



BEHAVIOUR OF PARTIALLY PRESTRESSED CONCRETE STRUCTURES
UNDER FATIGUE LOADING

by

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ABSTRACT

This thesis describes a theoretical and experimental study of the behaviour of partially prestressed concrete flexural members under fatigue loading. The study deals with the progressive changes in beam response which occur with repeated loading, and with the prediction of fatigue life.

The experimental work consisted of constant-cycle and cumulative damage fatigue tests on 6 partially prestressed beams. Associated material tests on concrete and steel were also performed. A significant increase in beam deformation and deflection was observed at low loads as well as at high loads during repeated loading. The effect was most pronounced in the early load cycles, but diminished in the later cycles. Beam response in the first cycle was found to be inelastic with considerable hysteresis. The permanent residual deformations increased with repeated loading.

Strain measurements on the test beams indicated that deformations occurred primarily in the tensile region. The increased permanent deformations at low loads were considered to be caused by the non-closure of flexural cracks which resulted from the mis-match of crack surfaces and the formation of debris in cracks. At high loads, increased deformations were caused by deterioration in the concrete tensile stiffening effect, due to progressive cracking. In addition, cyclic creep of concrete and possibly also of the steels contributed to increased deformations.

A theoretical method of beam analysis has been developed, which takes these physical mechanisms into account. The stress dependent, non-linear material behaviour and time-dependent effects of creep and shrinkage are also considered. A 'regional' analysis is employed, whereby the overall beam response is modelled by a single equivalent section. Good prediction of beam deformations for the first and subsequent load cycles was obtained.

The fatigue life of the partially prestressed beams was found to be governed by failure of the reinforcing steel and prestressing strand. Fatigue failure of the concrete was not observed. Steel fractures in the reinforcement and tendon invariably occurred at primary concrete cracks. Beam fatigue life was estimated by evaluating steel stresses at critical sections and relating them to fatigue strength by material S-N curves. Fatigue calculations based on a simple no-tension analysis of cracked section stresses are considered to give good results. This procedure can be used to provide approximate fatigue life predictions.

Statement of Originality

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University and that, to my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text. I consent to the thesis being made available for photocopying and loan if accepted for the award of the degree.

~~Moo-Heck~~ Foo

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Principal Notations

$A_{i,p}$	Area of the i^{th} prestressing steel element
$A_{i,s}$	Area of the i^{th} reinforcing steel element
B	Total width of section
b_j	Width of the j^{th} concrete slice
C_c	Total concrete force in section
$C_{c,c}$	Total concrete compressive force in section
$C_{c,t}$	Total concrete tensile force in section
c_1	Level of $F_{e,q}$ at which the elastic recovery phase ends in the unloading direction
c_2	Level of $F_{e,q}$ where significant wedging commences
c_3	Parameter defining the effective stress where the cracks open for the N^{th} load cycle
c_n	Additional wedging force at the minimum load of the N^{th} cycle sustained through repeated loading
D	Total depth of section
$d_{i,p}$	Depth of the i^{th} prestressing tendon measured from the extreme top fibre
$d_{i,s}$	Depth of i^{th} steel element measured from the extreme top fibre
d_n	Depth of the strain neutral axis measured from the extreme top fibre
E_p	Tangent modulus for prestressing steel
E_s	Young's modulus for steel
E_t	Tangent modulus of concrete in tension
F_c	Ultimate concrete strength in compression
$F_{e,q}$	Concrete effective force in regional analysis
$F_{e,q,c}$	Concrete effective force at point C
$F_{e,q,d}$	Concrete effective force at point D
$F_{e,q,max}$	Maximum tensile force in concrete just before beam cracking
$F_{k,e,q}$	Concrete effective force in the k^{th} prism

$F_{r.e.}$	Equilibrating force required to maintain equilibrium
F_t	Ultimate concrete strength in tension
$f_{i.p.}$	Stress in the i^{th} prestressing steel element
$f_{i.s.}$	Stress in the i^{th} reinforcing steel element
$f_{i.p.m.}$	Measured stress in the i^{th} prestressing element with time-dependent effects
$f_{i.s.m.}$	Measured stress in the i^{th} reinforcing steel element with time-dependent effects
$f_{i.s.c.}$	Stress in the i^{th} compression steel element
$f_{i.s.t.}$	Stress in the i^{th} tension steel element
$f_{s,max}$	Maximum steel stress
f_y	Yield stress of steel
G_c	Function representing the concrete compression stress-strain relation
G_p	Function defining the tendon stress-strain relation
G_s	Function defining the ordinary steel stress-strain relation
G_t	Function representing the concrete tension stress-strain relation
$g(t)$	Experimentally obtained pure time creep function, increasing monotonically from 0 to 1
k_c & $k_{c.0}$	Local concrete strain transformation factors
k_s & $k_{s.0}$	Local reinforcing steel strain transformation factors
k_p & $k_{p.0}$	Local prestressing steel strain transformation factors
L	Crack spacing between primary flexural cracks
$l_{k.e.}$	Depth of the k^{th} prism measured from the top-fibre
$l_{r.e.}$	Line of action of the required force, $F_{r.e.}$ measured from top-fibre
M_c	Total concrete moment about the extreme top-fibre
$M_{c.c.}$	Moment for concrete compressive stress taken about the extreme top-fibre
$M_{c.r.}$	Initial cracking moment

M_{ct}	Moment for concrete tensile stress taken about the top concrete fibre
M_0	Moment at which the concrete effective force is zero
M_{dec}	Initial decompression moment
M_{ext}	Applied external moment
M_F	Moment at which the slope of F_s-M curve changes slope
M_p	Moment contribution of the prestressing steel taken about the extreme top fibre
M_{req}	Equilibrating moment required to achieve equilibrium
M_s	Moment contribution of the reinforcing steel taken about the extreme top fibre
M_{ult}	Ultimate flexural moment of a concrete member
m_0	Dimensionless parameter relating M_0 to M_{cr}
m_F	Dimensionless parameter relating M_F to M_{dec}
NA	Total number of concrete elemental slices in section
NE	Total number of concrete elemental prisms
NP	Total number of prestressing steel elements
NS	Total number of reinforcing steel elements
N_i	Predicted fatigue life at a sustained stress level
$N_{ipd},$ N_{imax}	Predicted lives at the predominant and maximum loads respectively in the i^{th} cycle partition
N_p	Predicted Life of beam
N_t	Total number of cycles sustained
n_{exp}	Total number of partitions
n_i	Number of cycles sustained at a certain stress level
$n_{ipd},$ n_{imax}	Number of cycles sustained at the predominant and maximum loads respectively in the i^{th} cycle partition
S_1	Parameter defining the initial stiffness of the effective force-strain relation for the N^{th} load cycle
S_2	Parameter defining the 'postcracking' stiffness of the relation for the N^{th} load cycle
T_p	Total force in prestressing steel elements
T_s	Total force in reinforcing steel elements

$T_{s,c}$	Total force in the compression reinforcing steel
$T_{s,t}$	Total force in the tension reinforcing steel
T_v	Time parameter of viscoelastic creep component
T_x	Total tension force in section
t	Time of loading in hours
t_j	Thickness of the j^{th} concrete slice
u_1	Parameter defining the stiffness of the elastic recovery unloading curve
u_2	Parameter defining the stiffness of the unloading slip recovery phase
u_3	Parameter defining the stiffness of the final unloading phase
w_e	Dimensionless parameter defining the maximum compressive force at zero moment after unloading
w_{eN}	Dimensionless parameter defining the maximum compressive force at zero moment at the N^{th} load cycle
w_f	Degree of tension stiffening at maximum load
y_{cr}	Distance from the stress neutral axis to the extreme top fibre
y_j	Depth of the centroid of the j^{th} concrete slice measured from the extreme top fibre
z	Concrete normalised elastic strain
z_{max}	Maximum concrete normalised strain
ϵ_0	Total concrete strain in extreme top fibre
$\epsilon_{0,cr}$	Concrete inelastic creep strain in extreme top fibre
$\epsilon_{0,e}$	Concrete elastic strain in extreme top fibre
$\epsilon_{0,etr}$	Concrete strain in extreme top fibre at transfer
$\epsilon_{0,in}$	Total inelastic concrete strain for extreme top fibre
$\epsilon_{0,sh}$	Concrete inelastic shrinkage strain in extreme top fibre
$\epsilon_{0,t}$	Total concrete strain in extreme top fibre at time t prior to loading
$\epsilon_{0,tm}$	Observed concrete strain in extreme top fibre at time t

ϵ_1	Total strain in concrete extreme bottom fibre
$\epsilon_{1 \text{ e t r}}$	Concrete elastic strain in extreme bottom fibre at transfer
$\epsilon_{1 \text{ t}}$	Total concrete strain in extreme bottom fibre at time t prior to loading
$\epsilon_{1 \text{ t m}}$	Observed concrete strain in extreme bottom-fibre
$\epsilon^d(t)$	Dischinger strain component at time t
ϵ_e	Elastic concrete strain
$\epsilon^e(t)$	Linear elastic strain component at time t
$\epsilon_{i \text{ c p}}$	Concrete strain at depth $d_{i \text{ p}}$ of the i^{th} prestressing steel element
$\epsilon_{i \text{ c p t}}$	Elastic strain shortening of concrete due to prestress at time t
$\epsilon_{i \text{ p}}$	Total strain in the i^{th} prestressing steel element
$\epsilon_{i \text{ p i}}$	Initial strain in the i^{th} prestressing steel element before transfer
$\epsilon_{i \text{ p n}}$	Cyclic creep strain in i^{th} prestressing steel element
$\epsilon_{i \text{ s}}$	Total strain in the i^{th} reinforcing steel element
$\epsilon_{i \text{ s n}}$	Cyclic creep strain in the i^{th} reinforcing steel element
$\epsilon_{j \text{ 0}}$	Average concrete strain at zero load before loading
ϵ_j	Total strain in the j^{th} concrete slice
$\epsilon_{j \text{ c r}}$	Long term creep strain in the j^{th} concrete slice
$\epsilon_{j \text{ e}}$	Instantaneous elastic strain in the j^{th} concrete slice
$\epsilon_{j \text{ i n}}$	Total inelastic strain in j^{th} concrete slice
$\epsilon_{j \text{ m}}$	Total concrete strain in the j^{th} slice obtained from the average strain distribution
$\epsilon_{j \text{ s h}}$	Long term shrinkage strain in the j^{th} concrete slice
$\epsilon_{j \text{ x}}$	Local concrete strain in the j^{th} slice at a section and horizontal distance x along the longitudinal axis
$\epsilon_{k \text{ e q}}$	Total strain in the k^{th} elemental prism
ϵ_m	Mean steel strain
ϵ_n	Cyclic creep strain
ϵ_p	Prestressing strain

ϵ_s	Steel strain
$\epsilon_{s,h}(t)$	Time dependent inelastic shrinkage strain
$\epsilon_{u,t}$	Concrete strain at maximum stress
$\epsilon^v(t)$	Viscoelastic strain component at time t
ϵ_y	Yield strain
$\epsilon^*_{s,h}$	Ultimate shrinkage strain at time infinity
σ_0	Concrete stress in extreme top fibre
$\sigma_{0,t}$	Concrete stress in extreme top fibre at time t prior to loading
$\sigma_{0,t,r}$	Concrete stress in extreme top fibre at transfer
σ_j	Concrete stress in the j^{th} slice
$\sigma_{j,av}$	Average concrete stress in the j^{th} slice
$\sigma_{j,e}$	Concrete stress in the j^{th} slice derived from experimental strain
σ_{max}	Maximum concrete stress sustained
σ_x	Average concrete stress in the section at position x and cross section area of A
$\Delta\epsilon_0$	Incremental local concrete strain in extreme top fibre
$\Delta\epsilon_1$	Incremental local concrete strain in extreme bottom fibre
$\Delta\epsilon_{s,c},$ $\Delta\epsilon_{s,t},$ $\Delta\epsilon_{s,p}$	Incremental local strains for the i^{th} element of the compression, tension and prestressing steel respectively
ΔF_{eq}	Increment concrete effective force
ϕ_*	Creep function at time infinity



CHAPTER 1

INTRODUCTION

1.1 Introduction

A concrete member is said to be partially prestressed if the initial precompression in the concrete is insufficient to prevent cracking in the service load range. Untensioned reinforcement is usually included to provide adequate strength.

Partial prestressing enjoys several advantages over the more traditional forms of concrete construction, ie. reinforced concrete and fully prestressed concrete. It has better deflection and cracking characteristics when compared with a reinforced member and yet is often more economical than a fully prestressed structure. The problems of large anchorage stresses and excessive camber associated with full prestressing are also reduced when partial prestressing methods are employed.

While the benefits of economy and improved structural performance are attractive in a partially prestressed structure, questions on its behaviour and strength under repeated loading have been raised in view of the significant cracking which occurs at design loads. The effect of opening and closing of cracks under many cycles of service load may be deleterious to fatigue strength. Less frequent overloads may also be harmful due to the high stresses involved. The possibility of fatigue failure is even greater in concrete offshore structures where the fatigue problem is compounded by corrosion and other effects.

1.2 Objectives and Scope

In this thesis, an investigation is made of the behaviour of partially prestressed concrete flexural members under static and repeated loading. The investigation consists of the following:

- a) a series of laboratory beam and material fatigue tests which provides data on first cycle and repeated load behaviour;
- b) a study of the factors influencing the behaviour of the test beams under loading, unloading and subsequent reloading conditions;
- c) development of a theoretical method of analysis to predict overall member behaviour and deformations under static and repeated loading, taking into account appropriate influencing factors such as the time-dependent effects of creep and shrinkage, prestressing losses, hysteretic and other inelastic effects in the concrete compression and tension zones;
- d) development of a method for estimating local stress cycles and hence fatigue life.

The thesis is concerned primarily with the repeated load behaviour of partially prestressed flexural beams. The experimental tests and theoretical analyses are limited to pre-tensioned beams. The behaviour of post-tensioned beams, although similar, is not considered here. Time-dependent effects of creep and shrinkage in the member prior to loading are taken into account in the beam analysis because of their significant influence on overall deflections as well as on local section stresses and strains, and hence fatigue life. However, a detailed theoretical treatment of these phenomena in cracked beams is beyond the scope of this thesis.

A detailed investigation of the fatigue properties of concrete and steel as individual materials is also outside the scope of this investigation. Beam fatigue behaviour under shear and torsion is not considered.

1.3 Layout and Contents of Thesis

A review of the static and fatigue behaviour of partially prestressed concrete members and their component materials is given in Chapter 2. The cyclic response and fatigue resistance of partially prestressed members are discussed.

Chapter 3 describes the experimental work, consisting of a series of flexural fatigue tests on partially prestressed beams together with relevant tests of the component materials. Test results are presented in this chapter, and also in detail in Appendix A.

The behaviour of partially prestressed members in the precracking range from the time of casting up to cracking is examined in Chapter 4. An evaluation of the early stress and strain distributions is required as the beam tests showed that initial conditions have a considerable influence on later beam response. A method of analysis is developed which traces the beam behaviour throughout the various stages of its early life, eg. immediately after transfer and in the subsequent time interval up to loading, and in the precracking load range.

Beam behaviour after first cracking, in the initial and subsequent load cycles, is studied in Chapter 5. Comparisons of test results are made with theoretical predictions from a simple no-tension theory. Other currently available methods of analysis are also investigated, and results from these are compared with experimental data for the first and subsequent loading and unloading cycles.

The physical factors influencing behaviour during loading, unloading and repeated loading are identified and studied in Chapter 6. An hypothesis is proposed to explain observed behaviour and a 'regional' analysis is developed, taking into consideration the influencing mechanisms, to provide accurate prediction of beam behaviour for first cycle and repeated loading. Comparisons between predicted and observed results

are made to support the analysis. A simplified analysis is then presented for design use.

The analysis of local beam behaviour and the calculation of beam fatigue life are dealt with in Chapter 7. A brief survey on studies of the variation of steel stresses along a member is made. Emphasis is placed on the accurate evaluation of local steel stresses at cracked sections for fatigue predictions. Several methods of calculating local steel stresses are examined and the results are used to determine beam fatigue life. Currently available methods for calculating fatigue life are also examined.

Chapter 8 contains conclusions and recommendations for further work.

Detailed fatigue test results of individual beams are given in Appendix A, while brief descriptions of the computer programs written for beam analysis, and flowcharts are presented in Appendix B. The Fortran program code for these are documented in microfiche in Appendix C.

CHAPTER TWO

LITERATURE REVIEW

This chapter reviews research into the fatigue behaviour of partially prestressed concrete members and their component materials. The static and fatigue properties of the materials are first discussed. The second section reviews, in greater detail, the behaviour of partially prestressed concrete structures under repeated loading. The cyclic response and fatigue resistance of beams are considered. The third section examines available methods of estimating fatigue life. To allow a more coherent presentation, analytical methods of predicting progressive beam response under first cycle and repeated loading are discussed separately in Chapter 5.

2.1 BEHAVIOUR OF COMPONENT MATERIALS

The response of structural members under both first and repeated loads is dependent upon the behaviour of the individual component materials and their interaction. In partially prestressed members, the materials comprise concrete, reinforcing steel and prestressing tendon. The behaviour of each of these components is considered in turn. A comprehensive review of material behaviour is outside the scope of this thesis, and only matters relevant to later parts of this thesis are discussed.

2.1.1 Behaviour of Plain Concrete under Static Loading

Concrete in Compression

Attention is restricted to the behaviour of plain concrete under uniaxial loading. The instantaneous response of concrete

is most commonly described by a non-linear stress-strain relation as shown in Fig 2.1. Non-linearity is attributed mainly to the progressive propagation of internal microcracks. The stress-strain relation is dependent on the type of cement, the mix proportions, the rate of loading, prior load history, test conditions and environment and presence of lateral reinforcement.

Various expressions have been proposed for the stress-strain curve, some of the earlier being reviewed by Popovics (1970). More recent expressions have been published by Carreira and Chu (1985), Ahmad and Shah (1979), Kaar et al (1978), Sangha and Dhir (1972).

Holmen (1982) recommended a power relation between stress and strain,

$$S = 1 - \left(1 - \frac{\epsilon_c}{3}\right)^{2.5} \quad (2.1)$$

where S is the normalised stress, ie $S = \sigma/\sigma_u$, σ_u is the ultimate concrete stress and ϵ_c is the longitudinal strain.

Balaguru (1980) developed a general expression for strains up to 0.4 percent,

$$f_c = A_1 \sin \left\{ \frac{\pi \epsilon}{.007} \right\} + A_2 \sin \left\{ \frac{\pi \epsilon}{.0035} \right\} + A_3 \sin \left\{ \frac{\pi \epsilon}{.00233} \right\} \quad (2.2)$$

where

$$\begin{aligned} A_1 &= 7.15 \sin[\pi(f_c' - 3.1)/13.1] \\ A_2 &= 12 - 10.3 \sin[\pi(14.5 - f_c')/17] \\ A_3 &= 5.3 \sin[\pi(f_c' - 1.85)/16] - 5 \end{aligned}$$

Wang, Shah and Naaman (1978) found the following expression, originally proposed by Sargin (1971), to be most suitable:

$$y = \frac{ax + bx^2}{1 + cx + dx^2} \quad (2.3)$$

where x and y are the respective normalised strain and stress

variables with respect to the maximum values, ϵ_u and σ_u , and a, b, c and d are constants which define the ascending and descending portions of the stress-strain relation.

Desayi and Krishnan (1964) proposed a simpler expression:

$$y = \frac{ax}{1 + x^2} \quad (2.4)$$

where $a = 2 = E_c \epsilon_0 / f_0$ and E_c is the secant modulus of elasticity.

Even though many stress-strain equations have been proposed, variations in the cement, aggregates and mix proportions, rate of loading, combined with the inherent variability of concrete suggests the need for experimental verification in any specific application. In view of this, concrete stress-strain relations derived from this author's material tests are used in the analytic work in later parts of this thesis.

Concrete in Tension

The tensile strength of concrete is much less than its compressive value - being approximately a tenth of the ultimate compressive value. Experimental evaluation of the tensile strength is prone to scatter and this is exacerbated by the fact that measurements can be made from three different test methods, namely direct tension, splitting tension and flexure. Raphael (1984) examined 12,000 individual test results to explain discrepancies among results from the three types of tests. A theoretical relationship was found between tensile strength and the modulus of rupture, as defined in a flexure beam test. The modulus of rupture was related to the compressive strength by:

$$f_r = 0.95 F_c^{2/3} \quad \text{in kg/cm} \quad (2.5)$$

With comparatively little tensile strength, concrete has commonly been regarded as brittle in tension, with a linear stress-strain relation. However, Hughes and Chapman (1966), using extremely stiff testing machines, produced a

stress-strain relation with strain softening properties. A typical relation is shown in Fig 2.2. Madu (1975) produced similar curves with reinforced concrete members. Reinhardt (1984) carried out tensile tests on concrete in the strain softening range, under static and repeated load conditions. Gopalaratnam and Shah (1985) reported tensile strain-softening tests on seven different mix proportions. Theoretical equations for concrete tensile strain-softening were developed by many researchers including Bazant (1984) and Gilbert and Warner (1978) for use in analytic prediction methods. These are described in detail in Section 5.2.2.

Creep and Shrinkage in Concrete

Recent reviews on creep and shrinkage were given by Bakoss (1985), Kilpatrick (1984), Neville et al (1983), and Bazant and Wittmann (1982). A detailed review of the phenomena of creep and shrinkage of concrete is outside the scope of this thesis. Generally, creep is defined as the time-dependent deformation of concrete under sustained loads, while shrinkage is considered as a time-dependent mechanism caused primarily by the loss of water.

The development of creep strains is dependent on the applied load, relative humidity, temperature, concrete strength, age, volume and type of aggregate, water-cement ratio and cement type, size and type of member.

Concrete shrinkage is caused by the evaporation of absorbed water into the atmosphere and also by carbonation. Factors influencing shrinkage include the water-cement ratio, concrete mix, chemical admixtures, aggregate content, member size and shape, and the surrounding environment.

A specific procedure for calculating creep and shrinkage strains in concrete with a time-varying stress history is discussed further in Chapter 4, where it is used to predict flexural behaviour.

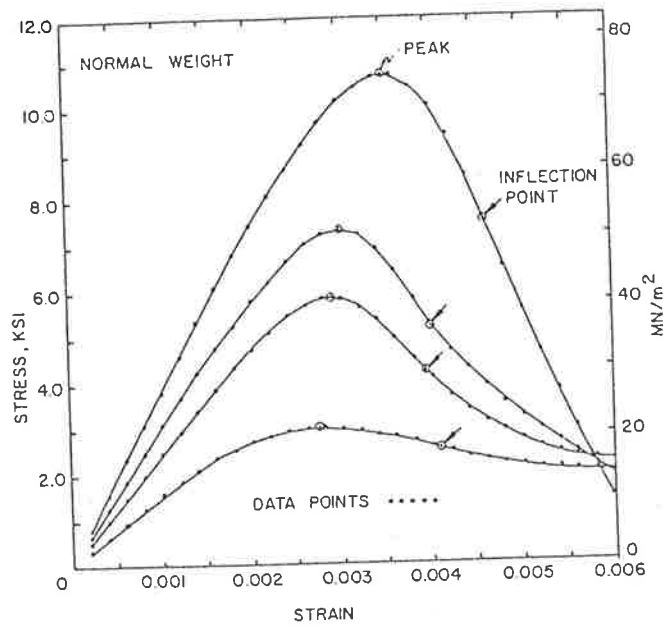


Fig. 2.1 Typical Concrete Compressive Stress-Strain curve (Wang et al, 1978)

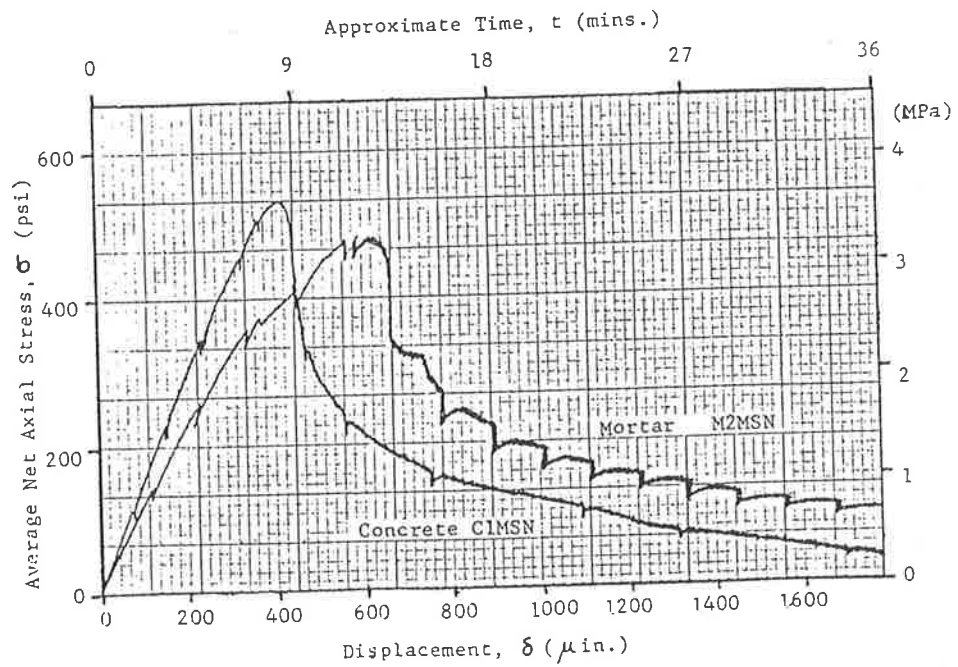


Fig. 2.2 Typical Concrete Tensile Stress Displacement curve (Gopalaratnam & Shah, 1985)

2.1.2 Fatigue Behaviour of Concrete

Extensive research has been conducted on the behaviour of plain concrete subjected to repeated loading. A comprehensive review of early work was reported by Nordby (1958) while a more recent review of flexural fatigue behaviour of plain concrete was published by Raithby (1979).

Fatigue Properties of Concrete in Compression

Concrete is a notch-insensitive material with an endurance limit for cyclic loading ranging from about 40 to 60 percent of the ultimate static strength. At lower loads, the damage induced by repeated loading is insignificant. The fatigue strength of concrete for a life of 10 million cycles, according to ACI Committee 215 (1974), is approximately 55 percent of the static strength in compression, or in tension, or in bending. The main factors affecting fatigue strength are:

- a) applied stress range
- b) loading history
- c) rate of loading
- d) stress gradient

Each of these factors will be dealt with briefly.

Stress Range

Karsan and Jirsa (1969) tested 46 short rectangular columns under different cyclic loads. Empirical stress-strain-cycle relations were obtained from the tests. It was noted that the envelope curve for the three-parameter relations coincided with the normal static stress-strain curve. This is shown in Fig 2.3. Their results also show a fatigue limit of approximately 63 percent of the ultimate static strength.

Aas-Jokobsen's investigation (1970) of the fatigue of beams and columns showed by means of a modified Goodman diagram the existence of a relationship between the maximum, minimum

non-dimensionalised stresses and the fatigue life for concrete subjected to repeated loading. Tepfers and Kutti (1979) in a later paper verified this relationship with results obtained from their own tests as well as data from published literature. The relationship is as follows:

$$\frac{f_c}{f_{c'}} = 1 - 0.0685(1-R) \log_{10} N \quad (2.6)$$

where $R = \frac{f_{min}}{f_{max}}$

A typical S-N curve for concrete is shown in Fig 2.4 below, which is reproduced from Hanson (1983).

Hsu (1981) in a recent paper proposed a four-parameter (stress-strain-cycle-time) relationship for fatigue. He recommended that time was an important variable to be considered as concrete strength is both time and rate-dependent. Two separate sets of equations were derived, one for high cycle fatigue and another for low cycle fatigue.

Load History

Hilsdorf and Kesler (1966) carried out one of the first variable-amplitude load tests to study the applicability of the Palmgren-Miner cumulative damage rule in concrete fatigue calculations. It was determined that short rest durations were helpful in prolonging the fatigue life and that low-level repeated loads below the endurance limit also improved fatigue resistance. Miners' rule gave better correlation for cumulative damage loadings where the differences between the stress ranges were small and gave either conservative or unsafe estimates otherwise.

Holmen (1982) tested 480 concrete specimens in compression in his study of fatigue under constant and variable amplitude loading. Due to the inherent scatter of fatigue results and variability of concrete properties Holmen applied statistical methods to establish stress-cycle-probability relations from his test data. The secant modulus was found to

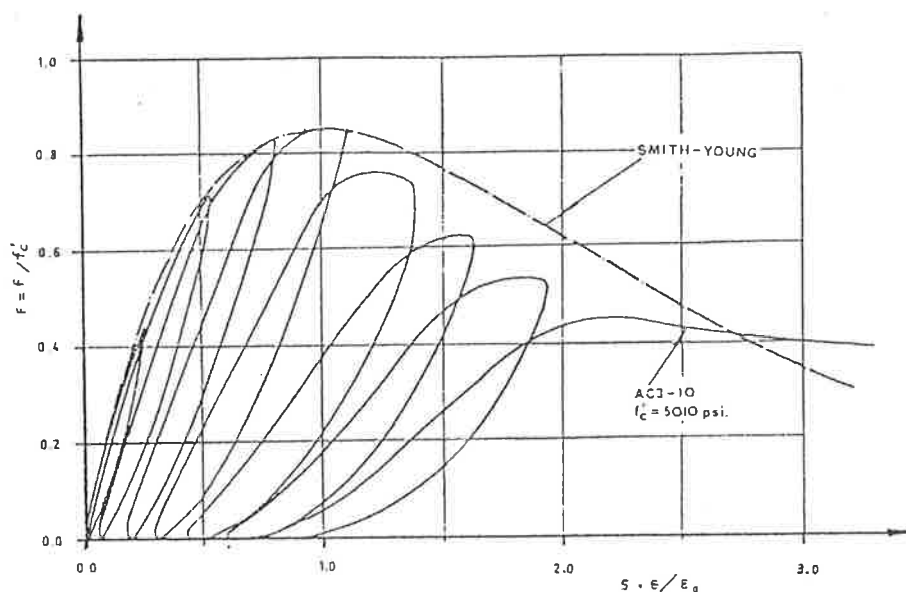


Fig. 2.3 Concrete Stress-Strain Envelope in Compression (Karsan & Jirsa, 1969)

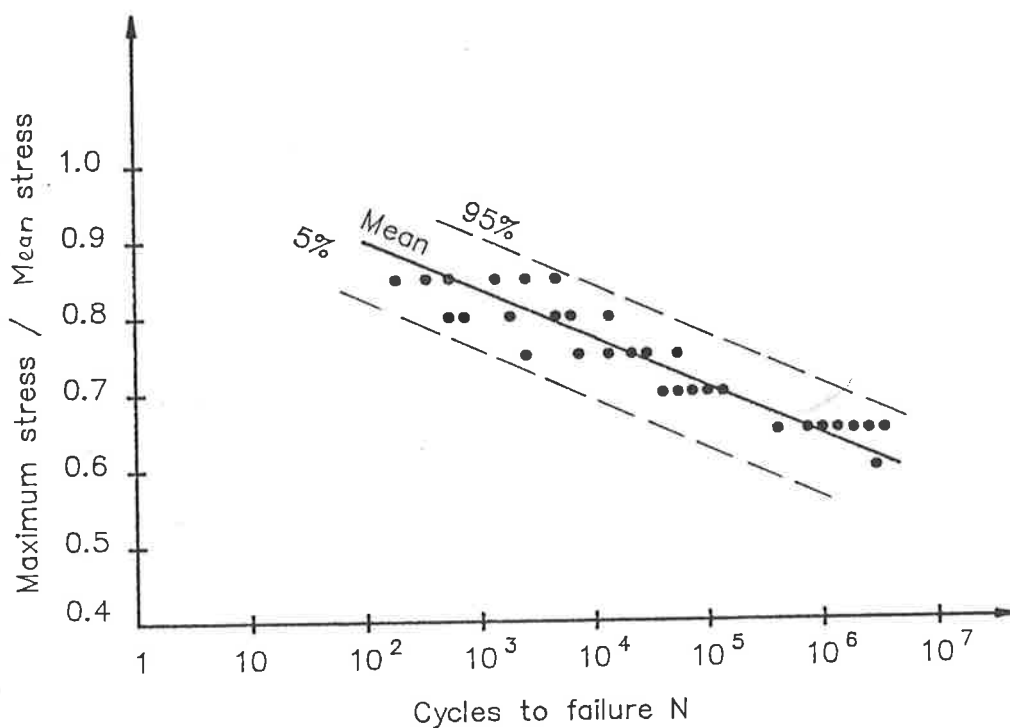


Fig. 2.4 Concrete S-N curve in Compression (Hanson, 1983)

decrease with repeated loading in both constant and variable amplitude loading.

The influence of cyclic load history on the modulus of elasticity is an important point for consideration in fatigue analysis. Bennett and Raju (1969) reported that Young's modulus for concrete was basically unchanged for applied loads less than the fatigue limit. However, Probst (1925) found that the modulus decreased with repeated loading. Whaley and Neville (1973) on the other hand reported that strength as well as modulus increased. Cook and Chindaprasirt (1980) in their work on concrete, mortar and paste found that cyclic loading decreased both strength and stiffness of plain concrete.

Rate of Loading

It is generally believed that the loading rate for ranges between 70 and 900 cycles per minute has no influence on the fatigue strength provided the maximum stress level is less than 75 percent of the static ultimate strength. At higher stress levels a reduction in fatigue resistance has been observed. This was reported by Murdoch (1965) who based his conclusions on published data. Sparks and Menzies (1973) on the other hand reported conflicting results. It was found that fatigue resistance was enhanced by increased loading rates.

Stress Gradient

Ople and Hulsbos (1966) investigated the effect of compressive stress gradient on the fatigue life of plain concrete. Static and fatigue tests were conducted on 157 prismatic specimens. The main test variables studied were stress gradient and maximum stress level. The stress gradient was found to have a significant effect on fatigue behaviour. The fatigue strength of non-uniformly stressed specimens was approximately 15 to 18 percent higher than for uniformly stressed prisms. An approximate, empirical relationship between stress level, fatigue life, probability of failure and stress gradient was established.

Concrete Compressive Cyclic Response

Past studies of the fatigue of plain concrete have revealed that cyclic loading accelerates the non-elastic deformation of concrete. This inelastic deformation is commonly known as cyclic or dynamic creep. A typical stress-strain curve at various cycles of repeated loading is shown in Fig 2.5, which is reproduced from Holmen (1982).

Whaley and Neville (1973) conducted fatigue tests on concrete specimens to study behaviour under cyclic compression. The tests verified that repeated loads accelerated deformations which were largely irrecoverable. Concrete subjected to cyclic loading sustained larger deformations than those loaded under an equivalent mean stress. They suggested that if the total deformation due to repeated loading could be considered as being analogous to creep, then the resulting creep could be expressed as the sum of a static component, resulting from creep under a mean stress, and a cyclic component, resulting from a stress range, ie

$$\epsilon_n = 129 \sigma_m (1 + 3.87\Delta)t^{1/3} \quad (2.7)$$

where ϵ_n is the cyclic creep, σ_m is the mean stress expressed as a fraction of the static strength at the start of loading, Δ the stress range and t the time of loading in hours.

In later investigations, Neville and Hirst (1978) investigated the fundamental mechanism of cyclic creep. Again repeated load tests on concrete specimens were conducted with the main variables being applied mean stress, stress range and environmental conditions. The experimental results confirmed the validity of Whaley and Neville's equation. The cause of cyclic creep was attributed to two factors: increased movement of gel particles due to the breakdown of absorbed water layers and, increased microcracking at the aggregate-matrix interface.

Haugland et al (1976) proposed a different set of expressions

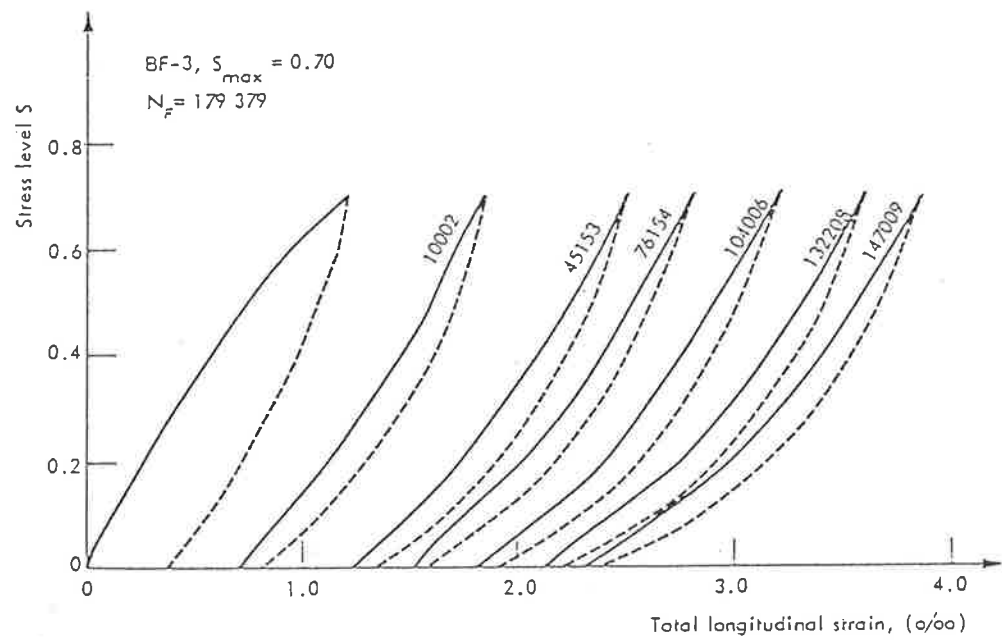


Fig. 2.5 Concrete Compressive Stress-Strain curves under Repeated Loading (Holmen, 1982)

for concrete deformations under repeated loading. It was reported that deformation increases were highest initially and then stabilised subsequently until just before fatigue failure of the concrete, when large deformations preceded failure. Two equations were proposed, one for low stress-range calculations and the other for higher stress ranges.

Holmen (1982) tested 462 cylinders in his investigation of the effects of load history on fatigue behaviour. The variation of concrete strain with repeated load has been shown above in Fig 2.5. Three distinct phases of response were observed. Rapid increases were recorded in the beginning and end stages of fatigue life, with a steady period in the middle. A set of equations for the concrete strain was proposed, with the total strain separated into two components, ϵ_e the 'endurance' component and ϵ_t the time dependent strain (creep) component:

$$\epsilon_{max} = \epsilon_e + \epsilon_t \quad (2.8)$$

$$\text{For } 0 < \frac{N}{N_f} \leq 0.10$$

$$\epsilon_e = \frac{1}{\tan \alpha} \left\{ S_{max} + 3.180(1.183 - S_{max}) * \left[\frac{N}{N_f} \right]^{0.2} \right\} \quad (2.9)$$

$$\epsilon_t = 0.413 S_c^{1.184} \ln(t+1) \quad (2.10)$$

$$\text{For } 0.10 < \frac{N}{N_f} \leq 0.80$$

$$\epsilon_e = \frac{1.11}{\tan \alpha} \left\{ 1. + 0.677 * \left(\frac{N}{N_f} \right) \right\} \quad (2.11)$$

$$\epsilon_t = 0.413 S_c^{1.184} \ln(t+1) \quad (2.12)$$

where ϵ_{max} is the maximum total strain, ϵ_e is the maximum total strain in the first load cycle, S_{max} is the maximum stress level, S_m is the mean stress, N is the number of load cycles, N_f is the number of load cycles to failure, t is duration of alternating load in hrs, RMS is the root mean square value and $\tan \alpha = S_{max} / \epsilon_e$ and $S_c = S_m + \text{RMS}$

Concrete under Tensile Repeated Loading

Recent investigations on the tensile fatigue properties of concrete were made by Oh (1986), Saito and Imai (1983) and Tepfers (1979). In Saito's direct tensile tests the minimum applied stress level was 8 percent of the ultimate strength and the maximum stress level was between 75 and 87.5 percent. The S-N curve for 50 percent probability was found to be

$$S = -4.12 \log N + 98.73 \quad (2.13)$$

where S is the maximum stress level, and N the number of cycles sustained. A typical S-N curve is reproduced in Fig 2.6. The stress-strain curves at different stages of loading are shown in Fig 2.7. Saito reported that the relations were approximately linear, with little change in slope with repeated load cycles. However, irrecoverable strains were accumulated with increasing load cycles and hysteresis loops were present.

2.1.3 Reinforcing Steel

Static Behaviour

Hot rolled high yield ductile reinforcing steel (Tempcore) has recently been introduced in Australia as the main type of reinforcement. As it was used in the construction of the test beams described in Chapter 3, we shall be concerned primarily with the properties of this relatively new material. A typical stress-strain curve for the steel obtained from this author's material tests is shown in Fig 2.8a.

Behaviour under Fatigue Loading

The behaviour of reinforcing steel under repeated loads has been well researched. A recent review has been presented by Tilly (1979). Other earlier reviews were reported by Helgason et al (1969), Menzies (1971) and the ACI Committee 215 (1974). Experimental investigations have included in-air tests as well

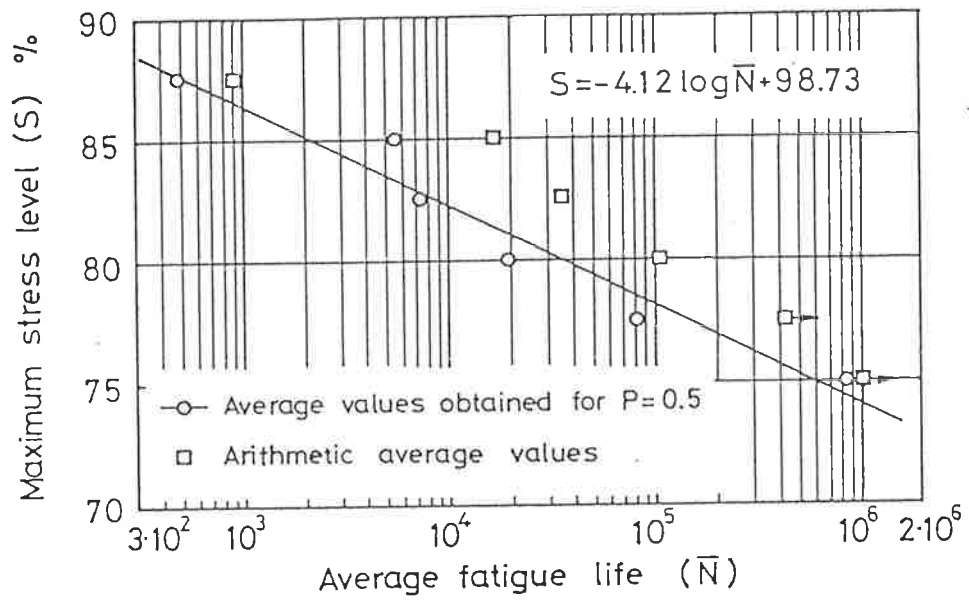


Fig. 2.6 S-N curve for Concrete in Tension (Saito & Imai, 1983)

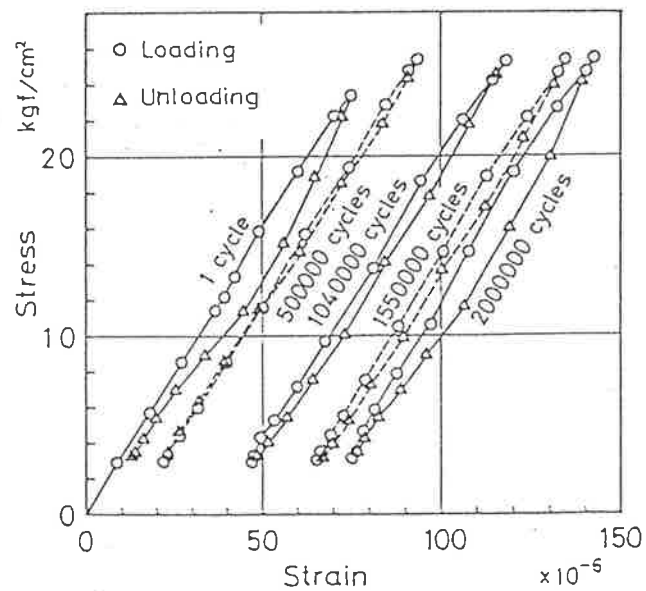


Fig. 2.7 Concrete Tensile Stress-Strain curve under Repeated Loading (Saito & Imai, 1983)

as beam tests. Most in-air tests were direct tension tests while concrete-embedded specimens have been tested in direct tension and in bending. Early beam tests were recorded by Ros (1950), Rehm (1960) and Soretz (1965). Wascheidt (1965) conducted extensive investigations on both in-air and embedded specimens. Recent in-air and embedded tests have been carried out by Tilly and Moss (1982). Nurnberger (1982) also investigated factors influencing the behaviour of embedded bars.

Fatigue behaviour is dependent on the chemical composition, material microstructure, inclusions, manufacturing process and other variables. Generally, the performance of different types of bars is approximately similar (Tilly, 1979). Physical properties, as opposed to chemical properties, also have to be considered, and these are:

- a) Stress range, minimum and maximum stresses
- b) Bar size and geometry of deformations
- c) Yield and ultimate tensile strength
- d) Presence of stress raisers, welding, bending etc
- e) Friction or fretting between concrete and steel at cracks
- f) Temperature
- g) Corrosion

Hanson, Somes and Helgason (1974) carried out a comprehensive test program involving 236 reinforced concrete beams under repeated flexural loading to investigate the effect of stress range, minimum stress level, bar diameter, specimen type and grade of steel on fatigue life. A statistical analysis was carried out and the results reported in a separate paper by Helgason and Hanson (1974). Stress range was found to be the predominant variable affecting fatigue strength. The minimum stress level was also found to be 'statistically' significant with increases in minimum stresses causing reductions in strength.

While extensive fatigue research has been conducted on steels manufactured by the classical processes such as hot rolling

and cold forming, little information is available for Tempcore steel. In fact, the in-air tests of Behan and Warner (1984) provided the only source of fatigue data on Tempcore bars. The S-N curve for 12mm bars is shown in Fig 2.8b. The steel was tested in a range from 10 percent compression to 80 percent tension. The mean curve is defined by the equation:

$$\log_{10} N = 6.1 - 1.4 \frac{R_{11.5}}{40} \quad (2.14)$$

$$\text{where } R_{11.5} = S_{max} - S_{11.5} \quad (2.15)$$

and the endurance limit $S_{11.5}$ is estimated from extrapolation of the experimental data to be:

$$S_{11.5} = 48 - 0.005 S_{min} + 0.75 S_{min} \quad (2.16)$$

where

$$S_{max} = \frac{f_{max}}{f_{ult}}$$

$$S_{min} = \frac{f_{min}}{f_{ult}}$$

In-Air versus Concrete-Embedded Tests

There is uncertainty as to whether in-air or embedded bars have higher fatigue strength. Rehm (1960) in his studies, found that embedded bars showed lower strength than naked specimens. However, Soretz (1965) found that hot-rolled bars in concrete beams exhibited higher strength.

Other studies indicated little difference between the two. A comprehensive test program was conducted by Wascheidt (1965) to investigate fatigue properties of in-air and concrete-embedded reinforcing steels. It was reported that for ribbed and twisted bars, the in-air fatigue strength was marginally less than the embedded strength. Wascheidt contended that the variance was so small that in effect, there was no difference between them. However, results of plain round bars showed a 17 percent reduction in the fatigue resistance of embedded specimens in comparison to naked bars. MacGregor (1971) reported similar results for deformed bars.

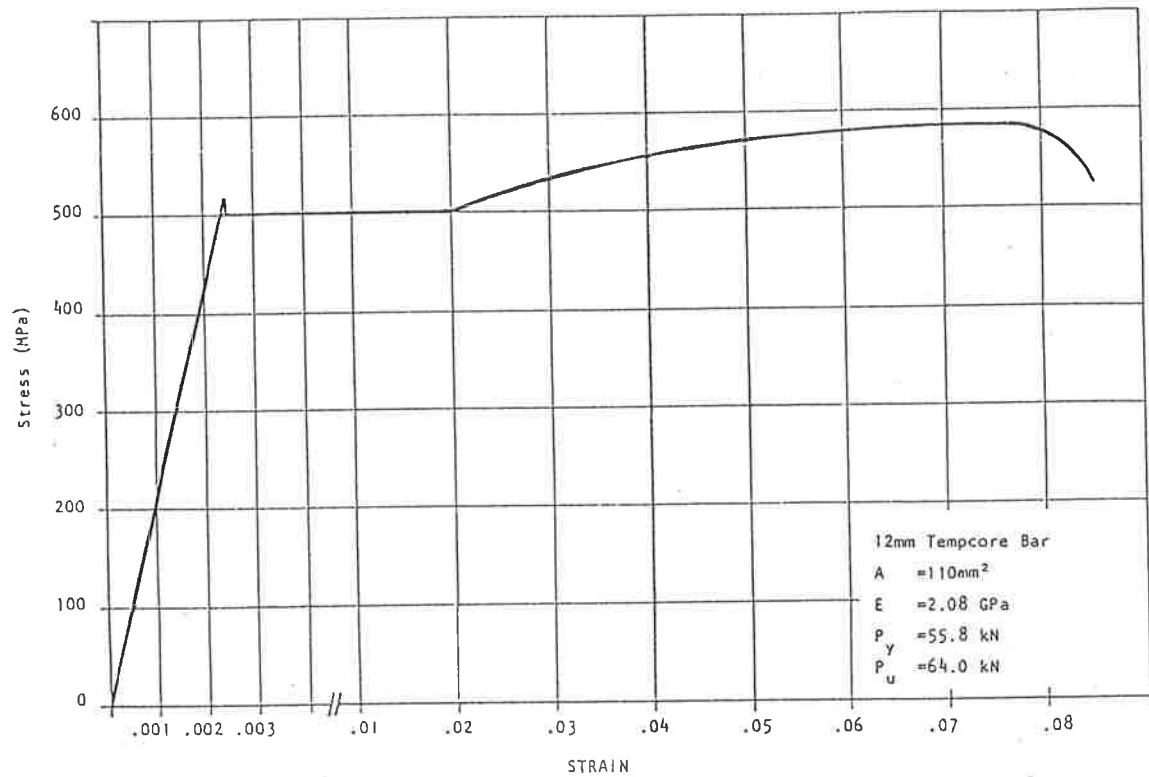


Fig. 2.8a Stress-Strain curve for 12mm Tempcore Reinforcing Steel

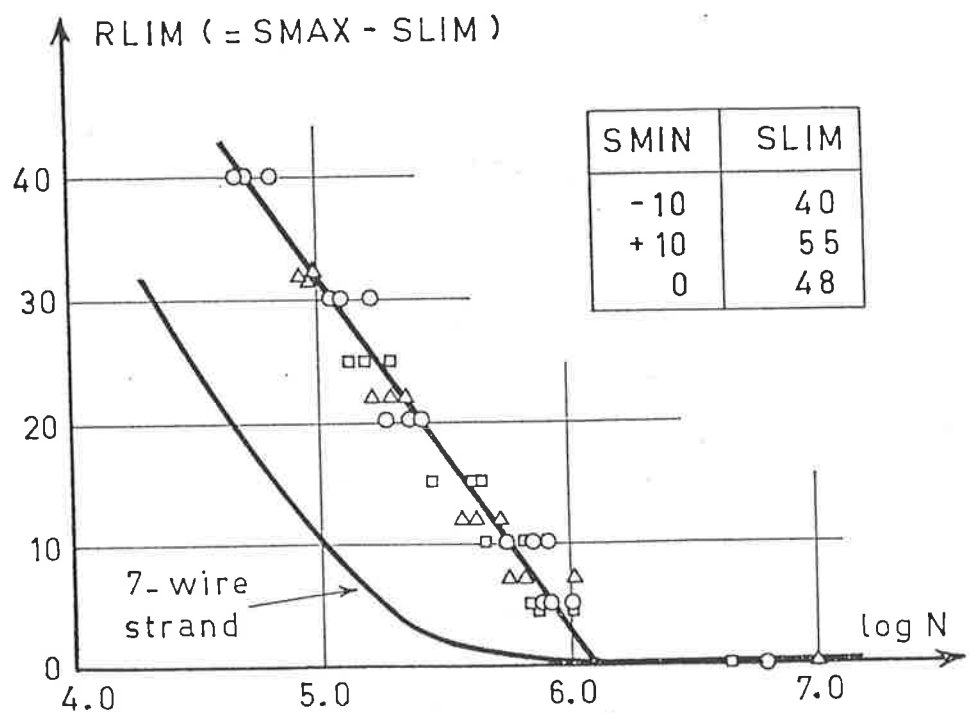


Fig. 2.8b S-N curve for 12mm Tempcore Reinforcing Steel (Behan & Warner, 1984)

Nagai, Yamagata and Karatsu (1982) from their tests on 6 beam-column joints reported no significant difference between the fatigue properties of in-air and concrete-encased bars. It was found that naked bars were marginally superior to embedded specimens. However, because of the small number of tests, no conclusive deductions were made.

Tilly and Moss (1982) also made comparisons between naked and embedded specimens. For the embedded bars, testing was carried out in bending under a 4 point load arrangement with a single bar as the main reinforcement. It was reported that the axial tests exhibited lower fatigue life. A minimum stress range of 200 MPa was required to cause fracture compared with 260 MPa for bending. Approximately 20 percent higher stresses were required to produce failure for the axial specimens. Tilly reported that fractures of the axial bars originated from imperfections in the surface whereas those of the bending tests occurred near the ribs. This is in conflict with in-air tests by Behan and Warner (1984) who reported that fatigue cracks typically initiate at the ribs.

One major problem encountered in comparing in-air and embedded results is in estimating steel stresses. In the case of bending tests, steel stresses vary along the beam and the precise evaluation of local stresses is virtually impossible. Comparisons of steel fatigue properties are thus meaningless if there is a large degree of uncertainty in the reference stresses.

2.1.4 Prestressing Steel

Static Behaviour

A description of the various types and properties of high tensile steel used for prestressing is found in a FIP report (1976). Prestressing steels are classified into wire, strand and bars. The main factors affecting fatigue performance are relaxation, creep, corrosion and temperature. The static

stress-strain curve is linear except near yield. A typical curve obtained from laboratory tests is shown in Fig 2.9.

Behaviour under Fatigue Loading

The factors influencing reinforcing bars generally apply also to prestressing steel. An early review of fatigue results for strand was presented by Baus and Brenneisen (1968). Factors influencing fatigue strength include the condition of prestressing steel, type of steel profile, fretting effects between steel and duct, degree of bond, type of anchorage and the size of wire in strand.

Tide and VanHorn (1966) in their statistical investigation of 156 strand specimens reported that fatigue strength was slightly greater at lower temperature (0 degrees F) than at room temperature (70 degrees F). An equation predicting mean fatigue life was proposed:

$$\log N_s = 8.213 - 0.02077 S_r + 0.00316 S_r^2 \quad (2.17)$$

where S_r is the stress range. The equation is only valid for

$$40\% \leq S_{min} \leq 60\%$$

$$12\% \leq S_r \leq 30\%$$

Warner and Hulsbos (1966) derived equations for the probable fatigue life of US made prestressing strand from 150 in-air tests. In a later publication, Warner (1982), found good correlation of test data for both wire and strand manufactured in Australia and in the United States. A relation between stress increment and the number of load cycles was given:

$$\log_{10} N = \frac{1.169}{R} + 5.227 - 0.031 R \quad (2.18)$$

$$\text{where } R = S_{max} - S_{min} \quad (2.19)$$

and S_{min} is defined as the fatigue limit of which values are found in Table 2.1 below. The S-N curve is shown in Fig 2.10

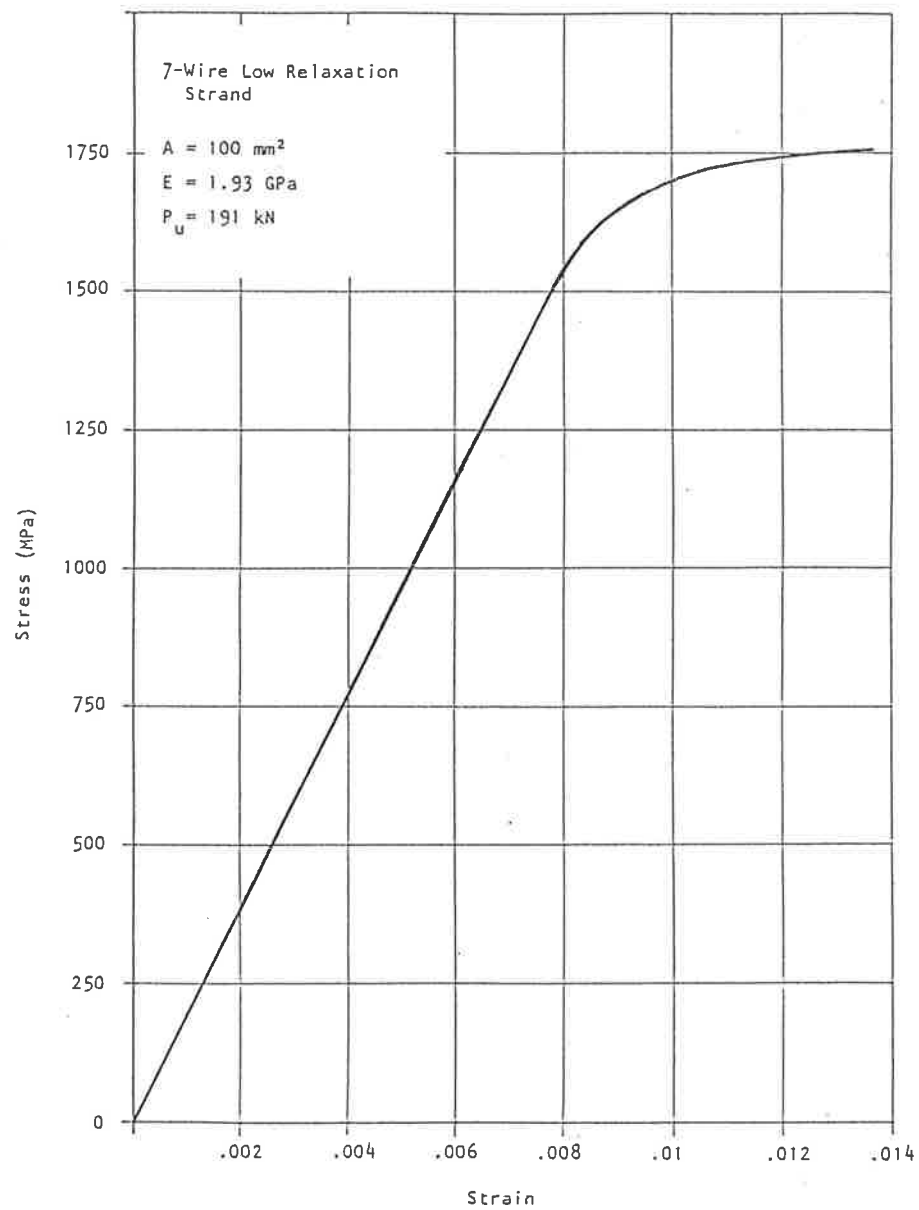


Fig. 2.9 Stress-Strain curve for 12.7mm Prestressing Strand

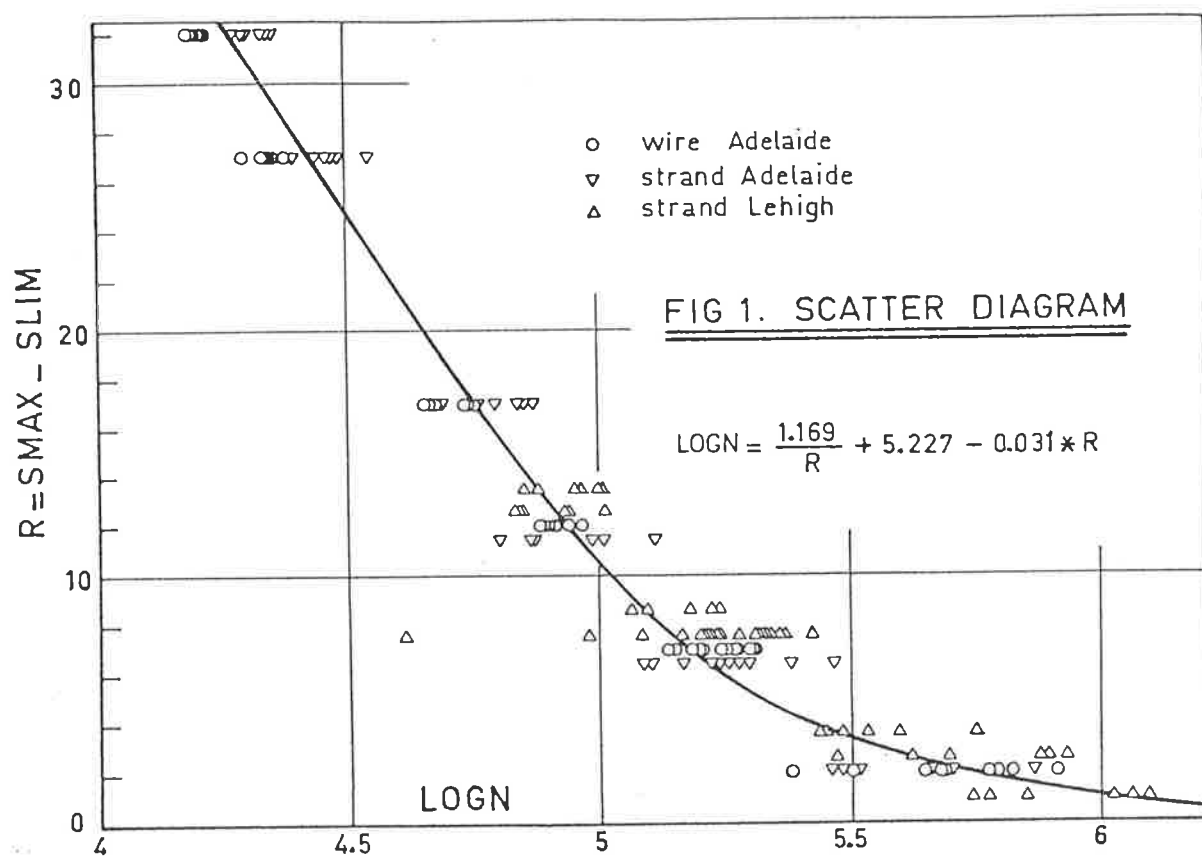


Fig. 2.10 S-N curve for Prestressing Strand
 (Warner, 1982)

Table 2.1 Fatigue Limits for Different Minimum Stress Levels

Minimum Stress Level %	Fatigue Limit %
40	53
60	73

Naaman (1982) derived a relationship from North American and European strand tests:

$$\frac{f_r}{f_{pu}} = -0.123 \log_{10} N + 0.87 \quad (2.20)$$

where f_r is the stress range and f_{pu} is the ultimate prestressing strand stress.

2.2 FATIGUE BEHAVIOUR OF CONCRETE STRUCTURAL MEMBERS

Fatigue distress in concrete members may result from excessive flexural, shear or bond stresses or a combination of these. However, the scope of this investigation is limited to flexural fatigue. Partial prestressing is relatively new compared with the two other types of concrete construction and not a great deal of research has been conducted on fatigue in partially prestressed flexural members. This review has therefore been extended to cover briefly major contributions on fatigue in simply reinforced and fully prestressed concrete structures.

Only publications concerning repeated loading in the service load ranges are considered. The structural behaviour under overload or earthquake conditions (eg. reported by Sinha et al, 1964; Agrawal et al, 1965; Burns et al, 1966; Ruiz and Winter, 1969; Thompson and Park, 1980; Hawkins, 1977) is outside the scope of this thesis. However, considerable work has been conducted on the fatigue strength evaluation of offshore structures (Waagaard, 1982; Taylor and Sharp, 1978; Gerwick, 1981; Price et al, 1982) which is relevant to the present review.

Major Review Papers

Earlier investigations on fatigue of structural concrete members were reviewed by Rowe (1957), Nordby (1958) and Hawkins (1964). Later recommendations for fatigue design in concrete structures are contained in a report by ACI Committee 215 (1974). More recent reviews have been presented in the IABSE Colloquium on the Fatigue of Steel and Concrete Structures (1982) and the ACI Symposium on the Fatigue of Concrete Structures (1982). A general view of fatigue failures of concrete structures was reported by Lenschow (1982). Separate papers on fatigue-related studies of the ASCE and ACI were also presented by Galambos (1982) and Hawkins and Shah (1982). A brief overview of the phenomenon of fatigue has also been given by Hanson (1983).

2.2.1 Repeated Load Behaviour of Flexural Members

Reinforced Concrete Members

The occurrence of residual deformation in reinforced concrete structures in fatigue loading was one of the first observations obtained from experimental testing. Nordby reported that Berry (1908) observed increases in permanent set with repeated loads. Most of the set occurred in the first few thousand repetitions and increased with load and number of cycles. Van Ornum (1907) observed similar increases in deflections in his comprehensive test program involving 59 beams. He attributed the increase to the gradual destruction of bond between concrete and steel.

In later tests, Russell and Webber (1967) reported up to 60 percent increases in maximum deflections for reinforced concrete beams sustaining 2 million load cycles. The load range was from half to full design working load. It was observed that repeated loading increases the beam stiffness throughout the whole load range. Similar increases in stiffness were also reported by Saliger (1935), Bate (1963)

and Nordby et al (1957).

Sparks and Menzies (1973) also reported significant deflection increases in their testing of 20 reinforced concrete beams. The objective of their investigations was to determine the influence of load fluctuations on deflections. An empirical linear logarithmic relationship was found between the increase in maximum deflections and the number of repeated load cycles or the length of time under load. The increases in deflections under cyclic loading were about the same as for beams subjected to sustained load equal in magnitude to the maximum of the fluctuation:

$$\Delta = A N^b \quad (2.21)$$

where Δ is the increase in deflection, N the number of load cycles sustained and A and b constants defined empirically for each test beam.

Loo and Wong (1985 and 1984) reported similar increases in deflection in 30 reinforced concrete box beams tested under repeated loading. Five partially prestressed box beams were also tested. An effective stiffness approach was used to predict beam deflections and the authors reported good predictions for both reinforced and prestressed beams.

Prestressed Concrete Members

Increased deformations and permanent set were also observed for fully prestressed structural members. Venuti (1965) reported the presence of permanent set after the end of the first load cycle. In contrast with Russell and Webber's findings, flexural stiffness was found to decrease with repeated loading, albeit at a diminishing rate. Similar decreases have been reported by Rowe (1957), Hawkins (1964), Ople and Hulsbos (1966) and Harajli and Naaman (1985). Slight reductions were reported by Sawko and Saha (1968). Warner and Hulsbos (1966) also observed considerable changes in beam response in the initial ten percent of the beam life. A more

stable response was observed for the remaining life. The behavioural changes were attributed to creep effects, changes in the material stress-strain relations and progressive bond failure in the concrete tension zone.

An investigation into the causes of increased deformation and varying member stiffness was carried out by Sawko and Saha (1968). Nine prestressed concrete beams were tested. Hysteresis was observed in the load deflection curve for the first load cycle. With repeated loading these effects diminished rapidly. The authors suggested that fatigue hardening of the compressive zone and softening of the tension zone were the main contributors to increased deformation. An equivalent section with a larger transformed area at the top fibres and a smaller area at the bottom fibres (shown in Fig 2.11) was recommended for use in theoretical predictions.

However, deformations in prestressed structures only become significant when the applied load was sufficiently high to cause cracking of the concrete. Ozell and Ardaman (1956) observed that when a beam was loaded for a million cycles without exceeding the cracking level, no apparent ill effect was sustained. Increases in deformations were observed when repetitive loads greater than the cracking load were applied.

Partially Prestressed Members

Partially prestressed members have also been found to sustain residual deformation with repeated loading. Considerable increases were reported by Abeles et al (1968 and 1974) who tested 6 fully prestressed beams and 52 under-reinforced prestressed beams in two separate programs. A typical load-deflection diagram for various load cycles is shown in Fig 2.12. Increases in maximum deflections with repeated loads can be as high as 100 percent.

Kulkarni and Ng (1979) tested 8 beams with varying amounts of prestress to investigate the flexural and serviceability behaviour of limited prestressed beams under repeated loads. Increases in deformations with repeated loading were observed.

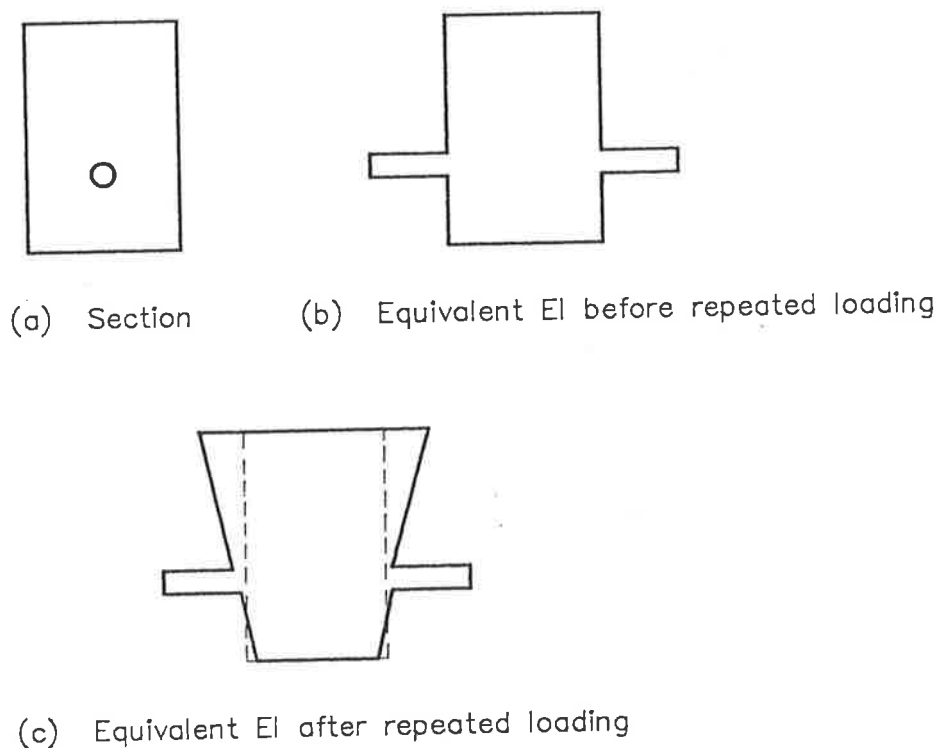


Fig. 2.11 Equivalent EI's for Repeated Loading (Sawko & Saha, 1968)

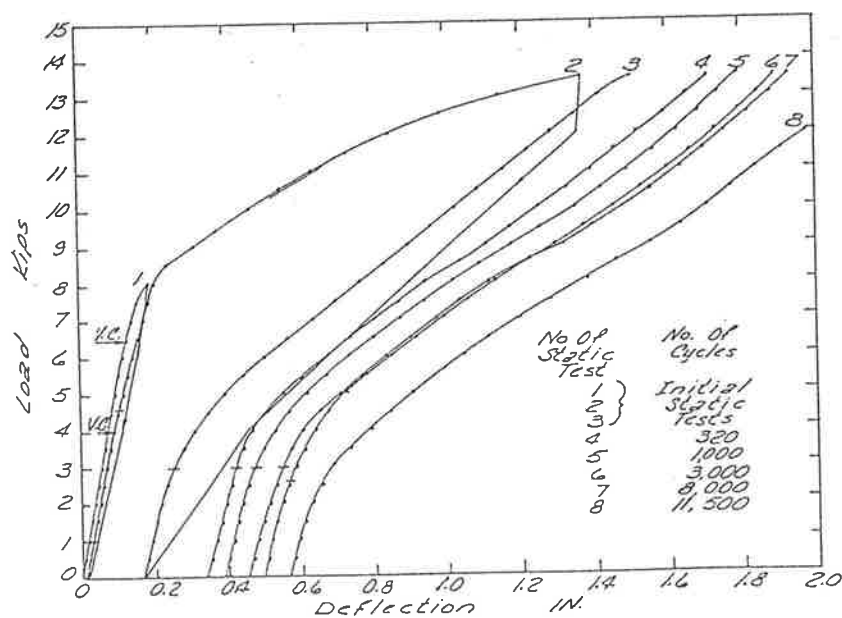


Fig. 2.12 Load Deflection Diagram under Repeated Loading (Abeles et al, 1974)

Widening and deepening of cracks were attributed to destruction of bond. A majority of the total deformation was sustained during the first million load cycles for beams which were loaded up to a maximum value of about 50 percent of ultimate.

Mansur (1982) tested 12 partially prestressed beams to investigate the influence of load history on beam behaviour and ultimate strength. Beam deformations and crack widths were found to increase with repeated loading, especially during the initial 33,000 cycles. No fatigue fractures were encountered and repeated loading stopped when the beams sustained over 100,000 load cycles.

Warner (1982) attributed the progressive changes in beam behaviour to shrinkage and static creep under minimum moment condition, dynamic creep under cyclic moment and permanent deformation caused by repeated loading. It was suggested that deformations resulting from cyclic loading could be caused by non-closure of flexural cracks and cyclic steel softening in addition to the generally accepted mechanisms of cyclic creep, loss of tension stiffening and progressive cracking of concrete in the tensile zone with bond breakdown. An analytic method of determining the stress cycle histories taking the above factors into consideration was called for.

Naaman (1982) suggested that concrete cracking and bond deterioration with accompanying slip between steel and concrete at cracks were the main causes of increased cracked widths and deflections. Slip in the prestressing was significant as it would cause loss of prestress and therefore increased deflections. This was also suggested by Abeles, Brown and Hu (1974). The influence of slip was later discounted by Gylltoft (1979) in his studies on bond properties of strand.

Recent tests by Harajli and Naaman (1985) (reported after the experimental work described in this thesis had been completed), also showed increases in beam strains. These were attributed to the effects of cyclic creep. The loss of

prestress due to slip of the prestressing strand was discounted as a possible cause as the authors measured no appreciable strand slip at the ends of the beams.

Summary

The reported behaviour of concrete structures under repeated loads is characterised by the following observations:

- a) hysteresis and residual deformations are present at the end of the first cycle;
- b) maximum and residual deformations increase with repeated loading;
- c) flexural stiffness may increase or decrease with repeated loading.

The increased deformation is probably more pronounced in reinforced and partially prestressed concrete structures, where cracks open under working loads. Similar conditions exist in fully prestressed members only if the loads are high enough to cause cracking. A major portion of the total deformation is sustained in the early loading stages. The structural behaviour of a member progressively changes during the initial stages but settles down to a consistent response for the rest of the member life. There is controversy as to whether member stiffness decreases or increases with repeated loading.

Cyclic creep of concrete, bond deterioration with accompanying slip, strain softening of steel, and loss of prestress have been suggested as the main factors influencing repeated load behaviour.

Analytic methods have been proposed for predicting the increase in deformation with repeated loading. These are discussed in detail in Chapter 5.

2.2.2 Fatigue Resistance of Flexural Members

Factors Influencing Fatigue Failure

Most of the early work conducted before 1958 on fatigue of reinforced concrete structures has been reviewed by Nordby (1958). Van Ornum (1907), Probst (1932), Saliger (1935), Lea (1940) and LeCamus (1946) all made significant contributions with extensive experimental work on reinforced concrete members. Abeles (1952 and 1954), Ozell and Ardaman (1956), Nordby and Venuti (1957) and Ekberg et al (1957) worked on prestressed structures.

In summarising past work Nordby reported that fatigue failure invariably occurred by fracture of the reinforcing steel, fatigue failure in the compressive concrete was non-existent except when it was specifically designed to do so (LeCamus, 1946). Prestressed beams seldom fail by fatigue of the concrete. Nordby observed that members which were subjected to cyclic loads of 60-70 percent of the static flexural strength failed after a million load cycles.

The majority of fatigue failures occur either in the reinforcing bars or prestressing strand in the vicinity of the cracks. Nordby noted that the failures seemed to be connected with severe cracking of the concrete and abrasion at the cracks. He reported that in Van Ornum's (1907) reinforced concrete beams, steel failure occurred at the location of tension or diagonal tension cracks.

The mode of failure was found to be also influenced to a certain extent by the level of loading. Chang and Kesler (1958) reported two different modes of failure from a set of 25 small reinforced concrete beams which were designed to fail in flexure under static loading. Fatigue failures in the reinforcement and also in concrete were reported. Low load cycles were observed to result in fracture of the steel whilst high load cycles resulted in shear failure.

This was supported by Venuti's (1965) tests on 90 nominally

identical pretensioned beams. Beams which were loaded at 50 percent of the ultimate capacity generally survived 5 million repeated cycles. Fatigue fracture of the prestressing strand occurred in beams loaded up to 60-70 percent of ultimate and for loads of 80-90 percent, failure occurred in the concrete compression zone. In one of the earliest extensive test programs on partially prestressed beams, Abeles observed brittle steel fractures when the load range was 30 to 80 percent of ultimate. Concrete crushing was observed when maximum loads were taken past 85 percent where yielding of steel may have taken place.

Stelson and Cernica (1958) on the other hand reported shear failure in all 11 test beams regardless of load range. However, their beams were intentionally overdesigned to ensure concrete failure. Beam fatigue predictions based on the fatigue properties of concrete cylinders under direct compression under-estimated beam life. This was attributed to the beneficial effect of stress gradient in the concrete stress block.

Loading parameters were found to influence fatigue behaviour in a theoretical parametric study by Rao and Babu (1979). Geometric and material properties such as cross-section details, steel percentage and concrete strength were considered along with loading parameters such as the 'fluctuating-load-to-permanent-load-ratio, working load level and final prestressing level'. An iterative, non-linear method based on equilibrium of sections was used to compute stresses and strains. Material and section properties were found to have insignificant influences. Loading parameters on the other hand were found to have positive influences.

Temperature effects did not seem to have much influence on the fatigue strength. Russell and Webber (1967) fatigue tested beams at sub-zero (-5 degrees Celsius) temperatures and found that the results were not significantly different from those tested at room temperature. Welding of rebars decreased resistance with rebars welded to stirrups showing significant reduction in fatigue strength. All fractures occurred at the

toe of the welds. Corrosion in the steel, and the effects of pore pressure and water pumping in and out of cracks (Waagaard, 1982) are additional factors which need to be considered in offshore structures.

Concrete Cracking and Steel Fatigue

Earlier tests carried out in the working load ranges showed that fully prestressed concrete beams were capable of sustaining large numbers of design loads without jeopardising serviceability or strength. It was only when applied cyclic loading was increased into the post cracking range above service load levels, that fatigue distress was observed.

One of the early investigations was carried out by Ozell and Ardaman (1956) who tested 8 fully prestressed beams under three point loading. When the applied loading was below the cracking moment, no apparent fatigue damage was observed with the beam sustaining more than 3 million cycles, but fracture in the strands was observed for beams loaded beyond cracking.

Bate (1963) reported that where beams are loaded in the normal working range with no cracking, little increase in deformation and no apparent fatigue distress were observed. When the beams were subjected to loads greater than cracking, significantly larger increases were recorded.

Hanson, Hulsbos and Vanhorn (1970) conducted tests on 6 fully prestressed pretensioned I-beams with load levels varying from approximately 20 to 55 percent of ultimate. Flexural cracks were observed to open with repeated loading and fatigue fractures were concentrated at the cracked sections. The flexural beam fatigue life was found to be less than that predicted from data for prestressing strands tested in air. Abrasion between concrete and steel, and significantly higher values of steel stresses at cracks were suggested to be the main causes.

Fifteen post-tensioned T-beams were tested by Rigon and Thurlimann (1985) in their investigation of factors

influencing steel fatigue. Steel fractures were observed in the vicinity of cracks and these occurred at a significantly lower life than predicted by in-air strand test data. It was suggested that fretting between the steel ducts and strands, and also between adjacent wires in strands were the main contributing factors to the strength decrease. This was confirmed by an improvement in the fatigue resistance when plastic ducts were used in later beams.

The influence of the presence of concrete tensile stresses in prestressed members under design load on fatigue resistance was also demonstrated by Rabbat, Kaar, Russell and Bruce (1979). Six full-sized standard pretensioned bridge girders were tested under static and fatigue loads. Fatigue fracture of the strands occurred in beams where tensile stresses were allowed in the concrete fibres even though the calculated stress range in the first load cycle was below the strand fatigue limit of 90 MPa.

These observations are supported by Dhondy (1982) who tested 2 full sized pretensioned precast twin-tee guideway girders. The 85-tonne girders were initially precracked with 60 percent overload and then subjected to 6 million cycles of repeated service loading under combined bending, shear and torsion. No fatigue fracture, deterioration of girder stiffness or crack propagation was observed. The evidently good resistance to fatigue was attributed to the 'zero service tension' design method. No distress was also reported by Magura and Hognestad (1966) for partially prestressed concrete girders where cracking did not take place. Concrete tensile stresses on the other hand, may cause fatigue problems as reported by Rabbat et al (1979).

Failure Modes in Partially Prestressed Members

Partially prestressed structures have been considered to be more prone to fatigue distress (Naaman, 1982) than either reinforced or fully prestressed members. This is because cracks in partially prestressed structures open and close during repeated loading. The stress ranges sustained under

such conditions were thought by Naaman to be larger than for comparable reinforced structures where the working load is already in the postcracking range or in fully prestressed structures where cracking is not even allowed to take place. Warner (1982) also suggested that fatigue resistance is a relevant design consideration for partially prestressed structures where significant cracking takes place. ^{Fatigue in} A fully prestressed structures, on the other hand, are not a practical design problem if no cracking takes place.

Stevens (1969) observed significantly larger changes in the reinforcing steel stress levels in his partially prestressed T-beams compared with those of simply reinforced beams. He concluded that fatigue could thus be critical in partial prestressing. Walther (1972), in a theoretical survey, reported that fatigue resistance of partially prestressed members was just as good as fully prestressed ones but that fatigue of the normal reinforcing steel may be a limiting factor.

Bennett and Dave (1969) found the fatigue resistance of 9 partially prestressed beams to be good when loaded with a moment range of half design load and maximum moment equal to the design level. All except one survived 3 million load cycles. Fatigue failure in the exception occurred in the prestressing strand. In later investigations on 'Class 3' post tensioned beams, Dave and Garwood (1975) reported similar results from tests using similar loading configurations.

In some cases the fatigue resistance of partially prestressed structures may be even better than reinforced or fully prestressed members. Harajli and Naaman (1982) tested 6 partially prestressed, 3 fully prestressed and 3 simply reinforced beams under constant amplitude loading between 40 and 60 percent of ultimate. Only the fully prestressed beams failed in fatigue, the others survived more than 5 million load cycles.

Fatigue failure can occur either in the reinforcing or prestressing steel. Fauchart, Kavyrchine and Trinh (1973)

recorded fractures in both reinforcing and prestressing steel in their fatigue tests on partially prestressed rectangular and T-beams. The prestressing steel level was varied to give different stress ranges for the reinforcing steel. Reinforcing steel which failed by fatigue fracture recorded high stress ranges which span both compression and tension regions.

Bennett and Joynes (1979) tested 14 partially prestressed beams whose normal reinforcement consisted of 10mm cold-worked deformed bars (Torbar). Fatigue failure occurred mainly in the untensioned steel. Abeles (1974) on the other hand only found brittle fractures in the prestressed steel, none were found in the untensioned strand. However, it must be noted that prestressing strand was used as untensioned steel in his beams.

Summary

Previous research has shown that fatigue failure of concrete structures generally occurs by steel fracture. Fatigue failure of the compressive concrete is extremely unlikely, except when especially designed for in a laboratory test.

The technical literature reveals a definite relationship between steel fatigue fracture and concrete cracking. Prestressed concrete members seldom experience fatigue distress unless they are loaded into the postcracking range. Beam cracking appears to lower overall fatigue resistance, resulting in lower fatigue lives compared with the theoretical value obtained from in-air strand test data. Abrasion between concrete and steel seems to be the main cause of premature failures.

In spite of this, no clear conclusions have been obtained concerning the superiority in fatigue resistance of one method of concrete construction over another. Partially prestressed members were considered to be more prone to fatigue compared with either fully prestressed or simply reinforced concrete members. However, this has yet to be confirmed.

Fatigue failure in partially prestressed members can occur in either steel type. Although there are a fair number of reports in the technical literature on the repeated loading of partially prestressed structures, not many tests have actually been continued until fatigue failure. For this reason it is difficult to conclude whether the prestressing or reinforcing steel govern in fatigue considerations. Failures in both steels have been reported. The situation is even less clear when wires or prestressing strands are used as non-tensioned steel in such structures.

2.3 METHODS OF PREDICTING FATIGUE LIFE

The fatigue strength of a concrete structure is to a large extent dependent on the resistance of its individual components. Fatigue life prediction therefore basically involves two essential operations, firstly estimation of the stress cycles sustained by the individual components at critical sections in the structural member, and secondly applying these stresses to appropriate S-N curves to obtain fatigue life estimates of each individual component material. If fretting and abrasion are considered to be important, the S-N curves need to be obtained from appropriate in-concrete tests, rather than in-air tests. The overall fatigue strength of the structure is then simply the lowest value of the individual concrete and steel lives calculated.

Precise evaluation of the moment-stress relation is a major obstacle in fatigue predictions, and therefore the majority of published prediction techniques are concerned with the accurate calculation of stress-cycles in critical sections.

2.3.1 Walls, Sanders and Munse

Walls, Sanders and Munse (1965) performed fatigue tests on 23 beams reinforced with single-V-butt welded reinforcement. Steel stresses in the beams were calculated using the straight line formula (a crude no-tension analysis - see Fig 2.13a)

analysis and comparisons made between experimental (using electric strain gauges stuck on bars) and calculated results. These showed that the measured strains were less than the theoretical values and this was attributed to tension stiffening effects. Theoretical calculations thus resulted in conservative estimates.

In order to obtain better estimates, the authors suggested a simple empirical two-staged procedure to obtain better stress ranges. A relationship between maximum steel stress and moment taking into account tension stiffening was first derived. This is shown in Fig 2.13b. The stress range is then obtained from a relationship (shown in Fig 2.13c) between maximum stress and stress ratio (defined as ratio of minimum stress to maximum stress) which was experimentally derived.

Though the method is approximate and the empirical relationships apply only to specific beams, this work is notable because it attempts to make a more realistic estimate of steel stresses for fatigue calculations.

2.3.2 Abeles, Brown and Hu

In their investigations on fatigue resistance of partially prestressed concrete beams, Abeles, Brown and Hu (1974) reported that the loss of prestress during fatigue loading may have a significant effect on the steel stresses. They suggested that repeated loading was accompanied by creep, bond deterioration and slip of the tendons, resulting in prestress losses. An increased stress range resulting in lowered fatigue strength was therefore envisaged. Cumulative damage calculations were recommended in view of the changing stress levels even though the test was carried out under constant amplitude loading.

To take into account creep and concrete stiffness changes during loading, a semi-empirical method was used to determine the steel stresses. The decompression moment at whatever load cycle was determined experimentally (by eye or load-strain

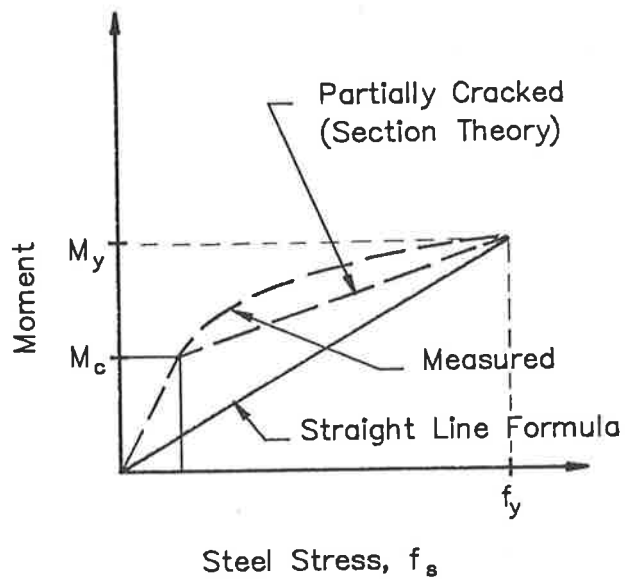


Fig. 2.13a Moment-Stress Relation (Walls et al, 1965)

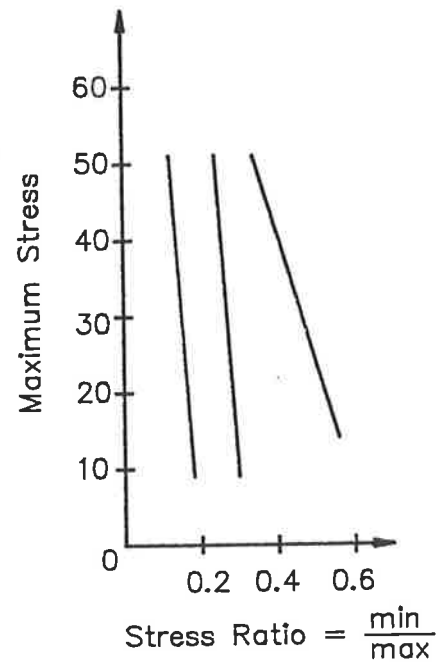


Fig. 2.13c Empirical Max Stress-Strain Range Relation

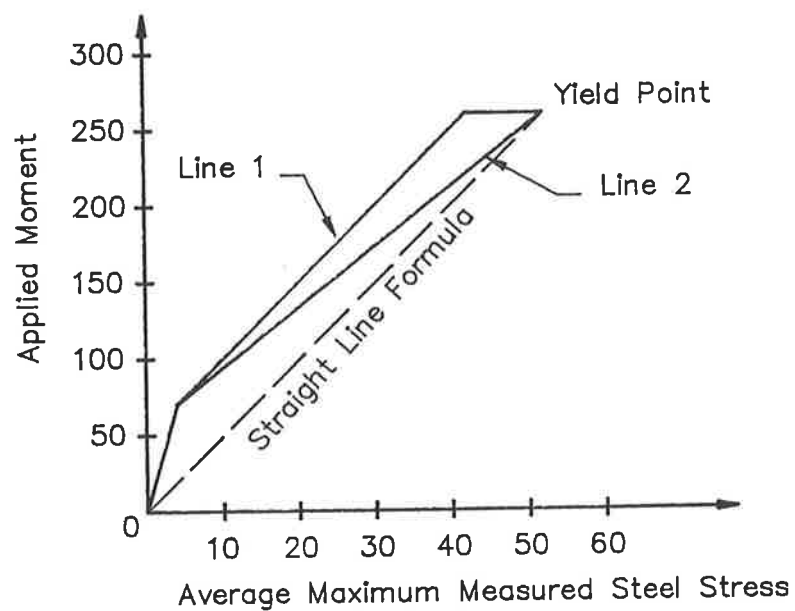


Fig. 2.13b Relationship between Applied Moment and Average Maximum Measured Steel Stress

measurements). The neutral axis was calculated using simple elastic theory with measurements of the bottom fibre strains. The location of the concrete compressive stress block was assumed and the steel stresses at decompression calculated as follows:

Prestressing steel stress at decompression:

$$f_{p,e} = \frac{M_o - \{A_{s,t} f_{s,t,e} (d_{s,t} - h/3)\} - \{A_{s,c} f_{s,c,e} (d_{s,c} - h/3)\}}{A_p (d_p - h/3)} \quad (2.22)$$

where M_o is the decompression moment, $f_{s,t,e}$ and $f_{s,c,e}$ are the untensioned tensile and compressive steel stresses, $A_{s,t}$ and $A_{s,c}$ are their respective areas. The depth of the prestressing and reinforcing steels are d_p , $d_{s,t}$ and $d_{s,c}$ respectively.

Prestressing and untensioned steel stresses at other loads were calculated using a measured neutral axis position, ie.

$$f_p = f_{p,e} + \Delta f_p$$

where

$$\Delta f_p = \frac{(d_p - ka) \{M - A_p f_{p,e} (d_p - ka) - A_{s,t} f_{s,t,e} (d_{s,t} - ka)\}}{A_p (d_p - ka)^2 + A_{s,t} (d_{s,t} - ka)^2} \quad (2.23)$$

Three different methods of calculating fatigue resistance were employed. The simplest method used the calculated stresses in the first load cycle to predict fatigue life. This was found to overestimate fatigue lives, with the observed lives approximately 88 percent of that predicted. A second method used mean stress values. The calculated stresses of the first and last load cycles were averaged and this was used in the computations. Slightly better agreement was noted with the observed strength approximately 91 percent of that calculated. In a third method, the authors calculated the theoretical resistance at various cycles of the fatigue loading. However, no mention was made of any attempt to use Miner's cumulative damage rule for the calculation of overall beam fatigue life.

Linear-elastic behaviour was assumed. Increased deformations were attributed to loss of prestress due to creep and strand slip, resulting in lowered steel stresses with repeated

loading. However, most experimental results (Warner, 1982; Venuti, 1965) indicated increased deformations and stresses with repeated loading. Slip of the prestressing steel (Haralji et al, 1985; Naaman, 1982; Gylltoft, 1979) was found to be negligible. The behaviour of prestressed members under fatigue loading can not be explained merely by the loss of prestress. Nevertheless, this approach in estimating fatigue life is interesting as it attempted to take into consideration some of the initial inelastic effects, and more importantly, recognised the possibility of varying steel stresses in a member subjected to constant amplitude loading.

2.3.3 Ekberg, Walther and Slutter

Abeles et al (1974) emphasised the importance of assessing steel stresses accurately at various stages of fatigue loading. However, it was apparently not realised that there might be significant differences between stresses obtained from average strain or deformation measurements and local stresses in the cracked section. Ekberg, Walther and Slutter (1957) were among the earliest to take this into consideration in their fatigue predictions. Calculation of the stresses by the simple cracked section analysis leads to error due to the invalidation of the assumptions used. Incremental strain measurements on the beam surface were used to describe the concrete stress block and hence locate the neutral axis position. The lever arm between the concrete and steel forces is then computed and knowing the applied moment, the total steel force and hence steel stresses can be calculated simply by:

$$T = \frac{M}{l} \quad (2.24)$$

where T is the total tensile steel force, M the applied moment and l the lever arm derived from strain measurements.

The fatigue strength is then checked by comparing the failure envelope (in the form of modified Goodman diagrams) of the component materials of concrete and steel with the sustained

stresses at the cracked sections.

Ekberg et al's method is notable in that it attempted to evaluate critical stresses at the cracked sections. The main criticism of this approach is its dependence on empirical measurements of strain. Another is that it does not take into account variations in stresses with repeated loading.

2.3.4 Bennett and others

Bennett (1974) further developed Ekberg's method to evaluate tensioned and non-tensioned reinforcement stresses in partially prestressed structures. The author assumed constant stress-cycles under repeated loading after 100,000 load cycles. However, it is unclear whether his calculations were made for the first or 100,000th load cycle. The theoretical results were then compared with published test data. Although the author concluded that 'the calculated range of stress appears to be a suitable criterion of fatigue failure, when compared with the modified Goodman diagram', his comparisons were vague and no definite conclusions could be derived.

Bennett and Veerasubramanian (1972) reported that the simple linear cracked section analysis produced results which were in good agreement with those obtained from Ekberg's method. Bennett and Joynes (1977) in another theoretical study analysed the stress ranges in normal and prestressed steel for various partially prestressed beams using a simple no-tension method and found that fatigue failure was more likely to occur in the ordinary reinforcement than in the tendons.

The simple cracked section analysis was reported to produce results comparable to those of Ekberg's. If this were so, then the proposed method of stress calculation is only as good as as the simple no-tension analysis which does not take into consideration the progressive changes in beam deformations with repeated loading. Bennett's method is thus considered to be inadequate for fatigue life predictions.

2.3.5 Venuti

Venuti (1965) was one of the earlier researchers to take into account the inherent variability of fatigue. A total of 90 nominally identical pretensioned beams were tested. The minimum load level was 10 percent and the maximum load varied from 50 to 80 percent of ultimate. Statistical means were used to derive a life-load-probability (N-R-P) relationship based on the normal distribution function. An empirical equation was derived to predict the beam fatigue life with a 50 percent probability of failure.

$$\log_{10} N = 1.143 - 19.060 \log_{10} R \quad (2.25)$$

where N is the predicted value of the fatigue life and R is maximum load expressed as a ratio of the ultimate strength.

The usefulness of this prediction equation is questionable as it is only applicable to this particular set of beams.

2.3.6 Warner and Hulsbos

Warner and Hulsbos (1966) proposed a probability based non-linear method of predicting fatigue life of prestressed concrete beams. A compatibility factor Ψ , which is dependent on the bond behaviour between concrete and steel, was used to account for the strain incompatibility at the cracked sections. The steel strain at the cracked section for a moment greater than the cracking value was expressed as:

$$\epsilon_{s,1} = \epsilon_{s,F} + \epsilon_{c,F} + \frac{1-k}{k} \epsilon_{c,1} \Psi \quad (2.26)$$

where $\epsilon_{c,F}$ is the elastic strain in concrete at the steel level due to prestress, $\epsilon_{s,F}$ is the steel strain due to prestress, $\epsilon_{c,1}$ is the concrete strain in extreme top fibre and k is the dimensionless factor defining the depth to the neutral axis at the cracked section.

The stress and strain conditions in the cracked section were calculated by considering compatibility and equilibrium conditions. The authors recommended a value of 1.00 for ψ at the mid-life stage of repeated loading. In stress calculations for fatigue, the authors assumed a constant stress cycle under constant cycle loading. The strand fatigue life was determined from material S-N-P curves, which were statistically derived from in-air strand tests.

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^X \exp \left\{ -\frac{(x-\mu)^2}{2\sigma^2} \right\} dX \quad (2.27)$$

$$\mu = \frac{1.4332}{R} + 5.5212 - 0.0486 R \quad (2.28)$$

$$\sigma = 0.2196 - 0.0103 R \quad (2.29)$$

where P is the probability of strand failure, $X = \log N$, μ is the mean of X , σ is the standard deviation of X and R is the stress interval.

The authors suggested that size effect and the number of strands used in the member influenced mean fatigue strength. This was taken into account by considering the probability of strand failure, P . For instance, for u similar strands in a section, the probability of beam failure was expressed as:

$$Q = 1 - (1-P)^u \quad (2.30)$$

For a 50 percent beam failure ie $Q = 0.5$, the mean beam fatigue life is equal to the number of cycles corresponding to the probability of strand failure of:

$$P = 1 - (1-0.5)^u \quad (2.31)$$

The failure life can be calculated using the three equations above and with an appropriate value of P .

The adequacy of this method is dependent on the ability of the compatibility factor ψ to account for increases in steel deformation caused by bond breakdown. However, other effects of repeated loading, for instance cyclic creep, were not

considered.

2.3.7 Rao and Babu

Rao and Babu (1979) recommended a semi-empirical method of calculating the mean steel strain. The steel strain at the cracked section was first calculated by a simple cracked section analysis. Allowance was then made for tension stiffening effects by subtracting a compensating amount, ie.

$$\epsilon_{s,m} = \epsilon_s - \frac{k \sigma_t}{E_s \rho_s} \quad (2.32)$$

where $\epsilon_{s,m}$ is the mean steel strain, ϵ_s is the steel strain derived from the simple cracked section analysis, k is a constant, σ_t is the flexural tensile concrete strength, E_s is the Young's modulus of steel and ρ_s is the gross reinforcement ratio.

The determination of the stress-moment behaviour under fatigue loading was performed by a semi-empirical procedure. The beam response under repeated loading was described by a linear relation, obtained by joining the point corresponding to maximum load to another point in the 'third quadrant' defined by the co-ordinates

$$\{ -\sigma_{u,c,r,r} (1-\lambda), -M_{c,r,r} (1-\lambda) \}$$

where $\sigma_{u,c,r,r}$ is the stress at cracking in the reverse direction, $M_{c,r,r}$ the corresponding cracking moment and λ defines the degree of prestress. An exact interpretation of λ is not given in this publication.

This is illustrated in Fig 2.14. It is not clear whether the steel stresses referred to here are mean stresses or stresses at the cracked section. The beam fatigue strength could be checked by comparing the calculated stress range with the steel Goodman diagrams.

