The Use of Fake Algebraic Riccati Equations for Co-Channel Demodulation

Garry A. Einicke, *Member, IEEE*, Langford B. White, *Senior Member, IEEE*, and Robert R. Bitmead, *Fellow, IEEE*

***Abstract*—This paper describes a method for nonlinear filtering based on an adaptive observer, which guarantees the local stability of the linearized error system. A fake algebraic Riccati equation is employed in the calculation of the filter gain. The design procedure attempts to produce a stable filter at the expense of optimality. This contrasts with the extended Kalman filter (EKF), which attempts to preserve optimality via its linearization procedure, at the expense of stability. A passivity approach is applied to deduce stability conditions for the filter error system. The performance is compared with an EKF for a co-channel frequency demodulation application.**

***Index Terms*—Communications, Kalman–Bucy filtering, local approximation, observer, robustness, time-varying estimation.**

I. INTRODUCTION

THE EXTENDED Kalman filter (EKF) [1] is used ubiquitously for state estimation within communication and aerospace applications. However, the EKF can exhibit poor performance when there are uncertainties in the problem assumptions. We seek to develop a suboptimal filter, determine the stability conditions, and present examples that demonstrate performance benefits compared with the EKF.

The nonlinear state estimation problem is formulated along similar lines to the application of the EKF described in [1] and [2]. The EKF uses the nonlinear plant update and measurement function to compute a prediction error, which is then multiplied by the Kalman gain matrix derived from the linearised system and added to the state estimate. The EKF is not guaranteed to be stable, and a so-called robust filter is desired in which optimality is traded off in return for increased stability. In a previous approach to robust nonlinear filtering [3], we have retained the structure of the EKF and sought a solution to a Riccati equation that achieves a compromise between least squares and H_∞ optimality criteria and accommodates some uncertainty in the input conditions and signal model. The extended H_∞ filter of [3] accommodates uncertainty by evolving an increased approximate error covariance at the cost of increased mean square error when uncertainties are absent. In this paper, a suboptimal "covariance" matrix is pursued via the fake algebraic Riccati techniques of [4]–[7], and stability conditions are obtained in a passivity framework [8].

This paper generalizes the fake algebraic Riccati approach from a one signal application [9] to a superimposed signal case. The nonlinear filter is developed in Section II. Section III describes the application of the fake algebraic Riccati technique to select the filter gain. The stability conditions for the error system are set out in Section IV. The frequency modulated (FM) signal tracking applications making use of the stability conditions are discussed in Section V.

II. DEVELOPMENT OF A NONLINEAR FILTER

Consider the following model comprising a stable, linear state evolution and a nonlinear output mapping

$$x_{k+1} = Ax_k + w_k \tag{1}$$

$$y_k = c_k(x_k) + v_k \tag{2}$$

where w_k and v_k are uncorrelated, zero mean, white, n- and m-order processes with known covariances Q and R, respectively. The matrix A and the matrix function $c_k(\cdot)$ are of appropriate dimensions. It is assumed that the components of $c_k(\cdot)$ are continuous and differentiable. A recursive filter is desired, which yields estimates of x_k, given measurements y_k for each $k > 0$. A nonlinear observer may be constructed having the form

$$\hat{x}_{k+1} = A\hat{x}_k + g_k(y_k - c_k(\hat{x}_k)) \tag{3}$$

where $g_k(\cdot)$ is a nonlinear gain function to be designed. From (1)–(3), the state prediction error may be written as

$$\tilde{x}_{k+1} = A\tilde{x}_k - g_k(\varepsilon_k) + w_k \tag{4}$$

where $\tilde{x}_k = x_k - \hat{x}_k$, and $\varepsilon_k = y_k - c_k(\hat{x}_k)$ is the output prediction error. The Taylor series expansion of the output mapping $c_k(\cdot)$ to terms linear in the state error yields $c_k(x_k) = c_k(\hat{x}_k) + C_k\tilde{x}_k$, where $C_k = [(\partial c_k(x))/(\partial x)]_{x=\hat{x}_k}$. It follows that $\varepsilon_k = y_k - c_k(\hat{x}_k) \approx c_k(\hat{x}_k) + C_k\tilde{x}_k + v_k - c_k(\hat{x}_k) = C_k\tilde{x}_k + v_k$. The objective here is to design $g_k(\varepsilon_k)$ to be a linear function of $\tilde{x}_k$ to first-order terms. It will be shown that for certain classes of problems, this can be achieved by a suitable choice of a nonlinear bounded matrix function of the states D_k, resulting in the adaptive gain function $g_k(\varepsilon_k) = K_k D_k \varepsilon_k$, where K_k is a gain matrix of appropriate dimensions. For example, consider an $(n \times 1)$ column vector x_k and an $(m \times 1)$ column vector y_k, which yield an $(m \times 1)$ vector ε_k and an $(m \times n)$ matrix C_k. Suppose that a linearizing $(p \times m)$ matrix D_k can be found so

Manuscript received November 8, 2001; revised April 4, 2003. The associate editor coordinating the review of this paper and approving it for publication was Prof. Zhi Ding.

