5.1 SUMMARY OF NUMERICAL ANALYSIS

The application of piezoceramic stack actuators as actuators for active vibration control has been examined for three types of physical structure; beams, plates and cylinders. For each type of structure, a theoretical model has been developed to describe the vibration response of the structure to excitation by point forces and to active vibration control using piezoceramic stack control actuators and vibration sensors. The effect of stiffeners on the vibration response of the structures has been included where appropriate.

The analysis of vibration in a beam was a one dimensional problem. An analytical model was developed from the one dimensional equation of motion for beams to describe the vibration response of an arbitrarily terminated beam to a range of excitation types. The numerical results indicated that flexural vibrations in beams can be effectively controlled using a single piezoceramic stack actuator and angle stiffener control source and a single vibration error sensor.

The dependence of the control force amplitude and phase and the attenuation of vibration level achieved on a number of parameters was investigated. It was found that the magnitude of the control force required for optimal control generally decreased with increasing stiffener flange length and increasing frequency. The control force amplitude required for optimal control was less when the beam was excited at a resonance frequency.
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When there was reflection from the beam terminations, the optimum control force was either in phase or $180^\circ$ out of phase with the primary source, but when there was no reflection from beam terminations (the infinite beam case), the control force phase cycled through $180^\circ$ as the excitation frequency was increased.

Comparison between the results obtained for infinite beam and finite beam cases indicated that the standing wave generated when there were reflections from beam terminations had a significant effect on the effectiveness of active vibration control. Maxima occurred in the control force amplitude required for optimal control when the separation between control and primary sources was given by $x = (c + n\lambda_\lambda/2)$ where $n$ is an integer and $c$ is a constant dependent on frequency and the type of boundary condition. These maxima occurred when the control source was located at a node in the standing wave generated by reflection from the beam terminations. Minima in the mean attenuation of acceleration level downstream of the error sensor also occurred when the control source was located at a node in the standing wave.

When the error sensor was located at a node in the standing wave in a finite beam, the attenuation achieved was less than that achieved with the error sensor located away from a node. However, locating the error sensor at a node did not affect the control force amplitude required for optimal control.

Increasing the separation between the primary and control source did not improve the
attenuation. The amount of attenuation achieved downstream of the error sensor increased with increasing separation between the error sensor and the control source.

Numerical results also indicated that it is possible to achieve reductions in acceleration level upstream of the primary source as well as the desired reduction downstream of the error sensor. The maximum mean attenuation in acceleration level upstream of the primary source was theoretically achieved with the separation between primary and effective control source locations given by $x = (c + n\lambda_p/2)$ for $n = 1, 2, 3...$ For error sensor locations outside the control source near field, the mean attenuation of acceleration level upstream of the primary source did not depend on error sensor location.

An analytical model was developed from the two dimensional equation of motion for plates to describe the vibration response of a plate to a range of excitation types. Two sets of boundary conditions were considered, both with simply supported sides. One plate was modelled as free at both ends, and the second was modelled as semi-infinite in length. The effects of the mass and stiffness of an across-plate stiffener were included in the analysis. The numerical results indicated that flexural vibrations in stiffened plates can be effectively controlled using the piezoceramic stack actuators and a line of vibration error sensors.

The numerical model showed that control of plate vibration is a similar problem to control of vibration in beams, but the added width dimension necessitated more than one control source and error sensor across the plate width to achieve optimum control.
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Like the corresponding beam case, the mean amplitude of the control forces required for optimal control of plate vibration generally decreased with increasing frequency. The optimum control forces were either in phase or 180° out of phase with the primary sources. This was true for the semi-infinite plate as well as the finite plate, because a standing wave was generated by the vibration reflections from the finite end and the angle stiffener. This was not so for the infinite beam case, as the beam was modelled as infinite in both directions and the mass loading of the angle stiffener was neglected.

Maxima occurred in the mean control force amplitude required for optimal control when the separation between control and primary forces was given by $x = (c + nx_s)$ where $n$ is an integer, $c$ is a constant dependent on frequency and the type of boundary condition, and $x_s$ is the separation between axial nodes in a standing wave. These maxima occurred when the control sources were located at a nodal line in the standing wave generated by reflection from the plate termination and the angle stiffener. Minima in the mean attenuation of acceleration level downstream of the error sensor occur when the control sources were located at a nodal line in the standing wave. Increasing the separation between the primary and control sources did not improve attenuation provided the control sources were located outside of the primary source near field.

