Procedures for Diagnosis and Assessment of Concrete Buildings

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TO MY MOTHER

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Summary

Given the enormous stock of existing concrete structures in Australia, the maintenance, rehabilitation and treatment of ageing structures (especially those potentially defective) is of national importance. Due to the lack of previous research in this field, there is an urgent need to develop systematic methodologies that can be used in the treatment of structural defects.

In this thesis, an attempt has been made to establish a practically useful process for dealing with existing concrete buildings based on sound theories and techniques. Three key procedures, i.e. diagnosis, condition evaluation and decision-making, are proposed, and relevant methods for implementing these procedures are described.

The proposed method for diagnosis is an *hypothesis-and-test* procedure through which the most likely explanations of the observed anomalies in structural behaviour can be identified. An hypothesis is a set of explanations that fully covers the observed pattern of anomalies, and a test is any information-gathering activity. To take account of the uncertainties involved, an informal probabilistic reasoning procedure is used to rank all possible hypotheses. The most likely hypothesis is the one which has the highest subjective probability. The diagnostic method can be continued until an hypothesis is identified which has an acceptably high level of probability.

Condition evaluation, as defined in this thesis, consists of procedures for the assessment of structural adequacy regarding safety, serviceability and durability. Both experience and structural reliability theory are used in the assessment procedure.

The decision-making procedure is used to plan appropriate corrective work for the structure. Based on the framework of probabilistic decision theory, the best action is selected from a set of alternatives according to the preferences of the owner of the structure, in regard to factors such as structural safety, serviceability, durability and incurred costs. The proposed method forms a multi-stage process in which the engineer can also decide on whether to gather more data or to take action at any stage, using the results obtained from diagnosis and condition evaluation.

By integrating systematically the iterative procedures of diagnosis, assessment and decision-making, a comprehensive process is developed for dealing with existing concrete structures, which allows the engineer to decide rationally on what to do about a given structure in a well-structured manner. Two examples are used to show how the proposed process is practically useful.

STATEMENT OF ORIGINALITY

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Signed (Wen-Gang Hua) Date 17/8/93

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Principal Notations

a_i	$=$ i^{th} course of action in a candidate set
A	$=$ a set of courses of action available
a^*	$=$ the best course of action in A at i^{th} stage of the multi-stage decision-making
a_U^*	$=$ selected urgent action to be taken
b_{ij}	$=$ j^{th} attribute of anomaly s_i
c_i	$=$ i^{th} causal factor of an explanation
C_{a+}	$=$ the maximum amount of money the owner is willing to spend on the improvement of the structure's safety condition
$CF(\cdot)$	$=$ confidence factor of $(\cdot)$
e_i	$=$ i^{th} explanation of an anomaly
$f_X(x)$	$=$ distribution function of random variable X
$F_X(x)$	$=$ cumulative distribution function of random variable X
$G(X)$	$=$ limit state function in reliability analysis
H	$=$ hypothesis set
H_C	$=$ candidate set of hypotheses
H_i	$=$ i^{th} hypothesis in H
H^*	$=$ the highest ranked hypothesis in H
O_i	$=$ i^{th} possible outcome of a test
$P(\cdot)$	$=$ probability of $(\cdot)$
$P'(\cdot)$	$=$ prior probability of $(\cdot)$
$P''(\cdot)$	$=$ posterior probability of $(\cdot)$
P^*	$=$ risk of action a^* regarding the safety attribute
P_{acpt}^H	$=$ acceptable probability value in diagnosis
s_i	$=$ i^{th} anomaly
T	$=$ set of tests to be considered

t^* = selected test to be conducted at i^{th} stage of the multi-stage decision-making

t_i = i^{th} test in T

$U(\cdot)$ = utility of $(\cdot)$

$\bar{U}(\cdot)$ = expected utility of $(\cdot)$

X_C = attribute of the objective – *to minimize cost*

X_D = attribute of the objective – *to provide adequate durability*

X_F = attribute of the objective – *to provide adequate safety*

X_S = attribute of the objective – *to provide adequate serviceability*

X^{a_i} = consequences of course of action a_i

β = reliability index

$\beta_{|H}(H_i)$ = reliability index conditional on hypothesis H_i

θ_j = j^{th} outcome of a course of action

ΔI = newly obtained information

ϵ = prior knowledge

μ_X = mean value of random variable X

$\mu_{\tilde{A}}(x)$ = membership function of fuzzy set $\tilde{A}$

σ_X = standard deviation of random variable X

Chapter 1

Introduction

1.1 Background

Since the end of the Second World War there has been an enormous increase in the construction of bridges, buildings, dams, roads, railways and the like, in most countries around the world. Today, the existing infrastructure represents an immense investment of our society.

Although these structures have been designed to last for a long time, they are hardly maintenance-free. One obvious reason is that any system decays with time, and materials used in the structure deteriorate. The functionality of a structure thus inevitably decreases as the structure ages, and maintenance is necessary to keep the structure's functions above the level of acceptance. In addition to structural deterioration, original inadequacies in design and construction occasionally occur in structures, and these "inherent deficiencies" together with various damage to the structure due to factors such as improper use, fire, overloading and earthquake, can cause structural inadequacy and safety problems with serious consequences. Again, appropriate actions are required in such cases to improve the condition of those structures to an acceptable level. Furthermore, it is usually an economic proposition to undertake a regular maintenance program to extend the service life of a structure. Without investment in relevant maintenance, the huge resource represented by the existing infrastructure will be depleted. It is therefore not surprising to see the increased attention now being paid to problems in existing structures, both by engineers and managers.

The scale of the problem of maintaining existing structures is already remarkably large in many countries, and is increasing because ever more structures are being erected. It is reported (Ahlskog, 1990) that about 40 per cent of the 578 000 highway bridges in the U S are classified as deficient by the Federal Highway Administration. In the U K, 3 per cent of houses needed major repairs in 1971; in 1976 it was 4.3 per cent and it was over 5 per cent in 1981 (Blakey, 1989). Although comprehensive data are not available in Australia, published information on the requirement of structural maintenance is not optimistic. An article appearing in *The Australian* (Stewart, 1992) says that "Australia's basic infrastructure is in serious decay, with an estimated $8 billion in extra spending required each year for the next decade to restore our crumbling roads, sewers, railways, schools and hospitals".

The task of maintaining such a huge stock of structures provides both owners and engineers with financial and technical challenges. On the one hand, owners of public structures, such as relevant authorities and government agencies, are faced with questions which relate to the *managerial aspect* of structural maintenance. "Which structures need to be repaired, which need to be demolished and which require minimum maintenance ?"; "How much money should be spent on maintenance-related work ?"; "How can the available funding be best used for maintenance ?" Such questions have to be answered. To this end, a comprehensive management system is needed to assist high level policy makers in allocating resources for maintenance, and for providing managers with relevant guidelines for developing detailed maintenance plans. Obviously such a system needs to consider the technical, social and economic factors involved, as well as the general condition of the stock of existing structures. However, the current situation is far from being ideal or even adequate. Many political leaders and top level decision makers do not recognize the importance and urgency of taking care of existing infrastructure, and hence available financial resources are always short. Although more and more researchers are now involved in the study of infrastructure management, and computerized management systems for bridge maintenance have been developed in several countries (Andersen, 1990; Sinha, *et al,* 1990), further research is needed to enhance the capability of these systems. It is also desirable to establish network-linked data bases with information on the condition of all existing structures, so that efficient management of the entire stock of any particular owner can be achieved. Furthermore, relevant legislation and standards for structural repair work are also needed.

In addition to the managerial aspect of the problem, structural engineers are responsible for solving the problems arising from an individual structure which is defective, deteriorated or damaged. Tasks to be fulfilled can be summarized briefly as: *to decide what to do about a given structure and how to do it efficiently.* Superficially, such decisions can be easily made on the basis of the condition of the structure and its ability to satisfy various structural and functional requirements. If the structure is considered inadequate to satisfy the structural requirements, appropriate action such as major repair can be undertaken. Otherwise, actions such as preventive maintenance may be enough. Unfortunately, it is not an easy task to evaluate accurately the condition of a structure, especially when safety is a problem. Firstly, the assessment of an existing structure usually suffers from the limited data available and imprecise theoretical models. Secondly, any experiment or test on an existing structure needs to be non-destructive. Thirdly, while the causes of defects usually need to be considered in assessing the structure and also in deciding on an appropriate repair method, determining the true causes is rarely a straightforward task. Due to these difficulties, two types of error can occur in evaluating an existing structure (Warner, 1981):

- A *type* I error occurs when the structure is assessed, incorrectly, as being inadequate, thereby incurring unnecessarily the costs of corrective work;
- A *type* II error is made if the structure is assessed as adequate when it is not, so that property, and possibly life, are endangered.

Since the possibility of committing such errors always exists due to various uncertainties, the best possible option is to have a systematic approach which can be used to rationally assess the condition of an existing structure using the information available, and to make relevant decisions on what to do about the structure in such a way that the risk of making type I or type II errors and the cost incurred by assessment/repair are balanced. However, such an approach has not been available partially due to the fact that dealing with the problems of existing structures has not to date attracted wide attention of researchers. The treatment of structural defects in practice is in many cases carried out on the basis of experience, without rigorous prior condition assessment. At the same time, engineers are now faced with ever more structures in need of maintenance, and more complex situations. It is time to study maintenance-related problems in the research, and to develop needed methodologies which can aid engineers in treating a concrete structure with symptoms of deficiency or inadequacy.

There is thus a need for research into two different areas regarding maintenance-related problems of existing structures, i.e. the establishment of effective management programs for building stock, and the development of systematic methods for dealing with individual defective structures. This thesis attempts to tackle the second problem. Problems related to the managerial aspect of structural maintenance will not be considered in this study.

Before defining the detailed scope of the work to be carried out, a brief overview will be given in the next section on relevant past work. This review provides a framework and context for the work described in later chapters of this thesis.

1.2 Dealing With Defective Concrete Structures — An Overview

This review is primarily concerned with the overall process of treating defective concrete structures and with the different steps and methods involved in this process. More detailed reviews of specific methods and techniques are presented in subsequent relevant chapters.

1.2.1 General Procedure

The remedial treatment of a defective concrete structure has not yet become a unified process, and different engineers tend to have developed their own individual strategies based on personal experience. Nevertheless, the general procedures adopted by many researchers and practical engineers all contain a number of similar steps. Firstly, a **condition survey** of the structure is carried out, which usually includes a preliminary inspection, detailed site inspection of the structure, examination of relevant documents, such as the original design calculations and drawings, construction records (if available). During the condition survey, any anomalies in structural behaviour such as excessive cracking or rusting of steel bars is marked and recorded with attention paid to features such as *location* and *severity*. Based on the results of the condition survey, the second step of the process is to identify the likely contributing factors or causes of the recorded anomalies in structural behaviour using a diagnostic method, and for convenience this may be called **diagnosis**. The third step, referred to as **condition evaluation** is to assess the real physical state of the structure with regard to relevant structural requirements such as safety, serviceability and remaining service life using the information gained up to date. Finally a decision has to be made on what course of action is appropriate for the given structure according to the result obtained from the first three steps. The last step mentioned above is **decision-making**. If the information available is not sufficient for making an appropriate decision, further activities such as site and laboratory testing, structural analysis, have to be performed, and the process goes back to the second step. Obviously this process of dealing with existing defective structures is iterative, and stops when a terminal decision is made. There are actually a number of researchers, such as Warner (1981), Rewerts (1985), Yao (1985), and Chung (1991), who have discussed general procedures similar to the one outlined above.

There are also a number of reported case studies which clearly demonstrate these steps, such as in the treatment of building floor cracking by Majid *et al* (1989), the rehabilitation of a long bridge by Beard *et al* (1988), the remedy of a corbel support failure by Aboobucker *et al* (1989).

In the context of the overall procedure summarized above, the review presented in the following sections will consider in turn the steps of condition survey, diagnosis, condition evaluation and decision-making.

1.2.2 Condition Survey of Existing Concrete Structures

The purpose of the condition survey is to gather relevant information about the structure for use in subsequent steps such as diagnosis and assessment. Rewerts (1985) summarized various approaches available and proposed that the following key items should be considered in the investigation of an existing structure:

- description of the structure;
- visual inspection and measurements;
- loading environment;
- construction details;
- environmental conditions;
- materials of construction.

For this purpose, appropriate procedures and tools can be employed. Visual inspection plays a very important role in identifying noticeable deflections, cracks, deterioration, etc. (Whittington *et al.*, 1988). Photography is also useful (Shroff, 1986), and can provide quick, accurate records of anomalies in structural behaviour, appearance and surface conditions. Relevant measurements are necessary in determining the dimensions of the structure, as well as the presence, location, width and depth of cracking, and corrosion (Domon *et al,* 1989). To obtain detailed information, some in-situ or laboratory testing may be needed for evaluating variables such as concrete strength, carbonation depth, chloride content (Tassios *et al,* 1990). Furthermore, new modern technologies have resulted in non-destructive tests which now make it possible to detect deterioration and damage inside the concrete, and obtain needed information without damaging the existing structure. Wiberg (1989) used ultrasonic testing to detect cracks and deterioration in concrete. Hillemeier (1989) applied radar technology to locate prestressing tendons in concrete structures. An active microwave imaging system was designed and used to determine the number, position, and diameter of the different steel bars in concrete (Pichot *et al,* 1990). New techniques from other fields have been reviewed by Nowak (1990) with regard to their application in structural engineering.

From the condition survey, a list of anomalies in structural behaviour can be assembled, which is indicative of possible defects. This list, together with other information obtained, provides the starting point for the diagnosis.

1.2.3 Diagnosis of Defective Concrete Buildings

The terminology relating to diagnosis is in itself the cause of some confusion, because researchers have used similar terms in different contexts. Hartog (1989) used a term *building diagnostics*, and quoted a definition for it given by the U S National Academy of Science as "*all activities involved in judging how well a building performs its function through an understanding of the building's purpose, present use, environment and history.*" Nowak (1990) defined the diagnostic procedure as to "*identify the critical parts or elements of the structure, identify the cause of distress, monitor structural performance, warn against failure, and provide statistical data for the development of design and evaluation criteria.*" However, CIB Working Commission 86 — *Building Pathology*, considers that the diagnostic procedure has the purpose of finding out the causes of defects (van den Beukel, 1991). In this chapter, the word *diagnosis* temporarily refers to finding out the causes of anomalies in structural behaviour as observed in the condition survey. A rigorous definition will be given in Chapter 2.

In the practice of treating defective concrete structures, diagnosis relies heavily on the experience of the investigator (Majid *et al,* 1989; Beard and Tung 1988), and the detailed diagnostic reasoning procedure which can single out the true causes of a defect from a set of candidates, using the information available, is lacking. Several publications have discussed diagnosis from the scientific researcher's point of view, and emphasised the need for a well-structured diagnostic procedure. Hartog (1989) stated: "*I have emphasised the importance of scientific method, of a structured approach to the investigation of building failures,*" and that "*Skill in problem-solving is often ascribed to an investigator's 'experience', 'insight', 'judgement' or even 'intuition', but building diagnostics is neither an arcane art nor a mysterious process. Many common problem-solving techniques are used every day by the building diagnostician, though he might not recognize them by name.*" The importance and need for a systematic approach to building diagnosis is also well recognized by the CIB Working Commission W86 (van den Beukel, 1991).

Due to the immaturity of building diagnostic methodology, Hartog (1989) suggested that the terminology and conceptual framework of medical diagnostics might be borrowed. He also discussed a number of problem-solving routines which are appropriate to building diagnosis. He particularly emphasised the importance of the recursive nature of scientific method, which is illustrated in Fig. 1.1.

Warner (1981) used a diagnostic chart to aid in the diagnosis of concrete structures, in which common symptoms are related to a list of possible causes based on experience. An initial, tentative diagnosis is then made with the help of this chart, and further confirmation is sought by retrospective analysis. However, the detailed methods for this purpose are lacking.

The advantages of fault-tree analysis in building diagnostics has also been well recognized, especially by members of CIB W86 (van den Beukel, 1986; Croce,

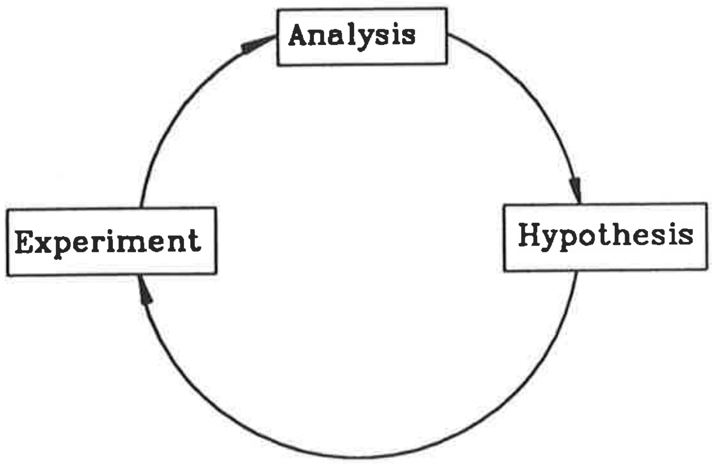


Figure 1.1: The Recursive Nature of Diagnostic Method

1986). *A fault-tree is a model that graphically and logically represents various parallel and sequential combinations of events resulting in the occurrence of a predefined top or main event* (Hadipriono and Toh, 1989). An *event* is defined as a dynamic change of state that occurs to a system element (Barlow and Lambert, 1975). Events are connected by **OR** gates and/or **AND** gates. This approach originally evolved in the aerospace industry in the early 1960's, and is one of the principle methods of systems safety analysis. It has been applied to concrete diagnosis recently and an example of such applications is illustrated in Fig. 1.2 (van den Beukel, 1986). From the figure, it can be seen that the anomalies in structural behaviour and their possible causes are represented by the *top event* and *basic events* respectively. Since there are only **or** gates presented in this particular case, the occurrence of any single basic event can cause the top event to occur. Therefore, the diagnostic problem is solved by identifying and confirming a continuous path starting at a basic event and ending at the top event. For a given top event, there are usually a large number of different paths available, which link up with various basic events. To prove a path, answers are needed to a series of linked questions regarding the *basic events, events*, and *circumstances* contained in the path. However,

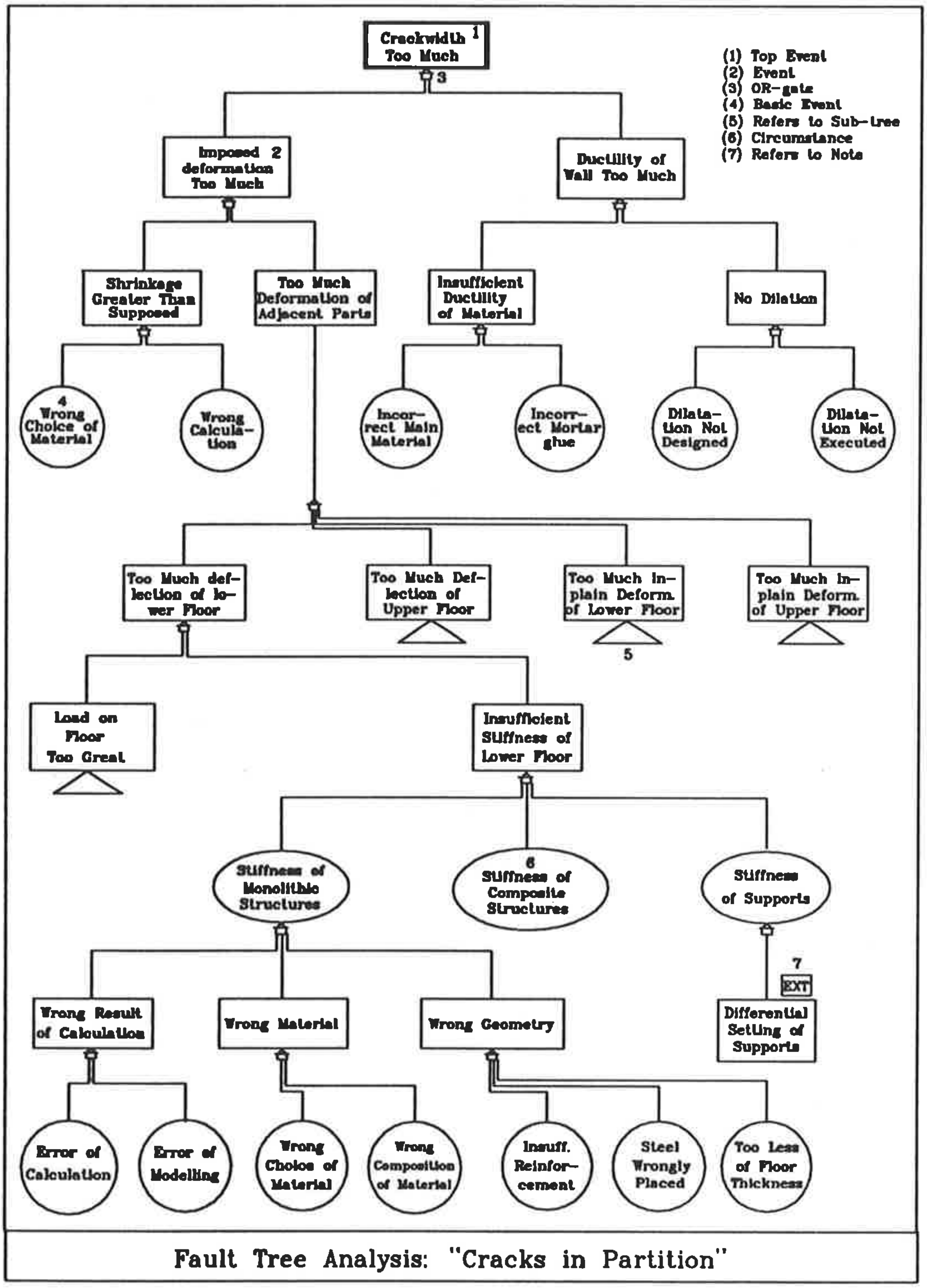


Figure 1.2: An Example of Fault-Tree for Diagnosis

these answers usually can not be obtained with certainty in practice, and hence it is difficult to obtain a conclusive diagnostic result based on the fault-tree if information available is incomplete. Applications of this approach to concrete diagnosis in the literature are still very limited, and detailed methods for reaching diagnostic conclusions are needed.

As can be seen from the above review, although the importance of having a systematic method for concrete diagnosis has been well recognized, and a number of publications regarding diagnosis have appeared, including the application of techniques in other fields, detailed methods for diagnostic reasoning specifically for concrete structures are still lacking.

After an existing structure has already undergone condition survey and diagnosis, in which all indications of structural deficiencies and defects are identified, and the true causes or most likely causes have been determined, condition evaluation is carried out next to determine the structure's capability of satisfying various requirements for its future use. This is also referred to as *structural evaluation*, *structural assessment*, *condition assessment* or *condition evaluation* in the literature, and relevant work on this subject is reviewed in the next section.

1.2.4 Condition Evaluation of Existing Concrete Structures

Depending on the characteristics and severity of identified anomalies in structural behaviour, an assessment is usually needed to evaluate the structural condition regarding various requirements such as durability, serviceability and, in particular, safety.

Durability Assessment

Unlike safety and serviceability requirements, durability is a vague concept. Strictly speaking, durability assessment can not be clearly separated from serviceability and safety related problems. However, the difference is that the study of durability problems has to include effects of time, while serviceability and safety requirements are usually evaluated at a specific time.

In the design of a new structure, durability requirements are achieved by obeying relevant specifications in adopted codes on factors such as materials, mix properties and workmanship. For an existing concrete structure, however, there is no code to apply, and the assessment of durability usually requires a study of factors such as deterioration, corrosion, and chemical attack. For this purpose, research into the individual deterioration mechanisms of construction materials exposed to various environmental conditions has been extensive (Tonini and Dean, 1976; Rostasy and Bunte, 1989; Gulikers, 1989; Hilsdorf, 1989; Page *et al.*, 1990). However, the effect of the degradation of materials on the analysis of structural behaviour requires further study. In practice, qualitative and descriptive approaches based on personal experience are widely used in durability assessment. For example, Sakai *et al* (1990) evaluated a concrete structure subjected to chemical attack. The steel bars inside the concrete were assessed on the basis of the carbonation depth of concrete, and the corrosion of the reinforcement was classified as grade I, II, III, or IV. Concrete properties such as chloride content were also measured, and the structural members were finally judged as being in *deterioration degree 1* or *degree 2* according to the severity of deterioration of steel bars and concrete. In a similar approach proposed by Chung (1991), the damage intensity of reinforced concrete due to corrosion is assessed from the condition of the concrete and steel in the member. The condition of concrete is classified as "satisfactory", "poor", "serious", or "very serious" according to the carbonation of the concrete cover and chloride content. Relevant repair techniques are then recommended for the concrete according to the deterioration level. These methods are obviously only concerned with the properties of the component materials regarding individual deterioration mechanisms, and are very much experience-based. A unified condition measure for a structural member or a structure concerning all aspects of deterioration based on a quantitative approach is obviously needed. For this purpose, Roper *et al,* (1985) developed a set of useful equations for assessing the durability of concrete members. Upon the study of 50 deterioration phenomena commonly present in defective concrete structures, three descriptors, called *Measurement, Intensity,* and *Distribution*, are defined for each of the phenomena. These descriptors are in turn calculated quantitatively by three unified eqations, and the durability condition of a structural member is then represented by these three values.

From the above brief review on durability assessment, existing approaches tend to evaluate only the severity of individual deterioration phenomena. Consequently the remedial recommendations are often determined from experience. Due to the lack of knowledge about the relationship between a deterioration phenomenon and the material's property change with time, quantitative measures of structural durability such as service life in terms of years based on accurate calculation seems impractical for the time being, and those existing experience-based methods are relatively useful from a practical point of view. However, a unique measure is obviously needed to describe the durability condition of a structure for various deterioration phenomena. Although linguistic words such as "good" and "satisfactory" are used in methods reviewed previously, they can not be compared on the same basis.

Serviceability Assessment

Serviceability problems in existing structures are in many cases noticeable on inspection, and the necessity of corrective work is relatively easy to decide based on the comparison of the intensity of observed anomalies in structural behaviour, such as excessive cracking, with various requirements in terms of relevant limits, such as limit on crack width specified by codes. Potential serviceability deficiency may be identified according to the causes of anomalies occurred. For these reasons, attention should be focused on diagnosis, and a rigorous assessment regarding the relevant serviceability requirements is not as important as in safety and deterioration problems.

There are few methods mentioned in the literature purely for assessing adequacy with regard to serviceability for existing structures. Instead, assessment of the general condition of the structure is carried out by considering both durability and serviceability. For example, in a rating system developed by Sabnis *et al,* (1990), structural members are placed in one of the following states: "good condition", "minor deterioration", "major deterioration", "hazardous" according to the severity of cracking, corrosion and humidity. The general condition of the structure can then be determined by the member conditions. These experience-based approaches have also been formulated using fuzzy set theory so that the non-numerical subjective data obtained in linguistic form can be processed systematically. Tee *et al* (1988) developed a fuzzy mathematical approach for bridge condition evaluation, in which the subcomponents of the bridge structure are first inspected, and their conditions and importance are then rated according to the physical states and related functions in the structure. The results for each component are represented in linguistic terms such as "good", "poor", and described by fuzzy sets. The overall condition of the bridge is represented by the weighted average, and obtained by:

$$\bar{R} = \frac{\sum_{i}^{n} \tilde{W}_i \tilde{R}_i}{\sum_{i}^{n} \tilde{W}_i} \qquad (1.1)$$

where $\tilde{R}_i$ and $\tilde{W}_i$ represent the rated condition and importance of the i^{th} structural member respectively.

It is important to note that serviceability and durability problems should not be mixed up. Time plays an important role in durability assessment, while time-dependent deterioration of materials is usually not considered in the evaluation of serviceability.

Safety Assessment

For safety assessment of a possibly defective concrete structure, there is a trend to use simple approaches in conjunction with experience-based judgement in practice, in order to avoid complicated structural analysis, especially when methods for such analysis are not well established. In a reported case study by de Brito *et al* (1989), tests were made to explore the strength of the concrete, and although the structure had deteriorated, the fact that the measured concrete strength was higher than the original design value led to the conclusion that the structure was safe. Generally, though, evaluation of materials is not a sufficient basis for structural adequacy.

Even if structural analysis is carried out, the code-specified design methods are usually adopted (Shroff, 1986; Majid *et al,* 1989). However, due to differences in the physical state of the structure between the design phase and the stage of assessment, the use of design methods for the safety evaluation of an existing structure may sometimes lead to incorrect results. Code design methods deal necessarily with the uncertainties of creating a structure at some time in the future. In an existing building, many of these uncertainties can be eliminated in the assessment process. Therefore, use of the partial safety factors of a design method may lead to over-conservative results. On the contrary, when a structure shows signs of being defective, the causes of the defects may play an important part in evaluating the safety of the structure. However, the design codes do not consider the influences of all specific causes, and assume that the design and construction of the structure are under some standard quality control scheme which results in an acceptably low probability of committing errors in the design and construction phase. If it happens that the true causes are not confirmed with certainty in the diagnosis (this is very likely in practice), and/or the effects of causes on the safety evaluation have to be accounted for by specifically modifying the model or relevant equations of structural analysis, code-based safety assessment may lead to unconservative results, and this is highly undesirable. For these reasons, particular care is needed if a code-based deterministic approach has to be used. More realistic methods for safety evaluation of existing defective structures are required, which can handle uncertainties rationally, and have a sound theoretical basis. For this purpose, probabilistic methods and fuzzy set-based approaches are well developed for structural safety evaluation (Thoft-Christensen, 1982; Melchers, 1987; Blockley, 1975; Brown, 1979). Related work in these areas will be reviewed in some detail in Section 3.2 and Section 3.4 of Chapter 3, and will not be discussed further in this overview.

1.2.5 Decision-making and Repair Techniques

After the completion of diagnosis and condition evaluation of a given structure, it is necessary to decide what action, if any, needs to be taken. Although different courses of action can be identified for specific problems, depending on the severity of defects and the owner's resources, as indicated by several researchers (Warner and Kay, 1983; Chung, 1991), the following broad categories are usually a subset of those to be considered in treating defective concrete structures:

- do nothing, on the judgement that the structure will be able to satisfy all relevant performance requirements;
- monitor the structure in service, in order to check more carefully on performance and deterioration;
- undertake repair or corrective work in order to bring the structure into an acceptable condition;
- undertake repair or corrective work after taking the structure temporarily out of service;
- take the structure out of service with the possibility of demolition and reconstruction;
- Other options in particular cases.

Choosing the most appropriate action for any particular problem is not an easy task. The decision usually needs to take account of both the technical and financial aspects of the problem. Technically, both the necessity and consequences of taking a particular course of action have to be considered with regard to durability, serviceability and safety conditions of the structure before and after the action is executed. Financially, it is desirable to choose the action which is the cheapest and also capable of producing a durable, serviceable and safe structure. However, due to the lack of systematic approaches for this purpose, courses of action regarding the remedy of defective structures are usually determined on an *ad hoc* basis in practice. Frequently, the decision to strengthen a structure is determined conservatively from experience (Beard *et al*, 1988; Mahamond *et al*, 1989; Pakvor *et al*, 1989; Tassios *at al*,

1990) without a rational evaluation of necessity and efficiency. In many cases, corrective work is planned solely according to the causes of defects (Majid, 1989). When difficulties arise from diagnosing the true causes, actions are occasionally taken arbitrarily. Warner and Kay (1983) tried to apply statistical decision theory (Raiffa, 1970) to the establishment of a rational approach in selecting relevant repair strategies for dealing with defective concrete structures. In the method, possible consequences of each course of action and the risk of failure are considered, and the final action is chosen in such a way that the expected cost is minimized. Although the approach has presented a framework for applying decision theory to the planning of appropriate actions in treating defective structures, detailed methods for estimating relevant probabilities and evaluating consequences were not discussed.

Basically the majority of the existing methods for making a decision in regard to a repair strategy are based on the engineer's personal experience. Some well developed approaches (Chung, 1991) are domain-dependent, and only applicable to very narrowly defined fields. Importantly, the risk of making a wrong decision and the related consequences such as structural failure are not rationally treated in most of those experience-based methods. Although Warner and Kay's approach seems promising, many detailed procedures involved are yet to be developed. For these reasons, a well structured and universal approach is needed, which can be used to find the most appropriate course of action for general problems using the information available.

If there is such an approach, a set of relevant courses of action with regard to repair have to be designed so that a choice can be made. For this purpose, depending upon the specific problem, various detailed repair techniques are available (O'Donnell, 1989). Generally speaking, repair of concrete structures is carried out to improve durability, strength, function and appearance. For these purposes, there exist a variety of different methods, techniques and materials which can be adopted in specific cases, for example, the use of epoxy-bonded steel plate for strengthening (Rostasy and Ranisch, 1982), strengthening and stiffening by external prestressing (Trinh, 1990), various epoxy resins and mortars for crack repair (ACI, 1978; Brøndum-Nielsen, 1978; Hewlett and Morgan, 1982). The detailed discussion of such techniques does not fall within the scope or purpose of this thesis, and hence will not be pursued here.

1.2.6 Non-destructive Test-based Approaches

In previous sections, the process of dealing with defective concrete structures has been briefly reviewed. With the application of new electronic devices to structural engineering, the use of non-destructive test-based methods for diagnosing and assessing defective concrete structures is becoming popular. *System identification* (Sage *et al,* 1971) is one such approach, which relies on the results of dynamic tests conducted on the existing structure. A theoretical model of the structure (system) is firstly assumed, and parameters involved in the model are then estimated in such a way that the dynamic response of the structure obtained from the test is similar to model predictions. The analytical model of the structure is thus tuned using the test data. This process is also referred to as *structural identification* (Hart and Yao, 1977; Liu and Yao, 1978).

The use of structural identification techniques in damage assessment of concrete structures has been discussed in various publications. Agbabian *et al,* (1990) conducted a test on a model bridge to study the structural changes by analyzing the response measurements and parameter variation. Flesch and Kernbichler (1990) used this technique to inspect the safety condition and detect damage in large bridges. The basic concept of these approaches is that damage, in general, decreases the structural stiffness and increases damping, and this in turn results in changes in the dynamic properties. Other similar studies have been carried out by Casas *et al,* (1990), Vestroni and Capecchi (1987), and a comprehensive review of this topic was given by Yao (1985).

Structural identification is a new technique in the evaluation of existing concrete structures. Several technical problems, however, are still to be solved. For example, the structural response from a dynamic test is recorded by sensors which always contain a certain amount of noise. The presence of such noise can influence the accuracy and reliability of various system identification algorithms. On the other hand, as a result of limited dynamic load applied on the tested structure, the measured responses may be insensitive to changes in some structural parameters of interest. Also, the method does not give reliable information on the strength of structural members. Due to these shortcomings, the application of this technique is still very limited in practice.

1.2.7 Summary and Conclusions

From this overview, it seems that researchers and engineers basically agree on the general process to be adopted for the maintenance, repair and rehabilitation of existing defective concrete structures. This process consists of a number of procedures including condition survey, diagnosis, condition evaluation and decision-making as illustrated here in Fig. 1.3.

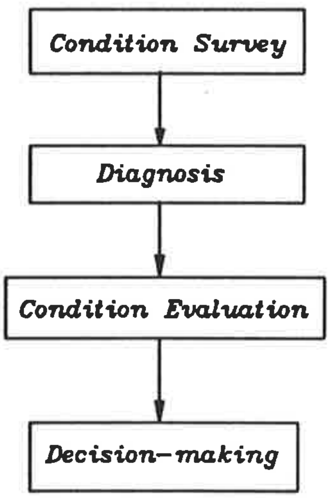


Figure 1.3: General Process of Dealing With Existing Structures

However, detailed methodologies for diagnosis, condition evaluation and decision-making have not been well established, and relevant literature available on these subjects is very limited. For these reasons, the task of dealing with an existing defective concrete structure in practice is usually carried out in an *ad hoc* manner. Although experienced engineers have implicit ability to assess a structure and make relevant decisions based on their own judgements, the decision of this type is too often made without the aid of formal models considering the balance between the imposed cost and the possible consequences (or outcomes). It is reasonable to argue that such a decision-making process may not always be suitable for dealing with existing concrete structures. This is particularly the case when new and difficult problems are encountered.

On the other hand, the existing process of dealing with defective structures itself needs to be further developed. Firstly, procedures involved in this process are highly inter-related, and the output of one procedure is the input of the next. Unfortunately outcomes of these procedures usually contain uncertainties due to the incompleteness of information available, and hence a practically useful process has to logically integrate these procedures in order to make decision under uncertainty. The simple process illustrated in Fig. 1.3 is not suitable for this purpose. Furthermore, the individual procedures involved can not be undertaken in the simple sequence as in Fig. 1.3 due to practical reasons. For example, if a structure shows signs of defects, after the inspection, an immediate decision has to be made as whether to take urgent action to protect the property and human life. In this case, the engineer can not afford to wait for detailed information to go through the intermediate steps of thorough diagnosis and assessment.

Based on the above discussion, it can be concluded that detailed methodologies for diagnosis, condition evaluation and decision-making need to be developed, but also that a comprehensive process of integrating these procedures in a methodical way is essential for the treatment of existing defective concrete structures. This thesis attempts to fulfill these tasks.

For this purpose, an overall process of treating existing structures is firstly proposed in the next section, and relevant details needed in the process are then developed in later chapters of this thesis.

1.3 The Overall Process for Dealing With Defective Concrete Buildings

An overall process for dealing with existing defective concrete structures, particularly for buildings, is proposed in Fig. 1.4. Although there are a lot of details lacking at this stage, an overall picture of the work to be carried out in this thesis can be seen from the flow-chart.

The process starts with the inspection of the structure, and a set of anomalies in structural behaviour can be obtained. With the information gained through various actions such as an interview with the owner or occupants and study of available documents, preliminary diagnosis is undertaken to identify possible causes of the obtained anomalies. Preliminary condition evaluation is carried out next to determine the real physical state of the structure with regard to structural requirements of safety, serviceability and durability. Based on these results, a decision is immediately to be made as to whether urgent action is necessary or not, using the available information. If the structure is in danger of immediate collapse, appropriate action is taken to protect human life and property. After the selected action is executed or if such an action is unnecessary, an immediate decision needs to be made as whether to continue the process of assessment. If the information available up to date is sufficient to make a terminal decision or if the process has to stop for whatever reasons, feasible courses of actions are then evaluated, and a final decision is made as

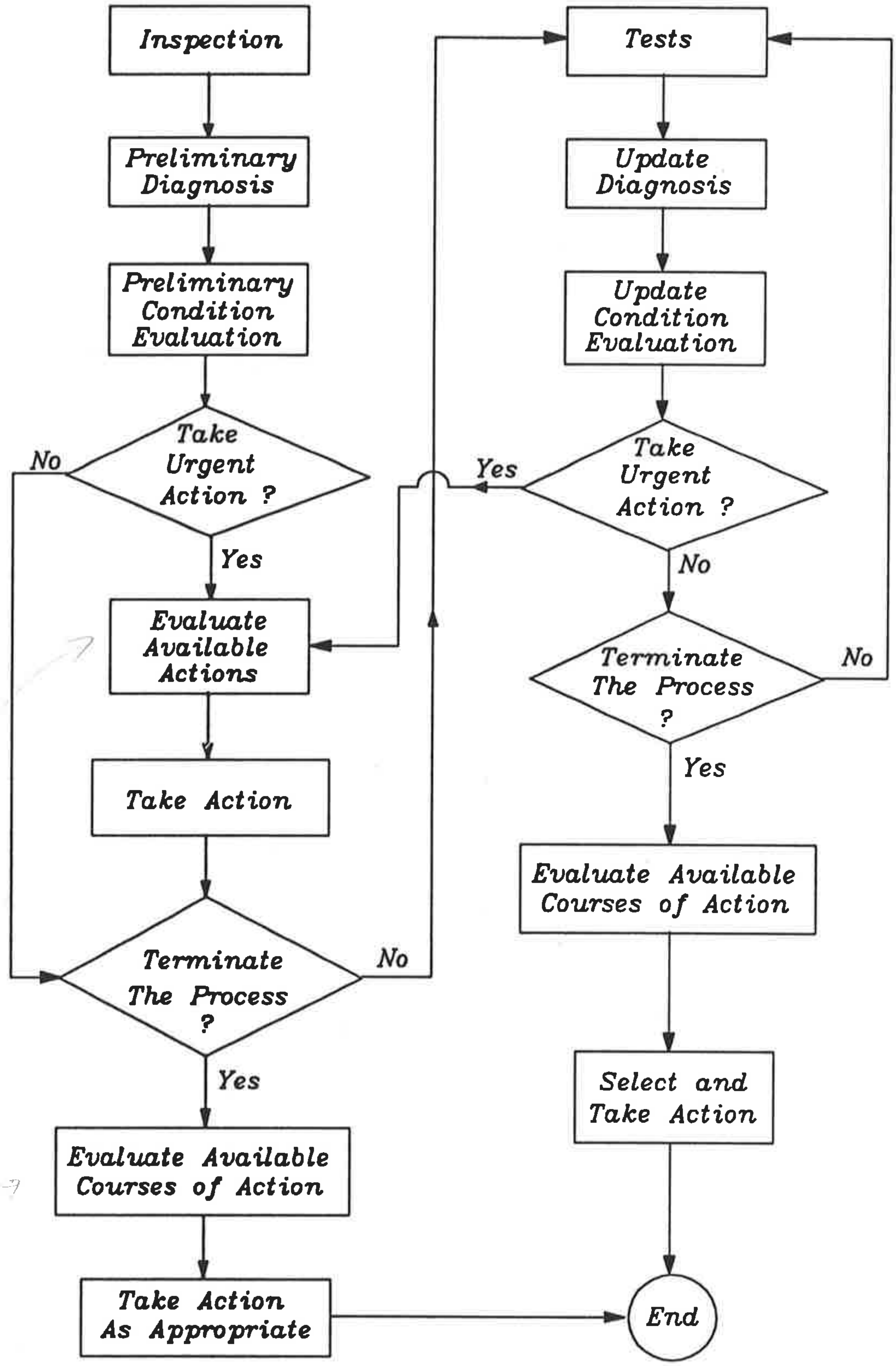


Figure 1.4: The Proposed Process for Dealing With Defective Concrete Buildings

appropriate. Otherwise, the process advances to the step of gathering more information by means of tests. Using the new information obtained from the tests, the diagnosis and condition evaluation can be updated. Based on the new results from the updated diagnosis and condition evaluation, it is necessary to decide again on the necessity of taking urgent action.

After all this, an appropriate course of action needs to be determined for the given structure. For example, if the structure is evaluated as having inadequate safety, a remedy for strengthening the structure would be required. However, due to the incomplete information available, outcomes of diagnosis and condition assessment are usually presented with some uncertainty. This can only be reduced at the expense of acquiring more information. Therefore, a decision has to be made at this stage on whether to continue the process by obtaining new data to improve the precision of diagnosis and assessment, or to take the chosen course of action based on the current information. If the latter is the case, the process ends when the action is executed. Otherwise, the process goes back to the information gathering step, and more tests have to be conducted. The term *test* here should be interpretated in a broad sense, i.e. it includes not only experimental work but also any other information gathering activities such as analysis or even study of relevant documents. Using the new knowledge obtained, diagnosis and condition evaluation are updated, and a new decision is made. The process thus works iteratively until a terminal decision is made.

The overall process of dealing with existing concrete structures has been proposed above. The objective of the rest of this thesis is to develop relevant methods for the steps in the process, and hence to further elaborate the flow-chart in Fig. 1.4 with details. Of particular importance are the decision-making steps, represented by diamonds in the flow-chart. The specific objectives and scope of the thesis are outlined next.

1.4 Objectives and Scope of Thesis

The general objective of this study is to develop a detailed, systematic process for dealing with defective concrete buildings based on sound theories. Specific objectives are:

1. development of a diagnostic procedure which can be used to find out the most likely causes of anomalies in structural behaviour observed in a concrete building;
2. development of a method to assess the condition of existing concrete buildings, in particular those which are damaged or defective;
3. development of a decision model which is able to decide "what to do" about a given concrete building using the results from diagnosis and condition evaluation, with incomplete information;
4. development of a detailed, comprehensive process for dealing with existing defective concrete buildings by integrating these three steps together.

The study is primarily concerned with problems related to existing concrete buildings which are damaged or become defective as the result of "ordinary" causes. The term *ordinary* is used to exclude other extreme causal events like fire or earthquake which are special problem areas. The fatigue phenomenon is also ignored because it rarely controls in building design.

1.5 General Terminology

Before specific procedures are described, it is necessary to define the terminology to be used throughout this thesis. Additional terms which are used for a specific method will be introduced in appropriate chapters.

To be consistent with concepts in structural design, some terminologies of diagnosis, condition evaluation and related decision-making in this thesis are defined in accordance with the philosophy of limit state design which specifies various structural requirements through relevant limit states. For example, the **serviceability limit states** define the requirements for structural performance when the structure is subjected to service loads and normal environmental conditions, while **ultimate limit states** define the requirements when extreme conditions of overload and environment are considered. In addition, a structure usually has to satisfy various non-structural requirements for **functional** purposes.

Based on this philosophy, a concrete building or one of its structural members will be said to be **defective** if it does not satisfy one or more of the limit state design requirements. Specifically, a **structural defect** is the non-satisfaction of a structural requirement, either serviceability or strength. In addition, a **functional defect** occurs when a functional requirement is violated.

Usually defects in a structure are initially indicated by various **anomalies**. An **anomaly** is an unusual or undesirable pattern of structural behaviour which is observed during the operation or inspection of the structure.

1.6 Layout and Contents of Thesis

In Chapter 2, a literature review on diagnostic techniques in other fields such as medical science is carried out. A diagnostic method for concrete buildings is then proposed, in which the most likely explanations of the observed anomalies can be identified through a *hypothesis-and-test* process using information available.

Chapter 3 describes the development of methods for condition evaluation of existing concrete buildings, following a literature review on structural reliability theory and other techniques such as fuzzy-based safety assessment.

Statistical decision theory is briefly reviewed in the early part of Chapter 4, and a procedure for deciding on the course of action to be taken for a defective concrete building is developed from this theory using the results of diagnosis and condition evaluation described in Chapter 2 and Chapter 3.

In Chapter 5, an extensive overall process of dealing with existing defective concrete buildings is established by assembling logically the procedures already developed.

Chapter 6 gives two examples of solving real problems using the process proposed in Chapter 5.

Chapter 7 contains a brief summary, and recommendations for future research work.

Finally, the appendices list some statistical information on basic variables, which may be needed in carrying out some of the procedures proposed. This information has been taken from existing literature.

Chapter 2

Diagnosis of Defective Concrete Buildings

2.1 Introduction

The identification of explanations for the observed anomalies is a crucial step in the whole process of dealing with potentially defective concrete buildings. Other procedures such as condition evaluation and decision-making with regard to the corrective work all rely on the output of the diagnosis. In this chapter, a method is developed with the aim of providing a systematic approach for engineers to diagnose existing buildings in a well-structured manner.

Although diagnosis has not yet been extensively studied for existing concrete structures in current research, various diagnostic techniques have been well-established in other fields such as medical science, and it is beneficial to use this experience of diagnostic research as a reference in the development of our own method for concrete diagnosis. For this reason, literature on research into diagnostics in related fields is briefly reviewed in the next section.

2.2 Diagnostic Methods In Other Fields

Extensive research has been conducted in medical diagnosis and mechanical and electrical trouble shooting in the last few decades. A major emphasis of this research has been placed on developing computer-aided automatic diagnostic systems. In these systems, diagnostic reasoning mechanisms and relevant expertise used by human experts are acquired and explicitly stored in computers so that the user of the system can diagnose like an expert. While relevant publications on diagnostic methods developed in these fields are briefly reviewed here, there is no intention to deal with the design and architecture of these systems. Instead, the primary emphasis is on the techniques for diagnostic reasoning used in these systems.

2.2.1 The Overall Process of Diagnosis

Although different approaches may be employed for diagnosis in various areas, existing methods seem to follow a common process (Weiss *et al*, 1978; Peng and Reggia, 1987a; de Kleer and Williams, 1987). This process is usually triggered by a set of *anomalous patterns of behaviour* observed in the system to be diagnosed. The anomalous behaviour has generally been referred to in the literature as **symptoms**, **signs** or **manifestations**. The task of diagnosis is to identify a set of **causes** which are responsible for the identified symptoms. These causes are usually **diseases** in medical diagnosis, and **faults** in mechanical and electrical trouble shooting.