G. A. Einicke is with the CSIRO, Division of Exploration and Mining, Kenmore, QLD 4069, Australia (e-mail: garry.einicke@csiro.au).

L. B. White is with the School of Electrical and Electronic Engineering, The University of Adelaide, Adelaide, Australia.

R. R. Bitmead is with Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411 USA.

Digital Object Identifier 10.1109/TSP.2003.815376

that $\bar{C}_k = D_k C_k$ is a $(p \times n)$ matrix possessing approximately constant terms; then, it will be shown that an $(n \times p)$ matrix K_k and an $(n \times 1)$ vector $g_k(\varepsilon_k)$ can be calculated. Note that the observer system (3) is adaptive because the $g_k(\varepsilon_k)$ is a function of the time-varying state estimates.

In view of $g_k(\varepsilon_k)$, the locally linearized error (4) may be written as

$$\tilde{x}_{k+1} = (A - K_k \bar{C}_k)\tilde{x}_k - K_k D_k v_k + w_k. \qquad (5)$$

Let $\lambda(A)$ denote the eigenvalues of A. If $|\lambda(A)| < 1$ and if the pair $[\bar{C}_k, A]$ is completely observable, then the asymptotic stability of (5) can be guaranteed by placing the eigenvalues arbitrarily to ensure $|\lambda(A - K_k \bar{C}_k)| < 1$. A method for choosing the gain K_k is described in the next section.

III. Gain Selection via Fake Algebraic Riccati Equations

A. Fake Algebraic Riccati Equation Technique

In the case of the EKF, the gain is specified by the solution to a Riccati difference equation (RDE) [1, p. 195]. We propose a procedure here that retains the familiar gain structure of the EKF but with a positive definite "covariance" matrix that is chosen by the filter designer rather than by the solution of an RDE. From (5), an approximate equation for the estimation error covariance $P_k = E\{\tilde{x}_k \; \tilde{x}_k^T\}$, neglecting all interdependencies, may be written as $P_{k+1} = (A - K_k \bar{C}_k) P_k (A - K_k \bar{C}_k)^T + K_k D_k R D_k^T K_k^T + Q$. The K_k is given by

$$K_k = P_k \bar{C}_k^T \left(\bar{C}_k P_k \bar{C}_k^T + D_k R D_k^T\right)^{-1} \qquad (6)$$

where P_k is found by solving the RDE

$$P_{k+1} = A P_k A^T - P_k \bar{C}_k^T \left(\bar{C}_k P_k \bar{C}_k^T + D_k R D_k^T\right)^{-1} \bar{C}_k P_k + Q. \qquad (7)$$

Note that in the linear case, the Kalman filter produces the smallest P_k [1, Th. 2.1], and the corresponding Cramér–Rao bound is identical to the Kalman filter error covariance [10].

In general, the solutions P_k, while positive definite, need not be stabilizing because of the impact of the nonlinearities $c_k(x_k), g_k(\varepsilon_k)$, and therefore, the resulting error system can lack stability. The fake algebraic Riccati equation (ARE) approach is motivated by the observation that Kalman filter gains can tend to a constant value [6, p. 54]. The technique is also known as "covariance setting" and relies on connections between the RDE and ARE stability results [4]–[7]. Using the approach of [4]–[7], the RDE (7) may be masqueraded by the fake ARE

$$\Sigma_k = A \Sigma_k A^T - \Sigma_k \bar{C}_k^T \left(\bar{C}_k \Sigma_k \bar{C}_k^T + D_k R D_k^T\right)^{-1} \bar{C}_k \Sigma_k + Q. \qquad (8)$$

We follow the design method of [4] and choose a suboptimal Σ_k rather than solve the RDE (7). That is, rather than finding a solution to (7), we select an arbitrary fixed positive definite solution Σ_k and then calculate the gain each time k from (6), using Σ_k in place of P_k. Section IV will show that selecting the gains in this manner can lead to the identification of stability conditions for the ensuing suboptimal filter.

