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Appendix A

An ecological approach to perceptual organisation: Nearest neighbour categorization of dot displays ranging from tightly clustered to highly regular

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Running head: An ecological approach to perceptual organisation

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ABSTRACT

An experiment is described in which participants rated the degree to which dot patterns (distributed within a square area, along a line, or around the circumference of a circle) appeared clustered, random, or regular. Ratings were unaffected by dot density and could not be accounted for in terms of absolute distance. However, ratings were well described as a linear function of the ratio between the mean nearest neighbour distance in each pattern and that expected in a random pattern with the same dot density. This finding is interpreted more generally in terms of a relational analysis of dot patterns. Applications of similar analyses to other visual tasks are described and challenges for the development of an ecological (or geometric) approach to perceptual organisation are discussed. A hierarchy of more powerful relational analyses and associated measures is briefly described. Evidence that the visual system has access to such information is presented, and it is argued that its use greatly simplifies the later processing that needs to be assumed. It is concluded that it would be fruitful to re-examine the interpretation of receptive fields in the light of both recent neurophysiological findings and of the geometry of relational structures in dot patterns.

A principal interest of Gestalt psychologists has been the ability of the human visual system to detect clustering and regularity in abstract patterns and one of the enduring legacies of the Gestalt school is the *principle of proximity*, enunciated by Wertheimer (1923; 1958). According to Wertheimer, "other things being equal, grouping occurs on the basis of small distance" (1958, p. 118).

This "factor of nearness" (or proximity) is usually interpreted to mean that the closer elements are to one another, the more likely they are to be seen as grouped or belonging together. In some cases, this factor could be interpreted in absolute terms. In others, it explicitly refers to the relative closeness of two sets of elements (e.g., Palmer, 1999, p. 257). Although Wertheimer's original article (in German) contained a detailed discussion of experimental techniques for the study of the principle of proximity, this was not included in the English translation with which most researchers are familiar. This is unfortunate because the obviousness of Wertheimer's examples seems to have discouraged all except a few subsequent researchers from examining the principle more closely.

Interpretations of the proximity principle in both absolute and relative terms seem to be assumed in different accounts of perceptual organisation. For example, Glass patterns (Glass, 1979) are generated by subjecting an array of randomly positioned dots to some geometric transformation and superimposing the transformed array on the original. The resulting pattern is seen as clearly structured along the trajectory of the transformation used to generate it. Maloney, Mitchison and Barlow (1987) proposed a model for the detection of structure in Glass patterns that depends on differentiating the number of pairs of dots related by the transformation in question from the number of random pairings. To qualify as a transformed pair, the dots in question must have an absolute separation and orientation that falls within some range of that expected from the transformation used. A very similar approach has been taken by Wilson, Wilkinson and Asaad (1997) and Wilson and Wilkinson (1998) in terms of hypothesised filtering units that are 'tuned' to specific orientations and separations.

The above interpretations appear to be strongly influenced by the assumptions of classical receptive field approaches (Bullier, 2002), in which the activity of a large number of similar, independent and precisely delineated anatomical units is assumed. On the other hand, approaches from computational geometry have tended to take the form of computational algorithms that are independent of such specific metric information. For example, Caelli (1981) suggested that the perception of structure in Glass patterns is based on detecting a difference between the distribution of nearest neighbour distances for those dots related by a common transformation and the distribution of nearest neighbour distances between dot pairs that are randomly positioned. Similarly, Zahn (1971) advocated the use of algorithms based on nearest neighbours and minimum spanning trees for the automatic detection of clusters in spatial point patterns. A minimum spanning tree is the shortest path that directly links all *n* nodes in an array with (*n*-1) edges and contains no loops (Ahuja, Magnanti, & Orlin, 1993). The minimum spanning tree contains all nearest neighbour links, although it may also contain additional edges that connect the closest members of nearest neighbour pairs.

More recently, van Oeffelen and Vos (1982; 1983) and Compton and Logan (1993) have developed and evaluated a so-called CODE algorithm, in which clusters of dots are detected by a smoothing process in which the strength of the relationship between dots is determined by a distribution function that is scaled to the distance between a dot and its nearest neighbour. In a parallel development, Kubovy and Wagemans (1995) and Kubovy, Holcombe and Wagemans (1998) have put forward a theory of grouping by proximity in dot lattices, based on the empirically supported assumption that the relative strength of grouping into strips of dots in a particular orientation is a decreasing exponential function of the relative distance between dots in that orientation (i.e., the distance scaled to the shortest distance in the lattice).

As Kubovy and Wagemans (1995) themselves point out, their model is designed to describe the relative strength of alternative perceptual organisations of lattice structures and cannot account for the identification of clusters of random dots in the plane. Kubovy and Wagemans point to other models, such as the CODE model of van Oeffelen and Vos (1982; 1983), that have been designed to describe such clustering. Conversely, however, the CODE algorithm does not seem to be readily applicable to the perceptual organisation of dot lattices, which may be viewed as multi-stable perceptions. In turn, neither of these approaches, in their current stages of development, seems to be immediately applicable to a problem that occurs commonly in a range of different practical situations. This is the problem of deciding whether a given spatial point array (such as the coordinates of a particular plant species) is evidence of a purely random process or whether it exhibits either more clustering or more regularity than would be expected by chance. In the latter two cases, there is also the more specific problem of assessing the degree of clustering or regularity exhibited by the array.

A set of techniques, that are closely related to those proposed by Zahn (1971) and by van Oeffelen and Vos (1982; 1983) and that directly address such problems, has been developed for the statistical analysis of spatial point patterns in several quite different fields, ranging from geophysics, through ecology up to astronomy. These techniques are concerned with the nearest neighbour analysis of spatial point processes in two- and one-dimensional situations. Points distributed over an area are most commonly addressed, but points distributed along an open or around a closed curve – such as a straight line or circle – can be analysed in a similar way. Boots and Getis (1998), Ripley (1981), Selkirk and Neave (1984), Smith (1975), Walford (1995), and Young (1982) all provide useful treatments. Analyses relevant to the present paper are based on mean nearest neighbour distances for points arrayed within a square area, along a straight line, and around a circle; and are summarised in the Appendix.

The observed mean nearest neighbour distance for an empirical distribution of n points may be denoted r_o. The expected mean for a uniformly random distribution of n points, often called complete spatial randomness (CSR), may be denoted r_e, and can be evaluated by means of the formulae presented in the Appendix. Dividing the observed by the expected mean gives the nearest neighbour statistic, $R = r_o/r_e$, which can be tested for significance. The nearest neighbour statistic, R, is a descriptive device, which provides a quantitative summary of the spacing between array elements. Average nearest neighbour distance decreases as points become more tightly *clustered*; hence, the closer to 0 the value for R becomes. The most tightly clustered situation is that in which all points are superposed, when the value of R equals 0. The closer the points are to being *randomly* dispersed, the more similar are the values of r_o and r_e, and the closer to 1 the value for R becomes. The value of R equals 2.149 for points that are spaced with perfect uniformity, as in a triangular lattice arrangement. Hence, the closer to 2.149 the value for R becomes, the more *uniformly spaced* are the points. (For points located on the intersections of a square grid R = 2.)

The significance of the difference between some R resulting from sample data and the R for complete spatial randomness can be examined by a z test,

$$z = \frac{r_o - r_e}{s_d},$$

where s_d is the standard error.

A literature search[1] suggested that, with the exception of Pickett's (1967) study of the discrimination of coarsely and evenly grained dot matrices and O'Callaghan's (1974) study of the differentiation of dot pattern areas, there has been little systematic study of human performance in detecting and discriminating clustering and regularity in point patterns. It also suggested that the potential of nearest neighbour analysis for application to pattern detection and discrimination generally appears to have been overlooked or underestimated in perceptual and cognitive psychology. Accordingly, the following experiment was designed to address this issue in a preliminary way.

METHOD

Stimuli. The stimuli consisted of areal, circular, and linear arrays of small, hollow circles, blue in colour, presented against a white background on a 19-inch computer monitor. Areal patterns contained hollow circles, each approximately 1 mm in diameter, distributed over two dimensions, and circular and linear patterns contained similar hollow circles, distributed over one dimension. Areal patterns appeared on a centrally located, 240×240 mm white square area. Circular patterns were distributed around the circumference of an (unseen) centrally located circle of diameter 223 mm. Linear patterns were distributed along an (unseen) centrally located, horizontal line 240 mm in length. They were viewed from a distance of approximately 700 mm.

For each type of pattern there were four categories of dot distribution: fixed clustered, Poisson clustered[2], uniformly random, and regular, with five levels in each category except for the random category, which, of course, can have only one level. The eleven levels were spread across the range of dot distributions from tightly clustered to highly regular. The levels were determined on the basis of nearest neighbour analyses in such a way that, for both the fixed and Poisson clustered categories, levels of R ranged in five approximately equal steps (to within 3%) from tightly ($R \cong 0.05$) to loosely clustered ($R \cong 0.80$). For the regular category, grades ranged in five approximately equal steps from barely regular ($R \cong 1.20$) to highly regular ($R \cong 2.00$). Nearest neighbour analysis for the most loosely clustered and the barely regular levels showed z to be several standard deviations from the mean of respective CSR distributions; hence all the levels were significantly different from CSR.

For the two clustered categories, each parent dot was randomly located and each offspring dot was located in any possible direction at a random distance that was limited by dot distribution level. For the regular category, each dot was perturbed from a square grid location, in any possible direction, over a random distance that was limited by the dot distribution level.

Two examples of each of the 16 levels of dot distribution were independently generated. To provide a range of dot densities, the number of dots in any one display was randomly determined over a range between 36 to 800 in areal displays, 15 to 80 in circular displays, and 20 to 67 in linear displays. The lower limits were chosen to ensure that z would be approximately normally distributed and the upper limit was set to a value at which the display was not obscured by overcrowding.

Examples of areal, circular and linear patterns characterised by fixed and Poisson clustering, randomness and regularity are shown in Figures 1 to 3.

INSERT FIGURES 1 to 3 HERE

Procedure. Participants were first provided with exemplars in a short practice session. (These did not duplicate any stimulus used in trial sets.) After practice, test displays were presented in separate blocks of trials, with one pattern type (areal, circular or linear) in each block. The three pattern types were presented in a different random order for each participant. Within each pattern type, the 32 alternatives were presented in an order that was random and that differed for each participant. Participants were required to respond by clicking with the computer mouse on an eleven-point scale at the foot of the screen. Immediately following each response, another display was presented.

The eleven-point scale was arranged as eleven squares abutted horizontally beneath the stimuli, and numbered consecutively from –5, through 0, to +5. From left to right, the five squares to the left of centre represented most to least clustered and the five squares to the right of centre represented least to most regular. The square at the centre represented randomness.

Participants. Participants were 8 males and 8 females, aged from 19 to 52 years. They were drawn opportunistically from the university student population. All had normal or corrected vision.

RESULTS

For each participant, and for each of the 16 dot distribution levels, the two raw ratings (which were in the range from –5 up to +5) were shifted upwards by 5 and divided by 5 to provide a range between 0 and 2. This was more convenient for statistical manipulations and corresponded to the range of the objective scale of values for R. A raw rating of 0, for example, corresponds to a rescaled rating of 1 and signifies a judgment that the distribution of elements in a pattern was purely random.

For each level, the two rescaled ratings were averaged, and the arithmetic mean of these averages was then calculated over the 16 participants. For each of the 32 stimuli, the values of the objective mean nearest neighbour distance, r_o, and of the nearest neighbour statistic, $R = r_o/r_e$, were calculated.

It might be thought that the simplest supposition would be that participants judge the degree of clustering or regularity by estimating the objective mean nearest neighbour distance, r_o. However, although there were reasonably strong correlations in the case of all three pattern types (average Pearson $r = .89$), scattergrams of the rescaled ratings against r_o in each case showed substantial deviations from linearity.

INSERT FIGURE 4 ABOUT HERE

In contrast, Figure 4 shows the mean rescaled ratings, produced by participants, for the areal, circular and linear patterns, respectively, plotted against values of R. Because the ratings for patterns with fixed and Poisson clustering were virtually identical, no attempt has been made to differentiate between them in the Figures or in the subsequent analyses.

For all three pattern types, the relation between the rescaled ratings and *R* is well described by a straight line, with an intercept close to zero and a slope approaching unity. For the areal, circular and linear pattern types, r^2 values were .97, .97 and .96, respectively. As illustrated in Figure 4d, data for all three pattern types are well described by a single linear function with an intercept of 0.12 and a slope of 0.92 (r^2=.96). It is clear that the theoretical values of *R* account for virtually all (at least 96%) of the variance in the empirical ratings

In contrast, when the differences between mean rescaled ratings and values of *R* were compared with the number of dots in each pattern, there was no significant correlation for any of the three pattern types. That is, dot density made no significant additional contribution to predicting response ratings. This reinforces the earlier conclusion that rescaled ratings were not satisfactorily accounted for by mean nearest neighbour distance alone.

DISCUSSION

We first discuss implications of the results for understanding the categorisation of clustered, regular and random arrays in terms of nearest neighbour relations. We then examine the application of similar relational analyses to performance in a range of different visual tasks. We conclude by considering some further challenges that such an approach needs to meet in order to qualify as a general ecological theory of perceptual organisation.

Nearest neighbour analysis and the perception of clustering, regularity and randomness

The finding that participants' ratings of the degree of clustering, regularity or randomness in dot patterns is not determined solely by the mean nearest neighbour distance is to be expected. For example, imagine a pattern, in which dots are located at the vertices of an (unseen) square grid, and in which the side of each grid element is 1 cm. We would be unlikely to judge such a pattern as more (or less) regular than a similar pattern, in which the side of each element is 2 cm, despite the fact that the mean nearest neighbour distance for the first pattern is half that for the second. That is, judgments of regularity are based on the structure of a pattern rather than the density of its elements. This has the advantage that such judgments can be expected to remain invariant (within broad limits) under the uniform dilation or contraction of a pattern (as well as under rotation and translation).