This method is clearly empirical and inadequate as it does not

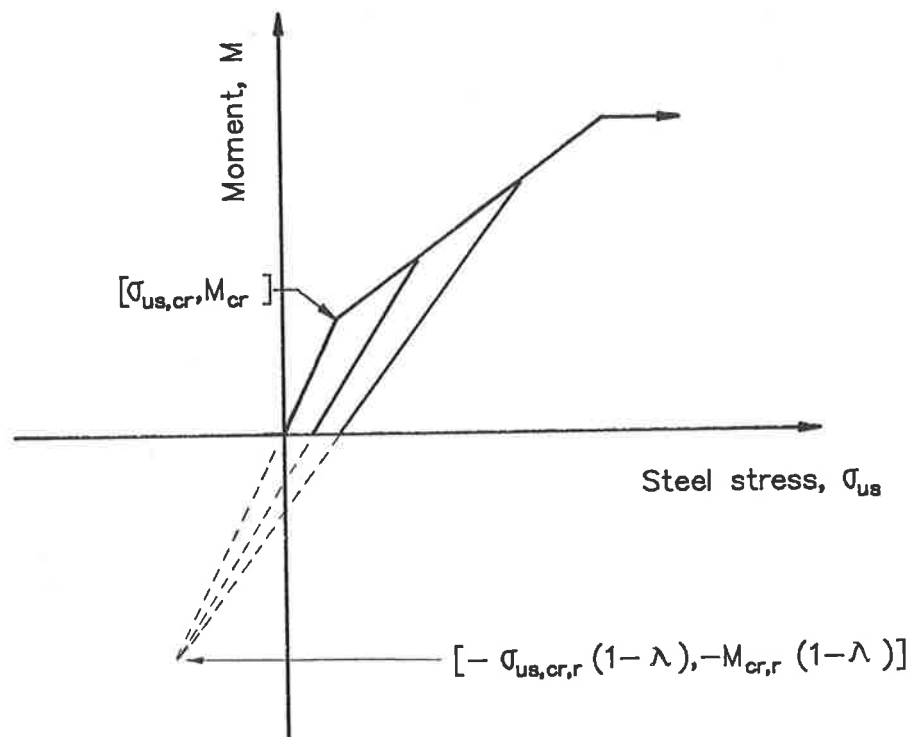


Fig. 2.14 Moment-Steel Stress Response
(Rao & Babu, 1979)

take into consideration the mechanisms involved in fatigue loading.

2.3.8 Balaguru

Balaguru (1981 and 1982) attempted to predict beam deflections and crack widths by the use of an analytical model which took into consideration cyclic creep and bond deterioration. These two factors, together with cyclic creep of the steels were considered to be responsible for the progressive changes in beam response. From work carried out by Whaley and Neville (1973) on cyclic creep of concrete, an expression for the apparent stiffness of concrete due to repeated loading was derived, ie.

$$E_{c,N} = \frac{\sigma_{max}}{\frac{\sigma_{max}}{E_c} + \epsilon_c} \quad (2.33)$$

where $E_{c,N}$ is the apparent cyclic modulus after N cycles and E_c is the initial secant modulus.

Bond deterioration was accounted for by the concept of a reduced modulus of rupture (ultimate tensile stress) which decreased the cracking moment and therefore reduced the effective moment of inertia defined in Branson's formula. The effective moment of inertia in the N^{th} load cycle was defined as:

$$I_{eff,N} = I_{cr,N} + \left\{ \frac{M_{cr,N}}{M_a} \right\}^2 (I_g - I_{cr,N}) \quad (2.34)$$

where $I_{eff,N}$, $I_{cr,N}$ and I_g are the effective, cracked and gross second moment of area for the N^{th} load cycle. M_a is the applied moment, and $M_{cr,N}$ is the cracking moment defined by:

$$M_{cr,N} = \frac{I_g f_{r,N}}{y_t} \quad (2.35)$$

where y_t is the distance of the neutral axis from the extreme fibre and $f_{r,N}$ the modulus of rupture is dependent on the number

of load cycles sustained,

$$f_{r,N} = f_r \left\{ 1 - \frac{\log_{10} N}{13} \right\} \quad (2.36)$$

Beam deflections in the N^{th} load cycle could hence be calculated using the effective flexural rigidity given by $E_{c,N}$ $I_{eff,N}$.

Balaguru suggested that the concrete and steel stresses required for fatigue predictions be obtained from equations developed by Naaman and Siriaksorn (1979). These equations were derived from simple, linear elastic theory and applied to the first cycle response only. The stresses were then checked against stress limitations to prevent fatigue failure.

The analysis for fatigue loading is applicable only to deflection and crack width predictions. In spite of the considerations given to cyclic creep and bond deterioration, no equations were recommended for stress-cycle calculations under repeated loading. Stress calculations were obtained from a simple first-cycle, no-tension analysis. Evidently, this is inadequate for precise fatigue predictions.

2.3.9 Summary

The importance of obtaining accurate estimates of steel stresses for fatigue calculations has long been recognised. Walls et al (1965) found poor agreement when they compared the S-N curves of their beam tests with those of their naked steel specimens. This was attributed to poor evaluation of beam steel stresses by the 'straight line formula'. More realistic estimates were attempted with a proposed empirical procedure.

Abeles et al (1974) suggested that beam fatigue response was influenced by slip in the tendons resulting in a significant loss of prestress. He took account of the varying steel stresses under repeated loading by using Miner's cumulative damage rule for fatigue calculations. Ekberg et al (1957), Bennet (1974), and Warner et al (1966) all recognised the need

for calculating stresses at the critical section. However, their methods were not suitable as they did not adequately account for the progressive changes in beam deformations under repeated loading.

Balaguru (1981 and 1982), on the other hand attempted to take into consideration cyclic creep and bond deterioration effects, but unfortunately used a global modified stiffness approach which was only useful for predicting increased deflections and crack-widths. The method of checking fatigue failure was based on a simple first-cycle, cracked section analysis. Venuti's fatigue prediction equation is purely empirical and is unsuitable for other members. Rao and Babu's method (1979) is also empirical and therefore inadequate.

There is still some uncertainty as to the actual mechanisms occurring in a beam under fatigue loading. Although it is accepted that changes occur which lead to increases in deformations, the physical nature of the changes has not been completely explained. On the one hand, Abeles emphasises loss of prestress, while Balaguru assumes a reduction in overall stiffness to be critical together with dynamic creep, but, precise qualitative treatment of these changes has been lacking.

A search of the literature reveals the lack of an adequate method of analysis for making accurate evaluations of the moment-stress relations in a member at critical sections under repeated loading. However, the precise estimation of stresses alone may not be sufficient for an accurate fatigue prediction. Premature failures of the steel may occur early through stress raisers brought about by abrasion, fretting and perhaps corrosion in the vicinity of cracks. Test results by Hanson, Hulsbos and Vanhorn (1970) and more recently by Rigon and Thurlimann (1985) support this. The in-air S-N curves of concrete and steel thus may not be appropriate for use in fatigue life evaluation.

Fatigue prediction thus involves more than just making stress-cycle calculations. It requires a comprehensive

assessment of all the major factors influencing the overall structural behaviour as well as the individual fatigue properties under repeated loading.

CHAPTER THREE

EXPERIMENTAL INVESTIGATION

A series of flexural fatigue tests was carried out on partially prestressed concrete beams, manufactured from Tempcore reinforcing bar, seven-wire prestressing strand and high strength concrete. Material tests were conducted concurrently in separate programs (Warner, 1982 and Behan and Warner, 1984) to investigate the fatigue properties of both the Tempcore reinforcing bar and the seven-wire prestressing strand.

One objective of the beam tests was to obtain reliable experimental data on the progressive changes which occur in the flexural behaviour of partially prestressed concrete members when loaded repeatedly into the cracking range. A second objective was to provide information on the final modes of fatigue failure and to provide test data on fatigue life for comparisons with theoretical predictions.

Results of the tests have been reported overseas (Foo and Warner, 1984b) and locally (Foo and Warner, 1984a).

3.1 DETAILS OF TESTS

3.1.1 Beam Load Details

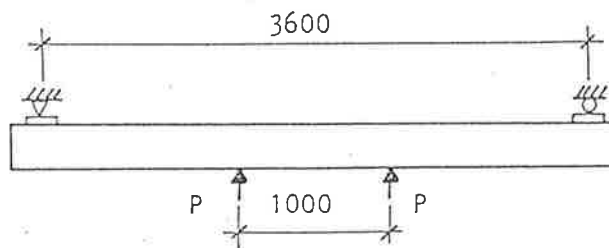
The main test variable was the load cycle applied to the beams. Two beams were tested with a constant load cycle, repeatedly applied until fatigue failure occurred. The remaining four beams were each subjected to cumulative damage loading. Two different load cycles were combined into a cumulative damage loading block of twenty thousand load cycles which were repeatedly applied until fatigue failure occurred. The load cycles for each beam are summarised in Table 3.1. In

Table 3.1 Details of Fatigue Load Cycles

Beam	Type of load	A/B	P(max) (kN)	P(pred) (kN)	P(min) (kN)	Age* (day)
PA 2	Constant	0.00	29.25	29.25	10.0	49
SA 1	Constant	0.00	33.13	33.13	10.0	77
PA 1	Cumulative	0.25	33.13	29.25	10.0	96
SA 2	Cumulative	0.25	33.13	29.25	10.0	56
SA 3	Cumulative	0.50	33.13	22.50	10.0	86
SA 4	Cumulative	0.50	33.13	29.25	10.0	71

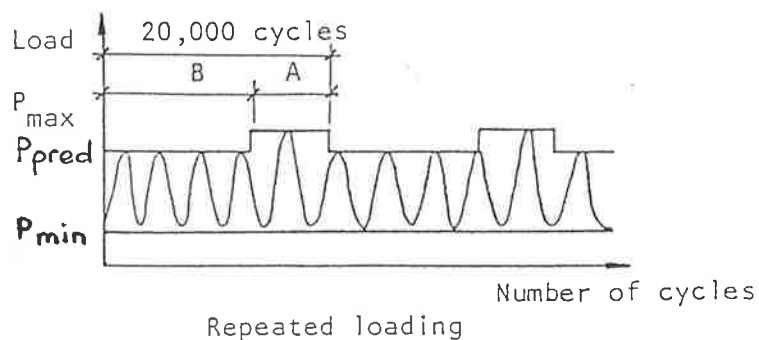
A/B: Ratio of number of maximum load to predominant load cycles in load block as shown below

* At test



Test arrangement

Note: All dimensions in mm



Repeated loading

the cumulative damage tests, the minimum load level was constant for both cycles. The cycle with the smaller maximum load was also the most frequently occurring (predominant) cycle. The ratio of the number of maximum load cycles to predominant load cycles in the load block is shown as A/B in Table 3.1.

Static load tests on the beams were carried out at regular intervals between the repeated loadings to record progressive changes in beam behaviour by the measurement of deformations, strains and deflections.

The beam tests were accompanied by specimen tests on the component materials. The material tests included a series of cyclic load tests on 12 plain concrete cylinders to provide data on the effect of cyclic creep on concrete. Standard static compressive and tensile strength tests on 150mm diameter concrete cylinders of 300mm height and 100mm X 100mm X 500mm beams were conducted at various ages and stress-strain curves of concrete obtained. Creep and shrinkage tests on a total of 9 concrete cylinders were also conducted. Direct tension tests on the reinforcing and prestressing steel were carried out to obtain stress-strain curves, yield and ultimate strength values.

3.1.2 Details of Beams

The beams were manufactured in two groups: the PA series (preliminary series comprising two beams) and the SA series (second series comprising four beams). The dimensions and section properties were nominally identical for all beams, which were four metres long with a rectangular cross section 160mm wide and 280mm deep. Both compressive and tensile reinforcing steel were included, in addition to the pretensioned prestressing steel. Shear reinforcement was provided over the length of the beam except in the central pure-moment test region. The nominal beam dimensions and cross-section details are shown in Fig 3.1. Details of the as-constructed beams are given in Table 3.2.

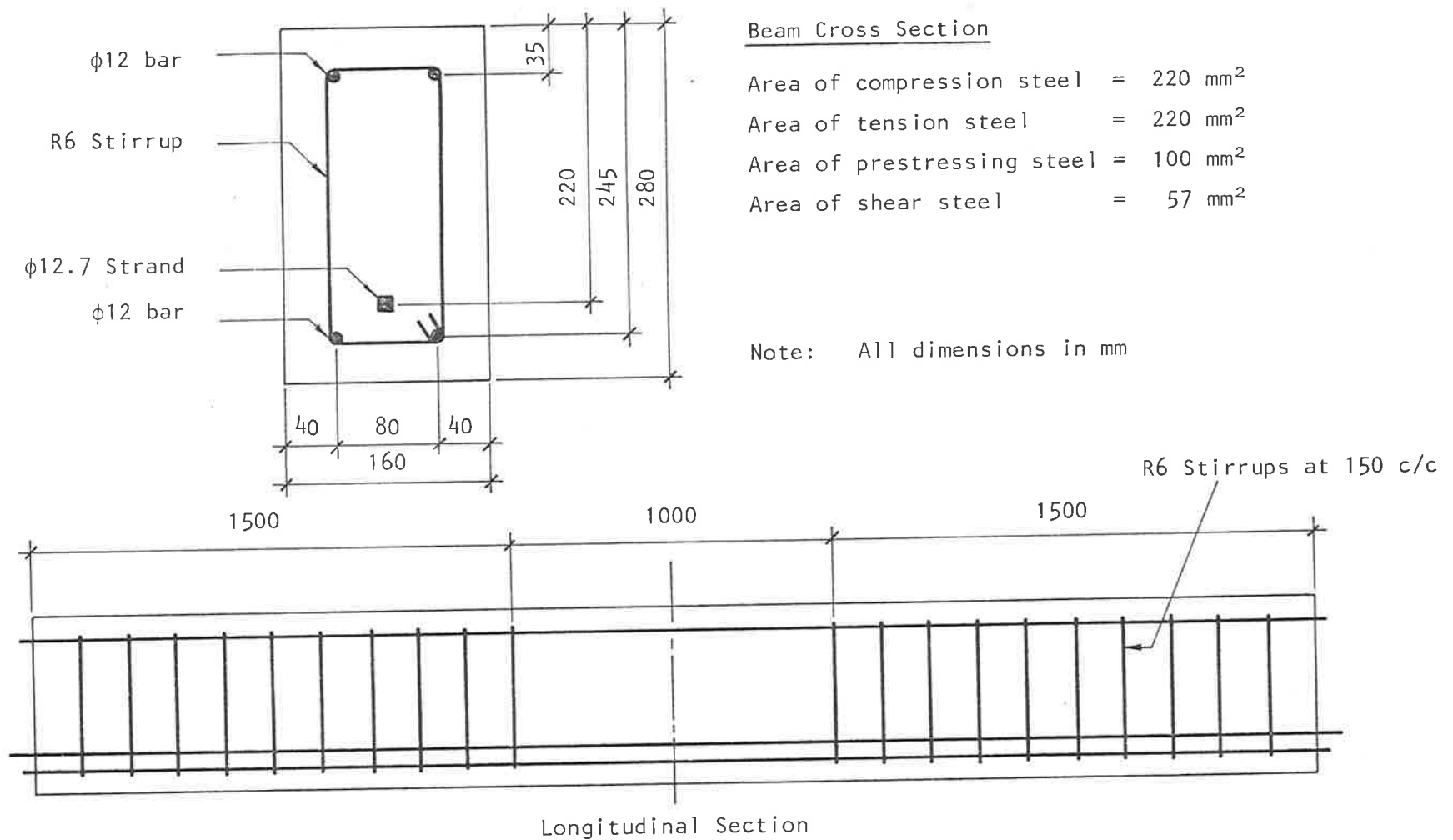


Fig 3.1 BEAM DIMENSIONS AND REINFORCING DETAILS

Table 3.2 Beam Details

Beam	Date Cast	Age days	F_c^* (MPa)	F_t^* (MPa)	P_i (kN)	P_e (kN)
PA 1	11/3/83	96	55.2	7.9	139.86	127.23
PA 2	11/3/83	49	57.5	6.4	139.86	131.25
SA 1	26/7/83	77	57.6	6.1	139.24	128.20
SA 2	26/7/83	56	59.2	6.6	139.24	129.34
SA 3	9/8/83	86	63.5	6.7	135.54	126.20
SA 4	9/8/83	71	60.3	7.8	135.54	126.68

P_i : Initial prestressing force
 P_e : Effective Prestressing force
 F_c : Ultimate concrete compressive stress
 F_t : Ultimate concrete tensile stress
 * at test

The beam section was selected to reproduce realistically the stress conditions in typical partially prestressed concrete members subjected to repeated loads. The level of partial prestressing was chosen with the aim of producing at least two different modes of fatigue failure, ie fracture in the prestressing steel or reinforcing steel, depending on the load cycle.

Two beams were cast at a time, together with the accompanying concrete test cylinders and flexure specimens. The prestressing strand was initially tensioned and anchored in a prestressing bed. The stressing force was measured by a steel load cell positioned at each end. Recordings in the load cells were made by two independent pairs of electrical strain gauges positioned diametrically opposite one another on each cell.

Concrete was poured in three successive layers in the beams and each layer was compacted by two immersion vibrators as well as with an external pneumatic form vibrator. The green concrete was kept moist by means of wet hessian cloth and plastic sheeting. Sideforms were removed after four days to allow Demec targets to be attached to the sides. Prestress was transferred to the concrete seven days after casting. The immediate elastic concrete shortening due to prestress was obtained from Demec readings before and after transfer. Creep and shrinkage tests were also set up to record the inelastic time dependent changes in the concrete.

3.1.3 Materials

Concrete

Commercially pre-mixed concrete was used in the manufacture of the test specimens. The concrete mix details are given in Table 3.3. Standard size cylinders (150mm X dia X 300mm) and flexure beams (100mm X 100mm X 500mm) were cast from each batch to determine the tensile and compressive strength at various ages. The mean strength (averaged from 6 cylinder or beam tests) of the concrete specimens are summarised in Table 3.4.

Table 3.3 Details of Concrete Mix

Material Property	Quantity
Characteristic strength	40 MPa
Specified Slump	80 mm
Maximum Size Coarse Aggregate	20 mm
Cement (Adelaide Type A)	425 kg/m ³
Pt Augusta Flyash	75 kg/m ³
RMC Dolomite 10mm	330 kg/m ³
RMC Dolomite 20mm	865 kg/m ³
Pedlars Creek Washed Concrete Sand	455 kg/m ³
Approximate Total Water (includes water absorbed by aggregates)	210 litres
Admixture - Eucon Retarder	450 mls/100kgs

Table 3.4 Concrete Strength at Various Ages

Beam	Age* (days)	F _c at 7 days	F _t at 7 days	F _c at 28 days	F _t at 28 days	F _c at test	F _t at test
PA 1	96	37.0	3.5	52.5	5.4	55.1	7.86
PA 2	49	37.0	3.5	52.5	5.4	57.5	6.42
SA 1	77	35.5	5.3	48.4	6.0	57.6	6.05
SA 2	56	35.5	5.3	48.4	6.0	59.2	6.63
SA 3	86	42.9	5.0	54.5	5.5	63.5	6.69
SA 4	71	42.9	5.0	54.5	5.5	60.3	7.75

F_c = ultimate compressive stress in MPa

F_t = ultimate tensile stress in MPa

* at test

Concrete Cyclic Load Tests

A program of direct compression fatigue tests was undertaken on 12 plain concrete specimens (104mm dia X 208mm deep cylinders). The objective was to provide information on the concrete constitutive relation at various stages of repeated loading and also to collect strain data on dynamic creep effects. Testing of the nominally identical cylinders was conducted under constant amplitude loading conditions. Five different load ranges were utilised in the program, varying from approximately 25 to 50 percent of ultimate strength. The minimum load was kept constant at about 5 percent of ultimate. These were chosen to approximate the compressive stresses in the beam sections under first cycle and subsequent loading. The loading details are shown in Table 3.5.

Table 3.5 Cyclic Load Details for Concrete Cylinders

Cylinder	Minimum Stress %	Maximum Stress %
A1,A2	5	24.5
B1,B2	5	34.3
C1,C2	5	40.2
D1,D2,D3	5	46.1
E1,E2,E3	5	50.0

The uniaxial compression tests were carried out in an Instron dynamic test facility with electronic feedback control, set to load control mode. Load eccentricities were avoided by careful centring and the use of sulphur capping to obtain true loading surfaces. A movable spherical head was also incorporated in the loading platens. Static tests for the first few load cycles were performed to permit load and strain measurements to be taken. Fatigue loading was then permitted to run continuously except for periodic interruptions when load and strain measurements were taken during a static load

cycle. Plate 3.1 shows a concrete specimen in the testing facility.

Measurements of concrete strains were effected with electrical strain gauges wired in pairs and placed diametrically opposite one another to counteract load eccentricities and bending. A minimum of two strain gauge pairs were used on each cylinder. Drift and temperature compensation were provided by dummy gauges.

Concrete Compressive Stress-Strain Behaviour

The typical fatigue behaviour of a plain concrete cylinder under compressive loading is given in Fig 3.2a. The first cycle stress-strain curve shows near-linear behaviour in the loading direction with some hysteretic effects and residual deformations in the unloading portion. The extent of hysteresis varied with the maximum sustained load and ranged from approximately 9 percent to 15 percent of the maximum strain sustained. Larger hysteretic effects were observed for those cylinders which experienced higher stresses.

Equations were derived from the concrete tests for the loading, unloading and subsequent reloading curves and are applicable within the test load range of between 0 to 30 MPa. For the first-cycle loading curve:

$$G\{z\} = 0.5 z F_c \exp(1 - 0.4z) \quad (3.1)$$

where G is the function defining the stress-strain relation and z is the normalised elastic strain.

$$G\{z\} = \frac{\sigma}{F_c} \quad (3.2)$$

$$z = \frac{\epsilon_e}{\epsilon_{ult}} \quad (3.3)$$

σ is the concrete stress, F_c is the ultimate stress, ϵ_e is the elastic concrete strain and ϵ_{ult} is the strain at maximum stress

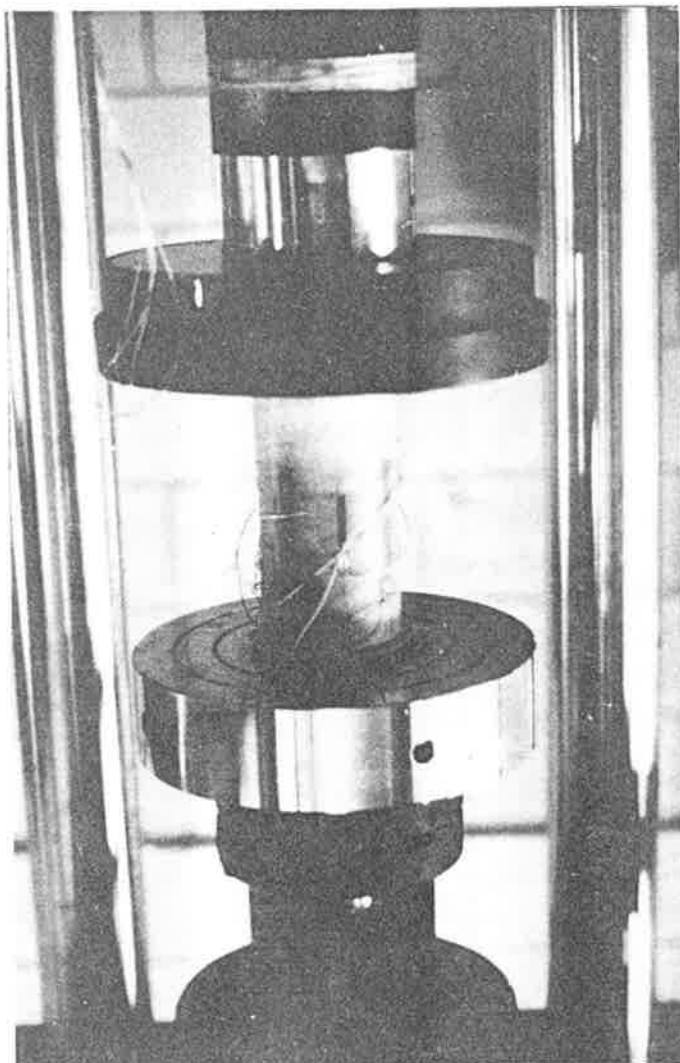


Plate 3.1 Concrete Specimen Under Compressive Fatigue Loading

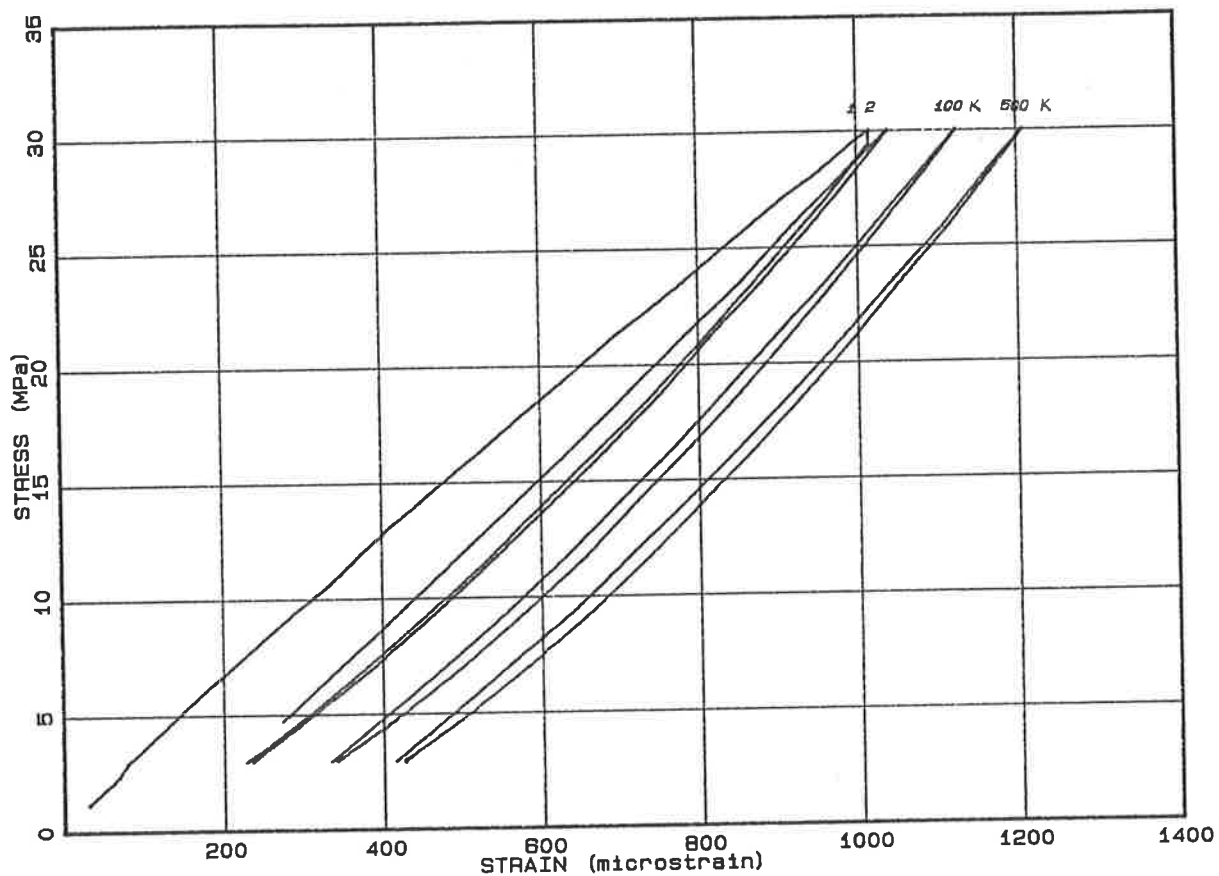


FIG 3.2a TYPICAL COMPRESSIVE CONCRETE STRESS-STRAIN RELATIONS
UNDER REPEATED LOADING
K : 1000 CYCLES

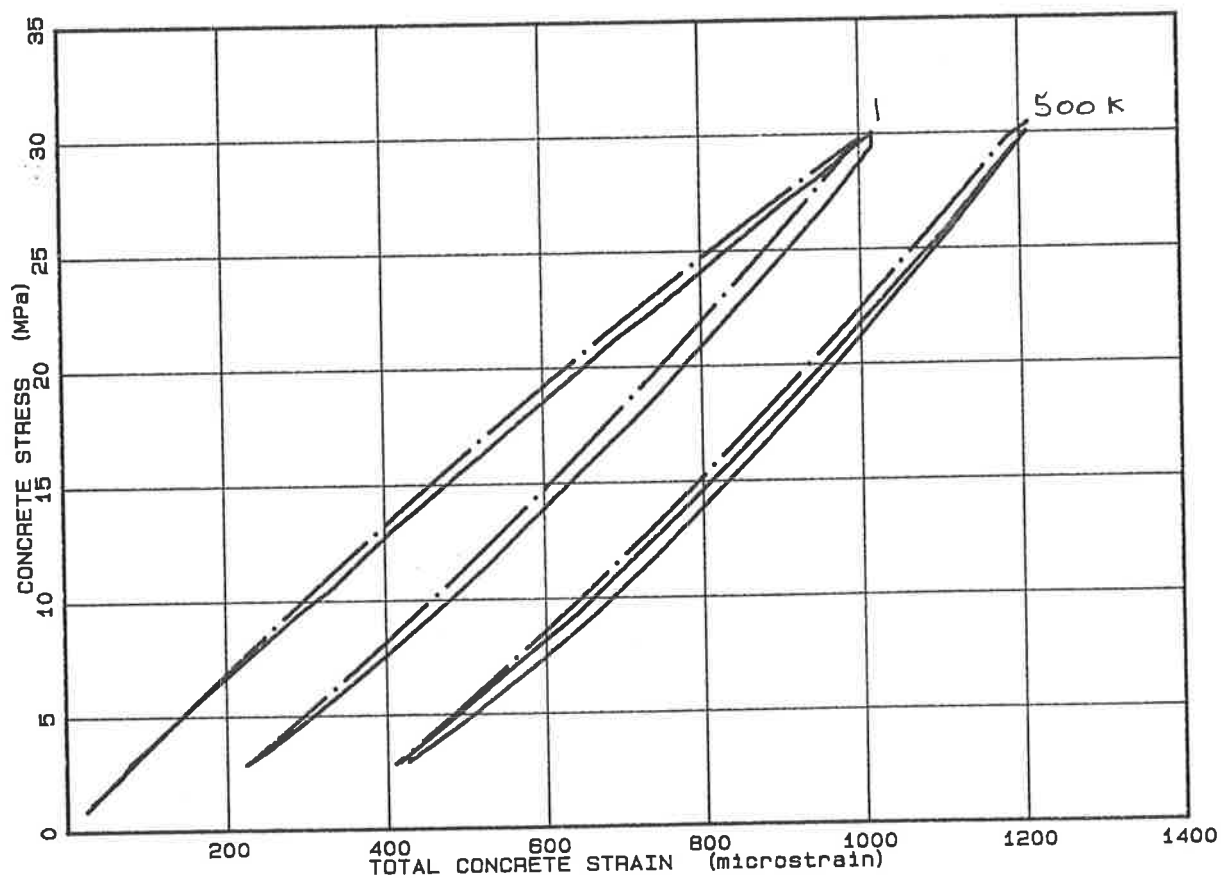


FIG 3.2b CONCRETE STRESS-STRAIN RELATIONS - CYLINDER E1
for first cycle and repeated loading

Observed ——— Theoretical - - -

For the unloading direction, the following equation was derived:

$$G_c \{z\} = \sigma_{max} - 0.58 (z_{max} - z) F_c \exp(1 - 0.5(z_{max} - z)) \quad (3.4)$$

where z_{max} is the maximum normalised strain and σ_{max} is the maximum stress sustained

The second cycle loading curve showed significantly less hysteretic effect. Slight increases in strain over the first cycle were recorded. With further load cycles, there appeared to be a small but constant increase in strain over the entire load range. The stress-strain curve shifted to the right. After several hundred load cycles, the shape of the loading curve changed from convex to concave and was similar to the first unloading curve. Hysteretic effects then stabilised to almost zero. While the tangent modulus of elasticity evidently changed (due to the change in shape), the secant modulus appeared to be relatively unchanged. With more load repetitions, there were further albeit more gradual increases in deformation. After a million load cycles the concrete appeared to stabilise to a fairly elastic response with negligible hysteresis. No fatigue failure of the concrete was observed in any of the cylinder tests.

The concrete stress-strain curve after many cycles of repeated loading is similar in shape to the first unloading curve and therefore expressed as:

$$G_c \{z\} = \sigma_{max} - 0.58 (z'_{max} - z') F_c \exp(1 - 0.5(z'_{max} - z')) \quad (3.5)$$

where z'_{max} and z' are the maximum and elastic normalised concrete strains taking cyclic creep strain into account.

A comparison of the observed and theoretical concrete compressive stress-strain relations for the first and 500000th

load cycle is shown in Fig 3.2b. Good agreement is shown for the first loading and unloading curve as well as for the 500000th load cycle.

Fig 3.3 shows the increases in residual strain (dynamic creep) with repeated loading for a typical cylinder. The increase was found to depend on the maximum load level as well as the number of load cycles applied. However, the number of tests conducted (12 cylinder tests) was insufficient to provide sufficiently comprehensive or reliable estimates of concrete permanent deformations for the entire stress range at various cycles of repeated loading. To obtain a more accurate evaluation, Whaley and Neville's (1973) expression for cyclic creep was considered and an empirical equation based on their equation was obtained. This is expressed as:

$$\epsilon_n = 65 \sigma_m (1 + 3.87\Delta)t^{1/3} \quad (3.6)$$

where ϵ_n is the cyclic creep strain, σ_m is the mean stress expressed as a fraction of the static strength at the start of loading, Δ is the stress range and t is the time of loading in hours.

Creep and Shrinkage Tests

The loads in the creep rigs were set so that the compressive stresses in the concrete test cylinders approximated the maximum stresses present in the unloaded, but prestressed beams. The creep and shrinkage test specimens and the test beams were all stored in the same laboratory environment. Average shrinkage and combined creep and shrinkage strains are shown in Fig 3.4. as function of time.

Prestressing Steel

The prestressing steel consisted of 12.7mm diameter seven-wire low relaxation strand, manufactured to Australian Standards AS 1310 and AS 1311. Results of fatigue tests on this steel have been reported (Warner, 1982). Static properties of the strand are summarised in Table 3.6. A typical stress-strain curve for the strand is shown in Fig. 3.5.

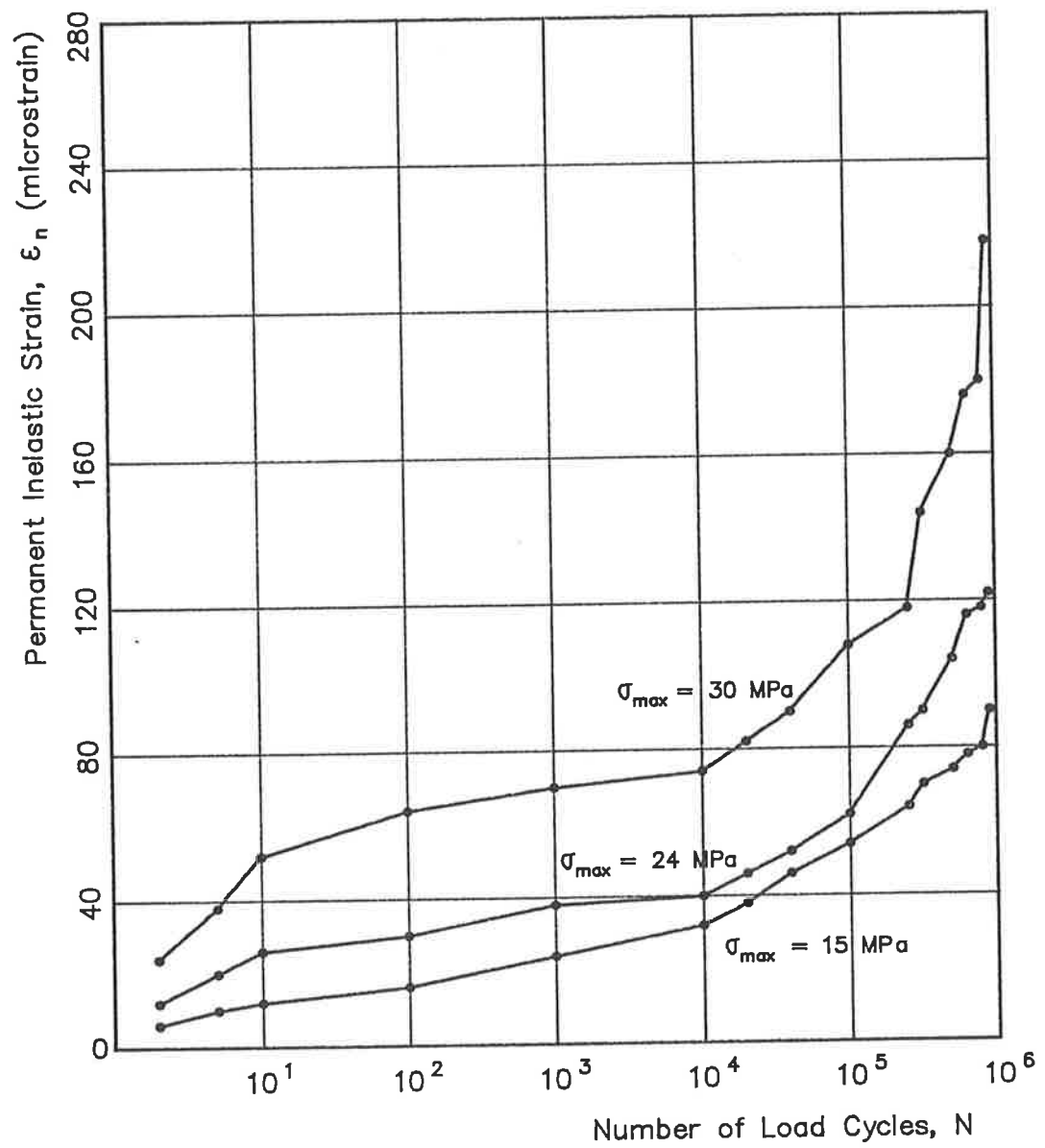


Fig. 3.3 Variation of Concrete Permanent Strain with Repeated Loading

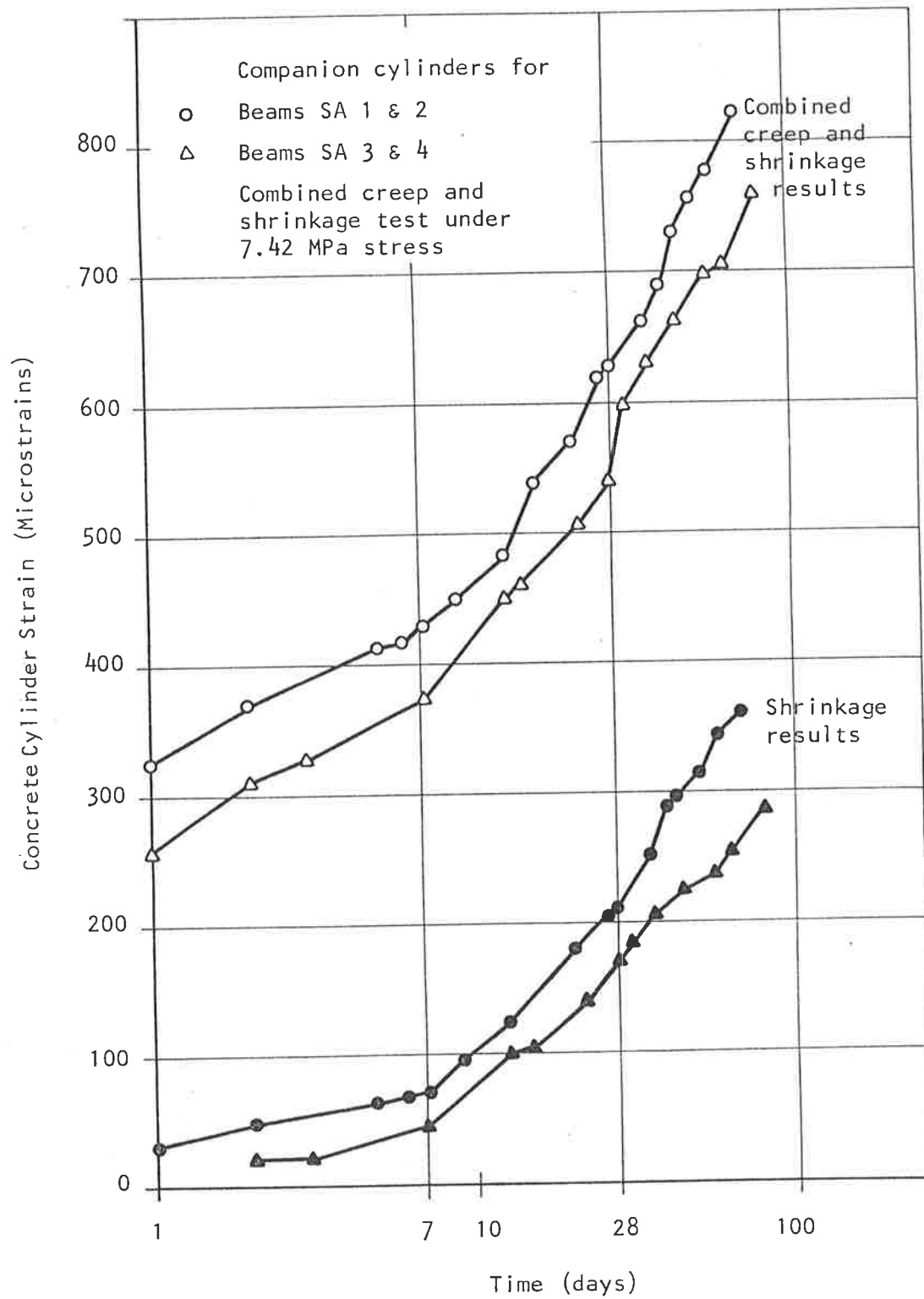


Fig 3.4 CONCRETE CYLINDER CREEP AND SHRINKAGE STRAINS VS TIME

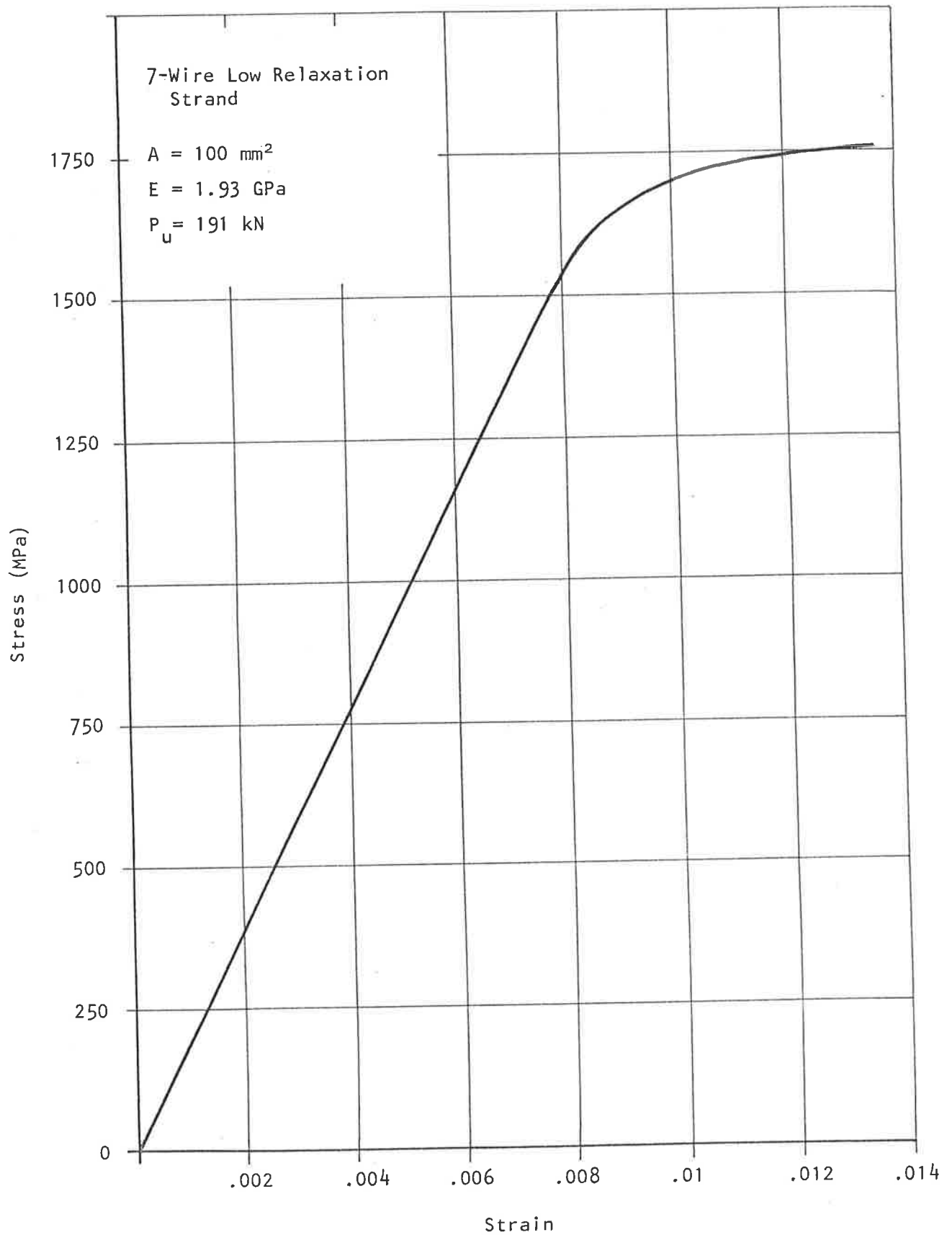


Fig 3.5 STRESS-STRAIN CURVE FOR 12.7mm PRESTRESSING STRAND

Table 3.6 Material Steel Properties

Steel	Type	Dia (mm)	Area (mm ²)	P(ult) (kN)	P(yield) (kN)	E (GPa)
Prestressing	7-Wire	12.7	100.1	191.0	—	1.932
Reinforcing	Tempcore	12.0	110.0	64.0	55.8	2.079

Reinforcing Steel

Hot rolled Tempcore reinforcing steel, which has been recently introduced into Australia, was used in the manufacture of the test beams. The fatigue properties of this new material are being investigated in a separate research program (Behan and Warner, 1984). The 12mm nominal diameter size bar was used for all test beams. The physical and material properties of this steel are given in Table 3.6. A typical stress-strain relation for the steel is shown in Fig 3.6.

3.1.4 Test Set-Up and Procedures

The beams were tested upside-down in a frame securely anchored to the laboratory strong floor. Two load points, one metre apart, provided a constant-moment zero-shear region on a test span of 3.6m length shown schematically in Fig 3.7. Plate 3.2 shows the actual laboratory test set-up with the beam in the background and the instrumentation in the foreground.

Beam deformation measurements were taken at regular intervals from the time of prestress transfer to the time of test to provide a strain record of the time-dependent effects. These are shown in Table 3.7. Deformation readings were also taken before and after placement in the test frame to note the effects of self-weight. A small load was then applied and initial deformation and deflection readings were taken. The beam was statically loaded in approximately thirteen equal increments up to the maximum fatigue load, and then unloaded in the same increments to the predetermined minimum.

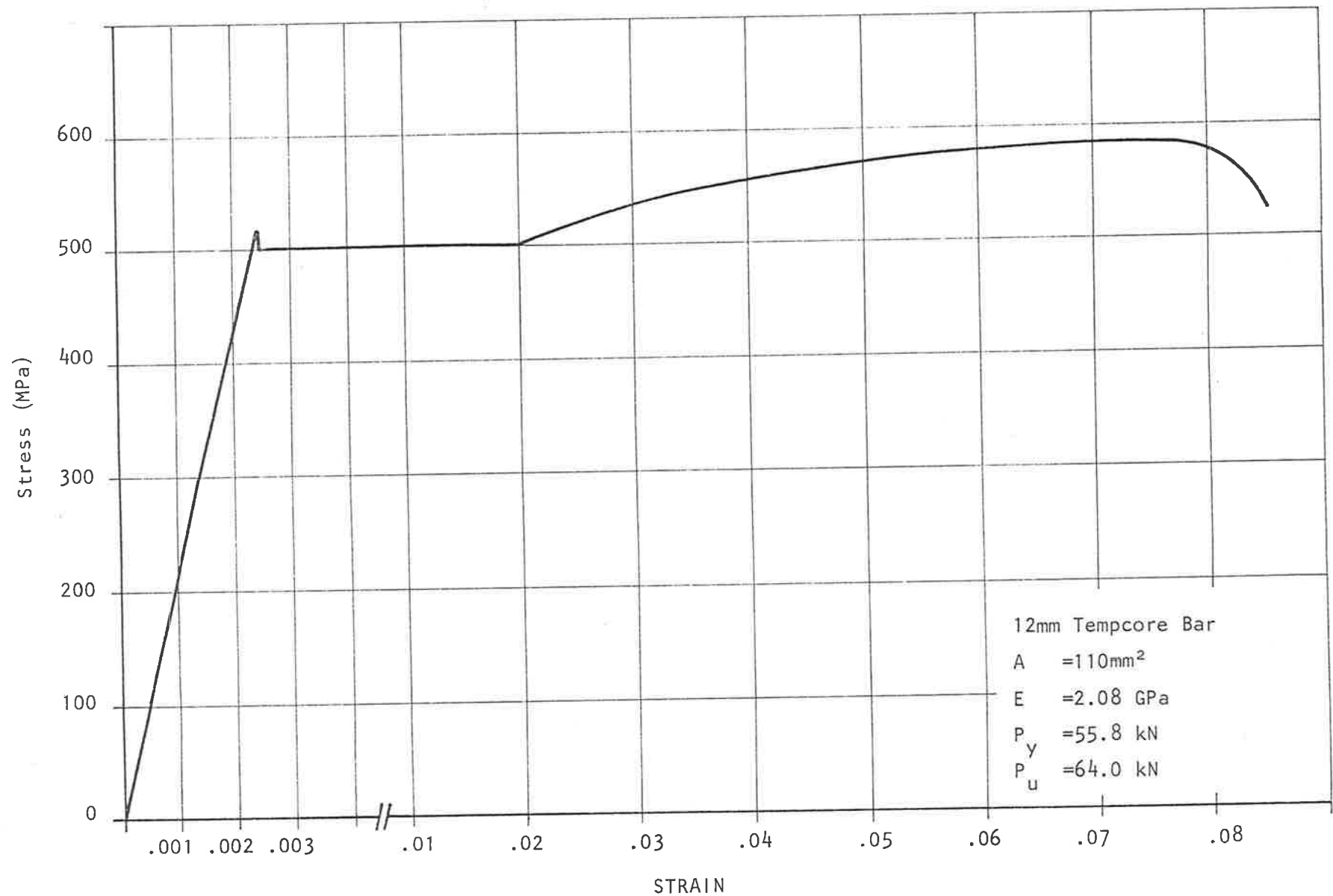


Fig 3.6 STRESS STRAIN CURVE FOR 12mm REINFORCING STEEL

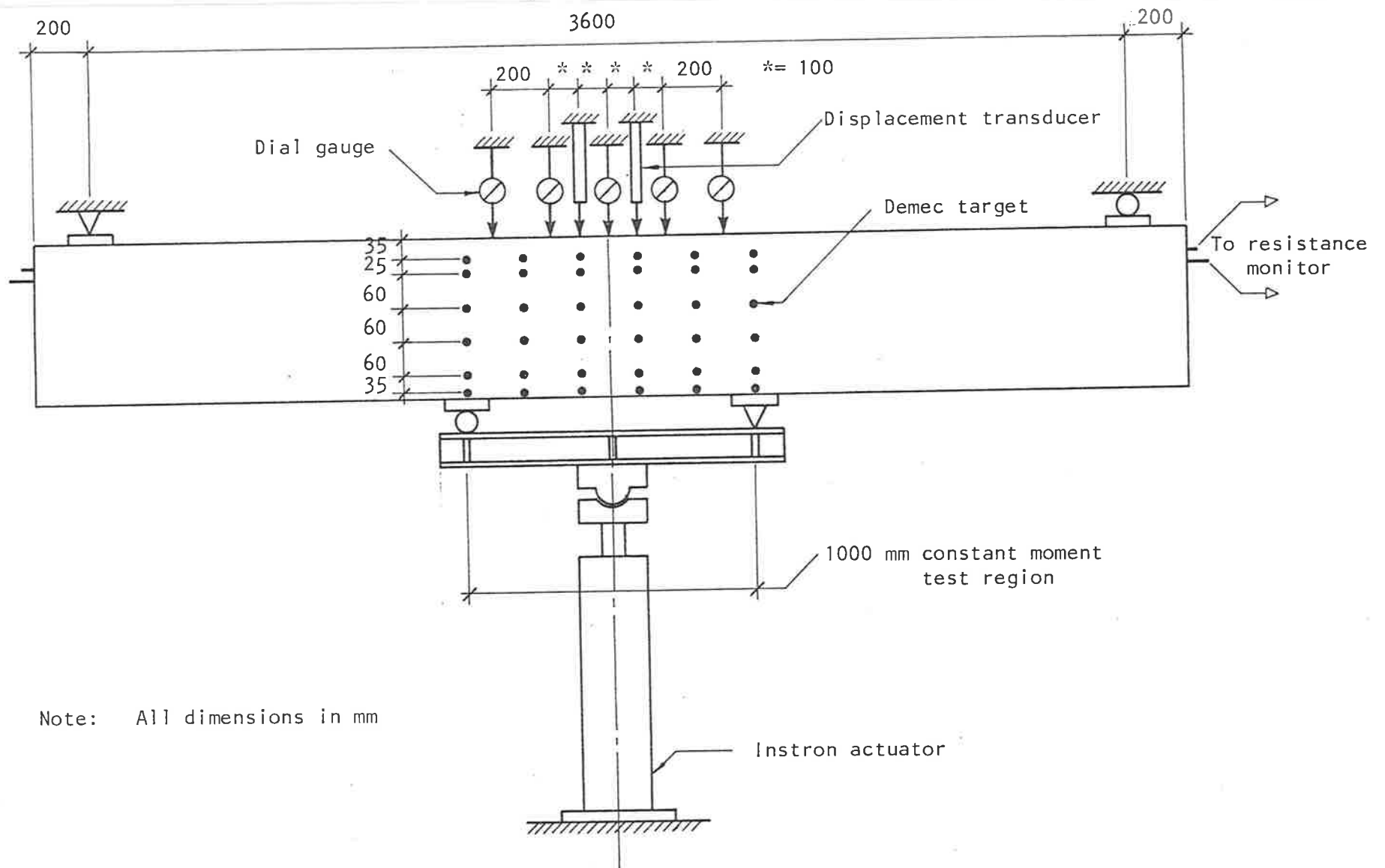


Fig 3.7 SCHEMATIC DIAGRAM OF TEST SET-UP

in the same increments to the predetermined minimum. Deformation, strain and deflection readings were taken at every increment. After a second static cycle of load with measurements of deformations and deflections, the load was cycled. Fatigue testing was interrupted regularly to allow static readings to be taken of strain and deflection during a static load cycle.

Table 3.7 Variation of Concrete Strain Distribution with Time

Beam	0 days		7 days		28 days		Test	
	Top*	Bot*	Top	Bot	Top	Bot	Top	Bot
SA1	34	-231	5	-406	- 76	-541	-133	-669
SA2	42	-243	7	-417	- 91	-585	-140	-622
SA3	46	-218	20	-313	- 94	-461	-102	-578
SA4	52	-203	19	-310	- 89	-479	-120	-550
PA1	69	-249	-12	-404	- 80	-653	-150	-850
PA2	41	-213	-41	-481	-121	-715	-107	-592

Note: Strains are in microstrains, tensile strains are positive
 Time is measured in days after transfer
 * Concrete top-fibre strain
 + Concrete bottom-fibre strain

The beams were loaded sinusoidally at a constant frequency of three cycles per second, by means of an Instron hydraulic test facility under electronic feedback control, run in the 'load control' mode. Maximum and minimum load and deflection limit controls were used to prevent accidental overloading. The minimum load, constant in all beams, was set at a value below the cracking load and was approximately equal to a quarter of the ultimate strength. The cracking load was half of ultimate and the maximum load was set high enough to ensure fatigue failure in each test.

3.1.5 Measurements

Deformation

Concrete deformations in the beams and cylinders were measured by a Demec deformeter over a 203mm (8in) gauge length. Demec readings were taken over the full one metre length test region on six vertical columns of six rows of targets placed on both sides of the beams. The target grid is shown in Fig 3.7.

Strain

Local concrete and steel strains in some specimens and beams were measured by electric strain gauges. Readings were taken using a half-bridge configuration with dummy gauges to provide drift and temperature compensation. The strains were recorded on a large capacity computer-controlled Solartron data logger and on a smaller electronic Kyowa strain logger.

Deflection

Deflections were measured at seven locations along the beam with dial gauges and electronic displacement transducers (DCDTs). The DCDTs were used to check dynamic deflections, while mechanical dial gauges were utilised to record deflection under static load. They are shown in Plate 3.3. The DCDT readings were monitored by a small digital microprocessor programmed to halt the fatigue test when preset deflection limits were exceeded. The locations of gauges, etc are shown in Fig 3.7.

Crack Width

Crack widths were monitored with an optical microscope. Measurements were taken of primary flexural cracks at the reinforcing steel depth.

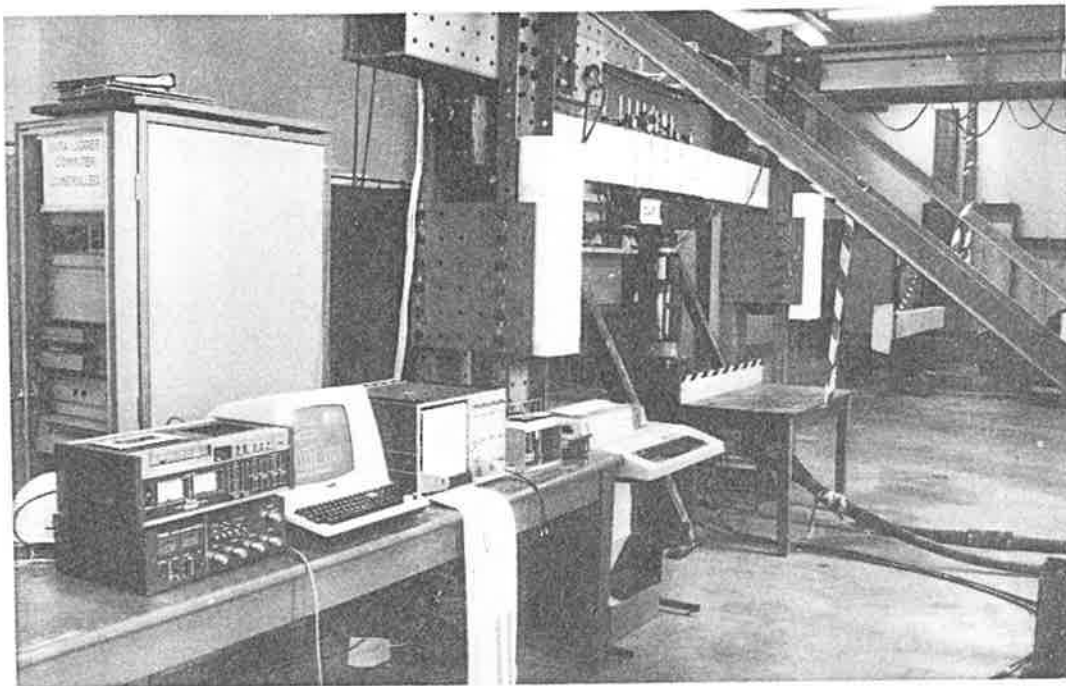


Plate 3.2 Beam Fatigue Test Set-Up

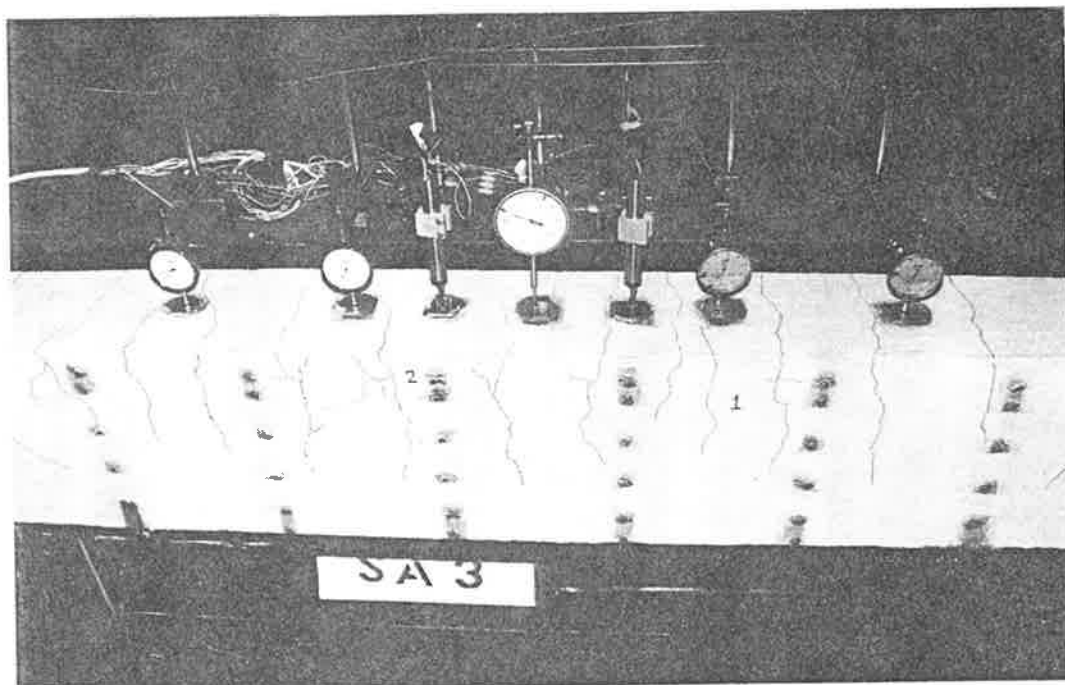


Plate 3.3 Beam Deflection Measuring Devices

Detection of Fatigue Failure

Detection of fatigue failure of individual prestressing wires is difficult, as the effect of a single wire fracture on the overall beam response can be quite insignificant. Fatigue failure of a prestressing wire involves the sudden release of energy, accompanied by sudden, if slight, acceleration changes and vibrations. A small, sensitive accelerometer attached to the beam surface at the level of the prestressing strand was therefore introduced to detect fatigue fracture. The transducer was connected to a detection circuit which continuously monitored the level of acceleration and switched the Instron test machine into 'hold' mode when the threshold level was exceeded.

Fatigue failure of a reinforcing bar also need not necessarily be accompanied by significant increases in the central beam deflection, especially if the failure occurs in a region remote from midspan. Moreover bar failure is not accompanied by any significant vibration or acceleration. The method used for detecting strand fatigue was not applicable to reinforcing bars. Use was made of the electrical conducting properties of the reinforcing bar to detect fracture. A current was periodically pulsed through the bars and a specially constructed microprocessor-controlled device monitored the level of electrical resistance of the bars. Bar fracture showed as a significant change in resistance, which threw a switch to set the testing machine on hold.

Continuous application of the current was avoided to eliminate the possibility of heat buildup in the beam. Negligible temperature increases in the steel and surrounding concrete were recorded even when a continuous current was applied. The same device was connected to the prestressing strand to provide a secondary detection system to the accelerometer device. The resistance readings and dynamic beam deflections were recorded on paper by a Hewlett Packard pen recorder at regular intervals. A schematic diagram of the electrical set-up for the fatigue detection devices is given in Fig 3.8. Plate 3.4 shows the fatigue detection set-up in the steels.

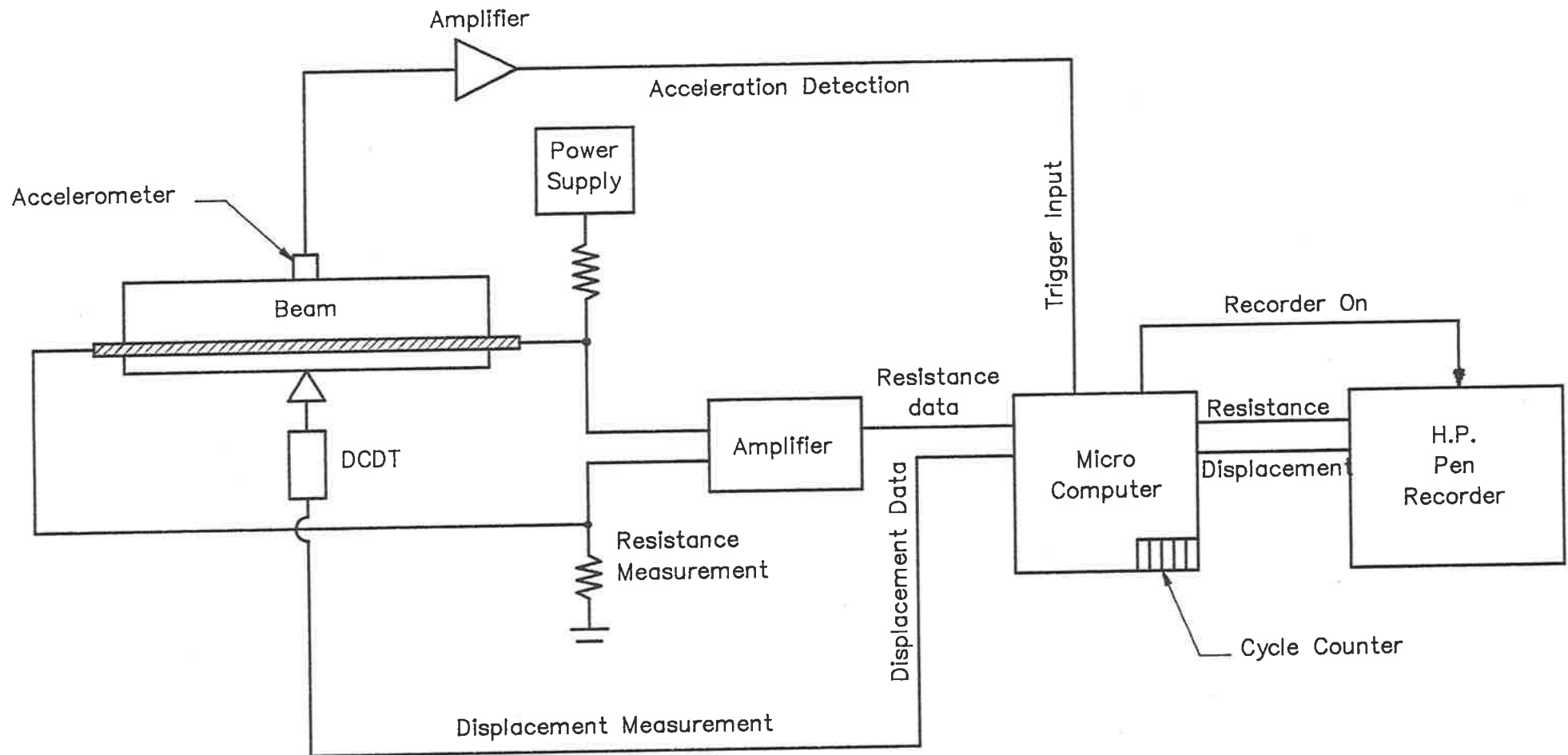


Fig. 3.8 Block Diagram of Fatigue Detection Circuits

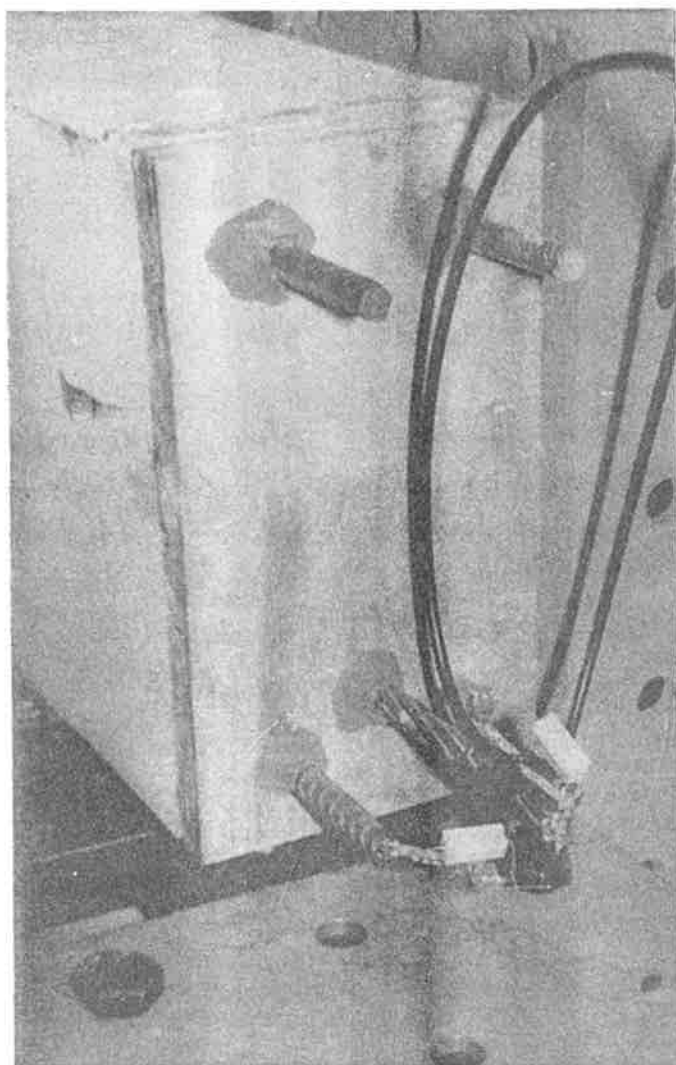


Plate 3.4 Fatigue Detection Transducers

3.2 BEAM BEHAVIOUR UNDER REPEATED LOADING

3.2.1 Beam Deformations

The average strain distribution in the pure moment test region was obtained by a linear regression of the strain readings averaged across the three central Demec gauge lengths (a total of 600mm) on both sides of the beam. Deformation readings from the two outermost gauge lengths near the load points were not used in order to avoid stress dispersion effects in the immediate vicinity of the load points. Fig 3.9 shows the measured average concrete strains (without regression adjustment) at various depths in the section for Beam SA1. The distribution of average strain across the section was found to be approximately linear, and was typical of all the test beams.

The strain in the concrete top fibres increased compressively with increasing load, while the concrete bottom-fibre strains increased in the tensile direction. Fig 3.9 shows that the strain increments increased significantly after cracking (M_{cr} is approximately 25 KNm), especially in the concrete bottom fibres. A regular crack pattern was developed after cracking with vertical flexural cracks in the constant moment region. This is shown in Fig 3.10 for Beam SA 1. Small sand particles and debris were observed in the vicinity of the cracks. Examination of crack widths by microscope showed incomplete closure of primary cracks at the end of the first load cycles.

Fig 3.11 shows a strain distribution for the same beam, obtained from a single set of Demec readings averaged over both sides of the beam. The measurements were taken from a region in which symmetric cracking occurred on both sides of the beam, and where the cracks did not straddle two gauge lengths. The local strain distributions shown in Fig 3.11 were found to be significantly less linear than the average strain distributions in Fig 3.9. Figs 3.12 and 3.13 show the

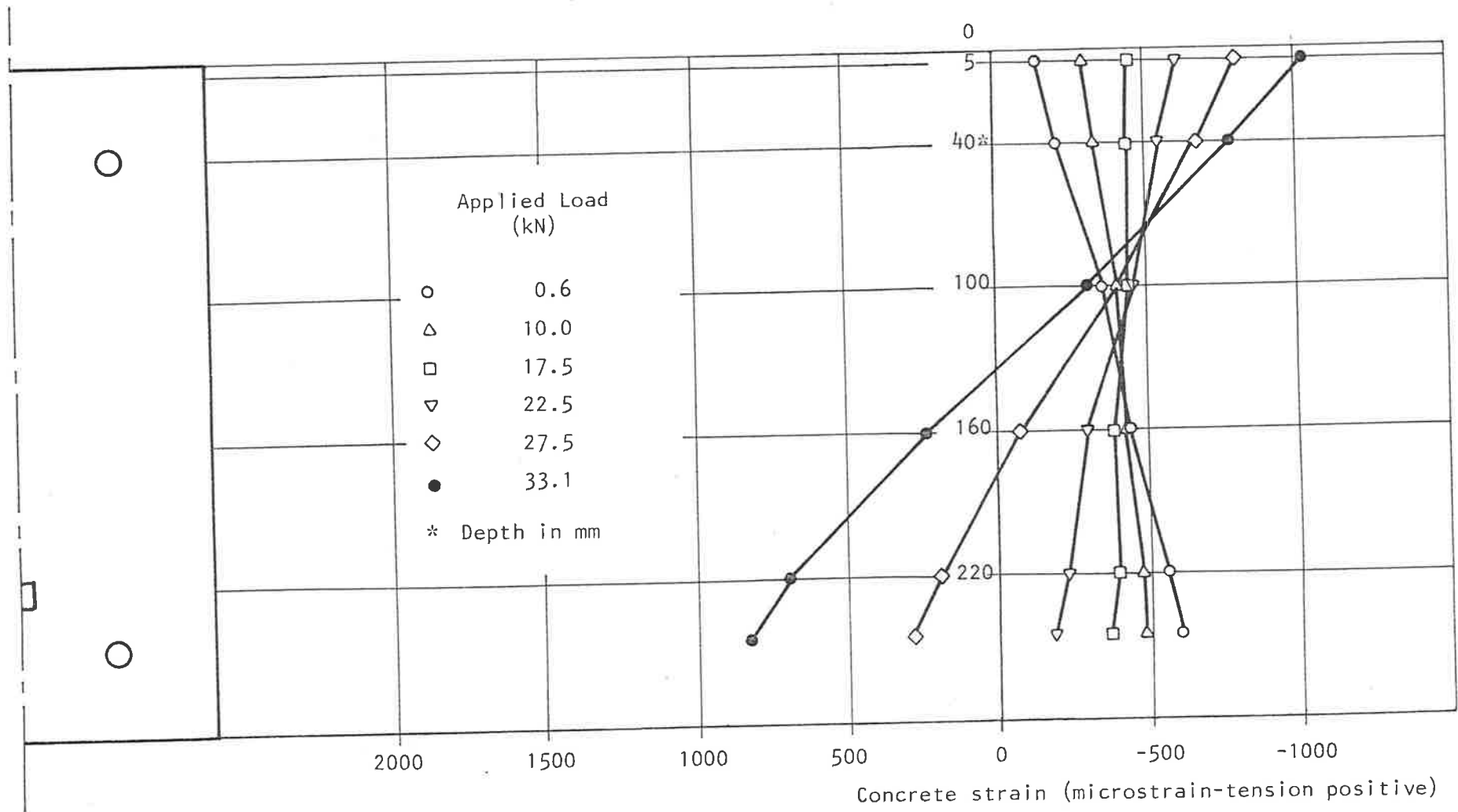


Fig 3.9 AVERAGE CONCRETE STRAINS AT VARIOUS LOADS FOR BEAM SA1

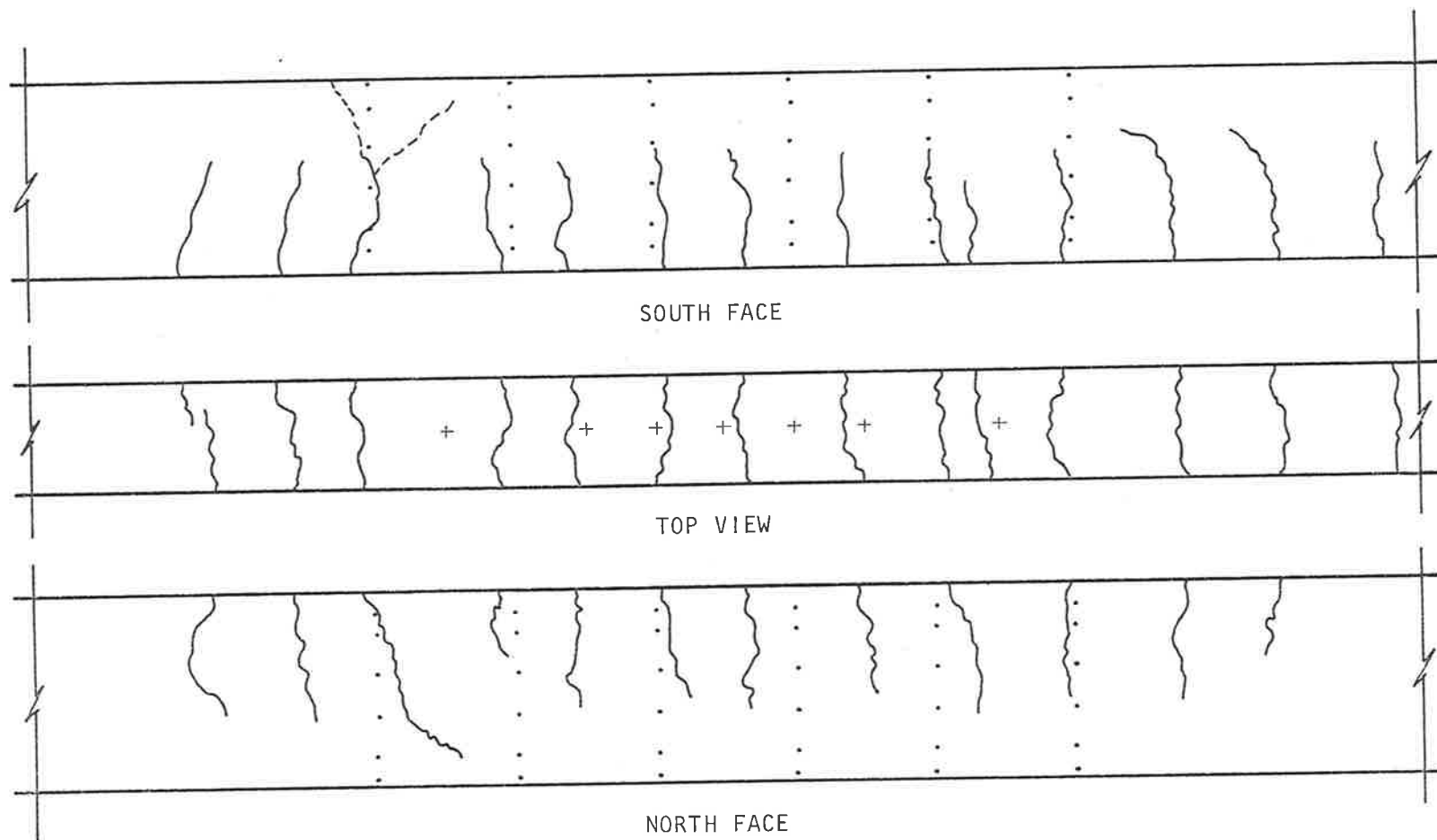


Fig 3.10 CRACK PATTERN FOR BEAM SA1

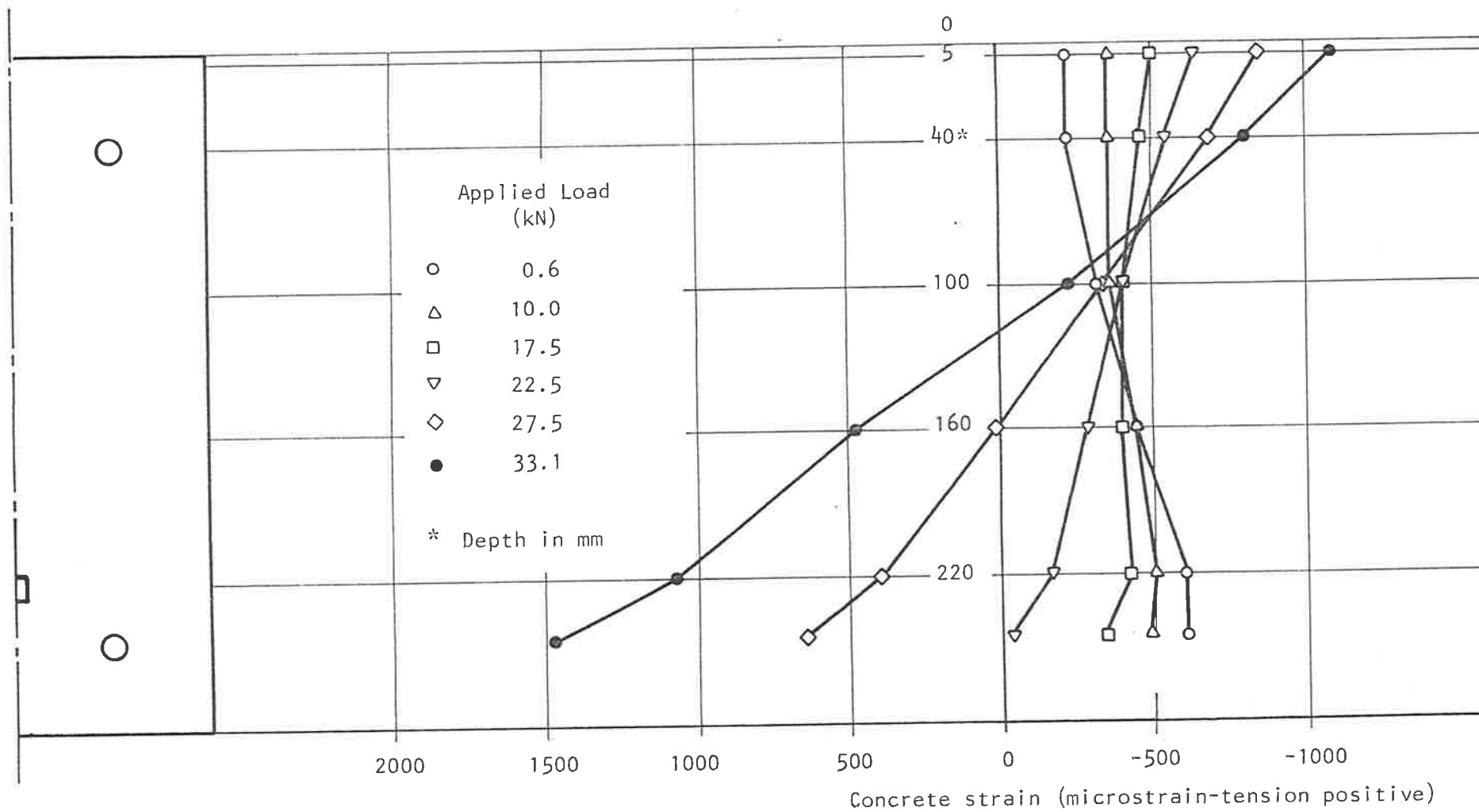


Fig 3.11 LOCAL CONCRETE STRAINS AT VARIOUS LOADS FOR BEAM SA1

variation of concrete strains with increasing number of load cycles, for average and local values.

3.2.2 Moment-Curvature Relations

The midspan curvature of the beam at a particular load was calculated as the ratio of the extreme fibre compressive strain to the depth of the neutral axis. A typical moment-curvature relationship (Beam SA 1) at various cycles is shown in Fig 3.14. Fig 3.15 shows the moment curvature relations obtained for Beam SA1, using strains from a single set of Demec readings averaged over both sides of the beam. Fig 3.16 shows plots of both the average and local moment-curvature relationships for Beam SA1. The comparison shows local curvatures to be larger than average curvatures with differences in the order of 20 percent.

First Load Cycle

The average moment-curvature curve shows a linear-elastic relationship in the precracking range for the first load cycle. The point of cracking is not sharply defined in the curve, but is indicated by a gradual decrease in stiffness. At loads higher than the immediate cracking region, the postcracking behaviour is fairly linear.

The unloading curve has three distinct phases. The first shows an initial steep fall-off in load with small decreases in curvature, suggesting near-elastic recovery. The stiffness is similar in magnitude to the uncracked value. The second phase is recognised by a reduced stiffness somewhat less than the first but greater than the postcracking stiffness. This is the predominant unloading phase. The last phase commences in the load range below first cracking. The stiffness increases once again as flexural cracks attempt to close together.

Beam behaviour in the first cycle can be seen to be inelastic with a large hysteretic effect. Increases in curvature of over 100 percent of the curvature change were recorded at the end

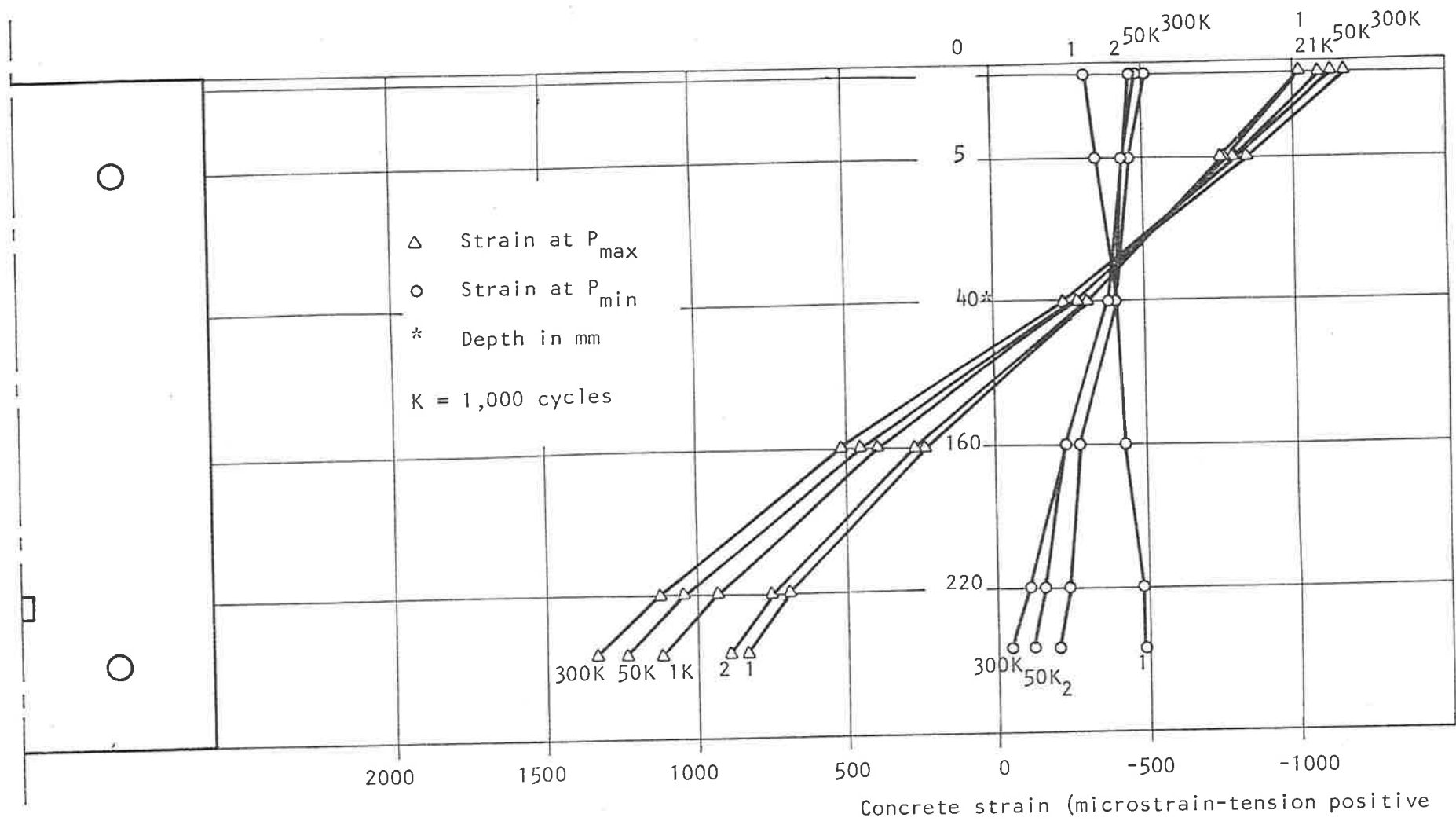


Fig 3.12 AVERAGE CONCRETE STRAINS AT VARIOUS LOAD CYCLES FOR BEAM SA1

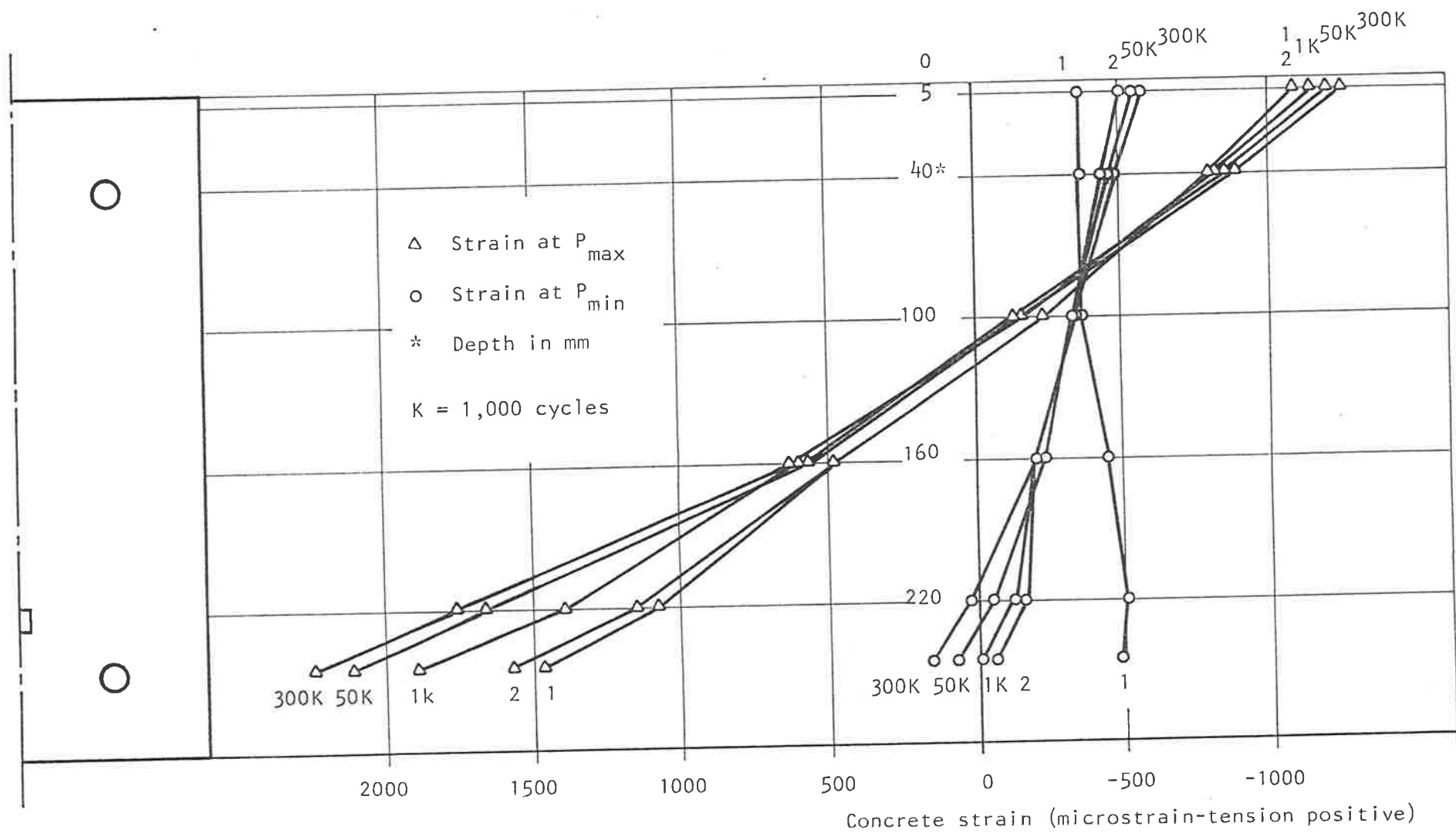


Fig 3.13 LOCAL CONCRETE STRAINS AT VARIOUS LOAD CYCLES FOR BEAM SA1

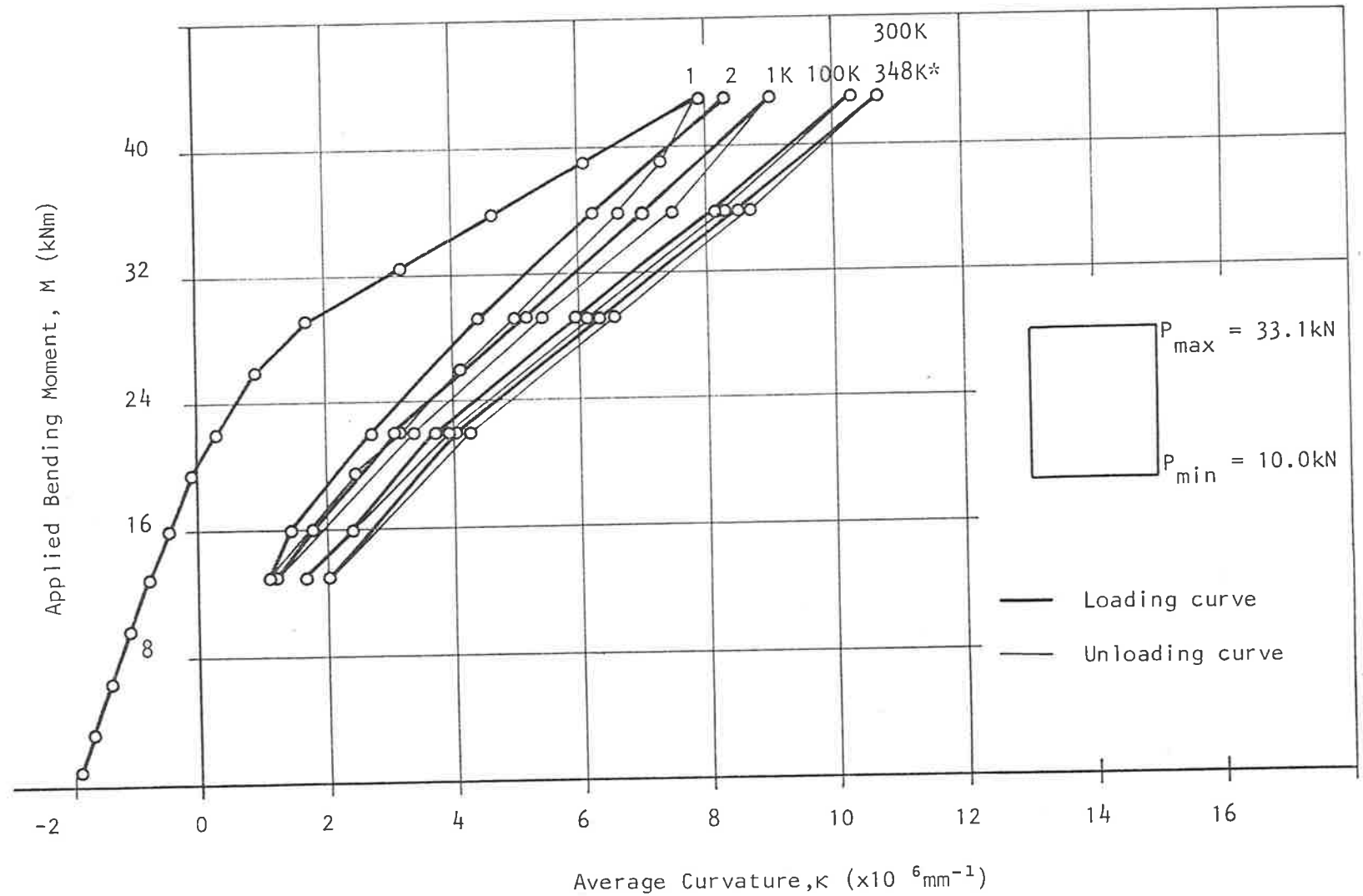


Fig 3.14 MOMENT - CURVATURE RELATIONS FOR BEAM SA1

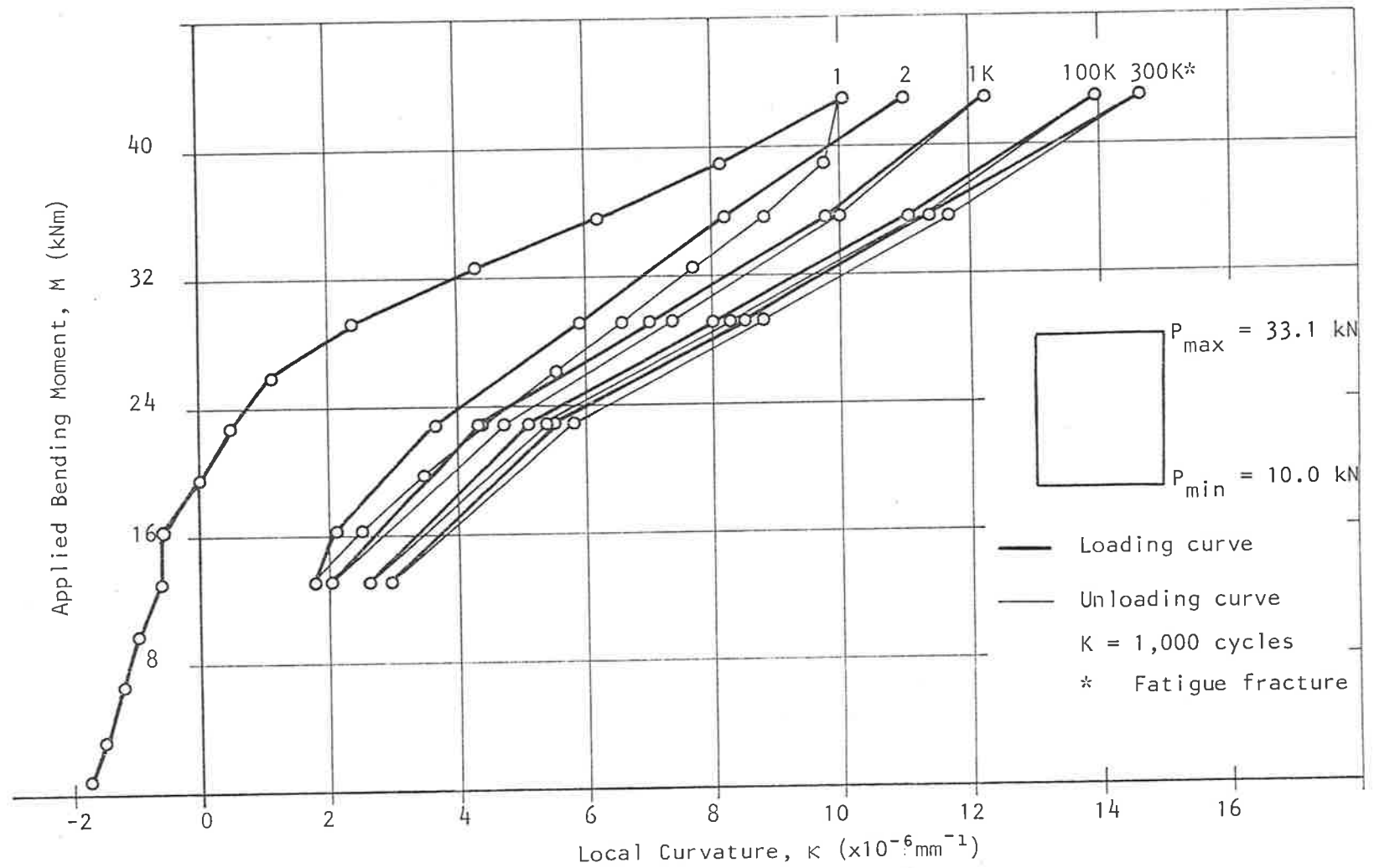


Fig 3.15 LOCAL MOMENT - CURVATURE RELATIONS FOR BEAM SA1

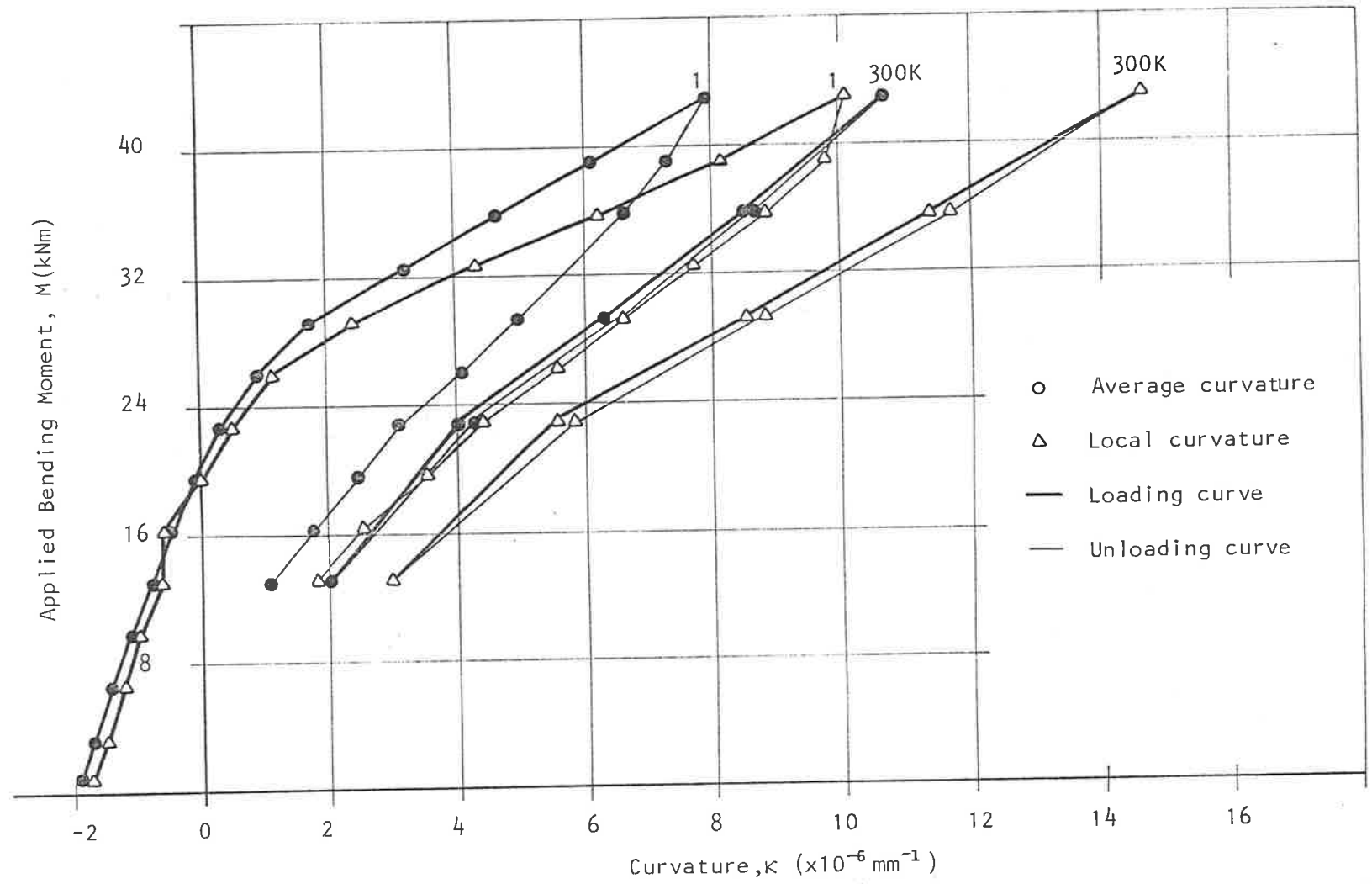


Fig 3.16 AVERAGE AND LOCAL MOMENT - CURVATURE RELATIONS FOR BEAM SA1

of the cycle (minimum load).

