THE APPLICATION OF CORRELATION TECHNIQUES

TO CHECKING AND ADJUSTING MATHEMATICAL MODELS

by

REYNOLD G. KEATS, B.Sc.

A Thesis submitted for the degree of Doctor of Philosophy in the University of Adelaide
Department of Mathematics
June, 1965.
CONTENTS

Summary (iii)
Signed Statement (v)
Acknowledgements (vi)

CHAPTER 1. INTRODUCTION
1.1 The development and use of a mathematical model 1
1.2 Antecedent and related work 7
1.3 Scope of the present work 13
1.4 Notation 16

CHAPTER 2. MATHEMATICAL STATEMENT OF THE PROBLEM 19
2.1 Adjusting the model 20
2.2 Causal transformations 21
2.3 An adequate model 21
2.4 Statement of the problem 22

CHAPTER 3. STATIONARY LINEAR SYSTEM AND MODEL 23
PRELIMINARY DEFINITIONS AND THEOREMS

CHAPTER 4. STATIONARY LINEAR SYSTEM AND MODEL 24
ERROR FREE RECORDS OF UNLIMITED LENGTH
4.1 Model correction not involving generalised functions 49
4.2 Model correction when generalised functions are involved 56

CHAPTER 5. STATIONARY LINEAR SYSTEM AND MODEL 73
ERROR FREE RECORDS OF FINITE LENGTH
5.1 Gauss Markov input. Single sample 76
5.2 Input derived from Gauss Markov process. 79
    Single sample
5.3 Several input and output samples 88
5.4 Correction by iteration 91
5.5 Corrections involving generalised functions 95
CHAPTER 6. STATIONARY LINEAR SYSTEM AND MODEL RECORDS WITH ERRORS
  6.1 Records of unlimited length 100
  6.2 Finite length records with errors 112

CHAPTER 7. UNCORRELATED MULTIPLE INPUTS 126
  7.1 Error free records of unlimited length 127
  7.2 Records of finite length 128

CHAPTER 8. SOME EXPERIMENTAL CONFIRMATION 132
  8.1 Experiment 1: comparison of cross correlation approximations to an exponential weighting function 134
  8.2 Experiment 2: adjusting the mathematical model of a servomechanism 147

CHAPTER 9. DISCUSSION AND SOME EXTENSIONS OF THE WORK 157
  9.1 Correlated stationary multiple inputs 159
  9.2 Non linear and other systems not of type L 165

CHAPTER 10. CONCLUSIONS 174

APPENDIX ROUTINE CALCULATIONS USED MAINLY IN CHAPTER 5 177

NOTATION 183

BIBLIOGRAPHY 187
SUMMARY

A mathematical approach to the problem of checking and adjusting a mathematical model of a physical device is presented. A measure of the adequacy of such a model is proposed and a detailed study is made of the case where both the system and model are linear transformations, while the inputs are realisations of stationary random processes with rational spectra. The difference between the weighting function of the system and that of the model is then the error in the weighting function of the model. If the records were error free and of unlimited length this difference could be found as the solution of a Wiener-Hopf integral equation. In the case of an input whose spectrum is sufficiently flat and of unit power density, the required solution may be found in terms of the cross-correlation function of the input and the difference between the outputs of the system and model. The particular inputs considered may all be derived by linearly transforming a realisation of a process having a flat spectrum; this problem may therefore be reduced to the determination of a cross-correlation function.

In practice the records are of limited length; the problem may then be reduced to one of estimating the above cross-correlation from a finite length of data. A formula is obtained for the variation of the expected adequacy of the corrected model with the length of data available for model
correction. The weighting function of the model should, in general, only be adjusted for an interval of its argument which is much smaller than the length of data available for model correction. A method for determining this interval is described and a simple formula, which will be a useful guide in most cases, is derived.

The effect of errors in the records is also investigated. It is shown that, providing certain statistical information concerning the errors is available, compensation for these errors may be made. The effect of recording errors on the expected adequacy of the corrected model then depends on the length of data available according to expressions derived in the text.

If the system has several inputs which are not cross-correlated, the problem may be reduced to the case of a single input with errors in the recordings. If the inputs are correlated the cross-correlation technique may be used iteratively to improve the adequacy of the corrected model, but the model weighting functions will not necessarily converge to the system weighting functions.

Experimental evidence is produced to support the theoretical work and extensions of the work to other types of system and input are discussed in the final chapter.