The finding that dot density makes no contribution to categorising the patterns as clustered or regular suggests that it would not be possible to explain the results in terms of a set of distances that exceed, or fall below, some critical absolute size. Such an explanation is, in any event, unlikely because by comparison with values of R for nearest neighbour distances, which range from 0.05 up to 2, equivalent values for all interdot distances show minimal variation; despite extreme changes in the structure of the patterns. Equivalent values for all interdot distances are invariant with respect to the density of random dot patterns and, as illustrated in Table 1, are fairly insensitive to constraints in such patterns. Hence there seems to be little prospect for attempts to account for the operation of a Gestalt principle of proximity by means of neural structures (like those of Maloney et al., 1987, Wilson et al., 1997, or Wilson & Wilkinson, 1998) that are sensitive to dot pairs (or dipoles) that are (anatomically) defined in terms of absolute distance.

INSERT TABLE 1 ABOUT HERE

On the positive side, the present results are consistent with the view that participants are capable of doing exactly what they are asked to do in this experiment (i.e., assess the degree to which patterns are clustered, random, or regular; with respect to completely random patterns). Moreover, their judgments accurately reflect a statistical summary of the relational structure in each pattern. This suggests that participants not only have information about the mean nearest neighbour distance within a pattern, but that they can also calculate (or have access to) some quantity that represents the mean expected nearest neighbour distance for a random pattern with a similar number of dots. In other words, we can frame an explanation of the proximity principle at the computational level distinguished by Marr (1982). At this level, participants appear to make their judgments in a way consistent with the calculation $R = r_o/r_e$.

Applications of similar relational analyses to other perceptual tasks

When presented with a different task, of course, the computational accomplishment of participants may well differ. For example, we detect structured clusters, even in random point arrays. In an unpublished study, Vickers, Preiss and Hughes (submitted) found that the numbers of links and clusters seen in random dot patterns, as dot density increased, was a linear function of the number of links and clusters detected by a simple nearest neighbour algorithm. That is, in a task where the degree of clustering was held constant (with $R \cong 1$), links and clusters were detected on the basis of nearest neighbour relations alone, without reference to the mean nearest neighbour distance for a random pattern with a given dot density. As with judgments of the degree of clustering or regularity, this implies that the detection of structured clusters is invariant with respect to such transformations as translation, rotation and dilation or contraction. As in the more successful variants of the CODE algorithm tested by Compton and Logan (1993), the important determinant of membership in a cluster appeared to be the relative distances between each dot and its immediate neighbours. A similar approach could be applied to the perception of predominant orientations in lattice structures and seems to be consistent with the analysis proposed by Kubobvy and Wagemans (1995) and Kubovy et al. (1998), though it does not include the metric specification assumed in the latter treatment.

In the stimuli employed by Vickers, Preiss and Hughes (submitted), the dots were distributed randomly, and without constraints, so that there was no directional anisotropy in the arrays. As a result, relational information is concentrated in lower-level structures, such as nearest neighbour links. Where directional anisotropy exists, then it is possible that the visual system is sensitive to information associated with higher-level geometric structures. For example, in a second unpublished study, Vickers, Dry, Lee and Hughes (submitted) found that correlations between the frequencies of alternative responses to translation Glass patterns, oriented in one or the other of two diagonal orientations, showed that participants based their judgments on first or second nearest neighbour links that fell within a ±15° tolerance of the positive or negative diagonals. As in the present experiment, it was not possible to account for the pattern of relative response frequencies by assuming selective sensitivity to dot pairs defined in terms of absolute distances. At the same time, our exploratory investigations suggest that, at much higher element densities, the visual system may make use of information provided by more comprehensive relational structures, as discussed below.

The analysis of relational information has also proved useful in explaining the human ability to produce near-optimal solutions to a variety of visually presented combinatorial optimisation problems. For example, Vickers, Butavicius, Lee and Medvedev (2000) found that participants, who were instructed to draw the most “natural, attractive or aesthetically pleasing” pathway that passed once only through each of a set of randomly distributed dots and returned to the starting point, produced pathways that were almost as short as participants who were explicitly instructed to produce the shortest pathway. The latter task is an instantiation of the so-called Travelling Salesman Problem (TSP: Lawler, Lenstra, Rinooy Kan, & Schmoys, 1985), a notorious problem in combinatorial optimisation, for which there exists no algorithm that can be guaranteed to arrive at the best solution once the number of dots becomes appreciable. In a further unpublished study, Vickers, Lee, Dry, Hughes and McMahon (submitted) found that ratings of the perceived goodness of visually presented TSP solutions were correlated with their relative optimality. Ormerod and Chronicle (1999), Vickers et al. (2000) and Vickers, Lee et al. (submitted) all conclude that the production of near-optimal solutions to visually presented optimisation problems appears to be due to the spontaneous operation of perceptual organising principles. If so, then studies of this ability may throw some light on the nature of these principles.

Consistent with this view, Vickers, Mayo, Heitmann, Lee and Hughes (2004) found that performance on three types of visually presented optimisation tasks was highly correlated with a measure of path complexity that indexed the extent to which participants made use of nearest neighbour links in arriving at a solution. A similar result was obtained by Vickers, Lee, Dry and Hughes (2003), who reported that human solutions to visually presented TSP instances were determined by the number of potential intersections in the array of dots through which they were instructed to draw the shortest pathway. Because the optimal solution can never contain any crossed links and because links between nearest neighbours never cross, this suggests that the almost universal avoidance of crossings in TSP solutions might be due to participants constructing solution pathways by first establishing clusters, based on nearest neighbours, and then sequentially linking these up. Some support for this hypothesis is provided by the results of Vickers, Bovet, Lee and Hughes (2003), who found that the best single predictor of performance on both ‘open’ and ‘closed’ visually presented TSP instances was the proportion of links that solutions shared with a nearest neighbour algorithm. Taken together, these three studies suggest that the impressive human performance on visually presented optimisation problems is due to spontaneous perceptual organising processes that are, in turn, based on a locally focussed analysis of relational information of the kind provided by nearest neighbour links (though, as explained below, not necessarily restricted to such links).

Future development of relational analysis in visual perception

We have presented evidence for a computational level of explanation of the categorisation of degrees of clustering or regularity in terms of a hypothesised comparison between the perceived mean nearest neighbour distance and that expected for a random pattern. We have also summarised evidence that some other aspects of the perception of structure in dot patterns can be explained in terms of relational information of the kind provided most simply by nearest neighbours, though not necessarily restricted to this level of analysis. On this basis, we would argue more generally for an ecological analysis of the perception of structure in multi-element patterns in terms of the relational information such patterns provide.

Ecological theorists in the Gibsonian tradition have pointed to many potentially useful geometrical constraints that operate on an image when the spatial relations are changed between a light source, an illuminated object and an observer. However, Gibson himself dismissed the potential structure available in self-luminous point arrays (such as the constellations) as meaningless and irrelevant for visual perception (1950, p. 197; 1968, p. 220). This is unfortunate because, although it is true that the constraints in such cases are not intrinsically geometrical, they can nevertheless be analysed and described geometrically. Moreover, the spatial analyses developed in a wide range of different disciplines, as cited earlier, provide strong justification for the view that the geometrical analysis of such constraints can yield information that is ecologically valid and useful.

If such an approach is to be applied generally to the study of perceptual organisation, then it will have to meet a number of challenges. These include demonstrating the adequacy of specific relational analyses as predictors of human perceptual performance, developing algorithmic explanations for performance in specific perceptual tasks, and describing the possible neurophysiological implementation of such explanations. We conclude with a brief discussion of each of these issues.

The adequacy of nearest neighbour and other relational analyses

First, it will not be sufficient for an ecological approach concerning perceptual organisation to point to the relational information contained in visual images; it will be necessary to demonstrate that it is this information that determines the way we perceive these images.

In this connection, it should be emphasised that, while we believe that nearest neighbour relations provide the simplest – and possibly the most salient – information about the structure in dot patterns, there is a hierarchy of other structures and associated variables, to which the visual system might be receptive. The maximum number of edges linking nearest neighbours in an n-dot array is (n-1). However, if all the n dots in an array (where n is even) have reflexive (i.e., mutual) nearest neighbours, then the number of edges linking nearest neighbours is n/2, which is a minimum. All nearest neighbour links are included in the *minimum spanning tree*, which is the shortest network of (n-1) edges that links all n dots in an array and contains no loops. The minimum spanning tree is a subgraph of the *relative neighbourhood graph*, which, in turn, may contain additional edges up to a limit of (3n-6), and in which two dots are linked if they are at least as close to each other as they are to any other dot. (Which means that pairs are linked if no dots are located in the intersection of two disks, each of which is centred on a member of the pair, and with the common radius equal to intra-pair distance.) In turn, the edges of the relative neighbourhood graph are included in the *Gabriel graph*, which contains an edge between two dots if the disc having that edge as its diameter is empty of dots. Finally, the Gabriel graph is a subgraph of the *Delaunay triangulation*, which is the dual of the *Voronoi diagram*. All these structures are dealt with by Preparata and Shamos (1985) and by a number of other texts on computational geometry. In view of the capacity of the visual system to perceive hierarchical organisation (Palmer, 1999, pp. 259-261; Compton & Logan, 1993), it would not be surprising to find that the system was capable of making use of information associated with more than one of these structural levels.

The most comprehensive of these structures is the Voronoi diagram (and its dual, the Delaunay triangulation). As illustrated in Figure 5, a Voronoi diagram is a tessellation of polygons that define a region around each dot in which every point in the region is closer to the dot than it is to any other dot. Thus, the edges of each Voronoi polygon describe the locus of all points that are equidistant from a pair of adjacent dots, and the polygon is formed from the intersection of the half-planes associated with these edges. Voronoi structures arise in various natural situations: they possess powerful mathematical properties and contain all of the proximity information defined by a given set of points (Aurenhammer, 1991).

The dual of the Voronoi diagram is the Delaunay triangulation (i.e., Delaunay triangle edges have a one-to-one correspondence with Voronoi polygon edges), illustrated in Figure 6. Every edge of a Voronoi polygon is the locus of all points that are equidistant from a pair of adjacent dots and the edges of a Delaunay triangulation correspond to the line segments that link these dot pairs (and hence are at right angles to their corresponding Voronoi polygon edges or to their continuation). The Delaunay triangulation is an optimal triangulation that maximises minimum angles. It contains the same amount of information as the Voronoi partitioning, but the significance of being a neighbour may be shown more clearly by the triangle edges.

INSERT FIGURES 5 AND 6 ABOUT HERE

These dual structures can be used to distinguish between different kinds of adjacency relations. For example, if a line is drawn from a dot to one of its neighbours, the line is either perpendicularly bisected by a Voronoi edge or it passes through two or more Voronoi edges (but would be perpendicularly bisected by an extension of the common boundary between the two dots). Thus, a dot can be directly or indirectly across the ‘fence’ from its neighbour. For edges linking direct neighbours, a circle imposed upon the edge as a diameter does not include any other dot, whereas, for edges linking indirect neighbours, it will. (The Gabriel graph, mentioned earlier, is the graph of all direct neighbour edges.)

There are a number of ways of defining neighbourliness relations and a variety of measures associated with each. It may well be that, for more complex tasks, and with more complex and highly populated arrays, analyses in terms of nearest neighbour links alone will turn out to be insufficient to account for human perceptual performance and that it will be necessary to consider the possibility that the visual system has access to information associated with these more powerful structures. For example, Pomerantz (1981) asked participants to connect all the dots in simple matrix arrays to indicate their perceived structure. This is tantamount to requiring participants to construct a spanning tree. Pomerantz found that participants connected the arrays using structures that had the shortest total length. That is, the most natural connected structures perceived by participants were minimal spanning trees.

A similar conclusion has been reached by Vickers, Preiss and Hughes (submitted). These authors examined the representation of thirty northern hemisphere constellations in a popular desktop planetarium (as well as in some half dozen star atlases). They found that, although such representations varied, and did not conform to any single algorithm, the odds against the degree to which constellation diagrams shared structure with the corresponding minimum spanning trees were over 200,000,000:1 against this being due to chance.

There is also some evidence that suggests that the human visual system can make use of higher-order structures, such as the Delaunay triangulation, which includes (but is not restricted to) all nearest neighbour links in an array, as well as the minimum spanning tree. In a study of performance with visually presented TSPs, Vickers, Preiss, Lee and Hughes (submitted) found that solution times were a linear function of $n \log n$, where n is the number of dots (nodes) in an array. Most serial algorithms for the computation of near optimal TSP solutions are of the order n^2 or higher. However, the Delaunay triangulation can be computed in a time proportional to $n \log n$. Although we do not wish to claim that the visual system employs a serial process to arrive at the equivalent of a Delaunay triangulation, the computational efficiency provided by such a structure could explain how human beings can arrive at near optimal solutions in a time that does not increase with n at anywhere near the rate for most computer algorithms. The number of possible pathways to be considered in arriving at a TSP solution is $(n-1)!/2$. Because there are at most $3n$-6 edges in a Delaunay triangulation, if the search for solution pathways is restricted to Delaunay edges, the serial task of constructing such pathways is enormously simplified. The utility of this restriction is underlined by the finding that, of the edges in known optimal solutions to the TSP instances employed by

Vickers, Bovet et al. (2003), 100% (in open tours) and 98.55% (in closed tours) turned out to be Delaunay edges. In the participants' solutions examined by Vickers, Preiss, et al. (submitted), 97.62% (in open tours) and 96.09% (in closed tours) of the edges employed by participants were also Delaunay edges. On this basis, Vickers, Preiss, et al. (submitted) proposed a model of human performance in visual TSP tasks based on using the Delaunay triangulation.

It should be emphasised that these results are still isolated and preliminary. A great deal of research remains to be carried out before the usefulness of an ecological approach to perceptual organisation can be considered as established. However, the point we wish to argue is that several powerful geometric analyses are available, much is known about various measures associated with them, and there seems to be at least a *prima facie* case for investigating their application to perceptual organisation.

