Infrared Behavior and Fixed Points in Landau-Gauge QCD

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We investigate the infrared behavior of gluon and ghost propagators in Landau-gauge QCD by means of an exact renormalization group equation. We explain how, in general, the infrared momentum structure of Green functions can be extracted within this approach. An optimization procedure is devised to remove residual regulator dependences. In Landau-gauge QCD this framework is used to determine the infrared leading terms of the propagators. The results support the Kugo-Ojima confinement scenario. Possible extensions are discussed.

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Charged particles have a long-range nature, whereas localized objects are neutral. For local quantum field theories one derives more generally that every gauge-invariant localized state is singlet with respect to the unbroken global charges of the gauge symmetry. Thus, in QCD without a Higgs mechanism any localized physical state must be colorless. This extends to all physical states only if a mass gap is present. Then, color-electric charge superselection sectors cannot arise, and every gauge-invariant state must also be a color singlet. These are the signatures of confinement.

Within covariant linear gauges, the necessary conditions for confinement were formulated more than 20 years ago by Kugo and Ojima [1]. They express the necessity of a mass gap and the conditions for avoiding the Higgs mechanism in a covariant continuum formulation of QCD in terms of local fields. Both conditions are encoded in the critical infrared exponents of gluonic and ghost correlations in Landau-gauge QCD [2]. For the mass gap, the massless transverse gluon states of perturbation theory must be screened nonperturbatively, leading to an infrared suppression for the gluon propagator. In turn, the global gauge charges entail the infrared enhancement and dominance of the ghost correlations.

The infrared behavior just described has been found in solutions of truncated Dyson-Schwinger (DS) equations [3]. Since then, the same qualitative behavior has emerged in a variety of nonperturbative approaches including an increasing number of lattice simulations [4] and investigations based on the Fokker-Planck–type diffusion equation of stochastic quantization [5]. Quantitatively, the latter leads to the same exponents as the DS equations, which reflects an equivalence between the two formulations under certain conditions [5,6]. Interesting in their own right, such interrelations are especially useful for finding ways to go beyond the particular approximations employed in each case.

Here we present an investigation of the infrared sector of Landau-gauge QCD by means of a nonperturbative flow equation [7,8]. Central to the approach is the effective action Γ_k, where quantum fluctuations with momenta $p^2 > k^2$ are already integrated out, and k denotes an infrared cutoff scale. The variation with k leads to a flow equation for Γ_k, thereby interpolating between the classical action in the ultraviolet and the full quantum effective action in the infrared where the cutoff is removed. Moreover, it has been shown that truncations can be systematically improved by means of an optimization procedure [9]. The formalism has its particular merits in gauge theories as it is amiable to approximations being nonlocal in momenta and fields, precisely the behavior we expect in QCD. So far it has been applied to Landau-gauge QCD for a determination of the heavy quark effective potential [10] and effective quark interactions above the confinement scale [11]; for a brief review and further applications in Yang-Mills theories, see [12–14].

Conceptually, the flow comprises a system of coupled differential equations for all Green functions which involves only dressed vertices and propagators. Hence, the correct renormalization group (RG) scaling is displayed by the full flow and truncations with the correct symmetry properties. This is one of the main contradistinctions to DS equations or stochastic quantization which involve both bare and dressed Green functions. This makes it often difficult in the latter two approaches to implement the correct RG scaling within truncations.

In this Letter we explain how the full infrared momentum behavior of Green functions is extracted within a fixed point regime. In Landau-gauge QCD, this allows for a simple determination of infrared coefficients for gluon and ghost propagators as well as the running coupling. Results for general regulators are presented and interpreted in the context of an optimization of the flow. The systematic extension of the truncation is discussed.

We begin by introducing the classical gauge fixed action for QCD in a general covariant gauge in four dimensions. Including also the ghost action, it reads

$$S_{\rm cl} = \frac{1}{2}\int {\rm tr} F^2 + \frac{1}{2\xi}\int (\partial_\mu A^\mu)^2 + \int \bar{C}\partial_\mu D_\mu C. \quad (1)$$

The flow equation approach relies on a momentum cutoff for the propagating degrees of freedom. Here, the momentum regularization for both, the gluon and the ghost, is achieved by $S_{\rm cl} \to S_{\rm cl} + \Delta S_k$ with

$$\Delta S_k = \frac{1}{2} \int A_a^\mu R_{\mu\nu}^{ab} A_b^\nu + \int \bar{C}_a R^{ab} C_b. \qquad (2)$$

