Active control of sound in a small single engine aircraft cabin with virtual error sensors

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Abstract

The harmful effects of aircraft noise, with respect to both comfort and occupational health, have long since been recognised, with many examples of sound control now implemented in commercial aircraft. However, the single engine light aircraft cabin is still an extremely noisy environment, which apparently has been side-lined by both cost and weight constraints, especially with respect to low frequency sound reduction. Consequently, pilots and passengers of these aircraft are still exposed to potentially damaging noise levels and hearing damage can only be avoided by the proper use of ear defenders. Minimisation of the noise around the occupants of the aircraft reduces the dependency of personal ear defenders and is conducive to a more comfortable, hygienic and less stressful environment. This thesis describes the basis of a theoretical and experimental project, directed at the design and evaluation of a practical active noise control (ANC) system suitable for a single engine light aircraft.

Results from initial experiments conducted in a single engine aircraft demonstrated the viability of ANC for this application. However, the extreme noise, the highly damped cabin, the multiple tone excitation, the severe weight limitations and the requirement of air worthiness certification severely complicated the problem of achieving noise reduction throughout the entire aircraft cabin. Compromising the objective to only achieving local control around the occupants still presented difficulties because the region of attenuated noise around the error sensors was so small that a nearby observer experienced no sound level reduction whatsoever.
The objective was therefore to move the control zone away from the error sensor and place a broad envelope of noise reduction immediately around the occupant’s head, through the use of “virtual sensors”, thus creating the perception of global noise control.

While “virtual sensors” are not new (Garcia-Bonito et al. (1996)), they are currently limited to acoustic pressure estimation (virtual microphones) via the initial measurement of an observer / sensor transfer function. In this research, new virtual sensor algorithms have been developed to:

1. minimise the sound level at the observer location
2. broaden the control region,
3. adapt to any physical system changes and
4. produce a control zone that may ultimately follow an observer’s head

The performance of the virtual sensors were evaluated both analytically and experimentally in progressively more complex environments to identify their capabilities and limitations. It was found that the use of virtual sensors would, in general, attenuate the noise at the observer location more effectively than when using conventional remotely placed error sensors. Such a control strategy was considered to be ideal for a light single engine aircraft, because it would only require small light speakers (possibly fitted into a head-rest) to achieve a broad control zone that envelopes the region around the occupants’ heads.
Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, everything that is presented in this thesis is my own original work and has not been presented previously for the award of any other degree or diploma in any University. If accepted for the award of the degree of Ph.D. in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

Colin D. Kestell.
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Chapter 1

Introduction

Before World War 2 the world’s commercial aircraft fleet was modest in size and consisted entirely of propeller driven aircraft. The idea of noise reduction in these early years was only of superficial interest or purely academic, addressing no commercial or regulatory requirement. In fact, during the war, the loud distinctive and familiar noise emissions of "friendly aircraft" comforted the nation at war (Smith (1989)). However, noise is now an increasing issue of environmental and occupational concern and is a recognised contributor to long term hearing damage, stress, fatigue and general annoyance.

Ignoring the continued health of employees, or the comfort of customers, can now cost employers or commercial organisations through civil actions or loss of custom. Although, the single engine light aircraft has been somewhat overlooked, noise reduction in large commercial airliners, in one form or another, is now common place. The single engine light aircraft remains a noisy environment for the pilot and an unpleasant prospect for potential passengers. While higher frequency noise may be addressed by passive means, the longer wavelengths associated with low frequencies means that effective damping materials would have to be impractically large and massive. Passive noise control is therefore not a practical solution for propeller noise or low frequency engine noise in the already cramped environment of a small, single engine aircraft with obvious reasons for weight restrictions.
The option at these lower frequencies is "anti-noise" or more correctly “active noise control (ANC)”. The concept is not new, in fact it is over 60 years old. The earliest published documents on ANC were patent applications, one of which was submitted in 1936, by Paul Lueg (1936). He proposed the basis of ANC, which is widely cited as the first publication of the concept. Lueg suggested that a transducer in the path of a noise source could be used to generate a secondary, canceling noise (figure 1.1).

Now a field of immense interest, active noise control is detailed comprehensively in a number of well known publications (Hansen and Snyder (1997), Fuller and Elliott (1996), Nelson and Elliott (1995), Tokhi and Leitch (1992) and Kuo and Morgan (1996)).

1.1 Objective

The low frequency, enclosed field and periodic nature of the light aircraft noise, make it an ideal candidate for adaptive feedforward active noise control. The objective of this research is therefore to design and evaluate a practical active noise control strategy for reducing the
perceivable noise in a single engine light aircraft cabin. It will be shown that actively controlling the noise throughout the entire cabin (global control) presents a number of practical difficulties, while limiting the extent of the control to small zones (local control) can result in increasing the noise in other locations. As stated, reducing the “perceivable” noise is the objective and it is the hypothesis of this thesis that this can be satisfied with a steerable control zone using virtual error sensors that may ultimately track the location of an observer’s head.

### 1.2 Scope

The practical application of active noise control in a single engine light aircraft cabin brings together many specific fields of acoustic research, all of which are considered in the following chapters. This thesis commences with a review of the most recent and applicable published literature, which addresses:

- active noise control in similar applications and its relevance to a single engine light aircraft,
- published aircraft noise data, to identify the principle mechanisms of cabin noise,
- modelling of the enclosed primary sound field and predicting the success of active noise control and
- active noise control mechanisms, to identify an appropriate control and sensing strategy.

Noise measurements conducted in the cabin of a Piper Archer validated the findings of the literature review and provided a basis for some initial active noise control experiments that were conducted both in the laboratory and in the aircraft.

Achieving global noise reduction throughout the aircraft cabin presented practical difficulties and minimising the noise at the sensor location did not necessarily reduce the noise at the observer location. This led to the design of a system that would steer a broad zone of attenuated...
noise away from the error sensors and towards the observer location by means of “virtual sensing”. The basic concept for these “virtual sensors” and their application to a pressure squared and energy density cost function are presented, developed and tested using a number of simplistic models of increasing environmental complexity. Each model was then experimentally validated and the results discussed. Finally, the virtual error sensors were evaluated in the cabin of a single engine light aircraft.

Following the results and conclusions, a direction for future research is suggested, with the intention of finalising the design and production of a commercially viable system.
Chapter 2

Literature review

2.1 Applications of active noise control

2.1.1 Propeller aircraft

Noise generated from light aircraft is primarily periodic, consisting of a fundamental and multiple harmonics of the blade pass frequency. The constant speed, variable pitch blades produce relatively constant frequencies with an inherent predictability and stability. The provision of a reference source (from a tachometer signal, or flywheel pickup for example) makes the small single engine two (or four) seater aircraft a prime candidate for successful feedforward active noise control (ANC). The degree of success in controlling this type of noise depends on the environment, the noise transmission path, the wave complexity, the cabin modal density and the coherence between the reference and the error sensors. The frequency must also be low enough (below 500 Hz) to avoid the complexity of the sound field that is characteristic of higher frequencies and the corresponding physical limitations of the active noise control system. Thus, the control of the light aircraft cabin noise over the full audio frequency range could therefore be approached by a combination of active and passive control methods (Kestell and Hansen (1998)).
Fuller (1986) studied the phase relationship between the acoustic waveform produced by each propeller for twin engine aircraft and the effects of their relative timing to reduce structural vibration and interior noise. The mechanism of noise cancellation is similar to that of ANC and is most effective in the plane of the propeller with theoretical localised reductions of up to 40 dB. However, this method obviously cannot be used for a single engine light aircraft.

Active noise control in aircraft cabins is well documented and its application in larger commercial twin engine aircraft is becoming increasingly popular, with Ross (1999) reporting approximately 250 ANC aircraft and 55 ANVC (Active Noise & Vibration Control) aircraft in service in early 1999. Jackson and Ross (1996) detail the active noise control of corporate (King Air and Twin Commander) turboprop aircraft, with the demonstration of global tonal noise reductions of 8 dBA throughout the entire cabin and 20 dB in some locations at the fundamental blade-pass frequency. The Lord Corporation (1999) used Digisonix equipment and demonstrated active noise control success in the Beech King Air, the Cessna Conquest and the Turbo-Commander twin engine aircraft (figure 2.1).

![Figure 2.1: An example of the Lord Corporation active noise control in a twin engine aircraft for the blade pass fundamental only. The upper schematic illustrates no active noise control while the lower shows the effect of active noise control.](image)

Their published data initially reported reductions of up to 20 dB for the blade pass frequency and 10 dB for the overall noise. Closer examination of the data presented reveals an average
global reduction of between 9 and 11 dB of the blade passage fundamental frequency, with no
further conclusions possible for the overall noise spectrum. Elliott aviation (1999) also report
success with the King Air 90 using 24 microphones and 12 loudspeakers (figure 2.2) with
“better than 6 dB” noise attenuation.

![Figure 2.2: An example of the Elliott Aviation active noise control in a twin engine aircraft.](image)

the twin engine aircraft market: an “UltraQuiet seat” and an “UltraQuiet cabin”. The seat
is reported to locally reduce the noise around the head of seated passengers by 10 dB. The
generalised performance of the “UltraQuiet cabin” (shown in figure 2.3) is represented by two
dimensionless contour patterns of the sound field before and after control, but there are no
noise attenuation figures. NVS have installed systems in the deHavilland Dash 8, the SAAB
340, SAAB 2000, the Beech 1900D, the King Air 90, the King Air 200, the King Air 300 and
the Twin Commander.

All of these examples have shown that published data with a commercial focus can some-
times be ambiguous. In each case it is unclear whether the reported attenuation was measured
directly at the error sensor locations or at the location of the observers, or whether the reduc-
tions are at specific frequencies or over the entire noise spectrum. Never-the-less, even from
a skeptic’s perspective and considering only the more conservative figures, the results are en-
couraging with respect to the application of active noise control in the similar environment of
a single engine light aircraft.

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"Active control of sound in a light aircraft cabin with virtual error sensors." Colin D. Kestell.
Figure 2.3: The Ultra Electronics NVS “UltraQuiet cabin”. The red contoured regions represent a “high” noise level and the blue regions represent a reduced noise level.

However, it appears that the single engine aircraft has yet to be targeted for the commercial application of active noise control. This may be due to a combination of reasons that may include:

- Physical differences between the single and twin propeller aircraft (section 2.3),
- the severe space and weight restrictions of the single propeller aircraft and
- the absence (to date) of a market push for ANC in a single engine aircraft.

### 2.1.2 Automotive cabins

The cabin of a single engine light aircraft is perhaps more similar to an automobile compartment than the cabin of a twin engine aeroplane. While the excitation mechanisms are somewhat dissimilar (apart from the periodic nature of the engine) they warrant review with respect
to the enclosure geometric similarity, methods of control and error sensing. Automobile active noise control is too well documented to fully cite in the context of this thesis but the following articles represent a good sample of the work to date.

Bernhard (1995) described road noise inside automobiles as a "challenging low frequency problem" He identified a major source of noise as tyre interaction with the road, transmitted via the suspension to the enclosing panels of the cabin. He observed that while the revolving tyres produced some periodic frequency components, the irregular road surface generated an enveloping broad band signal. However, he stated that a reference taken from the wheel axle vibration was sufficiently far enough from the cabin (so that the feed forward controller could predict the sound field) to infer that road noise was a good candidate for adaptive feed forward control. In feasibility studies Benhard highlighted the importance of the reference signal with respect to control and has shown (after Ross (1980)) that the active noise control performance (NR) is limited by the error signal coherence ($\gamma^2$) such that:

$$NR = 10\log_{10}(1 - \gamma^2)$$  \hspace{1cm} (2.1)

In his case studies, using frequency response measurements in a variety of cars, he predicted that the tyre/road generated noise in the cabin could be reduced by up to 9.7 dB.

With respect to the periodic noise characteristic of light aircraft, a suitable reference signal is easily obtained using a tachometer signal. This results in a strong error / reference signal coherence and provides a great deal of optimism for successful active noise control in light aircraft.

Bernhard (1995) mainly addressed the potential of a feed forward system in a vehicular application, citing ISVR and Lotus (Sutton and S.J. Elliott (1994)) as an example of success, with a 7 dBA overall reduction between 100 and 200 Hz. Results for other vehicles were briefly tabulated with the noise attenuation ranging between 7 and 10 dB. One important similarity
to light aircraft, that he touched on, is that of damping. Cars are known to be heavily damped (more so than considered in previous analytical studies) which has "some system behaviour impact". However, he then questionably stated that the situation may still follow the philosophy of lightly damped applications. He then proceeded to show that this statement is limited to frequencies below 200 Hz, where modal overlap is small and most likely the prime reason for his (albeit limited) success. Bernhard cites Sutton et al., who confirmed the 200 Hz success limitation, but further analysis of their published data suggested a different explanation. The coherence function they presented demonstrates an overall good low frequency value interspersed with harmonic drop outs, which are not explained, but probably due to engine noise. At frequencies characterised by a wavelength shorter than the period of rotation of a tyre (>200 Hz), the coherence between the noise in the cabin and a reference signal taken from an accelerometer on the wheel axle becomes generally poor, and this is the most likely reason (not discounting excessive modal overlap) for poor control above 200 Hz.

In conclusion, while the light aircraft has geometric and damping similarities to automobiles, any further comparisons (particularly with respect to the methods of excitation) are weak. The airborne noise of a light aircraft is dominated by strong periodic tones that are extremely coherent with an easy to measure reference signal, in contrast to the structure borne "quasi-periodic" road noise of an automotive vehicle for which it is difficult to obtain a good reference signal above 200 Hz.

### 2.1.3 Active headsets

Within the environment of an aircraft cabin, headsets are the most obvious solution to the noise problem as they are normally worn by pilots and already fitted with loudspeakers for communication. Active noise control headsets are of value and typically provide an additional 10 dBA attenuation over regular headsets.
Salloway and Millar (1996) have reported on the use of feedback active noise control headsets designed by GEC Plessey for use in armoured vehicles. They have shown that in typical headsets, that are widely available, the passive reduction provided 20 dBA of attenuation while active noise control contributed to a further 10 dBA. Johanson and Winberg (1997) have shown that a combination of analogue feedback and digital feedforward control can be used in a helicopter headset. In their research, the feed forward control targeted the discrete tonal components (which are also evident in light aircraft noise) while the feedback control targeted the broadband noise (up to 400 Hz). When they used their ANC system to control replayed helicopter noise, the feed forward system component removed over 20 dB from the fundamental blade pass frequency and the first two harmonics, while the analogue feedback system component (operational up to 400 Hz) simultaneously removed 20 dB of the linearly weighted broadband noise.

While the use of active headsets can provide a high level of noise attenuation, there is however a downside to their use. For effective hearing protection and noise attenuation, headsets must be properly fitting (to avoid air leaks, which would impair the attenuation) and used throughout the period of noise exposure. Hygiene factors, comfort and the reduced ability to hear warning signals must also be considered. Noise reduction throughout the cabin would reduce the demand for personal hearing protection and offer a far more attractive alternative.

2.2 Single engine aircraft noise characteristics

2.2.1 Noise levels

While noise levels for turbo-prop or jet aircraft are well documented there is little literature that comprehensively quantifies the noise levels in the cabins of single engine light aircraft. However, De Metz (1988) stated that around the proximity of the pilot’s head, a single engine Cessna produces 97 dBA at cruise (220 km/h). This exceeds comfortable conversation
levels of 82 dBA by 15 dB. For the professional pilot this is an extremely noisy and dangerous occupational environment, especially when compared to more conventional work areas. For example, a light machine workshop is recommended by the Australian standard AS2017 (1987) to be no more than 70 dBA or a typical office 45 dBA.

A study conducted by the International Institute of Noise Control Engineering (INCE (1987)), to reduce the risk of long term hearing damage, suggested that to protect 90% of the people, noise environments should not exceed an 8-hour $L_{eq}$ of 85 dBA. Their recommended exchange rate of 3 dB per half / double of exposure time, implies that working in an environment of 97 dBA should be limited to 1/2 an hour. When allowing for taxiing, take off and landing, this corresponds to extremely short flight durations.

The level of 97 dBA reported by De Metz (1988) was also reported by Jha and Catherines (1978a) who presented a narrow band spectral distribution of the noise. The spectrum shows an underlying broadband noise with predominant distinct frequency components. These occur at the engine rotational frequency, a half sub-harmonic and at multiple harmonics, with the even harmonics of dominant amplitude. The periodic nature of the spectrum implies that the noise has a high potential for reduction using active noise control.

### 2.2.2 Noise generation and transmission

The characteristics of aircraft cabin noise were realised prior to 1940 by Bruderlin (1937), who identified airborne paths to be the dominant transmission medium from the source to the cabin interior via the excitation of the cabin enclosure. As the following section reveals, modern aircraft behave no differently, with a number of more recent papers drawing the same conclusion (von Flotow and Mercadel (1995), Wilby (1996), Jha and Catherines (1978a) and Kallergis (1987)).

While von Flotow and Mercadel (1995) defined the major contributors to the cabin noise, like the majority of most papers published on the subject, they were primarily concerned with
2.2. Single engine aircraft noise characteristics

Turbo-prop and jet aircraft. However, the close source and mechanism similarities to light single engine aircraft, still allow useful conclusions and comparisons to be made. von Flotow and Mercadel (1995) illustrated that aircraft noise sources are a combination of:

- aerodynamic noise
- structure-borne noise due to spool imbalance
- exhaust noise and
- propeller noise

Wilby (1996) stated that on the ground, a twin engine aircraft produces higher levels of the high order harmonics than in flight, whereas Jha and Catherines (1978a) concluded that for a single engine, both ground and flight conditions have similar characteristics, but with increased broad band noise during ground tests. There are two possible reasons for this difference:

1. More significant ground reflections of the blade pressure perturbations may be evident with the twin engine aircraft,

2. There may also be an increase in aerodynamic turbulence on the single engine aircraft fuselage with fore mounted propeller wash reflecting from the ground.

Whatever the reason, the example serves to highlight subtle differences between the single engine and twin engine aircraft and that while the noise sources may be common between the two, their relative contribution may vary.

Wilby (1996) stated that while general aerodynamic noise must be considered as an aircraft noise source, it is usually only significant in aircraft with higher cruise speeds than those associated with propeller aircraft. Typical speeds of single engine light aircraft (Cessna Textron
(1997)) do not exceed 150 mph (240 km/h), while aerodynamic noise (a combination of turbulence, separated flow and interaction with protuberances) only becomes significant at speeds above 200 mph (321 km/h).

Eliminating aerodynamic noise leaves structure borne noise, blade passage pressure noise and exhaust noise for consideration as principal primary noise source contributors. Lyle and Atwal (1988) studied the use of sound intensity to identify the transmission loss into a light aircraft cabin. However, while they demonstrated the technique on a number of panels on a light aircraft cabin, it merely showed that the method was feasible. Their work fell short of identifying the dominant airborne transmission path and ignored structure borne paths altogether. The field studies of light aircraft noise conducted by Jha and Catherines (1978a), showed that there was a linear increase of 'A' weighted noise by 17 dB per doubling of engine speed. This demonstrated that the propulsion system, either via structure borne or airborne paths, was a significant contributor to the cabin noise that they measured. The cabin noise was dominated by the discrete frequency components of the blade passage and engine firing order frequencies, as well as their harmonics. By comparing accelerometer and microphone spectra, they concluded that the majority of the cabin noise was of airborne origin, and not structure-borne, but admitted to encountering difficulties due to a twin bladed propeller sharing a blade pass frequency (BPF) with the engine firing (and hence exhaust) frequency. Their related paper (Jha and Catherines (1978b)) based on laboratory results concluded that both structure and airborne noise can contribute significantly to interior noise levels during flight. While the results are reliable, it is not clear how they determined the correct shaker force to represent the actual flight conditions. Their latter conclusion is therefore questionable, with their initial conclusion (based on flight data) of airborne dominance (as opposed to structure-borne), the most legitimate.

Eliminating aerodynamic noise sources and structure-borne noise, isolates airborne noise as the dominant transport medium, with engine exhaust noise and propeller noise as the dominant contributors. Kallergis (1987) distinguished between the two. He measured a pressure

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time history, at close proximity to the exhaust, and then subtracted a measurement taken at a
greater distance (which was adjusted for phase and distance attenuation). While the technique
was well demonstrated, it failed to arrive at a numerical ratio of contributions. Inspection of
the illustrations of the exhaust contribution to the overall level however, leads the reader to
conclude that the propeller noise (at the blade passage frequency and harmonics of the blade
passage frequency) was the most significant contributor.

In conclusion, with respect to the mechanism of dominant noise generation and transmission
paths, it appears that light single engine aircraft interior noise is dominated by propeller noise
at the blade passage frequency and its harmonics.

2.3 Modelling the cabin

To fully understand the mechanisms of noise generation and transmission, experimental work
may not be sufficient. The modelling or simulation of a system not only allows the perform-
ance to be predicted, but allows for a detailed analysis of all the applicable physical mech-
anisms which leads to a far greater level of understanding of the problem. With respect to
active noise control in aircraft, models have continued to progress from the simplistic geomet-
rical shape of a long cylindrical tube (which is relatively simple to relate to classical theory),
to the finite element analysis of irregularly shaped cavities, which is far more representative of
real systems. In section 2.4.2 it will also be shown that it is not only possible to use modelling
as a means of reproducing (and hence understanding) the enclosed sound field, but that it may
also be used to simulate and quantify the effectiveness of active noise control and assist in
determining the optimum placement of error sensors and control noise sources.

Pope et al. (1987a) modelled a twin engine aircraft cabin and then compared their results
with the results obtained from a scale model and a flight test (Pope et al. (1987b)). However,
their model was based on a stiffened cylinder, excited in a single plane by the propeller blade
pressure pulses. In common with many other papers (Bullmore et al. (1990) for example), neither the stiffened cylinder nor the sidewall excitation loading is applicable to the much shorter and irregular shape of a light aircraft cabin, which is located to the rear of the propeller wash and behind the plane of the blade passage pressure maximums.

Classical methods for predicting / calculating resonance frequencies and mode shapes can be applied with relative ease to enclosures of a typical regular shape, but irregularly shaped cavities become far too complex for this method of analysis. Missaoui and Cheng (1997) addressed predicting the response of an irregularly shaped cavity, by breaking it down into smaller regular and irregular sub-cavities. The responses of the regularly shaped sub-cavities were analytically obtained, while the irregularly shaped sub-cavity responses were defined as a function of their regularly shaped bounding sub-cavities. The adjacent sub-cavities were then coupled across a common membrane of zero mass and stiffness to ensure the continuity of pressure and velocity. They reported errors of less than 1.4%, but these were for very simplistic irregular enclosures. Their research limited this method to the uncoupled cavity response but will later address structure and cavity coupled systems. The errors that they have inherited in such a simple scenario, leave small hope for the far more complex nature of an aircraft cabin.

Notwithstanding the work of Missaoui and Cheng (1997), there are only a few enclosures (such as tubes, cylinders or rectangular cavities) that allow the exact prediction of their acoustic response to be obtained using analytical methods. Geometric complexities require the use of the approximate numerical method of finite element analysis (FEA), which divides a system into contiguous regularly shaped components of linear dimension, sharing common nodes with neighbouring elements (Craggs (1997)). The methods either allow for fully coupled system modelling or, separately modelled structures and cavities, which may be subsequently coupled at the common boundary by using the Green’s function between the structural velocities of the bounding structure and the pressure of the cavity immediately adjacent to the bounding structure (Craggs (1997)).
The finite element model can also be used to show the effect of active noise control. Miccoli and Sarti (1995) used the finite element method to model a cabin (albeit a tractor cab), with a view to simulating modal control via a single control source, to minimise the total sound field at the drivers ear. They discovered that minimisation of the dominant mode can result in control or modal spill over. Controlling either the first acoustic mode or the most significant mode, by introducing a single control source, resulted in a higher contribution from other modes and an overall total sound pressure level increase. Adding absorbent lining to their rigid model damped the resonance frequencies and caused further control problems. While they did not elaborate on an explanation, this would have been due to an increase in the number of modes that significantly contributed to the enclosed sound field at the control frequency.

The boundary element method (BEM) is similar in concept to the finite element method and warrants review with respect to its value in modelling the sound field of the single engine light aircraft cabin. With this method, approximations are only made at the boundary and all interior points are solved with a solution that satisfies the governing partial differential equations exactly (Beer and Watson (1992)). Coyette and Cremer (1997) compared both the boundary element method and the finite element method to an idealised benchmark, for calculating the sound radiated from a baffled elastic plate. They reported a good performance for both methods but stated that finite element analysis was more all encompassing and that the boundary element method was limited in its applications. They explained that while the finite element method may be used to model both the cavity and the structure, the boundary element method can only be used to generate a cavity model.

For problems involving the sound transmission from a structure into either an enclosed field or a free field, the boundary element method is favoured because only the radiating surfaces are modelled using two dimensional elements, which are less expensive in terms of computational time (Seybert and Wu (1997)). However, if the cavity of an aircraft cabin is modelled with BEM, then the consideration of structural / acoustic interaction will require the cavity model to be coupled with a model of the structure. The structural model may only be generated by
the FEA method and therefore where coupled structure / cavity models are concerned, it is more sensible to use finite element methods for predicting the uncoupled and coupled modes and the forced frequency response of the system.

The coupling or mutual effect of structural modes and cavity modes is complex, but may be simplified in certain situations. Where the cavity medium is air and the structure is massive enough that the air provides negligible acoustic loading, weak coupling may be assumed (Snyder and Hansen (1994a)). Under these conditions, the acoustic modes may be modelled with an assumed rigid enclosing structure (the fluid particle velocity at the boundary is zero) and the structural modes may be modelled assuming an interior vacuum (i.e. with no fluid loading across the entire frequency range). Modal coupling theory is then used to combine the independent models to determine the coupled system response.

The notion of separately modelling the cavity and structure and then coupling the two, while apparently cumbersome, does offer a distinct advantage over a fully coupled model. It serves to identify whether the modes are structurally driven or acoustically driven, which would otherwise be unclear, in both fully coupled models or experimentally derived data. In a light aircraft cabin, identification of the mode source can indicate whether the structural measurement or control is viable, or advantageous over more traditional acoustic methods (i.e. microphones and speakers). Using the modal coupling analysis method also helps to explain the differences in the modal frequencies between theory and experiment.

Sung and Nefske (1994) applied the methods of structural / acoustic coupling (as illustrated by Fahy (1998b)) to the finite element model of an automobile. From their theoretical verification, it is clearly visible that the response of the acoustic cavity is the summation of the individual coupled mode contributions. Their methods of using experimentally derived damping values to produce a good model / experimental agreement below 100 Hz would also be an appropriate method for modelling a light aircraft cabin. However, by excluding data above 100 Hz, because of poor model confidence, they have limited any practical observations of the limitations in their model. They have however demonstrated the relative contribution of structural
2.4 Aircraft cabin active noise control

2.4.1 Introduction

At low frequencies, the response of even a simplistic rectangular enclosure (Bies and Hansen (1996)) will be spatially irregular, characterised by nodes and anti-nodes (pressure peaks) of associated enclosure modes. In the presence of sound absorbing material, the standing waves of an enclosure are damped, broadening the slope of their response vs frequency curve, resulting in increased modal overlap. The implication of this, is that if a driving force has a frequency between two normal mode resonance frequencies and sufficient modal overlap exists, both modes will produce a significant contribution to the total response at the frequency of excitation. This physical mechanism is applicable to active noise control in light aircraft cabins, which incorporate damping material for higher frequency noise reduction. This simultaneous excitation of multiple modes is the cause of complications when considering the control of fundamental and harmonic excitation tones, that most likely do not correspond to
the resonance frequencies of the enclosure. Each excitation frequency may excite multiple overlapping room modes (Beranek (1996)). The associated structural / acoustical interaction further complicates the noise generation mechanisms, making it essential to:

- consider all of the possible means of error sensing and control source generation and
- to assess the suitability of each for this application to a small four seater highly damped aircraft cabin.

With the inherent geometric complexities of such a cabin, placement of the sensors and control sources also becomes a non-trivial key consideration. The options for determining the optimum transducer locations, the most effective means of error sensing, control sound field generation and the preferred methods for the practical implementation of active noise control in a single engine light aircraft are therefore discussed in the following sections.

### 2.4.2 Sensor / control location

Snyder and Hansen (1994b) state that it is impossible to obtain a closed form solution to the problem of optimising the physical placement of error and control source locations for multiple control and error sensor systems.

Using modelling methods to predict the sound field and associated modes (and hence nodes and anti-nodes) provides an indication for the optimum locations of control and error sensing transducers. A geometrically complex environment will demand multiple error sensors and control sources and will therefore require a comprehensive method to quantify the degree of control for each possible location. However, once the control source locations are selected, Snyder and Hansen (1991) have shown that the optimum locations for the error sensors are at the points where there is the greatest difference between the primary sound field and the controlled sound field.
Exhaustive studies, where all conceivable control source locations and error sensor combinations are considered, are computationally expensive. Methods that incrementally advance the position of control and error transducers in the direction of increasing noise reduction (continuous optimisation), are said to be ineffective for predicting a global optimum where multiple local regions of attenuated noise exist. Such methods continually compute the dynamic response of the system and are hence limited to simplistic scenarios.

Ruckman and Fuller (1993) have demonstrated that "subset selection" limits the search process to a number of permissible locations and substantially reduces the computational time. Poor location choice however (as is more likely in a limited choice mechanism), can be prone to collinearity, the numerical ill conditioning (due to one or more transfer functions that are not mutually orthogonal) invalidating the results. However, the fact that this method is based on limiting the choices is advantageous in the light aircraft cabin, where only a limited number of locations would be viable. In their example Ruckman and Fuller (1993) used a fluid loaded unstiffened cylinder and reported a noise attenuation of 12 dB for a single actuator control source and up to 70 dB when using multiple actuators.

Simpson and Hansen (1996) have claimed that "Genetic Algorithms" are an efficient method for optimising the placement of multiple control and error transducers in a complicated system with many modes. In its basic form a genetic algorithm generates random numbers that, when combined with historical results, predict improved values for the maturing iteration process. All the design variables are represented in a single alphanumeric string called a gene. A search commences with a population of randomly selected "breeding" strings. These breeding or "parent" strings, based on fitness (achieved attenuation), are then selected for the reproduction of the first generation. "Crossover" randomly copies information from each mated parent and "mutation" occasionally and randomly varies inherited values. The "children" subsequently replace their parents for a continuing and evolving iteration. Mutation is introduced to minimise the potential of converging on localised maxima of controlled noise attenuation,
that could otherwise occur with a pure "survival of the fittest" method. In the example used of a stiffened cylinder, the optimisation yielded a potential energy reduction of 28 dB.