The amount of attenuation achieved downstream of the error sensors increased with increasing separation between the error sensors and the control sources. When the line of error sensors was located at a nodal line in the standing wave that exists in finite plates, the
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attenuation achieved was less than that achieved with the error sensors located away from a node. Locating the error sensors at a node did not affect the amplitude of the control forces required for optimal control.

As more than one control source was used in the plate model, consideration was given to the effectiveness of active vibration control with the control sources driven by a common signal and with the control sources driven independently. At low frequencies, there was very little difference in the mean control effort required for optimal control and the mean attenuation downstream of the line of error sensors achieved, between control using independent control sources and control sources driven by a common signal. At higher frequencies when higher order across-plate modes became significant, very little attenuation was achieved with control sources driven by a common signal. Good reduction in acceleration level was achieved with independently driven control sources right across the frequency range considered. It was determined that use of three independently driven control sources and three error sensors was sufficient to give optimal attenuation.

Both the beam model and the plate model were extended to include a second angle stiffener and control source(s) downstream from the first, with the aim of controlling the vibration better when the first control source was located at a node in a standing wave. The magnitude of the first control force(s) was limited to a maximum value, and the second control source(s) used when the limit was reached. The maxima in control force amplitude and the minima in attenuation that occurred when the first stiffener and control source(s) were located at a
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standing wave node were eliminated in this way.

It was shown, for the beam case, that there was no simple practical method of using a second error sensor to eliminate the minima in attenuation that occur when the first error sensor is located at a standing wave node, because of the difficulty in determining when the first error sensor was located at a node without the introduction of several more error sensors.

The analysis of vibration in a cylinder was a significantly more complicated problem than the beam and plate cases. The Flügge equations for the vibration of a cylinder in the radial, axial and tangential directions were used to develop a model to describe the vibration response of a ring-stiffened cylinder to a range of excitation types, and in particular to point force primary excitation sources and angle stiffener and piezoceramic stack control sources. The numerical results indicated that flexural vibrations in cylinders can be actively controlled using piezoceramic stack actuators placed between the flange of an angle stiffener and the cylinder surface.

The cylinder vibration response was different to the simpler cases of the plate and beam in two main ways. In the cylinder, vibration in the axial, tangential and radial directions was significant, and the separation $x_3$ between axial nodes in standing waves was much longer than that in the plate and beam cases considered.

Vibration amplitudes in the radial, axial and tangential directions were of similar order.
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Modes of axial and radial vibration occurred at the same tangential locations while modes of tangential vibration were located out-of-phase relative to the axial and radial modes. Optimally controlling radial vibration also significantly reduced axial and tangential vibration levels.

Because the separation $x_s$ between axial standing wave nodes in the cylinder was so large, the mean amplitude of the control forces required for optimal control did not greatly depend on frequency, axial control source location or axial error sensor location. There were minor fluctuations, particularly for the finite cylinder, but no pattern or increasing or decreasing trends. There were no axial locations of control sources and error sensors that gave maxima in control force amplitude or minima in attenuation.

Like both the plate cases and all but the infinite beam cases, the optimum control forces were either in phase or $180^\circ$ out of phase with the primary sources. This was true for the semi-infinite cylinder as well as the finite cylinder, because a standing wave was generated by the vibration reflections from the finite end and the angle stiffener.

Increasing the separation between the primary and control sources did not greatly affect the attenuation achieved downstream of the ring of error sensors. Increasing the separation between the error sensors and the control sources improved the mean attenuation of acceleration level downstream of the ring of error sensors.
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Little or no reduction was achieved with control sources driven by a common control signal, because higher order circumferential modes of vibration contribute significantly to the vibration response of the cylinder even at low frequencies.

The circumferential location of the control sources was significant. Generally, for every control source, there were two locations at which placement of an additional control source required excessive control force amplitudes for optimal control. Four control sources and four error sensors were sufficient to achieve optimal attenuation of cylinder vibration.
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5.2 SUMMARY OF EXPERIMENTAL RESULTS

For at least one example of each of the three structure types considered, the theoretical model outlined was tested experimentally. In each experiment, the acceleration distribution resulting from measured force inputs was gauged and compared with the theoretical predictions, both with and without active vibration control.