Second and Subsequent Load Cycles

The second load cycle follows closely the first unloading curve. The opening of cracks is indicated by a bending over of the curve, and commences at an earlier load. Slight increases in curvature are observed at higher loads. Beam stiffness in the loading direction is greater compared to the first cycle. The unloading path is similar to the first cycle. The hysteretic effect of the second cycle is markedly less than the first.

The third load cycle is similar to the second but with slight increases in curvatures. A progressive response in beam behaviour is thus observed in these early load cycles. The largest changes occurred in the first load cycle and reduced with further loading. Increases in curvature at all load levels occurred as a consequence of repeated loading. Larger increases were recorded at the higher load levels, while smaller values were measured in lower levels. The result is a decrease in stiffness with repeated loading.

After application of thousands of load cycles, there is a tendency for the beams to stabilize and give a consistent, near-linear and near-elastic response. The more lightly loaded beams settled after twenty or thirty thousand cycles, and thereafter displayed a near-constant response to load. However, in the more heavily loaded beams, deformations and curvatures continued to increase, albeit at a slower pace. The experimental results show that the overall response at this stage is almost bi-linear in nature with a stiffer curve in the low load ranges and a less steep portion at higher levels.

Increases with curvatures under fatigue loading are approximately 36 percent at maximum load and 500 percent at minimum load. The unusually large increase at minimum load is due to the relatively small value of the initial beam curvature.

3.2.3 Load-Deflection Relations

A typical load-deflection relationship (Beam SA 1) is shown in Fig 3.17. Deflection measurements were taken at mid-span. The curves are similar in shape to the moment curvature curves. Both hysteresis and permanent deflection increases are present in all beams. The deflection increases sustained under repeated loading vary from approximately 30 percent at maximum load to 26 percent at minimum load. Decreasing beam stiffness with repeated loading is also reflected here as demonstrated by the decreasing slope of the load-deflection curves.

Similarly shaped curves were reported by Abeles et al (1974), Sawko and Saha (1968), Loo and Wong (1984), and Harajli and Naaman (1985). The extent of hysteresis and deflection increases may differ in view of different beam and load parameters but the general shape of the load deflection curve is similar.

3.2.4 Strain-Cycle Relations

The variation in concrete strains with increasing numbers of load cycles for Beam SA 1 is shown in Fig 3.18. Strains in the concrete at 5 mm from the top, at the prestressing and reinforcing steel levels are plotted for maximum and minimum load levels. The largest increase in strain occurred in the first cycle of loading after initial cracking of the concrete. Subsequent changes in strain are more gradual. The curves show that the strains at minimum load appear to remain relatively constant with increasing load cycles while the strains at maximum load, in particular strains at the steel levels, increase further with increasing load cycles. The total increase in top fibre compressive strains varied from 8 to 17 percent over the life of the test beams while increases in bottom tensile deformations ranged from 36 to 70 percent.

At minimum load, the strain at the reinforcing steel level is in compression and then increases to tension as load is

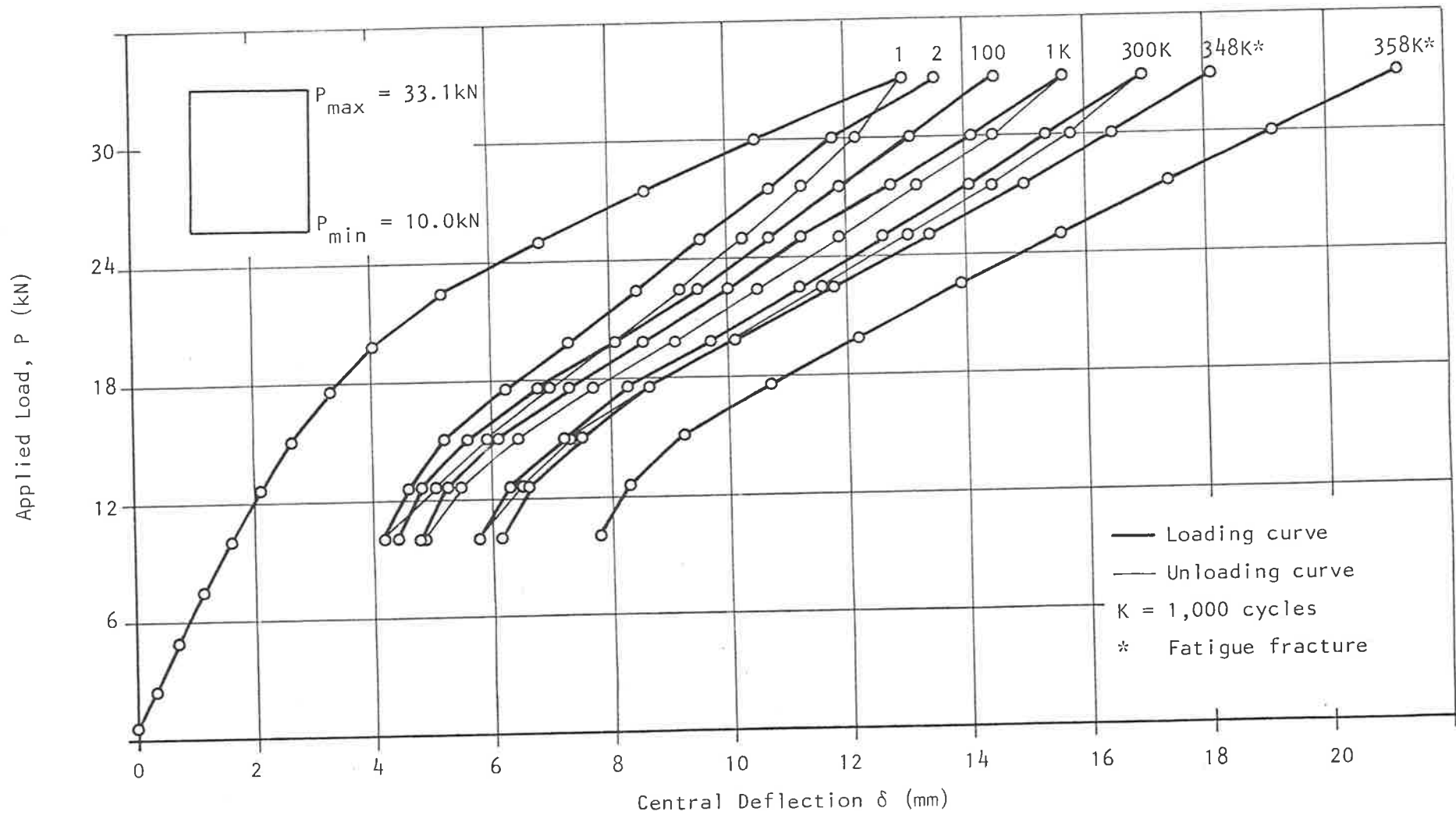


Fig 3.17 LOAD - DEFLECTION CURVE FOR BEAM SA1

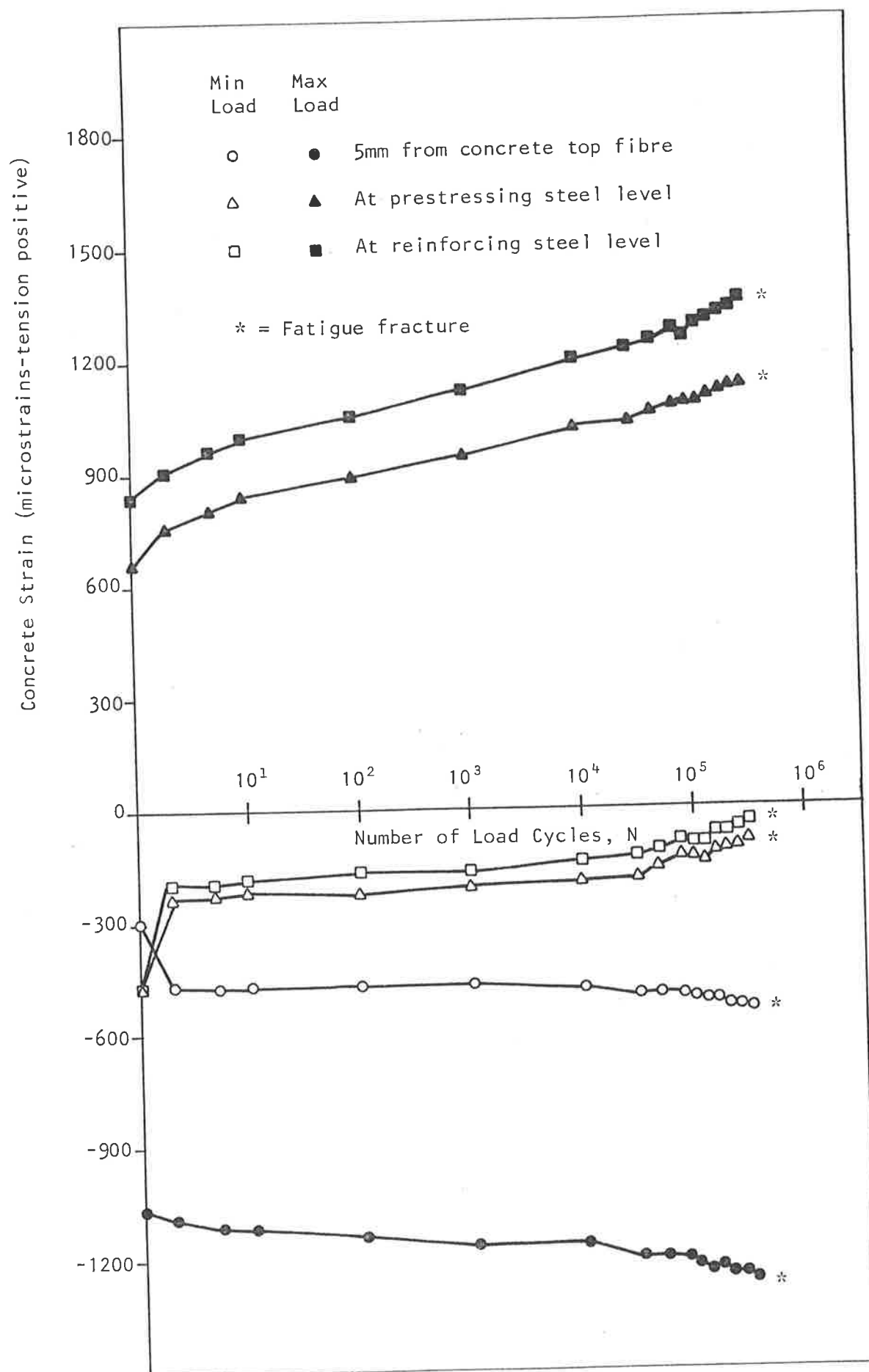


Fig 3.18 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM SA1

increased. The strain is compressive partly because of the large initial prestress applied and partly because of the effects of creep and shrinkage.

3.2.5 Curvature-Cycle Relations

The relationship between average curvature and the number of sustained load cycles for Beam SA 1 is shown in Fig 3.19. Values corresponding to maximum and minimum loads are plotted. All beams show an initial negative curvature resulting from prestressing. This then rises to positive values in the second cycle, brought about by the presence of substantial residual strains in the lower concrete region.

The curves show that curvature at minimum load remains relatively constant while curvature at maximum load level increases more appreciably with repeated loading. The difference in maximum and minimum curvature hence increases with further loading and beam stiffness decreases.

3.2.6 Deflection-Cycle Relations

The variation in midspan deflection with repeated loading for Beam SA 1 is shown in Fig 3.20. Midspan deflections corresponding to maximum and minimum loads are shown. The largest change in deflection occurs in the initial load cycle after first cracking. Subsequent loading brought more gradual increases in midspan deflection.

3.3 FATIGUE RESISTANCE

Of the six beams tested, five failed in fatigue by fracture of the reinforcing or prestressing steel or both. The number of load cycles sustained by the beams varied from over 1.24 million to a minimum of 348,430. One beam failed in flexure after it had resisted 339,837 load cycles when it was accidentally overloaded as a result of equipment malfunction.

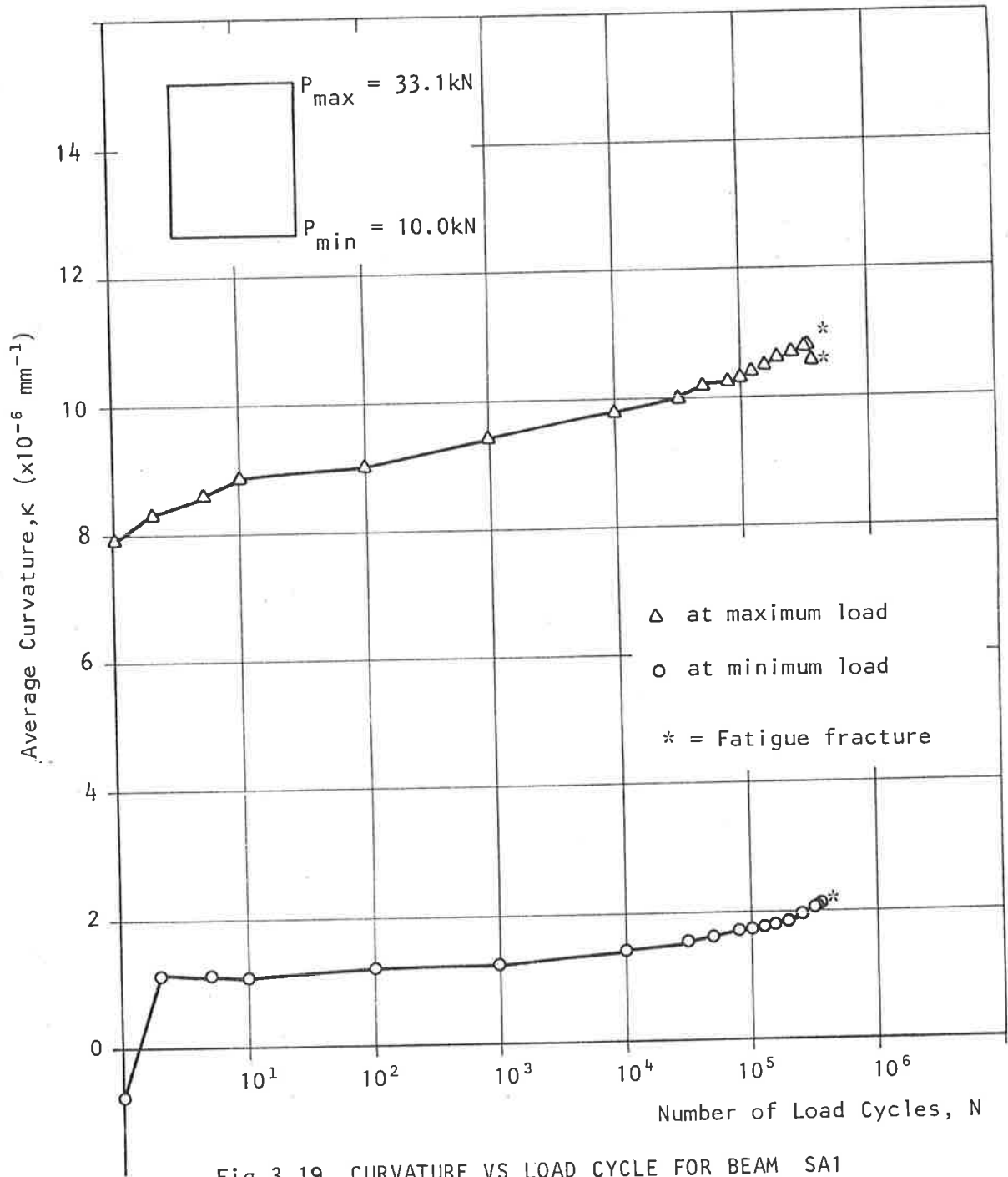


Fig 3.19 CURVATURE VS LOAD CYCLE FOR BEAM SA1

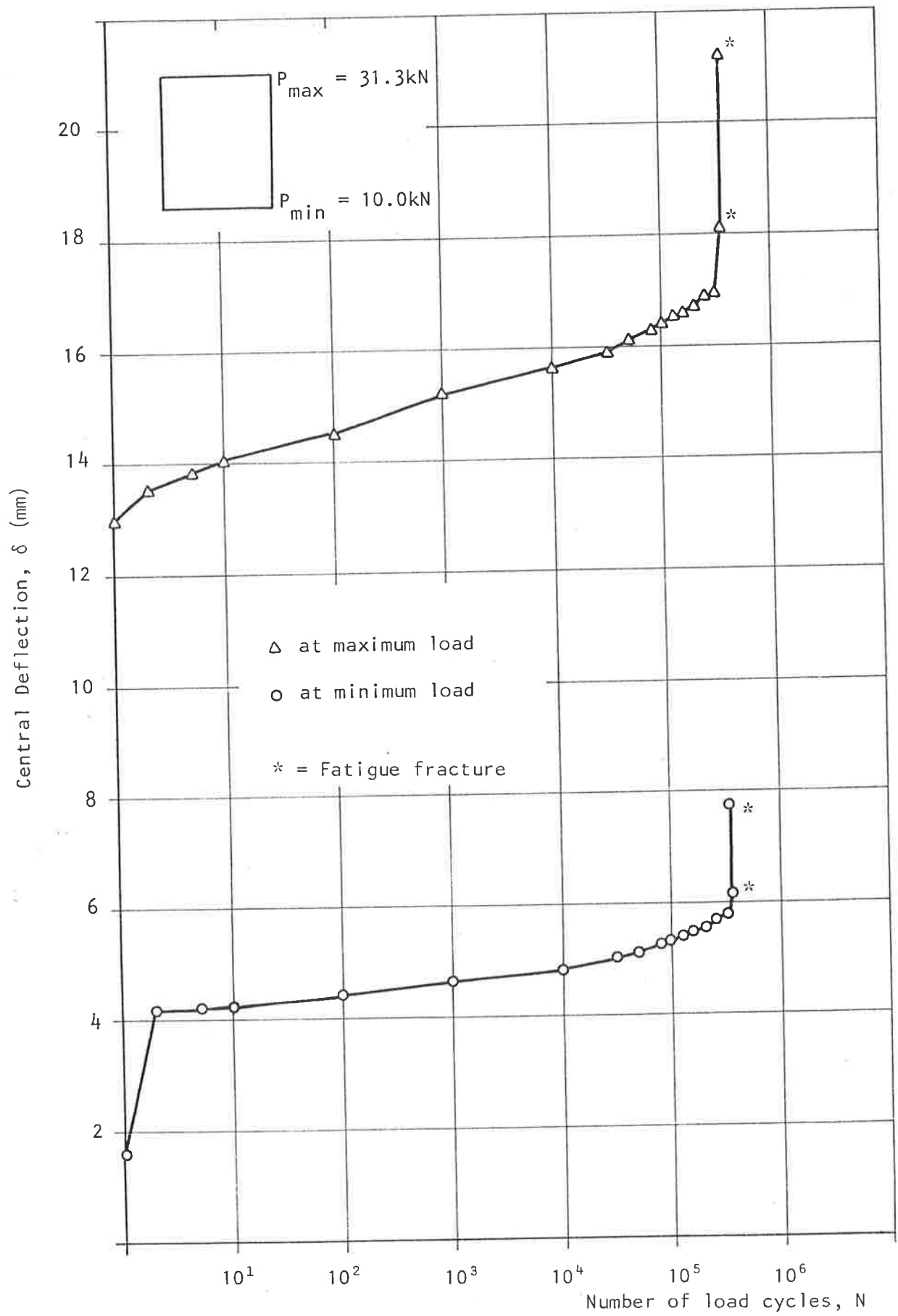


Fig 3.20 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM SA1

The fault was rectified and no further problems of this nature were encountered. Results of the fatigue tests are summarised in Table 3.8. Fig 3.21 contains further details of the test results.

In the beam tests, fatigue failure in one strand or reinforcing bar did not render the beam unserviceable. Testing was continued until progressive fractures had occurred to such an extent that either the beam was incapable of withstanding the maximum load or the deflections were judged to be excessive or unsafe. The number of load cycles between first steel fracture and final beam failure varied between 10,000 and 286,000 depending on the mode of failure and the type of fatigue loading. Significant and rapid increases in the deformations in the test region were measured as the beam was further cyclically loaded after first fracture, and corresponding changes in stresses evidently occurred.

The beam section was designed so that fatigue failure of both the prestressing and reinforcing steels were equally likely at the same loads. However, test results indicated that beams under relatively more severe loading failed in fatigue by fracture of the reinforcing bars while beams loaded under less severe loads failed by fracture of wires in the tendons. The occurrence of both bar and wire fractures in the same beam was also observed. A typical bar fracture is shown in Plate 3.5 and typical wire fractures in Plate 3.6.

Post-failure inspection of the strands and bars was undertaken by removing the cover concrete. It was found that all steel fatigue failures, whether in the strand or in the bar, took place at concrete flexural cracks. Subsequent steel fatigue fractures did not necessarily occur at the same crack. The presence of vertical stirrups did not appear to have any effect on the formation of fatigue cracks, and no fatigue cracks or signs of steel fretting were found at the stirrups.

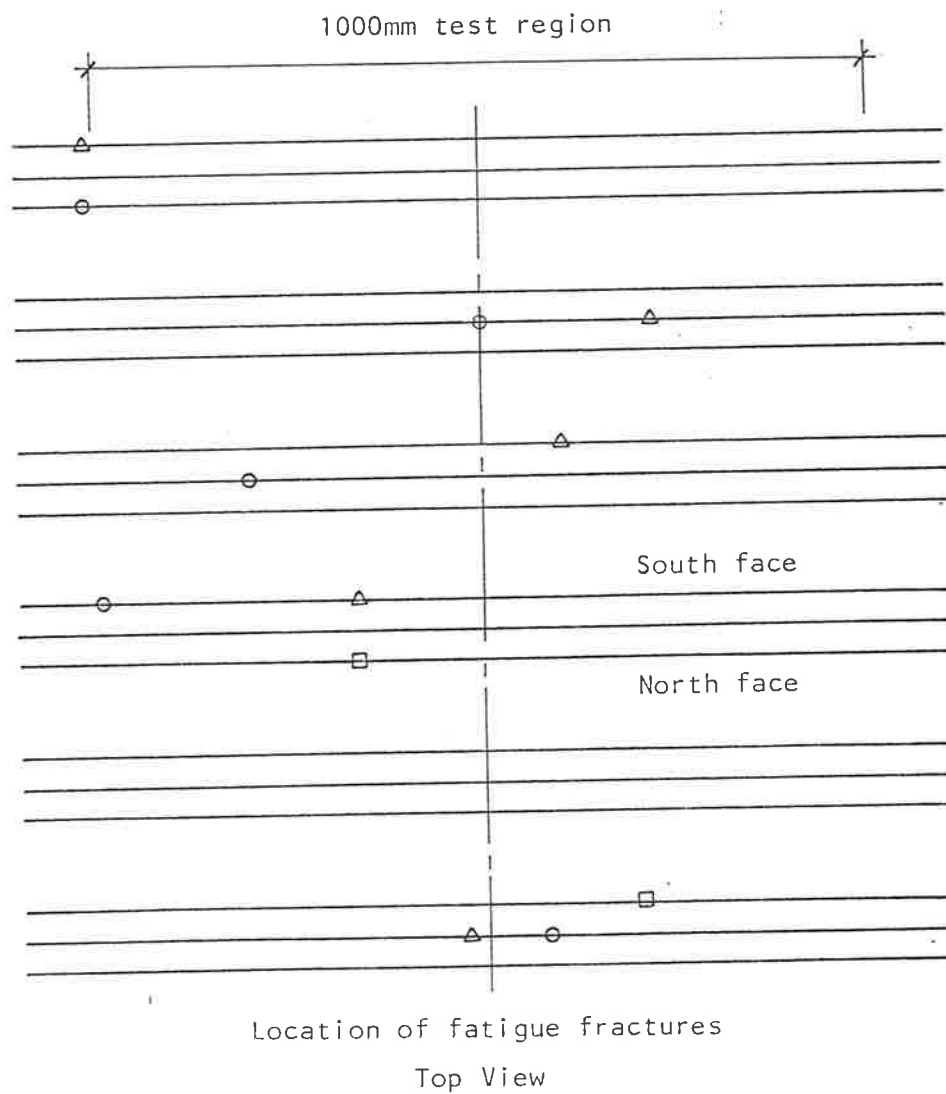
Wherever the concrete flexural cracks in a beam varied significantly in width, the first steel fracture tended to occur at the widest crack. This was observed in two of the

Table 3.8

Beam Fatigue Test Results

Beam	Load A/B	Configuration P(max)	P(pred)	N (first fracture)	N (second fracture)	N (third fracture)	N (fourth fracture)	N (fifth fracture)	Failure Mode
PA 1	0.25	33.13	29.25	339,837F	-	-	-	-	Flexural failure
PA 2	0.00	29.25	29.25	954,200W	982,600W	1,203,642W	1,236,000W	1,240,000 W,B	7 Wire & 1 Bar
SA 1	0.00	33.13	33.13	348,430B	358,200B	-	-	-	2 Bar fractures
SA 2	0.25	33.13	29.25	692,380W	692,383W	803,230W	803,340W	-	4 Wire fracture
SA 3	0.50	33.13	22.50	844,690W	956,430W	1,121,970B	-	-	2 Wire & 1 Bar
SA 4	0.50	33.13	29.25	505,740B	539,830B	573,330B			3 Bar fracture

Notes: B - Bar fracture
W - Wire fracture
F - Flexure failure



BEAM		NUMBER OF CYCLES AT FAILURE	
SA1	$P_{\max} = 33.1 \text{ kN}$	○	348,430
		△	358,200
SA2	$P_{\max} = 33.1 \text{ kN}$ $P_{\text{pred}} = 29.3 \text{ kN}$	△	692,380
		○	692,383
		○	803,230
		○	803,340
SA3	$P_{\max} = 33.1 \text{ kN}$ $P_{\text{pred}} = 22.5 \text{ kN}$	○	844,690
		○	956,430
		△	1,121,970
SA4	$P_{\max} = 33.1 \text{ kN}$ $P_{\text{pred}} = 29.3 \text{ kN}$	○	505,740
		△	539,830
		□	573,330
PA1	$P_{\max} = 33.1 \text{ kN}$ $P_{\text{pred}} = 29.3 \text{ kN}$	339,837 Flexure failure	
PA2	$P_{\max} = 29.3 \text{ kN}$	○	954,200
		△	982,600
		△	1,203,642
		△	1,236,000
		△	1,240,000 (3 wires)
		□	1,240,000

Fig 3.21 DETAILED BEAM FATIGUE TEST RESULTS



Plate 3.5 Typical Bar Fracture

test beams, where the critical cracks were about twice as wide as the others. In the remaining beams it was not possible to make similar observations as the cracks were found to be of the same width. Some difficulty was experienced in the accurate measurement of cracks, as the width was affected randomly by the presence of fine sand in the beam surface at the designated point of measurement.

For the reinforcing steel, the fatigue crack usually initiated on the side of a helical rib. The crack then propagated from both ends until fracture resulted. This behaviour was also observed in fatigue tests on in-air bars (Behan and Warner, 1984). In some cases it was found that a fractured bar had developed more than one fatigue crack and occurred at the location of primary concrete cracks. Two such fatigue cracks were observed. One of these is shown in Plate 3.7. With the seven-wire prestressing strand, there was no favoured location for crack initiation, although the fractures did occur in the outer six wires.

3.3.1. Beam PA 1

Beam PA1 was tested with a cumulative damage load pattern with a predominant load of 33.1 kN. Both maximum and predominant loads were greater than the cracking load. The test configuration was chosen to study beam response under combined cycles of high and low load. Testing proceeded without any difficulties until equipment malfunction caused the hydraulic actuator to overload the beam, thus causing the beam to fail in static flexure. The machine malfunction was traced to a faulty connection in the measuring displacement transducer in the load cell of the actuator.

3.3.2 Beam PA 2

Beam PA2 was tested with a constant load cycle. The maximum load of 29.3 kN continued until failure in fatigue took place. Failure first occurred at 954,200 cycles by fracture in one

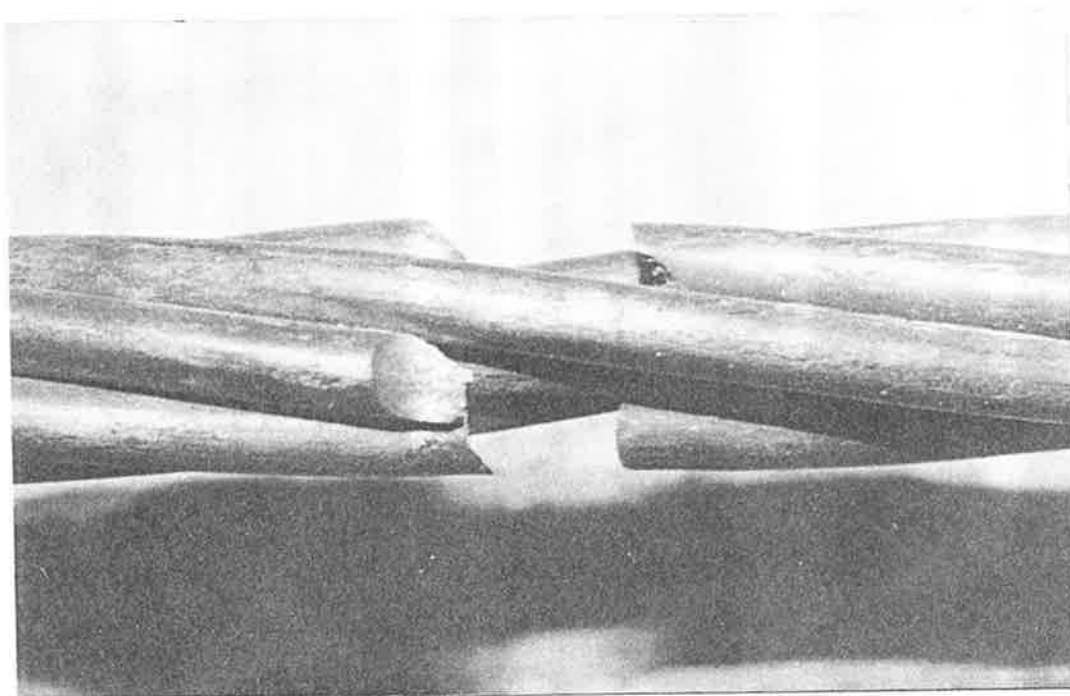


Plate 3.6 Typical Wire Fractures in Prestressing Strand

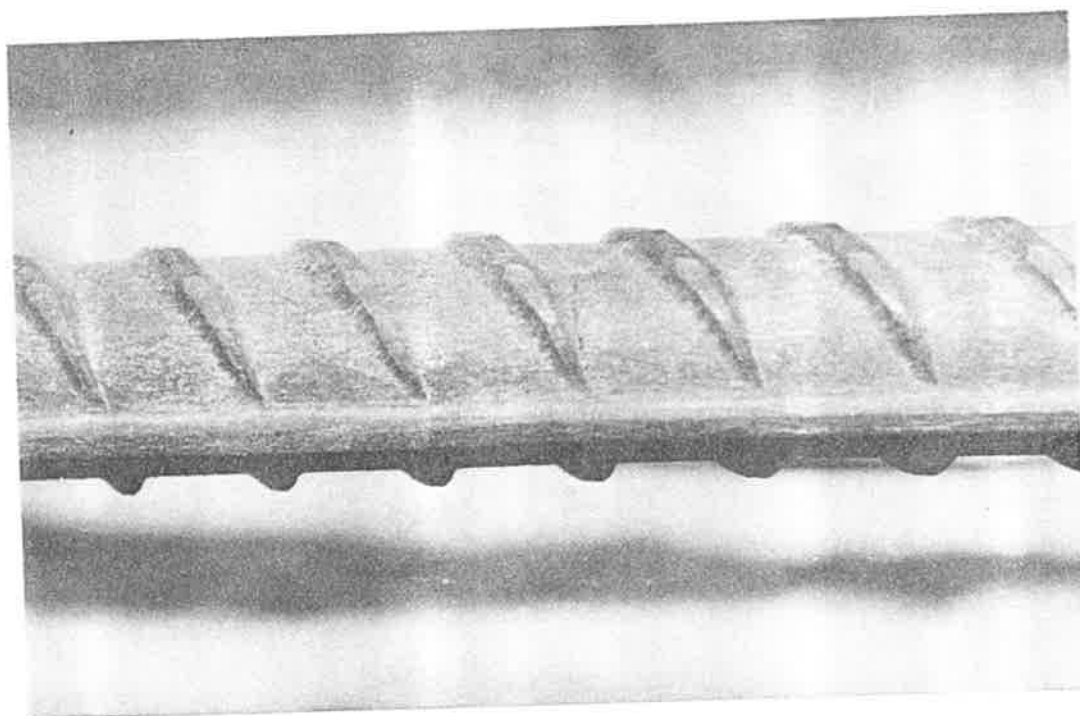


Plate 3.7 Early Fatigue Crack in Bar

wire of the strand. A second wire fractured at 982,600 cycles. The third and fourth wires fractured at 1,203,642 and 1,236,000 cycles. A bar and remaining wires ruptured at 1,240,000 cycles.

Unsuccessful attempts were made to investigate local tensile strains in this beam in the concrete between cracks. A total of six columns of six rows of electric strain gauges were placed on each side of the test beam. Three columns of gauges were used between two pairs of flexure cracks. The strain gauges were attached after the test beam had been initially loaded and cracks had formed.

The results showed poor agreement with the average demec strain readings. This was attributed in part to the use of strain gauges of relatively short (20mm) lengths and also to the influence of aggregate directly beneath the gauges. Use of longer strain gauges was not possible as the objective was to measure local strains. Furthermore the relatively small crack spacings (around 100 mm) prevented the placement of longer gauges.

3.3.3 Beam SA 1

Beam SA1 was tested with a constant load cycle similar to that of Beam PA2, but with the maximum load increased to 33.1 kN to study the effect of a larger load cycle. The first fatigue failure occurred at 348,430 cycles in the reinforcing steel at a flexural crack close to one of the load points. The second failure occurred in the other reinforcing bar at the same crack after a further 9,770 load cycles. Testing was stopped at this point as the beam was unable to withstand further loads safely.

3.3.4 Beam SA 2

Beam SA 2 was subjected to cumulative damage loading identical to that used for Beam PA1. Fatigue fracture occurred first in

the prestressing tendon at 692,383 load cycles with two wires fracturing within 3 cycles of each other. The fractures did not coincide but occurred in two adjacent concrete flexural cracks. This indicated that both cracks had formed at about the same time and developed in a parallel manner. Despite an appreciable change in concrete strain, the beam was still capable of withstanding further loads. The third and fourth wires fractured at 803,230 and 803,340 load cycles in the same section.

In addition to the four wire fractures in the strand, a small fatigue crack developed in one of the reinforcing bars, at the crack containing three of the fractured prestressing wires.

3.3.5 Beam SA 3

Beam SA 3 was subjected to cumulative damage loading. The predominant load of 22.50 kN caused the concrete flexural cracks to open up but was not sufficiently large theoretically to cause fatigue damage. The maximum load was set at 33.1 kN. The beam was tested to check whether the predominant load cycles had any effect on its fatigue beam. The load block differed from the other cumulative damage tests in that the higher load was applied first rather than second in the block. This caused more cracking initially with a larger residual inelastic deformation.

Fatigue failure of one wire took place at 844,690 cycles and a second was detected by the accelerometer detection device at 956,430 cycles. Final failure of the beam occurred by fatigue of a reinforcing bar at 1,121,970 cycles.

3.3.6 Beam SA 4

Beam SA4 was tested with a cumulative damage load pattern similar to that for Beam SA2, but with a higher percentage of maximum loads in the test block. First fatigue failure took place in the reinforcing steel at 505,740 cycles. The fracture

was located about 10 mm away from one of the load points, ie. the same distance away from the closest vertical shear stirrup. The second fatigue failure, at 539,830 cycles, was in the same reinforcing bar three crack spacings away from the initial fracture. The third and final failure occurred in the other reinforcing bar at a position close to the second fracture at 573,330 cycles.

The occurrence of two fatigue cracks along the same bar at a distance of 300mm apart (three crack spacings) suggested very good bonding between concrete and reinforcing steel. This was verified by distinct impressions of the bar deformations in the concrete (see Plate 3.8). The detection of two fatigue cracks in the same bar within 40,000 load cycles of each other, suggests that both cracks initiated at approximately the same time and developed independently, to reach one hundred percent damage at the end of the test.

3.3.7 Summary

Constant amplitude and cumulative damage fatigue tests on six partially prestressed concrete flexural members demonstrated significant changes in beam response in the post-cracking range. Increases in flexural deformation (curvature) strains and midspan deflections, of the order of 30 to 40 percent, occurred between the first load cycle and the first steel fracture. Overall beam stiffness was reduced correspondingly. Hysteretic effects were significant in the early load cycles, and were found to be greatest in the first loading-unloading cycle.

Fatigue failure took place by successive fracture of the elements of tensile steel reinforcement, the prestressing steel, or of both steels. None of the beams failed by fatigue of the compressive concrete.

Further discussions and analyses of the test data will be made in later chapters.

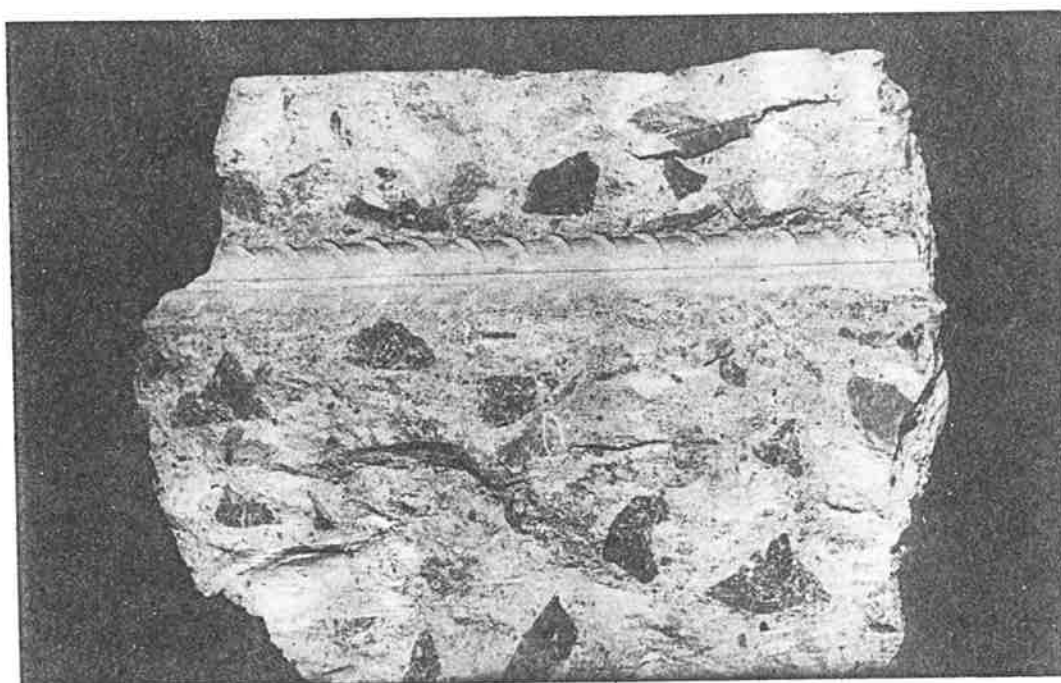


Plate 3.8 Imprints of Bar Deformation on Concrete

CHAPTER 4

BEAM BEHAVIOUR IN THE PRECRACKING RANGE

An analysis of precracking behaviour is needed to provide estimates of the stress and strain conditions in a member just prior to the application of repeated loading. This is important in prestressed structures where time-dependent effects, sustained prior to loading, can influence later behaviour. Factors such as creep and shrinkage are therefore critical in fatigue predictions, where accurate evaluation of stresses is essential.

The initial stress and strain conditions are also influenced by the type of prestressing. Pretensioned members tend to undergo more prestress losses than post-tensioned members as the transfer of prestress takes place earlier, and may result in larger inelastic deformations due to time-dependent effects. In the initial phases of a prestressed member, the normally 'tensile' reinforcement sustains compressive stresses and in some cases these stresses may be quite considerable even with no or nominal loads applied.

This chapter examines the behaviour of partially prestressed concrete flexural members in the precracking range, from the time of casting up to just before cracking. A theoretical method of analysis is developed which accounts for the time-dependent effects of creep and shrinkage as well as the non-linear, history dependent nature of the material properties. An empirical procedure for the calculation of the total inelastic concrete strain is also used to evaluate the accuracy of the analytic predictions of creep and shrinkage in the test beams.

4.1 UNCRACKED SECTION ANALYSIS

The following analysis is applicable mainly to pretensioned members. The analysis for post-tensioned members is outside the scope of this thesis and is not considered. However, the analysis can be easily developed along similar lines for use if needed.

The following basic assumptions are made in the proposed analysis:

- a) plane sections remain plane;
- b) full strain compatibility exists between steel and concrete;
- c) shrinkage effects from time of casting to transfer of prestress are negligible;
- d) temperature effects are negligible;
- e) creep and shrinkage effects _{after transfer} are significant.

4.1.1 Representation of Section

Partially prestressed concrete members are made up of concrete, reinforcing and prestressing steel. The concrete in the section is represented by a discrete number of horizontal elemental slices or layers which are symmetrical about a vertical axis. This is illustrated in Fig 4.1a. Stress and strain are determined for each slice. The method is applicable to both uniform and irregular sections. Fig 4.1b shows the layering technique applied to an irregular section. For such sections, the width and thickness of each individual slice can be varied to provide accurate representation of the actual geometrical shape. The accuracy of the analysis is, to a certain extent, dependent on the number of slices used. A larger number provides a more precise analysis but this is offset by the longer computational time. Because of computer capacity and time constraints, the maximum number of slices in this analysis has been limited to a hundred.

The steel reinforcement is represented by discrete steel

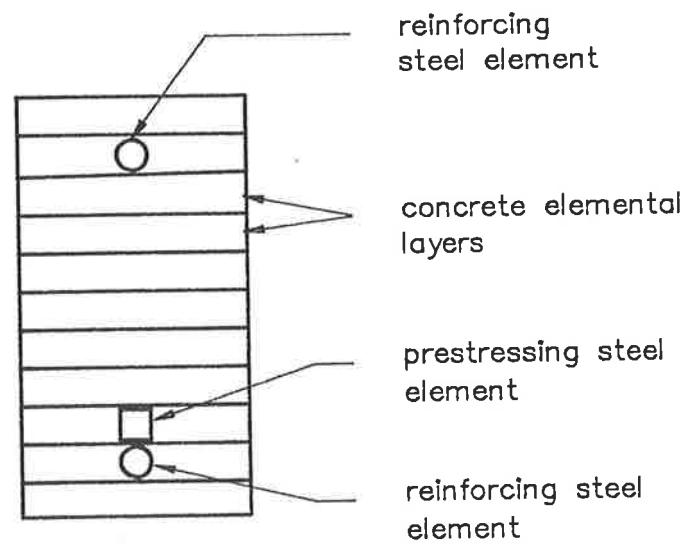


Fig. 4.1a Representation of a Regular Section

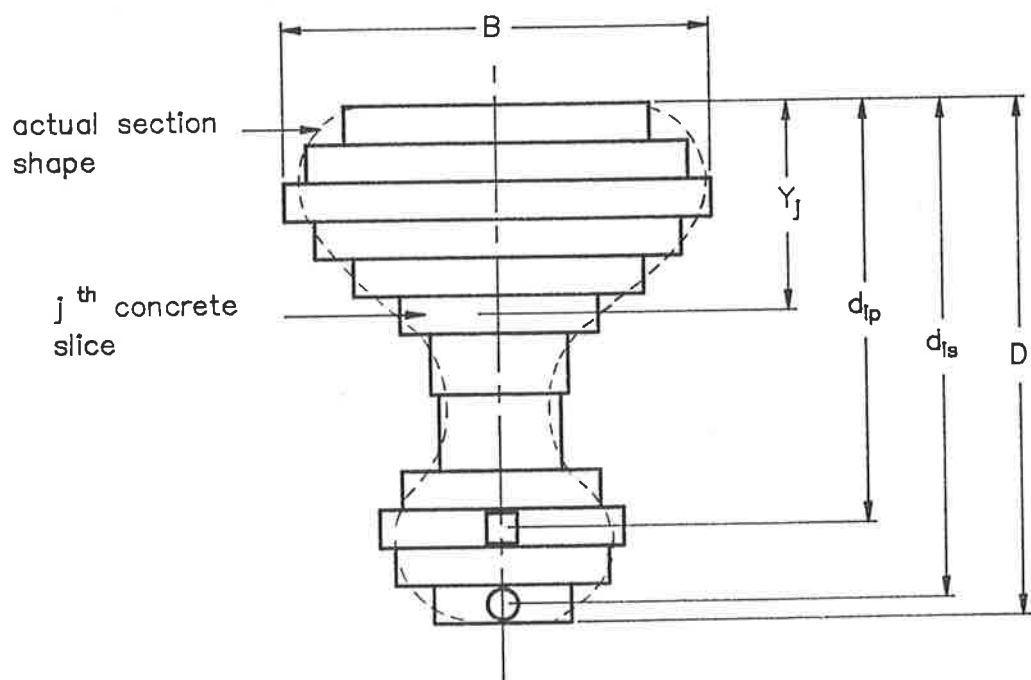


Fig. 4.1b Representation of an Irregular Section

elements. Reinforcement of the same type and located at the same depth are represented by a single element. The representation of prestressing tendons is similar to that for ordinary steel. Initial prestress is taken into account by introducing initial elastic steel strains (and therefore stresses) in the steel elements and also in the concrete. For simply reinforced concrete members, the number of prestressing elements can be set to zero.

4.1.2. Material Constitutive Relations

The properties of the component materials have been discussed in Chapter 2. Experimental stress-strain curves of concrete, prestressing and reinforcing steels have been described in Section 3.1.3. and are shown in Figs 3.2a, 3.5 and 3.6 respectively.

Concrete Stress-Strain Relations

Separate equations are used for compressive and tensile behaviour. For concrete in compression, a realistic non-linear relation which is derived from laboratory testing is used:

For compressive stresses in the first cycle - loading direction:

$$\sigma = G_c \{ \epsilon \} \quad (4.1)$$

where σ and ϵ are the concrete stress and strain and G_c the function defining the constitutive relation:

$$G_c(z) = 0.5 z F_c \exp(1. - 0.4z) \quad (4.2)$$

For the unloading direction, the expression is:

$$G_c(z) = \sigma_{max} - 0.58 (z_{max} - z) F_c \exp(1. - 0.5(z_{max} - z)) \quad (4.3)$$

where z is the normalised elastic strain,

$$z = \frac{\epsilon_s}{\epsilon_{ult}}$$

z_{max} is the maximum normalised strain, σ_{max} is the maximum stress reached in any previous or current load cycle, ϵ_s is the elastic concrete compressive strain, ϵ_{ult} is the strain at maximum stress, and F_c is ultimate concrete strength in compression.

For concrete in tension, a simple linear relationship is used,

$$\sigma = E_t \epsilon \quad (4.4)$$

where E_t is the tangent modulus of concrete in tension.

Prestressing Strand Stress-Strain Relations

The prestressing steel shows linear behaviour for the majority of the stress range until the near-yield region.

For $\epsilon_p \leq 0.007$,

$$f_p = E_p \epsilon_p \quad (4.5)$$

For $\epsilon_p \geq 0.007$,

$$f_p = E_p \epsilon_p - 22.2 * 10^6 (\epsilon_p - 0.007)^2 \quad (4.6)$$

where ϵ_p is the strain in the strand, f_p the sustained stress and E_p the tangent modulus.

Reinforcing Steel Stress-Strain Relations

Elastic-plastic behaviour is assumed for ordinary steel.

For $\epsilon_s \leq \epsilon_y$,

$$f_s = E_s \epsilon_s \quad (4.7)$$

For $\epsilon_s \geq \epsilon_y$,

$$f_s = f_y \quad (4.8)$$

where ϵ_s and ϵ_y are the steel strain and yield strain respectively and f_s and f_y the respective steel stress and yield stress.

Creep in Concrete

Creep in concrete is a stress and time dependent phenomenon. The inelastic concrete strain due to creep depends on the sustained stress level, the ambient surrounding conditions, the age at which first load was applied and the period of load sustained.

Although the numerical evaluation of creep strains is based on the work conducted by Warner and Lambert (1974), the method of analysis can cater for almost any desired creep law. A detailed review of creep theory and computational procedures is outside the scope of the current investigation. Only a brief description is given of the procedure employed.

A rate of creep model, consisting of three components is used:

- a) $\epsilon^d(t)$ Dischinger component which is linear, ageing and non-recoverable;
- b) $\epsilon^v(t)$ Viscoelastic component which is linear, non-ageing and recoverable;
- c) $\epsilon^n(t)$ non-linear component.

The rate of creep equation can be expressed as:

$$\dot{\epsilon}_{cr}(t) = \dot{\epsilon}^d(t) + \dot{\epsilon}^v(t) + \dot{\epsilon}^n(t) \quad (4.9)$$

The Dischinger component is:

$$\dot{\epsilon}^d(t) = \epsilon^s(t) \dot{\phi}^d(t) \quad (4.10)$$

where $\epsilon^e(t)$ is the linear-elastic component of the instantaneous strain and $\dot{\phi}^d(t)$ is the Dischinger creep function. The viscoelastic component is defined as:

$$\epsilon^v(t) = [\epsilon^e(t) \phi^{v*}(t)] \frac{1}{T_v} \quad (4.11)$$

Also, $\phi^{v*} = \alpha_v \phi_*$ (4.12)

$$\phi^{d*} = \alpha_d \phi_* \quad (4.13)$$

$$\phi^d(t) = \phi(t) - \phi^{v*}(1 - e^{-t/T_v}) \quad (4.14)$$

$$\phi_* = \frac{\epsilon_{cr}(\infty)}{\epsilon^e} \quad (4.15)$$

where ϕ_* is defined as the creep function at time infinity, and T_v is a time constant. Both quantities are determined experimentally. Warner (1969) recommended that:

$$30 \leq T_v \leq 60$$

$$\alpha_d \leq 1.0$$

$$\alpha_v \leq 1.0$$

The following values were used in creep calculations:

$$\phi^{v*} = 0.52$$

$$\phi^{d*} = 1.23$$

$$\phi_* = 1.75$$

$$T_v = 30.$$

A typical curve for the creep function is shown in Fig 4.2.

Shrinkage in Concrete

Shrinkage of structural concrete generally results from the loss of absorbed water from the cement paste. Temperature, humidity, concrete mix proportions and member size all influence concrete shrinkage. Sustained stresses however do not seem to have any significant effect on shrinkage strains.

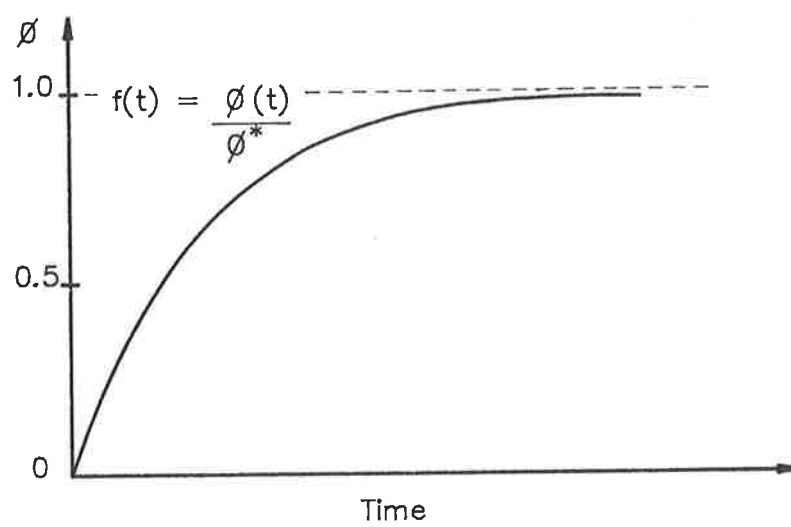


Fig. 4.2 Creep Function Curve

The time dependent inelastic shrinkage strains can be expressed simply as

$$\epsilon_{s,h}(t) = \epsilon^*_{s,h} g(t) \quad (4.16)$$

where $\epsilon^*_{s,h}$ is the maximum shrinkage strain at time infinity and $g(t)$ is an experimentally obtained pure time function increasing monotonically from 0 to 1. A value of 0.00035 was obtained experimentally for $\epsilon^*_{s,h}$. The shrinkage function curve is similar to the creep relation. Latham's (1981) parameters for the creep and shrinkage function curves are assumed in this analysis. For beam analysis, the shrinkage strain is assumed to be constant throughout the entire section at a particular time. Experimental data for creep and shrinkage strains have been reported by Bakoss (1983).

4.1.3. Overview of Computational Procedure

The proposed analysis is based on two conditions of equilibrium, moment and horizontal force equilibrium. Given the section properties and the loading details, the method calculates the stress and strain conditions at selected instants on the time scale by satisfying equilibrium and compatibility requirements and allowing free creep and shrinkage of the concrete to occur in the intervening intervals. A numerical method is used to search for the values of the top and bottom-fibre strains which satisfy both equilibrium and compatibility conditions at each time instant.

An initial value of the top-fibre strain is estimated. A trial value of the bottom strain is then chosen. These two values define the linear strain distribution across the section. Concrete and steel stresses are subsequently calculated from the strain distribution. The total tensile and compressive forces are calculated and force equilibrium is checked. If equilibrium is not satisfied then a new value of the bottom-fibre strain is chosen. Trial values are successively improved until equilibrium is obtained to within prescribed

tolerances. The search for the appropriate value of the bottom strain is performed by an efficient and systematic halving procedure.

Once the correct bottom-fibre strain is found, equilibrium for moment is checked. If equilibrium is not satisfied, the choice of the top-fibre strain is adjusted and the corresponding bottom-fibre strain required to maintain force equilibrium is again determined. Moment equilibrium is then re-checked. The whole procedure is repeated until both moment and force equilibrium are satisfied. The search for the top-fibre strain is carried out by a procedure similar to that used for the bottom strain.

Two levels of searches are therefore used in this analysis. The first involves seeking a bottom-fibre strain which satisfies force equilibrium. The second, at a higher level, searches for an appropriate top-fibre strain which satisfies moment equilibrium. The stresses at a section, corresponding to a given time point and applied moment in the precracking range and taking into account the previous load history, are thus computed.

4.2 ANALYSIS AT TRANSFER

4.2.1 Concrete and Steel Strains

Concrete Strains

A simple rectangular beam section with both prestressing and reinforcing steel, and the corresponding strain and stress diagrams at transfer are shown in Fig 4.3. Before transfer the concrete and reinforcing steel strains in the section are zero, assuming prior shrinkage in the concrete does not take place. After transfer, elastic deformations in the concrete are sustained resulting in a linear concrete strain distribution.

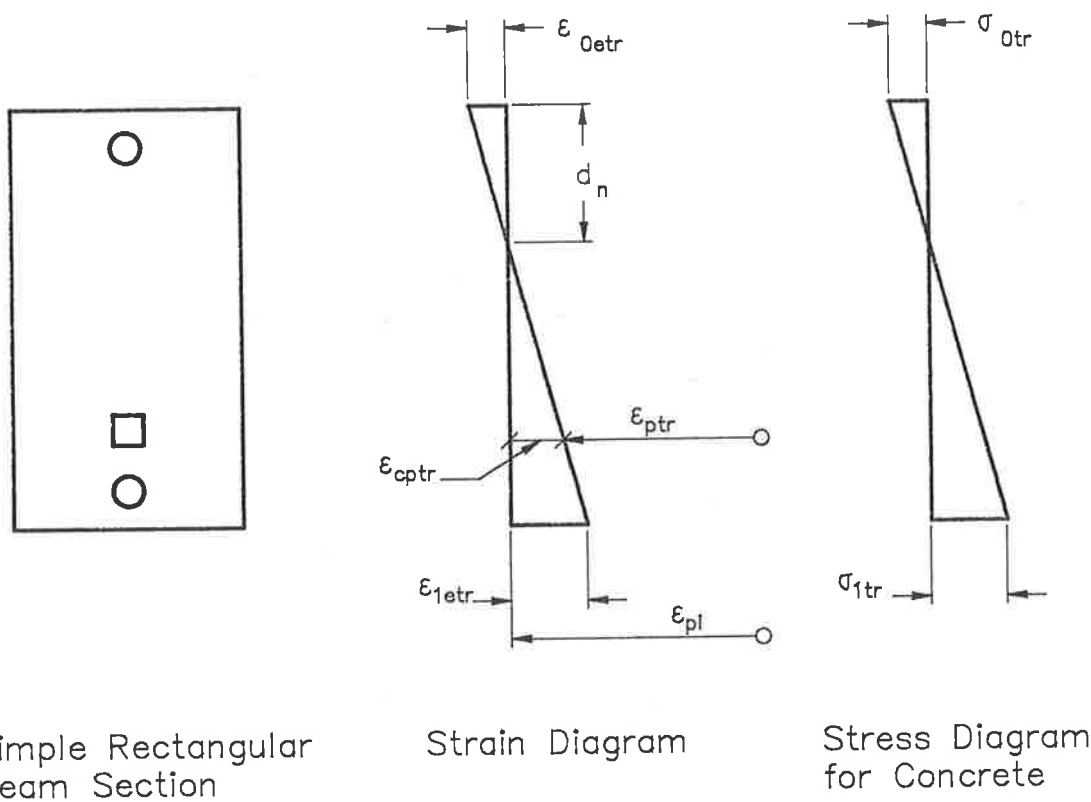


Fig. 4.3 Stress and Strain Conditions After Transfer

In prestressed members, the initial prestress may often be large enough to cause tensile strains in the top concrete fibres. The top-fibre tensile strain in the concrete is $\epsilon_{0 \text{ e t r}}$ and the compressive bottom-fibre strain is $\epsilon_{1 \text{ e t r}}$. The strain neutral axis is at depth d_n , where

$$d_n = \frac{\epsilon_{0 \text{ e t r}} D}{(\epsilon_{1 \text{ e t r}} + \epsilon_{0 \text{ e t r}})} \quad (4.17)$$

where d_n is the depth of the strain neutral axis from the extreme top-fibre and D is the total depth of the section.

From geometry, the compressive strain ϵ_j in the j^{th} concrete element at a depth y_j from the top-fibre is :

$$\epsilon_j = \frac{y_j}{D} \epsilon_{1 \text{ e t r}} - \frac{(D - y_j)}{D} \epsilon_{0 \text{ e t r}} \quad (4.18)$$

where y_j is the depth of the centroid of the j^{th} concrete slice measured from the extreme top fibre.

Steel Strains

As strain compatibility between concrete and steel exists, the steel strain is therefore equal to the concrete strain. For the i^{th} reinforcing steel element, the compressive steel strain is:

$$\epsilon_{i s} = \frac{d_{i s}}{D} \epsilon_{1 \text{ e t r}} - \frac{(D - d_{i s})}{D} \epsilon_{0 \text{ e t r}} \quad (4.19)$$

where $d_{i s}$ is the depth of the i^{th} steel element measured from the extreme top fibre

The compressive concrete strain at the i^{th} prestressing steel element is:

$$\epsilon_{i \text{ c p t r}} = \frac{d_{i p}}{D} \epsilon_{1 \text{ e t r}} - \frac{(D - d_{i p})}{D} \epsilon_{0 \text{ e t r}} \quad (4.20)$$

where $\epsilon_{i \text{ c p t r}}$ is the elastic shortening of the concrete due to prestress. The tensile strain in the i^{th} prestressing steel element is reduced by an equal amount and the strain

immediately after transfer is:

$$\begin{aligned}\epsilon_{i p t r} &= \epsilon_{i p i} - \epsilon_{i c p t r} \\ &= \epsilon_{i p i} - \frac{d_{i p}}{D} \epsilon_{i e t r} + \frac{(D - d_{i p})}{D} \epsilon_{e e t r}\end{aligned}\quad (4.21)$$

where $\epsilon_{i p i}$ is the initial strain in the i^{th} prestressing steel element before transfer and $\epsilon_{i p t r}$ is the tensile strain after transfer.

4.2.2 Stresses at Transfer

The concrete stress in the j^{th} slice is related to the elastic concrete strain by the material constitutive relation. No inelastic strain is assumed in the concrete immediately after transfer. The total concrete strain is therefore equal to the instantaneous elastic strain.

For compressive elastic strain,

$$\sigma_j = G_c \{ \epsilon_j \} \quad (4.22)$$

where G_c is the function representing the concrete compression stress-strain curve described in Section 4.1.2.

For tensile elastic strain,

$$\sigma_j = E_t \epsilon_j \quad (4.23)$$

where E_t is the tangent modulus of concrete in tension.

The steel stresses are also related to their material stress-strain relations. For prestressing steel,

$$f_{i p} = G_p \{ \epsilon_{i p} \} \quad (4.24)$$

For reinforcing steel:

$$f_{i s} = G_s \{ \epsilon_{i s} \} \quad (4.25)$$

where G_c and G_s are the respective functions for the material stress-strain relations.

4.2.3 Internal Forces and Moments

The total compressive force sustained by the concrete is obtained by summing the individual forces in the slices,

$$\begin{aligned} C_c &= \sum_{j=1}^{N^A} \sigma_j dA_j \\ &= \sum_{j=1}^{N^A} \sigma_j b_j t_j \end{aligned} \quad (4.26)$$

where b_j and t_j are the width and thickness of the j^{th} concrete slice. For prestressing steel elements, the total tensile force in the tendons is:

$$T_p = \sum_{i=1}^{N^P} A_{i,p} f_{i,p} \quad (4.27)$$

where $A_{i,p}$ is the area of the i^{th} prestressing steel element. For the reinforcing steel the total compressive force is:

$$T_s = \sum_{i=1}^{N^S} A_{i,s} f_{i,s} \quad (4.28)$$

where $A_{i,s}$ is the area of the i^{th} reinforcing steel element. The total concrete moment about the extreme concrete top-fibre is:

$$M_c = \sum_{j=1}^{N^A} \sigma_j b_j t_j y_j \quad (4.29)$$

where y_j is the distance of the centroid of the j^{th} slice from the top fibre. Similarly, taking moments about the top fibre for the prestressing steel elements,

$$M_p = \sum_{i=1}^{N^P} A_{i,p} f_{i,p} d_{i,p} \quad (4.30)$$

where $d_{i,p}$ is the depth of the i^{th} prestressing tendon measured

from the extreme top fibre. For reinforcing steel elements,

$$M_s = \sum_{i=1}^{N_s} A_{i,s} \cdot f_{i,s} \cdot d_{i,s} \quad (4.31)$$

where $d_{i,s}$ is the depth of the i^{th} steel element

4.2.4 Equilibrium Conditions at Transfer

Applying the horizontal force equilibrium condition,

$$C_c + T_s = T_p \quad (4.32)$$

Substituting,

$$\sum_{j=1}^{N_A} \sigma_j \cdot b_j \cdot t_j + \sum_{i=1}^{N_s} f_{i,s} \cdot A_{i,s} = \sum_{i=1}^{N_P} f_{i,p} \cdot A_{i,p} \quad (4.33)$$

Similarly taking moments about the extreme top fibre,

$$M_c + M_s = M_p \quad (4.34)$$

$$\sum_{j=1}^{N_A} \sigma_j \cdot b_j \cdot t_j \cdot y_j + \sum_{i=1}^{N_s} f_{i,s} \cdot A_{i,s} \cdot d_{i,s} = \sum_{i=1}^{N_P} f_{i,p} \cdot A_{i,p} \cdot d_{i,p} \quad (4.35)$$

4.2.5 Stress and Strain Conditions at Transfer

The two equilibrium equations can be expressed in terms of the top and bottom-fibre strains $\epsilon_{o,t,r}$ and $\epsilon_{i,t,r}$. The stress and strain distribution across the section after transfer can be found by solving equations 4.33 and 4.35. A numerical approach can also be employed to calculate the top and bottom-fibre strains.

The calculated and measured top and bottom-fibre strains immediately after transfer of prestress are given in Table 4.1. Comparison between theoretical and observed results show very good agreement for all 6 test beams. A maximum error of 25 microstrains was observed. This is considered to be within

limits of experimental error. Tensile strains were sustained in the concrete top fibres and compressive strains in the lower fibres.

Table 4.1 Comparison of Concrete Strains Immediately after Transfer

Beam	Top-fibre strain (μs)		Bottom-fibre strain (μs)	
	Cal	Obs	Cal	Obs
SA 1	59	34	-231	-231
SA 2	59	42	-225	-243
SA 3	55	46	-206	-218
SA 4	57	52	-219	-203
PA 1	61	69	-241	-249
PA 2	60	41	-232	-213

Note: Cal - Calculated
 Obs - Observed
 μs - microstrain
 tensile strains are positive, compression negative

4.3 THEORETICAL ANALYSIS CONSIDERING TIME-DEPENDENT EFFECTS

An incremental method of analysis is used to determine the non-linear, time-varying response of partially prestressed member sections under any prescribed moment history. The analysis is based on a method developed by Warner and Lambert (1974).

The incremental approach, in which the time-varying moment is represented by a series of moment values acting in a sequence of small time intervals, is used to predict accurately the creep and shrinkage strains. Discretization of the moment history is shown in Fig 4.4. The total concrete strain is assumed to consist of instantaneous and inelastic components. The time-varying stress history of individual concrete slices

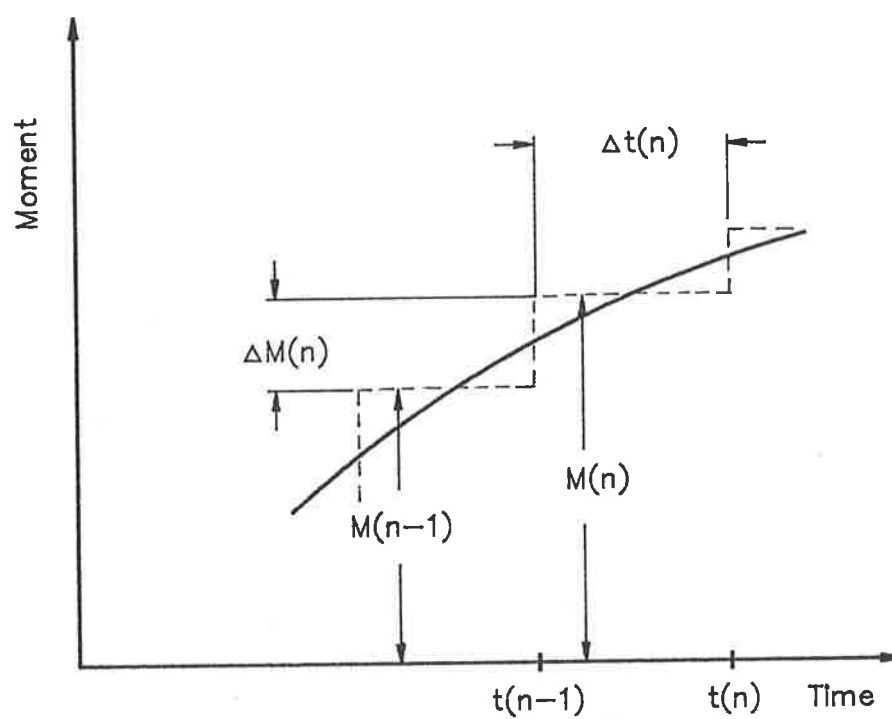


Fig. 4.4 Discretization of Moment History
(Warner & Lambert, 1974)

is discretized by assuming that stress changes occur only at the end of the time intervals. Stresses are assumed to remain constant during the intervals.

Calculations for stresses and strains are conducted for each interval. The time interval $\Delta t(n)$ commences at time $t(n-1)$ after application of the external moment increment $\Delta M(n)$. The state of stress and strain in each concrete and steel element at $t(n-1)$ is known from the previous computational cycle. The concrete elements are allowed to creep and shrink freely during the time interval. Corresponding inelastic strains are then calculated from the appropriate creep and shrinkage equations, using the previously described instantaneous stresses. Because the concrete elements are allowed to deform freely during the interval strain, incompatibilities arise between adjacent concrete elements or between concrete and steel elements. The instantaneous concrete strain and stresses have to be adjusted so that both conditions of compatibility and equilibrium are maintained at the end of the computational cycle.

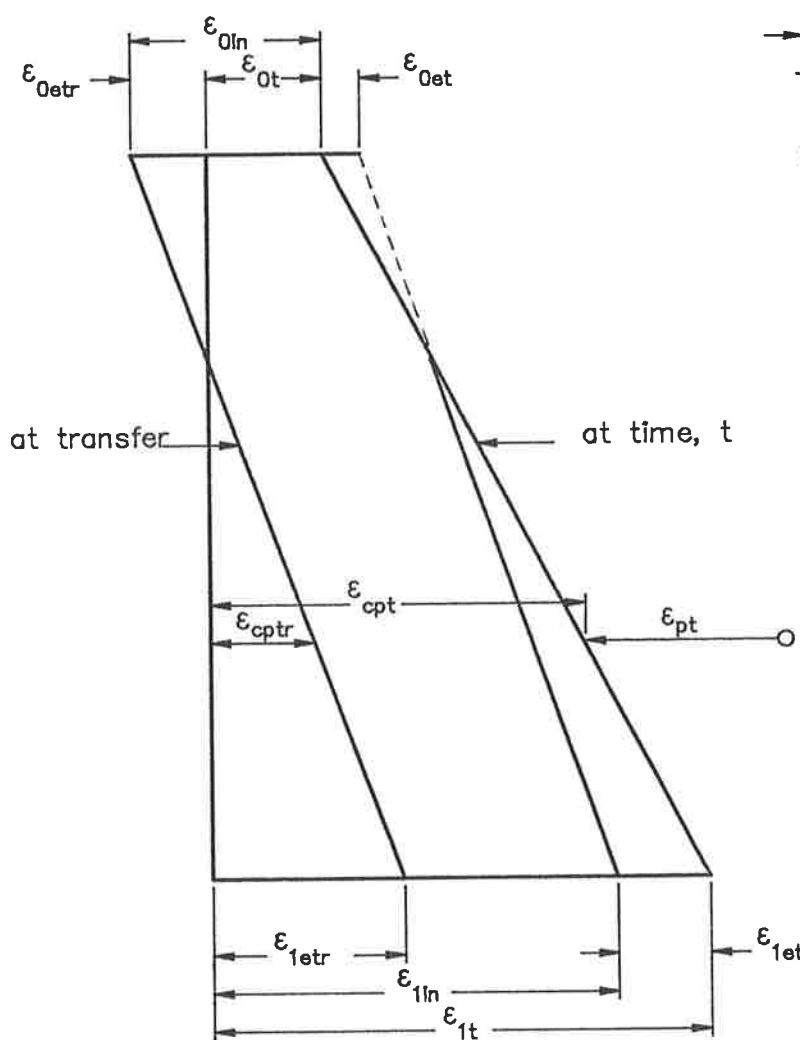
Given the initial conditions after transfer and the availability of the empirical creep and shrinkage function curves, the incremental analysis can be used to predict stress and strain conditions in concrete members under the influence of time-dependent inelastic effects.

4.3.1 Creep and Shrinkage Effects just Prior to Loading

The state of stress and strain in the beam section under the effects of creep and shrinkage, but with no applied load (self-weight being negligible for the test beams), is now considered.

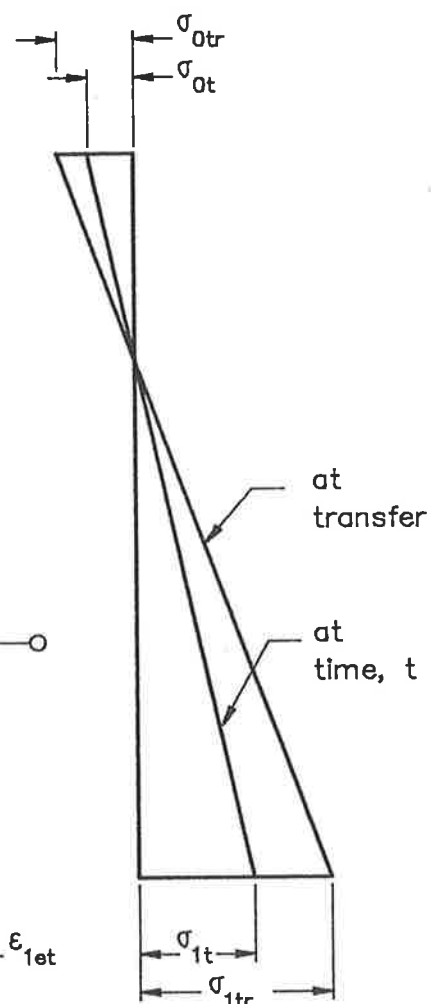
Concrete Strains

The concrete strain distributions in the section at time t , and at transfer with no load acting, are shown in Fig 4.5a. At transfer, the total concrete strain is equal to the elastic



Strain Diagram

Fig. 4.5a



Concrete Stress Diagram

Fig. 4.5b

Stress and Strain Conditions with
Time-Dependent Effects

strain. At time t , the total concrete strain comprises both elastic and inelastic components. The strain in the extreme top concrete fibre can be expressed as:

$$\begin{aligned}\epsilon_{et} &= \epsilon_{e,e} + \epsilon_{e,ln} \\ &= \epsilon_{e,e} + \epsilon_{e,cr} + \epsilon_{e,sh}\end{aligned}\quad (4.36)$$

where $\epsilon_{e,e}$ is the instantaneous elastic component, $\epsilon_{e,ln}$ is the total inelastic concrete strain, $\epsilon_{e,cr}$ and $\epsilon_{e,sh}$ are the inelastic creep and shrinkage strains respectively in the extreme top fibre.

Similarly, the instantaneous strain for the j^{th} concrete slice is expressed as:

$$\epsilon_{j,e} = \epsilon_j - \epsilon_{j,cr} - \epsilon_{j,sh} \quad (4.37)$$

where ϵ_j is the total strain in the j^{th} concrete slice and $\epsilon_{j,cr}$ and $\epsilon_{j,sh}$ are the creep and shrinkage strains respectively.

Assuming that the total concrete extreme top and bottom-fibre strains are compressive, the compressive concrete total strain ϵ_j is:

$$\epsilon_j = \frac{y_j}{D} \epsilon_{1,t} + \frac{(D - y_j)}{D} \epsilon_{et} \quad (4.38)$$

At time t , the assumptions of plane sections and strain compatibility still hold. The instantaneous compressive elastic strain at the j^{th} concrete slice is therefore:

$$\epsilon_{j,e} = \frac{y_j}{D} \epsilon_{1,t} + \frac{(D - y_j)}{D} \epsilon_{et} - \epsilon_{j,cr} - \epsilon_{j,sh} \quad (4.39)$$

With full strain compatibility between concrete and steel, the tensile prestressing steel strain at time t is:

$$\epsilon_{1,p} = \epsilon_{1,p,i} - \epsilon_{1,cr,t}$$

$$\epsilon_{i,p} = \epsilon_{i,p,t} - \frac{d_{i,p}}{D} \epsilon_{i,t} - \frac{(D - d_{i,p})}{D} \epsilon_{o,t} \quad (4.40)$$

where $\epsilon_{i,p}$ is the prestressing strain at time t and $\epsilon_{i,c,t}$ is the concrete strain at depth $d_{i,p}$ of the i^{th} prestressing steel element. The compressive strain in the i^{th} ordinary steel element is:

$$\epsilon_{i,s} = \frac{d_{i,s}}{D} \epsilon_{i,t} + \frac{(D - d_{i,s})}{D} \epsilon_{o,t} \quad (4.41)$$

Concrete Stresses

The stress in the j^{th} concrete slice is related to the instantaneous elastic strain by the normal stress-strain relation, i.e.

$$\begin{aligned} \sigma_j &= G_c \{ \epsilon_{j,e} \} \\ &= G_c \{ \epsilon_j - \epsilon_{j,i,n} \} \end{aligned} \quad (4.42)$$

where G_c is the stress-strain relation defined earlier in Section 4.1.2.

Steel stresses are calculated from the appropriate stress-strain relations. With no externally applied load, formulation of the force and moment equilibrium conditions is similar to that described in Section 4.2.3.

Stress and Strain Conditions with Creep and Shrinkage Effects under no Load

The strain conditions immediately after transfer and after time t are shown in Fig 4.5a. The concrete strain profile can alter significantly, even without the application of any load, i.e. due only to inelastic creep and shrinkage strain components. In the concrete top fibres, tensile creep and compressive shrinkage strains are sustained. Due to the small initial tensile stresses, the shrinkage strains are larger than the creep strains and may cause the top fibre strain to change from tension to compression. In the bottom concrete

fibres, compressive inelastic strains are induced by both creep and shrinkage. While the shrinkage strains remain constant throughout the section, the creep strains sustained in the bottom fibres are larger due to higher concrete stresses. The total inelastic strain components in the bottom fibres are therefore larger and result in a larger increase in overall concrete strain.

The observed strain conditions for all 6 test beams at transfer and at time t are given in Table 4.2 below. Considerable increases in compressive strains were observed for all beams after time t . Evidently, larger increases were observed for a longer period of time. For the extreme top fibre, the compressive strain increment varied from 148 to 219 microstrains while in the extreme bottom fibre, the increment ranged from 360 to 601 microstrains. This represented a maximum increase of 490 percent in the top-fibre strain and 241 percent in the bottom-fibre strain.

Table 4.2 Observed Concrete Strains with and without Time-Dependent Effects

Beam	Time t (days)	Top-fibre strain (μs)		Bottom-fibre strain (μs)	
		$t=0$	$t=t$	$t=0$	$t=t$
SA 1	77	34	-133	-231	-669
SA 2	56	42	-140	-243	-622
SA 3	86	46	-102	-218	-578
SA 4	71	52	-120	-203	-550
PA 1	96	69	-150	-249	-850
PA 2	49	41	-107	-213	-592

Note: μs - microstrain
tensile strains are positive, compression negative

The concrete stress distribution at transfer and at time t are shown in Fig 4.5b. Although there may be a large increase in the overall deformation, creep and shrinkage results in a

reduction of the instantaneous concrete stresses. A decrease in the top-fibre stress from $\sigma_{0,t}$ to $\sigma_{0,t}$ and from $\sigma_{1,t}$ to $\sigma_{1,t}$ for the bottom-fibre stress is illustrated. There is an accompanying loss of prestress due to the increased concrete deformation. Changes in the ordinary steel stresses also occur. Figs 4.5a and b also show that the location of the stress and strain neutral axes are no longer coincident.

The calculated and observed concrete strain distribution at time t is given in Table 4.3. Good agreement is obtained between the theoretical and observed strains with an average error of 27 microstrains for the extreme top fibre and 37 microstrains for the extreme bottom fibre. The above analysis of creep and shrinkage is therefore adequate. However, improved calculations may be obtained by the use of more accurate creep and shrinkage parameters.

Table 4.3 Comparison of Concrete Strains with Time-Dependent Effects

Beam	Top-fibre strain (μs)		Bottom-fibre strain (μs)	
	Cal	Obs	Cal	Obs
SA 1	- 89	-133	-662	-669
SA 2	-120	-140	-596	-622
SA 3	-100	-102	-617	-578
SA 4	- 80	-120	-610	-550
PA 1	-133	-150	-774	-850
PA 2	- 66	-107	-608	-592

Note: Cal - Calculated
 Obs - Observed
 μs - microstrain
 tensile strains are positive

4.3.2 Empirical Estimation of Total Inelastic Strains in Test Beams

Instead of using the theoretical creep and shrinkage function curves, an alternative procedure can be used to estimate the total inelastic strains. The concrete instantaneous stress distribution after time t is determined by first calculating the reinforcing and prestressing steel forces using the measured concrete strain distribution at time t , ie.

$$T_{p,m} = \sum_{i=1}^{NP} A_{i,p} f_{i,p,m} \quad (4.43)$$

where $A_{i,p}$ is the area of the i^{th} prestressing steel element and

$$f_{i,p,m} = G_p \left\{ \epsilon_{p,i} - \frac{d_{i,p}}{D} \epsilon_{1tm} - \frac{(D - d_{i,p})}{D} \epsilon_{otm} \right\} \quad (4.44)$$

G_p is the function defining the tendon stress-strain relation, ϵ_{otm} is the observed concrete top-fibre strain at time t . For the reinforcing steel the total compressive force is:

$$T_{s,m} = \sum_{i=1}^{NS} A_{i,s} f_{i,s,m} \quad (4.45)$$

where $A_{i,s}$ is the area of the i^{th} reinforcing steel element and

$$f_{i,s,m} = G_s \left\{ \frac{d_{i,s}}{D} \epsilon_{1tm} + \frac{(D - d_{i,s})}{D} \epsilon_{otm} \right\} \quad (4.46)$$

where G_s is the function defining the reinforcing steel stress-strain relation

The values of $T_{p,m}$ and $T_{s,m}$ are different to those at transfer. A search routine, similar to that described previously in Section 4.1.3, is then used to locate an elastic strain distribution which satisfies the equilibrium conditions. The total inelastic concrete strain is then simply the difference between the total strain distribution at time t and the calculated elastic strain.

For the j^{th} concrete slice the total inelastic strain is:

$$\epsilon_{j.in} = \epsilon_{j.t} - \epsilon_{j.elf} \quad (4.47)$$

where $\epsilon_{j.t}$ is the measured total strain and $\epsilon_{j.elf}$ is the estimated elastic strain.

A linear distribution of the total inelastic strain across the section is assumed. This assumption is valid in view of the relatively low concrete stresses involved and the results obtained from this procedure are therefore considered to be accurate. The method is unable to separate the total inelastic strain into creep and shrinkage components, but this is of no consequence to the analysis as the investigation is primarily concerned with the evaluation of member stresses. The evaluation of individual concrete inelastic strain components is of little interest.

Comparison of Total Inelastic Strains

The creep and shrinkage strains calculated from the theoretical and empirical methods are given in Table 4.4. Good agreement of the total inelastic strains is obtained, with differences in strain estimations of approximately 30 microstrains. Table 4.5 shows the concrete top and bottom-fibre stresses obtained from both methods of analysis. Good agreement is also obtained for these with a maximum variation of 0.5 MPa.

Table 4.4 Comparison of Total Inelastic Concrete Strains with Time-Dependent Effects

Beam	Top-fibre strain (μs)		Bottom-fibre strain (μs)	
	Theor.	Emp.	Theor.	Emp.
SA 1	-151	-181	-474	-500
SA 2	-129	-140	-407	-448
SA 3	-156	-145	-448	-422
SA 4	-140	-173	-430	-364
PA 1	-199	-200	-589	-675
PA 2	-128	-155	-413	-413

Note: tensile strains are positive, compression negative

Table 4.5 Theoretical and Empirical Concrete Stresses considering Time-Dependent Effects

Beam	Top-fibre stress (MPa)		Bottom-fibre stress (MPa)	
	Theor.	Emp.	Theor.	Emp.
SA 1	1.6	1.2	-6.0	-5.4
SA 2	1.6	1.2	-6.2	-5.7
SA 3	1.7	1.2	-5.9	-5.5
SA 4	1.6	1.4	-5.9	-5.8
PA 1	1.7	1.2	-5.6	-4.7
PA 2	1.7	1.2	-6.1	-5.7

Note: tensile stresses are positive, compression negative

The steel stresses, obtained from the empirical method, after transfer and at time t are given in Table 4.6. The table shows increases of 200 percent or more in reinforcement stresses. The compressive stresses sustained in this normally 'tensile' steel are considerable and are approximately 25 percent of the

yield value. On the other hand, a decrease in the tendon stresses of about 5 percent is observed. Table 4.6 also shows a reduction in the bottom concrete compressive stresses. Similar reductions in the tensile top-fibre concrete stresses were observed but not shown here. Creep and shrinkage in partially prestressed members therefore result in reduced concrete instantaneous stresses, increased compressive steel stresses and a reduction of the effective prestress.

Table 4.6 Section Stresses at Transfer and after Time-Dependent Effects

Beam	Time t (days)	Concrete* Stress (MPa)		Reinforcement Stress (MPa)		Tendon Stress (MPa)	
		t=0	t=t	t=0	t=t	t=0	t=t
SA 1	77	-7.4	-5.4	-40	-125	1358	1284
SA 2	56	-7.5	-5.7	-39	-113	1359	1295
SA 3	86	-7.3	-5.6	-36	-107	1325	1262
SA 4	71	-7.2	-5.8	-40	-102	1322	1266
PA 1	96	-7.3	-4.7	-46	-158	1359	1262
PA 2	49	-7.5	-5.7	-41	-110	1364	1303

Note: tensile stresses are positive

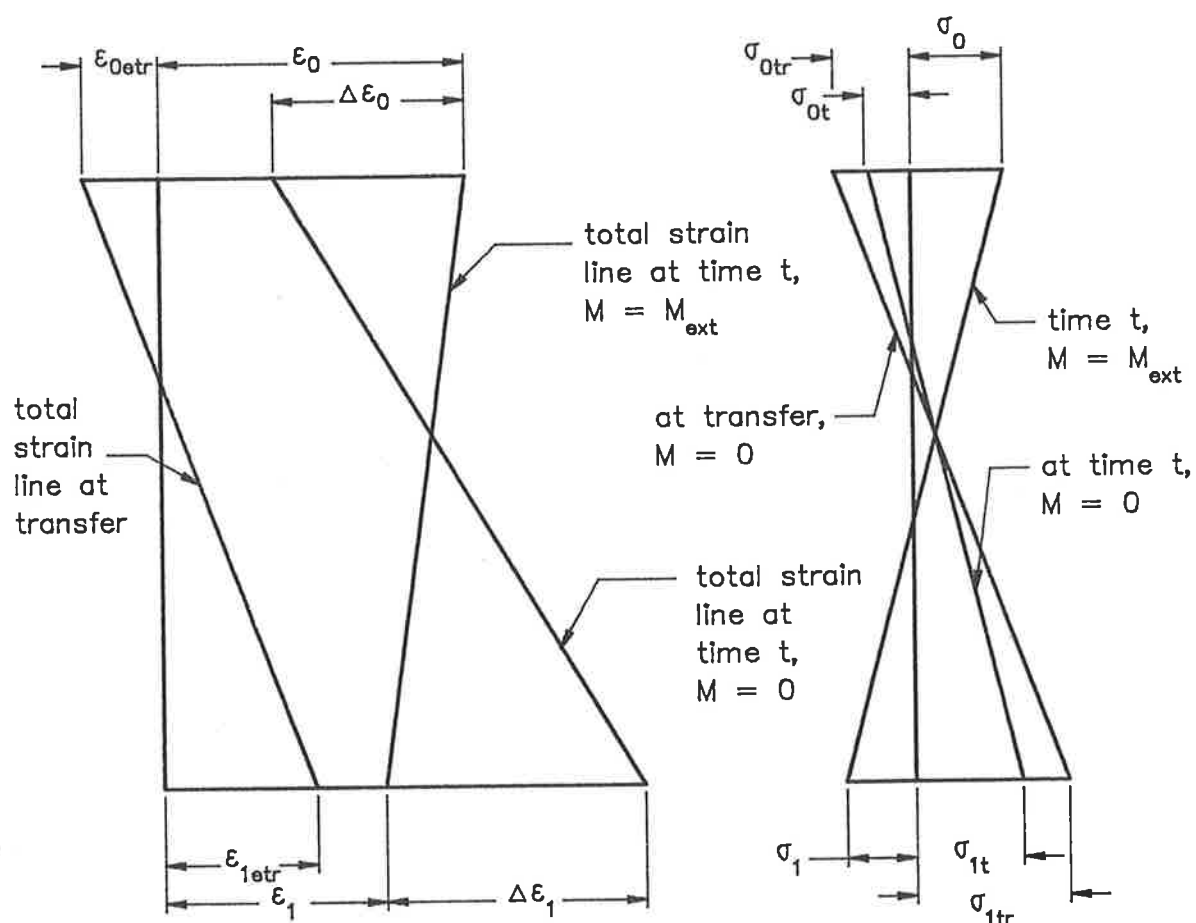
* extreme bottom-fibre

4.3.3 Loading in the Precracking Range

Beam behaviour in the precracking range is likely to be linear-elastic in view of the relatively low stresses. The analysis is similar to the previous case at time t except that an external moment is now applied.

Stress and Strain Calculations

The concrete strain distribution in a section at an applied moment is shown in Fig 4.6a. The concrete stress distribution is shown in Fig 4.6b. The compressive strain in the top and bottom extreme concrete fibres are ϵ_0 and ϵ_1 respectively at



Strain Diagram

Fig. 4.6a

Stress Diagram

Fig. 4.6b

Stress and Strain Conditions in Precracking Range

an applied external moment M_{xt} . The compressive concrete strain ϵ_j in the j^{th} slice is:

$$\epsilon_j = \frac{y_j}{D} \epsilon_t + \frac{(D - y_j)}{D} \epsilon_o \quad (4.48)$$

The steel strains can be calculated in a similar way. The concrete and steel stresses are related to the elastic strains by the material stress-strain relations.

Considering force equilibrium,

$$C_c + T_s = T_p \quad (4.49)$$

where C_c and T_s are the compressive concrete and ordinary steel force resultants and T_p is the tensile tendon force.

Taking moments about the extreme top fibre,

$$M_p - M_c - M_s = M_{xt} \quad (4.50)$$

where M_{xt} is the applied moment.

The calculation details are similar to those already discussed in this chapter.

Stress and Strain Conditions in the Precracking Range

The change in the state of stress and strain in a partially prestressed concrete flexural member with increasing moment is shown in Fig 4.7. Increasing moment is accompanied by increases in compressive strains and stresses in the concrete top fibres and increases in tensile strains and stresses in the bottom fibres. Due to the initial precompression and inelastic creep and shrinkage effects, the inelastic strains sustained in the concrete can be so high that cracking occurs when the total strain is still compressive, as shown in diagram (f) in Fig 4.7.

Analysis of the precracking beam behaviour has shown that considerable changes in the stress and strain conditions take

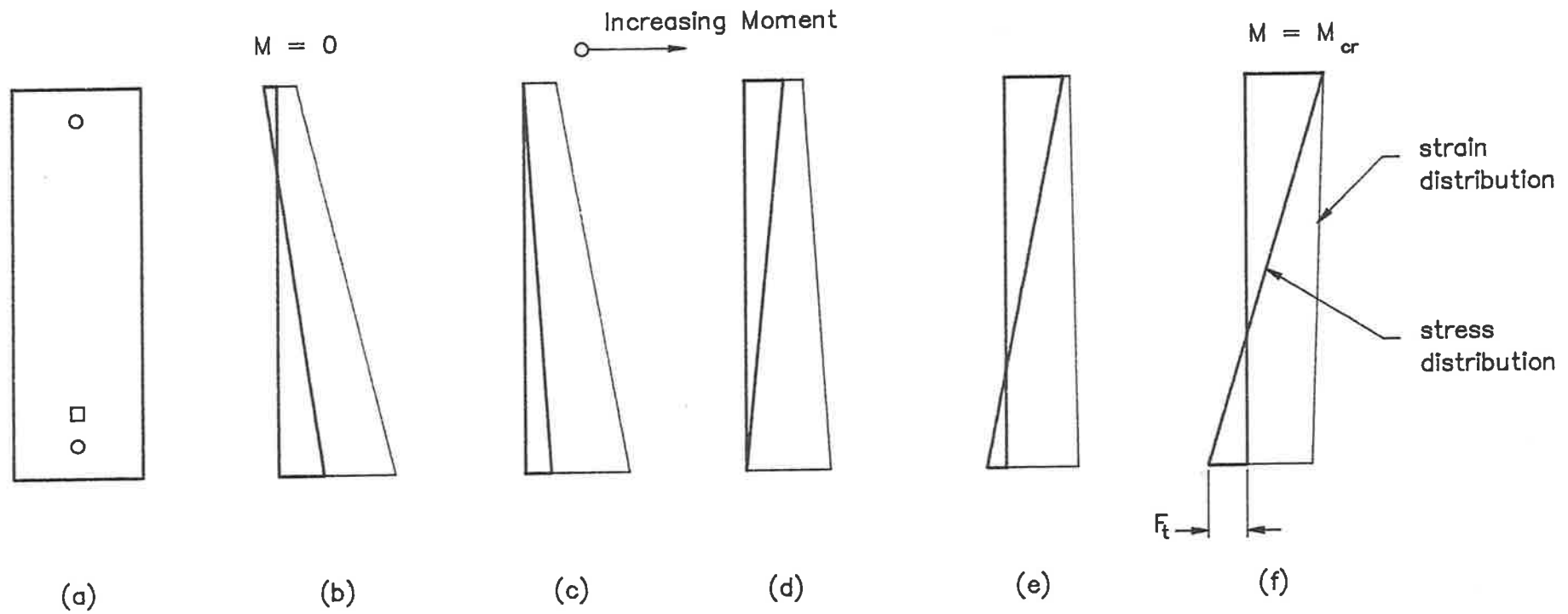


Fig. 4.7 Concrete Stress and Strain Distributions for Increasing Moment in the Precracking Range

place due to the time-dependent effects of creep and shrinkage as well as due to the application of external load. The postcracking behaviour of flexural members is studied next.

CHAPTER 5

REPEATED LOAD BEHAVIOUR IN THE POSTCRACKING RANGE - COMPARISONS WITH PRESENT METHODS OF ANALYSIS

In this chapter, the static and fatigue behaviour of the test beams are examined in relation to available theoretical methods of analysis. While the analysis of partially prestressed concrete members in the precracking range is relatively straightforward, beam behaviour after cracking is complex, even for the first load cycle. A non-linear no-tension section analysis which takes into consideration the effects of prior creep and shrinkage effects is first used for comparing predictions with observed results. Other improved methods of flexural analysis are then reviewed and theoretical estimations from these are compared with test results.

5.1 SIMPLE CRACKED SECTION ANALYSIS

The simple cracked section analysis is an extension of the uncracked section analysis applied to the postcracking load range. The computational procedure is similar to that of the uncracked method. The main difference is that after cracking, the concrete is assumed to have no tensile strength. Tensile forces in the section are carried solely by the steels while compressive forces are provided by concrete and compression steel. The assumed strain and stress distributions for concrete, and steel stresses are shown in Fig 5.1.

5.1.1 Detailed Description

The following assumptions are made:

- a) plane sections remain plane

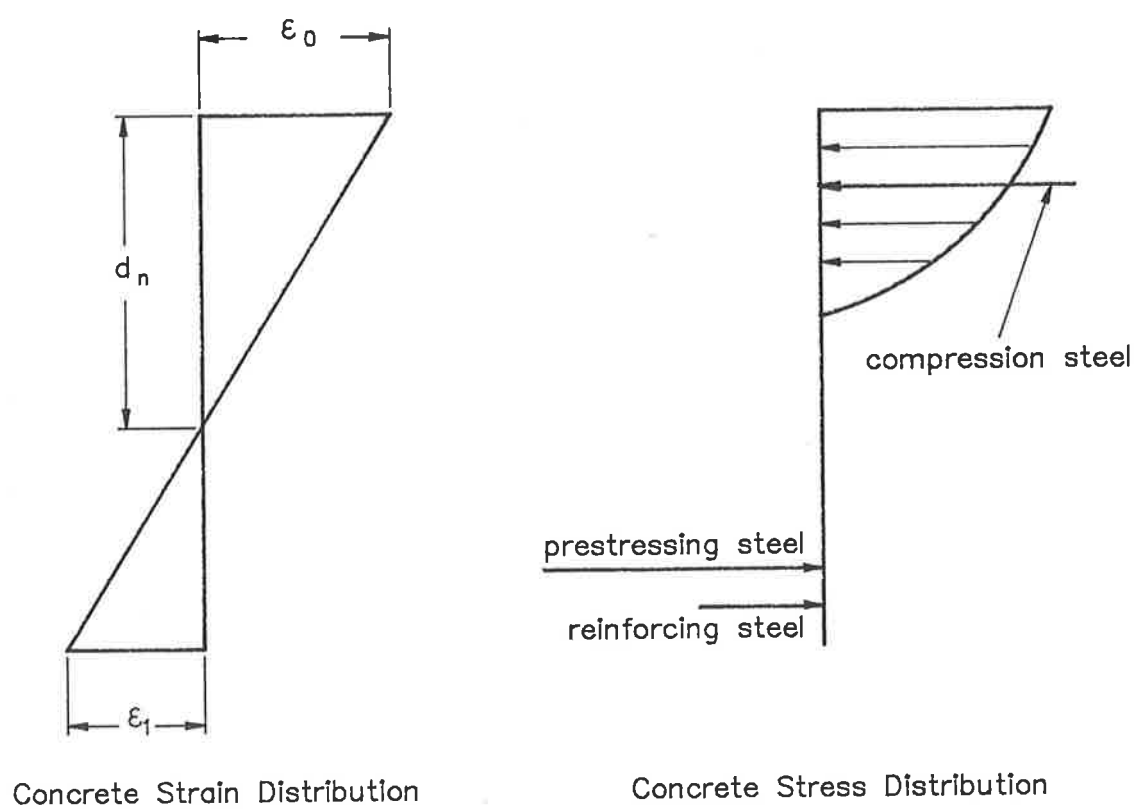


Fig. 5.1 Stress and Strain Conditions in Simple Cracked Section Analysis

- b) full strain compatibility exists between steel and concrete
- c) concrete has no tensile strength once cracking commences
- d) the inelastic creep and shrinkage strains remain constant after cracking

Non-linear, history-dependent material stress-strain curves are employed in the analysis.

Stress and Strain Calculations

The computational procedure is similar to the uncracked analysis. The same method of section representation is used. The compressive strain in the j^{th} concrete slice is calculated using the compressive top-fibre strain ϵ_0 and the tensile bottom-fibre strain ϵ_1 (as shown in Fig 5.1).

$$\epsilon_j = \frac{-y_j}{D} \epsilon_1 + \frac{(D - y_j)}{D} \epsilon_0 \quad (5.1)$$

The reinforcing and prestressing strain can be calculated in a similar manner.

The concrete elastic strain component for the first load cycle is obtained by subtracting the inelastic components from the total strain i.e.

$$\begin{aligned} \epsilon_{j,e} &= \epsilon_j - \epsilon_{j,cr} \\ &= \epsilon_j - \epsilon_{j,cr} - \epsilon_{j,sh} \end{aligned} \quad (5.2)$$

where $\epsilon_{j,e}$ is the elastic strain in the j^{th} concrete slice and $\epsilon_{j,cr}$ and $\epsilon_{j,sh}$ are respectively the long-term creep and shrinkage components. The concrete stress is related to the instantaneous elastic strain by the stress-strain relation:

$$\sigma_j = G_c \{ \epsilon_{j,e} \} \quad (5.3)$$

where G_c is the function defining the relation.