Genetic algorithms therefore should be a definite consideration with respect to optimising transducer placement in the active noise control of light aircraft cabins for two main reasons. First, the evolutionary iterative approach is far more cost effective (with respect to computational time) than exhaustive search routines. Second, the mutation reduces the possibility of convergence on localised areas of attenuated noise, as is likely with continuous optimisation.

In the small confined cabin of a single engine light aircraft, the permissible locations for either the error sensors or the control source transducers are limited. For this particular application, the best method for determining the optimum control source (or error sensor) locations may therefore be to use a combination of subset selection and a genetic algorithm search routine.

However, the occupants of the cabin are so confined and restricted in their movements, that achieving global noise reduction may be a somewhat unnecessary objective. The pilot and passengers are always in a seated position and are only able to move their heads around in a relatively small region. It is therefore possible that controlling a small (local) zone around the occupants’ heads is all that is required. The following sections therefore present a discussion regarding the size of control zones that can be achieved with local control, for either a pressure squared cost function or an energy density cost function and whether the resulting zones of noise attenuation are sufficiently large enough to envelop the heads of the aircraft occupants.

### 2.4.3 A pressure squared cost function

In an enclosure, the most common and simplistic error signal is from a microphone that directly measures the acoustic pressure. The signal is then squared to become the cost function (or the quantity that is to be minimised). However, as this section will show, while the acoustic pressure squared cost function is easy to measure and may be minimised at the error sensor lo-
cation, the region over which the sound pressure level is reduced may be so small and confined to the immediate vicinity of the error sensor, that it is of no practical value.

In an aircraft cabin, microphones are easily placed and are unobtrusive (often incorporated into head-rests or trim). But for global control (just like the use of speakers as control sources), their position with respect to the modal behaviour of the enclosure is of prime importance. Bullmore, Elliott and Nelson discussed the minimisation of harmonic enclosed sound fields in a series of three papers (Bullmore et al. (1987), Elliott et al. (1987) and Nelson et al. (1987)). They demonstrated analytically, computationally and experimentally, that provided an enclosure is excited at a resonance, then control sources (and error sensors) placed at primary pressure maxima (anti-nodes) substantially reduce the sound field on a global basis. While the multi-mode anti-nodes of a rectangular room would be spatially irregular, they all share a common anti-node at a corner location. However, while excitation (or measurement) of a mode is most efficient at an anti-node, it should be remembered that in practical situations, such as in an irregularly shaped light aircraft cabin, a corner (or even an obvious anti-node location) may not be an option. Bullmore et al. (1987) also only considered the excitation of enclosure modes with an excitation frequency common to the modal resonance frequency. Excitation specifically at the modal resonance would only occur as a result of unfortunate design, or in other words sheer bad luck. While useful in demonstrating the cancellation mechanism, it is therefore not representative of many practical applications. Although a flat broadband noise may excite the resonance frequencies of the enclosure, which may in turn characterise the noise spectrum experienced by the occupants, a light aircraft cabin is excited by dominant multiple tones, most likely to be (as previously described) between the resonance frequencies of the enclosure. Direct control of the excitation frequency is therefore likely to be localised, unless the participation of each enclosure mode can be controlled individually.

Fuller and von Flotow (1995) demonstrated that controlling sound pressure via a microphone at a single location may produce a high level of attenuation over a very small region at the expense of increased noise in other regions. The region of attenuation may be so small that it becomes...
an impractical solution to a noisy environment. Elliott and Garcia-Bonito (1995) demonstrated that the cancellation of pure tone sound pressure at a point in a diffuse sound field produces greater than 10 dB of attenuation in a spherical region with a radius of only $\lambda/20$, where $\lambda$ is the acoustic wavelength. Taking into account a typical practical distance between the error sensor and an observers’ ear (to avoid the transducer becoming a nuisance), the resulting attenuation measured at the error sensor may not be experienced at all by the observer. Any degree of head movement could also expose the observer to large variations in sound pressure as well as cause physical interference with the error sensor. In such an example it may be necessary for the observer to literally place his ear immediately adjacent to the microphone and place a finger in the other. A system that is accompanied with these operational requirements would obviously not be commercially attractive. Multiple error sensor and control source pairs would simply result in multiple pockets of localised sound attenuation, although if there were a sufficient quantity, then the regions may overlap and globally reduce the noise. Such quantities of transducers, when considering the weight and size of speakers with sufficient power to cancel the primary noise, would not be a viable consideration in a small light aircraft. It therefore becomes apparent that the success of global sound minimisation in an enclosure, where pressure squared is the cost function, is extremely limited by the practical realities and that even local control may not result in the noise being attenuated at the observer’s ear. An alternative more spatially consistent cost function may however produce large enough regions of attenuated noise to be of practical use.

### 2.4.4 An energy density cost function

The previous section established that although the most simplistic strategy for active noise control in an enclosure is to use a single microphone error sensor, the resulting zones of attenuation may be so small that the acoustic pressure squared cost function may be of no practical value. However, energy density is a more spatially uniform cost function than squared pressure and can also be measured at single locations (although with at least two microphones). In
numerical simulations Sommerfeldt and Nashif (1991) found that minimising the energy density at a discrete location significantly outperformed the minimisation of the acoustic pressures squared in terms of the size of the attenuation zone around the error sensor. Cook and Schade (1974) also showed that for 3-D systems with high modal densities, the spatial variance of energy density is also significantly less than the potential energy variance. Sommerfeldt and Nashif (1992) compared the use of three methods:

1. Spatially integrated potential energy,
2. Energy density at a discrete location and
3. The use of a microphone, where acoustic pressure squared is the cost function.

The theoretical spatially integrated potential energy method is impractical, but it is an idealised global control benchmark. Sommerfeldt and Nashif (1992) concluded that an energy density cost function was far superior to an acoustic pressure squared cost function, improving the overall global attenuation and reducing the localised control effect that is a common characteristic associated with the use of the pressure squared cost function.

By definition, minimising the pressure gradient between two microphones minimises the spatial variation in acoustic pressure between them. Therefore, in a single control source system, energy density becomes a compromise between the attenuation achieved at the error sensor and the size of the attenuation zone. However, a second control source would allow the independent control of pressure at the two microphone locations and hence the minimisation of the acoustic pressure as well as the maximisation of the control zone around the two sensors.

Total energy density, the summation of kinetic energy density and potential energy density, can be easily calculated via the measurement of particle velocity (proportional to the acoustic pressure gradient) and pressure. Elliott and Garcia-Bonito (1995) showed that minimising the pressure and pressure gradient (hence energy density) between two locations (effectively
minimising pressure at two locations) results in a 10 dB attenuation zone resembling a cylinder with a radius of $\lambda/20$ and a length of $\lambda/2$. The same attenuation zone is limited to only a sphere with a radius $\lambda/20$ when using a squared pressure cost function. Similar levels of local control using energy density were achieved in a heavily damped modally dense reactive enclosure above the Schroeder frequency by Cazzolato (1999). In such heavily damped acoustic systems, global control can be very difficult to achieve and subsequently local control may be the only practical alternative.

Nashif (1992) applied the principle of energy density minimisation to a one dimensional enclosure, also demonstrating that the use of an energy density probe can overcome the observability difficulties (such as pressure nodes) that are inherent in the use of microphones as error sensors in active noise control systems. Since a phase difference of 90° exists between the velocity nodes and pressure nodes of any enclosure resonance, the weighted summation of pressure and velocity is much more spatially uniform than either pressure squared or velocity squared. Sommerfeldt et al. (1995) and Parkins (1998) extended the work to a three dimensional enclosure, verifying the previous findings.

Sommerfeldt et al. (1995) also built a 3 axis energy density sensor made from 6 electret microphones mounted in a wooden sphere. Cazzolato (1999) later simplified this design to a four microphone device. Preliminary results indicate that controlling energy density with this energy density sensor has the potential to achieve greater global control than controlling squared pressures at the same error sensor location. It has also been found that the success of energy density cost functions is not limited to tonal or harmonic noise. Applying the method to broadband noise Park and Sommerfeldt (1997) reiterated the benefits of energy density minimisation and concluded from a numerical analysis and a two microphone implementation, that in the frequency domain:

- Energy density provides greater global control than can be achieved by minimising the square of pressure (as with microphone error sensors) and
2.4. Aircraft cabin active noise control

- It is not highly dependent on transducer location.

As far as implementation in a light aircraft is concerned, energy density probes (consisting of grouped multiple microphones) are easy to position and move, while offering distinct advantages over the use of multiple microphones (requiring the same amount of controller input channels). Thus energy density is the preferred cost function to pressure squared in such an environment, especially if the objective is to control noise throughout a broad region.

While the cited examples illustrate that a controlled zone is larger with an energy density cost function than a pressure squared cost function, the level of attenuation still diminishes as the proximity from the sensor increases. Remotely placed energy density sensors can extend the zone of local control sufficiently to encompass the head space but this remains limited to low frequencies, with the optimum position still located at the sensor.

2.4.5 Acoustic control sources

To minimise the cost function at the error sensor location(s) a method of generating the control signal must also be chosen. Loudspeakers are obviously an extremely efficient radiating source and an obvious first choice for generating the secondary (cancelling) sound field. While they are usually designed for broad band audio applications, the speaker performance may be optimised for specific frequency bands. Beranek (1996) broadly categorises loudspeakers as:

- Direct Radiators, where the vibrating cone, or diaphragm couples directly with the air and
- Horns, where the geometry of the horn is designed to optimise the coupling between the driver and the air for a specific application.

Although the latter is the most efficient and best suited for large scale systems, it is the most space intrusive and expensive (Beranek (1996)). In small enclosures such as a light aircraft...
where space is an issue, the cost and satisfactory response of a direct radiator system makes it the preferred option. Free-air direct-radiating speakers allow low frequency, destructive interference to occur between the front and the rear radiating sound fields, reducing the radiation efficiency. Low frequencies also require greater coil displacement, and hence are more prone to exceeding either the physical limitations of the speaker or the linear range of the magnetic field, hence causing harmonic distortion.

Fuller (1997) stated that in a commercial aircraft (as would also be applicable to a light aircraft environment) the need to use the existing space behind the trim of the aircraft, necessitated more units to achieve the required volume velocity for control. While the trim in automobiles and larger aircraft is often used for such purposes, light aircraft trim is often no more than 25mm in depth, requiring a speaker enclosure of adequate volume to stand proud from the existing surfaces and encroach into an already cramped environment. Multiple independent speakers would also demand multiple control channels.

Enclosing and venting a speaker optimises the low frequency performance by preventing the destructive interference between the front and the rear radiating sound fields and by adding designed low frequency resonance (Starobin (1997)). Blondel (1995) used a single vented (low pass) enclosure in his studies of helicopter active noise control. A double vent design (band pass) would have also further optimised his speaker for the low frequencies of interest. However, the size of his enclosure is extremely space intrusive if considered for a light aircraft application. The large size of the speaker enclosure may not be a problem during research, to establish if ANC is viable, but the size of such speakers would be unacceptable in a normal operational environments. Never-the-less his case study provides a perfect example of the difficulties associated with producing a powerful enough control source to cancel the primary noise field.

Electro-static speakers are lightweight and generally much thinner than conventional loud-speakers, consisting of a lightweight moving diaphragm that is electro-statically driven from a closely separated stationary electrode (SoundLab (2000)). While the lightweight and narrow
dimensions offer obvious advantages over the more conventional electromagnetic loudspeakers, they also have a number of disadvantages. A very high DC bias (up to 5000 V) is required as well as high AC voltages for excitation. Apart from the safety issues of high voltages, this also means that transformers are required, which detract from the afore mentioned weight advantages. The other issue, is that the surface displacement of the moving diaphragm is limited to only a couple of millimeters and so a large surface area would be required to generate high sound pressure levels at low frequencies. Electro-static speakers are also very expensive. However, Heydt et al. (2000) state that electro-restrictive polymer film (EPF) loudspeakers are far less expensive, driven by much lower voltages and have a larger surface displacement compared to electrostatic speakers. This means that they would also offer an improved low frequency response. Heydt et al. (2000) report on the performance of a 5 cm x 5 cm speaker and show that it has a generally flat frequency response between 1.5 KHz and 20 KHz with a sound pressure level of approximately 80 dB measured at a distance of 1 m. However there is a sharp 20 dB / octave roll off below 1.5 KHz and the sound pressure level at 70 Hz (the blade pass frequency of a single engine light aircraft) is only 25 dB. Larger speakers may of course produce higher sound pressure levels at lower frequencies and it will be of great interest to see how the research of Heydt et al. (2000) progresses.

Li et al. (1997), through their studies with electric power transformers, suggested the use of piezo patches to excite curved panels, which form one side of an otherwise rectangular enclosure. This initially presented the possibility of using secondary panels in the aircraft cavity. However, the panels are mainly designed to combat the environmental difficulties of transformers, which are open to the elements. With respect to their power to weight ratio and their ability to excite multiple frequencies at the required amplitudes, they offer no advantages over conventional speakers. However, the use of piezo patches to excite a vibrating surface to radiate sound does lead to the idea of direct structural control, as discussed in the following section (section 2.4.6).
2.4.6 Structural control and error sensing

The previous two sections discussed the direct control or sensing of the acoustic field. Since the noise in an enclosure is generated by the enclosing vibrating structure, the use of devices that either control or sense the vibration of the enclosing structure, become an option. Pan and Bies (1990) demonstrated that thin-walled structures and poorly-damped enclosures transmit internally generated noise out into the surrounding environment, especially if the structural modes are well coupled with the cavity modes. Reciprocity dictates that the same would also be true for external sound entering an enclosure (Fahy (1998a)).

Altering the response of the structure via actuators demonstrates two mechanisms of control (Snyder and Hansen (1994a) and Snyder and Hansen (1994b)). The first, "modal control", is the minimisation of the amplitudes of the dominant structural modes. The second, "modal rearrangement" is the modification of the relative amplitudes and phase between structural modes, reducing the extent of coupling with the acoustic modes.

Rossetti and Norris (1996) compared various configurations of actuators and sensors (both structural and acoustic) for active noise control in a deHavilland DASH-7 turboprop mock up. Their model predictions and experiments demonstrated:

- the use of structural actuators and error sensors is superior by approximately 10 dB for frequencies below 70 Hz,
- where strongly coupled modes dominate (the structural deformation corresponds to a cavity mode shape) between 70 Hz and 200 Hz, the results obtained with structural actuators and sensors are similar to those obtained using acoustic actuators and sensors and
- at higher frequencies (>200 Hz) where the acoustic dynamics dominate, acoustic sources demonstrate a superior performance to structural actuators.
They therefore concluded that structurally based noise control systems significantly outperform speaker-based noise control systems for reducing interior noise in turboprop aircraft, and should therefore be a serious consideration for the control of similar noise in light aircraft.

The global minimisation of the structural vibration of the enclosing panels of a light aircraft cabin (or the minimisation of all of the structural modes that contribute to the enclosed sound field) is a non-trivial task that would demand a very large number of structural error sensors and control sources. Attempting to simplify the task by minimising specific structural modes that have a high acoustic radiation efficiency may not necessarily reduce the radiated acoustic power. While radiation efficiencies may be used to calculate the acoustic power that radiates from a specific structural mode in isolation, the total acoustic power, radiated by a number of modes excited simultaneously on a vibrating structure, is not merely the sum of the acoustic power that radiates from each individual structural mode (Snyder and Tanaka (1995)). It is well known that if the amplitude of a specific structural mode is reduced, sometimes the radiated power may increase. In other words, the sound that radiates due to a specific structural mode may inherently cancel the sound that radiates due to another structural mode. Control of one of these structural modes may therefore effectively increase the sound power radiation.

Elliott and Johnson (1993) state that the acoustic power radiating from a surface may be expressed in terms of independently radiating surface velocity distributions, which are collectively termed radiation modes (or transformed modes Snyder et al. (1993)). Minimising a radiation mode (that is orthogonal with all other radiation modes) ensures that the radiated acoustic power from a vibrating surface panel is always reduced (Hansen (1997)). Because the radiation modes each contribute independently to the total radiated acoustic power they are sometimes also referred to as power modes (Tanaka et al. (1996)).

Cazzolato and Hansen (1998) considered the transmission of sound into a stiffened cylinder (representative of an aircraft fuselage). The cylinder was modeled with finite element analysis and the model was subsequently experimentally validated. They compared the results of total acoustic potential energy attenuation within the cylinder that resulted from:
• structurally sensing the radiation modes,

• estimating the acoustic potential energy at discrete locations and

• estimating the structural kinetic energy at discrete locations.

Cazzolato and Hansen (1998) demonstrated that, providing an adequate number of structural sensors are used to properly define the radiation modes, minimisation of only the first two radiation modes outperformed minimising an acoustic potential energy estimate from four sensor locations and minimising an estimate of the surface kinetic energy of the cylinder at ten locations. When minimising the first two radiation modes there were no increases in the sound pressure level at any frequency, but at some low frequencies there were attenuations of up to 40 dB. They concluded that sensing and minimising radiation modes reduced the required number of control channels and thus increased control stability.

Maillard and Fuller (1998) used “Discrete Structural Acoustic Sensing (DSAS)” to measure radiation modes and control the sound that radiated from a plate. In an array of multiple optimally located structural point accelerometers, the signal from each individual sensor was weighted, so that when all of the signals were summed together, the resulting cost function demonstrated a high sensitivity to a particular radiation mode. In a multi-channel control system, multiple signals, each proportional to a different radiation mode, were fed to separate controller input channels. This method was then compared to “Discrete Structural Volume Acceleration Sensing (DSVAS)” which used the same array of accelerometers, but the unweighted signals were summed to estimate the net volume acceleration of the plate. While both error sensing methods reduced the requirement for the number of control channels (compared to methods that require a control channel for each sensor) and demonstrated a good reduction of radiated sound power over the entire bandwidth (between 10 dB and 80 dB), DSAS (the method that senses radiation modes) demonstrated:

• a high sensitivity to particular radiation modes,
a reduced measurement of the non-radiating vibrational component,

an improved performance at higher frequencies (compared to DSVAS) and

a reduced modal spillover.

Snyder et al. (1993) and Tanaka et al. (1996) have shown that appropriately shaped polyvinylidene difluoride film (PVDF), bonded to the surface of a vibrating plate, can discriminate between radiation modes (by nature of their geometric profile). For low frequency (tonal or broadband) excitation they concluded that only the first few radiation modes are important and that because the sensor shape has only a weak frequency dependence, a broad and practical frequency range may be controlled. The continuous nature of the sensors also reduced the observability problems that are inherent with discrete sensors. Their example of controlling the first three radiation modes of a panel show acoustic power attenuations of up to 15 dB at some frequencies, with no increase in acoustic power at any frequency.

Fuller et al. (1998) discuss the concept of “Smart foam active skins”, which use PVDF elements embedded into foam as a control source that completely covers the radiating surface forming a controllable boundary. The foam passively absorbs noise at higher frequencies and the embedded control sources actively reduce the lower frequency noise. Using a plate as an example it was shown that between 500 Hz and 750 Hz, the active element of the smart foam produced an additional attenuation of up to 10 dB, when compared to the use of foam alone. Fuller et al. (1998) also introduce the concept of a “Piezoelectric double amplifier active skin.” In this concept, bimorph legs, of PZT and brass, support a paper diagram so that the low displacement that is normally associated with piezoelectric devices is mechanically amplified into the radiating paper diaphragm. A plate mounted into a baffle was used as an example in which the total radiated sound power was seen to reduce by approximately 10 dB when using the active skin. Both methods show promise for introducing an active noise control boundary in the cabin of a single engine light aircraft and may replace the existing trim.
While these methods may demonstrate performance advantages, considering the practical aspects of installation (with respect to a single engine light aircraft) reveal some of their weaknesses. To properly design a structural sensing / control strategy requires the identification of the structural modes that contribute to the radiating noise by measuring the frequency response function of the structure. These measurements and final installation of the actuators and sensors will require the removal and re-fitting of significant amounts of the aircraft trim (removal of the trim also applies to active boundary methods). Interfering with the structural response of an aircraft, without a comprehensive study on how the dynamic stresses of the aircraft may be changed, may not be well received by safety conscious authorities that certify air worthiness. Direct acoustic control will be perceived to be less likely to adversely affect the structural integrity of the aircraft. There is also the consideration of the ancillary equipment that is required to power the structural sensors and actuators, which may detract from any weight benefits. Where aircraft are to be initially supplied with active noise control equipment (prior to commissioning) these afore mentioned problems may be easily overcome and hence structural methods should be a definite consideration. However, single engine light aircraft are most likely to be retro-fitted with active noise control in the form of an accessory that the aircraft owner may later purchase. Installation must therefore be as simplistic as possible in order to keep the installation cost to a minimum and avoid any issues that may effect air worthiness certification. Direct acoustic sensing and control (that also allow an element of versatility for initial research) should therefore not be too hastily overlooked as a strategy for active noise control in a single engine light aircraft application.

2.4.7 Filtering

Fuller (1997) explained that while impressive overall noise reductions (of primarily low frequency components) may be reported in examples of active noise control applications, the "A-weighted" observers will have a lower perception of improvement. He suggested that by "A-weighting" the error and reference signal, or band limiting the control, the perceived im-
improvement will increase. While pre-filtered error signals would reduce the control performance for the fundamental and the lower order harmonics, apparently handicapping the controller in the range that it best performs, it would increase the error amplitude ratio of the higher order harmonics resulting in an improved perception of reduction.

2.5 Conclusions from the literature review

The literature review has shown that the cabin of a single engine light aircraft is noisy to the extent of posing an occupational hazard. The noise is characterised by distinct harmonics of the propeller blade pass fundamental frequency, for which a good reference signal can be obtained easily, suggesting that the noise is an ideal candidate for active noise control. While active noise control is now evident in many of the larger twin engine aircraft, with many papers and articles describing its success, it is evident that the small single engine aircraft has been overlooked. Application similarities were drawn to automotive cabins but were seen to be limited to the geometric attributes and the damping. Unlike noise in automobiles, the noise in single engine propeller aircraft cabins has a much higher level of coherence with the reference signal.

Active headsets have been demonstrated successfully and have been shown to definitely reduce the wearer’s exposure to aircraft noise. However, hygiene factors, and the importance of timely use infer that reduction of the cabin noise offers an advantage.

Modelling has been shown to be a valuable tool for predicting the performance of active noise control in simply shaped enclosures, whereas an accurate ANC model of a single engine light aircraft with a fore mounted propeller and irregularly shaped cabin would be extremely computationally expensive.

It was concluded that sensing and controlling the vibration of an enclosing structure, or producing an active boundary to reduce the noise that radiates into the enclosure, were more effec-
tive methods for active noise control than using loudspeakers and direct acoustic error sensing. This may well lead the reader to question the consideration of direct acoustic measurement and control for a light single engine aircraft. While structural sensing / control (or active boundary control) may improve active noise control performance, it may involve high installation costs and installation difficulties. Single engine light aircraft are most likely to be retro-fitted with active noise control and so installation must be simplistic and the associated costs kept minimal. An active noise control system that uses direct acoustic sensors and control sources (loudspeakers) offers a low cost noise solution. Speakers and microphones also provide the researcher with the required versatility for initial experiments, where a somewhat ad-hoc approach can prove the initial capabilities of the system within a particular environment. While localised control is initially most likely, this can be improved by the use of energy density measurement, or by increasing the number of control and error sensor pairs. Once an element of control has been established it is sensible to model the system (which may be validated by the experimental data) and optimise the placement of the control sources and error sensors. Direct acoustic control offers a further advantage in that it is not handicapped by the potential problems that are associated with structural control modifying the structural behaviour of the aircraft. Interfering with the structural response of an aircraft without a comprehensive study on how the dynamic stresses of the aircraft may be changed may not be well received by safety conscious authorities that certify air worthiness. Direct acoustic control will be perceived to be less likely to adversely affect the structural integrity of the aircraft.

It should also be realised that the objective of the global control of noise within the cabin may be an overkill approach in such an environment where any movement of the confined occupants is restricted. The local control achieved through acoustic control source and acoustic error sensing may be sufficient, as long as the zone of local control remains sufficiently broad and near to the location of the occupants’ ears. However, the use of traditional error sensors (microphones) has demonstrated that even this can be difficult to achieve. The following chapter (chapter 3) therefore shows the results of initial experiments that were conducted
to validate the findings of the literature review and to assess the viability and limitations of ANC in a single engine aircraft. Chapter 4 then presents a hypothesis for how a broad zone of attenuated noise may be centered directly at the occupant’s ear and how any small amount of head movement may be tracked.
Chapter 3

Initial experiments

3.1 Introduction

Noise measurements within the cabin of a single engine Piper Archer aircraft (figure 3.1) were conducted at Southern Aircraft Maintenance, to quantify the sound pressure amplitude and to determine the characteristics of the primary noise field in the cabin. The nominal dimensions of the fully furnished and carpeted cabin interior were 2.5 m x 1.5 m x 1.2 m. The noise measurements were also recorded to allow them to be reproduced under laboratory conditions and to establish the viability of using an active noise control system. A simple active noise control experiment with one speaker and one error sensor, was then conducted in the aircraft cabin, with the engine running at full throttle.

Figure 3.1: A Piper Archer, registration VHPOQ.
3.2 Equipment

3.2.1 General

Table 3.1 lists the equipment that was used in all of the noise measurements and experiments that are discussed in this chapter. A Hewlett Packard type 35665A dynamic signal analyser was used for all of the narrow band frequency measurements and a Bruel and Kjaer type 2260 hand held analyser for all of the 1/3 octave measurements. Recording the audio data was conducted via a Teac type CS RD100T digital tape recorder, which was calibrated by recording a 94 dB (reference 20 microPascals), 1 KHz tone from a microphone calibrator. The active noise control experiments were conducted using a six channel Causal Systems EZ ANC feedforward active noise controller, with microphone error sensors, a custom built reference signal generator and loudspeaker enclosures as detailed in the following section.

<table>
<thead>
<tr>
<th>Description</th>
<th>Make</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic signal analyser</td>
<td>Hewlett Packard</td>
<td>35665A</td>
</tr>
<tr>
<td>Hand held analyser</td>
<td>Brul and Kjaer</td>
<td>2260 “Investigator”</td>
</tr>
<tr>
<td>4 channel digital tape recorder</td>
<td>Teac</td>
<td>CS RD100T</td>
</tr>
<tr>
<td>Microphone</td>
<td>Generic brand</td>
<td>various</td>
</tr>
<tr>
<td>Microphone amplifier</td>
<td>Custom built</td>
<td>-</td>
</tr>
<tr>
<td>Microphone calibrator</td>
<td>Brul and Kjaer</td>
<td>4231</td>
</tr>
<tr>
<td>Feedforward active noise controller</td>
<td>Causal Systems</td>
<td>EZ ANC</td>
</tr>
<tr>
<td>Speakers</td>
<td>Custom built (section 3.2.2)</td>
<td>various</td>
</tr>
<tr>
<td>Phase locked reference signal generator</td>
<td>Custom built (section 3.2.3)</td>
<td>-</td>
</tr>
<tr>
<td>Power amplifier</td>
<td>Promaster</td>
<td>Pro series 3</td>
</tr>
</tbody>
</table>

*Table 3.1: A list of the equipment that was used for the initial experiments*

3.2.2 Loud speaker enclosures

The requirement for active noise control in a single engine aircraft, and (more immediately) the need to replay the recorded aircraft noise in the laboratory, brought forward a need for loudspeakers with a good low frequency response. More specifically, the speakers were required to reproduce the noise that was generated by the fundamental of the propeller blade
pass frequency (BPF) and each of the harmonics with minimal levels of distortion, at amplitudes equivalent to those measured in the aircraft cabin. For aircraft installation, the overall weight of the system must remain at a minimum, but the reproduction of this high amplitude low frequency noise meant that the speaker drivers would have to produce large volume velocities, by either using a large diameter or long displacement cone (Fry and Fryer (1990)). The loudspeakers discussed here however, were not intended for aircraft installation, but were required to provide an adequate control source for the initial round of experiments and demonstrate that ANC in a single engine light aircraft was feasible, if the sound power to weight ratio of the control sources was not an issue. Never-the-less, the performance of the loudspeakers had to be optimised for low frequency performance.

Three speaker enclosures were modelled, designed and fabricated (figure 3.2). The first was a 6 inch, 4 Ω, 80 Watt driver in a single vented tube, the second was an 8 inch, 4 Ω, 150 Watt driver in a double vented band pass enclosure and the third was a 12 inch, 4 Ω, 200 Watt driver in a single vented enclosure. The design, modelling, fabrication and test procedures for all three were identical so for brevity, only the performance of the second speaker (figure 3.2(b)), which provides a good example of the performance limitations, is discussed in the following section.

**Double vented band pass enclosure with an eight inch driver**

Dedicated application software LEAP (Linear X Systems Incorporated (1994)) was used to model and predict the response of an 8 inch, 4 Ω, 150 Watt driver in the following four configurations:

1. free air,

2. with an infinite baffle,

3. in a sealed enclosure and
Chapter 3. Initial experiments

Figure 3.2: The three speakers that were modelled, designed and built for the initial experiments.

(a) Single vented drainpipe speaker with an 6” driver

(b) Double vented band pass enclosure with an 8” driver

(c) Single vented enclosure with a 12 ” sub woofer
4. with a band pass double vent.

As expected and as shown in figure 3.3, the modelled free air condition of the drivers demonstrated extremely poor low frequency performance, due to destructive front and rear wave interference. An infinite baffle demonstrated an improved low frequency response with a 20 dB gain at 100 Hz and enclosing the speaker added approximately a further 5 dB at 50 Hz. A band pass enclosure, with vents in the front and rear cavities, provided the most favourable result with an average sensitivity of 90 dB between 60 Hz and 400 Hz and a 93 dB peak at 300 Hz. Consequently, this was the preferred design for this size driver.

Figure 3.3: The predicted frequency response of a speaker driven at 1 Watt and measured at a distance of 1 m. The black trace is the free air condition, the green trace is with an infinite baffle, the blue trace is a sealed enclosure and the red trace is for a band pass vent.

The enclosure was fabricated from 3/4” medium density fibre board, glued and screwed at all edges. The thickness was chosen to minimise any structural motion, care was taken to seal all
joints and the ports were telescopic for fine-tuning. Figure 3.4 shows the frequency response of the enclosed speaker in an anechoic chamber, measured 1m away from the front surface. The loudspeaker was driven with a 50 Watt constant power sine wave swept through the frequency range. The measurements were then repeated for a 100 Watt power input.

Figure 3.4: The measured frequency response of the band pass speaker compared to the model. The red trace is the 100 Watt response and the blue trace is the 50 Watt response, both measured at 1 m. The frequency response from the model (+20dB for a 100 Watt output) is shown as a background water mark.