Algorithmic explanation

To extend the explanatory power of the present analysis, it will be necessary to develop algorithmic explanations of how participants arrive at their judgments. This is difficult with the restricted data from the present experiment. For this purpose, a timed task, like that of Pickett (1967), in which participants discriminated between clustered, random and regular patterns would be more informative. Using the data from such experiments, it should be possible, for example, to design a sequential decision process (like that examined by Pickett, 1967, or by Vickers, 1979; 1998), in which randomly sampled nearest neighbour distances in a given pattern are compared with the mean nearest neighbour distance for a random pattern with the same dot density. Sampling would continue until a threshold accumulated positive or negative difference is reached, whereupon the corresponding response would be triggered. The probability of making responses of each type, the time taken to make them – and possibly also the confidence with which each is made – could all be used to constrain the selection of an appropriate process model. For example, Vickers, Dry, et al. (submitted) showed that response frequencies and confidence ratings in two experiments on discriminating Glass patterns could be successfully modelled by a decision process that simply counted the number of first and second nearest neighbour links that were positively or negatively oriented within a tolerance of $\pm 15^{\circ}$.

Other algorithmic explanations for such data will no doubt suggest themselves – possibly involving parallel as well as serial processing. However, the point we wish to argue here is that, if we assume that the visual system makes efficient use of the relational information available in the stimulus, then the complexity of the later processing that needs to be assumed is greatly reduced.

Neurophysiological implementation

On the present approach, the relation between any two dots hinges upon what other dots are in their vicinity and depends on the relative distances between dots rather than upon some absolute distance. It will be obvious from these properties and from the discussion so far that the ecological approach to perceptual organisation we advocate is at odds with the notion of classical receptive fields, according to which the outputs of a large number of independent, anatomically defined simple units are progressively integrated by successively more general units.

At the same time, it is also the case that the classical receptive field notion has undergone considerable qualification in recent years, and that many recent neurophysiological findings are consistent with the implications of relational analyses. For example, Motter & Belky (1998) found that target detection occurred only within a restricted area, determined by element density, and concluded that attention operates within an area having a radius of twice the average nearest neighbour distance. Brady et al. (1997) and Bex and Dakin (2003) showed that the visual system is sensitive to motion information at widely separate spatial frequencies and over a range of spatial scales. Similarly, Dakin & Herbert (1998) found that the spatial integration region for the perception of reflective symmetry varied inversely with peak spatial frequency. Such scale-invariant properties correspond well with recent evidence that dot density is the property that the visual system uses to implement scale invariance in the perception of symmetry and Glass pattern structure (e.g., Rainville & Kingdom, 2002). Both scale-invariance and the ‘tuning’ of receptive fields by element density are consistent with an analysis in terms of nearest neighbours. Thus, the relational perspective, advocated in this paper, could provide a fruitful conceptual framework, consistent with recent neurophysiological findings, which indicate a much wider interconnection among the elements of a visual field than previously assumed, and which emphasise receptive field plasticity, context effects and normalisation by element density.

In light of these developments, it would seem important to examine the possible contribution of relational structures to visual perception and their possible correspondence with neurophysiological structures. In particular, Voronoi tessellations and Delaunay triangulations possess considerable potential relevance. Such structures are not only appropriately responsive to element density, but, in addition to distributions of distances between variously defined neighbours, they possess many two-dimensional properties that are informative regarding direction-sensitive changes in density and other forms of structure in dot patterns. (In the case of Voronoi polygons, these properties include area and perimeter, completeness, elongatedness or compactness, direction of principal axis, variance of side lengths, and eccentricity.) A detailed treatment of the application of such analyses to the identification and description of structure in dot patterns, which aims to mimic the achievements of human perception, has been presented by Ahuja (1982) and by Ahuja and Tuceryan (1989), while Aurenhammer (1991) gives a complete historical, mathematical and applied survey of Voronoi diagrams.

In many ways, Voronoi tessellations behave like plastic, context-sensitive, density-tuned receptive fields. Such structures can be generated by uniform diffusion processes that proceed outwards from salient features, such as dots, and can be considered as corresponding to the first meeting points of these radial diffusion processes. This conceptualisation is closely related to Blum's (1973) 'grassfire' model for the recognition of biological shape. Blum (1967) argued for a reinterpretation of the receptive fields of Hubel and Wiesel (1962; 1965) in terms of radial diffusion processes. On the basis of the results reviewed above, we would argue that, although we should of course be chary of simplistic correspondences, it would nevertheless be fruitful to re-examine the interpretation of receptive fields, not only in the light of recent neurophysiological findings, but also in the light of the geometric properties of relational structures in dot patterns and multi-element arrays generally.

CONCLUSIONS

Ratings of the degree to which dot patterns appear to be clustered, random, or regular were independent of dot density, and could not be accounted for in terms of absolute distances. However, such ratings were well described as a linear function of the ratio between the mean nearest neighbour distance in a pattern and that expected for a random pattern with the same dot density. This finding is inconsistent with an explanation of the perception of clustering and regularity in terms of absolute distances and classical receptive fields. Instead, it suggests that the human visual system computes proximity information in this task in terms of relations among pattern elements. Several successful applications of similar relational analyses to a variety of visual tasks were described, and three challenges for the development of an ecological (i.e., geometric) approach to perceptual organisation considered. These include the adequacy of nearest neighbour and other relational analyses, the need for algorithmic (as well as computational) explanations of perceptual phenomena, and the prospects for a neurophysiological implementation of the proposed ecological (or geometric) approach. A hierarchy of more powerful analyses was described and the potential of various associated measures briefly indicated. Some evidence that the visual system has access to such relational information was presented, and it was argued that, if the visual system does make use of this information, then the later processing that needs to be assumed is greatly simplified. On the basis of the experimental results, and the findings reviewed in the discussion, it was concluded that it would be fruitful to re-examine the interpretation of receptive fields in the light, not only of recent neurophysiological findings, but also of the geometric properties of relational structures in multi-element arrays.

ACKNOWLEDGMENTS

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FOOTNOTES

1. A bibliography of 1809 references to experimental studies of the visual perception of spatial point (random dot) patterns is available as a Word document, a searchable Endnote list, or as a text file at https://www.psychology.adelaide.edu.au/members/staff/dougvickers/index.html. This bibliography was compiled by Matthew Dry and was last updated in April 2004.

2. A fixed clustered pattern has the same number of dots in any cluster, but the number of dots in clusters differs randomly from pattern to pattern. Fixed clustered displays constrained to 2, 3, *or* 4 dot clusters were used in the experiment. A Poisson clustered pattern has a random number of dots in any cluster. Poisson clustered displays constrained to 2, 3, *and* 4 dot clusters were used in the experiment.

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FIGURES

Figure 1. Examples of areal patterns with (a) fixed clustering, (b) Poisson clustering, (c) a random, and (d) a regular distribution. Values of R were 0.37, 0.59, 1.03, and 1.41 for (a) to (d), respectively.

Figure 2. Examples of circular patterns with (a) fixed clustering, (b) Poisson clustering, (c) a random, and (d) a regular distribution. Values of R were 0.23, 0.57, 0.96, and 1.58 for (a) to (d), respectively.

Figure 3. Examples of linear patterns with (a) fixed clustering, (b) Poisson clustering, (c) a random, and (d) a regular distribution. Values of R were 0.43, 0.65, 1.02, and 1.37 for (a) to (d), respectively.

Figure 4. Mean subjective ratings of the degree to which patterns were perceived as clustered, random, or regular, plotted against objective values of R. (The raw ratings have been rescaled from the range –5 to +5 to the range 0 to 2 by adding 5 and dividing by 5.) From (a) to (d), respectively, the Figures show the results for areal, circular and linear patterns, and those for all three pattern types combined.

Figure 5. Examples of Voronoi tessellations of sparse areal dot patterns with (a) fixed clustering, (b) Poisson clustering, (c) a random, and (d) a regular distribution. Values of R were 0.37, 0.62, 1.03, and 1.61 for (a) to (d), respectively.

Figure 6. Examples of Delaunay triangulations of sparse areal dot patterns with (a) fixed clustering, (b) Poisson clustering, (c) a random, and (d) a regular distribution. Values of R were 0.37, 0.62, 1.03, and 1.61 for (a) to (d), respectively.

