

Analytic Modelling of Agent-Based Network Routing Algorithms

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Thesis submitted for the degree of

Doctor of Philosophy

in

Applied Mathematics

at

The University of Adelaide

(Faculty of Engineering, Computer and Mathematical Sciences)

School of Applied Mathematics



November 4, 2002

Contents

Signed Statement	iv
Acknowledgements	v
Dedication	vi
Abstract	vii
1 Introduction	1
1.1 Network routing	4
1.2 Background information	6
1.3 Thesis overview	7
2 Agent-Based Network Routing	10
2.1 Biological ants and collective problem-solving	10
2.2 From biological ants to artificial ants	12
2.3 Overview of agent-based routing	14
2.3.1 Some proposed algorithms	16
2.4 An agent-based routing algorithm	20
2.5 Modelling and analysis	23
3 Fixed Link Delays	25
3.1 The Agent Model	25
3.1.1 Time-homogeneous Markov chain analysis	29

3.1.2	Agent looping	31
3.1.3	Existence of a fixed point	31
3.2	Numerical investigation	32
3.2.1	The simulation model	33
3.2.2	Collection of data from the simulation model	34
3.2.3	Verification of the Agent Model	35
3.2.4	The effects of exploration on Q -values	37
3.2.5	A limiting case	40
3.2.6	Probability of agent loops	52
3.3	Asynchronism in the agent-based routing algorithm	53
3.4	Summary	59
4	Agents and Reinforcement Learning	61
4.1	Shortest path Markov decision problems	62
4.2	Reinforcement learning	65
4.2.1	An on-policy Monte Carlo learning algorithm	66
4.2.2	A deterministic shortest path MDP	69
4.2.3	A model-free Monte Carlo learning algorithm	84
4.2.4	An off-policy Monte Carlo learning algorithm	87
4.2.5	Convergence properties	89
4.2.6	Q-learning and bootstrapping	93
4.3	Monte Carlo reinforcement learning and the agent-based routing algorithm	94
4.3.1	Discussion	97
5	Analysis of Traffic Flows	98
5.1	Traffic optimization and equilibrium	99
5.2	Necessary and sufficient conditions for user equilibria	102
5.3	Derivation of Wardrop equilibrium conditions	112

6	Flow-Dependent Link Delays	117
6.1	The Flow-Dependent Agent Model	118
6.1.1	Existence of a fixed point	125
6.2	The simulation model	127
6.3	Verification of the FD Agent Model	130
6.4	Numerical investigation	137
6.4.1	The case $\beta < \infty, \sigma < \infty$	137
6.4.2	The case $\beta < \infty, \sigma \rightarrow \infty$	147
6.5	An off-policy version of the agent-based routing algorithm	149
6.6	The Off-Policy FD Agent Model	152
6.7	The Off-Policy FD Agent Model and Wardrop equilibria	160
7	Adaptive Behaviour	165
7.1	Example A	165
7.2	Example B	167
8	Conclusion	172
8.1	Thesis summary	172
8.2	Conclusions	175
8.3	Further research	177
	Bibliography	180

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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Acknowledgements

I would like to warmly thank my supervisors, Professor Peter Taylor and Dr. Nigel Bean, for their strong encouragement and high level of support throughout the entire period of my postgraduate studies.

I would like to thank Dr. Peter Gill, who as head of the Department of Applied Mathematics, invited me to do honours, and has encouraged and facilitated all of my postgraduate studies.

I greatly appreciate the assistance generously offered by Donald Oats, who taught me about object-oriented programming. I am indebted to Dr. Nigel Bean, who worked well beyond the call of duty to install software that I required for my research. I would also like to thank Greg Lewis, Paul McCann, and Justin Viiret for their computing help.

I acknowledge the financial support provided by the receipt of an Australian Postgraduate Award, as well as a Premier's Scholarship for Postgraduate Research in Information Technology.

Dedication

This thesis is dedicated to my gorgeous wife Kat, and our beautiful son Harry. Thankyou for your immense love and support. To my wonderful parents, thankyou for all that you have done for us.

Abstract

Interest in adaptive and distributed systems for routing control in networks has led to the development of a new class of algorithms, which is inspired by the shortest path finding behaviours observed in biological ant colonies. This class utilizes ant-like agents, which autonomously traverse the network and collectively construct a distributed routing policy. Agent-based routing algorithms belonging to this class do not require a complete model of the network, and are able to adapt autonomously to network changes in dynamic and unpredictable environments. Previous studies of these algorithms have been carried out exclusively via the use of simulation-based models. In this thesis, we apply analytic modelling techniques to the study of agent-based routing algorithms. Our aim is to broaden the research in this field, as well as to gain a greater theoretical understanding of some fundamental properties of this class of algorithms.