The functions R implement the infrared momentum cutoff at the momentum scale $p^2 \approx k^2$. The scale dependence of the regularization in (2) induces a flow equation for the effective action. With $\phi = (A, C, \bar{C})$ and $t = \ln k$, the flow equation takes the form

$$\begin{aligned} \partial_t \Gamma_k[\phi] = \frac{1}{2} \int d^4 p G_{ab}^{\mu\nu}[\phi](p,p) \partial_t R_{\mu\nu}^{ba}(p) \\ - \int d^4 p G^{ab}[\phi](p,p) \partial_t R^{ba}(p), \end{aligned} \qquad (3)$$

where $G[\phi](p,q) = (\Gamma_k^{(2)}[\phi] + R)^{-1}(p,q)$ denotes the full regularized propagator. The flow is finite in both, the infrared and the ultraviolet, by construction. Effectively, the momentum integration in (3) only receives contributions for momenta in the vicinity of $p^2 \approx k^2$. Consequently, it has a remarkable numerical stability. The flow solely depends on dressed vertices and propagators leading to consistent RG scaling on either side of (3). The flow equations for propagators and vertices are obtained from (3) by functional derivatives.

To verify the Kugo-Ojima confinement criterion we have to determine the momentum behavior of the propagators in the deep infrared region

$$k^2 \ll p^2 \ll \Lambda_{\rm QCD}^2. \qquad (4)$$

In this domain, quantum fluctuations are already integrated out and *physical* quantities, as well as general vertex functions upon appropriate normalization, show no k dependence. The inequality $p^2 \ll \Lambda_{\rm QCD}^2$ entails that we are already deep in the infrared regime. No new physics arises and the flow equation (3) constitutes only the shifting of the transition regime at momenta about k^2 towards smaller scales. Accordingly, Green functions $\Gamma_k^{(n)} \equiv \delta^n \Gamma_k / \delta\phi_1 \cdots \delta\phi_n$ can be parametrized as

$$\Gamma_k^{(n)}(p_i^2 \ll \Lambda_{\rm QCD}^2) = z_n \hat{\Gamma}^{(n)}(p_i, p_i^2/k^2), \qquad (5)$$

where p_i stands for the momenta $p_1, \ldots, p_n$. Here z_n are possibly k-dependent prefactors accounting for an RG scaling that leaves the full action invariant. This reduces the remaining k dependence in $\hat{\Gamma}^{(n)}$ to dimensionless ratios $x = p^2/k^2$. It thus provides a k-independent interpolation between the physical infrared regime (4) and the regularized behavior for $p^2 \ll k^2$. Equation (5) implies a fixed point behavior. So far, fixed point investigations within the exact renormalization group have been put forward within derivative expansions. Here we provide a new truncation scheme for accessing the full momentum dependence of Green functions in the infrared limit. Taking advantage of their simple structure in the physical regime (4), we introduce the following parametrization for the gluon and ghost two-point functions

$$\begin{aligned} \Gamma_{k,A}^{(2)}(p^2) &= z_A Z_A(x) p^2 \Pi(p) \mathbb{1} + \text{longitudinal}, \\ \Gamma_{k,C}^{(2)}(p^2) &= z_C Z_C(x) p^2 \mathbb{1}, \end{aligned} \qquad (6)$$

where

$$Z(x) = x^\kappa (1 + \delta Z(x)).$$

Here $\Pi_{\mu\nu}(p) = \delta_{\mu\nu} - p_\mu p_\nu / p^2$, $\mathbb{1}_{ab} = \delta_{ab}$, and Z, δZ, κ without subscripts stand for $Z_{A/C}$, $\delta Z_{A/C}$, $\kappa_{A/C}$. The longitudinal part of the gluon two point function in (6) does not contribute to the Landau-gauge flows considered below. The momentum dependence of (6) in the asymptotic regimes $x \gg 1$ and $x \ll 1$ fixes δZ in both regimes: the prefactor x^κ is to encode the leading behavior in the momentum regime (4) where $1 \ll x \ll \Lambda_{\rm QCD}^2/k^2$. Hence $\partial_t \kappa = 0$ and δZ tends to zero, apart from possible logarithms. The k independence of (6) at $k = 0$ implies $\partial_t z = 2\kappa z$. In the limit of small x, we recover the trivial momentum behavior $\Gamma_k^{(2)}(p^2) = m_k^2 + z_1 p^2 + O(p^3)$ as a consequence of the cutoff. Accordingly,