At 100 Watts the frequency response of the speaker was similar to that predicted by the model, although 20 dB higher because of the 20 dB increased power input. The sound pressure level (SPL) was generally higher than 102 dB (between 60 Hz and 400 Hz) with a maximum peak of 113 dB at 330 Hz. At 77 Hz (the fundamental BPF of the aircraft noise) the SPL was 106 dB, demonstrating that sufficient acoustic power was available to cancel the primary sound field of the aircraft cabin at a distance of 1m from the speaker.
3.2. Equipment

Figure 3.5 shows the distortion associated with the enclosed speaker for a 50 Watt and a 100 Watt, 77 Hz sine wave input. The distorting harmonics are shown compared in magnitude to the fundamental (the 0 dB reference) and are due to the increase in constant-power displacement at lower frequencies. Large coil displacements travel outside of the linear range of the magnetic field and become flattened, generating harmonics. While the measured speaker harmonics are >22 dB lower than fundamental, they are similar in magnitude to the BPF harmonics of the aircraft propeller (refer figure 3.9) and hence likely to cause control difficulties in some instances.

![Diagram showing harmonic distortion](image)

*Figure 3.5: The speaker response showing harmonic distortion for excitation at 77 Hz.*

When the control source is driven at an appropriate cancelling amplitude and at the blade pass fundamental frequency, the speaker distortion will create coincident frequencies of similar magnitude to the primary BPF harmonics. If the BPF harmonics are not included in the reference signal, then their phase relationship with each corresponding speaker harmonic will be unpredictable and the resulting sound field at these harmonics may be either attenuated or increased. However, if the BPF harmonics are also targeted for cancellation and included
in the reference signal, the control system will adjust the phase of the output signal for each referenced frequency, to ensure the minimisation of the cost function within the limits of the system. Even so, a reduction in the speaker distortion, so that the control system does not have to compensate for the speaker non-linearity, would improve the dynamic range of the control system, and possibly the level of achievable noise attenuation.

The low frequency performance of the speaker may be improved by one of the following three methods:

1. Use a similarly sized speaker, but with a larger allowable coil displacement,
2. Use a higher powered speaker with a larger cone,
3. Redesign the enclosure to produce a sharper response at 77 Hz, to minimise the cone displacement.

The first two methods would allow a greater volume velocity and ensure that the speaker would safely operate well within the limits of the moving coils. The latter option, designed to optimise the performance of the speaker at the BPF fundamental, would however reduce the sensitivity of the speaker at the higher frequencies. Either way, the methods discussed will all add mass to the active noise control system and it may therefore be more practical to consider locating the speakers within a close proximity to the observer, possibly in a head rest.

### 3.2.3 Phase locked reference signal generator

For an active noise controller to operate most efficiently and remove only the noise at the required frequencies, a clean reference signal must be supplied to the controller. A “Phase locked reference signal generator” (figure 3.6 and figure 3.7) was therefore designed and built, which operates by measuring the signal from a magnetic pickup located adjacent to the toothed flywheel of the aircraft engine. The resulting pulses (149 per revolution) were electronically
converted (via a micro-controller and a “look up” table) into either a pure tone output signal at the blade pass fundamental frequency, the blade pass harmonic frequencies, or at any combination of fundamental and harmonic frequencies.

![Figure 3.6: The phase locked reference signal generator.](image)

![Figure 3.7: The active noise control system with the phase locked reference signal generator.](image)
3.3 Noise measurements

The interior noise of a Piper Archer small single engine aircraft (registration number VHPOQ, figure 3.1) was recorded while the engine was running at both idle and for full throttle. This was so that the noise could be replayed and controlled under laboratory conditions.

The noise was also measured with third octave filters, to estimate the power requirement for the speaker system (figure 3.8) and again with narrow band FFT filters to identify the specific noise characteristics (figure 3.9).

Measurements from inside the aircraft cabin, adjacent to the pilot’s ear, with the engine at an idle condition, showed a linearly weighted sound pressure level of 105 dB and an "A" weighted sound pressure level of 83 dBA (figure 3.8). At full throttle, the measured sound levels were 113 dB (97 dBA) and identical to those measured by De Metz (1988) and Jha and Catherines (1978a). Control sources capable of producing similar levels were required to cancel the sound field at the error sensor locations.

![Graph showing noise levels at different frequencies for idle and full engine speed.](image)

Figure 3.8: The third octave analysis of the noise measured at the pilot’s ear in the Piper Archer aircraft cabin, for both full throttle and idle engine conditions.

Figure 3.9 shows the narrow band spectrum of the noise in the region of the pilot’s ear, which is also in agreement with data measured by Jha and Catherines (1978a). The spectrum is...
-dominated by discrete frequency components, with the fundamental BPF evident at 77 Hz and its harmonics at 155 Hz, 232 Hz and 310 Hz, progressively reducing in magnitude by steps of approximately 7 dB. The Piper Archer has a twin blade propeller and a four cylinder engine, so this frequency also corresponds to the exhaust (or the engine firing) frequency. But as previously established by work reported in the literature review, the propeller source is known to be much louder than the engine exhaust.

![Figure 3.9: The narrow band analysis of the noise measured at the pilot’s ear in the Piper Archer aircraft cabin for a full throttle condition.](image)

Figure 3.9: The narrow band analysis of the noise measured at the pilot’s ear in the Piper Archer aircraft cabin for a full throttle condition.

Figure 3.10 shows the frequency response function of the noise measured at the pilot’s ear for a white noise source located in the rear of the cabin. From this, identification of modes is extremely difficult, illustrating that a high level of damping is evident throughout the noise spectrum. The presence of this high damping implies that multiple modes are likely to be significantly excited by any one excitation frequency and complicate any chance of global control. Calculating the modal overlap index and the Schroeder frequency would identify if the sound field in the cabin was diffuse (Sum and Pan (1998)) and hence whether global control or only local control was possible. However, this would either require knowledge of the cavity reverberation time or the half-power modal bandwidths and since neither are known and can vary considerably, the result would be a guess at best.
Chapter 3. Initial experiments

3.4 Active noise control in the laboratory

The tape recorded internal cabin noise, for full throttle conditions, was replayed through a loudspeaker in the laboratory. To establish the optimal ANC performance and evaluate the perceived improvement as a result of active noise control, the BPF fundamental and each harmonic were incrementally removed by a digital filter. Informal subjective audio comparisons were then made to establish the perceived improvement and the value of removing each frequency component. It was agreed in general that only the removal of the fundamental and the first three harmonics were found to have an appreciable effect and that there was no perceivable improvement in removing any of the higher order harmonics.

In an anechoic chamber, the tape recorded, un-filtered, full throttle, aircraft noise was replayed through a 12” enclosed speaker and controlled via a second. At the mid positioned microphone, significant attenuation was visible at the fundamental and the next three harmonics (figure 3.11). The fundamental (first harmonic) was reduced by 33 dB, the second harmonic

---

1The informal subjective audio comparison was conducted by Colin D. Kestell and Dr. Ben Cazzolato.
by 17 dB, the third by 12 dB and the forth by 13 dB. The noise increases slightly between the second and third harmonics, most probably because of system noise, since the acoustic properties of the anechoic chamber are characteristically similar to a free field. Evidence from the speaker design investigation indicates that a higher degree of harmonic reduction may have been possible with superior speakers. Never-the-less the findings, although for an extremely simple case, were encouraging.

![Graph showing active noise control](image)

**Figure 3.11:** The active noise control of the replayed flight data in the anechoic chamber, showing control of the BPF fundamental and the first three harmonics. The upper trace (which uses the left hand scale) shows the uncontrolled noise as a dashed line and the controlled noise as a red solid line. The lower blue trace (which uses the right hand scale) shows the attenuation that resulted from active noise control.

### 3.5 Active noise control within the aircraft

The first experiment conducted within the aircraft was intended to be simplistic and evaluate how well the blade pass fundamental could be controlled at a single location with a single control source active noise control system. The phase locked reference signal generator, as
described in section 3.2.3, was used to gain a pure tone phase locked reference signal, corresponding to the blade pass fundamental frequency of the aircraft (77 Hz).

The aircraft was parked on the runway apron (figure 3.12(a)) and a single control source loudspeaker was placed in the rear of the cabin, on the rear passenger seat (figure 3.12(d)). A single microphone error sensor was placed at head height between the front two seats, approximately 5 cm from the pilot’s ear (figure 3.12(d)).

(a) The aircraft on the apron, connected to the experimental equipment in the rear of the wagon

(b) The experimental equipment, operated from the rear of the wagon

(c) The cramped interior of the Piper Archer cabin

(d) The measurement microphone, with the secondary source speaker located on the rear seat

Figure 3.12: The equipment used for the Piper Archer active noise control experiment and the interior of the aircraft.
Prior to the aircraft operating, the user defined parameters of the controller were set to the most appropriate values. These were established in the active noise control experiments that were previously conducted in the laboratory. The phase locked reference signal generator was set to produce only a reference signal at the propeller BPF (77Hz). To ensure that the speaker and power amplifier were powerful enough to cancel the primary noise field at the propeller BPF, it was sinusoidally driven at 77 Hz to generate a 105 dB sound pressure level at the error sensor (figure 3.9). This was also taken as an opportunity to ensure that the microphone amplifier and the controller input attenuation were optimised for the anticipated noise levels. The engine was then started, taken to full throttle and once conditions were stable, narrow band measurements of the sound spectrum were taken with the active noise controller turned off and then again with it turned on. Figure 3.13 shows the primary noise, the controlled noise and the noise attenuation that resulted from the active noise control.

![Figure 3.13](image)

*Figure 3.13: The active noise control of real time aircraft noise where only the BPF (77 Hz) is controlled with one control source and one error sensor. The upper trace (which uses the left hand scale) shows the uncontrolled noise as a dashed line and the controlled noise as a solid line. The lower trace (which uses the right hand scale) shows the attenuation that resulted from active noise control.*

The controlled sound field apparently demonstrates a shift of the fundamental frequency under active control. This however would simply be a remaining side lobe of a somewhat damped...
primary signal partially cancelled with a sharper 77 Hz control source signal. While the 22 dB attenuation at the fundamental BPF was extremely encouraging and would be an easily perceivable reduction in sound, the pilot who was no more than 5 cm from the error sensor (microphone) heard no improvement as a result of active noise control.

Figure 3.14 shows why, with the results of a separate experiment in which an additional measurement microphone was incrementally moved further away from the error sensor to measure how the resulting attenuation varied with distance. It shows that the attenuation due to active noise control, rapidly decreases as the distance between the measurement location and the error sensor is increased. It should be noted that only the attenuation shown at 77 Hz is relevant, since this was the only frequency controlled. The variation at the other frequencies was merely due to temporal noise variations while multiple measurements (at an increasing separation distance from the observer) were made with a single microphone.

![Figure 3.14: The reduction in attenuation as the distance between the measurement location and the error sensor is increased.](image-url)
3.6 Conclusions from the initial experiments

The measured noise levels and characteristics of the Piper Archer cabin, under full throttle conditions, confirmed the findings of the literature review in as much defining the cabin to be an extremely noisy environment characterised by discrete tonal frequencies.

The design and fabrication of the control source loudspeakers revealed the difficulties associated with optimising a speaker for low frequency noise cancellation while keeping size and weight to a minimum. The band pass enclosure driven by an 8” speaker was shown to be capable of cancelling the cabin noise at a distance of 1m from the speaker, but demonstrated significant harmonic distortion. The use of more powerful or highly tuned speakers may alleviate the problem of harmonic distortion, but would also increase the system size and weight.

The high level of damping that was evident in the cabin implies that there will be little advantage in optimising the control source performance by attempting to locate it at pressure anti-nodes (which may also not be a practical consideration). Therefore to keep the speaker size to a minimum and ensure that it operates within acceptable limits of performance, it must be placed in close proximity to the pilot or passengers.

The experiments conducted in the laboratory demonstrated that noise in a multiple tone sound field (such as that in a small aircraft cabin) can be significantly reduced by active noise control. An experiment in the aircraft demonstrated that the operational aircraft noise can also be reduced by ANC. However, even though the noise was cancelled in close proximity to the pilot’s ear, the zone of attenuated noise was so small that the observer experienced no noise improvement at all. Consequently, rather than evaluating the noise reduction that is achievable at the error sensor, the focus of further research must be directed towards ensuring that the zone of noise reduction either envelops the observer or occurs exactly at his or her location. The following chapter therefore addresses how these objectives may be met.
Chapter 4

Evolving a hypothesis to reduce the observers’ experience of noise

4.1 Introduction

While structural control and sensing may be better suited to global cabin noise reduction, the literature review concluded that acoustic excitation and sensing methods offer distinct advantages with respect to aircraft certification and the practical and cost aspects of installation, especially if ANC is a retrofit option. To design an effective and practical control system however, still requires the precise definition of a design objective. While apparently obvious in as much as this objective must be: “To minimise the observers’ experience of cabin noise” it allows focus without an “overkill, cure all” approach.

Nelson and Elliott (1995) stated that minimising the potential energy in an enclosure (the theoretical optimum for global noise reduction) will always reduce noise in the higher noise regions, but at the expense of increasing the noise in regions where it was previously lower than the global optimum level. The spatial sound pressure variation of such a control strategy would be dependent on the physical and geometrical attributes of the enclosure, not the
location of the observers. It may therefore follow that if the occupant is already positioned at one of the quieter locations in the cabin, global noise reduction may in fact increase the noise at that particular location. Global control may therefore be a somewhat misguided goal towards the ultimate objective of reducing the observers’ experience of noise. Nelson and Elliott (1995) also continue to explain that global control within an enclosure can be achieved, even with a control source remotely placed from the primary source, but only providing that the enclosure is lightly damped and excited close to a lightly damped and isolated resonance.

Multiple sources can control multiple modes. This limitation infers that if the enclosure is highly damped and modally dense, as in the case of the single engine aircraft cabin, excited with multiple frequencies (which are unlikely to occur at resonance frequencies), pockets of localised sound reduction zones will at best be achieved (figure 4.1). This effect of localised control was observed in the initial experiments that were conducted in the Piper Archer aircraft (chapter 3).

![Figure 4.1: A schematic representation of the local region of control that may occur around a single microphone error sensor.](image)

The objective must therefore be to ensure that these pockets of localised sound reduction zones occur precisely at the observer location. However, placing an error sensor exactly at the observer location is not practical, since it will most certainly ensure interference between the sensor and the observer. But since the occupants of a small single engine aircraft are constrained to a seated position, with only a limited amount of upper body movement, a cost function that is more spatially constant than acoustic pressure squared, might sufficiently extend the attenuation zone from around the error sensor, to fully envelop the observer (figure 4.2). In a four
seater aircraft it would only be necessary to control the noise around eight soccer ball sized zones (one zone for each ear).

![Figure 4.2](image)

Figure 4.2: A schematic representation of the broad region of control that may occur with a cost function that is more spatially constant than acoustic pressure squared.

Alternatively, it is conceivable that the attenuation zone may be moved away from the error sensor and to the observer, by estimating the cost function at the observer location from the signal measured by the remote error sensor (figure 4.3).

![Figure 4.3](image)

Figure 4.3: A schematic representation of the region of control that may occur by estimating an acoustic pressure squared cost function at the observer location. The ghosted sensors represent the “virtual sensor” location.

It may also be possible to fully exploit the advantages of each method by designing an algorithm to predict a spatially constant cost function at the observer location and centre a broad region of control at the observer’s ear (figure 4.4). An appropriately designed algorithm, may also allow any movement of the observer’s head to be tracked and ensure that the attenuation zone always remains centered around the observer.
Figure 4.4: A schematic representation of the region of control that may occur by estimating an energy density cost function at the observer location. The ghosted sensors represent the “virtual sensor” location.

The following sections present a discussion of the advantages and disadvantages of each method and how each may be achieved.

4.2 Extending the zone of attenuation

As previously detailed in chapter 2, energy density is known to be more spatially uniform than acoustic pressure squared, resulting in larger regions of attenuation when it is used as an active noise control cost function. However, for a multi-channel control system, the maximum attenuation in acoustic pressure is still likely to occur at the error sensor location and the size of the zone of local control will be inversely proportional to frequency (Cazzolato (1999)).

4.3 Moving the zone of attenuation

Estimating the acoustic pressure squared cost function at an observer location, or simulating the presence of a microphone at the observer location (via the signal measured from remotely placed microphone) has been referred to as using a “virtual microphone”. The innovative concept of the virtual microphone was first introduced by Elliott and David (1992) and developed by Garcia-Bonito et al. (1996) and Garcia-Bonito et al. (1997). Rafaely et al. (1999)
showed that virtual microphones could be applied to the active control of broadband noise and Carme and De Man (1998) also used the same virtual microphone technique to improve the performance of a pair of ANC ear defenders.

Garcia-Bonito’s idea is based on measuring the acoustic pressure transfer function between a permanently placed remote microphone and a microphone temporarily located at the observer location. With the temporary microphone subsequently removed, the signal from the permanent microphone was modified with the transfer function to create the virtual microphone. The assumption is implicit that the transfer function does not alter with time and while it is possible that the control system will become ineffective if it changes significantly, Garcia-Bonito et al. (1996) report a good degree of stability when a spherical shape is added to the environment.

The theory is summarised as follows:

The total pressure field \( p \) is the sum of the primary \( p_p \) and control pressure fields \( p_c \)

\[
p = p_p + p_c
\]  

The control contribution may also be written as the product of the complex acoustic transfer impedance \( Z \), between the acoustic pressure measured at the control source location and the error sensor location) and the control source strength \( q_c \). This can be applied to both the actual and virtual microphone locations where the suffixes \( a \) and \( v \) apply to the actual and virtual locations respectively:

\[
p_a = p_{pa} + Z_a q_c
\]  

\[
p_v = p_{pv} + Z_v q_c
\]

The pressure difference between the two positions then becomes:
Garcia-Bonito et al. (1996) state that at low frequencies the spatial rate of pressure change due to the primary field is small enough to assume that the primary source pressure component is the same at both the virtual and actual location. Close to the control sound source the actual sensor and the virtual error sensor to control source transfer impedance functions are significantly different. The prior measurement of this difference can then be used as an operator on the actual error signal to estimate the pressure at the virtual location.

\[ p_a - p_v = (p_{pa} + Z_a q_c) - (p_{pv} + Z_v q_c) \] (4.4)

However, the modified acoustic pressure squared cost function will still tend to produce a relatively small zone of local control, that may limit the observer’s movement if they are to remain in the zone of attenuation.

Cazzolato (1999) suggests that future research may use an alternative method of forward wave prediction to estimate a cost function remote from the error sensor. The novel concept stops short of any development, but provides food for thought with respect to designing spatially adaptive virtual error sensors for a variety of cost functions.
4.4 Combining the benefits (the hypothesis)

Estimating an acoustic pressure squared cost function at the observer location may produce a highly attenuated region at the observer’s ear, but the control zone may be so narrow that the observer may be exposed to dramatic variations in sound pressure level with any small amount of head movement. This experience may be even more unpleasant than being subjected to the uncontrolled (yet stable) primary noise. As previously stated, an energy density cost function is more spatially uniform, but the acoustic pressure squared minimum is still likely to occur at the sensor location.

There is therefore an obvious benefit in combining the advantage of extending the zone of attenuation outwards from the error sensor (using an energy density cost function) with the idea of moving the attenuation zone towards the observer with a virtual microphone. This would mean that a cost function more spatially uniform than acoustic pressure squared would be centered around the observer and allow for a limited amount of head movement without exposure to dramatic sound pressure level variations.

Cazzolato’s suggestion of using forward wave prediction may also offer an additional benefit and evokes the hypothesis that:

*forward wave prediction virtual sensors may not only combine the benefits of the two afore mentioned methods, but may also adapt to any observer movement.*

This would further reduce the observer’s experience of sound pressure level variations with head movement.

The following research commences with this hypothesis and continues by developing equations for “*forward wave prediction virtual microphones*” and “*forward wave prediction virtual energy density sensors*”. It is anticipated that the use of these virtual sensors as active noise control error sensors will possess the following advantages with respect to the more conventional active noise control error sensors:

Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
1. The high attenuation associated with local control may be brought directly to the observers’ ear without immediately obtrusive transducers.

2. The attenuation zone around the observer may be broadened with a virtual energy density sensor.

3. With an algorithm based on separation distance, it may be possible to track the distance between the observer and allow the noise attenuation zone to follow the observer.

4. Controlling immediately around the head of the observer will allow the use of much smaller speakers (than required for global control) and minimise the additional weight in the aircraft.

The following sections introduce the theory as well as the development of a variety of new virtual sensor algorithms. These virtual sensors were then modelled and experimentally evaluated, under more progressively complex conditions to identify their potential and limitations for use in a practical application, such as that of a single engine light aircraft.
Chapter 5

Forward wave prediction theory

5.1 Introduction

An alternative approach to the Garcia-Bonito et al. (1996) transfer function based virtual microphone is to use forward difference extrapolation (figure 5.1). For low frequency noise, where the sensor spacing is much less than a wavelength, the sound pressure at the observer (virtual microphone) location may be estimated in real time by extrapolating the signal from remotely placed microphones. This eliminates the need for the prior measurement of the complex acoustic transfer impedance function and allows the prediction technique to adapt to any physical system changes such as head movement or any other mechanism that may alter the error sensor to control source complex acoustic transfer impedance function.

The theory for a first-order (2 microphone) and a second-order (3 microphone) forward difference extrapolation virtual microphone will now be derived. The same approach will be extended to a first-order (2 microphone) and a second-order (3 microphone) virtual energy density sensor with the intention of achieving a broader zone of local control to allow a practical amount of comfortable head movement.
Chapter 5. Forward wave prediction theory

Figure 5.1: Forward wave prediction.

(a) First-order forward prediction

(b) Second-order forward prediction

Figure 5.1: Forward wave prediction.
5.2 Virtual microphone

At low frequencies, the spatial rate of change of the sound pressure between relatively closely
spaced locations (in terms of wavelength) in free space, in a duct, or in an enclosure will be
small and hence predictable. By fitting either a straight line or a curve between pressures
measured at fixed locations, the pressures at other locations may be estimated by either inter-
polation or extrapolation. Since in this research it is intended that an observer is to be remote
from the physical sensors, rather than between them, the following equations are extrapolation
based.

5.2.1 Two microphone first-order pressure prediction

Figure 5.1(a) illustrates that the pressure at location \( x \) can be approximated by the first-order
finite difference estimate \( p = \left( \frac{dp}{dx} \right) x + c \), where \( \frac{dp}{dx} \) is constant) from two remote microphones,
separated by a distance of \( 2h \) by measuring pressures \( p_1 \) and \( p_2 \) respectively at two microphone
locations as follows:

\[
p_x = \frac{(p_2 - p_1)}{2h} x + p_2
\]  
(5.1)

To maintain continuity when comparing this method to the three microphone second-order
method (discussed later), \( 2h \) is chosen as a separation distance.

To confirm, if \( x = 0 \), or if \( x = -2h \) then equation (5.1) reduces to \( p_x = p_2 \) or \( p_1 \) respectively. But
more practically, if the separation distance \( x \) between the observer and the nearest transducer
is equal to \( +h \) then equation (5.1) reduces to:

\[
p_x = \frac{p_2 - p_1}{2} + p_2 = \frac{1}{2} (3p_2 - p_1)
\]  
(5.2)
if the separation distance is increased to \( x = 2h \) then:

\[
p_x = 2p_2 - p_1 \quad (5.3)
\]

### 5.2.2 Three microphone second-order pressure prediction

The use of a third intermediately placed microphone allows an estimation of the rate of change of the pressure gradient, which enables a greater prediction accuracy (figure 5.1(b)). This second-order approximation, where \( \frac{d^2p}{dx^2} \) is assumed constant (a constant rate of pressure gradient change), can be integrated to determine the relationship between the pressure \( (p_x) \) at the virtual location \( (x) \) and the pressures measured at the three actual microphone locations \( p_1, p_2 \) and \( p_3 \) as follows:

\[
p_x = \int \int \frac{d^2p}{dx^2} . (dx)(dx)
= \int \int k_1 . (dx)(dx)
= \int (k_1x + k_2) . (dx)
= \frac{k_1x^2}{2} + k_2x + k_3 \quad (5.4)
\]

The constants of integration \( k_1, k_2 \) and \( k_3 \) can be found by applying equation (5.4) to the pressure and location of each of the three microphones at \( x_3 = 0, x_2 = -h \) and \( x_1 = -2h \); thus:

\[
P_3 = k_3
P_2 = \frac{k_1h^2}{2} - k_2h + k_3
P_1 = 2k_1h^2 - k_2h + k_3 \quad (5.5)
\]
Simultaneously solving the equations allows the value for each of these constants to be calculated. When substituted into equation (5.4) this yields:

$$p_x = \left( \frac{p_1 - 2p_2 + p_3}{h^2} \right) \frac{x^2}{2} + \left( \frac{p_1 - 4p_2 + 3p_3}{2h} \right)x + p_3$$

(5.6)

Collecting like terms to calculate the weighting factors for each actual microphone (which is more practical for hardware design), results in the pressure at location $x$ being expressed as:

$$p_x = \frac{x(x+h)}{2h^2}p_1 + \frac{x(x+2h)}{-h^2}p_2 + \frac{(x+2h)(x+h)}{2h^2}p_3$$

(5.7)

Once again, in order to confirm the equation, if $x = 0$ or $x = -h$ or $x = -2h$ then equation (5.7) reduces to $p_x = p_3$, $p_2$ or $p_1$ respectively.

If $x = +h$ then equation (5.7) reduces to:

$$p_x = p_1 - 3p_2 + 3p_3 = p_1 + 3(p_3 - p_2)$$

(5.8)

If $x = 2h$ then equation (5.7) reduces to:

$$p_x = 3p_1 - 8p_2 + 6p_3 = p_1 - 2(p_2 - p_1) + 6(p_3 - p_2)$$

(5.9)

It could also be possible to have two closely spaced “virtual microphones” which when controlled would create a larger zone of quiet than would be achieved with a single “virtual microphone”. It may also be possible to define a virtual microphone array to extend the control region even further. In the next section this approach is adapted to derive a virtual energy density sensor, with the aim of achieving broad zones of reduced noise centered around the head of the observer.
5.3 Virtual energy density sensor

Using the same forward difference prediction method as introduced in the previous section, the pressure gradient at some point located away from the sensing microphones may also be estimated. Therefore, this section will now show the derivation of the equations for a first-and second-order “virtual energy density sensor”.

5.3.1 Two microphone first-order prediction

For the two microphone sensor shown in figure 5.1(a), the best estimate of pressure at a distance \( x \) from the second microphone is given by equation (5.1), ie. \( p_x = \left( \frac{p_2 - p_1}{2h} \right) x + p_2 \) and the particle velocity is obtained from the best estimate of the pressure gradient given by:

\[
\frac{dp}{dx} = \frac{p_2 - p_1}{2h}
\]  

Euler’s equation relates particle velocity to spatial pressure gradient with the general relationship:

\[
\frac{dp}{dx} = -\rho \frac{dv}{dt}
\]  

which for monotone sound fields reduces to:

\[
v = -\frac{1}{j\omega \rho} \frac{dp}{dx}
\]  

Therefore at any given frequency \( \omega \) (radians/s), the particle velocity estimate for a two microphone sensor is obtained by multiplying the pressure gradient in equation (5.10) by \( -\frac{1}{j\rho \omega} \), ie:

\[
v_x = -\frac{1}{j\rho \omega} \left( \frac{dp}{dx} \right) = \frac{p_2 - p_1}{j2\rho \omega}
\]
5.3. Virtual energy density sensor

The instantaneous energy density (Nashif and Sommerfeldt (1992)), which is the sum of instantaneous potential and kinetic energy density, at a point \( x \) is given as:

\[
E_{Dx} = \frac{p_x^2}{2\rho c^2} + \frac{\rho v_x^2}{2}
\]

\[
= \frac{1}{2\rho c^2} \left[ p_x^2 + \rho^2 c^2 v_x^2 \right]
\]

(5.14)

It should be noted that the terms outside of the square brackets equally weight both the pressure and the velocity. The terms inside the square brackets define the relative magnitude of the pressure and velocity components and must therefore be accurately incorporated in the design of any equipment.

Substituting the pressure (equation (5.1)) and velocity (equation (5.13)) estimates into equation (5.14) and simplifying with \( k = \omega / c \), an estimate for the energy density at some virtual location \((x)\) in terms of the pressures of the two microphones is obtained:

\[
E_{Dx} = \frac{1}{2\rho c^2} \left[ \left( \frac{(p_2 - p_1)}{2h} \right)x + p_2 \right]^2 + \rho^2 c^2 \left( \frac{(p_2 - p_1)}{j2\rho \omega} \right)^2
\]

\[
= \frac{1}{2\rho c^2} \left[ \left( 1 + \frac{x}{2h} \right)^2 \frac{p_2^2}{2} - \frac{x}{h} \left( 1 + \frac{x}{2h} \right) p_1 p_2 + \left( \frac{x}{2h} \right)^2 p_1^2 \right]
\]

(5.15)

From this equation it can be seen that in a two microphone system, the pressure gradient estimate is assumed to be constant and the pressure at the observer is estimated to be zero when the pressure at both sensors is also zero. Therefore, minimising an estimate of energy density at an observer location is, in this method, no different to minimising energy density at the sensor locations.
For a separation distance between the nearest transducer and the observer of $x = 0$ this reduces to:

$$E_{D_x} = \frac{1}{2 \rho c^2} \left[ p_2^2 - \frac{1}{(2hk)^2} \left( p_2^2 - 2p_1p_2 + p_1^2 \right) \right]$$  \hspace{1cm} (5.16)

or for $x = h$:

$$E_{D_x} = \frac{1}{2 \rho c^2} \left[ \left( \frac{3}{2} \right)^2 p_2 - \left( \frac{3}{2} \right) p_1 p_2 + \left( \frac{1}{4} \right) p_1^2 \right] - \frac{1}{(2hk)^2} \left( p_2^2 - 2p_1p_2 + p_1^2 \right)$$  \hspace{1cm} (5.17)

or if $x = 2h$:

$$E_{D_x} = \frac{1}{2 \rho c^2} \left[ \left( 4p_2 - 4p_1p_2 + p_1^2 \right) - \frac{1}{(2hk)^2} \left( p_2^2 - 2p_1p_2 + p_1^2 \right) \right]$$  \hspace{1cm} (5.18)

### 5.3.2 Three microphone second-order prediction

For the three-microphone sensor shown in figure 5.1(b), the pressure gradient estimate is obtained by differentiating the pressure estimate shown in equation (5.6), ie:

$$\frac{dp}{dx} = \frac{d}{dx} \left[ \frac{(p_1 - 2p_2 + p_3)}{h^2} x^2 + \frac{(p_1 - 4p_2 + 3p_3)}{2h} x + p_3 \right]$$

$$= \left[ \frac{(p_1 - 2p_2 + p_3)}{h^2} x + \frac{(p_1 - 4p_2 + 3p_3)}{2h} \right]$$

$$= \frac{1}{h^2} \left[ \frac{2x+h}{2} p_1 - (2x+2h) p_2 + \frac{2x+3h}{2} p_3 \right]$$  \hspace{1cm} (5.19)

In the same manner as the two microphone method, the particle velocity estimate for the three microphone sensor is obtained by multiplying the pressure gradient in equation (5.19)
by $1/j\rho_0$. This can then be substituted into equation (5.14) along with the virtual pressure estimate (equation (5.7)), to estimate the virtual energy density (equation 5.20).

$$E_{D_v} = \frac{1}{2\rho c^2} \left[ \left( \frac{x(x + h)}{2h^2} p_1 + \frac{x(x + 2h)}{-h^2} p_2 + \frac{(x + 2h)(x + h)}{2h^2} p_3 \right)^2 + \rho^2 c^2 \left( \frac{1}{j\rho_0 h^2} \left( \frac{2x + h}{2} p_1 - (2x + 2h) p_2 + \frac{2x + 3h}{2} p_3 \right) \right)^2 \right]$$

$$= \frac{1}{2\rho c^2} \left[ \left( \frac{x(x + h)}{2h^2} p_1 + \frac{x(x + 2h)}{-h^2} p_2 + \frac{(x + 2h)(x + h)}{2h^2} p_3 \right)^2 - \frac{1}{k^2} \left( \frac{2x + h}{2h^2} p_1 - \frac{2x + 2h}{h^2} p_2 + \frac{2x + 3h}{2h^2} p_3 \right)^2 \right]$$

(5.20)

This may also be customised for various set distances of $x$ in terms of $h$.