B. Application to Signal Demodulation

Consider the problem of tracking two frequency or phase-modulated signals present in the communication channel. A demodulator follows by constructing an EKF for an augmented state space system that includes the multiple signal components [2]. The signals may be modeled by (1), where $x_k = [a_k^{(1)}, \omega_k^{(1)}, \phi_k^{(1)}, a_k^{(2)}, \omega_k^{(2)}, \phi_k^{(2)}]^T, A = \mathrm{diag}[A^{(1)}, A^{(2)}]$, in which

$$A^{(i)} = \begin{bmatrix} \mu_a & 0 & 0 \\ 0 & \mu_\omega & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

and $\mu_a, \mu_\omega \in \mathbf{R}$. The states $a_k^{(i)}, \omega_k^{(i)}, \phi_k^{(i)} \in \mathbf{R}$ represent the instantaneous amplitude, frequency, and phase components. Let $y_k = \sum_{i=1}^{2} a_k^{(i)} e^{j\phi_k^{(i)}} + v_k$ denote the complex, baseband observations, where $y_k, v_k \in \mathbf{R^2}$. Expanding the prediction error to terms linear in the estimation error yields $C_k = [C_k^{(1)}, C_k^{(2)}]$, where

$$C_k^{(i)} = \begin{bmatrix} \cos\hat{\phi}_k^{(i)} & 0 & -\hat{a}_k^{(i)} \sin\hat{\phi}_k^{(i)} \\ \sin\hat{\phi}_k^{(i)} & 0 & \hat{a}_k^{(i)} \cos\hat{\phi}_k^{(i)} \end{bmatrix}.$$

This form suggests

$$D_k = \begin{bmatrix} D_k^{(1)} \\ D_k^{(2)} \end{bmatrix}$$

where

$$D_k^{(i)} = \begin{bmatrix} \cos\hat{\phi}_k^{(i)} & \sin\hat{\phi}_k^{(i)} \\ -\frac{\sin\hat{\phi}_k^{(i)}}{\hat{a}_k^{(i)}} & \frac{\cos\hat{\phi}_k^{(i)}}{\hat{a}_k^{(i)}} \end{bmatrix}.$$

An examination of the EKF asymptotic error covariance for the above problem under low measurement noise conditions suggests a structure for the fake ARE solution, namely, $\Sigma = \mathrm{diag}[\Sigma^{\{1\}}, \Sigma^{\{2\}}]$, where

$$\Sigma^{(i)} = \begin{bmatrix} \Sigma_a^{(i)} & 0 & 0 \\ 0 & \Sigma_\omega^{(i)} & \Sigma_{\omega\phi}^{(i)} \\ 0 & \Sigma_{\omega\phi}^{(i)} & \Sigma_\phi^{(i)} \end{bmatrix}$$

in which $\Sigma_a^{(i)}, \Sigma_\omega^{(i)}, \Sigma_\phi^{(i)}, \Sigma_{\omega\phi}^{(i)} \in \mathbf{R}$.

In the multiple signal component case, the linearization $\bar{C}_k = D_k C_k$ does not result in perfect decoupling. While the diagonal blocks reduce to

$$\bar{C}^{(i,i)} = \begin{bmatrix} 1 \; 0 \; 0 \\ 0 \; 0 \; 1 \end{bmatrix}$$

the off-diagonal blocks possess the time-varying quantities, shown at the bottom of the next page, which prevent an *a priori* solution for the adaptive gain. Instead, the gain may be evaluated at each time k from (8).

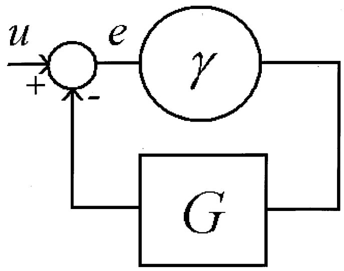


Fig. 1. Nonlinear system model.