For this purpose, a *hypothesis-and-test* procedure is usually adopted whereby the *hypothesis* is a set of diseases or faults that can explain the observed symptoms, while the test is an experimental action that can confirm a hypothesis or provide additional relevant information. This procedure is schematically illustrated in Fig. 2.1.

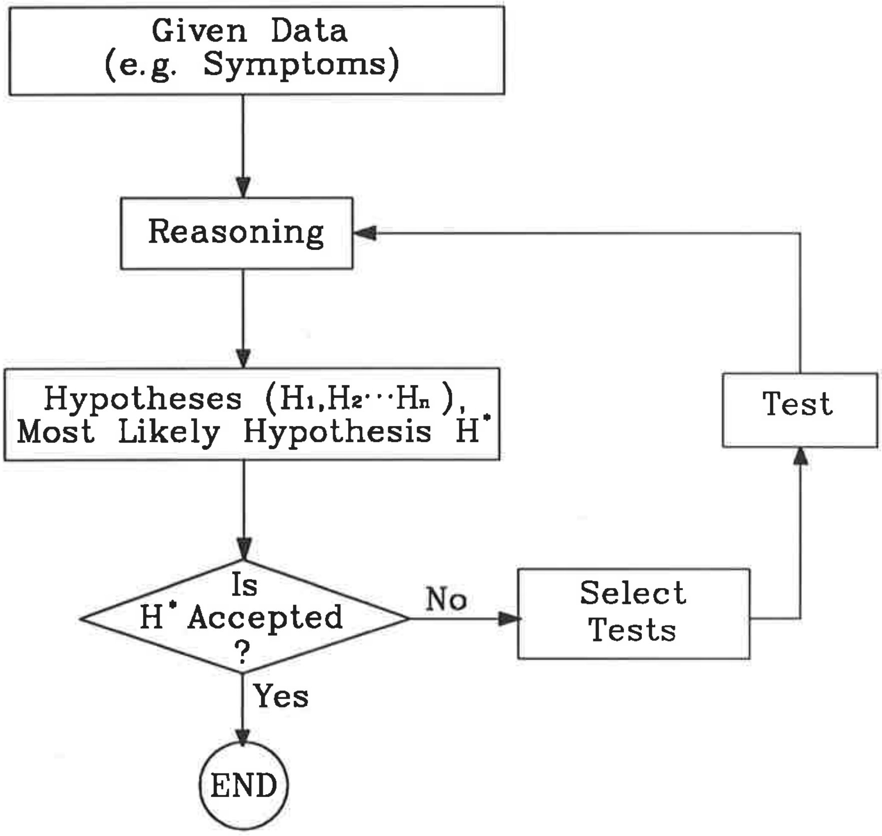


Figure 2.1: The Process of Diagnosis

As can be seen from the figure, after valid symptoms are identified from the given system, a reasoning procedure is used to generate a set of possible solutions – *hypotheses*, denoted by H, and to rank these hypotheses according to their likelihood of being true. The most likely hypothesis H^* is distinguished from others using the obtained rankings. A decision is then made to accept or not accept H^*, using an appropriate criterion. If H^* is confirmed by the information available and hence accepted, the process of diagnosis terminates at this stage. Otherwise, additional tests have to be chosen and conducted. The hypothesis set H and the ranking of its members are accordingly updated using the new information resulting from the testing, and the diagnostic process continues.

In the *hypothesis-and-test* process discussed above, a major task of the reasoning procedure is to rank all possible hypotheses using techniques available such as probability or fuzzy set theory. Criteria for accepting the most likely hypothesis H^* and selecting a test are usually dependent on the ranking method adopted. Obviously the reasoning mechanism is the most important part in the diagnosis, and therefore will be the focus of this literature review.

2.2.2 Probabilistic Reasoning for Diagnosis

Many existing systems for medical diagnosis (Ben-Bassat, 1980a; Reggia *et al*, 1985a, 1985b; Lauritzen and Spiegelhalter, 1988) and electrical or mechanical trouble shooting (de Kleer and Williams, 1987) have been developed using probabilistic reasoning. Although various detailed procedures can be adopted for this purpose, the reasoning mechanism used in all these approaches is more or less Bayesian in style. Specifically, possible symptoms of human patients or mechanical systems are related to a set of diseases or faults, with the strength of these relations defined in terms of relevant conditional probabilities. With the evidence such as identified symptoms, diagnostic reasoning is used to evaluate the updated posterior probabilities of those diseases or faults from their prior probabilities. Diseases or faults are hence ranked by their probabilities, and the most likely disease or fault is the one which has the highest probability.

For this purpose, assuming that there are m possible causes $C = C_1, C_2, \cdots, C_m$ to be considered for an observed pattern of symptoms, the prior probability of C_i is assessed and denoted by $P'(C_i)$. After the set of symptoms $S = (s_1, s_2, \cdots, s_n)$ are observed from the given system (patient or machine), the posterior probability of the i^{th} cause C_i is given by:

$$P(C_i \mid s_1, s_2, \cdots, s_n) = \frac{P'(C_i) \cdot P(s_1, s_2, \cdots, s_n \mid C_i)}{\sum_{i=1}^{m} P'(C_i) \cdot P(s_1, s_2, \cdots, s_n \mid C_i)} \tag{2.1}$$

where $P(s_1, s_2, \cdots, s_n \mid C_i)$ is the joint probability of $s_1, s_2, \cdots, s_n$ conditional on C_i. However, this formula is only suitable for diagnostic problems in which only one cause can be true. Although a multi-cause problem, in principle, can be transformed into a single-cause problem by defining a new set of possible causes which consists of all the 2^m possible intersections of $C_1, C_2, \cdots, C_m$, e.g. (C_1, C_2, C_1C_2, ϕ) for $m = 2$, where ϕ is an empty set, the exponential increase in the number of causes makes this approach impractical as m increases.

For this reason, Ben-Bassat (1980a) assumed that all causes in C are mutually *independent*, i.e. the probability of C_i being true does not affect the probability that C_k is true given a pattern of symptoms observed. It has to be mentioned that if $C = (C_1, C_2, \cdots, C_m)$ are mutually exclusive and exhaustive, members in C are then mutually *dependent*, since the existence of any one cause will exclude the occurrence of all others. Under this independence assumption, Ben-Bassat applied Eq. 2.1 to determine the probability of each cause C_i in the following manner: Let C_i denote the event that the cause C_i is responsible for the observed symptoms, and $\bar{C}_i$ denote the complementary event that the cause C_i is not responsible for the occurrence of identified symptoms. C_i and $\bar{C}_i$ thus constitute a mutually exclusive and exhaustive set, and hence the posterior probability that C_i is the cause of an observed symptom pattern $S = (s_1, s_2, \cdots, s_n)$ is given by:

$$P(C_i \mid s_1, s_2, \cdots, s_n) =$$

$$\frac{P'(C_i) \cdot P(s_1, s_2, \cdots, s_n \mid C_i)}{P'(C_i) \cdot P(s_1, s_2, \cdots, s_n \mid C_i) + (1 - P'(C_i)) \cdot P(s_1, s_2, \cdots, s_n \mid \bar{C}_i)} \tag{2.2}$$

In this way, by applying Eq. 2.2 for every element of C, the probability of each cause $P(C_i)$ can be obtained, and these $P(C_i)$'s constitute the solution of the problem. Alternatively, since $C_1, C_2, \cdots, C_n$ are mutually independent, the probability of any multi-cause set can also be evaluated as the product of the corresponding probabilities of the relevant causes involved. This method has been applied to real problem- solving (Ben-Bassat, 1980b) and demonstrated its usefulness in certain domains.

Obviously due to the difficulty of handling too many members in a multi-cause set, Ben-Bassat's approach transformed the multi-cause diagnostic problem into a single-cause problem based on the independence assumption. Actually the multi-cause diagnostic problem has been tackled by many researchers in different ways. Reggia *et al* (1985a, 1985b) proposed a so-called *generalised set covering* model which deals with the multi-cause problem in a very meaningful manner. To demonstrate this method, an example is given in Fig. 2.2 in which there are two symptoms $S = [s_1, s_2]$ and three diseases $D = [d_1, d_2, d_3]$. Causal relationships between diseases and symptoms are denoted in the figure by appropriate lines, e.g. s_1 can be caused by d_1 and/or d_2, and s_2 can be caused by d_1 and/or d_3. From the assumed relations, the hypothetical solution – *the hypothesis* can be found using the concept of set covering. For this purpose, a hypothesis is firstly defined as *a set of diseases which can fully explain, or cover, all of the presented symptoms.* Under this definition, hypotheses for the example in Fig. 2.2 will include $H_1 = (d_1)$, $H_2 = (d_2, d_3)$, $H_3 = (d_1, d_3)$, $H_4 = (d_1, d_2, d_3)$.

It has to be mentioned that there are only four valid hypotheses in this particular example, but for a complicated problem in which the number of possible diseases is large, the hypothesis set H generated in this way could become too big to handle. To overcome this difficulty, Reggia *et al* used a *minimal cardinality principle* to restrict the valid number of hypotheses, i.e. *in the hypothesis set H, only those members which contain the minimum number of diseases will be considered at the next stage of investigation.* In fact, this is another application of **Occam's Razor**, which reflects the idea of *the simplest solution being the best among all possible solutions.* Using this principle, the hypothesis to be considered for the example in Fig. 2.2 will be $H = H_1 = (d_1)$, since H_1 contains the smallest number of diseases among $(H_1 = d_1, H_2 = d_2d_3, H_3 = d_1d_3, H_4 = d_1d_2d_3)$, and H_1 thus identified is called the tentative hypothesis.

With the hypotheses H obtained, the set covering model proceeds by confirming or eliminating some members of H using information available, and the tentative hypotheses are used to guide further data gathering. If additional symptoms are found as a result of new information, the tentative hypotheses have to be modified so that all symptoms are covered with the simplest H set. A set of experience-based rules are finally used to rank the tentative hypotheses and no formal probability values are obtained for elements in H.

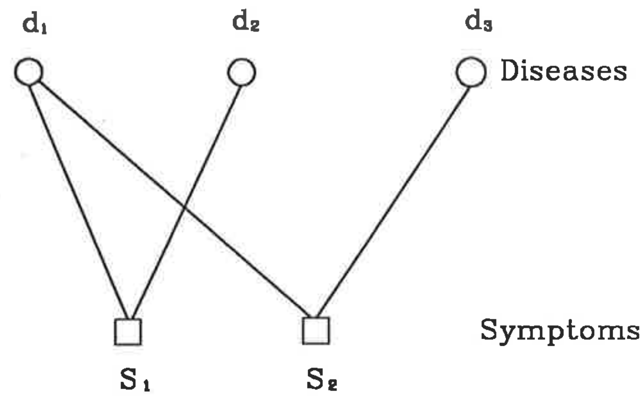


Figure 2.2: Causal Relations Used by Reggia *et al*

Probably the weakest part of the set covering model is the use of the minimum cardinality principle, because the tentative hypotheses obtained using this principle may miss the inclusion of the true hypothesis. For this reason, Peng and Reggia (1987a, 1987b) further improved the set covering model in a comprehensive approach developed using Bayes' theorem. In the method, a *hypothesis graph* is firstly constructed from the established relations between diseases and symptoms. For a given set of diseases d^*, instead of finding all hypotheses which can cover d^*, a modified heuristic state space search method is used to generate only those promising hypotheses which, however, guarantee that the most probable hypotheses are included. The diagnosis is then focused on evaluating the probability of each member in the hypotheses thus generated.

For this purpose, given a symptom s_j and a disease d_i, the *causation event* $s_j : d_i$ is defined as *the event that d_i actually causes s_j*. In other words, the event $s_j : d_i$ is true if and only if both d_i and s_j occur and s_j is actually caused by d_i. The probability $P(s_j : d_i \mid d_i)$ is then called the *conditional causal probability of s_j given d_i*, which represents the strength of the relation between s_j and d_i. Also, the prior probability of each disease, $P(d_i)$, is assumed to be known. Thus for a diagnostic problem with observed symptoms $s^* = (s_1, s_2, \cdots s_m)$ and a set of hypotheses $H = (H_1, H_2, \cdots, H_n)$, the probability of each hypothesis in H can be evaluated through Bayes' theorem:

$$P(H_i \mid s^*) = \frac{P(s^* \mid H_i)P(H_i)}{P(s^*)} \tag{2.3}$$

where the prior probability $P(H_i)$ is to be calculated from the prior probability of each disease d_i, and the conditional probability $P(s^* \mid H_i)$ have to be calculated from $P(s_j : d_i \mid d_i)$; $P(s^*)$ is a constant for a particular diagnostic problem. However, since the hypothesis set H is not completely generated, the constant $P(s^*)$ can not be obtained, and hence only the relative likelihood of each H_i is obtained in this approach.

Peng and Reggia's method provides a good framework for dealing with multi-cause diagnosis using formal probabilistic manipulation. However, due to the large amount of computational work required, the approach is suitable for building a computer-based automatic diagnostic system, but not particularly useful as a practical procedure to be used by a diagnostician.

In diagnostic reasoning previously reviewed, probabilistic relations are defined between only causes and symptoms. In real cases, however, a cause or fault may be attributed to further causes, and symptoms can also have correlated sub-symptoms. These *further causes* and *sub-symptoms* constitute a very important part of information other than the previous defined causes and symptoms. To include this knowledge in the probabilistic reasoning, Pearl (1986), and Kim and Pearl (1987) used a tree structure called *a causal network* or *a belief network* to represent the relationship between various pieces of information. The strength of these relations are also expressed in terms of conditional probabilities. Two examples of the network are shown in Fig. 2.3, in which

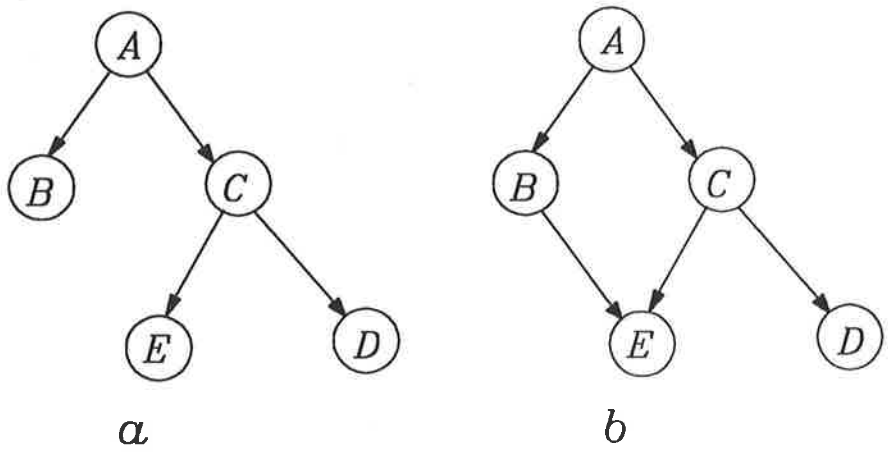


Figure 2.3: An Example of Causal Network

each node represents a variable or proposition, e.g. symptom, disease, or sub-symptom. The root node (node A in the figure) usually stands for the original cause, e.g. disease or fault, and nodes on the tips of the directed graph (nodes B, D, E) are data nodes which represent various manifested symptoms. Those nodes which are needed to tie together the root node and the data node are called *intervening nodes*. All nodes are not necessarily binary variables, and hence each node can have many possible values. Directed arcs are used to represent causal relations between two nodes, and the strength of this relation is quantified by a conditional probability, e.g. $P(c \mid a)$ denoting the probability of node C, given the occurrence of node A.

After the prior probabilities of those root nodes and relevant conditional probabilities have been assessed for an established network, the probability P_i of each node is evaluated and updated using new information available, while the conditional probabilities are kept unchanged. Since P_i thus dynamically changes, the probability of any node, say node B, is represented by the overall belief in B based on the data D available up to date, and denoted by $BEL(B)$, i.e. $BEL(B) = P(B \mid D)$. $BEL(B)$ can be obtained by:

$$BEL(B) = \alpha\lambda(B)\pi(B) \tag{2.4}$$

where α is a constant; $\lambda(B) = P(D_B^- \mid B)$, $\pi(B) = P(B \mid D_B^+)$ in which D_B^- represents the data contained in those nodes of a sub-network rooted at node B, and D_A^+ stands for the data contained in nodes of the rest of the whole network. Obviously Eq. 2.4 is also derived from Bayes' theorem, where $\pi(B)$ represents the causal support attributed to B by the ancestors of B, and $\lambda(B)$ represents the diagnostic support B receives from descendants of B.

Using Eq. 2.4, the probability of each node can be obtained and stored. If new information is gathered on a node i, the effect of this data on the belief of any node in the network can be evaluated, and the node-probability is then updated through a propagation procedure.

Exploring new information to reduce the uncertainty of a particular node is called *node expansion*, and which node to be further expanded is decided by the benefit of this expansion. For this purpose, the benefit of a node N in resolving the uncertainty of node T is defined as:

$$BEFIT(T, N) = IMP(T, N) \cdot E(N) \tag{2.5}$$

where $IMP(T, N)$ is called "important factor" determined by the strength of dependency between T and N; and $E(N)$ is the uncertainty of the node N which is measured by its entropy:

$$E(N) = -\sum P(n_i) \cdot \log P(n_i) \tag{2.6}$$

Kim and Pearl's work provides a good tool for solving diagnostic problems in a well-structured way under the context of probability theory. However, the network used in this method is restricted to a special kind of *singly connected causal network* (SCCN). A SCCN is defined as the network in which *one (undirected) path, at most, exists between any two nodes* (Pearl, 1986). For example, the network in part *a* of Fig. 2.3 is a SCCN, while the one in part *b* of Fig. 2.3 is not. The latter is usually referred to as a network with "loops". This restriction certainly limits the applicability of the approach, since many real problems can not be represented by a simple SCCN.

Probabilistic reasoning using a more general causal network has been proposed by Lauritzen and Spiegelhalter (1988) in which loops are permitted. For a given causal network with known conditional probabilities between linked nodes and prior probabilities of root nodes, the ideal representation of our belief within the network is the construction of a joint probability distribution over all those nodes involved in the network. Relevant marginal probabilities on interested nodes can then be obtained for the purpose of diagnosis. However, the computation for this purpose is usually a formidable task when the number of nodes is large. To overcome this difficulty, Lauritzen and Spiegelhalter developed a method which is capable of calculating the marginal probabilities on individual nodes efficiently without obtaining the joint probability distribution. For this purpose, some topological changes are made to the original network, and the resulting new network consists of a number of *cliques* which are defined as sets of nodes that are all joined directly in the network. With this change, the marginal probabilities on these cliques are evaluated, and the marginal probability on a single node can be easily calculated from the marginal probabilities of relevant cliques.

By using this algorithm of efficient probability manipulation within the network, one can determine the marginal probabilities of interested nodes that are considered as causes of observed symptoms. This can be carried out for any state of available information. If new data are gathered, the approach is capable of absorbing the knowledge through the network and hence updating the node probabilities. The use of strict probabilistic methods also ensures that the effect of multiple pieces of evidence does not depend on their order of arrival.

When deciding what test is to be conducted next for gathering more information on a particularly interesting node, say node A, "mutual information" is used to determine how valuable another node is to node A. For example, the mutual information between node A and B, $M(B, A)$, is defined as:

$$M(B,A) = \sum_{B}\sum_{A} \log\{\frac{P(B \mid A)}{P(B)}\}P(B \mid A)P(A) \tag{2.7}$$

This approach provides probably the most comprehensive method for rigorous probability calculation within a causal network, and is very suitable for constructing computer-based diagnostic systems. The application of it in medical diagnosis can be found in the work by Andreaseen *et al* (1987).

In addition to the above formal probability-based diagnostic reasoning, a large number of computer-aided diagnositic systems have also employed the subjective Bayesian inference (Lesmoet *et al*, 1982) using some rules in the form of:

$$If \;\; E \;\; then \;\; H \;\; with \;\; probability \;\; p, \qquad i.e. \; E \rightarrow H$$

where E can be evidence such as symptoms, and H is an hypothesis, e.g. a disease. The probabilistic nature of this kind of rule has been summarized by Duda *et al* (1976). Specifically, if the prior probability of H, $P(H)$, and the conditional probability $P(E \mid H)$ are assessed, the posterior probability of H after the evidence E is observed can be easily evaluated through Bayes' theorem, i.e. :

$$P(H \mid E) = \frac{P(H) \cdot P(E \mid H)}{P(E)} \tag{2.8}$$

in which $P(E) = P(H) \cdot P(E \mid H) + P(\bar{H}) \cdot P(E \mid \bar{H})$, and $\bar{H}$ is the complement of H. However, sometimes the evidence E may not be identified with certainty. For example, the conclusion "I am 70 percent certain that E is true" from a diagnostician only indicates that $P(E \mid \textit{relevant observations}) = 0.7$. If E' is used to denote the relevant observations, we can denote $P(E \mid E') = 0.7$. The probability of H given E' is obtained as:

$$\begin{aligned} P(H \mid E') = & \quad P(H, E \mid E') + P(H, \bar{E} \mid E') & (2.9) \\ = & \quad P(H \mid E, E') \cdot P(E \mid E') + P(H \mid \bar{E}, E') \cdot P(\bar{E} \mid E') & (2.10) \end{aligned}$$

where $\bar{E}$ is the complement of E. If it is assumed that when E is known with certainty, the observations E' relevant to E provide no further information about H, Eq. 2.10 becomes

$$P(H \mid E') = P(H \mid E) \cdot P(E \mid E') + P(H \mid \bar{E}) \cdot P(\bar{E} \mid E') \qquad (2.11)$$

where $P(H \mid E)$ and $P(H \mid \bar{E})$ can be obtained from Eq. 2.8. Thus the posterior probability of H given the vague evidence E' is represented by Eq. 2.11.

If there are several rules of the form $E_1 \rightarrow H, \cdots, E_n \rightarrow H$ all concerning the same hypothesis H, the diagnostic problem becomes to evaluate the probability $P(H \mid E_1, \cdots, E_n)$. For this purpose, based on the assumption that those pieces of information are conditionally independent, i.e. $P(E_1, \cdots, E_n) = \prod_{i=1}^{n} P(E_i \mid H)$ and $P(E_1, \cdots, E_n \mid \bar{H}) = \prod_{i=1}^{n} P(E_i \mid \bar{H})$, an analogous solution can be obtained from Eq. 2.8.

Although probability-based diagnostic reasoning as reviewed above has been very popular, fuzzy set theory is also widely applied in many existing diagnostic systems, and this is briefly reviewed in the next section.

2.2.3 Diagnosis Using Fuzzy Reasoning

Fuzzy set theory (Zadeh, 1965, 1973; Dubois and Prade, 1980) has been successfully used in treating unclearly defined linguistic variables, and proves to be a good tool for handling uncertainties. The application of fuzzy set theory in diagnosis can be frequently found in the literature (Gupta and Sanchez, 1982).

A *fuzzy set* is a collection of objects, in which there is a degree of membership related to each object. For illustration purpose, the linguistic term "integers close to 10" can not be described by an ordinary set, but can be easily represented by a fuzzy set $\tilde{A}$ with its membership function in the form: $\sum \mu_{\tilde{A}}(x) \mid x$, e.g.:

$$\tilde{A} = 0.1 \mid 7 + 0.5 \mid 8 + 0.8 \mid 9 + 1 \mid 10 + 0.8 \mid 11 + 0.5 \mid 12 + 0.1 \mid 13 \qquad (2.12)$$

where $x \in X$; X is the ordinary set representing all integers, and is referred to as the *reference set*; $\mu_A(x)$ denotes the degree of membership of x in the fuzzy set $\tilde{A}$. For example, $0.8 \mid 9$ means: "integer 9 belongs to $\tilde{A}$ with the degree of membership of 0.8".

In diagnosis, many variables such as symptoms can be described by fuzzy sets. For example, a symptom S_i can be defined as a fuzzy subset $\tilde{S}_i$ of a reference set $X = (X_1, \cdots, X_n)$, characterized by a membership function $\mu_{\tilde{S}_i}(x)$. Set X contains all possible values which can be taken by S_i. The membership function $\mu_{\tilde{S}_i}(x)$ defines the strength of x being in $\tilde{S}_i$. In this context, if fuzzy sets $\tilde{S}$ and $\tilde{C}$ represent symptoms and causes respectively in a diagnostic problem, the causal relation *if* $\tilde{C}$ *then* $\tilde{S}$ can be described by a fuzzy operation called *fuzzy relation* denoted by $\mathbf{R}$ (Zadeh, 1965; 1975a):

$$\mathbf{R} = \tilde{C} \times \tilde{S} \qquad (2.13)$$

in which the membership function of $\mathbf{R}$ can be obtained from that of $\tilde{C}$ and $\tilde{S}$ using relevant fuzzy calculations. With the observed symptom pattern defined by a fuzzy set $\tilde{S}^*$, the causes responsible for the occurrence of $\tilde{S}^*$, also described by a fuzzy set $\tilde{C}^*$, is obtained through another fuzzy operation, called *fuzzy composition*:

$$\tilde{C}^* = \tilde{S}^* \circ \mathbf{R} \tag{2.14}$$

where the membership function of $\tilde{C}^*$ can be easily evaluated from that of $\tilde{S}^*$ and $\mathbf{R}$.

Many fuzzy diagnostic reasoning techniques in existing approaches are based on the *fuzzy relation* and *fuzzy composition* described above (Zadeh, 1975b; Sanchez, 1979a, 1979b; Soula *et al*, 1980; Baldwin, 1981; Zadeh, 1983), although various forms of relations may be defined for specific problems.

For example, Adlassnig and Kolarz (1982b) used fuzzy relation to build a medical diagnostic system in which symptoms and diseases of a particular kind are collected and represented by two ordinary sets $S = (S_1, S_2, \cdots, S_m)$, $D = (D_1, D_2, \cdots, D_n)$. Each S_i and D_j are in turn defined as fuzzy subsets of relevant reference sets, and denoted by $\tilde{S}_i$ and $\tilde{D}_i$ respectively. Symptom-disease relationships are described in terms of two dependencies. The first dependency is the **occurrence** of a symptom S_i in case of a disease D_j, which represents the knowledge about the possibility of the presence of S_i given D_j. This "occurrence relation" is further represented by a fuzzy relation $\mathbf{R_O}$ defined on the reference set $S \times D$. The second dependency is the **confirmability** relation which indicates the power of an observed symptom S_i to confirm a certain disease D_j. The confirmability is also defined by a fuzzy relation $\mathbf{R_C}$ on $S \times D$. If the fuzzy set $\tilde{S}^*$ describes the observed symptoms on a particular patient, four fuzzy indications can be calculated by means of fuzzy composition (Eq.2.14):

1. S_i, D_j occurrence indication: $\tilde{R}_1 = \tilde{R}^* \circ \mathbf{R_O}$;
2. S_i, D_j confirmability indication: $\tilde{R}_2 = \tilde{R}^* \circ \mathbf{R_C}$;
3. S_i, D_j non-occurrence indication: $\tilde{R}_3 = \tilde{R}^* \circ (\mathbf{1} - \mathbf{R_O})$;

4. S_i, D_j non-symptom indication: $\tilde{R}_4 = (1 - \tilde{R}^*) \circ \mathbf{R_O}$;

The final decision regarding the diagnostic result is to be determined based on these four indications.

A similar approach using fuzzy relations is also used by Johnston (1985) in diagnosing faults of the wastewater treatment process, in which a different algorithm for fuzzy composition is adopted.

Although there are many other applications of fuzzy set theory to diagnostic reasoning, methods reviewed in this section represent the most popular approaches in this domain, and demonstrate the relevant concepts clearly. More literature on existing fuzzy-based diagnostic systems can be found elsewhere such as Gupta, Saridis and Gaines (1977), Esogbue and Elder (1980), Gupta and Sanchez (1982), Adlassnig (1982a), and will not be further reviewed here.

Techniques using probability and fuzzy set theory thus reviewed constitute the majority of existing methods for diagnostic reasoning. In addition, some other approaches are also frequently used in medical diagnosis and mechanical trouble shooting, and are briefly discussed next.

2.2.4 Other Approaches for Diagnostic Reasoning

In either probability or fuzzy reasoning discussed in previous sections, symptoms are related to relevant diseases or faults in terms of conditional probabilities or fuzzy relations assessed from *experience* or *qualitative analysis* of the physical system to be diagnosed. In troubleshooting mechanical devices or electrical circuits, however, some other approaches employ diagnostic reasoning based on the actual physical model of the devices or circuits, in which the relationship between symptoms and faults is established according to the measured behaviour of the model.

De Kleer and Williams (1987) proposed a domain-independent method for diagnosing multiple faults. For a device to be diagnosed, if its behaviour can be predicted without uncertainty from an established model which is a description of the physical structure of the device, a symptom can then be defined as *any difference between a predicted and observed behaviour.* From relevant measurements and records of the model behaviour, a set of such symptoms can be identified. These symptoms are further related to one or more "candidates" which are defined as *a set of components of the device which are possibly faulted.*

Under these definitions, a set of possible candidates are created through a *candidate generating procedure* which is basically similar to Peng and Reggia's set covering model described on page 34. The difference is that in this procedure every possible cover is considered in generating the candidates, while Peng and Reggia's approach considers only the hypotheses having minimal elements. The task of diagnosis is to confirm a particular candidate or to reduce the number of elements in the candidate set until only the true candidates are left. For this purpose, testing or measurements have to be carried out so that the candidate set can be re-generated using the new information obtained. However, the concept of probability is again employed to determine the most valuable parameter to be measured in the next testing. Suppose there are a set of parameters $x = x_1, x_2, \cdots, x_l$ to be measured in the testing. To choose one from x, each member of the "candidate set" $C = (C_1, C_2, \cdots, C_n)$ obtained above has to be ranked by its probability p_i, which is subjectively estimated using the knowledge currently available. The entropy EH is in turn defined as:

$$EH = -\sum_{i=1}^{n} p_i \log p_i \tag{2.15}$$

If a test is conducted to measure variable x_i, which has possible values $v_{i1}, \cdots, v_{im}$, the expected entropy for measuring x_i is evaluated by

$$\overline{EH}(x_i) = \sum_{k=1}^{m} p(x_i = v_{ik}) EH(x_i = v_{ik}) \tag{2.16}$$

where $p(x_i = v_{ik})$ is the probability that the variable x_i equal v_{ik}, and $EH(x_i = v_{ik})$ can be computed from Eq. 2.15 conditioned on $x_i = v_{ik}$. According to the calculated expected entropy, the best choice is to measure x_j, if x_j has the minimum expected entropy among all variables to be measured. However, the approach did not specify how the probability p_i was estimated.

Methods using well-established models are also found in medical diagnosis. For example, Weiss *et al* (1978) constructed a computerized diagnostic system in which a detailed model is established to represent various pathophysiological states in the form of a network. The causal relation between two states is defined for all possible states with the deep understanding of a particular kind of diseases. For instance, if n_i and n_j are two pathophysiological states, the mapping $n_i \stackrel{a_{ij}}{\rightarrow} n_j$ indicates that state n_j is caused by state n_i with the numerical strength a_{ij}. The belief in the truth of each state can be represented by a confidence factor $CF(n_j)$.

The diagnostic procedure is carried out in sequences, and new information about the pathophysiological states is obtained through experiments. The confidence factor of each state can be updated accordingly. If a symptom or a piece of evidence is observed from a test t_i, a confidence measure is used to indicate the degree of belief in the presence of specific states. For instance, $t_i \stackrel{Q_{ij}}{\rightarrow} n_j$ means that the i^{th} test or evidence supports the presence of state n_j with confidence Q_{ij}.

To select a test to be conducted, relevant *weights* are defined for various states presented in the causal network. For this purpose, a pathway can be identified between any two states in the network. A "*forward weight*" of entering a state n_j from a single pathway starting at state n_i is defined as:

$$w_F(j \mid i) = \prod_{k=1}^{j-1} a_{k,k+1} \tag{2.17}$$

where $a_{k,k+1}$ is the causation strength between two adjacent states n_k, n_{k+1} in the pathway as defined above. The total forward weight $w_F(j)$ of state n_j is the sum of all weights $w_F(j \mid i)$ for those pathways entering node n_j starting at the nearest confirmed state n_i in the network. With a set of interested states identified, the weight of these states can be evaluated from Eq. 2.17, and the state with the highest weight should be focussed in the next stage of information gathering. To do this, a set of tests can be designed, and the best choice is the one which is cheaper than all others.

After a test is conducted, confidence factors of relevant states are updated. For each state n_j, if its confidence factor $CF(n_j)$ is greater than a certain value, the state n_j is considered true. If several confirmed states form a continuous path in the causal network, a particular pattern of pathophysiological states can be identified and prove to exist. This pattern of states is then related to a specific disease by a set of pre-defined rules derived from experience and other relevant knowledge. Finally, the disease which has the highest confidence factor is considered true, and the relevant treatment suggested accordingly.

The methods reviewed in this section either require a well-established model to represent the physical structure of the system to be diagnosed, or need a deep understanding of the system so that accurate prediction of relevant behaviour is possible. These conditions are usually difficult to meet in dealing with existing concrete buildings, and hence it is probably not convenient to directly apply these methods for our purpose. Nevertheless, the strategies used for test selection as well as the reasoning mechanisms are domain independent, and could be useful in diagnosing defective concrete structures depending on particular problems at hand. Many more similar approaches can be found in the literature (Szolovits and Panker, 1978; Davis, 1984; de Kleer, 1986; Reiter, 1987), but will not be further discussed.

2.3 Summary and Conclusions

It can be seen from the above discussion that an iterative process of *hypothesis-and-test* has been widely adopted in most of the computerized diagnostic systems. This procedure works in cycles using the mechanism of abductive reasoning, i.e. it begins with a set of observed symptoms, and tries to find out the best hypothesis which can fully explain the given pattern of symptoms through a relevant process of information gathering, such as testing. Although diagnostic problems are very diverse, the hypothesis-and-test procedure seems to be suitable for a wide variety of diagnostic tasks. For this reason, the hypothesis-and-test procedure will be adopted in the development of a method for diagnosing defective concrete buildings.

In most existing approaches for diagnostic reasoning, relations between symptoms and causes are defined and measured by using either probability or fuzzy sets. Probabilistic reasoning proves to be a very meaningful and practical tool to deal with uncertainties in diagnosis, especially by the use of Bayesian subjective probability, while fuzzy reasoning is more capable of treating variables or symptoms which are vaguely defined or in linguistic forms. In this work, probabilistic reasoning will be used.

However, available approaches for probabilistic reasoning are generally developed and used for the construction of computerised diagnostic systems. These methods are not suitable for direct application to the diagnosis of concrete buildings due to the following reasons. Firstly, a computerised automatic diagnostic system has a large data base created by collecting all possible causes (diseases/faults) and symptoms for a specific kind of problem. In order to carry out probabilistic reasoning, relations between the causes and symptoms of the entire set have to be pre-defined in terms of conditional probabilities. In the diagnosis of existing buildings, it is very difficult for the engineer to create a complete set of diagnostic knowledge for a specific kind of problem.

Secondly, as a result of using the large data base, complicated computation procedures are usually needed to reach a diagnostic conclusion from the given evidence. This is a big disadvantage for the engineer in diagnosing concrete buildings, where simple calculations are obviously preferred. Therefore, the method of diagnostic reasoning adopted for concrete structures will have to overcome this difficulty.

For this purpose, consider the fact that unlike computers which need a complete set of pre-defined rules to proceed diagnostic reasoning, engineers can intelligently carry out relevant reasoning quickly in their mind using only the information currently available. Then an attractive way is to have a simple but well-structured procedure which can aid the engineer to process consistently the knowledge obtained, and to update his belief when new data become available. This procedure has to be less rigorous mathematically than that used in the computerised diagnostic systems, and easier to be carried out. Obviously, an informal probabilistic approach based on Bayes' theorem would be ideal for this purpose. If so, a complete, yet simple and practical method for diagnosing concrete buildings can be obtained by integrating this informal probabilistic reasoning into the *hypothesis-and-test* process. It is for this purpose that such a method is developed in the next section, for use in solving real problems in practice.

Finally, terms like *diseases*, *faults* are frequently used and are clearly defined in medical diagnosis and mechanical trouble shooting. However, similar terminologies are not well defined for concrete diagnostics. For this reason, pertinent terminology to be used will be defined whenever it becomes necessary.

2.4 A Method for Diagnosing Defective Concrete Buildings

2.4.1 Introduction

For a defective concrete building, diagnosis is usually triggered by a set of observed **anomalies**. They are defined as *the unusual or undesired aspects of structural behaviour.* Depending on the purpose of diagnosis, there can be different goals to be achieved by carrying out the diagnosis. If liability-related legal issues or insurance assessment are involved, the diagnosis is to find out who is responsible for the occurrence of the observed anomalies and any ensuing repair. On the other hand, if the purpose of diagnosis is to identify causes of the anomalies so that an appropriate course of action can be chosen to eliminate the causes and to rehabilitate the structure, then it is sufficient for the diagnosis to search for a set of explanations which can fully explain the occurrence of the anomalies within the context of concrete technology. The latter is the case in this work.

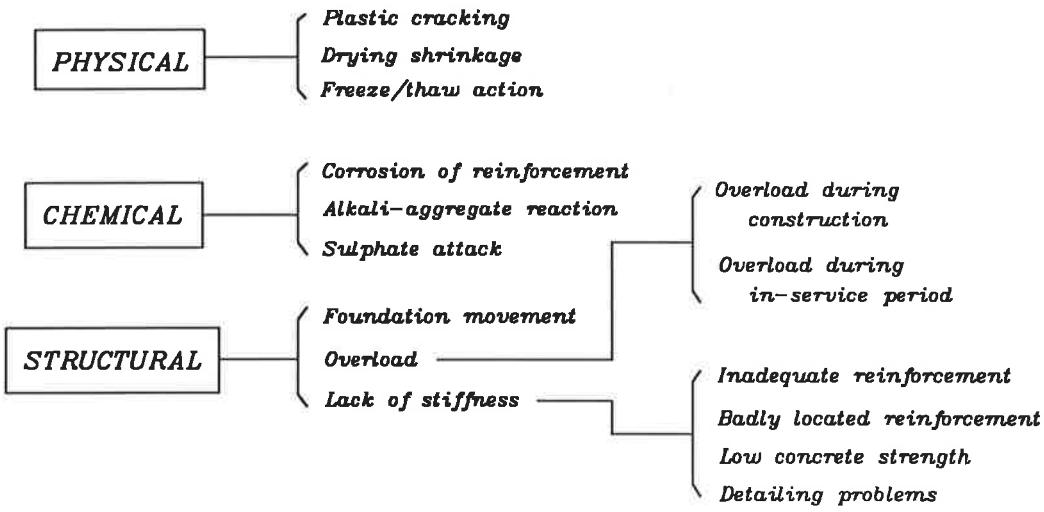
However, although the term *cause* was already used by some researchers in concrete diagnostics (Warner, 1981; ven den Beukel, 1991), it has not been clearly defined. Existing evidence indicates that the cause of an anomaly such as *excessive cracking* can in general be the consequence of some preceding causes, e.g. $\cdots \cdots$ **human error→improper curing→shrinkage→cracking**, and therefore it is almost impossible to go back to the original cause. Fortunately, the identification of the original cause is rarely necessary in structural assessment. For example, in the case of the cracking mentioned above, if *shrinkage* can be confirmed as responsible for the occurrence of *cracking,* the decision on the remedy of the cracking can be confidently made, and further investigation on why the shrinkage has occurred becomes unnecessary. In this case, the cause *shrinkage* suffices for the purpose of structural evaluation and repair planning. This kind of cause will be called an **explanation** in the rest of this thesis to distinguish it from other causes. In general, a satisfactory explanation can always be identified from a given chain of causes for an anomaly.

Based on this definition, the term **diagnosis** in dealing with potentially defective buildings is defined here as *the process of determining a set of technical explanations for the occurrence of the observed anomalies.* To identify explanations, relevant diagnostic data first have to be gathered and assembled.

2.4.2 The Representation of Diagnostic Data

At the beginning of diagnosis, the pattern of anomalies can be obtained from the condition survey. For each anomaly, a set of causes can be identified drawing on the engineer's experience together with some common sense and relevant knowledge of concrete structures. These causes are most likely to form a set of causal chains. From each chain, an explanation can be obtained according to the definition described above. For example, given an anomaly "excessive cracking on the bottom surface of a concrete beam", two possible cause-chains are as "···**design error→inadequate reinforcement →cracking**", "···**construction error→ early over-loading →cracking**". Obviously the explanations in these cases are "inadequate reinforcement" and "early over-loading".

Although there is probably no text book from which one can draw possible explanations for given structural anomalies, the following general approach may be helpful. Explanations of an anomaly can usually be categorised, and more specific explanations can always be figured out by dividing each category into further sub-categories. For example, the explanation of "cracking" may be worked out as:



In fact, for most common structural anomalies in concrete buildings, related explanations can usually be deduced from the three broad categories listed in the above example, i.e. "physical", "chemical" and "structural". This can be used as a general guide in the identification of explanations for most structural anomalies.

The observed anomalies and their related explanations can be represented by a tree structure. An example of such a tree is shown in Fig. 2.4 in which an anomaly such as "excessive crack #1" is represented by a *circle node*, and relevant explanations are included in various *rectangles*; the directed lines indicate the causal direction between different nodes. For each anomaly, there usually exist a set of **attributes** which represent the characteristics of the anomaly. For example, attributes for "cracking" may include "the time of appearance", "crack orientation", "crack width and length", etc. These attributes, when their values are known, can prove to be valuable information in diagnostic reasoning, and are represented by *ellipse nodes* in the tree. Also, according to our previous definition, an explanation is only one representative cause in a chain of causes of an anomaly. Therefore, an explanation in the tree can usually be further explained by other causal factors which are represented in the tree by

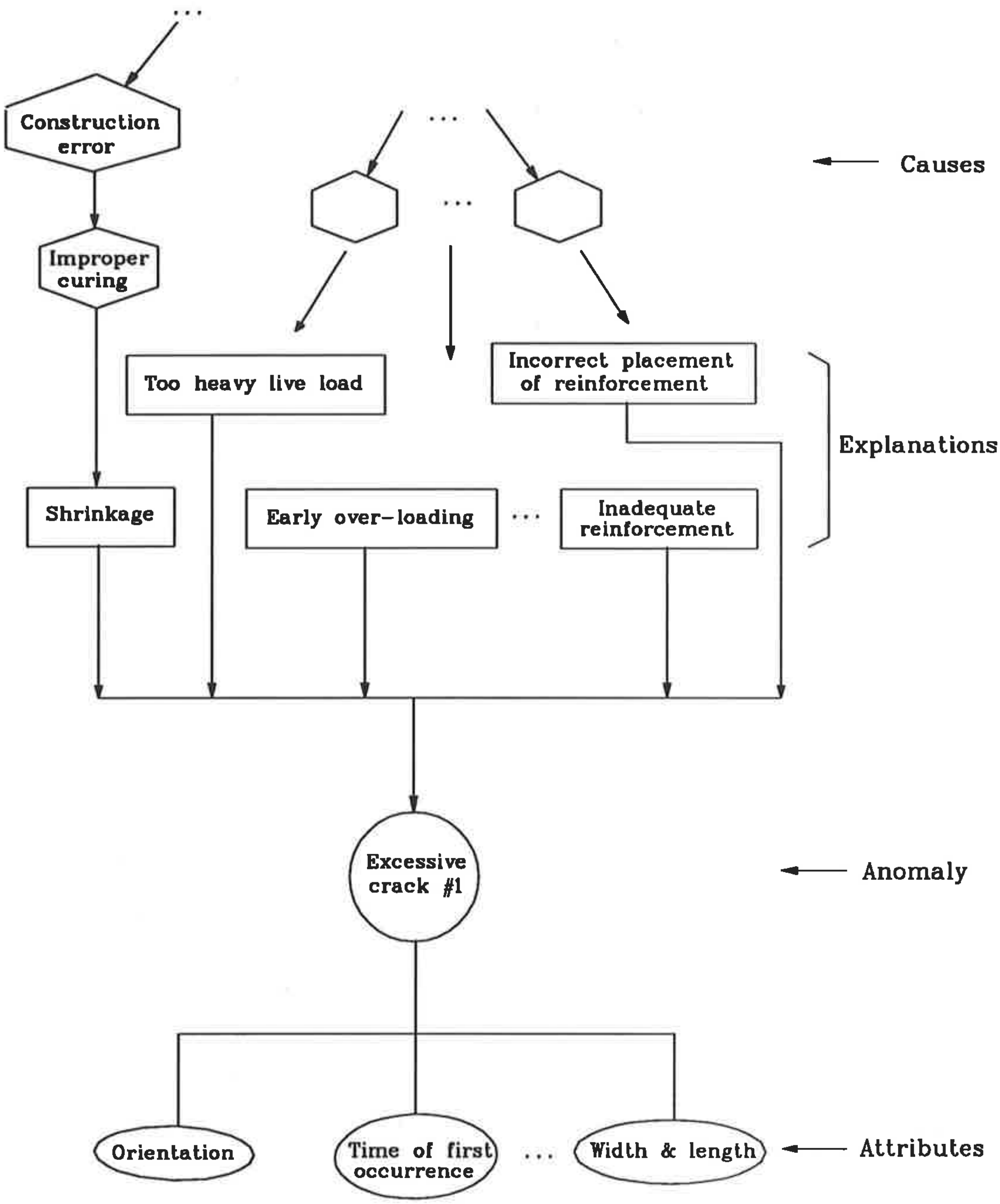


Figure 2.4: An Example of Explanation Tree

polygons. In diagnostic reasoning, confirming the existence or non-existence of these causal factors will be in general very helpful for the identification of the true explanations. The true explanations are those which really caused the occurrence of the observed anomalies. The tree structure thus described can be used to represent available diagnostic information in a well-organised manner, and will be called an **explanation tree** throughout this thesis. For a given pattern of anomalies, the task of diagnosis is to find out a set of true explanations with the help of the explanation tree.

It has to be pointed out that although there is only one anomaly presented in the explanation tree shown in Fig. 2.4, no restriction is placed on the number of anomalies in such a tree. In fact, many diagnostic problems have more than one anomaly presented, and all observed anomalies have to be included in the explanation tree. For each anomaly node, the related explanation nodes have to be as complete as possible in order to avoid missing the true explanations.

At different stages of the diagnostic process, the explanation tree can be modified according to new information available. For example, the causes in polygon nodes do not necessarily need to be exhaustive, and new causal nodes may be added any time when new causal factors are identified. Explanation nodes can also be modified if necessary. For instance, at the early stage of diagnosis, explanations like "inadequate amount of reinforcement" and "wrong location of reinforcement", denoted by e_1, e_2 respectively, can be represented by a single explanation "wrong reinforcement" to reduce the total number of explanation nodes. At a later stage, however, when structural evaluation such as safety assessment has to be carried out using the diagnostic results, the explanation "wrong reinforcement" may need to be replaced by e_1 and e_2. Obviously, the identification of explanation nodes at any stage of the diagnostic process has to relate to the purpose of the diagnosis.

In practice, the explanation tree can be constructed by an engineer who has an understanding of structural behaviour and some previous experience in concrete diagnostics. Of course, the people without such experience, but who are familiar with basic structural analysis can also build the tree, but diagnostic experience would certainly be helpful in creating a complete explanation tree without missing any important explanation nodes. For example, existing experience strongly suggests the possible explanation for diagonal cracking on a wall — *differential foundation settlement.*

With the explanation tree thus constructed for a given pattern of anomalies, it is possible to create a set of hypotheses, and this is described in the following sections.