The theoretical compressive concrete stress-strain relations

for first and repeated load cycles are shown in Fig 5.2. Three equations are used to define G_c . Two are used to describe the loading and unloading response in the first cycle and a third is used for repeated load behaviour.

For the loading direction,

$$G_c(z) = 0.5 z F_c \exp(1. - 0.4z) \quad (5.4)$$

For the unloading direction,

$$G_c(z) = \sigma_{max} - 0.58 (z_{max} - z) F_c \exp(1. - 0.5(z_{max} - z)) \quad (5.5)$$

where z is the normalised elastic strain defined as:

$$z = \frac{\epsilon_e}{\epsilon_{ult}} \quad (5.6)$$

and z_{max} is the maximum normalised strain, σ_{max} is the maximum stress sustained, ϵ_e is the elastic concrete compressive strain, ϵ_{ult} is the strain at maximum stress, and F_c is the ultimate concrete strength in compression.

For repeated loading, the concrete stress-strain curve is similar in shape to the first unloading curve but is shifted laterally due to the addition of a permanent inelastic cyclic creep strain component ϵ_n which is measured from the end of the first cycle as shown in Fig 5.2. The concrete elastic strain in the j^{th} slice is then:

$$\epsilon_{j,e} = \epsilon_j - \epsilon_{j,cr} - \epsilon_{j,sh} - \epsilon_{j,n} \quad (5.7)$$

where $\epsilon_{j,n}$ is the inelastic dynamic creep strain which can be determined from an empirical equation which was obtained from laboratory tests (Section 3.1.3) and expressed as:

$$\epsilon_{j,n} = 65 \sigma_n (1 + 3.87\Delta)t^{1/3} \quad (5.8)$$

where $\epsilon_{j,n}$ is the cyclic creep strain, σ_n is the mean stress expressed as a fraction of the static strength at the start of

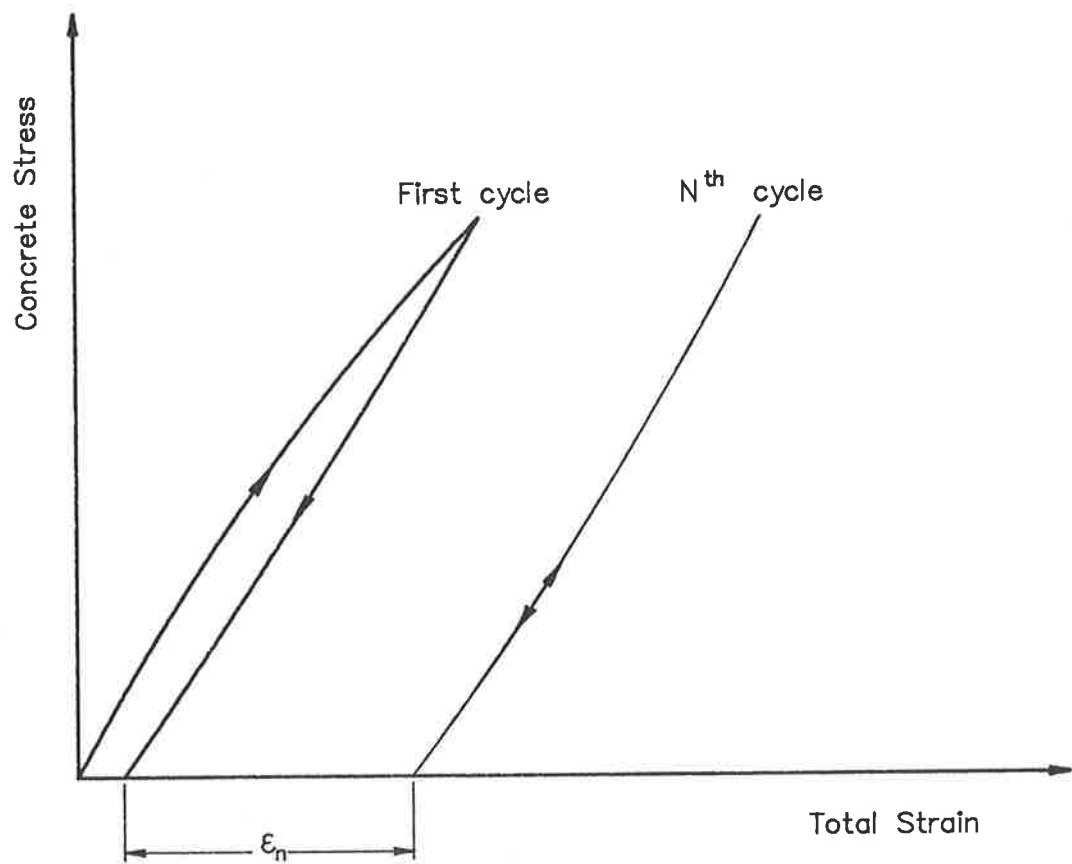


Fig. 5.2 Compressive Concrete Stress–Strain Relations for Cyclic Loading

loading, Δ is the stress range and t is the time of loading in hours. The concrete stress-strain relation is expressed as:

$$G_c(z) = \sigma_{max} - 0.58 (z_{max} - z) F_c \exp(1 - 0.5(z_{max} - z)) \quad (5.9)$$

where z and z_{max} are the normalised maximum and elastic concrete strain after taking into account cyclic creep.

No concrete tensile stresses are allowed in the postcracking range, and for elastic tensile strains:

$$\sigma_t = 0$$

The force and moment equilibrium equations can be written in a similar manner to previous analyses. The usual numerical search procedures are used to find the the appropriate top and bottom-fibre strains.

5.1.2 Comparisons with Test Data

First Load Cycle - Loading

The observed and theoretical first cycle moment-curvature relations of two typical beams (SA 1 & 2), one tested under constant amplitude loading and the other under cumulative damage conditions are given in Figs 5.3a and b. The observed relations after many load repetitions are also shown (300000 and 580000 for SA 1 and 2 respectively). Comparison of the theoretical and observed curves indicates good correlation in the precracking range but very poor agreement after cracking. The experimental curves show a gradual increase in curvature after cracking while the theoretical curves show a sudden jump in curvature at cracking. The sharp increase is caused by the computational curtailment of all concrete tensile stresses in the theoretical analysis at the onset of cracking. The calculated and observed stiffness after cracking show good agreement for both beams.

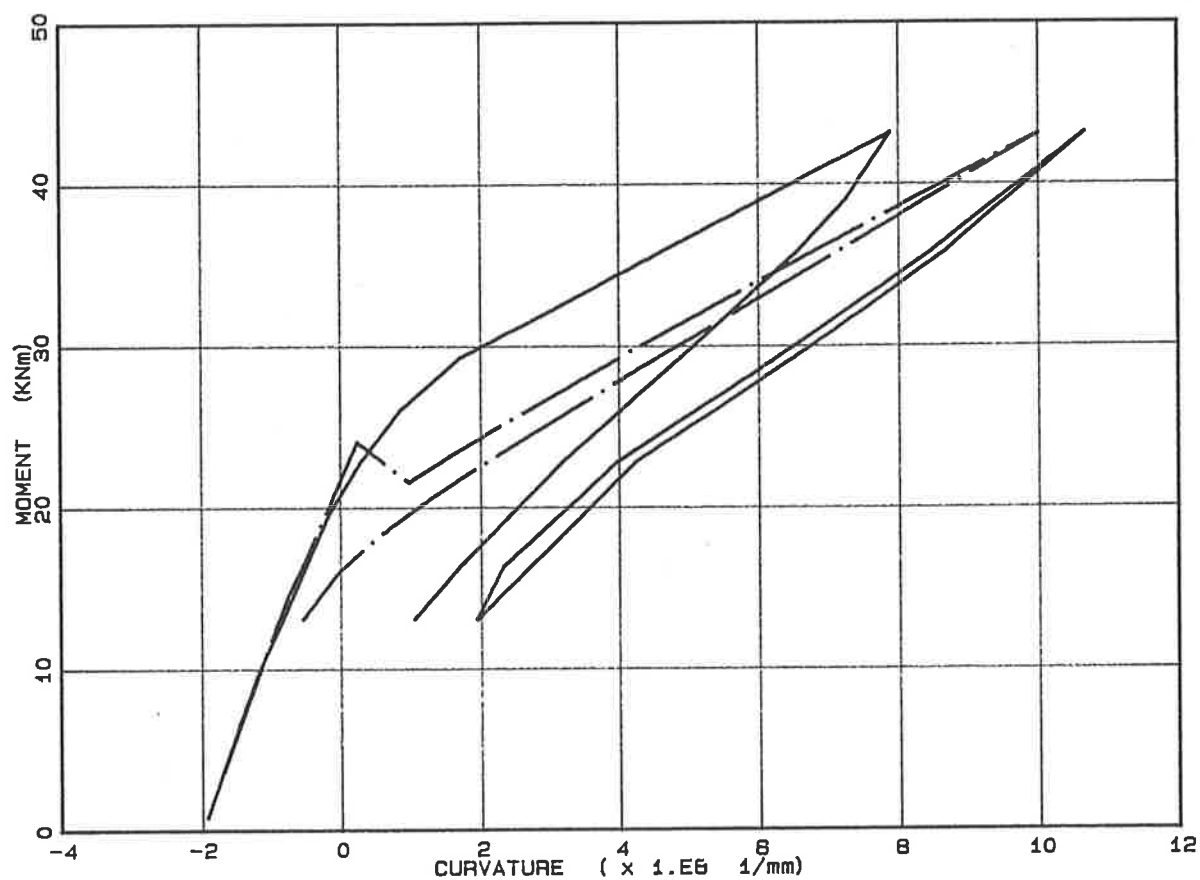


FIG 5.3a MOMENT-CURVATURE RELATIONS - BEAM SA1
(Using simple cracked section analysis)
Observed ——— Theoretical - - -

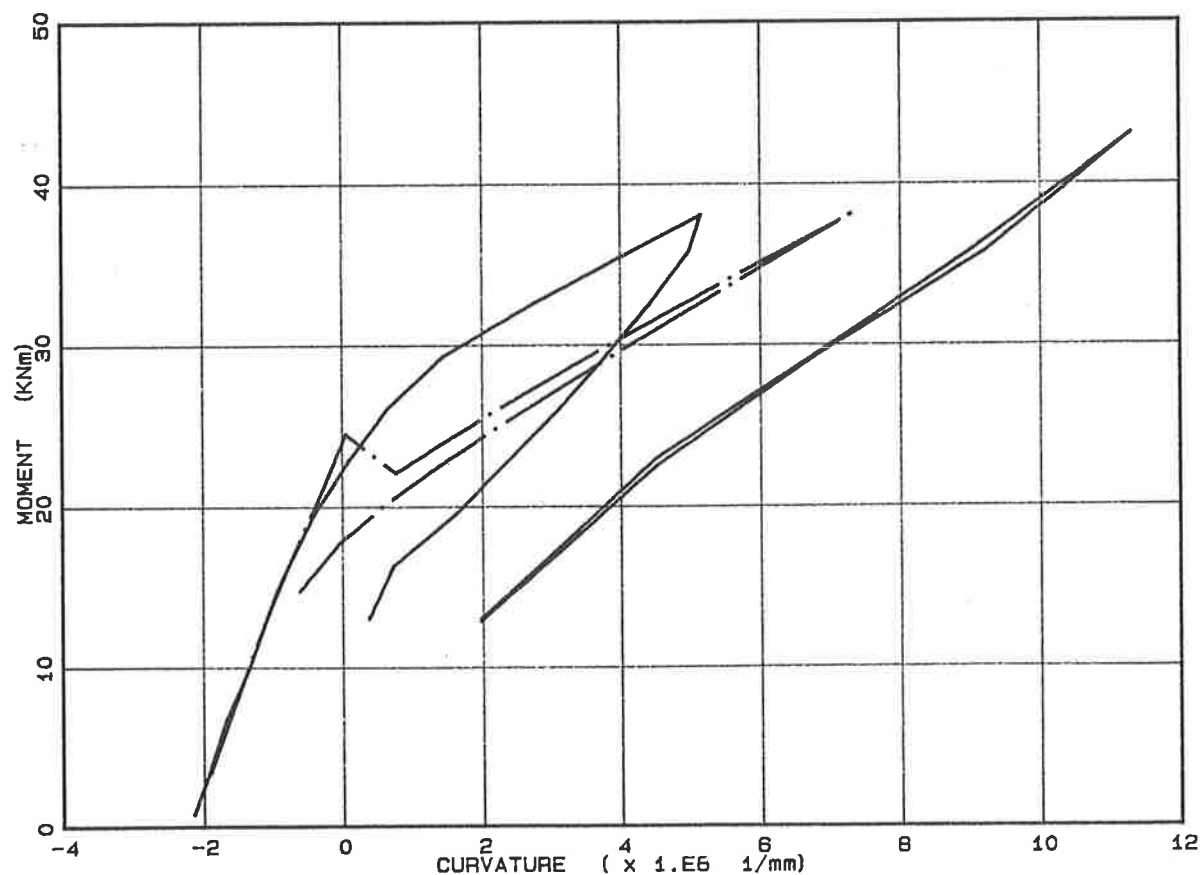


FIG 5.3b MOMENT-CURVATURE RELATIONS - BEAM SA2
(Using simple cracked section analysis)
Observed ——— Theoretical - - -

At loads higher than cracking, the simple analysis overestimates beam curvature with the theoretical curve consistently lower than the loading curve. There appears to be a relatively constant difference between the calculated and observed curvatures. The error ranges from approximately 50 percent of the curvature change at cracking to 20 percent at maximum load.

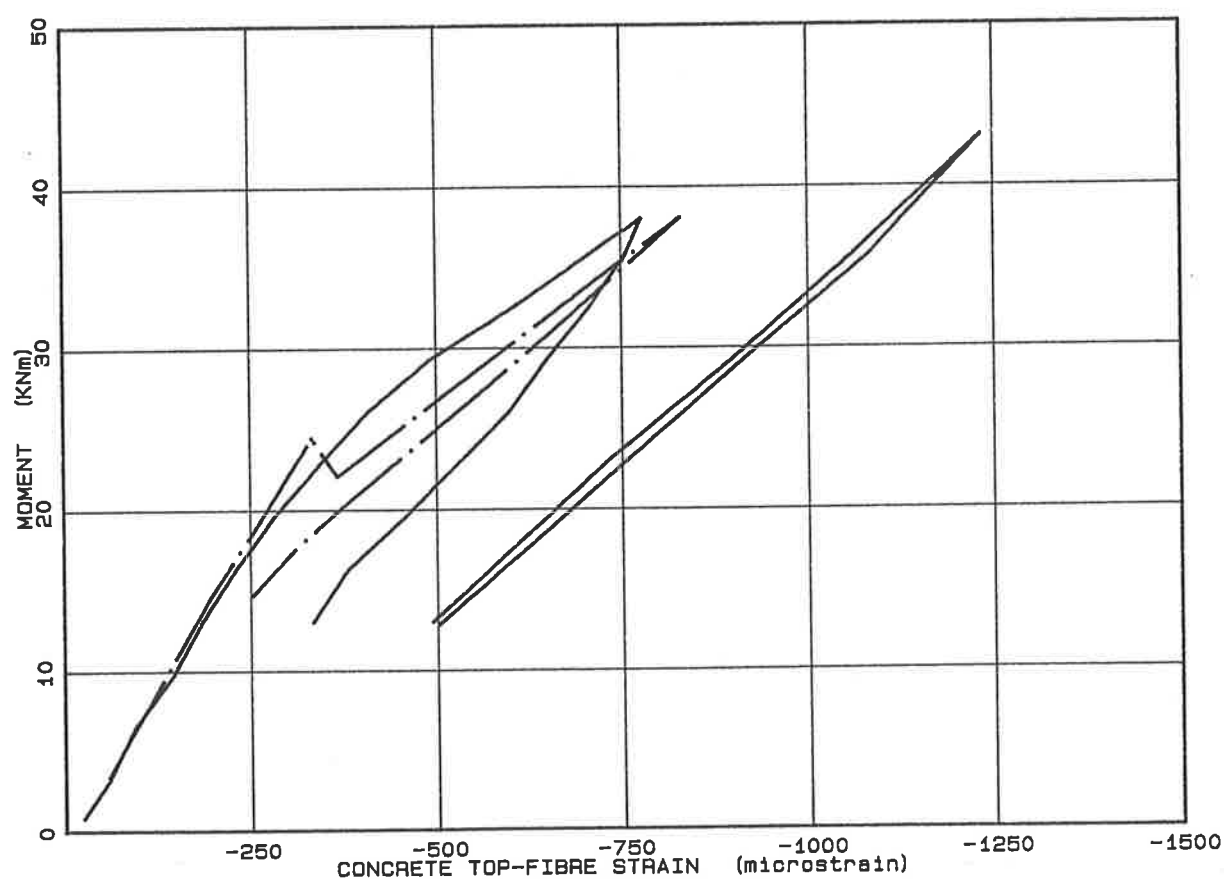
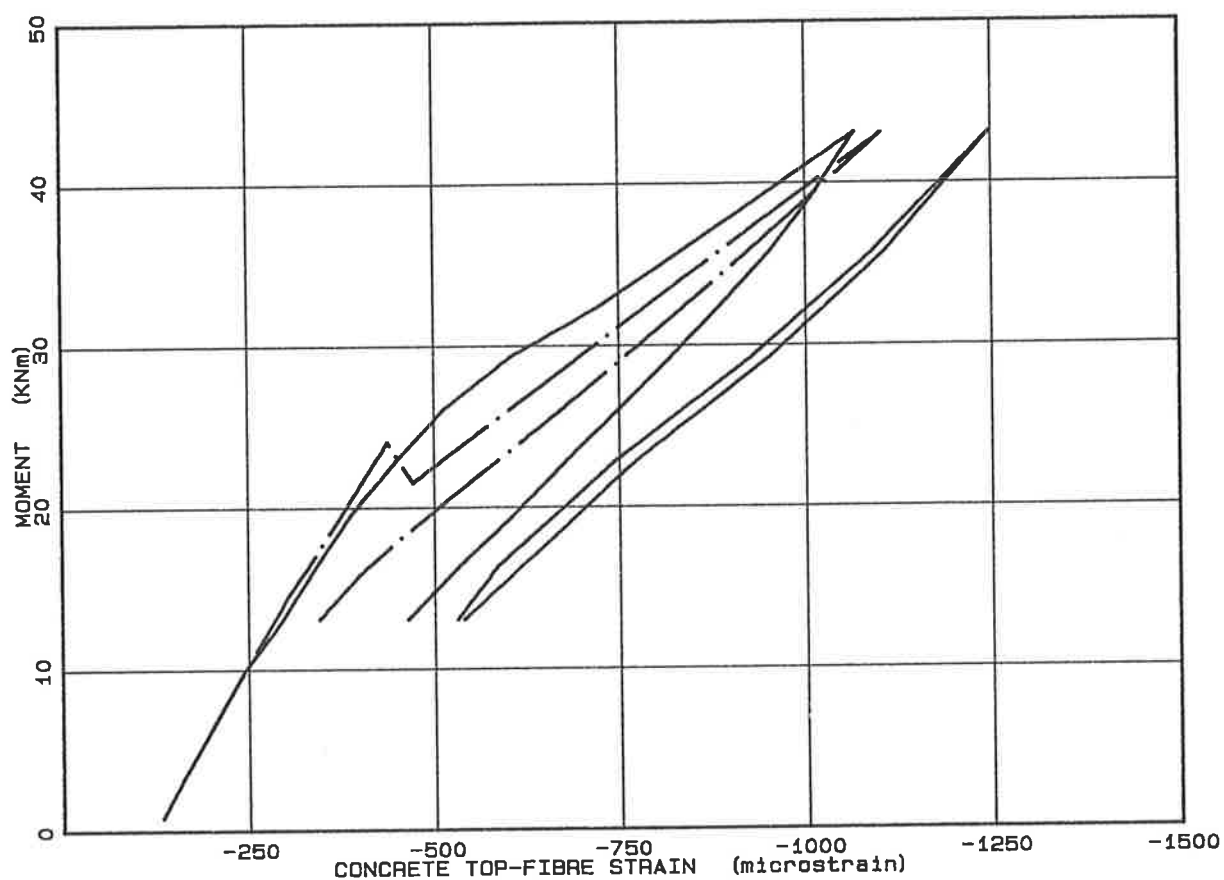
The predicted and experimental concrete top-fibre strains for the same beams are given in Figs 5.4a and b. Good agreement occurs in the precracking range. The discrepancy between the theoretical and observed top-fibre strains is not as large as that for the moment-curvature relation. It varies from approximately 80 microstrains (15 percent of the strain increment) in the immediate postcracking range to about 40 microstrains (4 percent) at maximum load.

The variation of the theoretical and observed extreme bottom-fibre strains with applied moment is shown in Figs 5.5a and b for beams SA 1 and 2. The curves show that the simple analysis grossly overestimates the bottom strains by as much as 500 microstrains (35 percent of the change in strain).

The discrepancy between observed and calculated results for the loading direction is generally attributed to tension stiffening effects. This is evident in the moment-strain comparisons which show much larger discrepancies in the bottom fibres and suggest that the cause of the discrepancy lies in the bottom concrete region. The simple analysis ignores the presence of concrete tensile stresses and overestimations of deformations and curvatures result. The gradual decrease in stiffness at cracking is caused by progressive concrete cracking in contrast to the more instantaneous phenomenon assumed by theory.

First Load Cycle - Unloading

Comparisons of the unloading and theoretical moment-curvature curves (Figs 5.3a and b) show very poor agreement. Large



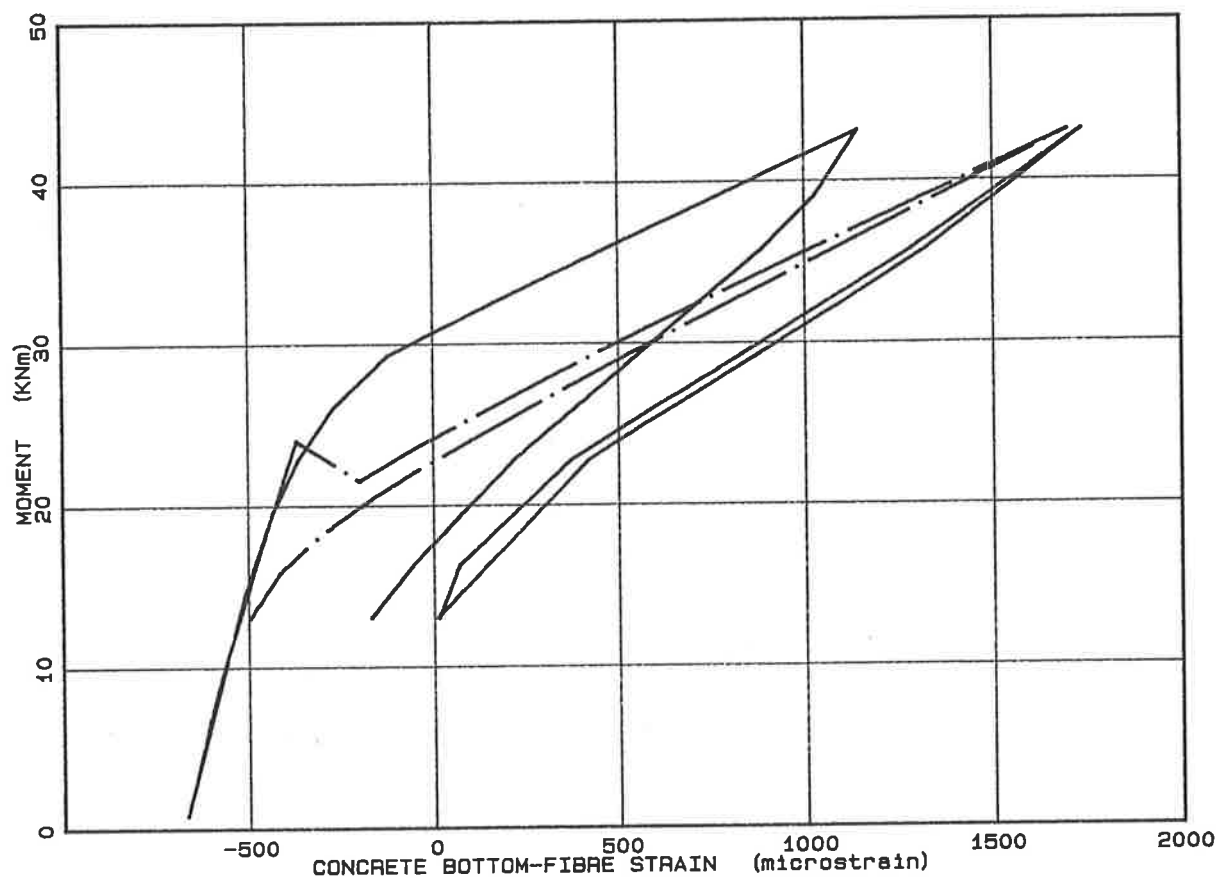


FIG 5.5a MOMENT-BOTTOM FIBRE STRAIN RELATIONS - BEAM SA1
(Using simple cracked section analysis)
Observed ——— Theoretical — . —

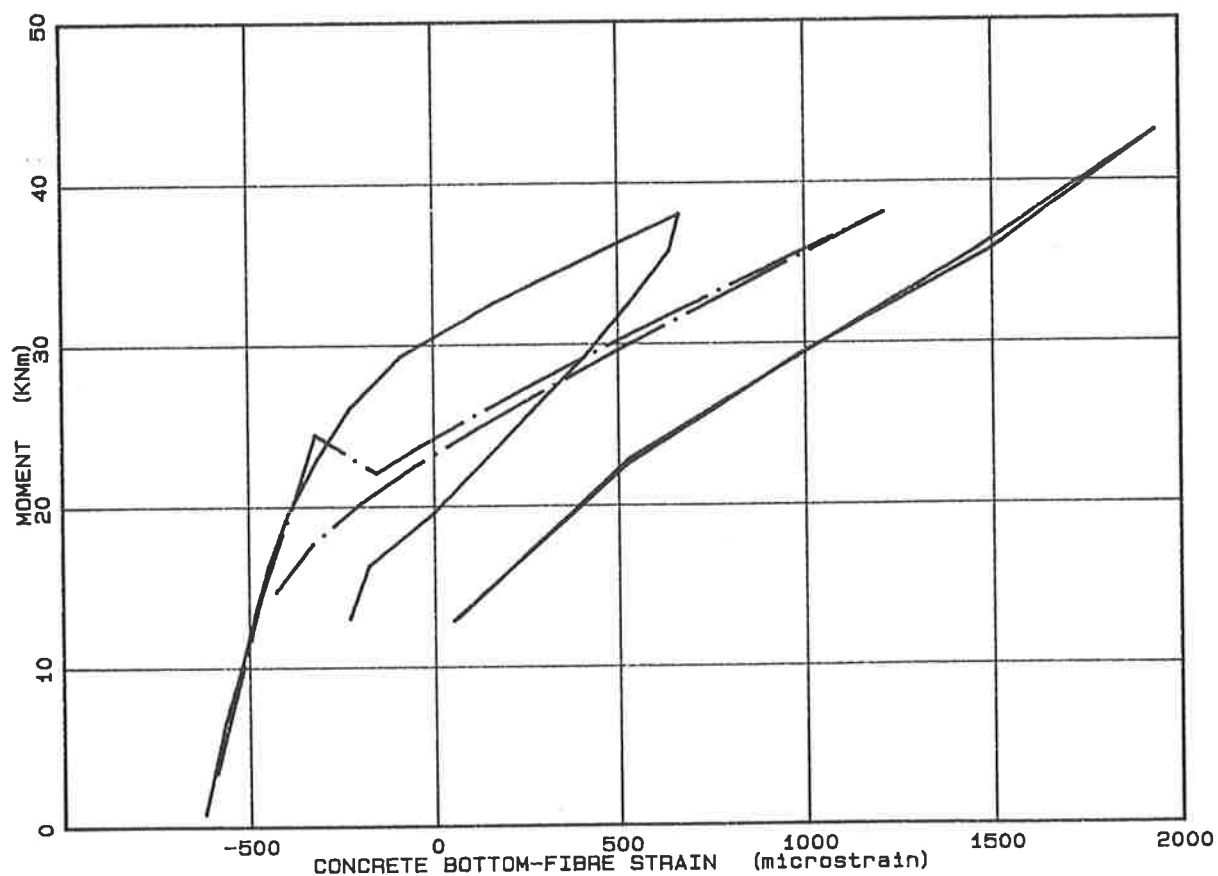


FIG 5.5b MOMENT-BOTTOM FIBRE STRAIN RELATIONS - BEAM SA2
(Using simple cracked section analysis)
Observed ——— Theoretical — . —

hysteretic effects are observed, but only marginal effects are predicted by the simple cracked section analysis. The latter are caused by the non-linear and plastic unloading material response of compressive concrete and prestressing tendon. The theoretical and experimental curves intersect just after the commencement of unloading, at relatively high loads. At lower loads, the observed curvatures are considerably larger than the theoretical values. The error becomes bigger with further unloading until at the minimum load, the measured curvature change is approximately 100 percent more than the theoretical value.

The observed strains in the concrete top fibres show a much stiffer unloading response than the predicted curve. Worst correlation is observed at minimum load, when the measured strain is 120 microstrains larger than the theoretical value.

Comparison of the observed and theoretical bottom-fibre strains shows even poorer agreement. At maximum load, there is overestimation of approximately 30 percent while upon unloading, the error compounds to an underestimation of 100 percent. There are thus even greater discrepancies in the unloading curve than in the first loading curve.

The simple analysis in the first load cycle therefore overestimates beam deformations and curvatures at high loads and underestimates these values at low loads. This implies that there are at least two separate factors which the theoretical analysis failed to consider. One of these is the commonly known concrete tension stiffening effect. Other influencing factors are discussed later.

Second and Subsequent Load Cycles

Poor agreement is found when the second cycle loading curve is compared with the theoretical curve. Curvatures are badly underestimated at low loads and overestimated at high loads. However, the theoretical predictions are marginally closer to the second cycle because of slight increases in beam deformations in the second cycle.

With the application of further load cycles, the experimental curve shifts laterally along the curvature axis. After application of a sufficiently large number of load cycles, the beam curvature may be greater than the theoretical value. Fig 5.3a shows that at the 300,000th load cycle, the measured curvature for Beam SA 1 at maximum load compares reasonably well with the predicted value. In fact, the simple analysis slightly underestimates beam curvature. But the relatively good agreement is applicable only at high load levels. At lower load levels, the correlation between experimental and theoretical values is still poor. At this stage of cyclic loading, the simple analysis underestimates overall beam curvatures throughout the whole load range.

While the simple analysis overestimates top-fibre strains for the first load cycle, Figs 5.4a and b show that after a large number of load cycles, the measured strains are larger than the theoretical values. This is valid for the entire load range. The discrepancy is in the order of 200 microstrains or 20 percent of the measured strain at maximum load.

Comparison of the bottom-fibre strains produces similar results. In the early load cycles, the simple analysis overestimates beam deformation. After a considerable number of load repetitions, the measured deformation increases and may even exceed the calculated values. Fig 5.5a shows that the concrete bottom-fibre strain in the 300000th load cycle for beam SA 1 is marginally greater than the theoretical value at maximum load. Larger differences are observed for beam SA 2 as shown in Fig 5.5b.

5.1.3 Simple Cracked Section Analysis with Cyclic Creep Effects

Concrete Cyclic Creep

The comparisons show that the simple cracked section analysis overestimates overall beam behaviour in the early cycles of

fatigue loading, but underestimates beam deformations and curvatures in the later cycles.

While the discrepancy between predicted and measured values for the first cycle might be attributed mainly to tension stiffening, this explanation cannot be used after application of many load cycles. The theoretical curve represents the limiting case where tension stiffening is completely eliminated. If the shift of the actual $M - \chi$ curve with increasing load cycles is assumed to be caused by the gradual loss of tension stiffening, then the test curvature values should approach but not exceed those calculated. However, the measured curvatures are larger than the calculated values and suggest that factors other than the gradual loss of concrete tension capability must influence beam behaviour.

The behaviour of the concrete compressive zone under repeated loading is considered to be one such factor. The cyclic creep effect was taken into account in a modified cracked section analysis. This was achieved by the use of a concrete compressive cyclic stress-strain relation as described in Section 5.1.1.

Moment-curvature relations for Beams SA 1 and SA 2, calculated from the simple analysis taking concrete cyclic creep effects into account are shown in Fig 5.6a and b. This results in a shift of the theoretical $M - \chi$ curve to the right, giving better estimates of curvatures. For Beam SA 1, at maximum moment the theoretical curvature shows good agreement with the observed value. For Beam SA 2, improved predictions are also obtained in the higher moment ranges. However, the theoretical analysis still underestimates curvatures. Similar calculations for the other test beams also show improved predictions but with some underestimation of beam curvatures at maximum moment. However, at low moments, beam curvatures and deformations are still grossly underestimated.

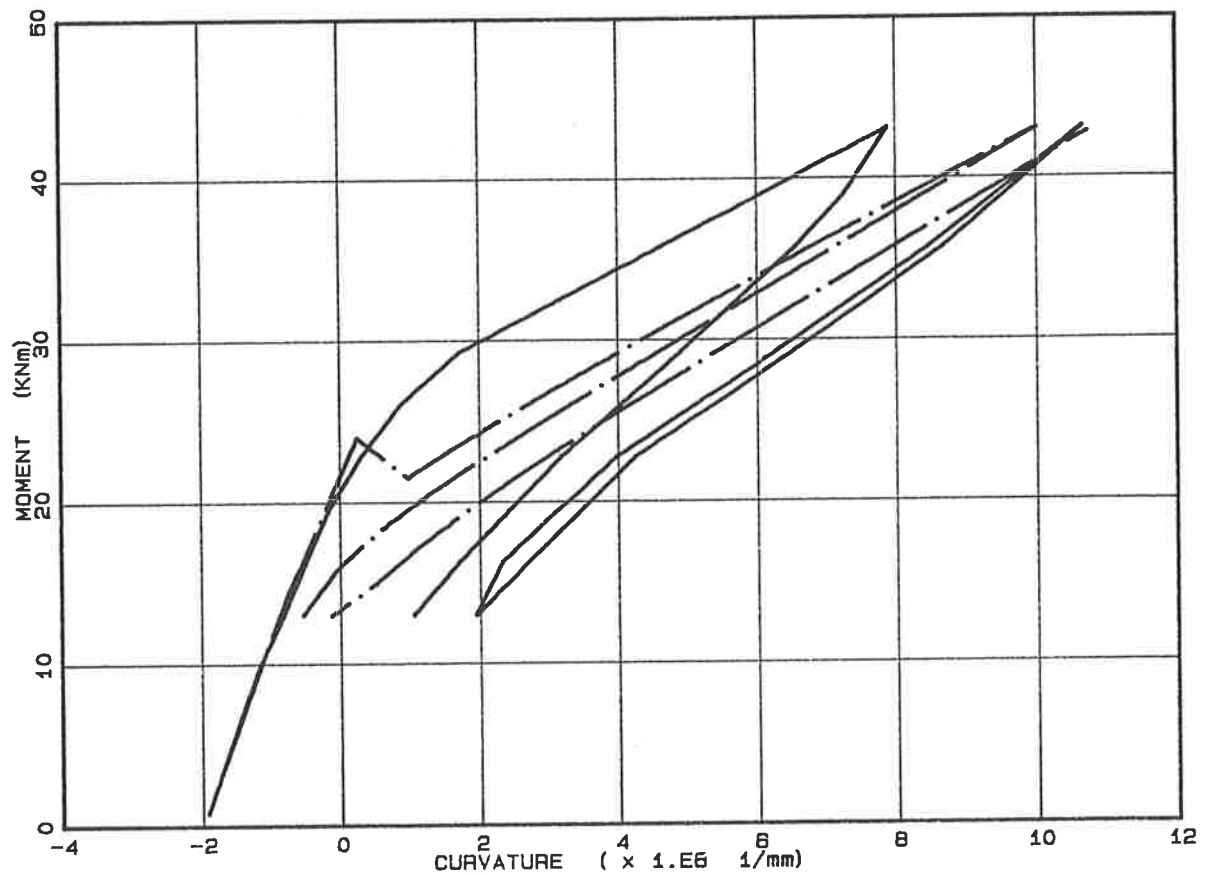


FIG 5.6a MOMENT-CURVATURE RELATIONS - BEAM SA1
(Using simple analysis with concrete dynamic creep)
Observed ——— Theoretical — . —

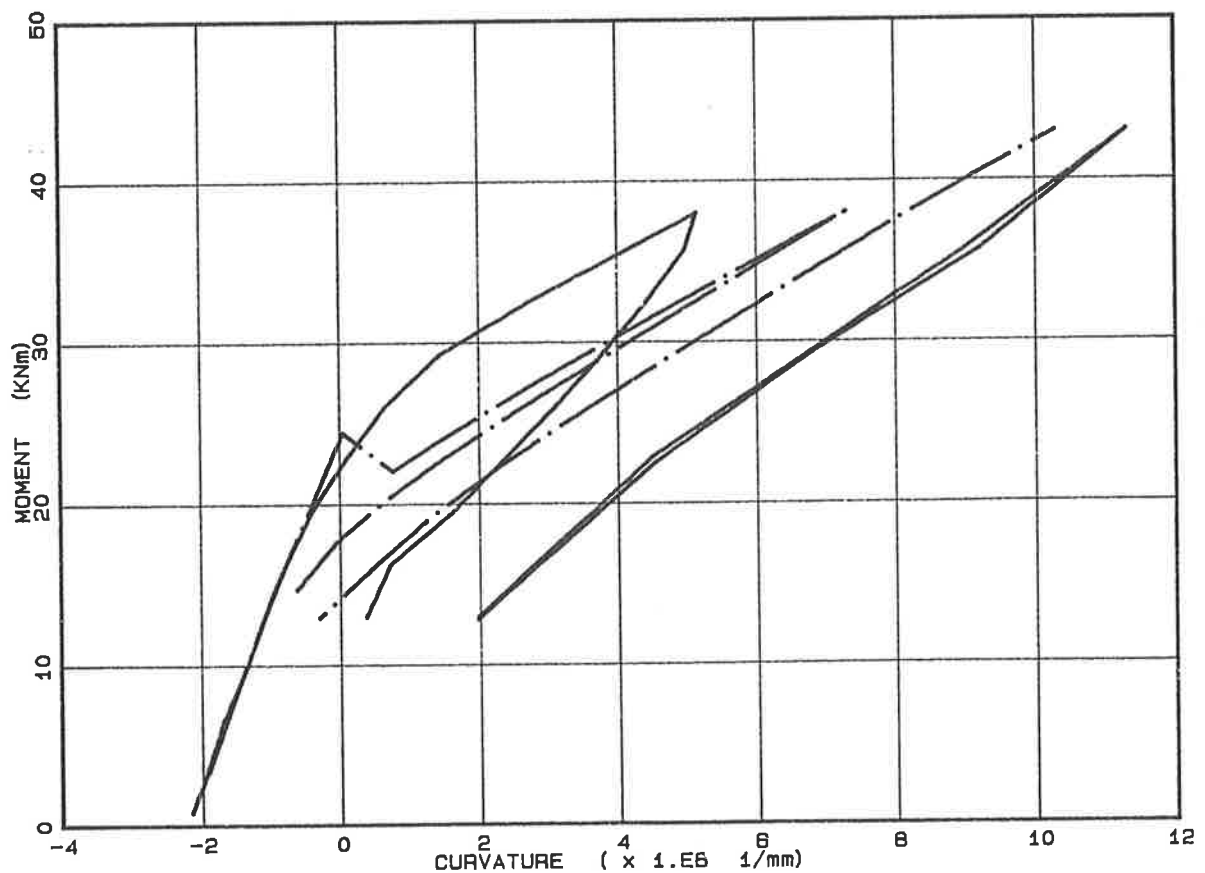


FIG 5.6b MOMENT-CURVATURE RELATIONS - BEAM SA2
(Using simple analysis with concrete dynamic creep)
Observed ——— Theoretical — . —

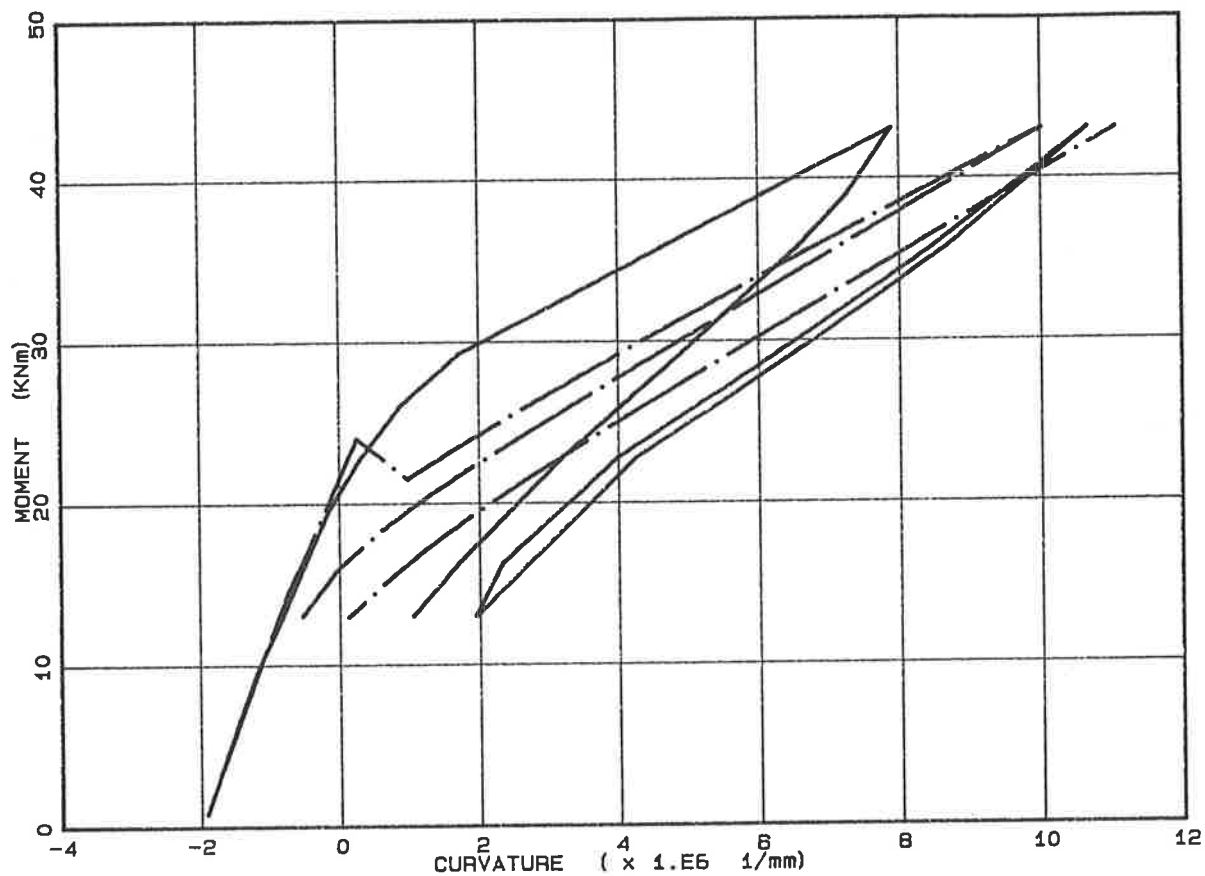


FIG 5.6c MOMENT-CURVATURE RELATIONS - BEAM SA1
(Using simple analysis with concrete & steel dynamic creep)

Observed

Theoretical

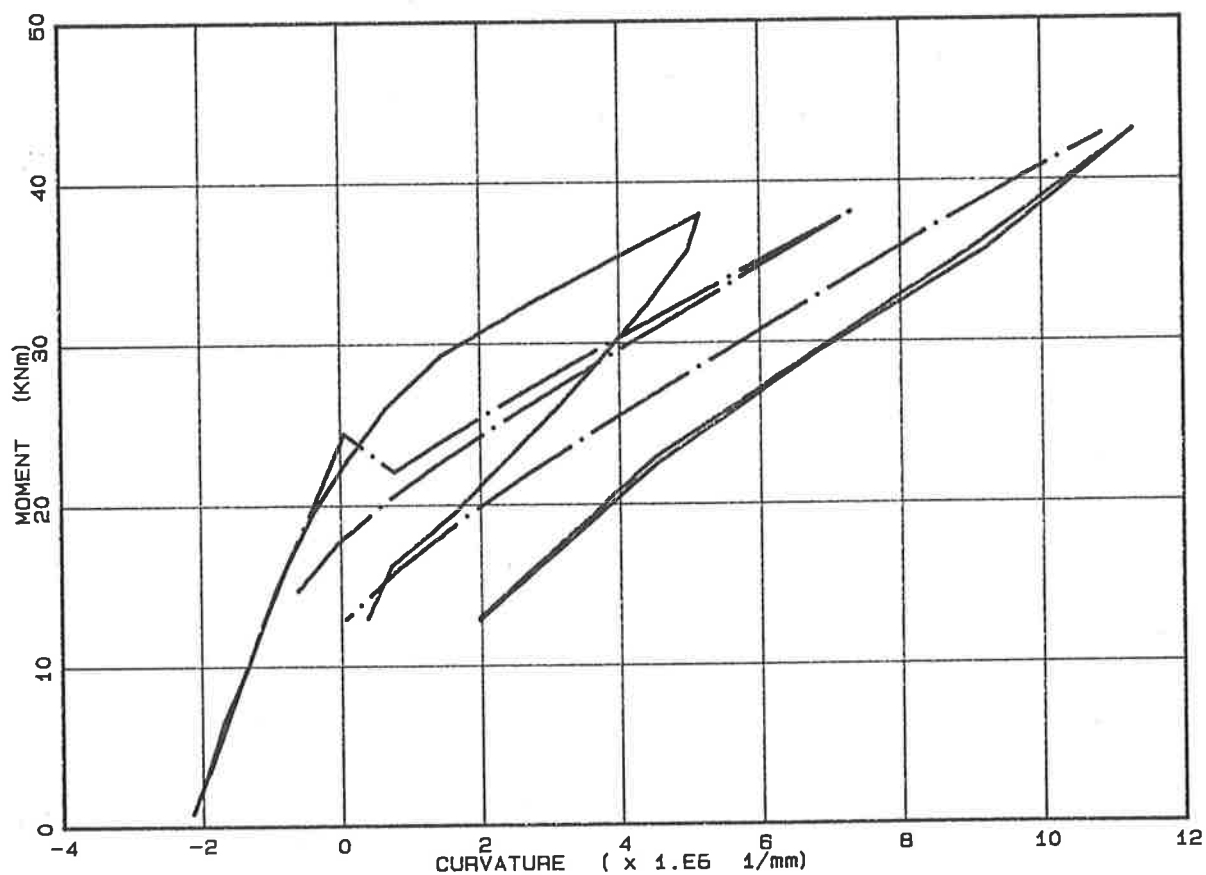


FIG 5.6d MOMENT-CURVATURE RELATIONS - BEAM SA2
(Using simple analysis with concrete & steel dynamic creep)

Observed

Theoretical

Cyclic Creep of Concrete and Steel

Although improved results were obtained, the calculations show that the cyclic creep effect in concrete is insufficient to account for all of the permanent inelastic deformation sustained during repeated loading, and suggest that there may be other contributing factors involved. The effect of cyclic creep and relaxation in steel is considered to be a likely possibility. However, little is known about the cyclic creep characteristics of reinforcing and prestressing steel under high cycle repeated loading (Balaguru, 1981). A crude approximation was therefore used to simulate the cyclic steel response. This was achieved by introducing an inelastic steel strain component. Evidently the inelastic strain is dependent on the steel strain range and on the number of load cycles sustained.

As no numerical data is available, values were chosen simply to give a good fit to the theoretical moment-curvature relation at maximum moment. More investigation is needed to provide reliable data on this phenomenon. The values of the steel cyclic creep strain are given in Table 5.1.

Table 5.1 Steel Cyclic Creep Strains

Beam	Reinforcing Steel (microstrain)	Prestressing Steel (microstrain)
SA 1	28	24
SA 2	77	65
SA 3	127	108
SA 4	85	72
PA 1	21	18
PA 2	111	94

The elastic steel strains in the normal and prestressing steel are expressed as:

$$\epsilon_{i,s,e} = \epsilon_{i,s} - \epsilon_{i,s,n} \quad (5.10)$$

$$\epsilon_{i,p,e} = \epsilon_{i,p} - \epsilon_{i,p,n} \quad (5.11)$$

where $\epsilon_{i,s,n}$ and $\epsilon_{i,p,n}$ are the cyclic creep strains in the i^{th} element of the reinforcing and prestressing steels. The corresponding steel stresses are:

$$f_{i,s} = G_s \{ \epsilon_{i,s,e} \} \quad (5.12)$$

$$f_{i,p} = G_p \{ \epsilon_{i,p,e} \} \quad (5.13)$$

where G_s and G_p are functions defining the respective stress-strain relations.

The theoretical moment-curvature relations of Beams SA 1 and 2, taking cyclic creep into account, are given in Figs 5.6c and d above. They show good agreement with the observed values at maximum loads but still give gross underestimation at low loads.

5.2 IMPROVED METHODS OF ANALYSIS

The inadequacies in the simple cracked section analysis of first cycle loading have long been recognised, and a variety of modifications have been suggested in order to achieve improved predictions.

The inadequacy of the simple cracked section analysis in the

first load cycle is due to the non-consideration of tensile stresses in the concrete. Different methods have been used to take into account tension stiffening. These can be classified generally into three main approaches, namely,

- a) local analysis
- b) regional smeared approach
- c) global approach

In the first approach, a detailed analysis is made of local behaviour in the tension zone with the use of bond and stress-slip relations. In the regional smeared approach, overall account is taken of the additional tensile forces carried by the concrete in the region surrounding the tensile steel. In the global approach, empirical adjustments are made to the slope of the moment-curvature relation or even to the load-deflection relation to obtain better predictions. The latter is used mainly in calculating service deflections for design.

5.2.1 Local Approach - First Load Cycle

A description of the steel and concrete stresses along the beam axis at the macroscopic level can be obtained by using appropriate bond stress-slip relations. Typical concrete and steel stress distributions along a cracked member are shown in Fig 5.7. Overall deformations and deflections can then be computed by integrating the local strain over the entire load span. Because of the complex equations involved, the numerical finite element method is usually employed. A State-of-the-Art report (ASCE,1982) on the finite element analysis of reinforced concrete has recently been published. Bond behaviour and stress-slip relationships under static and cyclic loads together with cracking models were discussed in detail.

A recent non-linear analysis was proposed by Plauk and Hess (1981) which took into account progressive cracking and local failure. Good agreement was reported with the use of

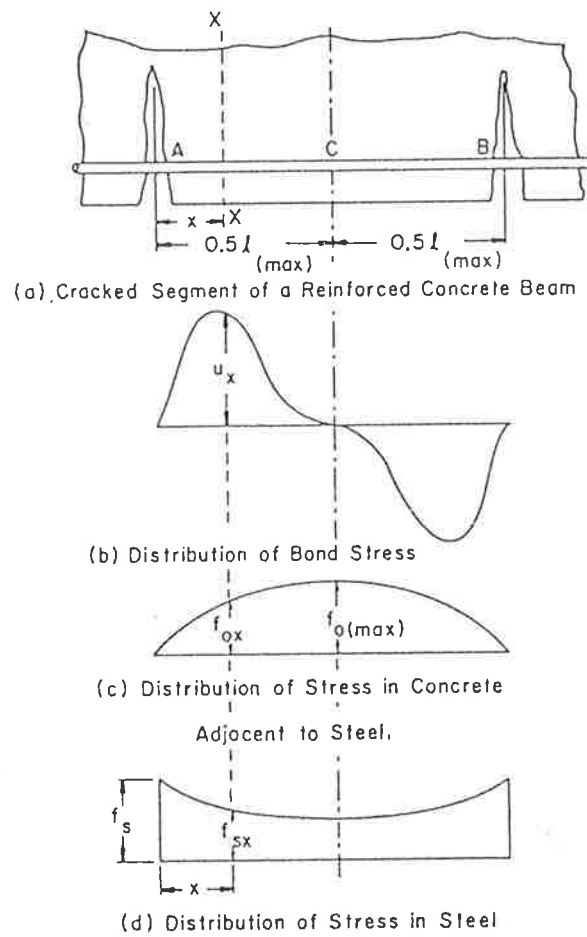


Fig. 5.7 Concrete and Steel Stress Distributions Along Cracked Member (Venkateswarlu et al, 1972)

'realistic' stress-slip relations. Similar investigations were made by Edwards and Picard (1972) and Venkateswarlu and Gesund (1972). These analysis were performed for the first load cycle. In view of the difficulties presented in modelling the extremely complex concrete 'tensile' zone and the need for treating not only the initial loading stage but also many repeated cycles of load, it was felt that this approach was not practical for the present investigation.

5.2.2 Regional Smeared Approach - First Load Cycle

Tension stiffening is often accounted for by the introduction of additional equivalent tension forces in the lower portion of the section. This can be achieved in a number of ways, of which four general methods can be identified. The first method uses the tensile strain-softening characteristics of the concrete, ie. the tensile stress-strain curve is revised to include a descending tail. This enables some of the concrete tensile stresses, which would normally be eliminated after beam cracking, to be retained. The second method modifies the steel stress-strain relation instead of concrete. The third method caters for the necessary stiffening force by the introduction of an additional area of tensile concrete or steel. The fourth uses a mean stress or strain concept with corrections for tension stiffening.

Concrete Tensile Strain-Softening Method

Scanlon and Murray (1974), Lin and Scordelis (1975), Madu (1975), Kabir (1976), Schafer (1976), Gilbert and Warner (1978), Kristjansson (1977), Chitnuyandondh et al (1979) and Cope et al (1979) all used the modified concrete tension stress-strain curve. Various strain-softening curves were also examined by Leibengood et al (1986) in their investigation of parameters affecting Finite Element analyses of concrete structures. The shape of the concrete descending tail is important as it defines the tensile stress distribution, which in turn is dependent on factors such as the proximity to steel bars, the steel ratio, the level of loading and the

geometrical properties of the member.

Different modifications have been used and these are shown in Fig 5.8 which is reproduced from Gilbert and Warner (1978). Gilbert and Warner proposed a discontinuous piece-wise linear unloading tail in their work on reinforced concrete slabs. A stepwise unloading path was examined by Scanlon while Lin and Scordelis made use of a gradually softening path. Madu also used a gradually softening curve which was expressed as:

For $0 \leq x \leq 1.3$

$$y = -0.667 x^4 + 2.74 x^3 - 4.46 x^2 + 3.40 x \quad (5.14)$$

For $1.3 \leq x \leq 14$

$$y = -9.03 * 10^{-4} x^3 + 2.60 * 10^{-2} x^2 - 0.274 x + 1.290 \quad (5.15)$$

where x and y are the normalised concrete tensile strain and stress with reference to the maximum concrete strength values.

More recently Bazant and Oh (1984) employed a linear softening path in their analysis of reinforced concrete beams. The bi-linear stress-strain relation for concrete in tension was expressed as:

For $\epsilon_c \leq \epsilon_{t,p}$,

$$\sigma_c = E_c \epsilon_c \quad (5.16)$$

For $\epsilon_{t,p} < \epsilon_c < \epsilon_{t,r}$,

$$\sigma_c = f_t' - (\epsilon_c - \epsilon_{t,p})(-E_t) \quad (5.17)$$

For $\epsilon_c > \epsilon_{t,r}$,

$$\sigma_c = 0$$

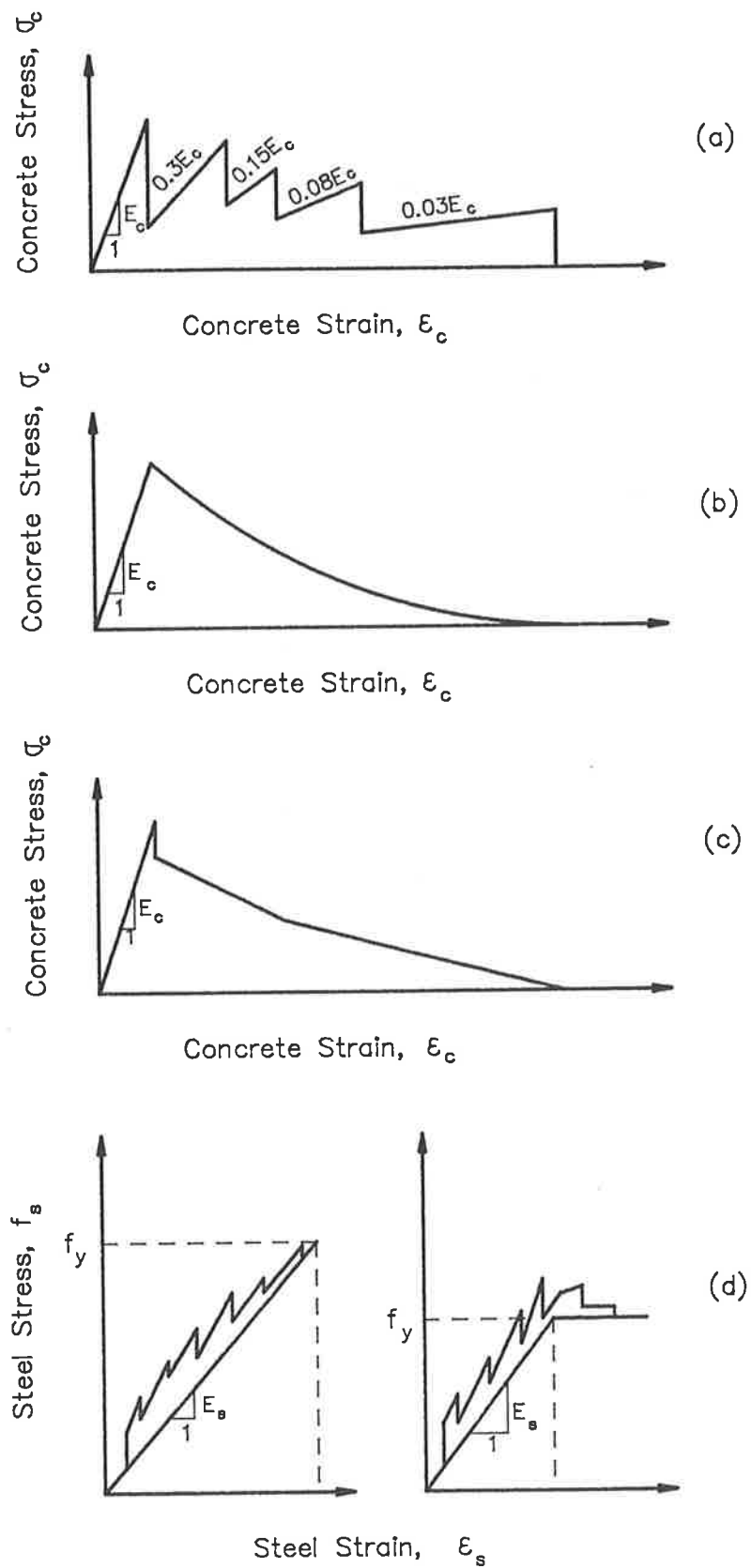


Fig. 5.8 Modified Stress—Strain Diagrams for Concrete and Tension Steel (Gilbert & Warner, 1978)

where σ_c and ϵ_c are the uniaxial stress and strain of concrete, E_c is the Young's modulus of elasticity, f_t' is the ultimate tensile strength, E_t is the tangent strain-softening modulus (negative), ϵ_{tp} is the strain at peak tensile stress, and ϵ_{tf} is the final strain when the tensile stress is reduced to zero at full fracture. Fig 5.9 shows the assumed constitutive relations as well as the stress and strain distributions for a section.

Bazant recommended an empirical equation for the slope of the linear unloading curve in tension E_t :

$$E_t = \frac{-70 E_c}{57 + f_t'} \quad (5.18)$$

where E_c , f_t' and E_t are in psi.

The analysis was linear and adopted all fundamental assumptions used in the simple no-tension analysis. The two analyses differed in the lower concrete tension zone where a tensile stress block was defined for the strain-softening method.

Modified Stress-Strain Relations for Steel

The tension stiffening effect may also be modelled by allowing the reinforcing steels to provide more tensile forces. This can be achieved by increasing the elastic modulus.

Moosecker and Grasser (1981) reported that the concept of a modified steel stiffness was used as early as 1950 when Murashev multiplied the elastic modulus by an empirical factor to take into account tension stiffening effects. Gilbert and Warner (1978) proposed a modified stress-strain diagram for steel. Concrete tensile stresses were represented by additional steel stresses introduced by the modification of the stress-strain diagram of the tension steel after the surrounding concrete has cracked. This is shown in Fig 5.8d above. The authors found that the modified tensile steel stress method required considerably less computer time than the concrete strain-softening method.

Perdikaris, Hilmy and White (1985) used a similar approach in their work on the extensional stiffness of precracked concrete panels. The total surface axial deformation was measured in the direction of applied tension. From the measured extensional stiffness and by introducing an average effective steel tensile strain, an expression for an 'average effective modulus' of steel $E_{s,e}$ was derived:

$$E_{s,e} = \frac{f_s}{\epsilon_{s,e}} \quad (5.19)$$

where f_s is the applied steel stress and $\epsilon_{s,e}$ is the measured average axial strain between two primary cracks. The postcracking effective tangent modulus of steel $E'_{s,e}$ can be expressed as:

For uniaxial tension,

$$E'_{s,e} = 123 \rho E_s \quad (5.20)$$

where ρ is the steel reinforcement ratio and E_s is the usual Young's Modulus for steel

Floegl and Mang (1982) also used this approach in their work on reinforced concrete panels. 'Tension stiffening factors' based on bond slip between concrete and steel, were applied to the total and incremental equilibrium equations to account for the concrete tensile forces. These factors were assumed to be dependent on the average bond stress, the crack spacings, the angle of the cracks, the average steel stress and type of reinforcement used. The analysis was carried out by means of a finite element program and the tension stiffening factors were incorporated in the material matrix of the reinforcing steel.

Bridge and Smith (1982) accounted for tension stiffening by a hypothetical polynomial stress-strain curve for the reinforcing steel. A fully cracked rectangular section was analysed assuming:

- a) plane sections remaining plane

- b) no tension stresses in the concrete
- c) non-linear constitutive relations for concrete
- d) full strain compatibility between concrete and steel

Equilibrium equations were first written involving section strains and stresses. The authors then derived an hypothetical stress-strain curve for steel which would give predictions that are in good agreement with the experimental values of applied moment, average curvature and concrete compressive strain. Tension stiffening effects were therefore accounted for by adjusting the steel stress-strain relations to match the observed beam test results. This method is empirical and is only applicable to the authors' beams.

Effective Area in Tension

The third regional method involves the use of an effective area of concrete in tension or an additional fictitious steel area. In either method, the effective stiffness of the cracked section is increased by the additional area and the overall behaviour can be computed by the elastic modular ratio method. A modified section is shown in Fig. 5.10.

Clark and Speirs (1978) made recommendations on the size of effective area and tensile forces for their reinforced concrete beams and slabs. The authors assumed a crack spacing of 1.3 times the crack height, and proposed that the effective concrete area can be estimated between two bounds:

$$b(h-x) \geq A_{ct} \geq b h_c$$

where A_{ct} is the area of effective concrete and

$$h_c = 0.325(h-x) + c + \phi \quad (5.21)$$

b is the width of the section, h is the total section depth, x is the neutral axis depth measured from the top compressive fibre, c is the cover and ϕ is the bar diameter.

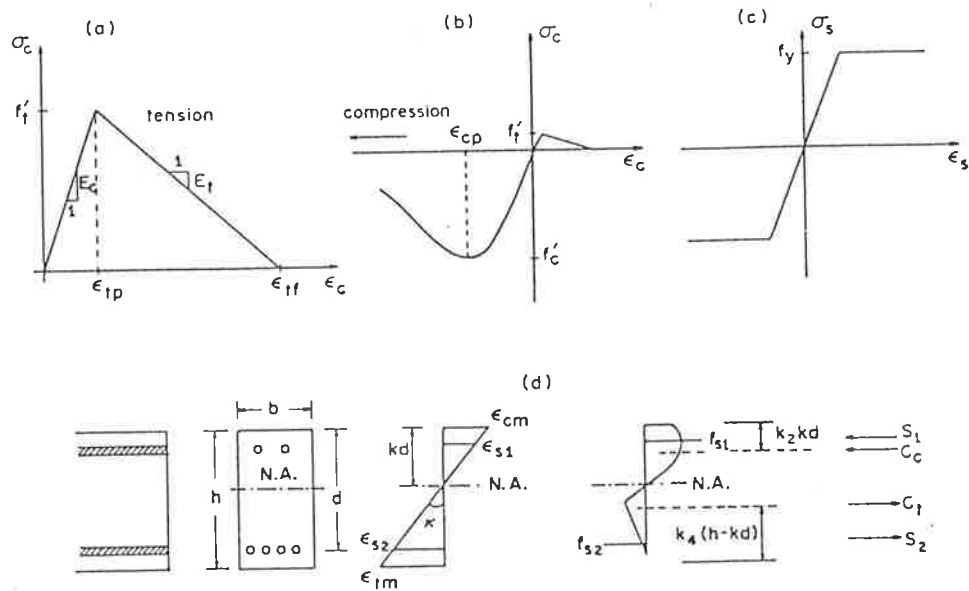


Fig. 5.9 Assumed Stress-Strain Relations for Concrete in Tension and Compression (a,b) and for Steel (c); and Stress and Strain Distributions in the Cross Section of Beams (d) (Bazant & Oh, 1984)

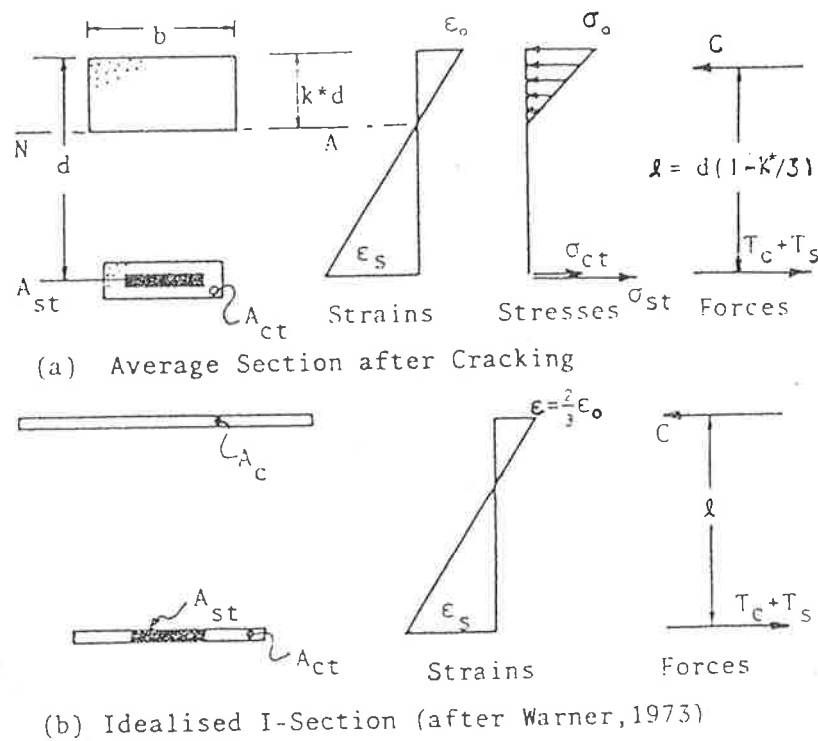


Fig. 5.10 Idealised Section with Effective Concrete Area in Tension (Gilbert, 1983)

The maximum tension stiffening force in the member is expressed as:

$$F_{s,x} = \beta_2 f_t A_c \quad (5.22)$$

where $A_c = b(h-x)$, f_t is the tensile strength of concrete and β_2 is a measure of the amount of tension stiffening present. From their experimental results, an empirical expression for β_2 was derived:

$$\beta_2 = 0.4 - \frac{s}{3b} \quad (5.23)$$

where s is the spacing of reinforcing bars.

The proposed equations were written for the particular set of test beams and slabs and may not apply to others.

Gilbert (1983) also proposed an empirical equation for the estimation of the effective area of tensile concrete which was assumed to be dependent on the magnitude of applied loading, the area of tensile reinforcement, the amount of concrete below the neutral axis and the tensile stress of the concrete. This was expressed as:

$$A_{ct} = (0.21 b_w d - n A_{st}) \left\{ \frac{M_c}{M} \right\}^2 \quad (5.24)$$

where b_w is the beam width, d is the section depth to main steel, n is the steel-concrete modular ratio, M_c is the cracking moment and M is the applied bending moment.

The tensile steel area is transformed into an equivalent concrete area, and the total equivalent concrete area at the steel level is:

$$\begin{aligned} A_{eq} &= n A_{st} + A_{ct} \\ &= A_{st} \left(n + \frac{A_{ct}}{A_{st}} \right) \\ &= n^* \rho \end{aligned} \quad (5.25)$$

where ρ is the reinforcing steel ratio and n^* is the modified

modular ratio.

Prediction of overall behaviour was obtained from a section analysis using the simple modular ratio theory with a modified modular ratio. The effective moment of inertia for a singly reinforced beam was expressed as:

$$I_e = \frac{bd^3}{12} \left\{ 4k^3 + 12n^*p(1-k)^2 \right\} \quad (5.26)$$

$$\text{where} \quad k = \frac{\sqrt{[(n^*p)^2 + 2n^*p]} + n^*p}{2} \quad (5.27)$$

An effective moment of inertia was defined for each load increment in the postcracking range. The moment-curvature relationship can then be determined for the whole load range. Good agreement was reported with test data for service load ranges.

The problems encountered in this method were in the determination of the size and shape of the effective concrete area and in the definition of the stress levels. In Gilbert's analysis, the effective concrete area was used to determine an increased stiffness which represented the tension stiffening effect.

Instead of introducing an effective concrete area, the effective steel area could be increased to provide the necessary tensile force. The German concrete code (DIN 1045) suggested an increase of 10 percent in effective steel area to account for tension stiffening effects. Cauvin (1978) recommended an additional area of fictitious reinforcement which is dependent on the magnitude of steel stress as well as the depth of the neutral axis,

$$\Delta A_s = \frac{c f_{tc} b (d-x)}{\sigma_s} \quad (5.28)$$

where ΔA_s is the area of fictitious reinforcement, c is constant taken to be 0.1667 and σ_s is the stress in the total steel reinforcement ($A_s + \Delta A_s$).

Mean Steel Strain Approach

The mean steel stress can also be used to compute average member curvatures. The average steel stress in a constant moment region can be approximated by:

$$\sigma_s = \sigma_s - \Delta \sigma_s \quad (5.29)$$

where σ_s is the steel stress obtained from a simple cracked section analysis and $\Delta \sigma_s$ is the 'average' reduction in stress due to tension stiffening effects.

Yu and Winter (1960) recommended an expression for the steel stress reduction:

$$\Delta \sigma_s = 0.02 f_c^{2/3} \frac{b h (h-x)}{A_s z} \quad (5.30)$$

where f_c is the compressive strength of concrete, b is the section width in the tension zone, h is the total section height, x is the depth of neutral axis and z is the lever arm of internal forces.

The CEB-FIP Model Code (1978) for Concrete Structures recommended a reduction in stress of:

$$\Delta \sigma_s = c \frac{\sigma_{s,r}^2}{\sigma_s} \quad (5.31)$$

where c is a constant depending on bond characteristics and type of loading, $\sigma_{s,r}$ is the steel stress in a cracked section immediately after cracking and σ_s is the steel stress in a cracked section at the load level considered

Beeby et al (1972) proposed a simple method of predicting average curvatures for partially prestressed members. Curvatures and strains are first calculated assuming no tension in the concrete and then modified by subtracting an appropriate correction term ie.,

$$\chi_{av} = \chi_{cr} - \frac{4 b_r d}{0.6 A_s h} * 10^{-6} \quad (5.32)$$

where χ_{av} is the average curvature and χ_{cr} the curvature calculated from the simple cracked section analysis, b_r is the width of section, d is the distance from concrete top fibre to the prestressing steel level, A_s the area of tension steel and h is the total depth of the section.

The correction is approximately 20 percent of the calculated curvature. A similar term is also given for the bottom concrete strain:

$$\epsilon_{av} = \epsilon_{cr} - \frac{4 A_s}{0.6 h b_r d} * 10^{-6} \quad (5.33)$$

where ϵ_{av} and ϵ_{cr} are the bottom-fibre strains for the concrete with and without tensile stresses respectively.

The equations above apply only to rectangular sections and assume linearity of the concrete compression stress block. These corrections were independent of the level of loading and contradicted the general consensus that tension stiffening effects decreased with increasing load.

Rao and Subrahmanyam (1973) employed a semi-empirical method to calculate the mean steel strain which was expressed as:

$$\epsilon_s = \epsilon_{cr} - 0.18 \left\{ \frac{f_{s,cr}}{f_s} \right\} \frac{f_r}{E_s \rho} \quad (5.34)$$

where ϵ_s is the mean steel strain, ϵ_{cr} is the steel strain at cracks, f_r is the concrete tensile strength, E_s is the modulus of elasticity of steel, ρ is the reinforcing steel ratio, $f_{s,cr}$ is the steel stress at the cracked section at first crack and f_s is the cracked section steel stress.

The authors proposed a 'trisegmental moment-curvature' relationship to simulate member behaviour. Different equations involving top and bottom-fibre strains were used to estimate average curvatures. Calculations for the precracking range were obtained from normal elastic theory. The tension stiffening effects were accounted for in the postcracking range. The average concrete top-fibre strain was derived from a simple cracked analysis:

$$\chi_{av} = \frac{\epsilon_c + \epsilon_s}{d} \quad (5.35)$$

where ϵ_c is the top-fibre concrete strain at the cracked section.

For the third stage in the overload range, tension stiffening effects were eliminated and the average and local strains were assumed to be equal. The theoretical curvature is therefore:

$$\chi_{av} = \frac{\epsilon_c + \epsilon_s}{d} \quad (5.36)$$

The authors claimed better correlation from their trisegmental relations than a simple bi-linear approach. However, the method is empirical and does not take into consideration the actual mechanisms influencing beam behaviour, and is thus considered inadequate.

5.2.3 Global Approach - First Load Cycle

The local and regional smeared approaches both require some kind of analysis of the stress and strain conditions in the flexural member. The modified stiffness approach needs no detailed analysis. Member behaviour is estimated by the use of effective stiffnesses.

The overall behaviour is described by the moment-curvature relation as defined by:

$$M = g \{ \chi \} \quad (5.37)$$

where g is a function defining the relation. For linear responses,

$$M = EI \chi \quad (5.38)$$

where EI is the flexural stiffness

Different rigidities have been used to describe various phases of beam behaviour. Tension stiffening in the postcracking range is accounted for by the use of an effective flexural stiffness. This approach is primarily used for the prediction of overall deflections under service load conditions. Section stresses and strains can not be easily computed from this method.

Yu and Winter (1960) used a multiplying factor on the cracked second moment of area to take into account the concrete tensile stiffening effect in the calculations of short-time deflections.

$$I_{eff} = \left\{ 1 - b' \frac{M_t}{M_{max}} \right\} I_{cr} \quad (5.39)$$

where M_{max} is the moment under working loads and M_t is defined as:

$$M_t = 0.1 (F'_c)^{2/3} D (D - kd) \quad (5.40)$$

and F'_c is the ultimate concrete compressive strength, D is the total section depth and kd is the distance of the neutral axis from the extreme top fibre for a cracked transformed section.

Branson (ACI Committee 435, 1966; Branson, 1968) recommended the use of an average effective moment of inertia I_{eff} :

$$I_{eff} = \left\{ \frac{M_{cr}}{M} \right\}^3 I_o + \left\{ 1 - \left(\frac{M_{cr}}{M} \right)^3 \right\} I_{cr} \quad (5.41)$$

where M_{cr} is the cracking moment and M is the moment for which

the moment of inertia is calculated, I_g is the gross moment of inertia and I_{cr} is the moment of inertia for the fully cracked section.

The effective stiffness is dependent on the ratio of the cracking moment to the applied moment. Branson and Trost (1982) later proposed a modified stiffness equation for use in partially prestressed members :

$$I'_{e} = \left\{ \frac{M'_{cr}}{M'} \right\}^3 I_g + \left\{ 1 - \left(\frac{M'_{cr}}{M'} \right)^3 \right\} I_{cr} \quad (5.42)$$

where M'_{cr} and M' are moment increments relative to the moment at which beam curvature is zero. The proposed moment-curvature relation is shown in Fig 5.11.

Beeby (1968) suggested a theoretical bilinear $M - \chi$ curve for reinforced concrete members. He made use of three limits, namely the conditions at zero load, at cracking and at ultimate load, to define the relationship. This is shown in Fig 5.12. A similar approach was also used by Kraemer, Thielen and Grasser (1975) to produce piecewise linear moment-curvature predictions. Warner(1980) also recommended a bi-linear $M-\chi$ curve.

5.2.4 Unloading Behaviour - First Load Cycle

The technical literature has revealed numerous investigations into beam behaviour in the loading direction. However, few studies have been reported on the unloading response of concrete members.

Kripanarayanan and Branson (1972) made brief references to the first unloading curve in their discussion on repeated load behaviour. Fig 5.13a shows the predicted unloading behaviour of a cracked reinforced concrete beam. The authors recommended an upper bound for the unloading curve; line BD with stiffness equal to the precracking stiffness, which assumes non closure of cracks, and approximates the case where the beam is severely cracked. Where severe cracking has not taken place

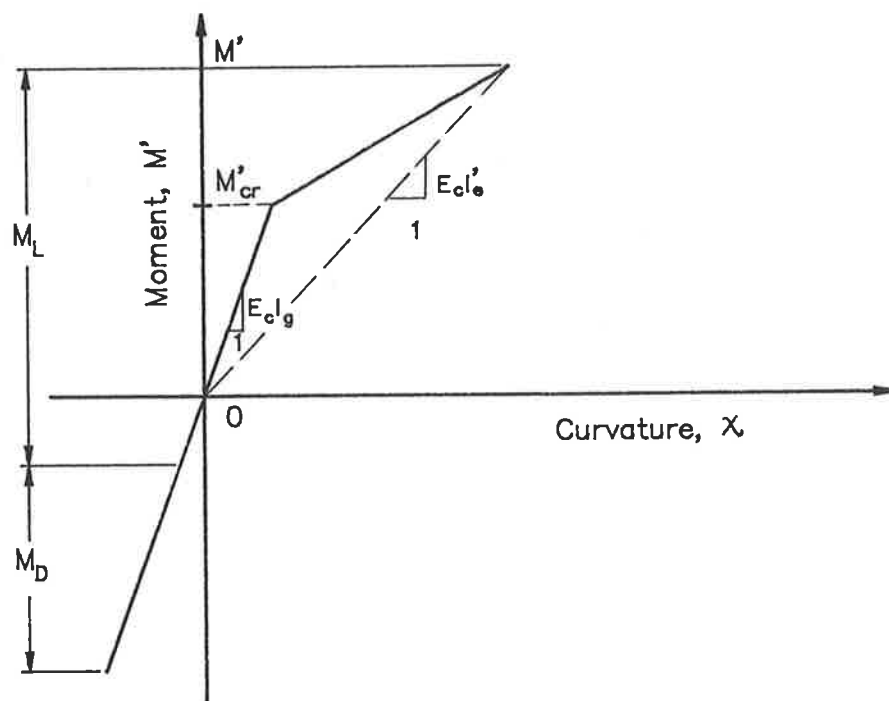


Fig. 5.11 Branson's Idealised Moment-Curvature Relation for a Prestressed Member (Branson & Trost, 1982)

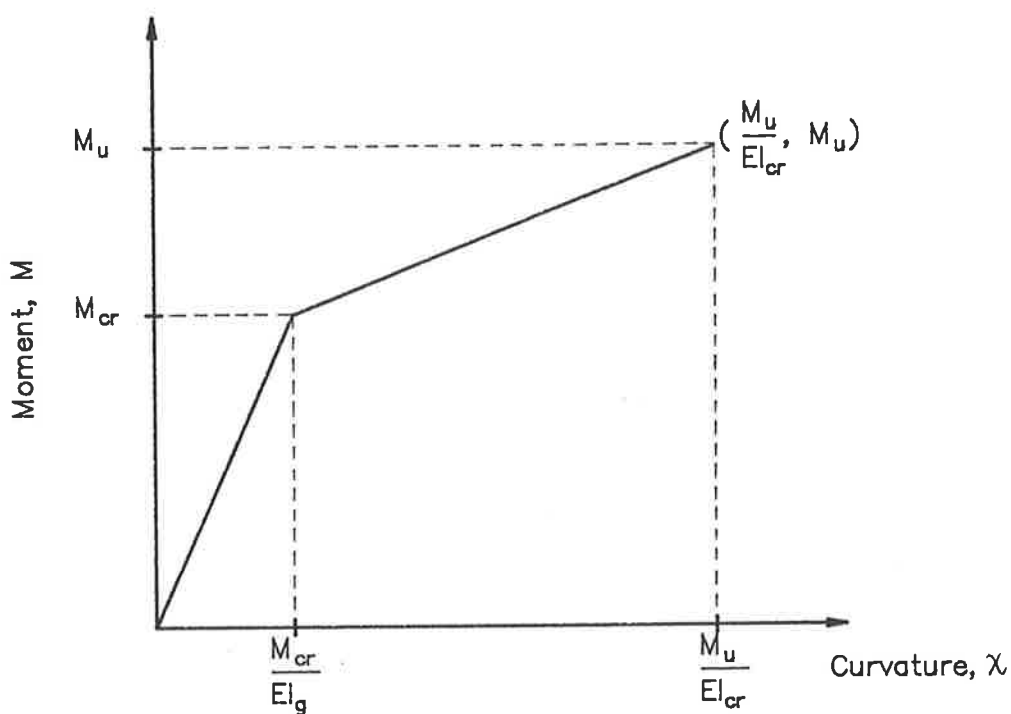


Fig. 5.12 Beeby's Moment-Curvature Relation (Beeby, 1978)

with some closure of cracks, it was suggested that unloading path will follow line BE which shows a small amount of elastic recovery indicated by DE. Total recovery in the first load cycle is shown by path BO. Although bounds were proposed by the authors, no quantitative recommendations were made regarding the actual unloading curve. Reloading behaviour was also assumed to be identical to the first unloading response.

Moosecker and Grasser (1981) reported that Schlaich, Schober and Koch (1979) obtained almost total recovery in steel deformations in tests on reinforced concrete tubes under axial load and bending. The unloading behaviour demonstrated a linear response similar to line BO in Fig 5.13a. Moosecker suggested that total recovery was only possible where the steel has not suffered plastic deformations. Where there is a possibility of steel yield, plastic strains were assumed as shown in Fig 5.13b. However, Kripanarayanan and Branson (1972) suggested that residual deformations may occur even without steel yield so long as there is incomplete crack closure. This is supported experimentally by Monnier (1969) in his tests on reinforced continuous beams. Warner and Pulmano (1980) also observed incomplete recovery in their work on partially prestressed beams.

5.2.5 Comparisons of Prediction Methods - First Load Cycle

Strain-Softening Approach

Fig 5.14a shows the theoretical moment-curvature relations for Beam SA 1 calculated using two different strain-softening curves; a linear curve proposed by Bazant and Oh (1984) and a non-linear curve recommended by Madu (1975). The experimental first cycle curve is also shown together with the simple no-tension theoretical relation. Although no consideration for creep and shrinkage effects were suggested by Bazant or Madu, these effects were nevertheless accounted for to provide a common point of origin for comparison purposes. Worst correlation between observed and theoretical values is obviously observed for the simple cracked section analysis.

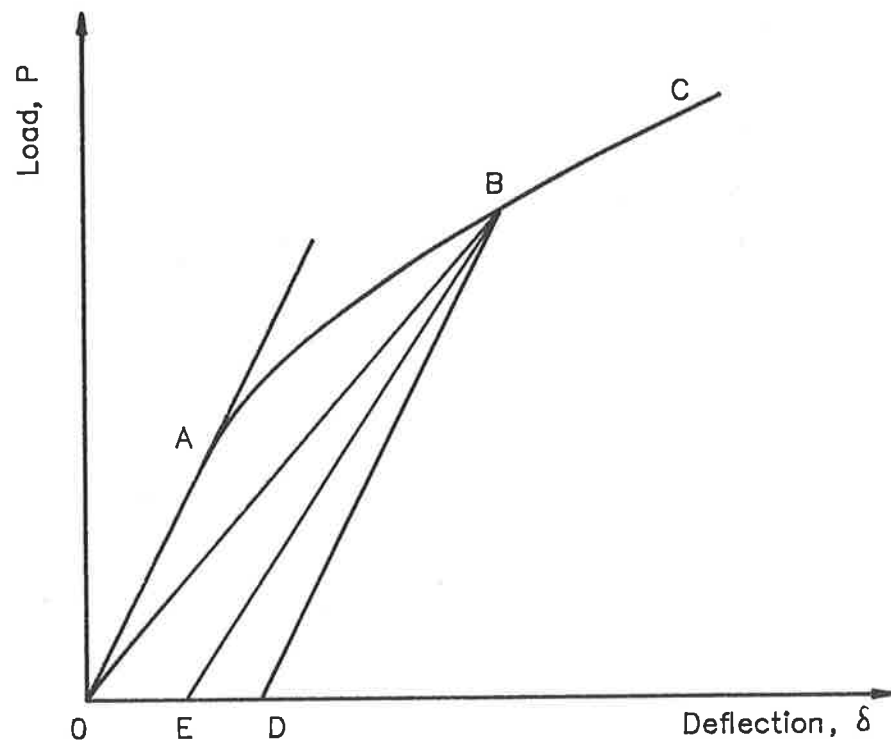


Fig. 5.13a Theoretical Load-Deflection curves (Kripanarayanan & Branson, 1972)

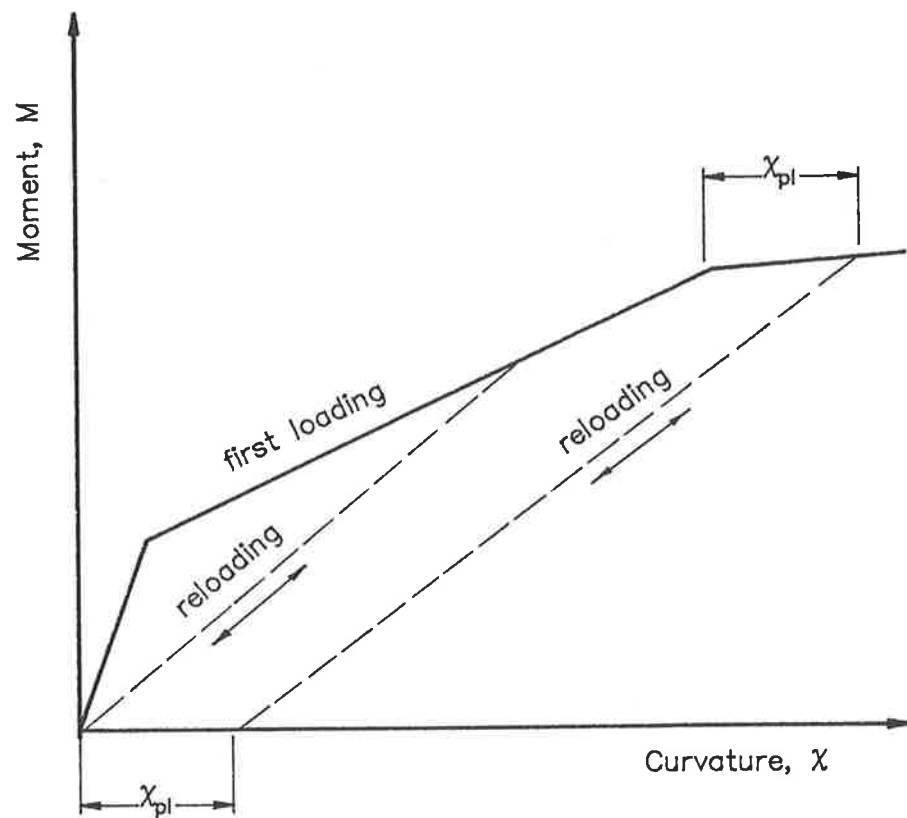


Fig. 5.13b Theoretical Moment-Curvature Relations for Reloading (Moosecker & Grasser, 1981)

The strain-softening methods produced good agreement in the immediate post-cracking region, but poorer agreement in the higher load ranges. Madu's non-linear strain-softening curve appears to give better results than Bazant's at higher moments. This is due to the gentle descending tail in Madu's tensile stress-strain relation.

Examination of Bazant's moment-curvature relation shows that the curve approaches the simple cracked section relation at high moments, resulting in overestimations of deformations and curvatures, suggesting that the theoretical tension stiffening contribution is negligible. This is not surprising as the concrete tensile stress block decreases with increased loading due to large increases in bottom-fibre deformations. The concrete tensile stiffening contribution therefore becomes smaller at higher load levels. Although the effect is not as noticeable for Madu's curve, plots at higher loads (not shown) give similar characteristics. The strain-softening method thus appears to work well in the immediate post-cracking region but does not perform satisfactorily at higher moments.

Other Current Methods of Analysis

Fig 5.14b shows the theoretical results using three different methods. The effective concrete area approach was represented by Gilbert's method (1983), while the mean strain approach was represented by Beeby (1972). Branson's global effective stiffness method was also examined. Again, initial creep and shrinkage effects were accounted for to provide a common reference point for comparisons. The loading portion of the first load cycle is accurately predicted by both Beeby and Branson's methods. Less accurate predictions were made by Gilbert's method where an overcompensation for the concrete tension stiffening effect was observed, resulting in underestimation of beam deformations. However, Branson's and Gilbert's methods do not make provisions for the calculation of the stress and strain conditions and as such their usefulness as a comprehensive method of section analysis is limited.

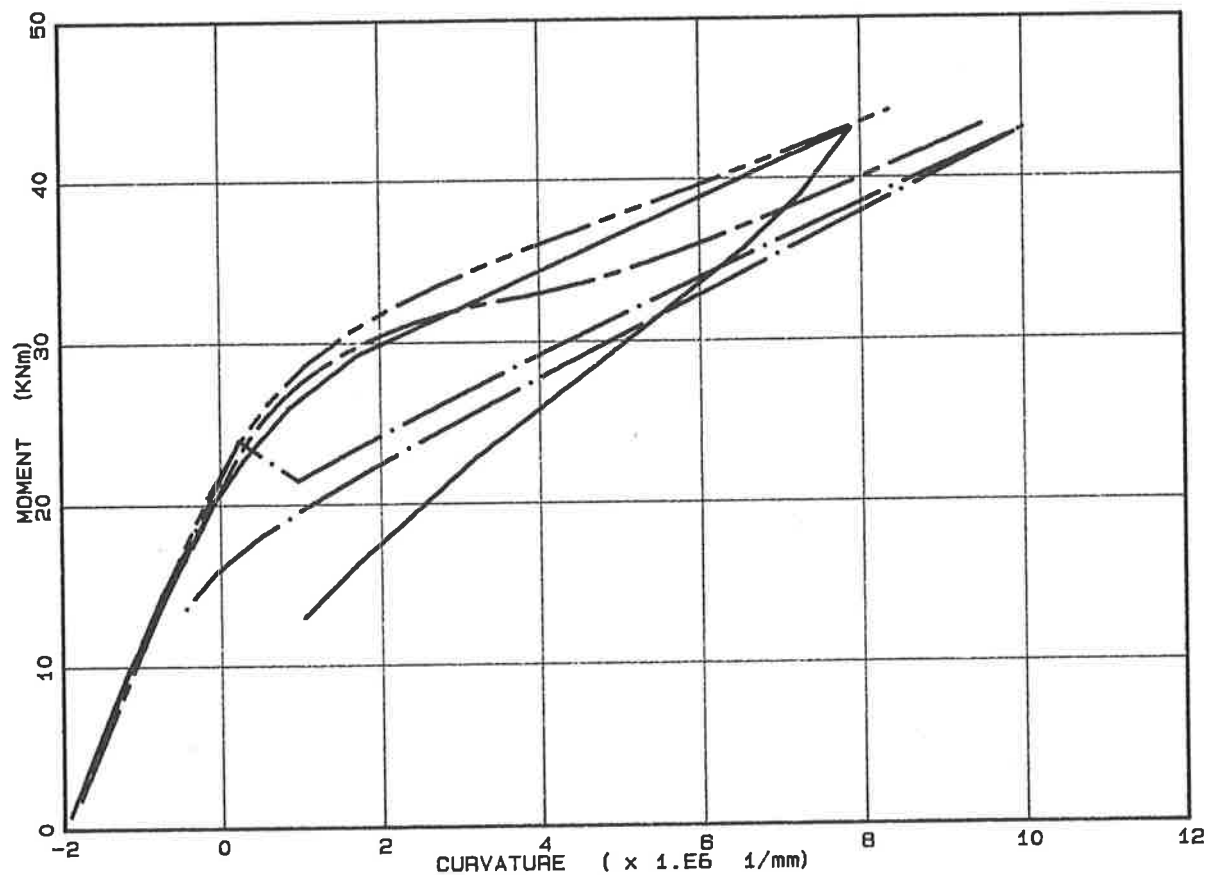


FIG 5.14a STRAIN-SOFTENING PREDICTION METHODS FOR FIRST CYCLE LOADING
 observed ——— cracked section
 bazant - - - - - madu - . - . -

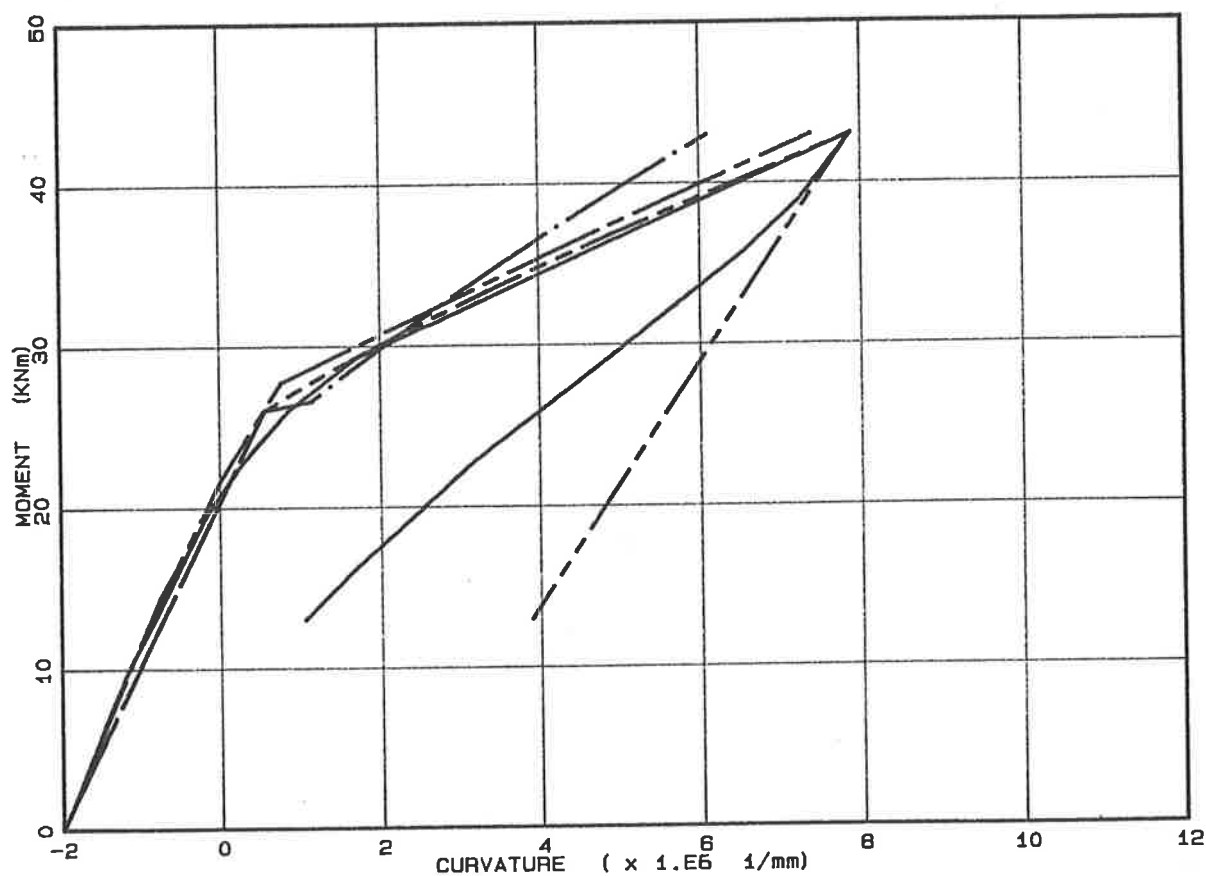


FIG 5.14b PREDICTION METHODS FOR FIRST CYCLE LOADING - BEAM SA1
 observed ——— gilbert
 beeby - . - . - branson - - - - -

The unloading curve in Fig 5.14b was calculated using the unloading equation proposed by Kripananrayaran and Branson (1972). The authors assumed that the unloading and reloading curves after first cracking were identical. Comparison of the theoretical and experimental unloading curves shows that the predicted relation is much stiffer than the actual behaviour.

The single available theoretical unloading curve typifies the extent of research expended on studies of the unloading response. Precise evaluation of the unloading response is important in fatigue calculations and is especially so in the case of partially prestressed members where considerable permanent inelastic deformations are sustained.

5.2.6 Behaviour under Repeated Loading

Relatively few theoretical investigations have been performed on repeated load response, and these deal mainly with deflection estimations. The most common approach involves the use of a modified effective stiffness for repeated loading. A typical theoretical moment-curvature relation involving the cracked, effective and repeated load stiffnesses is shown in Fig 5.15.

Kripanarayanan and Branson (1972) extended Branson's effective stiffness concept to take into account repeated loading. The repeated load stiffness $I_{r.e.p.}$ is written as:

$$I_{r.e.p.} = \psi I_e + (1-\psi) I_u \quad (5.43)$$

$$\text{where} \quad \psi = \frac{P_{ult} - P_{r.e.p.}}{P_{ult} - P_{cr}} \quad (5.44)$$

and I_e is the effective moment of inertia defined in Branson's (1968) equation, I_u the uncracked effective moment of inertia, P_{ult} is the ultimate load, $P_{r.e.p.}$ the maximum cyclic load and P_{cr} the cracking load.

The authors reported insignificant increases in residual

deformations after 3 load cycles. While they conceded that residual deformations could be caused by creep and cracking effects, the authors assumed no increases in deformation with repeated loading. Therefore, no allowances were made for increased deformations with repeated loading. The proposed effective stiffness is independent of the number of load cycles sustained and beam stiffness is assumed to be constant during repeated loading.

A similar approach was taken by Warner and Pulmano (1980) in their analyses of 16 test beams. A bilinear $M - \chi$ relationship was assumed for the overall behaviour and the slope of the curve during unloading and reloading for the first 30 cycles and I_{eff} was expressed in a similar equation as Kripanarayanan and Branson (1972). However I_e and ψ were modified to give better agreement,

$$\psi = \frac{M_{\text{max}} - M_{cr}}{M_y - M_{cr}} \quad (5.45)$$

$$E_c I_{\text{eff}} = E_c I_g - (E_c I_g - E_c I_r) \frac{M - M_{cr}}{M_y - M_{cr}} \quad (5.46)$$

A slightly different approach was used by Monnier (1970) and later Subrahmanyam (1972), with minor alterations where repeated loading was represented by a modified stiffness. Monnier's proposed equation was:

$$(EI)_r = \frac{M + M_{cr}}{\chi + \chi_{cr}} \quad (5.47)$$

where M_{cr} and χ_{cr} are the cracking moment and curvature, and M and χ are the respective values at maximum cyclic load.

Balaguru (1981 and 1982) proposed a fatigue analysis which was supposed to take account of the cyclic creep effects of concrete and bond deterioration. A description of his analysis has been given in Chapter 2. The analysis is based on a combination of a simple elastic linear analysis and Branson's modified stiffness equation under repeated loading. The cyclic creep effect was represented by a decrease in concrete stiffness and bond deterioration was simulated by a reduction

in the concrete modulus of rupture and hence cracking moment. The method is cumbersome to use and appeared to be global in nature despite claims of a comprehensive method of analysis.