The simplifications for the one signal case follow with $i = 1$, yielding the gains

$$K_k = \begin{bmatrix} K_a & 0 \\ 0 & K_\omega \\ 0 & K_\phi \end{bmatrix}_k$$

where $K_a = (\Sigma_a/(\Sigma_a + \sigma_v^2)), K_\omega = (\Sigma_{\omega\phi}/(\Sigma_\phi + \sigma_v^2 \hat{a}_k^{-2}))$, and $K_\phi = (\Sigma_\phi/(\Sigma_\phi + \sigma_v^2 \hat{a}_k^{-2}))$. The nonlinear observer then becomes $\hat{a}_{k+1} = \hat{a}_k + ((\Sigma_a(y_k^{(1)} \cos\hat{\phi}_k + y_k^{(2)} \sin\hat{\phi}_k))/(\Sigma_a + \sigma_v^2)), \hat{\omega}_{k+1} = \hat{\omega}_k + ((\Sigma_{\omega\phi}(y_k^{(1)} \cos\hat{\phi}_k + y_k^{(2)} \sin\hat{\phi}_k))/(\hat{a}_k\Sigma_\phi + \sigma_v^2 \hat{a}_k^{-1}))$, and $\hat{\phi}_{k+1} = \hat{\phi}_k + \hat{\omega}_k + ((\Sigma_\phi(y_k^{(1)} \cos\hat{\phi}_k + y_k^{(2)} \sin\hat{\phi}_k - \hat{a}_k))/(\hat{a}_k\Sigma_\phi + \sigma_v^2 \hat{a}_k^{-1}))$. It can be seen that the observer is adaptive since the gains are a function of $\hat{a}_k$.

IV. Stability Conditions

In this section, we seek to identify conditions for the error system (5) to be asymptotically stable. The problem is recast in a passivity framework in which there is a cascade of a linear system and a block of memoryless nonlinearities shown in Fig. 1. This requires that (5) be reformulated as

$$e = u - G\gamma(e) \tag{9}$$

where G is a stable, linear system, $\gamma(\cdot)$ is a nonlinear function matrix satisfying specified sector conditions, and e and u denote the vectors $[e_k^{(1)}, e_k^{(2)}, e_k^{(3)}, \ldots, e_k^{(r)}]^T$ and $[u_k^{(1)}, u_k^{(2)}, u_k^{(3)}, \ldots, u_k^{(r)}]^T$, respectively. Let ∇ denote a forward difference operator with $\nabla e_k^{(i)} = e_k^{(i)} - e_{k-1}^{(i)}$. We set out the generalization of the discrete-time Popov criterion [8] for the multiple-input–multiple-output case. Since we are motivated by the problem of tracking superimposed signals, our attention is confined to $\gamma(\cdot)$ consisting of r identical, noninteracting nonlinearities. Namely, for $i = 1, 2, \ldots, r$, the nonlinearities $\gamma(e_k^{(i)})$ depend only on $e_k^{(i)}$. It is assumed that $\gamma(\cdot)$ satisfies sector conditions that may be interpreted as bounds existing on the slope of the components of $\gamma(\cdot)$ [8, Th. 14, p. 7]. In addition, let $\langle x, y\rangle$ denote the inner product of x and y.

Lemma: Consider the system (9), where u, e maps $\mathbf{R^r} \to \mathbf{R^r}$. Suppose that $\gamma(\cdot)$ consists of r identical, noninteracting nonlinearities, with $\gamma(e^{(i)})$ monotonically increasing in the sector $[0, \beta], \beta \geq 0, \beta \in \mathbf{R}$, i.e.,

$$0 \leq \gamma\left(e^{(i)}\right)\Big/ e^{(i)} \leq \beta \tag{10}$$

$\forall e^{(i)} \in \mathbf{R}$ with $e^{(i)} \neq 0$. Let G be a causal, stable, time-invariant map $\mathbf{R^r} \to \mathbf{R^r}$ having finite gain, and suppose that G has a z-transform $\hat{G}(e^{j\theta})$, which is bounded on the unit circle. Let I denote an $r \times r$ identity matrix. Suppose that for some $q \geq 0, q \in \mathbf{R}$, there is a $\delta > 0, \delta \in \mathbf{R}$ such that

$$\langle (G + q\nabla G + I\beta^{-1})e, e\rangle \geq \delta\langle e, e\rangle \tag{11}$$

$\forall e^{(i)} \in \mathbf{R^r}$. Under these conditions, then, $u \in \ell_2 \Rightarrow e, \gamma(e) \in \ell_2$.