## 5.4 Higher-order prediction methods

From equation 5.1 and figure 5.1(a) it is evident that the two sensor prediction method is fundamentally based on the assumption of a constant pressure gradient between the sensor locations and the observer, or that the estimate has a constant first-order derivative. The two microphone forward wave prediction method is therefore described as a “first-order” estimate. In the three sensor prediction method (equation 5.7 and figure 5.1(a)), it is assumed that that the pressure gradient changes at a constant rate, or that the second-order derivative is constant. Three sensor forward wave predictions are therefore termed “second-order” estimates. From figures 5.1(a) and (b) it can be seen that the second-order method is theoretically more accurate than the first-order method at defining a waveform that is remote from the sensors. This indicates that theoretical prediction accuracy increases with the order of the prediction method. In fact, any waveform ($f(x)$) can be explicitly defined by a Taylor's series expansion:
\[ f(x + h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(0) + \ldots \]  
(5.21)

While equation 5.21 has an infinite amount of terms, it can be estimated by using \( n \) terms in the following equation.

\[ f(x + h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(0) + \ldots + \frac{h^n}{n!} f^{(n)}(0) \]  
(5.22)

Introducing higher-order terms to theoretically improve the accuracy of a cost function estimate at the observer location, will require that the number of sensors proportionally increase to \( n + 1 \). However, this theory does not consider any practical issues associated with practical implementation that may adversely effect the prediction accuracy. Therefore, in the chapters that follow, only first-and second-order virtual error sensors shall be evaluated and compared, in order to observe the practical benefits of using higher-order terms for the estimation of the cost function at the observer location.

### 5.5 Movement tracking

The first-and second-order methods of forward wave prediction presented in the previous sections of this chapter all depend on the measurement of pressure from physical sensors placed at some distance \( x \) from the observer. It therefore follows that if a varying separation distance were measured (via an ultrasonic sensor for example), then the prediction algorithm could be continually updated to effectively make the sound field minimum follow the observer. Although only a fixed virtual location is considered here, tracking an observer’s head movement may provide the basis of future research.
Chapter 6

Evaluating the performance of forward prediction virtual error sensors

6.1 Introduction

In this chapter the zone of local control around a forward prediction “virtual energy density sensor” and a forward prediction “virtual microphone” is compared with that achieved when using an actual energy density sensor and a single microphone. It will be shown that in general the forward prediction virtual energy density sensor outperforms the actual energy density sensor, and that the forward prediction virtual microphone outperforms the actual microphone in terms of centering a practically sized zone of local control around an observer who is remotely located from any of the physical sensors. The superiority (or otherwise) of any particular forward prediction virtual error sensor will be shown to be dependent on its environment.

The systems analysed commence with the most simplistic scenario of a single primary sound source in a free field environment. An analytical model is used to predict the performance of the virtual sensors as error sensors and compare their control performance with their physical counterparts. Experimental verification validated the performance of the algorithms under
actual test conditions. Noise in a one-dimensional rigid-walled duct was then modelled using FEA to predict the performance of the sensors in a reactive environment. The model was then validated using a real duct which demonstrated the performance of the sensors in an actual reactive environment.

6.2 Method

The free field and rigid walled duct examples that follow share common methods for simulating the error sensor performance in a model, acquiring the experimental data and comparing the results. Any specific methods unique to each example are discussed with the description of the relevant system.

In each of the models and experiments, the primary noise was generated from a single noise source and the secondary (cancelling) noise was generated from either one or two other independent control sources as required. Transfer functions (between 0 and 400 Hz) were then calculated (or measured in the experiments) between the primary noise source signal and the signal from a measurement microphone at each of 21 measurement locations. These locations were at 25 mm intervals \(h\) along a 0.5 m length. The procedure was then repeated for each individually driven control source.

The equipment used for the noise generation and measurement is listed in table 6.1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Make</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microphones</td>
<td>Generic brand</td>
<td>-</td>
</tr>
<tr>
<td>Microphone amplifiers</td>
<td>Custom built</td>
<td>-</td>
</tr>
<tr>
<td>Dynamic signal analyser</td>
<td>Hewlett Packard</td>
<td>35665A</td>
</tr>
<tr>
<td>Enclosed loudspeakers</td>
<td>Custom built</td>
<td>-</td>
</tr>
<tr>
<td>200 W power amplifier</td>
<td>Promaster</td>
<td>Pro series 3</td>
</tr>
</tbody>
</table>

Table 6.1: A list of the equipment that was used in the experiment to validate the predicted performance of the virtual sensors.
To allow a direct comparison between the active noise control results from each model and its respective experiment and to eliminate any further differences that may be associated with the performance of a practical ANC controller, the cost functions (either from the physical sensors or the virtual sensors), for both the numerical and experimental examples, were all minimised via quadratic optimisation (Appendix A.1).

In theory, infinite attenuation may be achieved at an error sensor. When presented graphically this would either result in data truncation (figure 6.1(a)) or a reduction in detail if all of the data were presented within the limits of the vertical axis. In practice, active noise control systems would have inherent errors that would limit the amount of noise attenuation that could be achieved. Therefore, to force the optimisation results to a more realistic magnitude and emulate the uncertainty that occurs in practice, transfer function errors were incorporated into the control algorithm. Initially, a transfer function magnitude error of 1% and a phase error of \( 1^\circ \) \( (1.01e^{j\pi/180}) \) were to be applied to the control signal. However, the phase error was found to sometimes force the cost function minimisation away from the error sensor (figure 6.1(b)) and cloud the issue of assessing the performance of the virtual sensors (figure 6.1(c)). Therefore the limitation of attenuation was purely due to an applied 1% magnitude error (figure 6.1(a)), thereby limiting the amount of control to <40 dB.

To control pressure at a single location only requires one control source; additional sources are redundant (figure 6.2(a)). In a single control source system however, there is little advantage in trying to minimise energy density (Cazzolato (1999)). Attempts to minimise the pressure gradient can result in a local sound pressure increase. Using the free field model data as an example, figure 6.2(b) shows that while the control source magnitude and phase (with respect to the primary noise source) can be optimised to produce a zero pressure gradient between the two sensors, it is at the expense of constructive wave summation that results in an increase in the sound pressure level throughout the region of interest.

Therefore, to effectively minimise energy density, the independent control of pressure and pressure gradient (ie. independent control of pressure at two locations) requires the use of a
(a) The achievable attenuation with various magnitude errors in the control algorithm for a single microphone error sensor. The microphone location is shown as a single circle on each curve. The infinite attenuation associated with no magnitude error is obviously outside of the scale of the graph, but more data points would have brought the descending lines closer.

(b) The achievable attenuation with various phase errors in the control algorithm for a single microphone error sensor (each case includes a -40dB magnitude error). The microphone location is shown as a single circle on each curve.

(c) The achievable attenuation with various phase errors in the control algorithm for a second-order virtual microphone (each case includes a -40dB magnitude error). The physical sensors are shown as three circles on each curve.

Figure 6.1: The effect of errors introduced into the control algorithm to limit the achievable attenuation. In each example the error sensor is separated from the observer by a distance of $4h$. In each sub-figure the vertical line is the observer (desired control) location and the physical sensor locations are shown with a circle.
second control source.

(a) A single microphone error sensor, with increasing separation distances from the observer

(b) Energy density control, with increasing separation distances from the observer

Figure 6.2: Comparing the pressure minimisation at a single location with energy density control using only one control source. The vertical line is the observer (desired control) location and the physical sensor locations are shown with a circle.

Minimising energy density in a two control source and two sensor system, is the result of minimising the acoustic pressure gradient between the two sensors and the mean acoustic pressure measured between them (equation 5.14, page 71). In the simplistic systems considered here, where it is possible to equally minimise the signal from the two sensors; energy density control estimated via two microphones (figure 6.3(a)) is identical to simply minimising the pressures at the two microphone locations (figure 6.3(b)). Practical systems would have more complex wave interaction and inherent errors and it is therefore unlikely that both pressures would be significantly equally minimised while the pressure gradient is also minimised. In a first-order virtual energy density sensor the pressure gradient is assumed to be spatially constant, ie. the same at the sensor and observer locations (figure 5.1). Because the pressure at the virtual location is also at a minimum when the two error sensor pressures are at a minimum (equation (5.1)), first-order virtual energy density control (figure 6.3a)) will be identical to minimising the energy density at the sensor location (figure 6.3(b)) or the pressure at the two error sensor locations (figure 6.3(c)).
Figure 6.3: A comparison of the primary and controlled sound pressure levels for first-order virtual energy density control at the observer location, first-order energy density control at the sensors and minimising the acoustic pressure at the two error sensors. There are two control sources and a primary noise tone at 100 Hz. The vertical line is the observer (desired control) location and the physical sensor location is shown with a circle. The minima are shown between two of these in each case.
A second-order system that involves estimating the remote pressure and pressure gradient with three sensors, results in a more accurate prediction of both pressure and pressure-gradient (in the absence of noise). The pressure at the observer location is estimated by extrapolating the pressure profile from three sensors and the pressure gradient is no longer considered constant, but assumed to have a constant rate of change. The use of only two control sources however is still pertinent since (in these examples) the energy density has two independent contributors; namely pressure and pressure-gradient.

In the examples that follow, the use of a single control source will therefore be limited to observing the acoustic pressure minimisation via a single microphone, a first-order virtual microphone and a second-order virtual microphone. Energy density minimisation (identical to two point pressure control and first-order virtual energy density control) and second-order virtual energy density control will be evaluated with two control sources.

The performance of the error sensors, with increasing separation distances between the physical sensors are compared and discussed:

1. A single microphone,
2. A first-order virtual microphone,
3. A second-order virtual microphone,
4. First-order virtual energy density control (or two point pressure minimisation or energy density control) and
5. second-order virtual energy density.

6.3 A free field model

This section investigates the most simplistic scenario of a single primary sound source in a free field model and is used to illustrate the performance of the error sensors in ideal conditions.
in the absence of any environmental effects. The performance of the virtual sensors as error sensors is compared to the control performance of their physical counterparts. It is shown here that in general the virtual energy density sensor outperforms the actual energy density sensor, the actual microphone and the virtual microphone in terms of centering a practically sized zone of local control around an observer who is remotely located from any physical sensors.

6.3.1 The system

The system investigated here consists of a single frequency primary noise source located in a free field. The 21 measurement locations (detailed in section 6.2) were located 2 m away, along a 0.5 m length at 25 mm intervals (referred to as $h$) and the control noise sources (control speakers) were positioned at 4.5 m and 5 m from the primary source (figure 6.4).

The system was modelled with the primary and control sources represented as point monopoles with a spherical pressure amplitude radiation pattern (Hansen and Snyder (1997)), which is defined by:

$$p_r = \frac{j\omega \rho_o q e^{-jk r}}{4\pi r}$$

(6.1)

where $p$ is the pressure amplitude measured at $r$ distance from the source, $\omega$ is the rotational frequency, $\rho_o$ is the air density, $q$ is the source signal strength and $k$ is the wave number.

6.3.2 Control of a 100 Hz sinusoidal wave

Figure 6.5 shows the results obtained when controlling a 100 Hz tone in a free field with various control strategies. The results obtained by controlling the pressure at the physical error sensor location are shown in figure 6.5(a). This demonstrates that as the error sensor is moved away from the observer, the attenuation at the observer location decreases from 40 dB for a
zero separation distance, to 8 dB for a 4h separation distance. Figure 6.5(b) shows the results obtained using a first-order virtual microphone to control at the observer location. Since the algorithm adapts to an increasing separation distance there is only a negligible reduction in attenuation at the observer location. What small error there is, is due to the decrease in estimation accuracy as the separation distance increases. But this error is reduced by the more accurate second-order estimation technique as illustrated in figure 6.5(c). Results obtained by either controlling the pressure at two sensor locations or controlling direct or virtual first-order energy density (section 6.2) are shown in figure 6.5(d). Controlling energy density produces a broader region of control and as the separation distance between the sensors and observer increases, the attenuation only reduces from 40 dB at 0h to 18 dB at 4h. This is a significantly better result than minimising the pressure at one real error sensor, but is still out-performed by the first and second-order virtual microphones. The second-order virtual energy probe, however, continues to produce an attenuation of 40 dB at the observer location while maintaining a broad region of control (figure 6.5(e)).

6.3.3 Control of a 200 Hz sinusoidal wave

The results obtained when controlling a 200 Hz tone in a free field are shown in figure 6.6. Figure 6.6(a) shows the results obtained by controlling the pressure at the microphone location...
Chapter 6. Evaluating the performance of forward prediction virtual error sensors

Figure 6.5: The primary and controlled sound pressure level from a 100 Hz single sound source along a 0.5 m length in a free field. The actual sensors are marked with a circle and the observer location by a vertical line.

(a) Pressure control at one microphone location

(b) First order virtual microphone

(c) Second order virtual microphone

(d) Pressure control via 2 microphones, energy density control or first order virtual energy density control. The curves are identical for all three cases.

(e) Second order energy density sensor
for increasing separation distances between the error sensors and the observer. The attenuation at the observer location decreases from 40 dB for a 0\(h\) separation distance, to 4 dB for a 4\(h\) separation distance. The control zone is noticeably narrower than the 100 Hz case which is attributed to the shorter wavelength associated with the 200 Hz tone. The results obtained by controlling at the observer location using a first-order virtual microphone are shown in figure 6.6(b). Similarly to the 100 Hz example, the attenuation at the observer location remains high (24 dB at 4\(h\) separation) but is slightly reduced (compared to the 100Hz case) due to increasing prediction errors with the shortening wavelength. The second-order virtual microphone (figure 6.6(c)) again illustrates a higher prediction accuracy and a very encouraging level of attenuation at the observer location (30 dB at 4\(h\)). But once again due to the shorter wavelength, the performance is slightly reduced when compared to the 100 Hz example.

Results obtained by either controlling the pressure at two sensor locations (which is identical to first-order energy density control by interpolation or extrapolation estimates) is shown in figure 6.6(d). This produces a broader region of control than merely minimising pressure at one location. Consequently, as the separation distance between the sensors and observer increases, attenuation reduces from 40 dB at 0\(h\) to 6 dB at 4\(h\). Once again, this is significantly better than reducing the pressure at one real error sensor, but is still out-performed by the first and second-order virtual microphones. The second-order virtual energy probe however continues to produce an attenuation of 40 dB at the observer location while maintaining a broad region of control, albeit slightly narrower than the 100 Hz example (figure 6.6(d)).

### 6.3.4 Control of a 400 Hz sinusoidal wave

Figure 6.7 shows the results that are obtained when controlling a 400 Hz tone in a free field.

The reducing wavelength as the frequency increases, continues to reduce both the control zone and the prediction accuracy (where relevant) for all of the various control strategies. The results obtained by controlling the pressure at the microphone location are shown in figure 6.7(a).
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Figure 6.6: The primary and controlled sound pressure level from a 200 Hz single sound source along a 0.5 m length in a free field. The actual sensors are marked with a circle and the observer location by a vertical line.
6.3. A free field model

Figure 6.7: The primary and controlled sound pressure level from a 400 Hz single sound source along a 0.5 m length in a free field. The actual sensors are marked with a circle and the observer location by a vertical line.
As the separation distances between the error sensors and the observer increases, the attenuation at the observer location decreases from 40 dB for a 0h separation distance, to become a gain of 4 dB for a 4h separation distance. Once again the control zone has been reduced in size when compared to the previous example. Figure 6.7(b) shows the results obtained by controlling at the observer location by a first-order virtual microphone. Attenuation reduces from 40 dB to 8 dB as the separation distance is increased to 4h, remaining significantly better than controlling directly from a microphone with an equivalent separation distance. The second-order virtual control microphone again illustrates a higher prediction accuracy, with the level of attenuation at the observer location of 12 dB for a 4h separation distance (figure 6.7(c)).

Figure 6.7(d) shows the results for controlling the pressure at two sensor locations. While this produces a broader region of control than merely minimising pressure at one location, the attenuation at the observer location reduces to become a gain of 5 dB as the observer / sensor separation increases to 4h. Results obtained by second-order virtual energy control (figure 6.7(e)) show that an attenuation of 40 dB is maintained at the observer location when the observer / sensor separation distance is increased to 4h. The zone of attenuation however is narrower than the 200 Hz example and implies that the size of the attenuation zone is inversely proportional to frequency.

### 6.3.5 Conclusions

Table 6.2 summarises the attenuation that resulted at the observer location in the free field model for various error sensors that were separated from the observer by 4h (100mm). However, the summarised results presented in the table, while indicative of the performance of the error sensors, should not be interpreted in isolation since the zone of attenuation is also important. This however was difficult to quantitatively summarise because the shape and location of the control zone varied significantly from one example to another. It has been demonstrated...
that for all of the methods of control investigated, the size of the zone of attenuation reduces as the primary sound source frequency increases. This implies (and is shown in table 6.2) that as the physical error sensors are moved further from the observer the attenuation rapidly diminishes for higher frequencies.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Single microphone</th>
<th>1st order virtual microphone</th>
<th>2nd order virtual microphone</th>
<th>Energy density</th>
<th>2nd order virtual energy density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free field - 100 Hz</td>
<td>8 dB</td>
<td>38 dB</td>
<td>40 dB</td>
<td>18 dB</td>
<td>40 dB</td>
</tr>
<tr>
<td>Free field - 200 Hz</td>
<td>4 dB</td>
<td>24 dB</td>
<td>30 dB</td>
<td>6 dB</td>
<td>40 dB</td>
</tr>
<tr>
<td>Free field - 400 Hz</td>
<td>-4 dB</td>
<td>8 dB</td>
<td>12 dB</td>
<td>-5 dB</td>
<td>40 dB</td>
</tr>
</tbody>
</table>

Table 6.2: A summary of the attenuation levels at the observer location (separated from the sensors by 4h) that resulted from using various error sensors in the free field model. Negative attenuation values show a gain in the sound pressure level.

It has also been shown that first-order prediction methods for energy density estimation at a remote location (the observer) offer no advantage to controlling energy density directly at the remote sensor (section 6.3.1).

All of the virtual microphone systems investigated show the potential to outperform their physical counterpart, offering a higher level of attenuation at the observer location than by minimising pressure at an equivalent observer / sensor separation distance. While the highest attenuation at the observer location was in general achieved by using a second-order virtual microphone, the size of the attenuation zone was narrow. In terms of both a high level of attenuation and a broad control zone around the location of the observer, the second-order virtual energy density probe produced the most favourable results. The following section will demonstrate how the analytical model discussed here can be applied in practice and will begin to identify the physical limiting factors.
Chapter 6. Evaluating the performance of forward prediction virtual error sensors

6.4 Experimental verification of the free field model

In the previous section an analytical model was used to compare the performance of various new “virtual sensor” cost functions with the performance of conventional sensors in an ideal free field environment. This section reports the results of experimental work that was conducted to validate the results of the analytical study and to observe the performance of the virtual sensor algorithms under the actual test condition of a single primary sound source in an anechoic chamber. As in the previous section, it is shown here that in general the “virtual energy density sensor” outperforms the actual energy density sensor, the actual microphone and the virtual microphone in terms of centering a practically sized zone of local control around an observer that is remotely located from any physical sensors. The virtual sensor algorithms however, are shown to be sensitive (by varying degrees) to short wavelength spatial pressure variations of the primary and control sound fields.

6.4.1 The system

The physical system investigated here (representative of the previously discussed model) consists of a single frequency primary noise source located in an anechoic chamber. The location of the measurement sensors and the 150 mm diameter enclosed primary source and control source speakers, were identical to those locations used in the model (figure 6.8).

The following sections compare the various experimental error sensing strategies and also show how each method compares with its analytical counterpart. For ease of comparison, the analytical results are presented alongside the graphs from the experiment.
6.4. Experimental verification of the free field model

(a) The primary sound source and the measurement microphone

(b) The two secondary sound sources and the measurement microphone (foreground)

Figure 6.8: The experimental configuration in the anechoic chamber.
6.4.2 Control of a 100 Hz sinusoidal wave

Figure 6.9 shows the results that are obtained when controlling a 100 Hz monotone in both the free field model and the anechoic chamber experiment. Figures 6.9(a) and (b) compare the modelled and experimental results for a conventional pressure squared cost function, where the sensor is incrementally moved further from the observer location. Both are similar and show that the attenuation at the observer location reduces from 40 dB to 8 dB as the observer / sensor separation distance increases from $0h$ to $4h$ (100 mm). In figures 6.9(c) and (d) the modelled and experimental results for the first-order virtual microphone are compared. As the sensor is moved further away, the sound pressure at the observer location is estimated by way of extrapolation. Both the model and the experiment show similar results, although the experiment shows that as the observer / sensor separation distance increases to 100 mm, attenuation at the observer location reduces from 40 dB to 22 dB compared to only a negligible reduction in the models performance. However, this control strategy still demonstrates a practical advantage over the conventional remotely placed single microphone (figure 6.9(b)).

Figure 6.9(e) illustrates that in theory second-order prediction is more accurate for the forward wave prediction of a pressure squared cost function, but in practice the experiment shows it is less accurate than the first-order method. The model (theory) is based on sound fields that smoothly reduce at a rate of 6 dB per doubling in separation distance from the source. Figure 6.10 shows that both the primary and control sound fields actually reduce slightly more erratically, introducing errors into the extrapolation (for reasons later discussed in section 7). In this example, the effect of these errors are amplified in the more sensitive second-order method (figure 6.11(b)), resulting in poorer performance in practice than achieved when using the first-order method.

Figures 6.12(a) and (b) illustrate the achievable active noise reduction of a 100 Hz monotone achieved using an energy density cost function and a second control source. Direct energy density control, or first-order virtual energy density control (section 6.2) produces a broader region
6.4. Experimental verification of the free field model

Figure 6.9: A 100 Hz primary sound source controlled via one control source. Measured along a 0.5 m length in an anechoic chamber, the actual sensors are marked with a circle and the observer location by a vertical line.

Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
Chapter 6. Evaluating the performance of forward prediction virtual error sensors

(a) The scale used throughout the chapter

(b) An enlarged scale to illustrate the spatial irregularities in the primary sound field

Figure 6.10: The primary sound field.

(a) An example of the greater theoretical accuracy of a second order extrapolation technique in the absence of short wavelength noise

(b) An example of when the first order method may be more accurate in the presence of short wavelength noise.

Figure 6.11: Examples of extrapolation (prediction) error.

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6.4. Experimental verification of the free field model

of control (when compared to that obtained using a single microphone) and hence maintains an attenuation envelope around the observer location as the sensors are moved further away. This attenuation however reduces from 35 dB to 18 dB (at a separation distance of 100 mm) at the observer location in both the model and experimental results.

The combination of the primary and secondary sound fields in the experiment, which results

Figure 6.12: A 100 Hz primary sound source controlled via two control sources. Measured along a 0.5 m length in an anechoic chamber, the actual sensors are marked with a circle and the observer location by a vertical line.

Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
from minimising the cost function, produces a slightly more erratic control profile than predicted by the model. In figures 6.12(c) and (d) the performance of the modelled and actual second-order virtual energy density sensors are compared. The experiment shows that despite the slightly erratic spatial decay of the sound fields (figure 6.10) the more rugged energy density cost function maintains the maximum attenuation at the observer location.

Deciding whether the size of this attenuation zone is practical remains somewhat subjective, but if 10 dB is chosen to be an acceptable level of attenuation, then for this example, the zone of acceptable attenuation extends between \( \frac{x}{\lambda} = 0.06 \) and 0.075 for observer/sensor separation distance of \( 4h \) (the worst case). At 100 Hz this is equivalent to the defined zone extending to ±25 mm either side of the observer. When the observer/sensor separation distance is reduced to 3h, the defined attenuation zone significantly increases to ±85 mm. While this alone may not allow for much observer movement, the zone of attenuation at the observer location should be compared to that achieved with a remotely located energy density sensor (6.12(b)). It should also be remembered that this method of cost function estimation has the potential to adapt to observer movements.

### 6.4.3 Control of a 200 Hz sinusoidal wave

The results that are obtained when controlling a 200 Hz monotone in both the model and the experiment are shown in figure 6.13. Figures 6.13(a) and (b) again illustrate that the conventional pressure squared cost function produces similar results in both the model and the experiment. As the observer/sensor separation distance increases to 100 mm, the attenuation at the observer location reduces from 40 dB to approximately 4 dB in the model and 5 dB in the experiment. For both cases and as expected, the size of the attenuation zone is reduced when compared to the 100 Hz example, due to the halving of the acoustic wavelength.

In figures 6.13(c) and (d) the modelled and experimental results for the first-order virtual microphone are compared. The results of both the model and experiment are not dissimilar,
although the experiment shows that as the observer / sensor separation distance increases to 100 mm, attenuation at the observer location reduces from 40 dB to 12 dB compared to the models predicted reduction from 40 dB to 24 dB. In a similar way to the 100 Hz example, this control strategy still demonstrates a practical advantage over the conventional remotely placed single microphone (figure 6.13(b)). Figure 6.13(e) again shows that in theory second-order prediction is more accurate for forward wave prediction. However in practice (figure 6.13(f)) this method’s sensitivity to shorter wavelength spatial pressure variations, results in only a 5 dB improvement to the pressure squared cost function of figure 6.13(b).

In figures 6.14(a) and (b) the achievable sound pressure reduction for a 200 Hz monotone with an energy density cost function and two control sources is shown. Again (as in the 100 Hz case), energy density control produces a broader region of control (when compared to using a single microphone) and allows the same conclusion to be made as for the 100 Hz example. In figures 6.14(c) and (d) the performance of the modelled and actual second-order virtual energy density sensor are compared. Once again (as in the 100 Hz example) the maximum attenuation remains at the observer location with a narrower yet still practically sized attenuation zone. From the observer’s perspective second-order virtual energy density remains (as for the 100 Hz case) the superior cost function.

6.4.4 Control of a 400 Hz sinusoidal wave

In figure 6.15 the results obtained when controlling a 400 Hz monotone in both the model and the experiment are shown. Figures 6.15(a) and (b) again illustrate that the conventional pressure squared cost function produces similar results in both the model and the experiment. The size of the attenuation zone has been further reduced with the increased frequency, so that the attenuation at the observer location now becomes a gain of 4 dB for a observer / sensor separation distance of 100 mm.
Figure 6.13: A 200 Hz primary sound source controlled via one control source. Measured along a 0.5 m length in an anechoic chamber, the actual sensors are marked with a circle and the observer location by a vertical line.
6.4. Experimental verification of the free field model

Figure 6.14: A 200 Hz primary sound source controlled via two control sources. Measured along a 0.5 m length in an anechoic chamber, the actual sensors are marked with a circle and the observer location by a vertical line.
The modelled and experimental results for the first-order virtual microphone are compared in figures 6.15(c) and (d). Once again the results of both the model and experiment show similar attenuation. For a observer / sensor separation distance of 100 mm, this control strategy shows an experimental improvement of approximately 10 dB when compared to the conventional single microphone error sensor (figure 6.15(b)).

In figure 6.15(f) it is shown that the second-order virtual microphone control strategy is sensitive to higher order spatial pressure variations, with no resulting improvement in control when compared to results obtained using the conventional pressure squared cost function.

In figures 6.16(a) and (b) the achievable active noise control results for a 400 Hz monotone with an energy density cost function and two control sources is shown. Once again (as in the 100 Hz and 200 Hz examples), energy density control produces a broader region of control than achieved when using a single control microphone. The experiment shows that as the observer / sensor separation distance increases to 100 mm, the attenuation at the observer location is still approximately 8 dB, compared to an observer gain of 4 dB that occurs when using a single remotely placed microphone. The experimental performance appears somewhat better than theoretically possible (when compared to the model). This is merely due to the more fortunate destructive wave interference of the experiment’s slightly more erratic pressure profiles for both the primary and secondary sound fields. The second-order virtual energy density sensor continues to contribute towards a superior control strategy (figures 6.16(c) and (d)) with the maximum attenuation remaining at the observer location for relatively large observer / sensor separation distances.

