Mathematical and Computational Techniques for Predicting the Squat of Ships

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Contents

Abstract

This thesis deals with the squat of a moving ship; that is, the downward displacement and angle of trim caused by its forward motion. The thesis is divided into two parts, in which the ship is considered to be moving in water of constant depth and non-constant depth respectively. In both parts, results are given for ships in channels and in open water.

Since squat is essentially a Bernoulli effect, viscosity is neglected throughout most of the work, which results in a boundary value problem involving Laplace's equation. Only qualitative statements about the effect of viscosity are made.

For a ship moving in water of constant depth, we first consider a one-dimensional theory for narrow channels. This is described for both linearized flow, where the disturbance due to the ship is small, and nonlinear flow, where the disturbance due to the ship is large. For nonlinear flow we develop an iterative method for determining the nonlinear sinkage and trim. Conditions for the existence of steady flow are determined, which take into account the squat of the ship.

We then turn to the problem of ships moving in open water, where one-dimensional theory is no longer applicable. A well-known slender-body shallow-water theory is modified to remove the singularity which occurs when the ship's speed is equal to the shallow-water wave speed. This is done by including the effect of dispersion, in a manner similar to the derivation of the Korteweg-deVries equation. A finite-depth theory is also used to model the flow near the critical speed.

For a ship moving in water of non-uniform depth, a linearized one-dimensional theory is derived which is applicable to unsteady flow. This is applied to simple bottom topographies, using analytic as well as numerical methods. A corresponding slender-body shallow-water theory for variable depth is also developed, which is valid for ships in channels or open water. Numerical results are given for a step depth change, and an analytic solution to the problem is discussed.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I consent to this copy of my thesis being available in the University Library for loan and photocopying.

SIGNED: Tim Gourlay DATE: 23/2/2000

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General Introduction

When a ship is travelling forward in calm water, it sits in a different vertical position to when it is at rest. This phenomenon is known as “squat”. It is characterized by an overall vertical displacement of the ship, called the sinkage, as well as a certain angle of trim.

Squat can be thought of as a simple Bernoulli effect, caused by the increased flow speed beneath the ship’s hull, which decreases the fluid pressure on the hull. The vertical displacement of the ship is therefore usually downward.

This means that when a ship is travelling through shallow water, squat may in fact cause it to strike the bottom, despite the fact that it may have had sufficient clearance when at rest. In fact, sinkage usually increases with speed, so that the faster a ship is moving, the more it is at risk of grounding. The problem is exacerbated by the fact that the trim of the ship will make either the bow or the stern particularly vulnerable to grounding.

If a ship’s squat can be accurately predicted under given conditions, sufficient allowance can be made for the ship to pass through shallow water in safety. Conversely, since the cargo capacity of a ship is often limited by conservative squat predictions for its passage through shallow water, accurately predicting the squat allows the ship to carry as much cargo as is safe. At the same time, the ship’s captain can know the maximum speed at which the ship can travel for each depth of water.

Our aim is therefore to be able to predict accurately the squat of any ship at any given speed and water depth. This will be done by building on existing techniques and developing some new ones.

Historical Summary

The phenomenon of ships settling downwards in the water when they are under way has been known to mariners for centuries. What has not been well known, however, is how the magnitude of this sinkage varies with water depth and vessel speed. It is only in quite recent times that these have been included in prediction of a ship's sinkage. Indeed, in former times, a fixed allowance, say one foot, would often be allowed for this sinkage whatever the water depth or vessel speed.

Early analytic methods

Early attempts to predict ship squat mathematically were made by assuming water of infinite depth, using methods previously developed for studying ship waves. Havelock (1939) determined analytically the sinkage of an elliptical hull form in infinite depth of water. This theory was of limited usefulness, however, as the problem of most practical importance is that of a ship travelling in shallow water. This is the situation in which the ship is most likely to hit the sea floor and do serious damage, so its squat needs to be accurately predicted under these circumstances. As it was known that the presence of the sea floor has a profound effect on the hydrodynamics, subsequent squat prediction methods attempted to take the water depth into account.

A subsequent important early work on squat considered a very different physical problem: that of a ship travelling in a shallow, narrow channel. Under these assumptions, Constantine (1961) asserted that flow around the ship could be studied in much the same way as open-channel flow, using one-dimensional "hydraulic" theory. He considered the ship to be stationary and the water to be moving past it, which meant that the flow was usually steady in this frame of reference. Constantine found that three different types of flow were possible in this situation.

At low speeds the flow was predicted to accelerate past the ship, producing a pressure decrease beneath the hull and a downward sinkage of the ship. However, once the ship reached a certain speed, which is dependent on the ship and channel dimensions, no such steady flow could be found. In this "critical" speed range, Constantine predicted that a one-dimensional bore would be formed, travelling ahead of and faster than the ship. At higher speeds the ship would catch up to this bore, resulting in a new type of steady flow. This "supercritical" flow would be characterized by a decelerated flow past the ship, causing the ship to rise in the water.