Proof: From (9), we have $\nabla u = \nabla e + \nabla G\gamma(e)$ and

$$u + q\nabla u = (G + q\nabla G + I\beta^{-1})\gamma(e) + e - I\beta^{-1}\gamma(e) + q\nabla e. \tag{12}$$

Then

$$\langle u + q\nabla u, \gamma(e)\rangle = \langle e - I\beta^{-1}\gamma(e), \gamma(e)\rangle + \langle q\nabla e, \gamma(e)\rangle + \langle (G + q\nabla G + I\beta^{-1})\gamma(e), \gamma(e)\rangle. \tag{13}$$

Consider the first term on the right-hand side of (13). Since the $\gamma(e)$ consists of noninteracting nonlinearities, $\langle \gamma(e), e\rangle = \sum_{i=1}^r \langle \gamma(e^{(i)}), e^{(i)}\rangle$, and $\langle e - \gamma(e)I\beta^{-1}, \gamma(e)\rangle = \sum_{i=1}^r \langle e^{(i)} - \gamma(e^{(i)})I\beta^{-1}, e^{(i)}\rangle \geq 0$. Using the approach of [8] together with the sector conditions on the identical noninteracting nonlinearities (10), it can be shown that expanding out the second term of (13) yields $\langle \nabla e, \gamma(e)\rangle \geq 0$. Using $\|\nabla u\| \leq 2\|u\|$ [8, p. 192], the Schwartz inequality, and the triangle inequality, it can be shown that

$$\langle (u + q\nabla u, \gamma(e)\rangle \leq (1 + 2q)\|u\|. \tag{14}$$

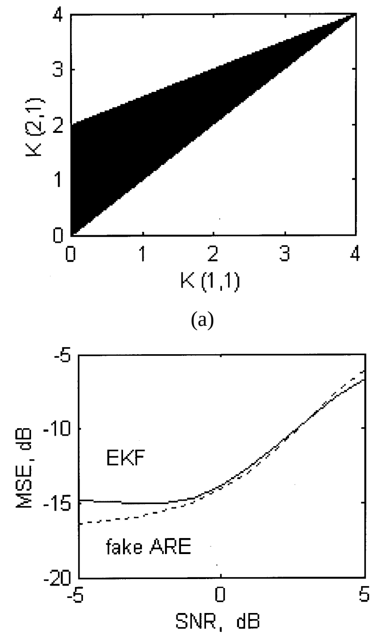
It follows from (11), (13), and (14) that $\|\gamma(e)\|^2 \leq (1 + 2q)\delta^{-1}\|u\|$; hence, $\gamma(e) \in \ell_2$. We also have $G\gamma(e) \in \ell_2$ since the gain of G is finite. △

If the linear part is stable and bounded on the unit circle, then (11) becomes

$$\lambda_{\min}\{[I + q(I - z^{-1}I)][\hat{G}(e^{j\theta}) + \hat{G}(e^{j\theta})^*] + \beta^{-1}\} \geq \delta \tag{15}$$

for $z = e^{j\theta}, 0 \leq \theta \leq \pi$ [8, pp. 175 and 194]. The Lemma, together with (15), are used to precalculate a stability region for the censuring the components of the K_k (see Section III-B) in a one-signal demodulation example below. In a subsequent two-signal demodulation example, the K_k possesses a larger

$$\bar{C}^{(i,j)} = \begin{bmatrix} \cos\left(\hat{\phi}_k^{(i)} - \hat{\phi}_k^{(j)}\right) & 0 & \hat{a}_k \sin\left(\hat{\phi}_k^{(i)} - \hat{\phi}_k^{(j)}\right) \\ -\frac{1}{\hat{a}_k^{(i)}} \cos\left(\hat{\phi}_k^{(i)} - \hat{\phi}_k^{(j)}\right) & 0 & \frac{\hat{a}_k^{(j)}}{\hat{a}_k^{(i)}} \cos\left(\hat{\phi}_k^{(i)} - \hat{\phi}_k^{(j)}\right) \end{bmatrix}$$



(a)

(b)

Fig. 2. (a) Stable gain space for Example 1. (b) Demodulation performance for Example 1.

number of components, in which case, it is more convenient to evaluate (15) at each time k.