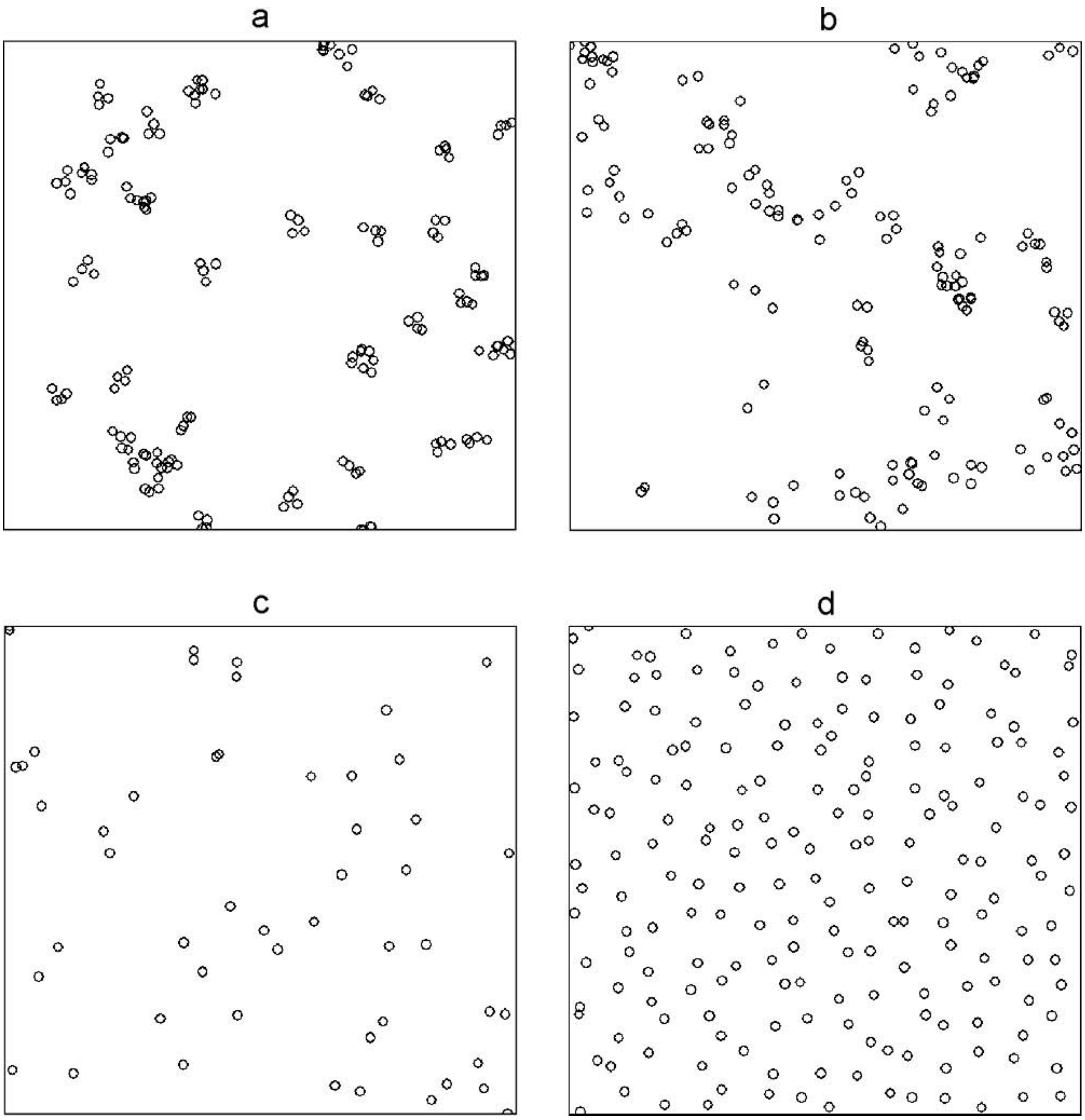

Figure 1. Preiss & Vickers

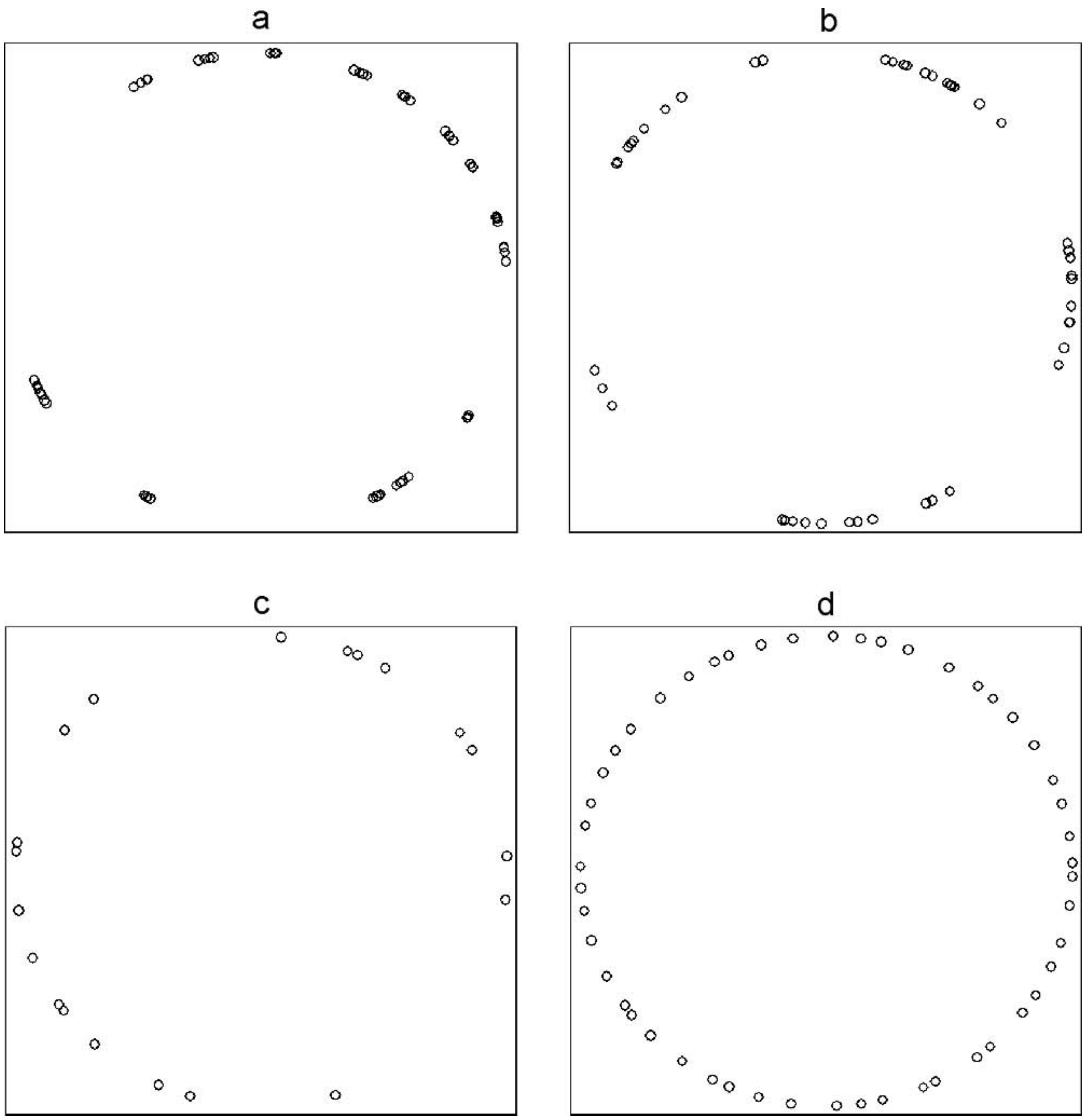

Figure 2. Preiss & Vickers

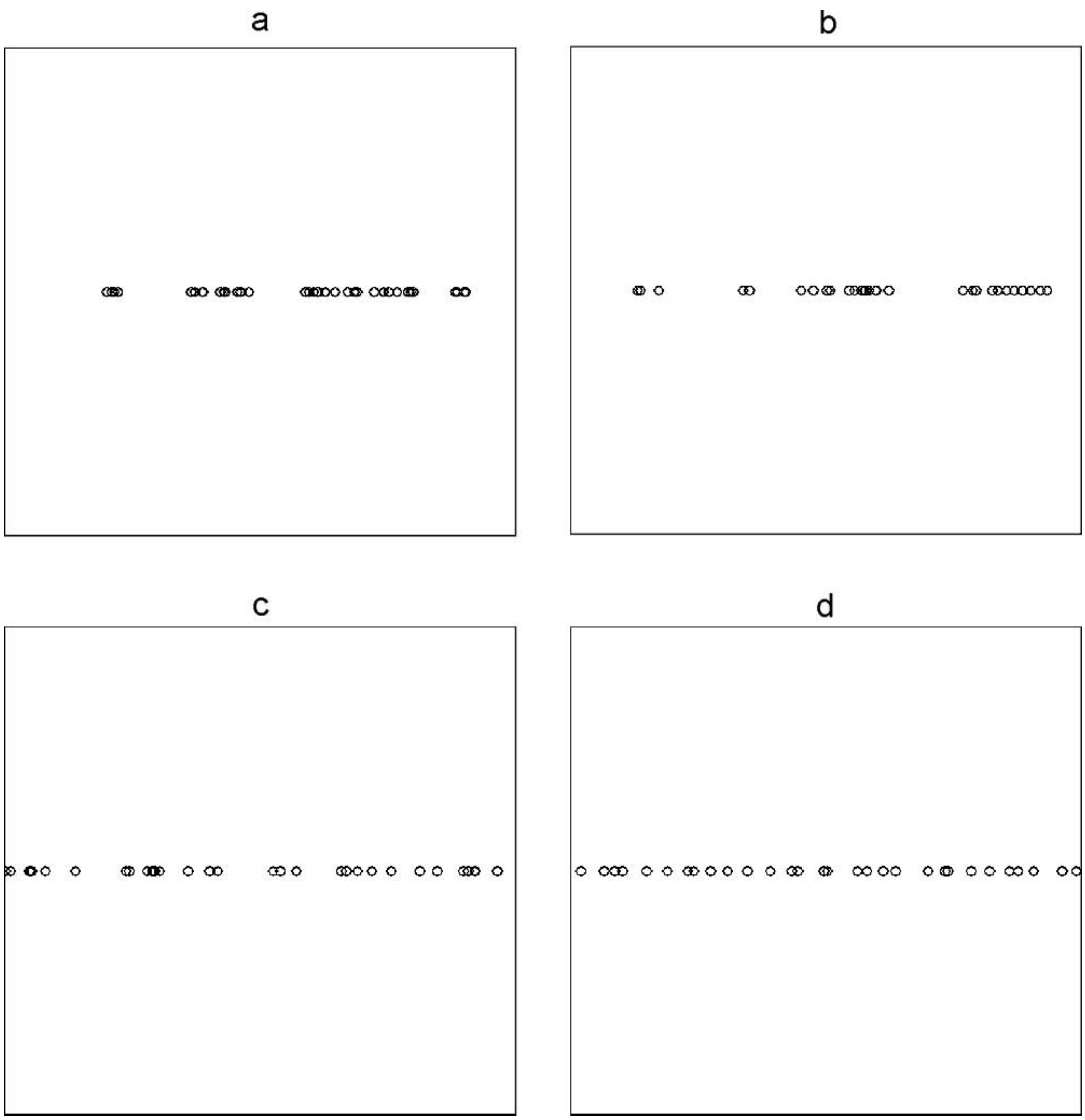

Figure 3. Preiss & Vickers

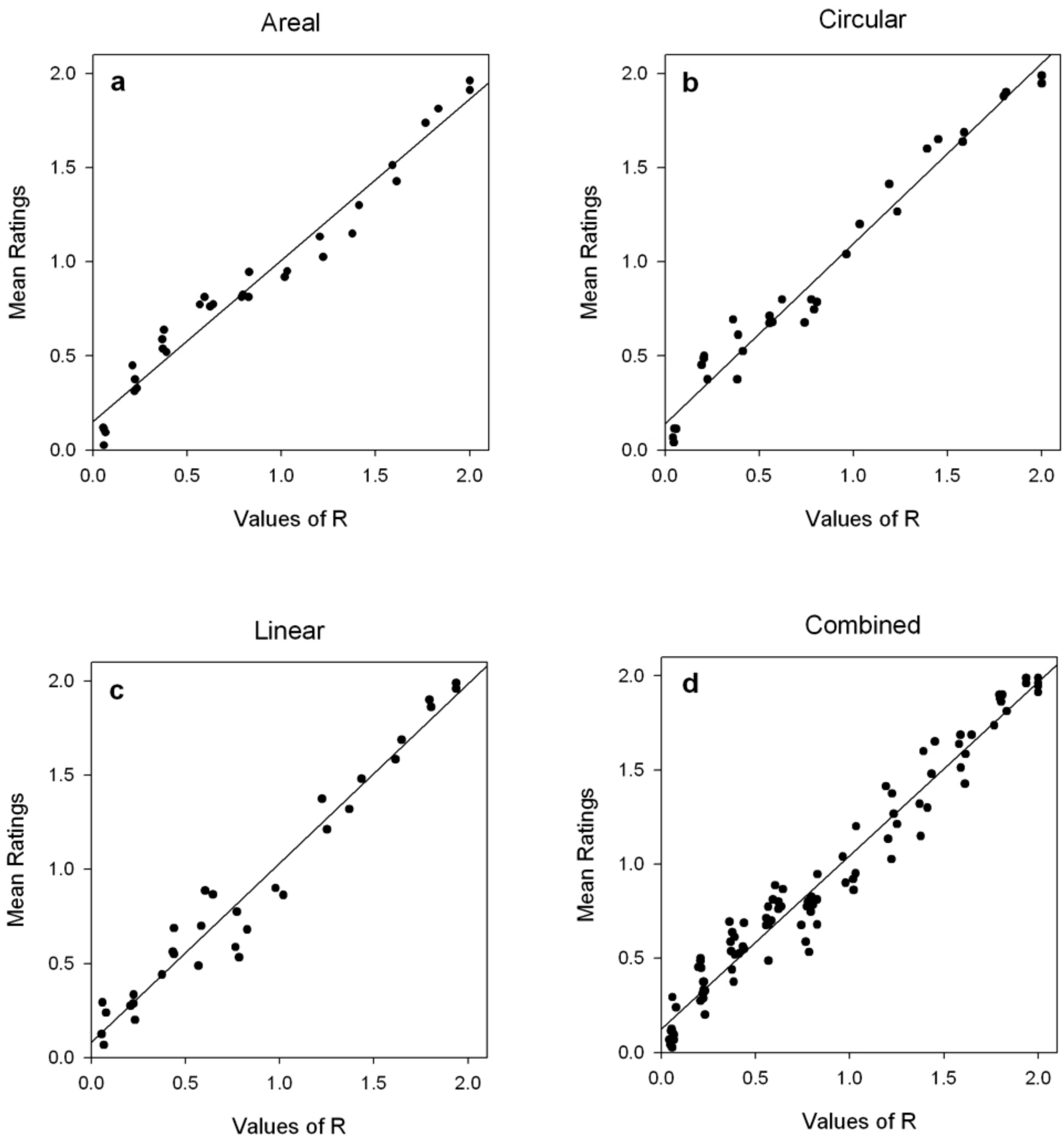

Figure 4. Preiss & Vickers

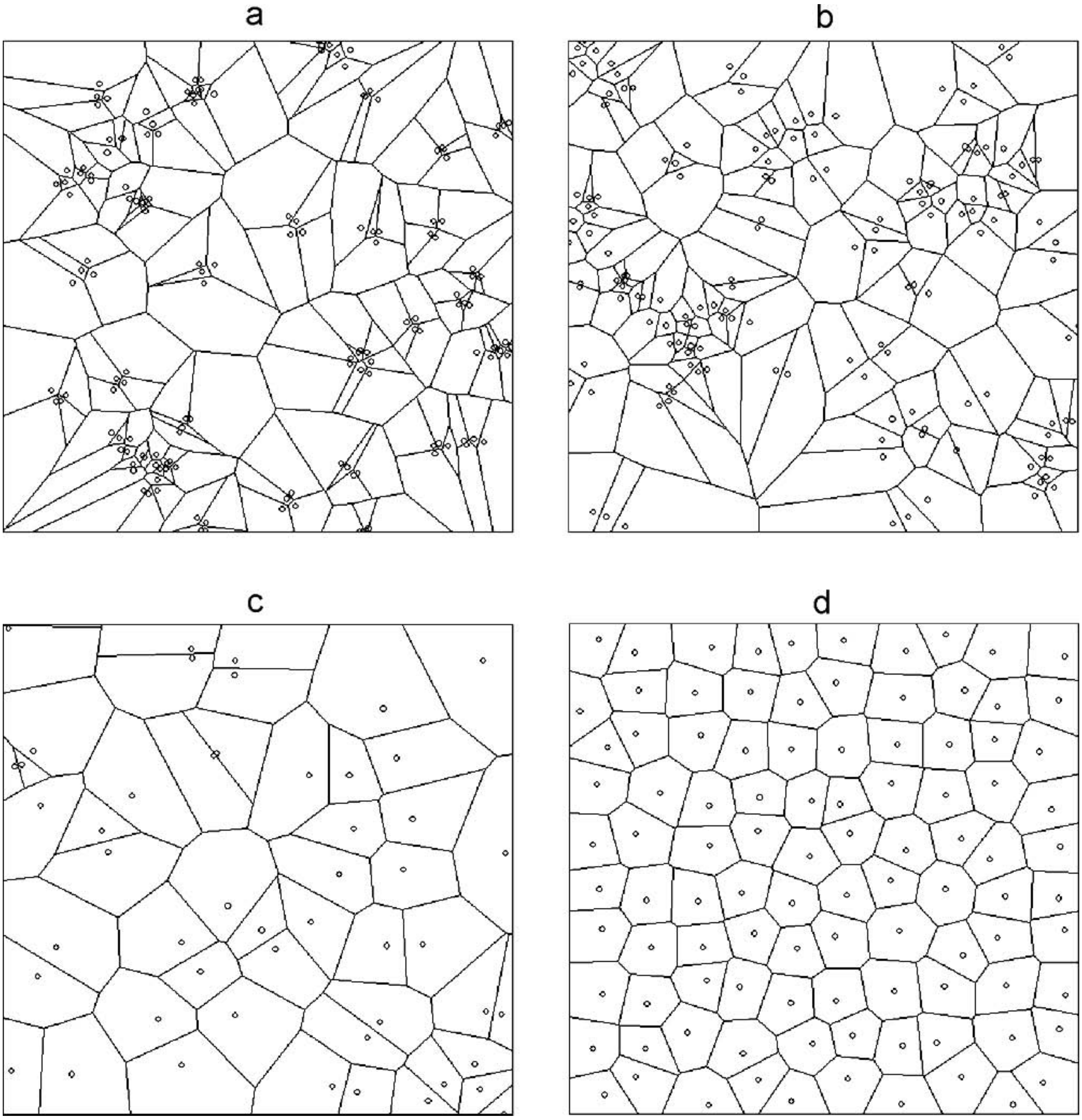

Figure 5. Preiss & Vickers

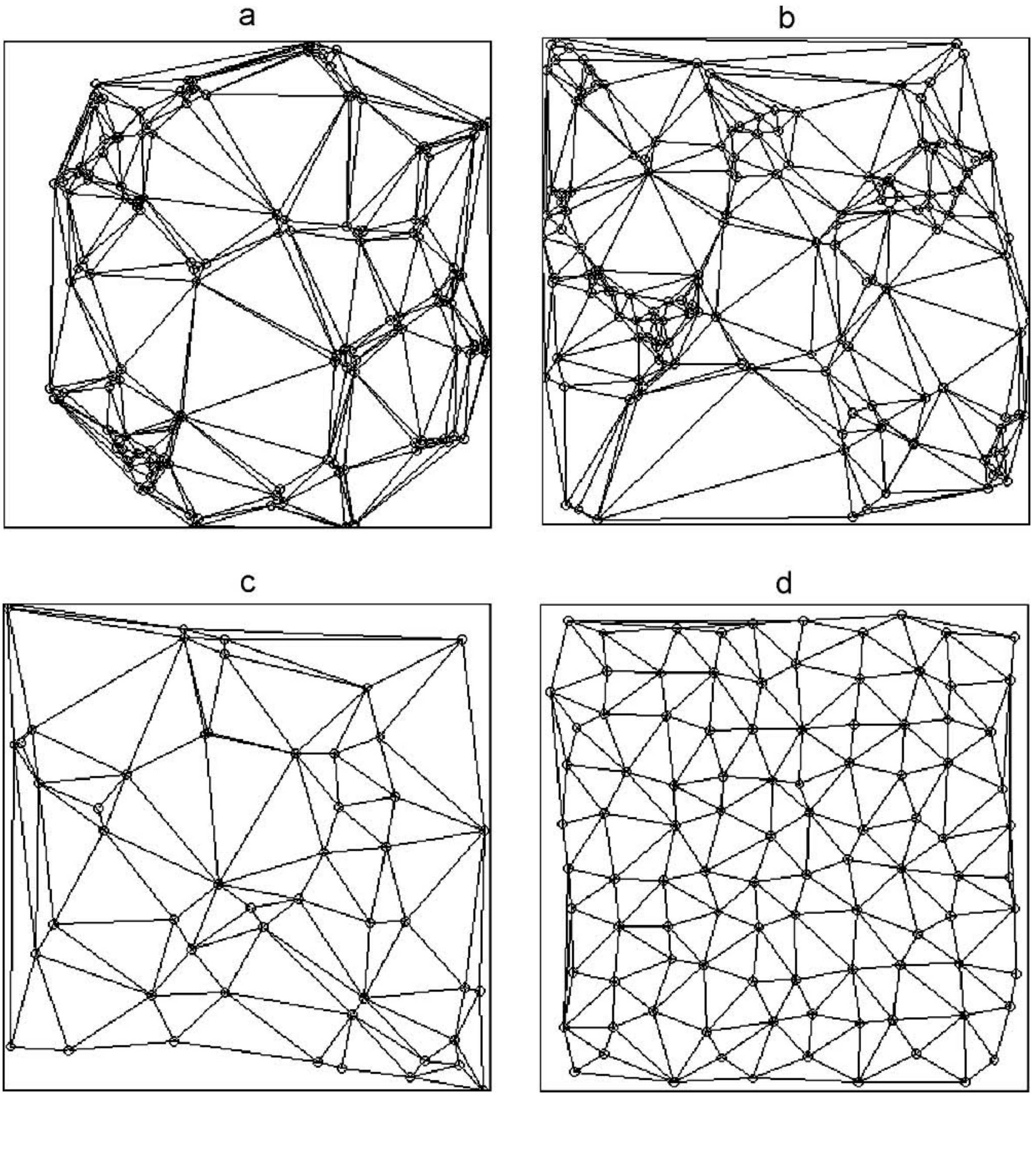

Figure 6. Preiss & Vickers

TABLES

	R values for nearest neighbour distances			Equivalent values for all interdot distances		
	Areal	**Circular**	**Linear**	**Areal**	**Circular**	**Linear**
Most clustered	0.06	0.05	0.07	0.92	0.86	0.94
Most regular	2	2	1.94	1.02	1.01	1.15

Table 1. *R* values for nearest neighbour distances at most clustered and most regular employed in this experiment, compared to equivalent values at most clustered and most regular for all interdot distances.

APPENDIX

For a set of points where the distance between the *i*th point and the *j*th point is u_{ij}, the observed mean nearest neighbour distance is

$$r_O = \frac{1}{n}\sum_i \min_{j \neq i}\{u_{ij}\}.$$

Complete spatial randomness for *n* points in an area with volume *A* is described by the Poisson process, with probability density function

$$p(d) = 2\pi\lambda d \exp\left(-\pi\lambda d^2\right),$$

where $d > 0$ is the distance between neighbouring points, and $\lambda = n/A$ is the point density (i.e., the mean number of points per unit area). The basic assumptions underlying this process are that a point is equally like to fall at each location in the area, and that multiple points are chosen independently.

The expectation of this distribution,

$$r_E = \frac{1}{2}\sqrt{\frac{A}{n}},$$

gives the average distance between nearest neighbours for a random process, with standard error

$$s_d = \sqrt{\frac{4-\pi}{4\pi\lambda n}}.$$

The expected nearest neighbour distance for complete spatial randomness around a closed curve of length *w* is

$$r_E = \frac{w}{2n}$$

with standard error

$$s_d = \sqrt{\frac{w^2}{6n^2(n+1)}}.$$

The expectation under complete spatial randomness for *n* points on an open curve is

$$r_E = \frac{w(n+2)}{2n(n-1)}$$

with standard error

$$s_d = \sqrt{\frac{w^2\left(2n^2 + 7n - 36\right)}{12n^3\left(n-1\right)^2}}$$

in the case where points are located at both ends of the curve, and by the expectation

$$r_E = \frac{w(n+2)}{2n(n+1)}$$

and standard error

$$s_d = \sqrt{\frac{w^2\left(2n^2 + 17n + 12\right)}{12n^2\left(n+1\right)^2\left(n+2\right)}}$$

in the case where there are no points at the ends of the curve.

In all of these cases, a standard null hypothesis test comparing the expected and observed mean nearest neighbour distances can be made using the Z statistic

$$Z = \frac{r_E - r_O}{s_d},$$

which, for large enough n, is Normally distributed with zero mean and unit variance.

Appendix B

Unpublished manuscript, 2003

The role of nearest neighbours in the perception of structure and motion in dot patterns

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Abstract. Evidence for the importance of nearest neighbours in the perception of visual structure is reviewed and their role investigated in three studies. In the first, nearest neighbours, and minimum spanning trees incorporating them, accounted for the representation of constellations in a desktop planetarium. In the second, nearest neighbour relations predicted mean link and cluster lengths, as well as the numbers of links and clusters detected in random dot patterns with different numbers of dots. In the third, the extent of motion seen in random dot kinematograms varied inversely with dot density and was closely related to the mean distance between nearest neighbours. None of these results was consistent with a proximity principle or a detector mechanism based on absolute distance. Implications are drawn for the perception of apparent motion and structure in dot patterns, including Glass patterns. The potential of nearest neighbour analysis to contribute to a unifying account of the perception of structure and motion in dot patterns is discussed.

1 Introduction

The use of dot (point) patterns for the study of visual form perception was pioneered by French (1953) in an adaptation of earlier experiments on the discrimination of visual number (Taves, 1941). Significant innovations were introduced by Glass (1969) and by Marroquin (1976; see Earle, 1991) in the form of random and regular dot patterns, respectively, that are subjected to a geometrical transformation that is then superimposed on the original. Other novel applications were devised by Julesz (1964) and by Barlow and Reeves (1979) in studies of the perception of stereoscopic depth perception and of symmetry. Successions of such patterns, dubbed random dot kinematograms, were first employed by MacKay (1967), Anstis (1970) and Braddick (1973) in the study of form and motion perception. To date, an estimated 850 papers have been published using some form of dot pattern to investigate these different aspects of visual perception.

Although there seems to be little explicit recognition of this in the psychology or neurophysiology of human visual perception, the statistical properties of both random and constrained dot patterns have been extensively studied in a wide variety of other contexts where spatial statistics are of interest. Such contexts include geophysics, ecology, anthropology, botany, geography, and zoology, as well as the unifying theoretical fields of geometric probability and the statistics of spatial point processes (e.g., Bartlett, 1975; Boots & Getis, 1988; Cressie, 1993; Diggle, 1983; Kendall & Moran, 1963; Pielou, 1977; Ripley, 1981; Upton & Fingleton, 1985; 1989). In particular, one form of well-developed relational analysis, termed *nearest* *neighbour analysis*, appears to be highly relevant to the human ability to detect structure and motion in dot patterns, random or otherwise. The purpose of this paper is to present a preliminary exploration of the importance of nearest neighbour relations for such perceptual achievements.

1.1 Previous evidence for the importance of nearest neighbours in the perception of visual structure

The hypothesis that the perception of organisation in spatial point patterns is largely determined by nearest neighbour relations has been foreshadowed in a number of papers. These include studies of the discrimination of random dot patterns, the detection of structure in Glass patterns, and the perception of organisation in arbitrary dot patterns.

1.1.1 *Discrimination of dot patterns.*

Instead of varying the number of randomly positioned dots, French (1953) varied the location of a single dot between the two members of a pair of otherwise identical random dot patterns. Over successive pairs, the number of dots in each pattern was varied from 2 up to 7 and participants were required to categorize the two patterns of each pair as “Same” or “Different”.