Loo and Wong (1984) recommended a new equation for the fatigue deflection prediction of singly-reinforced concrete box beams. The total deflection sustained in the N^{th} cycle is assumed to comprise of two components, namely:

$$\delta_T = \delta_{da} + \delta_i \quad (5.48)$$

where δ_i is the instantaneous live load deflection at the N^{th} cycle, and δ_{da} is the accumulated dead load deflection (permanent set) caused by dead load and effects due to repeated loading. Fig 5.16 shows the theoretical moment-deflection relationship.

The equation can be rewritten as:

$$\delta_T = a_k \delta_{di} + \frac{\alpha M_i L^2}{E_c I_{re,p}} \quad (5.49)$$

where

$$a_k = \frac{\delta_{da}}{\delta_{di}} \quad (5.50)$$

and δ_{di} is the initial dead load deflection, α is a multiplication factor governed by the loading condition, L is the span, E_c is the elastic modulus of the concrete, $I_{re,p}$ is the equivalent moment of inertia at the N^{th} load cycle.

The response to fatigue loading is separated into two distinct effects, namely the increase in residual deformation and the progressive loss of stiffness. Increases in residual deformation are accounted for by the amplification factor a_k which was derived from experimental results:

$$a_k = 1.18 + \frac{.0343}{\rho} \frac{(M_t - M_d)}{(M_y - M_{cr})} \frac{N^{0.6}}{(0.3 + N^{0.6})} \quad (5.51)$$

where M_t is the maximum moment, M_{cr} the cracking moment, M_y the moment at yield and M_d the dead load moment and ρ the

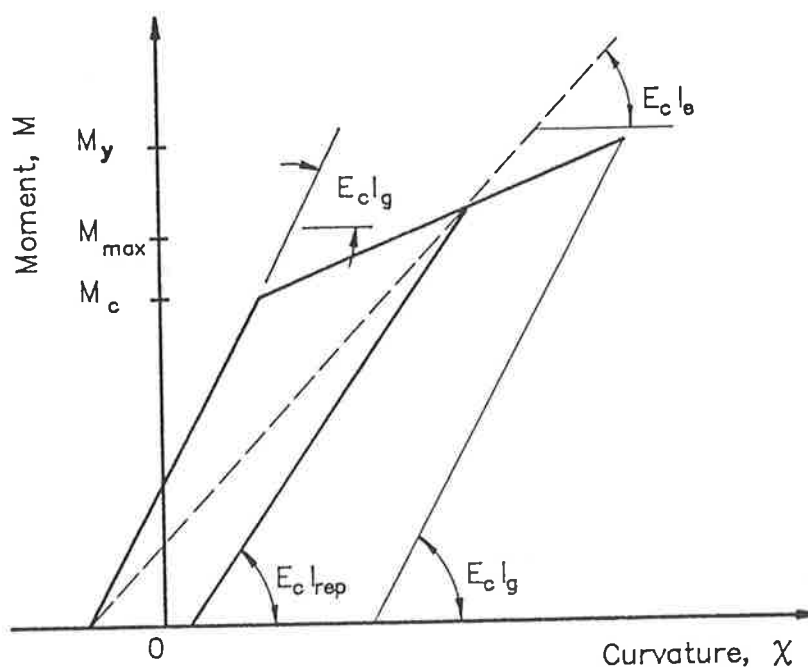


Fig. 5.15 Moment-Curvature Relation Involving Loading, Unloading and Repeated Loading

δ_{di} = Initial dead load deflection. δ_L = Live load deflection.
 δ_{d1} = 1st cycle dead load deflection. δ_{Ti} = Initial total deflection.
 δ_{d2} = Accumulated dead load deflection. δ_T = Total deflection.

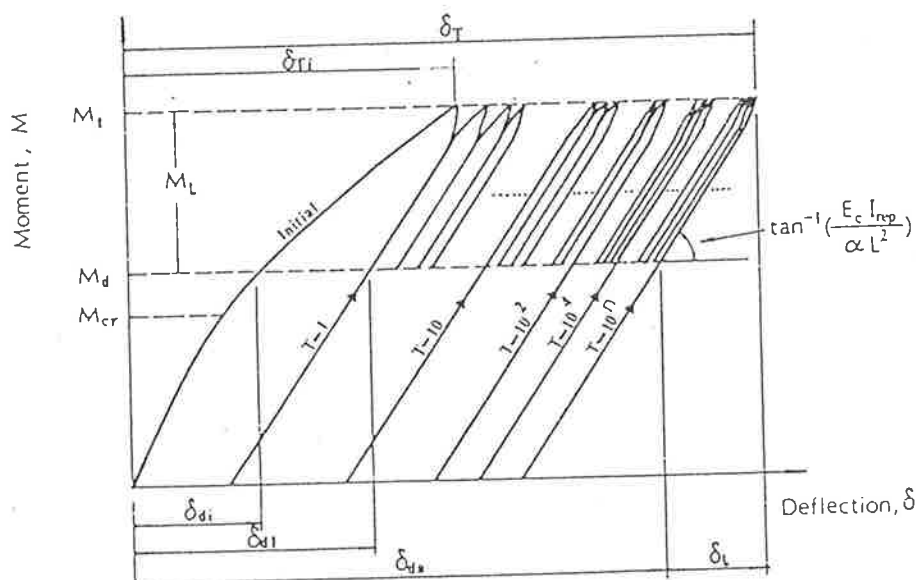


Fig. 5.16 Deflection of a Concrete Beam Under Repeated Loading (T = number of loading cycles) (Loo & Wong, 1984)

reinforcing ratio

The loss of stiffness with repeated loads is represented by an effective moment of inertia I_{eff} :

$$I_{eff} = \left\{ \frac{M_{cr}}{M_x} \right\}^m I_g + \left\{ 1 - \left(\frac{M_{cr}}{M_x} \right)^m \right\} I_{eff} \quad (5.52)$$

where

$$M_x = \frac{(M_t - M_{cr})^2}{(M_y - M_{cr})} + M_{cr} \quad (5.53)$$

I_{eff} is the effective moment of inertia as defined in Branson's equation and $m = 2, 3$ or 4 . A value of 2 is recommended by the authors for use in box beams.

The analysis is global in nature and based on Branson's concept of I_{eff} . The first cycle load deflection or moment curvature curves are calculated using Branson's equations. Subsequent cycle deformations are then calculated using the proposed empirical relationships. Prediction is limited to $100,000$ cycles. The proposed amplification factor a_k , responsible for the increases in residual deformation, does not seem to be sensitive to repeated loading for large values of N (N may be in the order of $100,000$ cycles) when it is dependent on the following expression:

$$\frac{N^{0.6}}{0.3 + N^{0.6}}$$

Branson and Trost (1982) accounted for the increased deflection due to further cracking under repeated load cycles by a multiplying factor. The overall deflection is assumed to consist of a creep component and an instantaneous component which took into account the increased deformations under a small number of repeated load cycles. The time dependent deflection is:

$$\delta = k_t C_t \delta_i + F_{rep} \delta_i \quad (5.54)$$

where k_t is a reduction factor for the effect of the

compression steel and downward movement of the neutral axis with time. C_t is the ratio of creep strain to initial strain, δ_1 the deflection at first load and $F_{r,1}$ the amplification factor for repeated loading.

Lovegrove and El Din (1982) proposed two empirical equations for calculating the long term cyclic deflection and curvature for reinforced concrete members sustaining a large number of repeated load cycles. The equations are derived from experimental testing of twelve reinforced concrete beams:

At maximum deflection,

$$\delta_n = 0.225 \delta_0 \log N \quad (5.55)$$

At maximum curvature,

$$\chi_n = 0.225 \chi_0 \log N \quad (5.56)$$

Where δ_n and χ_n are the N^{th} cycle deflection and curvature and δ_0 and χ_0 are the corresponding first cycle values. N is the number of load cycles sustained.

They recommended that for theoretical predictions, the initial deflections and curvatures be estimated using methods specified in the British CP 110 code or the ACI Committee 318 publication. Cyclic deflections and curvatures can then be computed from the above equations.

5.2.7 Comparison of Prediction Methods for Repeated Loading

Fig 5.17 shows the theoretical moment-curvature relations for Beam SA 1 derived from Loo and Wong's method and from Kripanarayanan and Branson's method for the first and 30000th load cycle. The simple cracked section analysis relations (taking into account concrete cyclic creep effects) are also plotted together with the experimental curve. The first cycle predictions from Loo and from Kripanarayanan are identical as both use Branson's effective stiffness equation. No increase

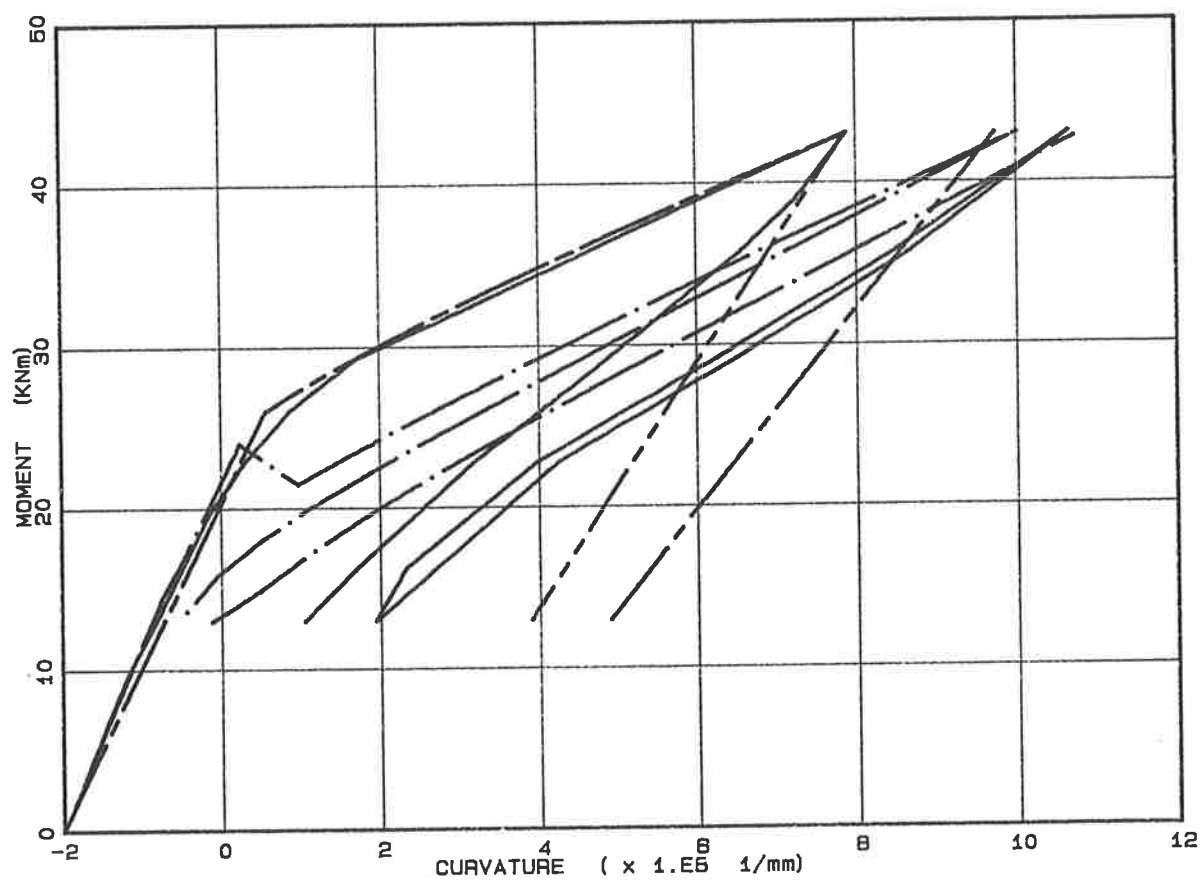


FIG 5.17 PREDICTION METHODS FOR REPEATED LOADING - BEAM SA1

observed	———	cracked section	——— · ———
loo & wong	———	kripanarayanan	——— · ———

in residual deformation was observed in Kripanarayanan and Branson curve. The authors assumed that the unloading and reloading curves were identical. Loo and Wong on the other hand did not consider the first unloading curve but accounted for increased permanent deformations by an amplification factor. However, the calculation of curvatures at low moments is overestimated by approximately 100 percent while there is a slight underestimation of curvatures at high moments. The simple no-tension analysis appears to give similar estimates as Loo at maximum load.

5.2.8 Summary

A review of the current literature has shown various methods for predicting overall beam behaviour. For accurate predictions, the concrete tensile stiffening effect needs to be taken into consideration. The modified stiffness approach, first proposed by Branson, appears to be most popular and gives good predictions in the postcracking load range for the first loading cycle. However, this approach is global and does not take into account the underlying mechanisms involved. The strain-softening method gives good predictions in the immediate postcracking range but is not as good at high moments. The mean strain method appears to give good predictions in the load range tested. However, the method is semi-empirical in nature. The initial effects of creep and shrinkage have not been considered in most of the prediction methods mentioned above. This is unsatisfactory as the time-dependent inelastic effects can have considerable influence on later beam response.

Fewer investigations on behaviour under repeated loading were found in the technical literature. The modified stiffness approach is again favoured by most researchers. A reduced effective moment of inertia I_{eff} is used to represent the decreased stiffness caused by repeated loading. Increased residual deformation is represented simply by an amplification factor. The method therefore does not qualitatively account for the various physical mechanisms occurring in the beam.

Furthermore, the method is inadequate for use in fatigue life calculations as no provision is made for the evaluation of section stresses.

A comprehensive method of predicting beam behaviour under static and repeated loading is thus lacking. The presently available methods are inadequate in defining the state of stress and strain in a flexural member during loading, unloading and reloading conditions. The development of such a method is called for and is considered in the following chapter.

CHAPTER 6

ANALYSIS OF REPEATED LOAD BEHAVIOUR IN THE POSTCRACKING RANGE

The methods of analysis reviewed in Chapter 5 provide neither an adequate explanation nor an accurate quantitative account of repeated load behaviour in the postcracking range. An attempt is made in the present chapter to explain the observed behaviour and then to develop a simple and accurate method of analysis which allows deformations and stresses to be estimated.

Sections 6.1 and 6.2 re-examine and evaluate repeated load behaviour observed in the beam tests while Section 6.3 describes the fundamental concepts of a proposed 'regional' analysis. A detailed description of the method is given in Section 6.4, and comparisons of theoretical and experimental results are made in Section 6.5. A more general and simplified method of analysis is also described in Section 6.6.

6.1 STUDY OF OBSERVED STRAIN AND STRESS PROFILES

As the first step in developing a more accurate method of predicting repeated load behaviour, the test data presented in Chapter 3 was re-examined and analysed.

6.1.1 Detailed Description

The variation of average observed concrete strain across a typical cracked section with increasing load is reproduced in Fig 6.1. The typical average strain and stress distributions, obtained for a specific applied load, are shown in Fig. 6.2. The stresses in the concrete and in the steels were obtained

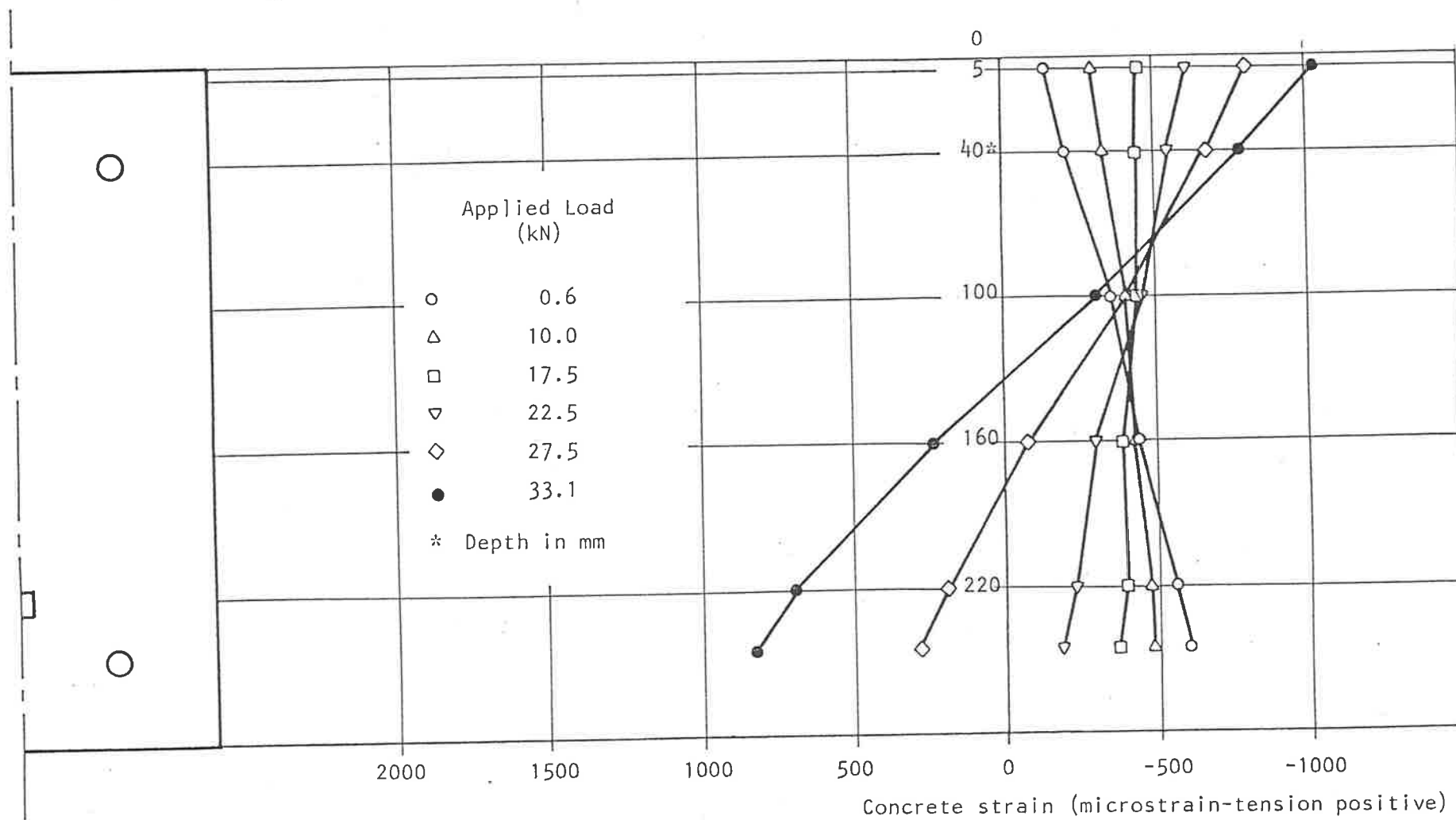
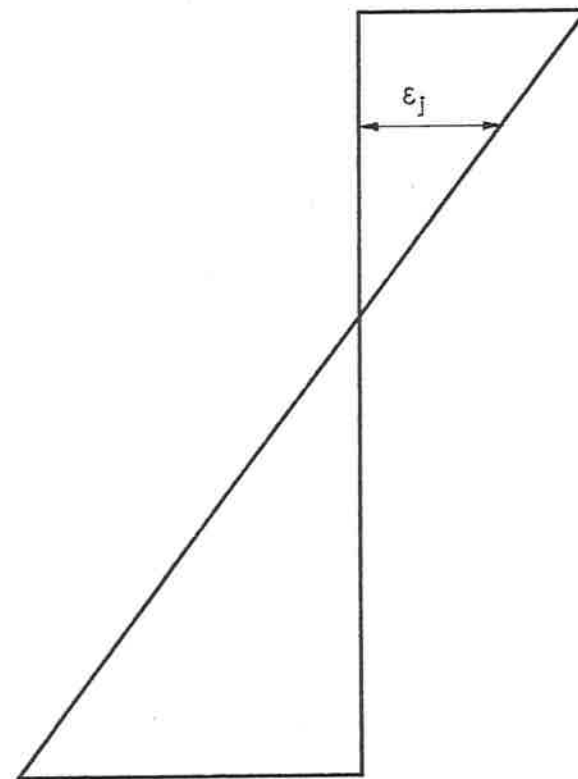
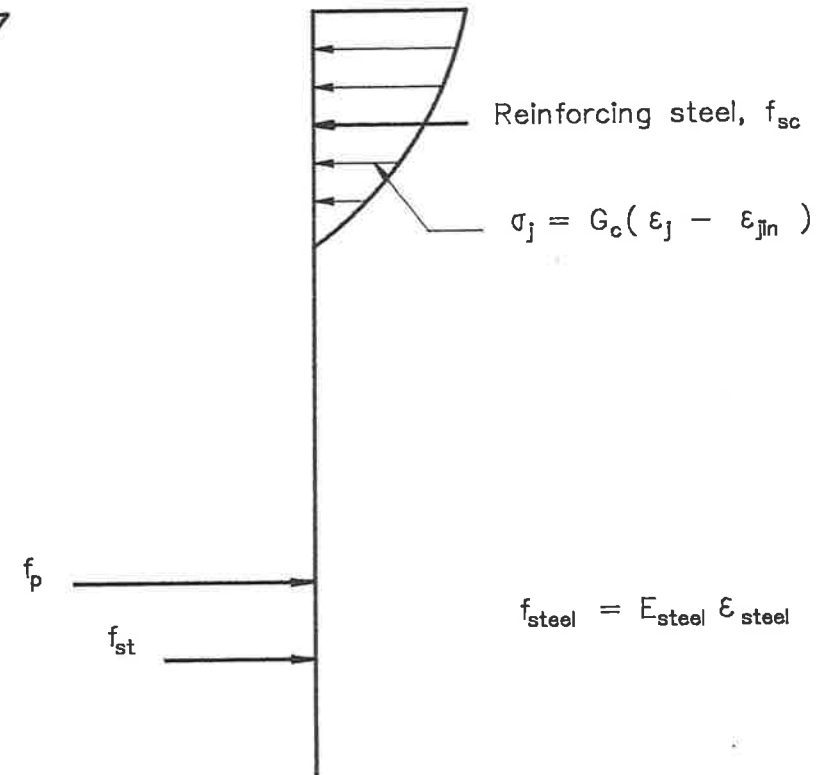


Fig 6.1 AVERAGE CONCRETE STRAINS AT VARIOUS LOADS FOR BEAM SA1



Average Concrete Strain Distribution



Average Stress Distribution

Fig. 6.2 Observed Stress and Strain Conditions

from the measured strain profile using elastic stress-strain relations for the steels and a concrete constitutive relation which ignores tensile stresses after cracking. The initial creep and shrinkage strains were estimated using the method described in Section 4.3.2 and were assumed to remain constant during the first cycle and subsequent repeated loadings. The material constitutive relations defined earlier in Section 5.1.1 were assumed to be valid for average stress calculations.

Concrete Stresses from Observed Strains

The layering technique of section analysis, already described in Chapters 4 and 5, was used. The elastic strain component in the j^{th} concrete slice is expressed as:

$$\epsilon_{j,e} = \epsilon_{j,t} - \epsilon_{j,i} \quad (6.1)$$

where $\epsilon_{j,t}$ is the total concrete strain obtained from the observed average strain distribution, and $\epsilon_{j,i}$ is the estimated total inelastic strain in the layer.

For elastic compressive strains:

$$\sigma_{j,c} = G_c \{ \epsilon_{j,e} \} \quad (6.2)$$

For elastic tensile strains:

$$\text{before cracking: } \sigma_{j,t} = G_t \{ \epsilon_{j,e} \} \quad (6.3)$$

$$\text{after cracking: } \sigma_{j,t} = 0$$

Stress Profiles and Force Resultants

The calculated average concrete stress and measured strain distributions for a typical test beam (SA 1) at different levels of applied moment in the first, second and 300,000 load cycles respectively are shown in Figs 6.3a,b and c. The stress diagrams are drawn in blue while the strain lines are shown in green. The magnitude and location of the quantities are

plotted to scale. However, where the line of action lies outside the section, the distance measured from the extreme top-fibre (down is positive) is shown. The magnitude and location of the resultant force of the concrete compressive and tensile stress block as well as the tensile and compressive forces in the reinforcing and prestressing steels, are also shown (in red). Tensile stresses and forces are shown as positive (left) and compressive values negative (right).

A check was made to determine whether these calculated forces were in equilibrium with the applied moment. In cases where the equilibrium requirements were not satisfied, the additional tensile force $F_{r.e.}$ and moment $M_{r.e.}$ (taken about the extreme top fibre) required to maintain equilibrium in the section were calculated, ie

$$F_{r.e.} = C_{c.c} + T_{s.c} - C_{c.t} - T_{s.t} - T_p \quad (6.4)$$

$$M_{r.e.} = M_{c.c} + M_{s.c} - M_{c.t} - M_{s.t} - M_p \quad (6.5)$$

where $C_{c.t}$ and $C_{c.c}$ are the total concrete tensile force and compressive force respectively, and $M_{c.c}$ and $M_{c.t}$ are the moments for these forces taken about the top concrete fibre. $C_{c.t}$ and $M_{c.t}$ exist in the precracking range only and are zero after cracking. $T_{s.c}$ is the compressive steel force and $T_{s.t}$ and T_p are the respective tensile reinforcing and prestressing steel forces. $M_{s.c}$, $M_{s.t}$ and M_p are the respective moments taken about the extreme top-fibre.

The total concrete compressive force and corresponding moment about the extreme top fibre are defined as:

$$C_{c.c} = \sum_{j=1}^{N_A} (\sigma_{j.c} \cdot b_j \cdot t_j) \quad (6.6)$$

$$M_{c.c} = \sum_{j=1}^{N_A} (\sigma_{j.c} \cdot b_j \cdot t_j \cdot y_j) \quad (6.7)$$

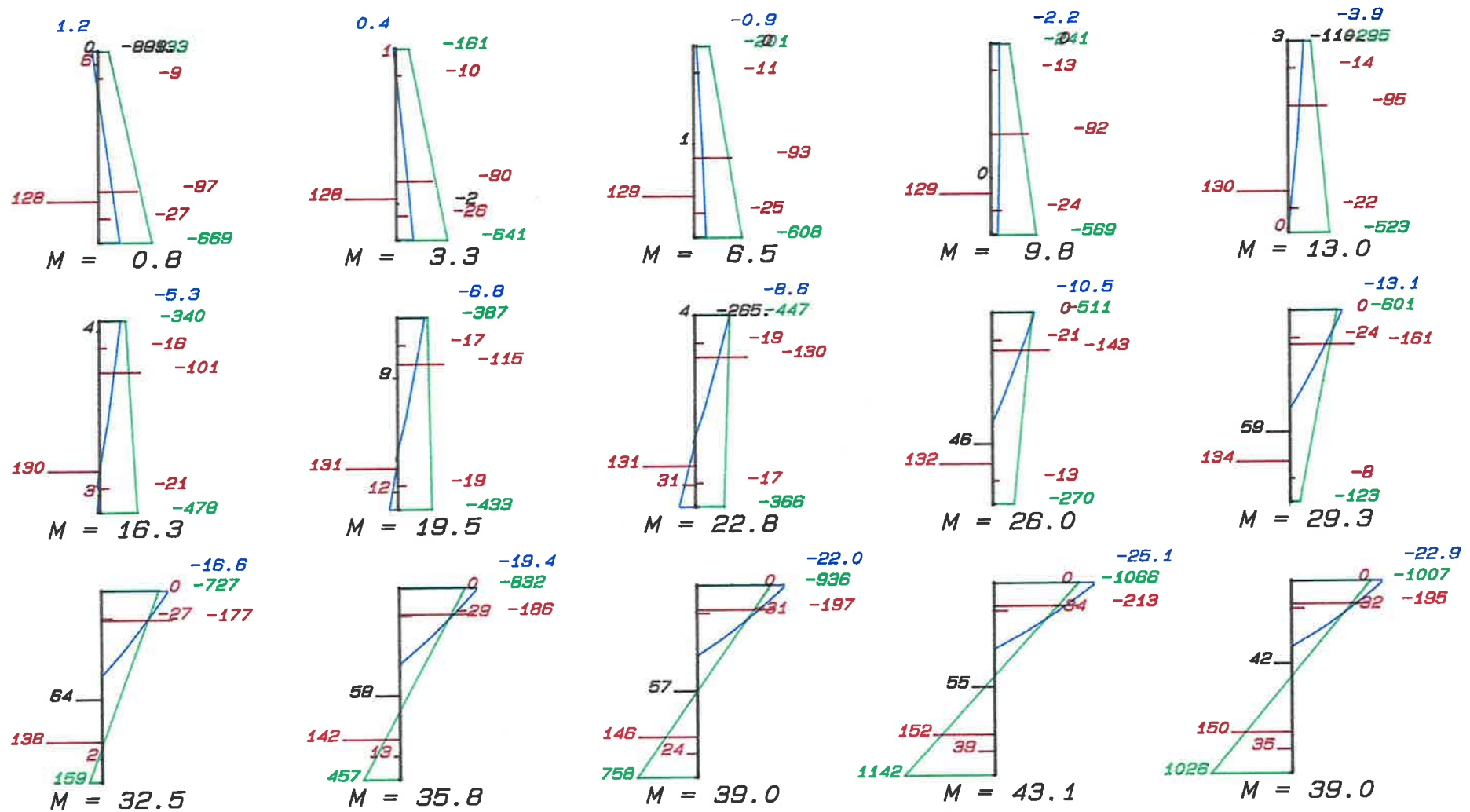


FIG 6.3a OBSERVED STRESS AND STRAIN DISTRIBUTIONS - BEAM SA1
 First load cycle, applied moment (KNm)-black, Freq (KN)-black
 concrete stress (MPa)-blue, strain (microstrain)-green, force resultants (KN) -red

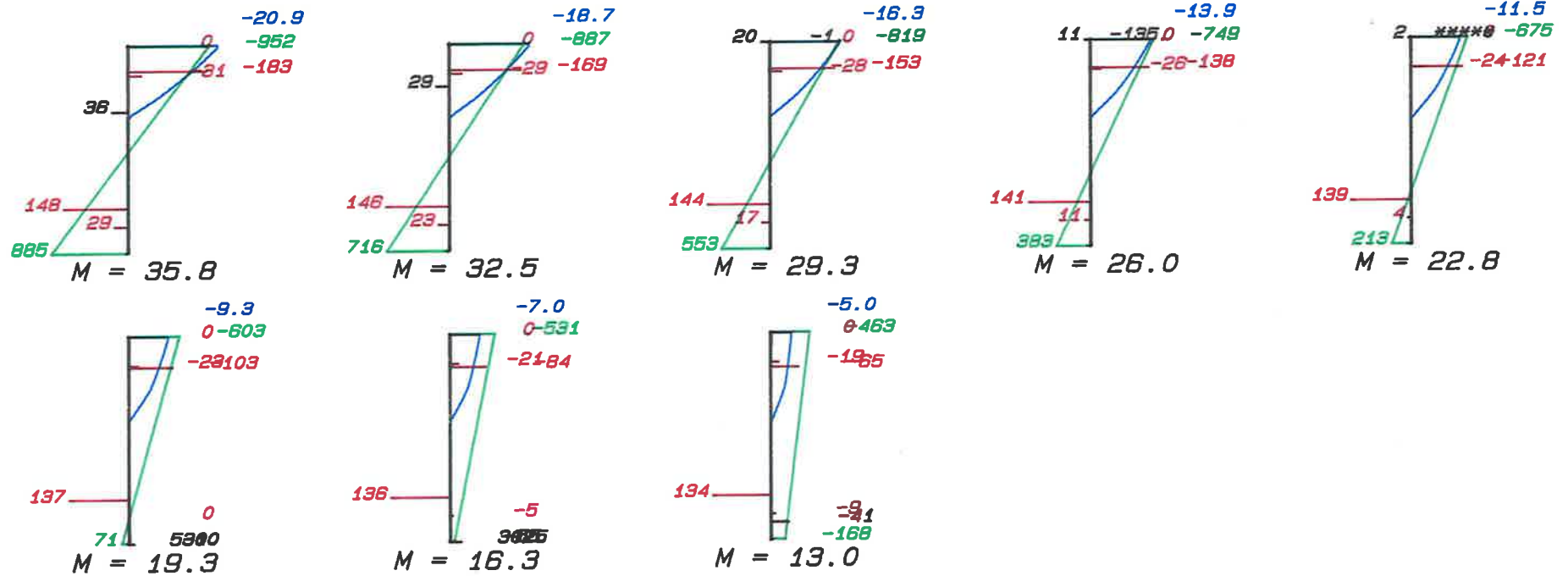


FIG 6.3a cont'd OBSERVED STRESS AND STRAIN DISTRIBUTIONS - BEAM SA1
 First load cycle, applied moment (KNm)-black, Freq (KN)-black
 concrete stress (MPa)-blue, strain (microstrain)-green, force resultants (KN)-red

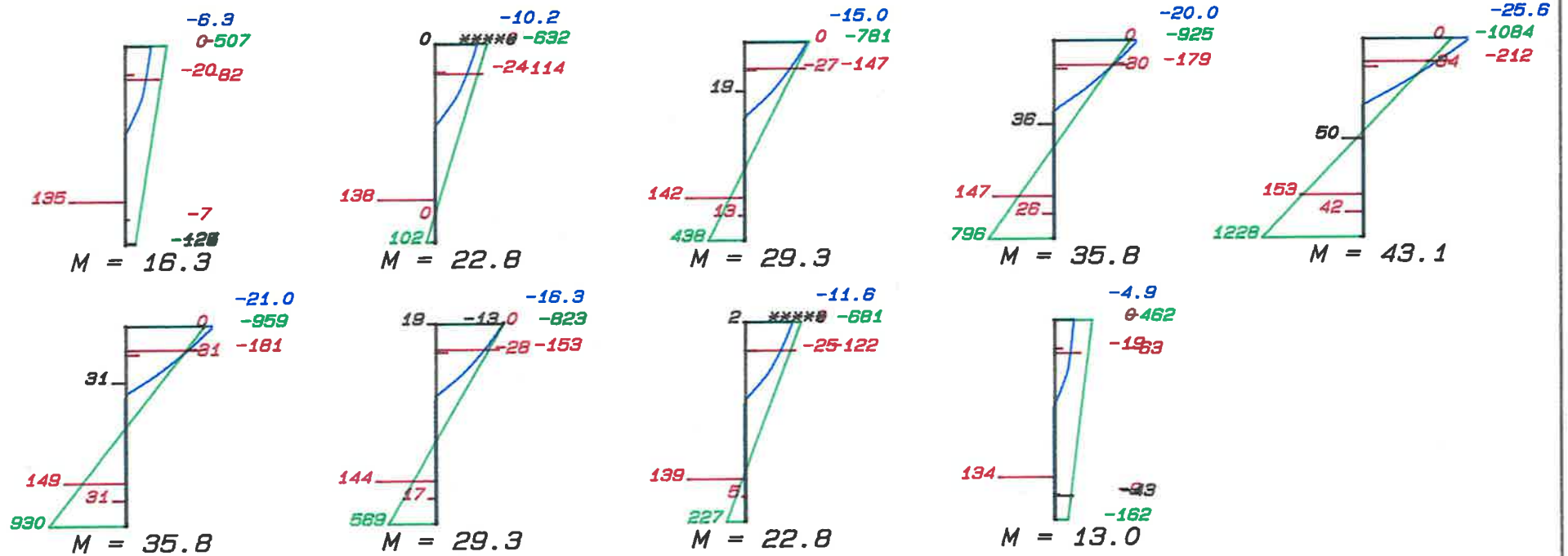


FIG 6.3b OBSERVED STRESS AND STRAIN DISTRIBUTIONS - BEAM SA1
 Second load cycle, applied moment (KNm)-black, Freq (KN)-black
 concrete stress (MPa)-blue, strain (microstrain)-green, force resultants (KN)-red

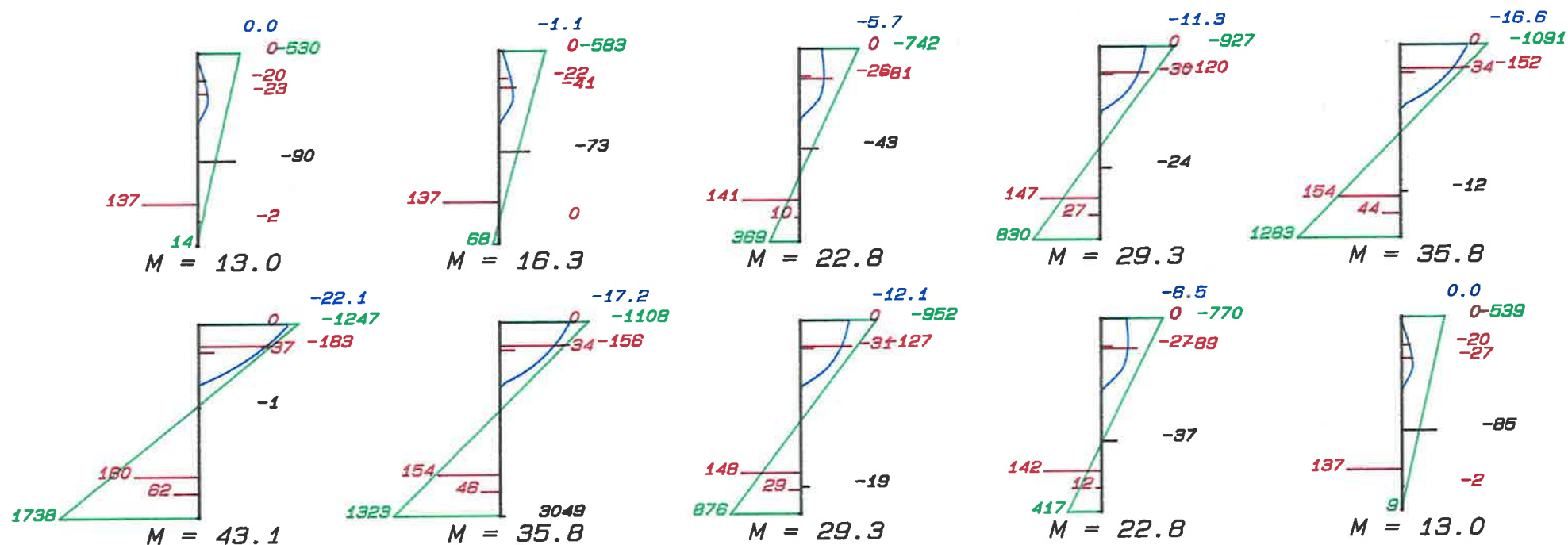


FIG 6.3c OBSERVED STRESS AND STRAIN DISTRIBUTIONS - BEAM SA1
 300000th load cycle, applied moment (KNm)-black, Freq (KN)-black
 concrete stress (MPa)-blue, strain (microstrain)-green, force resultants (kN)-red

The line of action of the required equilibrating tensile force $F_{r.e.}$ is determined from moment equilibrium conditions ie.,

$$l_{r.e.} = \frac{M_{r.e.}}{F_{r.e.}} \quad (6.8)$$

where $l_{r.e.}$ is measured from the extreme top fibre. The location and magnitude of $F_{r.e.}$ are plotted and shown (in black) in Figs 6.3a, b and c. When $F_{r.e.}$ is close to zero and $M_{r.e.}$ is large, $l_{r.e.}$ can become large and is inherently sensitive in these circumstances. When $l_{r.e.}$ is excessively large, its magnitude is indicated by asterisks.

6.1.2 Observations from Experimental Data

First Cycle - Loading

Fig 6.3a shows that the concrete and steel stresses are compressive at low moments while the prestressing tendons are of course in tension. First cracking occurs between the applied moments of 22.8 and 26.0 KNm when the total concrete strain in the extreme bottom fibre is still compressive (300 microstrains). Further increases in load causes the stress neutral axis to rise and the compressive concrete stresses and the tensile bar and tendon stresses to increase.

Before cracking, both force and moment equilibrium conditions are satisfied within expected errors. After cracking, equilibrium is not satisfied, and the additional force $F_{r.e.}$ and moment $M_{r.e.}$ required to maintain equilibrium are approximately 30 percent and 25 percent of the total tensile force and moment in the section. These values are too large and consistent to be attributed to random or experimental errors. The force $F_{r.e.}$ decreases slightly as the moment increases to its maximum value. During this phase of loading, the location of $F_{r.e.}$ remains relatively constant at a position approximately midway between the stress neutral axis and the prestressing steel. Clearly, $F_{r.e.}$ is a measure of the tension stiffening effect in the tensile concrete in the bottom fibres of the beam, while $M_{r.e.}$ is a measure of the location of the

tension stiffening effect.

First Cycle - Unloading

A comparison of the stress profiles in the loading and unloading directions, in Fig 6.3a, shows that for the same applied moment the top-fibre concrete and steel stresses are larger during unloading. The stress neutral axis is higher, and in contrast with the loading direction, falls comparatively slowly during unloading. The concrete compressive force is small compared with the corresponding force obtained in the loading direction because of the relatively high neutral axis position. The strain profiles show similar behaviour with the top and bottom-fibre strains being larger during unloading. Considerably larger increases are observed in the bottom fibres than in the top.

The tensile force $F_{r..}$ reduces quickly from a tensile value at maximum moment to zero, approximately at the initial cracking moment, and then becomes compressive at lower loads. At the end of the first load cycle, the magnitude of this compressive force is as large as its previous maximum value in tension. Fig 6.4 shows the variation of the required equilibrating force as a function of applied moment. The variation of the required additional moment $M_{r..}$ with applied moment is also presented in Fig 6.5. The curve is similar in shape to that of Fig 6.4.

Significant residual deformations are incurred at the end of the first load cycle. Fig 6.3a show that the strain conditions (in green) at the beginning and at the end of the load cycle are quite different. The corresponding stress conditions are also shown (in blue). There is an increase in the compressive concrete extreme top-fibre strain of 168 microstrains (57 percent of the precracking value) and an even larger increase (tensile) in the bottom-fibre strain of 355 microstrains (68 percent).

The large residual tensile strains in the bottom fibres ensure that the elastic concrete strains in the bottom fibres are

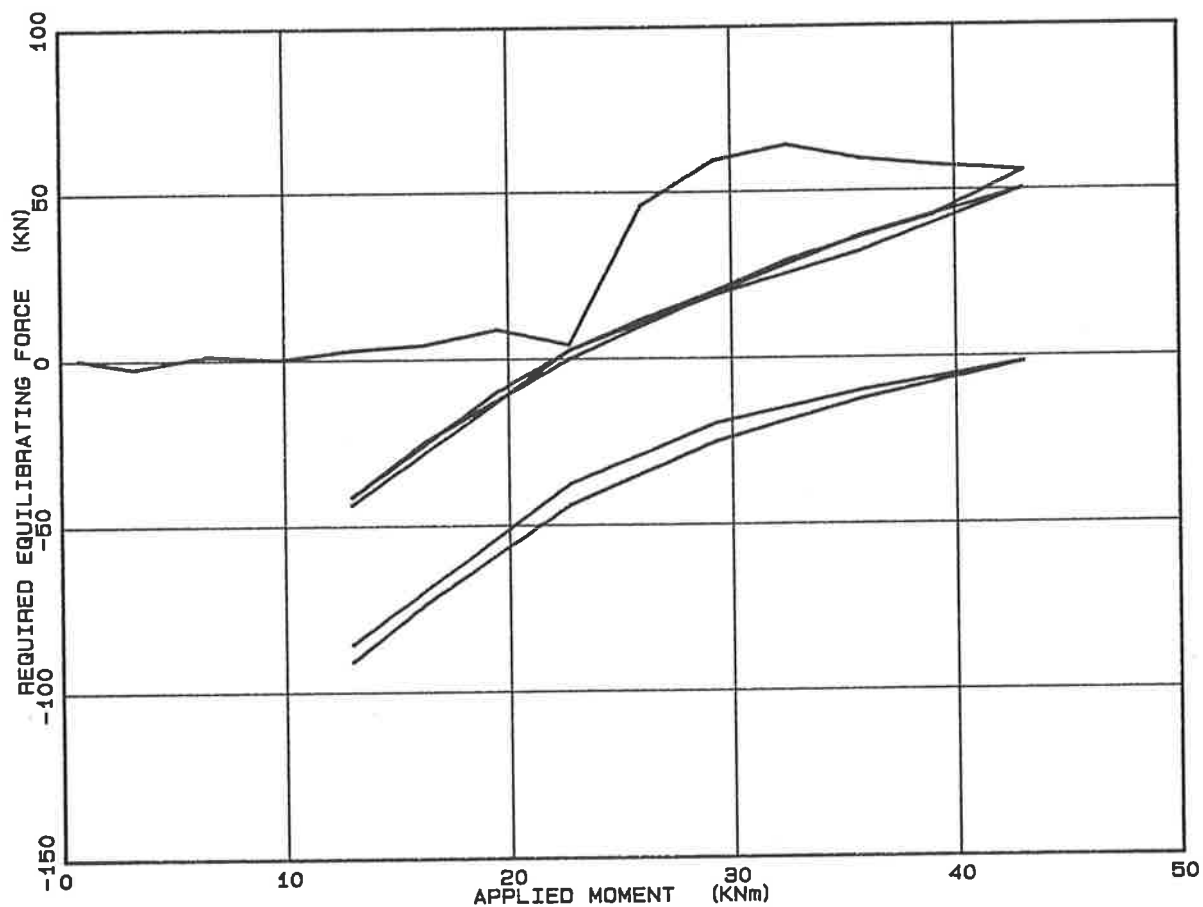


FIG 6.4 EQUILIBRATING FORCE-MOMENT RELATIONS
BEAM SA 1

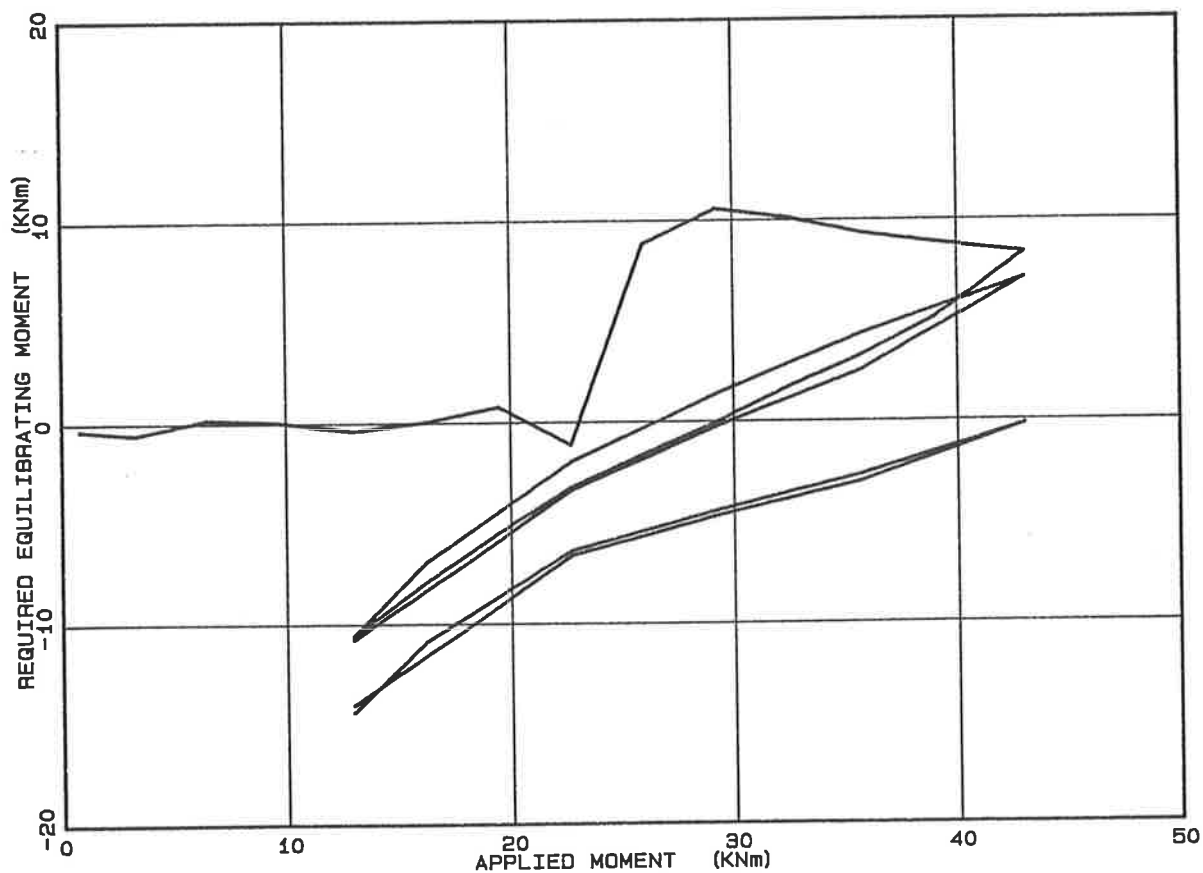


FIG 6.5 EQUILIBRATING MOMENT-MOMENT RELATIONS
BEAM SA 1

tensile at the end of the first cycle. The stress neutral axis is located high up in the section and because of this, the total concrete compressive force is small. A large compressive force $F_{r,c}$, is thus required to maintain force equilibrium.

Second and Subsequent Load Cycles

The stress conditions in the section calculated for the second load cycle follow closely those for the first unloading curve, as seen in Fig 6.4. There is a slight decrease in the tensile force $F_{r,t}$, at high moments, and an increase in the compressive force $F_{r,c}$, at low moments. The hysteresis effect is much smaller compared with the first cycle, with the loading and unloading curves virtually following the same path.

After a large number of load cycles (300,000 for beam SA1), Fig 6.4 shows a significant decrease in $F_{r,t}$ at high loads and also an increase in the compressive force $F_{r,c}$ at low moments. The reduction in the tension stiffening force at maximum moment is marked, with $F_{r,t}$ decreasing almost to zero.

Fig 6.6 & 6.7 are scatter diagrams which show the $F_{r,t} - M$ and $M_{r,c} - M$ curves for four test beams. The curves are all similar, indicating a systematic relationship between the level of applied moment (and load history) and the requirement of an additional force and moment to maintain equilibrium.

While the large tensile values of $F_{r,t}$ at high moments are in keeping with current concepts of tension stiffening, the large compressive forces at low moments are unexpected and need to be explained.

6.2 EXPLANATION OF BEAM BEHAVIOUR

An hypothesis is introduced to explain beam behaviour and in particular the unusual results presented in Section 6.1. In simple terms, the large equilibrating compressive forces are assumed to occur at low moments in association with non-closure of the tensile cracks. The non-closure is brought

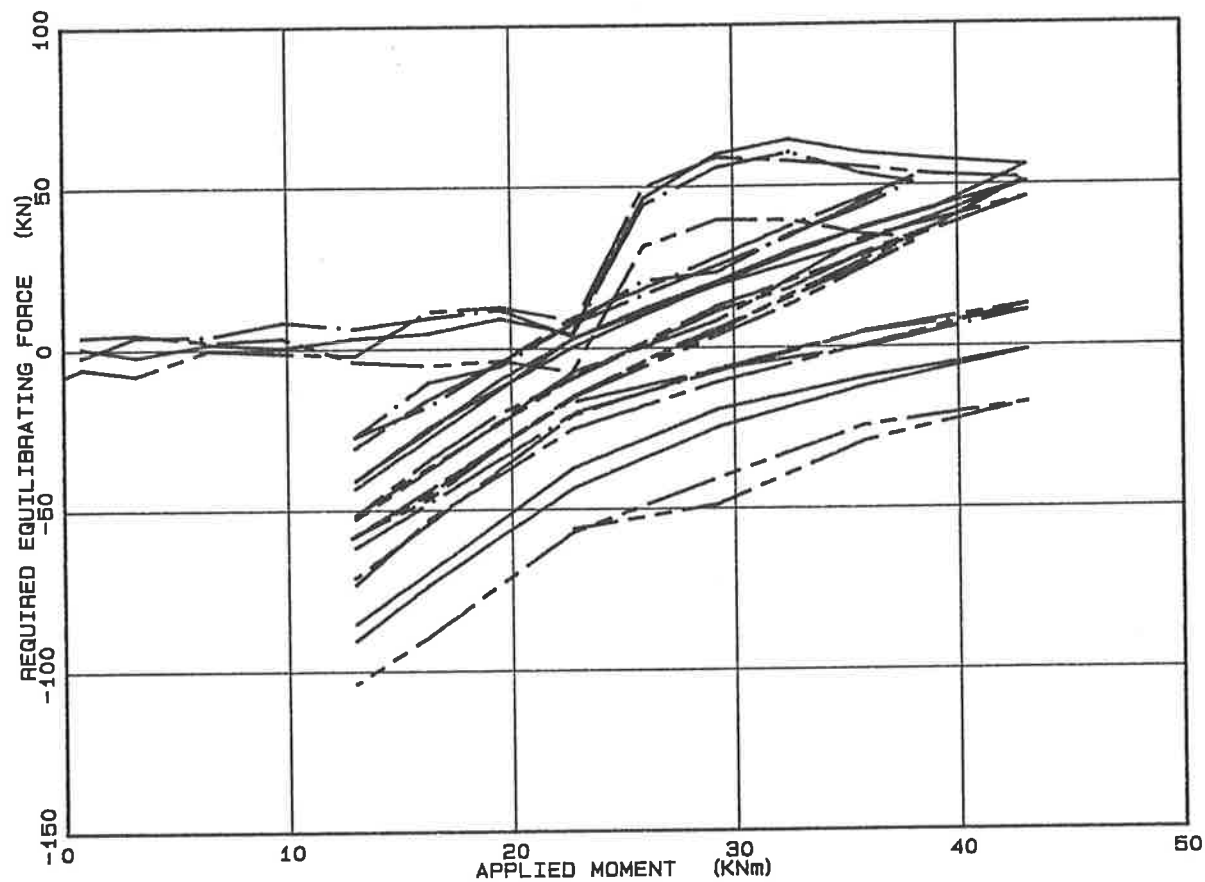


FIG 6.6 EQUILIBRATING FORCE-MOMENT RELATIONS
 Beam SA 1 ——— Beam SA 2 — · —
 Beam SA 3 - - - - - Beam SA 4 - - - - -

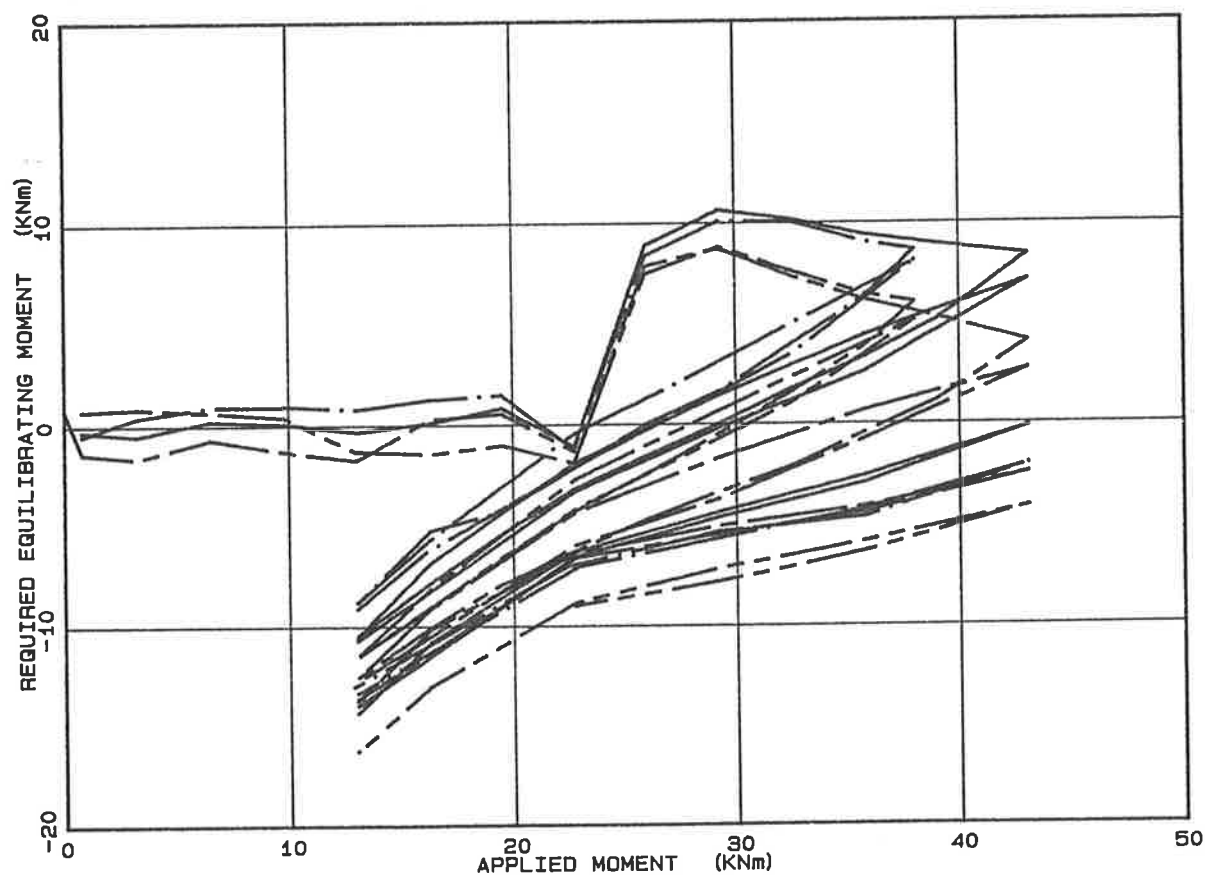


FIG 6.7 EQUILIBRATING MOMENT-MOMENT RELATIONS
 Beam SA 1 ——— Beam SA 2 — · —
 Beam SA 3 - - - - - Beam SA 4 - - - - -

about by mis-match of the crack surfaces and some formation of debris within the crack. This produces a permanent inelastic strain increment, or wedging effect, in the concrete. At low moments, the tensile force in the tendon is increased, with corresponding increased compressive force in the concrete. With increasing moment during a subsequent loading cycle, the wedging action decreases and at high moments disappears. However, at low moments, its presence is responsible for the large compressive equilibrating force $F_{r,c}$. A more comprehensive discussion of these effects is now given, step by step.

6.2.1 First Cycle - Postcracking, Loading

While the mechanisms of cracking in flexural members may not be identical to those in direct tensile specimens, much can be learnt from previous studies on the cracking and bond behaviour of pure tensile elements in reinforced concrete structures. The behaviour of the concrete in the immediate vicinity of the steels under flexural loading is, to a certain extent, similar to the response of a concrete prism which contains a reinforcing bar.

The phenomenon of cracking in flexural members (Beeby, 1968; Broms, 1964; Base and Beeby, 1966; Clark and Speirs, 1978) as well as in direct tensile specimens (Goto, 1971; Bresler and Bertero, 1968; Popov, 1984) have been well researched. Fig 6.8 shows a typical pattern of internal cracks and 'comb-like' concrete keys in the vicinity of the reinforcing steel, as reported by Goto (1971). It was suggested that due to the tensile stresses in the steel, large bearing stresses are sustained under the steel lugs and produce compressive struts in the intact concrete. The compressive stresses are in turn balanced by a combination of longitudinal tensile stresses (which produce the tension stiffening effect) and tensile hoop stresses in the concrete.

In addition to the above mechanisms, two other mechanisms have to be considered which also influence behaviour in the

post-cracking range.

Microcracking

The actual stress field in the concrete is much more complex than that presented above. Many localised tensile and compressive stress paths exist in the concrete even though the resultant effect is a tensile stiffening force. In addition to the primary and internal cracks, it is likely that microcracking in random directions takes place due to the very high tensile stresses. Further cracking may develop from the tip of inclined internal cracks and extend to the primary cracks.

With extensive cracking in the concrete surrounding the steel bars, a form of shear-lag is considered to take place and results in a loss of force transfer capability from the steel to the concrete even though there is no relative movement of the concrete immediately adjacent to the steel. The concrete close to the cracks therefore becomes ineffective in transmitting the tensile stresses to the relative uncracked outer zone. Fig 6.9 shows the two concentric zones of concrete surrounding the steel. Microcracking and the resulting loss of tensile stiffening force increases with increasing load.

Slip Mechanisms

Slip is defined as relative movement between adjacent concrete and steel surfaces. It occurs when either the concrete in front of the steel lugs crushes (Rehm, 1961; Lutz and Gergely, 1967; Kemp and Wilhelm, 1976), which is unlikely under working loads, or when the concrete teeth are pushed away by the lugs through wedging action (Mirza and Houde, 1979). Mirza reported no evidence of crushing of concrete due to rib pressure or polishing of the surface due to sliding of the bar. This author's beam tests also showed little cracking or crushing of concrete in the vicinity of steel (Section 3.4.6). Some slip also occurs with the opening of primary and internal cracks as the applied moment increases. With extensive microcracking, it is likely that the predominant mechanism is not slip between

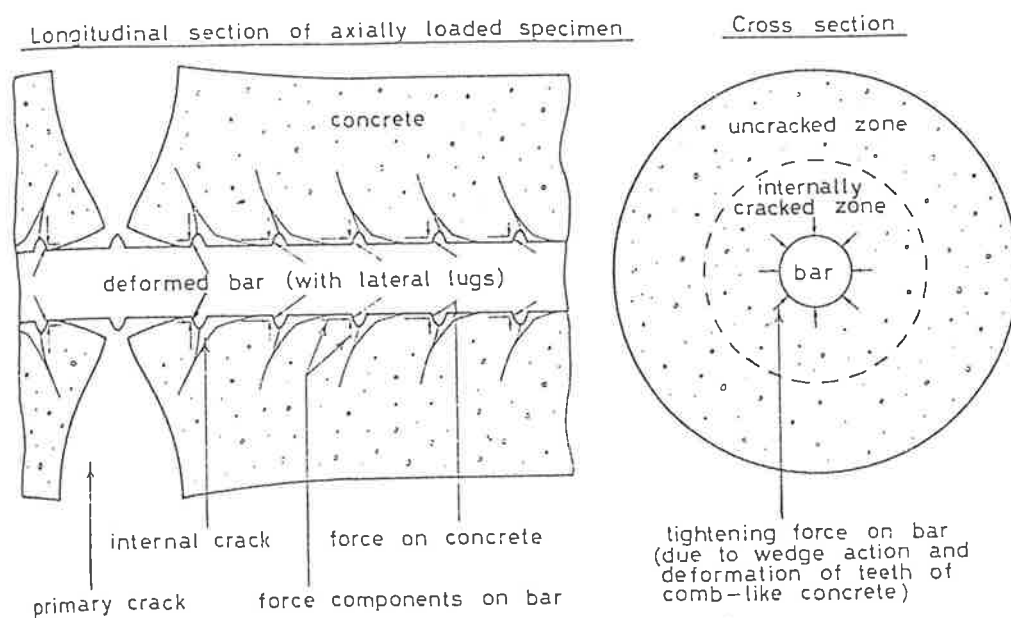


Fig. 6.8 Schematic Diagram of Concrete Deformation around Reinforcing Steel (Goto, 1971)

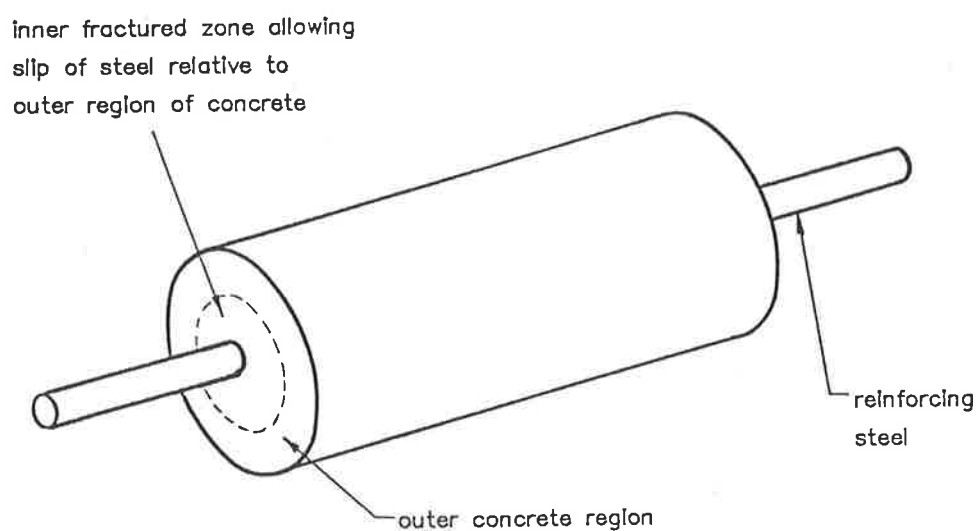


Fig. 6.9 Schematic Diagram of Concrete Slip Zone

steel and the immediate concrete, but rather relative sliding between the two different concrete rings (as pictured in Fig 6.9). This results in a relative slip between the outer, relatively uncracked concrete and the reinforcing steel.

The combination of progressive microcracking and slip results in a deterioration of the tension stiffening effect with increased deformations under increasing load.

6.2.2 First Cycle - Unloading

Three distinguishing stages of unloading, - initial near-elastic recovery, slip recovery and crack closure, were exhibited in the beam tests. Large residual strains were observed during unloading and these occurred predominantly in the bottom concrete fibres. This suggests that the factors responsible for the increased permanent deformations act in the lower concrete region. Different mechanisms may be present at different sections along the beam. Therefore, two distinct regions will be considered. The first is in the vicinity of the primary flexural cracks, and the second in the concrete region in-between.

First Phase - Initial Elastic Recovery

During unloading, elastic recovery is the initial response of the tension zone. This occurs in the intact concrete between cracks. Slip in the opposite direction between concrete and steel in this region is resisted by friction and interlock of the cracked surfaces. Bresler and Bertero (1968) suggested that slip is resisted primarily by the wedging action of the rugged surfaces. With no relative movement between concrete and steel, the compressive stresses in the concrete teeth (Fig 6.8) reduce elastically with decreasing load. The result is that relatively large fall-offs in tensile stresses are accompanied initially by small decreases in overall deformation. The section behaves elastically with the effective member stiffness somewhat smaller than the gross uncracked stiffness, but greater than the cracked stiffness in

the last stage of loading. This effect is shown in path CD of Fig 6.10.

The initial unloading response is also influenced by short-term creep effects. As the beam tests are performed under conditions where the load is controlled and deformations unrestrained, a reversal in loading direction initially causes relatively little reduction in deformation. This phenomenon has been attributed by some researchers to short-term creep. At primary cracks, the initial unloading behaviour is very similar to that in the loading direction. Very little tensile concrete is present and no resistance is offered against the recovery of steel strains. The near elastic unloading action observed in the relatively uncracked sections is absent.

The overall initial unloading behaviour, being a combination of the responses of all zones, shows an unloading stiffness somewhat less in magnitude than that observed in the uncracked loading range. This first phase of unloading represents a small portion of the total unloading curve and may be ignored in a simplified analysis.

Second Phase - Slip Recovery

The stress field in the relatively intact concrete surrounding the steel is complex, and elastic recovery of the concrete stresses occurs at different rates with some stresses recovering earlier than others. In those areas where tensile stress recovery is completed, it is possible for localised compressive stresses to be set up even though the resultant force is still tensile. Localised slipping commences as soon as resistance in the opposite direction is overcome. This heralds the start of the second unloading phase and occurs at a relatively high load. Point D in Fig 6.10 indicates the commencement of the second unloading phase.

Micro-sliding and slipping of both concrete and steel surfaces occur with progressive closure of the cracks. This is the dominant unloading phase with the majority of the 'recoverable' concrete and steel deformations being recovered.

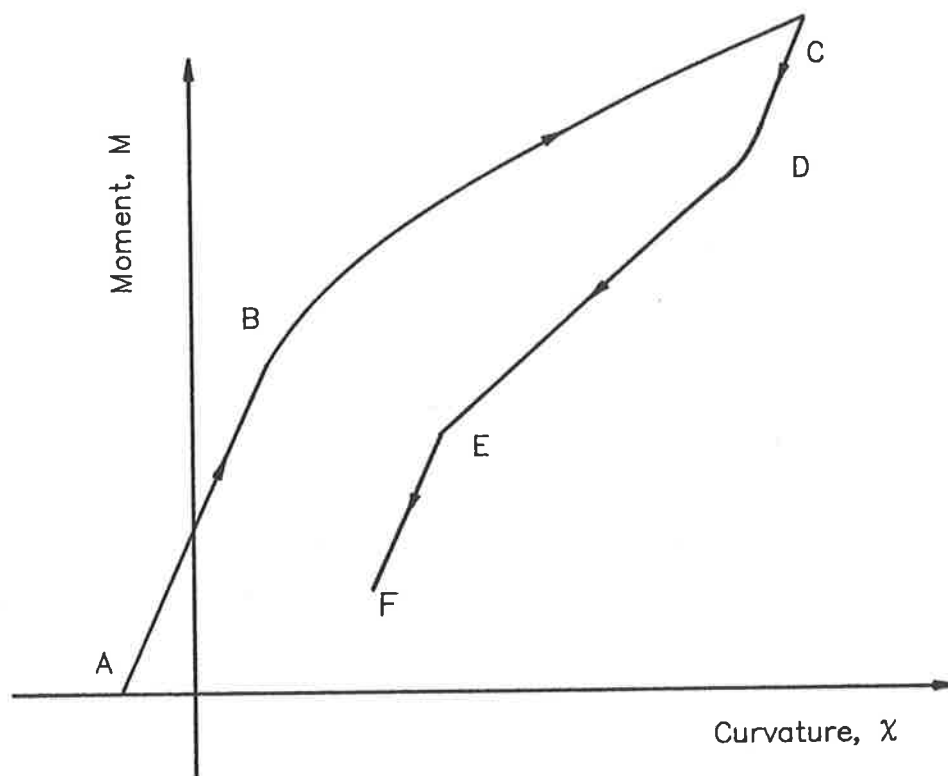


Fig. 6.10 Beam Behaviour Under Loading and Unloading Conditions

The overall behaviour of the beam shows a flattening of the slope of the moment-curvature curve. With further unloading, tensile stresses throughout the concrete reduce to zero, while compressive stresses are set up as some crack surfaces begin to come together.

At sections in the vicinity of primary cracks, the unloading behaviour is probably not dissimilar to that of the initial loading phase. Recovery of steel deformation is effected with little opposition. The stress profiles here are therefore similar to those in the loading direction.

The second phase continues until the concrete cracked surfaces start to come together and are able to carry compression stresses. The moment at which this commences is lower than the first cracking moment.

Final Unloading Phase - Incomplete Closure of Cracks

Path EF in Fig 6.10 shows the final unloading phase in the observed moment-curvature plot. The final phase is characterised by an increase in member stiffness. The primary cracks are prevented from closing properly because of mis-match of the opposing crack surfaces and the inclusion of debris within the crack interstices. The crack surfaces are not smooth or regular but probably consist of a number of irregular, overlapping surfaces. Debris inclusion was indicated in the beam tests (reported in Chapter 3) by the presence of dust and fine particles in the cracked regions during beam testing. Permanent inelastic deformations in the cracked concrete can thus be induced, so that as the prestressing force reimposes compression into the concrete, wedging effects result.

The inelastic deformations may also occur in the regions between primary cracks. It is not unreasonable to expect mismatch of adjacent crack surfaces and even debris formation to take place in the microcracks. The effect is even more complicated with the presence of different types of cracks.

There is also a possibility that these effects may be larger than at the primary cracks due to the bigger intact concrete area, more widespread pattern of internal and microcracking and the fact that the cracks are small in size - mismatch effects increase inversely with size. Recovery of the steel deformation is resisted in part by the frictional resistance of the intact concrete surfaces and by incomplete closure of cracks. This results in residual tensile steel stresses which may possibly be higher than those in the primary cracks. The concrete stress conditions are shown in Fig 6.11 below.

Overall, the stiffness of the unloading curve in the the final phase is similar to that found in the precracking range. Residual deformations are present in the member resulting in compressive stresses in the concrete and corresponding tension stresses in the steels. A simplified conceptual model of this third phase of unloading is of small wedges being placed within the primary, secondary and micro-cracks, thus producing permanent tensile deformations in the tensile zone, with a large resultant compressive force in the concrete and an increased tensile force in the tendon.

6.2.3 Behaviour in the Second and Subsequent Load Cycles

The initial loading behaviour in the second load cycle is elastic and retraces the path of the unloading curve in the first cycle. The compressive concrete stresses in the bottom fibres near the steel reduce elastically with increasing applied moment. The variation of the effective concrete force with applied moment for the second cycle is shown in Fig 6.4 above.

With further loading, local compressive stresses near cracks reduce to zero, and cracks begin to open. In addition, slipping between concrete and steel surfaces and also sliding between adjacent crack surfaces commence. Increased deformation is experienced with decreasing effective compressive stresses. When sufficient load has been applied to the member, tensile stresses are now sustained in the concrete

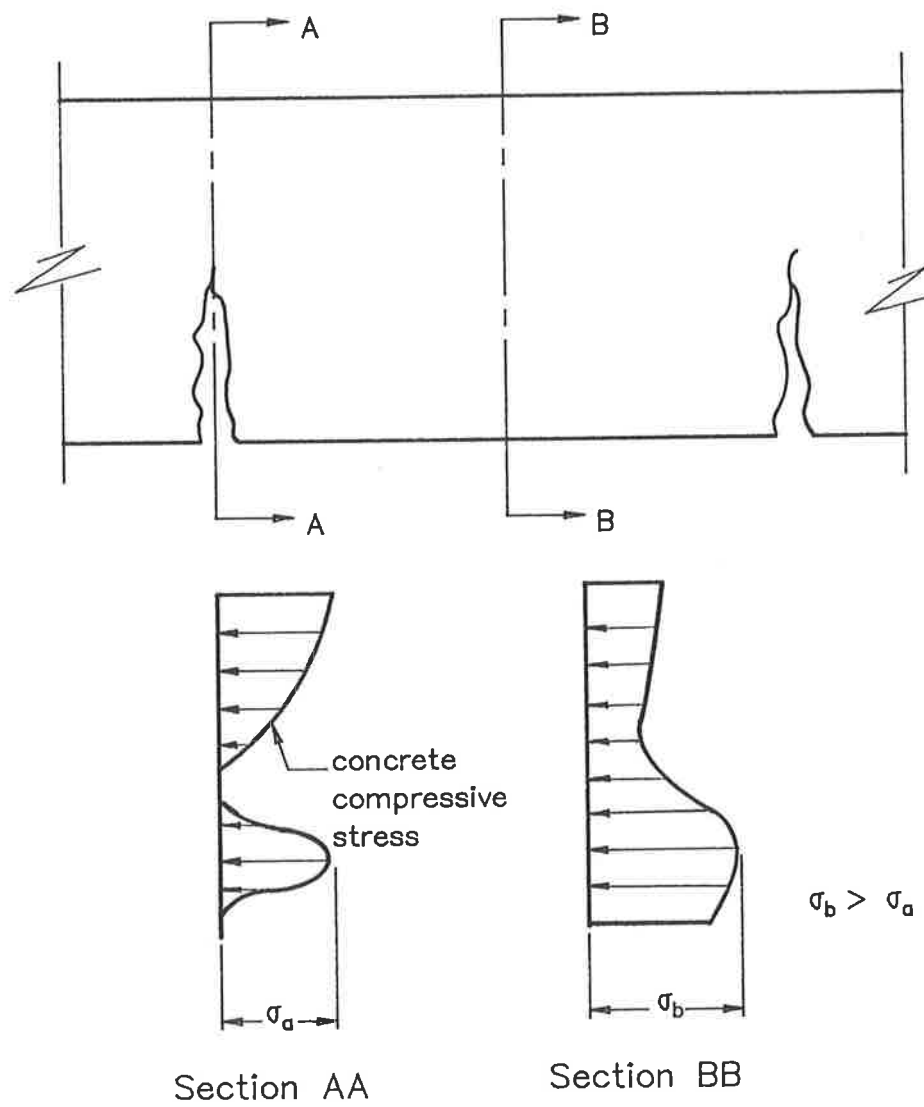


Fig. 6.11 Local Stress Conditions at Low Moments

between the primary cracks. Further microcracking may take place with additional increases in deformation. This continues up to the maximum moment where the deformation at the second cycle is slightly larger than that of the first. The total effective concrete tensile force is reduced slightly due to the further microcracking.

The unloading path of the second cycle is similar to the first unloading curve. Hysteresis is negligible when compared with the first cycle. The initial recovery is slightly less elastic than before. Resistance to slip between concrete and steel in the reverse direction is reduced by further cracking. Some slip may even occur on the onset of unloading. Creep effects are also reduced. The unloading curve shows a less distinct delineation between the first two phases. Additional wedging action may be encountered as applied load decreases, with more debris falling into cracks and higher mismatch of cracked surfaces. Residual deformation at the end of the second load cycle is greater than in the first, and correspondingly, compressive concrete stresses and tensile steel stresses are increased.

Beam behaviour in the third load cycle is similar to the second. Further microcracking and loss of tensile stiffening effects take place at high loads, accompanied by increases in slip and overall deformation. Increases in concrete compressive stresses and residual deformations in the lower load ranges are expected. A slight decrease in beam stiffness is also expected to take place.

The twin effects of progressive tensile stiffening deterioration and increased crack wedging continue at a decreasing rate with repeated loading. After application of many load cycles, the tensile stiffening effect reduces to zero while increases in permanent deformation at low moments due to wedging occur at a slower rate.

6.2.4 Summary

Tension stiffening is the main factor influencing beam behaviour in the postcracking range. Progressive cracking accompanied by an indirect form of slip (Fig 6.9) are the predominant effects at high moments. However, examination of the test data has shown that additional unloading mechanisms come into play as the moment decreases in each cycle. Incomplete crack closure is suggested to be a major factor influencing unloading behaviour. Consequently, large concrete compressive stresses due to wedging occur and this is accompanied by increased tensile steel stresses. Repeated loading produces further microcracking, resulting in a gradual reduction of tension stiffening effects and increased deformations at high moments, and increases in wedging effects at low moments.

6.3 REGIONAL ANALYSIS - BASIC CONCEPTS

The physical factors influencing beam behaviour under static and fatigue loading have been described in Sections 6.1 and 6.2. A method of analysis taking into consideration the above factors is now proposed. The fundamental concepts of the analysis are discussed in this section and the detailed development of the method will be made in Section 6.4.

The overall behaviour of a structural member is made up of the response of individual sections and regions. Ideally, the analysis should involve the integration of the individual local responses in each region. These include sections of primary cracking as well as intermediate regions of secondary cracking. The analysis of any local section is difficult due to the uncertainty of the local strain and stress distributions. Furthermore, a detailed analysis of numerous sections along the member would require extensive computer modelling.

The finite element method is a powerful analytical tool and has been used extensively in structural concrete analysis. But

this numerical method, when applied to the present problem, is only as good as its representation of the complex behaviour of the concrete tension zone. Modelling of the concrete-steel interface and the complex cracking system is extremely complicated in view of the non-continuous mechanisms involved. Without accurate representation and coupled with the inherent variability of concrete, this method is no better than any other available methods.

In view of these difficulties, the concept of a regional analysis based on a single 'equivalent' section to represent the overall member behaviour is attractive because of its simplicity and speed in computation. The effects of internal cracking, mechanical interlock, frictional resistance, slipping and sliding of surfaces, crack mismatch and debris inclusion need to be taken into account.

The problem of defining the local strain distribution is avoided by the use of an equivalent section for analysis. The average strain across the section is linear by virtue of symmetry even though local distributions may differ drastically. The analysis is thus 'regional', and stresses and strains considered are average values. Local stresses, required for fatigue calculations, are derived from these via another procedure which will be discussed later in Chapter 7.

6.3.1 Representation of Section

The section is divided into two regions with the concrete compressive zone in the upper fibres and the complex concrete zone around the main steels in the lower fibres. Fig 6.12a shows the representation of the two distinct zones. The compressive zone is divided into elemental slices similar to Section 4.1.1 while the lower concrete 'tensile' zone is represented by a series of elemental prisms. An elemental prism is shown in Fig 6.12b. For computational purposes, slicing of the concrete is extended throughout the entire section. However, only slices with elastic compressive strains are allowed to carry stresses.

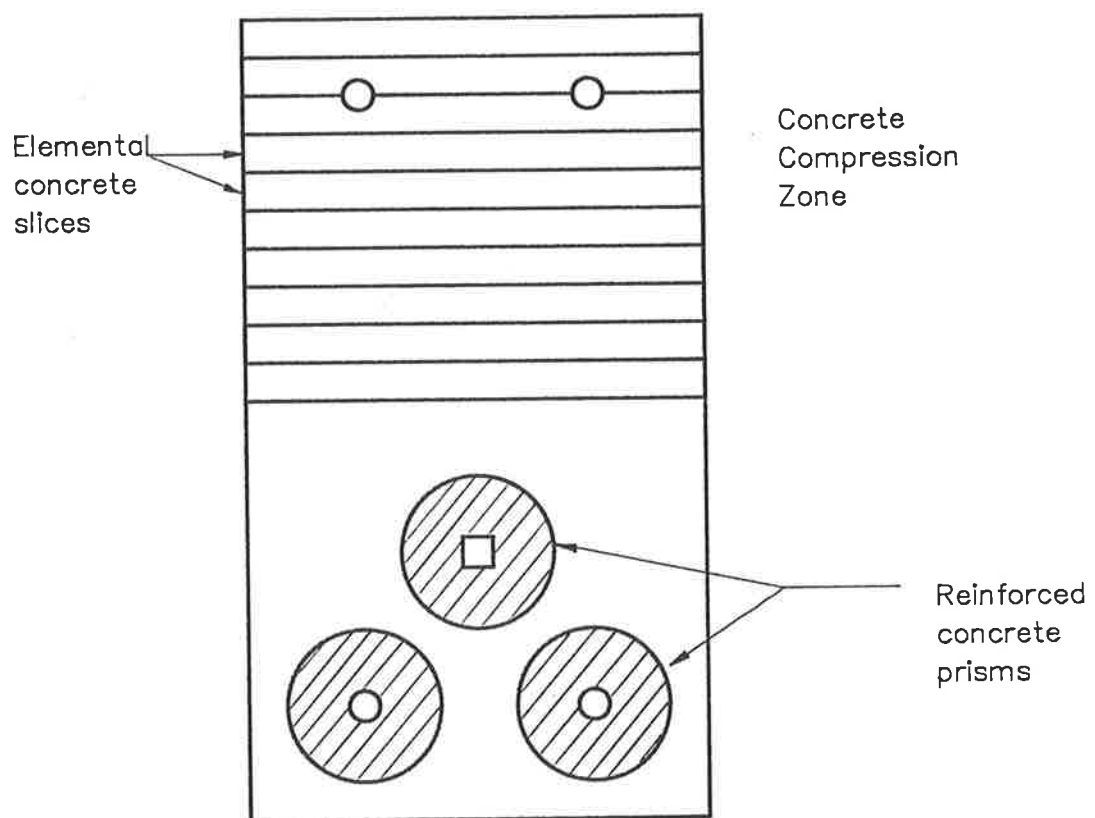


Fig. 6.12a Proposed Representation of a Beam Section

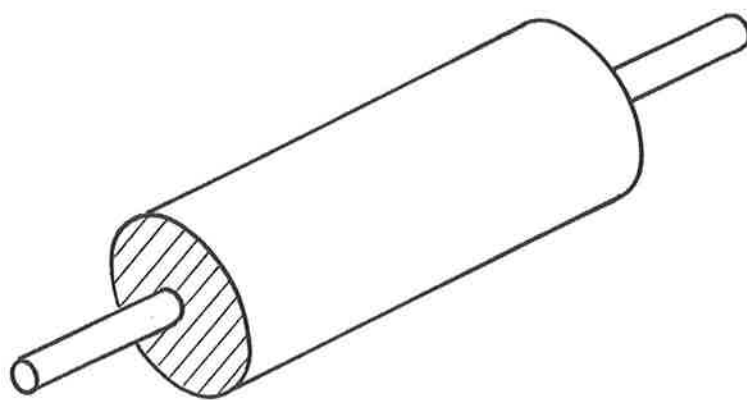


Fig. 6.12b A Reinforced Concrete Prism

The behaviour of the top compressive zone under load is generally straightforward, but the response of the bottom region is more complicated and is represented by the behaviour of the elemental prisms. Fig 6.13 presents a schematic illustration of the derivation of concrete compressive top-fibre stresses and bottom-fibre equivalent forces from concrete strains. In the upper region, concrete stresses are related to elastic strain by the normal stress-strain relations. In the lower region, the effective concrete force is related to the total strain by the average effective force-strain curve.

6.3.2 Concrete Compression Zone

Assuming symmetry about the vertical axis and therefore uniform strain in the horizontal concrete slice, the average strain in the equivalent section is:

$$\epsilon_{j,av} = \frac{1}{L} \int_L \epsilon_{j,x} dx \quad (6.9)$$

where $\epsilon_{j,x}$ is the local concrete strain in the j^{th} slice at a section at a horizontal distance x along the longitudinal axis, as shown in Fig 6.14. L is the crack spacing.

The average concrete stress in the j^{th} fibre in the equivalent section is:

$$\sigma_{j,av} = G_c \{ \epsilon_{j,av} \} \quad (6.10)$$

where G_c is the normal concrete constitutive relation.

6.3.3 Concrete Tension Zone

The concrete tension zone is represented by elemental prisms subjected to uniaxial loading. The overall behaviour of the zone is the sum of the individual prism responses. This approach reduces the difficult problem of analysing the

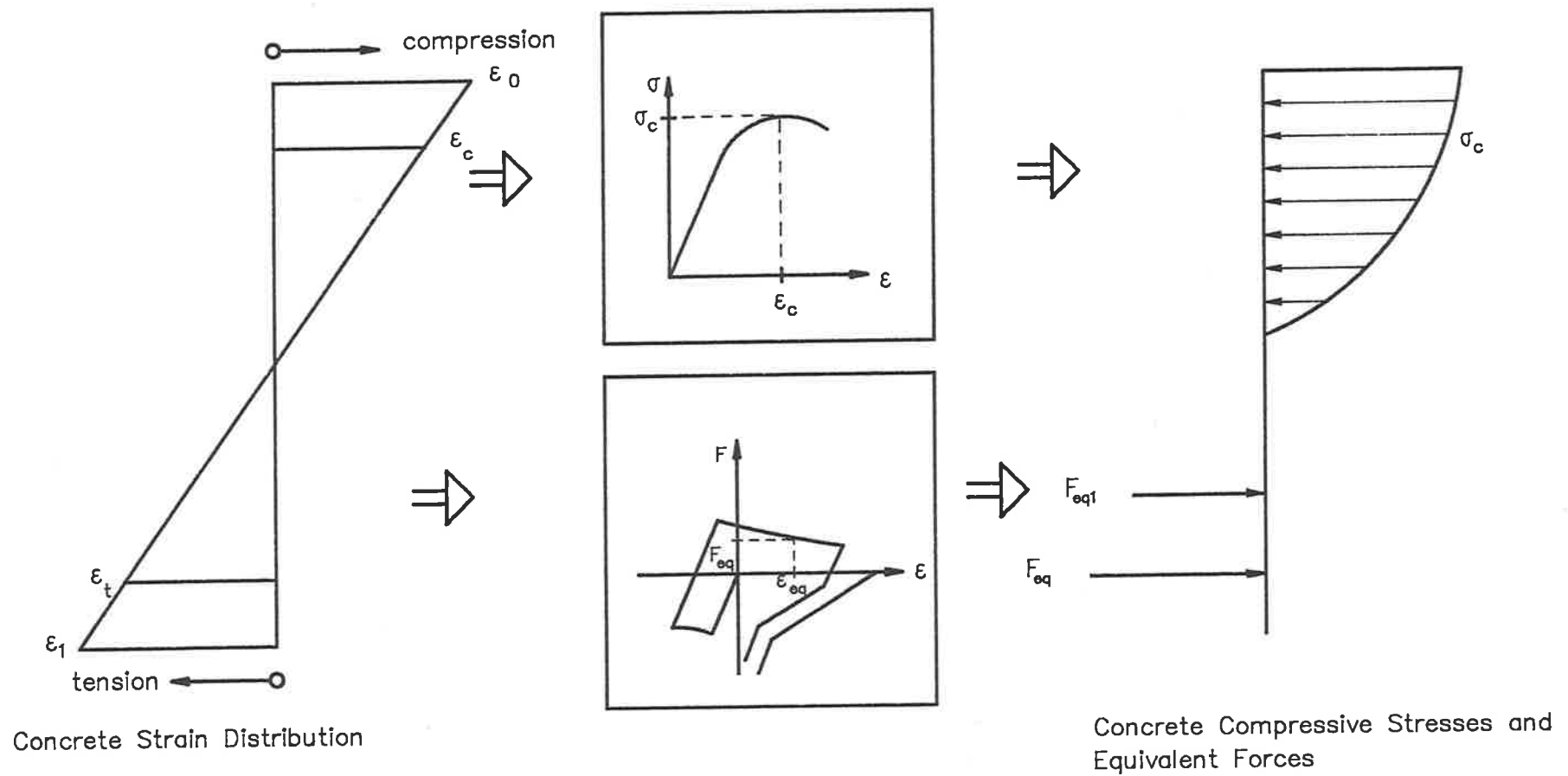


Fig. 6.13 Derivation of Concrete Stresses and Forces

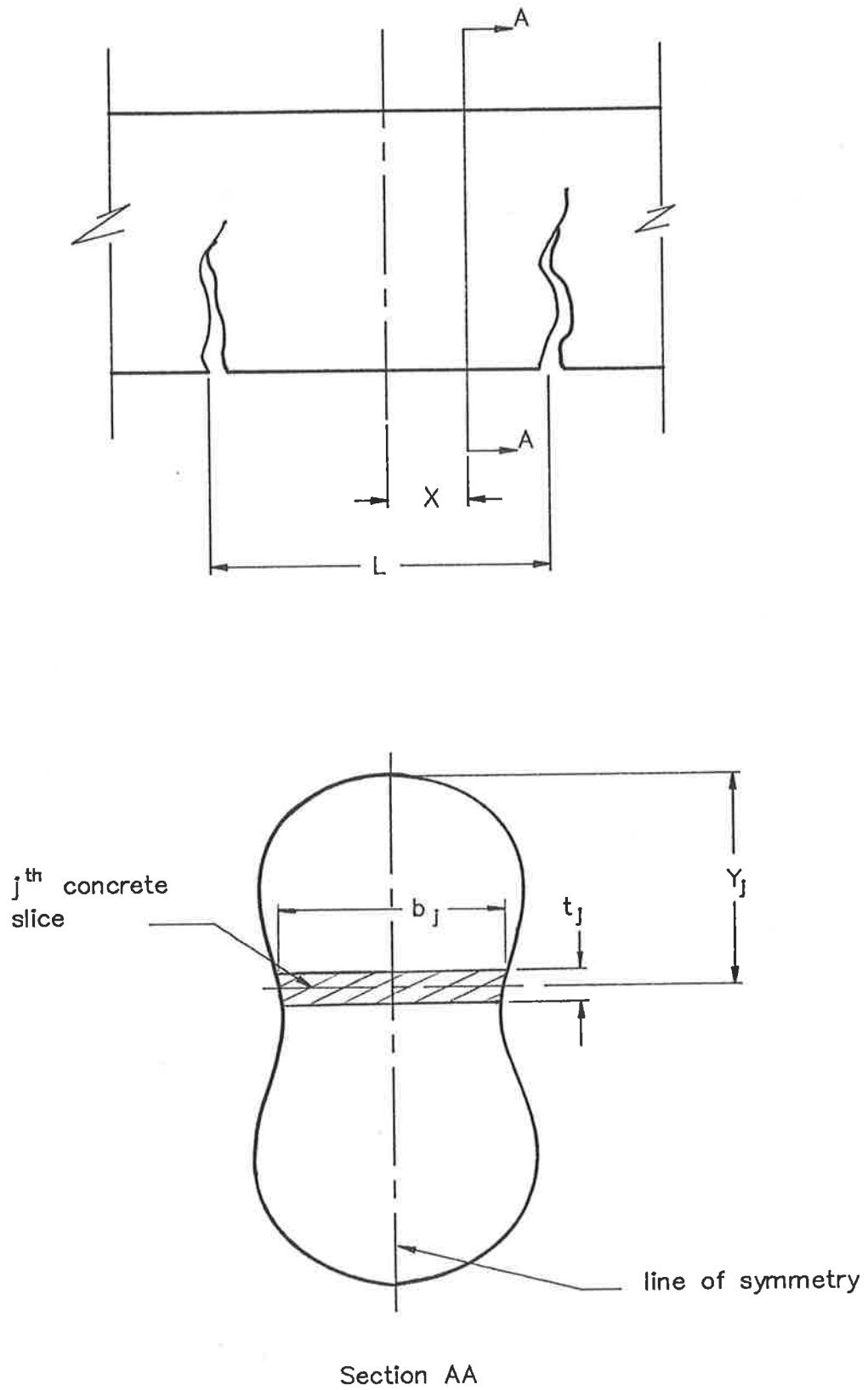
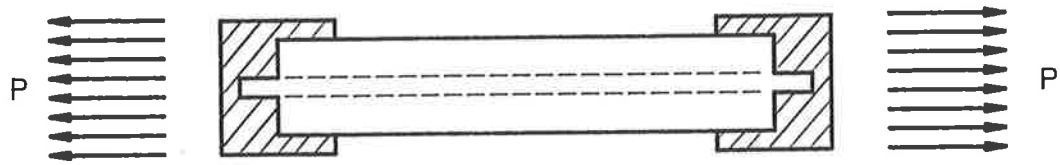


Fig. 6.14 Front Elevation and Cross-Section of Beam

complex flexural behaviour of the lower concrete region to the simpler one of prescribing the response of the elemental prisms under uniaxial loading. While the mechanisms influencing cracking in flexural and direct tension members may not be entirely identical (Base, 1982), these prisms under uniaxial loading can still be usefully employed in flexural beam analysis, provided the response is modified to take this into consideration.

The technical literature reveals a large number of direct tensile tests on reinforced concrete elements in connection with cracking and bond studies. Recent reviews are given in a State-of-the-Art report by the CEB (1982) and in the International Bond Conference (Bartos, 1982). A more specific review on bond under cyclic loading has been published by Tassios (1979). In spite of the numerous pull-out tests reported, surprisingly little information is available regarding the extensional stiffness of uniaxially tensioned elements. So far, the experimental emphasis had been placed on bond stress and slip measurements. Also, the data collected is applicable mainly to the loading direction and little information is available for unloading and repeated loading.

The behaviour of a typical prism prior to cracking under uniaxial tensile load is near linear-elastic, depending on the response of plain concrete. During this stage of loading, no slip takes place. Fig 6.15 shows the response of a prism under direct tension loading. P is the total tensile load, and P_c and P_s are the respective tensile forces sustained by concrete and steel. Cracking at localised sections occurs when the concrete tensile capacity is exceeded. The average crack spacing and width is dependent on the concrete cover and size of the steel reinforcement. Tensile stress in the intact concrete is transferred by bond at the steel and concrete interface, and is dependent on the tensile capacity of concrete and the area of concrete surrounding the steel. These stresses reduce progressively with further cracking and bond deterioration as load increases. The result is a reduction in the overall stiffness and is reflected by a gradual bending over of the overall curve (actual behaviour curve). As the



$$P = P_c + P_s$$

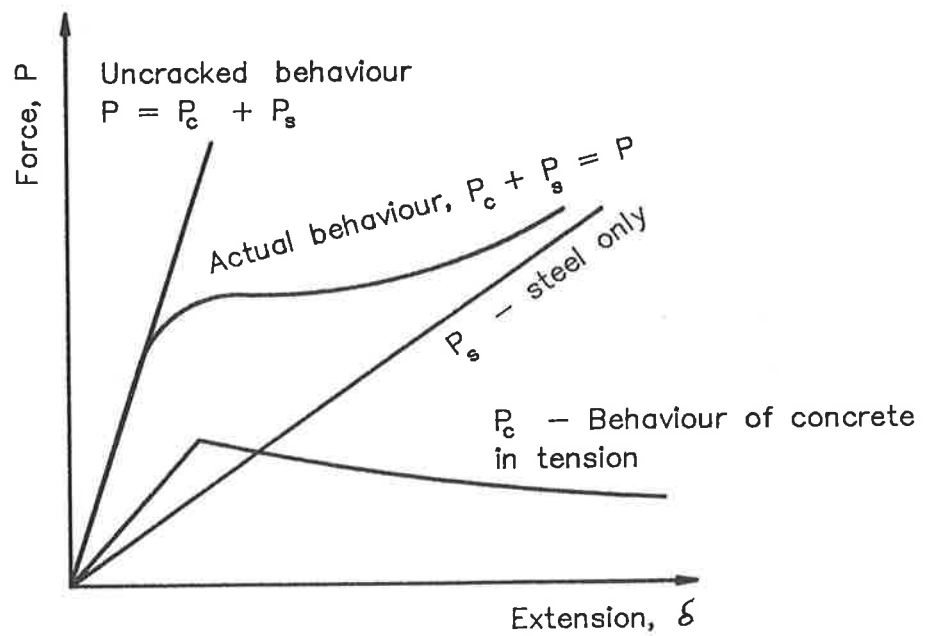


Fig. 6.15 Behaviour of Reinforced Concrete Prism Under Uniaxial Tension

load increases to approach ultimate, the stiffness decreases and approaches that of the unbonded steel.

The variation of local concrete tensile strain in the prism is shown in Fig 6.16 which is reproduced from Perdikaris (1985). The tensile concrete strain is zero at the primary cracks and builds to a maximum midway between cracks. Local steel strains are maximum at primary cracks while the reverse is true for concrete strains. The variation of steel stress with load is also shown. A greater stiffness for the unloading curve is observed for the unloading curve compared to the postcracking loading stiffness.

Cyclic load tests by Bresler and Bertero (1968), Ismail and Jirsa (1972), Tassios and Yannopoulos (1981) and Popov (1984) showed that stiffness decreases with repeated loading and that bond deterioration is dependent on the previous stress history. Edwards and Yannopoulos (1978) reported increases in residual slip with load cycling at a decreasing rate. The variation of bond stress with slip reproduced from Edwards is shown in Fig 6.17.

Behaviour of Proposed Elemental Prism

The prism response under initial prestress is shown in Fig 6.18. The average concrete strain is defined as the total strain experienced averaged over a sufficiently large distance L . Uniform strain is assumed throughout the each prism section. The average concrete strain in the prism is therefore expressed as:

$$\epsilon_{av} = \frac{1}{L} \int \epsilon_x dx \quad (6.11)$$

where ϵ_x is the strain in the prism at distance x along the prism axis.