2.4.3 The Candidate Set of Hypotheses

An **hypothesis** is defined here as *a set of explanations which can fully account for the observed pattern of anomalies.* The generation of an hypothesis from given anomalies can be easily made using an explanation tree. This is demonstrated by an example with the related explanation tree shown in Fig. 2.5. S_1

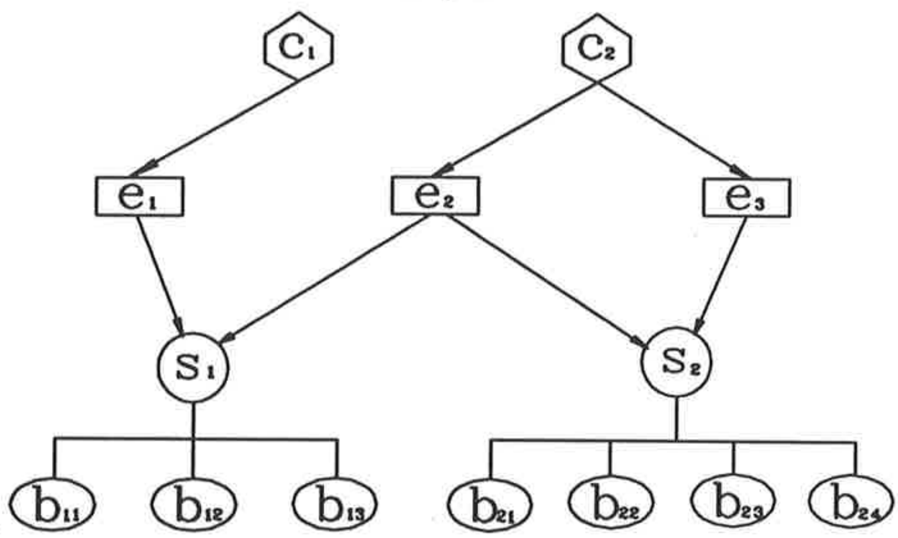


Figure 2.5: The Explanation Tree of An Example

and S_2 are two observed anomalies which can possibly be caused by explanations e_1, e_2 and e_3 with relevant causal relations denoted by directed lines; the three explanations can in turn be caused by other factors/events denoted by $C_1, C_2, \cdots$; some common attributes of S_1 and S_2 are also listed in the tree and denoted by b_{ij}. From the explanation tree, it can be seen that the single explanation e_2 can be responsible for both the two observed anomalies, and hence e_2 is obviously an hypothesis. In addition, multi-explanations such as e_1e_2, e_1e_3 are also feasible hypotheses. Based on the definition of hypothesis given above, all possible hypotheses for S_1 and S_2 can be obtained from Fig. 2.5 as $H_c = (e_2,\ e_1e_2,\ e_1e_3,\ e_2e_3,\ e_1e_2e_3)$.

The set H_c thus generated is exclusive and exhaustive. However, most real problems will have explanation trees much larger than the one in this example, and consequently a complete set H_c may be very large. This can produce difficulties for diagnostic reasoning, since the engineer is usually not very good at handling hundreds of hypotheses at one time. Fortunately, some members in H_c can usually be eliminated when information on relevant attributes and/or causal factors of explanations are available. For example, in Fig. 2.5, if e_2 = "over-loading during construction", and s_1 = "excessive cracking" with its attribute b_{11} = "the time of appearance", e_2 is certainly a valid hypothesis before the value of b_{11} is known. However, if new information reveals that s_1 appeared 5 years after the structure was erected, and there was no cracking observed right after the construction was completed, e_2 can then be eliminated from H_C.

Therefore, the hypotheses in H_c can be updated at each stage of the diagnostic process using new information available. Even at the beginning of diagnosis, information together with relevant experience are usually enough to eliminate some members from H_c, and hence to only consider a limited numer of hypotheses in the diagnostic reasoning. For this reason, the set H_c obtained above is called the *candidate set* of hypotheses. The valid *hypothesis set* at any stage is a subset of H_c, and only contains those elements which are realistic possibilities. The hypothesis set is thus dynamic, and can be obtained from H_c depending on the information available.

2.4.4 Formation of Hypotheses

The valid hypotheses are obtained from the candidate set H_c by making intelligent use of all available information. However, if the data available are not sufficient to eliminate enough elements from H_c, the hypothesis set, if denoted by H, could still be too large to be practically handled. More importantly, at the beginning of the diagnosis, if no particular useful knowledge is gathered, the hypothesis set H would be theoretically equivalent to H_c. In these cases, it is necessary to limit the number of hypotheses to be considered in H, as long as the potential true hypothesis is included in H.

For this purpose, some useful principles and existing experience can be jointly applied. For example, one widely used rule for forming hypotheses in many computer-based diagnostic systems is the so called *Occam's Razor* mentioned on page 34, i.e. a simple solution is better than a complicated one. Considering the example shown in Fig. 2.5 in which the candidate set of hypotheses is $H_c = (H_1, H_2, H_3, H_4, H_5) = (e_2,\ e_1e_2,\ e_1e_3,\ e_2e_3,\ e_1e_2e_3)$. The application of Occam's Razor would result in $H = (H_1) = (e_2)$, since H_1 has the minimum numer of explanations to cover the anomalies S_1, S_2, and hence is simpler than any others. However, this seems too arbitrary, and could lead to oversimplification. In order to avoid missing the true hypothesis in obtaining H from H_C, the following rule can be used as a guide: *the hypothesis containing more explanations is less likely to be true than the one having fewer explanations, unless existing information suggests otherwise.* However, in applying this rule, the hypotheses to be eliminated from H_C have to be determined by careful evaluation of all existing evidence so that the promising members remain in H. Experience can also play an important role for this purpose. For instance, $H_5 = e_1e_2e_3$ may be eliminated in the above example, because past experience indicates it is rare that all possible explanations are true simultaneously.

However, the hypothesis set H thus obtained is not *complete*, and there is a risk of missing the true hypothesis. To guarantee the true hypothesis is included in H, a *pseudo hypothesis* $H^0 = others$ is added to H, which represent all those hypotheses being ignored. In other words, if $H_1, H_2, \cdots, H_l$ are valid hypotheses obtained from the candidate set H_c, the hypothesis set H will be formed as $H = (H_1, H_2, \cdots, H_l, H^0)$. At a later stage of diagnosis, if any revealed evidence tends to eliminate members in $H_1, H_2, \cdots, H_l$, but support some elements in H^0, the set H then has to be revised, and relevant hypotheses previously ignored have to be included. The hypothesis set H created in this way is complete, and can be *considered* exclusive and exhaustive.

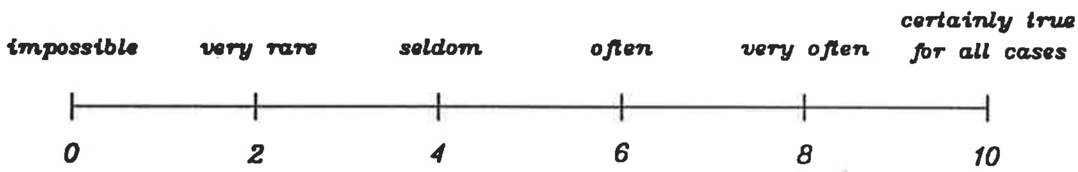
As the process of diagnosis proceeds, and more information is gathered through experiments and other means, the hypothesis set H is usually reduced in size.

With the hypothesis set H thus generated from the given pattern of anomalies, diagnosis must identify the true hypothesis H^* from H. However, until complete information is available, the confirmation of H^* is difficult. It is alternatively useful to rank the members of H in a rational way so that the most likely hypothesis can be obtained under uncertainty. This can be achieved through a procedure of diagnostic reasoning to be described next.

2.4.5 Diagnostic Reasoning

Prior Confidence Factor

To rank the elements in a hypothesis set $H = (H_1, H_2, \cdots, H_n)$ obtained using the procedure described in the last section, before any pertinent information is gathered, the engineer has to estimate his/her degree of belief in how often H_i is true in practice from past experience and existing knowledge. It is convenient to represent this degree of belief by a numerical scale called the **prior confidence factor** denoted by $CF(H_i)$. A real number between 0 and 10 can be assigned to $CF(H_i)$ for each hypothesis of H using a scaler shown below:



where $CF(H_i) = 0$ indicates that the hypothesis H_i is impossible in practice; a value of 10 represents that H_i is true for all cases; and all other possibilities fall between 0 and 10.

For a single-element hypothesis, the assessment of the confidence factor $CF(H_i)$ can be easily made. However, for a multi-explanation hypothesis, such as $H_i = e_j e_{j+1} \ldots e_{j+6}$, the assessment of $CF(H_i)$ may not be so straightforward. For this reason, in addition to relevant experience, the principle used in the hypothesis formation can be applied, i.e. "a hypothesis which contains more explanations is usually less likely to occur in practice than the one which has fewer explanations." For example, for two hypotheses $H_1 = (e_1 e_3 e_4)$ and $H_2 = (e_5)$, $CF(H_1)$ would usually be smaller than $CF(H_2)$.

After $CF(H_i)$ is thus assessed, it can be normalized over all members of H as:

$$P'(H_i) = \frac{CF(H_i)}{\sum_{i=1}^{n} CF(H_i)} \tag{2.18}$$

where $\sum_{i=1}^{n} P'(H_i) = 1$. Since the hypothesis set H is considered exclusive and exhaustive according to its definition, the normalized value $P'(H_i)$ can be loosely interpreted as the subjective probability of H_i. For convenience, $P'(H_i)$ is called the prior probability of H_i.

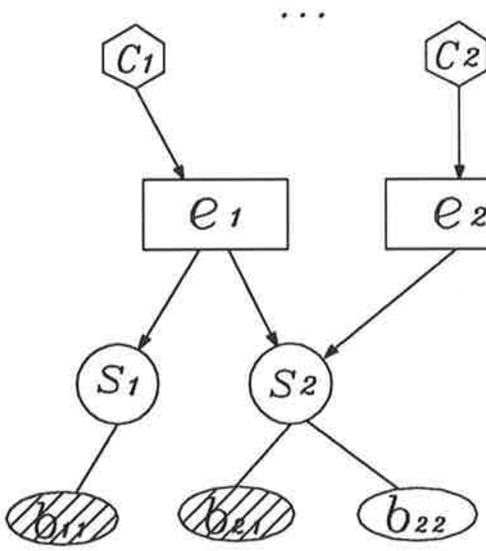


Figure 2.6: The Explanation Tree of An Example

The numerical ranking of H_i in terms of the prior subjective probability thus obtained only reflects our degree of belief in H_i before using any specific information about the observed anomalies and associated attributes. Therefore, it is not sufficient to decide on the truth of H_i purely from $CF(H_i)$. When new knowledge, denoted by ΔI, is gathered through relevant means such as detailed site inspection or testing, our belief in each H_i according to both the prior information and ΔI has to be re-assessed. This can be done using the procedure described below.

Partial Confidence Factors

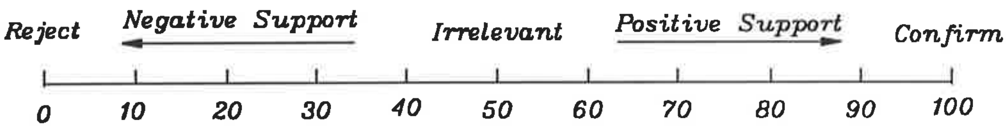
First of all, the general principle for confirming or rejecting an hypothesis used in this thesis is that (1) *an hypothesis H_i, consisting of explanations $e_i \cdots e_{i+k}$, is true if and only if there is sufficient evidence available to support the occurrence of $e_i, \cdots, e_{i+k}$;* **and** *$e_i \cdots e_{i+k}$ can completely explain the attributes and other properties of the observed anomaly pattern;* (2) *H_i is false if there is evidence which can reject the existence of any explanation contained in H_i;* **or** *H_i can not fully explain the observed anomalies and associated properties.*

The above principle can be clearly demonstrated with the assistance of the explanation tree shown in Fig. 2.6 on Page 60, in which, for simplicity, only two possible explanations e_1 and e_2 are considered for the two observed anomalies s_1, s_2 with associated attributes denoted by b_{11}, b_{21} and b_{22}. The hypothesis set is $H = (H_1, H_2) = (e_1, e_1e_2)$. Thus if the information available can confirm the existence of a factor or event C_1 which surely causes the occurrence of e_1, **and** if e_1 can completely explain s_1, s_2 and b_{11}, b_{21}, b_{22}, the hypothesis H_1 can be considered true. On the contrary, if c_1 contradicts the existence of e_1, **or** any revealed attribute tends to reject e_1, then H_1 is false.

In the above example, apparently two types of information are used separately in two different ways, i.e. (1) the information about C_1 is used to confirm the existence of C_1, and this in turn confirms the occurrence of explanation e_1; (2) the information on anomalies S_1, S_2 and their attributes b_{11}, b_{21} and b_{22} is used to explore how well e_1 explains the observed anomalies. The total information available ΔI can thus be divided into two parts: ΔI^+ and ΔI^-. ΔI^+ represents the information on the explanation nodes and their parent-nodes in the explanation tree, and ΔI^- denotes the evidence regarding the anomaly nodes and attribute nodes. For example, in the explanation tree shown in Fig. 2.6, ΔI^+ will represent the knowledge relevant to e_1, e_2 and their further causes such as c_1, c_2, while ΔI^- denotes the information regarding

s_1, s_2 and b_{11}, b_{21}, b_{22}.

To assess the impact of the newly obtained information ΔI on the belief in a hypothesis H_i, two partial confidence factors $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ are introduced. $CF(\Delta I^+, H_i)$ describes how likely explanations contained in H_i really exist according to the information ΔI^+. $CF(\Delta I^+, H_i)$ is assumed to be a real number between 0 and 100. Similarly, $CF(\Delta I^-, H_i)$ is also real number of [0,100], and reflects how well H_i explains the evidence revealed by ΔI^-. These two partial confidence factors can be assessed with the assistance of the numerical scaler shown below:



in which values of 0 and 100 indicate the rejection and confirmation of the explanations in H_i respectively. If the information available is irrelevant to H_i, the confidence factor is then 50. The values of $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ increase in the range of (50,100) with the increasing of the strength of positive support to H_i by ΔI^+ and ΔI^- respectively. On the contrary, $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ decrease in the range of (0,50) with the increasing of the strength of negative support from ΔI^+ and ΔI^- respectively.

Total Confidence Factor

The total strength between H_i and $\Delta I = \Delta I^+ + \Delta I^-$ is then represented by a **total confidence factor** denoted by $CF(\Delta I, H_i)$ which is defined as:

$$CF(\Delta I, H_i) = CF^n(\Delta I^+, H_i) \cdot CF^n(\Delta I^-, H_i) \qquad (2.19)$$

in which $CF^n(\Delta I^+, H_i)$ and $CF^n(\Delta I^-, H_i)$ are obtained by normalizing $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ over all elements in the hypothesis set $H = (H_1, H_2, \cdots, H_n)$

respectively.

Finally, the obtained total confidence factor $CF(\Delta I, H_i)$ is also normalized and denoted by $P(\Delta I, H_i)$:

$$P(\Delta I, H_i) = \frac{CF(\Delta I, H_i)}{\sum_{i=1}^{n} CF(\Delta I, H_i)} \tag{2.20}$$

With prior subjective probabilities $P'(H_i)$ and the normalized total confidence factor $P(\Delta I, H_i)$ thus defined for each hypothesis H_i, the ranking of $H_i \in H$ is represented by the product of these two values, i.e. $P'(H_i) \times P(\Delta I, H_i)$. A loosely defined total subjective probability of H_i can then be evaluated by:

$$P(H_i) = \frac{P'(H_i) \times P(\Delta I, H_i)}{\sum_{i=1}^{n} P'(H_i) \times P(\Delta I, H_i)} \tag{2.21}$$

The obtained $P(H_i)$ represents the degree of truth of H_i based on both the prior information and newly obtained information $\Delta I = \Delta I^+ + \Delta I^-$. In this way, any element in the hypothesis set $H = (H_1, H_2, \cdots, H_n)$ can be ranked by its subjective probability $P(H_i)$. In other words, the higher the subjective probability of a hypothesis H_i is, the more likely H_i is going to be true. The hypothesis H^* which has the highest probability is called the most likely hypothesis.

The procedure of obtaining these subjective probabilities is called **tentative diagnosis**. Although the true hypothesis is yet to be figured out, the tentative diagnosis results in the "best explanation" for the observed anomalies at this stage of the diagnostic process using the information currently available. Further information may also be gathered, and the obtained ranking can be updated using the same procedure. The procedure of this updating may be called **sequential diagnosis**, and is discussed later in Section 2.4.7.

2.4.6 The Bayesian Interpretation of Diagnostic Reasoning

The method of ranking hypotheses described in the last section is in fact a loose application of Bayes' theorem. For a hypothesis set $H = (H_1, H_2, \cdots, H_n)$, H is exclusive and exhaustive. The normalized confidence factors $P'(H_i)$ and $P(\Delta I, H_i)$ are analogous with the prior probability $P'(H_i)$ and conditional probability $P(\Delta I \mid H_i)$ respectively. At the beginning of diagnosis, if there is no specific information available, the ranking of a hypothesis can be represented by $P'(H_i)$. After the new information ΔI is obtained, the posterior probability of H_i can be evaluated by Bayesian theorem as:

$$P(H_i \mid \Delta I) = \frac{P'(H_i) \cdot P(\Delta I \mid H_i)}{\sum_{i=1}^{n} P'(H_i) \cdot P(\Delta I \mid H_i)} \tag{2.22}$$

As can be seen, if $P(\Delta I, H_i)$ substitutes for the term $P(\Delta I \mid H_i)$, the right hand side of Eq. 2.22 equals to that of Eq. 2.21 in the last section. The subjective probability $P(H_i)$ obtained in the last section is thus equivalent of $P(H_i \mid \Delta I)$ in Eq. 2.22.

2.4.7 Sequential Diagnosis

Using the method described in Section 2.4.5, the tentative diagnosis for a given pattern of anomalies can be represented by a set of hypotheses H whose members are ranked in terms of subjective probabilities, $P(H_i)$, for every $H_i \in H$. However, this ranking is obtained using only the information available at the time. When more knowledge is gathered, the subjective probabilities of hypotheses can be updated. In this way, the whole process of diagnosis works iteratively through sequential diagnosis. The updated $P(H_i)$ can be calculated using the same method as in the tentative diagnosis, and is demonstrated here by an example.

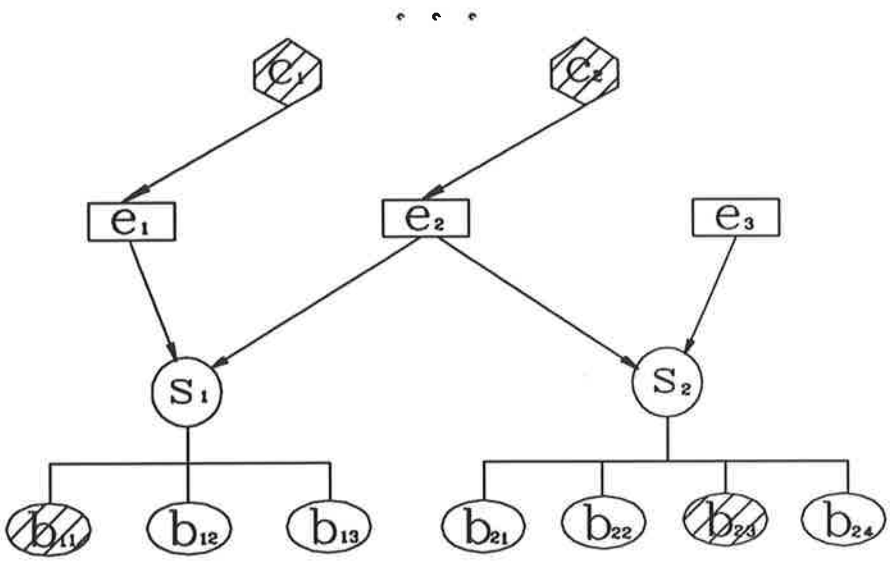


Figure 2.7: An Example of Sequential Diagnosis

The explanation tree of the example is shown in Fig. 2.7, where s_1 and s_2 are two observed anomalies with their associated attributes denoted by b_{ij}; there are three possible explanations denoted by e_1, e_2, e_3 which can cause the occurrence of s_1, s_2. An explanation e_i can also be attributed to further causes such as c_1, c_2. At the beginning of diagnosis, the values of attributes b_{11}, b_{23} are known, and the two causes c_1, c_2 are not revealed until some later stage of the diagnosis. The candidate set of hypotheses is obtained as $H_c = (e_2,\ e_1e_2,\ e_1e_3,\ e_2e_3,\ e_1e_2e_3)$. From past experience and the judgement of the engineer, $e_1e_2e_3$ is not considered likely to be true, and hence is ignored temporarily. The hypothesis set is then $H = (H_1, H_2, H_3, H_4, H^0) = (e_2,\ e_1e_2,\ e_1e_3,\ e_2e_3,\ others)$ at this stage.

Relevant confidence factors and probabilities for the members in H in the first cycle of the diagnostic reasoning are listed in Table 2.1. The prior confidence factor $CF(H_i)$ for each hypothesis before further analysis is assessed as 6, 5, 5, 4, 2 respectively based on how often these hypotheses happen in practice. Since there is no evidence yet available regarding the causal factors C_1, C_2

at this stage, i.e. $\Delta I^+ = \phi$, where ϕ is an empty set, the value of $CF(\Delta I^+, H_i)$ is assessed as 50 for all hypotheses. Suppose the known attributes b_{11}, b_{23} favour explanations e_1 and e_2, and the confidence factors $CF(\Delta I^-, H_i)$ are estimated as 80, 65, 55, 55 and 45 for H_1, H_2, H_3, H_4 and H^0 respectively. The subjective probabilities of H are finally evaluated and given in the last column of Table 2.1. This is the first cycle of the iterative diagnostic process.

Table 2.1: Diagnostic Results in the First Cycle

Hypothesis	$CF(H_i)$	$CF(\Delta I^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_2$	6	50	80	0.267	0.346
$H_2 = e_1e_2$	5	50	65	0.217	0.234
$H_3 = e_1e_3$	5	50	55	0.183	0.197
$H_4 = e_2e_3$	4	50	55	0.183	0.158
$H^0 = others$	2	50	45	0.150	0.065

Table 2.2: Diagnostic Results in the Second Cycle

Hypothesis	$P'(H_i)$	$CF(\Delta I^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_2$	0.346	50	70	0.298	0.443
$H_2 = e_1e_2$	0.234	50	70	0.298	0.300
$H_3 = e_1e_3$	0.197	50	35	0.149	0.126
$H_4 = e_2e_3$	0.158	50	35	0.149	0.101
$H^0 = others$	0.065	50	25	0.106	0.030

After the first cycle of reasoning, further information ΔI reveals other unknown attributes of S_1, S_2, and results of updated reasoning using ΔI are given in Table 2.2. Firstly, the subjective probability $P(H_i)$ obtained in the first cycle is used here as the prior probability $P'(H_i)$. Other confidence factors are then re-assessed according to ΔI. Suppose the revealed attributes favour the explanation e_2, but tend to negatively support the explanation e_3. The re-assessed confidence factors of $H_3 = e_1e_3$ and $H_4 = e_2e_3$ are thus lower than

50. The updated values of $P(H_i)$ are listed in the sixth column of Table 2.2. This is the second cycle of the diagnostic process.

Further investigation confirms the existence of c_1 which slightly favour the explanation e_1, but is irrelevant to e_2 and e_3. Based on this information, relevant subjective probabilities are updated and listed in Table 2.3.

Table 2.3: Diagnostic Results in the Third Cycle

Hypothesis	$P'(H_i)$	$CF(\Delta I^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_2$	0.443	50	50	0.185	0.408
$H_2 = e_1e_2$	0.300	60	50	0.222	0.332
$H_3 = e_1e_3$	0.126	60	50	0.222	0.139
$H_4 = e_2e_3$	0.101	50	50	0.185	0.093
$H^0 = others$	0.030	50	50	0.185	0.028

In the fourth round of diagnosis, the information available confirms the existence of c_2 which strongly supports e_2 but tends to negatively support e_1. Therefore, newly assessed values for $CF(\Delta I^+, H_i)$ are 80, 20, 20, 50 and 20 respectively. The updated probabilities are listed in Table 2.4.

At this stage, it is obvious that the hypothesis $H_1 = e_2$ is much more likely

Table 2.4: Diagnostic Results in the Fourth Cycle

Hypothesis	$P'(H_i)$	$CF(\Delta I^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_2$	0.408	85	50	0.436	0.704
$H_2 = e_1e_2$	0.332	20	50	0.102	0.134
$H_3 = e_1e_3$	0.139	20	50	0.102	0.056
$H_4 = e_2e_3$	0.093	50	50	0.256	0.094
$H^0 = others$	0.028	20	50	0.102	0.011

to be true than others using the information available. However, the engineer still has the choice of conducting more tests or taking e_2 as the acceptable explanation. Actually, the diagnostician has to make a similar decision at each round of diagnosis before the true hypothesis is confirmed. Some issues regarding this will be further discussed in the next section.

2.4.8 Stopping Rule and Test Selection

Based on our approach, the process of diagnostic reasoning can continue from stage to stage until eventually a true hypothesis is identified. A practical question is then when to stop gathering more information and to make a final conclusion so that the iterative diagnostic procedure can terminate. Obviously, if this decision is made too early, risks of making errors are high, because diagnostic results are usually not very reliable in the early stages due to the limited knowledge available. On the other hand, if the decision is made too late, financial resources may be wasted since it is usually expensive to gather ever more information. Therefore, any stopping rule has to consider both of these two factors.

However, the risk of making errors in the diagnosis has to be evaluated with regard to the impact of accepting a wrong hypothesis on selecting a course of action for the structure and related consequences, because the diagnostic results are to be used in procedures of condition evaluation and decision-making for remedy planning. Therefore, the rule for stopping further information-gathering for the purpose of diagnosis has to consider the results of condition evaluation and possible consequences of relevant actions taken. For these reasons, such a rule simply can not be determined at this stage. Instead, it will be developed and integrated into a comprehensive decision-making procedure to be described in Chapter 4.

On the other hand, the diagnostic procedure can be simply stopped in a special case where the true hypothesis is found. Mathematically, this means that there is a hypothesis whose probability reaches unity. From a more realistic and practical point of view, if the probability of the most likely hypothesis H^* is high enough, the engineer would take H^* as the acceptable set of explanations for the observed anomalies. For this purpose, if there is an acceptable value of probability P^H_{acpt} available, the diagnostic procedure can be stopped when the probability of H^* is not less than P^H_{acpt}.

The use of P^H_{acpt} to stop the diagnostic procedure means that the engineer will accept the risk of $1 - P^H_{acpt}$, regardless of what would happen if H^* is false. Therefore, the value of P^H_{acpt} should be reasonably conservative. P^H_{acpt} can be determined by the engineer for specific problems by taking into account of factors such as *the importance of the structure*. The value around 0.90 may be suggested here from the experience of using the method of diagnostic reasoning described in this chapter.

Since the summation of probabilities of all members in the hypothesis set $H = (H_1, H_2, \cdots, H_n)$ has to be unity, the use of a fixed value of P^H_{acpt} usually leads to the fact that it is very difficult for a $H^* \in H$ to satisfy the condition of $P(H^*) \geq P^H_{acpt}$, if n is large. This is reasonable and conservative. According to our diagnostic reasoning, the number of valid hypotheses is closely related to the amount of information available. Limited information would result in a large number of hypotheses, and certainly is not sufficient to stop the sequential diagnosis.

The criteria for terminating the iterative diagnostic procedure have been discussed. In short, if the probability of the highest ranked hypothesis H^* reaches the acceptable value P^H_{acpt}, H^* can be considered true. Otherwise, a more sophisticated criterion has to be used to decide whether it is necessary to continue the diagnosis by gathering more data or not. Such a criterion is elaborated through a decision-making procedure to be described in Chapter 4.

If doing more tests is adopted, it is necessary to determine which test is to be conducted next. In most of those existing diagnostic systems reviewed in the early part of this chapter , test selection is usually determined by the incurred cost and the probability changes of interested hypotheses before and after the test. However, the purpose of testing here is to reduce the uncertainty and hence to decrease the risk of making errors in the selection of the repair method. Therefore, the most appropriate test has to be determined with regard to both the financial cost and the consequence of taking a course of action. For this reason, as for the stopping rule, the choice of a test is also to be addressed in the decision-making procedure described in Chapter 4.

It can be seen from the above discussion that different procedures of diagnosis, condition evaluation and decision-making involved in the process of treating existing structures are highly inter-related, and the diagnosis is usually carried out before the others. A final decision regarding what to do about the structure has to be made based on the results obtained from all these procedures. The inter-relationship among the involved procedures will be described further in the overall process later in Chapter 5.

2.5 Summary of This Chapter

A method for diagnosing potentially defective concrete buildings has been developed using the procedure of *hypothesis-and-test*. A set of hypotheses $H = (H_1, H_2, \cdots, H_n)$ is firstly created for a given pattern of observed anomalies with the help of an *explanation tree*, and each hypothesis H_i is then ranked by its subjective probability assessed using the information available. The approach allows for the use of the engineer's personal experience as well as the pertinent data on the given problem. The procedure works iteratively with the subjective probabilities in each cycle updated according to the new knowledge obtained from relevant testing. The process becomes complete when a simple stopping rule is satisfied, or alternatively when a terminal decision is made regarding the remedy of the structure. The diagnostic results obtained in this chapter will be part of the input to the procedure for condition evaluation described in the next chapter.

Chapter 3

Condition Evaluation of Existing Concrete Buildings

3.1 Introduction

Condition evaluation is the process of determining the structural adequacy of an existing building or component for its intended use. Depending on the relevant structural and functional requirements, this process usually includes the assessment for safety, serviceability, durability, and other matters such as appearance.

In the design of a new building, structural requirements are fulfilled by complying with specifications in relevant codes. Since the real structure is never exactly the same as its original design, the condition evaluation of an existing building has to be carried out for its present physical state, and design codes are not necessarily sufficient for this purpose. Although some standards such as *Guideline for Structural Condition Assessment of Existing Buildings* (ASCE, 1991) have been published in response to the increasing need to deal with existing defective concrete structures, rigorous assessment codes are still unavailable and current practice in condition evaluation is very much based on the individual's experience and expertise. Occasionally, different interpretations of the assessment results can be obtained somewhat arbitrarily by different engineers, and it makes the situation even worse when these inconsistent interpretations are influenced by factors such as financial costs and political pressure. It is the purpose of this chapter to develop a systematic approach for evaluating existing concrete buildings using not only the engineer's experience but also available theories and sound methodologies. The results from such an approach are to be suitable for use in relevant decision-making techniques, and the methods adopted are to be as simple and realistic as possible, yet able to make use of all sorts of information available, as well as the diagnostic result obtained from Chapter 2.

For this purpose, procedures for assessing safety, serviceability and durability of existing buildings will be described in separate sections with particular emphasis on the safety issue, simply because safety is likely to dominate decision-making in regard to maintenance and repair. To study the problem of structural safety, sound theories are available, and a vast amount of references can be found in the literature. Therefore, before the procedures for condition evaluation are proposed, a brief literature review will be carried out on the theory of structural safety and its relevant application. On the other hand, literature available on the serviceability and durability assessment of existing structures, which is very limited, was already briefly reviewed in Chapter 1, and will not be discussed in this chapter.

For the analysis of structural safety, reliability theory has been one of the most popular methods, and is accepted by more and more people in the engineering profession. Fuzzy set approaches have also been introduced into engineering in recent years, and could be a potentially useful tool. The following literature review therefore focuses on these two theories.

3.2 Structural Reliability Analysis

In the conventional design procedure, the requirement of structural safety is achieved by assuring that the "maximum" load effect is less than the "minimum" resistance: in this way, the structure is considered to be safe. However, absolute limits on the "largest" load and the "weakest" strength do not exist because of the large uncertainties involved. It has been widely recognized that random variations in structural resistance and load effects are important sources of uncertainties and need to be treated in a rational way in designing a structure. For this reason, structural reliability theory (ASCE, 1972; Thoft-Christensen, 1982; Melchers, 1987) based on the concept of probability has been well developed over the past few decades, and structural safety can now be quantified in terms of probability. This theory not only allows checks to be made of the safety of structures; it also provides a basis for rational probabilistic design. In this section, the theory of structural reliability is firstly reviewed, and the application of this method to the assessment of existing structures is then discussed.

3.2.1 Classical Reliability Method

The classical method for structural reliability analysis is largely derived from the work of Freudenthal (1956; 1966) and Borges (1971). For a structural member or a system drawn from a defined population, the resistance R and load effect S are considered to be random variables to account for the variations in R and S. Failure occurs when S exceeds R, and the probability of failure, denoted by P_f, is a very useful quantity for measuring structural safety. When the probability distributions of R and S are available, P_f can be defined and evaluated through a *limit state function* G. For example, if G is defined as $G = R - S$, the event $G < 0$ will represent failure, and the failure probability can be evaluated as:

$$P_f = P(G < 0) = P(R < S) \tag{3.1}$$

where $G = 0$ represents the failure boundary. Related reliability is then defined as $\mathcal{R} = 1 - P_f$.

To evaluate the failure probability P_f, probability density functions of resistance R and load effect S are plotted in Fig. 3.1, and denoted by $f_R(r)$ and $f_S(s)$ respectively. If the x axis represents the resistance r and the load effect

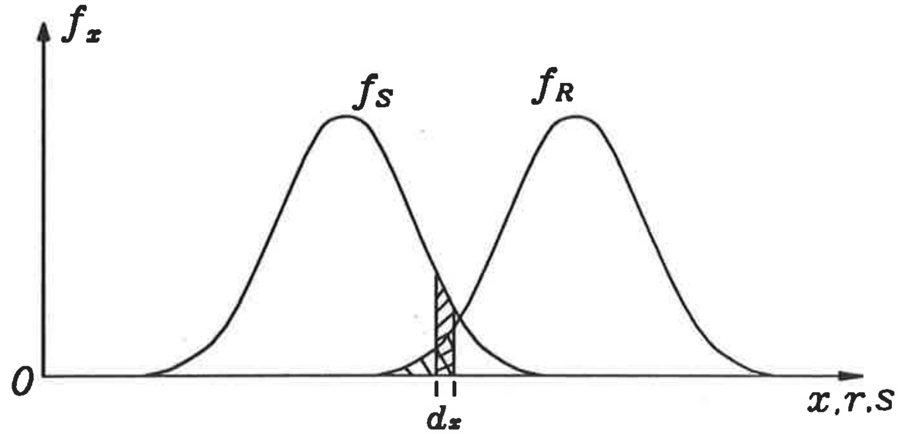


Figure 3.1: Definition of the Failure Probability

s, then for a given value of x and infinitessimal limit length dx, the probability of the event that the load effect S lies in the domain $[x, x + dx]$, is approximately $f_S(x)dx$, and the probability of another event that resistance R is less than the applied load effect $s = x$, is $F_R(x)$. If R and S are assumed statistically independent, the probability that these two events occur simultaneously is $F_R(x)f_S(x)dx$. The failure probability P_f can be obtained by integrating $F_R(x)f_S(x)dx$ over the whole axis:

$$P_f = \int_{-\infty}^{+\infty} F_R(x)f_S(x)dx \tag{3.2}$$

In practice, however, the limit state function G can not always be expressed in terms of only R and S. In fact, G is usually a function of a set of variables, denoted by $X = (X_1, X_2, \cdots, X_n)$, where $X_1, \cdots, X_n$ are *fundamental and functionally independent variables that may be used in the structural design process*, such as material properties, dimensions and applied loads. These variables are so-called *basic variables.* For a particular limit state with n basic variables involved, the function G can be defined in such a way that $G(X_1, X_2, \cdots, X_n) < 0$ represents the failure. The probability of failure is then evaluated by:

$$P_f = \int_{G(X)<0} f_X(x)dx \tag{3.3}$$

where $f_X(x)$ is the joint probability distribution function of the basic variables $X = (X_1, \cdots, X_n)$. To clearly understand Eq. 3.3, it is helpful to point out that the random variables X form an n-dimensional space, and the failure equation $G(X) = 0$ which defines a *failure surface* divides the whole space into a "safe region" and a "failure region". P_f obtained by Eq. 3.3 thus represents the weighted volume of the failure region.

Although classical reliability theory is clear in concept, the evaluation of Eq. 3.3 needs multiple integrations and a closed-form solution does not exist in most cases. For this reason, numerical methods are usually used to evaluate the failure probability, and Monte-Carlo simulation is widely adopted for this purpose in practice.

3.2.2 Monte-Carlo Simulation Technique

This technique was probably first used to study structural safety by Warner and Kabaila (1968), and its main idea is as follows. If the failure criterion is defined by the limit state function $G(X)$ as:

$$G(X)\begin{cases} >0 & safe \\ =0 & boundary \\ <0 & failure \end{cases}$$

the failure probability $P_f = P(G < 0)$ can be evaluated by carrying out the following steps:

1. Generating a set of values randomly for the basic variables $(X_1, \cdots, X_n)$ according to appropriate probability density functions;
2. Evaluating function $G(X)$ using the values of X obtained in step 1;
3. Repeating steps 1 and 2 many times until a sufficiently large sample size is achieved;
4. Estimating the probability of failure P_f:

$$P_f = \frac{1}{N}\sum_{i=1}^{N} I[G(X_1, X_2, ..., X_n)] \tag{3.4}$$

where N is the number of cycles; $I[G(X_1, X_2, ..., X_n)]$ is a function which takes 1 for $G < 0$ and 0 for $G \geq 0$.

The above procedure is called the conventional Monte-Carlo technique, and is widely used in structural reliability analysis. For example, Allen (1970) used the method to study the ultimate strength and ductility ratio of reinforced concrete beams. Grant *et al.* (1978) conducted a comprehensive study of the strength of concrete columns using the Monte-Carlo technique. Other similar works are such as those by Ramsay *et al.* (1979) and by Monnier and Schmalz (1981).

Although the conventional Monte-Carlo technique proves very useful in many cases, the method is usually time consuming in order to evaluate a reliable P_f. It is reported (Melchers, 1988) that the error involved in P_f from the procedure is proportional to $N^{-\frac{1}{2}}$, where N is the number of samples. Most structures are designed with a very low failure probability, say less than 10^{-4}, so that if the error is controlled to be less than 10^{-5}, at least 10^{10} repetitions are needed. This is too time consuming even for a high speed computer, especially when the structural analysis procedure is itself complicated and time consuming. To overcome this difficulty, a more efficient algorithm can be adopted (Warner and Kabaila; 1968), and a recently developed smart Monte-Carlo technique has been suggested by Melchers (1988) in which the required number of repetitions is greatly reduced, while the accuracy of P_f remains.

Although the classical method of structural reliability discussed above is conceptually important, the computations required for obtaining P_f make the method too complicated for practical purposes. Also, the joint probability distribution function $f_X(x)$ in Eq. 3.3 is usually not available in practice. In recognition of this difficulty, the so-called *First Order Second Moment* method has been developed as a means of using probabilistic methods in dealing with structural safety, while remaining simple in application. This approach is reviewed next.

3.2.3 First Order Second Moment Method

For the random resistance R and load effect S with means μ_R, μ_S and standard deviations σ_R, σ_S respectively, failure occurs when the function $G = R - S$ is less than zero. If we consider G itself as a random variable, its distribution can be obtained via R and S as illustrated in Fig. 3.2. In the absence of information on distributions of R and S, it is difficult to evaluate the probability of failure P_f. However, the quantity

$$\beta = \frac{\mu_{R-S}}{\sigma_{R-S}} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{3.5}$$

defined by only the mean and deviation of $G = R - S$ is a very useful measure of the degree of safety or reliability of a structure, and is called the **safety**

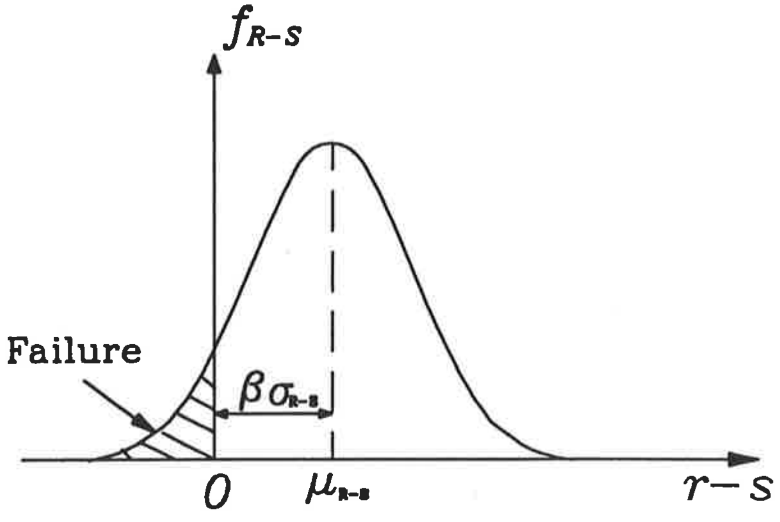


Figure 3.2: Definition of Safety Index β

index or **reliability index** (Cornell, 1969; Ravindra, *et al.* 1969). Referring to Fig. 3.2, β is the number of standard deviations of $(R-S)$ between its mean and zero. If the mean μ_{R-S} is fixed, the value of β is inversely proportional to σ_{R-S}, i.e. the smaller the uncertainty is, the safer the structure is. The safety level of a structure is thus very much related to the dispersions of R and S. If the full distributions of R and S are known, for any given value of β, the failure probability P_f can be calculated. Because only first and second moments of random variables involved are used in estimating β, this approach is referred to as the *second moment method.*

The above procedure treats an idealized case where the statistical properties of R and S are known, and the failure function is linear. In more general cases, G may be a complicated non-linear function of a set of randomly varying basic variables, i.e. $G = G(X_1, X_2, ..., X_n)$, and it is then impossible to use Eq. 3.5 to evaluate β. Therefore, in order to estimate the mean and variance of G, an approximate linearization procedure can be used to expand the function G in a Taylor series at the mean values of $X_1, X_2, ..., X_n$ and ignoring the higher order terms:

$$G \approx G(\mu_{X_1}, \mu_{X_1}, ..., \mu_{X_n}) + \sum_{i=1}^{n} \frac{\partial G}{\partial x_i} \frac{(x_i - \mu_{X_i})}{1!} \tag{3.6}$$

$$\mu_G \approx G(\mu_{X_1}, \mu_{X_1}, ..., \mu_{X_n}) \tag{3.7}$$

$$\sigma_G^2 \approx \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial G}{\partial x_i} \frac{\partial G}{\partial x_j} Cov(X_i, X_j) \tag{3.8}$$

where μ and σ are relevant mean and standard deviation respectively; $Cov(X_i, X_j)$ is the *covariance* of X_i and X_j. If the basic variables are assumed to be statistically independent, Eq. 3.8 becomes

$$\sigma_G^2 \approx \sum_{i=1}^{n} (\frac{\partial G}{\partial x_i})^2 \sigma_{X_i}^2 \tag{3.9}$$

With the mean and deviation of G approximately estimated in this way, the reliability index β is calculated by:

$$\beta = \frac{\mu_G}{\sigma_G}. \tag{3.10}$$

Since the approximation used in Eq. 3.6 only considers the first order terms in the expanded Taylor series, the procedure to evaluate β thus discussed is then called *first order second moment* (FOSM) method. Although the approximate linearization simplified the evaluation of β, errors are unfortunately induced by ignoring the second-order and higher terms in the Taylor expansion. On the other hand, the safety index so obtained is not invariant for different definitions of the function G of the same problem. For example, consider the following failure function:

$$G = A \times f_{sy} - P \tag{3.11}$$

which applies to a simple steel member in axial tension. The variables are independent and we take the following given data:

$$P = 100 \qquad (constant)$$

$$\mu_A = 50 \qquad \sigma_A = 8$$

$$\mu_{f_{sy}} = 10 \qquad \sigma_{f_{sy}} = 2$$

the mean and deviation of G can then be obtained using Eq. 3.7 and Eq. 3.9 as:

$$\mu_G = 50 \times 10 - 100 = 400$$
$$\frac{\partial G}{\partial A}|_{A=\mu_A,\, f_{sy}=\mu_{f_{sy}}} = \mu_{f_{sy}} = 10$$
$$\frac{\partial G}{\partial f_{sy}}|_{A=\mu_A,\, f_{sy}=\mu_{f_{sy}}} = \mu_A = 50$$
$$\sigma^2 = 10^2 \times 8^2 + 50^2 \times 2^2 = 16400$$
$$\beta = \frac{400}{\sqrt{16400}} = 3.12$$

However, the failure function can also be rewritten as $G = f_{sy} - \frac{P}{A}$, and the safety index β' based on this definition is calculated as:

$$\frac{\partial G}{\partial A}|_{A=\mu_A,\, f_{sy}=\mu_{f_{sy}}} = \frac{P}{A^2} = \frac{100}{2500} = 0.04$$
$$\frac{\partial G}{\partial f_{sy}}|_{A=\mu_A,\, f_{sy}=\mu_{f_{sy}}} = 1$$
$$\mu_G = 10 - \frac{100}{50} = 8$$
$$\sigma^2 = 0.04^2 \times 8^2 + 1^2 \times 2^2 = 4.1$$
$$\beta' = \frac{8}{\sqrt{4.1}} = 3.95$$

Obviously there is a considerable difference between β and β' for the same problem. This is a serious drawback of the method. To solve this problem, a significant contribution was made by Hasofer and Lind (1974) to achieve invariance in β while maintaining the advantage of the method. The improved method is also referred to as the *advanced FOSM method*, and is discussed in the following section.

3.2.4 Advanced First Order Second Moment Method

In the previous discussion, basic variables were assumed to be statistically independent. For more general cases where the X_i terms are statistically correlated, an orthogonal transformation (Hasofer and Lind, 1974) can be carried out so that the variables X are replaced by a new set of variables, $Y = (Y_1, \cdots, Y_n)$, which are uncorrelated. In addition, the new basic variables are further transformed so that they are comparable in the same coordinate system, i.e.

$$Z_i = \frac{Y_i - \mu_{Y_i}}{\sigma_{Y_i}} \tag{3.12}$$

where μ_{Y_i} and σ_{Y_i} are the mean and standard deviation of Y_i respectively. As mentioned previously, for a limit state function $G(X_1, \cdots, X_n)$, the failure surface, defined by $G = 0$, will divide the n-dimensional space of basic variables into a safe region and a failure region. Thus under these transformations, Hasofer and Lind (1974) showed that the shortest distance from the origin to the failure surface in the transformed coordinate system of $Z_1, \cdots, Z_n$ is equivalent to the reliability index β defined in the FOSM method, i.e. $\beta = \min(\sum_{i=1}^{n} Z_i^2)^{\frac{1}{2}}$, if the function G is linear. If the failure boundary is non-linear, a linearization similar to the Taylor expansion used in the FOSM method can be made. The two dimensional case is illustrated in Fig. 3.3, where (z_1^*, z_2^*) is the so-called *design point* or *checking point* (Paloheimo and Hannus, 1974), which is located at the point where the failure is most likely to occur, and $G(z_1^*, z_2^*) = 0$.

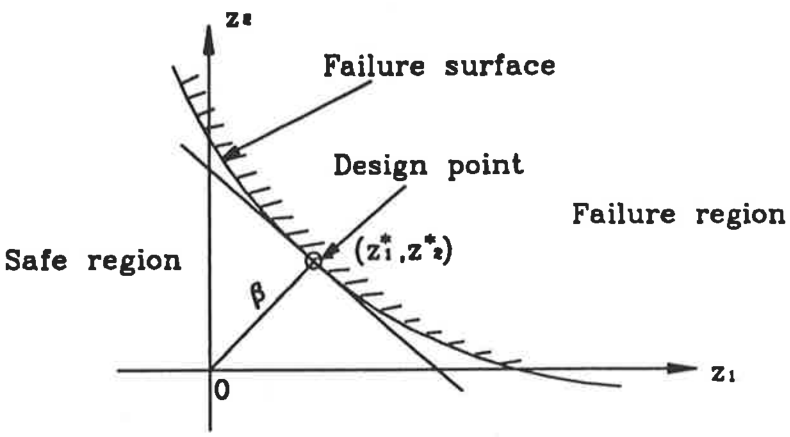


Figure 3.3: Definition of Hasofer-Lind's Safety Index

It should be noted that the Taylor expansion of $G(X)$ is made here at the design point $(x_1^*, \cdots, x_n^*)$. The error due to this approximation is very small in comparison with that in the FOSM method in which the same expansion is made at the mean point $(\bar{x}_1, \cdots, \bar{x}_n)$. The design point can be found through an iterative procedure:

$$x_i^* = \mu_{X_i} - \alpha_i \beta \sigma_{X_i} \tag{3.13}$$

where α_i is the so-called *sensitivity factor* for X_i and can be evaluated by:

$$\alpha_i = \frac{G_i'(x^*)\sigma_{X_i}}{[\sum_{j=1}^{n}(G_j'(x^*)\sigma_{X_j})^2]^{\frac{1}{2}}} \tag{3.14}$$

in which $G_i'(x^*) = \frac{\partial G}{\partial x_i}$ evaluated at the design point $x^* = (x_1^*, \cdots, x_n^*)$.

