)$	Method (b) $(w_i = \frac{1}{\ell_i^2})$
A	-25.66	-24.69
B	-51.52	-54.56
C	-43.23	-44.72
D	-18.68	-18.93
E	-33.50	-33.02
F	-54.84	-56.40
G	-58.36	-59.74
H	-53.38	-53.94
I	-57.63	-58.66

TABLE 4.6: Comparison of Weighting Methods

4.4.4 Summary of Methods of Interpolation

The method of least square collocation is often used for interpolation in large areas where there is sparse data but does not prove useful in areas of relatively dense data such as the South Australian region. Moritz (1975) considered three simple cases of interpolation

(a) between two data points,

(b) within a triangle formed by three points, and

(c) within an area contained by four data points

using least square collocation techniques. The results demonstrated that the procedure is equivalent to a simple geometric interpolation method provided the known data points were within a small distance of one another. After testing this method of interpolation in the north west of New South Wales, Kearsley (1976) concluded that the covariance approach to predication is unsuitable in local areas where the density of discrete observations is relatively high.

Using the height correlation method of interpolation, Table 4.4 shows that for a maximum error of ±5.6 mGal in the predicted value of Δg, approximately 70% of the 0.25° x 0.25° equi-angular blocks would provide adequate representation. The selection of blocks that are suitable for this type of interpolation can be ascertained from a Bouguer anomaly map of the area or by analysis of existing data, if sufficient exists. Care must be taken with this analysis, for as can be seen in Table 4.4, some individual blocks can have standard deviations as large as ±20 - 40 mGal. Using 0.1119 as an approximate value of the gradient of the line of regression for this area, increases the mean value of $\sigma_{\Delta g}$ by less than one mGal and is generally preferred to Method (1) as it greatly reduces the computer storage and time required for the evaluation of the a-value and hence the predicted anomaly.

The method of weighted means is a simple method to apply and has been shown to be reliable for the interpolation of point gravity values. Kassim (1980) compared this method to the least square collocation method for predicting point gravity values in flat, undulating and mountainous terrain in Canada and found in all terrain types that the method of weighted means gave better results.

Ideally, before either weighted means or least squares collocation methods are used for gravity anomaly interpolation, the height dependant element of the anomaly should be removed and then added after, to the predicted value. This assumes a knowledge of the terrain heights is available. Thus all three methods of interpolation depend to some extent on height information. Unfortunately, for the majority of the South Australian region, height information at the density required to interpolate between existing stations, is not available. Thus the method of weighted means is used for interpolation because, although not ideal without height data, it has been shown to give better results than the method of least square collocation in areas of relatively dense data, and the method using height correlation is not applicable due to the lack of height information. It must be stressed that the discussion here is on the interpolation of gravity over small areas (less than 1 degree square) where the existing data is reasonably dense (of the order of 250 values per block).

4.5 Mean Gravity Anomalies of the Compartments

From the available gravity data described in section 4.2, mean gravity anomalies are determined for the compartment sizes shown in Table 4.1. For the 5° equal area blocks in the Outer Zone, the Rapp 5° data is used with no interpolation required. The mean anomaly for the 1° x 1° equal angular or geographical blocks is obtained from the Rapp 1° data set but not all 1° x 1° geographical blocks within the Middle Zone have assigned mean gravity values. The missing values are linearly interpolated from the neighbouring compartments and given a standard deviation of ±30 mGals. Since only 78 of the 1° x 1° mean gravity anomalies are available from the Rapp 1° data, compared with a total number of approximately 3450 situated within the Middle Zone, and the great majority of these are located near the outer limits of this zone, any error in the interpolated values would have a minimal effect on the results obtained.

The surface gravity data described in sections 4.2.1 and 4.2.2 is used to determine the mean gravity anomalies of the 0.5° x 0.5° geographical blocks situated in the Near Zone. The gravity station data is standardised to the International Gravity Standardization Net, 1971 (Dooley & Barlow, 1976). The free air anomaly for each station is computed using the GRS-67 gravity formula and then the mean gravity anomaly $\overline{\Delta g}_{0.25}$ for each 0.°25 x 0.°25 geographical block is obtained from the mean of all free air anomalies within that geographical location. The mean gravity anomaly for the 0.°5 x 0.°5 block, $\overline{\Delta g}_{0.5}$, is then taken as the mean of the four $\overline{\Delta g}_{0.25}$ mean anomalies. This ensured a reasonably even distribution of data that is contributing to the final mean value of the gravity anomaly representing the 0.°5 x 0.°5 block.

In order to estimate the errors in the resulting mean gravity anomalies, the error of representation $E_{0.25}$ of $\overline{\Delta g}_{0.25}$ is determined for each 0.°25 x 0.°25 block using the formulae (Hirvonen, 1962, p.4).

$$E^2_{0.25} = \frac{1}{n} \sum_{i=1}^{n} (\overline{\Delta g}_{0.25} - \Delta g_i)^2 \quad \text{..........} \quad (4.14)$$

where n is the number of gravity stations and Δg_i is a gravity anomaly within the 0.°25 x 0.°25 block. The error of representation $E_{0.5}$ of $\Delta g_{0.5}$ is then determined using the formula (Ibid, p. 5):

$$E^2_{0.5} = \frac{1}{4} \sum_{i=1}^{n} E^2_{0.25} + \frac{1}{4} \sum_{i=1}^{n} (\overline{\Delta g}_{0.5} - \overline{\Delta g}_{0.25_i})^2 \quad \text{...............} \quad (4.15)$$

In the distant part of the zone in the Southern Ocean area where no point gravity values existed in some 0.°5 x 0.°5 blocks, the value of the mean gravity anomaly from 1° Rapp data, and the corresponding standard deviation is used to represent $\overline{\Delta g}_{0.5}$ and

$E_{0.5}$ respectively.

The mean gravity anomalies $\Delta g_{0.1}$ of the 0.°1 x 0.°1 blocks in the Inner and Innermost Zones are computed from the same surface data set as used in the computation of $\Delta g_{0.5}$ values. From this data set the values of each $\Delta g_{0.1}$ is computed as the simple mean of all the free air anomalies situated within the block with no allowance being made for the actual location of the anomalies. For the purpose of this study any change that may occur to the computed mean value due to the use of some location dependant algorithm would not appreciably alter the results when the magnitude of the random errors are considered.

If no values existed within the 0.°1 x 0.°1 geographical block then an estimation of the mean anomaly is obtained using equation (4.13) with n being the total number of known $\Delta g_{0.1}$ values within a grid distance of 0.5 degrees and w_i is the weight as defined in Method (b) in section 4.4.3.

CHAPTER 5

THE FREE AIR GEOID OF SOUTH AUSTRALIA

The contributions to the free air geoid separation and the deflections of the vertical from each zone are computed separately for each half degree latitude and longitude grid point in South Australia, and then from a summation of the contributions of each zone, the values of N, ξ and η are obtained. Thus:

$$N = \sum_{i=1}^{n} N_i, \quad \xi = \sum_{i=1}^{n} \xi_i,$$

$$\eta = \sum_{i=1}^{n} \eta_i \qquad \text{..............} \quad (5.1)$$

where n = 5 is the total number of zones as shown in Table 4.1, and N_i, ξ_i and η_i are the contributions from each zone. Computer programs suitable for use on a Cyber 173 have been written and used to evaluate these zone contributions.

In the same programs, an estimate of the standard deviations of the contributions to N, ξ and η for each zone are computed using (Gilliland, 1982):

$$\mu_N^2 = \left(\frac{R}{4\pi\bar{\gamma}}\right)^2 \sum_i S(\psi)_i^2 \, \mu_{gi}^2 . \Delta\sigma_i^2 \qquad \text{............} \quad (5.2)$$

$$\left.\begin{matrix}\mu_\xi^2\\ \mu_\eta^2\end{matrix}\right\} = \left(\frac{1}{4\pi\bar{\gamma}}\right)^2 \sum_i \left(\frac{dS(\psi)_i}{d\psi}\right)^2 \mu_{gi}^2 \left\{\begin{matrix}\cos^2 \alpha\\ \sin^2 \alpha\end{matrix}\right\} \Delta\sigma_i^2$$

.... (5.3)

where μ_N, μ_ξ and μ_η are the standard deviations of the contributions to N, ξ and η respectively from the zone, and μ_{gi} is the standard deviation of the mean gravity anomaly Δ_{gi} for each compartment $\Delta\sigma_i$.

5.1 Evaluation of the Zone Contributions

5.1.1 Outer Zone

This zone is the largest and most distant zone from the computation grid points. The inner irregular boundary as shown in Figure 4.1 is coincident with selected boundaries of the Rapp 5° equal area data and remains unaltered for all grid point computations for ease of computation. In this zone $\Delta\sigma_i$ used in equations (4.1), (4.2), (5.2) and (5.3) are not defined by equation (4.3) since the surface areas of these equal area compartments are defined in the Rapp 5° data in units of steradians.

The contribution to N from this zone ranges from 12 metres in the south west corner of the state to 20 metres in the north east corner. The gradient of the change is smooth and has maximum gradient in a north east direction. The values obtained for the deflections of the vertical ξ and

η are approximately -1".2 and -1".0 respectively. The variations in both deflection components about their mean values are less than 0".3 and much of this is accounted for by the set position of the inner boundary of this Outer Zone. The contributions to the geoid undulations have similar effects to those computed by Kearsley and Van Gysen (1979), but a direct comparison is not possible because the inner boundary of the Outer Zone differs in actual geographical location.

Using the standard deviation of each mean anomaly of the 5° equal area blocks, the standard deviation of the components of N, ξ and η, were calculated. They changed very little with location and the mean values obtained are ±1.1 metres, ±0".06 and ±0".04 respectively.

5.1.2 Middle Zone

As with all zone evaluations other than the Outer Zone, the value of $\Delta\sigma_i$ is evaluated using equation (4.3) and in this zone the values of n and m are both equal to one. The geographical location of the inner boundary of this zone changes with the computation point as explained in section 4.5.

The range in the contribution to N is from -13 metres in the south west corner, to +4 metres in the north east corner with a general geoid change in the same direction as the

Outer Zone. The ξ component ranged in value from $-3''.6$ in the north to $-1''.1$ in the south, and the η component ranged in value from $-2''.2$ in the west to $-3''.2$ in the east. Again, part of this variation is explained by the set position of the outer boundary of this zone. On combination of the Outer Zone and Middle Zone effects, the range in the ξ component is from $-3''.0$ in the south with a gentle gradient to $-4''.7$ in the north of the state, and the η component varies from $-3''.6$ to $-4''.4$, with the lower value occurring near the central meridian of the state.

The standard deviation of the contribution to N, ξ and η again did not vary appreciably with position, and the average values are ±0.5 metres, $\pm0''.07$, and $\pm0''.08$ respectively.

5.1.3 The Near Zone

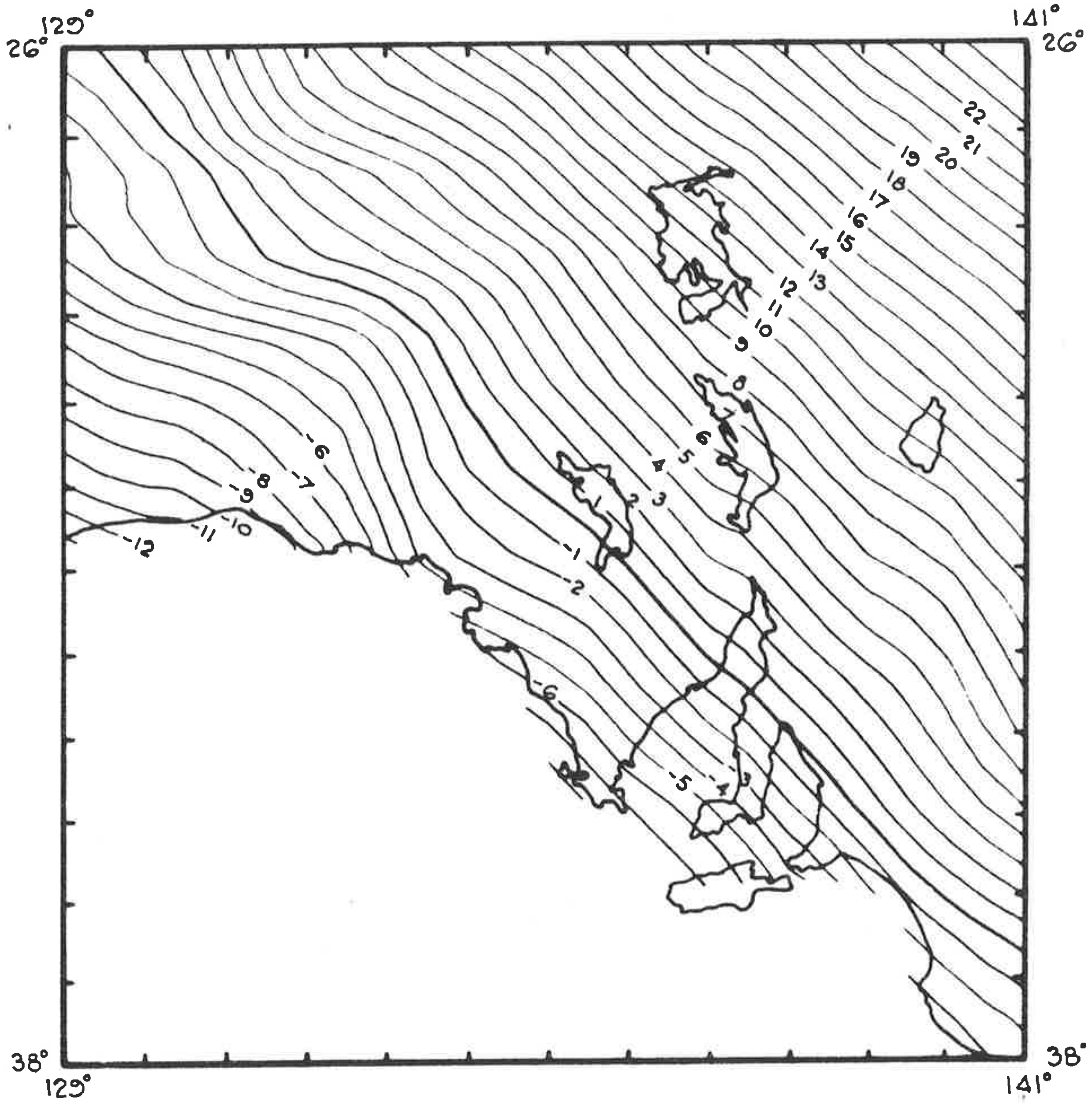
The values of n and m used to define $\Delta\sigma_i$ in equation (4.3) are both 0.5 for this zone, and the inner boundary is defined by the 3° x 3° geographical block with the computational grid point as its centre.

The Near Zone contribution to N ranges in value from -12 metres to 0 metres, with a general gradient increasing from west to east across the state. The changes in the Near Zone effects on the deflections of the vertical are more varied than the Outer and Middle Zones, showing the sensitivity of the ξ and η with respect to closer zones and blocks. The contribution to ξ varies from $-2''.6$ to $+1''.4$ with the lower values occurring in the south east, the north west, and the north east of the state and the contribution to η varies from $-2''.6$ to $+1''.0$ with the higher values occurring in the north west corner and the north east region of the state.

The standard deviations from this zone are computed using the error of representation as the standard deviation for each mean gravity anomaly. The mean values obtained for the contributions to N, ξ and η were ±0.3 metres, $\pm 0''.15$ and $\pm 0''.17$ respectively, with ranges about these values of ±0.1 metres, $\pm 0''.07$ and $\pm 0''.10$. The highest values occur in the north west corner of the state where there are rapid changes in the free air gravity field.

5.1.4 The Combined Effects of the Outer, Middle and Near Zones

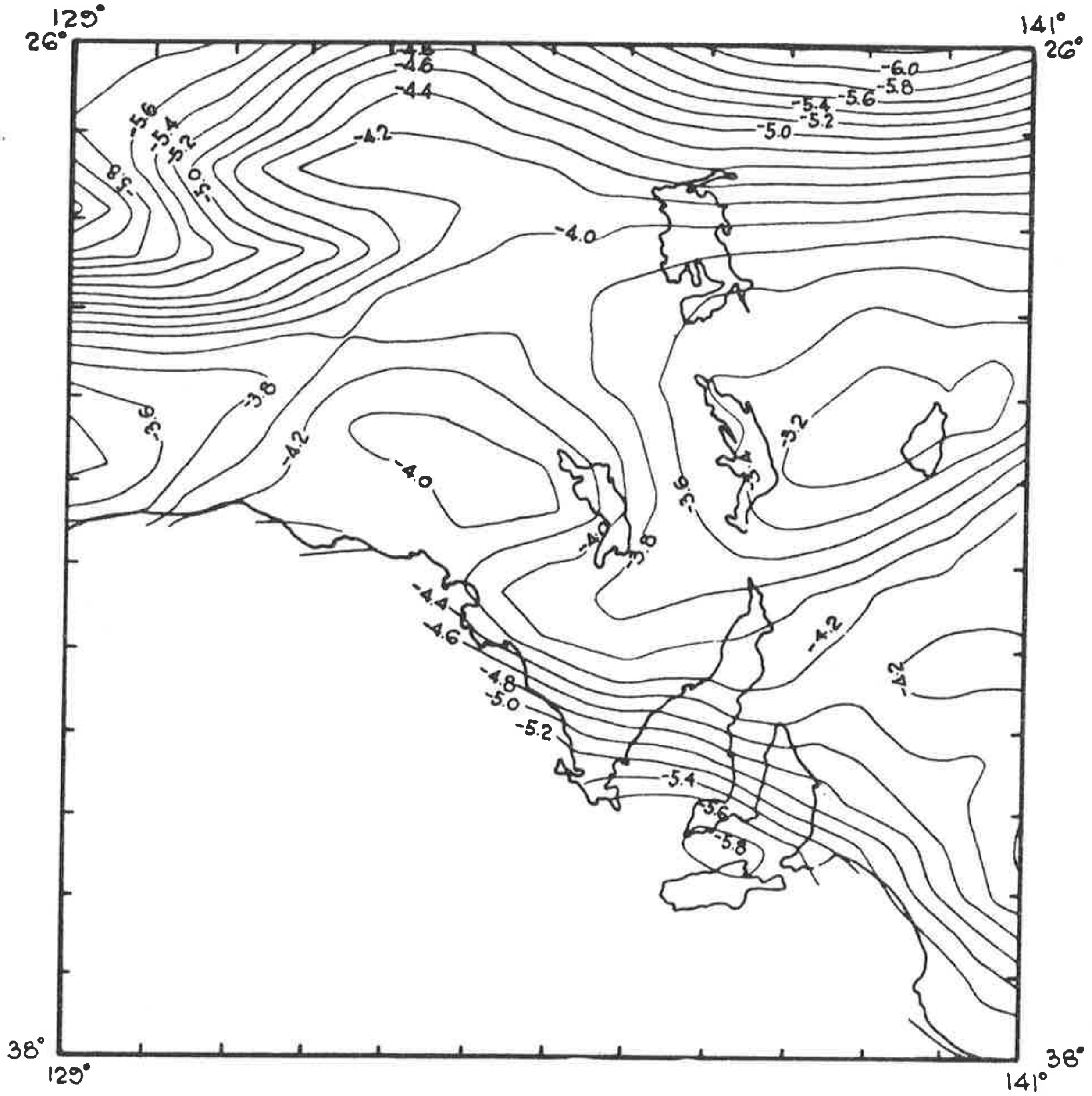
The total contributions to N, ξ and η from the Outer, Middle and Near Zones are shown in Figures 5.1, 5.2 and 5.3 (Gilliland, 1981). These give a graphical representation of the combined effects from the "distant" zones. In astro-gravimetric levelling (section 2.1), it is the contributions to ξ and η of these zones that are considered to have linear variations between adjoining astro-geodetic stations if the local gravity survey is extended to approximately 150 km.