In 1966, Tuck developed a slender-ship theory that was valid for shallow water. In fact, it was valid *only* for shallow water, as one of the assumptions was that the water depth must be small compared to the ship's length. This revolutionary approach yielded expressions for the ship's sinkage and trim that were roughly quadratic in the ship's speed. For practical speed ranges, the theory generally gave good results for sinkage and trim, and was used extensively over the next decades. However, a problem with the theory was its behaviour as the ship speed approached the natural speed of waves in shallow water. In this case the theory became singular and predicted infinite sinkage and trim.

Whereas the original theory was developed for water of infinite lateral extent, it was later modified (Tuck 1967) for channels of arbitrary width. Provided that the ship's beam remained small compared to the channel width, the final solution was found to approach the linearized version of Constantine's solution in the narrow channel limit. Thus this theory was uniformly valid for any width of channel, and provided a link between Constantine's one-dimensional theory and Tuck's infinite-width theory.

There still remained the problem of the singularity when the ship approached the shallow-water wave speed, for which Tuck and Taylor (1970) proposed a remedy. They found the velocity potential for a ship moving in a *finite* depth of water by rejecting the shallow-water assumption completely. However, the resulting formulas were extremely complicated and were only studied for some simple test cases.

Meanwhile, in the same article and a subsequent article by Tuck (1974), the one-dimensional theory for a ship in a narrow channel was also being extended. The limits of steady flow were more accurately defined for either block-like ships free to squat or general-shaped ships fixed vertically.

Empirical methods

The sinkage formula given by Tuck (1966) was too complicated for routine use by mariners, as it required computation of a double integral. In an attempt to find a simpler expression that would be easier to implement in practice, Tuck & Taylor (1970) studied the properties of this double integral. They found that over a wide range of hull shapes it was roughly proportional to the ship's displacement divided by the square of its length, and used this to produce a much simpler expression for ship sinkage.

Experiments performed by Hooft (1974) showed excellent agreement with the sinkage formula of Tuck & Taylor. However, a compilation of experimental results from several testing laboratories was made by Huuska (1976) which showed that the formula tended to underestimate the sinkage. This was later confirmed by Millward (1992). Since the general speed dependence was accurate, Huuska proposed that a corrective multiplicative constant be introduced into the formula.

This empirical squat prediction method started a new phase in squat research. Subsequent researchers attempted to fit a universal curve to the experimental results for different ships, which had as its parameters the ship speed, water depth and ship dimensions.

Some of these formulas, such as those of Huuska (1976) and Millward (1992) used the speed dependence calculated by Tuck (1966) but with a simplified dependence on hull shape. Others matched experimental results to some power of the ship speed: Eryuzlu & Hausser (1978), Eryuzlu et al (1994), and Barrass (1979) put forward empirical expressions for squat with very different dependences on speed and draught-to-depth ratio.

These empirical methods have been shown to perform well for ships and channels in the range that they were designed for. Despite having little or no physical basis, they give accurate squat predictions in the case of ship types, channel types and speed ranges for which experimental data are available. However, they cannot be expected to perform well for all hull forms, channel types and speed ranges.

The critical region

For ships travelling in wide channels, the experiments of Graff et al (1964) showed that sinkage and trim were not unbounded as the ship approached the shallow-water wave speed. Instead the sinkage reached a maximum before becoming quite small thereafter.

The finite-depth theory of Tuck & Taylor (1970), which was developed for open water, predicted the correct overall form of the sinkage as a function of speed. A shallow-water approach by Lea & Feldman (1972) concentrated on a ship travelling at exactly the shallow-water wave speed, and included the leading nonlinear term to solve for sinkage and trim. Mei (1976) developed a slender-ship theory for open water similar to that of Tuck (1966) but including the leading nonlinear and dispersive terms. However, his paper concentrated on wave resistance and did not calculate sinkage and trim.

Although the flow around a ship travelling in water of constant depth is steady at any speed if the ship is in *open water*, a very interesting type of flow was found to occur experimentally for a ship travelling in a *channel* at close to the shallow-water wave speed. Thews & Landweber (1935) observed in model experiments that in this case the flow was unsteady, with waves propagating ahead of the model. Huang et al (1982) found that these waves had the same form as solitary waves, or "solitons", which are single crests capable of travelling along a channel unchanged in form. However, these solitons were produced almost periodically, breaking away from the bow of the ship and travelling ahead of and faster than the model. This was a more complicated scenario than the simple bore solutions predicted by Constantine (1961) although the generating mechanism was the same. In fact, at higher speeds in the critical region, where solitary wave production was impossible, Constantine's bores were witnessed in the experiments.