### 6.4.5 Conclusions

The results of control using the various error sensors in an anechoic chamber are summarised in table 6.3 for an observer / error sensor separation distance of $4h$ (100mm). The table omits a quantitative summary of the size of the control zone (which is also important) because of
Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
(a) Analytical model - Energy density control
(and first order virtual energy density control)

(b) Experimental results - Energy density control
(and first order virtual energy density control)

(c) Analytical model - Second order virtual
energy density control

(d) Experimental results - Second order virtual
energy density control

Figure 6.16: A 400 Hz primary sound source controlled via two control sources. Measured along a 0.5 m length in an anechoic chamber, the actual sensors are marked with a circle and the observer location by a vertical line.
the variety of control zone shapes and locations. For the frequencies analysed and in this particular environment, it has been demonstrated that the first-order virtual microphone (based on forward difference prediction) outperforms a conventional microphone (in terms of noise reduction at the observer location) with an equivalent observer/sensor location separation distance. The theoretically more precise prediction method of a second-order virtual microphone was found to be more sensitive to shorter wavelength spatial variations in a real sound field, offering only a small advantage to using a conventional microphone.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Single microphone</th>
<th>1st order virtual microphone</th>
<th>2nd order virtual microphone</th>
<th>Energy density</th>
<th>2nd order virtual energy density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free field - 100 Hz</td>
<td>8 dB</td>
<td>22 dB</td>
<td>10 dB</td>
<td>-10 dB</td>
<td>38 dB</td>
</tr>
<tr>
<td>Free field - 200 Hz</td>
<td>5 dB</td>
<td>12 dB</td>
<td>8 dB</td>
<td>8 dB</td>
<td>38 dB</td>
</tr>
<tr>
<td>Free field - 400 Hz</td>
<td>-4 dB</td>
<td>6 dB</td>
<td>1 dB</td>
<td>8 dB</td>
<td>38 dB</td>
</tr>
</tbody>
</table>

Table 6.3: A summary of the attenuation levels at the observer location (separated from the sensors by 4h) that resulted from using various error sensors in the experimental verification of the free field model. Negative attenuation values show a gain in the sound pressure level.

In section 6.3 it was demonstrated that first-order virtual energy density is the same as controlling energy density directly at the sensors. The second-order virtual energy density sensor however continued to produce the most favourable results (theoretically and experimentally) in terms of both a high level of attenuation and a broad control zone around the location of the observer, showing promise for more complex and practical environments.

### 6.5 Virtual sensors in a long narrow duct.

In this section the zone of local control around a “virtual energy density sensor” is compared with that associated with an actual energy density sensor, a single microphone and a virtual microphone in a long narrow duct which approximates a one dimensional waveguide. The sensors are numerically evaluated in a finite element model of the duct, with control implemented...
via quadratic optimisation. Experimental verification validates the algorithm performance under actual test conditions. It is shown that virtual sensors can outperform actual sensors in terms of placing the centre of a practically sized zone of local control around an observer location in the duct.

### 6.5.1 The system

The reverberant system discussed here consisted of a long narrow (0.205 m x 0.205 m x 4.800 m) rigid walled duct (figure 6.17). In both the model and in the experiment, the 21 measurement locations were at 25 mm (h) intervals along a 0.5 m length, with the end microphone 0.25 m from the nearest control noise source (figure 6.17). A primary noise source was positioned at one end of the duct with two control sources at 0.5 m and 1.0 m from the opposite end. With the ends of the duct rigidly terminated, the resonance quality factors ($Q$) for all of the modes were approximately 50. Acoustically absorbent material added to one end of the duct, to observe the effect of damping, reduced the resonance quality factors to approximately 10.

The duct was modelled (for both examples of damping) with the dedicated finite element software package “ANSYS” to observe the theoretical performance of the virtual sensors. The model’s transfer functions (section 6.2) were calculated using a modal superposition technique (ANSYS Incorporated (1998)) with 386 modes (the maximum permissible, limited by the model’s degrees of freedom). In a study such as this, where high order modes are of concern and significant, the model must be capable of reproducing as many as possible.

The results from the model were then validated experimentally with the methods discussed in section 6.2. The graphical analysis of the modelled results are presented alongside the graphs from the experiment.
6.5. Virtual sensors in a long narrow duct.

(a) Schematic system representation for both the model and the experimental verification.

(b) The Ansys finite element model, showing the elements and an example of a resonance mode.

(c) The experimental configuration. The primary noise source and the rod for adjusting the location of the measurement microphone are in the foreground and the two secondary noise sources are in the background.

(d) The movable measurement microphone, used to measure the transfer functions between each of the three noise sources and the 21 microphone locations.

Figure 6.17: The long narrow duct.
6.5.2 Results for the rigidly terminated duct

The sum of the squared pressures at the sensor locations when the system is driven by only the primary sound source, is shown in figure 6.18(a). The difference between the measured data and that of the model at low frequencies is because the experimental data were measured using a microphone and normalised against the signal from second microphone located in the rear chamber of the primary noise speaker. All of these transducers (including the speaker) had poor low frequency sensitivity. The remaining differences are best categorised as resulting from the slight physical differences between the model and reality.

The system was investigated at resonance and at an anti-resonance to observe the effect of the varying dominance of a resonant mode. The axial pressure responses for the chosen test frequencies are shown in figure 6.18(b). The observer location, common to both cases is identified by a vertical line. Each test case was investigated with either 1 or 2 control sources, depending on whether the objective was to merely minimise a single pressure, or two pressures and hence pressure gradient and energy density.

6.5.2.1 On-resonance

Figure 6.19 shows the primary sound field of a duct resonance (250 Hz in the model and 252 Hz in the experiment) compared to the controlled sound fields when using a variety of error sensing strategies and one control source. The left hand column sub-figures show the results for the numerical simulation and the right hand column shows the results from the experiment. Figure 6.19(a) shows the control that can theoretically be achieved with a single microphone error sensor. As the microphone is moved away from the observer’s location by increasing separation distances of \( h \) (where \( h = 25 \text{mm} \)), the observer’s experience of noise attenuation reduces. When the sensor is at the observer location \( (0h) \), control is only limited (to 40 dB) by the imposed practical limitation detailed in section 6.5.1. However as the microphone is moved further away to a separation distance of \( 4h \), the attenuation at the observer location reduces to
6.5. Virtual sensors in a long narrow duct.

(a) Sum of the squared pressures at the microphone locations. The chosen resonance and anti resonance are highlighted by a circle.

(b) Pressure amplitude along the duct for the chosen resonance and anti-resonance. The vertical line shows the observer location.

Figure 6.18: A rigidly terminated duct response, comparing the results from the numerical model and the experimental verification. The chosen resonance occurs at 250 Hz in the model and 252 Hz in the experiment. The chosen anti-resonance occurs at 269 Hz in the model and 272 Hz in the experiment.
22 dB in the model and 15 dB in the experiment (figure 6.19(b)). In this ideal control example, the resonance standing wave dominates the primary field, so minimising the squared pressure at a location in the proximity of its anti-node produces a broad zone of attenuation.

Figures 6.19(c) and 6.19(d) show the control profile that results from minimising the acoustic pressure with a first-order virtual microphone. The numerical simulation shows a very small decrease in performance as the observer / sensor separation distance is increased to $4h$, while the experimental verification shows that control in this manner is only marginally better than using the conventional single microphone.

The results from the second-order virtual microphone numerical simulation (figure 6.19(e)) demonstrate an even higher theoretical prediction accuracy, with almost no decrease in the observed attenuation over an increasing separation distance of up to $4h$.

However, the results from the experiment (figure 6.19(f)) reveal that, for this scenario, a second-order virtual microphone is useless as an error sensor. Figure 6.11 shows that the presence of short wavelength extraneous noise can have a much higher effect on the accuracy of a second-order prediction method than a first-order prediction method. This noise could result from transfer function errors, instrumentation errors, the effect of higher order modes and the effect of cross modes, all of which are not considered in the model and will be the subject of further research.

Figure 6.20(a) shows the theoretical results obtained when controlling the energy density which, as detailed in section 6.2, is identical to using a first-order virtual energy density sensor or independently controlling pressure at two locations. As previously stated, two control sources are required to allow the independent control of both pressure and particle velocity, thereby effectively controlling energy density. As to be expected, the level of control decreases as the separation distance increases. This is attributed to the extrapolation accuracy of a first-order system that assumes a constant pressure gradient. Figure 6.20(c) shows the numerical simulation of control via the more theoretically accurate second-order virtual energy sensor,
illustrating a much higher level of control at the observer location as the observer / sensor separation distance increases. The experimental validation of energy density error sensing however (figures 6.20(b) and (d)), shows that both methods of energy density control are no better than when using a conventional remotely placed single microphone (figure 6.19(a)).

At first glance, with this initial example, it would appear that virtual error sensors offer no significant benefit to conventional error sensing methods. In this particular scenario, this is as expected because a single acoustic mode is controlled close to its pressure anti-node and represents the ideal example for a single location, squared pressure cost function. However, it will be shown later that when a resonance is less dominant in the primary sound field, the virtual error sensors demonstrate a superior performance compared to using a single remotely placed microphone.

### 6.5.2.2 Off-resonance

Figure 6.21 shows the numerical and experimental results obtained by controlling an anti-resonance (250 Hz in the model and 252 Hz in the experiment) using one control source and a variety of error sensing methods. The controlled sound pressure profiles are shown compared to the primary sound field.

Since several modes dominate the response, the results obtained using direct pressure control from a single microphone (figures 6.21(a) and (b)) show an increase in modal spillover with a high level of very localised attenuation when compared to the resonance example. When the observer / sensor separation distance is increased to $4h$ (where $h = 25\text{mm}$), the observer in the numerical simulation experiences no improvement in noise reduction at all. With the reduced effect of a single resonant mode, the zone of attenuated noise is smaller than in the resonant example. Consequently, as the local control zone moves away from the observer, with an increasing observer / sensor separation distance, attenuation at the observer’s location rapidly reduces. Due to the slight differences between the model and the experiment, the results from
(a) Numerical simulation - control with a single microphone error sensor

(b) Experiment - control with a single microphone error sensor

(c) Numerical simulation - control with a first order virtual microphone error sensor

(d) Experiment - Control with a first order virtual microphone error sensor

(e) Numerical simulation - control with a second order virtual microphone error sensor

(f) Experiment - control with a second order virtual microphone error sensor

Figure 6.19: The uncontrolled and controlled sound pressure along a rigidly terminated duct at an acoustic resonance with one control source and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.
(a) Numerical simulation - control with an energy density error sensor.  
(b) Experiment - control with an energy density error sensor.  
(c) Numerical simulation - control with a second order virtual energy density error sensor.  
(d) Experiment - control with a second order virtual energy density error sensor.

Figure 6.20: The uncontrolled and controlled sound pressure along a rigidly terminated duct at an acoustic resonance, with two control sources and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.
the experiment produce even smaller zones of attenuation, resulting in an 8 dB gain at an observer/sensor separation distance of $4h$.

As previously observed (in the resonant example), figures 6.21(c) and (e) show that the second-order pressure estimate is theoretically superior to the first-order pressure estimate, with both techniques offering far better local control at the observer location than that achieved when using a single microphone located an equivalent distance away. However, while the second-order virtual microphone (figure 6.21(f)) outperforms the single microphone in practice, it is the first-order virtual microphone (figure 6.21(d)) that proves to be the superior one of all three. This supports the earlier mentioned theory (section 6.5.2.1) of short wavelength extraneous noise having a greater effect on second-order prediction estimates than on first-order prediction estimates (figure 6.11). When the observer/sensor separation distance is increased to $4h$ the first-order virtual microphone yields 16 dB of noise attenuation compared to the 8 dB of attenuation achieved with the second-order virtual microphone.

In figures 6.22(a) and (c) the theoretical advantages of energy density control and second-order virtual energy density control, over a single microphone error sensor when two control sources are used, become more apparent than in the resonance example. Although in practice, it is only the energy density sensor (figure 6.22(b)) that offers any advantage when compared to the conventional single microphone error sensor (a 12 dB improvement at $4h$). This again is due to the high sensitivity to short wavelength noise (figure 6.11) associated with the second-order prediction estimates. In this example the results obtained by using a second-order virtual energy density sensor show a larger error in the estimation of pressure and pressure gradient at the observer location, when compared to results obtained using a first-order virtual (or physical) energy density sensor.

For this example, the first-order virtual microphone proves to be the superior method for error sensing in terms of placing a zone of attenuated noise at the observer location.
6.5. Virtual sensors in a long narrow duct.

Figure 6.21: The uncontrolled and controlled sound pressure along a rigidly terminated duct at an acoustic anti-resonance with one control source and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.
Figure 6.22: The uncontrolled and controlled sound pressure along a rigidly terminated duct at an acoustic anti-resonance with two control sources and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.
6.5.3 Results for the damped duct

The investigation detailed in section 6.5.2 was repeated with damping material placed at one of the end terminations. This reduced the resonance quality factors from 50 to 10 for all of the modes. The sum of the squared pressures at the sensor locations, when the damped duct is driven by only the primary sound source, is shown in figure 6.23(a). As in the case of the rigidly terminated system, the low frequency difference between the model and the experiment is due to instrumentation limitations and the slight physical differences between the model and reality.

The damped duct was investigated at resonance and at an anti-resonance to observe the effect of the varying dominance of a resonant mode. Figure 6.23(b) shows the axial pressure response for the chosen test frequencies. The observer location is identified by a vertical line.

6.5.3.1 On-resonance

Figure 6.24 shows the numerical simulation and experimental results for the active noise control using a single control source, of the chosen duct resonance (261 Hz in the model and 262 Hz in the experiment) with a variety of error sensors. The primary sound field is initially compared to an actively controlled sound field with a single microphone error sensor (figure 6.24(a) and (b)). This illustrates a scenario where control is far more localised than for the rigidly terminated duct at resonance (figure 6.19(a) and(b)).

However, since the response is still dominated by a single acoustic mode, pressure control is generally good without any significant modal spillover. As in the case of the rigidly terminated duct at resonance, the numerical simulations for both virtual microphones, show that maximum noise attenuation remains at the observer location when the observer / sensor separation distance is increased to $4h$ (figures 6.24(c) and (d)).
(a) Sum of the squared pressures at the microphone locations. The chosen resonance and anti resonance are highlighted by a circle.

(b) Pressure amplitude along the duct for the chosen resonance and anti-resonance. The vertical line shows the observer location.

Figure 6.23: A damped duct response, comparing the results from the numerical model and the experimental verification. The chosen resonance occurs at 261 Hz in the model and 262 Hz in the experiment. The chosen anti-resonance occurs at 281 Hz in the model and 282 Hz in the experiment.
6.5. Virtual sensors in a long narrow duct.

With the good performance of a pressure squared cost function (either measured directly or by a virtual microphone), in an environment still sufficiently dominated by a single acoustic mode, energy density control with a second control source, provides no improvement over pressure squared control with a single control source. The numerical simulation for the energy density sensor (or first-order virtual energy density sensor) demonstrates that energy density control with two control sources is identical to simply minimising pressure at the two physical microphones, with two very localised zones of control, each centered around a physical sensor (figure 6.25(a)). The second-order virtual energy density sensor shows that the pressure gradient is more accurately estimated than in the case of the first-order virtual energy density sensor (figure 6.25(c)). While this results in a smoother spatial pressure profile, there is no advantage in the size of the control zone, or the amplitude of attenuation at the observer location, when compared to using a single microphone error sensor.

In the experiment, at an observer / sensor separation distance of 4h, the attenuation at the observer location is:

- 8 dB with a single microphone (figure 6.24(b))
- 24 dB with a first-order virtual microphone (figure 6.24(d))
- -6 dB (a gain) with the second-order virtual microphone (figure 6.24(f))
- 8 dB with the first-order virtual energy density sensor (figure 6.25(b))
- 15 dB with the second-order virtual energy density sensor (figure 6.25(d)).

As in the case of the rigid duct excited at an anti-resonance, the first-order virtual microphone proved to be the superior error sensor compared to all of the others considered. Experimental results obtained using the second-order virtual microphone again show a higher sensitivity to short wavelength extraneous noise (section 6.5.2.1 and figure 6.11). The first-order virtual
energy density sensor performs similarly to the numerical simulation but offers no advantage to using a single microphone with one control source.

The results obtained using a second-order virtual energy density sensor show a better level of control when compared to the results obtained using a first-order virtual energy density sensor. While in this example the errors due to extraneous noise are still evident (causing a more erratically shaped pressure profile than in the numerical simulation), there is no significant adverse effect on the pressure attenuation at the observer location. In this example, the combination of the second-order pressure estimate and the second-order pressure gradient estimate in the energy density cost function reduces the overall extraneous noise effect when compared to estimating pressure alone (figure 6.24(d)).

6.5.3.2 Off-resonance

The primary sound field for the damped duct anti-resonance is compared to the control that can be achieved with a single control source and various cost functions in figure 6.26. In the example the chosen anti-resonance occurs at 281 Hz in the numerical simulation and 282 Hz in the experiment.

In all of the examples considered this is the case that is least dominated by a single acoustic mode. Therefore, this is the worst case scenario for a single microphone error sensor. All methods of control show significant modal spillover with a narrow zone of control around the location of the sensors. Figures 6.26(a) and (b) show the active noise control results obtained due to a single microphone error sensor for which squared pressure is the cost function. As the microphone is moved further from the observer’s location, the attenuation decreases until at a sensor/observer separation distance of 4h the observer actually experiences a noise gain of 20 dB in the model (figure 6.26(a)) and 12 dB in the experiment (figure 6.26(b)).

The results from the numerical simulation for both virtual microphones (figure 6.26(c) and (e)) show that the maximum attenuation remains at the observer location for increasing observer
6.5. Virtual sensors in a long narrow duct.

Figure 6.24: The uncontrolled and controlled sound pressure along a damped duct at an acoustic resonance with one control source and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.

(a) Numerical simulation - control via a single microphone error sensor

(b) Experiment - control via a single microphone error sensor

(c) Numerical simulation - control with a first order virtual microphone error sensor

(d) Experiment - control with a first order virtual microphone error sensor

(e) Numerical simulation - control with a second order virtual microphone error sensor

(f) Experiment - control with a second order virtual microphone error sensor

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Chapter 6. Evaluating the performance of forward prediction virtual error sensors

Figure 6.25: The uncontrolled and controlled sound pressure along a damped duct at an acoustic resonance with two control sources and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.
sensor separation distances. Both perform similarly resulting in a significant level of local control at the observer location for all observer / sensor separation distances. More importantly, the experiment validates these results with both methods resulting in an attenuation of 18 dB at the observer location for an observer / sensor separation distance of 4h.

Figure 6.27 shows the results obtained using two control sources and an energy density cost function. The first-order energy density sensor, which is equivalent to two point pressure minimisation, is illustrated in figures 6.27(a) and (b) with a tight control zone around each acoustic pressure sensor. In the numerical simulation and at an observer / sensor separation distance of 4h, the second-order virtual energy density sensor shows a gain of 6 dB at the observer location. While this result is not as good as either of the virtual microphones it is still a 12 dB improvement on the use of a single microphone. The experimental validation shows an attenuation of 8 dB (fortuitously 14 dB better than the model). The model has subtle differences to the physical system and as previously observed (section 6.5.2.1) the physical system can be subject to short wavelength extraneous noise. Errors may bias results either way, hence in this example, the pressure minimisation obtained by using a second-order virtual energy density sensor in practice is better than theoretically possible.

Both virtual microphones (the first and second-order) prove to be equally superior error sensors in this example.

6.6 Conclusions

The attenuation that resulted at the observer location in the duct model, for various error sensors that were separated from the observer by 4h (100mm), are summarised in table 6.4. Table 6.5 summarises the results of the experimental verification of the duct model. As previously mentioned, while these tables indicate the attenuation resulting from using the error sensors, the size of the control zone is also important. This however, because of variety of control zone shapes and location, was difficult to concisely summarise in a table.
Figure 6.26: The uncontrolled and controlled sound pressure along a damped duct at an acoustic anti-resonance with one control source and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.
6.6. Conclusions

Figure 6.27: The uncontrolled and controlled sound pressure along a damped duct at an acoustic anti-resonance with two control sources and various error sensing strategies. The actual transducer locations are marked by a circle and the observer location with a vertical line.

<table>
<thead>
<tr>
<th>Environment</th>
<th>Single microphone</th>
<th>1st order virtual microphone</th>
<th>2nd order virtual microphone</th>
<th>Energy density</th>
<th>2nd order virtual energy density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigidly terminated duct at a resonance</td>
<td>22 dB</td>
<td>38 dB</td>
<td>40 dB</td>
<td>20 dB</td>
<td>32 dB</td>
</tr>
<tr>
<td>Rigidly terminated duct at an anti-resonance</td>
<td>0</td>
<td>20 dB</td>
<td>30 dB</td>
<td>15 dB</td>
<td>14 dB</td>
</tr>
<tr>
<td>Damped terminated duct at a resonance</td>
<td>8 dB</td>
<td>30 dB</td>
<td>35 dB</td>
<td>5 dB</td>
<td>15 dB</td>
</tr>
<tr>
<td>Damped terminated duct at an anti-resonance</td>
<td>-20 dB</td>
<td>11 dB</td>
<td>18 dB</td>
<td>-25 dB</td>
<td>-6 dB</td>
</tr>
</tbody>
</table>

Table 6.4: A summary of the attenuation levels at the observer location (separated from the sensors by 4h) that resulted from using various error sensors in the duct model. Negative attenuation values show a gain in the sound pressure level.

Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
Table 6.5: A summary of the attenuation levels at the observer location (separated from the sensors by 4h) that resulted from using various error sensors in the experimental verification of the duct model. Negative attenuation values show a gain in the sound pressure level.

It was concluded in section 6.3 that out of all the sensors considered in the free field system, the virtual energy density sensor produced the best control around an observer remotely located from any physical sensor. This conclusion was based on the size of the control zone in addition to the level of attenuation. The examples considered in this section led to a conclusion that the virtual sensors behave quite differently in the reactive environment of a one dimensional waveguide (a long narrow duct).

Various active noise control strategies were modelled and compared in a long narrow duct. The duct was excited both on-resonance and off-resonance with the effect of additional damping also considered. The results from the numerical simulations were validated experimentally and all the results compared.

When a system response is dominated entirely by a single resonant mode, cancelling the mode at its acoustic pressure anti-node with a pressure squared cost function will theoretically cancel all noise throughout the enclosure. Consequently, a microphone error sensor is extremely hard to improve upon in the first example of a rigidly terminated duct, excited at resonance with an error sensor in the proximity of the acoustic pressure anti-node. This is an ideal scenario for active noise control but unfortunately, not one that often occurs in practice. Periodic noise transmitted into enclosures usually excites multiple acoustic modes to varying degrees and these excited modes characterise the enclosure response. The remaining examples were therefore more representative of what may occur in a practical situation and should form the
basis of evaluating the virtual error sensors. The dominance of a single mode was reduced throughout the remaining examples, first by changing the control frequency to coincide with an anti-resonance and then by adding damping. As the performance of the single microphone worsened, the virtual sensors demonstrated their increasing value. The study progressed to show that in general the virtual sensors outperform the microphone in terms of placing a zone of localised control around the location of the observer for an equivalent separation distance between the observer and the physical transducers.

The first-order virtual energy density sensor was shown to be mathematically identical to remotely controlling energy density with a two sensor system, or merely the same as independently minimising pressure at the two sensor locations. The second-order virtual microphone was theoretically the most superior in terms of placing a high level of attenuation at the observer, but in practice it (and the second-order energy density sensor) demonstrated a sensitivity to short wavelength extraneous spatial pressure variations.

In the example of a long narrow duct, the first-order virtual microphone showed consistent results and continually proved to be a more robust “virtual” error sensor with a performance superior to all of the error sensors considered.

It was postulated that the pressure variations that were shown to adversely affect the second-order estimation methods were due to the presence of short wave length noise. The short wavelength noise may be more significant in the duct than in the free field because it would be contributed to by both higher order modes and short wavelength cross modes. It is anticipated that the response of a more practical damped three-dimensional enclosure (such as a light aircraft cabin) will be controlled by multiple highly damped modes with longer wavelengths (in all three axes), hence lessening the effect of the shorter wavelength noise.
Chapter 7

Forward wave prediction errors

7.1 Introduction

The previous chapter demonstrated that forward difference prediction can be used to create virtual active noise control error sensors, which are able to predict (and hence minimise) the cost function at some remote location from the physical sensors. The technique has been shown to work in the free field and in the reactive environment of a long narrow duct for both virtual microphones and virtual energy density sensors, although to varying degrees of success with noticeable performance differences between the model and the experimental results.

Measurements with any form of instrumentation will invariably contain errors, especially if the methods involve some form of estimation. Cazzolato (1999) discussed these errors in active noise control instrumentation, which he identified to be due to: finite difference approximation, phase mismatch, sensitivity calibration errors, sensor physical geometry, diffraction interference and environmental effects. Without exception, all of these errors are significant, but already well defined. Hence, this chapter shall only address the errors (or physical and environmental phenomena) that specifically affect the the accuracy of forward wave prediction sensors.
In extrapolation based forward wave prediction methods, the separation distance between the physical sensors and the separation distance between the virtual sensor and the physical sensors, will both obviously effect the estimation accuracy. Therefore the results of a numerical simulation, used to illustrate how the relationship between the sensor spacing and the wavelength of the measured signal effected the forward wave prediction accuracy, are shown in 7.2.

While second-order prediction methods are theoretically more accurate than first-order prediction methods, first-order virtual microphones were shown to be more rugged in the presence of short wavelength extraneous noise. It will also be shown how low amplitude, short wavelength noise can adversely effect the prediction accuracy of a virtual sensor.

The mechanism that caused the low amplitude, short wavelength noise, in both the free field and the reactive field experiments, was also investigated. In the experimental verification of the free field model, it was assumed that the measurements were conducted in the far field, where the sound pressure level reduces by 6 dB for every doubling of separation distance between the measurement location and the sound source. However, figure 6.10, page 94 shows that the actual sound pressure decay with respect to distance reduced more erratically, inferring that the measurements may have been made in the near field of the acoustic sources. The results of reducing the size of the acoustic source in the free field experiments are shown to enable the observation of whether the low amplitude, short wavelength noise was reduced as a consequence. The short wavelength noise that was evident in the long narrow duct, may have been due to the presence of high-order modes and cross modes and so it will be shown here mathematically, how high-order modes can significantly affect the estimation accuracy of forward wave prediction virtual sensors.
7.2 Sensor spacing with respect to wavelength

In chapter 5, section 5.4 it was shown that a waveform may be explicitly defined by a Taylor's series expansion. It was also shown that the waveform may be estimated, and that the accuracy of the estimation was dependent on the number of terms, or orders in the series. Therefore, prediction methods that only use a first or second-order estimation technique will have inherent errors. However, when a wavelength is long with respect to the physical sensor spacing and the spacing between the sensors and the the virtual sensor location, the spatial rate of change will be small and predictable with low-order estimation methods. However, it remains apparent that the accuracy of a forward wave prediction sensor is dependent upon the relationship between the sensor spacing and the wavelength of the signal that is to be estimated. The extent that the wavelength and sensor spacing affects the accuracy of a forward wave prediction sensor, may be shown numerically by either:

1. varying the sensor spacing to see how the change affects the prediction accuracy for a fixed wavelength signal or,

2. varying the signal wavelength to see how the prediction accuracy is affected for a constant sensor spacing.

Since there are two spacing variables (the distance between each physical sensor and the distance between the virtual sensor location and the nearest physical sensor), which must also satisfy installation practicalities, the latter option would be preferable and would also identify an operational frequency range. The first and second-order virtual sensor examples shown in the previous chapters, have a practical sensor spacing \( (h) \) of 25 mm (between each sensor) and predict the value of the waveform up to 100 mm \((4h)\) away. These separation distances shall therefore also be used in the following examples to illustrate the theoretical prediction accuracy of the virtual sensors with respect to sensor spacing and wavelength.
Figure 7.1 shows the compressional maxima and minima of a numerically simulated waveform passing the physical sensor locations of a virtual microphone. The shaded region (to the left of the virtual microphone) shows how the amplitude of the wave was predicted. The black vertical line at the virtual microphone passes through the value of the true waveform, the first-order estimate and the second-order estimate. The traces to the left are not intended to imply that the whole waveform can be estimated at any one moment in time, but shows how the estimation accuracies vary as the waveform propagates past the virtual sensor.

The black trace to the right of the virtual microphone location represents the true value of the passing waveform, the blue trace represents the first-order virtual microphone prediction and the red line represents the second-order virtual microphone prediction. It can be seen that the prediction accuracy also varied with the phase angle of the passing wave and so the maximum amplitude prediction error (as a percentage of true waveform maximum amplitude) is also shown. In this example the wavelength was 40 h (1 m, or \( f = 343 \text{ Hz} \) when \( h = 25 \text{ mm} \)) and the maximum amplitude prediction error was 29% when using a first-order virtual microphone but only 8% for a second-order virtual microphone.

![Figure 7.1: Tonal noise with a wavelength of 40 h (1 m). The shaded region shows the mechanism for prediction and the region to the right shows the trace of the waveform and waveform estimates as they pass the sensors.](image-url)
Figure 7.2 shows how the prediction accuracy of a first and second-order virtual microphone changed when the wavelength was reduced from $40h$ to $30h$. The maximum error for the first-order virtual microphone increased from 29% to 51% whereas the maximum prediction error for the second-order virtual microphone increased to 18% of the true waveform maximum amplitude. In these examples where $h = 25$ mm, $30h$ corresponds to a wavelength of 0.75 m, or a frequency of 457 Hz.

![Figure 7.2: Tonal noise with a wavelength of 30 h (0.75 m). The shaded region shows the mechanism for prediction and the region to the right shows the trace of the waveform and waveform estimates as they pass the sensors.](image)

The numerical simulations were repeated for a range of wavelengths and the results of the prediction errors for both the first-order and second-order virtual microphones are summarised in figure 7.3 and plotted as a function of $h/\lambda$, where $h$ is the sensor spacing and $\lambda$ is the wavelength. This shows that the prediction errors proportionally increased as $h/\lambda$ was increased, or as the wavelength with respect to the sensor spacing was reduced.

In chapter 3, subjective audio comparisons were described that concluded that it was only necessary to remove the tonal noise at the blade pass fundamental frequency and the first three higher harmonics. Removal of any higher-order harmonics made no perceivable subjective improvement in the noise. The fourth harmonic of the blade pass fundamental was at 310 Hz.
Figure 7.3: The effect of wavelength and sensor spacing on the amplitude prediction accuracy of a first and second-order virtual microphone. The separation distance between each physical sensor is \( h \) and \( 4h \) between the virtual microphone location and the nearest physical sensor.

\((\lambda = 0.9 \text{ m})\) and is the highest frequency at which the virtual sensors would need to operate. When \( h = 25 \text{ mm} \), this corresponds to a \( h/\lambda \) value of 0.027 and from figure 7.3 it can be seen that the prediction error for a first-order virtual microphone is approximately 20% and 6% for a second-order virtual microphone.

All of the numerical simulations illustrated that the prediction errors were always positive, or in other words predicted a higher amplitude than the amplitude of the true waveform. Since the sensors are intended to predict a cost function at an observer location and it is the objective of active noise control to minimise the cost function, the amplitude prediction errors may not necessarily always adversely effect the control performance. These amplitude prediction errors would have a similar effect to increasing the sensitivity of a virtual error sensor. While this suggests that it is not so critical for an acoustic pressure squared cost function, potential energy minimisation may be more biased than kinetic energy minimisation in an energy density cost function.