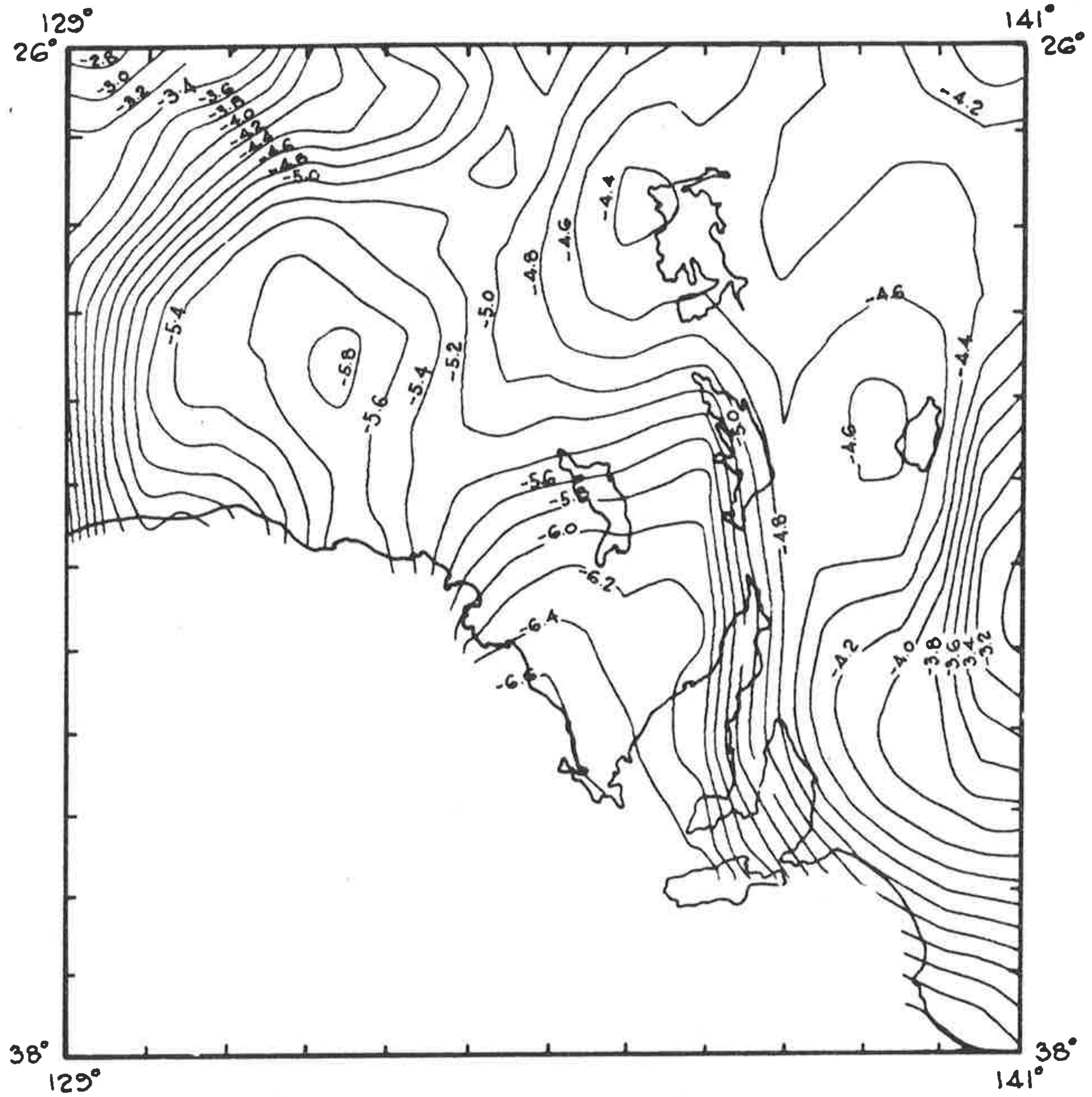
Contributions from the Outer, Middle & Near Zones to N

Contour Interval 1.0 m

Figure 5.1



Contributions from the Outer, Middle & Near Zones to ξ
Contour Interval 0.2′
Figure 5.2



Contributions from the Outer, Middle & Near Zones to η
Contour Interval 0.2"
Figure 5.3

The numerical value of the contribution to N changes smoothly in a north easterly direction from -13 metres to +25 metres. Both ξ and η contributions in Figures 5.2 and 5.3 demonstrates the more varied changes due to the contribution from the Near Zone.

The average standard deviations of the combined contributions from the three zones to N, ξ and η are respectively ±1.2 metres, ±0".17, and ±0".18. The standard deviations of ξ and η have variations of the order of ±0".1, due predominantly to the contributions of the Near Zone.

5.1.5 Inner and Innermost Zones

The Inner Zone is the 3° x 3° geographical block centred about the computation or grid point, but excludes the Innermost Zone which is the 0.°4 x 0.°4 geographical block centred on the computation point. The actual selection of the dimensions of this Innermost Zone were discussed in section 4.3.

Both the Inner and Innermost Zones are initially subdivided into 0.°1 x 0.°1 compartments and the mean free air anomaly is obtained as described in section 4.5. The average density of gravity readings within South Australia is one value per 42 km² (Dooley and Barlow, 1976). The mean gravity anomalies $\Delta g_{0.1}$ for these blocks are given a nominal Error of Representation of ±5 mGal. In much of the South Australian region the values of ±5 mGal seems too large, as the actual free air gravity field changes slowly, but in some parts, particularly the north west corner of the State, the gravity field changes rapidly and the value would appear to be small. It must be remembered that no detailed knowledge of the topographic heights is available for much of the total region under investigation, and hence no account can be taken of the correlation of height with free air anomalies. For similar reasons it would appear unwise to attempt to obtain mean gravity anomalies for geographical blocks smaller than 0.°1 x 0.°1 without closer gravity coverage or a detailed knowledge of the terrain heights.

For both the Inner and Innermost Zones for the computation of N, ξ and η contributions, the value of n and m in equation (4.) are equal, and each have the value of 0.1. The value of Stokes' and Vening Meinesz functions for the Inner Zone are computed to the geographic centre of each equi-angular block. For the Innermost Zone, equations (4.1) and (4.2) are rearranged to become:

$$N_{(\text{Innermost})} = \frac{R}{4\pi\bar{\gamma}} \sum_i \Delta g_i \, \Delta\sigma_i \sum_{j=1}^{100} \frac{S(\psi)_j}{100} \qquad \text{............... (5.4)}$$

and

$$\left.\begin{matrix}\xi \\ \eta\end{matrix}\right\}_{(\text{Innermost})} = \frac{1}{4\pi\bar{\gamma}} \sum_i \Delta g_i \, \Delta\sigma_i \sum_{j=1}^{100} \frac{dS(\psi)}{d\psi}_j \cdot \frac{1}{100} \cdot \begin{Bmatrix}\cos \alpha_j \\ \sin \alpha_j\end{Bmatrix} \qquad \text{............... (5.5)}$$

In this case the 0.°1 x 0.°1 blocks are subdivided into 0.°01 x 0.°01 sub-compartments. The values of the Stokes' function $S(\psi)_i$, Vening Meinesz function $\frac{dS(\psi)}{d\psi}_j$ and the azimuth α_j are computed and the means of these 100 sub-compartment values are used to represent the corresponding values of the 0.°1 x 0.°1 blocks. Thus, the errors incurred in using equations (4.4) and (4.5) to determine terms in equations (4.1) and (4.2) are greatly reduced.

The combined Inner and Innermost Zone contributions to N varied from -8 metres to +4 metres, with the greatest variations occurring in the north west region of the state. The contributions to ξ and η change rapidly, in some cases from one grid point to the next. Ignoring the contribution from the four innermost 0.°1 x 0.°1 blocks, the contribution to ξ varies from -15" to +10" with both extreme values occurring in the north west region of the state, and the contribution to η varies from -8" to +8" with the maximum changes occurring in the Flinders Ranges and north west region of the state.

Using an estimation of ±5 mGal for the standard deviation of the mean gravity anomalies within these zones, the resulting standard deviation of the contributions to N, ξ and η are respectively ±0.05 metres, ±0".53, and ±0".46.

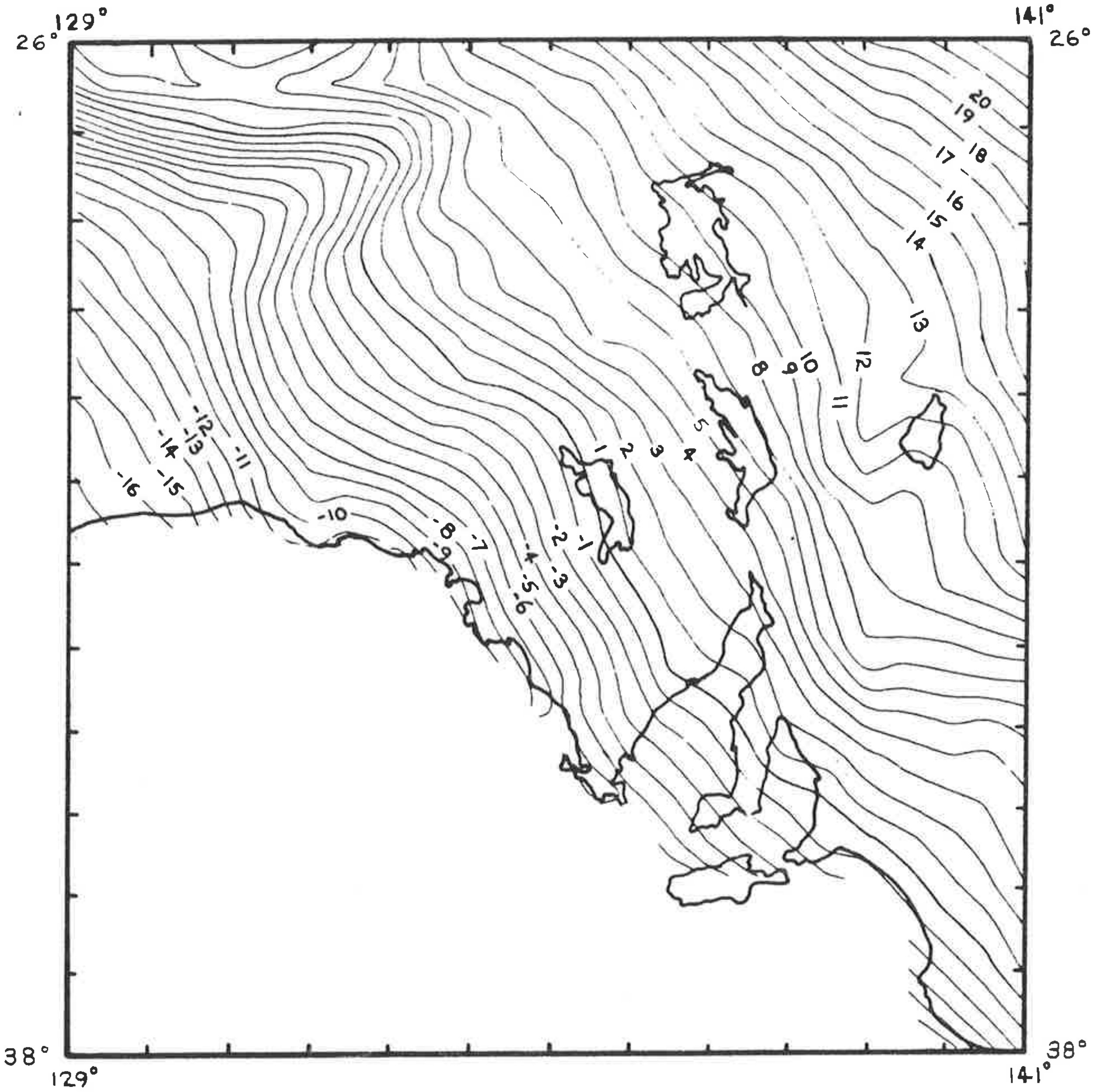
5.1.6 Combined Effects from All Zones

The combined effects of all zones on N, ξ and η are shown in Figures 5.4, 5.5 and 5.6 with the exception of the contribution to the deflections from the four innermost $0.^\circ 1 \times 0.^\circ 1$ blocks, which has been omitted. The contribution to the deflection of the vertical from these four innermost $0.^\circ 1 \times 0.^\circ 1$ blocks may have a magnitude of several seconds (see sections 3.1 and 6.1). As another example, in the north west of New South Wales Kearsley (1976) found the contributions to ξ and η for a similar area of influence defined by $0.14 \text{ km} < \psi < 8.6 \text{ km}$ to be 3".89 and -3".35 respectively. In a region where the general density of gravity stations is 1 per 100 - 200 km², Shimbirev (Brovar et al, 1964, p. 290) estimates that an additional gravity survey of the density shown in Table 5.1 is required to obtain a precision of ±0".15 in the deflections of the vertical.

Distance from Comp. Pt. (Km.)	Number of Stations
0	1
1.25	5
3.5	7
7.5	9

Table 5.1

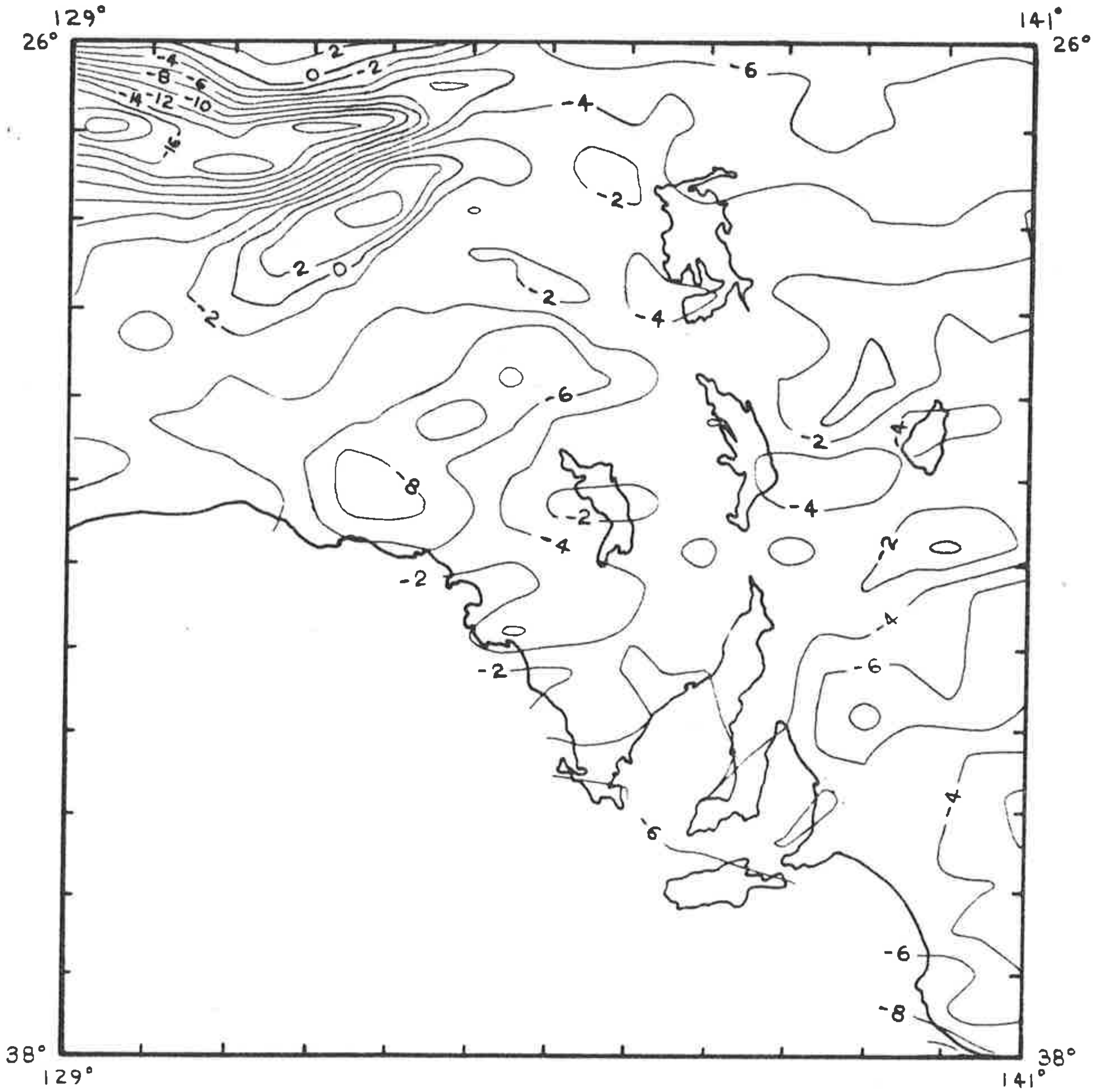
The density of gravity readings in this study is generally of the order of 2 stations per 100 km². Without a densifying survey about each grid point the contribution to the deflections of the vertical from these four innermost $0.^\circ 1 \times 0.^\circ 1$ blocks could have errors of a second or more.