Experiments were performed on a beam with four different sets of end conditions. The impedance corresponding to each termination was first calculated from experimental data. Comparison between experimental results and theoretical predictions showed that the accuracy of the theoretical model when compared to the experimental results was very high, both in predicting the control force amplitude and phase required relative to the primary force, and in determining the acceleration distribution occurring along the beam. The impedances calculated from experimental measurements gave more accurate results than the "classical" impedances corresponding to each termination. The theoretical model accurately predicted the amount of attenuation that could be achieved experimentally.

The theoretical model outlined for the plate was verified experimentally for a plate with simply supported sides, one end free and the other end anechoically terminated. A modal analysis of the plate indicated that the anechoic termination allowed some reflection and so did not exactly model the ideal infinite end, and that the angle stiffener made a significant difference to the vibration response of the plate. Comparison between experimental results and theoretical predictions for the vibration of the plate with and without active vibration
control showed that the theoretical model accurately predicted the vibration response of the plate for the uncontrolled case and the case with control sources driven by the same signal. The angle stiffener reflected more of the vibration and transmitted less than the theoretical model predicted. The theoretical model predicted more attenuation than could be achieved experimentally for the case with independently driven control sources. An error analysis indicated that an error in the control source signal of 0.1% would have produced a decrease in attenuation corresponding to the difference between the theoretical prediction and the experimental result. Nevertheless, around 25 dB attenuation was achieved experimentally for the case with independently driven control sources.

Experiments were performed on a cylinder with simply supported ends. A modal analysis of the cylinder indicated that the angle stiffener made a significant difference to the vibration response of the cylinder and that higher order circumferential modes contributed significantly to the overall cylinder vibration response. Comparison between experimental results and theoretical predictions for the vibration of the cylinder with and without active vibration control showed that the theoretical model accurately predicted the vibration response of the cylinder for the uncontrolled case and the case with control sources driven by the same signal. The theoretical model predicted more attenuation than could be achieved experimentally for the case with independently driven control sources. Again, an error analysis indicated that an error in the control source signal of 0.1% would have produced a decrease in attenuation corresponding to the difference between the theoretical prediction and the experimental result. Around 18 dB attenuation was achieved experimentally for the case with
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independently driven control sources. Finally, experimental measurements indicated that the amplitudes of axial and tangential vibration were of similar order to the radial vibration amplitude, as predicted by the theoretical model, for both uncontrolled and controlled cases.
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5.3 Conclusions

It is possible to actively control vibration in stiffened structures using actuators placed between a stiffener flange and the structure surface. Actuators placed across the width of a plate-like structure or around the circumference of a cylindrical structure at a single axial location can be used in conjunction with a line or ring of error sensors to significantly reduce the vibration transmission along the structure.

The theoretical models outlined in this thesis can be used to determine the maximum amount of vibration reduction that can be achieved under ideal conditions. It may not be possible to achieve the predicted level of reduction in practice, but high levels of attenuation are achievable.

When the separation between nodes in the axial standing wave in the structure is less than the length of the structure, for any given frequency of excitation, there will be discrete axial locations of control sources that will not yield high levels of attenuation, or, for given locations of control sources, there will be certain frequencies at which vibration cannot be controlled effectively. This occurs when the control source location or error sensor location corresponds to a node in a standing wave generated by reflections from the structure terminations and stiffeners. It is possible to overcome this difficulty simply with a second set of control sources at another axial location. The level of attenuation is also reduced, but not as much, when the error sensor location corresponds to a standing wave node. Introduction of a second set of error sensors does not yield a simple solution to this problem.
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The separation between nodes in the axial standing wave in typical cylindrical structures is very long, so axial location of control sources and error sensors is not critical.

Increasing the separation between the error sensors and the control sources increases the maximum reduction in acceleration level that can be achieved in each type of structure considered. Increasing the separation between the control sources and the primary sources does not greatly affect the amount of reduction in acceleration level that can be achieved.

The effect of the circumferential location of control sources in cylindrical structures has been briefly investigated in this thesis, and is significant. Control sources are most effectively used when located at different circumferential spacings where each control source affects different circumferential modes of vibration. The circumferential location of the error sensors has not been investigated here, but it is suggested that the error sensors should be similarly unevenly spaced circumferentially.
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