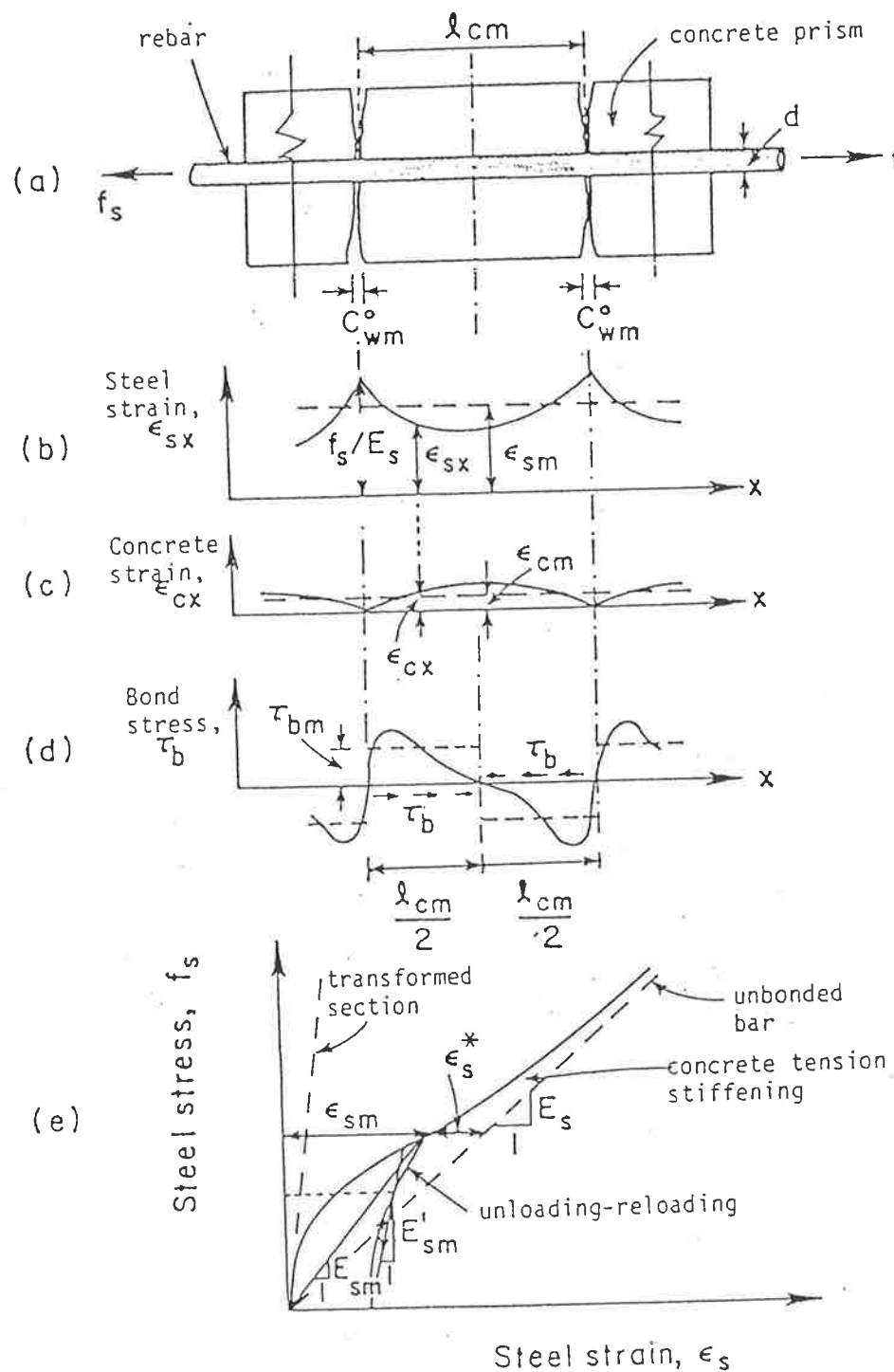


Fig. 6.16 Variation of Steel and Concrete Strain Along Concrete Prism Under Uniaxial Loading (Perdikaris, Hilmy & White, 1985)

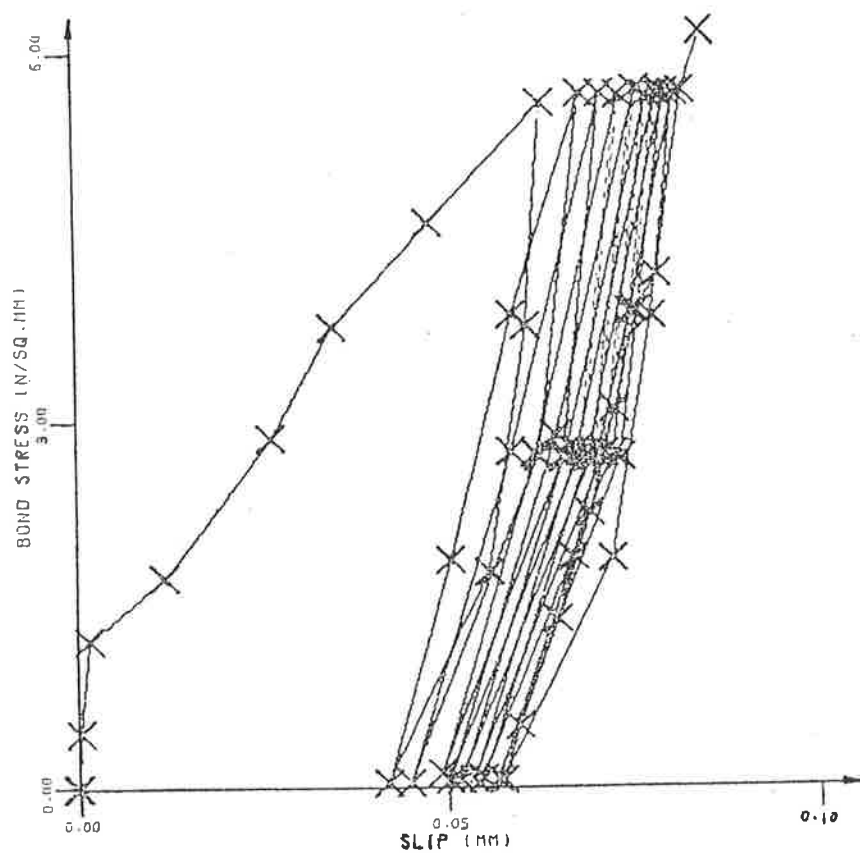


Fig. 6.17 Bond Stress — Slip Relation Under Repeated Loading (Edwards & Yannopoulos, 1978)

Similarly, the average effective force in the concrete under uniaxial loading is defined as the total force averaged over a sufficiently long length and is expressed as:

$$F_{e,x} = \frac{A}{L} \int_L \sigma_x dz \quad (6.12)$$

where σ_x is the average concrete stress in the prism at position x and A the cross-sectional area.

Path OA in Fig 6.18 shows the precompression force sustained by the concrete due to prestressing. The level of the compressive force sustained is dependent on the magnitude of the prestressing force and on the location of the prism. Path AB shows the increase in strain and decrease in prestress due to the time-dependent effects of creep, shrinkage and steel relaxation with little or no external loads applied. The response at this stage is non-linear with decreasing concrete force and increasing deformation.

Upon loading (path BC), the average effective force increases almost linearly from compression to tension with strain. Path BC represents the precracking response and C the point where first cracking takes place. CD reflects the effect of progressive cracking and corresponding loss of tension stiffening with increased deformation. D is the point of maximum load. The initial near-elastic unloading takes place along DE. Path EF shows the predominant unloading phase where the majority of slip and deformation recovery takes place. Path FG is the final unloading phase where cracks close.

Path GH shows the first loading phase of the second cycle. H indicates the commencement of slip as the cracks open and D² the position at maximum applied moment. The second unloading curve D²E²F²G² is similar to the first. With progressive cracking and wedging action due to crack mismatch, the behaviour of the Nth load cycle is represented by path GⁿHⁿDⁿ. Hysteresis effects are assumed to be negligible after many cycles of repeated loading.

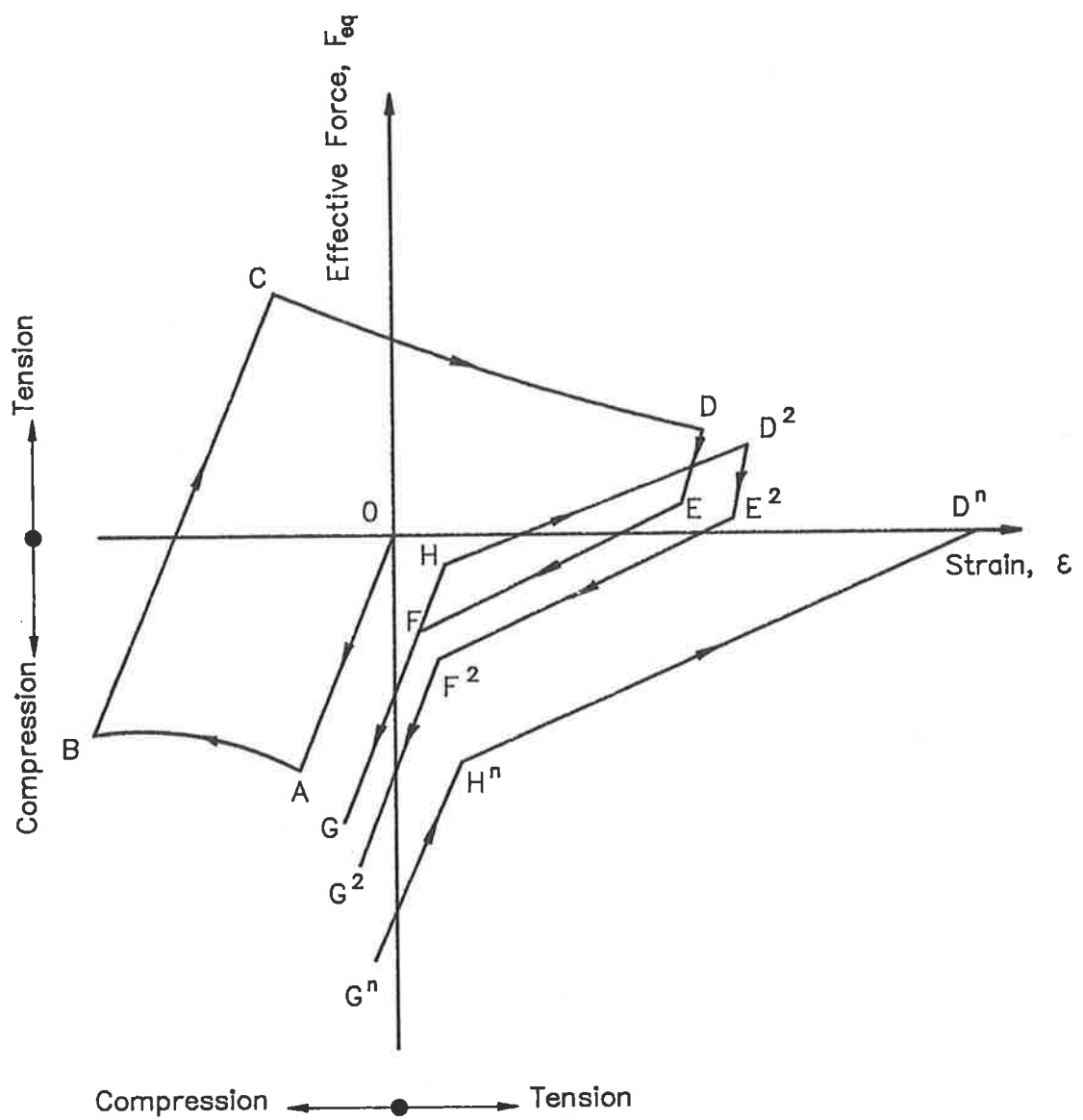


Fig. 6.18 Concrete Effective Force—Strain Relations

Tensile Strain-Softening Behaviour of Concrete

The proposed description of the behaviour of the prism is supported by the recent experimental work of Reinhardt (1984) who studied the tensile strain-softening characteristics of plain concrete under repeated loading. While the concrete cracking in a reinforced concrete prism is more complicated than that of plain concrete (there is a more complex pattern of cracks in reinforced concrete), the softening response of plain concrete can still be used as a basis for understanding the behaviour of reinforced concrete prisms.

The tensile stress-strain behaviour of plain concrete specimens in deformation-controlled uniaxial loading for both static and fatigue conditions, established by Reinhardt, are reproduced in Figs 6.19 and 6.20. The static tests show a relatively linear stress-strain relationship up to maximum stress. At maximum tensile strength, internal microcracks form which open up while the remainder of the concrete unloads elastically. The total deformation then consists of the instantaneous elastic strain and the differential displacement of the crack faces. Increasing deformation is therefore accompanied by a decrease in stress.

Fig 6.20 shows a typical concrete fatigue response from Reinhardt's cyclic load tests, which were conducted with a constant minimum compressive stress. The maximum tensile cyclic stress was defined by the static stress envelope. The curves show remarkable similarity to the concrete equilibrating force-moment relation (Fig 6.4), derived from this author's beam tests. Both show a monotonically decreasing region after cracking, with similar unloading and subsequent reloading characteristics such as reduced tensile carrying capacity and increased deformations at high loads and large compressive stresses at lower loads with repeated loading.

The unloading curve in Fig 6.20 consists of at least two, if not three different phases. According to Reinhardt, the cyclic response can be divided into four main stages: elastic elongation of specimen, small crack opening, elastic unloading

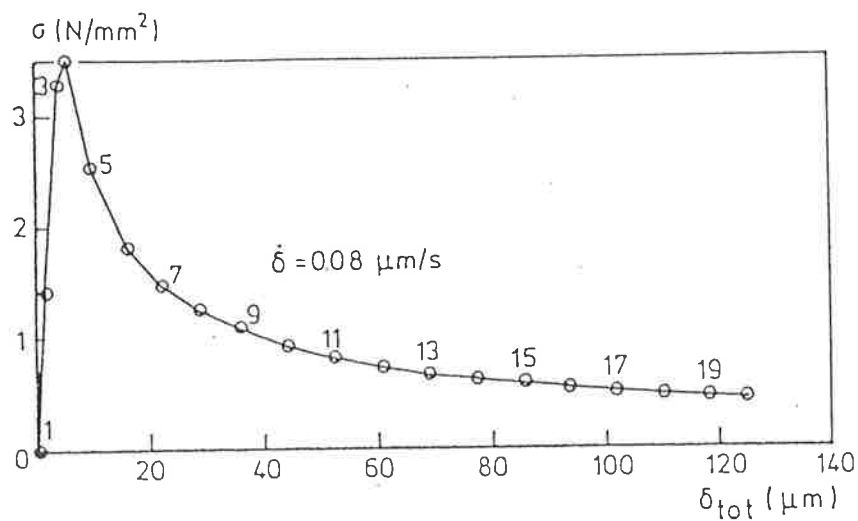


Fig. 6.19 Static Tensile Stress — Strain curve for Plain Concrete (Reinhardt, 1984)

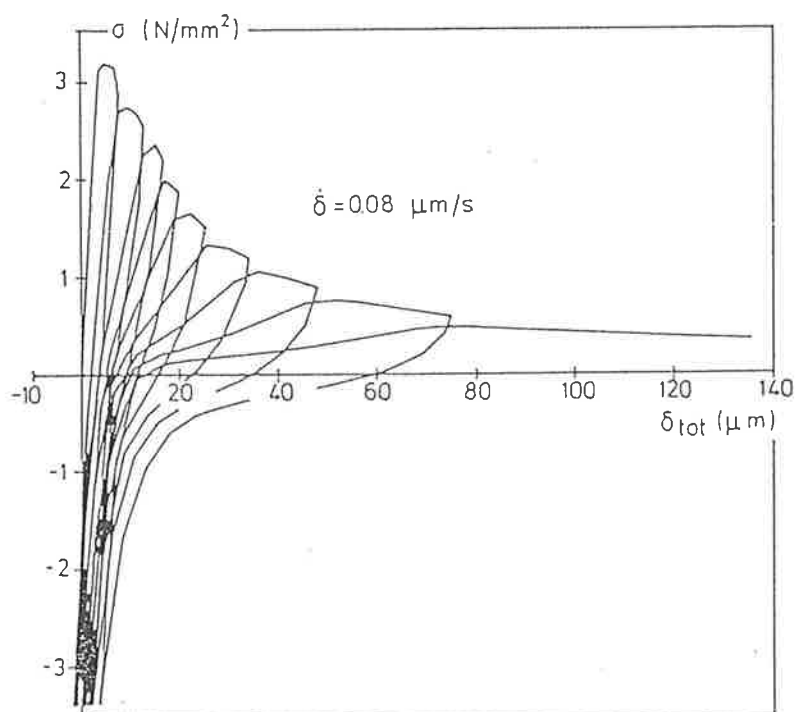


Fig. 6.20 Concrete Stress--Deformation Relation in a Post-Peak Cyclic test with a Large Compressive Stress (Reinhardt, 1984)

and crack closing. The initial stiffness is greater than the postcracking stiffness of the loading path. He suggested that the residual deformation arising from unloading in the softening stage may be caused by frictional effects on the region of damaged concrete, and that with repeated loading, the cracked zone increases, with possible mismatch of the crack faces. A schematic representation of the microcracked zone is given in Fig 6.21 for both the loaded and unloaded stage.

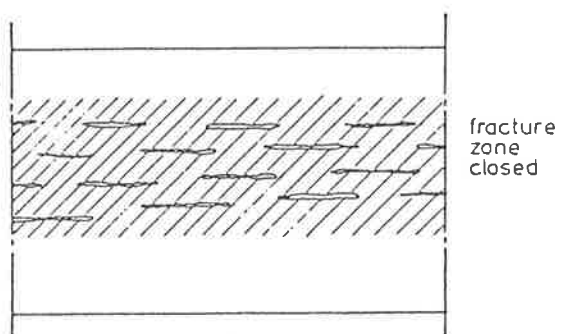
The similarity of the two independent experimental results adds credence to the proposed hypothesis on beam behaviour and substantiates the presence of distinct phases of elastic loading, crack opening, further cracking, initial elastic unloading, crack mismatch and wedging with localised compressive stresses occurring in static and fatigue loading.

6.4 REGIONAL ANALYSIS - DETAILED DESCRIPTION

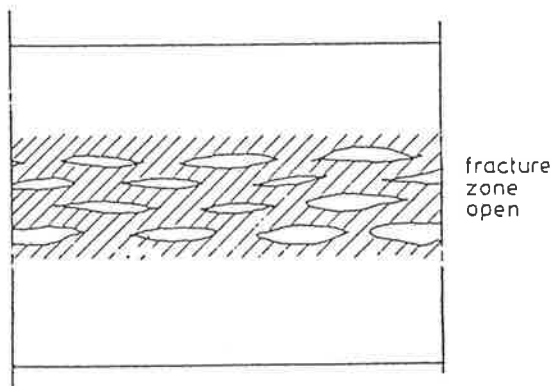
The behaviour of concrete members under static and repeated loading is a combination of the response of the upper concrete compressive zone and the lower tension zone. The analysis is divided into two parts - the response of the concrete compression region and the response of the lower tension zone.

In the compression zone, the concrete response is similar to its behaviour under uniaxial compression. The stress-history dependent nature of plain concrete results in inelastic residual strains during unloading. Repeated loading produces additional inelastic (cyclic creep) strains.

The behaviour of the lower concrete tension zone is more complex and has already been discussed at a qualitative level in Section 6.2. The overall beam response is an aggregate of the individual behaviour. Tensile stresses are sustained in the intact concrete at high loads while at low loads local compressive stresses may be present due to wedging effects. Repeated loading induces further cracking of the concrete



Unloaded Condition



Loaded Condition

Fig. 6.21 Schematic Representation of Microcracked Zone (Reinhardt, 1984)

resulting in increased deformation.

6.4.1 Concrete Compression Zone

While the local strain distribution across the section is no longer linear, the average strain across the equivalent section is still linear due to symmetry. The strain neutral axis can be defined in terms of the compressive top-fibre strain ϵ_0 and the tensile bottom-fibre strain ϵ_1 (as shown in Fig 6.13) as:

$$d_n = \frac{\epsilon_0}{(\epsilon_1 + \epsilon_0)} D \quad (6.13)$$

The compressive average strain in the j^{th} concrete slice is:

$$\epsilon_j = -\frac{y_j}{D} \epsilon_1 + \frac{(D - y_j)}{D} \epsilon_0 \quad (6.14)$$

where y_j is the depth of the centroid of the j^{th} concrete layer measured from the top fibre.

The elastic strain component is obtained by subtracting the inelastic components from the total strain i.e.

$$\begin{aligned} \epsilon_{j,e} &= \epsilon_j - \epsilon_{j,i n} \\ &= \epsilon_j - \epsilon_{j,c r} - \epsilon_{j,s h} - \epsilon_{j,n} \end{aligned} \quad (6.15)$$

where $\epsilon_{j,e}$ is the elastic strain in the j^{th} concrete slice and $\epsilon_{j,c r}$, $\epsilon_{j,s h}$ and $\epsilon_{j,n}$ are respectively the long-term creep, shrinkage and dynamic creep (short-term) strains.

The compressive concrete stress is related to the instantaneous elastic strain by the constitutive relation:

$$\sigma_j = G_c \{\epsilon_{j,e}\} \quad (6.16)$$

where G_c is the function defining the stress-strain relation and has been described in detail in Section 5.1.1

The compressive concrete stress-strain curves for first cycle and repeated loading are shown in Figs 6.22a and b. The stress-dependent characteristic of the concrete response is also illustrated. Equations for the loading, unloading and repeated loading response are as follows:

For first cycle - loading,

$$G_c\{z\} = 0.5 z F_c \exp(1-0.4z) \quad (6.17)$$

For first cycle - unloading,

$$G_c\{z\} = \sigma_{max} - 0.58 (z_{max} - z) F_c \exp(1-0.5(z_{max} - z)) \quad (6.18)$$

For repeated loading,

$$G_c\{z\} = \sigma_{max} - 0.58 (z_{max} - z) F_c \exp(1-0.5(z_{max} - z)) \quad (6.19)$$

where z is the normalised elastic strain defined as:

$$z = \frac{\epsilon_e}{\epsilon_{ult}} \quad (6.20)$$

and z_{max} is the maximum normalised strain, σ_{max} is the maximum stress sustained, ϵ_e is the elastic concrete compressive strain, ϵ_{ult} is the strain at maximum stress, and F_c is the ultimate concrete strength in compression.

Similar equations were defined for the unloading and repeated loading curves. In the case of repeated loading, the increased inelastic strain was accounted for by the cyclic creep strain component ϵ_n .

6.4.2 Concrete Tension Zone

The derivation of the effective concrete force from the $F_c - \epsilon$ relation is illustrated in Fig 6.13. The tensile total

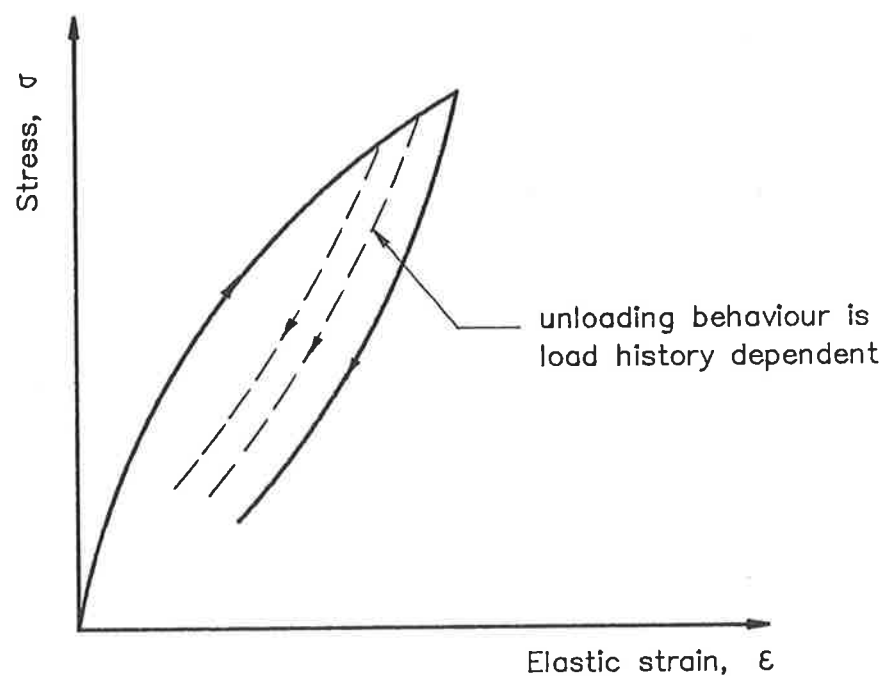


Fig 6.22a Compressive Concrete Stress-Strain curve under Static Loading

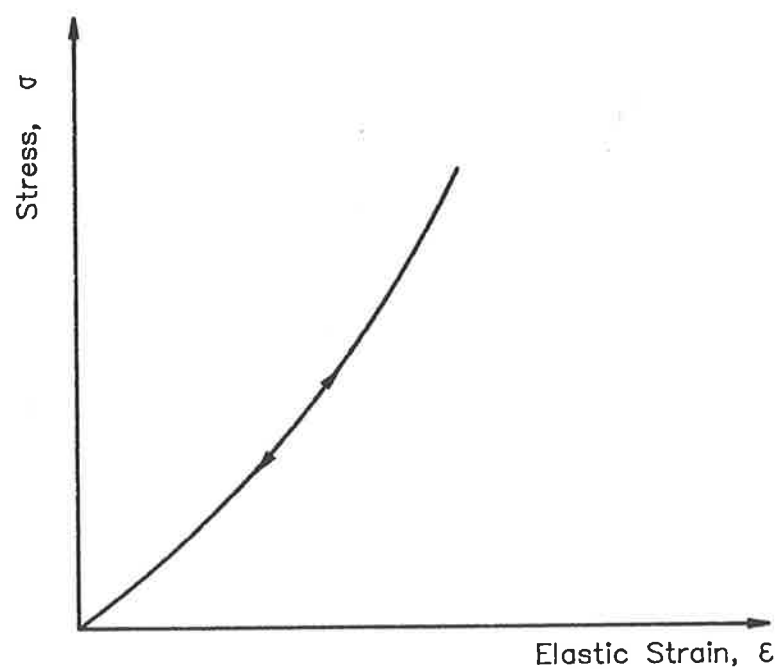


Fig. 6.22b Compressive Concrete Stress-Strain curve under Repeated Loading

concrete strain in the k^{th} elemental prism is expressed in terms of the compressive top-fibre strain ϵ_0 and tensile bottom-fibre strain ϵ_1 as:

$$\epsilon_{k,q} = \frac{l_{k,q}}{D} \epsilon_1 - \frac{(D - l_{k,q})}{D} \epsilon_0 \quad (6.21)$$

where $l_{k,q}$ is the depth of the k^{th} concrete prism measured from the top fibre.

The concrete effective force is related to the total concrete strain by the force-strain relation:

$$F_{k,q} = G_{q,q} \{ \epsilon_{k,q} \} \quad (6.22)$$

where $G_{q,q}$ is the function relating effective force to total strain. The total force in the lower concrete zone is:

$$C_t = \sum_{k=1}^{NE} F_{k,q} \quad (6.23)$$

where NE is the total number of elemental prisms. Taking moments about the extreme top fibre,

$$M_t = \sum_{k=1}^{NE} F_{k,q} l_{k,q} \quad (6.24)$$

Quantifying the $F_{q,q} - \epsilon$ curve

The $F_{q,q} - \epsilon$ relation is dependent on the size and shape of the prism, the proportion of concrete and steel area, the material properties and also the location of the prism. The number of prisms required is dependent on the placement of the steel reinforcement and geometry of the section. Where the steels are placed far apart, a prism may be defined for every bar or strand. Where bars or strands are bundled together, fewer are required. A larger number of elements will result in a more accurate representation, but this is offset by the necessity of defining separate $F_{q,q} - \epsilon$ relations for each element.

Defining the area of the concrete for each prism is a

difficult problem. It is dependent on the section properties of the beam as well as on the size and type of steel used. Clark and Speirs (1978) and Gilbert (1983) recommended effective areas to take account of tension stiffening. A similar approach can be used here. Alternatively, data for the $F_{eq} - \epsilon$ curve could be obtained experimentally by testing under direct tension, prestressed concrete prisms reinforced with additional normal reinforcing steel.

A simpler approach, which avoids these problems, is used. Where the steels are grouped fairly closely, the bottom concrete tension zone can be represented by a single prism. The total concrete tension force after cracking is therefore replaced by a single force located at the main steel level.

Fig 6.23 shows the state of stresses in the uncracked section just before cracking. The total concrete tensile force $F_{eq,max}$ is expressed as:

$$F_{eq,max} = \int_A \sigma_{jt} dA \quad (6.25)$$

where σ_{jt} is the concrete tensile stress in the j^{th} concrete slice. For rectangular sections:

$$F_{eq,max} = \frac{1}{2} B y_{cr} F_t \quad (6.26)$$

where y_{cr} is the distance from the stress neutral axis to the extreme bottom fibre, F_t is the concrete tensile strength and B is the section width. $F_{eq,max}$ is dependent on section properties and on the ultimate tensile capacity of concrete.

An empirical method is initially used to quantify the $F_{eq} - \epsilon$ curve. Parameters are used to define critical points shown in Fig 6.24. These parameters are related either to the concrete effective force or to the slope of the lines.

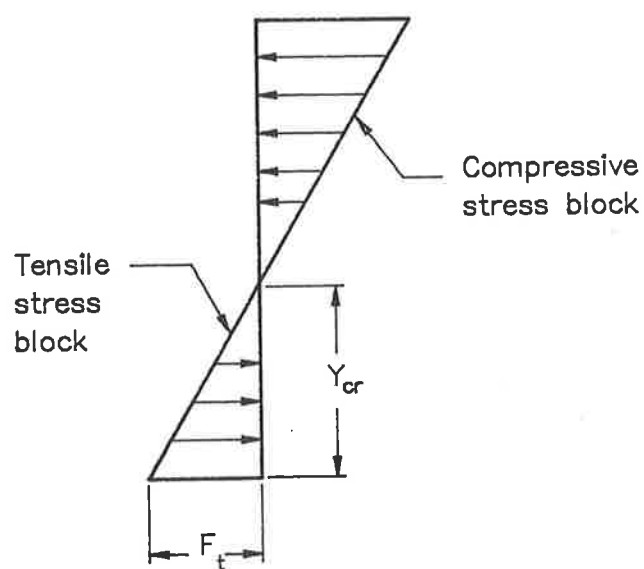


Fig. 6.23 Concrete Stresses in Section Just Before Cracking

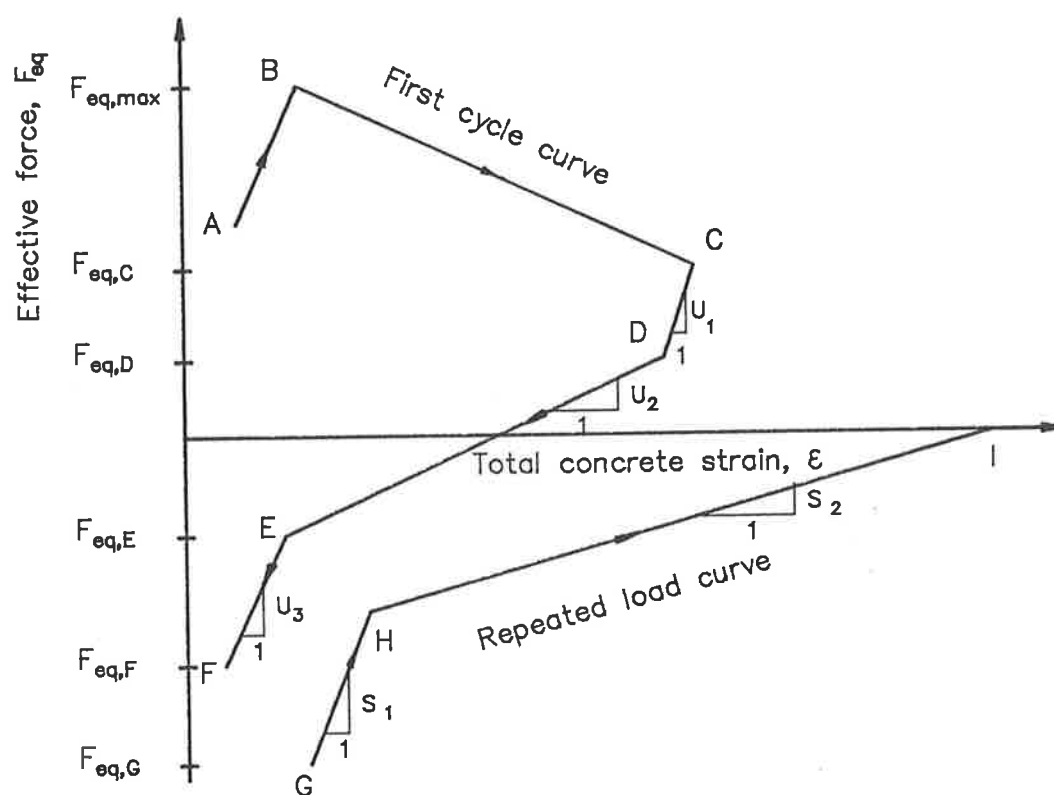


Fig. 6.24 Concrete Effective Force-Strain Relations

w_r indicates the degree of tension stiffening at maximum load,

$$\begin{aligned} w_r &= \frac{\text{effective concrete force at max load}}{\text{maximum effective concrete force}} \\ &= \frac{F_{eq,c}}{F_{eq,max}} \end{aligned} \quad (6.27)$$

c_1 defines the level of F_{eq} at which the elastic recovery phase ends in the unloading direction,

$$c_1 = \frac{F_{eq,D}}{F_{eq,c}} \quad (6.28)$$

$F_{eq,D}$ and $F_{eq,c}$ are the effective concrete forces at points D and C respectively

c_2 defines the level of F_{eq} where significant wedging commences,

$$c_2 = \frac{F_{eq,E}}{F_{eq,c}} \quad (6.29)$$

u_1 defines the stiffness of the elastic recovery unloading curve, path CD

$$\Delta F_{eq} = u_1 \Delta \epsilon_{eq} \quad (6.30)$$

u_2 defines the stiffness of the slip recovery phase, path DE in a similar manner to u_1

u_3 defines the stiffness of the final unloading phase, path EF

c_n introduces the additional compressive wedging force, sustained under repeated loading, at the minimum load of the N^{th} cycle, expressed as a fraction of the magnitude of the first cycle effective concrete force $F_{eq,c}$

$$\Delta F_{eq} = c_n F_{eq,c} \quad (6.31)$$

c_3 defines the magnitude of the effective force where the cracks open for the N^{th} load cycle (point H) which is expressed as a proportion of the effective force at minimum load in the same cycle,

$$F_{e,q,H} = c_3 F_{e,q,G} \quad (6.32)$$

s_1 defines the initial stiffness of the relation for N^{th} load cycle, path GH

$$\Delta F_{e,q} = s_1 \Delta \epsilon_{e,q} \quad (6.33)$$

s_2 defines the 'postcracking' stiffness of the relation for the N^{th} load cycle, path HI

A preliminary regional analysis was conducted for each test beam with trial values of the parameters. At the critical points, the theoretical moment-curvature values were compared with test results and the parameters were adjusted until good agreement was achieved. The parameters for first cycle loading are given in Table 6.1 below. The average values given in the table represent a general $F_{e,q} - \epsilon$ relation. Two values of w_r (defining the degree of tension stiffening) were defined for the two different maximum load levels. The parameters for repeated loading are shown in Table 6.2.

Table 6.1 Beam Parameters for First Cycle Loading

Beam	w_f	u_1 (N/microstrain)	u_2	u_3	c_1	c_2	M_{max} (KNm)
SA1	0.70	160.	44.	153.	0.5	-1.3	43.07
SA2	0.75	168.	57.	151.	0.6	-1.3	38.03
SA3	0.27	170.	30.	161.	0.6	-2.5	43.07
SA4	0.65	172.	60.	148.	0.6	-2.4	38.03
PA1	0.53	167.	50.	145.	0.5	-1.3	38.03
PA2	0.30	171.	28.	150.	0.5	-3.5	38.03
Ave*	0.50						43.07
	0.60	168.	45.	151.	0.55	-2.1	38.03

* Average values

Table 6.2 Beam Parameters for Repeated Loading

Beam	c_n	s_1 *	s_2 *	c_3	N^+
SA1	0.95	100.	20.	0.01	300
SA2	0.90	34.	12.	0.10	660
SA3	0.80	67.	13.	0.60	760
SA4	1.00	101.	13.	0.30	400
PA1	0.30	101.	24.	0.60	280
PA2	1.80	67.	20.	0.60	950
Ave	0.75				350
	1.17	84.	17.	0.37	800

* Units in N/microstrain

+ Number of load cycles in thousands

6.4.3 Steel Stresses and Strains

Full compatibility between concrete and steel strains exist for average values. The tensile strain in the steels are:

for the i^{th} reinforcing steel element,

$$\epsilon_{i,s} = \frac{d_{i,s}}{D} \epsilon_1 - \frac{(D - d_{i,s})}{D} \epsilon_0 \quad (6.34)$$

where ϵ_0 is the compressive extreme top-fibre concrete strain and ϵ_1 is the tensile bottom-fibre strain.

for the i^{th} prestressing steel element,

$$\epsilon_{i,p} = \epsilon_{i,p,i} - \frac{d_{i,p}}{D} \epsilon_1 + \frac{(D - d_{i,p})}{D} \epsilon_0 \quad (6.35)$$

where $\epsilon_{i,p,i}$ is the initial prestressing strain of the i^{th} element

The elastic steel strains are expressed as:

$$\epsilon_{i,s,e} = \epsilon_{i,s} - \epsilon_{i,s,n} \quad (6.36)$$

$$\epsilon_{i,p,e} = \epsilon_{i,p} - \epsilon_{i,p,n} \quad (6.37)$$

where $\epsilon_{i,s,e}$ and $\epsilon_{i,p,e}$ are the elastic steel strains for the reinforcing and prestressing elements, and $\epsilon_{i,s,n}$ and $\epsilon_{i,p,n}$ are the respective steel cyclic creep strains.

The steel stresses are related to the elastic strains by the normal stress-strain relations. The computations are similar to that described for the simple analysis in Chapter 5.

6.4.4 Equilibrium Conditions

Equilibrium conditions are prescribed for the equivalent cross section. The force equilibrium of the region can then be expressed as:

$$C_c = T_s + T_p + \Sigma F_{c,e} \quad (6.38)$$

where C_c is the total compressive concrete force, T_s and T_p are the total tensile reinforcing and prestressing steel forces and $\Sigma F_{c,e}$ is the total concrete effective force. The force equilibrium equation can be rewritten as:

$$\sum_{j=1}^{N_A} \sigma_j b_j t_j = \sum_{i=1}^{N_S} f_{i,s} A_{i,s} t + \sum_{i=1}^{N_P} f_{i,p} A_{i,p} + \sum_{k=1}^{N_E} F_{k,e} \quad (6.39)$$

where $f_{i,s}$ is the i^{th} element stress in the ordinary steel, $f_{i,p}$ the i^{th} tendon stress, σ_j the concrete stress in the j^{th} slice and $F_{k,e}$ the tensile concrete effective force in the k^{th} prism.

Considering moment equilibrium about the extreme top fibre, we have:

$$M_{ext} = \sum_{i=1}^{N_S} f_{i,s} A_{i,s} d_{i,s} + \sum_{i=1}^{N_P} f_{i,p} A_{i,p} d_{i,p} + \sum_{k=1}^{N_E} F_{k,e} l_{k,e} - \sum_{j=1}^{N_A} \sigma_j b_j t_j y_j \quad (6.40)$$

where M_{ext} is the applied moment.

6.5 COMPARISONS BETWEEN THEORETICAL AND OBSERVED RESULTS

6.5.1 First Load Cycle

Figs 6.25a and b show the predicted and observed moment-curvature relations for two test beams, SA 1 and 2, for the first load cycle. The 300000th and 580000th load cycle for Beams SA 1 and 2 respectively are also shown. The theoretical analyses were carried out using the average parameters (Table 6.1 and 6.2) to define the effective force-strain relation. The curves show good correlation in the loading and unloading direction for the first load cycle. Only a slight overestimation in curvature with a maximum error of about 4 percent of the curvature change was observed for the loading direction and 8 percent in the unloading direction. Generally, the theoretical and experimental curves show good agreement throughout the entire load cycle.

Figs 6.26a and b show the relationship between applied moment and concrete top-fibre strains for Beams SA 1 and 2. Correlation between the calculated and observed strains is excellent in the precracking range. After cracking, there is still good agreement with errors of underestimation of about 4 percent at maximum load.

For the concrete bottom-fibre strain, Figs 6.27a and b show good agreement before cracking. In the postcracking range, there is good agreement for the major portion of the loading curves. The maximum error in both beams occurred at maximum moment where there is an overestimation of about 10 percent.

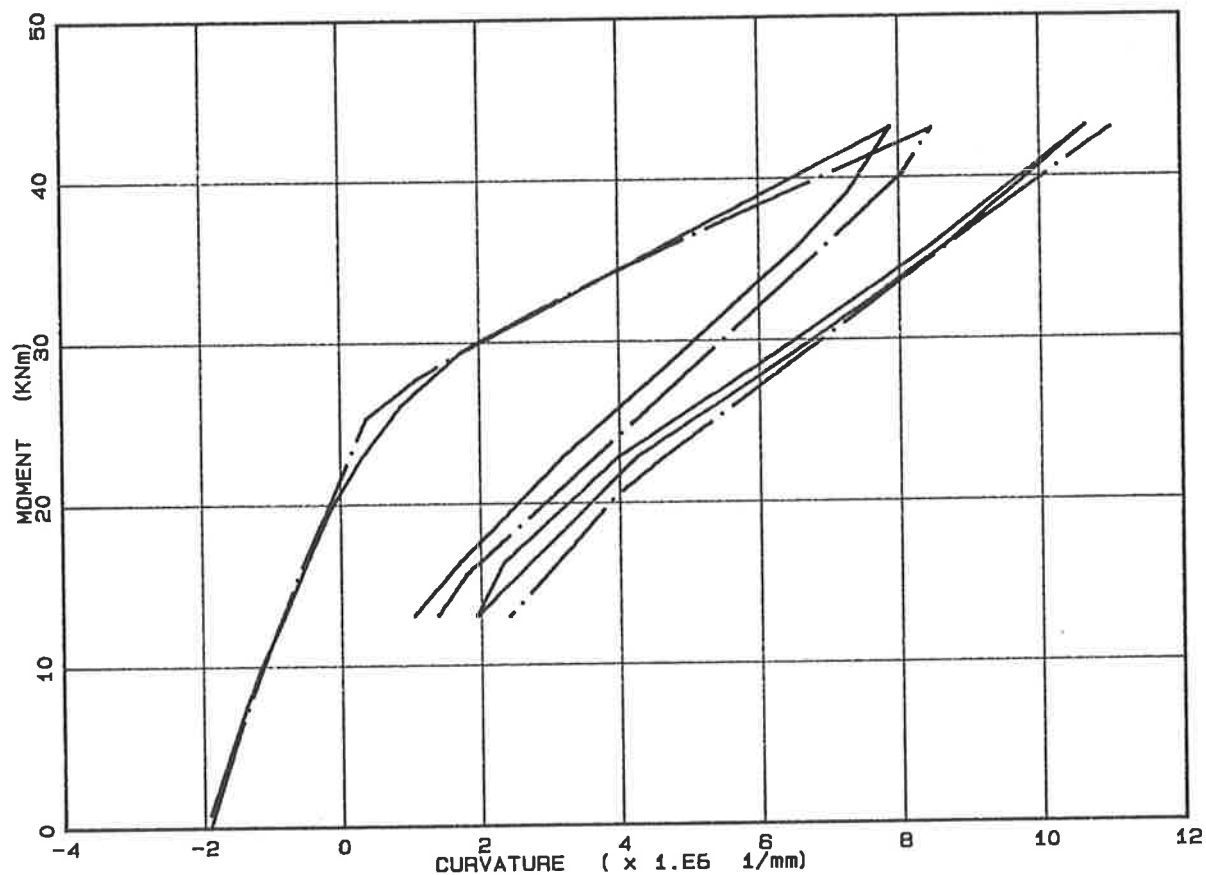


FIG 6.25a MOMENT-CURVATURE RELATIONS - BEAM SA1
(Using regional analysis)
Observed ——— theoretical — . —

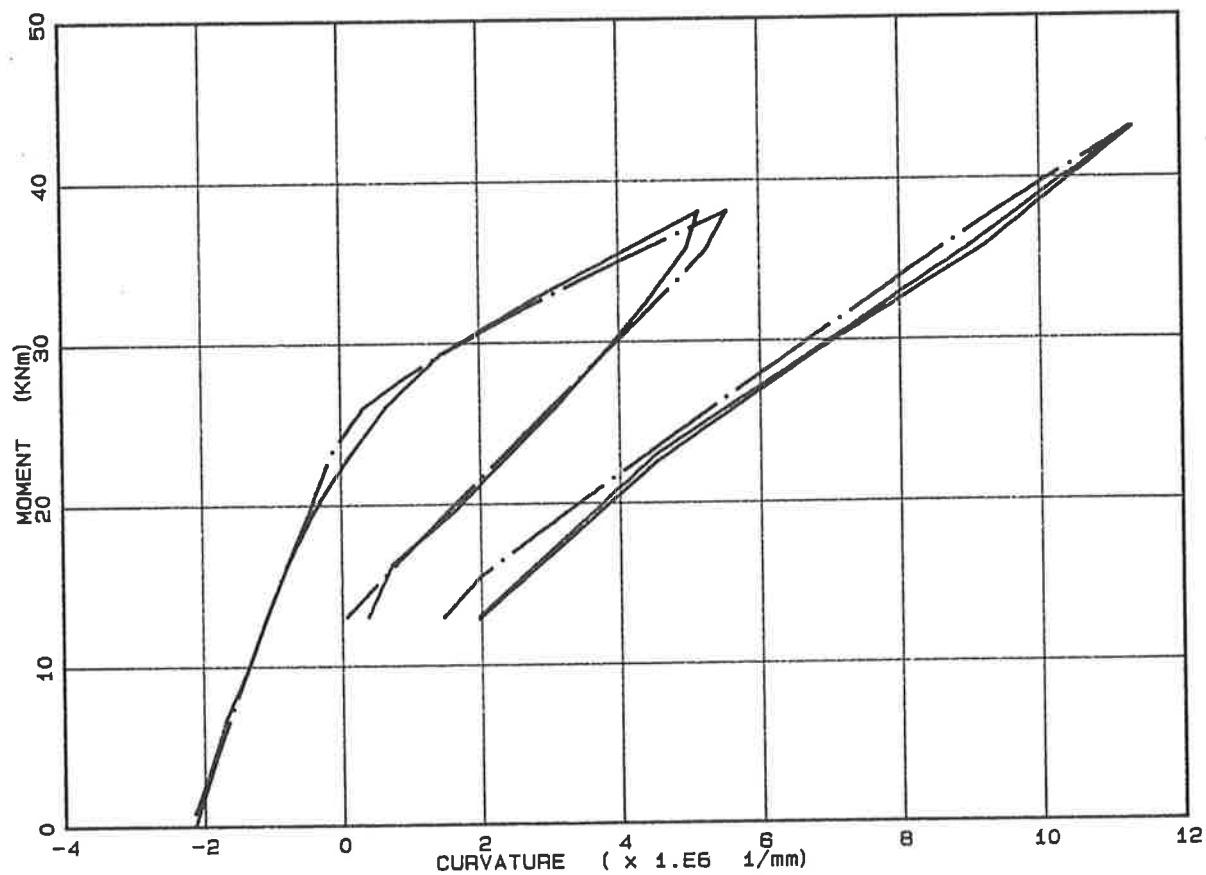


FIG 6.25b MOMENT-CURVATURE RELATIONS - BEAM SA2
(Using regional analysis)
Observed ——— theoretical — . —

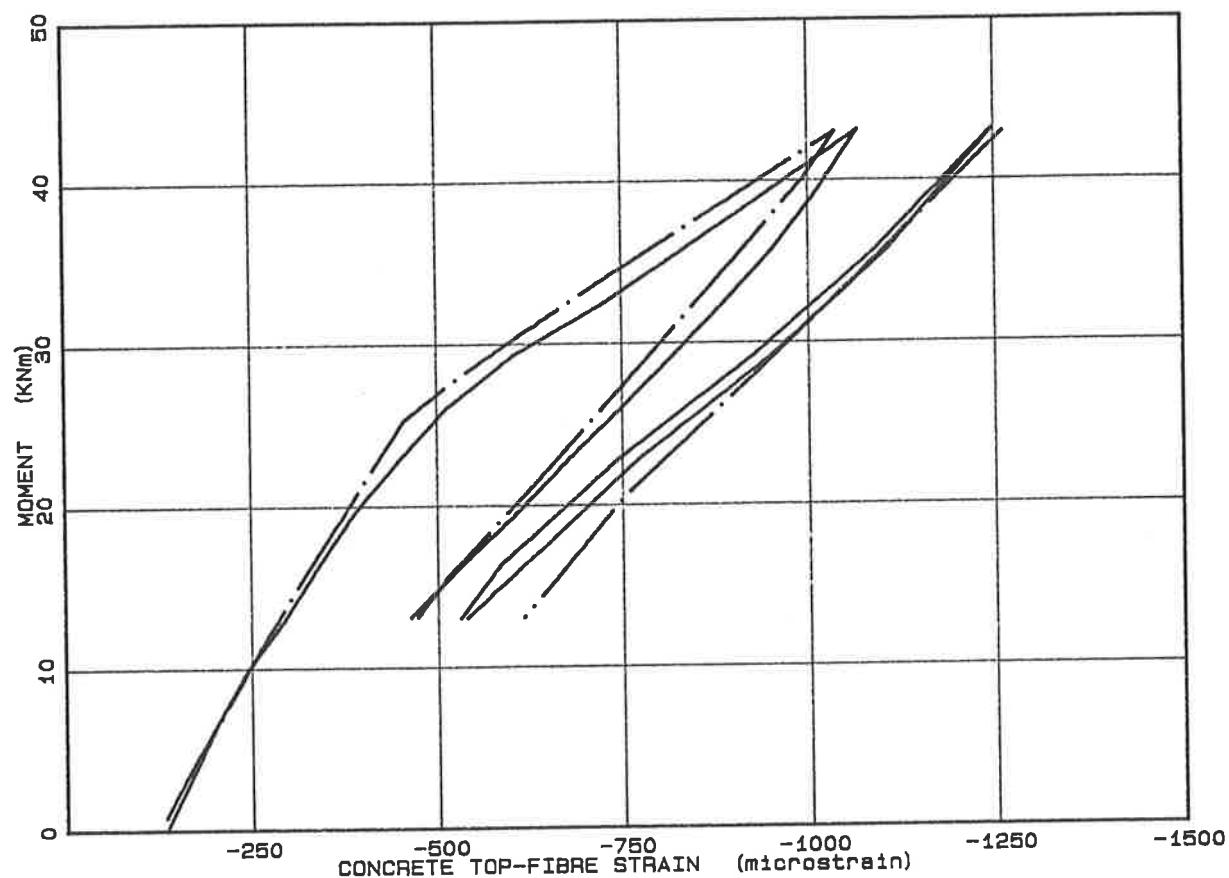


FIG 8.26a MOMENT-TOP FIBRE STRAIN RELATIONS - BEAM SA1
(Using regional analysis)

Observed ——— Theoretical ——— . ———

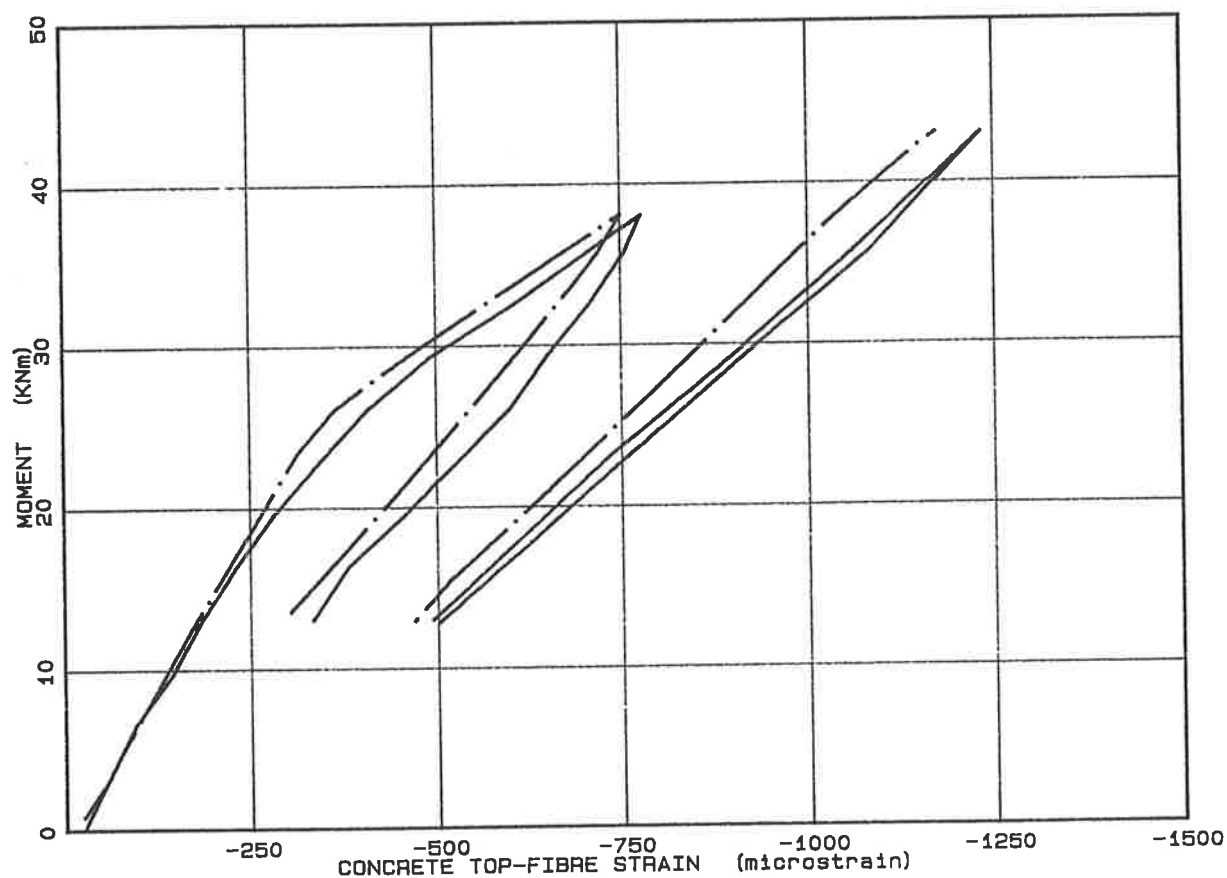


FIG 8.26b MOMENT-TOP FIBRE STRAIN RELATIONS - BEAM SA2
(Using regional analysis)

Observed ——— Theoretical ——— . ———

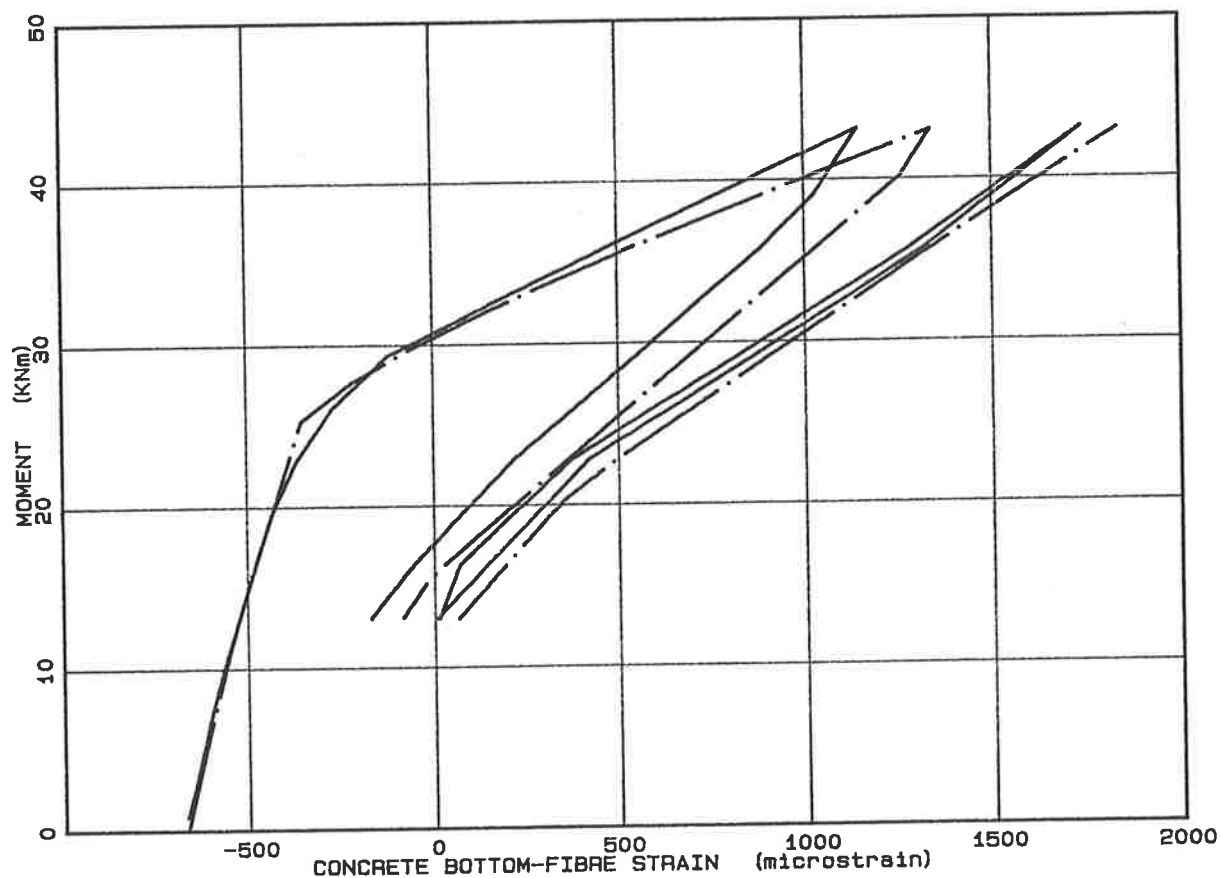


FIG 6.27a MOMENT-BOTTOM FIBRE STRAIN RELATIONS - BEAM SA1
(Using regional analysis)
Observed ——— Theoretical — . —

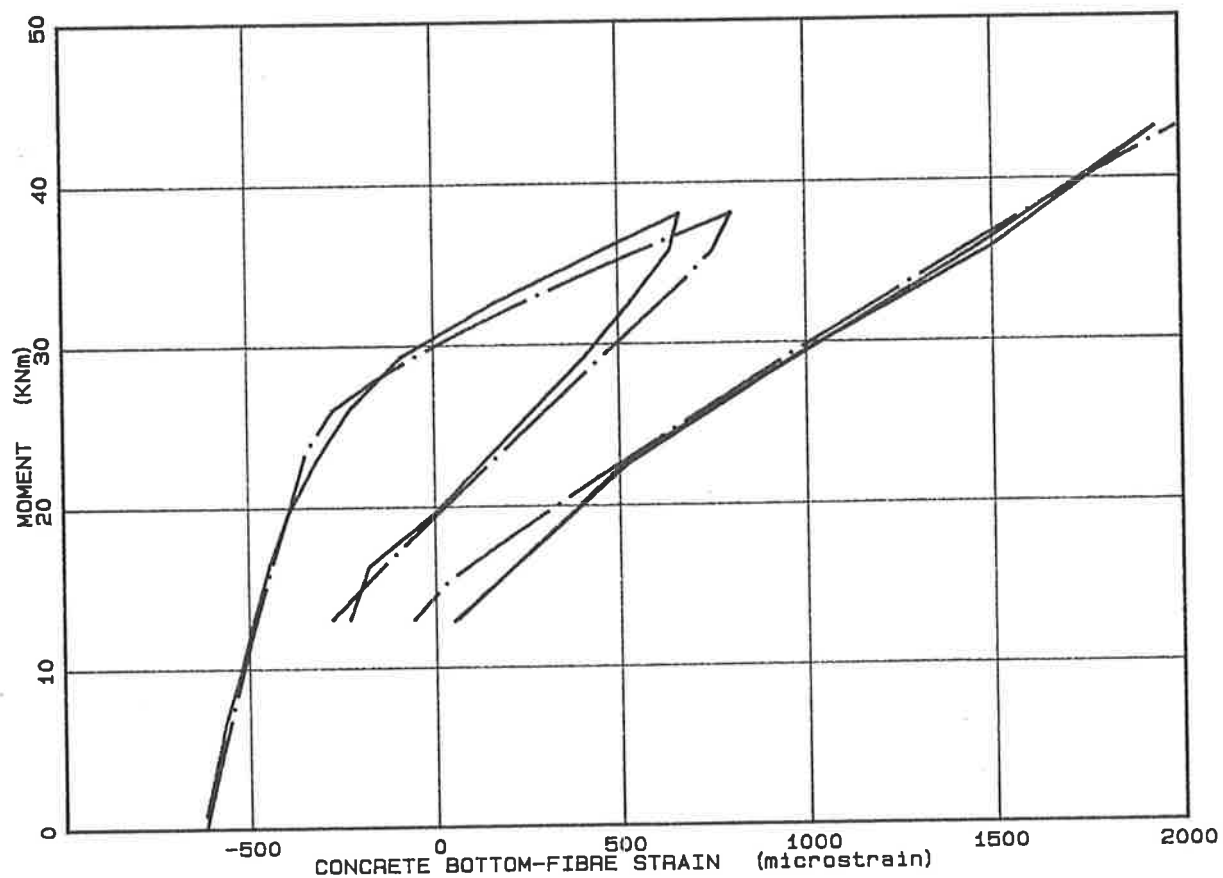


FIG 6.27b MOMENT-BOTTOM FIBRE STRAIN RELATIONS - BEAM SA2
(Using regional analysis)
Observed ——— Theoretical — . —

6.5.2 Repeated Load Behaviour

The theoretical and experimental moment-curvature relations in the 300000th and 580000th load cycle for Beams SA 1 and 2 are shown in Figs 6.25a and b. Close agreement for the entire load range is obtained and shows that the analysis can adequately account for the various physical mechanisms which occur at low and at high moments. Good agreement between predicted and observed results is also obtained for other load cycles, not shown here.

Comparison of the predicted and observed top-fibre strains for Beams SA 1 and 2, is considered good, with errors of about 7 percent at maximum load. Close agreement is also obtained for the concrete bottom-fibre strains of both beams under repeated load conditions. Maximum deviation between theoretical and test values is approximately 4 percent at maximum load.

Figs 6.28a and b show the plot of the experimental and theoretical $F_s - \epsilon$ relations for beam SA 1 and 2. The comparison shows similar shaped curves for both the experimental and theoretical values. The correlation is not as good as for the final moment-curvature relations and indicates that the $M - \chi$ predictions are not highly sensitive to the force-strain curve assumed.

6.5.3 Comments on the Regional Analysis

Advantages

The proposed analysis predicts beam behaviour in the first load cycle as well as in subsequent cycles. The physical mechanisms influencing beam response at high moments as well as at low moments are taken into consideration. Tension stiffening effects at high loads are accounted for, while at low loads, crack mis-match and the resulting wedging action in

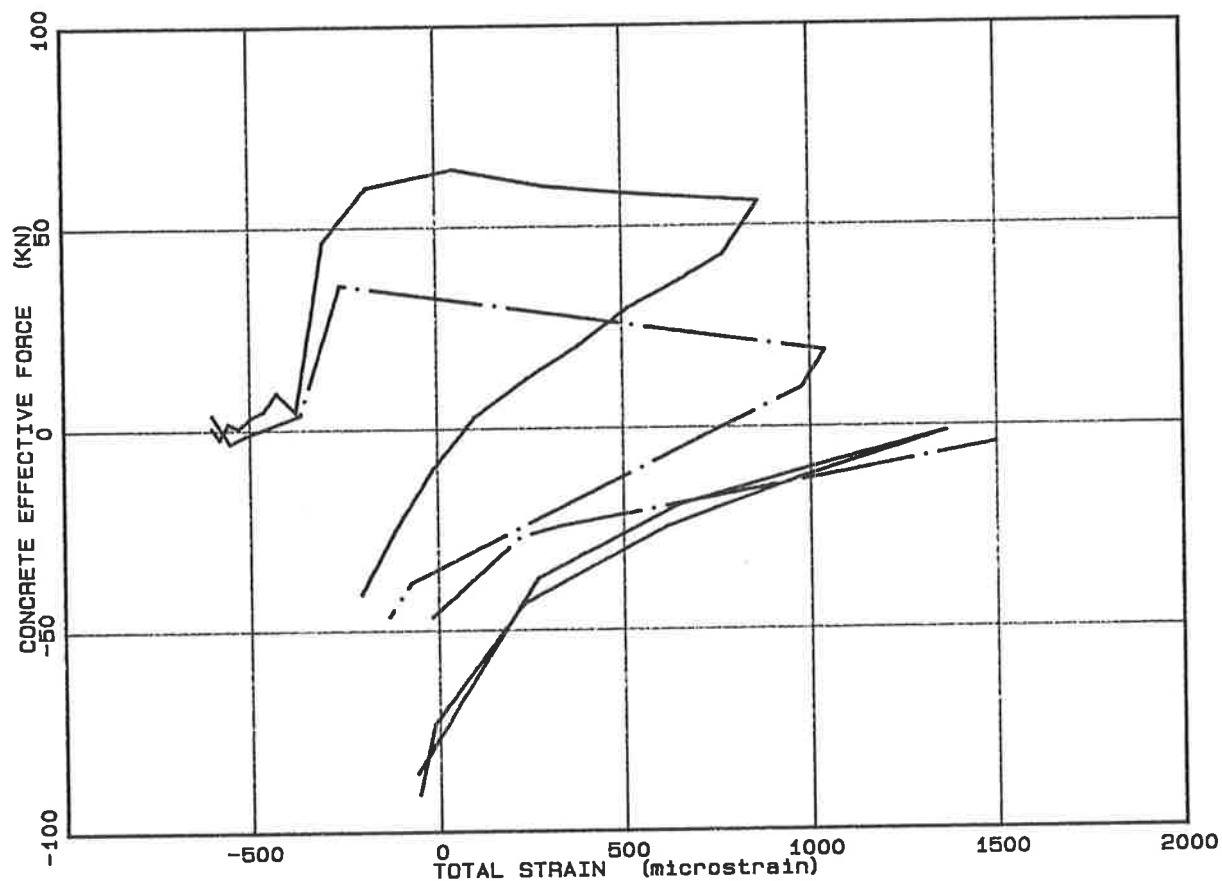


FIG 6.28a EFFECTIVE FORCE-STRAIN RELATIONS - BEAM SA1
(Using regional analysis)
Observed — Theoretical — . —

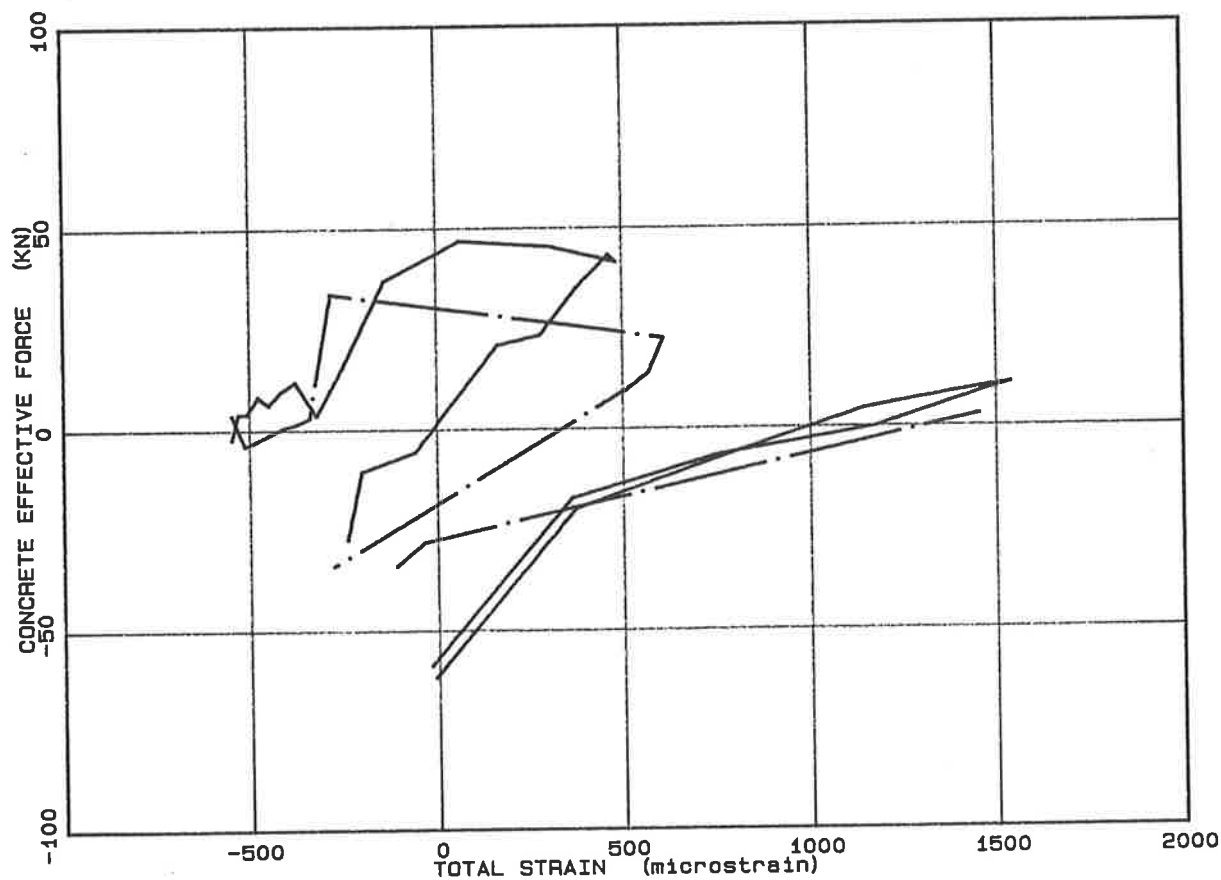


FIG 6.28b EFFECTIVE FORCE-STRAIN RELATIONS - BEAM SA2
(Using regional analysis)
Observed — Theoretical — . —

the first load cycle are allowed for. For beam fatigue loading, the cyclic response of concrete and steels are taken into consideration, together with the progressive loss of tension stiffening, increased mis-match and wedging action and increasing deformation.

The method is relatively easy and simple to use as a single section is employed for analysis. The analysis of individual local sections with the attendant problems of defining the appropriate local stress and strain distributions is avoided.

Disadvantages

A disadvantage of the analysis is that the proposed equivalent section is hypothetical and does not resemble any particular section. The analysis is regional and only average stresses and deformations are considered. Estimates of local stresses at certain beam sections cannot be determined directly.

The proposed concrete effective force-strain $F_{ce} - \epsilon$ relation is empirical, being derived from beam test results, and may not be directly applicable to other flexural members. The location of the concrete effective force F_{ce} at the level of the main reinforcing steel may not be appropriate in other types of sections.

6.5.4 Summary

A theoretical method of analysis which predicts beam behaviour under first cycle and repeated loading has been presented. Good results have been obtained by the proposed method. In particular, predictions of member curvatures are accurate. Estimations of top and bottom-fibre strains are also precise with average underestimations of about 5 percent for the top strains and overestimations of approximately 6 percent for the bottom-fibre strains. This is acceptable in view of the wide variability in concrete material properties.

The proposed method provides a simple means of predicting

overall structural behaviour and is a vast improvement over the simple cracked section analysis. It is also superior than any other currently available methods of analysis involving beam behaviour under repeated loading.

6.6 GENERALISED REGIONAL ANALYSIS

The regional method of analysis provides accurate predictions of beam behaviour under static and repeated loading. However, defining the effective stress-deformation relation for the lower concrete region is complicated because of the complex nature of the curve. A set of parameters has been recommended, but these are empirical and have limited applicability. There is a need for a more general method of prediction which can be applied to any concrete structural member.

A simplification is now introduced which is both easier to use, and more general in application. The approach is similar to the previous analysis in that an effective force is used to represent the concrete response in the lower regions under both first cycle and subsequent repeated load conditions. However, this effective force is now expressed as a function of the external moment.

6.6.1 Defining the Concrete Effective Force

The proposed generalised effective force-applied moment relation is shown in Fig 6.29.

First Load Cycle

As a simple approximation, a monotonic decrease in the effective force from cracking to ultimate moment can be assumed. Examination of the empirical force-moment curve shows fairly small reductions in the load range just after cracking. This suggests a parabolic type relationship with small initial decreases and then larger reductions later on, ie:

$$F_{e,} = F_{e, \max} \left\{ 1 - \frac{(M - M_{c, r})^2}{(M_{u, t} - M_{c, r})^2} \right\} \quad (6.41)$$

where M is the applied moment, and $F_{e,}$ is the corresponding concrete effective force, $F_{e, \max}$ is the tensile effective force at cracking, $M_{u, t}$ and $M_{c, r}$ are the ultimate and cracking moments respectively. Fig 6.29 shows the variation of $F_{e,}$ with respect to applied moment.

Although there are three distinct unloading stages (Section 6.2.2), only the two major phases (second and third) are considered here. The small initial 'near-elastic' effect caused by short-term creep and frictional resistances is considered to be insignificant and therefore ignored. A linear relation is used for the first dominant phase where unloading takes place from maximum moment to a moment near the initial cracking value. In this range the effective force decreases from a tensile value to zero as shown in Fig 6.29.

The moment at which the tensile force becomes zero is defined in terms of the cracking moment, ie.

$$M_0 = m_0 M_{c, r} \quad (6.42)$$

where $M_{c, r}$ is the initial cracking moment and m_0 is a dimensionless constant.

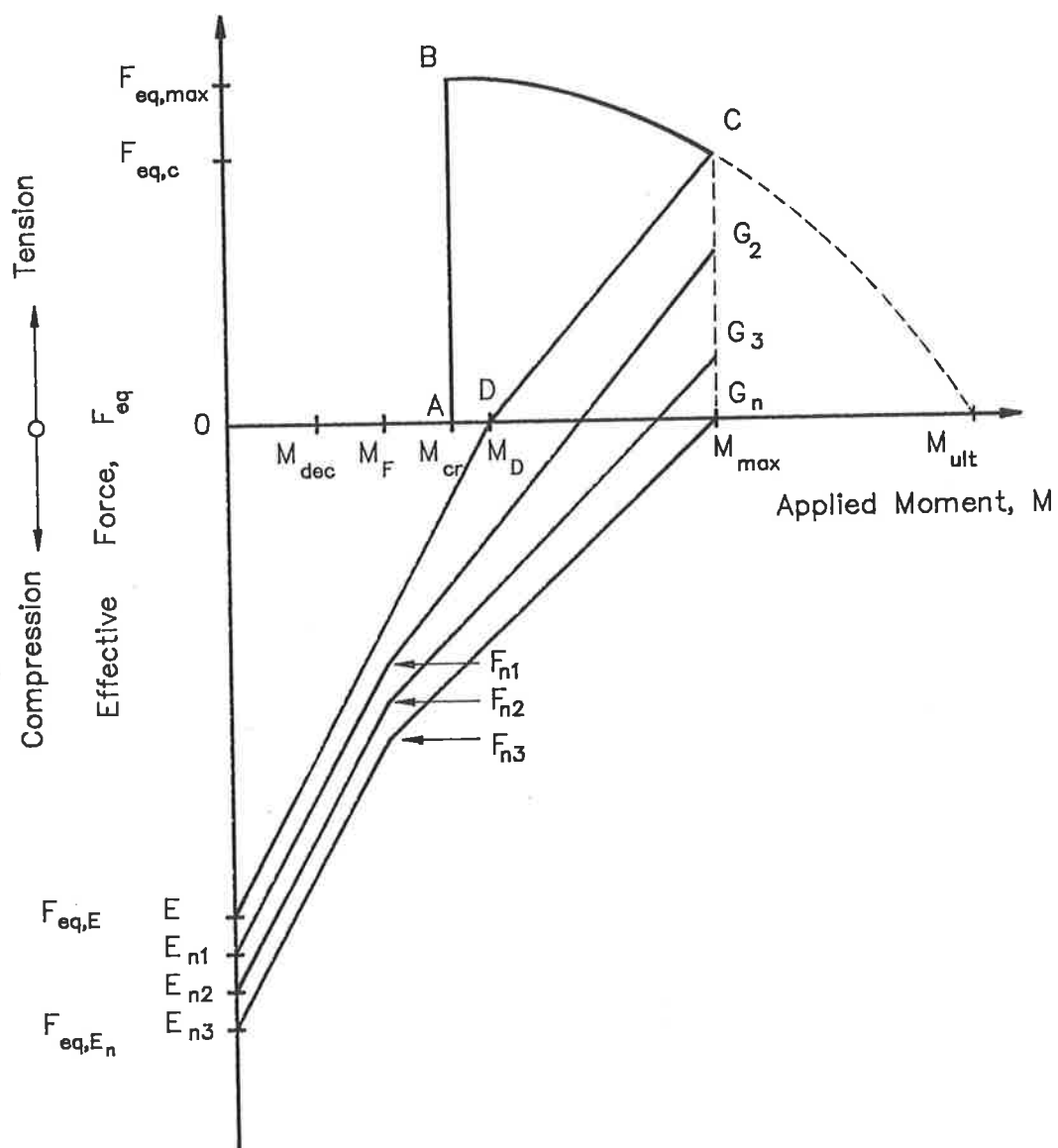


Fig. 6.29 Generalised Effective Force—Moment Relation

A second linear relation is assumed for the remaining load range. The effective force increases in compression from zero at cracking to a maximum at zero moment. The magnitude of the maximum compressive force can be defined in terms of the maximum tensile force,

$$F_{c,q,E} = w_E F_{t,q,max} \quad (6.43)$$

where w_E is the dimensionless parameter defining the maximum compressive force at zero moment after unloading.

Second and Subsequent Load Cycles

Increased compressive effective forces at low moments and reduced tensile forces at higher moments are sustained through repeated loading. The hysteresis effects for the second and subsequent load cycles are insignificant compared to the first. A series of successively lowering bi-linear curves can be used to describe the force-moment relation at various stages of repeated loading. The compressive force (which increases with load repetitions) at zero moment in the N^{th} load cycle is defined in terms of the magnitude of the compressive force in the first cycle.

$$F_{c,q,E,N} = w_{E,N} F_{c,q,E} \quad (6.44)$$

The slope of the force-moment relation in the lower load range is assumed to remain constant with repeated loading. A decrease in 'stiffness' occurs when the moment is in the vicinity of the first decompression moment as the cracks commence to open. This position, denoted by point F_n in Fig 6.29 is defined by a parameter m_F , such that the moment at which the cracks open is:

$$M_F = m_F M_{d,c} \quad (6.45)$$

where $M_{d,c}$ is the first decompression moment and M_F the moment at which the slope of the $F_{c,q} - M$ curve changes.

After applications of a large number of load cycles, it is

expected that the tension stiffening effect will be totally eliminated. The equilibrating force then becomes completely compressive throughout the entire load range, with a minimum value of zero at maximum moment. This is indicated by point G_n in Fig 6.29. At this stage of repeated loading, the analysis at maximum moment is identical to the simple cracked section analysis taking into account cyclic creep of concrete and steel. However, the two analyses are dissimilar in the lower load ranges where the latter does not consider unloading effects.

6.6.2 Computational Procedures

To carry out the proposed analysis for a given moment (and moment history), the effective concrete force is first determined directly from the generalised equilibrating force-moment relationship given in Section 6.6.1. A systematic search is then undertaken to find a linear concrete strain distribution and its accompanying stress profile which satisfies both force and moment equilibrium. A trial value of the top-fibre concrete strain is first chosen and a search technique is used to locate the bottom-fibre strain which defines the strain and therefore stress distribution that satisfies force equilibrium. Moment equilibrium in the section is then checked and if this is not satisfied, successive top fibre strains are used and the process repeated until both equilibrium conditions are satisfied within tolerances.

The procedure for determining the top-fibre concrete strains and steel strains is similar to that used in the regional analysis. The layering technique is used for the representation of concrete area while discrete elements are substituted for the ordinary and prestressing steels. The usual concrete stress-strain relation is employed. However, no concrete tensile stresses are allowed after first cracking, as the concrete tensile stiffening stresses are now represented by the effective force. The calculation of the concrete instantaneous strain from the total strains is similar to the previous analyses. As in the case of the regional analysis,

initial creep and shrinkage effects as well as cyclic creep effects of concrete and steel are accounted for.

After first cracking, the concrete stress in the j^{th} layer is:

$$\sigma_j = G_c \{\epsilon_{j,e}\} \text{ for compressive elastic strain} \quad (6.46)$$

$$\sigma_j = 0 \quad \text{for tensile elastic strain} \quad (6.47)$$

The steel stresses are related to the elastic strains by their normal stress-strain relations as described in Section 6.4.3.

6.6.3 Empirical Parameters

Empirical determination of the $F_{cu} - M$ parameters was achieved by conducting regional analyses of the test beams with trial values of these variables. These were then varied to provide the best possible fit between the theoretical and observed values. Table 6.3 below shows the values for all six test beams.

Table 6.3 Empirical Parameters for the $F_{cu} - M$ Relation

Beam	m_0	m_F	W_E	$W_E N$	N^+
SA 1	1.2	1.4	3.0	1.025	300
SA 2	1.1	1.4	2.5	1.180	660
SA 3	1.5	1.4	3.5	1.180	760
SA 4	1.1	1.4	3.0	1.050	400
PA 1	1.1	1.4	2.5	1.015	280
PA 2	1.2	1.4	2.5	1.050	950
Average	1.2	1.4	2.83	varies	-

+ Number of load cycles in thousands

Good correlation of the empirical parameters is observed from

Table 6.3. There appears to be little disagreement on the location where the tensile force reduces to zero (m_0) or at which the slope changes (m_F) for the bi-linear repeated load response curve. Good agreement is also observed for the parameter w_E which defines the maximum effective compressive in the first load cycle. However, there appears to be only fair agreement for w_{EN} , the parameter representing increases in the compressive force F_c , due to repeated loading. This can be explained by the fact that some beams sustained a greater number of load repetitions than others. The magnitude and type of repeated loading (constant amplitude or cumulative damage) is different for each beam. Table 6.4 shows the values of w_{EN} together with the number of load cycles sustained, grouped in ascending order. It can be seen that as the numbers of repeated load cycles increase, w_{EN} increases as well. The exception to this is beam PA 2 which showed relatively smaller increases. A likely explanation of this discrepancy is that the magnitude of the repeated loads sustained by this beam is less than the others.

Table 6.4 Variation of w_{EN} with Repeated Loading

	PA 1	SA 1	SA 4	SA 2	SA 3	PA 2
No of Cycles (*1000)	280	300	400	660	760	950
w_{EN}	1.015	1.025	1.050	1.180	1.180	1.050

6.6.4 Comparisons between Theoretical and Observed Results

Figs 6.30a and 6.30b show the predicted and observed moment curvature relations for two typical beams (SA1 and SA 2), one tested under constant amplitude and the other under cumulative damage loading conditions. The average parameters determined in Table 6.3 were used to describe the $F_c - M$ relation. Good agreement between the observed and the theoretical predictions in both beams was observed for static as well as repeated load behaviour. A very slight discrepancy was observed in the

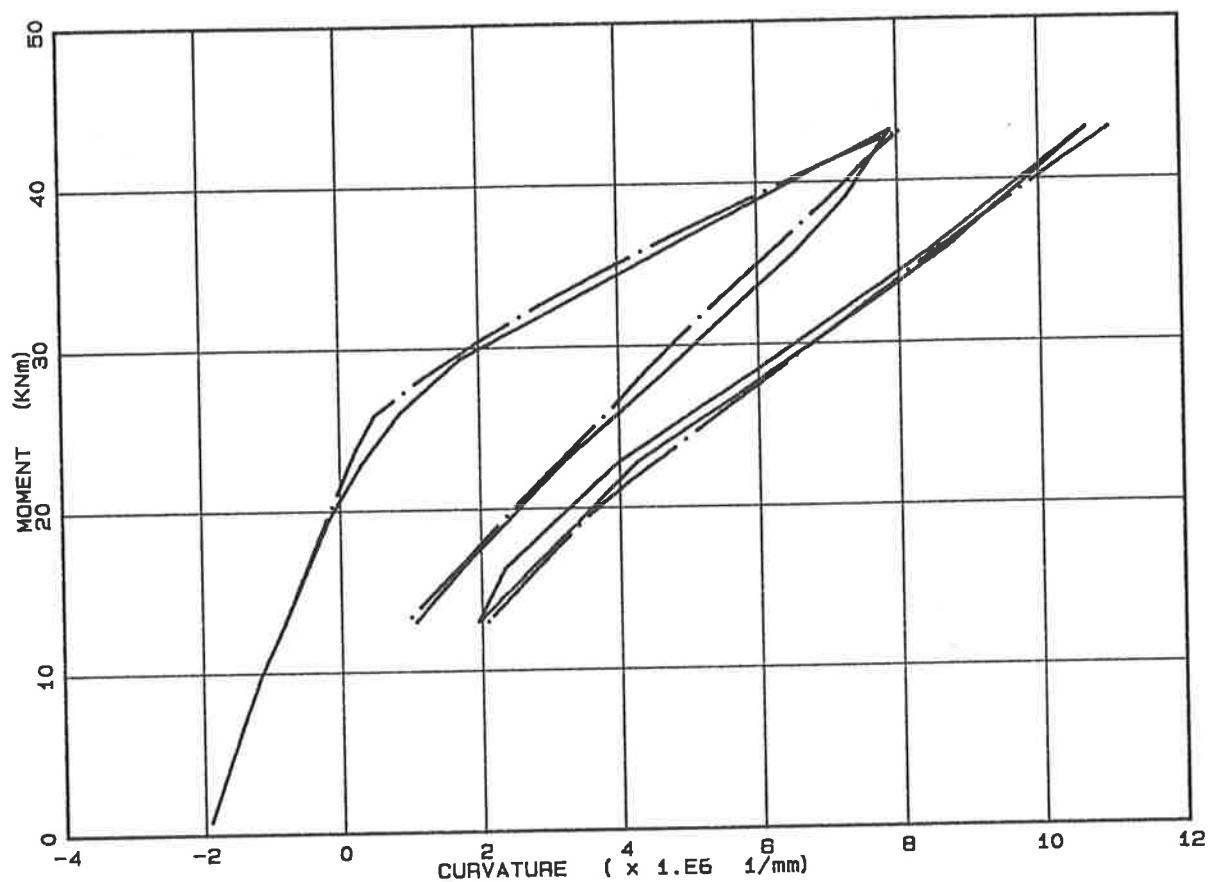


FIG 6.30a MOMENT-CURVATURE RELATIONS - BEAM SA1
(Using generalised regional analysis)
Observed ——— theoretical — . —

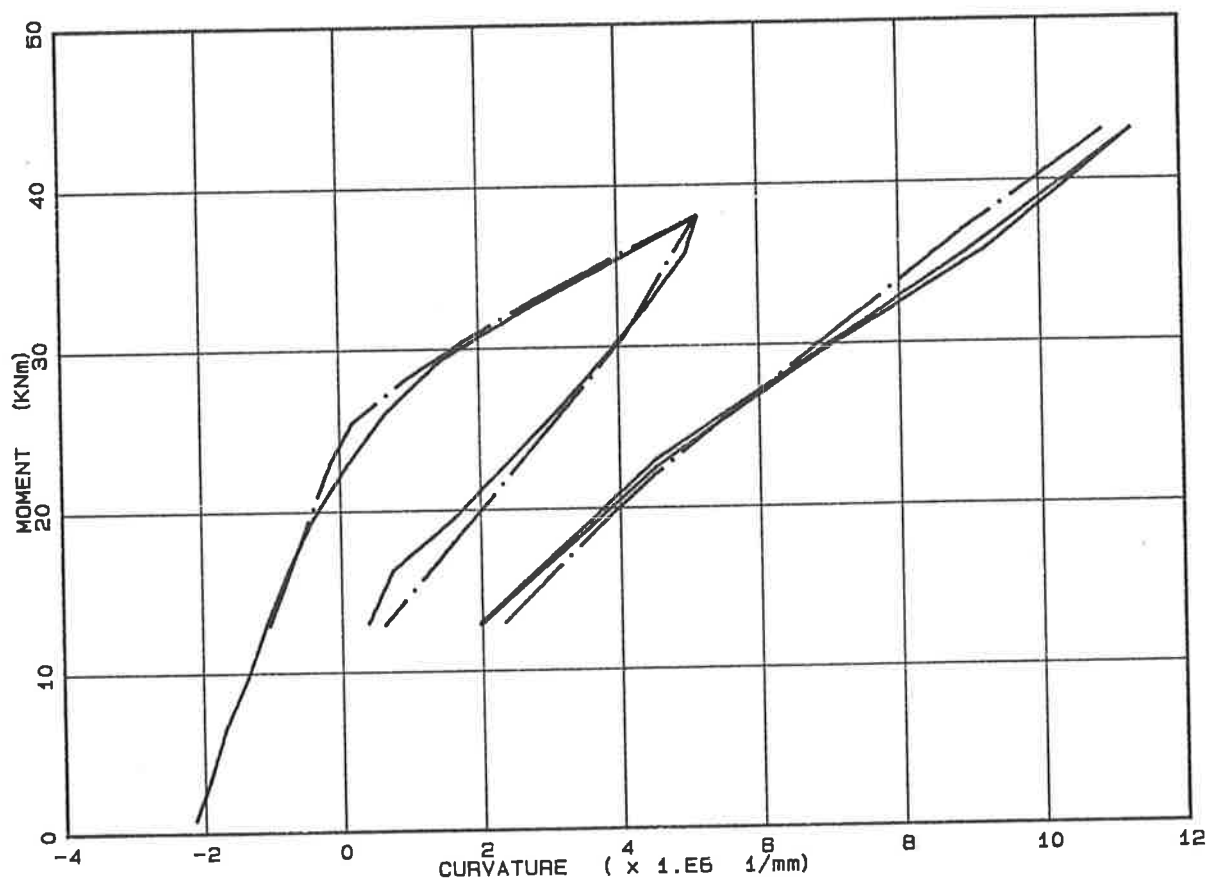


FIG 6.30b MOMENT-CURVATURE RELATIONS - BEAM SA2
(Using generalised regional analysis)
Observed ——— theoretical — . —

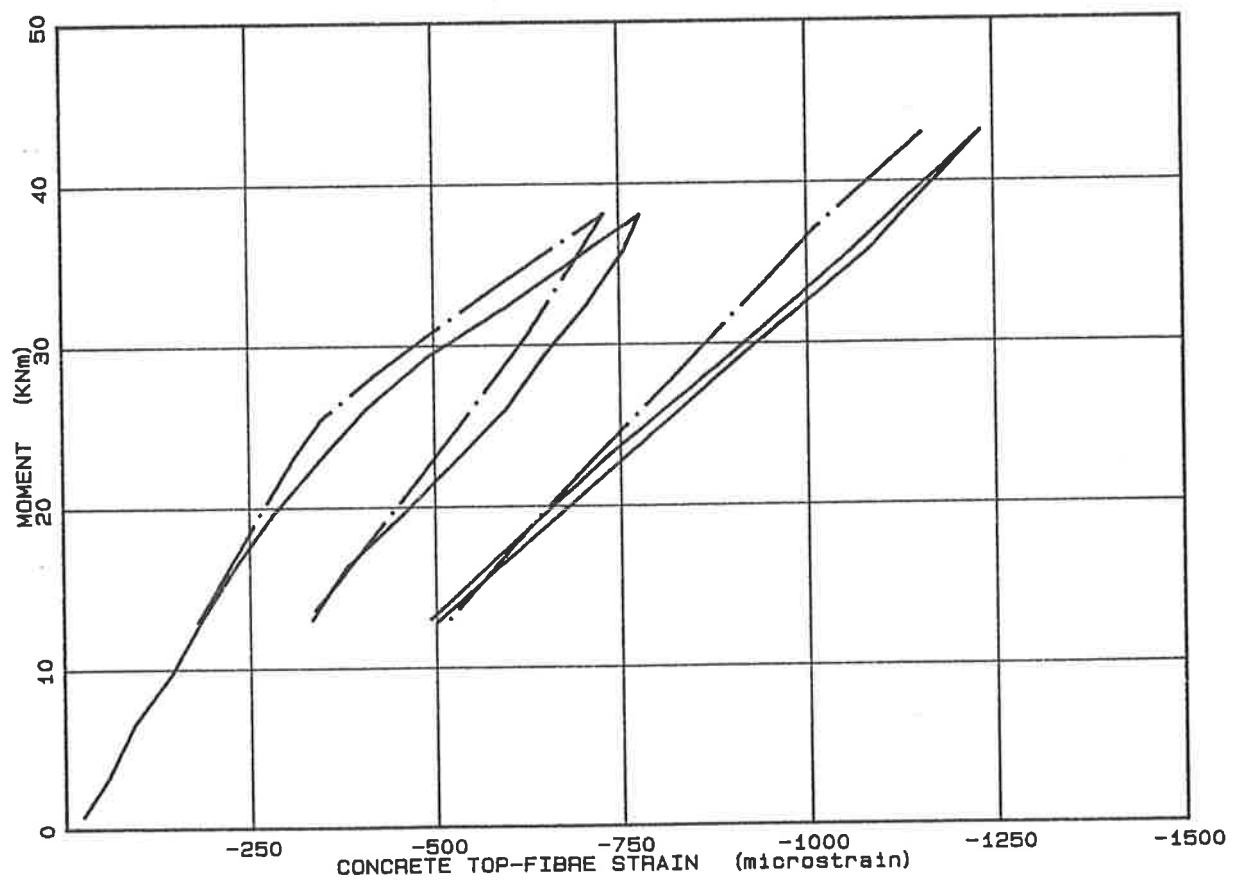
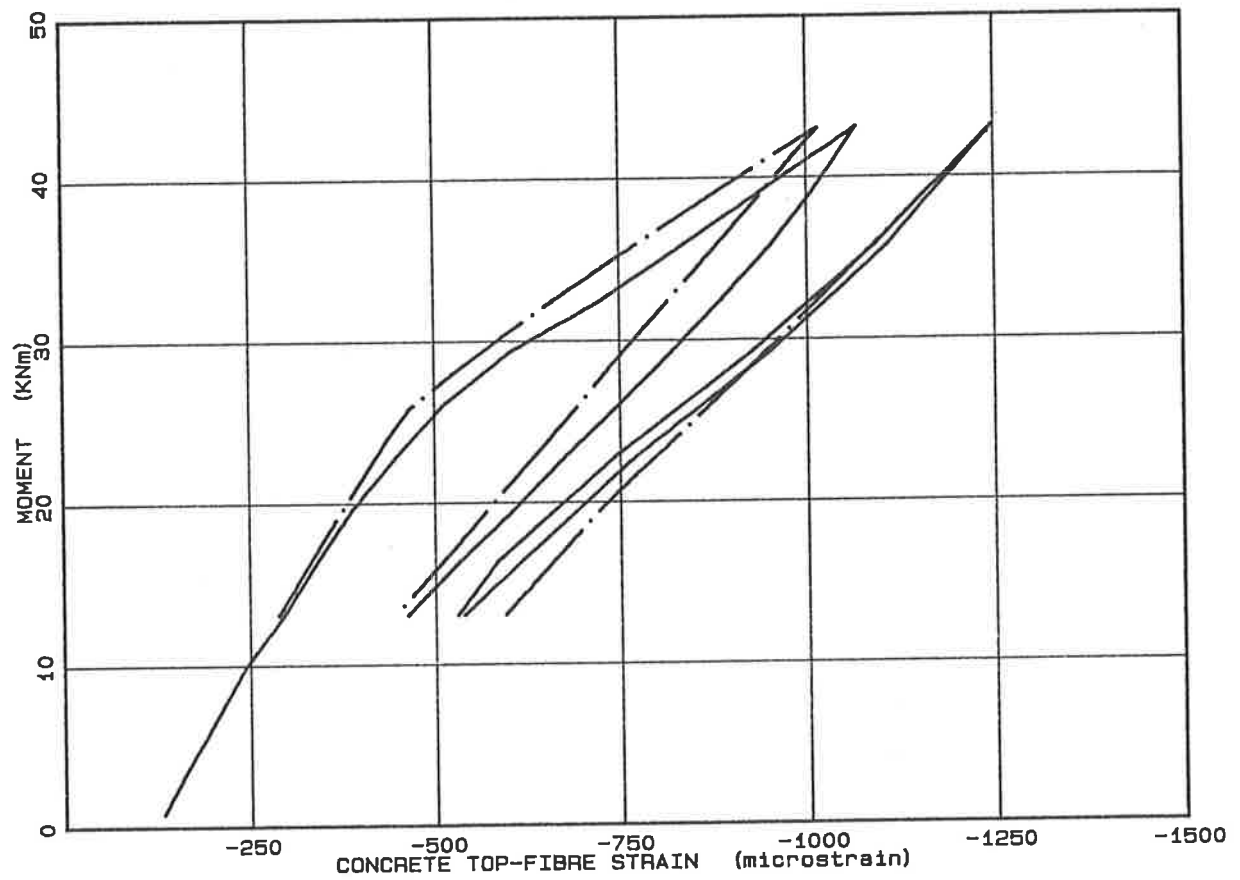
initial unloading portion of the first load cycle where possible influences of short-term creep and elastic effects were not allowed for.

Comparisons of the predicted and theoretical curves for the concrete top-fibre strains are shown in Fig 6.31a and 6.31b. Good agreement was observed for both beams in both static and repeated loading, although the correlation was not as good as that for the moment-curvature relations with slight underestimations in the postcracking range of the first load cycle. A maximum error of 50 microstrains or 7 percent was recorded for the initial cycle and 66 microstrains or 6 percent for the mid-life load cycle. This is considered to be well within acceptable limits, bearing in mind the variable nature of concrete. The minor discrepancy could also be caused partly by an incorrect positioning of the effective force - which has been assumed to act at the level of the main reinforcing steel.

The measured and predicted concrete bottom-fibre strains are also shown in Fig 6.32a and 6.32b. All three curves again show good agreement with a maximum error of 50 microstrain or less than 5 percent. The discrepancy, if any, appears to be on the conservative side.

6.6.5 Summary

A more general and simpler method of analysis has been presented here. The effective force representing the concrete 'tension' zone is related to the applied moment instead of the average tensile deformation. An advantage of this method is its simplicity. Fewer empirical parameters are required and the relation can potentially be applied to a wide range of structural members under flexural loading.



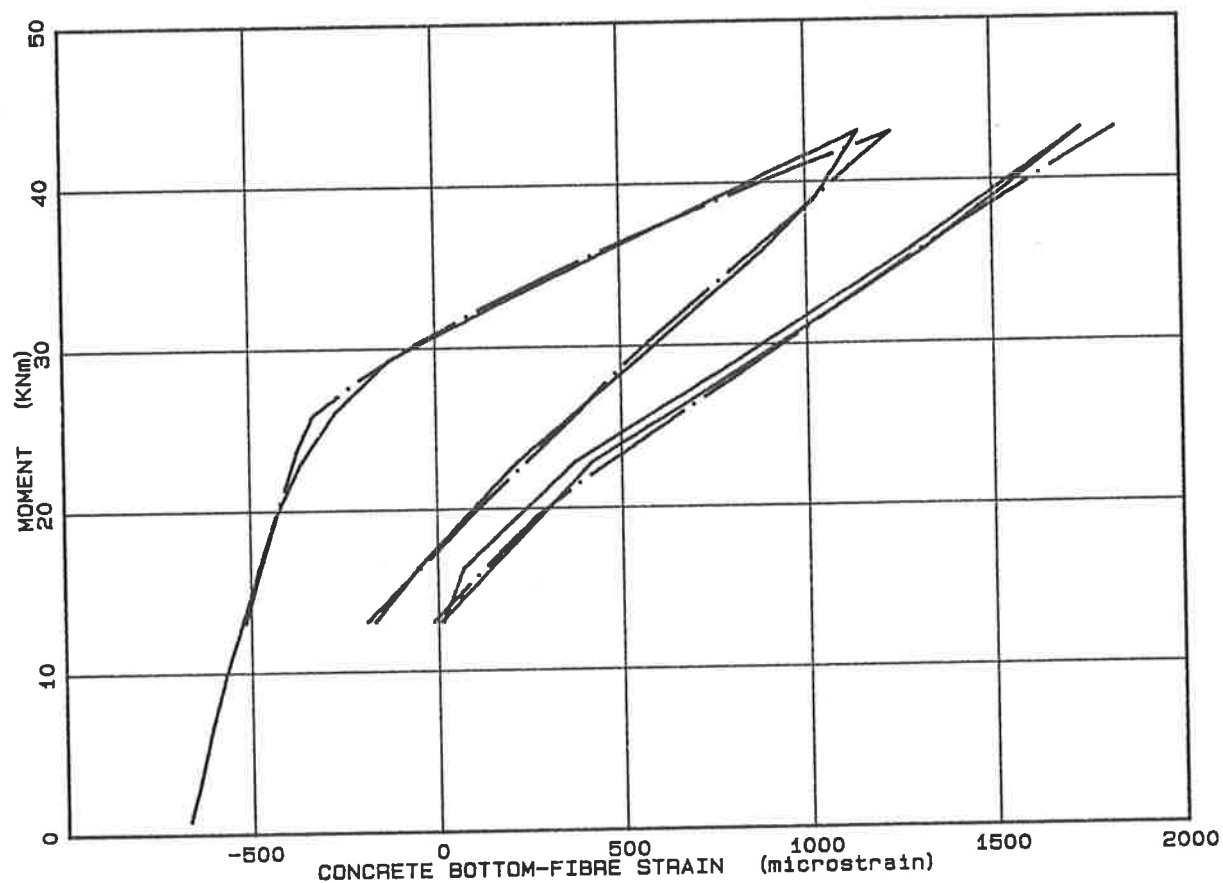


FIG 8.32a MOMENT-BOTTOM FIBRE STRAIN RELATIONS - BEAM SA1
(Using generalised regional analysis)
Observed ——— Theoretical ——— . ———

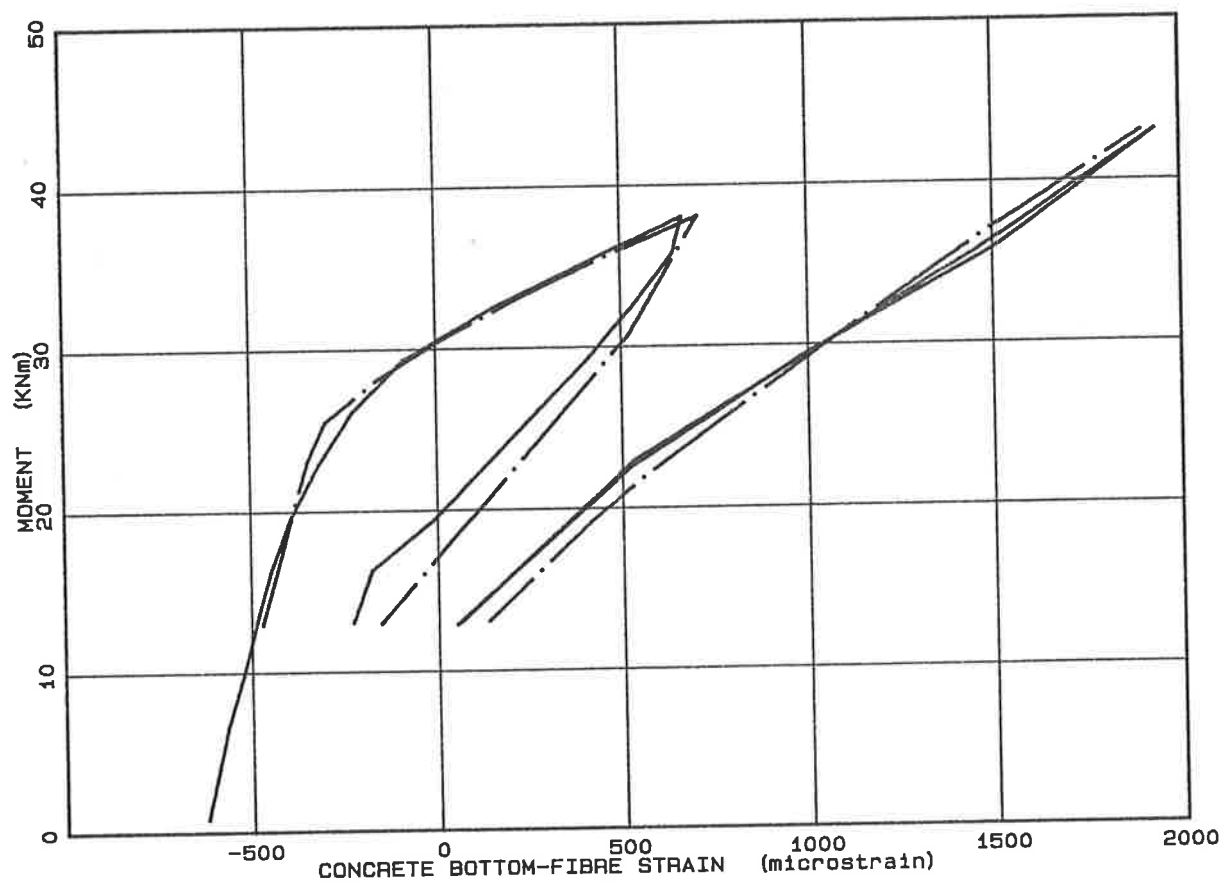


FIG 8.32b MOMENT-TOP FIBRE STRAIN RELATIONS - BEAM SA2
(Using generalised regional analysis)
Observed ——— Theoretical ——— . ———

CHAPTER 7

LOCAL STRESS AND FATIGUE CALCULATIONS

The behaviour of concrete members under repeated loading and unloading have been studied in detail in Chapters 5 and 6. These investigations were directed towards predicting overall service load behaviour under repeated loading. Studies of local behaviour are now required to provide accurate estimates of critical stresses for fatigue life. Methods of evaluating local steel stresses are first investigated, after which several procedures of calculating fatigue life are examined.