South Austalia : Free-air Geoid-Reference System 1967

N (Contour Interval 1.0m)

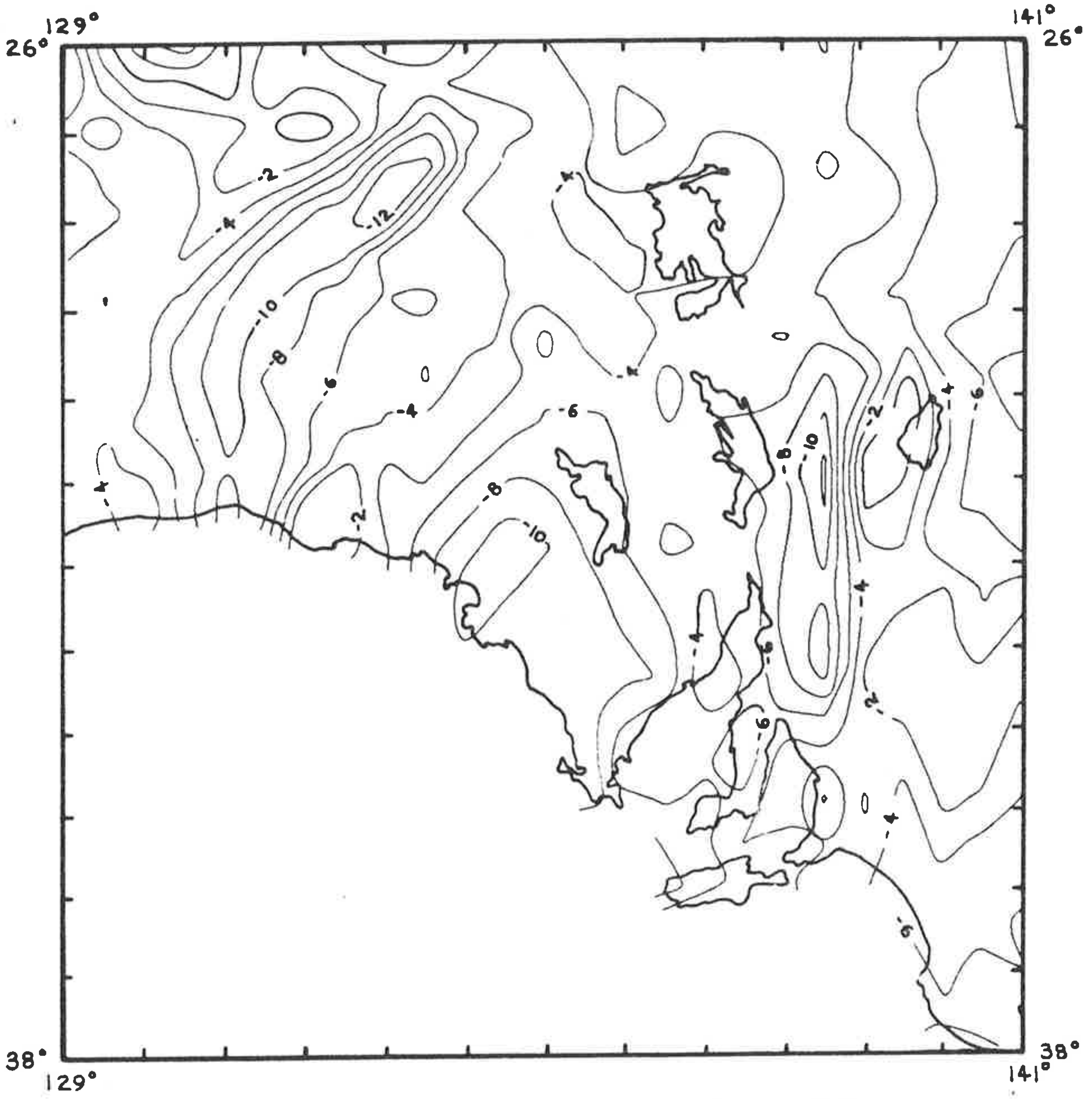
Figure 5.4



South Austalia : Free-air Geoid-Reference System 1967

ξ (Contour Interval 2")

Figure 5.5



South Austalia : Free-air Geoid-Reference System 1967

η (Contour Interval 2")

Figure 5.6

It is for this reason that local gravity stations have been established in the near vicinity of selected trigonometric stations for the purpose of geoid comparisons as discussed in Chapter 6.

The value of N for the free air geoid increases in a north easterly direction with numerical values ranging from -17 metres on the west coast, to +23 metres in the north east of the state. The change in the value of N is generally smooth and the irregular rates of change that have occurred are due to the contribution of the inner zones. This is clearly demonstrated when comparing Figure 5.1 with Figure 5.4.

The inner zones contribute the major variations to the deflections of the vertical shown in Figures 5.5 and 5.6 since variations due to the omitted innermost four blocks may have magnitudes of several seconds, particularly in the areas where the gravity field is rapidly varying, inclusion of these results would tend to obliterate the regional trends. The effects of the large variations of the gravity field in the north west region of the state are particularly evident in Figure 5.5 and to a lesser extent in Figures 5.4 and 5.6. Similarly the effect of the Flinders Ranges which lay north-south with an approximate mid-longitude of 138.5°E can be seen in Figure 5.6

The average standard deviations of the values of N, ξ and η

Latitude Longitude	26° 00'S 131° 00'E		28° 00'S 138° 00'E	
Zones	Contribution to N & the S.D. in metres			
Outer	15.642	±1.130	17.974	±1.127
Middle	-4.149	±0.442	-1.328	±0.432
Near	-7.805	±0.423	-2.392	±0.269
Inner & Innermost	-2.421	±0.043	-2.863	±0.043
Total	1.268	±1.285	11.391	±1.239
	Contribution to ξ & the S.D. in seconds			
Outer	-1.189	±0.060	-1.346	±0.062
Middle	-3.644	±0.068	-3.353	±0.063
Near	-0.950	±0.178	0.632	±0.104
Inner & Innermost	8.663	±0.532	1.170	±0.532
Total	2.879	±0.568	-2.897	±0.549
	Contribution to η & the S.D. in seconds			
Outer	-1.224	±0.043	-1.000	±0.044
Middle	-2.447	±0.064	-3.153	±0.062
Near	0.551	±0.234	-0.211	±0.123
Inner & Innermost	4.387	±0.462	0.246	±0.462
Total	1.267	±0.524	-4.118	±0.484

TABLE 5.2

Contribution to N, ξ and η at two grid points.

for all grid points are respectively ±1.25 metres, ±0$''$55, and ±0$''$50, and the total ranges of the standard deviations over the state are insignificant. The largest contribution of ±1.1 metres to the standard deviation of N comes from the Outer Zone, but as expected the maximum contributions to the standard deviations of ξ and η are derived from the Inner and Innermost Zones. These standard deviations are an indication of the effects of random errors in the gravity data but they do not reflect any systematic effects in either the gravity data or the computational procedures (Gilliland, 1982).

Examples of the contributions from each zone, their total contribution to N, ξ, and η, and the respective estimations of the standard deviations for two grid points are given in Table 5.2. The contributions to the deflection of the vertical from the four innermost 0.°1 x 0.°1 blocks in the Innermost Zone are not included.

5.2 Errors due to Approximations and Omissions in the use of Computational Formulae.

5.2.1 Spherical Approximations

As stated in Section 2.2, Stokes' integral is derived using spherical approximations. This does not infer that the reference surface is a sphere. The reference surface remains an ellipsoid, but the relationship between relatively small quantities such as the disturbing potential and the geoid separation are derived using spherical approximations. This results in errors of the order of the flattening of the ellipsoid. As the value of the geoid separation does not exceed 25 metres absolute over the area studied, the expected maximum effect would be less than 0.1 metres. The effect on the results obtained using Vening Meinesz formulae is negligible.

5.2.2 Indirect Effect

The free air geoid is a co-geoid and the correct solution of the geoid requires a solution for the effect of the removal of masses external to the geoid. This change of the co-geoid due to the external masses is known as the indirect effect.

The indirect effect can be subdivided into three resultant effects on the geoid separation N. These are a zero order effect, a Stokesian effect, and a non-Stokesian effect (Mather, 1968.b). The zero order effect is given by:

$$\frac{R}{\gamma} M\{\Delta g_{oc}\}$$

where $M\{\Delta g_{oc}\}$ is the world mean value of the gravity anomaly Δg_{oc} which is the free air anomaly corrected for the effects of the potential ϕ_E due to matter external to the geoid (this matter should include the atmosphere). The non-

Stokesian effect is caused by a change in potential at the geoid due to the external masses, and the Stokesian effect is due to a small change in the free air anomaly from the geoid to the co-geoid.

Formulae for these effects are given in Fryer (1970, p.95), and were used in an approximate form to compute both Stokesian, non-Stokesian and the combined effect for Australia. The Stokesian and non-Stokesian terms varied slowly over the continent of Australia, and the combined effect had a variation of less than 1 metre. In the area of South Australia, the mean value of the combined Stokesian and non-Stokesian terms was -6.6 metres with a total variation range of ±0.1 metres. This means a free air geoid solution for South Australia, apart from a near constant correction of approximately -6.6 metres and constant zero order corrections, is a good representation of the geoid.

The indirect effects on the deflections of the vertical are given by:

$$\delta\xi = -\frac{1}{R}\frac{\partial\delta N}{\delta\phi} \quad \text{................ (2.21a)}$$

$$\delta\eta = -\frac{1}{R\cos\phi}\frac{\partial\delta N}{\partial\lambda} \quad \text{................ (2.21b)}$$

where δN, $\delta\xi$ and $\delta\eta$ are the indirect effects on N, ξ and η respectively. No attempt has been made to analyse these equations, but an estimation of the indirect effect on the deflections of the vertical is given below using the solution proposed by Molodenskii (see Section 2.2).

Another possible method of assessing the indirect effect on N is to substitute $(N^c + \delta N)$ for N in Equation (2.24), and hence:

$$N^c + \delta N = \delta - \frac{(\bar{g} - \bar{\gamma}_0)}{\bar{\gamma}_0} \cdot H \quad \text{............} \quad (5.6)$$

where N^c is derived from Stokes' integrals and the height anomaly δ is derived from a solution of Molodenskii's boundary value problem, of which the major contribution is obtained from Stokes' integral using a gravity anomaly defined by:

$$\Delta g = g - (\gamma_0 + \frac{\partial \gamma}{\partial h} \cdot H^*) \quad \text{...............} \quad (5.7)$$

In this equation γ_0 is the normal gravity on the ellipsoid, H^* is the normal height, g is the observed gravity at the surface of the earth, and Δg is a free air anomaly referred to the earth's surface.

Using the same gravity anomaly as defined in Equation (5.7), the deflections of the vertical at the surface of the earth, ξ_s and η_s are given by:

$$\left.\begin{matrix} \xi_s \\ \eta_s \end{matrix}\right\} = \frac{1}{4\pi\bar{\gamma}} \iint_\sigma (\Delta g + G') \frac{dS(\psi)}{d\psi} \left\{\begin{matrix} \cos \alpha \\ \sin \alpha \end{matrix}\right\} d\sigma$$

$$\text{...............} \quad (5.8)$$

where

$$G' = \frac{R^2}{4\pi} \iint_{\sigma} \frac{(h - h_p)(\Delta g - \Delta g_p)}{r^3} d\sigma \qquad \ldots\ldots\ldots\ldots\ldots (5.9)$$

The subscript p refers to the point of computation, h is the height, and r is the distance between Δg and Δg_p. Thus the indirect effect on the deflections of the vertical can be assessed by comparing the results from Equation (5.8) with those obtained from Vening Meinesz formulae using free air anomalies. Allowance must be made for the curvature of the plumbline since Equation (5.8) determines deflections of the vertical at the surface of the earth. As previously discussed in Section 2.2, Kearsley (1976), obtained a root mean value of ±0".3 for the contribution of the G' term to ξ_s and a smaller value for the contribution to η_s for seven stations situated in "rough" terrain in New South Wales, and a negligible amount for a further five stations situated nearby in relatively flat terrain. Since the South Australian region is predominantly flat, and the inner four 0.°1 x 0.°1 blocks' contribution to the deflection of the vertical has not been assessed, the indirect effect will have very little effect on the results.

5.2.3 Atmospheric Effects

In the Geodetic Reference System 1967, the mass of the atmosphere is included in the mass of the earth. In the usual reductions to obtain the free air anomaly, the effect of the atmosphere is ignored. An approximate allowance for the atmosphere may be made by increasing the observed gravity value, and hence the resultant gravity anomaly, by 0.87 mGal. (Olliver, 1979). This value of 0.87 mGal is arrived at by condensing the mass of the atmosphere as a surface layer on the surface of the geoid and assuming a homogeneous atmosphere above all observed gravity stations, all of which are situated at Mean Sea Level. A more precise variable correction could be computed considering homogeneous ellipsoidal atmospheric shells above stations situated on the surface of the earth and not, as in the above case, at Mean Sea Level. This still assumes that no earth mass protrudes above these ellipsoidal shells.

Anderson, et al (1975), suggests an alternative approach called the Boundary Value Problem approach. In this method, the potential due to the atmosphere V, is separated from that of the solid earth, and is treated separately. The two principal effects produced by this approach is the replacement of the conventional gravity anomaly with an anomaly which is increased by an amount δg_a where:

$$\delta g_a = \frac{2\gamma}{R} + \frac{\partial V}{\partial h} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.9)$$

and

a numerical dominant contribution to the zero degree term approximating to $-\frac{R}{\gamma}$ $M'\{\delta g_a\}$ where $M'\{\delta g_a\}$ is the world mean value of δg_a.

The zero effect computed from either the Boundary Value method or the concentric shell approach allowing for terrain height, results in an almost identical contribution to N of approximately -5.5 metres (Ibid, 1975). This figure is obtained assuming a global distribution of surface gravity information. For this to be so, atmospheric effects must be correctly allowed for before surface gravity is derived from low degree orbital information. If the simple atmospheric model is used, and the gravity anomalies are increased by 0.87 mGal, the total result on the Stokesian term is zero. The more sophisticated approach of concentric shells and the B.V.P. approach gives similar results if a precision of the order of flattening is sought. Both methods showed a contribution to the Stokesian term of N in the range of 0-10 cm. (Ibid, 1975).

Rapp and Rummell (1975), show the effects of the atmosphere on the Stokesian term when limited cap sizes of ψ_0 are used. For $\psi_0 = 5°$ with a mean elevation of the cap at Mean Sea Level, the effect of the atmosphere on the value of the geoid separation is 0.56 metres, and for $\psi_0 = 30°$ the result is 2.97 metres. Similar results were obtained by the author. These corrections need to be applied when a mixture of computation techniques are being used to determine the value N, i.e. when Stokes' integral is being used to a radius of ψ_0 and the remaining effects are being determined using potential coefficients. No corrections are required to the deflections of the vertical when a mixture of computation techniques are being used since this simple model has only a zero order effect. Thus, for a solution to the order of flattening, the Stokesian atmospheric effect can be ignored provided the data being used is internally consistent.

5.2.4 Summation Errors

The use of Equations (4.1) and (4.2) in place of Equations (2.12a) and (2.16) for the practical evaluations of Stokes' integral and Vening Meinesz formula are approximations, but the resulting errors are negligible if the size of the surface elements are selected carefully using the following criteria:

: the gravity anomaly Δg_i adequately represents the gravity anomaly field of the compartment.

: the values of $S(\psi)_i$ and $\frac{dS(\psi)_i}{d\psi}$ are the mean of the values of the compartment.

Both these criteria have been discussed in Chapter 4. The results show that the errors in the values obtained for the deflections of the vertical and the geoid separation arising from the Vening Meinesz and Stokes' functions are less than 0".03 and 0.1 metres respectively. The assessment of the error in Δg_i is discussed later.

5.2.5 Zero Order Term

The free air geoid does not include the zero order term N_o (Equation 2.15), as this cannot be assessed by the gravimetric methods described. The zero order term does not effect the values of ξ and η as it is a constant independent of position. The atmospheric and indirect zero order effects are included in this term.

5.3 Systematic Errors in the Gravity Field

Systematic errors may exist in the gravity data used to derive the mean free air anomalies of the compartment. The existence of these errors may be due to systematic errors in gravity control stations, geographical location including heights, and the derived normalised potential coefficients in the case of satellite data. Systematic errors of ±0.1 mGal over regions of 5° x 5° can result in an error in N of 0.05 metres (Mather, 1973). If these figures are linearly extrapolated then systematic errors of 0.5 metres in N may result from a 1 mGal systematic error over the same regions. These figures are very pessimistic since their derivation assumes that these systematic errors would occur in the immediate vicinity of the computation point.

The B.M.R. data and other surface gravity data has been described in Chapter 4. The uncertainty in individual gravity values is estimated to be ±0.5 mGal and the location of the station is fixed to ±0.1 minutes of Latitude and ±5 metres in height within each survey. These errors are random in nature since these surveys are tied to the Australian Height Datum (AHD) and the Australian National Gravity Network (ANGN) which has in turn been tied to the International Gravity Standardization Net 1971 (IGSN 71). Systematic errors in the ANGN are unlikely to exceed 0.2 mGal with half wave lengths of 3.5×10^3 km (Mather, et al, 1976). The AHD is not a freely adjusted level network since during adjustment of the levelling data, the local mean sea level as determined at thirty tide gauges, was held fixed. The levelling data used in the adjustment was obtained using third order levelling techniques. This may result in systematic errors of 0.15 mGal with half wavelengths of 3.5×10^3 (Ibid, 1976). Considering the adverse case when these systematic errors are cumulative, it is unlikely that the total effect on N would exceed ±0.2 metres, with a half wavelength of 3.5×10^{-3} km. The effect on the deflections of the vertical would be negligible.