One main finding by French was that the proportion of correct “Different” responses correlated reliably with the logarithm of the average separation of the dots within each pattern. French pointed out that, with only two dots, a participant can do little more than detect a difference (in distance or direction) between the two dots. However, with three dots, there are three distance (and direction) relations that can serve as cues. As the number of dots in a pattern increases, the number of relations modified by the displacement of a single dot also increases. Specifically, where R_t is the total number of (distance and/or direction) relations, R_m is the number of modified relations, and n is the number of dots, the relative proportion, P_m, of modified relations is given by

$$P_m = \frac{R_m}{R_t} = \frac{n-1}{n(n-1)/2} = \frac{2}{n}$$

French’s (1953) analysis remains preliminary and speculative. However, we believe it is insightful and useful because it draws attention to the importance of the totality of relations within a point pattern and suggests that any dot displacement is seen relative to the entire distribution of inter-dot distances.

Recently, an effectively similar conclusion has been reached independently by Hirsch and Mjolsness (1992) and in apparent ignorance of French’s study. These investigators employed the same kind of discrimination task as French (1953), but varied not only the total number of dots but also the number of dots displaced and the display size. Hirsch and Mjolsness were able to reject a number of plausible, locally-based models, which can be interpreted as a strong indication that the mechanism underlying discrimination in this task depends on the entire distribution of relative dot distances and/or directions. Hirsch and Mjolsness found that their results were in agreement with predictions based on a globally computed centre-of-mass parameter – one of a number of parameters that depend on the entire distribution of dot locations – but stress that this agreement simply fails to rule out the centre-of-mass model, rather than confirming it.

1.1.2 *Perception of Glass patterns.*

A congruent suggestion was made by Caelli (1981) with reference to Glass patterns. Caelli proposed that the detection of structure in such patterns does not depend on absolute distances between dots but upon nearest neighbour distances for the corresponding members of each pair of dots, relative to the nearest neighbour distances for non-corresponding (random) dots. He argued that this explanation was consistent with the diminishing quality of perceived structure as the distance through which the dots were transformed is increased. He also argued that this explanation was *not* inconsistent with the observation by Glass (1969) that structure is still detectable even when several non-corresponding nearest neighbour distances occur below the distance between corresponding dot pairs.

Unfortunately, Caelli did not follow up this study with a formal, detailed analysis of the role of nearest neighbours in the perception of Glass patterns. As a result, his (1981) suggestion has received less attention than it deserves and a number of researchers have continued to argue (we believe mistakenly) that the perception of structure, despite the occurrence of intervening, randomly located nearest neighbours, shows that a mechanism based on nearest neighbours cannot account for Glass pattern perception (e.g., Dakin, 1997; Glass, 1979; Maloney, Mitchison, & Barlow, 1987; Stevens, 1978). As Caelli's (1981) Figure 1b shows (and as is intuitively obvious), the distribution of nearest neighbour distances for corresponding dots in Glass patterns (with perceptible transformational structure) has a different mean and variance from that for non-corresponding dots, and we have developed pattern-sensitive, cumulative statistics (Vickers, 2002; 2002), that enable us to differentiate sampled distances reliably as arising from one or the other distribution, even when several non-corresponding distances intervene.

1.1.2 *Perception of organisation in arbitrary patterns.*

Pomerantz (1981) asked participants to indicate how they saw various patterns, consisting of dots located at selected vertices of an underlying lattice structure (which was not shown). He found that the preferred organisation was almost invariably the one with the shortest total length for the connected structure. Each stimulus comprised a small number (<10) of dots, that were located at a regular subset of possible positions, and participants were required to connect all the dots. That is, participants were asked, in effect, to construct a spanning tree linking the dots. In computational geometry, a spanning tree is a structure that contains no loops and links all n vertices of a graph (or point pattern) with n-1 edges (Johnsonbaugh, 2001). Pomerantz's (1981) generalization can thus be interpreted as stating that participants organized the dots as if they were finding a minimum spanning tree (MST) (i.e., a spanning tree for which the sum of the edge lengths is a minimum).

Pomerantz (1981) went on to suggest that the organizing principle employed in this experiment is quite general and, for example, may underlie the way people organize clusters of stars into constellations. However, the MST hypothesis would not account for the segregation of constellations from the rest of the stellar background (since it requires that all stars be connected by a single structure). What seems to be required therefore is some principle that can lead to the identification of clusters as well as accounting for the internal structure of a cluster.

One answer to this problem is suggested by evidence that human beings are remarkably efficient at discerning another type of minimal structure in point patterns – finding a path that begins at one vertex, visits all vertices exactly once, finishes at a specified vertex, and is as short as possible. Human beings typically produce near-optimal solutions to visually presented realizations of this computationally intractable optimization problem, familiarly known as the Travelling Salesman (or Salesperson) Problem (TSP) (MacGregor, Ormerod, & Chronicle, 2000; Polivanova, 1974; Vickers, Butavicius, Lee, & Medvedev, 2001).

In a recent study, Vickers, Lee, Dry and Hughes (submitted) found evidence that human solutions to TSPs were consistent with the hypothesis that participants effectively avoided paths that cross. This conclusion is consistent with the notion that the perception of point patterns is largely determined by the configuration of nearest neighbours. Paths that cross cannot constitute least distances between neighbouring points, and hence would be avoided - in effect, at least, if not with explicit intention. In agreement with this interpretation, Vickers, Bovet, Lee and Hughes (submitted) found that performance in both open and closed versions of the TSP was well accounted for by the proportion of links that an algorithm, based on successively joining nearest neighbours, shared on average with the optimal solution. (In "open" versions of the TSP, the starting and finishing vertices are different while, in "closed" versions, they are the same.) Despite its simplicity and total absence of planning ahead, this nearest neighbour algorithm is known to provide quite good solutions to the TSP.