7.1 LOCAL STRESS VARIATIONS

Variations in stresses along cracked concrete structures have long been recognised (Broms, 1964; Beeby, 1979; Murashev, 1940). The distribution of local stresses along a flexural member is schematically shown in Fig 7.1 reproduced from Babu (1979). These variations can be quite large and are thought to be the principal cause of the consistent occurrence of steel fatigue fractures at primary concrete cracks (Abeles et al, 1974; Nordby, 1958) in spite of the inherent variability of fatigue properties of materials. There may also be other contributing causes such as fretting of the steel (Rigon, 1985). Whatever the cause, the precise estimation of local stresses at critical sections is of paramount importance in fatigue life predictions. Since fatigue of concrete is generally not a primary consideration, the present investigation concentrates on steel fatigue behaviour.

7.1.1 Variation of Steel Stresses along Member

Direct Tension Tests

Experimental investigations of steel stress variations have usually been linked with bond stress and slip studies on direct tension reinforced concreted prisms. Fewer studies have been performed on flexural members.

A brief description of research conducted on the bond behaviour between concrete and steel has already been given in Section 6.3.3. Internal strain gauging of the steel bar, using a method proposed by Mains (1951), has enabled measurements of local steel strains without affecting bond and has since been widely used by other researchers (Nilson, 1971; Scott and Gill, 1984; Jiang et al, 1984).

A typical variation of steel stress along the bar under static loading conditions is given in Fig 7.2 which is reproduced from Jiang et al, (1984). Maximum steel stresses were recorded at cracks and smaller values found elsewhere. Table 7.1 shows the results of a brief survey on stress and strain variations from recent static tensile tests on uniaxially loaded reinforced concrete prisms (Ismail and Jirsa, 1972; Jiang et al, 1984; Scott and Gill, 1984; Bresler and Bertero, 1968; Gijssbers et al, 1979)

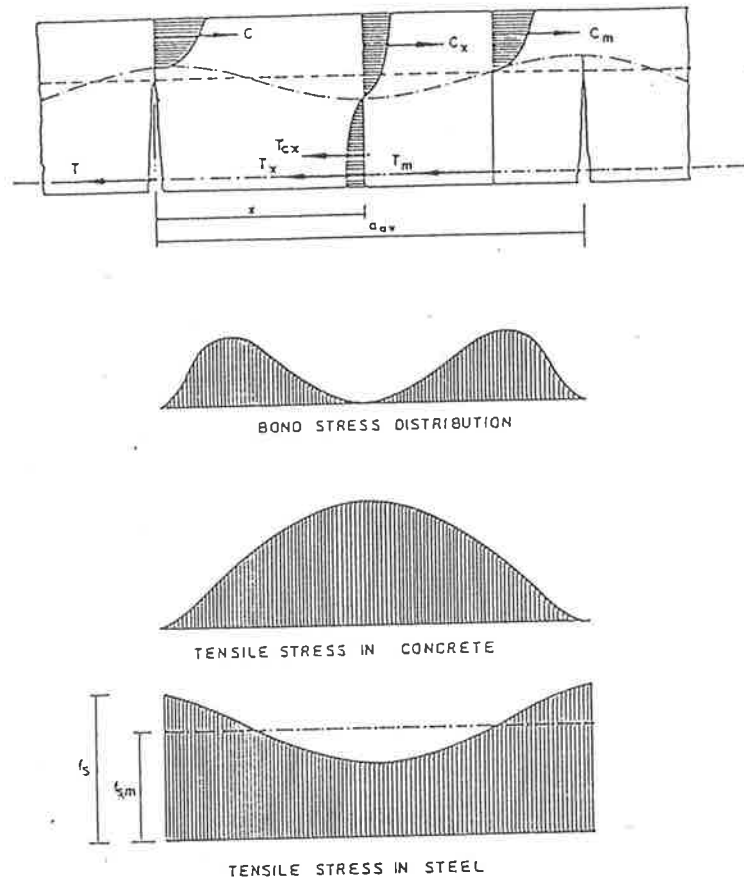


Fig. 7.1 Stress Distributions Along Cracked Flexural Member (Babu, 1979)

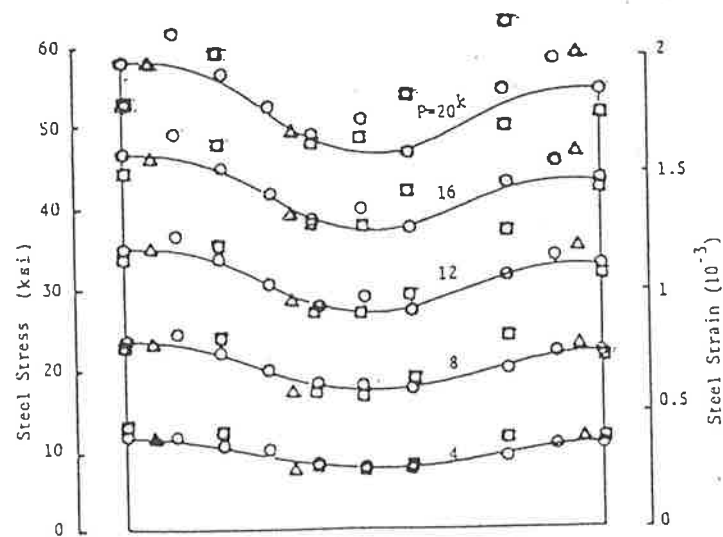


Fig. 7.2 Steel Strain Variation Along Concrete Tension Specimen (Jiang et al, 1984)

Table 7.1 Steel Strain Variations in Tensile Concrete Prisms under First Cycle Loading

Source	ϵ_{max} (microstrains)	ϵ_{min}	$\Delta\epsilon$ (MPa)	$\Delta\sigma$
Ismail & Jirsa	1461	1115	346	69
Jiang, et al	1148	888	259	52
Scott & Gill	1600	1325	275	55
Bresler & Betero	358	137	220	44
Gijsbers et al	700	200	500	100
Average	1053	733	320	64

The survey shows reasonable agreement among the results of various researchers with an average variation of 320 microstrains (or stress of 64 MPa) between minimum and maximum strains, for a wide range of maximum strain. The reported strain variation ranges from 220 to 500 microstrains. This is not unreasonable as load conditions and factors such as the area of concrete, steel to concrete ratio, and bond characteristics have considerable influence on the strain variation.

Beeby (1979) proposed an equation for calculating the average strain in a member subjected to pure tension. The strain at a cracked section is first calculated and an appropriate tension stiffening allowance is then subtracted from it, ie.

$$\epsilon_m = \epsilon_t - \frac{K f_t f_{s,cr}}{E_s \rho f_t} \quad (7.1)$$

where ϵ_m is the mean steel strain, ϵ_t is the strain calculated on the basis of a cracked section, f_t is the tensile strength of concrete, f_t is the steel stress corresponding to ϵ_t , $f_{s,cr}$ is the steel stress at cracking load, E_s is the Young's modulus for steel, ρ is the reinforcement ratio and K is a constant which depends on bar type.

The predicted strain variation between maximum and minimum values is thus:

$$\Delta\epsilon_m = \frac{2 K f_t f_{s,cr}}{E_s \rho f_t} \quad (7.2)$$

Unfortunately, no estimate of the constant K was given.

Strain Variations with Repeated Loading

Studies involving measurements of steel strains in prisms under repeated loading have been few. Bresler and Bertero (1968) reported that stress history was a major factor influencing stress transfer. Increased steel strains were observed with cyclic loading. Repetitions of load cycles with constant peak stress produced increases in the residual stresses due to the inelastic effects of creep, shrinkage and slip. Deterioration of bond transfer and axial stiffness were observed with increasing load cycles. Similar results were observed by Ismail and Jirsa (1972) although a lesser degree of deterioration was reported.

Beam Test Results

Measurement of longitudinal steel strains in flexural members is less common and little data is found in the literature. Kulkarni and Ng (1979) attached strain gauges on the outside surface of the steel. Distinct differences in strains were measured along the steel. However, there is some ambiguity in the reported strain diagrams which showed changes in average steel strain of only 10 microstrains between maximum and minimum loads.

Priestley, Park and Lu (1971) took into account variations in stresses in their studies on moment-curvature relations and produced local curvature measurements between cracks, but the actual strain variations were not reported.

7.1.2 Qualitative Evaluation of Steel Stresses at Cracked Sections

Local Stresses in the Immediate Postcracking Range

Primary cracks in concrete flexural members do not all occur simultaneously, because of the variable nature of the concrete tensile strength. Immediately after cracking, tensile stresses reduce to zero at a section where the primary crack occurs, while in the uncracked regions tensile stresses still remain. Large variations are therefore likely to exist in steel stresses in the immediate postcracking load range. This is illustrated in Fig 7.3.

The precise estimation of stresses for any particular section is difficult because of the ambiguity in the local strain distribution. The usual assumptions of plane sections remaining plane and full strain compatibility between concrete and steel are not valid. The simple cracked section analysis (Section 5.1), based on assumptions of linear strain distribution and compatibility does not truly predict the actual conditions in a cracked section.

Local Stresses in the High Moment Range - First Load Cycle

Increased loading is accompanied by further cracking resulting in the formation of primary, secondary and tertiary cracks. At sections where primary cracks are located, the stress profile is relatively simple with a compressive concrete stress block and steel stresses. In the regions between primary cracks, there is the additional effect of tension stiffening.

The variation in steel stresses is likely to be less at high moments because of the reduction in the concrete tension capability due to further cracking. This is shown in Fig 7.3. Predictions of concrete and steel stresses in the cracked sections by the simple no-tension theory will therefore give closer estimates.

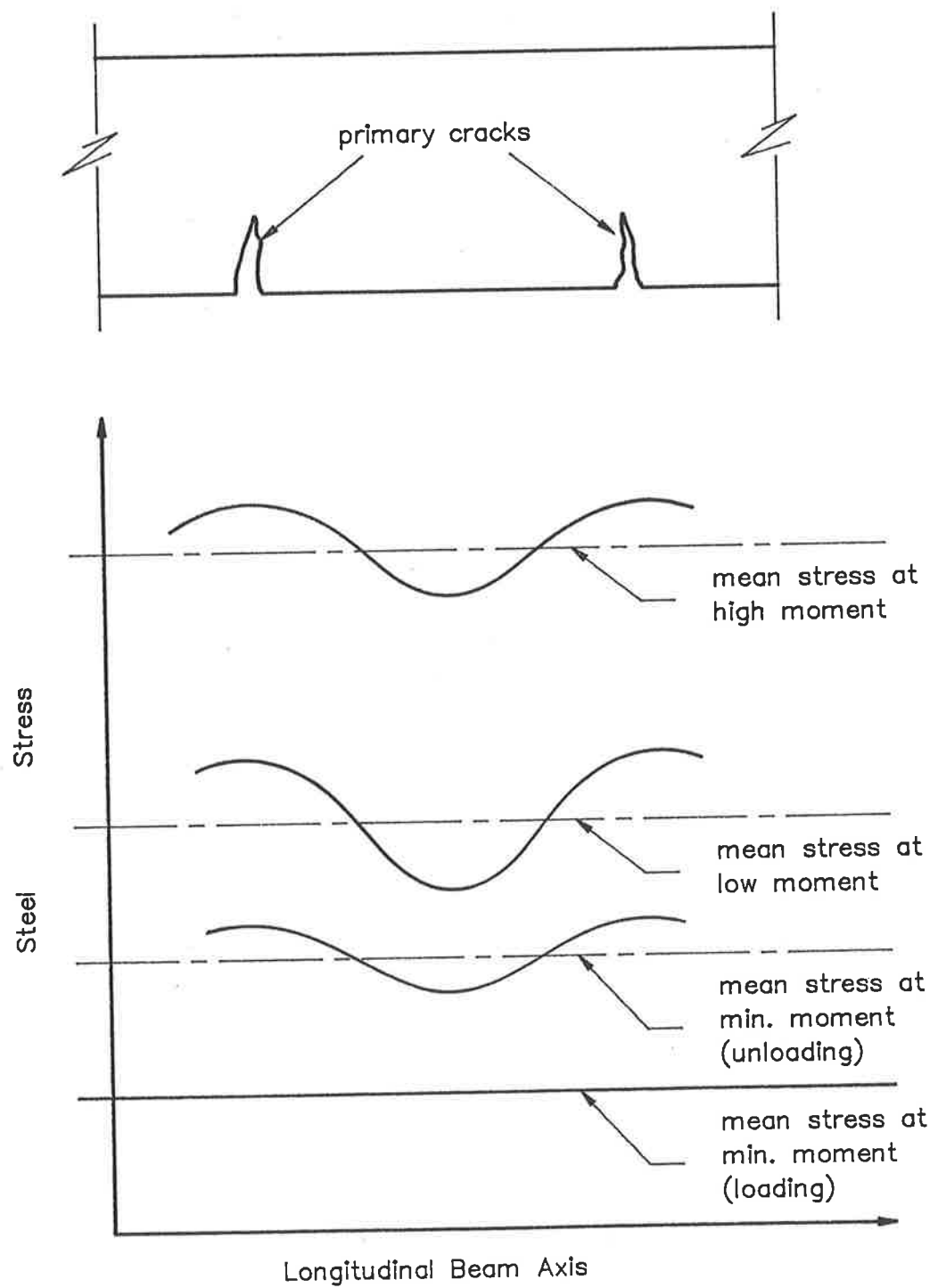


Fig. 7.3 Steel Stress Variation in Post-Cracking Region

Local Stresses During Unloading - First Cycle

The beam tests showed considerable differences between loading and unloading behaviour in the first load cycle. Stresses during unloading can therefore be expected to differ from those during loading. For fatigue prediction purposes, it is necessary to evaluate steel stresses at the minimum cyclic load (which is assumed to be lower than the initial cracking moment). Accurate estimations of stresses are important as beam fatigue life is dependent on stress range as well as on the maximum stress level. Earlier investigations have shown that the theoretical no-tension analysis underestimates deformations with errors in the order of 100 percent or more.

Test results show a definite increase in the mean steel stress at the end of unloading. This is illustrated in Fig 7.3 where the mean stress at minimum moment for the first load cycle in both loading and unloading directions are drawn. A uniform distribution is present in the loading direction and a non-uniform one in the unloading direction. There is uncertainty about the shape of this distribution. The stress variation is likely to be less than at maximum load with highest stresses at primary cracks and lowest in regions in-between.

Local Stresses in the High Moment Range - Repeated Loading

Test results in Chapter 3 have shown that increases in average steel stresses occur with repeated loading. There is also a slight reduction in beam stiffness indicating an increased steel stress range with cyclic loading. The steel stresses at various cycles of repeated loading hence need to be checked for accurate assessment of fatigue strength.

The generalised concrete effective force-moment relation (Fig 6.29) showed that the tensile capacity of concrete is virtually eliminated after many load cycles. Therefore, at maximum load it is expected that there would be little variation in steel stresses. Calculation of the local stresses then becomes straightforward and the simplified regional

analysis can be used to determine these stresses.

Local Stresses in the Low Moment Range - Repeated Loading

With no tension stiffening effects left, it might be expected that the stress variation along the member may be fairly uniform. However, there are the effects of crack mismatch and wedging to consider at low loads, and incomplete closure of cracks in certain sections will still cause variations in local stresses. On the other hand, the progressive deterioration of bond between concrete and steel may be so extensive that there is no facility for any stress transfer between concrete and steel, thus resulting in a uniform steel stress distribution along the beam. A uniform distribution is assumed and stresses at cracks can be approximated by the regional analysis.

Summary

Large differences between the local and average stresses are probably present in the early cycles of fatigue loading. At maximum moment, steel stresses at cracked sections are higher than the mean stress at maximum moment while at minimum moment there is some doubt as to whether the local stresses are actually higher or lower. After applications of many load cycles, it is expected that the tension stiffening effects will become insignificant resulting in a much more uniform stress distribution along the beam at high moment.

7.2 METHODS OF EVALUATING LOCAL STRESSES

Several methods of calculating stresses at the cracked section are investigated here. An approximate method, in connection with the proposed regional analysis (Chapter 6) is first examined. A more comprehensive alternative is then described, and modifications are suggested to provide better predictions.

7.2.1 Approximate Method

The steel stresses at a cracked section can be estimated by using the concrete effective force described in the proposed regional analysis. With no tensile stiffening effects at major cracked sections, the steel stress is equal to the mean stress and an additional component, assuming that the total tensile force remains constant along the beam.

The maximum steel stress is:

$$f_{s,max} = f_s + \frac{F_{eq}}{A_{st}} \quad (7.3)$$

where $f_{s,max}$ is the maximum steel stress, f_s the average steel stress and F_{eq} the average concrete stress defined in Chapter 6.

Where there are different layers or types of steel, the effective force must be distributed amongst them. A simple method is to proportion according to the steel areas. The maximum local stresses are:

for the i^{th} reinforcing steel element;

$$f_{is,max} = f_{is} + \frac{F_{eq}}{A_{is}} \frac{A_{is}}{(\sum_{NS} A_{is} + \sum_{NP} A_{ip})} \quad (7.4)$$

and for the i^{th} prestressing steel element;

$$f_{ip,max} = f_{ip} + \frac{F_{eq}}{A_{ip}} \frac{A_{ip}}{(\sum_{NS} A_{is} + \sum_{NP} A_{ip})} \quad (7.5)$$

Stress Calculations Using Approximate Method

Table 7.2 shows the calculated steel stresses for all six test beams in the first load cycles. The concrete effective force and average steel stresses were obtained from a regional analysis (described in Chapter 6). The calculated stresses are found to vary considerably from the average. At maximum load,

the local prestressing and reinforcing steel stresses are approximately 70 MPa higher (5 and 50 percent for strand and bar respectively) than the average steel stresses. This is also greater than the average reported variations of 32 MPa obtained from direct tension tests (Section 7.1.1). However, a comparison between the calculated stresses and values predicted by the simple no-tension theory shows good agreement, with slightly higher stresses from the approximate method. The differences are approximately 20 MPa (2 percent) for the prestressing steel and 5 MPa (3 percent) for reinforcing steel.

The procedure is not applicable at low loads as it does not account for increased steel stresses due to wedging effects.

Table 7.2 Local Stresses by Approximate Method
- First Load Cycle

Beam	Moment (KNm)	Average		Local		SCA*	
		f_p (MPa)	f_s (MPa)	f_p (MPa)	f_s (MPa)	f_p (MPa)	f_s (MPa)
SA1	43.07	1511	190.4	1591.8	271.2	1563	276.0
	13.00	1323	-47.9	-	-	1265	-97.9
SA2	38.03	1463	114.0	1543.2	194.1	1506	204.7
	13.00	1337	-46.0	-	-	1274	-87.7
SA3	43.07	1522	250.3	1554.6	282.8	1544	288.0
	13.00	1313	-12.8	-	-	1248	-83.0
SA4	38.03	1442	131.4	1516.5	205.1	1507	207.0
	13.00	1303	-45.4	-	-	1281	-76.5
PA1	38.03	1475	127.5	1537.2	189.7	1507	202.0
	13.00	1335	-49.3	-	-	1254	-114.9
PA2	38.03	1498	159.4	1532.4	193.8	1522	196.7
	13.00	1316	-69.4	-	-	1297	-85.0

* Simple Cracked Section Analysis

7.2.2 Strain Transformation Method

In the approximate method, the total tensile force is assumed to be constant throughout the member. This is not true as the stress neutral axis position varies, resulting in changes in the lever arm of the force resultants. The total tensile force is therefore not uniform in a constant moment region. The adequacy of the above method is thus dependent on the extent to which the tension force varies.

An alternative approach is to approximate the local strain distribution at the cracked section by a simplified piecewise-linear strain distribution which is related to the average strain by empirical parameters. To take into account initial prestressing and time-dependent effects, incremental strains are considered. This is defined as the difference between the total average strain and the reference strain, which is the strain at no any load just before loading. Fig 7.4a shows the average strain across the section at zero load, at cracking load and at a load greater than cracking. Compressive strain increments are observed in the concrete top-fibre strains while tensile strain increments are recorded in the concrete at the steel levels. The corresponding strain distributions at the cracked section are illustrated in Fig 7.4b.

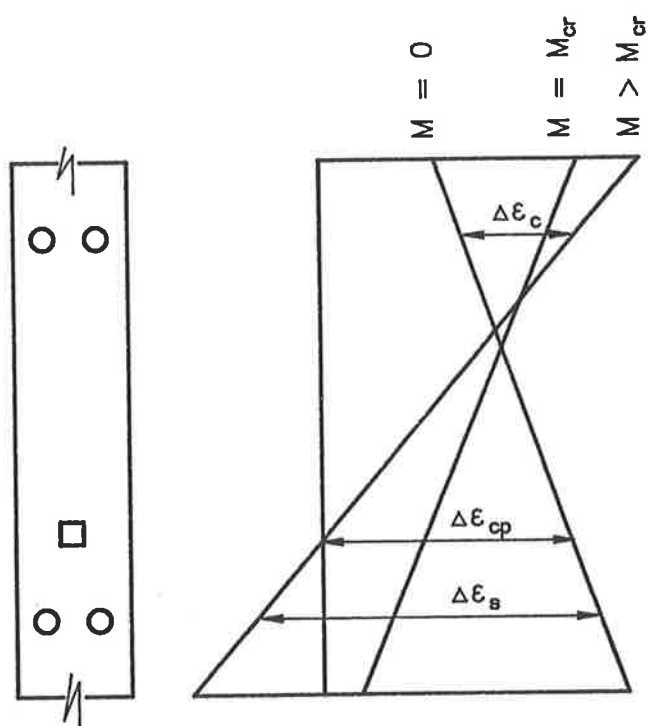
Relationship between Local and Average Incremental Strains

The local and average strain distributions are shown in Fig 7.5. The same method of section representation (described in Chapters 4, 5 and 6) is used. For concrete, the compressive incremental average strain for the j^{th} layer is expressed as:

$$\Delta \epsilon_j = \epsilon_j - \epsilon_{j0} \quad (7.6)$$

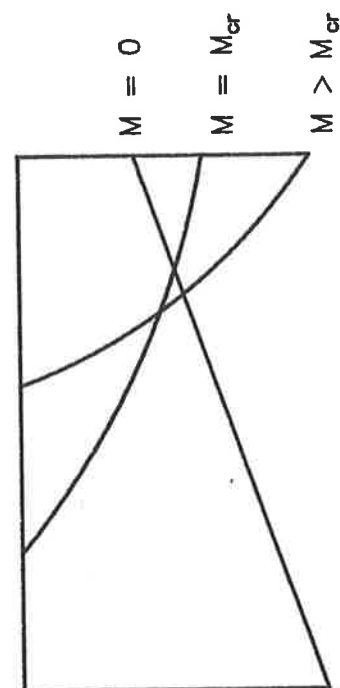
where ϵ_{j0} is the compressive average strain at zero load before loading, and ϵ_j is the compressive average strain at the applied load.

Assuming a linear relationship between average and local



Average Concrete Strain Distribution

Fig. 7.4a



Local Strain Distribution at Cracked Section

Fig. 7.4b

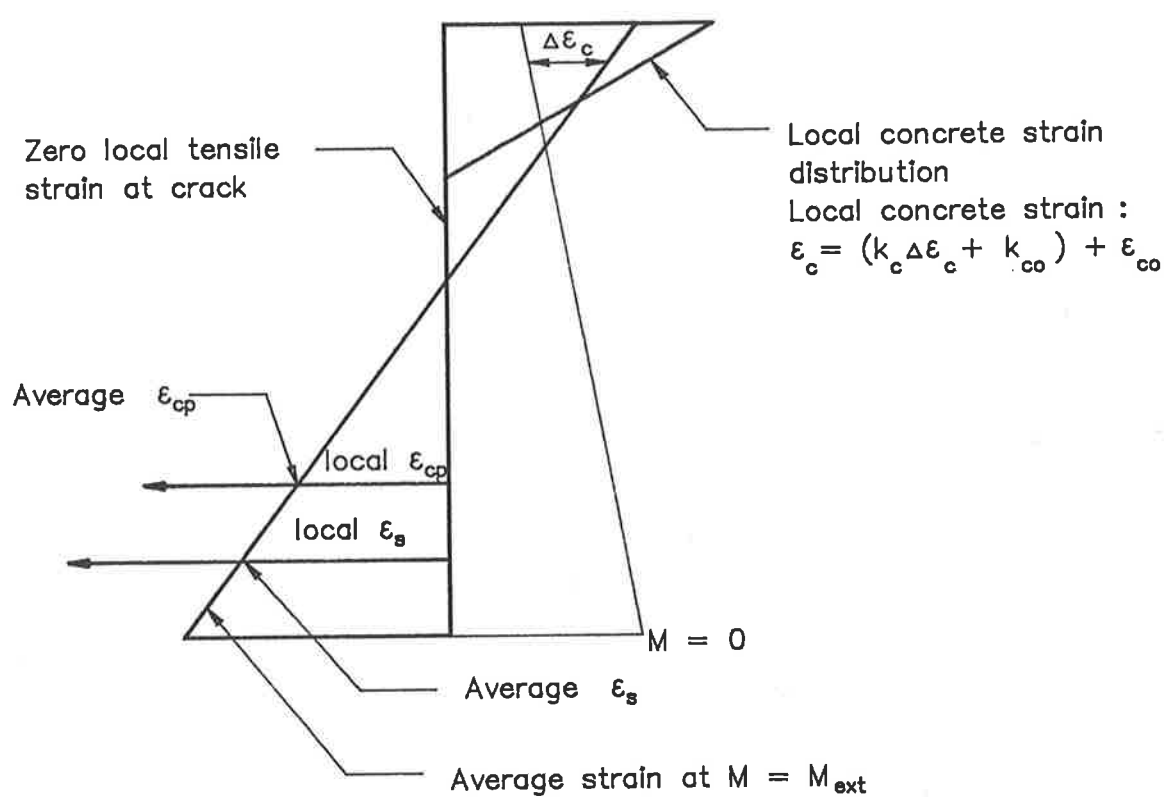


Fig. 7.5 Average and Local Strain Distributions

strains, the incremental local strain is expressed as:

$$\begin{aligned}\Delta \varepsilon_j &= k_c \Delta \varepsilon_j + k_{c0} \\ &= k_c (\varepsilon_j - \varepsilon_{j0}) + k_{c0}\end{aligned}\quad (7.7)$$

where k_c and k_{c0} are the concrete compression zone transformation factors.

For the steels similarly,

$$\Delta \varepsilon_{is} = k_s (\varepsilon_{is} - \varepsilon_{is0}) + k_{s0} \quad (7.8)$$

$$\Delta \varepsilon_{icp} = k_p (\varepsilon_{icp} - \varepsilon_{icp0}) + k_{p0} \quad (7.9)$$

where $\Delta \varepsilon_{is}$ and $\Delta \varepsilon_{icp}$ are the tensile incremental local strains for the i^{th} element of the reinforcing and prestressing steel respectively. k_s , k_{s0} , k_p and k_{p0} are the corresponding transformation parameters.

The compressive concrete strain at the cracked section for the j^{th} concrete slice is :

$$\begin{aligned}\varepsilon_j &= \Delta \varepsilon_j + \varepsilon_{j0} \\ &= k_c \varepsilon_j + (1 - k_c) \varepsilon_{j0} + k_{c0}\end{aligned}\quad (7.10)$$

Similar for the i^{th} reinforcing steel element,

$$\varepsilon_{is} = k_s (\varepsilon_{is} - \varepsilon_{is0}) + k_{s0} + \varepsilon_{is0} \quad (7.11)$$

And for the i^{th} prestressing steel element,

$$\varepsilon_{icp} = k_p (\varepsilon_{icp} - \varepsilon_{icp0}) + k_{p0} + \varepsilon_{icp0} \quad (7.12)$$

Computation of concrete and steel stresses is similar to the simple cracked section analysis. The usual material constitutive relations are used. Concrete tensile stresses are assumed to be insignificant.

Simplifying Assumptions

The above equations show 6 parameters, two for each of the component materials ie k_c and k_{c0} for concrete, k_s and k_{s0} for reinforcing steel ...etc. Simplifying assumptions are required to reduce the number of parameters.

Full strain compatibility is assumed between concrete and compression steel.

The term, k_{c0} , in the transformation equation influences the position of the local strain neutral axis. k_{c0} is set to 200 microstrains to approximate the actual strain neutral axis position.

In the tension zone, the relation between average and local strains is assumed to be similar for both reinforcing and prestressing steels. Hence,

$$k_p = k_s \quad (7.13)$$

$$k_{p0} = k_{s0} \quad (7.14)$$

Also, k_{s0} is assumed to be equal to k_{c0} .

$$(7.15)$$

$$k_{s0} = k_{c0} \quad (7.16)$$

$$= 200 \text{ } \mu\text{s}$$

With these simplifications, the number of parameters have been reduced to two, namely k_c and k_s .

Equilibrium conditions can now be written in terms of the two 'k' parameters and the average concrete strains. Average strains are obtained from experimental data and the parameters k_c and k_s can therefore be calculated together with the stresses at the cracked section.

Calculations were performed for all six beams using test data

in the postcracking loading and unloading range for the first load cycle. Results at minimum load conditions show that the cracked section stresses are much lower than average values. This anomaly is due to an inadequacy in the analysis which does not take into account the residual compressive stresses and inelastic concrete strain at the cracked section. The method is not applicable at low moments. The empirical 'k' parameters together with the ratios of the local to average strains are shown in Table 7.3. Table 7.4 shows the calculated stresses at the maximum and predominant moment. Steel stresses at minimum moment are assumed to approximate the average values.

Table 7.3 'k' Parameters for all Test Beams at Maximum Load

Beam	Moment (KNm)	Concrete k_c	r_j	k_s	Steels r_p	r_s
PA1	38.03	1.262	1.020	1.165	1.048	1.810
SA1	43.07	1.294	1.062	1.146	1.048	1.478
SA2	38.03	1.414	1.133	1.183	1.047	1.804
SA3	43.07	1.230	1.014	0.928	1.010	1.069
SA4	38.03	1.462	1.151	1.028	1.030	1.378

Note: r_j , r_s and r_p are the ratios of the local strain at the cracked section to the average strain for concrete, reinforcing and prestressing steel respectively.

Table 7.4 Cracked Section Stresses

Beam		Strain Transform.		Approximate Meth.	
		f_s (MPa)	f_p (MPa)	f_s (MPa)	f_p (MPa)
SA1	Max	265.0	1593.0	271.2	1591.8
	Pred	—	—	—	—
	Min	- 47.9	1323.0	-47.9	1323.0
SA2	Max	249.0	1584.0	297.0	1586.0
	Pred	180.8	1528.0	194.1	1543.2
	Min	- 46.0	1337.0	-46.0	1337.0
SA3	Max	255.6	1586.0	282.8	1554.6
	Pred	44.7	1426.0	188.0	1404.
	Min	- 12.8	1313.4	-12.8	1313.4
SA4	Max	246.9	1577.0	303.0	1588.0
	Pred	174.0	1524.0	207.0	1507.0
	Min	- 45.4	1303.0	-45.4	1303.0
PA1	Max	267.8	1601.9	277.0	1588.0
	Pred	185.2	1534.0	202.0	1507.0
	Min	- 49.3	1335.2	-49.3	1335.2
PA2	Max	189.4	1525.0	193.8	1532.4
	Pred	—	—	—	—
	Min	-69.4	1316.0	-69.4	1316.0

Good agreement is observed between results obtained from the 'k' factor analysis and the earlier approximate method. The difference in stresses for both steels was approximately 10 to 15 MPa (or 1 and 3 percent respectively). There appears to be little difference between the two methods. Both methods also do not work well at low moments.

7.3 FATIGUE LIFE CALCULATIONS

Three methods of fatigue predictions were considered. The first uses average strains to calculate stresses which are related to fatigue strength by material S-N curves. The simple no-tension theoretical analysis is employed in the second method to estimate critical stresses at the cracked section. The third approach attempts to make more accurate stress

calculations at various cycles of repeated loading.

7.3.1 Fatigue Calculations using Average Strains

Results of the beam fatigue tests show increases in overall deformations, especially bottom fibre deformations with increasing load cycles. The measured average bottom fibre strain of a typical beam (SA 1) at maximum and minimum loads under repeated constant amplitude loading is illustrated in Fig 7.6.

An increase in strain with load cycles (N) can be seen in Fig 7.7. The difference in strain at maximum and minimum load (and consequently stress range) also increases with N. Even though the applied loading is of a constant amplitude, the sustained stress cycles vary, with increases of up to 76 percent at maximum loads. Cumulative damage calculations should thus be employed in predictions of beam fatigue life.

Stress Calculations

Fatigue calculations are made on the assumptions of average strains over the life of the structure, and on the basis of the following assumptions:

- a) Stress is related to strain by normal constitutive relations
- b) Miner's hypothesis on cumulative damage is valid
- c) S-N curves for bar and strand are taken from Behan and Warner(1984) and Warner(1982) respectively

Fig 7.7 shows the variation of steel stresses with load cycles. The stress-cycle curve is divided into a number of small cycle periods. Stresses are assumed to be constant for each period. The partitions are made halfway between measurements points. The theoretical life of the steel for a certain stress level is simply obtained from the relevant S-N curves and the overall beam fatigue strength is computed using Miner's cumulative damage rule.

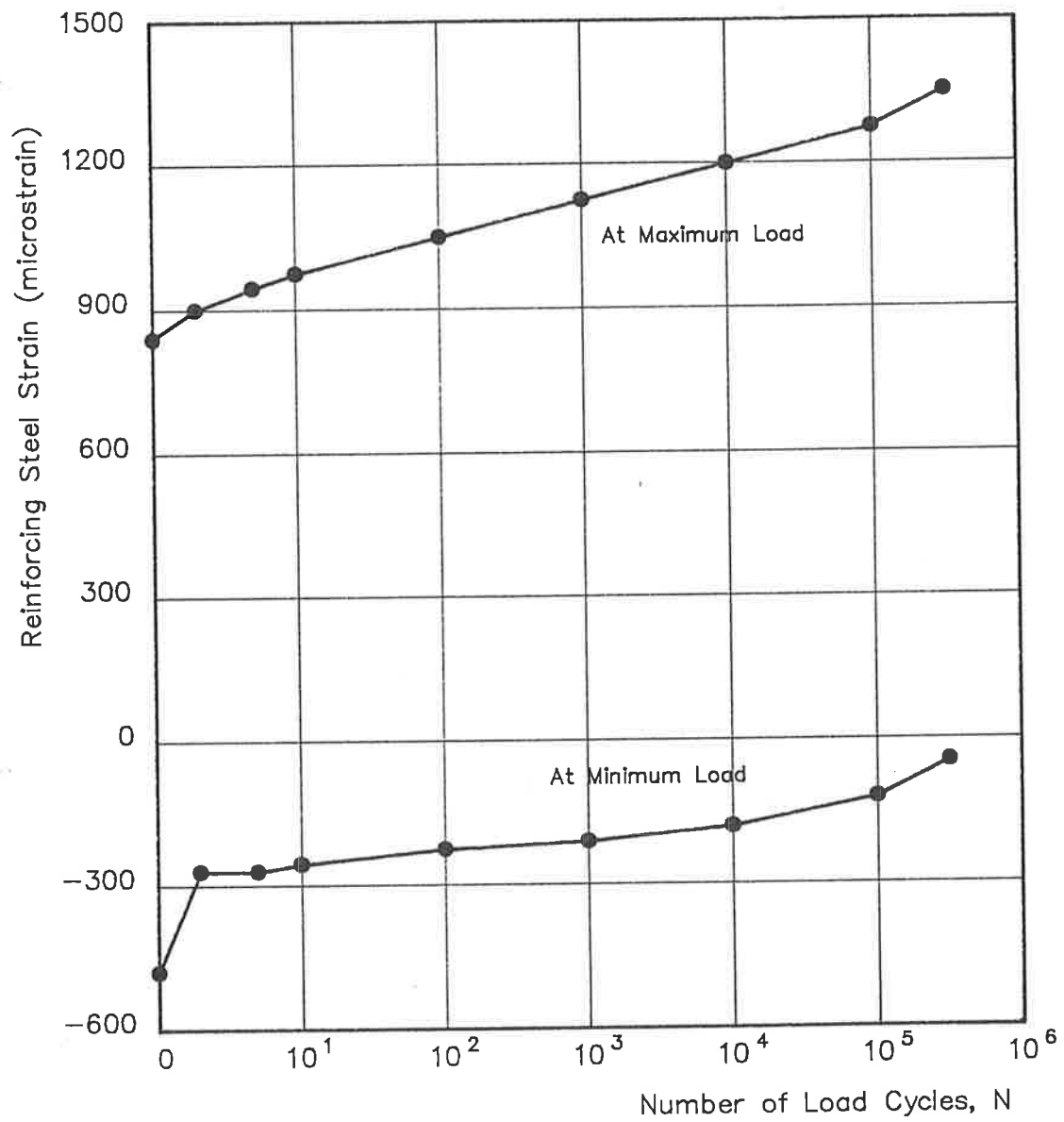


Fig. 7.6 Variation of Average Steel Strain with Repeated Loading for Beam SA1

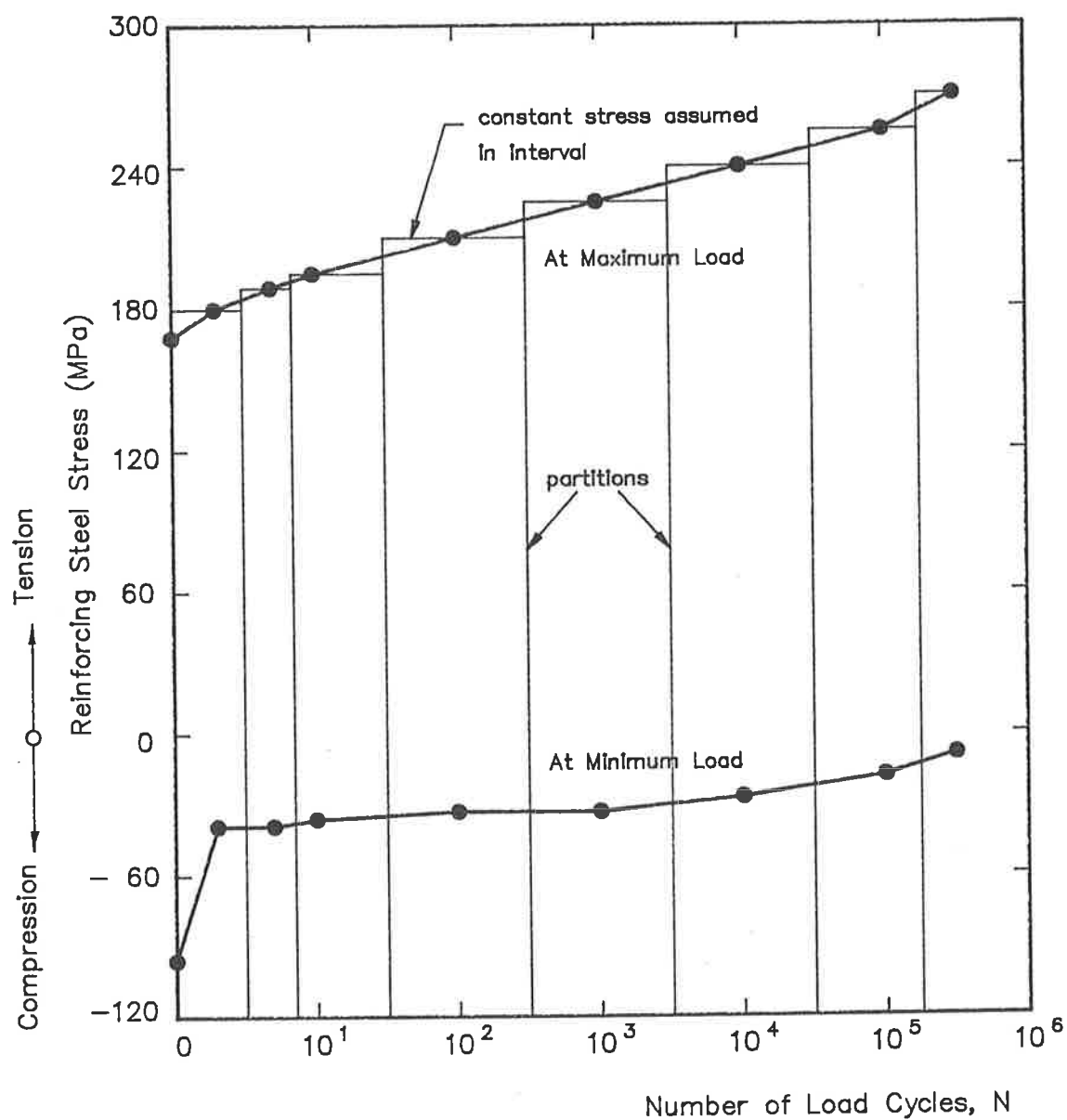


Fig. 7.7 Variation of Reinforcing Steel Stress with Repeated Loading for Beam SA1

Fatigue Calculations

Calculation of the total fatigue damage is carried out using Miner's rule. Because stress levels change with repeated loading even under constant amplitude loading, cumulative damage calculations are considered for all beams.

For constant amplitude loading,

$$\text{Damage, } D_{g.e} = \sum_{i=1}^{n_{exp}} \frac{n_i}{N_i} \quad (7.17)$$

where n_i is the number of cycles sustained at a certain stress level, N_i the predicted fatigue life, and n_{exp} the total number of partitions.

For variable amplitude loading,

$$\text{Damage, } D_{g.e} = \sum_{i=1}^{n_{exp}} \frac{n_{i.p.d}}{N_{i.p.d}} + \frac{n_{i.max}}{N_{i.max}} \quad (7.18)$$

where $n_{i.p.d}$ and $n_{i.max}$ are the number of cycles sustained at the predominant and maximum loads in the i^{th} partition, and $N_{i.p.d}$ and $N_{i.max}$ are the respective predicted lives.

The overall predicted beam fatigue life is then:

$$N_p = \frac{N_t}{D_{g.e}} \quad (7.19)$$

where N_p is the predicted life and N_t the total number of cycles sustained, i.e., for constant amplitude loading,

$$N_t = \sum_{n_{exp}} n_i \quad (7.20)$$

and for variable amplitude loading,

$$N_t = \sum_{n_{exp}} n_{i.p.d} + n_{i.max} \quad (7.21)$$

Table 7.5 shows the predicted and observed fatigue lives for the six test beams. They show an underestimation by a factor of 3 to 4 in the case of the reinforcing steel and by an even greater value for the prestressing steel. Hanson, Hulsbos and VanHorn (1970) reported underestimations of a factor of 2 for their prestressed concrete I-beams. Abeles et al (1974) obtained similar results. The use of average steel stresses in beam fatigue calculations therefore do not give good results.

Table 7.5 Predicted Fatigue Life using Average Stress

Beam	Steel	Observed N ⁺	Predicted N	Ratio = Pred/Obsv
SA1	Rebar Strand	348 —	1,017 ∞	2.92 —
SA2	Rebar Strand	692	3,880 23,450	— 33.88
SA3	Rebar Strand	— 845	1,727 3,765	— 4.46
SA4	Rebar Strand	506 —	2,119 16,960	4.19 —
PA1	Rebar Strand	F*	—	—
PA2	Rebar Strand	— 954	32,000 ∞	— ∞

Note: F* denotes flexural failure due to overload
 ∞ steel stress below fatigue endurance limit
 + Number of load cycles in thousands

The prediction errors could be caused by any of the following factors or by a combination of some or all of them:

- a) Underestimation of stresses
- b) Fretting effects and influences of concrete on steel resistance
- c) Inaccurate S-N curves

Underestimation of Stresses

This is likely to be the major cause of discrepancy. Variations between local and average stresses have been discussed in detail in Section 7.1.1 above.

Fretting Effects

Abrasion of concrete on steel may decrease fatigue resistance. This is most pronounced at major cracks where there is fretting between the jagged concrete surfaces and steel. Rigon and Thurlimann (1985) reported significant reductions in fatigue strength of post-tensioned beams with steel ducts. Fretting between the steel duct and prestressing wires was reported to be the primary cause of premature failure. The effect may not be as severe in pretensioned beams without steel ducts. Fretting between concrete and steel, and perhaps between the individual wires in the strand may still occur, but at a much lower level.

Inaccurate S-N curves

The material S-N curves used are applicable within certain stress levels and may not be suitable outside these limits. For instance, the stress range in the reinforcing steel extends from compression to tension in the beam tests, while Behan's curve applies only for bars experiencing slight compression to tension. In the case of prestressing steel, the minimum strand stress level employed by most researchers is approximately 60 percent (of ultimate) while the observed steel stress in the beam tests has a minimum level of about 70 percent. Fatigue strength decreases with increasing minimum stress for the same stress range and the use of Warner's curve outside the range tested may result in overestimations.

7.3.2 Fatigue Calculations using the Simple Cracked Section Analysis

The simple no tension analysis is used to approximate stresses at the cracked section. Table 7.6 shows the steel stresses at maximum, minimum and predominant values for the test beams. The stress ranges are also shown. Creep, shrinkage and initial prestress losses have been taken into account. The analysis does not take into account increases in the overall beam deformation with repeated loading. The stresses calculated are actually for the first load cycle. Even when cyclic creep of the compressive concrete is considered, the variation in stresses is not considerable and has negligible effects on the overall fatigue life of the member.

Table 7.6 Calculated Steel Stresses using Simple Analysis for the First Load Cycle

Beam		Applied Moment (KNm)	Tensile Stress (MPa)	Rebar Δ Stress (MPa)	Prestress Steel Stress (MPa)	Δ Stress (MPa)
SA1	Max	43.07	276.0	373.9	1563	298
	Pred					
	Min	13.00	- 97.9		1265	
SA2	Max	43.07	277.7	365.4	1566	292
	Pred	38.03	204.7	292.4	1506	232
	Min	13.00	- 87.7		1274	
SA3	Max	43.07	288.0	371.0	1544	296
	Pred	29.25	188.0	171.0	1384	136
	Min	13.00	- 83.0		1248	
SA4	Max	43.07	283.0	359.5	1568	287
	Pred	38.03	207.0	283.5	1507	226
	Min	13.00	- 76.5		1281	
PA1	Max	43.07	277.8	392.7	1568	314
	Pred	38.03	202.0	316.9	1507	253
	Min	13.00	-114.9		1254	
PA2	Max	38.03	196.7	282.3	1522	225
	Pred					
	Min	13.00	- 85.7		1297	

Table 7.7 gives the calculated (using Simple Cracked Section analysis) and observed beam fatigue lives. Good agreement in

fatigue prediction is shown, especially in the reinforcing steel. A slight overestimation of fatigue life is observed for the prestressing steel. The method appears to give good predictions, but this could be coincidental as the analysis does not take into consideration the progressive changes in the structural member under repeated loading. The presence of residual steel stresses caused by wedging action in the first load cycle is also ignored.

Table 7.7 Predicted Fatigue Life using Simple Analysis

Beam	Steel	Observed N ⁺	Predicted ¹ N	Predicted ² N
SA1	Rebar	348	365	335
	Strand	—	321	304
SA2	Rebar	—	845	805
	Strand	692	1,792	1,753
SA3	Rebar	—	1,000	995
	Strand	845	995	995
SA4	Rebar	506	780	724
	Strand	—	1,224	1,119
PA1	Rebar Strand	Flexural failure		—
PA2	Rebar	—	1,330	1,258
	Strand	954	10,000	10,000

Note: ¹ Cyclic creep effects ignored
² Cyclic creep taken into account
⁺ Number of load cycles in thousands

Comparisons between results obtained from the simple analysis and from beam tests have already been discussed in Section 5.1.3. With regard to stress conditions at the end of the first cycle, the test results show that:

- large residual steel stresses are observed;
- observed average steel stresses are higher now than in the loading direction;
- calculated local steel stresses (by simple no tension method) are lower than the observed, even allowing for concrete and steel hysteresis.

This is illustrated in Table 7.8 which shows the observed and theoretical steel stresses calculated by the simple no-tension method. The simple analysis hence appears to underestimate steel stresses at minimum load.

Table 7.8 Comparison of Average and SCA* Stresses for Beam SA 1 - First Load Cycle

	Reinforcing Steel (MPa)		Prestressing Steel (MPa)	
	average	SCA	average	SCA
Max stress	179.3	276.0	1519.7	1563
Min stress	- 42.4	- 97.9	1346.0	1265
Δ stress	221.7	373.9	173.7	298

* Simple cracked section analysis

At maximum loads, the estimation of steel stresses by the simple analysis may be more accurate. The stress conditions here are more straightforward compared to the unloading situation.

The simple no-tension method provides relatively good estimations of stresses at high moments and not as good estimations at lower moments. Effects due to repeated loading are also not considered. Despite these, the simple analysis appears to give accurate predictions. However, because the basic assumptions used by the simple analysis are invalid, the method in general can not be safely recommended as a design procedure for predicting fatigue. But it can be useful as a simple, preliminary check on fatigue life by the designer.

7.3.3 Proposed Fatigue Prediction Method

Discussions on the local stress conditions at various stages of static and repeated loading have been made in Section 7.1.2. The following assumptions, derived from the above

discussions, are made:

- a) Local steel stresses are significantly higher than average stresses at high moments in the early stages of repeated loading.
- b) The approximate analysis will be used to calculate stresses in the high moment range.
- c) Steel stresses at minimum moment are relatively uniform in the early stages of loading. Local stresses are then equal to average stresses which can be obtained from the regional analysis.
- d) After many repetitions of loading, steel stresses along the member are relatively constant. Local stresses are thus equal to average stresses at both high and low moments.

Table 7.9 shows the calculated local stresses for the first and N^{th} load cycle.

Table 7.9 Local Stresses by Approximate Method
- First Load Cycle

Beam	Moment (KNm)	1 st Cycle		N^{th} Cycle	
		f_p (MPa)	f_s (MPa)	f_p (MPa)	f_s (MPa)
SA1	43.07	1591.8	271.2	1568	286.5
	13.00	1323.0	-47.9	1347	-15.8
SA2	43.07	1586.0	297.0	1601	344.9
	38.03	1543.2	194.1	1571	273.0
	13.00	1337.8	-46.0	1380	8.3
SA3	43.07	1554.6	282.8	1582	354.6
	38.03	1404.0	188.0	1552	276.0
	13.00	1313.0	-12.8	1326	5.9
SA4	43.07	1588.0	303.0	1570	332.6
	38.03	1516.5	205.1	1547	277.6
	13.00	1303.0	-45.4	1324	-17.5
PA 1	43.07	1588.0	277.0	1570	284.4
	38.03	1537.2	189.7	1548	207.0
	13.00	1335.0	-49.3	1345	-35.5
PA2	38.03	1532.4	193.8	1541	299.8
	13.00	1316.0	-69.4	1335	-44.5

The theoretical and observed fatigue lives are shown in Table 7.10 below. Two separate calculations are made, the first using the first cycle stresses and the second using the N^{th} cycle stresses. Cumulative damage considerations are ignored at this stage. Average values of the two predictions are also listed below. Both methods provided overestimates of fatigue strength, although the use of the N^{th} cycle stress show slightly better predictions. Generally, prediction of the rebar fatigue life is overestimated by a factor of about 1.5. Larger errors are observed for the prestressing steel, some predictions show infinite fatigue lives in spite of fractures occurring in the tendons at less than a million cycles.

Although the discrepancy between the predicted and observed fatigue lives may appear to be considerable, the error is not great, bearing in mind the flat shape of the material $S - N$ curves in the stress ranges near the endurance limit. In this region, the fatigue life is very sensitive to stress variations, and a change of only 3 to 4 percent increase in stresses is sufficient to cause a corresponding increase in fatigue life of 200 percent. Table 7.11 shows the level of stress increases required to incur fatigue failure in the test beams at the observed number of load cycles. On the average, only a 4.6 percent change in the stress level is needed to provide good agreement between the theoretical and observed results.

Table 7.10 Predicted Fatigue Life

Beam	Steel	Observed N ⁺	Predicted ¹ N	Predicted ² N	Average N
SA1	Rebar Strand	348 —	732 927	852 ∞	792 ∞
SA2	Rebar Strand	692	1,336 ∞	986 ∞	1,161 ∞
SA3	Rebar Strand	— 845	1,836 ∞	741 10,970	1,289 10,970
SA4	Rebar Strand	506 —	926 1,294	679 ∞	820 1,294
PA1	Rebar Strand	Flexural failure		—	
PA2	Rebar Strand	— 954	1,701 ∞	507 ∞	1,104 ∞

∞ Infinite fatigue life

¹ Using first cycle stresses

² Using Nth cycle stresses

+ Number of load cycles in thousands

Table 7.11 Sensitivity Analysis of Fatigue Life

Beam	Steel stress (MPa)	Δ Stress (MPa)	Percent Δ %
SA1	271.2 — 47.9	13.6 2.4	5
SA2	1586.0 1543.2 1337.8	63.4 61.7 —	4
SA3	1554.6 1404.0 1313.0	62.2 42.1 —	4
SA4	303.0 205.1 — 45.4	24.2 16.4 —	8
PA2	1532.4 1316.0	30.6 26.0	2

The above calculations show that accurate deterministic predictions of fatigue life are difficult in view of the inaccuracy in evaluating steel stresses, variability in the material properties, inherent scatter of fatigue data and the 'flatness' of the S-N curves. A probalistic method of analysis is therefore required to allow for these effects. However, the consistent overestimation of fatigue life suggests the possibility of some steel fretting. The simple no-tension analysis appears to be suitable for use as an approximate method for checking beam fatigue life.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

This study of the behaviour of partially prestressed concrete members under fatigue loading has concentrated on two major areas, ie. beam behaviour under repeated loading and fatigue life prediction. The major contributions in these areas are described below.

Beam Behaviour and Deformations

The large inelastic changes which occur in the stress and strain conditions in a partially prestressed concrete beam prior to loading were found to have an important influence on the post-cracking behaviour. They need to be allowed for in any accurate analysis of structural behaviour. These effects, caused mainly by the time-dependent phenomena of creep and shrinkage, influence conditions in a beam to such an extent that flexural cracking is likely to take place when the strain in the concrete is still compressive. A computer-based analytic procedure has been developed to take these factors into consideration. The results obtained are in good agreement with test observations.

Significant permanent deformations occur during the loading and unloading cycles in the post-cracking range. While such deformations are maximum in the first load cycle, they occur progressively with repeated loading, albeit at a decreasing rate. The major cause of these permanent deformations at low load has been found to be non-closure of flexural cracks. This is considered to occur as a consequence of the mis-match of crack surfaces and the formation and inclusion of debris in the cracks. The resulting effect is a form of wedging action which leads to additional compressive stresses in the concrete and additional tensile stresses in the reinforcing and

prestressing steels.

Increased deformations occur not only at low loads but also at higher loads. The main cause of increased deformation at high loads is attributed to the deterioration of the concrete tensile stiffening effect, brought about by the progressive microcracking of concrete and accompanying loss of tensile force-transfer capability between steel and concrete under repeated loading.

In addition to these factors, there are other secondary influencing mechanisms which act throughout the entire load range. Cyclic creep of concrete is one such factor. Comparison of test and theoretical results also suggests that cyclic creep and relaxation of prestressing and reinforcing steel may also be possible secondary factors.

Previously reported methods of analysis have not adequately treated these inelastic changes in beam deformations under repeated loading. A method of analysis has therefore been developed which takes the physical mechanisms into account. The stress-dependent, non-linear material behaviour and initial time-dependent effects are also considered. Accurate predictions of deformations for the first and subsequent load cycles have been obtained.

Fatigue Resistance

The fatigue strength of partially prestressed concrete beams is governed by the fatigue characteristics of the reinforcing and prestressing steel. Fatigue of the concrete is not considered to be a problem under realistic load conditions. Fracture of the steel is likely to take place at the primary concrete cracks. Bar fracture tends to occur under severe loading while wire fracture is more likely under less severe load conditions. A combination of bar and strand fracture is also feasible.

Beam fatigue life was estimated by evaluating steel stresses and relating them to fatigue life by in-air material S-N

curves reported in the literature. Predictions based on measured average steel stresses were found to produce overestimates by a factor of 3 or more. Calculations made with more precise estimates of local steel stresses at cracked sections fared slightly better with overestimations usually by a factor of approximately 2 to 3.

This error is not great, bearing in mind the flat shape of the S-N curves near the endurance limit. In this stress range, the fatigue life is very sensitive to stress variations, and a change of only 5 percent in stress is sufficient to cause a corresponding change in fatigue life of 200 percent. Accurate deterministic forecasting of fatigue life is thus difficult, if not impossible. However, inaccuracy in stress evaluation and variability in fatigue data may not be the sole cause of error. The consistent overestimation of fatigue life suggests a causal effect. Fretting may be partly responsible for the discrepancy in prediction.

On the other hand, good predictive results were obtained by the use of the simple no-tension analysis to estimate cracked section stresses. For approximate fatigue life calculations, this method appears to be suitable.

The test beams sustained between 300,000 to a million cycles of what can be considered to be significant overloads (relative to service loads) where the maximum cyclic load was 77 percent of the flexural strength, with load ranges varying between 45 and 54 percent. This suggests that partially prestressed concrete members can be designed successfully to resist fatigue loading, provided the stress range is kept within normal code limits.

8.2 Recommendations for Further Research

Further research into the following areas are recommended:

- a) Additional beam fatigue tests on similar and different sections to verify the present test results, and to

provide additional data for the employment of statistical methods to account for the inherent variability of concrete and scatter in steel fatigue data.

- b) Experimental verification of the force-moment relation, used in the prediction of beam deformations, in the high overload region. The relation has only been checked against test results for load ranges up to 77 percent of ultimate.
- c) Investigation into the behaviour of the complex concrete 'tensile' zone in flexural members by fatigue testing of uniaxial concrete prisms containing tensioned and untensioned steels, and plain concrete specimens with stress ranges spanning both compression and tension.
- d) Theoretical evaluation of local steel stresses along flexural members under first cycle and repeated loading taking into account the physical mechanisms influencing beam behaviour, and experimental measurements of local steel strains by improved strain-gauging methods.
- e) In-air and 'in-concrete' fatigue tests of reinforcing steel and prestressing tendons, with emphasis on measuring the cyclic creep effects in the steel. To date, virtually no work has been carried out on the cyclic creep effects of steel. Additional in-air fatigue data is also needed for prestressing tendons in the practical higher load ranges with minimum stresses of about 70 percent of yield strength.
- f) Probabilistic methods of analysis to allow for variability in the material properties, local stress variations and the 'flatness' of the S-N curves. Fatigue calculations have shown that accurate deterministic forecasting of beam fatigue life is almost impossible.
- g) Investigation into the fatigue response of partially prestressed post-tensioned members, which is made more complicated by the presence of tendon ducts, grout, draped

cables and anchorage zones. The possibilities of significant fretting and steel corrosion effects need to be considered.

- h) Study of the behaviour of members subjected to a combination of shear, torsion and flexure under repeated loading.

Despite the need for much further work, the investigation has presented a qualitative description of the different mechanisms which influence a flexural member under repeated loading. A theoretical non-linear method of analysis has been developed to predict beam behaviour in the first and subsequent load cycles, taking into account these mechanisms as well as the time-dependent effects prior to loading. Deterministic predictions of beam fatigue life are difficult and the simple no-tension analysis is found to be just as good as more sophisticated methods. Test results indicate that partially prestressed concrete members can be designed to resist fatigue. While this investigation on the behaviour of partially prestressed flexural members under fatigue loading is by no means comprehensive, the findings nevertheless provide a basis for predicting both the behaviour and deformations under repeated loading, and the fatigue life of partially prestressed members.

APPENDIX A

Appendix A contains detailed individual fatigue test results of the 6 test beams. They are listed in the following order:

<u>Figures</u>	<u>Description</u>
Figs A1 - A6	Moment-Curvature Relations
Figs A7 - A12	Load-Deflection Curves
Figs A13 - A18	Concrete Strain-Load Cycle Relations
Figs A19 - A24	Curvature-Load Cycle Relations
Figs A25 - A30	Central Deflection-Load Cycle Relations