V. Frequency Demodulation Examples

Example 1: Consider the problem of demodulating a unity amplitude FM signal. With respect to the model (1, 2), let $x_k = [\omega_k \;\; \phi_k]^T$

$$A = \begin{bmatrix} \mu_\omega & 0 \\ 1 & 1 \end{bmatrix}$$

$y_k = \sum_{i=1}^{2} e^{j\phi_k} + v_k$, where ω_k, ϕ_k, y_k, and v_k denote the instantaneous frequency, instantaneous phase, observations, and measurement noise, respectively. Consider the error system

$$\begin{bmatrix} \tilde{\omega}_{k+1} \\ \tilde{\phi}_{k+1} \end{bmatrix} = \begin{bmatrix} \mu_\omega & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\omega}_k \\ \tilde{\phi}_k \end{bmatrix} - \begin{bmatrix} K_{11} \\ K_{21} \end{bmatrix} \sin \tilde{\phi}_k + w_k \quad (16)$$

for $K_{11}, K_{21} \in \mathbf{R}$. An appropriate reformulation of (16) is

$$\begin{bmatrix} \tilde{\omega}_{k+1} \\ \tilde{\phi}_{k+1} \end{bmatrix} = \begin{bmatrix} \mu_\omega & -K_{11} \\ 1 & 1 - K_{21} \end{bmatrix} \begin{bmatrix} \tilde{\omega}_k \\ \tilde{\phi}_k \end{bmatrix} + \gamma\left([0 \quad 1] \begin{bmatrix} \tilde{\omega}_k \\ \tilde{\phi}_k \end{bmatrix} \right) + w_k \quad (17)$$

where $\gamma(x) = x - \sin x$. The z-transform of the linear part of (17) is $\hat{G}(z) = ((K_{21}z + K_{21} + \mu_\omega K_{11})/(z^2 + (K_{21} - 1 - \mu_\omega)z + K_{11} + 1 - \mu_\omega K_{21}))$. The nonlinearity $\gamma(x)$ satisfies the sector condition (10): $0 \leq ((x - \sin x)/x) < 1.22 \;\; \forall x \in \mathbf{R}$. Candidate gains may be assessed by checking that $\hat{G}(z)$ is stable and then applying the test condition (15), which is simplified for the single-input–single-output case [8, p. 194]. The resulting gain space that ensures the local stability of (16), where $\mu_\omega = 0.9$, is shown in Fig. 2(a).

The solution to (17) also can be written as the sum of a "natural part" (due to non zero initial error) and the remaining "forced part" (due to state noise). In a linear time-invariant system, for example, the "forced part" of the output is the input process convolved with the impulse response. Clearly, the "forced part" is in ℓ_2 from the passivity argument. Since the "natural part" can be written as a product of the initial conditions and some function of the delta function, clearly, the "natural part" is in ℓ_2. Thus, the location of (K_{11}, K_{21}) within the region of Fig. 2(a) is sufficient for (17) to be asymptotically stable, independent of initial conditions.

A sample of 8-kHz speech (i.e., the phrase "Matlab is number one") was used to synthesize a unity amplitude FM signal. An EKF was constructed using the model (1) and $\sigma_\omega^2 = 0.02$. In an adaptive observer defined within Section III-B, it was found that suitable parameter choices were

$$\Sigma^{(1)} = \begin{bmatrix} 0.001 & 0.08 \\ 0.08 & 0.7 \end{bmatrix}.$$

Simulations were conducted for 100 realizations of additive Gaussian white noise at 3 dB signal-to-noise ratio (SNR) steps, with zero initial conditions: The gains were censured at each time k, according to the stable space of Fig. 2(a). A histogram of mean square error (MSE) exhibited by the two demodulators is shown in Fig. 2(b). In the one signal case, it can be seen that compared with the EKF, the fake ARE approach provides a slight improvement for low SNR, at the cost of degraded performance for high SNR.

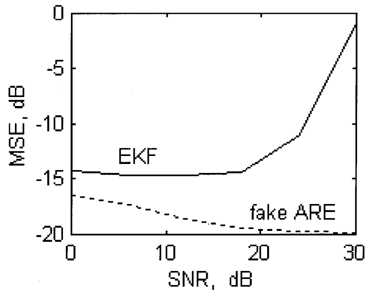
Example 2: Consider the problem of demodulating two superimposed FM signals present in the frequency channel. Two 8-kHz speech samples (i.e., "Matlab is number one" and "Number one is Matlab"), centred at ± 0.25 rad/s, were used to synthesize two superimposed, unity amplitude, FM signals. An EKF was constructed with $\mu_\omega^{(i)} = 0.9$ and $(\sigma_\omega^{(i)})^2 = 0.02$. A fake ARE filter (3) was constructed, and it was found that a suitable parameter choice for an arbitrary solution in (8) is $\Sigma = \text{diag}[\Sigma^{(1)}, \Sigma^{(1)}]$. Neglecting observation noise, a suitable approximation of the error system in the form (9) is

