



# Active Vibration Control of a piezoelectric laminated plate using Spatial Control Approach

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# Abstract

This thesis represents the work that has been done by the author during his Master of Engineering Science candidature in the area of vibration control of flexible structures at the School of Mechanical Engineering, The University of Adelaide, between March 2003 and June 2004. The aim of this research is to further extend the application of the Spatial Control Approach for two-dimensional flexible structures for attenuating global structural vibration with the possible implication of reduction in noise radiation. The research was concentrated on a simply supported thin flexible plate, using piezoelectric ceramic materials as actuators and sensors. In this work, active controllers were designed for the purpose of controlling only the first five vibration modes (0-500Hz) of the plate.

A spatial controller was designed to minimize the total energy of the spatially distributed signal, which is reflected by the spatial  $H_2$  norm of the transfer function from the disturbance signal to the vibration output at every point over the plate. This approach ensures the vibration contributed by all the in bandwidth (0-500 Hz) vibration modes is minimized, and hence is capable of minimizing vibration throughout the entire plate.

Within the control framework, two cases were considered here; the case when the prior knowledge of the incoming disturbance in terms of reference signal is available and the case when it is not available. For the case when the reference signal is available, spatial feedforward controller was designed; whereas for the case when the reference signal is not available, spatial feedback controller was designed to attenuate the global disturbance.

The effectiveness of spatial controllers was then compared with that of the standard point-wise controllers numerically and experimentally. The experimental results were

found to reflect the numerical results, and the results demonstrated that spatial controllers are able to reduce the energy transfer from the disturbance to the structural output across the plate in a more uniform way than the point-wise controllers. The research work has demonstrated that spatial controller managed to minimize the global plate vibrations and noise radiation that were due to the first five modes.

# Statement of Originality

To the best of my knowledge and belief, all the material presented in this thesis, except where otherwise referenced, is my own original work, and has not been published previously for the award of any other degree or diploma in any university. If accepted for the award of the degree of Master of Engineering Science, I consent that this thesis be made available for loan and photocopying.

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Yong Keat LEE

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*Dedicated*  
*to*  
*my father, my mother*  
*&*  
*my brother.*

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