The safety index β thus obtained from the advanced FOSM method will have an exact relation to the probability of failure P_f, if the failure function $G(X)$ is linear and the basic variables are normally distributed, i.e.

$$P_f = \Phi(-\beta) \tag{3.15}$$

where Φ is the normal cumulative distribution function. However, in more general cases, some of the basic variables are non-normal. It seems that modifications are needed so that the calculated β can be related to P_f, at least approximately. For this purpose, a further improved FOSM method, called the ***approximate full distribution approach*** can be used, in which the full distributions of each basic variable (if available) can be used in evaluating β using the advanced first order second moment method. For example, a non-normal random variable X_i with density and cumulative distribution functions f_{X_i} and F_{X_i} respectively can be approximately transformed into a normally distributed variable by letting the values of f_{X_i} and F_{X_i} equal the values of φ and Φ at the design point, i.e.:

$$F_{X_i}(x_i^*) = \Phi(\frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}}) \tag{3.16}$$

$$f_{X_i}(x_i^*) = \frac{1}{\sigma_{X_i}}\varphi(\frac{x_i^* - \mu_{X_i}}{\sigma_{X_i}}) \tag{3.17}$$

where Φ and φ are the cumulative and density functions of a unit normal distribution respectively; x_i^* is the ordinate of the design point; μ_{X_i} and σ_{X_i} are the mean and standard deviation of the new normal variable, and are to be evaluated. From Eq. 3.16 and Eq. 3.17, we can get:

$$\sigma_{X_i} = \frac{\varphi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(x_i^*)} \tag{3.18}$$

$$\mu_{X_i} = x_i^* - \Phi^{-1}[F_{X_i}(x_i^*)]\sigma_{X_i} \tag{3.19}$$

After μ_{X_i} and σ_{X_i} are obtained, the same procedure as in the advanced FOSM method can be used to calculate β value.

3.2.5 Brief Summary of Structural Reliability Theory

By treating parameters in structural design as random variables, the concept of structural safety is changed from the traditional "absolute" approach to the modern statistical approach whereby uncertainties encountered are considered in a rational way. With the approximate FOSM method available, structural reliability now serves not only as the theoretical foundation for safety analysis, but also as a tool for probabilistic design (Ang, *et al.* 1974; Ravindra, *et al.* 1974) and reliability-based code implementation (CIRIA, 1977; Ellingwood, *et al.* 1980; Leicester, *et al.* 1986).

As a useful tool to study structural safety, reliability theory has also been applied to evaluate existing structures. This will be reviewed in the next section.

3.3 Reliability Analysis Relevant to the Evaluation of Existing Structures

One of the differences between an existing structure and a future structure is the different amount of information available at design and re-design (assessment). This has been summarized by Tichy (1986), and is re-stated in Table 3.1.

Table 3.1: Differences Between Existing and Future Structures

Object of information	Existing structures	Future structures
Location	Known exactly	Often unknown, well known in many cases
Age and lifetime	Age known with some uncertainty, residual lifetime is estimated	Lifetime is always estimated
Environment	Known	Unknown
Loads	Types and intensities are usually well known, with some uncertainties	Estimated with uncertainties, except for particular cases
Material properties	Known with some uncertainties	Estimated with uncertainties
Subsoil	Known with uncertainties	Estimated with uncertainties
Use and purpose	Known	Estimated

As can be seen from the table, there are potentially many more data available for an existing structure than for a future structure. New information on a specific existing structure can usually be gathered from experiments during the assessment process. For this reason, many approaches in the literature have employed Bayes' theorem to revise the reliability of existing structures using the additional data.

3.3.1 Safety Revision Using Bayesian Updating

For a future structure, the intended nominal safety of the original design can be evaluated using any standard method of reliability analysis. For this purpose, properties of relevant basic variables and limit state function are usually estimated using both subjective judgement and test data available on the whole population of a specific kind of structure. The re-evaluation of the reliability of an existing structure at the time of assessment can be made by updating the probability distributions of those basic random variables in the limit state function $G(X_1, \cdots, X_n)$, and/or by improving the function G itself, considering the real physical state of the structure using new information acquired in the process of investigation.

The updating of the distributions of X can be made in two ways (CIRIA, 1977; Bosshard, 1979). For a random variable X_i, its true mean μ_{X_i} and standard deviation σ_{X_i} are usually unknown. However, if testing is conducted on a sample, the sample mean and standard deviation can be evaluated by:

$$\bar{x}_i = \frac{1}{n} \sum_{j=1}^{n} x_{ij} \tag{3.20}$$

$$s_{X_i}^2 = \frac{1}{n-1} \sum_{j=1}^{n} (x_{ij} - \bar{x}_i)^2 \tag{3.21}$$

where n is the sample size. If n is sufficiently large, a convenient way is then to let

$$\mu_{X_i} = \bar{x}_i \quad and \quad \sigma_{X_i} = s_{X_i} \tag{3.22}$$

However, this is usually not the case, and the sample size may be very limited, especially for existing structures, where any test is supposed not to damage the structure itself.

Another way to update the distribution of X_i is by means of Bayes' theorem. In this method, it is assumed that the mean and deviation of X_i can never be known exactly, and μ_{X_i}, σ_{X_i} themselves are random variables. When more and more data are available, the distributions of μ_{X_i} and σ_{X_i} will closely approach their true values. Generally, assuming that θ is a parameter in the distribution function of the basic variable X_i, such as μ_{X_i}, the distribution of θ after the test can be updated by:

$$f''(\theta) = kL(\theta)f'(\theta) \tag{3.23}$$

where $f'(\theta)$ and $f''(\theta)$ are the prior and posterior distributions of θ respectively; $L(\theta)$ is the so-called likelihood function, i.e. the probability of obtaining the specific data observed for a given value of θ, namely $P(data \mid \theta)$; k is a constant. The predictor distribution of X_i is obtained by:

$$f_{X_i} = \int_{-\infty}^{+\infty} f_{X_i}(x_i \mid \theta) f''(\theta) d\theta \tag{3.24}$$

After the distributions of basic variables are thus updated, safety revision of an existing structure can be easily carried out by calculating the reliability index β using the First Order Second Moment method (Diamantidis, 1987). For this purpose, it is well known that for a limit state function $G(X)$, there is a *design point* $X^* = (x_1^*, \cdots, x_n^*)$ at which failure is most likely to occur. The design value x_i^* for each random variable X_i with distribution function F_{X_i} can be defined by the following equation:

$$x_i^* = F_{X_i}^{-1}[\Phi(-\alpha_i \beta)] \tag{3.25}$$

where Φ is the standard normal distribution function, and α_i is a so-called *sensitivity* or *importance factor*. If the distribution of the basic variable X_i is updated from $F_{X_i}(x_i)$ to $F'_{X_i}(x_i)$, a new design value $x_i'^*$ can be defined such that:

$$F'_{X_i}(x_i'^*) = F_{X_i}(x_i^*) \tag{3.26}$$

The reliability change, denoted by $\Delta\beta_i$, due to the change of statistical properties of X_i can be evaluated by:

$$\Delta\beta_i = \alpha_i[\Phi^{-1}(F'_{X_i}(x_i'^*)) - \Phi^{-1}(F'_{X_i}(x_i^*))], \tag{3.27}$$

which represents the increase or decrease in safety compared to the original reliability index β, provided that the importance factor α_i remains the same.

With the above procedure available for evaluating the relative change in the reliability index β of a strucrture, Diamantidis (1987) suggested a number of steps to be adopted in the assessment of existing structures:

(1) quantification of the additional information based on the inspection results;
(2) identification of the major risk items;
(3) definition of risk scenarios;
(4) formulation of limit states;
(5) assessment of the relative change in reliability compared to the originally accepted safety goals;
(6) evaluation of the cost-effectiveness of possible courses of action;

In addition to the revision of statistical properties of basic variables, in some cases, the reliability of an existing structure can also be updated by the use of the recorded structural performance using Bayes' theorem. For this purpose, the parameter θ discussed in Eq. 3.23 can be generalized to denote a parameter in any mechanistic model, and its statistical properties can be updated from observations of the model. For example, Tang (1981) used this approach to refine the analysis of the settlement of a gravity platform from the recorded platform performances. The equation for predicting the amount of settlement was expressed by $\rho(t, \theta)$, in which t represents time, and θ is a random variable. According to the in-situ measured settlement in the history, the posterior distribution of θ, denoted by $f''(\theta)$, can be obtained from Eq. 3.23. If the maximum permissible settlement is d, the updated probability of satisfying the settlement requirement is then given by:

$$P''(\rho < d) = \int_{-\infty}^{+\infty} P(\rho < d \mid \theta) f''(\theta) d\theta \qquad (3.28)$$

Updating the reliability of existing structures using additional data on the structural behaviour can also be achieved in a slightly different way. For example, when a structure has resisted an external load Q, Andersen (1989) used the following equation to update the failure probability of the structure through Bayes' theorem:

$$P_f = P(L > R \mid R > Q) = \frac{P(L > R \bigcap R > Q)}{P(R > Q)} \tag{3.29}$$

where P_f is the probability of failure; L and R are the load effect and resistance of the structure respectively; Q is the external load resisted by the structure in the history.

Some concepts of reliability-based evaluation of existing structures are explicitly illustrated by Schneider (1992) using an example. An assessment procedure is also proposed based on the assumption that a correct application of the respective codes or standards results in a safe structure. Thus if an existing structure is correctly designed according to relevant standards, its reliability index achieved in the design, β_0, can be easily evaluated using the FOSM method. The safety index related to the actual condition of the structure, β, is secondly estimated using the current information available. If β is not less than β_o, the structure is assessed to be safe.

It should be noted that in all methods discussed above, the updated failure probability P_f is not an inherent characteristic attached to the structure itself. The quantity P_f simply represents the expert's degree of belief in the safety condition of the structure, and hence depends upon the state of knowledge available to the expert. To clearly understand the safety index β, various uncertainties involved have to be clarified.

Following Kiureghian (1989), the sources of uncertainty involved in engineering structures can be classified broadly into four kinds: (1) inherent variability; (2) estimation error; (3) model imperfection; and (4) human error. Inherent variability reflects the random nature of the characteristics of the structure and the loading environment such as material properties, load effects. Estimation error arises from the incompleteness of statistical data and our inability to accurately estimate the parameters of probability models that describe the inherent variability. Model imperfection exists in both mathematical models, such as the model for structural analysis, and probability models such as the chosen probability distribution functions for random variables. Finally, human error arises from mistakes made by various people during design, construction and operation of the structure. It is interesting to note that there are differences among these uncertainties. Inherent variability does not change with the amount of information available, while the second and third uncertainties can be reduced if more relevant data are obtained. Uncertainties due to human error are influenced by quality control measures.

When complete information is available and uncertainties arise only from the inherent variability, a strict measure of structural safety can be represented by the probability of failure P_f or the reliability index β with the relation:

$$\beta = \Phi^{-1}(1 - P_f) \tag{3.30}$$

where Φ is the standard normal distribution function; and β can be estimated using any standard reliability method. In this case, P_f and β are intrinsic statistical properties of a structure and its environment, and only vary if the structure and/or its environment are altered.

However, the exact value of the strict safety measure usually can not be evaluated in practice, since uncertainties other than inherent variabilities always present. We can only reduce these uncertainties by acquiring more data, and it is difficult to completely eliminate them. Assuming an imperfect state of knowledge, Kiureghian (1989) proposed an approach using Bayes' theorem to evaluate the possible values of the strict β. For this purpose, let the vector of random variables $\mathbf{X}$ describe the irreducible uncertainties such as inherent variability, and Θ represent the reducible uncertainties such as those arising from estimation errors. Specifically, $\mathbf{\Theta}$ includes the set of parameters that define the probability models of $\mathbf{X}$ and that of the limit state function $G(\mathbf{X})$. The probability distribution of $\mathbf{X}$ is expressed as a conditional distribution, $f_{\mathbf{X}|\mathbf{\Theta}}(\mathbf{x}, \theta)$, and the limit state function as $G(\mathbf{X}, \mathbf{\Theta})$. When perfect information, i.e. complete statistical data and perfect models, are available, $\mathbf{\Theta}$ is deterministically known. Otherwise, the degree of incompleteness of the available knowledge is characterized by the distribution of $\mathbf{\Theta}$, denoted by $f_{\mathbf{\Theta}}(\theta)$. The predictor distribution of $\mathbf{X}$, combining the inherent variability of $\mathbf{X}$ and the uncertainty in $\mathbf{\Theta}$ is given by:

$$\bar{f}_{\mathbf{X}}(\mathbf{x}) = \int f_{\mathbf{X}|\mathbf{\Theta}}(\mathbf{x}, \theta) f_{\mathbf{\Theta}}(\theta) d\theta \tag{3.31}$$

where $f_{\mathbf{\Theta}}(\theta)$ can be updated by the Bayesian rule if new information on $\mathbf{\Theta}$ is obtained.

For a given $\mathbf{\Theta} = \boldsymbol{\theta}$, the conditional failure probability can be evaluated as:

$$P_{f|\mathbf{\Theta}}(\theta) = \int_{G(\mathbf{X},\mathbf{\Theta})<0} f_{\mathbf{X}|\mathbf{\Theta}}(\mathbf{x}, \theta) d\mathbf{x} \tag{3.32}$$

and the conditional reliability index is obtained by $\beta_{|\mathbf{\Theta}}(\theta) = \Phi^{-1}[1 - P_{f|\mathbf{\Theta}}(\theta)$. For uncertain $\mathbf{\Theta}$, the reliability index $\beta_{|\mathbf{\Theta}}(\mathbf{\Theta})$ is also uncertain, and its distribution reflects the level of our knowledge available. If perfect data is gathered, $\beta_{|\mathbf{\Theta}}(\mathbf{\Theta})$ then coincides with the strict reliability index β.

One important advantage of this approach is that uncertainties from various sources are considered separately so that the level of knowledge available can be quantified through the distribution of β. Although this is a good idea, due to limited data available, the application of this approach is usually restricted.

In addition to Bayesian safety updating, there are also other reliability-based approaches available for existing structures which are discussed in the following section.

3.3.2 Other Approaches for the Revision of Structural Reliability

Motivated by recent advances in structural code development using probabilistic limit state design or load and resistance factor design, Shin *et al.* (1988) looked to the establishment of practical reliability-based bridge rating methods or rating codes. A rating criterion in the form of a Load and Resistance Factor Rating (LRFR) code is proposed which can take into consideration any data obtained on actual bridge conditions such as the "measured-to-calculated" load effects and other test results. In the approach, failure is defined as:

$$R - S_D - S_L < 0 \tag{3.33}$$

where R is a random variable representing the resistance; S_D and S_L are dead and live load effects respectively, which are both random quantities. These variables can be further modelled so that various uncertainties can be treated rationally. For example, R is represented by:

$$R = R_n MFPD \tag{3.34}$$

where R_n =nominal resistance of real section considering deterioration, damage etc. ;

M =random variable representing uncertainties about materials;

F =random variable representing fabrication or construction uncertainties;

P =random variable representing model uncertainties;

D =random variable representing uncertainties involved in the assessment of damages and/or deteriorations.

The mean value of R can be expressed as: $\bar{R} = R_n \bar{M}\bar{F}\bar{P}\bar{D} = R_n \eta_R$ where η_R is the mean-nominal ratio of resistance. From the data available in the literature and experience-based judgements on those uncertainties involved in R, S_D and S_L, the mean-nominal ratio and variance of resistance, dead and live load effects can be evaluated. Also, from the field inspection and relevant measurements, the nominal resistance in consideration of factors such as damage, and nominal load effects in consideration of measured-to-calculated responses are obtained by appropriate analysis. The reliability index β can then be evaluated using these data and the First Order Second Moment method. If the obtained β is larger than the target value, say $\beta_o = 3.0$, the bridge is classified as being safe. Otherwise, appropriate actions have to be undertaken for maintenance or repair regarding structural safety.

Hart *et al.* (1981) also used the reliability-based method to evaluate the expected damage to existing structures resulting from the future occurrence of natural hazards such as earthquakes. The reliability index or failure probability of the structure concerned is firstly evaluated from data available. A set of damage states such as *moderate, severe damage, partial collapse* are defined according to the cost of repairing the damage. These states are further related to different values of the reliability index for one component, one failure mode, or alternatively the average of a system of components. Therefore, the damage state of a particular structure can be obtained from the calculated reliability index of the structure. However, the relation between the damage states and reliability values are obtained arbitrarily from experience and subjective judgements, and there is no reasonable procedure available to establish this linkage consistently.

If a proof load test has been conducted on an existing structure, the reliability of the tested structure can also be updated using the test results. Early work on this topic was carried out by Shinozuka (1969).

A proof load test is a test in which some pre-determined load is applied. The strength of a structure which successfully passes the proof load test is clearly higher than the test load. If the structure is chosen from a population with strength distribution $f_R(r)$, it is possible to obtain the strength distribution of the tested structure, $f_R^*(r)$, by truncating $f_R(r)$ at the proof load Q:

$$f_R^*(r) = \frac{f_R(r)}{1 - F_R(Q)}; \qquad r > Q \tag{3.35}$$

$$f_R^*(r) = 0; \qquad r \leq Q \tag{3.36}$$

where $F_R(r)$ is the cumulative distribution of the strength R. This is schematically illustrated in Fig 3.4. The improved estimate of strength distribution

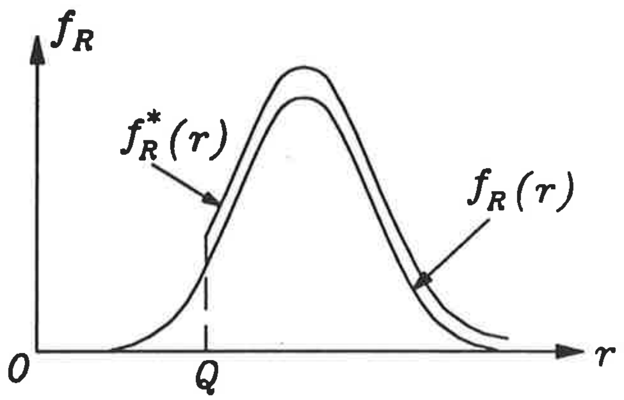


Figure 3.4: Strength Distributions before and after Proof Load Test

can be used to revise the reliability of the existing structure.

The truncation of the lower tail region of the strength distribution increases the mean and reduces the variance of R. This will increase the second moment reliability index β. The effects of proof load testing on the reliability level are not only due to the changes in μ_R and σ_R, but also the truncation of f_R. However, the β value obtained from the FOSM method can not well reflect the truncation effect. For this reason, Fujino and Lind (1977) defined a *proof load test safety index* β^* in which the effect of truncation can be deliberately taken into account. To evaluate β^*, the transformation of r to a new variable r^* is firstly made by:

$$r^* = \Phi^{-1}[\frac{\Phi(r) - \Phi(Q)}{1 - \Phi(Q)}], \quad Q \leq r < \infty, \tag{3.37}$$

where Φ is the normal distribution function; and Q is the applied load. β^* is secondly calculated using the standard reliability analysis and the limit state function of variables r^*.

These approaches for updating structural reliability based on proof load testing, however, have used only the information of survival or failure of the structure under testing. Veneziano *et al.* (1978) further improved the reliability-based analysis of proof load testing by using additional data such as the moment-curvature relation recorded during the test. In the approach, the posterior strength distribution $f''_R(r)$ after proof load testing is obtained through the Bayes' theorem:

$$f''_R(r) \propto f'_R(r) l(r \mid Q, C) \tag{3.38}$$

where $f'_R(r)$ is the prior strength distribution, and Q is the largest value of applied load; C denotes the moment-curvature relation; $l(r \mid Q, C)$ represents the likelihood of the strength r, given Q and C. The failure probability of the structure is evaluated as:

$$P''_f = \int_Q^\infty F''_R(s) dF_S(s) \tag{3.39}$$

in which $F_S(s)$ is the distribution function of load effect S.

However, practical difficulties of using this method arise from the construction of the likelihood function, because the value of Q is usually considerably smaller than the value of strength in practice. In a given example, the author tried to obtain the ultimate strength from an idealized parabolic moment-curvature curve. The accuracy of the procedure was discussed, and is questionable.

It is important to realized that prior service load in the service history of the structure can also be considered as a proof load, so that the reliability of an existing structure may be revised without the expense of conducting a load test. For an existing structure without severe wearout, Hall (1988) suggested that the service load can be treated as proof load with uncertainty. In this case, the proof load S is a random variable with distribution function $F_S(s)$. The posterior strength distribution $f''_R(r)$ in Eq. 3.35 changes to:

$$f''_R(r) = \frac{F_S(r) f'_R(r)}{\int_{-\infty}^{\infty} F_S(r) f'_R(r) dr} \tag{3.40}$$

The improvement in the reliability thus assessed depends on service load data collected. If the proof load has a high variance, then little improvement can be achieved in updating the strength distribution and hence the safety evaluation. Also, this approach can only be used when it is judged that the structural condition is not seriously deteriorated. Otherwise, the approach may lead to an unconservative result.

Reliability analysis relevant to the safety evaluation of existing concrete structures has been briefly discussed above. This review will be summarized in the next section.

3.3.3 Brief Summary on Reliability Updating

Generally, reliability analysis for future structures has to consider all possible sources of uncertainty. For an existing structure, more specific details can be obtained in the assessment procedure, and hence part of the uncertainty can be eliminated. However, there is no fundamental difference between reliability-based methods for analysis of future structures and existing structures.

Most of the existing approaches for reliability evaluation of existing structures are carried out by either updating the probability distributions of relevant basic variables or updating the model which describes the limit state function. The updating of basic variables usually uses the sample data available. Since sample testing is in many cases restricted to concrete and has only limited size due to practical difficulties in investigating existing structures, reliability revision using only sample data on basic variables seems not very efficient.

As Yao (1980) pointed out: *following the completion of the construction process, each structure has its own characteristics, which can no longer be described with the same initial mathematical models used in the design phase,* the uncertainty involved in the structural analysis itself, or model error, will play an important role in the reliability re-evaluation. However, appropriate methods for this purpose seem yet to be developed. Although proof load testing is a good way to justify structural resistance, its effectiveness is usually restricted due to the limited load Q applied. If Q is too large, the risk of structural failure under testing has to be considered.

In addition to reliability-based assessment, fuzzy set theory is another meaningful tool in dealing with uncertainties, and has proved useful in structural safety evaluation. Approaches of this kind will be discussed next.

3.4 Structural Safety Evaluation Using Fuzzy Sets

The concept of fuzzy sets was already discussed on page 42. More details on this theory can be found elsewhere (Zadeh, 1965; 1973; 1975a; 1975b; Zimmermann, 1985). The use of fuzzy sets in structural safety evaluation was triggered by the work of Pugsley (1973). He suggested that one way to study the proneness to accidents of a given structure, or a class of structures, is to look at a number of parameters that have been highlighted as relevant by experience in previous accidents. For this purpose, a set of parameters such as *experience of design and construction team*, *industrial climate*, *financial climate* are distilled from structural failures experienced in the past. A group of experienced engineers could then assess the accident proneness of a given structure in broad terms by grading these parameters. However, specific mistakes made in design or construction can not be identified by using the procedure. Instead, the method is only able to draw attention to some overall features of the structural design-construction process, and hence to reduce the possibility of structural failure.

It is obvious that those parameters selected by Pugsley (1973) can be assessed only subjectively. To have a consistent way of assessing and manipulating these parameters in evaluating structural safety, Blockley (1975) treated those parameters as linguistic variables using the concept of fuzzy sets. A linguistic variable is a variable whose values are words, phrases or sentences in a language. Each parameter i is thus quantified by its *size* P_i and *importance* or *weight* W_i, where both P_i and W_i have values like "small", "large", and are defined as fuzzy subsets of the interval $[0, 1]$. For m parameters, the total effect P_T is evaluated through appropriate fuzzy operations as:

$$(P_1 \cap W_1) \cup (P_2 \cap W_2) \cdots (P_n \cap W_n) \tag{3.41}$$

i.e.

$$P_T = \sum_{i=1}^{m} (P_i \cap W_i) \tag{3.42}$$

where $\cap$ and $\cup$ stand for the intersection and union respectively in the fuzzy calculation. The effect of P_T on the safety assessment is in turn considered through a fuzzification procedure. For this purpose, the engineer's *a priori* judgement of the probability of failure about the given structure is expressed in the form of 10^{-n}, in which n is an integer. A fuzzy kernel $K(n)$ is defined as a fuzzy subset of the interval [0,1]. To obtain $K(n)$, a fuzzy relation $\tilde{R}$ is defined as:

$$\begin{aligned} \tilde{R} \;=\; & if\ P_T\ is\ small\ then\ K(n)\ is\ small\ else \\ & if\ P_T\ is\ medium\ then\ K(n)\ is\ medium\ else \\ & if\ P_T\ is\ large\ then\ K(n)\ is\ large. \end{aligned}$$

Using the fuzzy relation, the kernel $K(n)$ can be evaluated from the fuzzy composition $P_T \circ \tilde{R}$. The final result of the safety assessment is obtained by fuzzifying the *a priori* failure probability 10^{-n} by the kernel $K(n)$.

The fuzzification of failure probability was further generalized by Brown (1979). In his approach, factors affecting structural safety are divided into two parts:

the objective and subjective factors. It is considered that the failure probability P_f in the form of 10^{-n} obtained from structural reliability theory using objective information is incomplete because the subjective part has not been considered. With the presence of additional subjective information, the belief in n of 10^{-n} wavers, and the attractions of $n-1$ and $n-2$ increase. For this reason, the combination of both objective and subjective knowledge will result in a range of beliefs in the set $n,\ n-1,\ n-2,\ \cdots,\ n-d$, where d is an integer and $d \leq n$. Considering the failure set $F = \{n, n-1, \cdots, n-d\}$, the support for the membership of $n, n-1, \cdots, n-d$ can be represented by:

$$F = \mu_0 \mid n \ + \ \mu_1 \mid (n-1) \ + \ \cdots, \mu_d \mid (n-d) \tag{3.43}$$

where μ_i is the membership function and $\mu_i \in [0,1]$. F can be obtained by a fuzzification procedure similar to that used by Blockley (1975). Under this definition, it is clear the failure probability 10^n obtained from ordinary reliability analysis using only objective information has the support of 1 for n, and no supports for others, i.e. $F_o = 1 \mid n \ + \ 0 \mid (n-1) \ + \ \cdots, \ + 0 \mid (n-d)$.

Similar approaches using fuzzy sets in structural engineering are also illustrated by Yao (1980) in several examples.

The safety measure obtained by previous approaches is a fuzzy quantity, and hence there are difficulties in interpreting the result and comparing it with the failure probability P_f. With the concept of fuzzy probability, Shiraishi and Furuta (1983) define failure as a fuzzy event so that the resulting safety measure is still in terms of probability or reliability index as that in reliability analysis, but is fuzzified according to the subjective information available. In reliability analysis, the failure probability P_f is defined as the probability of the event A that the structural resistance R is smaller than the load effect S, i.e.:

$$P_f = P(A) = \int_A dP \tag{3.44}$$

where A is expressed by the safety margin Z, e.g. $Z = R - S < 0$. This means a clear-cut definition of failure at $Z = 0$. Considering the existence of subjective uncertainties, it is preferable to define the failure event in a more flexible form. A fuzzy set $\tilde{A}$ is then used to represent the failure event with the function $\mu_{\tilde{A}}(x)$ indicating the degree of membership of x in $\tilde{A}$. The fuzzified failure probability P_f can be evaluated as:

$$P_f = \int_{-\infty}^{+\infty} \mu_{\tilde{A}}(z) f(z) dz \tag{3.45}$$

where $f(z)$ is the probability density function of function $Z = R - S$. It can be seen that the failure probability defined in ordinary reliability analysis, i.e. Eq. 3.44, can be rewritten according to Eq. 3.45 as:

$$P_f = \int C_A(x) dP \tag{3.46}$$

in which $C_A(x)$ is defined by:

$$\begin{cases} C_A(x) = 0 & x \notin A \\ C_A(x) = 1 & x \in A \end{cases} \tag{3.47}$$

Clearly, Eq. 3.44 is a special case of Eq. 3.45.

To evaluate the probability of failure defined by a fuzzy event, the membership function $\mu_{\tilde{A}}(x)$ in Eq. 3.45 can be obtained in the same way as proposed by Blockley (1975). For this purpose, a number of subjective uncertain factors regarding the failure event are modelled as linguistic variables. The total effect P_T of these factors is a fuzzy set. If the fuzziness involved in the definition of failure is K, a fuzzy relation can be defined as: $\tilde{R}$ =*if* P_T *is large, then* K *is large, else if* P_T *is medium, then* K *is medium, else if* P_T *is small, then* K *is small.* $\mu_{\tilde{A}}(x)$ finally can be obtained by the fuzzy composition of $\tilde{R} \circ P_T$.

The fuzzification concept can also be used to fuzzify the distributions of resistance R and load effect S, i.e. : $f_R(r) \rightarrow \tilde{f}_R(r)$, $f_S(s) \rightarrow \tilde{f}_S(s)$. The fuzzified failure probability is calculated in the same way as in reliability analysis, i.e.

$$P_f = \int\int_A \tilde{f}_R(r) \tilde{f}_S(s) dr ds \tag{3.48}$$

Itoh and Itagaki (1989) also used the concept of fuzziness in Bayesian updating to study the fatigue crack problem using inspection data. If fuzzy set $\tilde{A}$ is defined on the universe of X with membership function $\mu_{\tilde{A}}(x)$, the conditional probability of a fuzzy event can be defined as:

$$P(\tilde{A} \mid \theta) = \sum_{X} \mu_{\tilde{A}}(x) P(x \mid \theta) \tag{3.49}$$

where $P(x \mid \theta)$ is the conditional probability of x given θ. The posterior probability of θ given the observation of the fuzzy event is then:

$$P(\theta \mid \tilde{A}) \propto \sum_{X} \mu_{\tilde{A}}(x) P(x \mid \theta) P(\theta) \tag{3.50}$$

where $P(\theta)$ is the prior distribution of θ.

Fuzzy sets are also used to rate the condition of existing bridges by Tee *et al.* (1988). In the proposed approach, the i^{th} subcomponents of an existing bridge is inspected and rated by two factors $\tilde{R}_i$ and $\tilde{W}_i$. $\tilde{R}_i$ represents the physical condition of the subcomponent, and is obtained by the engineer's judgement. $\tilde{W}_i$ is called the *importance factor* which relates the component's condition to overall safety of the structure. Both $\tilde{R}_i$ and $\tilde{W}_i$ are modelled as linguistic variables and described by fuzzy sets, and are evaluated subjectively and imprecisely by the inspector. The weighted average $\tilde{R}$ defined by:

$$\tilde{R} = \frac{\sum_{i=1}^{n} \tilde{W}_i \tilde{R}_i}{\sum_{i=1}^{n} \tilde{W}_i} \tag{3.51}$$

is used to represent the overall condition of a component or a bridge. The evaluation of Eq. 3.51 requires operations of fuzzy addition, fuzzy multiplication and fuzzy division, and can be carried out using the *extension principle* in fuzzy set theory (Zadeh, 1965; Zimmermann, 1985).

3.5 Summary on Fuzzy-Based Safety Evaluation

Existing approaches for structural safety evaluation using *fuzzy relation* and *fuzzy composition* provide an alternative way to include experience-based subjective information in the assessment procedure. With the help of fuzzy kernel K, the objective safety measure β can be fuzzified by taking into account subjective data (Blockley, 1975; Brown, 1979). Under the concept of fuzzy probability (Shiraishi and Furuta, 1983), probability distributions of both state variables such as resistance and basic variables such as concrete strength can also be fuzzified, and the final safety measure using both objective and subjective information can be evaluated by the same procedure as in structural reliability analysis.

However, the major problem involved in these methods is the estimation of the fuzzy kernel K which is defined arbitrarily in those approaches reviewed. As a result, the safety measure thus obtained is somewhat arbitrary. For this reason, further development of this method seems needed before it can be widely accepted in engineering profession.

3.6 Summary and Conclusions on This Review

As can be seen from the above review, structural reliability theory and fuzzy safety analysis are two major approaches for structural safety evaluation of existing structures which are treated in the literature. The reliability method has been relatively well developed, and is becoming accepted by more and more engineers. Its application to engineering practice has led to important structural code implementation worldwide. However, fuzzy-based safety assessment needs further research before it can be widely applied in practice. For this reason, the reliability method will be used in the development of a rational procedure for the safety evaluation of existing defective concrete buildings.

The reliability method proves to be very useful in dealing with statistical variations. It is not surprising to see that most of those existing methods mainly employ the updating of probability distributions of basic variables using test data in assessing the existing structures. However, uncertainties arise not only from random variations but also other sources such as unknown faults which have occurred in the construction. Uncertainties of the second kind are in fact particularly important in the evaluation of defective structures. A structure is designed to behave properly within its planned service life under specified conditions. If it manifests structural anomalies, there are good reasons to believe that various errors, faults or harmful factors may have occurred during the process of design, construction and operation. Although the occurrence of these anomalies may also be attributed to the "low-strength and high-load" event due to random variations, the chance of this event is usually very small because of conservative design. Therefore, a realistic method for reliability assessment of existing structures has to be able to take into account these faults and harmful factors explicitly.

Existing methods are generally inadequate for this purpose. Although the study of effects of human errors on structural safety has attracted many researchers (Ditlevsen, 1980; Melchers, 1984a, 1984b, 1984c; Arafah and Nowak, 1986; Hadipriono and Lin, 1986), current research is primarily concerned with the effectiveness of quality assurance in controlling human errors, and the implementation of quality control in structural design, construction and use. However, for a specific existing structure which shows signs of defects, its safety assessment has to account for the particular faults and harmful factors which have occurred.

From the above discussion, it can be concluded that a realistic reliability-based safety assessment of an existing structure has to consider uncertainties arising from not only random variations but also various specific faults and harmful factors which may have occurred to the structure itself. Such a method is developed in the next section, in which possible faults/factors identified from the diagnostic procedure and their effects on structural safety are explicitly taken into account.

As mentioned at the beginning of this chapter, condition evaluation should include the assessment not only for safety but also for other structural requirements such as serviceability and durability. Therefore, suitable assessment procedures will also be developed for durability and serviceability in the following sections.

Finally, prognosis of structural behaviour regarding various requirements after a course of action is taken, is briefly described in Section 3.10.

3.7 Safety Study of Existing Concrete Buildings

Safety assessment is probably the most important part of condition evaluation due to the possible serious consequences of structural failure. An accurate evaluation of safety regarding various structural requirements will help the decision maker choose appropriate actions, and hence avoid the severe consequences of failure. For this purpose, reliability-based approaches can be used. In practice, however, experience usually also plays an important role in safety evaluation, and experience-based safety assessment is a good alternative to be considered, especially when existing information is insufficient to allow numerical analysis to be carried out, but a decision regarding the safety condition must nevertheless be made at the time. In usual cases, both of these two approaches are likely to be employed together to produce satisfactory results. In the following sections, detailed methods of assessing the safety of an existing defective concrete building, using both the engineer's experience and the reliability method are described.

3.7.1 Reliability-Based Safety Evaluation

Structural reliability analysis can be carried out for various purposes. For example, in the reliability-based code implementation, a large population of structures has to be considered, and the related reliability analysis is *generic.* In the assessment of an existing defective concrete building, the reliability evaluation is *specific*, and involves only one particular structure or even a single member.

Conceptual differences exist between generic and specific reliability evaluations. In structural reliability analysis, the safety margin can usually be expressed by $Q = R - S$, in which R and S are random variables representing resistance and load effect respectively. In generic reliability analysis, R represents the resistance of a large group of structural members of the same kind, and hence its probability distribution, denoted by $f_R(r)$, describes the frequency of the occurrence of all possible values of R. In this case, $f_R(r)$ is obtained from experimental data or numerical simulations.

In dealing with a specific defective structure, the reliability evaluation is unique to the member and a particular limit state, and the related resistance R is theoretically a deterministic variable. For example, if a concrete beam is loaded to bending failure, there is only one ultimate load to be recorded. If this deterministic value can be obtained, safety assessment would be much simplified. Unfortunately, the resistance R of a real structural element is never known exactly in practice, because the failure test usually can not be conducted on an existing structure which is still in service, and uncertainties unavoidably exist in evaluating R by whatever methods. The uncertain resistance R of an existing structure or member then has to be treated also as a random variable for the purpose of analysis. However, the probability distribution $f_R(r)$ in this case should be interpreted in a *Bayesian* or *subjective* sense rather than in a *frequency* sense. In other words, $f_R(r)$ here only reflects our degree of belief in the true value of the resistance, and hence its shape is closely dependent upon the amount of information available. The more information available on the given structure, the closer $f_R(r)$ will be to the true point value of R. Reliability-based safety assessment of an existing structure is very much directed to the evaluation of $f_R(r)$ using specific information about the physical state of the structure.

Since the resistance R is usually a function of a set of basic variables such as material properties, dimensions, and the function is dependent on the chosen model of structural analysis, the estimation of the probability distribution $f_R(r)$ involves determining the distributions of those basic variables and to assess the uncertainty involved in the model of structural analysis. In generic reliability analysis, each of these basic variables is represented by a population distribution obtained from available test data together with professional judgements, with its mean and variance estimated with reference to the nominal value of the variable assumed in the original design. For example, the compressive strength of concrete with "good workmanship" can be modelled by a normal distribution with the mean value of $1.16f'_c$ and a coefficient of variation of 0.10, in which f'_c is the nominal concrete strength (Ellingwood, 1977). Distributions of this kind for common basic variables are available in the existing literature, and are summarized in Appendix **A**. In the assessment of an existing structure, if sample data are not available, a good approximation is to use these population distributions to represent the relevant basic variables, provided that the structure is functioning properly and there is no serious deterioration. However, the real physical state of an existing concrete building at the time of assessment may be considerably different from that in the original design documents, mainly due to *factors* such as *errors or faults during design, construction and service periods, deterioration* and *damage.* This is particularly the case for potentially defective structures where the observed anomalies strongly suggest the occurrence of these errors or harmful factors. Therefore, the use of population distributions to represent the basic variables for an existing structure is irrelevant in many cases. The effects of those factors which cause the defects have to be taken into account. Procedures described below are specifically developed for this purpose.

Updating the Distributions of Basic Variables

Two types of basic variables regarding structural resistance are usually encountered in evaluating the reliability of an existing structure regarding an interested ultimate limit state. For those variables whose properties can be obtained with certainty in the condition survey, deterministic treatment is adequate. For example, the dimension of a concrete beam at the mid-span may be accurately measured during the investigation, and can be considered as a deterministic variable in the reliability analysis regarding the bending ultimate limit state. Other basic variables such as concrete strength, the amount of reinforcement (which may not be directly measured) have to be treated as random variables, and represented by their probability distributions.

Suppose there are m basic variables involved, i.e. $X = (X_1, X_2, \cdots, X_m)$. To estimate the distribution $f_{X_i}(x_i)$, the effects on the structure of any errors and/or harmful factors have to be taken into account. Since the observed anomalies are the manifestations of possible defects, it is convenient here to use the hypotheses obtained in the diagnosis to represent the errors and harmful factors when estimating $f_{X_i}(x_i)$. For this purpose, the distribution of each random basic variable X_i can be assessed conditional on a hypothesis H_j, and denoted by $f_{X_i|H}(x_i, H_j)$. Obviously each hypothesis $H_j \in H = (H_1, H_2, \cdots, H_n)$ contains a set of explanations which are possibly responsible for the occurrence of the identified anomalies, and therefore $f_{X_i|H}(x_i, H_j)$ represents the distribution of X_i with regard to the joint effects of all explanations contained in H_j. Thus if the diagnosis is incomplete, X_i has to be described by n conditional distributions.

Before sufficient sample test data are available, these conditional distributions have to be estimated using experience and judgement with reference to the original design documents available. Actually the incorporation of subjective information such as experts' opinion in engineering problem-solving is indispensable, and the subjective assessment of $f_{X_i|H}(x_i, H_j)$ is practically possible. For example, in a case where cracks were found in concrete beams of a newly erected structure, it is assumed, for simplicity, that there are only three hypotheses to be considered, i.e. H_0="null hypothesis", H_1="early overloading", H_2="the wrong use of lower grade concrete". To evaluate the variable "compressive concrete strength" X_c, it is necessary to estimate the three conditional distributions $f_{X_c|H}(x_c, H_0)$, $f_{X_c|H}(x_c, H_1)$ and $f_{X_c|H}(x_c, H_2)$. The concrete strength assumed in the original design documents is considered to follow a normal distribution with mean and deviation of m and σ respectively. Since H_0 and H_1 do not affect the properties of X_c, relevant distributions conditional on H_0 and H_1 can be approximately represented by the same population distributions as in the design, with their means and deviations being $f_{X_c|H}(x_c, H_0) = (m,\ \sigma)$, $f_{X_c|H}(x_c, H_1) = (m,\ \sigma)$ respectively. The conditional distribution $f_{X_c|H}(x_c, H_2)$ can be represented also by a normal distribution with mean and standard deviation as $(m - \Delta m,\ \sigma + \Delta\sigma)$, in which Δm and

$\Delta\sigma$ have to be judged by the engineer from his/her experience and any other information obtained in the condition survey.

Conditional distributions thus estimated can be further updated if new sample data are gathered in the assessment process. For this purpose, the subjectively evaluated $f_{X_i|H}(x_i, H_j)$ can be considered as the prior distribution and denoted by $f'_{X_i|H}(x_i, H_j)$, and the posterior conditional distribution $f''_{X_i|H}(x_i, H_j)$ using the new data D is obtained from Bayes' theorem:

$$P''(H_j) \cdot f''_{X_i|H}(x_i, H_j) \propto P'(H_j) \cdot f'_{X_i|H}(x_i, H_j) \cdot l(x_i, H_j \mid D) \qquad (3.52)$$

where $l(x_i, H_j \mid D)$ is the likelihood of the observed test result D given X_i and H_j; $P'(H_j)$ is the probability of H_j before D is obtained, and $P''(H_j)$ is the updated probability of H_j using D; both $P'(H_j)$ and $P''(H_j)$ are obtained from the diagnostic procedure described in Chapter 2. The updated conditional distribution $f''_{X_i|H}(x_i, H_j)$ then reflects our degree of belief in the value of the variable X_i conditional on H_j, based on knowledge currently available.

Updating the Estimate of Model Uncertainty

Uncertainties in the imperfect model of structural analysis (model uncertainty) are also crucial to the accuracy of reliability evaluation. In generic reliability analysis, structural behaviour regarding a particular limit state can be analyzed using appropriate models developed from either the standard codified design method or a suitable advanced approach. These models usually adopt the assumptions made in the design, and the uncertainty is represented by a random variable whose properties are estimated from test data and subjective judgements. For an existing structure showing various anomalies, however, assumptions made in the original design may not be still valid, and different models may have to be considered depending on various possible explanations of the observed anomalies. In fact, structural analysis can usually be improved at the time of assessment by applying a more specific and accurate model. Generally, if the hypothesis H_j has been confirmed from the diagnosis, a mathematical model particularly suitable for the relevant limit state can be determined by taking into account the effect of H_j, and denoted by $G_{|H_j}$. For the whole hypothesis set $H = (H_1, H_2, \cdots, H_n)$, there could be n models selected in this way, i.e. $G_{|H_1}, G_{|H_2}, \cdots, G_{|H_n}$. Apparently $G_{|H_j}$ is valid only if H_j is true.

For conditional models thus obtained, the error involved in each $G_{|H_j}$ regarding its own imperfection can be treated in the same way as that in the generic reliability analysis mentioned above.

It has to be pointed out that for a hypothesis set $H = (H_1, H_2, \cdots, H_n)$, theoretically there would be n models to be considered. However, since one model might well be suitable for a number of different hypotheses, the actual number of models to be considered in practice is far less than n. Therefore, the treatment of model uncertainties in this manner should not lead to a formidable task of computation.

Building Floor Loads

The reliability of a structure or structural member is evaluated in terms of the probability of not entering the ultimate limit state which is defined by the structural resistance and load effects. Large uncertainties are usually also present in the determination of applied loads, and hence appropriate probability models for various loads are crucial to the reliability analysis.

The important load component applied to a building is the floor load, which consists of dead and live loads. The dead load is produced by the weight of elements comprising the structure such as permanent equipment, partitions and installations, roofing and floor coverings. In the assessment of existing structures, since the weight of such elements can in many cases be obtained with reasonable accuracy, the dead load can usually be considered as a deterministic variable. However, if the uncertainty presented is relatively large, existing approaches in the literature often treat the dead load as a random quantity which follows a normal distribution with mean $m_D = 1.0D$ and variance V_D in the range 0.06 to 0.15, with a typical value of 0.10 (Ellingwood *et al.*, 1980), in which D is the nominal dead load. If there is no other specific model available, this normal distribution can be adopted in the assessment.

Live load contributes in a major way to the uncertainty involved in structural analysis, and its treatment is crucial to the reliability evaluation. Unlike the dead load which is assumed to remain constant throughout the service life of a structure, live load varies significantly with time, and therefore has to be modelled as a stochastic process. Based on available load survey data, probabilistic models for floor live load have been developed by researchers in the last few decades. Details of these models are summarized in Appendix B. The maximum live load L_t during the structure's lifetime (about 50 years) is found to be represented by a Type I extreme value distribution with moments given by (Ellingwood and Culver, 1977):

$$E(L_t) = 18.7 + \frac{520}{\sqrt{A_I}} \; psf \tag{3.53}$$

$$Var(L_t) = 14.2 + \frac{18900}{A_I} \; (psf)^2 \tag{3.54}$$

where A_I is the influence area whose definition is discussed in Appendix B. The parameters represented by Eqs. 3.53 and 3.54 were obtained jointly by judgements and fitting the load survey data into the probability models, and therefore may not necessarily be relevant for a specific structure's load condition. However, these parameters and the model have been used to produce design load specifications in various countries (Ellingwood *et al.*, 1980; Pham and Dayeh, 1986), and hence the use of this model in evaluating an existing structure will result in a reliability index which can be used in checking the structure's safety level in comparison with that required in the appropriate design codes.

The Formulation of Reliability Analysis

Since probability distributions of the basic variables regarding structural resistance are assessed conditional on a hypothesis H_j, the distribution of the resistance R can also be represented by a conditional distribution denoted by $f_{R|H}(r, H_j)$. Using $f_{R|H}(r, H_j)$ and the probability distribution of the applied load $f_S(s)$ discussed in the last section, the failure probability conditional on each hypothesis, $P_{f|H}(H_j)$, can be calculated and conceptually expressed by:

$$P_{f|H}(H_j) = \int_{-\infty}^{\infty} F_{R|H}(x, H_j) \cdot f_S(x)dx \tag{3.55}$$

where $F_{R|H}(r, H_j)$ is the cumulative probability distribution of resistance R conditional on a hypothesis H_j. In practice, it is more convenient to evaluate the conditional reliability index $\beta_{|H}(H_j)$ using the famous First Order Second Moment method, and the relation between the failure probability and the safety index can be approximately represented by

$$\beta_{|H}(H_j) = \Phi^{-1}[1 - P_{f|H}(H_j)] \tag{3.56}$$

in which Φ is the cumulative distribution function of the standard normal distribution. The conditional reliability index $\beta_{|H}(H_j)$ thus defined represents the engineer's belief in the safety condition of the structure if the hypothesis H_j is true. For simplicity, it is assumed in the above formulation that the load effect is independent of H_j.

Apparently there will be n conditional safety indices available for the hypothesis set $H = (H_1, H_2, \cdots, H_n)$. The probability that $\beta_{|H}(H_j)$ is true is equal to the probability of hypothesis H_j, i.e. $P[\beta_{|H}(H_j)] = P(H_j)$, in which $P(H_j)$ is a subjective probability, and obtained from the diagnostic procedure described in Chapter 2. This is schematically illustrated in Fig. 3.5.