$$\begin{aligned} \delta Z(x \to \infty) &\to 0 + \text{logarithmic terms}, \\ \delta Z(x \to 0) &\to -1 + \mu x^{-(1+\kappa)} + O(x^{-\kappa}), \end{aligned} \qquad (7)$$

where the limit $x \to \infty$ implies $k \to 0$ as $x \ll \Lambda_{\rm QCD}^2/k^2$; see Eq. (4). For $x \approx 1$ the propagators show a regulator-dependent interpolation between the physical infrared behavior and the trivial cutoff regime. Also

$$\partial_t \delta Z(x) = -2x \partial_x \delta Z \qquad (8)$$

reflects that the dependence of δZ on k stems from the interpolation property while its shape is independent of it. The truncation is completed by vertices $\Gamma_k^{(n)}$ with

$$\Gamma_k^{(n)} = z_n S_{\rm cl}^{(n)} \quad \text{for } n \geq 3, \qquad (9)$$

with renormalized coupling g. For vanishing cutoff $k = 0$ we deal with the full theory, implying vanishing δZ as $x \to \infty$. Thus, for $k = 0$ we can use the results of the DS analysis in [6]: nonrenormalization of the ghost-gluon vertex implies $z_{\bar{c}Ac} = 1$ and

$$\kappa_A = -2\kappa_C, \qquad \alpha_s = \frac{g^2}{4\pi} \frac{1}{z_A z_C^2}. \qquad (10)$$

It follows from (10) and $\partial_t z = 2\kappa z$ that α_s has a fixed point in the infrared, $\partial_t \alpha_s = 0$. The z factors for the three- and four-gluon vertices are proportional to $\alpha_s^{1/2} z_A^{3/2}$ and $\alpha_s z_A^2$, respectively. Also, within the truncation (9) there is no mass term for the ghost, corresponding to $\mu_C = 0$ in (7). In summary, the truncation (6) and (9) with (7) and (10) has the RG properties of Landau-gauge QCD and satisfies the truncated Slavnov-Taylor identities in the presence of the regulator.

We proceed with determining the strong coupling fixed point α_s and the infrared exponents κ from (3) in the truncations put down in (6). We fix the tensor structure of the regulators as $R^{ab}_{\mu\nu} = R_A \delta_{\mu\nu}\delta^{ab}$ and $R^{ab} = R_C \delta^{ab}$, leaving the scalar part $R = zp^2 r(x)$ at our disposal. Integrating $\partial_{t'}\Gamma^{(2)}_k$ from $k' = 0$ to $k' = k$ leads to an integral equation for $\Gamma^{(2)}_k(p) - \Gamma^{(2)}_0(p) = z\delta Z(x)p^2$ with

$$\delta Z_{A/C}(x) = \frac{\alpha_s}{\pi^2} N \int_x^\infty \frac{dx'}{x'^{2+\kappa}} f_{A/C}(x'), \tag{11}$$

where

$$f_A(x) = -\frac{1}{3}\int_0^\infty dy y \int_{-1}^1 dt \frac{y}{u} \frac{(1-t^2)^{3/2}\mathcal{T}_C(y)}{r_C(u) + Z_C(u)},$$
$$f_C(x) = \frac{x}{2}\int_0^\infty dy y \int_{-1}^1 dt (1-t^2)^{3/2} \frac{1}{u}\left(\frac{y}{u}\frac{\mathcal{T}_C(y)}{r_A(u) + Z_A(u)} + \frac{\mathcal{T}_A(y)}{r_C(u) + Z_C(u)}\right),$$

and $u = x + y + 2t\sqrt{xy}$. We have also introduced the abbreviation $2\mathcal{T} = k^2 G(\partial_t R)G$, which reads explicitly

$$\mathcal{T}(y) = \frac{1}{y}\frac{\kappa r(y) - y r'(y)}{(r(y) + Z(y))}.$$

In (11), the integration variable k' was traded for x', using $z = \hat{z}x'^{-\kappa}(\Lambda^2/p^2)^\kappa$ with $\partial_{t'}\hat{z} = 0$. The purely gluonic diagrams in (3) are suppressed by powers of $p^2/\Lambda^2_{\rm QCD}$ in the fixed point regime and do not contribute to (11).