MSTs necessarily contain all nearest neighbour links (though they may also contain other links that join separate nearest neighbour clusters). Therefore, the hypothesis that the perceived structure in a point pattern is determined by nearest neighbour relations among the points is also consistent with Pomerantz's suggestion that perceived structures (in very sparse, constrained point patterns) conform to what amount to MSTs. In fact, the *only* links in the patterns shown by Pomerantz (1981) are between nearest neighbours.

Within the field of automatic pattern recognition, a comprehensive analysis of the (machine) detection of Gestalt clusters in arbitrary point sets in terms of nearest neighbours and MSTs has been presented by Zahn (1971). Unfortunately, although Zahn called for a program of research in psychology to investigate the relationship between the human perception of simple organisation and graph-theoretical algorithms that achieve an economical description of a spatial point pattern, his thorough and suggestive analysis seems to have been overlooked by psychologists and neurophysiologists working in visual perception.

1.2 Nearest neighbour analysis

An important structural characteristic of an analysis of nearest neighbour (and other) relations in spatial point patterns is that it can differentiate between two major classes of theoretical explanation in visual perception: the prevalent one, based on classical receptive fields and scale-bound detector mechanisms that respond on the basis of absolute size; and a more recent alternative, based on receptive field plasticity, normalisation and relative distance. A critical instrument in this differentiation is the observation that the distribution of all inter-dot distances in a random dot array has different properties from those of nearest neighbour distances.

1.2.1 *Inter-dot distances.*

For example, for a circular pattern, with unit radius, in which n dots have been randomly located with uniform probability, there are $n(n$-1)/2 inter-dot distances. The cumulative probability distribution of inter-dot distances, t, is given by

$$1+\pi^{-1}\{2(t^2-1)\cos^{-1}(t/2)-t(1+t^2/2)\sqrt{(1-t^2/4)}\} \qquad (0 \le t \le 2)\,.$$

The mean, $E(t)$, is given by

$$E(t)=\frac{128}{45\pi} \tag{1}$$

and the standard deviation, σ_t, by

$$\sigma_t=\sqrt{\frac{-\frac{16384}{2025\pi}+\pi}{\pi}} \qquad (2).$$

This distribution is discussed by Diggle (1983, pp. 11-12 and by Kendall & Moran, 1963, pp. 41-42) and is illustrated in Figure 1a. The point of immediate importance is that the above expressions have no parameter that refers to the number of dots. Hence, the distribution of all inter-dot distances, and consequently both the mean and standard deviation, remain unchanged with increases in the dot density, $h = n/A$, where A is the area.

Insert Figure 1 about here

Because the distribution of inter-dot distances remains invariant with changes in dot density, it follows that, if participants identify links on the basis that dot pairs are closer than some absolute distance, d, then the number of links they identify should increase as a constant proportion of the total number of inter-dot pairs, $n(n$-1)/2. However, the mean length of the links identified should remain constant, irrespective of the number of dots in a pattern.

1.2.2 *Nearest neighbour distances.*

In contrast, for a similar pattern, there are n nearest neighbour distances, of which a proportion (approximately 0.62) are reflexive nearest neighbour distances, where a point P is the nearest neighbour of point Q and Q is the nearest neighbour of P (Clark & Evans, 1955). The probability, P_r, that the nearest neighbour of any point (in the centre of a circle of radius r), lies within an annulus between r and $r + \delta r$ is equal to the joint probability of there being no point within r and at least one point within the annulus. This probability has been shown by Clark and Evans (1954) to be given by

$$2\pi rhe^{-\pi r^2 h}\delta r$$

As shown by Thompson (1956), this result can be generalized to provide expressions for the probability that the k^{th} nearest neighbour is distant r_k from the centre. This probability, $P(r_k)$, is given by

$$P(r_k) = 2\lambda^k e^{-\lambda r_k^2} r_k^{2n-1} \delta r_k / (n-1)!$$

where $\lambda = \pi h$. The mean distance, $E(r_k)$, to the k^{th} nearest neighbour is given by

$$E(r_k) = \frac{1}{\sqrt{h}} \frac{(2k)!k}{(2^k k!)^2} \quad (3)$$

When $k = 1$, Equation (3) simplifies to

$$E(r_1) = \frac{1}{2\sqrt{h}} \quad (4),$$

and the variance is given by

$$V(r_1) = \frac{(4-\pi)A}{4\pi n^2} \quad (5).$$

The above equations for nearest neighbour distances assume unbounded patterns. Edge corrections are necessary and are discussed by Boots and Getis (1988, pp. 39-45). However, such corrections do not materially alter the shape of the distribution or its underlying logic.

A typical distribution is illustrated in Figure 1b. Equations 4 and 5 imply that both the mean and variance of the distribution of nearest neighbour distances decrease as dot density increases. Therefore, if participants link together nearest neighbours to form clusters, the number of links identified should increase as a constant proportion of n. However, the mean length of links identified should decrease as a function of $1/\sqrt{n}$, and should be a linear function of the mean nearest neighbour distance.

On an absolute distance hypothesis, the probability of identifying a cluster of size s is equal to the joint probability of encountering two or more inter-dot distances that are less than some critical amount, d. Because of this, the number of clusters identified of a given size (as well as the number of clusters overall) should be a constant proportion of $n(n-1)/2$. In addition, because the mean length of identified links should be independent of n, the mean length of identified clusters should also increase as a function of $n(n-1)/2$.

In contrast, on the nearest neighbour hypothesis, the number of clusters should increase as a linear function of n, while the mean length of each cluster should decrease as dot density is increased. Specifically, for a constant area, the mean length of identified clusters should decrease as a function of $1/\sqrt{n}$.

Against this background, we carried out three studies to explore the potential of an analysis in terms of nearest neighbours to account for the perception of structure and motion in random dot patterns. The first could be regarded as a ‘field study’, and was intended to examine the applicability of nearest neighbour analysis in a familiar situation that is often cited as a paradigmatic, real-life example of perceptual organisation (though never actually investigated). The two others were conventional laboratory experiments.

2 Experiment 1

Since 1928, the International Astronomical Union has recognized 88 constellations. These comprise the 48 mentioned by Ptolemy in his *Almagest*, together with some 40 minor and southern hemisphere constellations, added by navigators, astronomers and map makers between the 16th and 18th centuries (Bakich, 1995).

Whereas ancient civilizations projected mythological figures on to the constellations as a mnemonic device, in modern star atlases "it is preferred merely to join up the brightest stars" (de Callataÿ, 1958, p. 14). Although the way in which this done is rarely made explicit, a commonly avowed aim is to make the drawings in the atlas "correspond to those which [the] eyes form, naturally and instinctively, when they look at the starry sky" (de Callataÿ, 1958, p. 14). Even a cursory examination of a representative sample of current star atlases shows a fair degree of consensus as to what these 'natural and instinctive forms' are.

Because, under optimum conditions, there are some 9000 visible celestial objects, the 88 constellations can be viewed as the most readily identifiable configurations, or best Gestalten, in the celestial sphere. (An additional attraction is that each 'stimulus' has a unique history and a rich associated mythology.) It is therefore of some interest to attempt to articulate the algorithm(s) that might underlie such universally recognized configurations.

2.1 Method

A subset of 30 constellations, as represented in the Redshift 3 Desktop Planetarium from Maris Technologies, was selected from the 48 originally identified by Ptolemy. The criteria for selection of a constellation were that it should include a reasonable number of stars (8 or more), and that the constellations have a more complex structure than a simple, linear one. These criteria were applied because sparse and linear structures suggest few, if any, alternative patterns, and could be viewed as biased in favor of our experimental hypothesis.

For each constellation, the reflexive nearest neighbour, nearest neighbour, and second-nearest neighbour links were identified. The MST structure was also found, using Kruskal's algorithm (Johnsonbaugh, 2001). The links identified by these procedures were then compared with those represented in Redshift 3.

2.2 Results

The results are summarized in Table 1. The number of links in MST solutions is necessarily equal to the number of points (stars) minus one, but is included in Table 1 for completeness and convenience. Similarly, the number of links in the Redshift 3 representations is usually equal to the number of points (±1). Thus, the number of links in MST solutions corresponds very closely to the number in RS3 representations and is almost perfectly correlated with it (Pearson $r = 0.99$; N = 30).

Insert Table 1 about here

Table 1 also shows the numbers of links common to the RS3 representations and to various calculation procedures. The great majority (87%) of the MST links are also links in the RS3 representations and the great majority of RS3 links (83%) are also MST links.

To evaluate the significance of this overlap, consider that, for each constellation of n points, there are $k_t = n(n-1)/2$ possible links. Let k_c be the number of links that occur in RS3, so that the proportion of the total number of possible links that are RS3 links is given by $p = k_c/k_t$. Then the probability that m MST links will also be RS3 links is given by

$$\binom{n-1}{m} p^m (1-p)^{n-1-m}.$$

The probability of there being m or more links in common can be found by summing values of the above expression from m to n-1. Values of the logarithm of the odds against the observed results (i.e., the numbers of links common to RS3 and MST), for each constellation, are given in the right-hand column of Table 1. The statistical significance of the results can be gauged by considering the typical constellation, Gemini, which has 15 stars, and is represented by 15 links in RS3, of which 12 are shared with the corresponding MST. The logarithm of the odds against this degree of sharing arising by chance is 19.3 (i.e., over 200 million to 1).

Insert Figure 2 about here

A second feature of the results is that virtually all (96%) of the links that constitute reflexive nearest neighbours occur in the RS3 representations, while the proportion of nearest neighbour links that occur in RS3 is almost as high (91%). Similarly, 87% of the MST links are RS3 links. In other words, the RS3 representations use virtually all of the nearest neighbour links (even more proportionately than of MST links). As illustrated in Figure 2, the very small proportion of exceptions are additional links that form loops and seem likely to have been made in response to some perceived symmetry.

3 Experiment 2

The results of our first study provide evidence that human beings perceive natural clusters in patterns of random points on the basis of nearest neighbour relations and that they represent given configurations of points by constructing close approximations to MSTs that assemble individual clusters into a parsimonious larger configuration. In order to explore the role of nearest neighbours more widely, two further studies were carried out. Twelve participants carried out the two experiments in different random orders.

Experiment 2 investigated the role of nearest neighbours in the identification of clusters in a random dot pattern. There are two main theoretical explanations for the number and size of links and clusters identified by participants and for the selection of particular dots as belonging together. The first is that all dots within a certain absolute distance, *d*, from each other are seen as linked. The second is that the links perceived by participants correspond to nearest neighbour links, and hence depend on relative distance. The first hypothesis could be interpreted in terms of a Gestalt principle of proximity and would also describe the operation of a scale-bound detector mechanism. The second is derived from an analysis of nearest neighbour relations. As detailed above, the two hypotheses make very different predictions.

Insert Figure 2 about here

3.1 Method

3.1.1 *Stimuli*

The stimuli consisted of 21 random dot patterns, similar to those in Figure 4. For each pattern, the *x*- and *y*-coordinates were selected randomly and independently, according to a uniform probability distribution. Each pattern was presented on a 19-inch computer display as black dots on a white background.

3.1.2 *Design*

The 21 patterns consisted of 7 groups of 3. The patterns in different groups had 10, 20, 30, 40, 60, 80, or 100 dots, randomly located within a 7×7 inch square (considered as theoretically divisible into 5000×5000 units), according to a uniform distribution. Within each group, the 3 different patterns were independently generated. The 21 random dot patterns were presented as 3 successive blocks of 7. Within each block, each number of dots was presented once in an order that was random. Each successive block employed a different random order and the orders of presentation were different for each participant. The 21 presented patterns were different for each participant.

3.1.3 *Procedure*

Participants were instructed to examine each pattern, to identify any individual clusters of dots that were seen as belonging together and to indicate this by using the computer mouse to join up the dots constituting each cluster (like identifying constellations in the night sky). For this purpose, participants were provided with a second computer screen, side by side with the first, on which a separate copy of the pattern was presented. (The reason for this was to allow participants to revise later identifications without being determined by their earlier responses.) The experiment took 15-25 minutes.

3.1.4 *Participants*

There were 12 participants, recruited opportunistically from the university student population. Participants received a payment of A$10 for their participation.

3.2 Results

The number of links identified by participants and the number identified by the model both increase as a linear function of the number of dots (with *y*-intercepts of 0.687 and 2.876 and slope coefficients of 0.576 and 0.632 and r^2 values of .988 and .998, respectively). Figure 3a shows the mean number of links, identified by participants in each pattern group, plotted as a function of the mean number of links between nearest neighbours in each pattern group. The relation between the participants' and the nearest neighbour model's number of links is well described by a straight line (with *y*-intercept = -2.780, slope coefficient = 0.937, and with r^2 = 0.997). In contrast, a proximity principle, expressed in terms of some critical minimum distance between dot pairs, would predict that many more links would be perceived as dot density is increased. These data clearly rule out any such hypothesis.

Insert Figure 3 about here

Both the mean length of links identified by participants and the mean length of links identified by the nearest neighbour model increase as a function of $1/\sqrt{n}$ (with *y*-intercepts of 57.6 and 22.4 and slope coefficients of 2271 and 2234 and r^2 values of .996 and .992, respectively). Figure 3b shows the mean length of identified links (averaged over all clusters and all participants), plotted against the mean length of the links identified by the nearest neighbour model. The relation between the participants' and the model's mean link lengths is well described by a straight line (with *y*-intercept = 58.087, slope coefficient =0.752, and with r^2 = 0.993). These data are inconsistent with a scale-bound filter mechanism or proximity principle, which predict that the mean length of identified links should remain constant as the number of dots is increased.

Both the empirical and the predicted numbers of clusters increase as a linear function of *n* (with *y*-intercepts of 1.569 and 1.201 and slope coefficients of 0.146 and 0.284 and r^2 values of .993 and .998, respectively). Figure 3c shows the mean number of clusters (of two or more dots), identified by participants, plotted against the mean number of clusters identified by the nearest neighbour model in each pattern group. The relation between the participants' and the model's numbers of clusters is well described by a straight line (with *y*-intercept = 0.991, slope coefficient = 0.513, and with r^2 = 0.992). These data are not well accounted for by a proximity principle, based on absolute distance, which predicts that many more clusters should be perceived as dot density is increased.