The Rapp 1° and 5° data which is described in Chapter 4 is obtained by combining terrestrial and satellite derived data. When comparing 1° x 1° mean surface gravity anomalies during compilation of these data sets, Rapp (1978.b) reports discrepancies of 150 mGals were often found. In some cases, mean gravity anomalies were supplied by the same agency at different times for the same block and these varied by up to 100 mGals. The altimeter derived anomalies when compared to a set of terrestrial anomalies on a global basis, showed a Root Mean Square (RMS) difference of 15 mGals, and this was further reduced to 12 mGals if terrestrial anomalies with standard deviations greater than ±10 mGals were removed from the comparison. It is possible that these combined data sets do contribute systematic errors to the results obtained, but the effect of this can only be reliably assessed using another more precise data set, or comparing results with those obtained by other methods. The errors could be of the order of 1-2 metres in N, and 1 second in each of the deflection terms. These approximate values include any systematic effects due to missing data in the Outer and Middle Zones. These effects would have large wavelengths and hence small relative effects between neighbouring grid points. These effects are further discussed in Chapter 6.

One other source of systematic error is the use of the data from the Rapp 1° data set to represent values of some $\frac{1}{2}°$ x $\frac{1}{2}°$ geographical blocks in the ocean south of South Australia. This may have an effect on the values of N, ξ and η situated near the coast. The effects, if any, are difficult to assess without detailed comparisons of astrogeodetic and gravimetric determined values of N, ξ and η at selected trigonometric stations situated on or near the coast.

5.4 Random Errors

The standard deviation of the deflections of the vertical and the geoid separation are an estimation of the precision of the results. The derivation of these values assumes the error as represented by the standard deviation of the gravity anomaly Δg_i of each compartment, is random in nature and that no correlation exists between it and values of other compartments. It is a measure of the random errors and not the systematic errors. The results given in Section 5.1.6 differ from estimated values published by some authors (e.g. Sjoberg, 1977; Ihde, 1981).

To derive the standard deviations of N, ξ and η using Equations (5.2) and (5.3), an estimated value of the standard deviation of the mean gravity anomaly μ_{gi} is required for each block in each Zone. The Rapp 1° and 5° data sets both have a corresponding value of μ_{gi} for each mean gravity anomaly. The values of the Rapp 1° data set have been obtained in a variety of ways (Rapp, 1977, 1978.b). In some cases standard deviations were estimated on the basis of the field methods used to gather the original data, i.e. on the basis of ship gravity accuracy, line spacing, anomaly magnitudes, and in the case of satellite derived anomalies, on the adjusted undulations and data distributions. Other values were given with the data supplied by the agencies, e.g. data for Australia supplied by Mather. In other instances, when widely different estimates of Δg_i for the same compartment were encounted, a standard deviation of one half the difference between the two anomalies was adopted. The Rapp 5° data set is derived from the Rapp 1° data and this includes the estimation of the standard deviation of each Δg_i using least squares prediction procedures. In the Middle Zone where no data existed for a 1° x 1° block, a nominal value of ± 30 mGals was used. It is expected that this value is an over estimation of the magnitude of the standard deviation of a predicted mean gravity anomaly in this Zone.

The error of representation (Equations (4.14) and (4.15)) is used to estimate the a value of μ_{gi} for each of the gravity anomalies in the Near Zone. This error is a measure of the variability of the gravity field within the compartment and not a measure of the precision of measurement. Its value is generally larger in mountainous regions than in flat terrain. In the relatively few compartments in the distant part of the Zone where, due to the lack of gravity data a value of the error of representation could not be assessed, the corresponding value from the Rapp 1° data was substituted. This may be an under estimation but the effects on the final result will be small because of the location and the number of blocks involved.

The value of μ_{gi} for the 0.°1 x 0.°1 blocks was given a nominal value of ±5 mGal. The error of representation was not computed for these blocks because the small number of individual gravity stations within each block would tend to produce an under estimation of μ_{gi}. In addition to the considerations discussed in Section 5.1.5 the error of representation was computed for five 0.°1 x 0.°1 blocks randomly selected, which contained eight or more gravity stations. The mean value obtained was ±3.5 mGals which included one extreme value of ±9.5 mGals from a block situated in a rapidly changing gravity field in the Flinders Ranges. From a consideration of these results and their locations, the value of ±5 mGal as a mean value of μ_{gi} for the 0.°1 x 0.°1 blocks is a reasonable assessment.

Using these values of μ_{gi} an average value of ±1.25 metres is obtained as an estimation of the standard deviation of the geoid separation. Paul and Nagy (1973) used 5° x 5° blocks classified on the basis of their gravity coverage to obtain an estimation of the standard deviation of N. A value of ±4.09 metres was obtained for a station situated in the centre of a well surveyed area and using values of μ_{gi} of ±5, ±10, ±20 mGal respectively for surveyed areas, partially surveyed areas, and unsurveyed areas. The major contribution of ±3.77 metres comes from the region $5° \leqslant \psi \leqslant 180°$ corresponding to the Outer and Middle Zones, where combined average contribution for the South Australian region is ±1.2 metres. The discrepancies in the results are explained by the respective value of μ_{gi} that are used. The RMS values of μ_{gi} of the 5° equal area compartments used in the present study is ±4.0 mGal which is significantly less than the range in values used by Paul and Nagy in 1973.

A similar result to Paul and Nagy was obtained by Ihde (1981). In this study, a value of the error contribution to the height anomaly ($\simeq$ N) from prediction errors in the mean free air anomalies, was assessed at ±3.22 metres for the region $9° \leqslant \psi \leqslant 180°$. The error variances of the mean gravity anomalies are evaluated using covariance functions estimated by Tscherning and Rapp (1974) and are a function of the density of measurement within each compartment. The values for the 5° equal area anomalies were calculated from the given densities of measurement given in Rapp (1977).

The discrepancy in results arises because these densities are a measure of the number of known 1° x 1° mean anomalies within each 5° equal area block and not the actual density of gravity stations. If these densities were used then the estimation of the prediction error in N would possibly be reduced to a similar value as that obtained in the South Australian study.

The mean values of the standard deviations of ξ and η obtained in this study are $\pm 0''.55$ and $\pm 0''.50$ respectively, and these are for the region $0.1 \leqslant \psi \leqslant 180°$. The study of Ihde (1981), suggested a value of $\pm 0''.2$ for the region $9° \leqslant \psi \leqslant 180°$ which again is higher by a factor of approximately 2.5 for the same reasons as discussed above.

In another study of the accuracy of gravimetric deflections of the vertical obtained from combination techniques using GEM 7 potential coefficients and terrestrial gravity data, Sjoberg (1977), determined the errors in the total deflection of the vertical θ as defined by:

$$\theta = \sqrt{\xi^2 + \eta^2}$$

for the two outer components as used in Equation (2.50a) and (2.50b). A value of $\pm 0''.69$ was obtained for the region $4° \leqslant \psi \leqslant 180°$ with the GEM 7 potential coefficients being used to represent the area $\psi \geqslant 30°$ and 1° x 1° compartments being used for the remainder. This approximately corresponds to the Outer and Middle Zones in the present study which contributed a mean value of $\pm 0''.13$ to the estimation of the error in the total deflection of the vertical. The error due to lack of more detailed gravity data in the 1° x 1° blocks between $4° \leqslant \psi \leqslant 30°$ in Sjoberg results, is ±0".60. This figure is obtained using degree variances computed by Tscherning and Rapp (1974) to estimate a mean contribution to the error in θ from this zone of influence and as such does not take into account location of the computation point or improvements in the data coverage.

For the Inner and Innermost Zones, Kearsley (1976), using a Rice Rings system, obtained values of approximately ±0".3 for the error contribution to both ξ and η. These values closely agree with those obtained in the present study. The small differences are explained by the estimation processes used in the assessment of the gravity anomaly errors and the compartment sizes. This close agreement is a further justification in the adopted value of ±5 mGal for the value μ_{gi} in these Zones.

5.5 Summary of Errors

Ignoring for the moment the zero order effects, the combination of the systematic errors on the value of N arising from spherical approximation, atmosphere, and summation errors, are not expected to exceed 0.25 metres. The indirect effect has a variability of ±0.1 metres with a constant effect approximating to 6.6 metres. The systematic errors in the Rapp 1° and 5° data may have effects of 1-2 metres over the South Australian region, but this effect would have large wavelengths with minimum effects between adjoining grid points. The B.M.R. data is estimated to contribute a maximum error of ±0.2 metres to values computed.

The zero order effect is constant at any point on the earth's surface. This effect may be combined with other errors that are constant near the area studied, such that the results obtained for the free air geoid of South Australia give the relative geoid undulations but not the absolute values.

With the exception of the possible errors in the Rapp 1° and 5° data, the random error as estimated by the standard deviation of N is much larger than the variable systematic effects and should be a reasonable assessment of the precision of the final results.

Zero Order terms do not effect the deflections of the vertical. The systematic errors in the deflections of the vertical are generally small when compared with the random errors of approximately ±0$''\!.$5 for each components. The two possible exceptions are the terrain effect and the effects of the systematic errors in the gravity field. With a few exceptions in the north west of the state and the Flinders Ranges, the terrain correction is expected to be less than 0$''\!.$3.

The combined contribution to both values of ξ and η from the Outer and Middle Zones have gentle slopes with variations less than 2". Systematic errors in the Rapp 1° and 5° data may result in estimated errors of 1" in both ξ and η. This error may be considered as having two components; a constant error for all points within the study region and a variable component which has a small rate of change between neighbouring grid points.

To assist with the evaluation of the results obtained and the associated errors in the free air geoid study, some comparisons are made with other geoid determinations in Chapter 6.

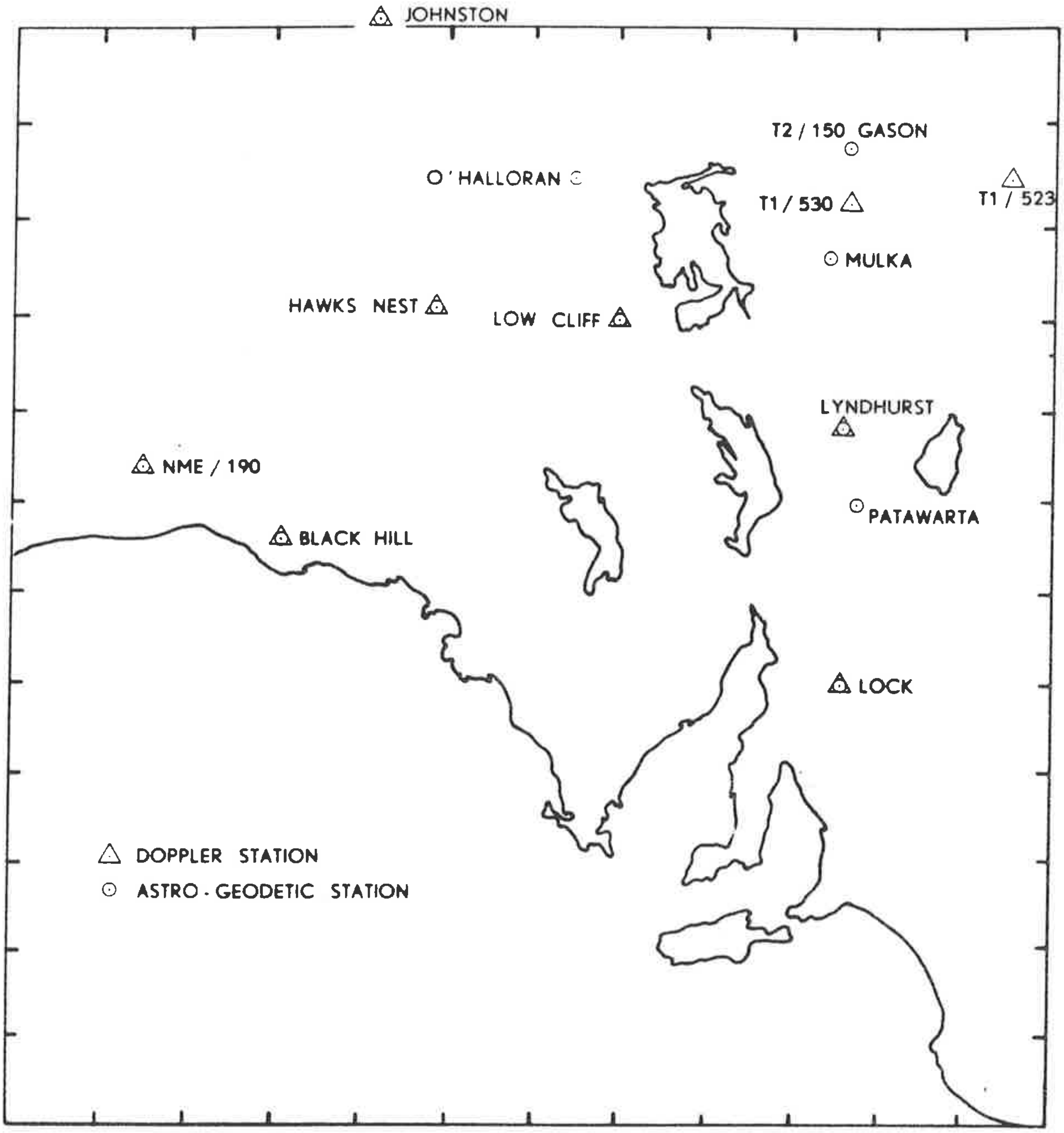
CHAPTER 6

GEOID COMPARISONS

As discussed in Chapter 2, there are various methods of determining the geoid separation and the deflections of the vertical. In this section the results for the free air geoid are compared with those obtained by astro-geodetic techniques, previous gravimetric investigations, Doppler methods, and the use of spherical harmonic coefficients.

6.1 Inner Zone Computations around Selected Trigonometric Stations

In order to facilitate the comparison between the free air geoid results and available astro-geodetic values or Doppler derived results, additional gravimetric stations were observed in the immediate vicinity of thirteen trigonometric stations. The methods used for these surveys have been described in Section 4.2.2. These survey stations have been selected because of their geographical location and available geoid information. Astro-geodetic determined values of N, ξ, and η had been previously derived for eleven of the stations and this information was obtained from either the Lands Department of South Australia or the



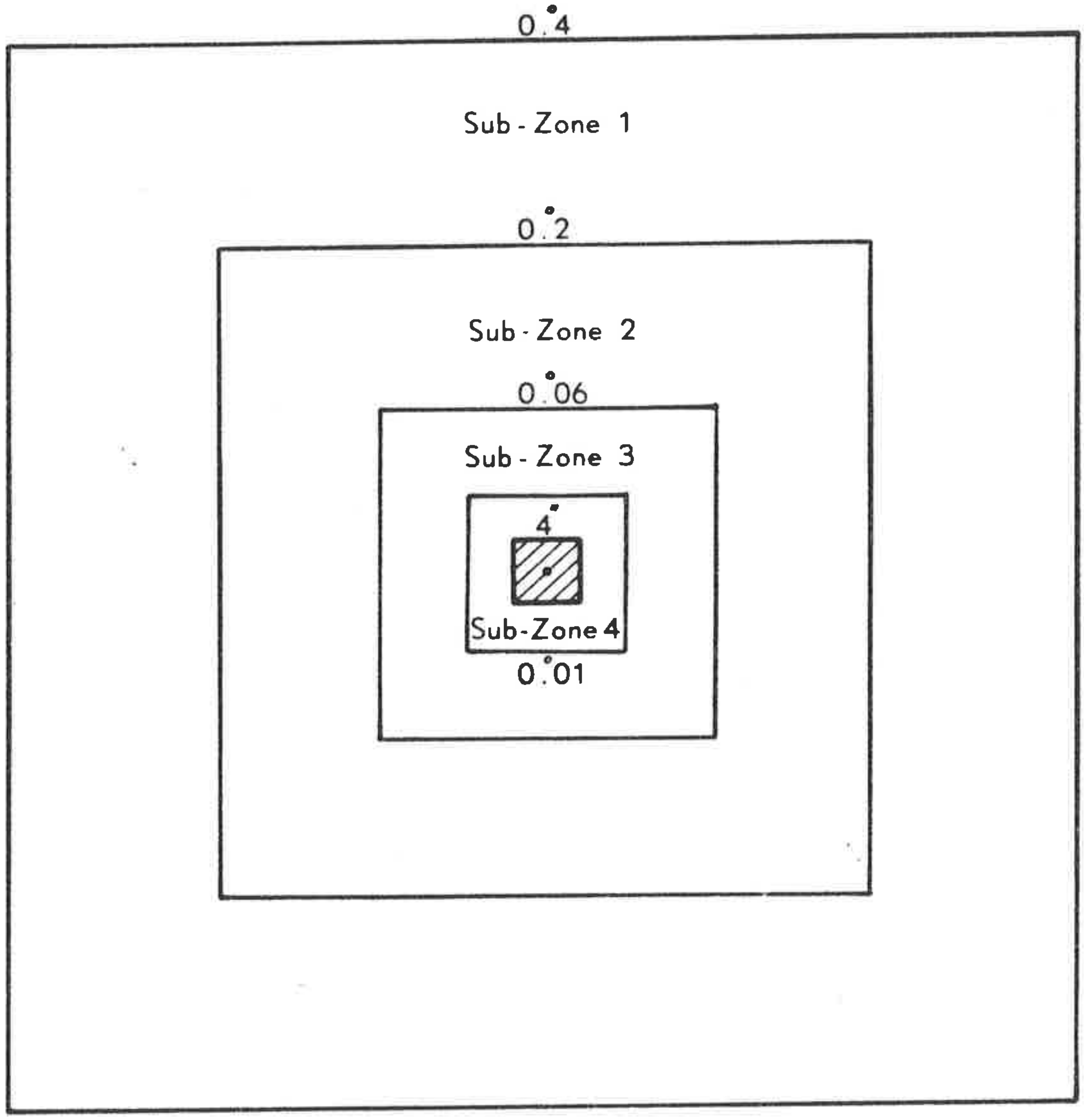
Trigonometric Station Locations

Figure 6.1

Division of National Development and Energy, Canberra. Doppler derived coordinates for nine stations were also available from the same sources. The location and information of each of these thirteen trigonometric stations are shown in Figure 6.1.