$$\begin{bmatrix} \tilde{\omega}_{k+1}^{(1)} \\ \tilde{\phi}_{k+1}^{(1)} \\ \tilde{\omega}_{k+1}^{(2)} \\ \tilde{\phi}_{k+1}^{(2)} \end{bmatrix} = (A - K_k \bar{C}) \begin{bmatrix} \tilde{\omega}_k^{(1)} \\ \tilde{\phi}_k^{(1)} \\ \tilde{\omega}_k^{(2)} \\ \tilde{\phi}_k^{(2)} \end{bmatrix} - K_k \begin{bmatrix} \sin \tilde{\phi}_k^{(1)} - \tilde{\phi}_k^{(1)} \\ \sin \tilde{\phi}_k^{(2)} - \tilde{\phi}_k^{(2)} \end{bmatrix} \quad (18)$$

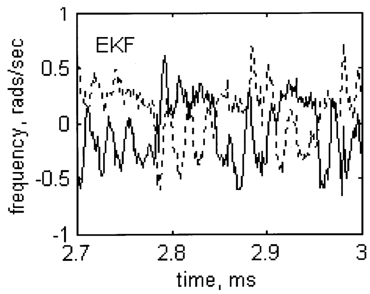
where

$$\bar{C} = \begin{bmatrix} 0\;1\;0\;0 \\ 0\;0\;0\;1 \end{bmatrix}.$$

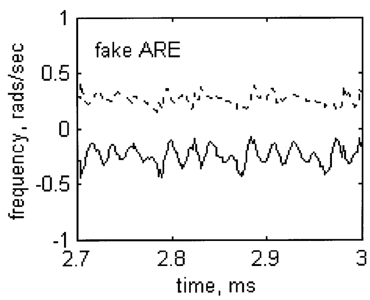
It follows that the linear part of (18) may be written as $G(z) = \bar{C}(zI - (A - K_k\bar{C}))^{-1}K_k$. From Section V, for the stability of (18), $\hat{G}(z)$ must be stable, and a $\delta > 0$ must be found satisfying (15) for a $q > 0$. In contrast to Example 1, where the stable gain space was precalculated, here, the test condition (15) was calculated at each time k and used to censure the gains. It was found that $\beta = 1.2$; then with $q = 0.001$, a $\delta = 0.82$ was sufficient to satisfy (15) for stable $\hat{G}(z)$.



(a)



(b)



(c)

Fig. 3. (a) Demodulation performance for Example 2. (b) . EKF frequency tracks for Example 2. (c) Fake ARE frequency tracks for Example 2.

Simulations were conducted with 100 realizations of additive Gaussian white measurement noise, from 0 to 30 dB SNR. A histogram of mean square error (MSE) exhibited by the two demodulators is shown in Fig. 3(a). It can be seen that the EKF performance degrades with increasing SNR. The presence of co-channel signals causes outliers in the frequency estimates. The locally stable fake ARE filter is seen to provide some robustness to outliers; in particular, at 30 dB SNR, the reduction in MSE approaches 20 dB.

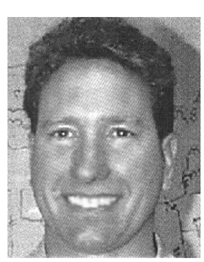
Two mechanisms have been observed for occurrence of outliers or faults within co-channel demodulators. First, errors can occur in the state attribution, i.e., there is correct tracking of some component speech message segments, but the tracks are inconsistently associated with the individual signals. This is illustrated by the example frequency estimate tracks shown in Figs. 3(b) and (c). The solid and dashed lines in both of Figs. 3(b) and (c) indicate two example co-channel frequency tracks. Second, the phase unwrapping can be erroneous so that the frequency tracks bear no resemblance to the underlying messages. These faults can occur without any significant deterioration in the error residual. An insight into co-channel fault behavior follows from an observability perspective. (Observability refers to whether or not the states can be uniquely reconstructed from the measurements.) It is conjectured that the phase ambiguities appear because the locally linearized system loses observability. The co-channel demodulators have been observed to be increasingly fault prone at higher SNR. This arises because lower SNR designs possess narrower bandwidths and are less sensitive to nearby frequency components. Fig. 3 illustrates the trade-off between stability and optimality. In particular, it can be seen from Fig. 3(b) that the example EKF speech estimates exhibit faults in the state attribution. This contrasts with Fig. 3(c), where the example fake ARE filter tracks exhibit stable state attribution at the cost of conservatively filtered speech estimates.