The production of solitary waves by a ship in a channel provoked much interest, and is still the topic of intensive research today. Huang et al (1982) found that the first soliton produced was invariably the largest, and the first one to break at higher speeds. Ertekin et al (1985) observed that small solitary waves could be formed at ship speeds well below the previously accepted critical region. They observed experimentally that the drag on the ship fluctuated almost periodically as solitons were produced. It was suggested that the amplitude of successive waves would gradually tend toward zero if the ship speed is less than the shallow-water wave speed; however, this issue still remains unresolved.

An attempt to model ship solitons mathematically was made by Wu & Wu (1982) using Boussinesq equations generalized to allow 2-D wave propagation. Ertekin et al (1985) compared this method to that of Naghdi (1978) and used both methods to solve numerically for the free surface profile ahead of the model as a function of time. The two methods, which are the same to leading order in the soliton amplitude, both predicted solitons of gradually decreasing amplitude for ship speeds less than the wave speed.

Since the Korteweg-DeVries equation was known to be able to model solitary waves accurately, most mathematical treatments of the problem used KdV-type methods. Wu (1986) derived a KdV-type equation with a forcing term from the generalized Boussinesq equations, and used this to calculate the evolution of solitons. He also used energy and momentum conservation to relate soliton and trailing trough amplitudes, as well as the components of wave drag.

Mei & Choi (1987) used matched asymptotics to study the solitary waves produced by a ship in a channel, and explicitly calculated sinkage and trim. The general form of their results compared reasonably well with the experimental results of Graff et al (1964).

Since the mid-1980's there has been a large quantity of research published on the subject of solitary waves; this has been only a brief review, as solitary waves will not be studied in detail in this thesis.

[*subsequent note*: An examiner has pointed out that some of this nonlinear research is more relevant to the topic than the author had appreciated. In particular, a significant body of research by Xue-Nong Chen (e.g. Chen & Sharma 1995) improves upon the linear transcritical results of Chapter 2 of the present thesis.]

Squat in water of non-uniform depth

Interest in this topic came about through the grounding of several ships in shoaling water. In 1992, for example, the QE2 grounded in Vineyard Sound, Massachusetts, causing serious damage (see USCG 1993, NTSB 1993). The accident occurred in shoaling water as a result of the ship's squat. In an investigation into the incident (NTSB 1993), researchers predicted the squat of the ship using constant depth formulae, as there were no methods available for non-uniform depth. Then, as now, the best estimate of squat in non-uniform depth was obtained by using the average depth under the ship in the constant-depth formulae. However, it was not known whether this method would over- or under-predict the squat for a ship moving towards shallower water, for example.

Even in recent years, little research has been done in this area. Renilson & Hatch (1998) and Duffield (1997) have carried out experimental investigations into the transient squat of a ship travelling through water of non-uniform depth. However, few serious attempts have been made to treat the problem mathematically.

Overview

In this thesis I shall develop some new mathematical methods for predicting the squat of a ship. This is done with a view to practical evaluation of the squat of a moving ship, so that with better squat estimates, ships can pass through shallow water in safety.

Part I of the thesis deals with a ship travelling in water of constant depth. This simplifies the analysis, because the flow can usually be considered steady in this case.

Chapter 1 introduces a simple one-dimensional theory for flow past a ship in a narrow channel. We first discuss the commonly-used procedure of solving for the flow with the ship held fixed in its rest position, rather than in its squatted position. A consistent nonlinear method for computing sinkage and trim is then developed, which takes into account the flow changes that occur when a ship squats.

According to this theory, there are conditions under which no steady flow exists, namely a finite range of speeds that includes the critical speed of shallow-water waves. Explicit forms of these conditions are found for a ship of arbitrary shape that is held fixed vertically, and also for a ship that is allowed to squat.

Chapter 2 describes two improved methods for determining the squat of a ship moving in open water. These methods are valid at all ship speeds, even those close to the shallow-water wave speed. Firstly, a generalized shallow-water theory is discussed that makes allowance for the variation of wave speed with wavelength. Then, a previously-developed finite-depth theory is improved upon and studied in detail. Computational techniques are developed for evaluating the resulting formulae, and the two theories are compared to experimental results for sinkage and trim. The predicted maximum sinkage is in close agreement with experiment.

Part II of the thesis attempts to solve the more difficult unsteady flow problem of a ship moving in water of non-uniform depth. This is first done in Chapter 3 using a one-dimensional theory similar to Chapter 1, but now modified to allow for unsteadiness. It is found that the sinkage and trim can be found analytically for a ship travelling in a channel with a sudden depth change to deeper or shallower water.

In Chapter 4, a slender-ship theory is described which is able to treat a general non-uniform water depth. It is found that the problem can be solved analytically for the case of a ship approaching a sudden depth change in open water or a channel. For the case of a ship in a channel, a stable finite-difference method is found for solving the governing equation. Numerical results are given for step depth changes.

Detailed mathematical proofs are left to the Appendix; these discuss the conditions under which the hydraulic theory is valid, as well as showing that the theories developed here match each other in the appropriate limits.