Gravity was observed at all trigonometric stations and ideally at seven or eight symmetrical points about the stations ranging in distance from 0.5 to 2.5 kilometres. For the stations Black Hill and Hawks Nest, this ideal requirement of observing seven or eight gravity stations was not met due to time restraints and logistics. At Hawks Nest, only three additional gravity stations were observed, ranging in distance from 0.5 to 1.0 kilometres from the station, and located in the north west to the north east sector. An additional four gravity observations were made at Black Hill, ranging in distance from 0.6 km to 1.2 km, and these were symmetrically located with respect to the trigonometric station. These additional gravity values were combined with the B.M.R. data and used to more precisely define the gravity field in the vicinity of the stations.

The free air geoid values of N, ξ, and η at the survey stations are computed using similar techniques with a few modifications to those described in Chapter 4. For the Outer, Middle, Near and Inner Zone contributions to N, ξ, and η the geographical coordinates to the nearest second of arc of the trigonometric stations are used as the computation point. The inner boundary of the Outer Zone remains fixed as before and the inner boundaries of the Middle and Near Zones are defined as in Chapter 4



Sub - Zones
Figure 6.2

from the nearest half degree geographical grid point. The inner boundary of the Inner Zone is defined from the nearest tenth degree geographical grid point to the trigonometric station.

Because the contribution to the deflections of the vertical from the Innermost Zone is critically dependant on a detailed knowledge of the gravity field in this region, a series of sub-zones with varying compartment sizes are used. The compartment or block size is constant within each sub-zone, but decreases in size as the location of the sub-zone approaches the computation point. The selection of block sizes and sub-zone location is further complicated by the "non-grid type" location of the trigonometric stations. It is possible to use a system of compartments based on Rice's Rings (Kearsley, 1976) to determine the contributions from this region, and this method has the advantage of symmetrical compartments about the computation point. The disadvantages are that the computer programming is more involved and a merge zone is required between the outer grid region and the inner ring system.

In the present study, four sub-zones as shown in Figure 6.2 are used. The compartments for the sub-zones 1, 2, 3 and 4 are respectively 0.°1 x 0.°1, 0.°01 x 0.°01, 0.°005 x 0.°005, and 01" x 01" geographical blocks. The inner boundary dimensions of the sub-zones are shown and these are centred about the computational point to the nearest appropriate block dimension. As an example, for Johnston trigonometric stations where geographical coordinates are given in Table 6.1, the centres of the boundaries of the sub-zones in latitude and longitude are:

Sub-zone 1	-25° 54' 00"	133° 12' 00"
Sub-zone 2	-25° 57' 00"	133° 12' 36"
Sub-zone 3	-25° 57' 00"	133° 12' 36"
Sub-zone 4	-25° 56' 55"	133° 12' 31"

The contributions N, ξ and η from the four innermost 01" x 01" blocks are not evaluated and hence are considered to be zero. In the case of the deflections of the vertical this means that the gravity field for a radius of approximately 55 metres is either constant or has symmetrical variations with respect to the computation point. Assuming the Bouguer anomaly is constant over this small distance, any changes to the free air gravity field will be due to height changes (Gilliland, 1978). Since the topography immediately surrounding trigonometric stations surveyed in this study is generally symmetrical (with the possible exception at Patawarta), it is feasible to assume a zero contribution to the deflection components. It should be noted that the study in New South Wales (Kearsley, 1976) of the deflections of the vertical at twelve trigonometric stations showed that the region within a radius of 100 metres (not 55 metres) could contribute 0".56 to a deflection component in an extreme case where the station was situated on the edge of a large escarpment. For eight stations in that study, the contributions to either deflection component was less than 0".1. The contribution to N from this region is dependent on the magnitude of the gravity anomaly field. For a mean gravity anomaly of 100 mGal, the contribution to N would be approximately 0.005 metres. Thus in summary, the assumption to assure zero contributions from the four innermost 01" x 01" blocks would result in negligible errors provided the topography is symmetrical about the computation point.

The mean gravity anomaly for each 0.°1 x 0.°1 block in sub-zone 1 is determined using the same techniques as those described in Section 4.5 for the Inner and Innermost Zones. Equations (5.4) and (5.5) are used to evaluate the contributions to N, ξ, and η from this sub-zone.

For sub-zones 2, 3 and 4, the mean gravity anomaly of the appropriate block size is evaluated using the techniques of weighted means as described in Section 4.4.3, and all available gravity data within a range of 0.°15 of the trigonometric station. The contributions from sub-zone 2 are derived from the use of Equations (4.1) and (4.2), with the Stokes' and Vening Meinesz functions defined by Equations (4.4) and (4.5).

For sub-zones 3 and 4 the contributions to N, ξ and η are evaluated using Equations (4.1) and (4.2), but the values of Stokes' and Vening Meinesz functions are defined by:

$$S(\psi)_i = \frac{2}{\psi} \qquad \ldots\ldots\ldots\ldots\ldots\ (6.1)$$

and

$$\frac{dS(\psi)_i}{d\psi} = -\frac{2}{\psi^2} \qquad \ldots\ldots\ldots\ldots\ldots\ (6.2)$$

These equations are approximations obtained by assuming $\sin\psi = \psi$ when ψ, the spherical distance between the computation point and the centre of the compartment, is small. As the relative error in both Stokes' and Vening Meinesz functions of these approximations is about one percent for a distance of 10 kilometres and three percent for 30 kilometres (Heiskanen and Moritz, 1969, p. 121), no significant errors in the final results will occur.

The contributions from all zones and the resulting values of N, ξ, and η are shown in Table 6.1. The sub-zones 2, 3 and 4 contribute less than 1 metre to values of N, and in all but three stations, less than 0.4 metres. These three stations are the only stations with elevations above 550 metres with Patawarta having the maximum elevation of 1008 metres. The contributions to ξ and η from the same three sub-zones vary greatly with maximum numerical values for both components occurring at Patawarta, which is situated in very rugged terrain in the centre of the Flinders Ranges.

6.2 Datum Shifts and Coordinate Transformations

A geodetic datum is defined by the dimensions of the reference ellipsoid and its orientation in space with respect to the earth or the geoid. Various geodetic datums may have different geometric reference ellipsoids whose centres and orientations of axes do not correspond. In order to compare values of N, ξ, and η referenced to different datums and ellipsoids, it is necessary to use datum shift and coordinate transformation techniques.

Station & Position	Comp.	A	B	C	D	Total
Johnston Origin	N	14.79	-5.17	-3.88	-0.32	5.42
-25°56'55"	ξ	-4.83	-0.09	5.52	2.09	2.69
133°12'31"	η	-3.81	-0.70	-3.20	-0.96	-8.67
Mt. Gason	N	19.36	-1.92	-2.60	-0.09	14.75
-27°21'40"	ξ	-4.64	0.01	-0.78	-0.44	-5.85
138°43'07"	η	-4.07	-0.05	-1.30	-0.50	-5.92
O'Halloran	N	14.60	-4.75	-2.30	-0.18	7.37
-27°30'32"	ξ	-4.68	0.37	2.80	0.64	-0.87
135°26'39"	η	-3.95	-0.70	1.76	-0.92	-3.81
T1/523	N	21.25	-0.89	-2.74	-0.19	17.43
-27°36'59"	ξ	-4.64	0.26	-1.44	-0.30	-6.12
140°28'36"	η	-3.98	-0.40	-0.30	0.40	-4.28
T1/530	N	18.50	-1.91	-2.71	-0.21	13.67
-27°43'30"	ξ	-4.59	0.13	-1.05	0.22	-5.29
138°44'31"	η	-4.08	-0.01	-1.55	-1.14	-6.78
Mulka	N	17.42	-2.59	-2.30	-0.09	12.44
-28°20'10"	ξ	-4.67	0.93	0.69	0.27	-2.78
138°39'17"	η	-4.11	-0.15	-0.61	0.19	-4.68
Hawks Nest	N	9.04	-6.98	0.77	0.30	3.13
-28°54'18"	ξ	-4.62	0.91	1.06	-0.24	-2.89
133°52'48"	η	-4.21	-1.10	-0.01	-0.14	-5.46
Low Cliff	N	11.55	-4.18	-0.99	-0.09	6.29
-29°00'50"	ξ	-4.69	0.95	-0.91	0.06	-4.59
136°02'23"	η	-4.31	-0.18	0.98	-0.86	-4.37
Lyndhurst	N	12.68	-2.51	0.49	-0.02	10.64
-30°12'12"	ξ	-4.53	1.26	3.72	0.19	0.64
138°34'24"	η	-4.24	-0.44	-4.56	-0.49	-9.73

TABLE 6.1

Free Air Geoid Values of N, ξ and η at the Trigonometric Stations

.../

TABLE 6.1 (contd.)

Station & Position	Comp.	A	B	C	D	Total
NME/190	N	2.22	-10.92	-4.38	-0.37	-13.45
-30°37'32"	ξ	-3.86	-0.06	-1.26	0.38	-4.80
130°20'41"	η	-3.58	-1.83	-0.47	-3.01	-8.89
Patawarta	N	10.63	-1.90	1.86	0.82	11.41
-30°57'37"	ξ	-4.38	1.27	-4.21	3.71	-3.61
138°42'47"	η	-4.24	-0.40	-2.56	4.90	-2.30
Black Hill	N	2.60	-10.24	-1.88	0.06	-9.46
-31°34'30"	ξ	-3.60	-0.77	-2.11	0.16	-6.32
132°04'51"	η	-3.75	-1.48	5.02	0.33	0.12
Lock	N	5.41	-1.66	2.35	0.58	6.68
-33°05'35"	ξ	-3.99	-0.20	-0.53	-1.24	-5.96
138°34'31"	η	-4.13	-0.22	-4.93	0.18	-9.10

Column A is the combined Outer & Middle Zone Contributions

B is the Near Zone Contribution

C is the Inner & Innermost Zone Contributions excluding the four innermost 0.°1 x 0.°1 blocks

D is the contribution of the innermost four 0.°1 x 0.°1 blocks

Total is the total contribution to N, ξ and η

ξ and η are in seconds of arc

N is in metres.

To transform cartesian coordinates between two different reference systems, a 7 parameter transformation may be used.

$$\begin{bmatrix} X^1 \\ Y^1 \\ Z^1 \end{bmatrix} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1 + S) \begin{bmatrix} 1 & -R_Z & R_Y \\ R_Z & 1 & -R_X \\ -R_Y & R_X & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots \; (6.3)$$

where X^1, Y^1, Z^1 are the required cartesian coordinates in one geodetic system.

X, Y, Z are the known cartesian coordinates in another geodetic system.

ΔX, ΔY, ΔZ are the three cartesian translation parameters between the two reference ellipsoid centres.

R_X, R_Y, R_Z are the clockwise rotations (in radians) about the corresponding axis,

and S denotes a scale change.

The assumption in this formula is that the rotations are differentially small, and hence the actual rotation R_X, R_Y, R_Z can be substituted for the corresponding elements in the well-known three dimension rotation matrix. It is possible to have a transformation model with less than 7 parameters if one or more of the parameters are assumed to be zero. Ideally, all geodetic reference systems are aligned with the Z-axis in the direction of the Conventional International Origin (CIO), and the X-axis in the direction of the Zero meridian as defined by the Bureau International de l'Heure (BIH) but these two criteria are subject to observational errors.

If the two systems are assumed to be commonly aligned within the tolerances of observational errors, then a 4 parameter transformation may be used. In this case, the parameter would be the translation parameters ΔX, ΔY, ΔZ and the scale parameter.

The transformation formula is often used in the first instance to determine the transformation parameters from known common point values of (X, Y, Z) and (X', Y', Z'), e.g. satellite datum values and national geodetic values. The selection of the number of parameters used depends on the size of the geographical region and the particular geodetic system. Ashkenazi and Sykes (1978), having done extensive tests on various parameter combinations, used a 4 parameter (ΔX, ΔY, ΔZ and S) to align the Doppler broadcast ephemeris derived coordinates with the OSGB-77 (Ordinance Survey - Great Britain, 1977). In Australia, Allman and Steed (1980), compared a 7 and a 5 parameter transformation model to convert NWL9D (precise Doppler ephemeris) derived coordinates to GMA80 (Geodetic Model of Australia, 1980). The 7 parameter solution was preferred since the 5 parameter solution appeared to introduce a small systematic error. Since systematic errors may exist in geodetic networks, it is possible that more than one set of transformation parameters are required for different regions within that network. Leppert (1978) gives charts of Australia showing correction "contours" for conversions of satellite data to the AGD, and suggests the use of these corrections in preference to one transformation model because of their variability. In a relatively small geographical region, a 4 parameter transformation formula with no rotational parameter would suffice, since any rotational effect could be considered as constant and computed as part of the translation parameters.

The cartesian coordinates used in the transformation formula may be converted to the equivalent geodetic coordinates (ϕ, λ, h), using the following formulae.

$$\tan \lambda = Y/X \qquad \ldots\ldots\ldots\ldots\ldots \quad (6.4)$$

$$\tan \phi = \frac{(Z + e^2 \nu \sin \phi)}{(X^2 + Y^2)^{\frac{1}{2}}} \qquad \ldots\ldots\ldots\ldots\ldots \quad (6.5)$$

$$h = N + H = \frac{X}{(\cos \phi \cos \lambda)} - \nu \quad \ldots\ldots \quad (6.6)$$

where H is the orthometric height, e is the eccentricity, ν is the radius of curvature in the prime vertical, and all other terms have been previously defined. The determination of the geodetic latitude is an iterative process.

The coordinate transformations and datum shifts may be directly expressed in the terms of the geodetic coordinates. Assuming the axis of the reference ellipsoid are parallel (usually aligned to the CIO) and the transformations are relatively small, the variations of $\delta\phi$, $\delta\lambda$, and δh at some arbitrary point, ϕ, λ, and h, may be obtained from (Heiskanen and Moritz, 1967, p. 207):

$$\delta\phi = (\cos \phi_1 \cos \phi + \sin \phi_1 \sin \phi \cos \Delta\lambda)\delta\phi_1 - \sin \phi \sin \Delta\lambda.$$
$$\cos \phi_1 \delta\lambda + (\sin \phi_1 \cos \phi - \cos \phi_1 \sin \phi \cos \Delta\lambda).$$
$$\left(\frac{\delta h_1}{a} + \frac{\delta a}{a} + \sin^2\phi_1 \delta f\right) + 2 \cos \phi(\sin \phi - \sin \phi_1)\delta f,$$
$$\ldots\ldots\ldots\ldots\ldots \quad (6.7)$$

$$\cos\phi\delta\lambda = \sin\phi_1 \sin\Delta\lambda\ \delta\phi_1 + \cos\Delta\lambda \cos\phi_1\ \delta\lambda_1$$

$$- \cos\phi_1 \sin\Delta\lambda \left(\frac{\delta h_1}{a} + \frac{\delta a}{a} + \sin^2\phi_1\ \delta f\right) \qquad \ldots\ldots\ldots\ldots\ldots\ (6.8)$$

$$\frac{\delta h}{a} = (\cos\phi_1 \sin\phi - \sin\phi_1 \cos\phi \cos\Delta\lambda)\delta\phi_1 + \cos\phi \sin\Delta\lambda .$$

$$\cos\phi_1\ \delta\lambda_1 + (\sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos\Delta\lambda) .$$

$$\left(\frac{\delta h}{a} + \frac{\delta a}{a} + \sin^2\phi_1\ \delta f\right) - \frac{\delta a}{a} + (\sin^2\phi - 2\sin\phi_1 \sin\phi)\delta f \qquad \ldots\ldots\ldots\ldots\ldots\ (6.9)$$

where

$$\Delta\lambda = \lambda - \lambda_1$$

In these equations, $\delta\phi_1$, $\delta\lambda_1$ and δh_1 are the known changes in the geodetic coordinates at a point defined by ϕ_1, λ_1 and h_1. δa and δf are the other respective changes in the dimensions of the semi-major axis and the value of flattening.

These formulae may be rewritten in terms ξ, η and N since

$$\delta\phi = -\delta\xi$$

$$\delta\lambda \cos\phi = -\delta n$$

$$\delta h = \delta N$$

and hence on substitution become:

$$\delta\xi = (\cos\phi_1 \cos\phi + \sin\phi_1 \sin\phi \cos\Delta\lambda)\delta\xi_1 - \sin\phi \sin\Delta\lambda . \delta\eta_1 - (\sin\phi_1 \cos\phi - \cos\phi_1 \sin\phi \cos\Delta\lambda) . \left(\frac{\delta N_1}{a} + \frac{\delta a}{a} + \sin^2\phi_1\ \delta f\right) - 2\cos\phi(\sin\phi - \sin\phi_1)\delta f \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.10)$$

$$\delta\eta = \sin\phi_1 \sin\Delta\lambda\ \delta\xi_1 + \cos\Delta\lambda\ \delta\eta_1 + \cos\phi_1 \sin\Delta\lambda . \left(\frac{\delta N_1}{a} + \frac{\delta a}{a} + \sin^2\phi_1\ \delta f\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.11)$$

$$\frac{\delta N}{a} = -(\cos\phi_1 \sin\phi - \sin\phi_1 \cos\phi \cos\Delta\lambda)\delta\xi_1 - \cos\phi \sin\Delta\lambda . \delta\eta_1 + (\sin\phi_1 \sin\phi + \cos\phi_1 \cos\phi \cos\Delta\lambda) . \left(\frac{\delta N_1}{a} + \frac{\delta a}{a} + \sin^2\phi_1\ \delta f\right) - \frac{\delta a}{a} + (\sin^2\phi - 2\sin\phi_1 \sin\phi)\delta f \quad \ldots\ldots\ldots\ldots\ldots\ldots (6.12)$$

If the point (ϕ_1, λ_1, h_1) is taken as the origin of the geodetic system, then Equations (6.10), (6.11) and (6.12) express the change in the values of ξ, η and N respectively in terms of the changes in values at the origin. As with the transformation formula using cartesian coordinates, these formulaes may be used to determine the changes in values of the deflections of the vertical and the geoid separation at the geodetic origin from observed values at other stations. This type of method was used by Mather to determine the orientation parameters between the free air geoid of Australia and the GRS-67 Reference System (Chapter 3).