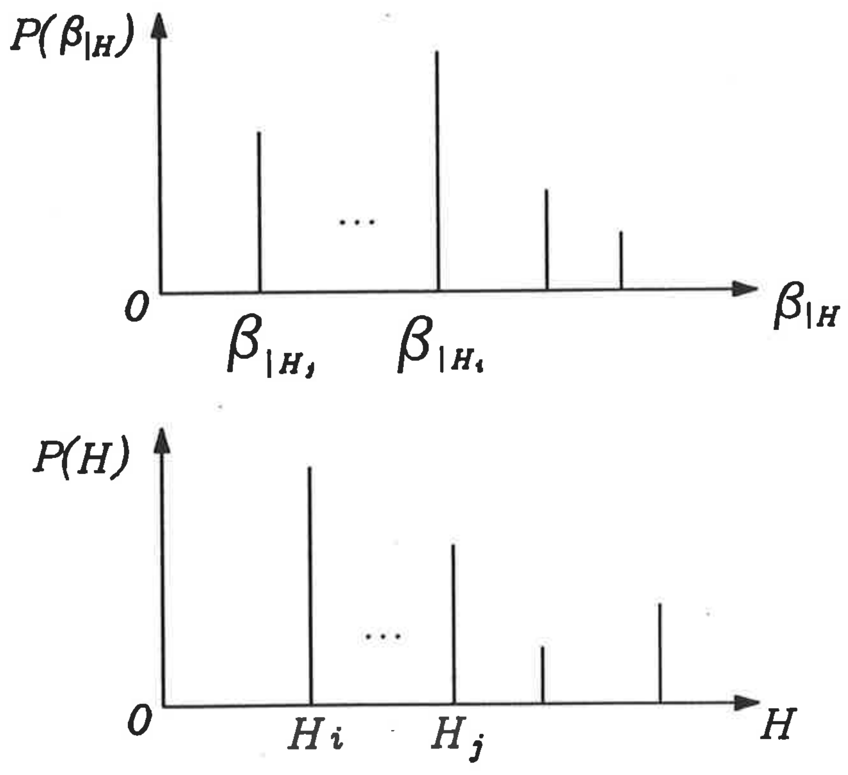


Figure 3.5: An Illustration of the Distribution of Index β

The distribution of H reflects the amount of information available for diagnostic purposes. However, the distribution of $\beta_{|H}$ represents both the effects of various hypotheses on the safety condition of the structure and the amount of information available. It is possible that several hypotheses correspond to the same β value. Since there is only one true hypothesis H^*, the complete diagnosis will result in a single reliability index β^*, which indicates the engineer's degree of belief in the safety of the structure using the complete diagnostic information.

It is important to realize that uncertainties considered in the reliability evaluation using the proposed approach fall into two categories: (1) uncertainties from inherent random variation and errors in estimating relevant models and parameters; (2) uncertainties from the effects of various unknown faults or events which have caused the damage and deterioration of the structure. Generic reliability analysis usually only considers the uncertainties of the first category. With the uncertainties from the second category treated by evaluating the safety index conditional on a hypothesis, the resulting $\beta_{|H_j}$ is at least as accurate as that in generic reliability analysis. This makes it possible to compare the safety condition of an existing defective structure with the legal safety level required in the relevant codes, which have usually been obtained using generic reliability analysis.

3.7.2 Assessment for Safety Conditions Using β Values

As is widely known, the reliability index usually does not correspond directly to any true failure probability, but nevertheless reflects consistently the safety condition of a structure. The accurate and practical interpretation of the values obtained for $\beta_{|H}$ is very important to the assessment of existing defective structures.

Although there is no absolute safety for engineering structures, it is frequently desirable to describe whether a structure is *safe* or *not safe* under specified conditions. Practically, a structure can be considered safe if its risk of failure is below the level which is widely accepted by the engineering profession and community, i.e. if the calculated failure probability P_f is not greater than an acceptable *target probability* P^0. Using the reliability index β in lieu of probability values, this $P_f \leq P^0$ is then equivalent to $\beta \geq \beta^0$, in which $\beta^0 = \Phi^{-1}(1 - P^0)$, and Φ is the function of standard normal distribution. Therefore, if a target reliability index β^0 is available, an existing structure can be assessed to be legally safe if its calculated β is not less than β^0.

However, difficulties exist in finding a suitable value of β^0 acceptable to all interested parties. In the reliability-based code implementation of recent years, the β value implied in current practice has been adopted as the target reliability for deriving the load factors. For example, it was found that β values for most flexural and compression concrete members tend to fall within the range of 2.5 to 3 for different load combinations in the United States, and hence the target safety index β^0 adopted for the combination of dead and live loads was 3.0 in ***The Building Code Requirements for Minimum Design Loads in Buildings and Other Structures*** (Ellingwood ***et al***, 1980). For the purpose of assessing the safety condition of an existing structure, one could naturally compare the calculated β with the code-implied target reliability. Specifically, if $\beta \geq \beta^0$, the structure can be considered safe. However, the fact that β values vary with different kinds of structural members and load combinations makes this approach a little clumsy in application.

In addition to the code-implied target reliability index β^0, based on the axiom that *a correct application of the relevant codes or standards results in a safe structure*, Schneider (1992) suggested that an existing structure will be safe if its re-evaluated β value at the time of assessment is not less than the value β^d which is the safety index achieved in a correct design of the same structure according to relevant codes. For any existing structure, if there is no error in its original design according to relevant codes, β^d can be easily evaluated using the design data of this structure. Otherwise, if there are design errors involved, β^d has to be calculated using the design data of the same structure after the error has been corrected. The comparison between β and β^d thus reflects the relative change of safety index of the same structure, and hence is reliable and meaningful. In this work, it is suggested that β^d be used to compare with the re-evaluated β for the the safety assessment, i.e. if $\beta \geq \beta^d$, the structure is considered legally safe.

If $\beta < \beta^d$, the structure's safety condition is deteriorated or damaged, and further examination is necessary. Generally, the higher the value of β, the safer the structure will be. If the calculated β using the information currently available is far smaller than β^d, the danger of structural failure is real, and hence the safety condition of the structure is certainly unacceptable. Therefore, the structure is "severely defective" regarding the relevant ultimate limit state. How much the β value is less than β^d has to be determined by the engineer based on experience with reference to typical values of β accepted in the current practice. For example, the typical value of β^d for flexible concrete members subjected to gravity loads is around 3.0 in current practice. If the evaluated β is less than 2.0, there is then little doubt that the structural safety condition has to be improved in order to keep the structure in service.

However, difficulties usually exist when the re-evaluated β is less than, but close to β^d. In this case, the assessment has to be made by evaluating factors such as likelihood and possible consequences of structural failure, using relevant judgements and information such as the diagnostic results. If it is judged that the danger of structural failure is a real concern, the structure is classified to be "severely defective"; if the structure is in a satisfactory condition, and no structural repair is needed, the structure is then classified to have "negligibly defective"; otherwise, the structure's safety condition falls into the category of being "moderately defective".

In summary, according to the reliability index β obtained, the safety condition of an existing structure concerning a specific limit state can be represented by the following four categories:

$$\begin{cases} If \ \ \beta \geq \beta_d^0 \ \ safe \\ If \ \ \beta < \beta_d^0 \ \ \begin{cases} negligibly \ defective \\ moderately \ defective \\ severely \ defective \end{cases} \end{cases} \tag{3.57}$$

in which the value of β is estimated conditional on a hypothesis H_j. In other words, this assessment has to be carried out for each hypothesis $H_j \in H$.

Outlined above is the procedure for evaluating the safety of existing concrete buildings based on structural reliability theory. It is certainly ideal to use this method in practice to assess accurately the safety condition of structures. However, the amount of data and knowledge needed in implementing the approach are not always available to the engineer in solving real problems. If this is the case, the experience-based procedure described in the next section can be adopted.

3.7.3 Experience-Based Safety Assessment

Safety assessment based purely on the engineer's experience has been occasionally criticized for its unreliability. Surely it is very difficult to judge whether a building is safe or not only using experience without carefully carrying out relevant calculations. However, by the use of diagnostic results obtained from Chapter 2, it is possible to assess the safety of an existing structure with reasonable accuracy.

In practice, we are usually only interested in a particular structural member or substructure regarding one specific ultimate limit state in the safety evaluation. Thus considering a potential safety problem triggered by a set of observed anomalies which can be related to a set of *explanations* defined in Chapter 2, if the true explanation has been identified, an acceptable judgement on the safety condition regarding the relevant limit state may be made by the engineer from has/her experience and relevant knowledge of concrete structures. For example, if it is confirmed that the explanation for the *cracking* on the bottom surface of a reinforced concrete slab is "early over loading during construction", it can be concluded that the slab will be *safe* regarding its bending ultimate limit state, provided that there are no other anomalies observed in the slab and the design complies with relevant codes. Since there is no absolute safety for engineering structures due to unavoidable uncertainties, in this kind of assessment the conclusion "safe" should be interpreted as "the member is at least as safe as the code requires", and hence can be considered as "legally safe" under the current code specification. For practical purposes, the experience-based procedure for safety assessment described in this section will use "safe", "moderately defective" and "severely defective" to indicate the safety condition of a structural member or substructure. The last two terms can be interpreted as "there is a potential danger" and "there is definitely a danger" respectively.

From the diagnostic procedure described in the last chapter, explanations for the pattern of anomalies observed in a structure are generally represented by a set of hypotheses $H = (H_1, H_2, \cdots, H_n)$. For a given H_i, it is possible to assess the safety condition of the structure according to explanations contained in H_i together with any other relevant information available. If it can be concluded by the engineer confidently that the structure can fulfill its intended functions satisfactorily, and that the risk of failure is acceptable, the safety condition is then assessed *safe*. Otherwise, if repair action has to be carried out in order to keep the structure in service, the structure is assessed to be *moderately defective* or *severely defective* depending on factors such as urgency of repair, repair cost and possible consequences of failure. In this way, safety assessment is carried out conditionally on each hypothesis $H_i \in H$.

Obviously, if the diagnosis is incomplete, the result of safety assessment is not unique. For a hypothesis set having n elements, i.e. $H = (H_1, H_2, \cdots, H_n)$, with each member ranked by its probability $P(H_i)$, the safety condition of the structure, if denoted by X_F, can be represented by $X_{F|H_i}$, where $i = 1, 2, \cdots, n$. The probability $P(X_{F|H_i})$ indicate how likely $X_{F|H_i}$ is true, and $P(X_{F|H_i}) = P(H_i)$.

In some cases, the judgement of safety will not be so straightforward, even if the explanation of the anomaly has been identified, simply because the information contained in the explanation is not sufficient to make such a judgement. For instance, if the explanation in the above example is "concrete compressive strength is low", it is then insufficient to conclude that the slab will be safe or unsafe. Under such circumstances, further information on the concrete strength needs to be gathered through experiments, and used in an analysis to provide new information for the assessment. The reliability-based safety assessment procedure given in previous sections may be adopted as appropriate.

It should be pointed out that the safety evaluation discussed above is carried out regarding the present physical state of the structure. Therefore, the obtained results are valid only if there are no chronic factors involved. Upon the examination of the outcomes of the condition survey and diagnosis, if time-dependent factors play important roles in the safety conditions, then it is necessary to carry out a durability assessment. For example, if the observed anomaly is "corrosion of the reinforcing steel", the safety evaluation regarding any ultimate limit state of the structural member or substructure will be carried out as part of the durability assessment, to be discussed in section 3.9.

3.8 Assessment for Serviceability Conditions

Since serviceability problems are usually manifested by various anomalies such as excessive cracking and deflection, the extent of damage regarding a interested serviceability limit state such as the crack width may be determined at the time of assessment. If this is the case, by comparing the obtained measurements with various limits on serviceability requirements specified in relevant codes, the engineer can decide whether the structure is *satisfactory* or *inadequate* regarding the relevant serviceability. Of course, owners of the structure may have their preferred requirements on particular serviceability limit states, and in this case the assessment has to allow for the owner's demand.

However, the assessment made in this way is valid only at the time of investigation. Requirements regarding any serviceability limit state have to be satisfied during the whole period of service life. For this reason, the structural performance regarding the interested limit state in the future has to be predicted and assessed as to whether it will be satisfactory or inadequate. For this purpose, numerical calculation using reliability method , if possible, is ideal. Unfortunately, due to poor mathematical models available for predicting the serviceability behaviour, relevant calculations for serviceability performance, such as the estimation of crack width, usually prove only effective in determining the dimensions of structural members in the design phase, and usually are not accurate enough for re-assessment. Therefore, serviceability assessment using numerical calculations is not always meaningful.

On the contrary, experience-based approaches can be very useful. Since the assessment of a serviceability limit state is usually triggered by the observed anomalies, the future serviceability performance of the structure, in most cases, can be well-predicted from experience, if explanations of the anomalies are identified. Specifically, for a given limit state X_S, the engineer can find out the code-specified limit on the structural performance concerning X_S, and is able to judge whether the structure is going to satisfy the serviceability requirement or not by assessing the effects of the identified explanations on the concerned serviceability. Because the explanations of anomalies are represented by a set of hypotheses in the diagnostic procedure, the result for serviceability assessment can be represented by $X_{S|H_i}$ =*satisfactory* and $X_{S|H_i}$ =*inadequate*, where H_i is the i^{th} hypothesis. Thus for n hypotheses available, i.e. $H = (H_1, H_2, \cdots, H_n)$, an assessment for any serviceability limit state can be made conditional on H, and denote by $X_{S|H}(H_j)$. The probability distribution of $X_{S|H}$ can be easily obtained from that of H, i.e. $P(X_{S|H_i}) = P(H_i)$, where $P(H_i)$ is obtained from the diagnostic results.

3.9 Assessment for Durability Conditions

Durability problems, in this work, refer to those chronically developing anomalies which are, or will, be indications of violation of ultimate and/or serviceability limit states. Examples of this kind are corrosion of reinforcing steel and carbonation of concrete cover.

Regarding the durability problem, repair becomes necessary when the anomalies have developed to such an extent that the structural safety is threatened, or the serviceability, functionality of the structure are impaired. Since the problem is time-dependent in nature, the decision on when to carry out the repair ideally should be made with an accurate relationship between the structure's performance and time represented by a curve. However, due to limited knowledge available on the mechanism of development of common anomalies, such a relationship can hardly be established. In practice, realistic solution is usually to determine the necessity of repair according to factors such as the owner's preferences and financial situation together with the structure's condition judged by the engineer. Generally, when the problem is firstly identified in the condition survey, owners of the structure have different attitudes to the repair strategy. Some would like to take a repair action only when it is definitely needed, while others may prefer to have cheaper but regular maintenance activities. No matter which situation it might be, the engineer has to provide the owner with an approximate estimation of the structure's remaining service life both with and without taking a course of action. The period of this remaining service life may be called *satisfactory period*, during which the structure is supposed to behave satisfactorily regarding the identified durability problem. Since the satisfactory period is a very important and useful parameter in the practical approach of selecting the repair strategy, it will be used here to represent the result of durability assessment. In other words, an existing structure's durability condition can be assessed in terms of its satisfactory period, denoted by X_D. X_D can be measured in terms of *years*, and for practical purposes is represented by the relevant ranges such as *5 years*, *15 years* and *20 years*. Of course different ranges can be used depending on the specific problem at hand.

The satisfactory period of a particular problem has to be assessed using experience by taking into account the physical state of the structure and ambient conditions. Information and test data such as chloride content, corrosion intensity are all valuable materials for this purpose. The causes or explanations of the deterioration may also be important in determining X_D. If this is the case, like the assessment for safety described in Section 3.7.2, X_D can also be evaluated conditional on a hypothesis H_j. For the hypothesis set $H_j \in H = (H_1, H_2, \cdots, H_n)$, X_D can be represented by $X_{D|H}(H_i)$.

Outlined above is the assessment of safety, serviceability and durability which are usually compulsory. In addition, depending on the owner's preference, other cosmetic concerns such as *appearance* may also be non-trivial issues. Therefore, the content of condition assessment has to be determined by the engineer for specific problems.

3.10 Prognosis Procedure

Prognosis here refers to the process of predicting the structural adequacy of an existing structure or component in the future after the corrective work is executed. Methods for assessing structural safety, serviceability and durability described in previous sections can be directly applied to the prognosis by taking into account the effect of the chosen course of action.

Thus in experience-based safety prognosis, the engineer has to judge whether the structure is going to be "safe" or not from his/her experience, with regard to a hypothesis and a chosen action a_j. If reliability-based prognosis has to be carried out, the statistical properties of basic variables as well as the model uncertainty have to be modified by taking into account the effect of a_j. The resulting conditional reliability index has to be denoted by $\beta^{a_j}_{|H_i}$. Finally, the result of the safety prognosis is represented by $X^{a_j}_{F|H}(H_i)$, where $H_i \in H$.

Similarly, the results of a serviceability prognosis can be represented by $X^{a_j}_{S|H}(H_i)$ with two linguistic values: "satisfactory" and "inadequate". Both the safety and serviceability prognoses have to be carried out for the entire period of the planned service life of the structure.

Durability prognosis can be performed in exactly the same way as that in durability assessment except that the effect of a course of action a_j has to be considered over time. For example, the result can be denoted by $X^{a_j}_{D|H}(H_i)$ with three values: "5 years", "15 years" and "20 years."

3.11 Summary of This Chapter

A procedure for condition evaluation of existing concrete buildings has been outlined, in which the structural conditions regarding safety, serviceability and durability before and after a course of action is taken can be assessed using both experience and reliability analysis. The set of hypotheses obtained from the diagnostic procedure described in Chapter 2 are deliberately used in the proposed approach, and the result of this chapter forms the basis on which relevant decisions with regard to maintenance and repair can be made through a rational decision-making procedure which will be developed in the next chapter.

Chapter 4

A Method of Decision-making for Dealing With Existing Concrete Buildings

4.1 Introduction

In previous chapters, our attention was focused on the development of suitable procedures for identifying and explaining observed anomalies, and assessing structural conditions regarding safety, serviceability and durability. However, the final purpose of dealing with existing concrete structures is to determine what course of action is appropriate for the structure. To do this, sound decision-making techniques need to be employed which can cope with the inherent difficulties of the situation.

One major difficulty is how to deal with uncertainty in making a relevant decision. The main purpose of taking a course of action is to restore the structure's performance to an acceptable level regarding various requirements such as structural safety, serviceability and durability at an acceptable cost. If the structure can be assessed for these requirements with certainty, the selection of a course of action from a set of candidates available can be simply made by comparing consequences of each action according to the decision-maker's preferences. However, as can be seen from the procedures of diagnosis and condition assessment, uncertainties are always presented unless perfect information is obtained. It is usually practical only to predict possible outcomes of a course of action. It is therefore difficult to make a decision using any deterministic approach or experience alone. This is especially the case when the consequence of making an inappropriate decision is very serious, such as structural collapse. Under such circumstances, a well-structured procedure combining suitable decision-making techniques together with practical concerns is needed to help the engineer consistently make rational decisions under risk and uncertainty, and hence reduce the chance of committing mistakes. It is the aim of this chapter to develop such an approach using sound theories.

Depending on specific problems concerned, many techniques are available for making decisions under risk and uncertainty. Probability-based modern decision theory is one of a number of reasonable approaches, and has been widely applied in engineering and other fields. The outcomes of diagnosis and condition evaluation described in previous chapters have been presented in terms of the concepts of probability, and can be easily incorporated with this theory. For these reasons, statistical decision theory will be used in this chapter as the framework for the development of a method of making rational decisions in the treatment of existing defective concrete buildings. A preliminary review of decision theory is therefore presented in Sections 4.2 and 4.3. Following a short summary given in Section 4.4, the proposed method is described in Section 4.5.

4.2 A Brief Review of Statistical Decision Theory

Decision-making is a matter of choosing a course of action from a set of alternatives. This choice is made in decision theory by comparing different courses of action using an appropriate criterion according to the decision maker's preferences. Before going more deeply into this theory, some concepts and basic steps involved are briefly summarized in the next section.

4.2.1 Basic Steps in Simple Decision Analysis

Statistical decision analysis (Pratt *et al*, 1965; Raiffa, 1970; Robinson, 1981) is usually carried out through a number of steps which decompose a complex problem into small subtasks and allow the decision analyst to solve them sequentially. Although the detailed steps are usually dependent upon the specific problem, broadly speaking, there are four basic steps in decision analysis.

The first step is to structure the problem, i.e. to work out feasible courses of action or alternative actions and their possible outcomes, to identify the nature of risk and uncertainties involved, and to identify the decision maker. The result of this step may be best represented by a tree structure called a *decision tree* as shown in Fig. 4.1. The square node D is the decision node, and the branches emanating from it denoted by $a_1, a_2, \cdots, a_m$ represent the courses of action which have been identified. Since the outcome of each a_i can not be pre-determined with certainty at this stage, all possible outcomes of a_i are presented in the decision tree and denoted by $\theta_1, \theta_2, \cdots, \theta_n$. The likelihood of occurrence of each outcome θ_j is measured by its probability $P(\theta_j)$ which is estimated by the decision analyst using the information available. Since the decision maker may have different preferences for the various outcomes of an action, each pair (a_i, θ_j) is ranked in terms of a numerical value denoted by $V(a_i, \theta_j)$ which has to be assessed from the decision maker.

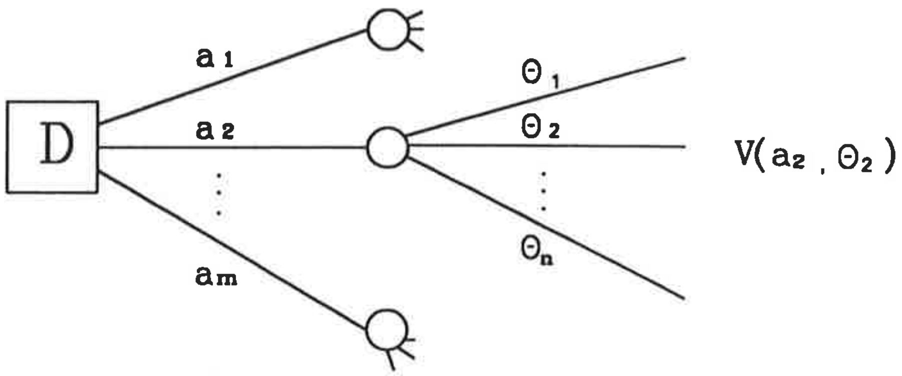


Figure 4.1: An Example of Decision Tree

The second step in the decision analysis is to estimate the probability of an outcome, $P(\theta_j)$, and the value function $V(a_i, \theta_j)$. The accuracy of the final decision will be very much dependent upon the result of this step. The probability $P(\theta_j)$ reflects the state of knowledge available, and is to be assessed by the decision analyst. It should be noted that the decision analyst and the decision maker are not necessarily the same person. The former is the person who carried out the decision analysis, and the latter is the one whose preferences regarding possible consequences are to be used to compare different alternatives.

After relevant probabilities and value functions have been estimated, the third step in the decision analysis is to choose an appropriate criterion on which the decision can be made. The most common criterion for this purpose is the *expected value criterion,* which is to choose the course of action that maximizes the expected value. However, depending on the specific problem, other criteria such as *Maximin* and *Maximax rules,* may also be applied. A number of decision rules widely used in practice are discussed later in section 4.2.6.

The final step is to carry out necessary numerical calculations for making the decision. As a simple example, the decision problem illustrated in Fig. 4.1 can be solved by firstly evaluating the expected value for each course of action a_i as:

$$E(a_i) = \sum_{j=1}^{n} P(\theta_j) \times V(a_i, \theta_j), \tag{4.1}$$

and choosing the action a^* which has the maximum expected value, i.e.

$$E(a^*) = Max\{E(a_1), E(a_2), \cdots, E(a_m)\} \tag{4.2}$$

The process of decision analysis has been briefly summarized above, and good references on decision theory can be found elsewhere (von Neumann, 1947; Howard, 1968; North, 1968; Benjamin and Cornell, 1970; Raiffa, 1970; Bradley, 1976).

As can be seen from the above discussion, uncertainties in decision analysis are represented by probability distributions which are estimated from information available. Therefore, the quality of a decision made at any time is closely related to the amount of knowledge available at that time. It is also necessary to realize that, unless uncertainties are completely eliminated, there is always a risk of facing bad outcomes from a decision thus made. In other words, a decision maker can make a good decision, but can not guarantee the best results.

In order to compare different courses of action, appropriate values have to be assigned to their outcomes. For this purpose, there exist a number of different ways to assess the value function over possible consequences from the decision maker's preferences, and some concepts regarding this are addressed in the next section.

4.2.2 Concepts of Preference and Utility

Generally, preferences are of two types, i.e. direct preference and the preference reflecting attitude to risk. For instance, "*I would prefer red to black*" is an example of direct preference. An example of the second type is : "*I would rather take course of action A and accept outcome A_0 for certain, rather than adopt course of action B, giving me a twenty percent chance of outcome B_1 and an eighty percent chance of outcome B_2*". In practice, preferences appear mostly as the combinations of these two types.

Various preferences may be simply scaled using monetary value, e.g. the cost of an outcome of a course of action in terms of dollars. However, this approach does not allow for decision-making in situations where non-monetary outcomes are present. Also, existing evidence indicates that most decision makers are averse to risk (Pratt, 1964; Benjamin and Cornell, 1970; Raiffa, 1970; Dandy and Warner, 1989), and the phenomenon of risk aversion can not be accounted for in the decision analysis if the monetary value is used to scale the preferences. For example, given two options A and B with the following properties:

- A=win \$ 100,000 for sure;
- B=win \$ 1,000,000 with probability of 0.1 or \$ 0 with probability of 0.9,

it is reasonable to expect that different people would have different choices depending on factors such as their financial resources, and attitude to risk. However, based on the expected value criterion using monetary values, there will be no difference between choosing A and B.

For these reasons, monetary value is not always appropriate for ranking consequences in decision-making, and better quantities are certainly necessary for scaling preferences when uncertainties are involved. According to Hull *et al.* (1977), utility is probably the most satisfactory, and generally accepted, scale for preferences. Utility theory derives from von Neumann's four basic assumptions (von Neumann, 1947). Briefly, these are:

1. outcomes resulting from a course of action can be compared. In other words, preferences can be evaluated for outcomes or prizes, and these preferences are transitive;
2. preferences can be assigned in the same way to lotteries involving prizes as they are assigned to the prizes themselves;
3. there is no fun in gambling;
4. continuity holds for preferences.

If these assumptions are accepted, there is then a mathematical function U, called a *utility function*, that assigns a real number to each prize or outcome with the following properties:

- utility function U is defined on the set of possible outcomes;
- outcome θ_A is preferred to outcome θ_B if and only if $U(\theta_A) > U(\theta_B)$;
- a course of action resulting in possible outcomes $\theta_1, \theta_2, \cdots, \theta_n$ with probabilities $p_1, p_2, \cdots, p_n$ is preferred to another one resulting in possible outcomes $\psi_1, \psi_2, \cdots, \psi_m$ with probabilities $q_1, q_2, \cdots, q_m$ if and only if

$$\sum_{i=1}^{n} p_i \times U(\theta_i) > \sum_{j=1}^{m} q_j \times U(\psi_j) \tag{4.3}$$

  where $\sum_{i=1}^{n} p_i = 1$ and $\sum_{j=1}^{m} q_j = 1$. The utility function allows preferences to be consistently described by means of a real number scale.

One interesting property of the utility function which needs to be mentioned is that its absolute magnitude is not important, since the preference scaled by its utility is unchanged if the numerical value of the utility is multiplied by a positive constant or added to any constant, i.e. if $U(x, y)$ is a utility function, then $U'(x, y) = k_1 + k_2 U(x, y)$, where $k_2 > 0$, is also a utility function. In other words, the utility function is invariant to positive linear transformations. This indicates that with preferences, utilities of consequences are measured relative to an arbitrary origin and unit of measure. This property greatly simplifies the estimation of utility values in practice.

4.2.3 The Estimation of Utilities

Utility values over possible outcomes are usually extracted from the decision maker by the decision analyst through an appropriate acquisition procedure which varies with the type of utilities. In practice, there are broadly two kinds of utility according to the number of dimensions in describing the consequences. If outcomes of a course of action can be described by a single parameter, the related utility is *unidimensional.* If more than one parameter are needed to fully describe the outcome, the resultant utility is then *multi-dimensional*, and is also referred to as a *multi-attribute* utility. Depending on the dimensionality, different techniques are required to estimate various utility values.

Assessing Unidimensional Utility

The simplest approach to evaluate unidimensional utility is direct rating (Torgerson, 1958). This method involves assigning quantitative scales to discrete outcomes according to the decision maker's judgement.

Perhaps the most widely used approach in assessing unidimensional utility is the gamble method which was described by von Neumann (1947) and Raiffa (1970). It involves establishing the certainty equivalents of a number of simple gambles, and utility values can be easily obtained for discrete outcomes. The method has been successfully applied to many practical decision problems (Marguardt *et al*, 1965; Ginsberg *et al*, 1968; Betaque and Gorry, 1977).

However, if utility values are required for a wide range of outcomes in a decision situation, the utility estimation using the above methods may become very tedious. This difficulty triggered the development of some simplified methods which usually reduce the utility evaluation from obtaining all points on a curve to determining a few parameters in a functional form. The commonly used function for this purpose is a quadratic one (Markowitz, 1959). However, it seems that this approach was mostly developed and used in the context of the utility of investors for money, and few applications are reported in other fields.

Assessing Multidimensional Utility

No fundamental differences exist between unidimensional and multi-dimensional utilities, since both of them are a means of describing preferences and attitude to risks over a set of outcomes involved in a decision problem. Theoretically, multi-dimensional utilities can also be evaluated using the same methods as for assessing unidimensional utilities. However, there exist practical difficulties for this purpose. Since a complete multi-dimensional outcome space is formed from the combination of different dimensions, if the involved dimensions are continuous, the total number of outcomes will be too large to be handled in the utility assessment. Therefore, various methods specifically suitable for estimating multi-attribute utilities have been developed (Fishburn, 1966; Farquhar, 1977), which can simplify the assessment procedure based on some assumptions.

Of these methods, one simple approach (Hull, 1977) is to assume that the utility is a linear function of values of different dimensions of the outcomes. For this purpose, if each dimension X_i can be measured by its own quantitative scale x_i, and if it is assumed that there is a constant rate of trade-off between one dimension and another, the utility over n-dimensional outcomes can be represented by the following function:

$$U(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} a_i x_i \tag{4.4}$$

where $U(x_1, x_2, \cdots, x_n)$ is the multi-dimensional utility function; x_i is the numerical scale of i^{th} dimension and a_i is a constant representing the trade-off between different dimensions. However, the application of this approach is very limited because, in many cases, not all dimensions can be measured by their own numerical scales.

Perhaps the most widely used method is to assume that multi-dimensional utilities follow an *additive utility function* (Adams, 1959; Fishburn, 1965; 1967; Pollak, 1967), i.e.

$$U(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} U_i(x_i) \tag{4.5}$$

where $U_i(x_i)$ is the unidimensional utility function on the i^{th} dimension X_i.

Fishburn (1965) has proven that the necessary and sufficient condition for using an additive utility function is that the evaluation of a course of action under risk depends only on the marginal distribution over each dimension, i.e. for an outcome space $X = (X_1, X_2, \cdots, X_n)$, Eq. 4.5 holds only if "the individual is indifferent between the two gambles in any pair of gambles that have the same total probability for each $x_i \in X_i (i = 1, \cdots, n)$ that appears in either." He also developed theorems which can be used to test for this condition. Pollak (1967) also described this condition in terms of preferences.

The function $U_i(x_i)$ in Eq. 4.5 can be evaluated using the same method as used in estimating unidimensional utility functions, conditional on some fixed element of outcomes. However, it is not sufficient to obtain $U(x_1, x_2, \cdots, x_n)$ by simply summing $U_i(x_i)$ thus assessed. Because each $U_i(x_i)$ is defined up to a positive linear transformation, linear transformations of each $U_i(x_i)$ do not necessarily result in a linear transformation of $\sum_{i=1}^{n} U_i(x_i)$. Different dimensions have to be ranked by a set of scale factors c_i. These factors reflect the decision maker's trade-offs between different dimensions. Methods for evaluating c_i are available in (Eckenrode, 1965; Keeney, 1974), and Eq. 4.5 then becomes:

$$U(x_1, x_2, \cdots, x_n) = \sum_{i=1}^{n} c_i U_i(x_i) \tag{4.6}$$

The major shortcoming of additive utility is the restrictiveness of its requisite condition which is usually difficult to justify. A less restrictive, but more practical alternative was originated in Keeney's work (Keeney, 1972) which is referred to as *quasi-additive* utility. The foundation of this method is the concept of utility independence. Given a utility function $U(x, y)$, one can say x is utility independent of y if the decision maker's preferences over any gambles on x, for a fixed y_0 in y, are the same regardless of the amount of y_0. Then, if x and y are mutually utility independent, $U(x, y)$ can be evaluated as:

$$U(x, y) = U(x, y_0) + U(x_0, y) + kU(x, y_0)U(x_0, y) \tag{4.7}$$

where k is a constant, and $U(x, y_0)$, $U(x_0, y)$ are conditional utility functions for x and y, for arbitrary y_0 and x_0 respectively. The same concept can be extended to the n-dimensional case (Keeney,1972; 1977; Keeney and Raiffa, 1976). The method has been illustrated by using a two dimensional example (Keeney, 1972), and it was shown that the approach has great practical importance.

Using relevant methods reviewed above, utility values over various outcomes can be assessed. To make a decision under incomplete information, probabilities of all possible outcomes have to be estimated, and depending on the amount of knowledge available, making a decision can fall into a number of different categories.

4.2.4 Decision-making With Incomplete Knowledge

For a set of courses of action $a_1, a_2, \cdots, a_n$, if perfect information has been acquired, the outcome of each a_i can be pre-determined, and the decision can be easily made by choosing the alternative which has the highest utility. This is called *decision-making under certainty.* However, decisions usually have to be made without knowing exactly the outcome of a_i. Rather, the information available allows us to assign probabilities to the possible consequences of each alternative. Decision-making in this situation is referred to as *decision-making under risk.* If there is no knowledge available at all about the occurrence of outcomes, the decision is then to be made *under uncertainty.*

In addition to the above three categories, decisions may fall in between *decision under uncertainty* and *decision under risk*, i.e. the information available is able to rank the possibilities of outcomes, but is insufficient to obtain the specific probability values. Fishburn (1964) studied this problem and developed an equation which can be used to make decisions under such circumstances. Firstly, he assumed that each course of action has n possible outcomes, and the decision maker has sufficient knowledge to rank the probabilities of these n outcomes, e.g. $P_1 \geq P_2 \geq \cdots \geq P_n$. Then, the expected utility of a course of action a_1, $\bar{U}(a_1)$, is larger than or equal to that of alternative a_2, $\bar{U}(a_2)$, i.e. $\bar{U}(a_1) \geq \bar{U}(a_2)$, if

$$\sum_{k=1}^{j} U(a_1^k) \geq \sum_{k=1}^{j} U(a_2^k) \tag{4.8}$$

for all $j = 1, \cdots, n$, where $U(a_1^k)$ and $U(a_2^k)$ are the utilities of the k^{th} outcomes of a_1, a_2 respectively. In other words, given $P_1 \geq P_2 \geq \cdots \geq P_n$, where P_i is the probability of i^{th} outcome, if Eq. 4.8 is satisfied, alternative a_1 is preferred to a_2.

However, a major disadvantage of this approach is that Eq. 4.8 is very difficult to satisfy in practice. To overcome this difficulty, Kmietowicz and Pearman

(1981) improved the above method by employing the linear programming technique in which the maximum and minimum expected utility of each course of action can be obtained by only knowing that $P_1 \geq P_2 \geq \cdots \geq P_n$. The best course of action is the one whose maximum expected utility is larger than any others'.

Regarding the common decision situations discussed above, the problem treated in this thesis falls into the second category, i.e. decisions under risk. Therefore, the assessment of probabilities for various outcomes is an important task. Probabilities involved in many existing decision analysis are usually subjectively assessed. Subjective probability is the measure of a person's belief as to the occurrence of an event. The evaluation of this probability is based on the state of mind, rather than independent trial experiments, and reflects the state of knowledge available. Many techniques for estimating subjective probabilities are available, and choosing an appropriate one is dependent on the specific decision problem. In this work, relevant probabilities of consequences are evaluated from the results of diagnosis and condition evaluation, and no separate probability assessment is necessary. Therefore, techniques of probability assessment will not be discussed here in detail, as references are available elsewhere (Winkler, 1970; Hampton *et al.*, 1973; Tversky, 1974).

With utilities and relevant probabilities obtained, the best course of action can be easily chosen using the expected utility criterion, and a single-stage decision-making is thus complete. However, it is usually possible and desirable to improve the quality of decision-making by gathering new information through various means such as experimental work, refined analysis. Therefore, different decisions can be made in sequence depending on the information available. In this way, the decision-making becomes a multi-stage process. Since gathering new information is costly, and sometime can be very expensive, whether to acquire more data or to take a terminating action at any stage is a crucial decision to be made. The most commonly used methods for this purpose will be reviewed briefly in the next section.

4.2.5 Multi-stage Decision-making

Multi-stage decision-making can be carried out in different situations. For a physical system whose states change stochastically, a sequence of maintenance/repair actions regarding its whole service period can be determined through a multi-stage analysis which is usually modelled as a Markovian process (Eckles, 1968; Satia *et al*, 1973). In this case, each stage represents one state of the system, and a course of action is usually chosen for each stage. Alternatively, in the situation where time factor is ignored, the multi-stage decision-making is to determine a sequence of information-gathering activities such as experiments, and finally to choose a course of action using the information available at each stage. In this case, various stages represent different states of information available. Decision-making considered in this thesis falls into the second category, and therefore the review here does not include the literature on stochastic process models.

Generally, a decision at the first stage is made by choosing a course of action a^* which has the highest expected utility calculated using the prior information. If there are a set of tests from which more data can be obtained, there is then a choice to be made between taking a^* or making the decision after a selected test is conducted. The key point in this multi-stage process is obviously to decide the necessity of transition from one stage to another. This is usually determined based on the *value of information* obtained from an experiment e (Howard, 1966; Matheson, 1968). To illustrate this, a two-stage decision tree is plotted in Fig. 4.2 where i and $i+1$ denote the i^{th} and $(i+1)^{th}$ stage respectively; e denotes the experiment to be conducted and C_e is its cost; the possible outcomes of e are represented by $d_1, d_2, \cdots, d_l$; all other symbols have the same meanings as in Fig. 4.1 except that U is now the utility function instead of the value function used before. At the i^{th} stage before test e is

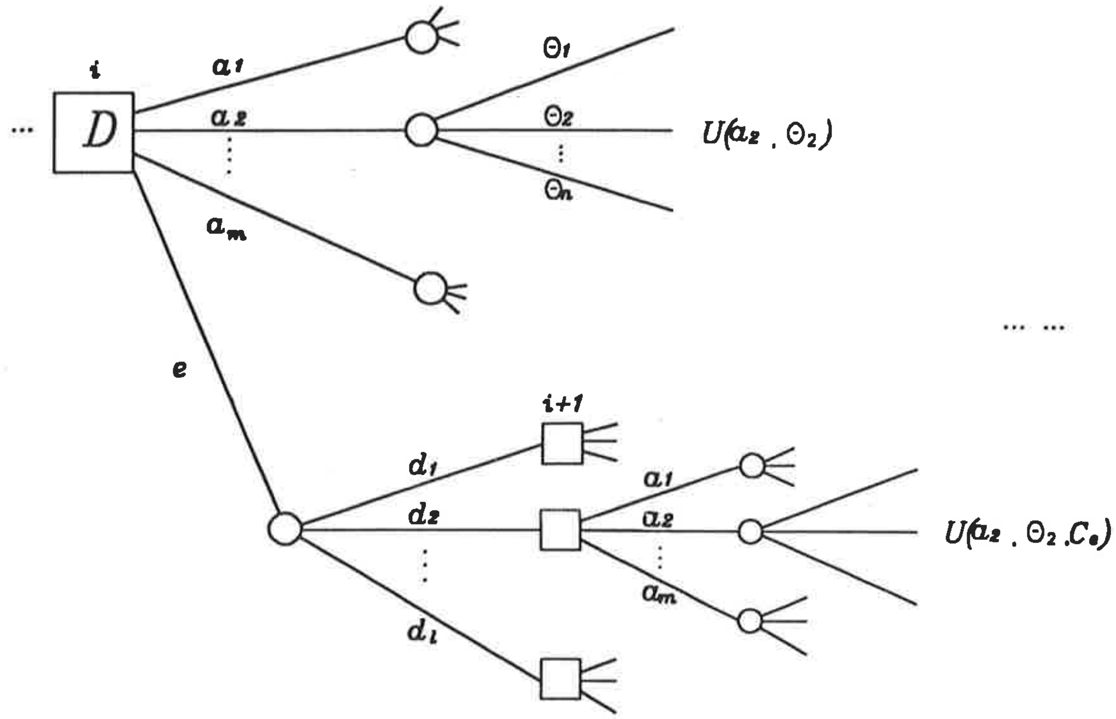


Figure 4.2: An Example of Multi-stage Decision Tree

carried out, the total information available to the decision analyst is known as prior knowledge, denoted here by ϵ. Therefore, at stage i, the expected utility of a course of action $a_i \in A$ is evaluated conditional on ϵ, i.e.

$$\bar{U}(a_i \mid \epsilon) = \sum_{j=1}^{n} P(\theta_j \mid \epsilon) U(a_i, \theta_j) \tag{4.9}$$

in which $P(\theta_j \mid \epsilon)$ is the probability of consequence θ_j estimated according to the prior data ϵ. The best alternative at the i^{th} stage is a^* which is determined by: $\bar{U}(a^* \mid \epsilon) = \max_{a_i \in A}[\ \bar{U}(a_i \mid \epsilon)\ ]$.

After test e is carried out, the probability $P(\theta_j)$ can be updated by the newly obtained data, and consequently an improved decision can be made. However, since test e will incur a cost C_e, and the outcome of e can not be predetermined, in order to decide whether to take a^* as the terminal decision or to conduct e, it is necessary to compare the expected utility of a^* with that of e. Assuming an outcome d_k for test e for sure, the expected utility of a_i

changes to:

$$\bar{U}(a_i \mid \epsilon, d_k) = \sum_{j=1}^{n} P(\theta_j \mid \epsilon, d_k) U(a_i, \theta_j, C_e) \tag{4.10}$$

where $P(\theta_j \mid \epsilon, d_k)$ is the probability of consequence θ_j conditional on the newly obtained data d_k. To evaluate $P(\theta_j \mid \epsilon, d_k)$, it needs to estimate the probability $P(d_k \mid \theta_j, \epsilon)$ subjectively from experience and other relevant knowledge. $P(\theta_j \mid \epsilon, d_k)$ can then be obtained through Bayes' theorem:

$$P(\theta_j \mid \epsilon, d_k) = \frac{P(d_k \mid \theta_j, \epsilon) P(\theta_j \mid \epsilon)}{P(d_k \mid \epsilon)} \tag{4.11}$$

The expected utility of test e conditional on a single outcome d_k can now be evaluated as:

$$\bar{U}(e \mid \epsilon, d_k) = \max_{a_i \in A} [\bar{U}(a_i \mid \epsilon, d_k)] \tag{4.12}$$

Finally, since the test e has l possible outcomes $d_1, d_2, \cdots, d_l$, its expected utility has to be calculated by:

$$\bar{U}(e \mid \epsilon) = \sum_{k=1}^{l} P(d_k \mid \epsilon) \bar{U}(e \mid \epsilon, d_k) \tag{4.13}$$

in which $P(d_k \mid \epsilon)$ is the probability of outcome d_k, and evaluated by:

$$P(d_k \mid \epsilon) = \sum_{j=1}^{n} P(d_k \mid \theta_j, \epsilon) P(\theta_j \mid \epsilon) \tag{4.14}$$

Up to this point, the decision at the i^{th} stage can be made by comparing the expected utility of experiment e with that of the best course of action a^* determined without performing the test. That is, if $\bar{U}(e \mid \epsilon) > \bar{U}(a^* \mid \epsilon)$, test e is preferred to a^*, and the decision process continues. In this case, the future decision at the $(i+1)^{th}$ stage will be made according to the result of experiment e. Otherwise, if $\bar{U}(e \mid \epsilon) < \bar{U}(a^* \mid \epsilon)$, it is not worth conducting e to gather more data, and the decision is to take a^*.

Instead of using the expected utility of an experiment, a different approach also widely used in multi-stage decision-making is to compare the expected utility of a piece of information with that of a^*. For this purpose, given a decision problem with courses of action $A = [a_1, a_2, \cdots, a_m]$, and possible consequences $\Theta = [\theta_1, \theta_2, \cdots, \theta_n]$ for each a_i, the best action a^* in A at the i^{th} stage can be firstly identified as the one which has the highest expected utility using the information ϵ, i.e. $\bar{U}(a^* \mid \epsilon) = \max_{a_i \in A}[\bar{U}(a_i \mid \epsilon)]$. Since the evaluation of the consequences of an action usually involves a number of variables, it is assumed here that each consequence θ_j is dependent upon a set of variables $X = (X_1, X_2, \cdots, X_m)$, among which those unknown are treated randomly with the probability distribution denoted by $\{x_i\}$. If there is a test e which can reveal pertinent information on X_i, the distribution of X_i is updated, and consequently an improved decision can be made using the updated $\{x_i\}$. To do this, the question is then which variable among X is so valuable that the expected utility of searching more data on it is higher than that of a^*. For this purpose, assuming that the true value of X_i can be obtained from an experiment e at the cost C_{x_i}, the probability of the consequence θ_j is then updated from $P(\theta_j \mid \epsilon)$ to $P(\theta_j \mid x_i, \epsilon)$ where x_i is the assumed true value of X_i. The expected utility of spending C_{x_i} to gather this information on X_i is evaluated by:

$$\bar{U}(C_{x_i} \mid \epsilon) = \int_{X_i} \bar{U}(a^* \mid x_i, \epsilon)\{x_i \mid \epsilon\} \tag{4.15}$$

where $\bar{U}(a^* \mid x_i, \epsilon)$ is the expected utility of a^* using the prior knowledge ϵ together with x_i, i.e. $\bar{U}(a^* \mid x_i, \epsilon) = \max_{a_j \in A} \bar{U}(a_j \mid x_i, \epsilon)$; in which $\bar{U}(a_j \mid x_i, \epsilon)$ can be evaluated in a routine manner:

$$\bar{U}(a_j \mid x_i, \epsilon) = \sum_{\theta_k \in \Theta} U(a_j, \theta_k) P(\theta_k \mid x_i, \epsilon) \tag{4.16}$$

With the expected utility of gathering information on X_i thus evaluated, the transition of the decision process from stage i to $i + 1$ can be easily decided, i.e. if $\bar{U}(C_{x_i} \mid \epsilon)$ is larger than the expected utility of a^*, we should design and carry out an appropriate test program which can result in particular data on X_i.

Actually by evaluating $\bar{U}(C_{x_i} \mid \epsilon)$ for all $i = 1, 2, \cdots, m$, the most valuable variable X^* in X can be found by $\bar{U}(C_{x^*} \mid \epsilon) = \max_{i=1,\cdots,m} [\bar{U}(C_{x_i} \mid \epsilon)]$. However, it is usually difficult to identify X^* because of large amount of computation required.

Theoretically, this approach can also be used to determine whether it is worth gathering new information on more than one variables $X_i \cdots X_k$ simultaneously by considering the probability $P(\theta_j \mid x_i \cdots x_k, \epsilon)$ and the joint distribution $\{x_i \cdots x_k\}$.

Approaches based on the *value of information* as reviewed above are the basis of multi-stage decision analysis. However, both of these two methods have the same shortcoming: in order to find the optimal test, computations involving an extremely large decision tree is needed. Therefore, application of the approach to real problem-solving is limited.

In approaches discussed above, all decisions are made using the expected value/utility criterion. In practice, there exist many other rules which can be adopted depending upon the specific problem at hand. In the next section, several commonly used decision rules as well as disadvantages of the expected value/utility criterion are discussed.

4.2.6 Different Decision Rules

Expected value/utility criterion has been most widely used in decision analysis. In the case of decision-making under risk, this is a very useful and meaningful criterion. However, its shortcomings are obvious. Since decisions made using this rule can not guarantee a good result, in an important decision situation, the approach may lead to severe consequences for the decision maker if the worst outcome of the selected action unfortunately occurs. On the other hand, the expected value/utility criterion only considers the mean value, and ignores the other properties of probability distributions such as deviation. This may result in an irational decision. For example, in the decision problem illustrated in Fig. 4.3, a_1 and a_2 are two courses of action available to the decision maker, and each of them has three possible consequences denoted by $x^{a_i}(1)$, $x^{a_i}(2)$, and $x^{a_i}(3)$ respectively. The utility of each consequence and related probability are represented by the bar chart in the figure. The expected utilities of a_1 and a_2 are $E(a_1) = 61$ and $E(a_2) = 56$ respectively. Using the expected utility criterion, apparently a_1 is preferred to a_2. However, it is reasonable to assume that many decision makers would choose a_2 instead of a_1 if they are averse to risk, because the consequences of a_2 have less deviation than that of a_1.