Both the empirical and the predicted mean cluster lengths increase as a function of $1/\sqrt{n}$ (with *y*-intercepts of 973.972 and –239.988 and slope coefficients of 4498.537 and 8557.842 and r^2 values of .782 and .984, respectively). Figure 3d shows the mean length of the clusters identified by participants, plotted against the mean length of clusters identified by a process of linking nearest neighbours. The relation between the participants' and the model's mean cluster lengths is approximately described by a straight line (with *y*-intercept = 1153.625, slope coefficient = 0.483, and with r^2 = 0.778). These data are inconsistent with a proximity principle, which predicts that the mean length of clusters should *increase* as the number of dots is increased.

The data summarized in Figures 3a through 3d are all clearly inconsistent with an organizing process based on a detector mechanism or proximity principle that is defined in terms of absolute distance. In contrast, the data are very close to what is predicted by a simple algorithm that joins dots that are nearest neighbours. For example, Figure 4 shows responses made by a typical participant to patterns containing 30 and 100 dots.

Insert Figure 4 about here

At the same time, as just noted, there are certain systematic differences between some data and the predictions of a nearest neighbour algorithm. First, the numbers of links and of clusters, identified by participants, falls about 17% and 39% short, respectively, of the numbers predicted by the theoretical algorithm. A likely explanation is suggested by the finding that the mean length of the nearest neighbour links identified by participants was 412 (sd = 178). In contrast, the mean length of the nearest neighbour links that participants failed to identify was 720 (sd = 316). The latter figure is about 1.75 times as large as the former. This suggests that participants may not perceive these links because they fail to distinguish them from other, random inter-dot distances.

A slightly different possibility is that participants *in this task* consider closer nearest neighbours to be more salient than more distant nearest neighbours. Because the task was to identify (an unspecified number of) individual clusters in each pattern, rather than to provide a link for every point in the pattern, it may be that participants achieve this by discounting those points that are situated at markedly greater distances from their nearest neighbours. There are quite large individual differences in the extent of this discounting, but it is not possible, in this experiment, to tell whether these are determined by differences in perception or in motivation.

A second difference, seen in Figure 3d, is that observers tend to produce longer clusters than predicted when the number of dots is medium or high. In part, this seems due to observers including a greater number of second-nearest neighbours (possibly in error). However, it also seems due to observers including more distant points that are collinear or that make a cluster symmetrical.

4 Experiment 3

In addition to the perception of static stimuli, random dot patterns (presented in rapid succession) have been used widely to study the perception of regular movement. For example, Williams and Sekuler (1984) studied sequences of RDKs in which the dots took independent random walks of constant size from one frame to the next. These authors investigated the effects of varying the step size of the random walks, dot density, display duration, and the range of directions taken by the steps on the probability of reporting the perception of coherent motion in the direction of the mean of the distribution of directions. Apart from the brief early report by MacKay (1967), however, no one, so far as we know, has investigated the ‘purely random’ motion that is experienced when a rapid succession of independently generated random dot patterns is viewed.

Nevertheless, it is instructive to examine such displays. Although the motion experienced seems well described as ‘random’, this shorthand description is misleading. It is most true when there are two or three dots only that dart around the entire display like frantic flies. It is least true when there is a large number of dots in each successive frame. In this condition, the dots are seen as ‘jiggling around’ within relatively restricted spaces. Logically, there is no reason why any dot in one frame should not be seen as moving to the location of any dot in a successive frame. If we perceived this, we should indeed be perceiving truly random motion. However, just as we perceive clustering in static dot patterns that are generated by a uniform random process, what we perceive in the case of dynamic random dot patterns is motion that, although irregular, is nevertheless highly constrained. In particular, as noted much earlier by Kolers (1972, p.77), the random paths followed by different dots are never seen as intersecting.

Insert Figure 5 about here

Since the paths between nearest neighbours cannot intersect (otherwise they would not constitute least distances), this suggests that nearest neighbours may also play an important role in the perception of motion in dynamic random dot patterns. Equation 4 implies that the mean nearest neighbour distance is a decreasing function of dot density, and, for a constant area, should vary as a function of $1/\sqrt{n}$. If motion is perceived between a dot in one frame and its nearest neighbour in an immediately subsequent frame, then we should expect the extent of the motion seen to decrease as the number of dots in each frame is increased. In contrast, a set of motion detectors that respond to motion over a range of distances, but that are sensitive to motion between *any* pair of successive dots, would (on average) register movement through the same mean distance, independent of the dot density. Thus, this situation constitutes a boundary condition that has important implications for accounts of the perception of apparent motion generally. The following experiment was carried out to examine the basis for these implications.

4.1 Method

4.1.1 *Stimuli*

Participants viewed sequences of random dot patterns, consisting of black dots on a square white background. As in the previous two experiments, the sequences were presented within an 7×7 inch square on a 19 inch computer screen. Each frame was presented for 200 msec, with less than 20-30 msec between successive frames.

4.1.2 *Design*

Each participant viewed a total of 70 separate sequences. There were 7 different types of sequence, with each type containing either 5, 7, 10, 14, 30, 80, or 650 dots, and there were 10 different examples of each type of sequence. The different numbers of dots were chosen to give approximately equal differences in the mean distance between nearest neighbours in the different types of pattern.

Within each sequence type, the 10 different sequences were independently generated. The sequences were presented as 10 successive blocks of 7. Within each block, each number of dots was presented once in an order that was random. Each successive block employed a different random order and the orders of presentation were different for each participant. The 70 presented sequences were different for each participant.

4.1.3 *Procedure*

Participants were asked to watch each sequence and to judge how far they saw the dots move on average within that sequence. Participants indicated the perceived distance by clicking the mouse at any position within a rectangular window at the foot of the display, dragging the mouse through the perceived distance and then releasing the mouse button. Because the motions seen were in random directions, there was no requirement to draw horizontal distances. The sequences were continued until the participant made a response.

4.1.4 *Participants*

Ten of the participants who took part in Experiment 2 also took part in Experiment 3.

4.2 Results

Both the distance, through which the dots are seen to move, and the theoretical mean nearest neighbour distance both increase as approximately linear functions of $1/\sqrt{n}$ (with y-intercepts of 1149.702 and –59.985 and slope coefficients of 2746.844 and 3167.332 and r^2 values of .956 and .996, respectively). Figure 5 shows the relation between the participants' mean judged motion and the theoretical mean nearest neighbour distance. The relation is quite well described by a straight line (with y-intercept =1210.936, slope coefficient = 0.855, and with r^2 = .932). This relationship is inconsistent with the predictions of an array of scale-bound motion detectors, according to which the extent of perceived motion should be independent of dot density.

5 Discussion

We begin by discussing the more general interpretation of our results and their implications for the perception of structure in random dot patterns and in Glass patterns. We then go on to discuss the relevance of our results for the perception of 'random' and coherent motion in RDKs.

5.1 The detection of structure in random dot patterns

The approach to visual perception that is suggested by our results has points of similarity both with the holistic emphasis of Gestalt theorists and with Gibson's (1950; 1966) theory of ecological perception. In common with the former, our approach focuses on the intrinsic structure among the stimulus elements (as opposed to the optical structure studied by Gibson) and it considers the perception of such structure to depend on the totality of distance relations among the elements constituting an array, but particularly on nearest neighbour relations. Both Experiments 1 and 2 provide evidence that human beings perceive clusters in patterns of random points on the basis of nearest neighbour relations, and that, when required, they represent larger configurations of clusters in the form of close approximations to MSTs that include most nearest neighbours. What is perceived as a 'natural' link depends upon what other links are possible in the pattern.

In agreement with the nearest neighbour hypothesis, the number of links and the number of clusters identified by participants increased as a linear function of the number of dots, the mean link length was determined by the mean distance between nearest neighbours, and the mean length of each cluster decreased as dot density was increased. None of these results is consistent with a detector mechanism or proximity principle based on absolute distance.

As pointed out by Zahn (1971) nearest neighbour relations and MST solutions have several advantages as accounts of the visual perception of structure. First, they are consistent with the Gestalt notion that the visual system organizes stimulus information in a way that is optimum in a clearly definable sense. Second, they lead to algorithms that tend to be independent of the order in which the dots are considered (and hence of the pattern of eye movements and the way in which a pattern is scanned). Third, nearest neighbour structures usually have a considerable degree of redundancy, and so are relatively insensitive to small to moderate amounts of noise introduced uniformly over the point array. Finally, nearest neighbour distances and MST solutions are invariant under similarity transformations (translations, rotations, reflections and dilations) and are effectively invariant under shears.

The property of invariance under transformation distinguishes the nearest neighbour approach from neurophysiological approaches that assume detectors that are tuned to specific distances and/or directions. At the same time, however, it underlines a correspondence with the ecological approach to perception that many (especially Gibson) might find surprising. As briefly indicated at the beginning of this paper, spatial point processes are of central interest in a wide variety of scientific fields, particularly those concerned with our physical environment. Sensitivity to clustering, regularity and other forms of structure in such processes is therefore an achievement that, we would argue, has long had considerable biological utility. Moreover, it is particularly advantageous for this sensitivity to be linked to invariant relations among the elements of the relevant physical process. Because of this, the perception of structure can be independent of accidental variations in the way a pattern is sampled, can survive the intrusion of other random effects, and is robust in the face of a variety of transformations of the seen image. In other words, we are proposing that, like optical structure, the visual perception of intrinsic (Gestalt) structure also has a strong ecological basis.

5.2 The perception of Glass patterns

As noted earlier, a number of researchers have concluded (we believe mistakenly) that the perception of structure in Glass patterns cannot depend on an analysis in terms of nearest neighbours. An adequate analysis of Glass patterns in terms of nearest neighbours would require a substantial article in its own right, and we are currently working on this. However, it may be helpful to indicate here why we believe that an explanation of Glass pattern perception in terms of nearest neighbours has particular advantages.

Human beings can perceive structure in Glass patterns that have been generated by a wide range of transformations. These include translations in the vertical, horizontal and diagonal directions (Wilson, Loffler, Wilkinson, & Thistlethwaite, 2001), as well as rotations, expansions from a centre, and spiral and hyperbolic transformations (Wilson & Wilkinson, 1998). In the case of dilations and rotations, human observers are sensitive to transformations by a constant amount, as well as by amounts that increase with distance from the centre of dilation or rotation.

Moreover, people can discriminate and identify all of these transformations directly and spontaneously, without training or practice. That is, our perceptual abilities in this respect are much more impressive than might be inferred from most experiments in the field, in which participants often merely indicate whether a given stimulus is structured or random.

We believe that our ability to discriminate and identify such a rich variety of transformational structure is difficult or impossible to model parsimoniously by means of different families of overlapping detector units that then send information to higher-order units responsible for detecting global structure. Such models tend to be scale-bound and to require the tuning of many different parameters even to discriminate between random patterns and those with a single type of transformational structure, such as rotation (Wilson & Wilkinson, 1998; Wilson, Wilkinson, & Assad, 1997). In contrast, the information contained in the distribution of nearest neighbour distances will always serve to differentiate a structured pattern from a random one, and to identify the type of structure in question. Moreover, this information is resistant to noise, is invariant over similarity transformations, and is not scale-dependent. All that is required to model performance in a particular experimental task is to assume that this information is fed into an appropriately designed decision process.

More specifically, from a nearest neighbour perspective, the perception of structure in Glass patterns depends crucially upon discriminating the distribution of nearest neighbour distances for corresponding dots from that of nearest neighbour distances for non-corresponding dots. Because this involves a process of detecting a signal in noise, and because the mean and variance of the distribution of nearest neighbour distances decrease as dot density is increased - but increase as the transformation distance is increased - we believe, in agreement with Caelli (1981), that this approach is capable of explaining the interacting effects of dot density, noise, and the separation between corresponding dots in Glass patterns.

5.3 The perception of motion in random dot kinematograms

The results of Experiment 3 demonstrate that the distance through which dot elements are seen to move in RDKs depends upon the mean distance between nearest neighbours in successive random dot patterns in an RDK. In other words, motion is perceived that follows a path from an arbitrary dot in one RDK frame to the position of the nearest dot in the immediately subsequent frame. This means that, in agreement with Kolers (1972, p.77), Attneave (1974), Navon (1976) and Ullman (1979, pp. 102-104), no crossed motion paths will be experienced. The dependence of perceived motion on nearest neighbour relations also means that the extent of the motion experienced will decrease as a function of dot density, as was observed.

The results in Figure 5 also raise two questions about the process relating nearest neighbour separations to the judged extent of movement. The first is why participants should indicate that the dots appear to move through a greater distance than the mean nearest neighbour separation (around 150% on average). The second question is why the relation depicted in Figure 5 appears to be slightly nonlinear.

It is beyond the intended scope of the present paper to develop a detailed model of the perception of motion in RDKs. However, we can suggest two factors that provide an answer, in general terms, to these two questions. First, some confusion seems bound to occur between nearest neighbour and other inter-dot distances (i.e., the perceptual system does not infallibly identify nearest neighbours). Second, for any pair of random dot displays, the mean distance between nearest neighbours can approach, but cannot exceed, the mean distance between all dots (2607 theoretical display units). The first factor can be expected to lead to some exaggeration in the extent of perceived motion (relative to the mean nearest neighbour distance), while the second factor may produce some non-linearity by imposing an upper limit to the distance through which the dots can be seen as moving – particularly in sparsely populated displays, where there is little difference between the distribution of nearest neighbour distances and that of all inter-dot distances.

Braddick (1973; 1974) has argued that the perception of apparent motion may be mediated by two distinguishable processes, which he termed the *short-range* and the *long-range* motion systems. One reason for this distinction is that, in classical studies of apparent motion, when a single, large-scale object is presented in two different alternating locations against a homogeneous background, smooth motion is perceived for displacements of many degrees of visual angle and for frame durations up to 300 msec (Anstis, 1980; Braddick, 1974; Neuhaus, 1930). For example, when the stimulus is a spot of light, apparent motion is experienced up to a stimulus onset asynchrony of up to 500 msec and over large angular distances up to tens of degrees.