6.3 Astro-Geodetic Comparisons

As discussed in Section 2.1, deflections of the vertical and geoid separations can be defined from a comparison of astronomical and geodetic observation. The results thus obtained are referenced to the "local" geodetic system and not an earth centred ellipsoid.

The great majority of first order astronomical observations for latitude and longitude in Australia have been made between the years 1955 and 1970. Before 1966, some longitude determinations were made with theodolites with fixed vertical wires and stop watches. Since this period, moving impersonal micrometer eyepieces and improved timing equipment have been used to determine the astronomical longitude. The time signal origin was WWV(H), Hawaii, and more recently VNG, Lyndhurst. The almucantar method was used to determine longitude, and meridian or circum-meridian altitude of stars were used to determine latitude (Bomford et al, 1970).

The average standard deviation from a single pair of stars for latitude was ±0".6 and if sixteen pairs of stars were observed, the average mean standard deviation was frequently less than ±0".2. For longitude, the standard deviation from a single star pair averaged 0".9 with the final mean standard deviation of sixteen pairs usually less than ±0".3. These standard deviations are a measure of the internal precision and they do not take into account the effects of external systematic errors such as refraction and errors in timing. An indication of the total error in the results is obtained by comparing observations made at the same survey station, but at different times. At 110 stations in the Australian network, astronomical latitudes and longitudes have been observed twice. The comparison of the results gave an average difference in the latitude of 0".43 and for the longitude 0".86 (Ibid, 1970).

As shown in Figure 6.1, astronomical latitudes and longitudes have been observed at eleven trigonometric stations by either the Lands Department of South Australia or the Division of National Mapping. The observations for these stations were made between the years 1957 and 1968. All latitude and longitude results have been corrected to the CIO. Some of the longitude results have been obtained using a stop-watch, but Bomford et al (1970) found no significant variation between the use of the impersonal micrometer eyepiece and the stop-watch methods in their error analysis. At Low Cliff, observations had been made both in 1957 and 1967. The 1967 results, which differed by 0".36 in latitude and 0".75 in longitude when compared to the earlier observations, were used in the analysis described.

The observed astronomical values are reduced to the geoid using Equation (2.2c), and deflections of the vertical are then obtained using Equation (2.1a) and (2.1b), with latitudes and longitudes being defined positively north and east respectively. These values are referenced to the Australian Geodetic Datum (AGD).

Values of the geoid separation N_{AGD} referenced to the AGD were obtained from the Division of National Mapping. These values are not directly obtainable from observation data, but are derived using formulae similar to Equation (2.5). The procedures used to obtain values of N_{AGD} are documented in Chapter 3 and the values are based on a value of N_{AGD} = 0.0 metres at the Johnston Origin. A point to be remembered in the later analysis of the results is that the average misclosure of the astro-geodetic levelling loops without respect to sign is 2.0 metres and the maximum misclosure value in the study region is 4.68 metres.

To enable comparisons between the astro-geodetic derived values and the free air geoid values of N, ξ, and η the transformation Equations (6.10), (6.11), and (6.12) are used. Ideally, the changes $\delta\xi_0$, $\delta\eta_0$ and δN_0 at the origin may be computed from known differences at one other point, and these derived differences may then be used to determine the changes at any other point in the the geodetic network. Since both the astro-geodetic and gravimetric values are subject to errors, an improved solution is obtained from a consideration of all known information. If the problem was to find the best values of $\delta\xi$, $\delta\eta$ and δN at the Origin, the information from each of the eleven trigonometric stations could be used separately in the right hand side of the transformation equations to determine eleven separate assessments of each of the changes. The mean of these values would then represent the changes at the Origin. In this work the problem is to find values of $\delta\xi_0$, $\delta\eta_0$ and δN_0 at the Origin that gives the best fit at the known stations.

To achieve this the values of the unknown changes at the Origin $\delta\xi_0$, $\delta\eta_0$, and δN_0 are substituted for the corresponding values $\delta\xi_1$, $\delta\eta_1$, and δN_1 in the right hand side of Equations (6.10), (6.11), and (6.12) and the values $\delta\xi$, $\delta\eta$, and δN on the left hand side are the known values determined from differences between the corresponding astro-geodetic and gravimetric quantities. Correspondingly, the values of δ_1 and λ_1 refer to the geodetic Origin's coordinates and δ and λ refer to survey station coordinates. Since there are eleven survey stations, the formation of the solution results in 3 unknowns and 33 equations. The solution is then obtained using least square adjustment techniques. Because the values of the semi-major axis of the two reference ellipsoids are identical and for practical purposes the flattening values are the same, all terms containing δa and δf in the transformation equation are set to zero.

For the best fit at the eleven trigonometric stations, the changes at the Origin to N, ξ, and η are respectively

$$\delta N_0 = -5.49 \text{ metres}$$

$$\delta\xi_0 = 4''.29$$

$$\delta\eta_0 = 3''.88$$

with a sign convention defined by

$$\begin{aligned} \delta N &= N_{AGD} - N_G \\ \delta\xi &= \xi_{AGD} - \xi_G \\ \delta\eta &= \eta_{AGD} - \eta_G \end{aligned} \qquad \text{................ (6.13)}$$

where the subscripts AGD refer to the astronomically determined values referenced to the Australian Geodetic Datum, and G refers to the gravimetrically determined values referenced to the GRS 67 system. These orientation parameters will be compared with other previously determined values in the following sub-section.

Table 6.2 shows the residuals after the application of Equation (6.13) at each of the survey stations. The residuals R_N, R_ξ and R_η are determined from

$$R_N = N_{AGD} - (N_G + \delta N) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.14)$$

$$R_\xi = \xi_{AGD} - (\xi_G + \delta\xi) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.15)$$

$$R_\eta = \eta_{AGD} - (\eta_G + \delta\eta) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6.16)$$

Station	Residuals		
	R_N (metres)	R_ξ (seconds)	R_η (seconds)
Johnston	0.07	0.51	0.55
Mt. Gason	-0.41	0.33	1.61
O'Halloran	0.24	1.25	1.62
Mulka	0.19	0.70	1.65
Hawks Nest	-0.63	-0.84	-0.60
Low Cliff	0.28	-0.28	0.28
Lyndhurst	0.45	0.66	-0.39
NME/190	0.74	0.56	1.25
Patawarta	-0.37	0.56	0.17
Black Hill	-0.78	-0.34	0.75
Lock	0.22	1.63	-0.97

TABLE 6.2

Residuals: Astro-geodetic - Gravimetric
(Oriented to the AGD)

The RMS of the residuals R_N, R_ξ and R_η are respectively 0.46 metres, 0".80 and 1".04.

The residuals in both deflection components show a systematic bias whereas the residuals in the separation, mean to zero. No weighting was used in the least square adjustment. If the adjustment was weighted so that the RMS of R_N was 1.0 metres, then the values of the RMS of R_ξ and R_η are reduced to 0".67 and 0".85 respectively, with the mean value of the residuals in each case being zero. Mather (1970) estimated the mean errors in ξ_{AGD} and η_{AGD} to be respectively ±0".50 and ±1".00 which are in close agreement with the values given above for the average difference in results from observations at different times. Using these values and the average standard deviations of ξ_G and η_G (section 5.1), an estimation of the expected error in the results would be respectively ±0".74 and ±1".12, which shows close agreement with the RMS's obtained. The RMS of R_N is less than expected considering the misclosures that exist in astro-levelling loops. Generally, the results show agreement between the two solutions, but it should be noted that the sample is limited, although well dispersed geographically.

6.4 Comparisons with other Gravimetric Solutions

The 1968 and 1970 Mather free air geoid solutions (Sections 5.1 and 5.2) are compared with the present solution. The 1968 solution is inferior and has been superseded by the 1970 but some comparisons are still made. The general results from this work have been presented in a form similar to that shown in Figures 5.4, 5.5 and 5.6, and referenced to both the International Spheroid 1930 with corresponding gravity formulae, and also to GRS-67 system. From a comparison of the figures with the present study, the geoid undulations have the same general shape and gradients, but the 1968 solution has values of N approximately 6 metres higher. When comparing both deflections of the vertical, the values agree to the 1"-2" level in the regions where the gravity field was reasonably defined in 1968. In the other regions the general trends agree, but the numerical values may differ by 8-10". This is particularly so in the north west region of the state where little or no data was available for the 1968 solution.

Details of the solution at the two grid points corresponding to those given in Table 5.2 are given (Mather, 1968.a), but these are referenced to the International Spheroid 1930. In the 1968 solution the grid point 26°00'S and 131°00'E is situated in a comparatively well surveyed gravity area. After conversion to the GRS-67 system, the resulting N values obtained from the 1968 solution are respectively 5.7 and 7.3 metres larger than those presently obtained. The solutions to both components of the deflection of the vertical differ by more than 2" for the grid point 28°00'S and 131°00'E. This is to be expected because the local gravity within one degree of the grid point was obtained in the 1968 solution by the application of interpolation techniques; no gravity data was available in this region. The values for both components ξ and η at the other grid point agree within 0".2, although the contributions from the various zones differ by as much as 1". No definite conclusions can be drawn from these comparisons.

The 1970 solution for Australia was obtained using a Rapp Data Set for the Outer Zone, an improved set of terrestrial data for the inner zones, and compatibility between data sets (Section 5.2). It should be noted that in this solution there were ten 1° x 1° geographical blocks in South Australia in which no observational gravity data was available. In the following paragraphs the results from the present study are compared with those of the 1970 free air geoid solution obtained from Mather (1970), unless stated otherwise.

Three types of comparisons with the 1970 solution can be made. They are a graphical comparison of the value of N, a comparison of orientation parameters to the AGD, and a comparison at individual trigonometric stations.

The 1970 solution is referenced to the GRS-67 system and hence direct graphical comparisons of the value of N can be made. Again, both results have the same general appearance with values of N changing in both cases from approximately -17 metres in the south west to +23 metres in the north east. The main discrepancy occurs in the north west region of the state. This discrepancy and other less significant ones are most likely due to the lack of gravity data in the South Australian region in the 1970 solution.

No graphical comparisons were made for the deflection components since no suitable figures for the 1970 solution are available.

Using the 1970 free air geoid solution, Mather determined a set of orientation which gave the best fit to the AGD. With the same sign convention as defined in Equation (6.13), these parameters referred to the Origin were (shown in Section 3.2 as ΔN_0, $\Delta\xi_0$ and $\Delta\eta_0$ with opposite signs):

$$\delta N_0 = -10.00 \pm 0.2 \text{ metres}$$

$$\delta\xi_0 = 4''.2 \pm 0''.2$$

$$\delta\eta_0 = 4''.5 \pm 0''.2$$

The value of δN_0 does not include any zero degree terms. These values were obtained from a combination of solutions (Section 3.2). Astro-geodetic derived values of N, ξ and η at 693 one degree grid points Australia wide, were compared with the free air geoid values to determined orientation parameters referred to the Origin. Another set of orientation parameters was obtained from comparisons of the same values at 38 trigonometric stations. The values of δN_0, $\delta\xi_0$, and $\delta\eta_0$ as defined above were obtained as a composite of these two methods. The estimation of the errors of these orientation parameters as quoted above are obtained from an average of the mean residuals at each comparison point. The approximate value of the RMS's of the residuals are respectively 2.4 metres, 1''.0 and 1''.8.

The orientation parameters determined in the present study

(Section 6.3), are for the South Australian region and not for the Australian continent, and have been determined from a small number of comparisons. They have primarily been used to orientate the free air geoid results to the AGD for comparisons with astro-geodetic derived values and not define the AGD in an earth centred system.

Comparison of the two sets of parameters shows close agreement in the value of $\delta\xi_0$, reasonable agreement in the value of $\delta\eta_0$ and poor agreement between the two values of δN_0. The disagreement in the values of $\delta\eta_0$ may possibly be explained by the limited number of comparisons used in the present study, but this does not explain the difference in the δN_0 values. Without detailed information of the contributions from each Zone to the free air geoid values computed in 1970, it is difficult to assess the reasons for these discrepancies. One contributing factor is the lack of gravity information in regions of South Australia and Western Australia in the 1970 solution, and the improvements in the Rapp data sets used in the present study. Another contributing factor is the discrepancy in the astro-geodetic determined values of N used in each comparison. The present study uses values of N based on the 1971 geoid solution of Australia (Section 3.4), with N=0 metres at the Origin, while the values in the 1970 solution are based on the study of Fischer and Slutsky (1967). This is demonstrated by a later comparison (Mather et al, 1971) between astro-geodetic values of N obtained from the 1971 geoid solution and the 1970 free air geoid values of N at 1133 stations which resulted in (Section 3.4):

$\delta N_0 \;=\; -8.3$ metres

$\delta \xi_0 \;=\; 4''.0$

$\delta \eta_0 \;=\; 4''.1$

Although the difference between the value of $\delta\xi_0$ in the present study and that quoted above is greater than that in previous comparison, the discrepancies in the remaining orientation parameters are appreciably reduced.

Table 6.3 shows five trigonometric stations which were used both in the 1970 solution and the present study to obtain orientation parameters. Table 4 in Mather (1970) gives the astro-

Stations	1970 Free air values			Residuals		
	N	ξ	η	R'_N	R'_ξ	R'_η
Mt Gason	15.49m	-5".60	-5".28	-0.74m	-0".25	-0".64
Mulka	13.44m	-2".31	-4".88	-1.00m	-0".47	0".20
Low Cliff	6.18m	-4".52	-5".39	0.11m	-0".07	1".02
O'Halloran	7.23m	-1".08	-4".81	0.14m	0".21	1".00
Johnston Origin	4.30m	3".90	-9".40	1.12m	-1".21	0".73

TABLE 6.3

Comparison with the 1970 Free Air Geoid Solution.

geodetic values of N, ξ and η used, and the residual between these values and the equivalent free air geoid values oriented with respect to the AGD. From these results the 1970 free air geoid values of N, ξ and η are computed using the Mather orientation parameters and Equations (6.10), (6.11) and (6.12). These results and their residuals R'_N, R'_ξ and R'_η when compared to the free air geoid values (Table 6.1), are given in Table 6.3. The residuals in the deflections of the vertical are generally small and show close agreement in locations where the gravity field was surveyed in 1970. The larger discrepancies at Johnston Origin and O'Halloran are explained in the 1970 solution by the lack of gravity data in the western region of the state and the lack of detailed gravity information in the rapidly changing field of the north western region of the state.

The values of R'_N do tend to show a systematic difference in the slope of the geoid, but the available sample is small and the results again are not conclusive.

6.5 Comparisons with Doppler Results

During the period 1957-1977, a Doppler satellite survey of junction points in AGD and other survey stations was carried out in Australia. The majority of points were surveyed using JMR-1 satellite survey receivers and the observations were reduced in terms of the precise ephemeris NWL-9D provided by the U.S. Defence Mapping Agency Topographic Command. These JMR-1 results were reduced to produce X, Y, Z coordinates of the point positions of the stations referenced to the NWL-9D datum (Leppert, 1978) and indirectly to the NWL-10E gravity model which was used to determine the precise ephemeris until June 15th, 1977 (Hotham, 1979).

The empheris used determines the scale and orientation of the Doppler coordinate system. The system of coordinates should be geocentric although Lachapelle and Kouba (1981) have recorded an incompatibility of 4 metres in the Z axis when comparing Doppler derived geoid undulations with those obtained from GEM, SAO and SE earth models at 290 globally balanced Doppler stations. The Z axis is orientated to the CIO, and the longitude origin, although arbitrary in theory, has been aligned as nearly as practically possible to the zero longitude as defined by BIH. The scale of the system is defined through the use of a particular value of GM (Strange & Hothem, 1980).

Tests on the NWL-9D Doppler system of coordinates by various authors (e.g. Hothem, 1979; Strange & Hothem, 1980) indicate a scale decrease of 0.4 ppm and a longitude rotation of 0".8 eastward should be applied. The Z axis corresponds to the CIO within 0".05 (Hothem, 1979).

In the present study the a value of the geoid separation N_D is determined at the nine trigonometric stations which have been surveyed using Doppler techniques. Knowing the orthometric height H and the ellipsoidal height h, the value of N_D is determined from (Section 2.3.2):

$$N_D = h - H \qquad \text{................} \quad (6.17)$$

The value of h along with the values of σ and λ are determined from Equations (6.4), (6.5) and (6.6) and are referenced to the GRS-67 system. This permits direct comparison with the free air geoid derived results.

Station	N_D metres	(N_D-N) metres
Johnston	-16.512	-21.932
T1/523	- 3.561	-20.991
T1/530	- 6.575	-20.245
Hawks Nest	-18.702	-21.832
Low Cliff	-15.599	-21.889
Lyndhurst	-10.980	-21.620
NME/190	-35.554	-22.104
Black Hill	-30.064	-20.604
Lock	-15.374	-22.054

TABLE 6.4

Comparisons with Doppler Results.

After applying the scale decrease of -0.4 ppm the Doppler derived values of N_D and the difference between these values and the corresponding free air geoid values $(N_D - N)$ are shown in Table 6.4. The mean value of the difference is 21.475 metres with a RMS of 0.60 metres about this mean. Considering that spheroidal heights derived from Doppler point positioning methods are estimated to have a precision of the order of 1-2 metres, the results are in close agreement. The mean difference may be considered as a measure of the N_0 term in Equation (2.15). This suggests a value of the semi-major axis of the mean earth ellipsoid is 6378138.5 metres, which is in close agreement with that adopted in the GRS-80 system of 6378137 metres. The derivation of this value is not an attempt to confirm or repudiate an accepted value of the semi-major axis but rather it is used as a check on systematic errors in the gravimetric solution.