It can also be seen in figure 7.1 and figure 7.2 that the phase angle estimation error increases with the amplitude estimation error. Figure 6.1 (chapter 6) shows that a phase error can adversely effect the control performance and alter the location of where the acoustic pressure is minimised. Therefore, phase errors were also observed in the numerical simulations and
the results are shown summarised in figure 7.4, where the phase difference between the true waveform maxima and the predicted waveform maxima is shown as a function of $h/\lambda$ for both the first-order and the second-order virtual microphone.

![Figure 7.4: The effect of wavelength and sensor spacing on the phase prediction accuracy of a first and second-order virtual microphone. The separation distance between each physical sensor is $h$ and $4h$ between the virtual microphone location and the nearest physical sensor.](image)

Figure 7.4 shows that as the wavelength was decreased, the phase error between the predicted location of the waveform maxima and the true location of the waveform maxima increased. However, at $h/\lambda = 0.027$ (the highest value of concern for active noise control in a single engine light aircraft cabin), the phase error for the first-order virtual microphone was only $5^\circ$ and $-2^\circ$ for the second order virtual microphone. For $h = 25\text{mm}$ and a wavelength of $0.9$ m, this corresponded to a location error of $12.5$ mm and $-5$ mm respectively.

### 7.3 Short wavelength noise

It was previously shown in chapter 6 that second-order prediction methods demonstrated a high accuracy for noise free signals of relatively long wavelength (with respect to transducer separation distance), whereas the first-order methods were more rugged in the presence of short wavelength extraneous noise. Figure 6.11 on page 94, shows how the assumption of a
either a constant first or second-order differential can lead to errors in the presence of short wave length extraneous noise. This section elaborates on why short wavelength noise, even at a very low amplitude is significant, regardless of its source.

With reference to figure 7.5 and for a given moment in time, consider the following sinusoidal acoustic pressure waveform in the spatial domain that is representative of the frequency to be controlled:

\[ p_c = A_c \sin(k_c x) \]  

(7.1)

where \( p_c \) is the amplitude of the wave at location \( x \), \( A_c \) is the maximum amplitude of the wave and \( k_c \) is the wavenumber (where \( k = \frac{2\pi}{\lambda} \)).

Now consider the superposition of a second tonal wave where \( k_n >> k_c \) and \( A_n << A_c \), representative of the high frequency extraneous noise on the primary tone component:

\[ p_n = A_n \sin(k_n x) \]  

(7.2)

The total waveform is now represented by the following equation:

\[ p_t = A_c \sin(k_c x) + A_n \sin(k_n x) \]  

(7.3)

The first-order differential is therefore:

\[ \frac{dp_t}{dx} = k_c A_c \cos(k_c x) + k_n A_n \cos(k_n x) \]  

(7.4)

and the the second differential is:
7.3. Short wavelength noise

\[ \frac{d^2p_t}{dx^2} = -k_e^2 A_e \sin(k_e x) - k_n^2 A_n \sin(k_n x) \]  \hspace{1cm} (7.5)

Figure 7.5: The increasing dominance of high frequency noise with higher-order differentials.

In equation 7.4, figure 7.5 and figure 7.6, it can be seen that the amplitude of the first-order differential is not only proportional to the amplitude of each acoustic pressure component, but also to the wavenumber (or frequency) of each component. Figure 7.5 shows how the first-order differential \( \frac{dp}{dx} \) varies in amplitude in the spatial domain, and figure 7.6 shows the variation in the frequency (or wavenumber) domain. It can be seen that while the significance of the extraneous noise has increased, the spatial pressure variations due to the primary tone are still evident and that in the frequency domain, the first-order derivative only varies slightly from what is assumed to be a constant amplitude in first-order estimation methods.

Figure 7.5 and equation 7.5 show how the high frequency extraneous noise can dominate the second-order spatial derivative of acoustic pressure and figure 7.7 shows how the second-
Figure 7.6: The variation of an assumed constant amplitude first-order differential in the presence of constant amplitude noise, with an increasing wavenumber (or frequency).

order differential significantly varies from a constant value and how this variation increases proportionally to the wavenumber of the constant amplitude extraneous noise.

Figure 7.7: The variation of an assumed constant amplitude second-order differential in the presence of constant amplitude noise with an increasing wavenumber (or frequency).

It can be seen in figure 7.7 that when \( k_n \gg k_c \) and \( A_n \ll A_c \) the amplitude of the second-order differential is significantly affected by the extraneous noise and that the prediction accuracy of an estimation method, where the second-order differential is assumed to be constant, can produce large errors.

The compressional maxima and minima of a numerically simulated waveform passing a virtual microphone are shown in figure 7.8. The mechanism of prediction is shown in the shaded
region (also shown enlarged) and the black trace to the right of the virtual microphone location represents the true value of the passing waveform. The blue trace represents the first-order virtual microphone prediction and the red trace represents the second-order virtual microphone prediction. The maximum amplitude prediction error is also shown, since the prediction accuracies vary with the phase angle of the passing wave. In this example the wavelength of the dominant tone is 40\text{h} (1\text{ m}, or f = 343 \text{ Hz} \text{ when } h = 25 \text{ mm}). Extraneous noise has been added to the waveform with a wavelength of 10\text{h} and with an amplitude that is 30\text{dB} less than the dominant tone. Figure 7.8 shows that when the wavelength of the extraneous noise is still relatively long with respect to the sensor spacing, the first-order virtual microphone has an amplitude prediction error of 39% compared to only a 20% amplitude prediction error for the second-order virtual microphone.

Figure 7.8: A tonal waveform with a wavelength of 40h superimposed with constant amplitude tonal noise that is 30dB lower in magnitude and has a wavelength of 10h. The shaded region shows the mechanism for prediction and the region to the right shows the trace of the waveform and waveform estimates as it passes the sensors.

However, in figure 7.9 where the wavelength of the extraneous noise has been reduced to 5h, the first-order virtual microphone prediction error of the waveform amplitude has increased to 43% and the second-order virtual microphone prediction error has substantially increased to 65% of the maximum value of the true waveform amplitude.
Figure 7.9: A tonal waveform with a wavelength of 40h superimposed with constant amplitude tonal noise that is 30dB lower in magnitude and has a wavelength of 5h. The shaded region shows the mechanism for prediction and the region to the right shows the trace of the waveform and waveform estimates as it passes the sensors.

Figure 7.10 shows a result summary for the virtual microphone amplitude prediction errors that were calculated for a range of extraneous noise wavelengths (all with an amplitude of 30 dB less than the dominant tone). The prediction was also found to be marginally affected by the phase relationship between the dominant tone and the extraneous noise tone. It was also observed (and expected) that as the extraneous noise wavelength approached the Nyquist limit of the sensors (ie. twice the length of the sensor spacing) the prediction errors were also found to vary considerably with only slight changes in either the noise wavelength, or the phase relationship between the noise and the dominant tone. Therefore, the curves shown in figure 7.10 represent the maximum prediction error as a function of $\lambda_n/\lambda_c$, where $\lambda_n$ is the wavelength of the extraneous noise and $\lambda_c$ is the wavelength of the dominant tone.

Figure 7.10 shows that when $\lambda_n/\lambda_c$ is larger than 0.15 (or the wavelength of the noise is no less than 0.15 of the dominant tone wavelength) the accuracy of the second-order virtual microphone is higher than that of the first-order virtual microphone at predicting the waveform amplitude at the observer location. In these examples the wavelength of the dominant tone is 40h and so when $\lambda_n/\lambda_c = 0.15$ the wavelength of the extraneous noise is 6h and corresponds to...
7.4 The near field effect of speakers

In section 6.4, which details the experimental validation of the virtual sensor performance in a free field model, it was postulated that the near field effect of both the primary source and separation distance between the virtual microphone location and the furthest physical sensor. For each virtual microphone, the prediction error is at a maximum when the wavelength of the extraneous noise approaches the Nyquist limit of the sensors ($4h$ or $\lambda_n/\lambda_c = 0.1$ for the first-order virtual microphone and $2h$ or $\lambda_n/\lambda_c = 0.05$ for the second-order virtual microphone).

While this illustrates how low amplitude short wavelength extraneous noise, that is superimposed onto a single frequency dominant tone, can adversely affect the prediction accuracy of virtual sensors, it is the relationship of the extraneous noise wavelength to the sensor spacing (rather than the noise wavelength to the dominant tone wavelength) that is important. It should also be realised that as active noise control begins to reduce the referenced component of the signal, the extraneous shorter wavelength noise will increase in significance and have a greater affect on the prediction accuracy of forward prediction virtual sensing.

**7.4 The near field effect of speakers**

In section 6.4, which details the experimental validation of the virtual sensor performance in a free field model, it was postulated that the near field effect of both the primary source and
the control source speakers contributed to short wavelength spatial pressure variations that affected the accuracy of the forward wave prediction.

The experiments were therefore repeated using speakers that had a smaller characteristic dimension, to observe if the spatial pressure variations were reduced and if the forward wave prediction accuracy of the virtual sensors improved. Horn speakers (figure 7.11), with a diameter of 25 mm, were used to replace the 150 mm diameter enclosed speakers, that were initially used for the free field experiments (section 6.4.1).

![Figure 7.11: The 25mm diameter horn speakers.](image)

Figure 7.12 shows how using the small horn speakers had only a small effect on the spatial variation of the primary field, when compared to using the larger enclosed speakers.

The results that were obtained when controlling a 200 Hz monotone in the free field using a second-order virtual microphone error sensor (which has been shown to be sensitive to short wavelength spatial pressure variations) are shown in figure 7.13. Figure 7.13(a) shows the results of actively controlling the primary sound field when both the primary and control sources were 150 mm enclosed speakers. Figure 7.13(b) shows the active noise control results using 25 mm horn speakers. Comparing the active noise control results for each speaker configuration and an observer/sensor separation distance of $4h$, shows that the use of the smaller speakers only slightly improved the attenuation by an additional 4 dB. Apart from this improvement,
7.4. The near field effect of speakers

![Graphs showing acoustic pressure amplitude vs. non-dimensional distance for different conditions.]

(a) The primary noise field with a 150 mm diameter enclosed speaker.  
(b) The primary noise field with a 25 mm horn speaker.

Figure 7.12: A comparison of the primary noise fields.

The results were similar for both cases and it was not possible to conclude that the free field conditions had been improved by using a smaller primary source and smaller control sources.

![Graphs showing acoustic pressure amplitude vs. non-dimensional distance for different separation distances.]

(a) 150 mm enclosed speakers  
(b) 25 mm diameter horn speakers

Figure 7.13: Comparing the active noise control results using a second order virtual microphone error sensor and either 150 mm diameter enclosed speakers or 25 mm diameter horn speakers. A 200 Hz primary sound source was controlled via one control source. Measurements were made along a 0.5 m length in an anechoic chamber, the actual sensors are marked with a circle and the observer location by a vertical line.

The zone of a near field and how far it extends from a radiating sound source and the location at which the far field commences, is a function of the characteristic length (the maximum radiating dimension) of the radiating surface and the wavelength of the noise. The location where a near field changes to a far field is not clear cut and a transition range exists between

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the far field, the geometric near field and the hydrodynamic near field. However, Bies and Hansen (1996) show that the location at which the far field commences \( r \) must satisfy the following three criteria:

\[
\begin{align*}
1. & \quad r \gg \frac{\lambda}{2\pi} \\
2. & \quad r \gg l \\
3. & \quad r \gg \frac{\pi l^2}{2\lambda}
\end{align*}
\]

(7.6)

Where \( \lambda \) is the acoustic wavelength and \( l \) is the characteristic dimension of the acoustic source.

The combination of these criteria are shown graphically in figure 7.14

![Figure 7.14: A schematic definition of the near and far field (after Bies and Hansen (1996)).](image)

In the previous examples, the characteristic acoustic source dimensions, the distance of the measurement zone from the nearest acoustic source and the frequency range of interest were all known and can therefore be compared to figure 7.14 to establish if the measurements were in fact made in the near or far field of the either the primary source or control sources.
7.4. The near field effect of speakers

Figure 7.15 shows how the acoustic field is categorised, in terms of near or far field, for noise between 100 Hz and 400 Hz that was measured at a location 2 m from the acoustic source. It is shown that while reducing the characteristic source dimension moved the measurement zone further away from the geometric near field, the distance from the hydrodynamic near field remained the same. Further examination shows that reducing the acoustic characteristic dimension $(l)$ will always move the measurement zone along a line that remains parallel to the zone of the hydrodynamic near field. Only higher frequency noise (with a reduced wavelength, $\lambda$) moved the measurement zone further away from the hydrodynamic near field region. Nevertheless, figure 7.15 shows that measurements made 2 m from either source over a frequency range between 100 Hz and 400 Hz, were in the free field. It can therefore be concluded that the measurements conducted in the anechoic chamber, were all in the far field of the primary acoustic source and the control sources and that reducing the size of the acoustic characteristic source dimension offered no advantage.

Further analysis of the near field of a baffled piston (which behaves similarly to a loudspeaker at low frequencies) shows that even if the experiment were conducted in the near field, the presence of short wavelength spatial pressure variations would not be attributed to the near field effects of the source\(^1\). Kinsler et al. (1982) show that the spacing between successive minima in the near field of a baffled piston is defined as:

$$r_m - r_{m+2} = \frac{a^2}{\lambda} \frac{2}{m(m+2)} + \frac{\lambda}{2}$$  \hspace{1cm} (7.7)

where $m$ is the number of the peak (progressing towards the source, with odd numbers representative of maxima and even numbers of minima), $r_m - r_{m+2}$ is the space between successive minima and $a$ is the distance from the source.

It can be seen that even for a large value of $m$ (close to the source) the spacing between successive peaks is always greater than $\frac{\lambda}{2}$. The cause of the low amplitude, high wavelength spatial

\(^1\)As suggested by Scott. D. Sommerfeldt in his examiners comments.
pressure variations (±0.5 dB) must therefore be attributed to uncorrelated extraneous noise resulting from a combination of system noise, instrumentation errors, measurement inaccuracies and low amplitude reflections from the anechoic room walls.

Figure 7.15: Extending the range of the schematic shown in figure 7.14, to show the effect of reducing the characteristic dimension of the speakers. The lines that are labeled for each source, represent a frequency range of between 100Hz and 400Hz, at a distance of 2m from the speakers.
7.5 High-order modes

So far it has been shown how high frequency (or short wavelength) noise can affect the prediction accuracy of high-order forward wave prediction estimates regardless of its source. If this noise is uncorrelated (such as that caused by a poor signal to noise ratio) it may be simply removed by low pass filters that are set to allow only the portion of the signal with frequencies at or below the control frequency to pass. However, in the examples that have been shown so far, uncorrelated noise is not a consideration since it is effectively removed by the FFT analysis. The short wavelength noise that remains evident in the free field and duct examples must therefore be due to a mechanism that is not sensitive to temporal filtering. In a reactive modal environment (which includes the anechoic chamber to some small degree), higher-order modes can still significantly contribute to short wavelength spatial pressure variations even when the modal resonance frequencies occur above the cut off frequency of the low pass filter. Figure 7.16 shows a schematic of a modal system (a long duct for example) in which two damped overlapping modes are considered, with resonance frequencies of \( f_1 \) and \( f_2 \) respectively. At an arbitrary mid point in the frequency spectrum \( (f_c) \) the spatial response is a combination of the two modes but still oscillates with a frequency of \( f_c \) Hz. Hence, even if a low pass filter were set to exclude frequencies above \( f_c \), short wavelength spatial noise, directly resulting from the higher-order mode in forced response would still be present.

7.5.1 Mass and stiffness controlled modes

Consider an enclosed sound field that is driven off resonance at some frequency \( \omega \). The expression for the transfer function between two points, 1 and 2, in a reactive acoustic enclosure is given by

\[
\frac{p_2}{q_1} = \sum_{i=1}^{\infty} \frac{\rho_0 \omega \phi_i(1) \phi_i(2)}{\Lambda_i \left( \omega_i^2 + \eta_i \omega \omega - \omega^2 \right)}
\]  

\( (7.8) \)
where \( p \) is the acoustic pressure, \( q \) is the source volume velocity, \( \rho_0 \) is the density of air, \( \omega \) in the angular frequency, \( \phi_i \) is the \( i^{\text{th}} \) acoustic mode shape, \( \Lambda_i \) is the modal volume of the \( i^{\text{th}} \) mode, \( \omega_i \) is the natural frequency of the \( i^{\text{th}} \) mode and \( \eta_i \) is the modal loss factor of the \( i^{\text{th}} \) mode. Here it can be seen that the frequency response of the sound field is purely due to the sum of the modal contributions.

If the resonant contributions of the closest modes to a chosen frequency \( f_c \) are neglected, then all the modes can be considered stiffness or mass controlled (Bies and Hansen (1996)). Figure 7.17 shows that the amplitudes of the mass controlled modes (with natural frequencies lower than \( f_c \)) decrease proportionally to \( \frac{1}{f} \), and that the amplitudes of the stiffness controlled modes (with natural frequencies higher than \( f_c \)) decrease proportionally to \( f \).

Figure 7.18 presents the same information in a different format, where the amplitude of a particular mode \( (A_c) \) at a chosen frequency \( (f_c) \) is plotted as a ratio to the maximum amplitude \( (A_n) \) of the mode at its natural frequency \( (f_n) \). \( A_n \) is assumed to have a constant amplitude of \( \phi = 1 \).
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Figure 7.17: The mobility plot of modes in a reactive system showing the mass and stiffness residues.

Figure 7.18: The relative amplitude of modes as a function of their natural frequency $f_n$ with respect to $f_c$. 

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Hence, the expression for the modal amplitude at the chosen frequency can be approximated by:

\[
\frac{A_c}{A_n} \approx \begin{cases} 
\frac{f_c}{f_n} & f_n < f_c \\
\frac{f_n}{f_c} & f_n \geq f_c 
\end{cases}
\] (7.9)

This expression shows how the spatial acoustic pressure profile at a chosen off-resonance frequency is contributed to by modes with a natural frequency of \( f_n \).

Equation 7.9 may also be expressed in terms of the wavenumber \( k \), (where \( k = \frac{2\pi f_c}{c_0} \)):

\[
\frac{A_c}{A_n} \approx \begin{cases} 
\frac{k_c}{k_n} & k_n < k_c \\
\frac{k_n}{k_c} & k_n \geq k_c 
\end{cases}
\] (7.10)

### 7.5.2 First order derivative

Section 7.3 shows how high-order spatial derivatives exhibit a much greater sensitivity to high frequency noise, particularly for frequencies with wavelengths near the Nyquist limit. In the first-order systems so far considered, the spacing of the sensors is \( 2h \), and so the Nyquist limit can be defined as:

\[
(2h) = \frac{\lambda_N}{2}
\]

Or in terms of the wavenumber \( k_N \), where \( k = \frac{2\pi}{\lambda} \):

\[
2k_N h = \pi
\] (7.11)
This sensitivity of high-order spatial derivatives to the high-order modes and the relevance of the Nyquist limit, can now be shown by considering a one-dimensional reactive environment with a pressure response defined by:

\[ p_c = A_n \sin(k_n x) \]  \hspace{1cm} (7.12)

where \( p_c \) is the amplitude of the wave at location \( x \), \( A_n \) is the maximum amplitude of the wave and \( k_n \) is the wavenumber.

Therefore, the first-order differential of the pressure response is:

\[ \frac{dp}{dx} = A_n k_n \cos(k_n x) \]  \hspace{1cm} (7.13)

This spatial derivative of the pressure waveform (the pressure gradient) is maximum when \( x = 0 \) and so for small values of \( k_n h \) the acoustic pressure gradient between locations at \( \pm h \) can be approximated to:

\[ \frac{\Delta p}{\Delta x} \approx A_n k_n \quad (\text{for small values of } k_n h) \]  \hspace{1cm} (7.14)

From equation 7.12 it can be seen that for each sensor (in a two sensor system) the acoustic pressure maximum is \( \pm A_n \) and occurs when \( \sin(k_n x) = 1 \). The maximum pressure difference between the two sensors of \( \pm 2A_n \) occurs when the wavelength of a waveform is such that its maxima are \( \pi \) (180°) out of phase between each sensor, or when \( \frac{\lambda}{2} = 2h \). This can be rephrased in terms of \( k_n \) where \( 2k_n h = \pi \) which, as should already be evident, is equal to the Nyquist limit (equation 7.15). The upper magnitude limit of the pressure gradient between the two locations is therefore \( \left| \frac{\Delta p}{\Delta x} \right|_{\text{max}} = \frac{2A_n}{2h} \) and is shown schematically in figure 7.19 where the envelope of the pressure gradient with respect to the wavenumber of the mode is plotted.
Figure 7.19: The spatial derivative sensitivity as a function of the non-dimensional wavenumber $(2k_h)$. The relationship shown in figure 7.19 can also be expressed mathematically as:

\[
\frac{\Delta p}{\Delta x}_{max} \approx \begin{cases} 
A_n k_n & 2k_nh < \pi \quad \text{small } k_h \\
\frac{2A_n}{2h} & \pi \leq 2k_nh \quad \text{large } k_h 
\end{cases}
\]  

(7.15)

where $2k_nh = \pi$ is the Nyquist limit of the sensor.

Of course the derivation is a gross simplification of the phenomenon since the derivative is also a function of sensor position within the sound field, and the magnitude of the gradient estimate does not asymptote to unity but oscillates between zero and unity. However, it does give an indication of the maximum gradient that could be expected.

If the two effects shown in Figures 7.18 and 7.19 are combined, then an expression for the spatial derivative as a function of wavenumber can be obtained. From equation 7.10 it can be seen that:

\[
A_n \approx \begin{cases} 
\frac{A_{k_n}}{c} & 2k_nh < 2c_h \\
\frac{A_{k_n}}{c} & 2k_nh \geq 2c_h 
\end{cases}
\]  

(7.16)

Substitution of this into equation 7.15 reveals:
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\[
\left. \frac{\Delta p}{\Delta x} \right|_{max} \propto \begin{cases} 
  k_n^2 & 2k_nh < 2k_ch \\ 
  k_c & 2k_ch \leq 2k_nh < \pi \\
  k_n \pi & \pi \leq 2k_nh 
\end{cases} \quad \text{Mass controlled, small } kh \\
\text{Stiffness controlled, small } kh \\
\text{Stiffness controlled, large } kh (above the Nyquist limit)
\]

(7.17)

This is shown graphically in figure 7.20, where it is shown that for the region bounded by \(2k_ch \leq 2k_nh < \pi\) the magnitude of the spatial derivative is constant.

![Figure 7.20](image)

Figure 7.20: The spatial derivative sensitivity to different acoustic modes (with respect to the non-dimensional modal wavenumber \(2k_nh\)) in a reactive enclosure.

This implies that even though the high-order modes will have smaller amplitudes than modes with natural frequencies near \(f_c\) they will contribute equally to the finite-difference approximation to the pressure gradient estimate in the region up to \(2k_nh < \pi\), after which the contributions begin to roll off.

### 7.5.3 High-order derivatives

The situation is even worse in the case of high-order derivatives. Consider the following pressure field defined by:
\[ p = A_n \cos(k_n x) \] (7.18)

The first order-differential is:

\[
\frac{dp}{dx} = -A_n k_n \sin(k_n x)
\] (7.19)

and the second derivative is:

\[
\frac{d^2p}{dx^2} = -k_n^2 A_n \cos(k_n x)
\] (7.20)

For small values of \( k_n h \) (or as \( k_n h \to 0 \)) this can be approximated by:

\[
\frac{\Delta^2 p}{\Delta x^2} \approx -k_n^2 A_n
\] (7.21)

With the same method used in section 7.5.2, it can be seen that for each sensor (in a two sensor system) the acoustic pressure maximum is \( \pm A_n \) and the upper limit of pressure gradient between the two locations is \( \left| \frac{\Delta p}{\Delta x} \right|_{\text{max}} = \frac{2A_n}{h} \)

Therefore the upper limit of the second-order differential is \( \left| \frac{\Delta^2 p}{\Delta x^2} \right|_{\text{max}} = \frac{4A_n}{h^2} \) and the entire relationship can be summarised as:

\[
\left| \frac{\Delta^2 p}{\Delta x^2} \right|_{\text{max}} \approx \begin{cases} -k_n^2 A_n & 2k_n h < \pi \text{ small } kh \\ \frac{4A_n}{h^2} & \pi \leq 2k_n h \text{ large } kh \end{cases}
\] (7.22)

where \( 2k_n h = \pi \) is the Nyquist limit of the sensor.

Substitution of equation 7.16 into equation 7.22 gives the following expression for the second order spatial derivative as a function of wavenumber:
where \( k_c = \frac{2\pi f_c}{c_0} \) is the wavenumber at the chosen frequency and \( k_{\pi} = \frac{\pi}{2h} \) is the Nyquist limit of the sensor. This is shown graphically in figure 7.21.

\[
\frac{\Delta^2 p}{\Delta x^2} \mid_{\text{max}} \propto \begin{cases} 
  k_n^3 & 2k_n h < 2k_c h \\
  k_c^2 k_n & 2k_c h \leq 2k_n h < \pi \\
  k_{\pi} & \pi \leq 2k_n h
\end{cases} \quad \text{(7.23)}
\]

**7.6 Conclusions**

It was demonstrated in the previous chapter that the cost function at an observer location (remote from any physical sensor) can be estimated by forward difference prediction. The second-order virtual sensors (the microphone and the energy density sensor) were theoretically the most superior error sensors, but in practice demonstrated a sensitivity to short wavelength extraneous spatial pressure variations. The first-order virtual microphone showed consistent results and continually proved to be a more robust “virtual” error sensor. In this chapter the errors that directly affected the accuracy of the forward wave prediction sensors were considered.
A numerical model of tonal noise was used to demonstrate how the sensor spacing affected forward wave prediction accuracy. It was shown that in the absence of extraneous noise, second-order forward prediction was more accurate than first-order forward prediction and that the prediction errors proportionally increased as the wavelength of the measured signal was reduced with respect to sensor spacing. However, since the amplitude estimates were consistently larger than the true waveform, the errors would have the same effect as a calibration or sensitivity error and should not adversely affect the cost function minimisation. It was also found that estimation errors of the waveform phase relationship with the observer location, proportionally increased as the wavelength of the measured waveform was reduced with respect to sensor spacing. This would affect the location of where a cost function was minimised.

It was also shown mathematically and numerically how short wavelength, low amplitude extraneous noise, directly affected the prediction accuracy of the virtual sensors. As expected, from the results of the experiments that were detailed in the previous chapter, the second-order prediction sensors were shown to be more sensitive to short wavelength noise.

Mechanisms that generate the short wavelength extraneous noise were also considered and the near field effect of the sound source was initially thought to be significant contributor. However, reducing the characteristic source dimension of the primary source and the control source did nothing to improve the spatial acoustic pressure irregularities that were apparent in the experimental validation of the free field model. Examining the geometric parameters of the acoustic sources and the frequency range of the sound field, showed that the measurement locations in the initial experiments were already in the far field and that reducing the size of the acoustic sources could not move the measurement locations any further from the hydrodynamic near field. For the experiments that were conducted in the anechoic chamber, the low amplitude, high wavenumber spatial pressure variations were therefore concluded to be the result of system noise, instrumentation errors, measurement inaccuracies and boundary reflections.
In a reactive environment higher-order modes were mathematically shown to significantly contribute to the short wavelength spatial pressure variations and degrade the estimates of the spatial derivatives. While first-order estimation methods were shown to have some sensitivity to low amplitude high-order modes, the second-order estimation methods were shown to be much more sensitive.

Reducing the sensor spacing would reduce the prediction errors that result from either measuring a short wavelength tone or from the presence of low amplitude short wavelength extraneous noise. If the sensor spacing were reduced, but the observer/sensor separation distance remained practically large, higher-order virtual sensors would be required to predict the waveform. However, comparing first and second-order virtual sensors demonstrated that high-order prediction methods are highly sensitivity to short wavelength extraneous noise and would therefore not be a practical solution. While the sensor spacing will dictate the upper frequency limit of the waveform that may be accurately predicted, one possible solution to reduce the effect of short wavelength extraneous noise may be to incorporate a method of spatial filtering which would involve using more sensors than necessary for the first or second-order derivative to be approximated.
Chapter 8

Forward prediction virtual error sensors
in an aircraft cabin

8.1 Introduction

In the previous chapters, the performance of forward wave prediction virtual sensors were evaluated both analytically and experimentally in simplistic systems to highlight the factors that affected and limited their use as active noise control error sensors. However, neither the free field example nor the long narrow duct were representative of a highly damped and modally dense aircraft cabin. This chapter shows the results from experiments that were conducted to demonstrate the performance of the virtual sensors, as active noise control error sensors, in a single engine light aircraft cabin (figure 8.1(a)) using methods similar to those discussed in chapter 6.

To observe the effect of the proximity of the control source with respect to the observer location, two configurations were considered. In the first, the control sources were remote from the observer and in the second they were located in the observer’s head-rest. In the same manner as the previous chapters, only one dimension is considered (in this three dimensional
example), along an axis that passes through the observer’s ear, because the virtual sensor algorithms developed in this thesis can only be applied to a single dimension. Consideration of all three dimensions is obviously more practical, but would require more sensors and further development of the virtual sensor algorithms. This is therefore a subject for future research.

8.2 Method

The stripped fuselage of a Cessna 150 was refurbished to approximate the environment of a fully commissioned aircraft (figure 8.1). Damping material (automotive carpet), typical of that used in light aircraft, was attached throughout the cabin and an instrumented mannequin upper body (the observer) placed on the pilot’s seat.

![The Cessna 150 fuselage and cabin.](image)

The equipment used in both experiments is listed in table 8.1 and shown in figure 8.2.

