ACTIVE CONTROL OF VIBRATION
IN STIFFENED STRUCTURES

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ABSTRACT

Active control of vibration in structures has been investigated by an increasing number of researchers in recent years. There has been a great deal of theoretical work and some experiment examining the use of point forces for vibration control, and more recently, the use of thin piezoelectric crystals laminated to the surfaces of structures. However, control by point forces is impractical, requiring large reaction masses, and the forces generated by laminated piezoelectric crystals are not sufficient to control vibration in large and heavy structures.

The control of flexural vibrations in stiffened structures using piezoceramic stack actuators placed between stiffener flanges and the structure is examined theoretically and experimentally in this thesis. Used in this way, piezoceramic actuators are capable of developing much higher forces than laminated piezoelectric crystals, and no reaction mass is required. This thesis aims to show the feasibility of active vibration control using piezoceramic actuators and angle stiffeners in a variety of fundamental structures.

The work is divided into three parts. In the first, the simple case of a single actuator used to control vibration in a beam is examined. In the second, vibration in stiffened plates is
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controlled using multiple actuators, and in the third, the control of vibration in a ring-stiffened cylinder is investigated.

In each section, the classical equations of motion are used to develop theoretical models describing the vibration of the structures with and without active vibration control. The effects of the angle stiffener(s) are included in the analysis. The models are used to establish the quantitative effects of variation in frequency, the location of control source(s) and the location of the error sensor(s) on the achievable attenuation and the control forces required for optimal control. Comparison is also made between the results for the cases with multiple control sources driven by the same signal and with multiple independently driven control sources. Both finite and semi-finite structures are examined to enable comparison between the results for travelling waves and standing waves in each of the three structure types.

This thesis attempts to provide physical explanations for all the observed variations in achievable attenuation and control force(s) with varied frequency, control source location and error sensor location. The analysis of the simpler cases aids in interpreting the results for the more complicated cases.

Experimental results are given to demonstrate the accuracy of the theoretical models in each section. Trials are performed on a stiffened beam with a single control source and a single error sensor, a stiffened plate with three control sources and a line of error sensors and a ring-stiffened cylinder with six control sources and a ring of error sensors. The experimental
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results are compared with theory for each structure for the two cases with and without active vibration control.
STATEMENT OF ORIGINALITY

To the best of my knowledge and belief all of the material presented in this thesis, except where otherwise referenced, is my own original work, and has not been presented previously for the award of any other degree or diploma in any University. If accepted for the award of the degree of Doctor of Philosophy, I consent that this thesis be made available for loan and photocopying.

Andrew J. Young
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