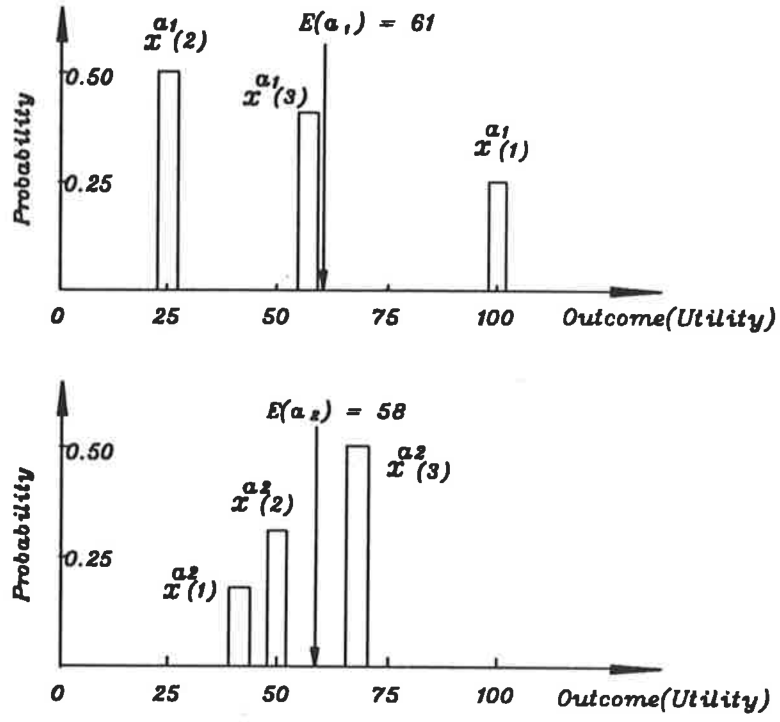


Figure 4.3: An Illustration of the Range of Outcomes

Due to the inadequacy of the expected utility criterion, a number of other decision rules are also widely adopted in practice. One of these is the so-called Maximin rule, which tends to select the course of action a^* whose possible minimum value is the greatest among all available courses of action, i.e. (according to Fig. 4.1 on page 127):

$$U(a^*) = \max_i\{\min_j[U(a_i, \theta_j)]\} \tag{4.17}$$

This rule is usually used by conservative or pessimistic decision makers. For example, it may be appropriate in structural assessment when the decision maker wishes to avoid a catastrophe at all costs.

In contrast to the conservative decision maker, optimistic decision makers usually hope that the uncertain future develops favourably for them, and hence try to select the course of action a^* whose possible maximum value is larger than that of any others, i.e. :

$$U(a^*) = \max_i\{\max_j[U(a_i, \theta_j)]\} \tag{4.18}$$

and this is called Maximax rule.

The disadvantage of both the Maximin and Maximax criteria is that they rely on extreme utilities or values, and this can lead to over conservative or over risky. Nevertheless, these rules are still very useful in some situations, especially when the decision is to be made under uncertainty.

For decision-making under uncertainty, there exist another criterion which is to assign the most unbiased probabilities for all possible outcomes and change the problem into decision-making under risk. Logically, the most unbiased judgement without any information on the chance of occurrence of consequences is that all outcomes have equal probability of occurrence (Jaynes, 1968), i.e. $P(\theta_j) = 1/n$, where θ_j denotes the j^{th} outcome of n possible outcomes. The expected utility principle can then be used to choose the alternative which has the highest expected utility:

$$U(a^*) = \max_i\{\frac{1}{n}\sum_{j=1}^{n} U(a_i, \theta_j)\} \tag{4.19}$$

These decision criteria represent most of those commonly used in practice.

Statistical decision theory has thus been briefly reviewed. While decision theory has been successfully applied in fields such as medical science (Ginsberg *et al.* 1968; Betaque and Gorry, 1977), economics (Keeney and Raiffa, 1976), its application in solving civil engineering problems can also be found in the literature. In the next section, previous work relevant to decision-making in structural assessment is briefly discussed.

4.3 Decision Analysis in Relevant Engineering Problems

There are a large number of applications of decision theory in engineering problem-solving. Of these approaches, however, very limited work on dealing with structural defects of existing concrete structures has been found in the literature. For this reason, the review in this section contains only a few applications which are more or less relevant to our problem.

Folayan, Höeg and Benjamin (1970) employed decision theory to study the problem of foundation settlement. The amount of settlement was considered as a function of a set of deterministic quantities as well as a random variable which represents the soil property. The probability distribution of this random variable is firstly estimated by an experienced engineer, and then updated through Bayes' theorem using sampling data obtained from further testing. The decision to be made is how many samples are needed to make a good prediction of the settlement. For this purpose, a utility loss function was defined as a quadratic curve of the prediction error ϵ, which is the difference between the calculated settlement and the true settlement. The optimal number of samples is obtained by minimizing the expected loss through a simple optimization procedure.

In a similar study, decision tree analysis is employed by Wu (1974) to decide whether sample testing is worth being conducted to explore soil properties for improving the prediction of foundation settlement and slope stability, in which a cost function was used in lieu of utility values.

In addition to the unidimensional decision analysis, in which the consequence of an action is represented by a single parameter, Nessim and Jordaan (1989) applied multi-dimensional approach to study error-control in structural design. For this purpose, probabilistic models are firstly developed for estimating the error content in a structure after the application of a checking strategy with given efficiency. The error content is then related to structural safety and serviceability through relevant structural analysis. Consequences of a course of action in structural design are represented by a set of attributes such as *total cost, number of deaths and injuries in case of structural failure.* The utility function is assessed over these attributes. The expected utility given an efficiency of quality control are next evaluated, and the best level of efficiency is obtained when the expected utility reaches the highest value. However, in order to calculate the expected utility it is necessary to assess the joint probability distribution over those involved attributes, and this is usually difficult in practice.

To study the maintenance and repair related problems of existing concrete structures, Warner and Kay (1983) have made a pioneering attempt to apply the decision theory. However, as already mentioned in Chapter 1 on Page 19, their approach lacks a great deal of details needed to implement the proposed procedure.

4.4 Summary on This Review

Probabilistic decision theory provides a good framework for consistently making choices from alternatives according to the decision maker's preferences, even when the consequences of courses of action can not be predicted with certainty. With the use of utility values, the decision maker's attitude toward risks can be considered in the decision-making. Since risks regarding the occurrence of various bad consequences are explicitly expressed in terms of probabilities, the decision maker can have a clear understanding of the risk scenarios to be faced. It is therefore potentially very helpful to use decision theory in the treatment of existing concrete structures, especially when involved risks of having severe consequences are high in choosing courses of action.

Although decision theory has been extensively applied to engineering as well as other areas, a systematic approach particularly suitable for dealing with existing concrete structures has not yet been available. Such a method is to be developed in the next section, by use of which questions such as *what to do about a given defective concrete building* can be answered rationally using results of diagnosis and condition evaluation obtained from previous chapters.

4.5 A Method of Decision-making for Dealing With Structural Defects

4.5.1 Introduction

We shall be concerned with a decision-making situation in which the engineer and the owner of the structure work together to decide on how to handle problems in an existing concrete building. When a building shows noticeable anomalies, the engineer is usually called upon by the owner of the structure to carry out an assessment and to advise on possible remedies. A final decision on adopting a course of action is to be made based on the owner's requirements and financial resources with references to the condition of the structure. Therefore, the owner of the structure will be considered here as the **decision maker,** whose preferences on different choices are to be used in the decision-making, and the engineer serves as the **decision analyst** who is responsible for carrying out relevant analysis required for making such choice, and advising the decision maker of technical options available, etc.

In dealing with potentially defective concrete buildings, decisions to be made usually include answers to questions such as "is any repair action needed ?", "what is the best action to cure the problem ?" Although different people have their own ways to find out satisfactory solutions, the primary principle probably adopted by most decision makers is *to minimize the incurred cost while trying to satisfy various requirements for the structure.* In other words, when a number of courses of action have been identified by the engineer, the problem is to choose one alternative which can best satisfy the abovementioned principle. For this purpose, the principle to be adopted for a given problem is usually defined in terms of a set of **objectives** which are to be achieved in the decision-making. The identification of relevant objectives for our problem are discussed in the next section.

4.5.2 Identification of Objectives and Their Associated Attributes

It is usually up to the decision maker to specify a set of objectives. However, in solving real problems, the decision analyst and the decision maker have to work closely together so that the decision maker clearly understands the essential features of the problem, and all important aspects are taken into account in determining the objectives. For dealing with existing concrete buildings, although objectives tend to vary with different concerns from the owner, the following four objectives are suggested for most problems:

1. to provide adequate structural safety;
2. to provide adequate structural serviceability;
3. to provide adequate structural durability;
4. to minimize the cost of achieving the first three objectives.

Of course for specific problems, relevant objectives other than those listed can be included if necessary. On the other hand, some items in the list may be dropped off.

To assess the degree to which a particular objective is satisfied by a course of action, each objective is associated with an **attribute** which has values. An objective can then be assessed using the value of its associated attribute. For example, the *objective* "to minimize the total cost" can be assessed according to its *attribute* X_{TC} in terms of "dollars", where X_{TC} represents the total cost. Obviously, the value of X_{TC} in this case is a real number.

To define an appropriate attribute for an objective, it is important to ensure that the attribute is *measurable* and *comprehensive* (Keeney and Raiffa, 1976). An attribute is measurable if it is possible to obtain a joint probability distribution over its range of values. An attribute is comprehensive if, by knowing a particular value of the attribute, the decision maker has a clear understanding of the extent to which the associated objective is achieved.

Considering these two requirements, the *objective* "to provide adequate safety" will now be associated with an attribute X_F which is a discrete linguistic variable with four possible values: $x_f(1)$="safe", $x_f(2)$="negligibly defective", $x_f(3)$="moderately defective" and $x_f(4)$="severely defective". For a given structure and a specific ultimate limit state, these four values can be obtained from the safety assessment procedure described in Section 3.7.2 of Chapter 3, and the interpretation is the same as before. Apparently any value of X_F is a clear indicator of the safety condition of the structure either before or after a course of action is taken. The probability distribution of X_F can also be estimated from the result of condition evaluation. Therefore, X_F is both comprehensive and measurable.

The second *objective* "to provide adequate serviceability" can be treated in a similar way. Using the results of serviceability assessment described in Section 3.8, an attribute X_S is defined for this objective, which is a linguistic variable with two values: $x_s(1)$="satisfactory" and $x_s(2)$="inadequate".

The *objective* "to provide adequate durability" is measured in terms of an attribute X_D which takes the values of the *satisfactory period* defined in durability assessment in Section 3.9, i.e. $x_d(1)$="5 years", $x_d(2)$="15 years", and $x_d(3)$="20 years". For example, $x_d(1)$="5 years" means that the structure will perform satisfactorily for at least 5 years regarding the concerned durability problem.

The last *objective* "to minimize the total cost" can be measured by the *attribute* X_C in "dollars", which represents all costs incurred in the process of structural assessment, i.e. :

$$X_C = C_I + C_R + C_L \tag{4.20}$$

in which C_I and C_R denote the costs incurred by the investigation (e.g. diagnosis, experiments) and repair respectively; C_L represents the cost which is equivalent to the profit loss due to any inconvenience to normal business caused by any action involved in the assessment and/or repair process. Obviously, X_C is a continuous real number.

In summary, relevant objectives and their associated attributes for making decisions regarding the treatment of existing concrete buildings have been defined, and are shown schematically in Fig. 4.4, where objectives are presented in those boxes, and the mathematical symbols under each box are related attributes.

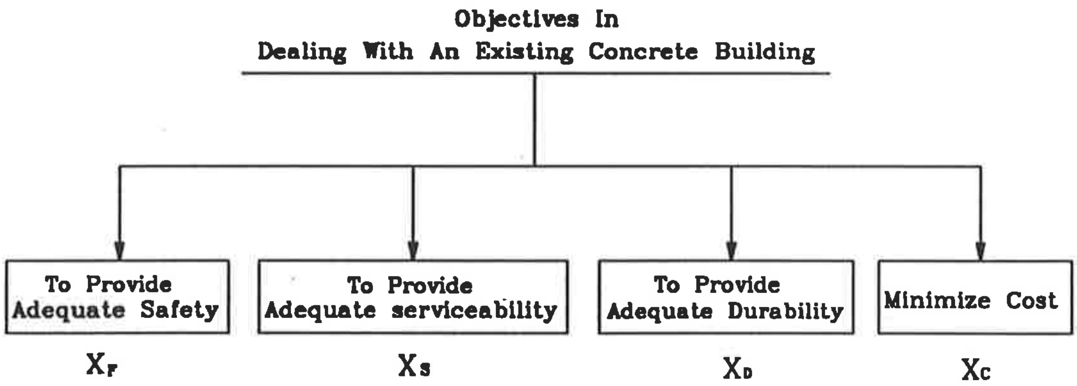


Figure 4.4: Objectives and Associated Attributes

Any individual objective measured in terms of its attribute as defined above can only cover one aspect of the decision maker's concern. Consequently, a particular value of an attribute only indicates how well the single related objective is satisfied. To assess the degree to which all interested objectives are satisfied by executing a course of action, it is necessary to combine the values of the four attributes together to form an overall indicator which is called the *consequence*. This will be discussed next.

4.5.3 The Consequence Space

Since any consequence is the result of a course of action, a specific consequence of an action a_i is denoted by x^{a_i}. The value of x^{a_i} is expressed in terms of the four attributes obtained in the last section. For example, if the action a_i is to "strengthen the structure" which totally costs 100 000 dollars (including the cost of assessment and other fees), and it is assessed that the structure will be "safe", "satisfactory" regarding the safety and serviceability performance respectively, and "will satisfactorily last about 15 years" in regard to the identified durability problem, the consequence of a_i is then represented by $x^{a_i} = [$ "$safe$", "$satisfactory$", "15 $years$", 100,000]. In this case, the value of each attribute is known with certainty, and hence the consequence x^{a_i} is determined as a single point in the consequence space.

However, when uncertainties are involved in evaluating the attributes, all possible consequences have to be considered. For this purpose, the *consequence space* of a course of action a_i, denoted by X^{a_i}, can be obtained by combining all possible values of the four attributes of a_i, i.e. $X^{a_i} = X_F^{a_i} \times X_S^{a_i} \times X_D^{a_i} \times X_C^{a_i}$, where $X_F^{a_i} = [x_f^{a_i}(1), x_f^{a_i}(2), x_f^{a_i}(3), x_f^{a_i}(4)]$, $X_S^{a_i} = [x_s^{a_i}(1), x_s^{a_i}(2)]$, $X_D^{a_i} = [x_d^{a_i}(1), x_d^{a_i}(2), x_d^{a_i}(3)]$ and $x_c^{a_i} \in X_C^{a_i} \subseteq R$, R is the set of real numbers.

In many cases, the number of dimensions of the consequence space can be reduced, depending on the specific problem. For example, after the condition survey and preliminary assessment of a concrete structure, if it can be judged that there will be no durability problem for a long period of time, the attribute X_D can then be eliminated from X, and the consequence space is reduced to $X = X_F \times X_S \times X_C$.

It has to be pointed out that consequences can be defined in many different ways, depending on how the objectives and their attributes are specified. For example, many objectives can usually be further broken down into a set of more detailed sub-objectives, and the consequences are in turn represented by the attributes of these sub-objectives. However, the number of dimensions in the consequence space resulting from this specification will be very large, and it is therefore very difficult to handle the large set of consequences in the utility and probability assessment to be discussed later. For this reason, the consequence space defined in this section aims at being as practical as possible on the one hand, and as comprehensive as possible on the other.

In order to compare two courses of action in terms of their consequences, it is convenient to rank or scale all possible consequences according to the decision maker's preferences. If numerical values can be used for the ranking, this comparison can be made with mathematical expressions. As discussed previously in Section 4.2.2, utility is a very meaningful tool for this purpose, which can represent not only the decision maker's preferences on different consequences, but also the attitude toward risks, and hence will be adopted in this work. In the next section, a method for assessing utility values over the defined consequence space is described.

4.5.4 Assessment of Utility Values

Utility values of various consequences can be assessed by asking the decision maker relevant questions in a well-designed procedure. For the problem considered in this thesis, utilities have to be assessed over the entire consequence space $X = X_F \times X_S \times X_D \times X_C$.

Utility values defined on a unidimensional consequence space can be easily assessed using the standard lottery-based method (Raiffa, 1970). For this purpose, if a lottery L yields consequences $x_1, x_2, \cdots, x_n$ with probabilities $p_1, p_2, \cdots, p_n$ respectively, the **certainty equivalent** of L is defined as a consequence x^* such that the decision maker is indifferent between the lottery L and the certainty of x^*. Therefore, the utility of x^* equals the expected utility of this lottery, i.e. :

$$U(x^*) = \bar{U}(L) = \sum_{i=1}^{n} p_i U(x_i) \tag{4.21}$$

where $U(x_i)$ is the utility of the single consequence x_i.

Based on the concept of certainty equivalent, unidimensional utilities can be evaluated in a routine manner. To do this, let $Y = (y_1, y_2, \cdots, y_m)$ be discrete unidimensional consequences, and y_1, y_m be the *least preferred* and the *most* *preferred* consequences, respectively. Since utility values are relative, we can arbitrarily assign utilities to y_1 and y_m, e.g. $U(y_1) = 0$ and $U(y_m) = 1$. To assess the utility of other consequences in Y, say y^*, a lottery $\langle y_m, p, y_1 \rangle$, yielding a p chance for y_m and $1 - p$ chance for y_1, is firstly designed. If we can find a value between 0 and 1 for p such that y^* is the certainty equivalent of $\langle y_m, p, y_1 \rangle$, i.e. we are indifferent between y^* for certain and the lottery $\langle y_m, p, y_1 \rangle$, the utility of y^* can be obtained as:

$$U(y^*) = pU(y_m) + (1 - p)U(y_1) = p \tag{4.22}$$

This is schematically illustrated in Fig. 4.5. By applying the same procedure, utilities of all consequences in Y can be determined. If Y is continuous, the assessed utility values can be approximately fitted to a curve or a function denoted by $U(y)$. The above procedure may be called *direct method.*

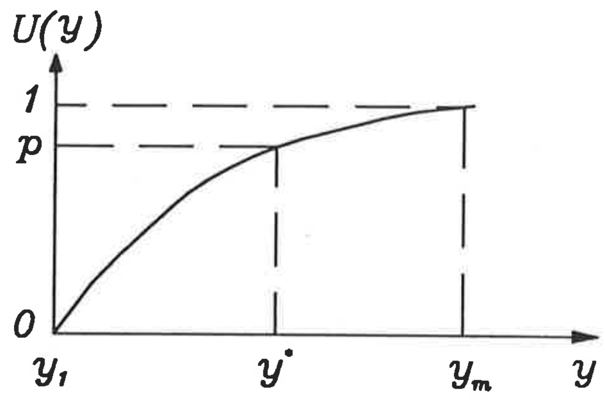


Figure 4.5: Illustration of Unidimensional Utility Assessment

However, to assess multi-dimensional utilities, the direct method is generally not suitable, and more comprehensive approaches such as that proposed by Keeney based on the concept of utility independence may have to be employed (Keeney, 1972; 1973; 1974; Keeney and Raiffa, 1976). Fortunately, due to the feature of the consequence space defined in this thesis, the assessment of the multi-dimensional utilities on $X_F \times X_S \times X_D \times X_C$ can be simplified to a unidimensional utility estimation, and carried out using the direct method just described.

For this purpose, let us consider the four attributes in the consequence space $X = X_F \times X_S \times X_D \times X_C$. The first three attributes are discrete variables with very limited numbers of possible values; however, X_C is continuous and represents the total cost in terms of dollars. Therefore, the possible combinations of $X_F \times X_S \times X_D$ will produce a limited number of elements, and can be represented by a finite set denoted by Y. For example, in a decision problem, if these three attributes – safety (X_F), serviceability (X_S) and durability (X_D) – are all relevant, the maximum number of combinations from $X_F \times X_S \times X_D$ is $4 \times 2 \times 3 = 24$. In many cases, if not all these three attributes have to be considered, the dimension of Y can be reduced, and consequently the total elements in Y could be much smaller than 24.

Having $X_F \times X_S \times X_D$ represented by Y with a limited number of discrete elements, the original four dimensional consequence space $X = X_F \times X_S \times X_D \times X_C$ is reduced to $X = Y \times X_C$ with two dimensions. Thus for a fixed value in Y, say y^*, the utilities on $y^* \times X_C$ can be fully described by a unidimensional utility function defined on the continuous variable X_C conditional on y^*, i.e. $U(x_c, y^*)$, where $x_c \in X_C$. The utility over the whole space X can be represented by a set of such conditional unidimensional utility functions as schematically illustrated in Fig. 4.6.

Obviously if there are n elements in Y, there will be n conditional utility functions available, i.e. $U(x_c, y_1)$, $U(x_c, y_2), \cdots, U(x_c, y_n)$. An important point is that these functions can be easily assessed using the direct method.

To do this, the possible maximum and minimum values of the attribute X_C have to be identified first, and denoted by x_{cmax}, x_{cmin} respectively. Utility values on the consequences formed by $Y \times x_{cmax}$ and $Y \times x_{cmin}$, i.e. those points marked by small circles and triangles in Fig. 4.6, are then assessed simply from the direct method. For a fixed value of Y, say y^*, the certainty equivalent of any point between $[x_{cmin}, y^*]$ and $[x_{cmax}, y^*]$, say $[x_c^0, y^*]$, can be found as the

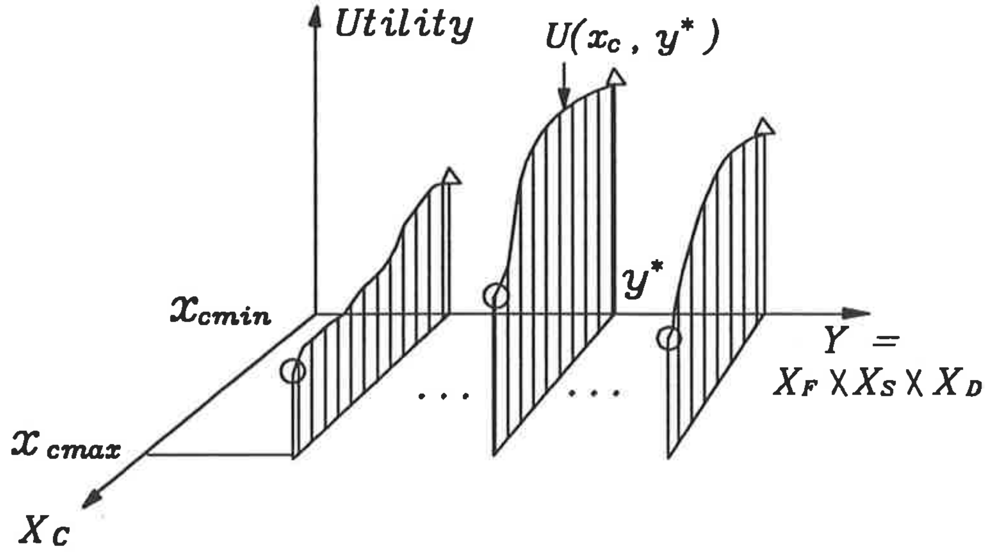


Figure 4.6: Illustration of Utilities Over the Consequence Space

lottery $\langle(x_{cmin}, y^*), p, (x_{cmax}, y^*)\rangle$. The utility of this point can be obtained by

$$U(x_c^0, y^*) = p \cdot U(x_{cmin}, y^*) + (1-p) \cdot U(x_{cmax}, y^*). \tag{4.23}$$

By assessing utilities on a set of points between $[x_{cmin}, y^*]$ and $[x_{cmax}, y^*]$, a curve or a function denoted by $U(x_c, y^*)$ can be obtained approximately.

In this way, the utilities over the multi-dimensional consequence space $X = X_F \times X_S \times X_D \times X_C$ have been simplified to assess a set of unidimensional utility functions conditional on Y, using the simple direct method.

Up to this point, relevant objectives and their associated attributes have been defined specifically for making decisions in the treatment of existing concrete buildings. Using these attributes, possible consequences of any course of action can be defined and measured in terms of utilities. Thus we have completed the preparation for choosing a course of action from a set of alternatives which can be created using the procedure described in the next section.

4.5.5 Creating Courses of Action

Although specific courses of action for different problems are not necessarily the same, the following broad categories cover most of those common courses of action to be considered in practice (Warner and Kay, 1983):

- do nothing, on the judgement that the structure will be able to satisfy all relevant requirements;
- monitor the structure in service, in order to check more carefully on adequacy of performance and any deterioration;
- undertake repair or corrective work which maintain the structure in service;
- undertake repair or corrective work after taking it temporarily out of service;
- take the structure out of service with the possibility of demolition and reconstruction.

Superficially, by modifying and amplifying this list, a set of courses of action for a particular problem can be identified. Since decision theory is basically nothing but a tool to aid the decision maker to choose one alternative from a set of candidates in a rational way, clearly, the set of candidates thus created has to be *complete* so that all potential effective courses of action can be considered in making the choice. However, the creation of such a complete set of candidates is not always practical. Each course of action itself is usually not necessarily an independent entity, and may be divided further into smaller parts. Various combinations of these parts can in turn produce a large number of valid courses of action which will make the decision tree extremely large. Consequently, the decision analysis becomes too complicated for practical purpose. The set of alternatives therefore has to be kept as small as possible, and at the same time has to contain all potentially good actions. For convenience, such a set will be called the *effective set* in this thesis. The procedure described below can be used as a guide to create the effective set of actions in dealing with existing concrete structures.

Considering an existing building with a set of anomalies identified, possible methods for remedy are usually determined following the results of diagnosis and condition evaluation. If the explanations of the observed anomalies are confirmed, it must be possible to plan appropriate courses of action for the defects, unless current technology is incapable of doing so. Since the diagnostic result is represented by a set of hypotheses $H = (H_1, H_2, \cdots, H_n)$, for each H_i, the engineer can design a set of relevant courses of action A_i which can cure the defects explained by H_i. If we can identify a course of action a_i from A_i such that a_i is the best in A_i regarding factors such as costs, effectiveness, availability of materials, a_i could be used as the effective alternative for H_i. Suppose we can identify such an effective option for each hypothesis in H, then the effective set is obtained as $A = (a_1, a_2, \cdots, a_n)$. Theoretically, there are n elements in A thus created. In this way, the total number of actions to be considered in the decision analysis is greatly reduced, while the completeness remains.

It has to be mentioned that sometimes an hypothesis may have more than one effective alternatives. For example, if H_i = "bad environmental conditions", and the observed anomaly is "corrosion of reinforcement", there are two options available: $a_i(1)$ = "repair immediately", $a_i(2)$ = "regular maintenance". Since it is difficult to compare these two actions by subjective judgement without detailed decision analysis, both $a_i(1)$ and $a_i(2)$ have to be considered as effective options for H_i. Therefore, the creation of the effective set of options A is closely related to the explanations contained in the hypothesis set H.

The effective set of alternatives created in this way is *dynamic* in the sense that A has to be re-generated at each stage of the multi-stage decision-making process, because the hypothesis set H itself is updated regularly. The iterative nature of the decision-making process will be shown later in section 4.5.7. The dynamic property of A will improve the efficiency of decision-making, and help to avoid missing any best course of action in making the final decision.

However, the effective set A only contains the kind of actions regarding repair/maintenance activities. Actions of other kinds, such as *conduct experiment* or *wait and see,* also have to be considered in any rational decision-making. Such actions can be added to the set A in the multi-stage decision-making whenever it is necessary, since A is modified at each stage.

After courses of action have been created, it is now possible to choose an alternative from A through a simple decision analysis.

4.5.6 Single-stage Decision-making

In order to choose an alternative from the effective set of actions $A = (a_1, a_2, \cdots, a_n)$, it is necessary to compare the utilities of each course of action. Since utilities are previously defined on the consequence space, the possible consequences of every $a_i \in A$ have to be specified first. For this purpose, a decision tree is shown in Fig. 4.7 in which H_i is the i^{th} hypothesis in H, and $X = X_F \times X_S \times X_D \times X_C$ represents the consequence space. As defined in Section 4.5.2, X_F, X_S, X_D and X_C are attributes regarding structural safety, serviceability, durability and the total cost respectively. X_F has four possible values: $x_f(1) = safe$, $x_f(2) = negligibly\ defective$, $x_f(3) = moderately\ defective$ and $x_f(4) = severely\ defective$; X_S has two: $x_s(1) = satisfactory$, $x_s(2) = inadequate$; X_D is measured in terms of the satisfactory period, and has been classified into $x_d(1) = 5\ years$, $x_d(2) = 15\ years$ and $x_d(3) = 20\ years$. Finally, X_C is a continuous real number. These attributes are assessed conditional on a hypothesis $H_j \in H$ for a given course of action. In other words, for an hypothesis H_j and an alternative a_i, the values of the four attributes can be deterministically obtained and denoted by $x^{a_i}_{f|H_j}$, $x^{a_i}_{s|H_j}$, $x^{a_i}_{d|H_j}$, and $x^{a_i}_{c|H_j}$ respectively. Therefore, the consequence of course of action a_i is determined if the hypothesis H_j is known, and can be represented by $x^{a_i}_{|H_j} = [x^{a_i}_{f|H_j}, x^{a_i}_{s|H_j}, x^{a_i}_{d|H_j}, x^{a_i}_{c|H_j}]$, e.g. $[x_f(3), x_s(2), x_d(3), x^{a_i}_{c|H_j}]$. The utility of any consequence is then denoted by $U(x^{a_i}_{|H_j})$, and can be obtained from the utility functions (curves) already assessed.

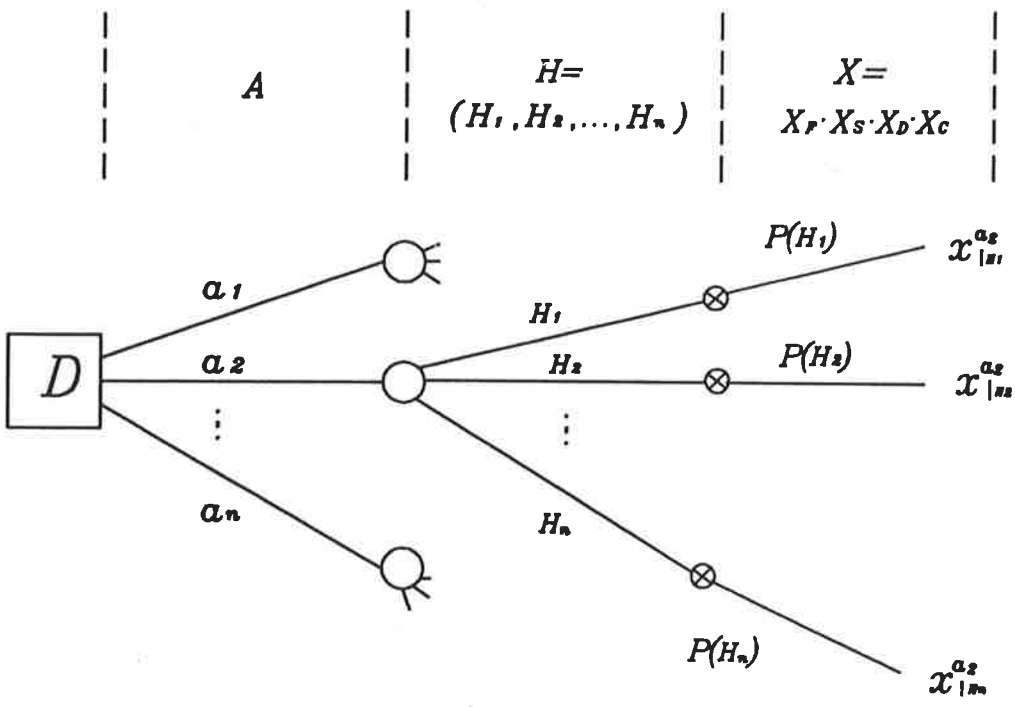


Figure 4.7: The Decision Tree for Single-stage Decision-making

It should be noted that the presence of the hypothesis set H in the decision tree is not totally necessary. However, the inclusion of H makes it easier to understand the procedure of evaluating expected utilities of each course of action to be described below. This also models the intuitive thought of the engineers in this selection.

Using these notations and from Fig. 4.7, the expected utility of each course of action can be evaluated easily by:

$$\bar{U}(a_i) = \sum_{j=1}^{n} U(x^{a_i}_{|H_j}) \cdot P(H_j) \tag{4.24}$$

where the bar "–" denotes the expectation, and $P(H_j)$ is the subjective probability of the hypothesis H_j, which can be obtained from the result of diagnosis in advance. At this stage, the best decision is to choose the course of action a^* which maximizes the expected utility over A, i.e.:

$$\bar{U}(a^*) = \max_{a_i \in A}[\ \bar{U}(a_i)\ ] \tag{4.25}$$

The decision thus made has used the *expected value* criterion. This rule is used because it is a very meaningful tool in decision-making under risk, and has been widely accepted in practice. However, due to its shortcomings, some practical modifications are made to this principle in making the final decision, and this will be discussed in Sections 4.5.7 and 4.5.8.

It is also to note that the course of action a^* chosen from Eq. 4.25 is the *best* among A only according to the information currently available. Therefore, it does not necessarily mean that a^* is the real best action and should be executed without further consideration. In reality, information can be gathered in sequence at different stages of the assessment process, and often a good decision can be made using a multi-stage decision process. This is described in the next section.

4.5.7 Multi-stage Decision-making

In dealing with existing structures, decision-making is a multi-stage process in the sense that when new information is available, related diagnosis and condition evaluation have to be updated, and consequently the choice of a course of action needs to be reviewed. For this purpose, the procedure for single-stage decision-making is perfectly suitable for making the choice within one cycle. However, to decide whether the process should continue or not, a number of tasks have to be fulfilled at the beginning of each stage.

Firstly, the effective set of alternatives A obtained in the previous stage need to be updated using the new information available. Secondly, since the effective set contains the kind of actions only regarding repair and maintenance, alternatives of other kinds such as "postpone the decision-making, and closely monitor the structure", have to be added to A. Thirdly, a set of feasible tests need to be worked out for information-gathering purposes.

With the updated courses of action A and feasible tests T, a decision usually has to be made on whether to stop the process by choosing an alternative from A using the information currently available or to acquire more information by conducting an experiment in T. The problem can be represented by the decision tree shown in Fig. 4.8 where $t_1, t_2, \cdots, t_m$ are feasible candidates of tests, and $O_1, O_2, \cdots, O_l$ are possible outcomes of t_j. All other symbols have the same meanings as in Fig. 4.7. Theoretically, the complete decision tree should include all future stages, and therefore is usually very large as can be seen from Fig. 4.8. At the i^{th} stage, a best course of action a^* can be firstly chosen from A in such a way that the expected utility of a^* is the largest among all members in A using the information available up to date. If the expected utility of each test in $T = (t_1, t_2, \cdots, t_m)$ is now calculated, and if the expected utility of the best test t^* is larger than that of a^*, then the decision is to take test t^*. After t^* is carried out, a further decision will be made at the $(i+1)^{th}$ stage using the new information resulting from the outcomes of t^*, and the process continues. If this is the case, the decision made at the i^{th} stage is referred to as *intermediate decision.* Otherwise, if no such test t^* can be found, which means a^* is preferred to any test in T, the decision is to choose a^*, and the decision-making process terminates. Thus a^* is called a *terminal* *decision.*

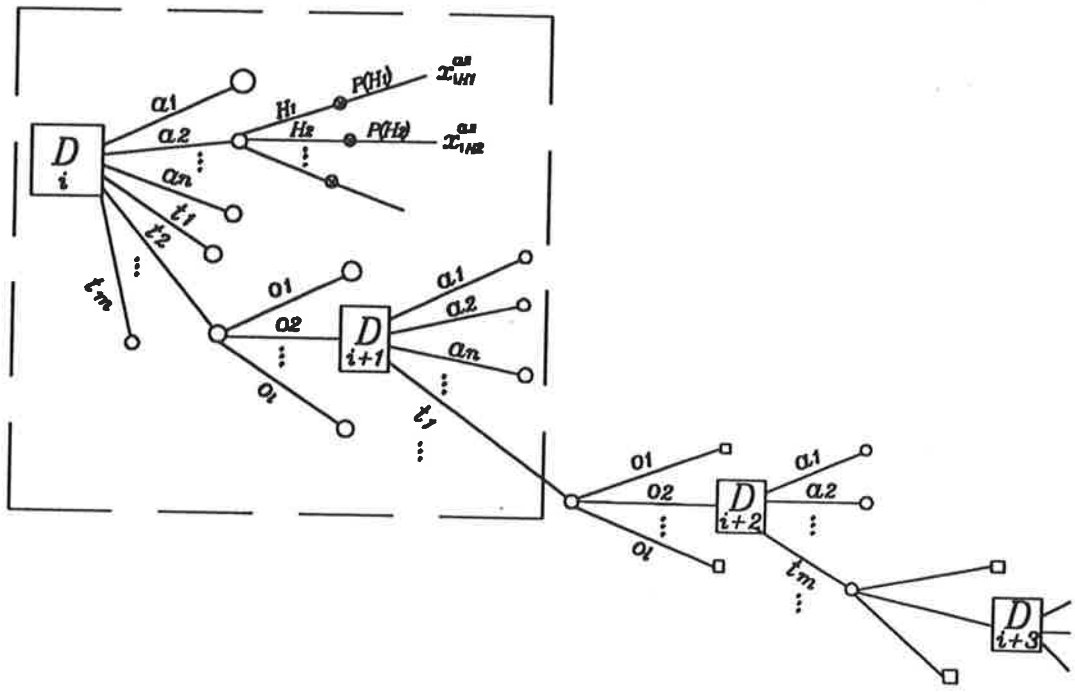


Figure 4.8: A Multi-stage Decision Tree

However, evaluating the expected utility of every test in T could be very tedious. Since any combination of different elements in T can also be valid test candidates, the number of tests to be considered can be theoretically very large, and this makes the above approach impractical. For this reason, in lieu of applying the routine decision-making procedure, a more practical approach is proposed here, in which the decision analyst, i.e. the engineer, subjectively selects the next test. Specifically, at the i^{th} stage, the decision analyst starts with calculating the expected utility of only a single test t_j which has been selected from T based on experience by considering factors such as "cost", "usefulness of possible outcomes". If the calculated $\bar{U}(t_j)$ is greater than that of a^*, this test is then taken as the intermediate decision without evaluating expected utilities of other candidate tests in T, and the decision-making process moves to the next stage. Otherwise, it is necessary to select another test and evaluate its expected utility. If all tests in T have been tried, and the condition $\bar{U}(t_j) > \bar{U}(a^*)$ is not satisfied, a^* may be taken as the terminal decision, and the process stops. This procedure will be called the *step-by-step approach* in the rest of this thesis.

Since the multi-stage decision tree is usually very large, it is extremely tedious to calculate the expected utility of a test from the entire tree. In the evaluation of expected utilities in the above step-by-step approach at any stage, consequences of tests contained in the child-branches in the decision tree are therefore ignored as a practical measure. For example, in Fig. 4.8, the evaluation of expected utilities of any test at the i^{th} stage only considers the part of the decision tree contained in the dashed square.

Using the *step-by-step* approach, the best course of action a^* at any stage can be identified from the candidate set A using the information available at the time, and the decision-making process advances to the next stage if a test t^* is adopted by the decision maker. The ideal case is that this process continues until sufficient information is gathered so that the true hypothesis H^* can be identified. In reality, however, the information-gathering process may stop much earlier if:

1. there is no such test $t^* \in T$ whose expected utility is larger than that of $a^* \in A$; or
2. it is impossible to continue the information-gathering process due to reasons such as financial, political factors; or
3. the decision maker prefers to take a terminating course of action right now.

If this is the case, the question is then "is it rational to take a^* as the terminal decision ?" To answer this question, the risk implied by a^* has to be explicitly estimated. In our problem, the risk involved in a decision is closely related to the diagnostic result. If the probability of the most likely hypothesis H^* reaches the acceptable probability value P^H_{acpt} as defined in Section 2.4.8 of

Chapter 2, i.e. $P(H^*) \geq P^H_{acpt}$, H^* can be considered acceptable. Under such circumstance, other hypotheses in H can be ignored with acceptable risk, and the decision can be made with regard to only H^*. Therefore, by carrying out the prognosis regarding the structure's future behaviour conditional on H^* and a^*, the terminating decision can be determined in the following way:

- if various structural requirements can be satisfied by executing a^*, the decision is to take a^*, and the multi-stage process terminates; Otherwise, it is necessary to design a set of feasible courses of action A^* which are specifically suitable for curing problems caused by H^*. Because the situation in this case is decision-making under certainty, the final decision is to choose action a^* from A^* such that a^* is the cheapest among those that can meet the relevant structural requirements.

However, if the information at this stage is insufficient to identify such an acceptable hypothesis H^*, the consequences of a^* can not be obtained with certainty, and have to be evaluated conditional on all elements in H. Therefore, the choice of any action will lead to unavoidable risks of bad consequences. Logically, a^* simply can not be taken as the terminal decision if the involved risk of catastrophic consequence such as structural collapse is unacceptably high. On the other hand, if the risk of structural failure is acceptable to the decision maker, but not acceptable to the public community, a^* can not be taken as the terminal decision either, because the disastrous consequence will be faced not only by the owner but also by the engineer and the community. Therefore, the risk related to a^* regarding the severe consequences has to be acceptable to all parties involved if a^* is taken as the terminal decision.

In choosing a course of action, consequences of having a dangerous structure can be very severe, and hence related risk has to be limited. This risk implied by course of action a^* will be explicitly evaluated and controlled through a procedure to be proposed in the next section.

4.5.8 Risk Control for Making a Terminal Decision

The risk of having a dangerous structure after a^* is taken can be evaluated with the help of a partial decision-tree shown in Fig. 4.9 (a), in which H_i is the i^{th} valid hypothesis in H at the stage a^* is obtained; $x^{a^*}_{f|H_i}$ is the value of safety attribute X_F evaluated conditional on H_i for course of action a^*. $P(H_j)$ is the subjective probability of the hypothesis H_j, obtained from the diagnostic procedure.

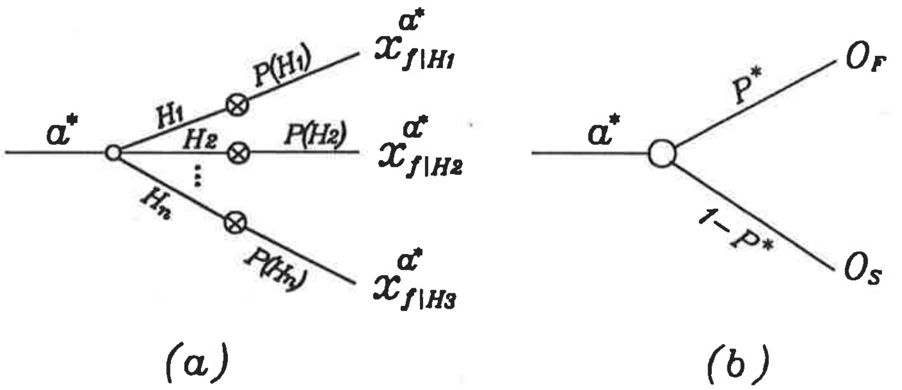


Figure 4.9: Possible Consequences of a^* Regarding Structural Safety

As defined in section 4.5.2, $x^{a^*}_{f|H_i}$ has four possible values, i.e. $x^{a^*}_{f|H_i}(1) = safe$, $x^{a^*}_{f|H_i}(2) =$ *negligibly defective*, $x^{a^*}_{f|H_i}(3) =$ *moderately defective* and $x^{a^*}_{f|H_i}(4) =$ *severely defective*, which can be assessed from either experience or the calculated reliability index $\beta^{a^*}_{|H_i}$, using the procedure described in Section 3.7.2 of Chapter 3. If we define here that consequences $x^{a^*}_{f|H_i}(4) =$ *severely defective* and $x^{a^*}_{f|H_i}(3) =$ *moderately defective* imply that "the structure is dangerous", these four values can be lumped into two, i.e. $O_F = x^{a^*}_{f|H_i}(4) \cup x^{a^*}_{f|H_i}(3)$ indicating *potential failure*, and $O_S = x^{a^*}_{f|H_i}(1) \cup x^{a^*}_{f|H_i}(2)$ indicating *non-failure*. In this way, the possible consequences of the course of action a^* regarding the safety attribute can be fully represented by the two outcomes denoted by O_F and O_S. This is illustrated in Fig. 4.9 (b), where P^* is the probability of the outcome O_F after a^* is taken, and can be evaluated by

$$P^* = \sum_{i=1}^{n} P(H_i) \cdot F(x^{a^*}_{f|H_i}) \tag{4.26}$$

in which $F(x^{a^*}_{f|H_i})$ is a function which takes 1 when $x^{a^*}_{f|H_i} \in O_F$, and is 0 if $x^{a^*}_{f|H_i} \in O_S$. This P^* can then be interpreted as "the subjective probability that the structure will be dangerous according to the current practice," and hence represents the risk of a^*. It has to be pointed out that P^* obtained here is different from the failure probability defined in structural reliability theory.

Due to the reasons discussed in the last section, this risk P^* must be controlled if a^* is to be taken as the terminal decision. For this purpose, an acceptable value of P^* has to be determined. In the diagnosis, the engineer adopts a value of P^H_{acpt} to accept a hypothesis. This implies that a risk of $1 - P^H_{acpt}$ of making wrong decisions is acceptable. If there is no other suitable value available, the probability $1 - P^H_{acpt}$ will be used as the acceptable risk level. In other words, the calculated risk P^* has to be not higher than $1 - P^H_{acpt}$ for the terminal decision.

With P^* and $1 - P^H_{acpt}$ thus determined, we can now continue the multi-stage decision-making following the last section. At the stage where further testing is impractical, the risk P^* of course of action a^* has to be evaluated first. If $P^* \leq 1 - P^H_{acpt}$, a^* can be taken as the terminal decision, and the decision-making process stops. Otherwise, if the safety requirement is not met by a^*, it means that none of the actions in the candidate set A is satisfactory to both the engineer and the owner simultaneously, because a^* is determined using the owner's utilities, and $1 - P^H_{acpt}$ is specified by the engineer. However, as mentioned in Section 4.5.5, although set A is representative, it is incomplete. In other words, it is still possible that a course of action a^+ may exist outside A, which can result in the acceptable risk level and has the expected utility not less than that of a^*. In fact, in multi-attribute decision-making, it is usually possible to keep the utility unchanged by obtaining gains in one attribute and sacrificing in others. Therefore, such an action a^+ might be found by modifying the existing course of action in A with more cost spent on reducing the involved risk of "structural failure". If such an a^+ exists, the decision is obviously to take a^+, and the process terminates. If there is no such an action available, then no agreement can be reached between the engineer and the owner at this stage. Under such circumstances, in order to carry on the decision-making process, the owner has to compromise by either doing more tests to acquire new data or selecting a course of action from A which is not as good as a^* in terms of expected utility but can satisfy the safety requirement. If the owner is not willing to compromise, the decision-making has to be postponed.

The complete procedure of the multi-stage decision-making process is therefore summarized as follows:

1. at any stage i, a candidate set of courses of action $A = (a_1, a_2, \cdots, a_n)$ is firstly obtained by modifying that from the previous stage using the information available up to date. According to the expected utilities of each $a_j \in A$, a best action a^* is identified such that the expected utility of a^* is the largest;
2. if the most likely hypothesis H^* satisfies $P(H^*) \geq P^H_{acpt}$, carry out the prognosis for a^* conditional on H^*. If relevant structural requirements are satisfied by a^*, the terminal decision is to take a^*. Otherwise, re-design a candidate set A^* according to H^*, and choose a new a^* which is the cheapest among those that can meet the relevant structural requirements;
3. if $P(H^*) < P^H_{acpt}$, and conducting more tests is feasible, choose a test t^* which satisfies $\bar{U}(t^*) > \bar{U}(a^*)$, and the decision-making process advances to the $(i+1)^{th}$ stage;
4. if $P(H^*) < P^H_{acpt}$, but performing more tests is impractical, due to whatever reasons, evaluate the risk P^* resulting from a^* using Eq. 4.26, and

(a) if $P^* \leq 1 - P^H_{acpt}$, take a^* as the terminal decision, and the process stops;

(b) if $P^* > 1 - P^H_{acpt}$, creat a new candidate set A^+ by designing actions which obtain gains in the attribute of "structural safety" but have losses in other attributes such as financial cost. If there exists an action $a^+ \in A^+$ such that the risk implied by a^+ is not larger than $1 - P^H_{acpt}$, and $\bar{U}(a^+) \geq \bar{U}(a^*)$, the decision is to take a^+. If a^+ can not be found, the terminal decision can not be made by satisfying both the decision maker's preferences and the risk control policy. Three possibilities then exist: (*i*) persuade the owner to adopt a course of action which results in a satisfactory risk level, at higher cost; (*ii*) persuade the owner to conduct further testing, if feasible; (*iii*) if items *i* and *ii* can not apply, the decision-making has to be postponed.

