Nonperturbative Approaches to Quantum Chromodynamics

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Abstract

Various nonperturbative methods are employed in investigations of Quantum Chromodynamics (QCD). An important theme in these explorations is the approximate chiral symmetry of QCD which provides a number of important constraints on our analyses.

We undertake a detailed study of the structure of the nucleon based on extant numerical simulations of QCD, exploring the methods necessary to connect these results to the physical world. We incorporate key constraints from chiral symmetry and the heavy quark limit into the extrapolation of the moments of the nonsinglet parton distributions from the large quark masses at which lattice simulations are performed to the physical regime. Using these moments, the Bjorken $x$ and quark mass dependencies of the underlying distributions are analysed and the implications for meson masses lying on certain Regge trajectories are studied.

Using the Dyson-Schwinger equation (DSE) framework, we examine the thermodynamic behaviour of various QCD based models. In this study, we place particular emphasis on the quark density at which nuclear matter undergoes a chiral symmetry restoring phase transition to a quark-gluon plasma. In addition, we investigate the behaviour of various pion properties. The DSE method is also used to study a novel class of vertex truncations which recursively includes an infinite class of diagrams. Having solved the DSEs in a simple model, we also derive a consistent truncation of the Bethe-Salpeter equations in the various meson and diquark channels. This truncation is consistent with chiral symmetry, preserving the axial-vector Ward-Takahashi identity and Goldstone's theorem. A simple model based on this scheme produces a mass spectrum that is in reasonable agreement with experiment, exhibiting no diquark bound states.
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