For the sharp cutoff the kernel $\mathcal{T}$ reads $\mathcal{T}(x) = \delta(x - 1)/(zZ(1))$. Then the t and y integrals in (11) can be performed analytically leading to a lengthy sum of hypergeometric functions. For generic smooth cutoff, κ and α_s are computed from (11) in the limit $x \to 0$ as follows. On the right-hand side of (11), the integrands $f_{A/C}$ are suppressed for small momenta x and y due to $\mathcal{T}_{A/C}$. This implies that δZ can contribute only for sufficiently large arguments where it tends to zero due to (7). Therefore we can safely neglect δZ under the momentum trace on the right-hand side of (11) in a first step. Then, we proceed to fully consistent solutions δZ of (11) by iteration. On the left-hand side of the gluon equation the term $\delta Z_A(x \to 0)$ is determined by (7). In the ghost equation, we have to carefully identify the terms. For $\kappa_C > 0$ we read off from (7) that $\delta Z_C(x \to 0)$ diverges. However, $\Gamma^{(2)}_{k,C}(p^2) = z_C x^{\kappa_C}(1 + \delta Z_C(x))$ has a Taylor expansion about $x = 0$ due to the IR regularization. Hence the coefficient of the constant term in δZ_C is -1.

The integral equation (11) can be solved numerically for κ and α_s. We have restricted ourselves to the domain $\kappa_C \in (\frac{1}{2}, 1)$ [15]. For *general* regulator $r(x)$ we obtain

$$\kappa_C = 0.595\,35\ldots, \qquad \alpha_s = 2.9717\ldots, \tag{12}$$

which agrees with the analytic DS result [6]. This might seem surprising given the completely different representations. However, the equivalence can be proven as follows: in the derivation of (12) we employed the truncation (6) and (9) with the additional constraint $\delta Z \approx 0$ under the momentum trace in (3). Integrating the flow (3) in this setting from 0 to k leads to

$$\begin{aligned} z_A p^2 x^{\kappa_A}\delta Z_A &= \mathrm{Tr}[GS^{(3)}GS^{(3)} - \tfrac{1}{2}GS^{(4)}G]^k_0, \\ z_C p^2 x^{\kappa_C}\delta Z_C &= \mathrm{Tr}[GS^{(3)}GS^{(3)}]^k_0, \end{aligned} \tag{13}$$

for the gluon and ghost two-point functions. Equation (13) already implies an UV renormalization at $p^2 = k^2$. This is reflected by (11) being finite. Within the present truncation the renormalized DS equations used in [6] are equivalent to (13). Beyond this truncation the DS equation and the flow equation differ. Apart from additional t-derivative terms, all vertices in (13) get dressed as distinguished from the DS equation, where both bare and dressed vertices are present. Consequently, the flow equation (3) carries the correct RG properties if the truncation for Γ_k does. In particular, Eq. (13) with the truncation (6) and (9) allows for a fully self-consistent renormalization. Both the left- and right-hand sides of (13) transform the same way under RG scalings, including the purely gluonic terms dropped in (11). Indeed, it is the latter fact which makes their neglection fully consistent. The RG properties are most obvious for regulators proportional to the wave function renormalizations z, $R \simeq z\tilde{R}$; see [13]. Then, the propagators can be written as $G = z^{-1}G_0$, where G_0 does not involve further wave function renormalization factors. Also, the vertex functions involve the appropriate multiplicative powers of z.

We proceed by iterating the above solution, also taking into account $\delta Z \neq 0$ on the right-hand side of the flow. Then (11) can be used for a numerical iterative determination of δZ and κ, α_s. We have computed α_s and κ for various classes of regulators. The exponent κ_C takes values in an interval bounded by the values (12) from above and the iterated sharp cutoff result $\kappa_C \simeq 0.539$ from below. However, the values (12) are singled out by their physical relevance: κ and α_s depend on the regulator. This reflects the fact that the truncation (6) and (9) *and* the choice of R define the approximation to the full problem. We expect that global extrema of $\kappa(R)$, $\alpha_s(R)$ represent an optimized approximation based on (6) and (9); see [9]. In general, it is very difficult to translate this idea into properties of optimal regulators or even in statements about the values of optimal observables. Here we present a new idea how to extract this information: let us assume that we have identified a regulator R_0 which is an extremum in the space of regulators in the sense that

$$\left[\mathrm{Tr}\left(\delta R \frac{\delta}{\delta R}\right)\Gamma^{(n)}_k\right]_{R=R_0} = 0 \tag{14}$$

for all δR. Equation (14) implies that $\kappa(R_0)$, $\alpha_s(R_0)$ are extrema, reflecting an optimization of physical quantities. Note that the $z_n(R_0)$ by themselves are not physical quantities, but they can be extrema up to RG scalings. However, (14) is more restrictive. For $n = 2$, (14) is the