The primary noise was generated using a single enclosed loudspeaker in the foot-well of the aircraft (figure 8.3(a) and (b)). In the first experiment, one of the control sources was placed on the wind-screen shelf of the cabin (figure 8.3(a) and (b)) and the second behind the seat of the observer (figure 8.3(a) and (c)). In the second experiment both of the control source loudspeakers were located in the head-rest of the cabin seat (figure 8.4).
### 8.2. Method

<table>
<thead>
<tr>
<th>Description</th>
<th>Make</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microphone</td>
<td>Generic brand</td>
<td>-</td>
</tr>
<tr>
<td>Microphone amplifier</td>
<td>Custom built</td>
<td>-</td>
</tr>
<tr>
<td>Dynamic signal analyser</td>
<td>Hewlett Packard</td>
<td>35665A</td>
</tr>
<tr>
<td>3 enclosed loudspeakers</td>
<td>Custom built</td>
<td>-</td>
</tr>
<tr>
<td>200 W power amplifier</td>
<td>Promaster</td>
<td>Pro series 3</td>
</tr>
<tr>
<td>Adjustable filter</td>
<td>Krohn Hite</td>
<td>3322</td>
</tr>
</tbody>
</table>

*Table 8.1: A list of the equipment that was used in the Cessna 150 experiments.*

*Figure 8.2: The measurement equipment configuration.*
Figure 8.3: The speaker and measurement locations when using remotely placed control sources.
8.2. Method

In both of the experiments, while only the primary noise source was driven with broad band random noise (50-400 Hz), transfer functions were measured with a Hewlett Packard 35665A spectrum analyser between the amplified signal from a microphone in the rear chamber of the primary noise source and the amplified signal from a measurement microphone at each of 21 measurement locations. These locations extended outwards from the observer’s ear at 25 mm intervals along a 0.5 m length (figure 8.3 and figure 8.4). The microphone signals were low pass filtered (< 400 Hz) to minimise the effect of short wavelength extraneous noise.

With the primary noise source turned off, each control source was individually driven and the procedure repeated until all 63 transfer functions between each measurement location and each noise source were measured and recorded.

To eliminate the inherent errors of a practical ANC controller, control was simulated numerically via quadratic optimisation with the same method as used in chapter 6. In both experiments the performance of a single microphone error sensor was compared to:

- a first order virtual microphone error sensor,
- a second order virtual microphone error sensor,
- a first order virtual energy density error sensor and
- a second order virtual energy density error sensor.

As previously illustrated in chapter 6, minimising pressure at a single location only requires one control source, while two control sources are required to effectively minimise energy density in one dimension. It is also shown in chapter 6 that the performance of a first-order virtual energy density sensor is identical to simply minimising either energy density at the physical sensor location or acoustic pressure at two microphone locations, in a two control source system. Therefore using a single control source is limited to observing the performance of a single microphone, a first-order virtual microphone and a second-order virtual microphone.
Figure 8.4: The speaker and microphone locations when the control source speakers are located in the observer’s head-rest.
Two control sources are used to observe energy density minimisation directly at the sensors and at the observer location with a second-order virtual energy density sensor. In all of the results that follow, the physical sensors are located 100mm ($4h$) from the observer location.

### 8.3 Results

#### 8.3.1 Remotely placed control source speakers

##### 8.3.1.1 One control source

Figure 8.5 shows the results of actively controlling the primary noise between 50 Hz and 400 Hz with one control source loudspeaker located on the wind-screen shelf of the cabin. The uncontrolled noise levels at the observer location were compared to the controlled noise levels (using various error sensors) and the results are shown in figure 8.5(a). With all of the physical sensors located 100 mm from the observer, it is shown that, in general, each error sensing strategy reduced the noise at the observer location across the entire spectrum. There are one or two exceptions with the second order virtual microphone, which previously (chapter 6) demonstrated a high sensitivity to short wavelength noise and measurement errors.

Figure 8.5(b) shows how the noise attenuation was improved at the observer location when using the virtual microphone error sensors, with respect to using a single microphone error sensor (the 0 dB reference). It can be seen that estimating the cost function at the observer location with a first order virtual microphone generally resulted in a higher level of noise attenuation when compared to using a single microphone error sensor placed at an equivalent separation distance to the sensor of the first order virtual microphone. However, using the second order virtual microphone resulted in extremely erratic noise attenuation across the spectrum, which may be attributed to a continually changing ratio of short wavelength noise to the amplitude of the control frequency noise and subsequent prediction inaccuracies.
Chapter 8. Forward prediction virtual error sensors in an aircraft cabin

(a) The uncontrolled and controlled noise spectrum for various error sensing strategies

(b) Comparing the attenuation achieved with virtual microphones to a single remote microphone (0dB)

Figure 8.5: ANC spectra at the observer location with only one control source, located on the windscreen shelf. In each case the physical sensors are located $4h$ (100mm) from the observer’s ear.
Figure 8.5 (b) shows that the first-order virtual microphone error sensor generally resulted in a higher noise attenuation across the frequency spectrum than was achieved with either a single microphone or a second-order virtual microphone, but gives no indication of how the sizes of the control zones compare. To analyse the performance of the error sensors at each and every frequency in the spatial domain would generate an indigestible quantity of plots. Therefore, since figure 8.5 (b) shows, that in this case, the first-order virtual microphone is generally the superior error sensor, three example frequencies were chosen where its performance excels. For each frequency (174 Hz, 228 Hz and 358 Hz) the size of the control zones are are shown in figure 8.6(a), (b) and (c) respectively.

Figure 8.6 (a) shows that when a conventional microphone was used as an error sensor in controlling noise at 174 Hz in the cabin, a significant noise attenuation of 40 dB was achieved at the location of the sensor. However it is also evident that the control zone was very small and that the noise attenuation at the observer location was only 5 dB.

Physically located at the same position as the microphone, the first-order virtual microphone was shown to predict the pressure squared cost function at the observer location, where 31 dB of sound pressure level attenuation was achieved (an improvement compared to the single microphone of 26 dB). The second-order virtual microphone, previously shown to result in an erratic control spectrum, could only estimate the cost function at the observer location accurately enough to result in 10 dB of noise attenuation. This was still an improvement on using a single remotely placed microphone, but figure 8.5 (b) suggests that slight changes in frequency significantly affect the ability to accurately predict the cost function at the observer location with a second-order virtual microphone. The conclusions and patterns of the control zones are similarly repeated at 228Hz (figures 8.6 (b)) and 358Hz (figure 8.6 (c) ).
Figure 8.6: The primary and controlled sound pressure level from at various frequencies using a single control source remotely located from the observer. The actual sensors are marked with “o” and the observer location is at the far left hand side of each graph.
8.3. Results

8.3.1.2 Two control sources

Figure 8.7 shows the uncontrolled primary noise spectra and the controlled noise spectra when using a control source loudspeaker located on the wind-screen shelf of the cabin and a second located behind the observer’s seat. It can be seen in figure 8.7(a) how the uncontrolled noise level spectrum at the observer location compares to the controlled noise level spectrum when using various error sensors. Identically to the previous example of using a single control source, all of the physical sensors were located 100 mm from the observer. In figure 8.7(a), it is shown (in general) that using either the first or second-order virtual energy density sensors reduced the noise at the observer location across the entire spectrum to a greater extent than using a single microphone error sensor. However, the question now arises whether it is the control algorithms or using the second control source that provided the additional benefit, merely by minimising pressure at the two sensor locations.

Figure 8.7(b) shows a more direct comparison between the virtual energy density sensors and using a single microphone (with one control source), in which control via the single microphone is the 0 dB reference. From this it can be seen that both the first and second-order virtual energy density sensor perform similarly, with only a slightly more erratic performance from the second-order virtual energy density sensor. It has been previously established that in these ’two control source’ examples, where only one dimension is considered, minimising energy density at the observer location with a first-order virtual energy density sensor is identical to either minimising energy density at the observer location or pressure at the two sensor locations (chapter 6). Therefore identical performance would be expected when the pressure is minimised at two locations separated from the observer location by 100 mm and 150 mm respectively. Hence it must be concluded that in this example that there is no advantage in minimising energy density at the observer’s location via a first-order estimation technique, but that using a second control source (where the pressures at each sensor location can be independently controlled) is responsible for the distinct control improvement. The similarity between the control spectrum when using a first-order virtual energy density sensor and the
control spectrum when using a second-order virtual energy density sensor, would also suggest that there is no advantage in using a second-order virtual energy density error sensor.

At three frequencies selected from 8.7(b), where the control field is improved with using a second control source, figure 8.8 shows the spatial variation of the uncontrolled primary noise and the controlled noise. All three examples (162 Hz, 209 Hz and 397 Hz) show that the addition of a second control source, to allow independent pressure control at each sensor, significantly improved the size of the control field around the observer location when compared to using a single microphone error sensor with a single control source. It can be seen in the 169 Hz example (figure 8.8(a)) that the second-order virtual energy sensor can improve upon using a first-order virtual energy density sensor, but these instances are only at a few isolated frequencies of small bandwidth (figure 8.7(b), figure 8.8(b) and 8.8(c)) and are not indicative of a pattern of improvement.
8.3. Results

(a) The uncontrolled and controlled noise spectrum for various error sensing strategies.

(b) The attenuation achieved with virtual microphones compared to single remote microphone (0dB).

Figure 8.7: ANC spectrums at the observer location with two control sources. One is located on the wind-screen shelf and the other behind the observer’s seat. In each case the physical sensors are located 4\(h\) (100mm) from the observer’s ear.
Figure 8.8: The primary and controlled sound pressure level from at various frequencies using two control sources remotely located from the observer. The actual sensors are marked with “o” and the observer location is at the far left hand side of each graph.

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8.3.2 Control source speakers fitted into a head-rest

8.3.2.1 One control source

The results of actively controlling the primary noise between 50 Hz and 400 Hz with a single control source loudspeaker located in the observer’s head-rest are shown in figure 8.9. Figure 8.9(a) shows how the uncontrolled noise levels at the observer location compared to the controlled noise levels using various error sensors. The second-order virtual microphone is once again shown to be extremely sensitive to short wavelength noise and produces an erratic control profile across the entire frequency range.

In general, each error sensing strategy is shown to reduce the noise at the observer location across the entire spectrum, although not to the same extent as when the single control source speaker is located on the wind-screen shelf (figure 8.5(a)). This statement not only applies to using the virtual microphones, but also for when using a single microphone, hence illustrating an effect of speaker proximity, rather than control algorithm performance. While this implies that controlling the noise across the considered frequency range is improved by increasing the separation distance between the observer and the control source, this would require higher powered speakers with an increased size and mass. Consequently, for the single engine light aircraft the advantages in locating the speakers close to the observer must not be hastily overlooked.

The noise attenuations at the observer location when using the first and second-order virtual microphone error sensors are compared directly to using a single microphone error sensor (the 0 dB reference) in figure 8.9(b). It is shown, that in this single control source example, using a first-order virtual microphone improves upon using a single microphone, but not to the same extent as in the example with the speaker placed on the wind-screen shelf (figure 8.5(b)). This however is as expected since it has already been established in the previous paragraph, that none of the error sensing strategies with a single control source located in the headrest.
perform as well as when the single control source is located at a more remote location on the wind-screen shelf.

Figure 8.10 shows the spatial variation of the uncontrolled primary noise and the controlled noise for each error sensor, at the three example frequencies selected from figure 8.9 (b), where using virtual microphones error sensors improved the active noise control performance, compared to using a single microphone.

In all of the examples (188 Hz, 270 Hz and 381 Hz) it can be seen that when the error sensor was a single microphone, the high level of noise attenuation achieved at the sensor did not extend to encompass the observer 100 mm away. Each illustrates that the virtual microphones improved the noise attenuation at the observer location.

With the single microphone error sensor as a basis of comparison (since the performance of the virtual sensors is very much frequency dependent), there was little difference in the size of the control zone that was achieved when the control source speaker was either remote from the sensor (figure 8.6) or in the head-rest (figure 8.10). This indicates that in this example of a single engine aircraft cabin (where the separation distance between the control source speaker and the observer is limited), the speaker separation distance (from the observer) does not affect the size of the control zone for a pressure squared cost function at a single location.
(a) The uncontrolled and controlled spectrum for various error sensing strategies

(b) The attenuation achieved with virtual microphones compared to 1 real microphone

Figure 8.9: ANC spectrums at the observer location with one control source located in the observer’s head-rest. In each case the physical sensors are located 4h (100mm) from the observer’s ear.
Figure 8.10: The primary and controlled sound pressure level from at various frequencies using a single control source located in the observer’s head-rest. The actual sensors are marked with “Æ” and the observer location is at the far left hand side of each graph.
8.3.2.2 Two control sources

Figure 8.11 illustrates the results of actively controlling the primary noise between 50 Hz and 400 Hz with two control sources located in the observer’s head-rest. The spectrum for the active noise control when using a single microphone, a first-order virtual microphone and a second-order virtual microphone are compared to the uncontrolled noise spectrum at the observer location. Figure 8.11(a) shows that all of the control strategies considered here, reduced the noise at the observer location across the entire frequency range of interest. In figure 8.11(b) the error sensing performance of both virtual energy density sensors are directly compared to using a single microphone (with one control source) in which control via the single microphone is the 0 dB reference. By comparing these results with those obtained using a single control source in the head-rest (figure 8.9(b)), it can be seen that in terms of noise attenuation at the observers location, the first-order virtual energy density sensor behaved similarly to using a first-order microphone (although slightly more erratically). While it will be shown later (figure 8.12) that the zone of control increases with a first-order virtual energy density sensor (compared to using a first-order virtual microphone), it has been established (section 8.3.1.2) that this is a result of the second control source (and the independent control of pressure at two locations) and not the cost function.

Figure 8.11(b) and figure 8.9(b) show that the second-order virtual energy density sensor shows a superior error sensing performance when compared to using all of the error sensing methods considered with the control sources located in the head-rest. In this example, the second-order virtual energy density error sensor performs similarly to using the first or second-order virtual energy density error sensors with two remote control sources, but carries the added benefit of the potential to use smaller lightweight speakers.

Figure 8.12 shows how the control zones compare in the spatial domain around the observer location at the three example frequencies (154 Hz, 237 Hz and 360 Hz) from figure 8.11 (b).
Figure 8.11: ANC spectrums at the observer location with two control sources both located in the observer’s head-rest. In each case the physical sensors are located 4h (100mm) from the observer’s ear.
Figure 8.12: The primary and controlled sound pressure level from at various frequencies using two control sources remotely located from the observer. The actual sensors are marked with “•” and the observer location at the far left hand side of each graph.
In all three cases, it is shown that the second-order virtual energy density error sensor not only resulted in the highest noise attenuation at the observer location, but produced a broad zone (compared to a single microphone) of attenuated noise centered around the observer location. The first-order virtual energy sensor, direct energy sensor, or two location pressure sensor, as is obviously apparent in figure 8.12, produced the widest control zone, but one that was centered around the physical sensors and not sufficiently large enough to fully envelop the observer.

8.4 Conclusions

Two methods of control have been analysed in a Cessna aircraft cabin. In the first, the control sources were located remote from the observer on the wind-screen shelf and behind the seat. In the second, the control sources were located in a prefabricated head-rest. For each method of control, the performance of the virtual error sensors (predicting the cost function at the observer location) was compared to that of traditional sensors, with all physical sensors located at the same separation distance (100 mm) from the observer.

Only one dimension was considered, because of the current limitations of the virtual sensor algorithms that were developed in this thesis. However, considering only one dimension was considered to be a valuable precursor to considering all three dimensions. In future research the virtual sensor algorithm will be developed for use in three dimensional applications.

Table 8.2 summarises how the various virtual error sensors improved upon the use of a remotely located single microphone. The table only indicates the maximum performance of the virtual sensors and so it is important to refer to the graphs in the result sections to observe how each sensor performs with respect to frequency and to observe the size of the resulting control zone.

It was found that, in general, all methods of error sensing would reduce the noise at the observer location. However, while using a single microphone error sensor resulted in high noise...
8.4. Conclusions

Table 8.2: A summary of the maximum attenuation improvement (when compared to using a single microphone error sensor, 0 dB) in the Cessna cabin at the observer location using various error sensors. The frequencies where the maximum attenuation improvement occurs are shown in brackets. The sensors are separated from the observer by 4h (100 mm).

<table>
<thead>
<tr>
<th>Control sources on the wind-screen shelf</th>
<th>1st order virtual microphone</th>
<th>2nd order virtual microphone</th>
<th>Energy density</th>
<th>2nd order virtual energy density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control sources on the wind-screen shelf</td>
<td>26 dB (174 Hz)</td>
<td>15 dB (375 Hz)</td>
<td>26 dB (274 Hz)</td>
<td>21 dB (274 Hz)</td>
</tr>
<tr>
<td>Control sources in the headrest</td>
<td>16 dB (381 Hz)</td>
<td>20 dB (254 Hz)</td>
<td>14 dB (184 Hz)</td>
<td>25 dB (233 Hz)</td>
</tr>
</tbody>
</table>

Tableau 8.2: Résumé de l'amélioration maximale de l'atténuation (par rapport à l'utilisation d'un capteur d'erreur microphone unique, 0 dB) dans la cabine de l'hydravion Cessna à la localisation de l'observateur utilisant divers capteurs d'erreur. Les fréquences où l'amélioration de l'atténuation maximale se produit sont indiquées entre parenthèses. Les capteurs sont séparés de l'observateur par 4h (100 mm).

The use of an energy density cost functions resulted in broadening the zone of control centered around the sensors or in the case of the virtual energy density sensors, centered around the observer location. In these two control source examples, estimating and controlling energy density at the observer location by a first-order estimation method was known to be the same as merely independently controlling pressure at the two sensor locations. The advantages in these examples are therefore not attributed to the cost function, but the addition of a second control source.

Energy density at the observer location was also estimated with a second-order virtual energy density sensor. In the example of remotely located control source speakers, the second order virtual energy density sensor behaved similarly to the first-order virtual energy density sensor, with no apparent benefits of predicting the energy density cost function at the observer location.

The system behaved differently with the control source speakers moved closer to the observer and located in the head rest. With only one control source speaker (and a pressure squared cost function at one location), the noise attenuation achieved with each sensor was less than
when using a remotely placed control source speaker. However, the implication of improving control performance by not locating the speakers as close to the observer would require larger heavier speakers than those used in a head rest. Adding another control source speaker to the head rest however, highlighted the benefits of estimating an energy density cost function at the observer location with a second-order virtual energy density sensor. The sensor generally resulted in a superior attenuation level at the observer location (when compared to using the other sensors), produced a broad and practically sized control zone and retained the benefit of using small lightweight speakers. This again shows that the advantage is gained through the second control source that allows pressure to be minimised at two locations, rather than energy density minimisation at the observer. Since the acoustic particle velocity must be zero at a solid object (such as the dummy’s head), only the potential energy (pressure) component of energy density can be further reduced. Hence, the broad zone must be attributed to sound pressure reduction at two locations close to the dummy’s ear.
Chapter 9

A comparison of virtual sensing methods

9.1 Introduction

In chapter 4 it was shown how Garcia-Bonito et al. (1996) introduced the concept of the virtual microphone to minimise a pressure squared cost function at an observer’s ear. Their idea was that the acoustic pressure transfer function between the location of a permanently placed remote microphone and the location of an observer could be used to modify the signal from the microphone and hence estimate a pressure squared cost function at the observer location. While the methods derived in this thesis are directed towards the same aim of predicting the cost function at the observer location, they are very different in application, as multiple sensors are used to predict the cost function at the observer location by forward wave prediction. Forward wave prediction continually measures the changing signal at the physical sensors to predict the cost function at the observer location and so will adapt to any acoustic pressure transfer function (between the location of a sensor and the observer) change, or any environmental changes. The forward wave prediction of an energy density cost function will also produce a wider zone of attenuated noise than the attenuation zone associated with a microphone error sensor (or a pressure squared cost function). The limitation of forward wave
prediction is that the spacing between the observer and the sensors must be small with respect to wavelength.

This chapter (using the example from the previous chapter, where the control sources were located in the head rest of a Cessna 150) shows the results of comparing the performance of Garcia-Bonito et al’s transfer function based virtual microphone, with the performance of the forward wave prediction virtual sensors that were developed in this thesis.

9.2 Method

In section 8.3.2, the performance of forward wave prediction virtual sensors were compared to traditional sensing methods in a Cessna 150 cabin, with control sources located in the head rest of the seat. In this chapter, the same example, with the same experimental data was used to generate results that show how a transfer function based virtual microphone performs in the same environment and compares to the forward wave prediction virtual sensors. Using the same method that has been used in all of the previous examples, the cost functions were numerically minimised by quadratic optimisation to simulate active noise control. These cost functions were either minimised at the sensor location or estimated and minimised at the observer location by using either forward wave prediction virtual sensors or a transfer function based virtual microphone.

A further experiment was also conducted to observe the stability of the transfer function based virtual microphone when an additional object was brought near to the observer’s head. Once the required transfer functions were measured, a panel was placed on top of the head rest (figure 9.1). This altered the acoustic pressure transfer functions between the acoustic sources and the measurement locations and the resulting change in the value of the cost function measured by the transfer function based virtual microphone was observed.
Figure 9.1: An additional reflective surface added near to the observer to change the frequency response measured at the observer location and the sensor location.

9.3 Results

9.3.1 Comparing the performance of the virtual sensors

Figure 9.2 shows the acoustic fields that were individually generated by the primary source and the control source. The graph compares the difference between the frequency response of the acoustic fields at the observer location (location 21) and the sensor location (location 17). It can be seen that across the entire frequency spectrum, the acoustic field that is generated by the control source has a higher amplitude at the sensor location (compared to the observer location) because of the closer proximity to the acoustic source. It can also be seen that the primary source acoustic field, measured at the observer location, is different to the primary source acoustic field measured at the sensor location.

Figure 9.3(a) shows how the uncontrolled primary source acoustic field around the head rest in a Cessna 150 compares to the controlled field that results from using active noise control and a variety of error sensing methods. The high level of noise attenuation achieved at the single microphone only extended over a narrow region and only 4 dB of attenuation was experienced at the observer location.
Chapter 9. A comparison of virtual sensing methods

While both the first-order and the second-order virtual microphone error sensors (discussed in more detail in section 8.3.2) outperformed the single microphone error sensor, in terms of reducing the noise level at the observer location (resulting in attenuations of 7 dB and 13 dB respectively), the performance of both were surpassed by the transfer function based virtual microphone, developed by Garcia-Bonito et al. (1996), which resulted in 38 dB of attenuation at the observer location.

However, it was previously proposed that the forward-prediction virtual microphone could be developed further, to track an observer’s head movement and continue to minimise acoustic pressure squared at the moving observer’s ear. Closer examination of figure 9.3(a) shows that if the observer were to move to the right by only 50 mm ($x/\lambda = 0.04$), the attenuation experienced using a transfer function based virtual microphone error sensor, reduces to the same level of attenuation that might be achieved by a tracking second-order virtual microphone.

Figure 9.3(b) shows that a second-order virtual energy density error sensor also results in a highly attenuated noise around the observer location. While the attenuation (30 dB) is not as high as that achieved using the transfer function based virtual microphone, the extended zone of attenuation would result in a more comfortable environment, since the observer would not experience such dramatic variations in sound pressure levels with small amounts of head

Figure 9.2: The difference between the primary and control source acoustic fields (individually generated, without cancellation) at measurement location 21 (the observer location) and measurement location 17 (the sensor location) in the Cessna 150 fuselage.
9.3. Results

(a) Comparing the transfer function based virtual microphone to the first and second-order forward prediction virtual microphones.

(b) Comparing the transfer function based virtual microphone to the first and second-order virtual energy density sensors.

Figure 9.3: Comparison of the transfer function based virtual microphone to the various types of forward wave prediction virtual sensors. The sensor locations are marked with a circle and the observer location is at the far left of each graph.

movement. However, while this shows how a virtual energy density sensor can improve the size of the control around the observer, the performance advantages must be attributed to the use of a second control source and not the cost function estimation method.

It is also demonstrated in figure 9.3(b) that the first-order virtual energy density sensor is (as previously explained in chapter 5) identical to minimising energy density at the sensor location. The attenuation at the observer location when using the second-order virtual energy density sensor is higher than the attenuation that is achieved at the sensor location when using direct energy density sensing. This is due to an over estimation of pressure at the observer location with the second-order virtual energy density sensor, causing the potential energy to be reduced more than the kinetic energy. This can be seen to result in a narrower region of control when compared to the first-order energy density sensor. This also has an interesting implication in that the components of energy density may be weighted to achieve a desired compromise between the level of attenuation and the size of the region of attenuation.

From figure 9.3(a) it can be observed how the transfer function based method shifts the pres-
sure squared cost function away from the microphone to the observer location. One would therefore expect that if Garcia-Bonito et al were to develop their method to predict energy density at the observer location, then the resulting control zone would resemble shifting the trace of the ‘real and 1st order virtual energy density sensor’ in figure 9.3 so that the minima coincided with the observer location.

### 9.3.2 Changing the transfer function

Figure 9.4 shows the results of the change in the primary source and the control source noise fields when an additional acoustically reflective surface (a wooden panel) is placed near to the observer location (figure 9.1). Figure 9.4(a) shows how the acoustic fields change at the sensor location (location 17) and figure 9.4(b) shows how the acoustic fields change at the observer location (location 21). It can be seen that the addition of the panel has only a minor effect on both the primary and control source acoustic fields at both the sensor and the observer locations.

![Graph](image.png)

(a) Measurement location 17 (the sensor location)  
(b) Measurement location 21 (the observer location)

Figure 9.4: Comparing the change in the primary source and the control source acoustic fields when an additional reflective surface is placed near to the observer.
The acoustic pressure transfer function between the sensor location and the observer location, for the transfer function based virtual microphone, was measured prior to the addition of the wooden panel. The panel was then added to simulate the effect that a waving hand or a book might have and to observe how well the transfer function based virtual microphone could continue to minimise the noise at the observer location (using the initially measured transfer function).

Figure 9.5 shows that without the panel, the transfer function based virtual microphone resulted in 38 dB of attenuation at the observer location, which only reduced by 2 dB to 36 dB after the panel was added.

![Figure 9.5: The effect of an additional reflective surface, placed near to the observer, on the transfer function based virtual microphone.](image)

**Figure 9.5:** The effect of an additional reflective surface, placed near to the observer, on the transfer function based virtual microphone.

### 9.4 Conclusion

The transfer function based virtual microphone has been shown to place a high level of attenuation at an observer location, that is remote from a physical sensor. For a static observer, the method is shown to be excellent and superior to either a first or a second-order forward wave prediction virtual microphone. However, the zone of attenuation is extremely narrow, and even the smallest amount of head movement would cause the observer to experience large...
variations in sound pressure level. It was demonstrated that movements as low as 50 mm could vary the attenuation by as much as 25 dB.

The second-order energy density sensor was shown to accurately predict the cost function at the observer location, which resulted in a broad region of high noise attenuation. This demonstrated the advantage of using a second control source and would be conducive to a more comfortable environment, allowing the observer to move without immediately experiencing a high level of sound pressure variation. Forward wave prediction virtual sensors (in general) also have the potential to track any movement, since their algorithms are distance dependent and show promise for allowing additional head movements within a following region of attenuated noise.

The next chapter will summarise these conclusions and all of the observations that have been made throughout the chapters of this thesis, and suggest a future direction of research that will lead towards a fully operational system.
Chapter 10

Conclusions and future research

10.1 Conclusions

In the literature review and in the initial experiments it was established that the cabin of a single engine light aircraft was an uncomfortably noisy environment. The low frequency noise was dominated by tones that were mainly generated by airborne propeller noise. The noise source was easily referenced and so the low frequency noise in the cabin had the potential for minimisation with active noise control, even though only the larger twin engine aircraft appear to have used this technique to date. Active noise control in personal headsets was a considered option, but hygiene factors, and the need to wear the headsets correctly and in a timely manner, suggested that actively controlling the noise around an observer would be a more comfortable and attractive alternative. Similarities with other applications were also considered and modelling was shown to be a valuable tool for predicting the performance of active noise control, although in more simplistic applications and examples.

The use of structural error sensors and control sources was reported to result in higher levels of attenuation and larger regions of noise attenuation, but would have encountered difficulties in installation, both in terms of the practical implications and safety certification. However, it
was also reported (both in the literature review and in the initial experiments) that the acoustic pressure squared cost function often resulted in impractically small regions of control. Even though the level of the attenuation at the microphone error sensor could be very high, as a result of active noise control, a nearby observer may experience no noise reduction improvement at all. Controlling the noise throughout the cabin (global noise control) was concluded to be extremely difficult and somewhat redundant when there would be only a limited amount of head movement possible in this particular environment. An energy density cost function, in which the pressure and spatial pressure gradient are minimised, offered the potential to sufficiently enlarge the control zone around an error sensor to fully envelope the observer. Another solution was to move the zone of attenuation away from the error sensor by estimating the cost function at the observer location using “virtual error sensors”. Combining the two would result in a cost function, more spatially constant than acoustic pressure squared, that was centered around the observer to allow for a practical amount of head movement without exposure to dramatic sound pressure level variations.

Various algorithms were developed to predict either an acoustic pressure squared cost function, or an energy density cost function at the observer location and were all based on forward wave extrapolation. Since this method uses a variable that is based on the separation distance between the sensors and the observer, the potential exists for any head movement to be tracked and enable an even larger amount of movement without the experience of large sound pressure level variations.

The forward wave prediction virtual sensors were numerically evaluated in simulated models and the results were then experimentally validated. The first-order virtual energy density sensor was shown to be mathematically identical to remotely controlling energy density with a two sensor system, or merely the same as independently minimising pressure at the two sensor locations.

In the free field example, using a one dimensional second-order virtual energy density sensor produced the largest attenuation zone around an observer remotely located from the physical...
sensors. In the example of a long narrow duct, it was generally found that the second-order virtual microphone was theoretically the most superior error sensor, in terms of placing a high level of attenuation at the observer, but in practice it was shown to be extremely sensitive to short wavelength spatial pressure variations. The first-order virtual microphone was a consistently robust virtual error sensor and was, in practice, the most superior error sensor for the reactive duct example.

It was thought that short wave length pressure variations adversely affected the second-order estimation methods. In the experiments, the near field effects of the acoustic sources were discounted as a contributor to the short wavelength noise, but it was shown how in general, regardless of the mechanism, very low amplitude short wavelength extraneous noise can have a large effect on the cost function estimation accuracy of forward wave prediction virtual sensors. In a reactive environment higher-order modes were shown to significantly contribute to the short wavelength spatial pressure variations. The accuracy of the forward wave prediction virtual sensor was also found to be a function of sensor spacing with respect to the wavelength of the measured signal. However, it was considered that amplitude estimation was not as important as estimating the phase of the cost function at the observer location. The practical performance of the second-order virtual microphones demonstrated that high-order prediction methods are highly sensitive to short wavelength extraneous noise and that any higher-order prediction methods, although theoretically more accurate would not be a practical consideration.

The performance of the virtual sensors as active noise control error sensors were also evaluated in the cabin of a Cessna 150. It was found that active noise control, regardless of the error sensing mechanism, reduced the noise at the observer location. Reducing an estimate of energy density at the observer location, by using a second-order virtual energy density sensor and two small acoustic control sources in the head rest, resulted in placing a broad zone of noise attenuation at the observer location. This shows potential as a method for placing a broad...
region of attenuated noise directly at the ears of the occupants in a small single engine light aircraft, while keeping the system weight to a minimum.