VI. Conclusion

The fake ARE technique has been applied in the development of an adaptive nonlinear filter for tracking multiple signals. A passivity framework has been used to arrive at conditions for local error stability. The results of simulation studies for the problem of demodulating FM signals have been presented in which the fake ARE approach demonstrates improved stability at the cost of optimality.

References

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Englewood Cliffs, NJ: Prentice-Hall, 1979.
[2] P. J. Parker and B. D. O. Anderson, "Frequency tracking of nonsinusoidal periodic signals in noise," *Signal Process.*, vol. 20, pp. 127–152, 1990.
[3] G. A. Einicke and L. B. White, "Robust extended Kalman filtering," *IEEE Trans. Signal Processing*, vol. 47, pp. 2596–2599, Sept. 1999.
[4] R. R. Bitmead, A.-C. Tsoi, and P. J. Parker, "Kalman filtering approach to short time Fourier analysis," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-34, pp. 1493–1501, June 1986.
[5] M.-A. Poubelle, R. R. Bitmead, and M. Gevers, "Fake algebraic Riccati techniques and stability," *IEEE Trans. Automatic Contr.*, vol. 33, pp. 379–381, Apr. 1988.
[6] R. R. Bitmead, M. Gevers, and V. Wertz, *Adaptive Optimal Control*. Sydney, Australia: Prentice-Hall, 1990, pp. 86–87.
[7] R. R. Bitmead and M. Gevers, "Riccati difference and differential equations: Convergence, monotonicity and stability," in *The Riccati Equation*, S. Bittanti, A. J. Laub, and J. C. Willems, Eds. New York: Springer-Verlag, 1991, ch. 10.8.1.
[8] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input Output Properties*. New York: Academic, 1975.
[9] G. A. Einicke and L. B. White, "Estimation of frequency and phase modulated signals using fake algebraic Riccati techniques," in *Proc. IEEE Int. Symp. Signal Processing Applications*, vol. 2, Gold Coast, Australia, Aug. 1992, pp. 339–343.
[10] M. Simandl, J. Kralovec, and P. Tichavsky, "Filtering, predictive and smoothing Cramer-Rao bounds for discrete-time nonlinear dynamic systems," *Automatica*, vol. 37, pp. 1703–1716, 2001.

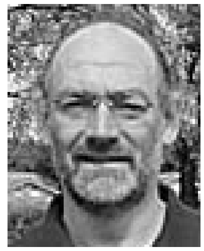


Garry A. Einicke (M'97) received the B.E., M.E., and Ph.D. degrees from the University of Adelaide, Adelaide, Australia, in 1979, 1991, and 1996 respectively.

He spent 1980 to 1996 with the Defence Science and Technology Organization, Canberra, Australia, where he was a Senior Research Scientist in signal analysis discipline. He has been with the Commonwealth Scientific and Industrial Research Organization, Kenmore, Australia, since 1997, where he is a Senior Research Scientist in Mine Automation Research.

Langford B. White (SM'00) received the B.Sc.(maths), B.E.(Hons), and Ph.D. (E.E.) degrees in 1984, 1985, and 1989, respectively, from the University of Queensland, Brisbane, Australia.

From 1986 to 1999, he worked for the Defence Science and Technology Organisation, Salisbury, Australia. Since 1999, he has been Professor with the School of Electrical and Electronic Engineering, the University of Adelaide, Adelaide, Australia, where he is also Director of the Centre for Internet Research. His research interests include signal processing, control, telecommunications, and internet engineering.



Robert R. Bitmead (F'91) was born in Sydney, Australia, in 1954. He received the B.Sc. degree in applied mathematics from the University of Sydney and the M.E. and Ph.D. degrees from the University of Newcastle, Callaghan, Australia, in 1976, 1977, and 1979, respectively.

He has been with the Cymer Endowed Chair with the Department of Mechanical and Aerospace Engineering, University of California at San Diego, La Jolla, since 1999 and has held faculty positions at the Australian National University, Canberra, from 1982 to 1999, and James Cook University of North Queensland from 1980 to 1982. He has held visiting faculty positions at Cornell University, Ithaca, NY; University of Louvain, Louvain-la-Neuve, Belgium; INRIA, Paris, France, and Kyoto University, Kyoto, Japan. He is the Editor of Adaptive and Intelligent Control for *Automatica*. His research interests are in the areas of adaptive systems, estimation, control design, modeling, and telecommunications.

Dr. Bitmead is a Fellow of the Australian Academy of Technological Sciences and Engineering.