It can be seen from the procedure that the terminal decision is actually made using both the expected utility criterion and the risk control policy. This means that the final course of action is chosen by the owner together with the engineer, and both of them jointly represent the interests of the community.

4.5.9 A Special Case

In a special case when the consequence space X consists of only the attributes X_F (safety) and X_C (cost), part (b) of step 4 of the multi-stage decision-making process described in the last section can be further elaborated.

Suppose at stage i, a "best" course of action a^* has been identified from A using the expected utility criterion, but the risk P^* is higher than $1 - P^H_{acpt}$. In order to find out how much the decision maker is willing to sacrifice in the cost attribute to gain in the safety attribute so that the safety requirement can be satisfied, assume that an action a^+ is available, which can result in the risk of just $1 - P^H_{acpe}$. The consequences of a^+ and a^* only regarding the safety attribute are plotted in Fig 4.10, where O_F and O_S, defined the same as before, represent consequences "dangerous" and "safe" respectively. Since only two attributes are considered, utilities of relevant consequences can be fully represented by $U(C_{a^*}, O_F)$, $U(C_{a^*}, O_S)$, $U(C_{a^+}, O_F)$ and $U(C_{a^+}, O_S)$, in which $C.$ is the cost of $(\cdot)$.

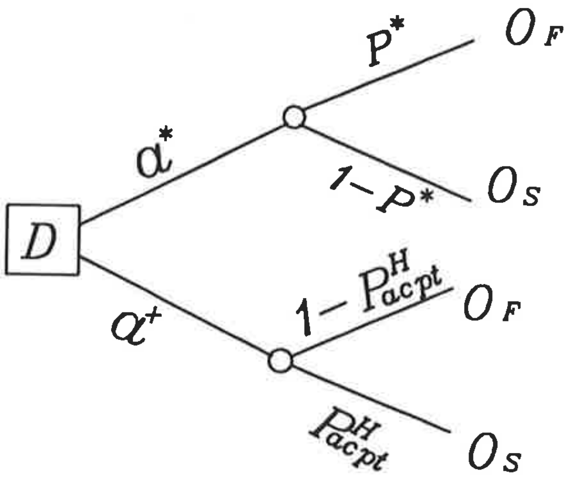


Figure 4.10: Outcomes of Terminal Decisions

With the value of P^H_{acpt} fixed, the expected utility of a^+ will be determined by its cost C_{a^+}. If C_{a^+} equals such a value that $\bar{U}(a^+) = \bar{U}(a^*)$, i.e. the decision maker is indifferent between a^+ and a^*, we have

$$P^* \cdot U(C_{a^*}, O_F) + (1 - P^*) \cdot U(C_{a^*}, O_S) = P^H_{acpt} \cdot U(C_{a^+}, O_S) + (1 - P^H_{acpt}) \cdot U(C_{a^+}, O_F). \tag{4.27}$$

Because a^* has been determined, P^* and C_{a^*} are fixed terms in Eq. 4.27. A relationship between the cost C_{a^+} and the acceptable risk $1 - P^H_{acpt}$ can be obtained as:

$$U(O_S, C_{a^+}) - U(O_F, C_{a^+}) = \frac{P^* \cdot U(O_F, C_{a^*}) + (1 - P^*) \cdot U(O_S, C_{a^*}) - U(O_S, C_{a_i^*})}{P^H_{acpt}} \tag{4.28}$$

For the known value of P^H_{acpt}, the value of C_{a^+} which satisfies Eq. 4.28 can be found according to relevant utility functions. C_{a^+} thus obtained is a very useful value. In fact, if there is an action a^+ which can result in a risk not higher than $1 - P^H_{acpt}$, and is less expensive than C_{a^+}, the decision is to take a^+ without evaluating its expected utility. Therefore, part (b) of step 4 of the complete procedure in the last section can be modified for this special case as:

- (b) if $P^* > 1 - P^H_{acpt}$, evaluate the value C_{a^+} from Eq. 4.28. If there exist a course action a^+ such that the P^+ is smaller than $1 - P^H_{acpt}$, and the cost of a^+ is lower than C_{a^+}, a^+ is then the terminal decision. Otherwise, if a^+ is not available, options (i), (ii) and (iii) listed in the last section apply.

4.6 Some Comments on the Proposed Method

It has to be mentioned that decisions thus made are not strictly optimal, because the expected utility is evaluated using the information currently available, and it is impossible to predict the future with certainty. At the i^{th} stage, if an optimal decision is to be made using the expected utility criterion, the complete multi-stage decision tree including all possible outcomes of future stages have to be considered. This seems too tedious because the evaluation of relevant expected utilities is a formidable task. Even if the calculations can be made by a computer, it is likely that the errors involved in predicting a larger number of probabilities of future outcomes will make the final result very unreliable.

Alternatively, an optimal decision may be made through an optimization procedure. In fact, decision-making aims to balance the cost and the risk of making wrong decisions. As illustrated in Fig. 4.11, the reduction of risk can only be achieved with more financial cost denoted by C_1, and the expected cost of accepting the risk, C_2, is proportional to the magnitude of the risk itself. The optimal decision can be made at the risk P_{opt} when the total cost $C_1 + C_2$ achieves its minimum value. Unfortunately, those relations in Fig 4.11 can not be evaluated in practice, and hence the approach is only an idealized model for the time being.

For these reasons, it seems that it is impractical to search for optimal decision. Instead, the practical procedure proposed previously is more useful.

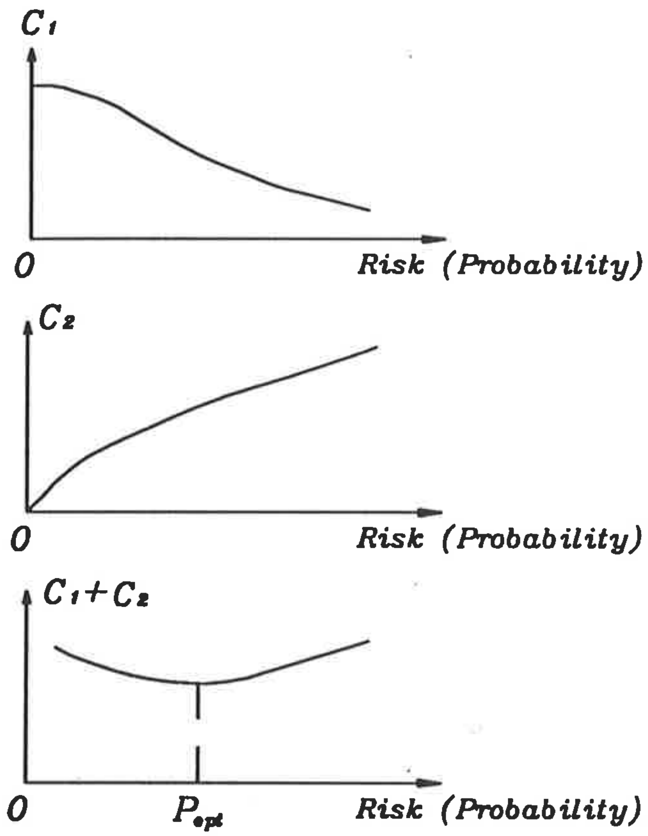


Figure 4.11: The Relation Between Cost and Risk

4.7 Summary of This Chapter

A multi-stage process for selecting a relevant course of action for an existing concrete building has been developed from decision theory using the results of diagnosis and condition evaluation obtained from previous chapters. The approach is purposefully developed in such a direction that it is practically applicable, and therefore any probability and mathematics concepts involved are treated in a simple way.

Chapter 5

The Detailed Process for Dealing With Defective Concrete Buildings

5.1 Introduction

In Chapter 1, an overall process of dealing with existing concrete buildings was proposed and represented by a flow-chart on page 24. In the chart, essential components including methods for diagnosis, condition evaluation and decision-making were not available at that time. These methods have been developed in Chapters 2,3 and 4, and it is the purpose of this chapter to establish an integrated procedure incorporating the results of those chapters.

5.2 The Detailed Process for Dealing With Existing Defective Buildings

The proposed process is shown in the flow-chart in Fig. 5.1 together with Fig. 5.2. It is developed from the framework of Fig. 1.4 in Chapter 1, but contains much more detail. In the chart, the rectangles in solid lines represent various actions to be executed in the process, and relevant decisions or choices to be made are included in rhombuses. For convenience, most components of the chart are numbered.

The process starts with the identification of anomalies through appropriate means such as site inspection. Information at this stage is then gathered by means such as interviews with occupants, analysis of inspection results, searching the design and construction documents.

With the anomalies and information thus obtained, a set of diagnostic hypotheses $H = (H_1, H_2, \cdots, H_n)$ can be created using the explanation tree approach described in Chapter 2. The method for diagnostic reasoning developed in Section 2.4.5 can be employed to rank the hypotheses in terms of their subjective probabilities.

After this, the process goes to step 5 in Fig 5.1 and an immediate decision is made as to whether urgent action such as *evacuating the building* is needed to protect the safety of occupants and contents. If the answer is yes, an urgent action a_u^* is selected, and the process moves to the next step. Otherwise, the process advances directly to step 6 in Fig. 5.1. The detailed method for deciding on the necessity of taking action a_u^* is to be described in Section 5.3.

After the initial consideration of urgent action, decisions regarding the structure's long term future have to be made. For this purpose, the most likely hypothesis H^* is identified from the hypothesis set H, and its subjective prob-

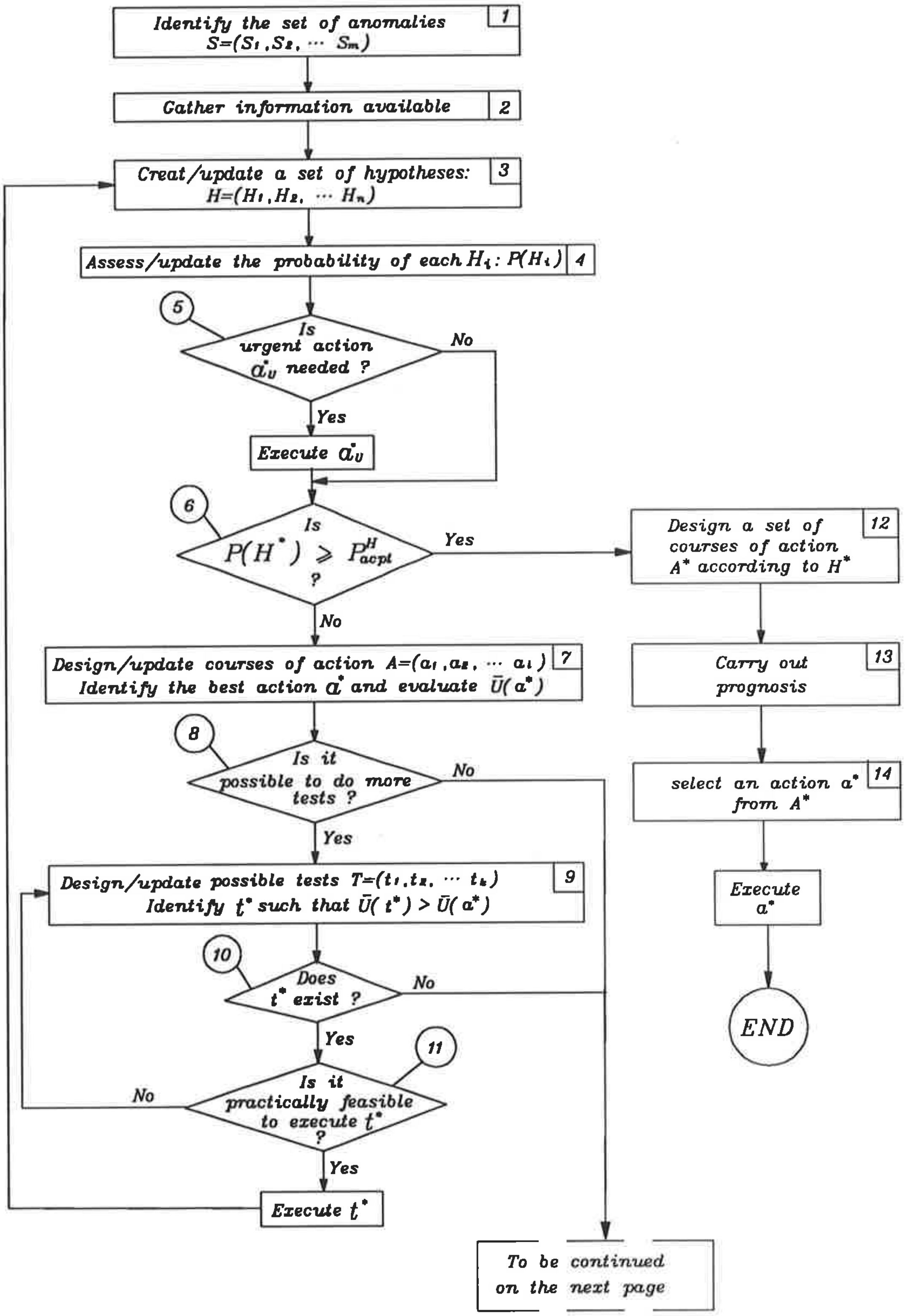


Figure 5.1: The Proposed Detailed Process — Part (I)

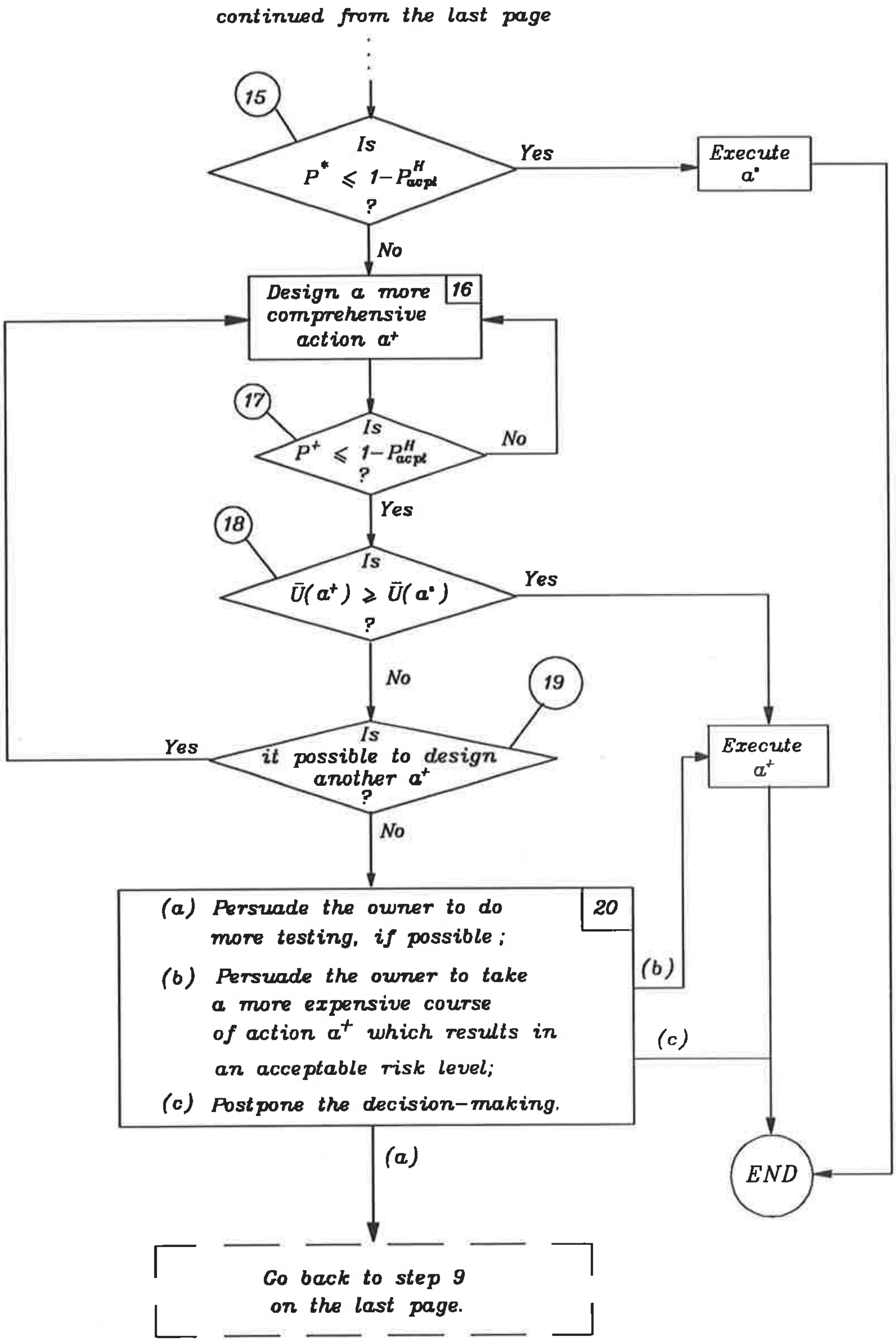


Figure 5.2: The Proposed Detailed Process — Part(II)

ability is $P(H^*)$. As mentioned in Section 4.5.7, if $P(H^*)$ reaches the value P^H_{acpt}, H^* can be considered acceptable, and the decision-making can be simply carried out according to H^*, where P^H_{acpt} is determined subjectively by the engineer based on his/her experience. Therefore, at step 6, if an hypothesis H^* is accepted, a set of actions A^* can be determined by specifically considering the implications of H^* in regard to the observed anomalies and future structural performance. To choose the best one from A^*, prognosis is carried out to predict the consequence of each member in A^* defined on relevant attributes such as safety and serviceability. Since only H^* is being considered, according to our definition, the consequence of each action $a_i \in A^*$ can be deterministically assessed. The decision is then to be made under certainty, and the best alternative a^* can be easily found in such a way that a^* is the cheapest among those that can meet the specified requirements. Finally, the process advances to the next step – execution of a^*, and is then complete.

However, if the answer to question No. 6 is *no*, the choice of an appropriate course of action has to be made under risk. For this purpose, possible courses of action $A = (a_1, a_2, \cdots, a_l)$ have first to be designed. An action a^* is identified as the one which has the highest expected utility $\bar{U}(a^*)$ in A. This means that if a terminal decision has to be made at this stage, a^* is the best candidate to the decision-maker concerning all attributes in the consequence space based on the information available at the time. If new information is gathered, an improved decision can be made.

However, acquiring more data is not always feasible even if there are obviously some promising tests available. Factors such as *political pressure to make a terminal decision right now, financial difficulties, the owner preference to take a terminal course of action* all can prevent the engineer from gathering more information. Question No. 8 is mainly concerned with these factors. If it is feasible to carry out further testing, a set of candidate tests have to be identified. These are denoted here by $T = (t_1, t_2, \cdots, t_k)$. A test t^* is then subjectively chosen by the engineer from T with regard to factors such as cost and importance of the test. Using the routine procedure of decision analysis, the expected utility of t^*, $\bar{U}(t^*)$, can be evaluated. If $\bar{U}(t^*) > \bar{U}(a^*)$, t^* is the candidate test to be conducted. If there are no technical difficulties, t^* can be executed, and the process goes back to step 3 in Fig. 5.1. Otherwise, if there are difficulties which prevent the conduct of t^*, a new t^* has to be identified from T, and the process returns to step 9. If there is no such test $t^* \in T$ with $\bar{U}(t^*) > \bar{U}(a^*)$, or if the answer to questions 8 is *no*, conducting more tests is impossible, and the possibility of taking a^* as the terminal decision has to be considered. If this is the case, the process switches to the second part of the flow-chart shown in Fig. 5.2 on page 176.

The risk of a^* regarding the safety issue is now evaluated in terms of the probability P^* of having a dangerous structure following action a^*. If P^* is not larger than the acceptable risk $1 - P^H_{acpt}$, the terminal decision is to take a^*, and the process is complete. Otherwise, if a^* can not meet the safety requirement, a new course of action a^+ has to be designed by paying more attention to the safety attribute with more cost. A prognosis is then carried out, from which the risk of a^+, P^+, is obtained. If the condition $P^+ \leq 1 - P^H_{acpt}$ is satisfied, and the expected utility of a^+ is greater than that of a^*, a^+ can be taken as the terminal decision. Otherwise, if $\bar{U}(a^+) \geq \bar{U}(a^*)$ is not met, the re-design of another a^+ is necessary, and the process goes back to step 16.

In a special case where the relevant attributes include the "safety" (X_F) and the "cost" (X_C) only, the determination of a suitable a^+ can be carried out using the procedure described on page 170 in the last chapter.

However, if there does not exist such an a^+ that satisfies both the safety requirement and the expected utility criterion, the process moves to step 20. This means that although the engineer has tried everything, an agreement on the terminal decision has not been achieved with the decision maker, using the information available up to date. In this case, three options can be considered: (i) if the engineer can persuade the owner to gather more information and let the terminal decision be made at a later stage, the process goes back to step 9 in the flow-chart; (ii) if there is no promising test available, or if the decision maker agrees to compromise by spending more money to take a satisfactory action a^+ which does not satisfy $\bar{U}(a^+) > \bar{U}(a^*)$, but nevertheless has the highest expected utility among those that satisfy the safety requirement, the process terminates after a^+ is executed; (iii) if no agreement can be achieved, the decision-making has to be postponed, and the process stops at this time.

Obviously the overall process works iteratively, and its recursive nature has to be emphasised. If test t^* is selected as an intermediate decision at the i^{th} cycle, further analysis has to be carried out at the $(i+1)^{th}$ cycle according to the actual outcome of test t^*. At each cycle, the hypothesis set H has to be updated using the procedure given in Chapter 2, and the probability of H_i is re-assessed using the new information available. The courses of action $A = (a_1, a_2, \cdots, a_l)$ and test candidates $T = (t_1, t_2, \cdots, t_k)$ also need to be updated. The dynamic nature of A is an important and indeed essential characteristic of the proposed procedure.

Although the process thus described is well structured and comprehensive, it appears to be tedious, especially if it has to run through a large number of cycles, especially when the number of hypotheses is also large. However, the intuitive thought processes of an engineer faced with this type of problem are exceedingly complex, and at least as involved as the logical procedure described here. Also, the analysis of each step in the procedure can be simplified as the amount of information increases. At the first cycle of analysis the knowledge available is very limited, and the number of hypotheses and courses of action may be very large. As the assessment process proceeds, more information is gathered, and many hypotheses will be eliminated from H, thus simplifying future cycles.

The complete process of dealing with existing defective concrete buildings has now been outlined. In the next section, a procedure is suggested for answering question No. 5 in Fig. 5.1, i.e. how to decide whether urgent action needs to be taken.

5.3 Taking Urgent Action

The objective of taking urgent action is to protect the safety of human life and property in case the structure is in danger of immediate collapse. Whether to adopt such an action or not has to be decided immediately after the site inspection and preliminary assessment have been completed. Any relevant analysis regarding this decision only needs to consider the structure's performance in a very short term.

Although any urgent action will have to be chosen in relation to the specific problem, those commonly used in practice will fall into the following categories:

- a_1 = evacuate the structure;
- a_2 = restrict the applied load;
- a_3 = provide temporary support or strengthening;
- a_4 = provide monitoring and a warning system but keep the building in service.

A relevant choice from a set of actions of this kind can usually be made based on the engineer's experience and expertise. For example, if the anomaly has been present for some time and the structure has not yet failed, it may be concluded from experience that the structure is not likely to fail in the near future unless circumstances change markedly, and hence no drastic action is needed. An experience-based approach is obviously appropriate.

However, if a decision can not be made in this way, and the danger of immediate failure is a real concern, a well structured procedure, which can balance the cost of taking action against the cost of serious consequences, is needed in order to reduce the chances of making unnecessary mistakes. In the following, a simple approach using decision analysis similar to that in Chapter 4 is proposed. It is aimed at helping the decision maker to identify rational choices in complex situations. For this purpose, a decision tree is shown in Fig. 5.3, in which a_i is an urgent action from the list on the previous page; a_0 is the action of "doing nothing and keeping the building in service"; H_i is the i^{th} hypothesis obtained in the preliminary diagnosis, and $P(\cdot)$ denotes the probability of $(\cdot)$. The possible consequence is defined as a binary event, i.e. *failure* or *non-failure* denoted by F and $\bar{F}$ respectively. Failure is further represented by *ductile failure* D and *non-ductile failure* $\bar{D}$. It has to be pointed out that the failure defined here is conceptually different from that used in Section 4.5.8 of Chapter 4 where failure only indicates an unacceptable risk level. Failure defined in this section means real structural failure regarding a particular ultimate limit state in a short term.

The probability $P(D)$ is to be estimated subjectively conditional on F, and $P(\bar{D})$ is then $1 - P(D)$. The failure probability $P(F)$ should be estimated by considering only the current loading condition. Either utilities or monetary cost can be used to scale the three consequences denoted by (a_i, D), $(a_i, \bar{D})$ and $(a_i, \bar{F})$.

With these notations, a routine decision analysis can be carried out to select an action a_u^* which has the highest expected utility (or lowest cost), i.e. $\bar{U}(a_u^*) = \max_i \bar{U}(a_i)$. If a_o is chosen, it means no urgent action is needed.

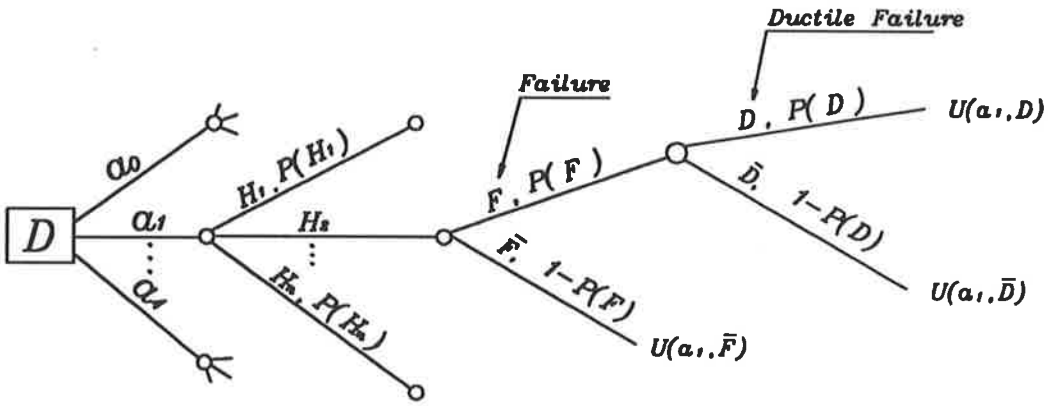


Figure 5.3: The Decision Tree for Choosing an Urgent Action

5.4 Summary of This Chapter

A comprehensive process for treating structural defects has been proposed, using relevant detailed procedures for tasks such as diagnosis, condition evaluation and decision-making which have been developed in previous chapters. To illustrate the use of the process in real problem-solving, two examples are given in the next chapter.

Chapter 6

Examples

6.1 Example 1

In a multi-story building, a permanent sag was found in the reinforced concrete slab floor at mid-panel of several panels. Extensive cracks were observed on first inspection on the underneath surface in mid-panel, and also around adjacent columns on the top surface. The owner of the structure was alerted by concerned occupants, and wanted to know whether there was a structural problem, and what, if any thing, needed to be done.

For this purpose, the procedure proposed in Chapter 5 is applied, and related details are given in the following sections.

6.1.1 Identification of Anomalies

The cracking pattern was recorded during the site inspection, and is shown schematically in Fig 6.1. According to our terminology, the observed cracks and excessive deflection are considered to be anomalies. For convenience, the cracking on the bottom surface of the slab in the mid-panel is called *anomaly No. 1* and denoted by S_1, and the cracks on the top surface around adjacent columns are referred to as *anomaly No. 2*, denoted by S_2, while S_3 represents *anomaly* No. 3 — the deflection. Attributes of S_1 to be considered are b_{11} = *orientation*, b_{12} = *time of occurrence*,, b_{13} = *appearance* and b_{14} = *width*. The same attributes are also considered for S_2. Two attributes are considered for S_3: b_{31} = *time of occurrence* and b_{32} = *endurance*.

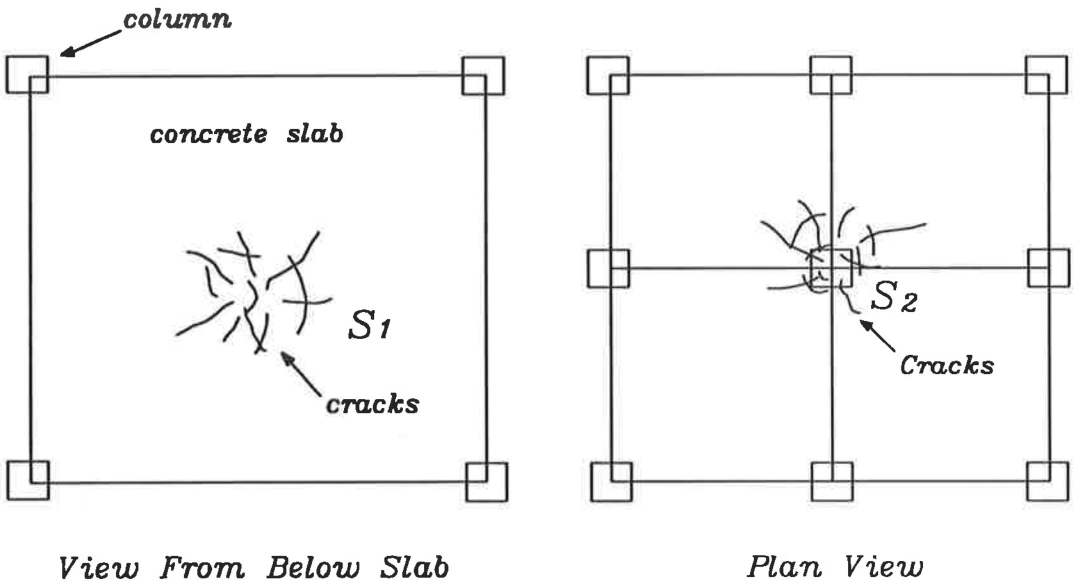


Figure 6.1: Recorded Crack Pattern

6.1.2 Available Information

From relevant sources, it was known that the building was 4 years old, and the concrete floors were cast in-situ. According to the appearance of the observed anomalies during the inspection, there was no evidence of rusting. The deflection has been present for quite a long time. Upon the examination of the design drawings obtained, no significant design error was found.

6.1.3 Identification of Possible Explanations

With the identified anomalies and information available, the explanation tree for this problem was established using the procedure described in Section 2.4.2, from which possible explanations were identified for the two anomalies S_1, S_2 and S_3, and listed in Table 6.1.

Table 6.1: Identified Explanations

Categories	Explanations
Physical	e_1: Shrinkage or temperature effects;
	e_2: Lack of bond or progressive loss of bond;
Chemical	e_3: Alkali-aggregate reaction;
	e_4: Corrosion of steel bars;
Overload	e_5: Overload during construction;
	e_6: Overload during in-service period;
Lack of stiffness	e_7: Detailing problems;
	e_8: Low strength concrete;
	e_9: Insufficient reinforcement at mid-span;
	e_{10}: Badly located reinforcement at mid-span;
	e_{11}: Insufficient reinforcement around columns;
	e_{12}: Badly located reinforcement around columns.

6.1.4 Creating the Hypothesis Set H

The explanation tree is shown in Fig. 6.2, in which e_i represents the i^{th} explanation, S_i denotes the i^{th} anomaly, and b_{ij} is the j^{th} attribute of the i^{th} anomaly; and the directed lines indicate the causal relations between explanations and relevant anomalies. From the information obtained so far, attributes b_{11}, b_{21} (orientation), b_{13}, b_{23} (appearance), and b_{32} (deflection endurance) are known, and represented by shaded ellipses.

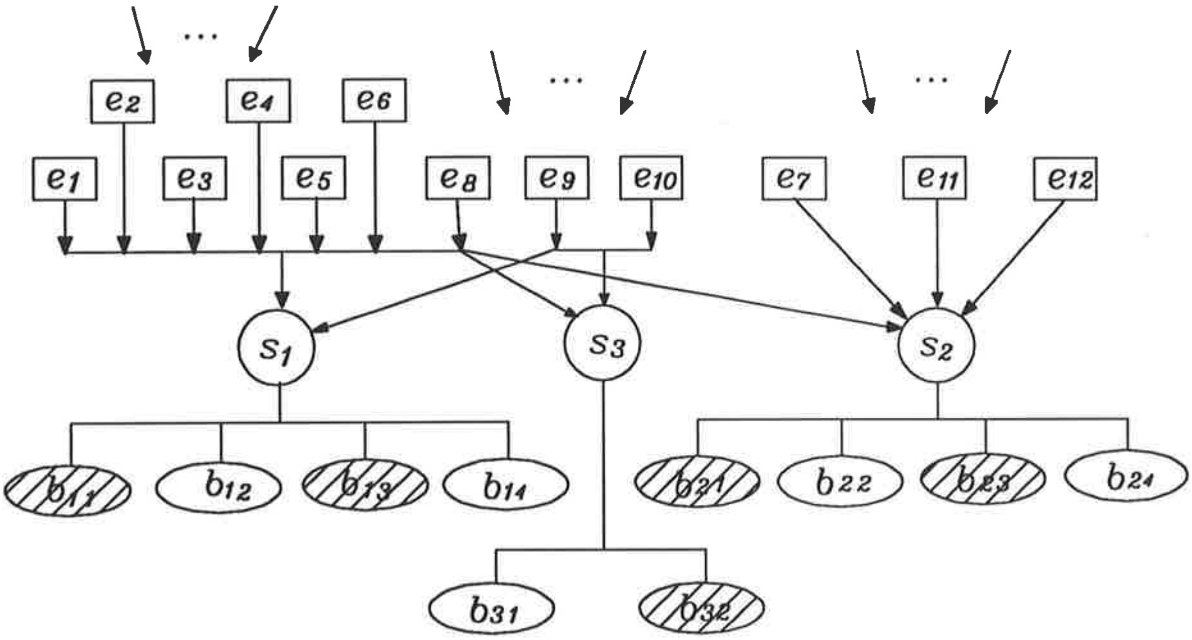


Figure 6.2: Explanation Tree of Example 1

It is obvious that the candidate set H_C of hypotheses consists of single-explanation elements $H_1^C = e_1$, $H_2^C = e_2$, $H_3^C = e_3$, $H_4^C = e_4$, $H_5^C = e_5$, $H_6^C = e_6$, $H_7^C = e_8$, two-explanation elements such as $H_8^C = e_7e_9$, $H_9^C = e_7e_{10}$, $\cdots$ and multi-explanation elements like $e_1e_2e_9$, $e_5e_6e_7, \cdots$. It is too tedious to list the complete set of H_C because of its large number of possible elements. However, as mentioned in Section 2.4.4, the hypothesis set $H \in H_C$ only comprises those promising members, and temporarily excludes the elements which are not very likely to be true, according to the information currently available. For this reason, the explanations $e_3 = Alkali - aggregate\ reaction$ and $e_4 = Corrosion\ of\ steel\ bars$ in our example do not need to be considered in the creation of hypotheses because the building was fairly new and not located in harsh environments, and there is no evidence of corrosion from the appearance of the cracks identified. Also, hypotheses containing too many explanations such as $H_j = e_1e_2e_5e_7e_8e_9$ are relatively much less likely to be true compared with those having small number of explanations. Therefore, hypotheses with three or more explanations are not considered in creating H at this initial stage.

Based on these considerations, the hypothesis set H is created as:

$H_1 = e_1$	$H_6 = e_7e_9$	$H_{11} = e_{10}e_{12}$
$H_2 = e_2$	$H_7 = e_7e_{10}$	$H_{12} = e_5e_7$
$H_3 = e_5$	$H_8 = e_9e_{11}$	$H_{13} = e_6e_7$
$H_4 = e_6$	$H_9 = e_9e_{12}$	$H_{14} = e_5e_8$
$H_5 = e_8$	$H_{10} = e_1e_{11}$	$H_{15} = e_6e_8$
		$H^o = others$

where H^o is the pseudo hypothesis which represents all other elements of H_C and otherwise ignored in H.

6.1.5 Assessment of Subjective Probabilities for Each Hypothesis

Using the method described in Section 2.4.5, the subjective probability of each hypothesis H_i can be obtained by assessing the prior confidence factor $CF(H_i)$ and two partial confidence factors $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$, where $CF(H_i)$ indicates how often H_i happens in practice; $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ represent the strength of relationship between the obtained information ΔI and the truth of H_i, ΔI^+ is the newly obtained information conveyed by the explanation nodes and their parent-nodes in the explanation tree; ΔI^- is the newly gathered information conveyed by the anomaly nodes and their associated attributes, and $\Delta I = \Delta I^+ + \Delta I^-$. $CF(H_i)$ ranges from 0 to 10, while both $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ takes values between 0 and 100. Relevant values of $CF(H_i)$, $CF(\Delta I^+, H_i)$ and $CF(\Delta I^-, H_i)$ are assessed for the hypotheses created in the last section, and listed in Table 6.2, where the subjective probability of H_i is evaluated by:

$$P(H_i) = \frac{P'(H_i) \times P(\Delta I, H_i)}{\sum_{i=1}^{n} P'(H_i) \times P(\Delta I, H_i)} \tag{6.1}$$

in which $P'(H_i)$ is obtained by normalizing the prior confidence factors; and $P(\Delta I, H_i)$ is the normalized value of the confidence factor $CF(\Delta I, H_i)$ which can be evaluated by Eq. 2.19 in Chapter 2.

As can be seen from Table 6.2, since there is no information available at this stage about any further causal factors on the explanations contained in the explanation tree shown in Fig. 6.2, i.e. $\Delta I^+ = \phi$ (an empty set), confidence factors $CF(\Delta I^+, H_i)$ are assessed as 50 for every valid hypothesis. Based on how well the observed cracking pattern and associated attributes confirm the existence of a hypothesis, values of $CF(\Delta I^-, H_i)$ are assessed and listed in the fourth column of the table. Finally, the subjective probability of hypothesis H_i is evaluated by Eq. 6.1, and listed in the sixth column of Table 6.2.

After $P(H_i)$ is estimated, the decision on the necessity of taking urgent action has to be made next before any further actions are considered.

6.1.6 Need for an Urgent Action ?

From the interview with occupants, it was known that the sag of the concrete slab was not a new problem. Considering the facts that currently applied dead

Table 6.2: Subjective Probabilities for Each H_i in The First Cycle

Hypothesis	$CF(H_i)$	$CF(\Delta I^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_1$	6	50	10	0.0119	0.0173
$H_2 = e_2$	5	50	30	0.0357	0.0432
$H_3 = e_5$	7	50	80	0.0952	0.1613
$H_4 = e_6$	5	50	70	0.0833	0.1008
$H_5 = e_8$	6	50	60	0.0714	0.1037
$H_6 = e_7e_9$	3	50	60	0.0714	0.0518
$H_7 = e_7e_1$	3	50	10	0.0119	0.0086
$H_8 = e_9e_{11}$	4	50	55	0.0655	0.0634
$H_9 = e_9e_{12}$	4	50	55	0.0655	0.0634
$H_{10} = e_{10}e_{11}$	4	50	55	0.0655	0.0634
$H_{11} = e_{10}e_{12}$	4	50	55	0.0655	0.0634
$H_{12} = e_5e_7$	3	50	65	0.0774	0.0562
$H_{13} = e_6e_7$	3	50	65	0.0774	0.0562
$H_{14} = e_5e_8$	3	50	65	0.0774	0.0562
$H_{15} = e_6e_8$	3	50	60	0.0714	0.0518
$H^o = others$	3	50	45	0.0536	0.0389

and live loads were not large in comparison with the design load, and the sag was not getting worse, it was suggested that no urgent action was needed, but the applied load must be kept no higher than the current level. This decision was also supported by the fact that, in the hypotheses obtained above, the three top ranked hypotheses H_3 = *Overloading during construction*, H_4 = *Overloading during in service period* and H_5 = *Low strength concrete* were not likely to result in catastrophic consequences in the short term if there is no sudden increase of applied loads.

6.1.7 Conducting More Tests ?

At this stage, since the highest ranked hypothesis H_3 has only the probability of 0.161, which is far lower than a reasonable acceptable value (say 0.85), it is too early to make a final diagnostic conclusion. Therefore, a decision has to be made on whether to do more tests or to take an appropriate course of action regarding the hypotheses. For this purpose, if following the proposed process described in Chapter 5, a set of actions $A = (a_1, a_2, \cdots, a_n)$ have to be decided on, and the best action a^* chosen from A using the expected utility criterion. Any test t to be conducted has to satisfy the condition of $\bar{U}(t) > \bar{U}(a^*)$, where $\bar{U}$ denotes the expected utility. Among the following feasible tests:

- t_1: search for information on loading during construction and in service period;
- t_2: carry out relevant structural analysis to calculate the deflection;
- t_3: detect locations and quantities of steel bars using appropriate devices;
- t_4: remove small regions of concrete to observe the steel reinforcement;
- t_5: take core samples of concrete to test for compressive strength;

Please note: Sections 6.1.7, 6.1.8 and 6.1.9 are superseded by relevant sections specified in the **Amendment** at the back of this thesis.

- t_6: conduct non-destructive tests to estimate compressive strength of concrete,

tests t_1 and t_6 should cost only a very limited amount of money, and therefore is almost certain to have expected utility larger than $\bar{U}(a^*)$, provided it can be done within a reasonable short period of time, and hence the cost of delay can be ignored. For this reason, t_1 and t_6 can be executed at this stage without identifying a^* and evaluating the expected utility $\bar{U}(a^*)$.

After these tests are carried out, the process goes back to the procedure of diagnosis.

6.1.8 The Second Cycle of Diagnosis

During the information search, it was confirmed by interview and knowledge from other sources that the cracking had already appeared when the construction was completed, and no excessive live loads were experienced during the 4-year service period. Also, from the construction documents, it was found that excessive loading was very likely applied mistakenly during the construction before the concrete reached its designed strength. Also, from the non-destructive test, it was found that the concrete was in good condition, and the compressive strength was acceptable.

Using this information, the probabilities of hypotheses H obtained in the first cycle of diagnosis can now be updated. According to the reasoning procedure, ***the time of first appearance of cracking*** is the value of the attributes b_{12}, b_{22} of the anomalies S_1 and S_2 respectively, and hence constitutes ΔI^-. The result of non-destructive test belongs to ΔI^+. All other information is about the causal factors of explanation nodes, (refer Fig. 6.2) and should be denoted by ΔI^+. Confidence factors $CF(\Delta I^-, H_i)$, $CF(\Delta I^+, H_i)$ are accordingly assessed, and listed, together with other calculated values of relevant probabilities, in Table 6.3.

Table 6.3: Subjective Probabilities for Each H_i in The Second Cycle

Hypothesis	$CF(H_i)$	$CF(\Delta^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_1$	0.0173	40	70	0.1786	0.0318
$H_2 = e_2$	0.0432	35	50	0.1116	0.0495
$\mathbf{H_3 = e_5}$	0.1613	95	80	0.4848	**0.8036**
$H_4 = e_6$	0.1008	10	0	0	0
$H_5 = e_8$	0.1037	0	40	0	0
$H_6 = e_7e_9$	0.0518	40	10	0.0255	0.0136
$H_7 = e_7e_1$	0.0086	30	20	0.0383	0.0034
$H_8 = e_9e_{11}$	0.0634	35	10	0.0223	0.0145
$H_9 = e_9e_{12}$	0.0634	40	10	0.0255	0.0166
$H_{10} = e_{10}e_{11}$	0.0634	40	10	0.0255	0.0166
$H_{11} = e_{10}e_{12}$	0.0634	40	10	0.0255	0.0166
$H_{12} = e_5e_7$	0.0562	45	15	0.0431	0.0249
$H_{13} = e_6e_7$	0.0562	10	10	0.0064	0.0037
$H_{14} = e_5e_8$	0.0562	40	0	0	0
$H_{15} = e_6e_8$	0.0518	10	0	0	0
$H^o = others$	0.0389	10	20	0.0128	0.0051

As can be seen from the table, four hypotheses H_1, H_2, H_3, H_{12} have probabilities much higher than any others, with H_3 by far having the largest probability, i.e. $P(H_3) = 0.8036$. Probabilities of all other hypotheses are comparatively very small. Therefore, all those hypotheses whose probabilities are less than 0.02 are considered unlikely to be true, and hence are subsumed into H^o.

Assuming the acceptable probability P^H_{acpt} is 0.80, it is obvious that hypothesis $H_3 = e_5 =$ *Overloading during construction* can be considered acceptable, and the diagnosis is completed. According to the flow-chart shown in Fig. 5.1 of Chapter 5, the process of treating structural defects can then advance from step 6 directly to step 12, i.e. the design of a relevant course of action with regard to explanations contained in H_3.

6.1.9 The Final Decision

Three courses of action are considered in this case:

- a_1=do nothing;
- a_2 =repair the cracks using relevant epoxy resin;
- a_3=increase the stiffness by epoxy-bonded steel plate.

Since there are no safety and durability problems, the choice among the available actions is quite simple and mainly dependent upon the owner's demand for serviceability behaviour as well as financial situation. a_1 was finally adopted as the terminal decision.

6.2 Example 2

A State Government Department is responsible for the management of its stock of highway bridges. Inclined cracking has been identified in 16 T-beam concrete bridges since October 1990. In fear of structural failure without warning, two of these bridges have already been replaced. Due to the limited funding, a value management study has been carried out in order to determine a cost-effective strategy for the management of the existing bridges. The proposed process described in Chapter 5 was employed for the assessment of the conditions of individual bridges and for the repair procedures (Hua, McGee and Warner, 1993).

In order to be applicable to this particular bridge problem, the flow-chart of Fig. 1.4 on page 24 was modified to the one shown below in Fig. 6.3. Although the approach has been developed in this thesis for concrete buildings, the application to bridges shows that the procedures are very useful and practical and, potentially, applicable to a wide range of problems.