The Cessna 150 was also used to compare the forward wave prediction virtual sensors to the transfer function based virtual microphone developed by Garcia-Bonito et al. (1996). For a static observer and in terms of the level of noise attenuation at the observer location, the transfer function based method was shown to be excellent when compared to the virtual sensor algorithms developed in this thesis. The method was also shown to be stable when other reflective surfaces were brought into the controlled sound field. However, while the resulting noise attenuation levels were impressive, the size of the attenuation zone was extremely narrow, and a small amount of head movement would expose the observer to extremely uncomfortable variations in the sound pressure level. The second-order virtual energy density error sensor was shown to be capable of producing similar attenuation levels at the observer location, but more importantly over a wider zone. As stated previously, forward wave prediction virtual sensors have the unique potential to track and adapt to head movement to enable an even larger amount of comfortable head movement. One further benefit is that forward wave prediction virtual sensors have no requirement for the initial measurement of the sensor / observer acoustic pressure transfer function, as is required for the transfer function based virtual microphone.

The research that has been conducted to date and detailed within the pages of this thesis, has demonstrated both the potential and the limiting factors of forward wave prediction virtual sensors. It has been shown how these sensors can be used to place a broad zone of attenuated low frequency noise around the location of an observer in the cabin of a single engine light aircraft in which weight and system size must be kept to a minimum.

To fully develop forward wave prediction virtual sensors into a commercially attractive method of error sensing, that may be used in a single engine light aircraft (or any other similar application), is beyond the scope and time-scale requirements for this thesis. Therefore, it is the
intention of the author to remain actively involved with the further development of these virtual sensors and the direction of future research is detailed in the following sections.

10.2 Future research

10.2.1 Virtual sensors in a practical control system

The University of Adelaide is currently designing and developing small, low weight multi-channel digital active noise controllers with programmable control algorithms. Therefore, the forward wave prediction virtual sensor algorithms may be programmed directly into the controller which will then have the capability to predict and hence minimise the cost function at an observer location. This thesis has also only considered control in one dimension as a valuable precursor to considering all three dimensions. Future research will therefore develop the virtual sensor algorithms for use in three dimensional applications and consider how this will affect the number of required sensors and control sources.

10.2.2 Movement tracking

In the previous chapters it has been suggested that forward wave prediction virtual sensors may track and adapt to any head movement of the observer, to ensure that they remain in the zone of attenuated noise for reasonable amounts of head movement. The virtual sensor algorithms are all extrapolation based and estimate the cost function an observer separated by a distance of \(x\) from the sensors. While the examples that have been used all assumed that the observer / sensor separation distance was constant, there is no reason why \(x\) can not be a variable. If the observer / sensor separation distance were continually measured, by an ultrasonic sensor for example, then the algorithm could adapt and continue to place the attenuation zone at a mobile observer location. Industrial ultrasonic sensors can be very....
expensive and are often designed to measure distances much larger than those that would need to be accurately measured in this particular application. However, the ultra sonic sensors that are used in inexpensive compact cameras may provide a commercially viable solution. It is therefore proposed that these proximity sensors may be used to measure movement in one dimension (figure 10.1) to observe whether the forward wave prediction sensors can adapt to a varying separation distance. The research would then culminate in developing a three dimensional tracking sensor.

![Figure 10.1: The concept of movement tracking](image)

**10.2.3 Spatial filtering**

The forward wave prediction virtual sensors, particularly the second-order virtual microphone, have all demonstrated a sensitivity to short wavelength extraneous noise. It is therefore proposed that future research also addresses methods of spatially filtering these short wavelength acoustic pressure variations, that adversely effect the prediction accuracy of forward wave prediction virtual sensors. One method proposed, is to derive a signal from the mean of two (or more sensors) as shown in figure 10.2.
Figure 10.2: A proposed concept for spatially filtering short wavelength noise

Figure 10.3 shows a numerical simulation of a first and second-order forward waveform prediction. The cyan coloured line is the true value of the waveform as it passes the sensors (marked by dotted vertical lines) and the observer location (marked by a solid vertical line). It can be seen that, in the absence of short wavelength noise and when the wavelength is long with respect to the sensor spacing, a second-order virtual sensor can accurately predict the waveform (as previously shown in chapter 7).

Figure 10.3: First and second-order forward wave prediction of a traveling waveform that is represented by the cyan coloured curve. The black curve shows the first-order estimate and how it is predicted at the observer location (blue vertical line) from the signals that are measured at the sensors (blue dotted vertical lines and circles). Similarly, the red curve shows the second-order prediction.

Figure 10.4(a) and (b) show the effect of when either short wavelength tonal noise or uncorrelated random noise is added to the numerical simulation. Figure 10.4(c) and (d) however, show spatial filtering estimation can improve the prediction accuracy. In these two figures a sensor has been added to left of the existing three sensors and the mean acoustic pressure was
calculated from the pressure measured at each adjacent location, such that:

\[
p'_1 = \frac{p_0 + p_1}{2}, \quad p'_2 = \frac{p_1 + p_2}{2} \quad \text{and} \quad p'_3 = \frac{p_2 + p_3}{2}
\]  

(10.1)

Where \(p'_1\) is the mean acoustic pressure of \(p_0\) and \(p_1\), \(p'_2\) is the mean acoustic pressure of \(p_1\) and \(p_2\) and \(p'_3\) is the mean acoustic pressure of \(p_2\) and \(p_3\).

The fourth sensor was added because a first-order prediction requires two mean values and a second-order prediction requires three mean values.

Figure 10.4(e) and (f) show the effect of adding a fifth sensor, where the mean acoustic pressure was calculated from the pressure measured either side of an existing sensor location, to replace its non-averaged acoustic pressure, as shown in equation 10.2.

\[
p'_1 = \frac{p_0 + p_2}{2}, \quad p'_2 = \frac{p_1 + p_3}{2} \quad \text{and} \quad p'_3 = \frac{p_2 + p_4}{2}
\]  

(10.2)

Where \(p'_1\) is the mean acoustic pressure of \(p_0\) and \(p_2\), \(p'_2\) is the mean acoustic pressure of \(p_1\) and \(p_3\) and \(p'_3\) is the mean acoustic pressure of \(p_2\) and \(p_4\).

Since the mean is taken from wider spaced locations it can be seen that the short wavelength tonal noise is reduced even further. However, since the random noise is still (as in the four sensor example) the mean of two random variables, there is no improvement.

These simple numerical simulations show the potential that spatial filtering has for reducing the prediction error in second-order virtual sensors, which result from short wavelength extraneous noise.
10.2. Future research

Figure 10.4: The addition of extra sensors to allow spatial filtering. A traveling waveform is represented by the cyan coloured curve, the black curve shows the first-order prediction and the red curve shows the second-order prediction. The actual sensors are marked with an open circle and the observer location by a solid vertical line.

Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
References


Appendix A

Fundamental Theory and Formulae

A.1 The Control Optimisation Method

Quadratic optimisation was used as a means of determining the unique control source signal strengths for minimising the cost function at the error sensor location. This is covered in detail by Nelson and Elliott (1995) and Hansen and Snyder (1997) and summarised here for the benefit of readers.

A.1.1 Hermitian transpose

It will first be of use to show the importance of the Hermitian transpose matrix.

If $X$ is a matrix defined by:

\[
X = \begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix}
\]  

(A.1)
The transpose of $X$ is then defined as:

$$XT = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$$  \hspace{1cm} (A.2)

The Hermitian transpose matrix is the complex conjugate of the transpose and defined as:

$$X^H = \begin{bmatrix} x_1^* & x_2^* & \cdots & x_n^* \end{bmatrix}$$  \hspace{1cm} (A.3)

The value of this becomes evident when observing the result of a complex number (which can represent coordinates of phase and magnitude) multiplied by its complex conjugate. For example: let $y$ be a complex number $(a + jb)$ where the complex conjugate $y^*$ is $(a - jb)$. Therefore:

$$y^*y = (a - jb)(a + jb) = a^2 - ajb - (jb)^2 = a^2 + b^2 \quad (\text{since } j^2 = -1) \hspace{1cm} (A.4)$$

If $y$ is the complex representation of a transfer function (for example), when it is multiplied by its Hermitian transpose (to ensure the inner matrix dimensions are identical) it reveals the sum of the real and imaginary magnitudes squared, which is equal to the transfer function magnitude squared.

### A.1.2 Quadratic optimisation

The Hermitian transpose matrix can be put to use in the concept of quadratic optimisation, where the objective is to minimise a cost function (squared pressure for example) and calculate the control source strengths required from each source to do so. In figure 6.4 the pressure component at the error sensors due to the primary source alone may be represented by:
At those same $m$ locations, the pressure due to one control source may be represented by:

$$p_T^c = \begin{bmatrix} p_{c1} & p_{c2} & \cdots & p_{cn} \end{bmatrix}$$  \hspace{1cm} (A.6)

However, in this example there is more than one control source. Let the number of control sources be defined as $n$ so that the strength of the control sources may be represented by:

$$q_T^c = \begin{bmatrix} q_{c1} & q_{c2} & \cdots & q_{cn} \end{bmatrix}$$  \hspace{1cm} (A.7)

Defining the acoustic transfer impedance $Z$ to be the transfer function (relationship) between the pressure measured at the error sensor location and each control source of strength $q_c$, then:

$$p_c = Z.q_c$$  \hspace{1cm} (A.8)

where:

$$Z = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & \ddots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ z_{1m} & \cdots & z_{mn} \end{bmatrix}$$  \hspace{1cm} (A.9)

Where each row is the transfer impedance function for a particular error sensor with respect to each control source ($1$ to $c$). The total pressure at any one location is, by superposition, the sum of the primary and the control source pressure components, and is represented as:
\[ p = p_p + Z.q_c \]  \hspace{1cm} (A.10)

With the number of control source control locations \( n \) equal to the number of error locations \( m \), the matrix is square (as many equations as variables) and hence becomes solvable. Therefore the total pressure \( p \) may equate to zero, or in other words be minimised.

Where squared pressure is to minimised at each error location \( (1 \text{ to } m) \), the cost function \( J_p \) can be written as:

\[ J_p = \sum_{m-1}^m |p_m^2| \]  \hspace{1cm} (A.11)

or, since the product of a matrix and its Hermitian transpose returns the magnitude squared:

\[ J_p = p^H p \]  \hspace{1cm} (A.12)

\[ J_p = (p_p + Z.q_c)^H (p_p + Z.q_c) \]  \hspace{1cm} (A.13)

\[ J_p = p_p^H p_p + p_p^H Z q_c + q_c^H Z^H p_p + q_c^H Z^H Z q_c \]  \hspace{1cm} (A.14)

Where the quadratic equation \( ax^2 + bx + c = 0 \) has the general form of solution:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]  \hspace{1cm} (A.15)

equation (A.14) also has the solution of:
\[ q_o = -A^{-1}b \]
\[ J_o = c - b^H A^{-1}b \]  \hspace{1cm} (A.16)

when:

\[ J = q^H A q + q^H b + b^H q + c \]  \hspace{1cm} (A.17)

By comparing equation (A.14) with the above relationships it can be seen that:

\[
\begin{align*}
A &= Z^H Z \\
b &= Z^H p_p \\
b^H &= p_p^H Z \\
c &= p_p^H p_p
\end{align*}
\]  \hspace{1cm} (A.18)

The optimum source strength \( q_o \) to return the minimum cost function value therefore becomes:

\[ q_o = (Z^H Z)^{-1} Z^H p_p \]  \hspace{1cm} (A.19)

which returns the minimum cost function:

\[ J_{po} = (p_p^H p_p) - (p_p^H Z) (Z^H Z)^{-1} (Z^H p_p) \]  \hspace{1cm} (A.20)
Having identified the optimum signal strength to minimise the cost function at the error sensor locations, the principle of superposition where \( p = p_p + Zq_e \) can be applied to calculate the sound pressure at any other location to fully describe the controlled sound field.
Appendix B

Control simulation code

B.1 An example of a plot routine

The following MATLAB code is an example of one used to generate the graphs within this thesis. This particular routine was used to compare the performance of a variety of error sensors at a set separation distance from the observer. The user parameters are defined in this routine, which in turn evokes the relevant control simulation code.

```matlab
% A PLOT ROUTINE FOR COMPARING CONTROL STRATEGIES—by CDK 29/6/99
%__________________________________________________________
% uncomment the following to evoke
%__________________________________________________________
%clear all
%loadfreemodel  % loads the free field model data
%loaddata      % loads the free field experimental data
%
h=2.5e-3;      % the sensor spacing
C=343;         % speed of sound in air
speed = C;
hold off;      % turns off what may have been on
dist=[];       % initialises the separation distance matrix
%
%USER DEFINED VARIABLES
%__________________________________________________________
```
shift=20 % rescale y axis to overlay data for comparison
obsv=21; % The observer location
dist=4 % sensors / observer separation
f=360 % the frequency for analysis

% ------------------------------------------
% w=2*pi*f; % rotational frequency
k=w/c; % the wave number
%
% DEFINE THE SENSOR GEOMETRY
% ------------------------------------------
pe = [obsv-dist,obsv]; % A microphone
p2e = [obsv-1-dist,obsv+1-dist,obsv]; % 2 microphones
Ede = [obsv-2-dist,obsv-dist,obsv]; % First order (2 mic) Energy density (interpolation)
vpe = [(obsv-2-dist:2:obsv-dist),obsv]; % First order virtual microphone (2 mics)
vp3e = [(obsv-2-dist:1:obsv-dist),obsv]; % Second order virtual microphone (3 mics)
vEd3e = [(obsv-2-dist:1:(obsv-dist),obsv]; % Second order virtual energy density (3 mics)

%------------------------------------------
% ONE CONTROL SOURCE
%------------------------------------------
%
% Plot the control profiles
%
% sensortype = 'p'; % a microphone
run algorithm1 % grabs the control algorithm
plot([0:1:20]*25,20*log10(abs(P(:,f)))+shift,'c-') % plots the primary field
hold on
plot([0:1:20]*25,CY+shift,'k-') % plots the controlled field for the chosen error
axis ([0,500,-40,40]) % sets the graph axes
%
% repeats the above with a different error sensor
sensortype = 'vp';
run algorithm1
plot([0:1:20]*25,CY+shift,'b-')
%
% repeats the above with a different error sensor
sensortype = 'vp3';
run algorithm1
plot([0:1:20]*25,CY+shift,'r-')
%
% Plot a circle at the observer location
%
sensortype = 'p';
run algorithm1
plot(AX,AY+shift,'ko')
B.1. An example of a plot routine

```matlab
% sensortype = 'vp';
run algorithm1
plot(AX,AY+shift,'bo')
%
% sensortype = 'vp3';
run algorithm1
plot(AX,AY+shift,'ro')
%
% Plot a square at the observer location
% sensortype = 'p';
run algorithm1
plot(VX,VY+shift,'ks')
%
% Graph axis titles
ylabel ('Acoustic pressure amplitude(dB)')
xlabel ('Sensor location (mm)')
%
% Plot a vertical line at the observer location
% vert=(min(axis):100:max(axis));
horiz=VX*ones(length(vert));
plot (horiz,vert,'k-')
%
% Change the thickness of some lines and markers
% ch=get(gca,'children'); % Get the values for the children of the current axis
set(ch(end),'linewidth',3); % change the first line width
set(ch(end-1),'linewidth',2);
set(ch(end-2),'linewidth',1);
set(ch(end-3),'linewidth',2);
set(ch(end-4),'linewidth',2);
set(ch(end-5),'linewidth',2);
set(ch(end-6),'linewidth',2);
set(ch(end-7),'linewidth',2);
set(ch(end-8),'linewidth',2);
set(ch(end-9),'linewidth',2);
```

Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
% Set up the legend
%------------------------------------------------
legend (
    ['Primary, f = ', int2str(f), ' Hz'],...
    '1 real mic.',..., 
    '1st order virtual mic.',..., 
    '2nd order virtual mic.',..., 
    3); %Places the legend
%
% Change some font sizes
%------------------------------------------------
set(gca,'fontsize',14);  % The graph axes font size
labelx = get(gca,'xlabel');
set(labelx,'fontsize',14);  % The X label font size
labely = get(gca,'ylabel');
set(labely,'fontsize',14);  % The X label font size
leg = get(legend,'children');
set(leg(end),'fontsize',14);  % The first line of the legend
set(leg(end-2),'fontsize',14);
set(leg(end-4),'fontsize',14);
set(leg(end-6),'fontsize',14);
%
% Save the plot to a postscript file
%------------------------------------------------
eval(['print -depsc c:\aa_data\virtmic\temp\cessna-4h-1\cnt-', int2str(f), '.eps']);
pause(1) % wait a sec. for it to do it's stuff
%
hold off;  % stops the next graph overlaying this one
%
% TWO CONTROL SOURCES (SEE PREVIOUS NOTES)
%------------------------------------------------
sensortype = 'p';
run algorithm1
plot([0:1:20]*25,20*log10(abs(P(:,f)))+shift,'c-')
hold on
plot([0:1:20]*25,CY+shift,'k-')
axis ([0,500,-40,40])
%
sensortype = 'Ed';
run algorithm2
plot([0:1:20]*25,CY+shift,'b-')
%
sensortype = 'vEd3';
run algorithm2
plot([0:1:20]*25,CY+shift,'r-')
%
% sensortype = 'p';
run algorithm1
plot(AX,AY+shift,'ko')
%
sensortype = 'Ed';
run algorithm2
plot(AX,AY+shift,'bo')
%
sensortype = 'vEd3';
run algorithm2
plot(AX,AY+shift,'ro')
%
sensortype = 'p';
run algorithm1
plot(VX,VY+shift,'ks')
%
sensortype = 'Ed';
run algorithm2
plot(VX,VY+shift,'bs')
%
sensortype = 'vEd3';
run algorithm2
plot(VX,VY+shift,'rs')
%
ylabel ('Acoustic pressure amplitude(dB)')
xlabel ('Sensor location (mm)')
vert=(min(axis):100:max(axis));
horiz=VX*ones(length(vert));
plot (horiz,vert,'k-')
ch=get(gca,'children');
set(ch(end),'linewidth',3);
set(ch(end-1),'linewidth',2);
set(ch(end-2),'linewidth',1);
set(ch(end-3),'linewidth',2);
set(ch(end-4),'linewidth',2);
set(ch(end-5),'linewidth',2);
set(ch(end-6),'linewidth',2);
set(ch(end-7),'linewidth',2);
set(ch(end-8),'linewidth',2);
set(ch(end-9),'linewidth',2);
%
legend (['Primary, f =',int2str(f), ' Hz'],... '1 real mic',... 'Real & 1st order virtual ED',... '2nd order virtual ED',...
3);  
%  
set(gca,'fontsize',14);  
xlabelx = get(gca,'xlabel');  
set(xlabelx,'fontsize',14);  
%  
labely = get(gca,'ylabel');  
set(labely,'fontsize',14);  
leg = get(legend,'children');  
set(leg(end),'fontsize',14);  
set(legend(end-2),'fontsize',14);  
set(legend(end-4),'fontsize',14);  
set(legend(end-6),'fontsize',14);  
%  
eval(['print -depsc c:\aa_data\virtmic\temp\cessna-4h-2cnt-'.int2str(f).eps']);

B.2 The optimisation routine for a single control source

The following routine provides the optimal solution for the error sensors that only require a single control source, or in other words a pressure squared cost function at a single location.

% A QUADRATIC OPTIMISATION ROUTINE - 1 SOURCE by CDK AND BSC  
%————————————————————————————————————  
% note - some variables are defined in the parent programme  
%———————————————————————————  
% This is the quadratic form of the cost function  
% J0 = pHp  
% = (pp+pc)H(pp+pc)  
%———————————————————  
%  
% Initialise the matrices  
Qopt_xfer = [];
J0_xfer = [];
c_xfer = [];
J0Ep_xfer = [];
cEp_xfer = [];
% DEFINE COST FUNCTIONS AND ASSOCIATE MATRICES  
%———————————————————  
%  
% Pressure at a point
B.2. The optimisation routine for a single control source

\[
\begin{align*}
\text{cp} &= P(p(1), f)^*P(p(1), f); \\
\text{bp} &= S(p(1), f)^*P(p(1), f); \\
\text{Ap} &= S(p(1), f)^*S(p(1), f); \\
\end{align*}
\]

% 2 element Virtual mic
% \[
\begin{align*}
x &= (vpe(3)-vpe(2)); \\
X &= ((1+x/2)*P(vpe(2), f)-x/2*P(vpe(1), f)); \\
Y &= ((1+x/2)*S(vpe(2), f)-x/2*S(vpe(1), f)); \\
cvp &= X^*X; \\
bvp &= Y^*X; \\
\text{Avp} &= Y^*Y; \\
\end{align*}
\]

% 3 element Virtual mic (arbitrary spacing = x)
% \[
\begin{align*}
x &= (vp3e(4)-vp3e(3)); \\
X &= ((x+2)*(x+1)/2*P(vp3e(3), f)-x*(x+2)*P(vp3e(2), f)+x*(x+1)/2*P(vp3e(1), f)); \\
Y &= ((x+2)*(x+1)/2*S(vp3e(3), f)-x*(x+2)*S(vp3e(2), f)+x*(x+1)/2*S(vp3e(1), f)); \\
cvp3 &= X^*X; \\
bvp3 &= Y^*X; \\
\text{Avp3} &= Y^*Y; \\
\end{align*}
\]

% Introduce a magnitude error to emulate that of a real controller
% \[
\begin{align*}
magerr &= 1/100; \\
\% \text{ A 1 percent error in magnitude} \\
\% \text{ Solve for the optimum source strengths} \\
\% \\
\text{Qopt} &= -inv(A)*b*(1+magerr); \\
J0 &= c + b^*Qopt; \\
\% \text{ J0Ep = Qopt^*AEp^*Qopt + Qopt^*bEp + bEp^*Qopt + cEp;} \\
\% \text{ Save the results to a set of matrices} \\
\% \\
\text{CX} &= []; \text{CY} &= []; \text{AX} &= []; \text{AY} &= []; \text{VX} &= []; \text{VY} &= []; \\
\% \text{ Initialise the results matrices} \\
\text{CX} &= \text{[CX, [0:1:20]’*25];} \\
\text{CY} &= \text{[CY, 20*log10(abs(P(:, f)+Qopt*S(:, f)))];} \\
\text{AX} &= \text{[AX, (errpos(1:length(errpos)-1))’*25];} \\
\text{AY} &= \text{[AY, 20*log10(abs(P(errpos(1:length(errpos)-1)], f)+Qopt*S(errpos(1:length(errpos)-1)], f))];} \\
\text{VX} &= \text{[VX, (errpos(length(errpos))-1)’*25];} \\
\text{VY} &= \text{[VY, 20*log10(abs(P(errpos(length(errpos)], f)+Qopt*S(errpos(length(errpos)), f))]);}
\end{align*}
\] Active control of sound in a light aircraft cabin with virtual error sensors. Colin D. Kestell.
The next routine provides the optimal solution for the error sensors that operate with two control sources, or in other words the independent control of pressure at two locations and hence energy density.

% A QUADRATIC OPTIMISATION ROUTINE - 2 SOURCES by CDK AND BSC
%
% note - some variables are defined in the parent programme
%
% This is the quadratic form of the cost function
% J0 = PHp
% = (pp+pc)H(pp+pc)
%
% Initialise the matrices
%
Qopt_xfer = [];
J0_xfer = [];
c_xfer = [];
J0Ep_xfer = [];
cEp_xfer = [];
CX=[]; CY=[]; AX=[]; AY=[]; VX=[]; VY=[];

% Pressure at a point
%
cp = P(pe(1:1),f)'*P(pe(1:1),f);
bp = S(pe(1),f)'*P(pe(1),f);
Ap = S(pe(1),f)'*S(pe(1),f);
%
% Pressure at 2 points
%
cp2 = P(p2e(1:2),f)'*P(p2e(1:2),f);
bp2 = [S1(p2e(1:2),f),S2(p2e(1:2),f)]'*/(2*h*w/speed);
Ap2 = [S1(p2e(1:2),f),S2(p2e(1:2),f)]'/([S1(p2e(1:2),f),S2(p2e(1:2),f)]);%

% 2 mic Energy density sensor
%
X1 = (0.5*P(Ede(1),f)+0.5*P(Ede(2),f))/(2*h*w/speed);
X2 = (P(Ede(2),f)-P(Ede(1),f))/(2*h*w/speed);
Y1 = ([0.5*S1(Ede(1),f)+0.5*S1(Ede(2),f)),(0.5*S2(Ede(1),f)+0.5*S2(Ede(2),f))];
Y2 = [(S1(Ede(2),f)-S1(Ede(1),f)),(S2(Ede(2),f)-S2(Ede(1),f))]*/(2*h*w/speed);
B.3. The optimisation routine for two control sources

\[
c_{\text{Ed}} = X_1^*X_1 + X_2^*X_2;
\]
\[
b_{\text{Ed}} = Y_1^*X_1 + Y_2^*X_2;
\]
\[
a_{\text{Ed}} = Y_1^*Y_1 + Y_2^*Y_2;
\]

% Two mic virtual energy density sensor (spacing = 2h)

X1 = \((2^*P(v_{\text{Ede}(2)},f)-1^*P(v_{\text{Ede}(1)},f))\);
X2 = \((P(v_{\text{Ede}(2)},f)-P(v_{\text{Ede}(1)},f))/(2^*h^*w/speed)\);
Y1 = \([(2^*S1(v_{\text{Ede}(2)},f)-1^*S1(v_{\text{Ede}(1)},f)),(2^*S2(v_{\text{Ede}(2)},f)-1^*S2(v_{\text{Ede}(1)},f))]\);
Y2 = \([(S1(v_{\text{Ede}(2)},f)-S1(v_{\text{Ede}(1)},f)),(S2(v_{\text{Ede}(2)},f)-S2(v_{\text{Ede}(1)},f))]/(2^*h^*w/speed)\);

\[
c_{\text{vEd}} = X_1^*X_1 + X_2^*X_2;
\]
\[
b_{\text{vEd}} = Y_1^*X_1 + Y_2^*X_2;
\]
\[
a_{\text{vEd}} = Y_1^*Y_1 + Y_2^*Y_2;
\]

% Three mic virtual energy density sensor (arbitrary spacing = x)

x = \((v_{\text{Ed3e(4)},v_{\text{Ed3e(3)}}})\); % Spacing between the virtual element and actual element
X1 = \[((x+2)^*(x+1)/2^*P(v_{\text{Ed3e(3)},f})-x^*(x+1)/2^*P(v_{\text{Ed3e(1)},f}))\];
X2 = \((2^*x^3)/2^*P(v_{\text{Ed3e(3)},f})-(x+1)^*(x+1)/2^*P(v_{\text{Ed3e(1)},f}))/(2^*h^*w/speed)\);
Y1 = \([(x+2)^*(x+1)/2^*S1(v_{\text{Ed3e(3)},f})-x^*(x+1)/2^*S1(v_{\text{Ed3e(1)},f})+x^*(x+1)/2^*S1(v_{\text{Ed3e(1)},f})]\);
Y2 = \([(2^*x^3)/2^*S1(v_{\text{Ed3e(3)},f})-(x+1)^*(x+1)/2^*S1(v_{\text{Ed3e(1)},f})+(2^*x^1)/2^*S1(v_{\text{Ed3e(1)},f})]\);

\[
c_{\text{vEd3}} = X_1^*X_1 + X_2^*X_2;
\]
\[
b_{\text{vEd3}} = Y_1^*X_1 + Y_2^*X_2;
\]
\[
a_{\text{vEd3}} = Y_1^*Y_1 + Y_2^*Y_2;
\]

% load the appropriate variables associated with their respective cost function

\[
\text{eval(['}A=A','sensortype','']);
\]
\[
\text{eval(['}c=c','sensortype','']);
\]
\[
\text{eval(['}b=b','sensortype','']);
\]
\[
\text{eval(['}\text{errpos} = ',sensortype,'e']);
\]
% Introduce a magnitude phase error to emulate that of a real controller

magerr = 1/100;
% Solve for the optimum source strengths

Qopt = -inv(A)*b*(1+magerr);
J0 = c + b'*Qopt;
J0Ep = Qopt'*AEp*Qopt + Qopt'*bEp + bEp'*Qopt + cEp;
%
% Save the results to a set of matrices

CX = [CX,[0:1:20]."*25];
CY = [CY,20*log10(abs(P(:,f)+[S1(:,f),S2(:,f)]*Qopt))];
AX = [AX, (errpos([1:length(errpos)-1])-1)"*25];
AY = [AY, 20*log10(abs(P(errpos([1:length(errpos)-1]),f)+ ...
    [S1(errpos([1:length(errpos)-1]),f),S2(errpos([1:length(errpos)-1]),f)]*Qopt ))];
VX = [VX, (errpos(length(errpos))-1)"*25];
VY = [VY, 20*log10(abs(P(errpos(length(errpos)),f)+ ...
    [S1(errpos(length(errpos)),f), S2(errpos(length(errpos)),f)]*Qopt ))];
Appendix C

Presentations and publications originating from this thesis

This sections lists the publications and international presentations that have directly resulted from the work undertaken to complete this thesis.

C.1 International journals


C.2 Conference papers


# Glossary

<table>
<thead>
<tr>
<th>Character</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Maximum amplitude</td>
</tr>
<tr>
<td>$c, c_o$</td>
<td>Sonic speed</td>
</tr>
<tr>
<td>$E_D$</td>
<td>Energy density</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$h$</td>
<td>The spacing between the sensors</td>
</tr>
<tr>
<td>$j$</td>
<td>$\sqrt{-1}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Wavenumber (or a constant of integration in chapter 5)</td>
</tr>
<tr>
<td>$l$</td>
<td>Acoustic characteristic dimension</td>
</tr>
<tr>
<td>$L_{eq}$</td>
<td>Equivalent noise level</td>
</tr>
<tr>
<td>$p_x$</td>
<td>Acoustic pressure (at location $x$)</td>
</tr>
<tr>
<td>$q$</td>
<td>Control source strength</td>
</tr>
<tr>
<td>$r$</td>
<td>Distance from the acoustic source</td>
</tr>
<tr>
<td>$v_x$</td>
<td>Acoustic particle velocity (at location $x$)</td>
</tr>
<tr>
<td>$x$</td>
<td>The observer / sensor separation distance</td>
</tr>
<tr>
<td>$Z$</td>
<td>Acoustic impedance transfer function</td>
</tr>
<tr>
<td>$\gamma^2$</td>
<td>Coherence</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Modal loss factor</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Modal volume</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Mode shape</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency</td>
</tr>
</tbody>
</table>