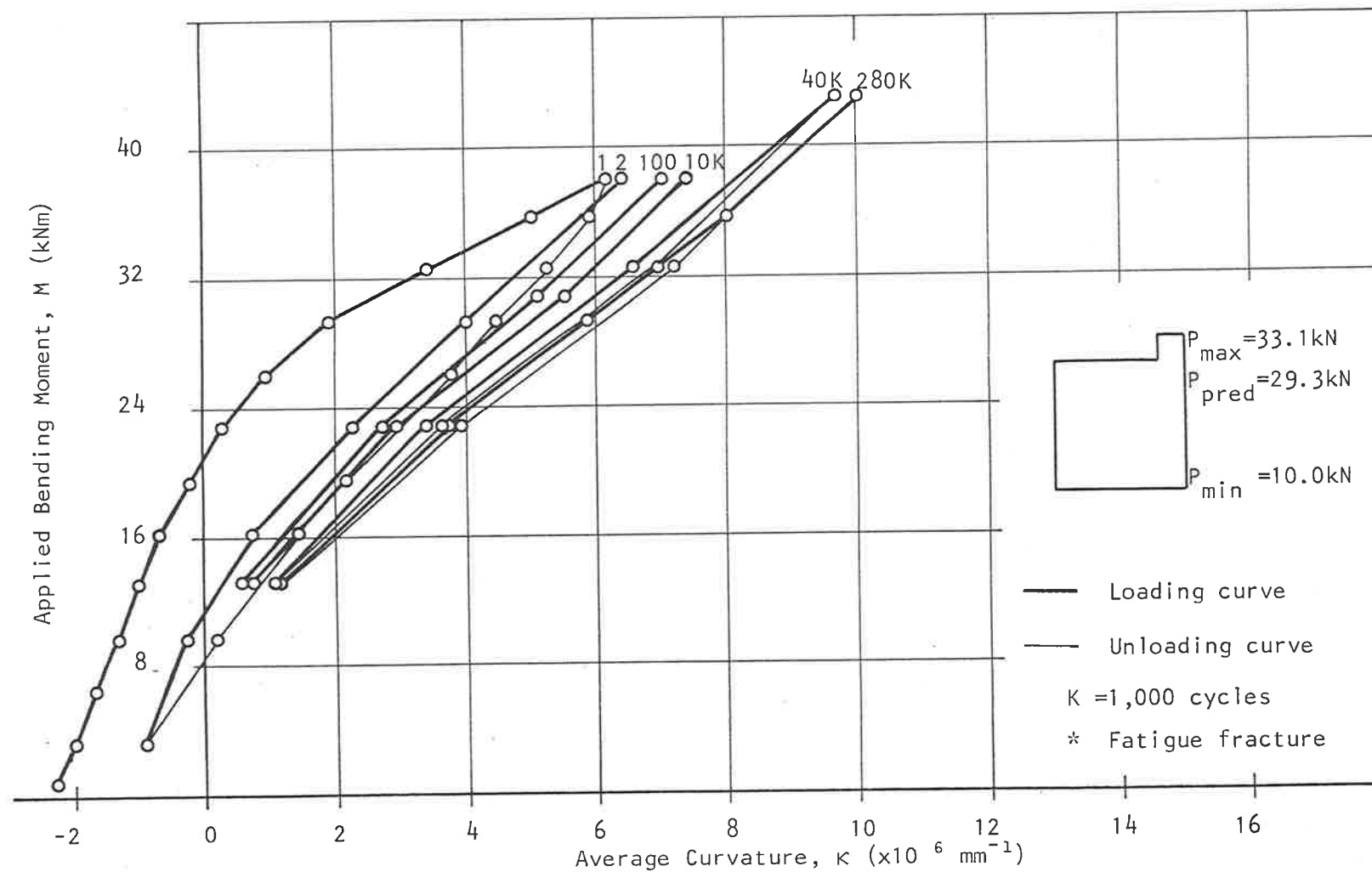


Fig A1 MOMENT - CURVATURE RELATIONS FOR BEAM PA1

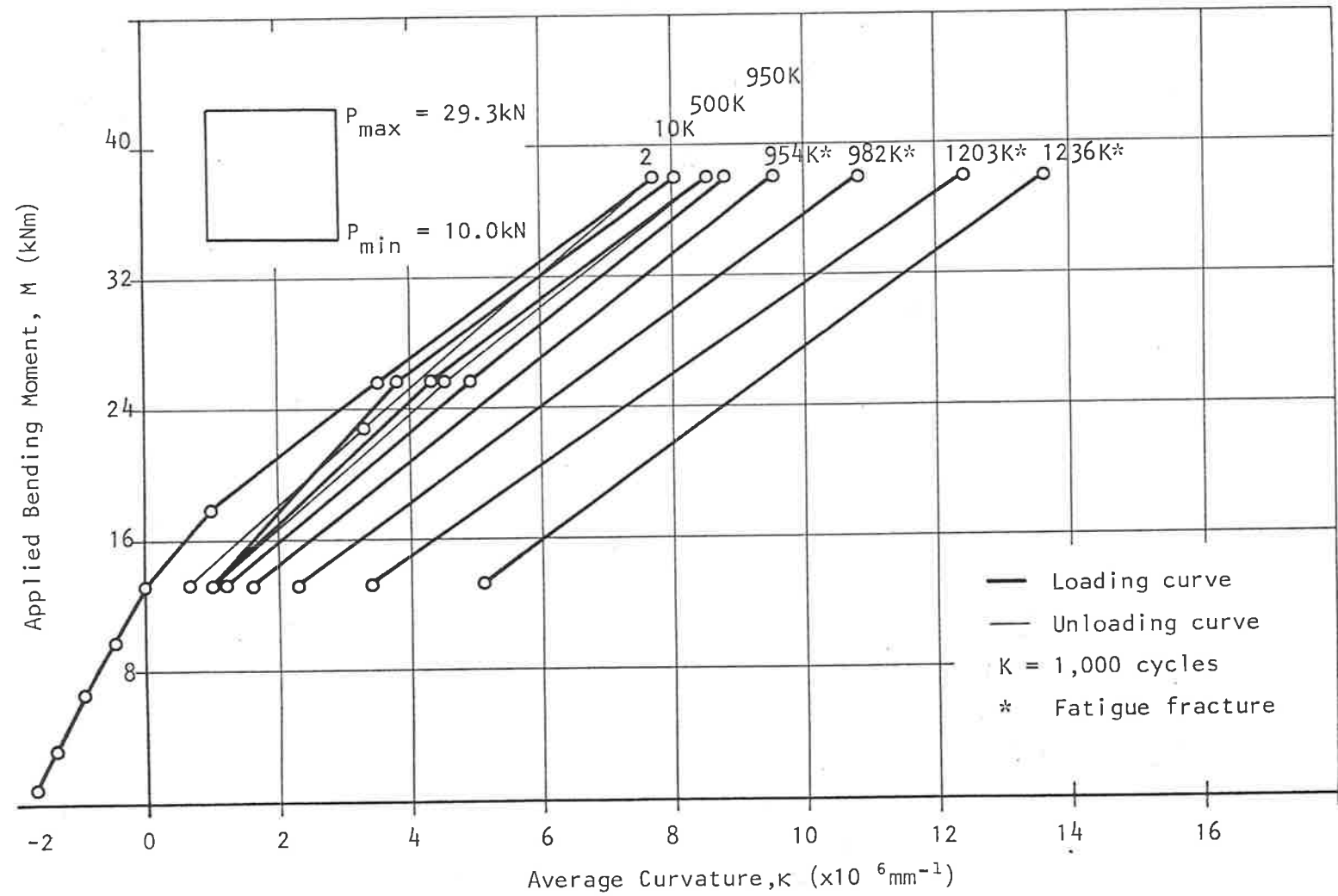


Fig A2 MOMENT - CURVATURE RELATIONS FOR BEAM PA2

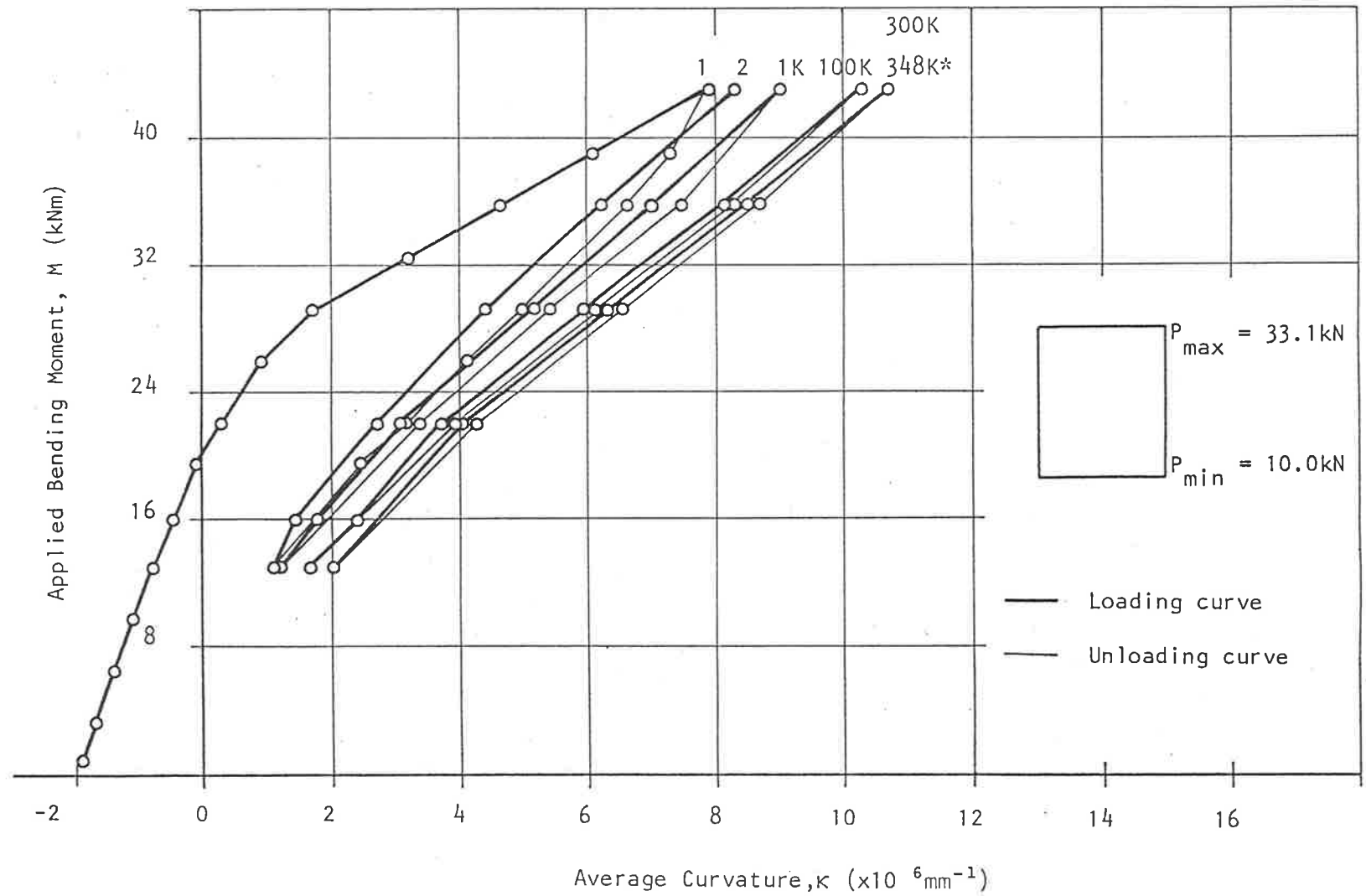


Fig A3 MOMENT - CURVATURE RELATIONS FOR BEAM SA1

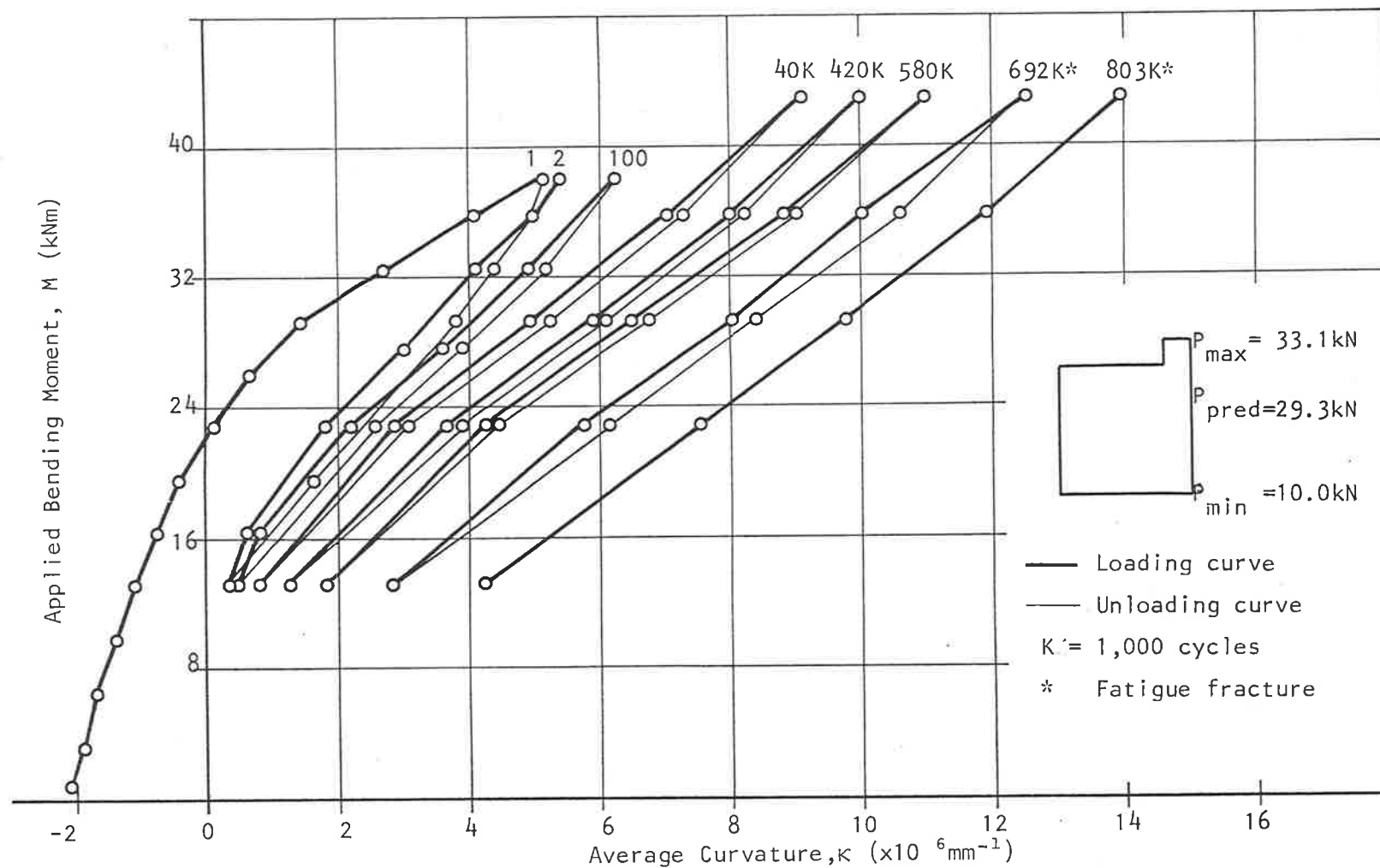


Fig A4 MOMENT - CURVATURE RELATIONS FOR BEAM SA2

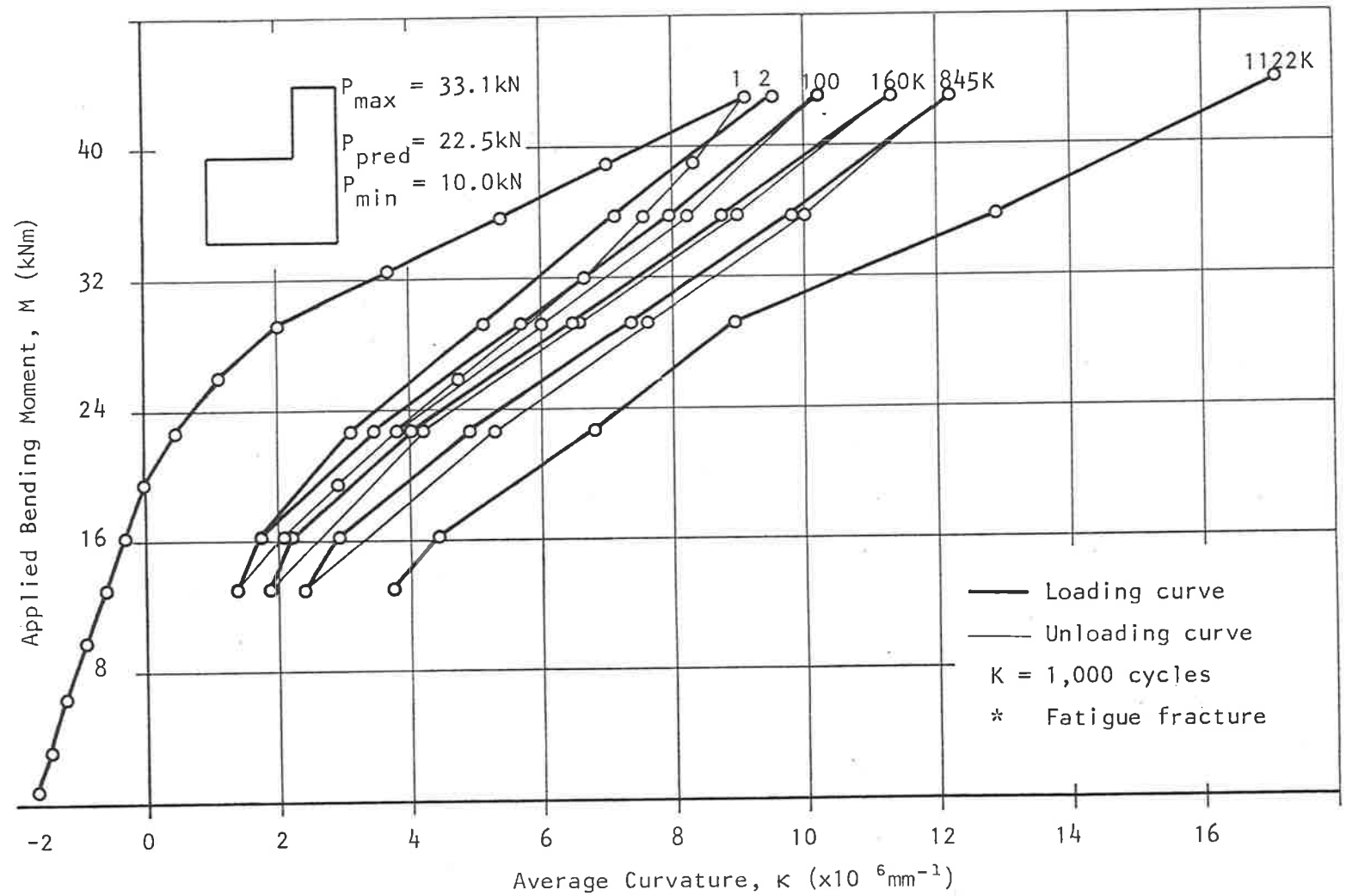


Fig A5 MOMENT - CURVATURE RELATIONS FOR BEAM SA3

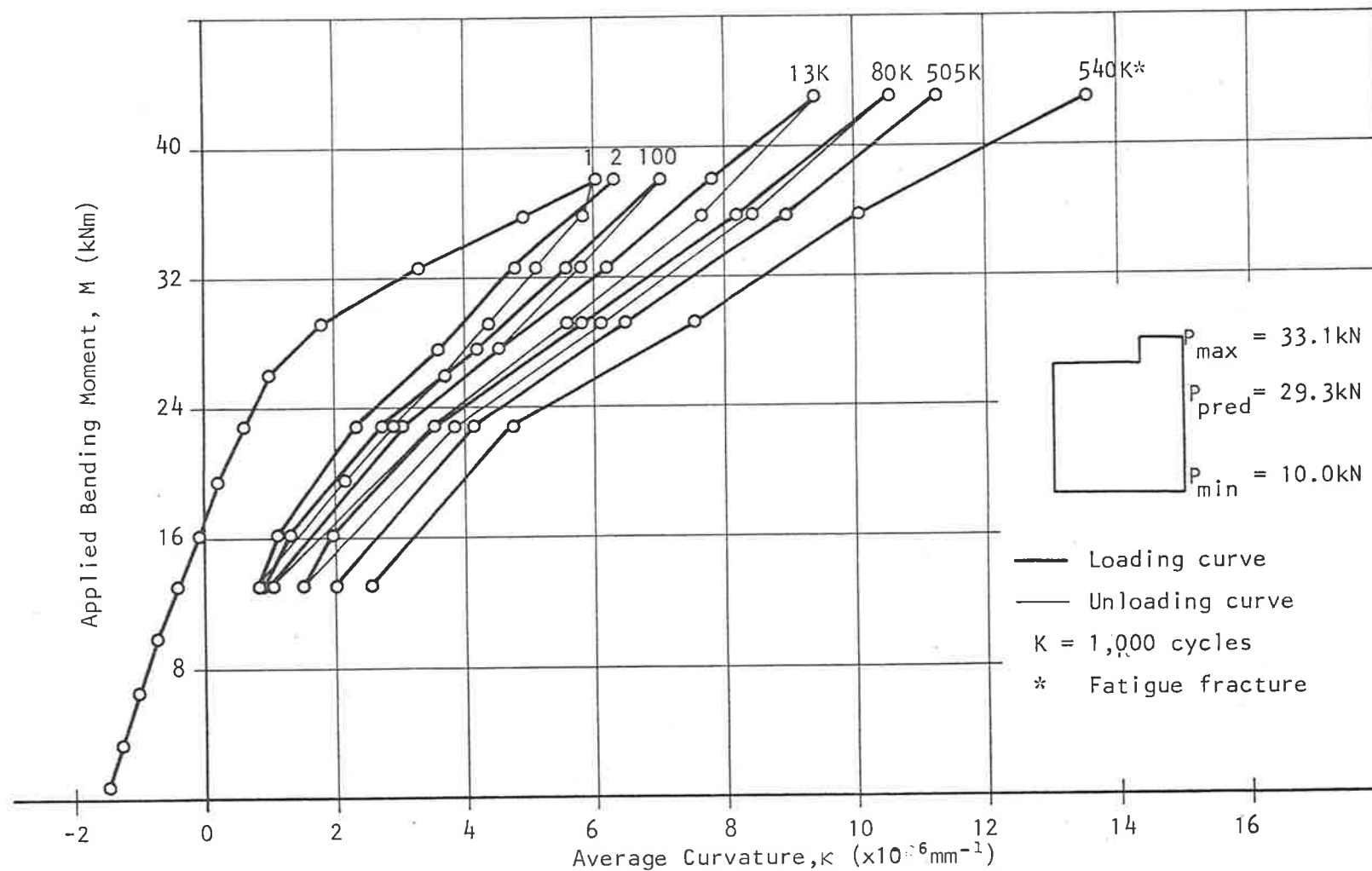


Fig A6 MOMENT - CURVATURE RELATIONS FOR BEAM SA4

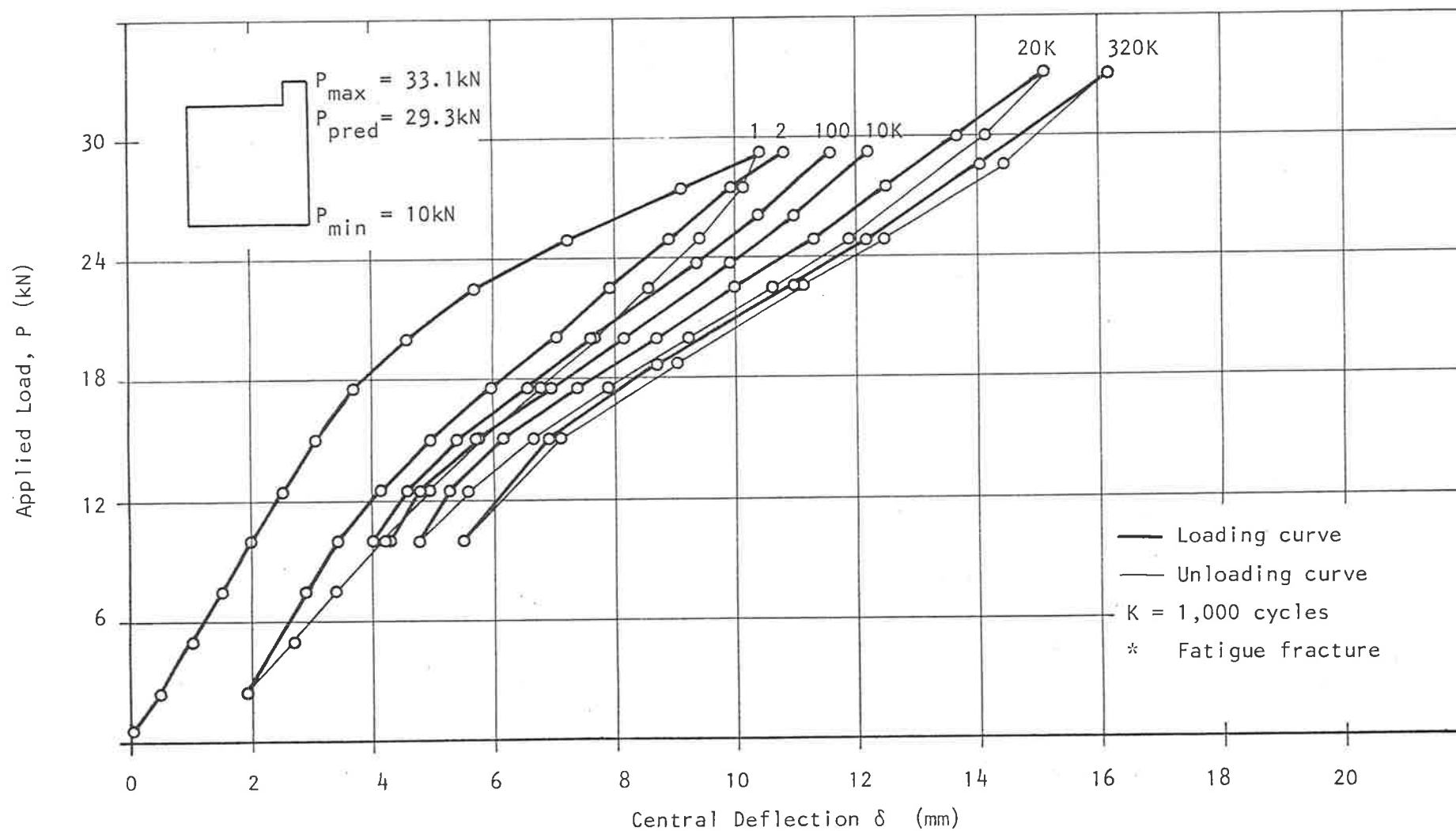


Fig A7 LOAD - DEFLECTION CURVE FOR BEAM PA1

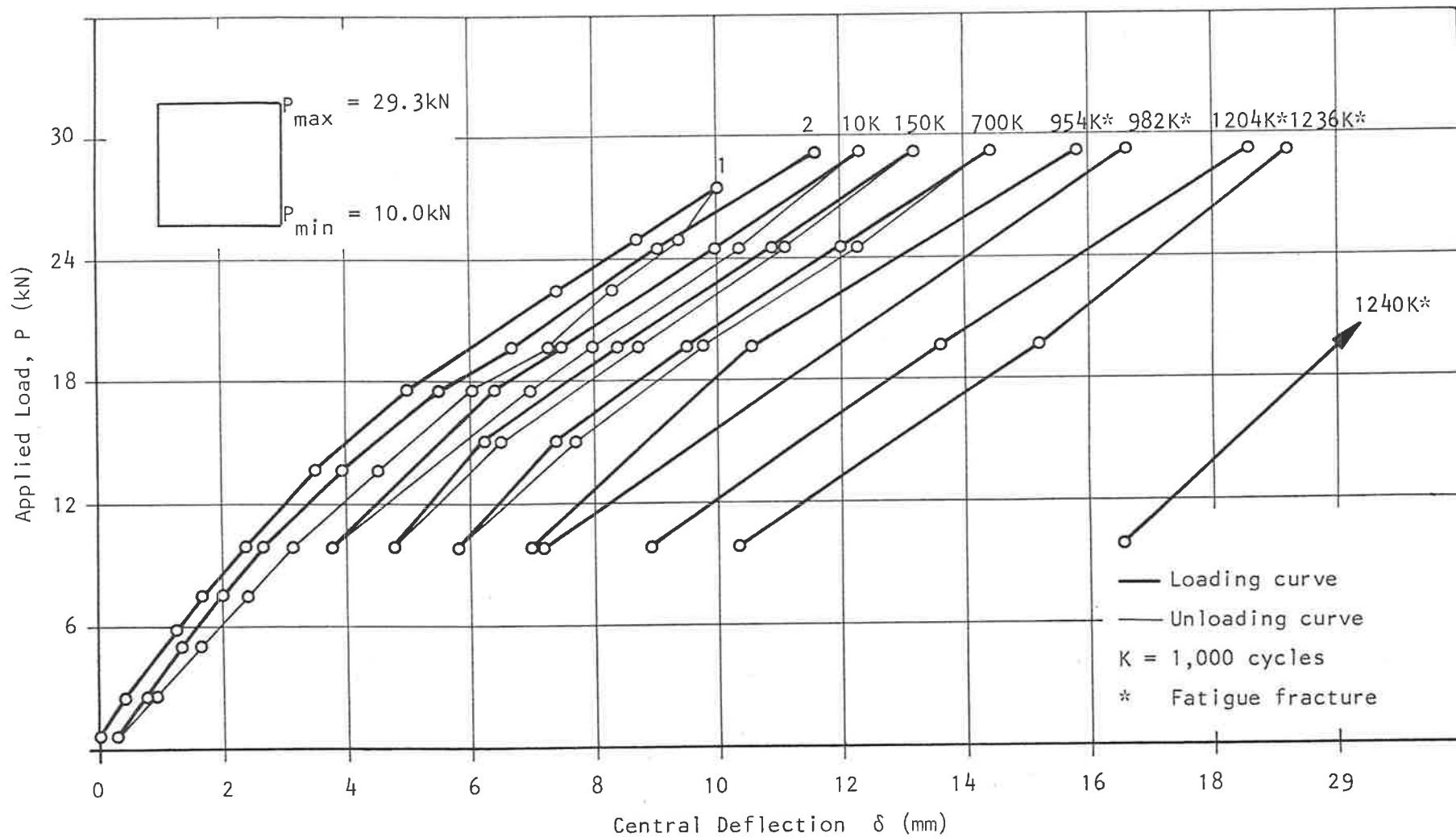


Fig A8 LOAD - DEFLECTION CURVE FOR BEAM PA2

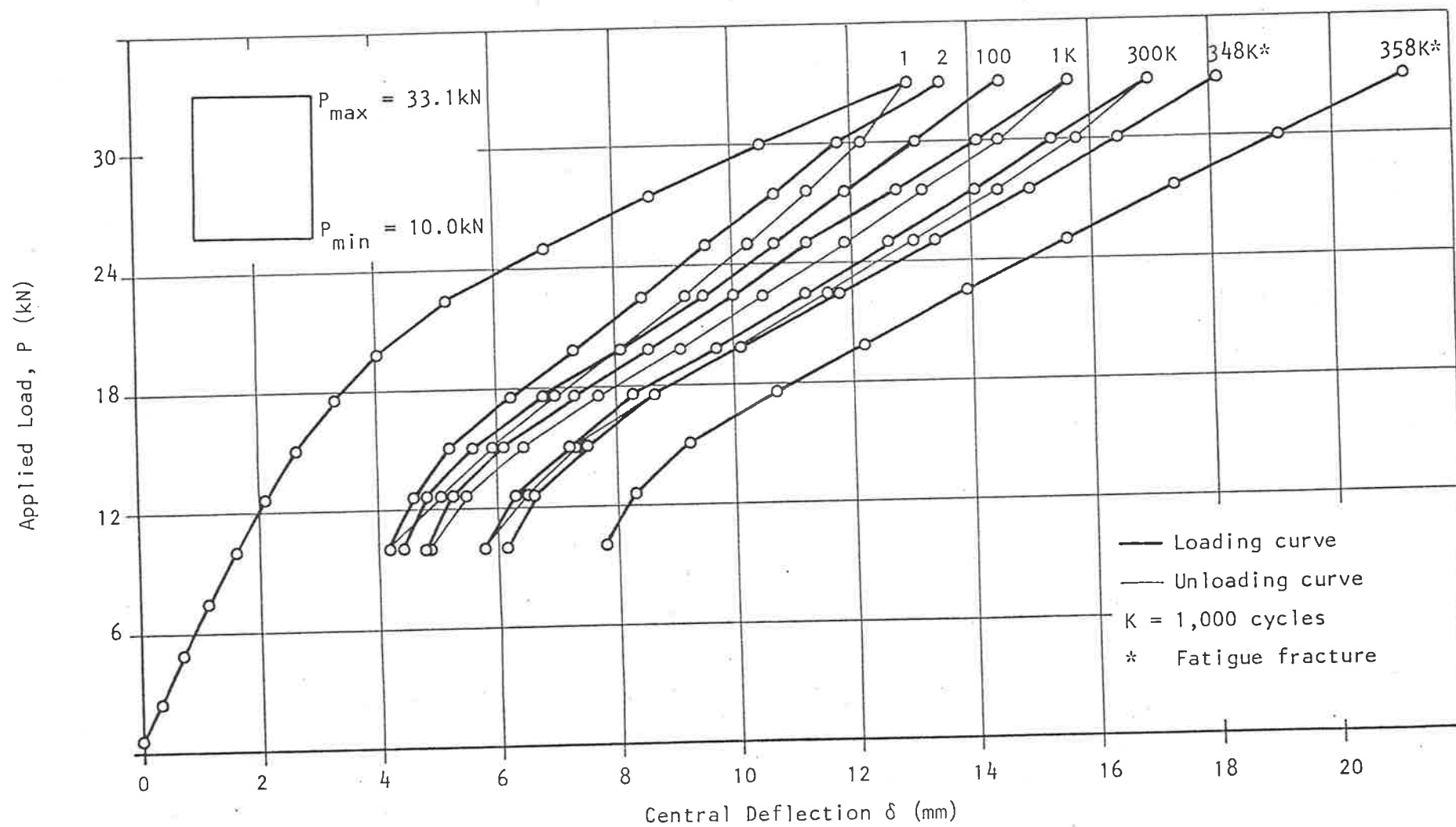


Fig A9 LOAD - DEFLECTION CURVE FOR BEAM SA1

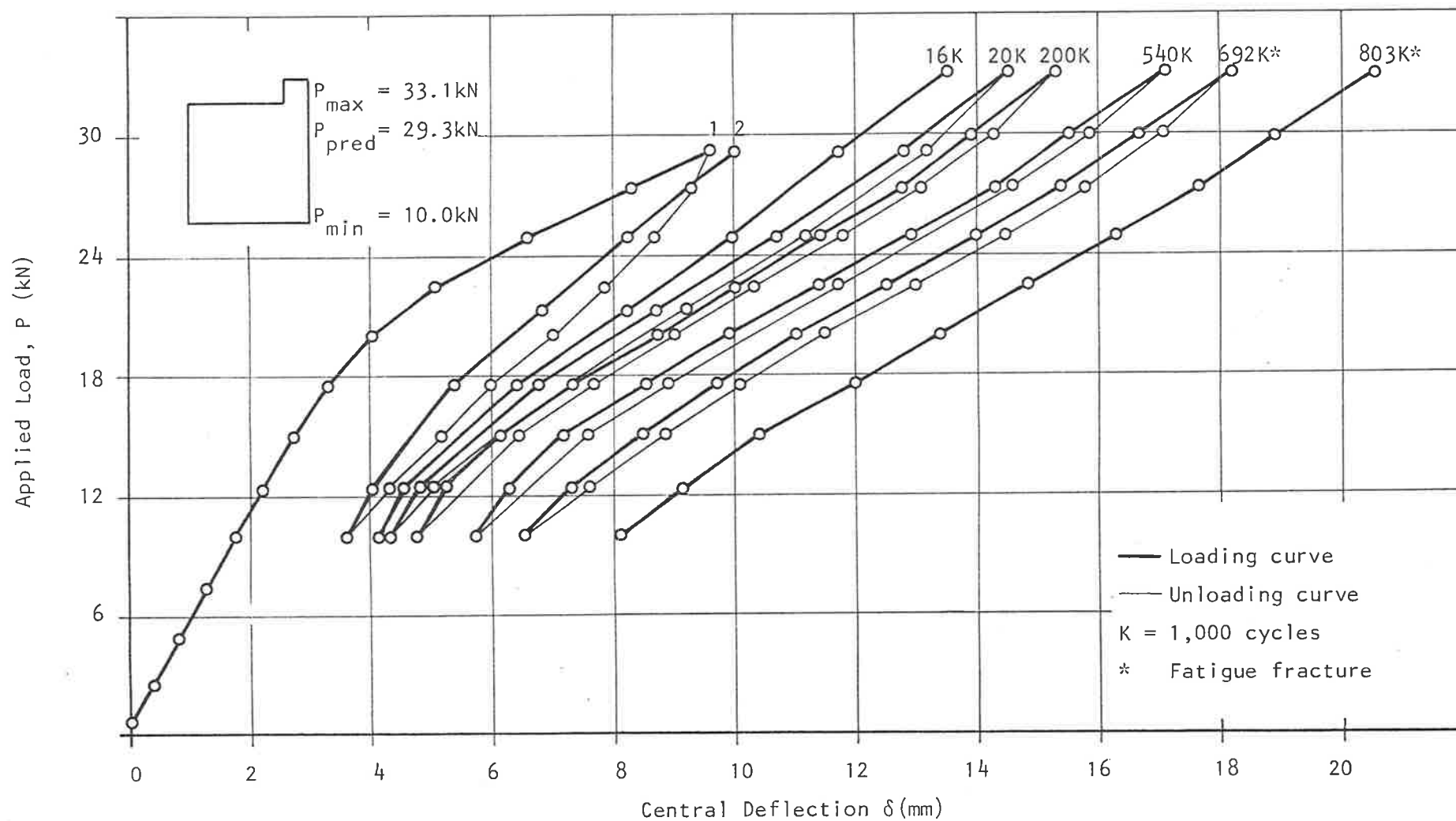


Fig A10 LOAD - DEFLECTION CURVE FOR BEAM SA2

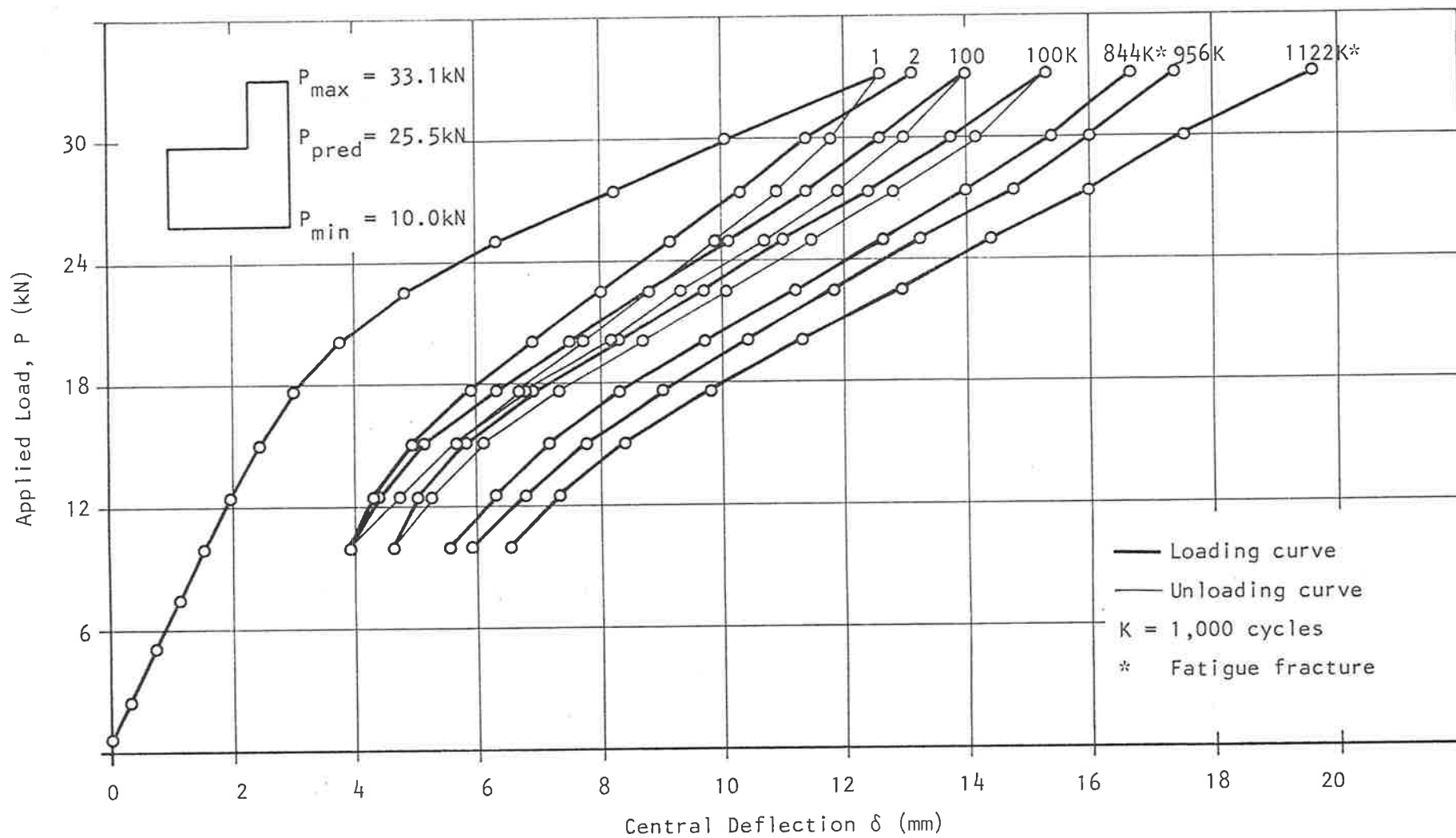


Fig A11 LOAD - DEFLECTION CURVE FOR BEAM SA3

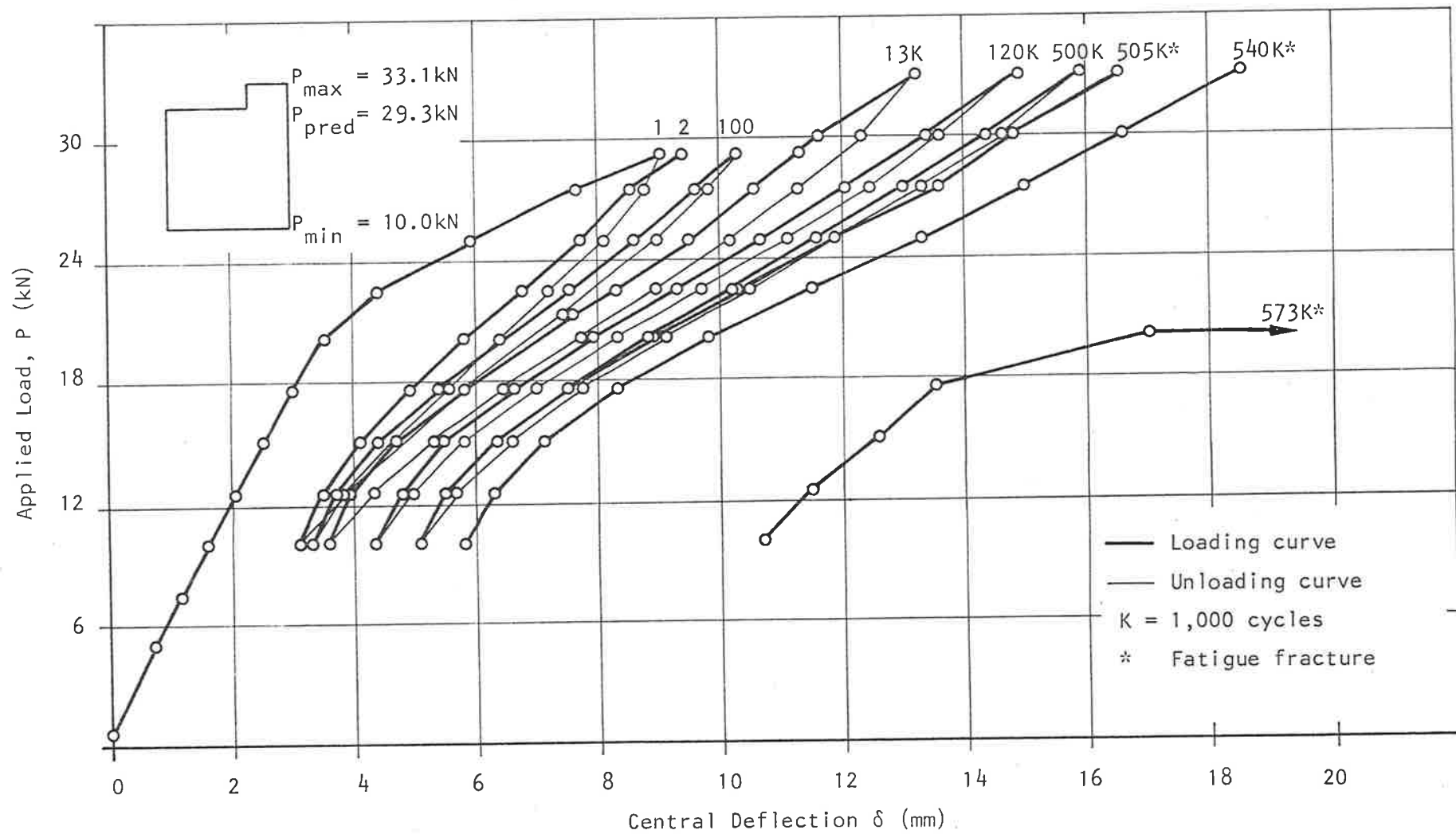


Fig A12 LOAD - DEFLECTION CURVE FOR BEAM SA4

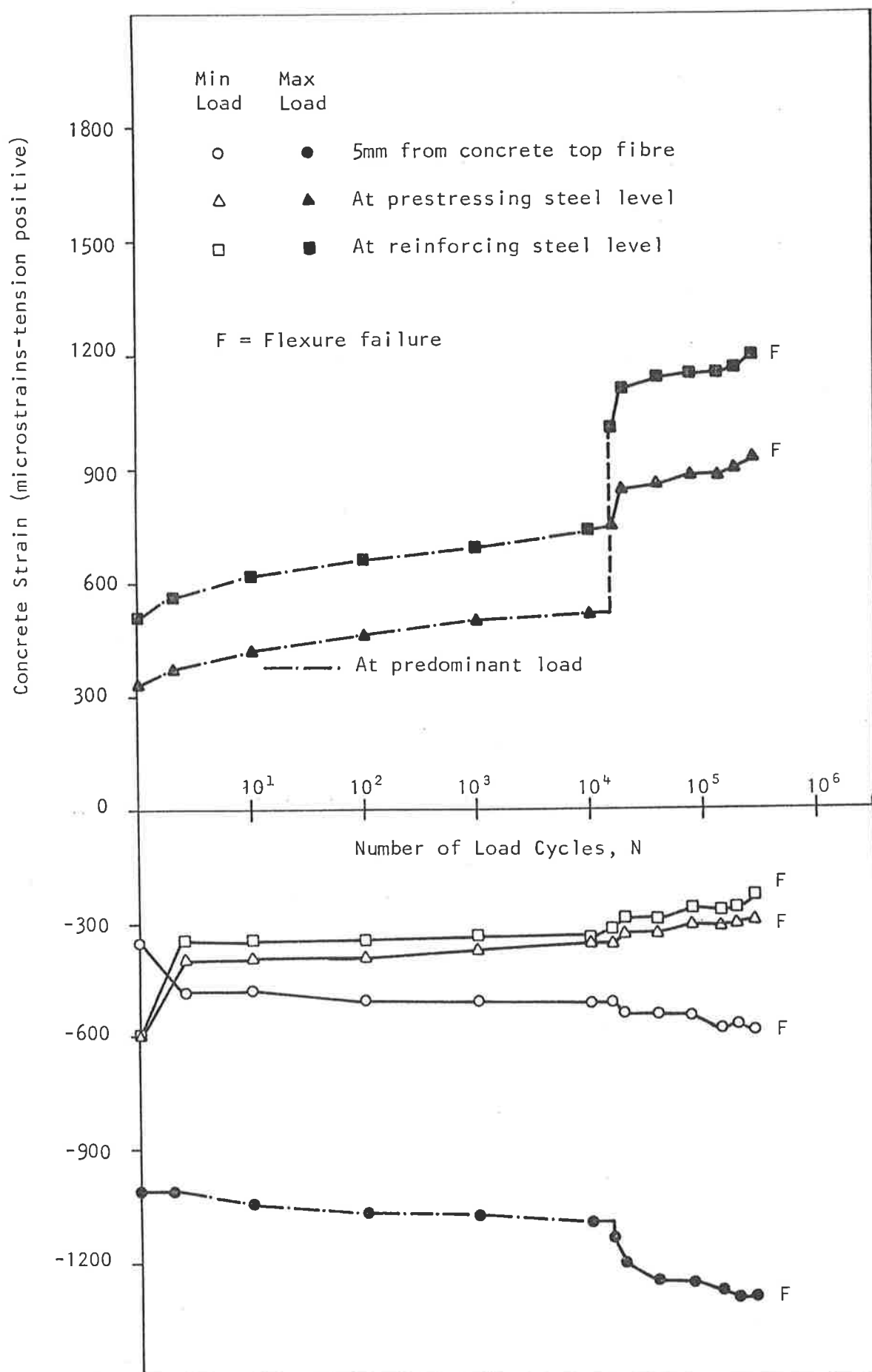


Fig A13 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM PA1

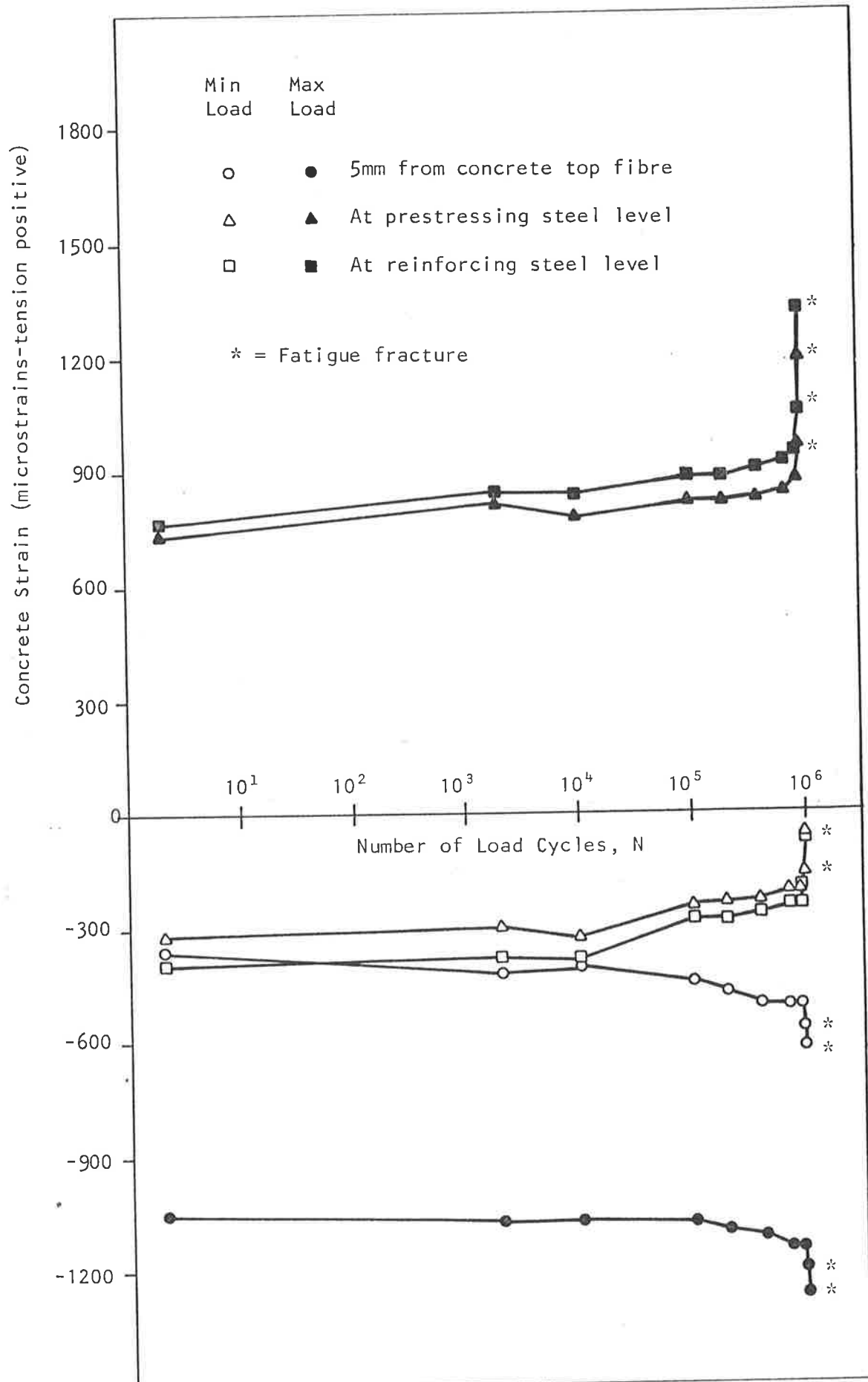


Fig A14 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM PA2

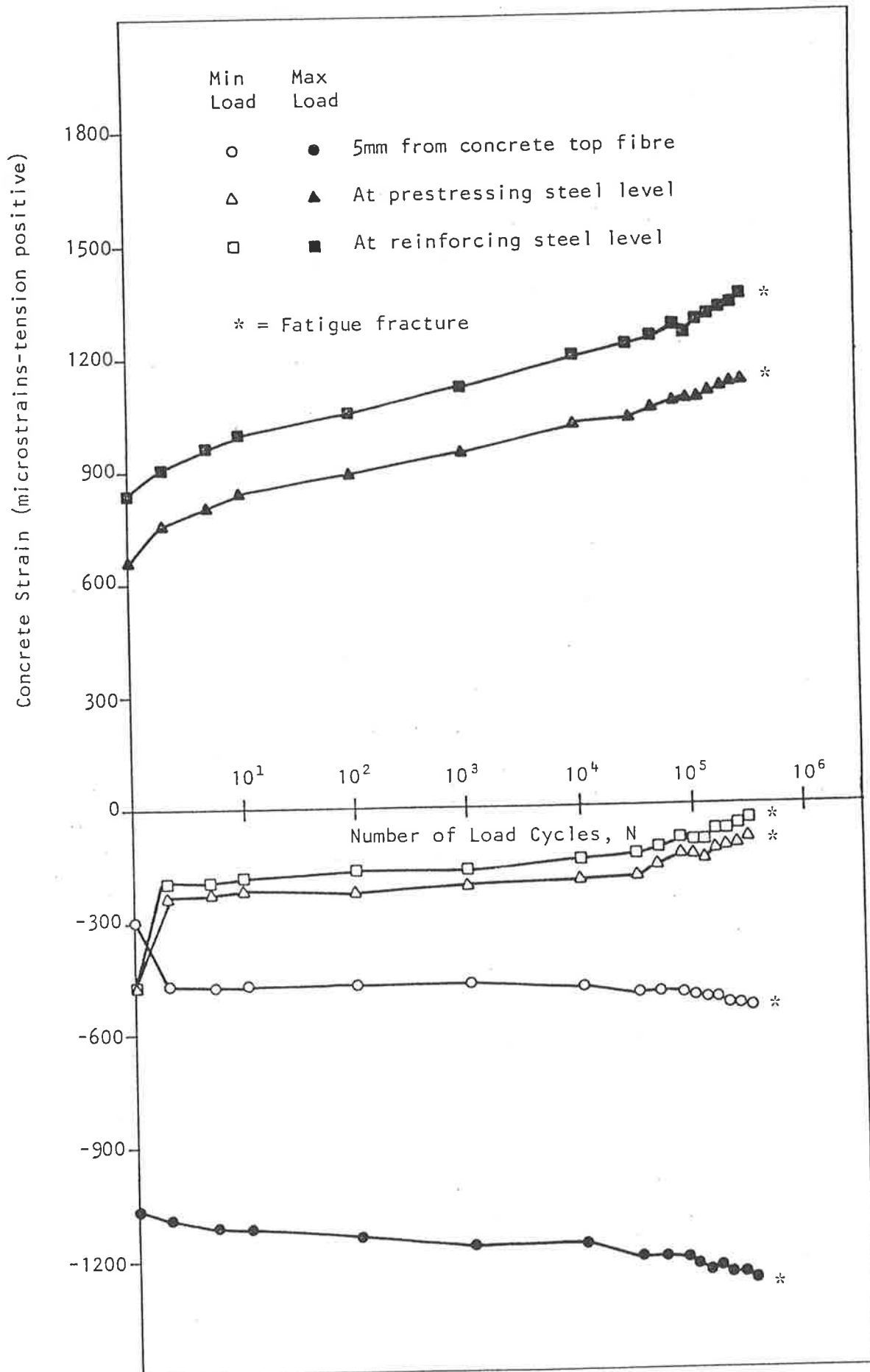


Fig A15 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM SA1

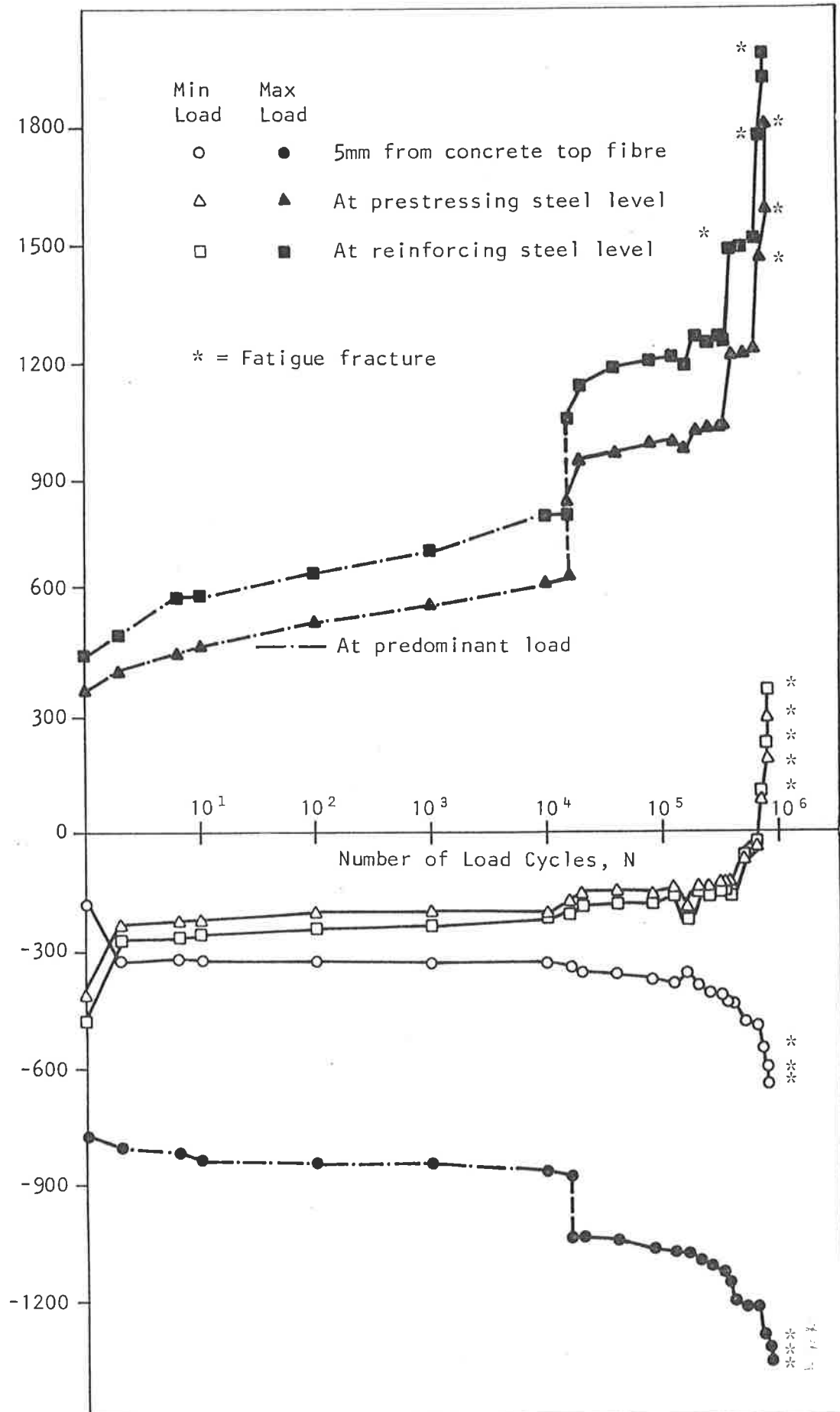


Fig A16 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM SA2

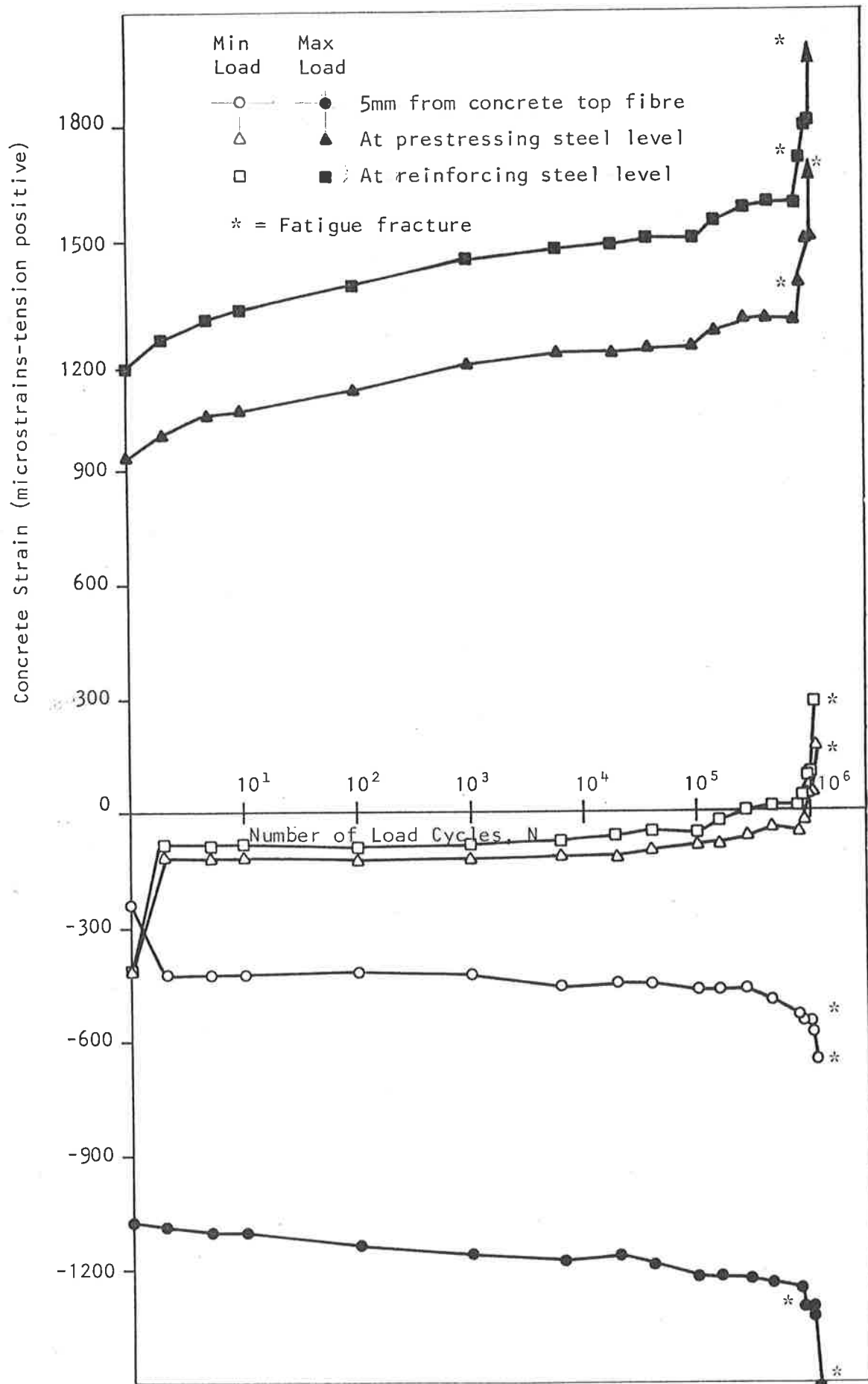


Fig A17 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM SA3

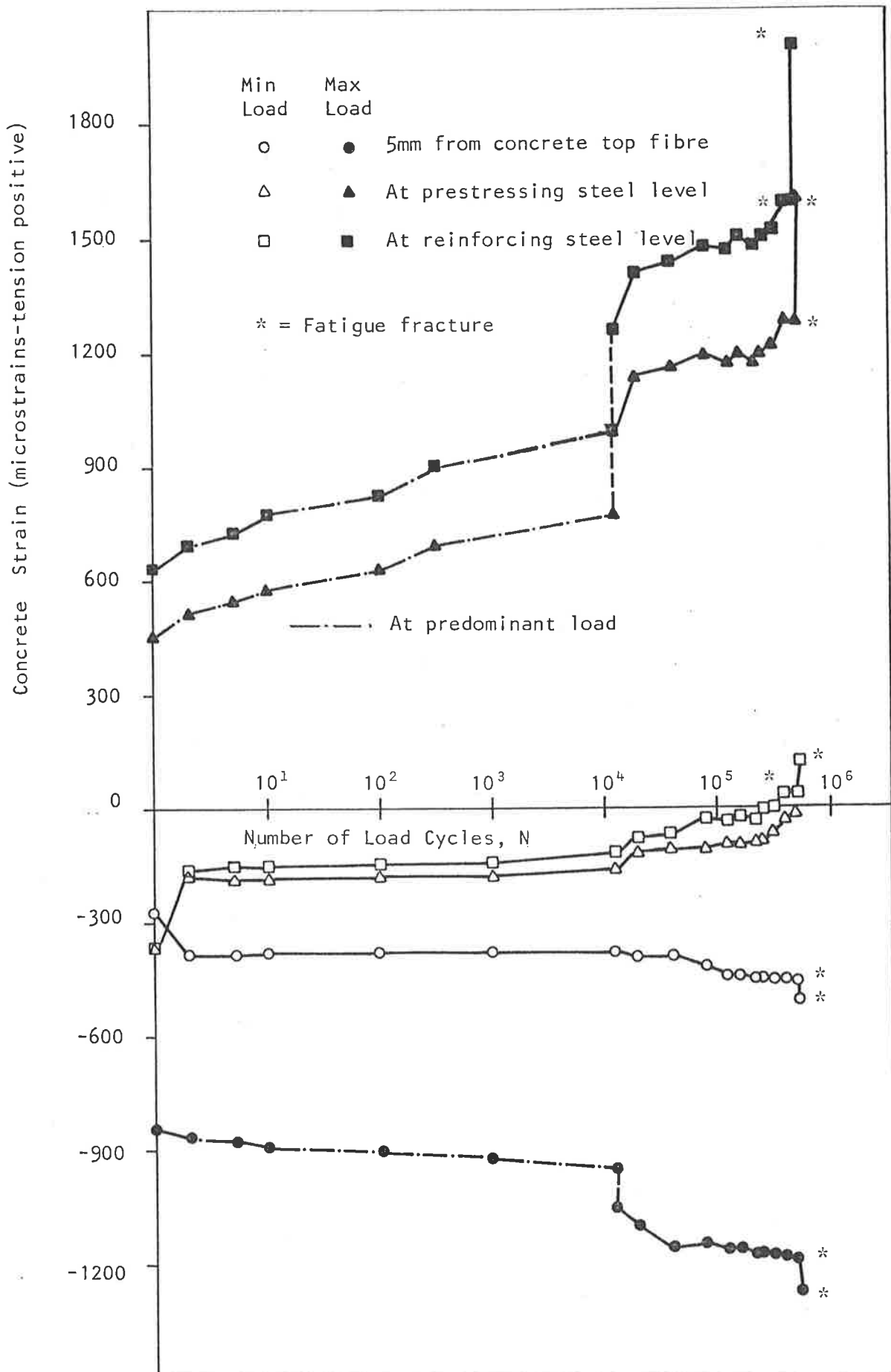


Fig A18 CONCRETE STRAIN VS LOAD CYCLE FOR BEAM SA4

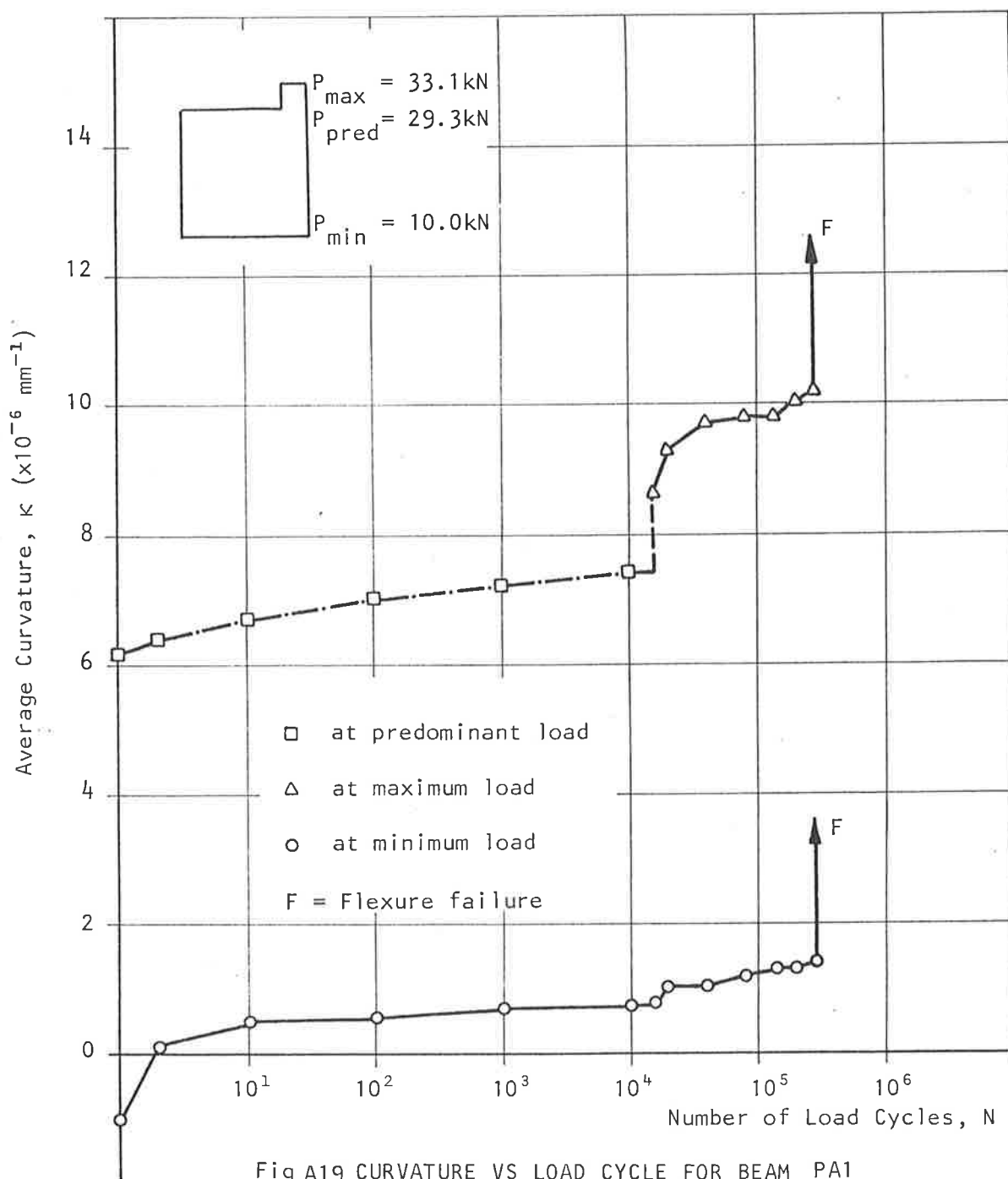


Fig A19 CURVATURE VS LOAD CYCLE FOR BEAM PA1

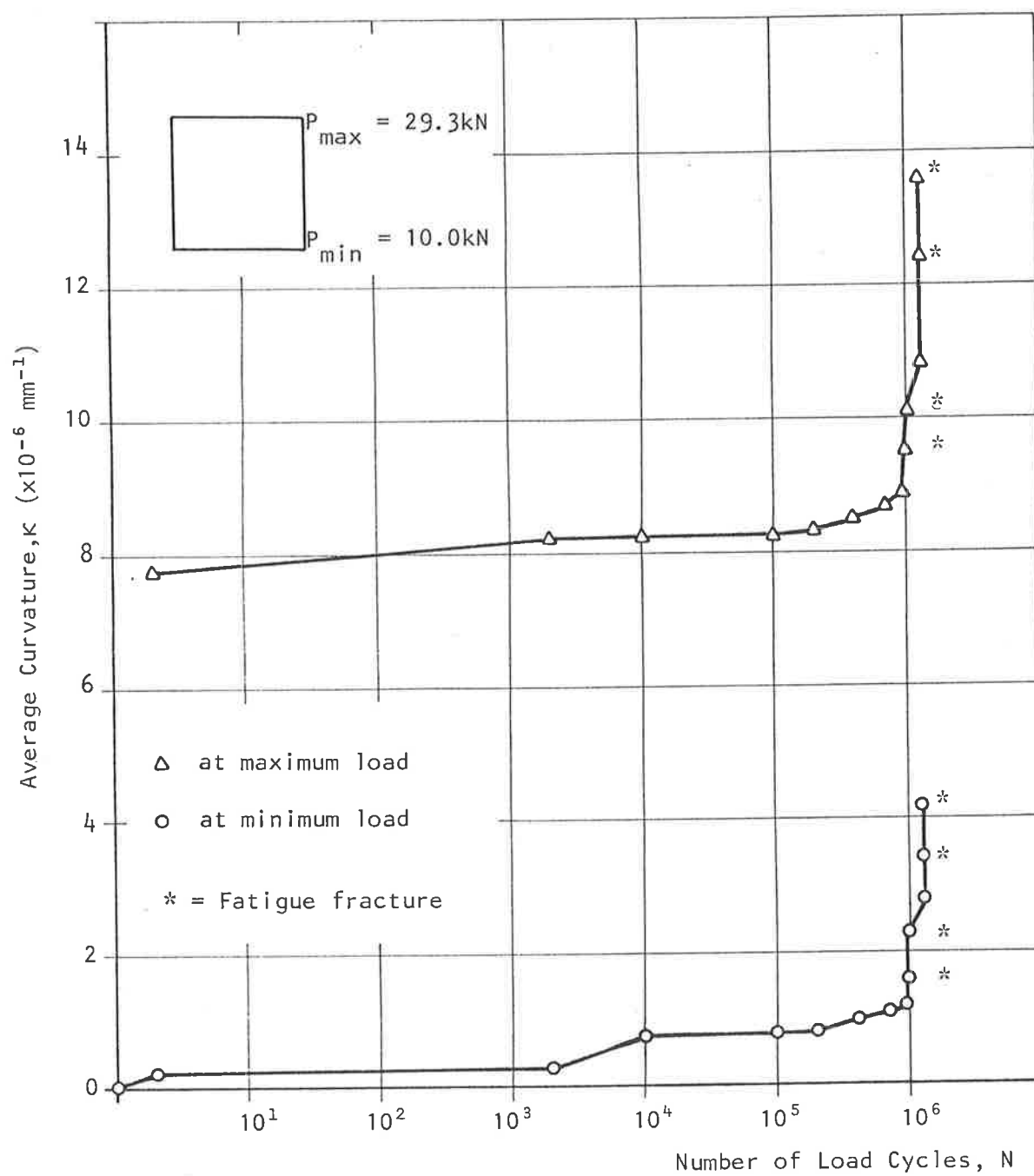


Fig A20 CURVATURE VS LOAD CYCLE FOR BEAM PA2

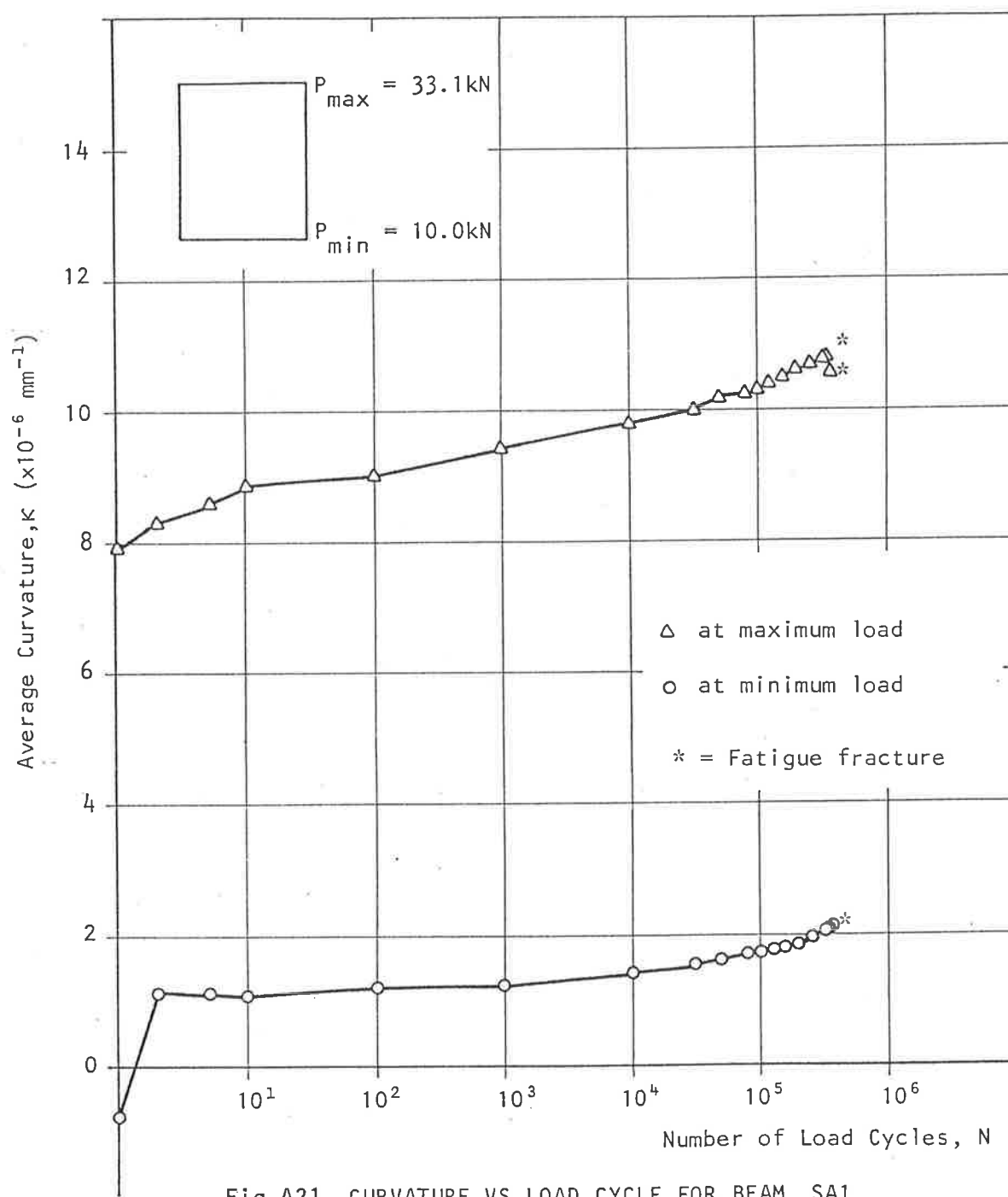


Fig A21 CURVATURE VS LOAD CYCLE FOR BEAM SA1

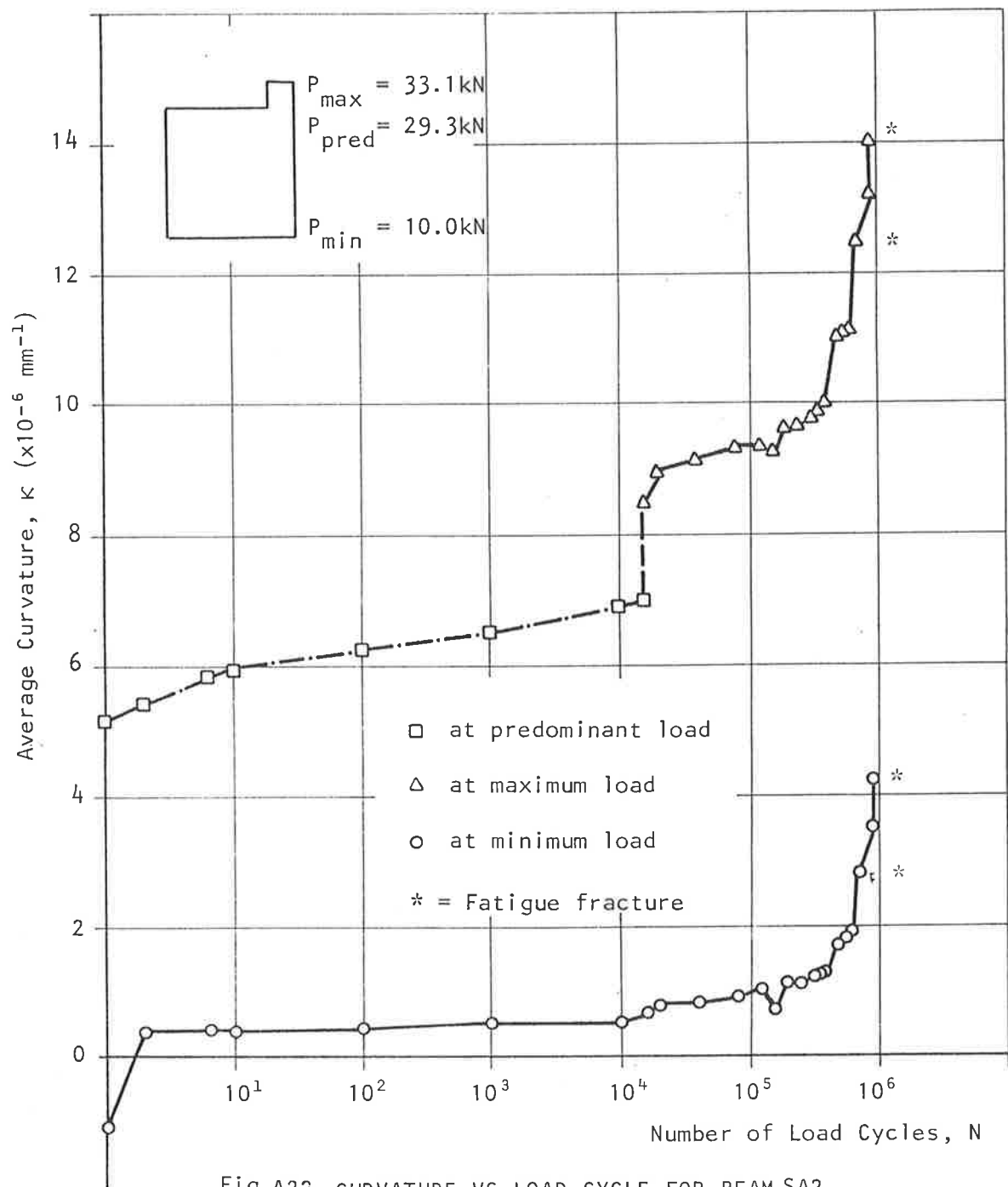
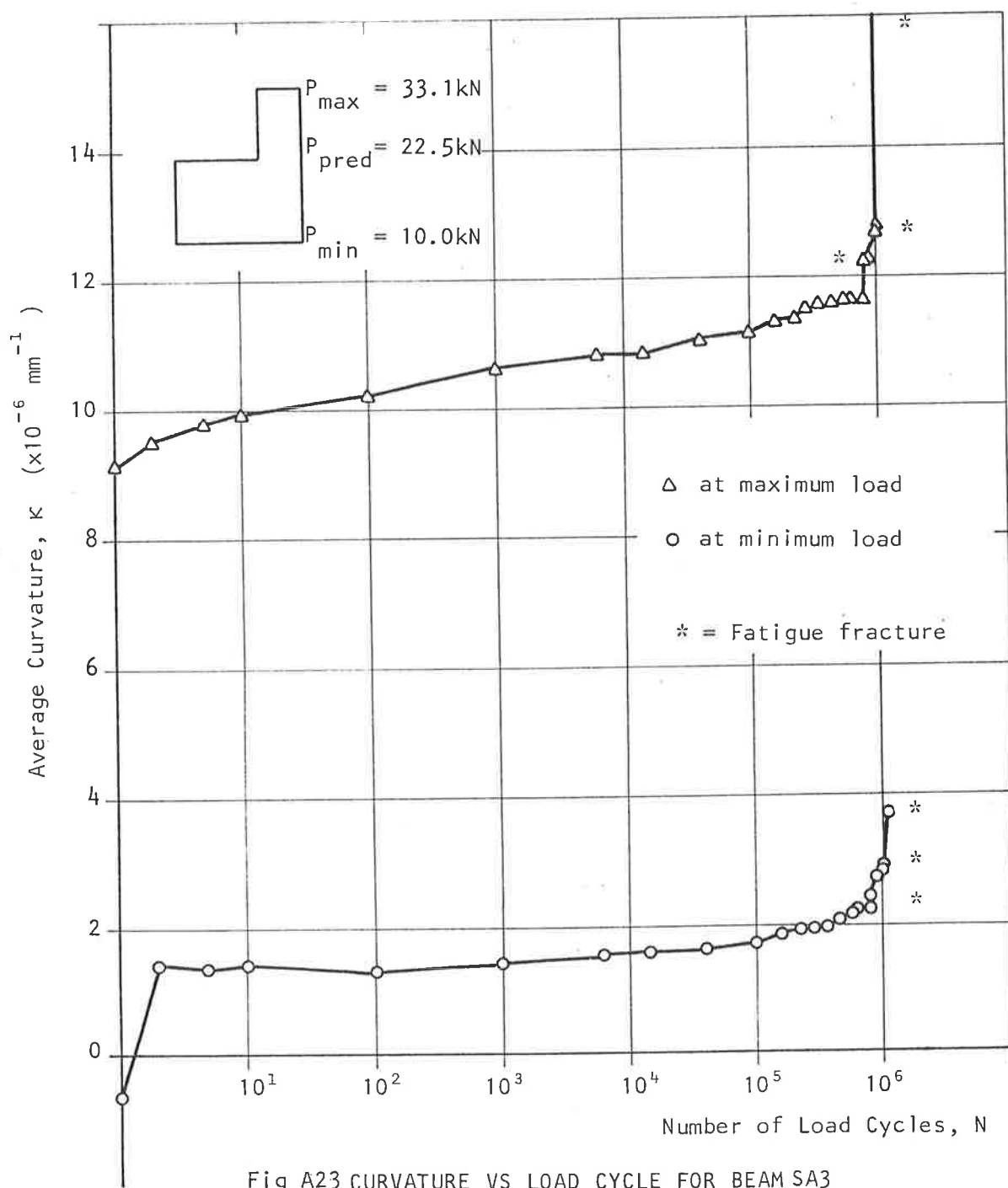


Fig A22 CURVATURE VS LOAD CYCLE FOR BEAM SA2



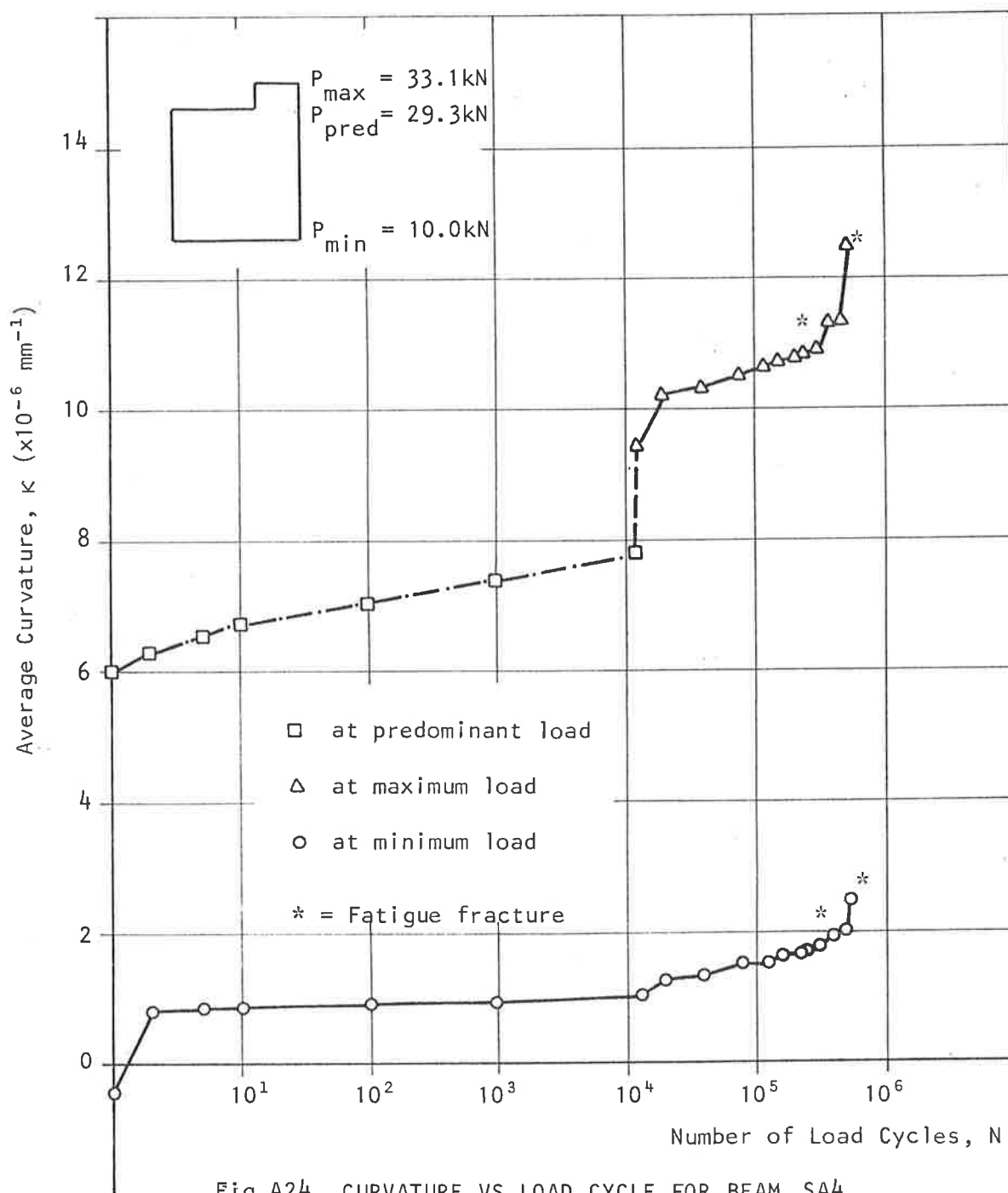


Fig A24 CURVATURE VS LOAD CYCLE FOR BEAM SA4

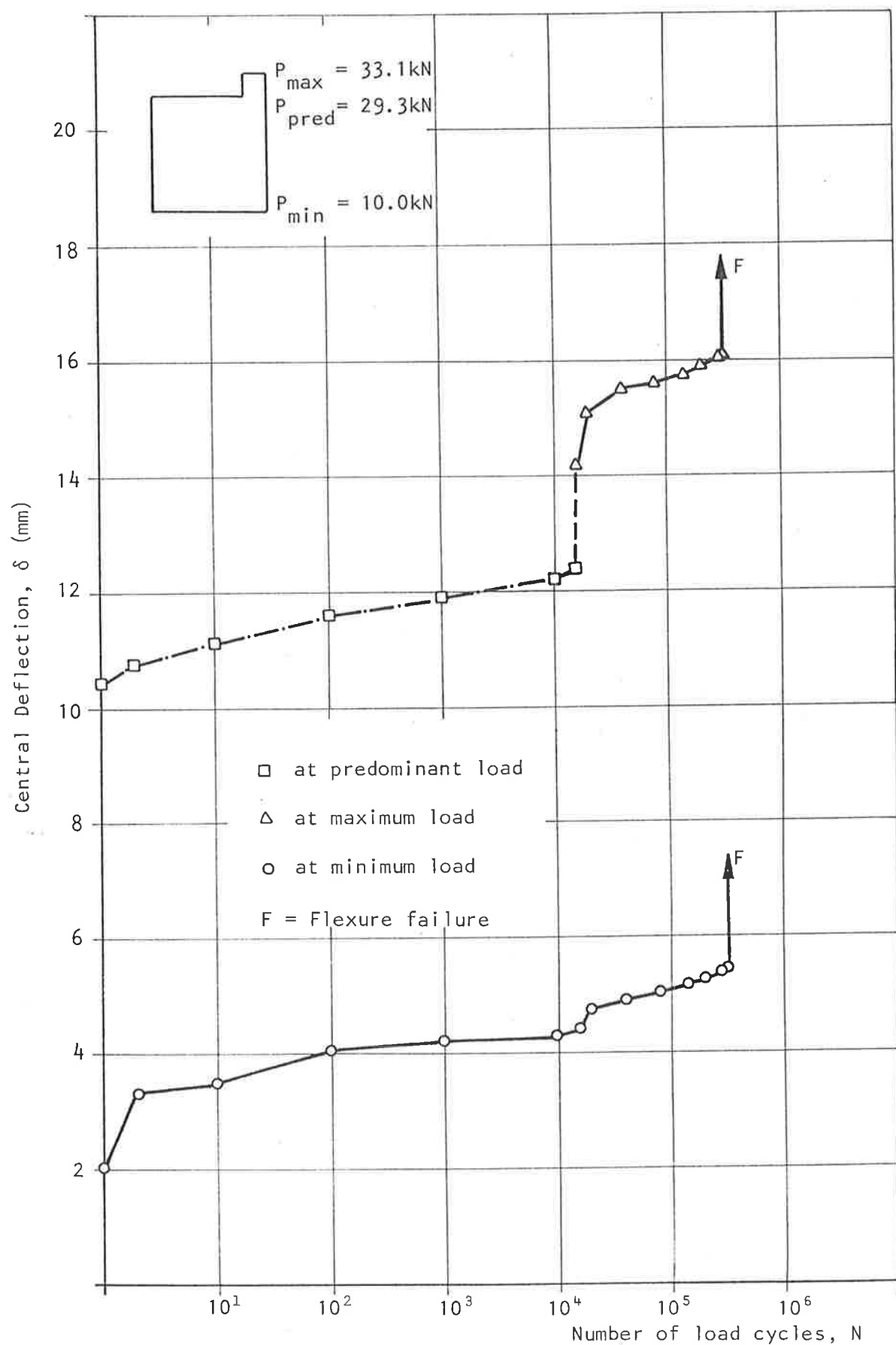


Fig A25 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM PA1

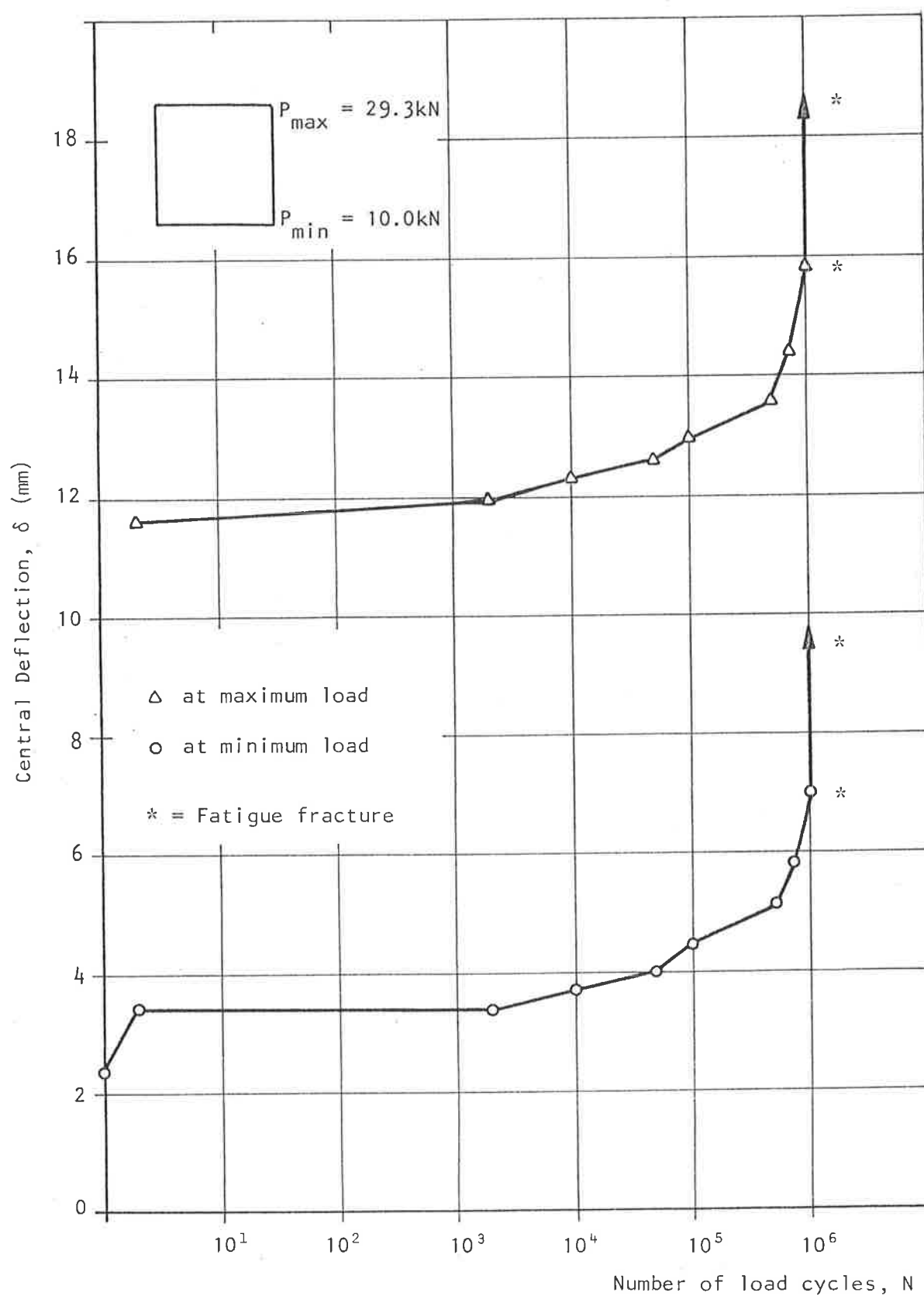


Fig A26 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM PA2

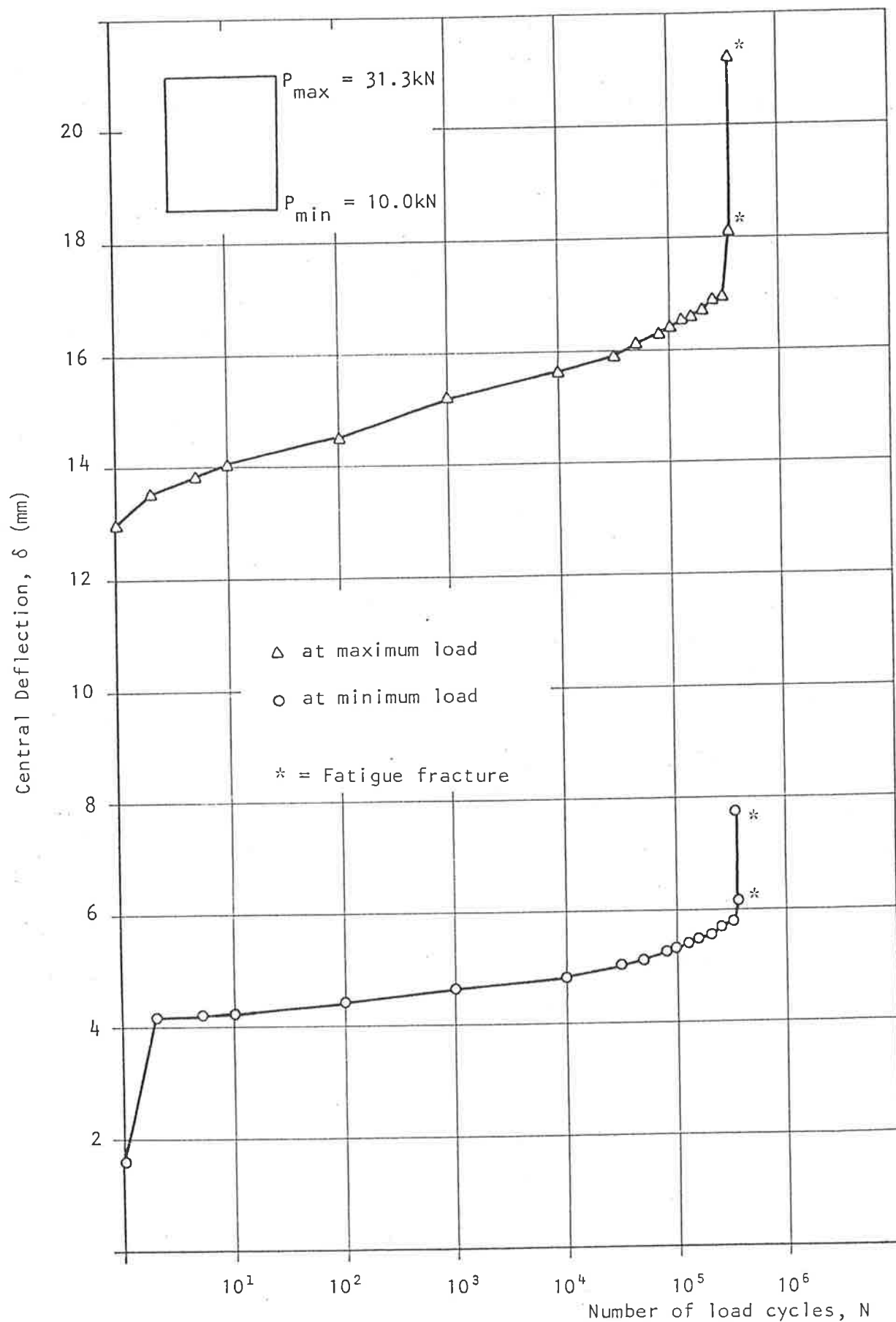


Fig A27 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM SA1

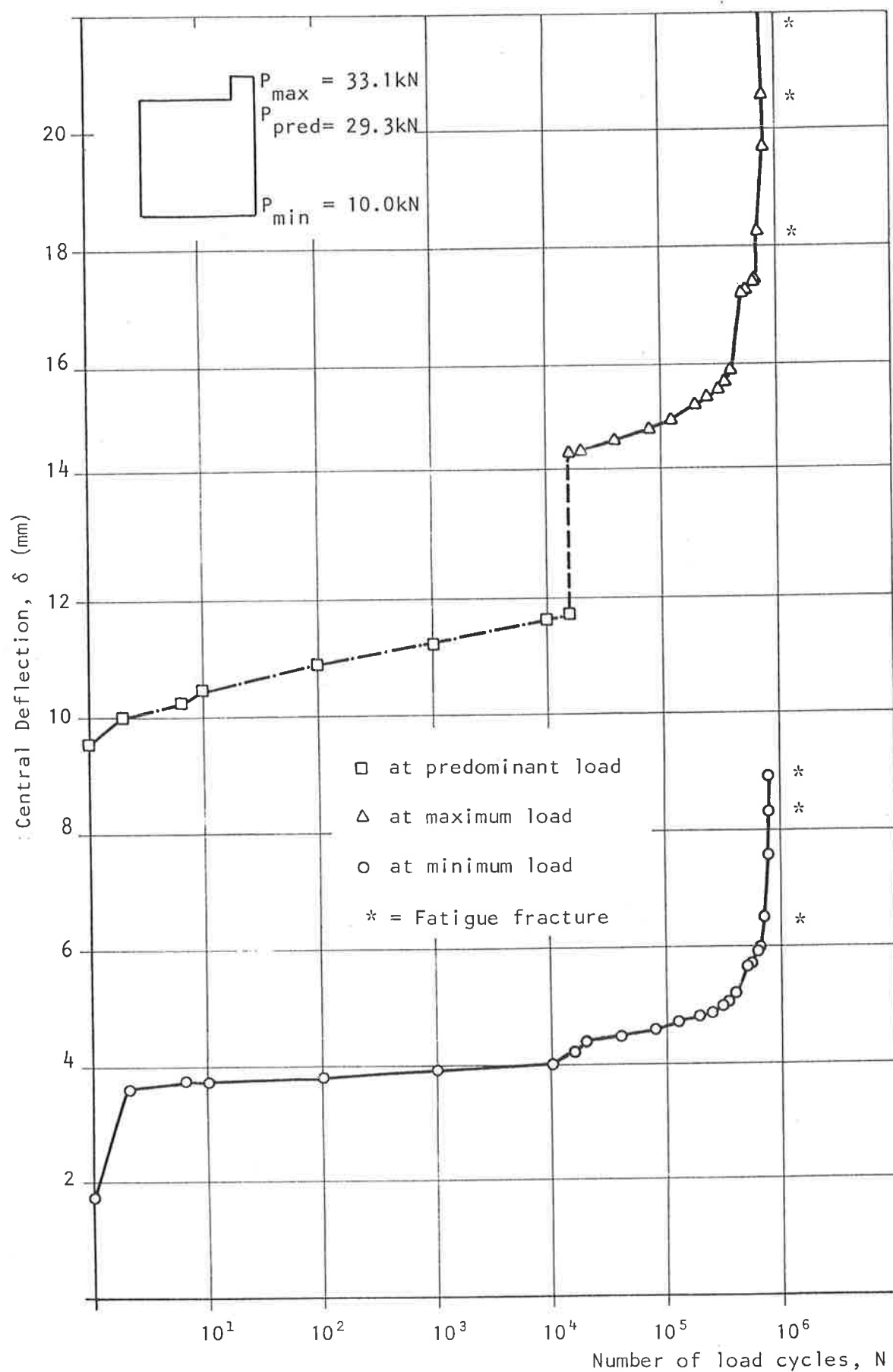


Fig A28 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM SA2

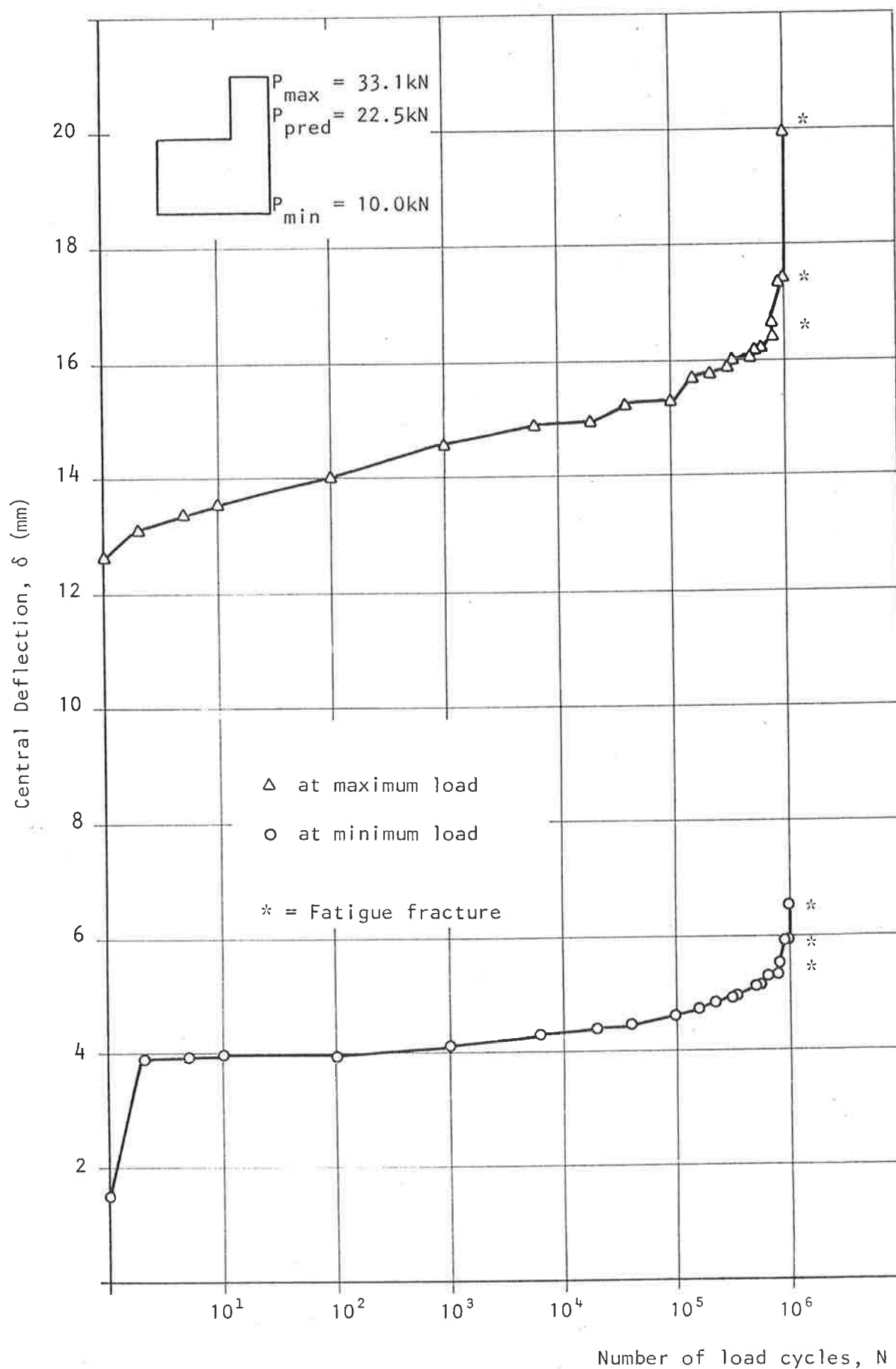


Fig A29 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM SA3

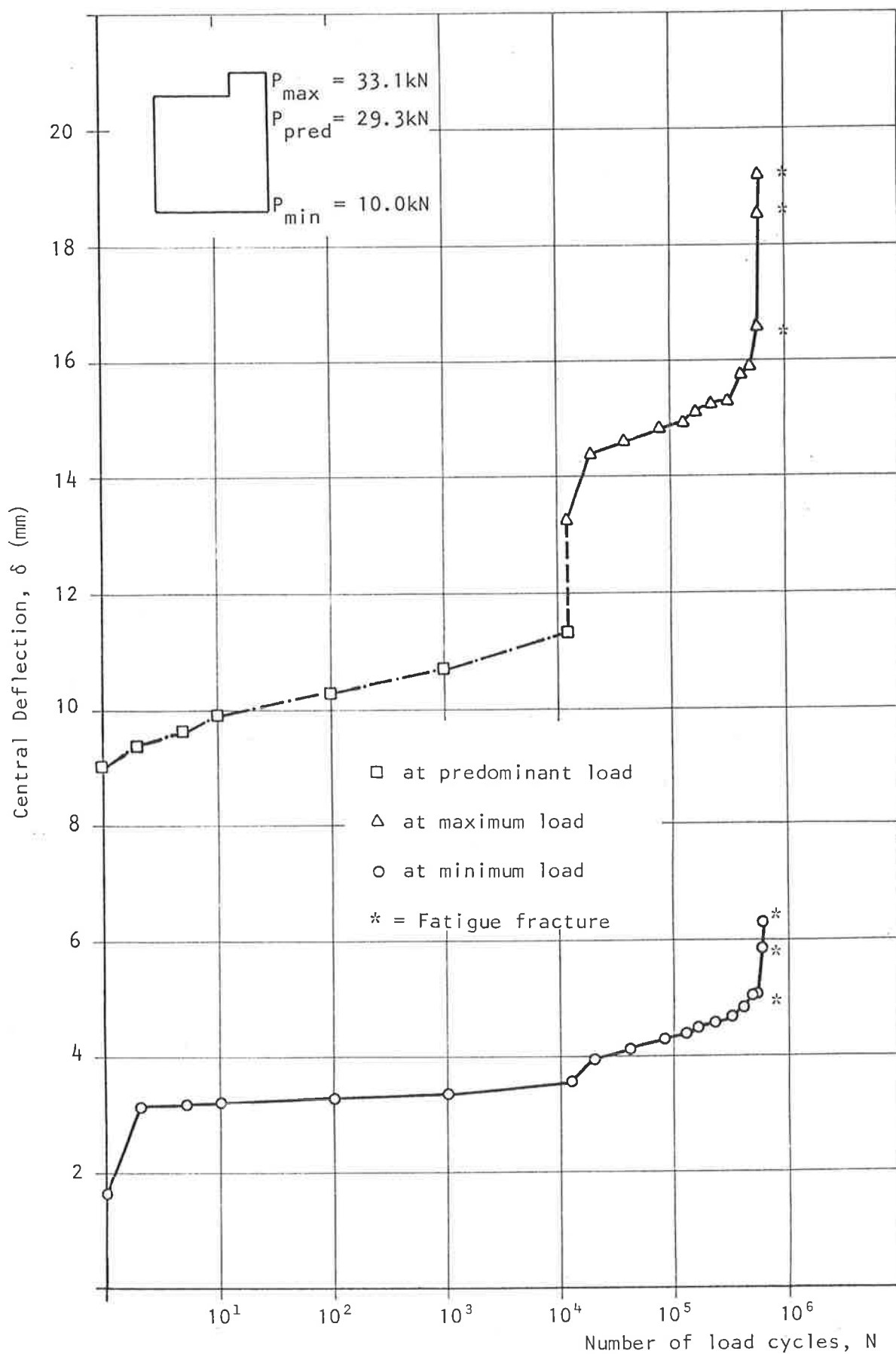


Fig A30 CENTRAL DEFLECTION VS LOAD CYCLE FOR BEAM SA4

APPENDIX B

DESCRIPTION OF COMPUTER PROGRAMS

A brief description of the computer programs developed for the analyses of partially prestressed concrete members under repeated loading is given here while the program listings are documented in Appendix C in microfiche.

The initial stress and strain conditions in a flexural member prior to loading under creep and shrinkage effects are analysed by program BINIT. A regional analysis of the beam behaviour under repeated loading is carried out by program REGION while a generalised and more simplified version of the analysis is contained in program GENERL.

An overview of the regional analysis is given in a flow chart in Appendix B1. Appendix B2 shows the creep and shrinkage calculation process while Appendices B3 and B4 describe the main search routines employed.

The major subroutines are briefly described below:

Subroutine INPUT

Section properties, including the division of concrete and steel areas into discrete elements, material properties, creep and shrinkage parameters, force and moment error tolerances, applied load cycles are read by INPUT from a data file.

Subroutine GEOMTY

GEOMTY calculates the geometrical properties such as the area of each concrete slice, centroidal locations and distances.

Subroutine BSEEK

For a given value of the extreme top-fibre strain ϵ_0 , BSEEK locates the extreme bottom-fibre strain ϵ_1 (hence defining the section strain distribution) which satisfies force equilibrium

in the section. A numerical halving search procedure is employed.

Subroutine ESEEK

Given the applied moment, ESEEK locates for the average concrete strain distribution which satisfies moment and force equilibrium. This is achieved by using two nested search routines. First, a trial value of ϵ_c is chosen and subroutine BSEEK called to locate ϵ_s to satisfy force equilibrium. Moment equilibrium is then checked. If equilibrium is not satisfied, ϵ_c is adjusted and the whole procedure is repeated until bottom force and moment equilibrium are satisfied.

Subroutine KSHIFT

KSHIFT calculates the permanent inelastic strains, increased stresses and forces sustained in the flexural member as a result of repeated loading.

Subroutines UFORCE, CFORCE and NFORCE

UFORCE and CFORCE are subroutines used for calculating the concrete and steel stresses as well as the sum of forces in the section, in the precracking and postcracking load range for the first load cycle. NFORCE is used for subsequent load cycles.

Subroutine MOMENT

Given the stresses and forces in the section, MOMENT calculates the moment contributions about the extreme top fibre.

Subroutine CCRSS, SCRSS, STRSS and PRSS

CCRSS describes the non-linear, history-dependent concrete stress-strain relation in compression and tension. SCRSS, STRSS and PRSS describes stress-strain relations for compression and tension reinforcing steels, and prestressing tendon respectively.

Subroutine ECRSH

With the current elastic stress and strain conditions, time interval and creep and shrinkage parameters, ECRSH calculates

the free-straining inelastic strain increments.

Subroutine CYCLES

CYCLES computes the cyclic creep strains in concrete and steel.

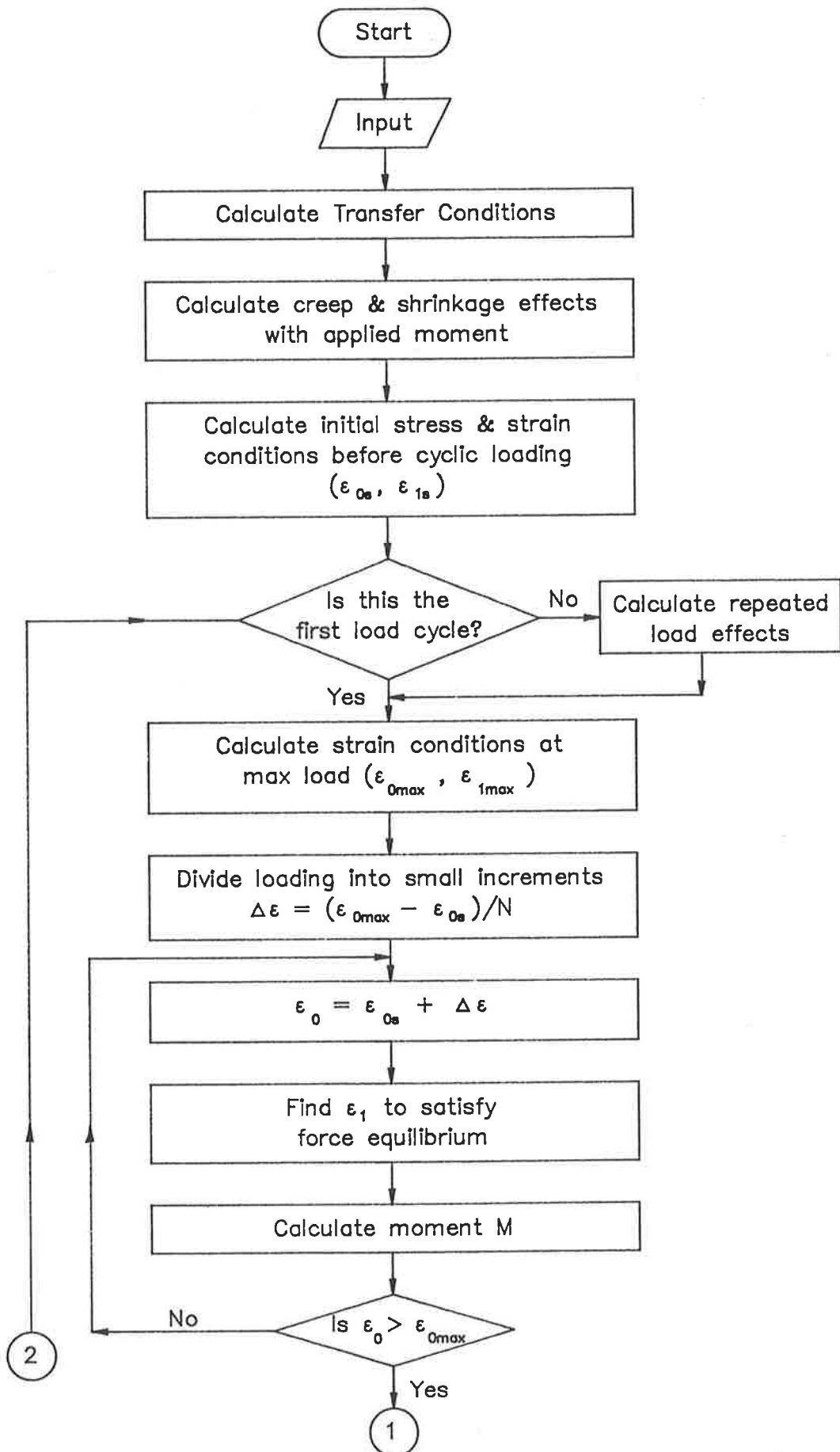
Subroutines WCRSS2, NFIRST and NORCE

These define the concrete effective force-strain relation in the regional analysis (program REGION).

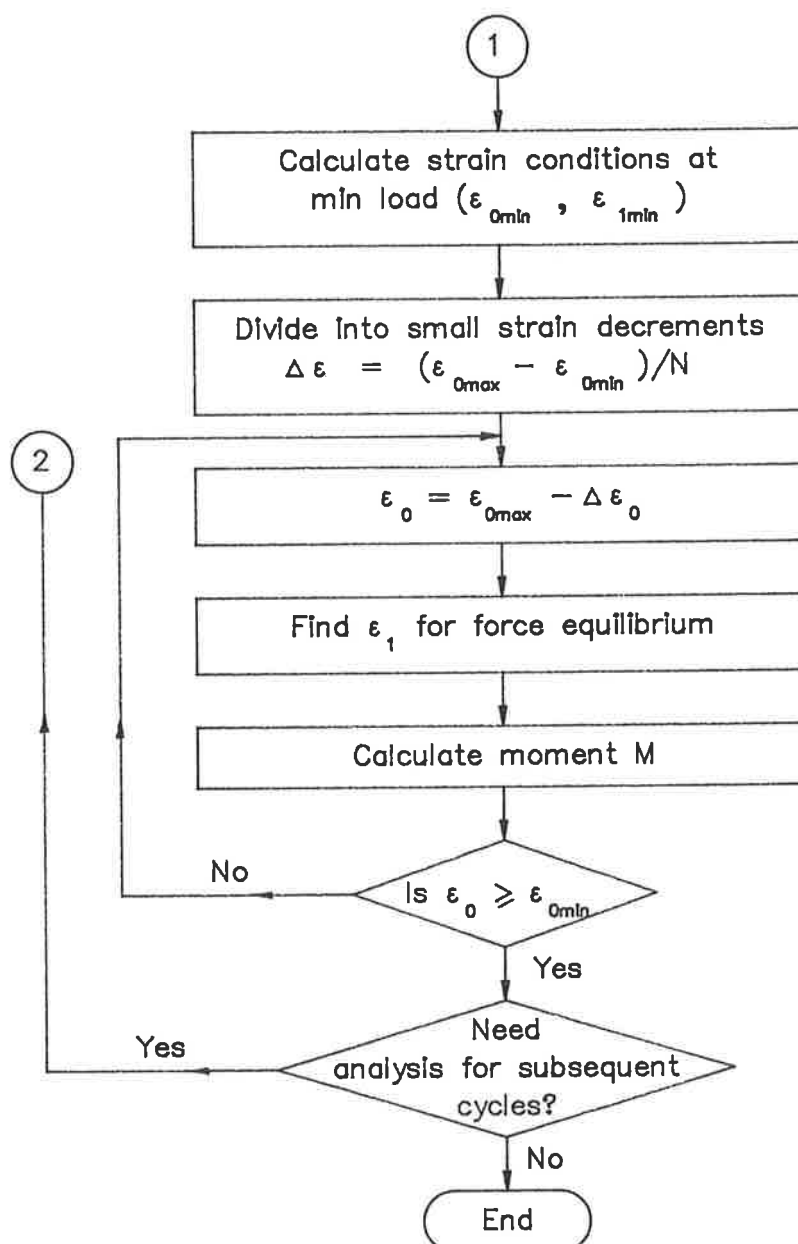
Subroutines MFORCE, MSEEK, MAXIS and FRCE

These subroutines are employed in the modified, generalised regional analysis (program GENERL). MFORCE calculates stresses and forces in the section (similar to NFORCE) while MSEEK and MAXIS are similar to ESEEK and BSEEK respectively. FRCE describes the theoretical concrete effective force-applied moment relation.

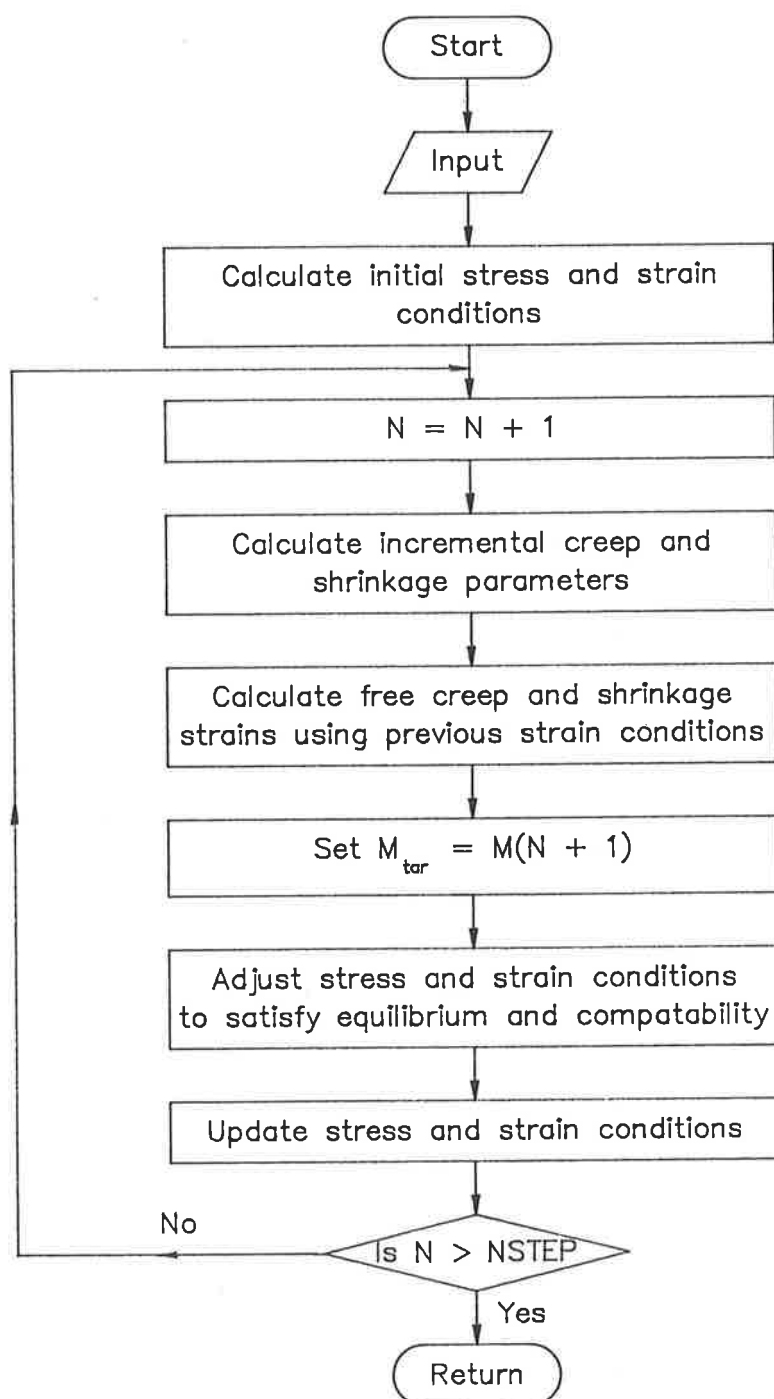
Overview of Analysis



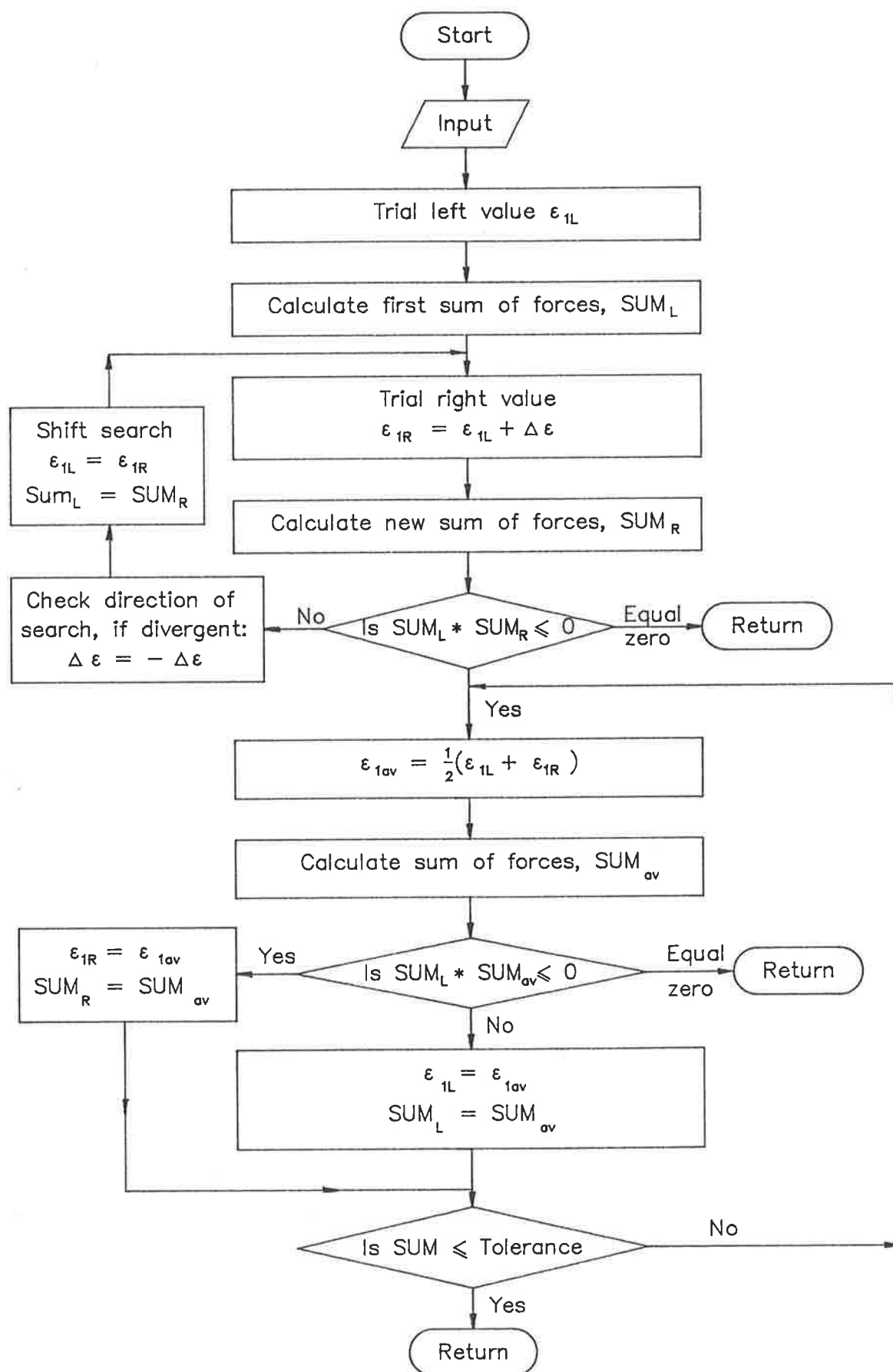
Appendix B1



Initial Creep and Shrinkage Calculations

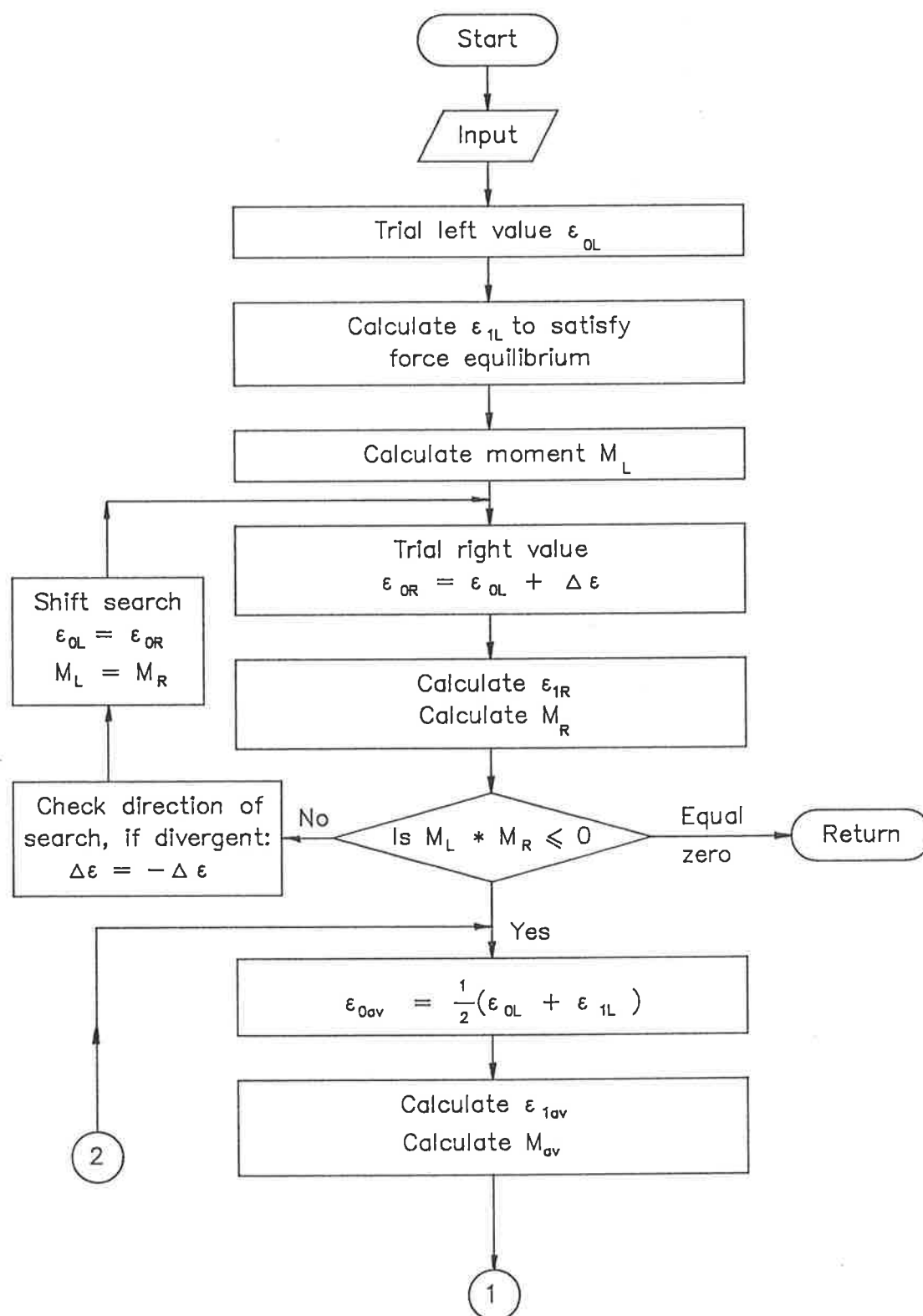


Subroutine BSEEK

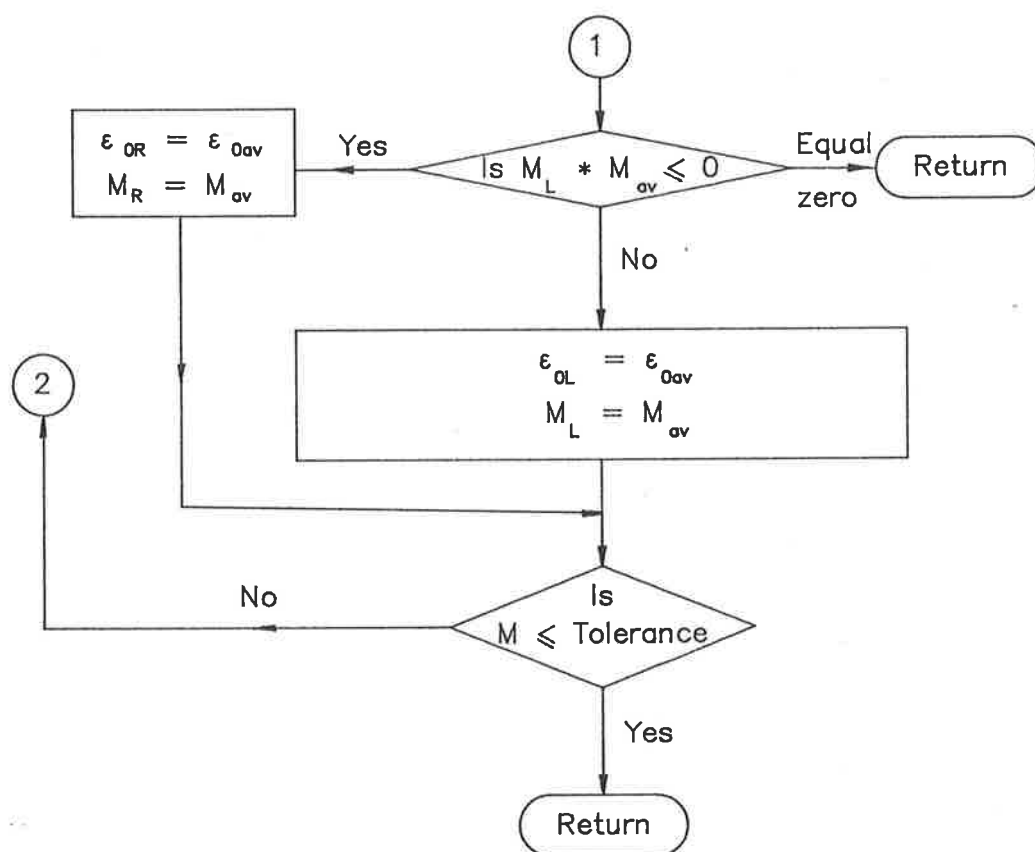


Appendix B4

Subroutine ESEEK



Appendix B4



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