At seven of the nine Doppler stations where astronomically determined values of latitude and longitude are available, the free air geoid values of the deflections of the vertical are compared with those deduced from the Doppler values using Equations (2.1a) and (2.1b). The comparison of the results shows a mean difference (astro-Doppler minus free air values) of -0".42 and 0".23 in ξ and η respectively, with RMS's about the mean of 0".79 and 0".72. The values are obtained after applying the longitude correction of 0".8 to the Doppler derived values. This small sample shows no significant systematic differences between the results which indicates the free air geoid values of ξ and η are not affected to the order of 0".5 by systematic error contributions from the outer zones.

6.6 GEM 10B Comparisons

The concept of the use or part use of spherical harmonics to determine the value of the geoid separation has been discussed in Sections 2.3.1 and 2.4.2. A computer program developed by C. Rizos is used to compute the values of N as defined in Equation (2.3) and the contributions to N from the region $\psi > 5°$ using Molodenskii's truncation functions (Section 2.4.2) at the one degree grid points in the study region. This program, which uses an efficient computer technique for the evaluation of the geopotential from spherical harmonic models (Rizos, 1979), permits the selection of the gravitational and the geometrical models to which the derived values of N are referenced. The parameters as defined by the GRS-67 system are used, thus allowing direct comparison between the results obtained and those computed for the free air geoid.

The normalised spherical coefficients used are those defined by the Goddard Earth Model 10B (GEM-10B). This model was obtained from the satellite and terrestrial data of GEM-10 which is defined by spherical harmonics to degree 22 and order 22, with selected higher degree terms, and has been extended and strengthened through the use of GEOS-3 altimetry (Marsh et al, 1980). The GEM-10B spherical coefficients are defined to degree 36 and order 36. The GEM-10B solution is a normal least square adjustment with no constraints. Since collocation techniques were not applied the model reflects the full sensitivity of orbital, altimeter and surface data.

Marsh et al (1980), have performed an assortment of tests to assess GEM-10B. Using 348 altimeter crossover points, GEM-10B was found to give a RMS of one metre for the radial positions for GEOS-3 which was significantly more accurate than GEM-10. Seasat and Skylab altimetry, both of which were not used in the determination of the spherical coefficients, were used as a completely independent check on the results. A Seasat derived geoid profile agreed to ±1 metre with that obtained from GEM-10B and the disagreements were thought to be due to high frequency geoid features.

The Skylab altimetry for one "round the world" pass was checked against most existing earth models and the GEM-10B derived values were found to have the least residuals with a RMS of 2.3 metres. Thus, from the results of these tests, the GEM-10B is an improvement on previous GEM models.

The contributions to N from the region $\psi > 5°$ were computed at 110 one degree geographical grid points using the GEM-10B coefficients and Molodenskii's truncation functions. These contributions to N should be equivalent to the combined contributions to the free air geoid from the Outer and Middle Zones after correcting for the atmospheric effects (Section 5.2.3). An atmospheric correction of 0.56 metres was added to the values derived from the GEM-10B coefficients. This value is derived using the simple atmospheric model and assuming a mean terrain elevation of zero. The actual mean height of the terrain will be larger than this but the effect on the atmospheric correction is negligible. As an example, for a mean terrain height of 400 metres, the correction becomes 0.54 (Rapp and Rummel, 1975).

After applying the atmospheric correction, the contributions were equated by subtracting the GEM-10B derived contribution from the free air value at each grid point to obtain the difference. The mean value of this difference is +0.50 metres with a RMS about this mean of 0.61 metres. The values of the differences are not random in nature but location dependent. The lower values generally occur between longitudes 135° and 139° East for all latitudes. This is probably due to the over smoothing in the GEM-10B coefficients of the rapidly changing gravity in the north west of the state. It must be remembered the earth model GEM-10B has coefficients to degree 36 and order 36 which will not reflect high frequency changes in the local gravity field.

These results demonstrate close agreement between the two solutions, particularly when the random error of approximately ±1.2 metres in the free air geoid contribution to N is considered.

The same 110 one degree geographical grid points were used to compare the free air geoid values of N and the corresponding values derived from the GEM-10B spherical coefficients. No atmospheric corrections are required since the evaluations are from $\psi = 0°$ to 180° and combined solutions are not used. The differences were computed in the same manner as above and the mean value of the differences of +0.49 metres with a RMS about this mean of 1.85 metres. The higher RMS value obtained shows the inability of the spherical harmonic solution to reflect localised rapid variations in the gravity field. The largest difference of 5.37 metres occurs in the north west corner of the state, and this difference reduces to 0.81 metres at the next one degree grid point south. This is an extreme case. Generally the rate of change of the differences is less than 1 meter per one degree latitude or longitude.

The consistent mean difference of 0.5 metres in both comparisons may suggest a systematic error in the representation of the Middle or Outer Zones by the Rapp 1° and 5° data, but it may also reflect a lack of detail in the GEM-10B derived solution. Without further evidence the systematic difference is too small to draw this type of conclusion. It does however confirm the free air geoid results obtained for the contributions to N from the Outer and Middle Zones within the estimated error tolerances.

CHAPTER 7

SUMMARY, CONCLUSION, RECOMMENDATION

7.1 The Free Air Geoid

The free air geoid of South Australia referenced to GRS-67 has been derived for each half degree geographical grid point using Vening Meinesz formulae for the deflections of the vertical and Stokes' Integral for the geoid separation. The results demonstrated in Figure 5.4 show a geoid separation of -17 metres in the south west, with a generally smooth gradient rising to +23 metres in the north east of the state. The influence of the rapidly changing gravity field in the north west and the relative high anomalies in the Flinders Ranges are indicated by the rate of change and direction of the geoid gradient in these locations.

The effect of these two gravity regions is more pronounced in Figures 5.5 and 5.6. Figure 5.5 shows the values of the deflection component ξ which changes approximately 20" in the north west region but does not show a rapid change in the Flinders Ranges region. This is to be expected because this mountainous region lies approximately north-south and will have little effect on this component. Figure 5.6 shows the values of the deflection component η which demonstrates the gravity change in both regions.

The values of ξ and η shown in these figures are compiled without the contributions from the gravity field of the four Innermost 0.°1 x 0.°1 geographical blocks which are centred on each half degree grid point. If the contribution were included it is expected that the changes would be more variable than demonstrated in the diagrams (see Section 5.1.6 and Table 6.1).

The values of ξ, η, and N were evaluated using combined terrestrial and satellite data to represent the mean gravity anomalies in the Outer and Middle Zones and terrestrial data obtained from the B.M.R. was used to represent the mean anomalies in the other Zones. The effects of systematic errors in the Rapp data sets were estimated to have a maximum effect of 1-2 metres on N and a possible effect of 1" on each of the deflection terms. The systematic errors are expected to have little, if any, effect on the relative changes from one grid point to the next. These systematic effects are demonstrated by the comparisons made with other methods of determining the geoid in Chapter 6. The comparisons with the Doppler derived values of N show no position dependent systematic differences. The limited comparison of combined Doppler-astronomical deflections of the vertical with the corresponding free air geoid values shows no systematic effects to the order of ±0".5 (Section 6.5). The GEM-10B solutions for the value of N and for the contributions from the Outer and Middle Zones give a systematic difference in both cases of approximately 0.5 metres but no position dependent variation can be verified because of the "smoother" gravity field inferred by the GEM-10B spherical harmonic coefficients. If systematic errors exist in the Rapp data their effects are much smaller than the original estimations quoted above.

The terrestrial data obtained from the BMR has sufficient density to determine the contributions to N from all inner zones without the use of extended interpolation or extrapolation techniques, except for a small region in the southern section of the Near Zone. In this region Rapp 1° data was used in preference to extrapolation methods. For the contributions to the deflections of the vertical an added problem occurs in the innermost four 0.°1 x 0.°1 blocks. Generally the data available is insufficient to enable the determination of the contribution. Where this information is required it may be obtained by additional gravity field surveys about the computation point (Section 6.1), or by interpolation methods using the correlation of the free air anomaly with height (Section 4.4.1). This latter method requires terrain height information which is not generally available for the majority of South Australia at the present time.

The mean anomalies for the Inner and Innermost Zones were computed as the mean of all the point free air gravity anomalies situated within each block or compartment. In the Near Zone, because of the larger compartment size (0.°5 x 0.°5) and the possible uneven distribution of data, the mean anomaly was determined from the mean of the four 0.°25 x 0.°25 mean anomalies which in turn were determined in a similar way to those in the Inner and Innermost Zones. Where no values existed in the 0.°1 x 0.°1 blocks they were interpolated using the weighted mean method and the surrounding mean values (Section 4.5).

The block sizes within each zone other than the Innermost Zone, have been selected so that any effects derived from the assumption that the mean value of Stokes' and Vening Meinesz functions for a block are those referred to the geographical centre, are negligible. This has been verified by computing the contribution to N, ξ, and η at the grid point 26°00'S and 129°00'E using the mean values of the functions referenced to 100 uniformly distributed points within each compartment. The results when compared to values obtained when the functions were referenced to the geographical centre, showed discrepancies of less than 0.1 metres in N and 0".03 in both ξ and η. Both the functions for the Innermost Zone have been computed as the mean of the values of the 100 uniformly distributed points within each block.

The average standard deviations of the computed free air values of N, ξ, and η at each grid point are ±1.25 metres, ±0".55 and ±0".50 respectively. These standard deviations were computed at each half degree grid but the range in values was insignificant being less than ±0.1 metres and ±0".1 for the standard deviations in N and both deflection components ξ and η. These standard deviations were obtained by summation of the effects of the standard deviations μ_{g_i} of each mean free air gravity anomaly.

The values of μ_{g_i} for the Rapp data were supplied with the data sets and have been discussed in Section 5.4. The error of representation was used to estimate the value μ_{g_i} in the Near Zone and a value of ±5 mGal was assumed for μ_{g_i} in the Inner and Innermost Zones. The adoption of this latter value is justified on the values of the error of representation obtained for five randomly selected 0.°1 x 0.°1 blocks which contained eight or more gravity stations, on the comparison of the contributions to the standard deviation of the deflections of the vertical from this region with the corresponding results obtained by Kearsley (1976), and for other reasons discussed in Section 5.1.5.

The standard deviations of N, ξ, and η derived in this manner are an estimation of the random errors and not the systematic errors. The values are obtained assuming the only error source to be the mean gravity anomaly and that no correlation of errors exists between the mean anomalies. Random errors in the mean anomalies include the effects of random errors in the latitude and the height of the individual gravity stations, and the random reading error of the gravity meter. Regional systematic errors in the height and gravity datums are not included. In the BMR data the local regional systematic errors are estimated to have an effect on N of less than ±0.2 metres with a half wavelength of 3.5×10^3 km in the adverse case when the individual errors are cumulative. The effect on the deflections of the vertical is negligible in the context of accuracy obtainable at this stage (Section 5.3).

The major contribution to the standard deviation N comes from the Outer Zone and is approximately ±1.1 metres. Any major improvement in the precision of the free air geoid determined value of N for a region such as South Australia situated in a comparatively well surveyed area, will be dependent on a better definition of the Outer Zone gravity field. In contrast to this, the major contribution of approximately ±0".5 to both ξ and η comes from the Inner and Innermost Zones. These figures could be an over-estimation if the value of μ_{g_i} is less than the estimated value of ±5 mGal.

The values of the standard deviations of N, ξ, and η obtained as an estimation of the random errors in the corresponding results, are less than those derived by some authors e.g. Idhe (1981), Paul and Nagy (1973), but of a similar value to those estimated by others, e.g. Olliver (1980), Wenzel (1979).

The approach used by Paul and Nagy to obtain an estimation of the standard deviation of N is the same as that used in this study, but the results differ because they used standard values of ±5, ±10, ±20 mGal respectively, to represent the standard deviations of the mean gravity anomaly in surveyed, partially surveyed and unsurveyed regions. The study of Idhe (1981) used covariance functions estimated by Tscherning and Rapp (1974), to estimate variances of the mean gravity anomalies. The results obtained depend on the density of measurement with each compartment. The discrepancy in results arises because the densities used were derived from the number of known 1° x 1° mean gravity anomalies in each 5° equal area block instead of the density of the original gravity data used to obtain the 1° x 1° mean gravity anomalies.

Wenzel (1979) computed the effects of the random errors on N from the region $\psi = 0$ to $\psi = 0$ to 25° with the computation points situated in the North Sea. The results varied from ±0.4 metres to ±0.8 metres depending on the density of gravity data. These results agree with the value of ±0.6 metres obtained in the present study for the combined contribution from the Middle, Near, Inner and Innermost Zones. A further confirmation of the results obtained in the present study is seen in Olliver (1980), in which a precision of ±1 metre is estimated for the gravimetric derived value of N after comparisons with astro-geodetic derived values.

7.2 Comparisons with other Geoids

Comparisons have been made with previous gravimetric solutions by Mather, astro-geodetic and Doppler derived values, and results obtained using GEM-10B spherical harmonic coefficients. These comparisons have been made to evaluate any possible systematic errors that may exist in any of the solutions and to verify the standard deviation determined in the free air geoid solution. In order to facilitate some of these comparisons, detailed gravimetric surveys were carried out in the immediate vicinity of thirteen trigonometric stations to enable the contribution to N, ξ, and η from the Innermost Zone to be fully evaluated (Section 6.1).

Using the astro-geodetic data available at eleven of the thirteen trigonometric stations and the free air values of N, ξ, and η, orientation parameters between the corresponding reference surfaces were determined using the principles of least squares. The values at the Johnston Origin are:

$$\delta N_0 = -5.49 \text{ metres}$$

$$\delta\xi_0 = 4''.29$$

$$\delta\eta_0 = 3''.88$$

and the sign convention is defined by equation (6.13). On comparison with the equivalent values determined by Mather (1970) for the orientation of the 1970 free air geoid with the AGD, the parameter $\delta\xi_0$ is in close agreement and the parameter $\delta\eta_0$ is in reasonable agreement, but the value of δN_0 differs by 4.5 metres. This discrepancy is thought to be due partly to the lack of a gravity data in large areas in South Australia and Western Australia in the 1970 solution. This is further reinforced by a comparison of free air values of N, ξ, and η at five trigonometric stations which are common to both studies (see Table 6.3). The other reason for the discrepancies is caused by the different astro-geodetic geoid used. In the case of the Mather study, the astro-geodetic data was obtained from the Fischer & Slutsky (1967) determination whereas the present study is based on the 1971 geoid (Section 3.4) with N = 0 metres at the Johnston Origin.

On orientation of the free air values of N, ξ, and η to the AGD, the comparisons with the equivalent astro-geodetic values gave RMS's of the residuals in the respective values of 0.46 metres, 0".80 and 1".14. The residuals in both deflection components show a bias whereas the residuals in the separation mean to zero.

This bias in the deflection components is eliminated if, by the use of weights, the RMS of the residual in the value of N is permitted to be 1.0 metres. This reduces the RMS of the residuals in ξ and η respectively to 0".67 and 0".85. The sample used to determine these orientation parameters is relatively small, although well dispersed geographically. The parameters are used as a means of comparing the values of N, ξ, and η referenced to two different geodetic datums. They are not meant as new sets of geocentric orientation parameters for the AGD.

At nine trigonometric stations where Doppler data was available, the geodetic coordinates (ϕ, λ, h) referenced to the GRS-67 system were derived from Doppler X, Y and Z coordinates. These cartesian coordinates were referenced to the precise ephemeris and obtained using point positioning Doppler techniques. The geoid separation N_D was then obtained using Equation (6.17) in which the orthometric height was assumed to be the height of the station referenced to the AHD. The mean difference between N_D and the corresponding free air geoid value at the nine trigonometric stations was 21.48 metres, with a RMS of 0.60 metres about the mean. Considering the precision of the Doppler derived spheroidal heights and the AHD, the results are compatible. The systematic difference of approximately 21.5 metres may be considered as a measure of the zero order effect on the free air geoid value of N. This suggests a value of 6 378138.5 metres for the semi-major axis of the mean earth ellipsoid which is in close agreement to the currently acceptable value, thus serving as a check on the presence of any unknown systematics. At seven of the nine Doppler geodetic coordinates, where astronomical values of latitude and longitude were available, the free air geoid values of the deflection components were compared to those derived from the Doppler geodetic coordinates. The results showed a mean difference of -0".42 in ξ and 0".23 in η with RMS's about these means of 0".79 and 0".72 respectively. Considering the size of the sample, these results show no significant systematic contributions (less than 0".5) from the gravity field in the Outer Zones.

The comparisons with results using GEM-10B spherical coefficients has been summarised in subsection 1 of this Chapter.

7.3 General Comments

From the comparisons made with other determinations, the free air geoid values of N, ξ, and η show no detectable position dependent systematic errors. Although the detailed comparisons were made at a relatively few well distributed points, the number and variety of comparisons verify the relativity of the results to within the range inferred by the derived standard deviations. Systematic errors that do exist, such as the zero order effects on N, have no effect on relative changes between computation points.

Previous studies (Chapter 3) have detected positional dependent errors when comparing gravimetric values to astro-geodetic solutions in Australia. These results were all obtained using a University of New South Wales data set (UNSW set) of gravity anomalies which was originally compiled in the late 1960's and a slightly modified version of 5° x 5° equal area mean anomalies' set compiled by Rapp in 1968 (Clarke, 1981). The orientation parameters that were used to align the GRS-67 to the AGD were those derived by Mather (1970). The present study uses much improved gravity data sets to represent both the Inner and Outer Zones and a set of orientation parameters derived from comparisons of data obtained within the region studied.