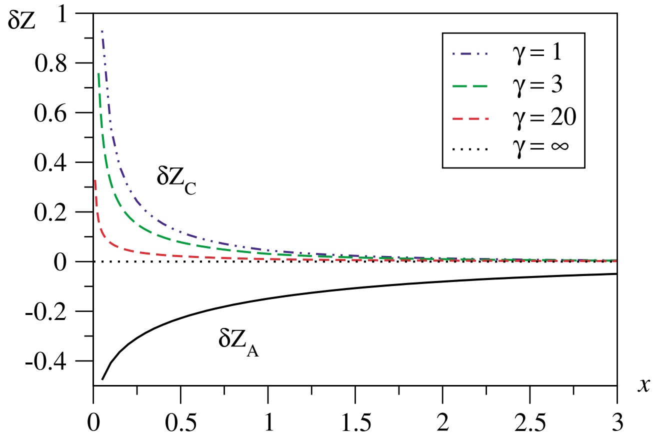

FIG. 1 (color online). δZ for regulators with (15).

additional demand, that $\delta Z(x)$ is an extremal curve. Hence the question arises whether solutions R_0 to (14) exist at all: the fixed point assumption underlying the whole analysis is valid for regulators R that lead only to a suppression of infrared modes. This implies a monotonous interpolation between the physical infrared regime $k^2 \ll p^2 \ll \Lambda_{\rm QCD}^2$ and the infrared regularized regime $p^2 \ll k^2$. Then, modulo RG scalings, extremal curves exist and are given by vanishing δZ.

The above considerations allow us to identify the optimal values for $\kappa_C(R)$ and $\alpha_s(R)$ without further computation. The correction terms proportional to δZ under the momentum trace are negligible near regulators R_0 and we are left with the noniterated Eq. (11) with regulator R_0. We have already shown that the noniterated solutions are independent of the regulator and we conclude that $\kappa_C(R_0)$, $\alpha_s(R_0)$ are given by (12). We also can construct an optimal regulator. For these regulators the effects of δZ on the right-hand side of (11) have to disappear. They need not satisfy (14) which applies only to a small subset of optimal regulators. In Fig. 1 we show $\delta Z_{A/C}$ for regulators $R = zp^2 r(x)$ with

$$r_C(x) = \theta^{-1}(x-1) - 1, \qquad r_A(x) = \gamma \frac{1}{x(1+x)}. \quad (15)$$

These regulators imply cutoff scales k for the ghost and $k_{\rm eff} = \gamma^{1/4} k$ for the gluon. As (4) also has to apply to $k_{\rm eff}$, the limit $\gamma \to \infty$ amounts to $k \to 0$. This limit yields the optimal values (12) as all contributions from δZ to the right-hand side of (11) tend to zero. Hence (15) with $\gamma \to \infty$ belongs to the class of optimal regulators. The regulator (15) is not in the extremal subset as only the ghost propagator satisfies (14) with $\delta Z_C(x \neq 0) = 0$. The γ dependence of δZ_A (via δZ_C) is negligible on the scale of Fig. 1.

In summary, we have shown how the nontrivial momentum dependence of Green functions in the infrared sector is extracted within the exact renormalization group. To that end we have developed new fixed point and optimization techniques applicable to this situation. This reasoning has been applied to the infrared behavior of ghost and gluon propagators in Landau-gauge QCD. Our results provide further evidence for the Kugo-Ojima scenario of confinement: the ghost propagator is infrared enhanced while the gluon propagator develops a mass gap. In a truncation with dressed propagators, we have recovered previous results from Dyson-Schwinger equations in a very simple manner. This is noteworthy also in view of the qualitative differences between these approaches. Our method allows for a straightforward improvement of the truncation by adding vertex corrections and further terms with the correct RG properties. The same applies to dynamical quarks. In both extensions the correct RG scaling as well as the inherent finiteness of the integrated flow are most important. Further results and details will be presented elsewhere.

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[15] The κ, α_s run from their perturbative values into the regime $\kappa_C \in (\frac{1}{2}, 1)$. We are interested in a more refined analysis of their infrared values rather than in the flow.