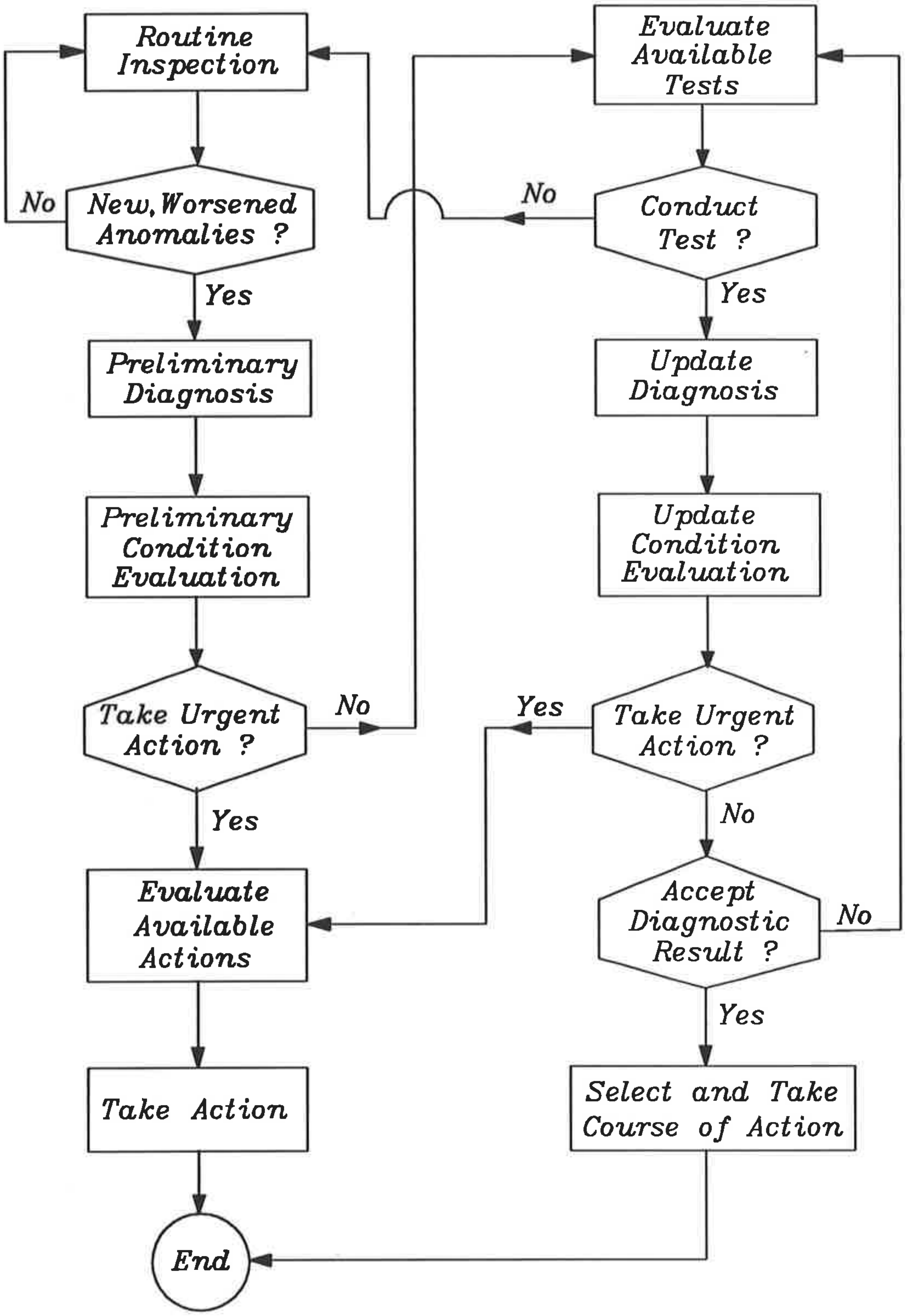


Figure 6.3: The Modified Process for Dealing With Concrete Bridges

Chapter 7

Summary and Future Work

7.1 Summary

In this thesis, an attempt has been made to establish a practically useful approach for dealing with existing concrete buildings from the academic research research point of view. For this purpose, a number of methodologies are developed for diagnosis, condition evaluation and decision-making which are essential to the treatment of structural defects. By integrating systematically the methods together, a detailed process has been proposed, which is capable of solving real problems in a well organized manner, and can be adopted by engineers as a guide in practice. The usefulness of the process has been demonstrated by examples.

7.1.1 The Method of Diagnosis

The proposed method for diagnosis is an *hypothesis-and-test* procedure through which the most likely explanations of observed anomalies can be identified. An hypothesis is the set of explanations that can fully cover the observed pattern of anomalies. Relevant hypotheses are firstly generated from an explanation tree constructed using observed anomalies and their associated attributes as well as other information obtained in the condition survey. An informal probabilistic reasoning procedure is then employed to rank the hypotheses in terms of subjective probabilities. If the probability of the most likely hypothesis H^* reaches a pre-defined value P^H_{acpt}, H^* is considered acceptable, and the diagnosis stops. Otherwise, gathering more information through testing has to be considered. Since the diagnostic results are used by other procedures to evaluate the structural condition and eventually to choose a course of action, the necessity of conducting more tests and the strategy of test selection are determined in a comprehensive decision-making procedure. Thus the diagnosis is closely related to other procedures in the whole process of dealing with structural defects.

7.1.2 The Method of Condition Evaluation

Condition evaluation of existing concrete buildings defined in this thesis consists of procedures for the assessment of structural adequacy regarding safety, serviceability and durability. For a specific hypothesis obtained from the diagnosis, the safety index β of an interested ultimate limit state concerning the real physical state of the structure at the time of assessment can be evaluated using structural reliability theory. The structure's safety condition is accordingly assessed to the status of "safe", "negligibly defective", "moderately defective" and "severely defective" varying with the value of β.

Experience-based approaches are employed for the assessment of serviceability and durability conditions. Using the proposed procedure, the assessment can be carried out in a well-organized manner.

7.1.3 The Method of Decision-making

To plan appropriate repair and maintenance strategies for an existing structure, a set of courses of action are firstly worked out by considering the effects of the explanations of the observed anomalies. Possible consequences of each option are defined in terms of the financial cost incurred and the structure's status regarding safety, serviceability and durability after the action is executed. Before adequate information is gathered, each alternative may have several possible consequences with different probabilities of being true. After consequences are scaled by utility values, the best action is chosen by comparing alternatives according to their expected utilities. The procedure forms a multi-stage process in which the choice can be made at any stage using the information available at the time. Refined decisions are made at the next stage if new data are gathered.

7.1.4 The Complete Process

The complete process of dealing with existing concrete buildings is obtained by assembling the proposed procedures for diagnosis, condition evaluation and decision-making. The process begins with a set of observed anomalies, and ends with a terminal decision regarding corrective work. When applied to real problems, the process usually results in a number of intermediate decisions such as conducting more tests before it terminates. The process works iteratively so that new information available at each stage is efficiently utilized in making choices. Since major factors regarding various structural requirements as well as financial cost are jointly considered from both the engineer and the owner's point of view, the final decision resulting from the process is rational and acceptable.

7.2 Concluding Remarks and Future Work

With the rapid expansion of the stock of concrete structures, it is expected that more and more attention will have to be paid to the maintenance and treatment of existing structures. Although there exist various experience-based approaches established by engineers in practice, this thesis has introduced what are thought to be new systematic methodologies. This was found to be necessary because of the lack of previous analytic research in this field. Although the methods have been proposed for concrete buildings, they should also be applicable quite generally.

To continue the research, time-dependent anomalies need further study. Durability problems caused by these anomalies usually dominate the treatment of structures exposed to harsh environments. With the wide application of computers, it is possible to build computerized *expert systems* (Levine *et. al*, 1987) to aid engineers in dealing with durability problems which need experience and expertise. This should prove to be a fruitful development area in the near future.

Appendix A

Statistical Data on Basic Variables

A.1 Introduction

Variations in the common basic variables involved in structural design such as properties of concrete and reinforcement need to be represented by probability distributions in reliability analysis. These distributions are usually obtained from test data available together with professional judgements, and reflect the current practice of concrete engineering. There are plenty of data reported in the literature for this purpose, and relevant distributions of material properties and dimensions are summarised in this section. However, the data presented here are mainly from probabilistic studies of concrete structures by a number of authors in North America, and hence its applicability to other areas is subject to further refinement.

A.2 Statistical Properties on Concrete Strength

In a Monte Carlo study of reinforced concrete beam column interactions conducted by Ellingwood (1977), the compressive strength of concrete f_c is modelled by normal distributions with the means and coefficients of variation (COV) listed in Table A.1 for different workmanships of the concrete, where f_c' is the nominal design value of concrete strength specified in relevant codes.

Table A.1: Data on Concrete Strength

variable	mean	COV		
		good	average	poor
f_c	$1.16f_c'$	0.10	0.15	0.25

A comprehensive study of the variation in concrete properties was conducted by Mirza *et al.* (1979b) based on a large amount of test data collected from various sources. It was pointed out that the strength of concrete in a structure tends to be somewhat lower than the strength of control cylinders molded from the same concrete, due to the differences in size and shape, and the effects of different stress conditions in the structure and specimens. The following equation is given to estimate the mean 28-day strength of concrete for minimum acceptable curing:

$$\bar{f}_{str28} = 0.675f_c' + 1100 \leq 1.15f_c' \ \ psi \tag{A.1}$$

where f_c' is the nominal design value. However, the in-situ strength of concrete is also affected by the differences in volumes of material under stress and the rate of loading. To allow for these effects, a probabilistic model of concrete strength was constructed as:

$$f_c = f_c' r_1 r_2 r_3 \tag{A.2}$$

where r_1, r_2 and r_3 are random variables to account for various influences. From the results of analyses based on this model, a normal distribution curve was proposed for the concrete compressive strength, f_c, with the mean and coefficient of variation as:

$$\bar{f}_c = \bar{f}_{str28}[0.89(1 + 0.08 \log R)] \;\; psi$$

$$V_c^2 = V_{cy}^2 + 0.0084$$

where R is the loading rate in psi/sec; $\bar{f}_{str28}$ is obtained by Eq. A.1 and V_{cy}^2 is the variability of cylinder tests.

For tensile strength, a similar procedure was applied and the mean value and coefficient of variation of flexural-tension strength of concrete are given by:

$$\bar{f}_t = 8.3\sqrt{\bar{f}_{str28}}[0.96(1 + 0.11 \log R)]$$

$$V_{cr}^2 = \frac{V_{cy}^2}{4} + 0.0421$$

A.3 Statistical Properties on Dimensions

The variation in geometric imperfection of concrete members depends, to a great extent, on the quality control. Large differences exist among various data reported in different sources. The available data were summarised by Mirza and MacGregor (1979c), and the recommended properties for concrete beams are normal distributions with means and standard deviations listed in Table A.2, in which the values in square brackets are for in-situ beams and others are for precast beams:

Table A.2: Variations in Beam Dimensions

Dimension description	Nominal range (in.)	Mean deviation from normal (in.)	Standard deviation (in.)
Rib width	14 [11–12]	0 $[+\frac{3}{32}]$	$\frac{3}{16}$ $[\frac{3}{16}]$
Flange width	19–24 [—]	$+\frac{5}{32}$ [—]	$\frac{1}{4}$ [—]
Overall depth	21–39 [18–27]	$+\frac{1}{8}$ $[-\frac{1}{8}]$	$\frac{5}{32}$ $[\frac{1}{4}]$
Top cover	2–2.5 [0.5–1]	0 $[\frac{1}{8}]$	$\frac{5}{32}$ $[\frac{5}{8}]$
Bottom cover	0.75 [0.75–1]	0 $[\frac{1}{16}]$	$\frac{5}{32}$ $[\frac{7}{16}]$
Span		0 [0]	$\frac{11}{12}$ $[\frac{11}{16}]$

A.4 Statistical Properties on Reinforcement

The in-batch variation in the yield strength of steel bars is relatively small. However, the variability of samples obtained from different batches and sources may be high, and has to be considered in safety analysis of concrete structures. According to the data obtained from various sources, Mirza and MacGregor (1979a) found that the Beta distribution provides the best fit to the existing data of steel strength with the density function given by:

$$PDF = 3.721\left(\frac{f_{ys} - 248.2}{220.7}\right)^{2.21}\left(\frac{468.9 - f_{ys}}{220.7}\right)^{3.82} \quad For\ Grade\ 40\ bars$$

$$PDF = 7.141\left(\frac{f_{ys} - 393}{351.7}\right)^{2.02}\left(\frac{744.7 - f_{ys}}{351.7}\right)^{6.95} \quad For\ Grade\ 60\ bars$$

The cross sectional areas of steel bars are also subject to variations. From the test data available, a normal distribution truncated at 0.94 with a mean value 0.99 and a coefficient of variation 2.4 percent is suggested for the ratio of measured to nominal areas, A_m/A_n. When the effect of variability of A_m/A_n is considered negligible, a single value of 0.97 may be used.

Data on the statistical properties of prestressing strands are relatively limited in comparison with that of steel bars. In a previous study of partially prestressed beams by Naaman and Siriaksorn (1982), the probability distributions of ultimate strength f_{pu}, yield strength f_{py}, modulus of elasticity E_p and cross-sectional area A_p of Grade 270 (12.7 mm) prestressing strand were taken as normal distributions with mean values $1.0387 f_{pun}$, $1.027 f_{pyn}$, $1.011 E_{pn}$ and $1.01176 A_{pn}$, and the corresponding coefficients of variation 0.0142, 0.022, 0.025 and 0.0125, respectively, in which x_n is the nominal value of x.

Properties of Grade 270 prestressing steel were also modelled as normal distributions by Mirza and MacGregor (1982) with the means and standard deviations listed below in Table A.3, in which ϵ_{pu} is the ultimate tensile strain.

Table A.3: Data on Grade 270 Prestressing Steel

Variable	Mean	Deviation
f_{pu} Mpa	1935	50
E_p Mpa	195 800	3900
ϵ_{pu}	0.05	0.0035

Appendix B

Probabilistic Models for Building Live Loads

B.1 Introduction

Applied loads on structures usually consist of dead and live loads. Live load contributes in a major way to the uncertainty involved in structural analysis, while the dead load has relatively smaller variations and could be assessed with satisfactory accuracy in many cases. In this section, only the office building live load will be discussed.

Live loads in buildings consist of **sustained** and **extraordinary** (or **transient**) components. The former represents the load due to the weight of things like desks, bookcases, which remain relatively constant for a long period. The latter represents short-term effects caused by events like gatherings of people and furniture storage.

Live loads are time-dependent, and the time history of building live loads is illustrated in Fig. B.1, in which the jumps in the sustained load represent the occupancy changes, and the sustained load is assumed being constant during one tenancy. For various design requirements, different values of live loads may be adopted. The load considered for ultimate limit states is usually the maximum combined sustained and extraodinary load that has a small probability of being exceeded within a specified period. The load for serviceability limit states, however, should be less conservative.

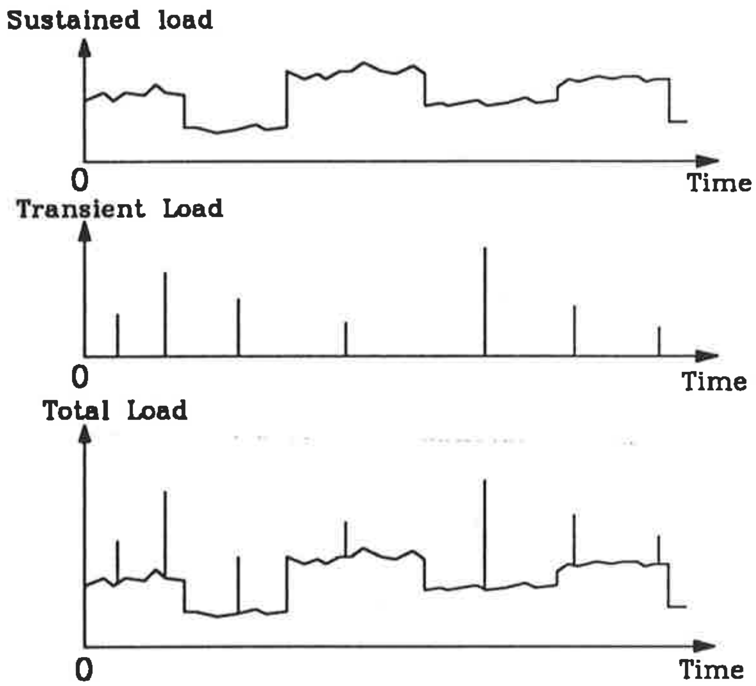


Figure B.1: The Time History of Live Loads

To treat building live load statistically, both survey data and probabilistic models are needed. The information obtained from most load surveys available is concerned with the sustained component. The earliest survey dates back to the last century, and a historical review of this information is reported by Corotis (1975). Although the information gathered in the earlier load surveys was not represented and analyzed on the basis of probability theory due to the lack of sufficient samples, it did provide data as an alternative to professional judgement for load code committees to work with. On the other hand, it was recognized from those surveys that the live load intensity in buildings decreases with increasing tributary area. This important feature of live load is now considered by many load committees in the world. In parallel with developments in structural reliability theory, extensive load survey data have been reported in the past decades (Dunham, 1952; Mitchell *et al.* 1971; Culver, 1976), and statistical models have also been developed (Corotis, 1977).

Because the extraordinary component of live loads is due to unusual actions, it is difficult to collect experimental data. A possible approach is to develop a reasonable probabilistic model with the parameters estimated from judgement, together with limited data available.

In the following, probabilistic models currently available (Peir and Cornell, 1973; McGuire and Cornell, 1974) for floor live loads in office buildings are presented, which has been used in developing design load specifications in different countries (Ellingwood *et al.*, 1980; Pham and Dayeh, 1986).

B.2 Probabilistic Models for Sustained Live Load

When concerning the sustained floor live loads, two aspects have to be considered. One is *spatial correlation*, which reflects the dependence between loadings at two different locations at the same point in time; Another is the *temporal correlation* which reflects the dependence between loadings at the same location at two different points in time. Unfortunately, it is difficult to determine these two correlations quantitatively from existing survey data, and statistical models for live loads have generally been chosen from a prior considerations by judgement. In the model proposed by Peir and Cornell (1973), the live load is represented in terms of load effect which is the structural response produced by the load acting on it other than nominal load, which is the sum of all loads on the floor. For analysis purposes, the terms **unit load** and **equivalent uniformly distributed load** (EUDL) are introduced. The former refers to the nominal load divided by floor area. The latter equals the load effect divided by the integral over the influence surface, and this is the uniform load intensity which would produce the same load effect if applied over the appropriate floor area. The influence surface introduced here is the shape of a function whose ordinate at any point (x, y) is the load effect of interest which would be caused by a unit load applied at that point.

The load intensity at point (x, y) on the floor of a building at an arbitrary point in time is expressed as:

$$W(x, y) = m + \gamma_1 + \gamma_2 + \epsilon(x, y) \tag{B.1}$$

in which m =the mean value of all unit live loads; γ_1 =a zero-mean random variable representing the deviation from m due to different buildings; γ_2 =a zero-mean random variable accounting for the variation of different floors in a building from $m + \gamma_1$; $\epsilon(x, y) = a$ stochastic process with nonzero spatial correlation, i.e. $Cov[\epsilon(x_1, y_1), \epsilon(x_2, y_2)] \neq 0$, representing the variation of load intensity on a given floor at different points.

With $w(x, y)$ determined, the unit load U and the EUDL load on area A_I can be obtained by:

$$U = \frac{\int\int_{A_I} W(x, y)dxdy}{A_I} \tag{B.2}$$

$$EUDL = \frac{\int\int_{A_I} I(x, y)W(x, y)dxdy}{\sum\sum_{A_I} I(x, y)dxdy} \tag{B.3}$$

in which $I(x, y)$ is the coordinate of the influence surface.

The properties of EUDL has to be determined from load survey data. Considering the live load acting on a influence area A_I with load intensity $W(x, y)$, the total load effect is

$$L(A_I) = \int\int_{A_I} I(x,y)W(x,y)dxdy$$

The mean and variance of $L(A_I)$ can be determined from stochastic theory:

$$E[L(A_I)] = \int\int_{A_I} I(x,y)E[W(x,y)]dxdy \tag{B.4}$$

$$Var[L(A_I)] = \int\int_{A_I}\int\int_{A_I} I(x,y)I(x_1,y_1)Cov[W(x,y),W(x_1,y_1)]dxdydx_1dy_1 \tag{B.5}$$

The mean and variance of EUDL are then:

$$E[EUDL] = \frac{E[L(A_I)]}{V_1} \tag{B.6}$$

$$Var[EUDL] = \frac{Var[L(A_I)]}{V_1^2} \tag{B.7}$$

in which V_1 is the volume under the influence surface.

The above model was further simplified by McGuire and Cornell (1974) for practical application. Firstly, the arbitrary point in time load intensity $W(x,y)$ is modified and represented by

$$W(x,y) = m + \gamma + \epsilon(x,y) \tag{B.8}$$

in which m is the mean sustained load; γ is a mean-zero random variable; $\epsilon(x,y)$ is a spatially varying random process. Secondly, it is assumed that the correlation between the values of $\epsilon(x,y)$ for two points is zero if the points are separated by any finite distance. The mean and variance of EUDL can then be simplified as:

$$E[EUDL] = m \tag{B.9}$$

$$Var[EUDL] = \sigma^2 + \frac{\sigma_s^2}{A_I}K \tag{B.10}$$

where σ^2=variance from m; A_I= influence area; σ_s^2 and K are experimental constants.

The preceding derivation of first and second-order moments does not depend on any form of probability distributions. Based on the study of Corotis (1977), it is assumed that the sustained EUDL conforms to a gamma distribution with moments given by Eqs. B.9 and B.10.

However, of prime interest in structural design is the life time maximum load, and this value for the sustained live load can be obtained by assuming that the occupancy changes occur as Poisson arrivals (see Fig. B.1). If the mean rate of this change is ν_s (per year), the distribution of the maximum sustained live load during the lifetime of T years is given by:

$$F_{Max-Sust}(x) = exp(-\nu_s T[1 - F_s(x)]) \tag{B.11}$$

in which $F_s(x)$ is the distribution function of arbitrary point in time sustained live load.

B.3 Probabilistic Models for Extraodinary Live Load

For extraodinary live load, a similar procedure was used by McGuire and Cornell (1974). The extraordinary component of live load was modelled by a series of unusual events. Each event is considered as a number of randomly distributed load cells, with one cell containing a cluster of loads. The mean and variance of equivalent uniformly distributed load associated with one extraodinary event can be evaluated by:

$$E(EUDL) = \frac{m_Q m_r \lambda_m}{A_I} \tag{B.12}$$

$$Var(EUDL) = \frac{k\lambda_m(m_Q^2 mR^2 + m_R\sigma_Q^2 + m_Q^2\sigma_Q^2)}{A_I^2} \tag{B.13}$$

in which A_I = influence area; m_Q = mean weight of single load Q; m_R = mean number ofloads per load cell; λ_m = a function of A_I; k = influence surface parameter; σ_Q^2 = variance of single load. It is assumed that the extraodinary

EUDL also corresponds to a gamma distribution. Due to the lack of data on extraordinary components, the parameters in Eq. B.12 and Eq. B.13 are decided mainly by judgement. The values given by McGuire and Cornell (1974) are listed here in Table B.1

Table B.1: Data on Parameters in the Extraodinary Live Load Model

μ_Q (pounds)	μ_R	σ_Q^2 $(pound)^2$	σ_R^2	λ_m	
145	5	900	4	1.43	For $A_I = 100\ ft^2$
				2.76	For $A_I = 200\ ft^2$
				3.99	For $A_I = 300\ ft^2$
				$\sqrt{\frac{(A_I-164)}{9}}$	$For A_I \geq 400\ ft^2$

Like the sustained live load, the extraordinary component is also time dependent. The maximum extraordinary load is the largest value of all extraordinary events happen during the lifetime T of the structure. If it is assumed that the extraordinary events occur as a Possion process with the rate ν_e, the distribution of lifetime maximum extraodinary live load is:

$$F_{Max-Extr}(x) = exp\{-\nu_e t[1 - F_e(x)]\} \qquad (B.14)$$

where $F_e(x)$ is the distribution function of one extraordinary event; t is the time period.

B.4 The Combined Maximum Live Load

The most important load value in code specification and design practice is the total lifetime maximum EUDL, L_t, which is the combination of sustained and extraodinary components. It is generally accepted that (McGuire and Cornell, 1974) the maximum load during a structure's life will almost certainly be one of the two cases or modes. The first mode is the maximum sustained load during the structure's life plus the largest extraodinary event which occurs during the random duration of this maximum sustained load. The second mode is the largest extraodinary event during the structure's life plus the sustained load acting at this (arbitrary) point of time.

Based on the first mode, the maximum total EUDL, L_t, was evaluated, and the results were fitted to a Type I extreme value distribution with the following properties (Ellingwood and Culver, 1977):

$$E[L_t] = 18.7 + \frac{520}{\sqrt{A_I}} \quad psf \tag{B.15}$$

$$Var[L_t] = 14.2 + \frac{18900}{A_I} \quad (psf)^2 \tag{B.16}$$

where the influence area A_I is different from the traditional tributary area A_T, e.g. $A_I = 2A_T$ for a beam, $4A_T$ for columns, and panel area for two-way slabs.

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Amendment

This amendment has been prepared to satisfy the comment numbered 2.4 which was made by the second examiner of this thesis. The main part of the amendment is actually an expansion of Example 1 described in Chapter 6 for the purpose of demonstrating how the proposed process can be applied in practice.

Section 6.1.1 on page 184: The first sentence beginning with "The cracking pattern" on line 1, and ending on line 2 with "Fig. 6.1" is superseded by the following sentence:

The cracking pattern was recorded during the site inspection, and is given schematically in Fig. 6.1, in which Part I (View From Below Slab) shows the typical cracking pattern on the bottom surface of the concrete slab at mid-panel, and Part II (Plan View) shows the typical cracking pattern on the top surface around the columns.

From page 190 to page 193 : Sections 6.1.7, 6.1.8 and 6.1.9 are superseded by the following sections.

6.1.7 What to Do Next ?

At this stage, since the highest ranked hypothesis H_3 has only the probability of 0.161, which is far below a reasonable acceptable value (say 0.80), it is too early to make a final diagnostic conclusion. Therefore, a decision has to be made under risk. Using the proposed process described in Chapter 5, a set of actions $A = (a_1, a_2, \cdots, a_n)$ have to be created first, and the best action a^* chosen from A based on the expected utility criterion. For this purpose, the assessment of utility functions is described in the next section.

6.1.8 Assessment of Utility Functions

The utility function represents the utility values over the whole consequence space of a decision problem. Using methods described in Sections 4.5.2 and 4.5.3 from page 147 to page 152, possible consequences for this example before or after a course of action is executed can be fully defined by three attributes, i.e. $X = X_F \times X_S \times X_C$, where X_F is the attribute of the *objective* "to provide adequate structural safety", and has four values: [*safe*, *negligibly defective*, *moderately defective*, *severely defective*] denoted by [SF, ND, MD, SD]; X_S is the attribute of the *objective* "to provide adequate serviceability", and has two values: [*satisfactory*, *inadequate*] denoted by [ST, IA]; and X_C is a real number representing the cost in monetary values.

As mentioned earlier in Chapter 4, it is tedious to assess utility values over a multi-dimensional consequence space. Using the method developed in Section 4.5.4 of this thesis, utilities over the above three-dimensional consequence space can be represented by a set of unidimensional utility functions. To do this, the two discrete attributes X_F and X_S in X can be combined into a single attribute Y, i.e. $Y = X_F \times X_S = [y_1,\ y_2,\ y_3,\ y_4,\ y_5,\ y_6,\ y_7,\ y_8]$, in which $y_1 = (SF,\ ST)$, $y_2 = (ND,\ ST)$, $y_3 = (MD,\ ST)$, $y_4 = (SD,\ ST)$, $y_5 = (SF,\ IA)$, $y_6 = (ND,\ IA)$, $y_7 = (MD,\ IA)$, and $y_8 = (SD,\ IA)$. The consequence space X then becomes $X = Y \times X_C$, and the utilities on X can be represented by a set of 8 utility functions denoted by $U(y_1, X_C)$, $U(y_2, X_C)$, $U(y_3, X_C)$, $U(y_4, X_C)$, $U(y_5, X_C)$, $U(y_6, X_C)$, $U(y_7, X_C)$ and $U(y_8, X_C)$, in which y_i is a fixed point and X_C is a real number in terms of dollars.

To evaluate these utility functions, utility values on the 16 discrete consequences formed by (y_i, x_{cmin}) and (y_i, x_{cmax}) have to be assessed, where x_{cmin} and x_{cmax} are the minimum and maximum values of the real number X_C respectively. In this example, x_{cmin} is assumed to be \$0 denoting no cost, and x_{cmax} is assumed 1 million dollars. To start the assessment, these consequences are firstly queued according to the decision maker's preference: $(y_1, 0) > (y_5, 0) > (y_2, 0) > (y_6, 0) > (y_3, 0) > (y_7, 0) > (y_1, 1m.) > (y_5, 1m.) > (y_2, 1m.) >$

$(y_6, 1m.) > (y_3, 1m.) > (y_7, 1m.) > (y_4, 0) > (y_8, 0) > (y_4, 1m.) > (y_8, 1m.)$, where the symbol $>$ denotes "is preferred to". Secondly, since consequences $(y_1, 0)$, representing "a safe and serviceable structure without any cost", and $(y_8, 1m.)$, representing "a severely defective and unserviceable structure after spending 1 million dollars on corrective work", are most and least preferred respectively, let $U(y_8, 1m.) = 0$ and $U(y_1, 0) = 100$, and utility values on the rest of the 16 discrete consequences can then be assessed easily using the *direct method* based on the concept of **certainty equivalent** described on page 152. For example, by asking the decision maker at what value of probability p, he/she is indifferent to $(y_7, 0)$ for certain and to the lottery $[(y_1, 0),\ p,\ (y_8, 1m.)]$, the utility of $(y_7, 0)$ can be obtained as $U(y_7, 0) = 100 \times p$.

With these utilities available, for a given y_i the utility value on the consequence $(y_i, x_c) \in X$ with any value of X_C can be assessed simply using the direct method by asking the decision maker at what value of probability p, he/she is indifferent to (y_i, x_c) for certain and to the lottery $[(y_i, x_{cmin}),\ p,\ (y_i, x_{cmax})]$. In this way, the utility function $U(y_i, x_c)$ is obtained, and the 8 unidimensional utility functions $U(y_i, x_c)$ $(i = 1, 2, ..., 8)$ can provide utility values for any point in the whole consequence space X. Utilities on X for this example have been assessed and are represented by diagrams in Fig. 1.

6.1.9 An Intermediate Decision

With the utilities assessed above, it is now possible to compare various courses of action, and to choose the "best" one. Since this is in the very preliminary stage of the decision-making process with limited information available, the following three broad actions are temporarily considered: a_1 = "do nothing", a_2 = "repair the cracks and increase the stiffness", a_3 = "undertake a major repair such as strengthening". Detailed repair methods can be designed according to these in a later stage of the decision-making process.

Among these actions, a_2 is obviously designed to satisfy the serviceability

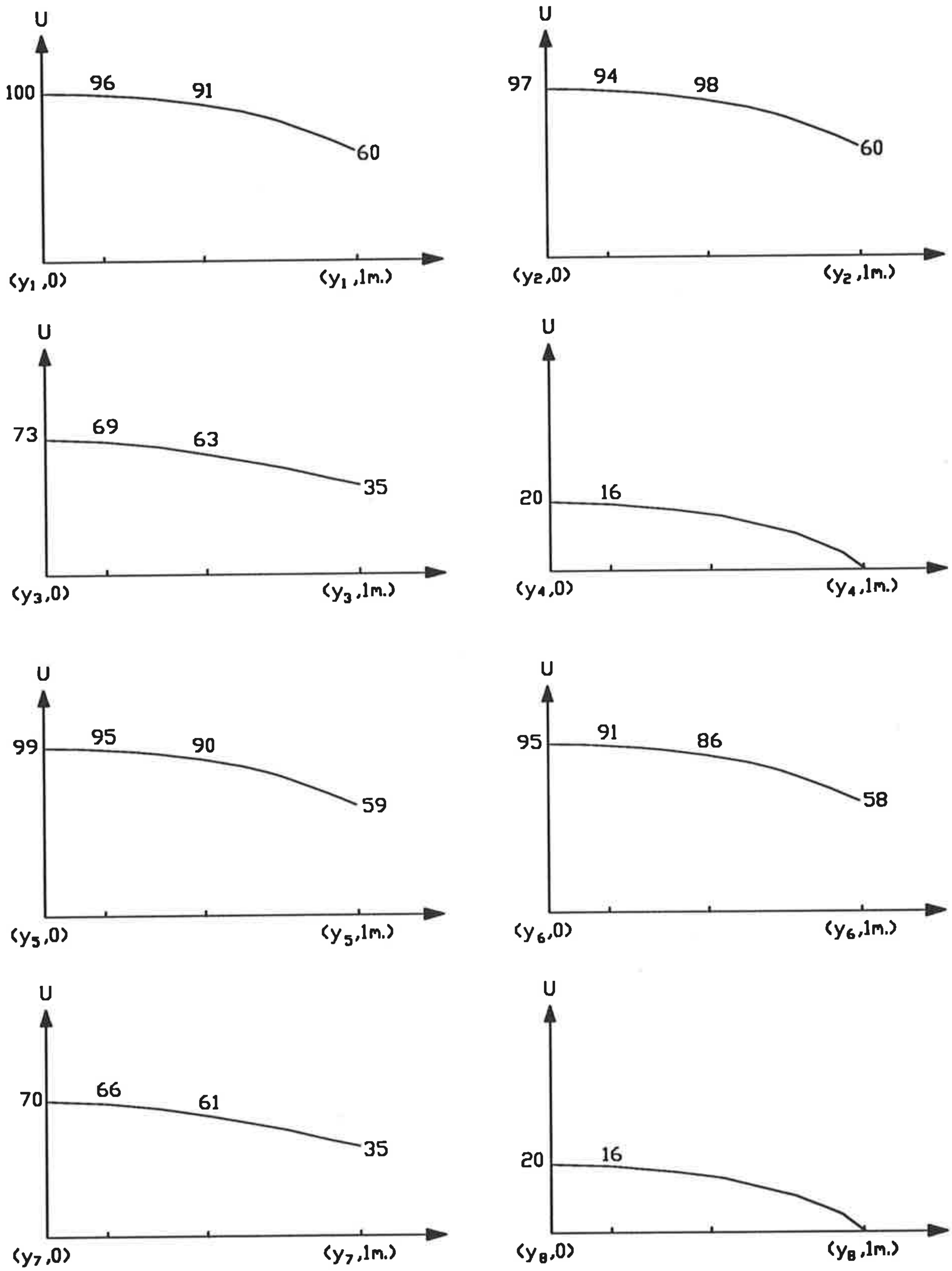

Figure 1: Assessed Utility Values Over the Consequence Space

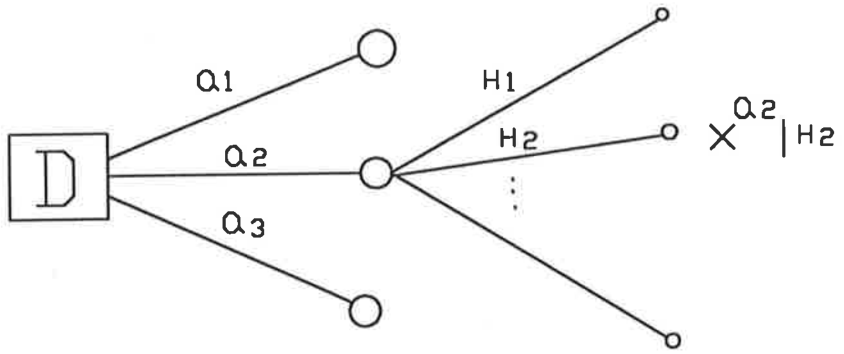

Figure 2: The Decision-tree for The Intermediate Decision

requirement, and a_3 is for safety. It is assumed that 200000 and 1 million dollars are required to carry out a_2 and a_3 respectively for all panels having anomalies in the building. The decision tree for this problem is shown in Fig. 2, in which $x^{a_i} \mid H_j$ denotes the consequence of action a_i if hypothesis H_j is true. Due to the limited information available, assessment of these consequences based on reliability theory is impractical at this stage, and hence the experience-based method described in Section 3.7.3 of this thesis is used. The result is listed in Table 1, in which related utilities have been obtained from Fig. 1.

Based on Fig. 2 and Table 1, the expected utilities of a_1, a_2 and a_3 can be calculated using Eq. 4.24 on page 160: $\bar{U}(a_1) = 62.8$, $\bar{U}(a_2) = 60.3$, and $\bar{U}(a_3) = 61.1$. Obviously a_1 is preferred, i.e. $a^* = a_1$.

However, a^* has been chosed using only the information currently available. Therefore, it is alternatively possible to conduct tests to obtain more information before the final decision is made. This is described in the next section.

6.1.10 Conducting More Tests ?

Feasible tests are listed below:

Table 1: Possible Consequences and Their Utilities

H	a_1		a_2		a_3	
	Consequence	Utility	Consequence	Utility	consequence	Utility
H_1	$(y_5, 0)$	99	$(y_1, 200000)$	96	$(y_1, 1m.)$	62
H_2	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H_3	$(y_5, 0)$	99	$(y_1, 200000)$	96	$(y_1, 1m.)$	62
H_4	$(y_5, 0)$	99	$(y_1, 200000)$	96	$(y_1, 1m.)$	62
H_5	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H_6	$(y_8, 0)$	20	$(y_4, 200000)$	16	$(y_1, 1m.)$	62
H_7	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H_8	$(y_8, 0)$	20	$(y_4, 200000)$	16	$(y_1, 1m.)$	62
H_9	$(y_8, 0)$	20	$(y_4, 200000)$	16	$(y_1, 1m.)$	62
H_{10}	$(y_8, 0)$	20	$(y_4, 200000)$	16	$(y_1, 1m.)$	62
H_{11}	$(y_8, 0)$	20	$(y_4, 200000)$	16	$(y_1, 1m.)$	62
H_{12}	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H_{13}	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H_{14}	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H_{15}	$(y_7, 0)$	70	$(y_3, 200000)$	69	$(y_1, 1m.)$	62
H^o	$(y^o, 0)$	72	$(y^o, 200000)$	68	$(y^o, 1m.)$	39

- t_1: search for information on loading during construction and the in-service period;
- t_2: carry out relevant structural analysis to calculate the deflection;
- t_3: detect locations and quantities of steel bars using appropriate devices;
- t_4: remove small regions of concrete to observe the steel reinforcement;
- t_5: take core samples of concrete to test for compressive strength;
- t_6: conduct non-destructive tests to estimate compressive strength of concrete,

To decide on whether a test is preferred to the action a^* chosen in the last section, it is necessary to evaluate the expected utility of the favoured test. For this purpose, let us try test t_6 (cost 5000 dollars) first, and the related decision-tree is shown in Fig. 3, in which d_1, d_2 and d_3 are three possible outcomes of the test. Since this test is mainly to get more information on hypothesis $H_5 = e_5 =$ *inadequate concrete strength*, for simplicity, it is assumed that $d_1 =$ *reject* H_5, $d_2 =$ *accept* H_5 and $d_3 =$ *information is irrelevant, and probabilities of* H_i *unchanged.* It is also assumed that the probabilities of these outcomes are $p(d_1) = 0.4$, $p(d_2) = 0.4$ and $p(d_3) = 0.2$.

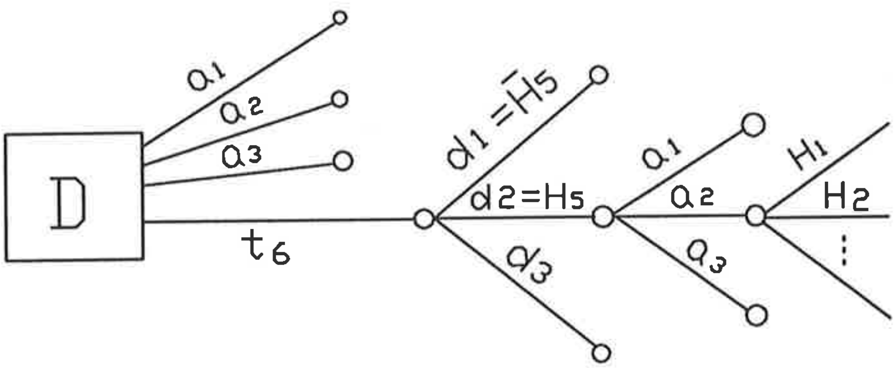

Figure 3: The Decision-tree for Test t_6

Since hypotheses in H are mutually dependent (mutually exclusive and exhaustive) if d_2 is true, the updated probability $p(H_5)$ is then 1, and the probability of any other element in H becomes 0. However, it is very important to note that $H_5 = e_5$ being true is different from the existence of e_5. According to the principle for diagnosis stated on page 61, if H_5 is true, then e_5 exists, but not vice versa.

If d_3 is true, the probabilities calculated for all hypotheses before the test is conducted will not change. If d_1 is true, $p(H_i)$ has to be re-evaluated conditional on d_1, and the result is given in Table 2.

Table 2: Updated Probabilities for Each H_i Assuming d_1 Is True

Hypothesis	$CF(H_i)$	$CF(\Delta I^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_1$	0.0173	50	50	0.0769	0.0220
$H_2 = e_2$	0.0432	50	50	0.0769	0.0548
$H_3 = e_5$	0.1613	50	50	0.0769	0.2047
$H_4 = e_6$	0.1008	50	50	0.0769	0.1279
$H_5 = e_8$	0.1037	0	50	0	0
$H_6 = e_7e_9$	0.0518	50	50	0.0769	0.0657
$H_7 = e_7e_1$	0.0086	50	50	0.0769	0.0109
$H_8 = e_9e_{11}$	0.0634	50	50	0.0769	0.0805
$H_9 = e_9e_{12}$	0.0634	50	50	0.0769	0.0805
$H_{10} = e_{10}e_{11}$	0.0634	50	50	0.0769	0.0805
$H_{11} = e_{10}e_{12}$	0.0634	50	50	0.0769	0.0805
$H_{12} = e_5e_7$	0.0562	50	50	0.0769	0.0713
$H_{13} = e_6e_7$	0.0562	50	50	0.0769	0.0713
$H_{14} = e_5e_8$	0.0562	0	50	0	0
$H_{15} = e_6e_8$	0.0518	0	50	0	0
$H^o = others$	0.0389	50	50	0.0769	0.0494

Based on Fig. 3 and Table 2, the expected utility of t_6 is calculated as:

$$\bar{U}(t_6) = 0.4 \times 61 + 0.4 \times 70 + 0.2 \times 62.8 = 64.6 > \bar{U}(a^*) = 62.8 \qquad (1)$$

Therefore, test t_6 is preferred to a^*. The intermediate decision is then to carry out test t_6 before a terminal decision is made.

6.1.11 The Second Stage of Decision-making

After t_6 was executed, it was found that the concrete was in good condition, and the compressive strength was acceptable. Using this information, the diagnosis, i.e. the probability of H has been updated, and the result is shown in Table 2. Using the updated $p(H_i)$, the utilities of the three courses of action are calculated as: $\bar{U}(a_1) = 61.0$, $\bar{U}(a_2) = 57.98$ and $\bar{U}(a_3) = 60.86$. Therefore, the most preferred action at this stage is again $a^* = a_1$.

However, the diagnostic result is still not conclusive after the outcome of t_6 is obtained. The same situation was faced in the first stage of decision-making, and the alternative option of conducting further tests has again to be assessed. For this purpose, let us try t_1 this time. The related decision-tree is shown in Fig. 4, in which d_1, d_2, d_3, d_4, d_5 and d_6 are possible outcomes of the test. Since this test is mainly to get more information on hypotheses H_3 and H_4, for simplicity it is assumed that $d_1 = accept\ H_3$, $d_2 = accept\ H_4$, $d_3 = reject\ H_3$, $d_4 = reject\ H_4$, $d_5 = reject\ both\ H_3\ and\ H_4$, $d_6 = information\ is\ irrelevant\ and\ probability\ of\ p(H_i)\ unchanged$. Probabilities of these outcomes are assessed as $p(d_1) = 0.2$, $p(d_2) = 0.2$, $p(d_3) = 0.2$, $p(d_4) = 0.2$, $p(d_5) = 0.1$ and $p(d_6) = 0.1$.

The expected utility of t_1 is then evaluated, and the result is:

$$\bar{U}(t_1) = 0.2 \times (99 + 99 + 60.43 + 60.58) + 0.1 \times 59.84 + 0.1 \times 61.0 = 75.89 > 61.0 \qquad (2)$$

Therefore, test t_1 is preferred to a^* at this stage.

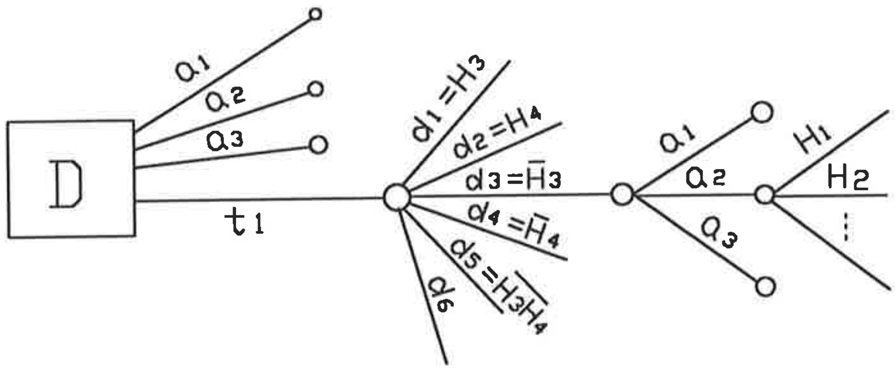

Figure 4: The Decision-tree for Test t_1

6.1.12 The Third Stage of Decision-making

From test t_1, it was confirmed that the cracking had already appeared when the construction was completed, and no excessive live loads were experienced during the 4-year service period. Also, from the construction document, it was found that excessive loading was very likely applied mistakenly during the construction before the concrete reached its designed strength.

Using this information, the probabilities of hypotheses H obtained in the second cycle of diagnosis can now be updated. According to the reasoning procedure, *the time of first appearance of cracking* is the value of the attributes b_{12}, b_{22} of the anomalies S_1 and S_2 respectively, and hence constitutes ΔI^-. All other information is about the causal factors of explanation nodes, (refer Fig. 6.2) and belongs to ΔI^+. Confidence factors $CF(\Delta I^-, H_i)$, $CF(\Delta I^+, H_i)$ are accordingly assessed, and listed, together with other calculated values of relevant probabilities, in Table 3.

Assuming the acceptable probability P^H_{acpt} is 0.80, it is obvious that hypothesis $H_3 = e_5 =$ *Overloading during construction* can be considered acceptable, and the diagnosis is complete. According to the flow-chart shown in Fig. 5.1 of Chapter 5, the process of treating structural defects can then advance from

Table 3: Subjective Probabilities for Each H_i in The Third Cycle

Hypothesis	$CF(H_i)$	$CF(\Delta^+, H_i)$	$CF(\Delta I^-, H_i)$	$P(\Delta I, H_i)$	$P(H_i)$
$H_1 = e_1$	0.0220	50	50	0.1356	0.0239
$H_2 = e_2$	0.0548	50	60	0.1626	0.0713
$\mathbf{H_3 = e_5}$	0.2047	95	95	0.4892	**0.8012**
$H_4 = e_6$	0.1279	0	0	0	0
$H_5 = e_8$	—	—	—	—	—
$H_6 = e_7e_9$	0.0657	50	10	0.0271	0.0142
$H_7 = e_7e_1$	0.0109	50	10	0.0271	0.0024
$H_8 = e_9e_{11}$	0.0805	50	5	0.0136	0.0088
$H_9 = e_9e_{12}$	0.0805	50	5	0.0136	0.0088
$H_{10} = e_{10}e_{11}$	0.0805	50	5	0.0136	0.0088
$H_{11} = e_{10}e_{12}$	0.0805	50	5	0.0136	0.0088
$H_{12} = e_5e_7$	0.0713	75	15	0.0610	0.0348
$H_{13} = e_6e_7$	0.0713	0	0	0	0
$H_{14} = e_5e_8$	—	—	—	—	—
$H_{15} = e_6e_8$	—	—	—	—	—
$H^o = others$	0.0494	20	40	0.0434	0.0172

step 6 directly to step 12, i.e. the design of a relevant course of action with regard to explanations contained in H_3.

6.1.12 The Final Decision

According to the accepted hypothesis $H_3 = e_5 =$ *overload during construction*, there are no safety and durability problems, and hence the following four detailed courses of action are considered:

- a_1=do nothing;
- a_2 =repair the cracks using relevant epoxy resin;
- a_3=increase the stiffness by epoxy-bonded steel plate;
- a_4=$a_2 + a_3$.

with costs of 0, 50000, 150000 and 200000 dollars respectively. Since the decision is to be made under certainty in this case, consequences related to each action are assessed as $x^{a_1} = (y_5, 0)$, $x^{a_2} = (y_5, 50000)$, $x^{a_3} = (y_5, 150000)$ and $x^{a_4} = (y_1, 200000)$, in which $y_5 = (SF, IA)$, $y_1 = (SF, ST)$, with SF representing "structure is safe", ST and IA denoting "serviceability requirement is satisfactory and inadequate respectively".

Utilities of these consequences have been found from the assessed utility functions shown in Fig. 1, and are: $U(a_1) = 99$, $U(a_2) = 98$, $U(a_3) = 96$ and $U(a_4) = 96$. Obviously, $a_1 =$ *doing nothing* is the best action according to the decision maker's preference.

Section 7.2 on page 199: Add the following sentence to the end of the first paragraph of Section 7.2:

However, it should be mentioned that methods proposed in this thesis are subject to further testing in practice and necessary refinement, before they can be used by engineers as a useful tool in dealing with structural defects.