Improved results in the value of N can only be obtained by an improvement in the gravity data in the Outer Zones. The gravity coverage in the Inner Zones is satisfactory. When comparing the gravimetric determined values of N with values determined by other methods in order to validate the results, the main difficulty that is encountered is the accuracy of the other methods. In Australia the definition of the height datum and its relationship to the geoid needs improvement. This is particularly apparent when deriving Doppler values of N from Equation (6.17). Assuming no error exists in the Doppler spheroidal height h (which is actually ±1-2 metres), the error in the results is still dependent on the error of the levelled height of the station. An anomaly occurs in the definition of the value of N at the Johnston Origin. If the levelled height (566.30) is assumed to be the orthometric height and the spheroid height is 571.2 metres (by definition), then N is equal to 4.9 metres. This is the value adopted by Allman and Steed (1980) when determining GMA 80 (Geodetic Model of Australia, 1980). If any other value is used, such as -6.0 metres, as adopted in the 1971 Australian Geoid or 0.0 metres, which is generally applied, then there is a conflict in the definition of terms.

The deflections of the vertical have proven precisions at least compatible with those obtained by astro-geodetic techniques. The limited comparisons with Doppler derived values show that any systematic errors contributed from the gravity field definition in the Outer Zones is less than $0''.5$. The relative changes between computation points within the study region would be much less than this. The terrain corrections have not been applied but these are expected to be small, particularly at points where the topography is symmetrical about the computation point. Although the gravity field in the Inner Zones is reasonably dense, additional gravity data is generally required about the computation point to adequately define the contribution from the Innermost Zone. Ignoring travelling time, this field work can be completed in less than one day.

7.4 Future Developments and Needs

This study has shown that with the existing gravity data available, the geoid can be defined by gravimetric methods as well as, if not better than, any other method in current use. Thus, whenever the values of N, ξ, or η, are required for geodetic purposes, it is feasible, if not desirable, to determine these by gravimetric methods.

There is a need for more detailed terrain information in South Australia; ideally this information could be stored as mean elevations of 0.°1 x 0.°1 blocks in a computer based data bank. This would permit the present gravity coverage to be interpolated more precisely and hence reduce the need for additional field operations. It would also permit simple calculations of the terrain corrections for both geodetic and geophysical exploration purposes. Many other applied scientific disciplines would also benefit from the use of such a data bank. For much of the state, this height information is not available in any form since topographic maps of scales larger than 1:250 000 do not exist for much of the state, and many of the 1:250 000 topographic maps do not have height information. As this mapping situation is being rectified, it would be a relatively simple matter for the appropriate Government Agencies to record on magnetic tape the appropriate height information.

With the increasing use of satellite positioning techniques, there is a need to obtain detailed information on the geoid undulation so that the derived spheroidal heights can be converted to orthometric heights. Using the methods derived in this study and additional comparison points, it may be possible to produce values of N on a tenth degree grid covering South Australia that are suitable for converting spheroidal heights referenced to the AGD to orthometric heights referenced to the AHD. It is anticipated that these derived heights would have a precision of ±1-2 metres, which would be satisfactory for 1:50,000 mapping, the detection of major blunders in levelling networks, and determination of height datums for many geophysical purposes.

It would be desirable to extend this study to the whole of the Australian continent. The information would enable a new set of orientation parameters to be determined between the GRS-80 and the AGD, detect the existence of any position dependent errors in N, ξ, or η in the much enlarged region of Australia, and permit the determination of orthometric heights from Doppler coordinates as described above.

A knowledge of the deflections of the vertical is becoming less significant with the introduction of space age technology. As an example, the use of Laplace's equation to control the azimuth in large geodetic networks (and hence the need for ξ and η) may be replaced by Doppler positioning methods or other satellite methods currently being developed. Though it must be remembered that any surveying work using conventional instruments such as theodolites, are aligned to the local potential surface and a knowledge of the deflections of the vertical is required to obtain the orientations between this system and the geodetic reference system (perhaps defined by satellite methods) being used.

The problem today is somewhat the reverse to that encountered using classical geodetic techniques. Using modern technology it is becoming easier to define the geodetic coordinates of a point, but more difficult to define the natural coordinates, i.e. ϕ, Λ, H, referenced to the geoid. A major requirement now, and in the future, is for an increasingly detailed knowledge of the geoid undulation to allow a simple transfer of data between the geoid and any defined geodetic reference surface.

BIBLIOGRAPHY

ANDERSON, E.G., RIZOS, C. and MATHER, R.S., 1975. *Atmospheric Effects in Physical Geodesy,* Unisurv G. No. 23, School of Surveying, U.N.S.W., Australia, 1975.

ANFILOFF, W., BARLOW, B.C., MURRAY, A.S., DENHAM, D. and SANDFORD, R., 1976. *Compilation and Production of 1976 1:5 000 000 Gravity Map of Australia.* BMR J. Aust. Geol. Geophys. 1(4), 1976.

ALLMAN, J.S. and STEED, J.B., 1980. *Geodetic Model of Australia 1980.* Technical Report 29, Nat. Map., Canberra, Australia, 1980.

ASHKENAZI, V. and SYKES, R.M., 1978. *Precise and Broadcast Ephemerides Determination over the U.K. and Europe.* 2nd International Symposium on the Use of Artificial Satellites in Geodesy and Geodynamics, Athens, June, 1978.

BOMFORD, A.G., 1963. *Woomera Geoid Surveys.* Technical Report No. 3, Nat. Map., Canberra, Australia, 1963.

BOMFORD, A.G., COOK, D.P., and McCLOY, F.J., 1970. *Astronomical Observations in the Division of National Mapping 1966-70.* Technical Report 10, Nat. Map., Canberra, Australia, 1970.

BOULANGER, Y.D., SHCHEGLOV, S.N., WELLMAN, P., COUTTS, D.A., and BARLOW, B.C., 1973. *Soviet-Australian Gravity Survey Along the Australian Calibration Line.* Bull. Geod., Vol. 110, pp. 355-66, 1973.

BROWN, D.C., 1976. *Doppler Surveying with JMR-2 Receiver.* Bull. Geod., Vol. 50, No. 1, 1976.

CLARKE, F.L., 1978. *Three Dimensional Geodetic Network Adjustment Incorporating Gravimetry.* Unisurv G. No. 29, School of Surveying, U.N.S.W., Australia, 1978.

CLARKE, F.L., 1981. *Modelling Position Dependent Errors in Gravimetric Deflections of the Vertical.* Bull. Geod., Vol. 55, No. 1, 1981.

DOOLEY, J.C., and BARLOW, B.C., 1976. *Gravimetry in Australia, 1819-1976.* BMR J. Aust. Geol. Geophys. 1(4), 1976.

FISCHER, I. and SLUTSKY, M., 1967. *A Preliminary Geoid Chart of Australia.* Aust. Surv., December 1967.

FRYER, J.G., 1970. *The Effect of the Geoid on the Australian Datum.* Unisurv. 20, School of Surveying, U.N.S.W., Australia, 1970.

FRYER, J.G., 1971. *The Geoid in Australia - 1971.* Technical Report No. 13, Nat. Map., Canberra, Australia, 1971.

GILLILAND, J.R., 1978. *Correlation of Free Air Anomaly with Height.* Unisurv G.29, School of Surveying, U.N.S.W., Australia, 1978.

GILLILAND, J.R., 1981. *The Outer, Middle, and Near Zone Effects on the Free Air Geoid of South Australia.* Aust. J. Geod. Photo. Surv., No. 34, June 1981.

GILLILAND, J.R., 1982. *The Free Air Geoid of South Australia.* Aust. J. Geod. Photo. Surv., No. 36, June 1982.

GRS-67, 1971. *Geodetic Reference System - 1967.* Special Publication No. 3, International Association of Geodesy, 1971.

GRS-80, 1980. *Geodetic Reference System - 1980.* Geodesists Handbook, Bull. Geod., Vol. 54, No. 3, 1980.

GRUSHINSKY, N.P. and SAZHINA, N.B., 1971. *The Gravitational Field and the Geoid of Australia.* Journ. Geol. Soc. Aust., Vol. 18, November 1971.

HEISKANEN, W. and MORITZ, H., 1967. *Physical Geodesy.* Pub. W.H. Freeman, San Francisco, 1967.

HOTHEM, L.D., 1979. *Determination of Accuracy, Orientation and Scale of Satellite Doppler Point Positioning Coordinates.* Proceedings of the Second International Geodetic Symposium on Satellite Doppler Positioning, Austin, U.S.A., 1979.

KASSIM, F.A., 1980. *An evaluation of three techniques for the prediction of gravity anomalies in Canada.* Department of Surveying Engineering Report No. 73, U.N.B.,Fredericton, Canada, 1980.

KAULA, W.M., 1966. *Tests and Combinations of Satellite Determinations of the Gravity Field with Gravimetry.* Journal of Geophysical Research, 71, 5303-5314, 1966.

KEARSLEY, A.H.W., 1976. *The Computation of Deflections of the Vertical from Gravity Anomalies.* Unisurv No. S.15, School of Surveying, U.N.S.W., Australia, 1976.

KEARSLEY, A.H.W., 1977. *Non-stationary Estimation of Gravity Prediction Problems.* Dep. of Geo. Sci. Rep. No. 208, The Ohio State University, Columbus, U.S.A., 1977.

KEARSLEY, A.H.W. and VAN GYSEN, H., 1979. *Outer Zone Effects on the Australian Gravimetric Geoid.* Aust. J. Geod. Photo. Surv. No. 31, December 1979.

LACHAPELLE, G., 1975. *Determination of the geoid using heterogeneous data.* Mitteilungen der geodatischen Institute der Technischen, Universitat Graz, 1975.

LACHAPELLE, G., 1976. *Physical Geodesy at the Geodetic Survey.* Presented at the Symposium on Solid Earth Geophysics in the Americas, Ottawa, Canada, 1976.

LACHAPELLE, G., 1978. *Estimation of the Geoid and Deflection Components in Canada.* Presented at the Second International Symposium on Problems related to the Redefinition of North American Geodetic Networks, Washington, D.C., 1978.

LACHAPELLE, G. and KOUBA, J., 1981. *Relationship between Terrestrial and Satellite Doppler Systems.* Reference Coordinate Systems for Earth Dynamics, pp. 195-203. Pub. D. Reidal Publishing Company, 1981.

LEPPERT, K., 1978. *The Australian Doppler Satellite Survey 1975-77.* Technical Report 21, Nat. Map., Canberra, Australia, 1978.

LERCH, F.J., WAGNER, C.A. and KLOSKO, S.M., 1980. *Goddard Earth Models for Oceanographic Applications (GEM-10B and 10C).* Presented at the Marine Geodesy Symposium, University of Miami, RSMAS, Florida, U.S.A., 1980.

MARSH, G.M. and CHANG, E.S., 1976. *Global detailed gravimetric geoid.* Presented at the Fall Annual Meeting of the American Geophysical Union, San Francisco, California, 1976.

MATHER, R.S., 1968.a. *The Free Air Geoid in South Australia and its Relation to the Equipotential Surfaces of the Earth's Gravitational Field.* Unisurv No. 6, School of Surveying, U.N.S.W., Australia, 1968.

MATHER, R.S., 1968.b. *The non regularised geoid and its relationship to the telluroid and regularised geoids.* Unisurv No. 11, School of Surveying, U.N.S.W., Australia, 1968.

MATHER, R.S., 1969. *The Free Air Geoid for Australia from Gravity Data Available in 1968.* Unisurv No. 13, School of Surveying, U.N.S.W., Australia, 1969.

MATHER, R.S., 1970. *The Australian Geodetic Datum in Earth Space.* Unisurv No. 19, School of Surveying, U.N.S.W., Australia 1970.

MATHER, R.S., 1972. *The 1971 Geoid for Australia and its Significance in Global Geodesy.* Journ. Geol. Soc. Aust., Vol. 19, July 1972.

MATHER, R.S., 1973.a. *A solution of the geodetic boundary value problem to order e^3.* Report X-592-73-11, Goddard Space Flight Centre, Greenbelt, Md, 1973.

MATHER, R.S., 1973.b. *Position from Gravity.* Report X-592-73-164 Goddard Space Flight Centre, Greenbelt, Md, 1973.

MATHER, R.S., 1975. *Gravimetric Investigations on the North American Datum, 1972-1973.* Report X-921-75-244, Goddard Space Flight Centre, Greenbelt, Md, 1975.

MATHER, R.S. and FRYER, J.G., 1970.a. *Orientation of the Australian Geodetic Datum.* Aust. Surv., March 1970.

MATHER, R.S. and FRYER, J.G., 1970.b. *Geoidal Studies in Australia.* Surv. Rev. No. 156, 1970.

MATHER, R.S., BARLOW, B.C. and FRYER, J.G., 1971. *A Study of the Earth's Gravitational Field in the Australian Region.* Unisurv. No. 22, School of Surveying, U.N.S.W., Australia, 1971.

MATHER, R.S., RIZOS, C., HIRSCH, B., and BARLOW, B.C., 1976. *An Australian Gravity Data Bank for Sea Surface Topography Determination (AUSGAD 76).* Unisurv. G.25, School of Surveying, U.N.S.W., Australia, 1976.

MOLODENSKII, M.S., EVEMEER, V., and YURKINA, M.I., 1962. *Methods for study of the external Gravitational Field and Figure of the Earth.* OTS.61-31207, translated from Russian, Israel Programme for Scientific Translations, Jerusalem, 1962.

MORITZ, H., 1963. *Interpolation and Prediction of Point Gravity Anomalies.* Ann Acad. Scient. Fenn. Ser. A111, Geologica, 69, 1963.

MORITZ, H., 1966. *Linear Solutions of the Geodetic Boundary Value Problem.* Dept. of Geod. Sci. Rep. No. 79, Ohio State University, Columbus, 1966.

MORITZ, H., 1968. *On the Use of the Terrain Correction in Solving Molodenskii's Problem.* Dept. of Geod. Sci. Rep. No. 108, Ohio State University, Columbus, 1968.

MORITZ, H., 1975. *Integral Formulae and Collocation.* Air Force Cambridge Research Laboratories, Massachusetts AFCRL-TR-75-0628, 1975.

OLLIVER, J.G., 1979. *A Study of the Geoid in Great Britain and Ireland.* D. Phil. Thesis, 1979, Oxford University, U.K.

OLLIVER, J.G., 1980. *The Gravimetric Geoid of Great Britain and Ireland.* Geophysical Journal Royal Astronomical Society No. 63, 1980.

PAUL, M.K. and NAGY, D., 1973. *The Accuracy of Geoidal Heights Obtainable from Gravity Data Alone.* Canadian Surveyor, Vol. 27, No. 2, 1973.

PELLINEN, L.P., 1964. *Expedient Formulae for Computing the Plumb-line Deflections in Mountainous Areas.* Bull. Geod. Vol. 89, pp. 345-354, 1964.

RAPP, R.H., 1964. *The Prediction of Point and Mean Gravity Anomalies through the use of the Digital Computer.* Dept. of Geod. Sci., Rep. No. 43, Ohio State University, Columbus, 1964.

RAPP, R.H., 1977. *Potential Coefficient Determinations from 5° Terrestrial Gravity Data.* Dept. of Geod. Sci., Rep. No. 251, Ohio State University, Columbus, 1977.

RAPP, R.H., 1977. *Incomplete Final Results for Sea Surface Topography and Gravity Anomalies from Geos-3 Altimeter Data.* Presented at Geos-3 Conference 1977, New Orleans, Louisiana.

RAPP, R.H., 1978.a. *Gravity Anomaly and Geoid Undulation - Results in Local Areas from Geos-3 Altimeter Data.* Presented at the 1978 Spring Meeting of the American Geoph. Union, Miami Beach, Florida.

RAPP, R.H., 1978.b. *A Combined Terrestrial-Altimeter 1° x 1° Mean Gravity Anomaly Field.* Presented at XVIIIth Meeting of IAG, Paris, 1978.

RAPP, R.H. and RUMMEL, R., 1975. *Methods for the computation of detailed geoids and their accuracy.* Dept. of Geod. Sci. Rep. No. 233, Ohio State University, Columbus, 1975.

RIZOS, C., 1979. *An Efficient Computer Technique for the Evaluation of Geopotential from Spherical Harmonic Models.* Aust. J. of Geod. Photo. and Surv., No. 31, December, 1979.

RIZOS, C., 1980. *The Role of the Gravity Field in Sea Surface Topography Studies.* PhD Thesis, School of Surveying, U.N.S.W., Australia, 1980.

RUMMEL, R., and RAPP, R.H., 1977. *Undulation and Anomaly Estimation Using GEOS-3 Altimeter Data Without Precise Orbits.* Bull. Geod. Vol. 51, 1977.

S.A. YEAR BOOK, 1981. *South Australian Year Book.* No. 16, 1981, Australian Bureau of Statistics, S.A. Office.

SJOBERG, L., 1977. *The accuracy of gravimetric deflections of the vertical as derived from GEM-7 Potential Coefficients and Terrestrial Gravity Data.* Dept. of Geod. Sci. Rep. No. 265, Ohio State University, Columbus, 1977.

SJOBERG, L., 1978. *The determination of gravity anomalies from geoid heights using Fourier Transforms.* NASA Contract Report 141442, Wallps Flight Centre, Virginia, 1978.

STRANGE, W.E., and HOTHEM, L.D., 1980. *Establishment of Scale and Orientation for Satellite Doppler positions.* Philosophical Transactions of the Royal Society of London, Series A, Vol. 294, 1980.

TORGE, W., 1980. *Geodesy.* Pub. Walter de Gruyter, Berlin, 1980.

TSCHERNING, C.C. and RAPP, R.H., 1974. *Closed Covariance Expressions for Gravity Anomalies, Geoid Undulations, and Deflections of the Vertical implied by Anomaly Degree Variance Models.* Dept. of Geod. Sci. Rep. No. 208, Ohio State University, Columbus, 1974.

UOTILA, U.A., 1960. *Investigations on the Gravity Field and Shape of the Earth*. Ann. Acad. Sci. Fenn. Ser. A111 Geologica, 55, 1960.

VYSKOCIL, V., 1970. *On the Covariance and Structure Functions of the Anomalous Gravity Field*. Studia Geophysica et Geodetica, Vol. 14, No. 2, 1970.

WENZEL, H.G., 1979. *Recent results of Geoid determination by combination techniques in the North Sea Test Area*. Presented at 17th General Assembly of IUGG, Canberra, Australia, 1979.