The findings with single-element stimuli differ markedly from what was found in Braddick's (1973, 1974) experiments with RDKs. In these RDK experiments, the observer perceives a regular subset of dots as moving to and fro in a coherent manner against a background of dots in 'random' motion. Braddick found that, in order for observers to detect coherent motion, the displacements between successive frames had to be small (less than about 0.25° of visual angle) and the alternation rate had to be rapid (with stimulus durations less than about 80msec per frame). He suggested that these contrasting results could be explained by postulating a short-range process, responsible for performance with the RDKs, and a long-range process, responsible for the classical apparent motion experienced with very simple stimuli.

Nearest neighbour analysis does not address possible differences in the optimal stimulus onset asynchrony for seeing 'short-range' and 'long-range' motion. However, it does suggest that it may not be necessary to differentiate between two separate motion systems on the basis of the displacements seen between successive views.

When there is only a single stimulus, there is no limit in principle to the extent of the motion that can be perceived. However, as soon as two or more similar stimulus elements change location from frame to frame, the extent of perceptible motion is limited primarily by the distribution of nearest neighbour distances between the elements. For example, a single stimulus element can be perceived as moving along the entire diagonal length ($\sqrt{2} = 1.41$) of a square of unit side. However, when as few as two elements change location randomly within a similar square, the *average* distance through which the dots are seen to move should be limited to about $1/2\sqrt{2} = 0.35$ (neglecting edge correction). Because the extent of perceived motion is determined by the mean of the distribution of nearest neighbours, and because this decreases as a function of dot density, short-range and long-range motion may be understood, not as reflecting the operation of separate perceptual processes, but as two extremes of a single continuum of stimulus conditions.

5.4 The perception of coherent motion in noise

A striking achievement by the human visual system is the perception of the apparent motion trajectory of a single dot, moving in a constrained manner in a field of randomly distributed dots that move by a similar amount, but in random directions, from one frame to the next. For example, if a single dot is set to move along a straight or curved path from frame to frame in a field of dots moving along random paths, then, provided the background is not too heavily populated, an observer can detect the constrained motion and can specify its direction (to the left or right, clockwise or counterclockwise). Systematic investigations of this phenomenon have been carried out by Watamaniuk, McKee, and Grzywacz (1995) and by Grzywacz, Watamaniuk, and McKee (1995).

According to a nearest neighbour approach, coherent motion will be experienced when the distribution of coherent nearest neighbour distances is distinguishable from the distribution of random (non-target) nearest neighbour distances. Thus, as with Glass patterns, the probability of making a nearest neighbour mismatch is determined by the step size of the movements (transformations) and by the number of noise dots.

Williams and Sekuler (1984) found that, when the dots in successive RDK frames moved by equal amounts in a restricted range of randomly selected directions, then coherent motion was experienced in the direction that corresponded to the mean of the distribution of directions used to generate the RDK. Williams and Sekuler also found that the probability of perceiving coherent motion depended on the interacting effects of the step size of the movements and the density of the dots. Williams and Sekuler claimed that a nearest neighbour analysis could not explain these results in their entirety, because variations in dot density appeared to have no effect on the probability of reporting coherent motion when the motion step sizes were small. However, they do not present a detailed analysis of nearest neighbour relations. This is unfortunate, because the perception of coherent motion can be expected to be determined by the relationship between the (highly constrained) distribution of least distances (and directions) for corresponding dots and the distribution of nearest neighbour distances (and directions) for non-corresponding dots, and a great deal depends on the specific values that determine dot movement and density. On the other hand, the generally interacting effects of step size and dot density are a clear indication that nearest neighbour distances are strongly implicated in their experiments.

Consistent with a nearest neighbour explanation, Watamaniuk et al. (1995) found that the probability of detecting coherent motion appeared to be controlled by the probability of making a mismatch between the target dot in one frame and one of the dots that moved close to it in the immediately subsequent frame. Although Watamaniuk et al. (1995) and Grzywacz et al. (1995) do not propose an analysis based on nearest neighbours, we have successfully used such an analysis to model the detection of coherent motion by a single dot, subject to the constraint that information is integrated over successive pairs of frames only (Todd, 1994, p.362; Williams & Sekuler, 1984). As in the case of Glass patterns, an adequate account requires a separate paper, and we are currently working on this (Vickers, 2001; 2002).

6 Conclusions

We have proposed an approach to the perception of structure and motion in random dot patterns that is based on relations among neighbouring dots. On this approach, the perception of clusters depends predominantly on the detection of nearest neighbours and especially on that of reflexive nearest neighbours. Nearest neighbour analysis correctly predicts that the number of links and clusters detected will increase as a linear function of the number of dots in a random dot display and that the mean link and cluster length should be a linear function of the mean distance between nearest neighbours. Nearest neighbour analysis also accounts for the very high degree of overlap between the links identified by participants and those determined by nearest neighbours. (The main qualification is that participants tend to omit some of the longer nearest neighbour links.) In turn, the representation of arbitrary dot patterns, in the form of constellations, is found to conform closely to MSTs, in which individual clusters, based on nearest neighbours, are linked into the shortest possible tree structure.

This theory of the perception of structure is consistent with data and suggestions put forward previously by Pomerantz (1981) and Zahn (1971), as well as with conclusions from studies of human attempts to solve visually presented TSPs (Vickers, Bovet et al., submitted; Vickers, Lee et al., submitted), that underline the importance of least distances in the solution process. This approach is also consistent with findings by French (1953) and by Hirsch and Mjolsness (1992) regarding the importance of the totality of inter-dot relations in the discrimination of random dot patterns.

From a nearest neighbour perspective, the perception of structure in Glass patterns depends upon discriminating the distribution of nearest neighbour distances for corresponding dots from that of nearest neighbour distances for non-corresponding (random) dots. In agreement with Caelli (1981), we argue that this approach can explain the interacting effects of dot density, noise, and the separation between corresponding dots in Glass patterns.

The average distance, through which dots are seen to move in RDKs, is also shown to be an approximately linear function of the mean distance between nearest neighbours. In other words, motion is perceived that follows a path from an arbitrary dot in one RDK frame to the position of the nearest dot in the immediately subsequent frame. This means that no crossed motion paths will be experienced and that the extent of the motion seen will decrease as a function of dot density, as was observed.

On this nearest neighbour account, the distinction between short-range and long-range motion is interpreted as a differentiation between two extremes of a single continuum of element densities, rather than as evidence for two separate perceptual processes. Further, if the motion paths seen between successive RDK frames are between nearest neighbours, this would account for the finding by Watamaniuk et al. (1995) that the probability of detecting coherent motion is determined by the probability of making a mismatch between the target dot in one frame and one of the (non-target) dots that moved close to it in the immediately subsequent frame. The same analysis would also account, in principle, for the dependence of the perception of coherent motion on the number of noise dots and the step size of the coherent movements.

Taking into account all these actual and potential explanatory advantages, we consider that nearest neighbour analysis is an incisive tool that has much to contribute to the study of human visual perception. We do not claim that it will apply universally or without qualification. However, we believe that it holds promise as a useful instrument for constructing a unifying account of the diverse phenomena, associated with the perception of structure and motion, that have been explored by means of dot patterns.

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TABLE

Table 1. Analysis of 30 constellations represented in the Redshift 3 Desktop Planetarium (RS3). For each constellation, successive columns show, from left to right: (1) the number of stars; (2) the number of links represented in RS3; (3) the number of links in the corresponding minimum spanning tree (MST); (4) the number of links between reflexive nearest neighbours (RNN); (5) the number of nearest neighbour links (1NN); (6) the number of nearest or second-nearest neighbour links (2NN); (7) the proportion of the MST links that are nearest or second-nearest neighbour links; (8) the proportion of the MST links that occur in the RS3 representation; (9) the proportion of RNN links that occur in RS3; (10) the proportion of nearest neighbour links in RS3; (11) the proportion of RS3 links that are nearest or second-nearest neighbour links; (12) the proportion of MST links that are both nearest or second-nearest neighbour links and RS3 links; and (13) the (natural) logarithm of the odds against the possibility that the degree of sharing of links between MST and RS3 (indicated in column 8) might arise by chance.

FIGURES

Figure 1. Probability density functions for spatial point processes. Figure 1a illustrates the theoretical distribution of all $n(n-1)/2$ inter-dot distances in a square random dot pattern of side 5000 units (and containing 256 dots). Figure 1b illustrates the theoretical distribution of the n distances between each point and its nearest neighbour in a similar pattern. Note the differences in scale between Figures 1a and 1b..

Figure 2. Two typical constellations, their representations in RS3, the corresponding MST solutions, and the links detected by a process of joining nearest neighbours (1NN). Hercules is an example of an RS3 representation that shares more than the average proportion of links with the MST and 1NN processes. Ursa Major is an example of an RS3 representation that shares less than the average proportion of links with the MST and 1NN processes.

Figure 3. Comparisons between measures observed in Experiment 2 and those predicted by a process of linking nearest neighbours. The observed measures are: the mean number of links detected (Figure 3a); the mean length of detected links (Figure 3b); the mean number of clusters detected (Figure 3c); and the mean length of detected clusters (Figure 3d). These measures are plotted against the values predicted by a nearest neighbour analysis of the stimuli. Distances are expressed in theoretical units for a 5000×5000 display. (Note that, in Figures 3b and 3d, values on the x-axis, from left to right, correspond to large to small numbers of dots, respectively.)

Figure 4. Clusters detected by a typical participant, and by the NN detection process in random dot patterns. The participant's responses are shown in solid lines and the NN links are shown in dashed lines. Figures 4a and 4c show the participant's responses to a pattern containing 30 and 100 dots, respectively. Figures 4b and 4d show the NN links detected in these respective dot patterns.

Figure 5. The mean distance through which dots were seen to move in Experiment 3, plotted against the theoretical mean distance between successive nearest neighbours in RDKs of varying dot densities. Distances are expressed in theoretical units for a 5000×5000 display. (Note that values on the x-axis, from left to right, correspond to large to small numbers of dots, respectively.)

	1	2	3	4	5	6	7	8	9	10	11	12	13
Constellation	**No. of Stars**	**No. of RS3 Links**	**MST Links**	**RNN Links**	**1NN Links**	**2NN Links**	**MST & 2NN Links**	**MST & RS3 Links**	**RNN & RS3 Links**	**1NN & RS3 Links**	**2NN & RS3 Links**	**MST & 2NN & RS3 Links**	**Log odds against**
Andromeda	15	14	14	5	10	20	0.93	0.79	1.00	0.90	0.86	0.79	16.6
Aquarius	15	14	14	4	11	20	0.93	0.86	1.00	0.91	0.86	0.86	20.0
Aquila	11	11	10	4	7	14	0.90	0.90	1.00	0.86	0.73	0.80	11.5
Auriga	9	9	8	3	6	13	1.00	0.88	1.00	1.00	1.00	0.88	8.1
Bootes	15	15	14	5	10	17	0.86	0.86	0.80	0.80	0.80	0.71	19.3
Canis Major	14	13	13	3	11	20	0.92	0.92	1.00	0.91	0.92	0.85	21.0
Capricornus	11	10	10	5	6	14	0.90	0.70	1.00	1.00	0.80	0.70	7.6
Centaurus	24	24	23	8	16	32	0.91	0.83	0.88	0.88	0.79	0.74	37.8
Cepheus	10	10	9	2	8	12	0.89	0.78	1.00	0.75	0.70	0.67	7.6
Cetus	16	17	15	4	12	20	0.93	0.93	1.00	0.92	0.88	0.87	25.1
Cygnus	9	8	8	3	6	12	1.00	1.00	1.00	1.00	1.00	1.00	12.0
Draco	15	15	14	5	10	18	0.93	0.86	1.00	1.00	0.93	0.86	19.3
Eridanus	10	9	9	3	7	13	1.00	1.00	1.00	1.00	1.00	1.00	14.5
Gemini	15	15	14	5	10	18	0.86	0.86	1.00	0.90	0.80	0.79	19.3
Hercules	22	22	21	9	13	29	0.95	0.95	1.00	0.92	0.91	0.90	44.2
Hydra	17	17	16	5	12	22	1.00	1.00	1.00	1.00	1.00	1.00	33.4
Leo	16	16	15	4	12	21	0.93	0.87	1.00	0.83	0.81	0.80	21.9
Lepus	11	10	10	4	7	14	0.90	0.90	1.00	0.86	0.90	0.80	13.1
Libra	8	8	7	2	6	9	1.00	1.00	1.00	1.00	0.88	1.00	9.2
Lupus	11	11	10	3	8	15	0.90	0.70	1.00	0.75	0.82	0.60	7.4
Ophiuchus	16	16	15	4	12	22	0.87	0.87	1.00	0.92	0.81	0.80	21.9
Orion	20	21	19	9	11	26	0.95	0.84	0.89	0.91	0.86	0.84	29.0
Pegasus	15	15	14	5	10	18	0.93	0.79	1.00	1.00	0.80	0.79	16.1
Perseus	17	16	16	6	11	21	0.94	0.88	1.00	0.91	0.94	0.88	25.3
Pisces	12	11	11	4	8	15	1.00	0.91	1.00	1.00	0.91	0.91	15.7
Sagittarius	23	23	22	8	15	28	0.91	0.91	0.88	0.93	0.87	0.86	42.8
Scorpius	18	18	17	6	12	23	0.94	0.94	1.00	0.92	0.89	0.88	31.8
Taurus	15	14	14	5	10	20	0.93	0.86	0.80	0.90	0.86	0.79	20.0
Ursa Major	19	20	18	7	12	23	1.00	0.78	0.86	0.83	0.80	0.78	22.8
Virgo	15	15	14	5	10	18	0.93	0.79	1.00	0.90	0.73	0.71	16.1
Average	**14.8**	**14.6**	**13.8**	**4.8**	**10.0**	**18.9**	**0.93**	**0.87**	**0.96**	**0.91**	**0.86**	**0.83**	**19.3**

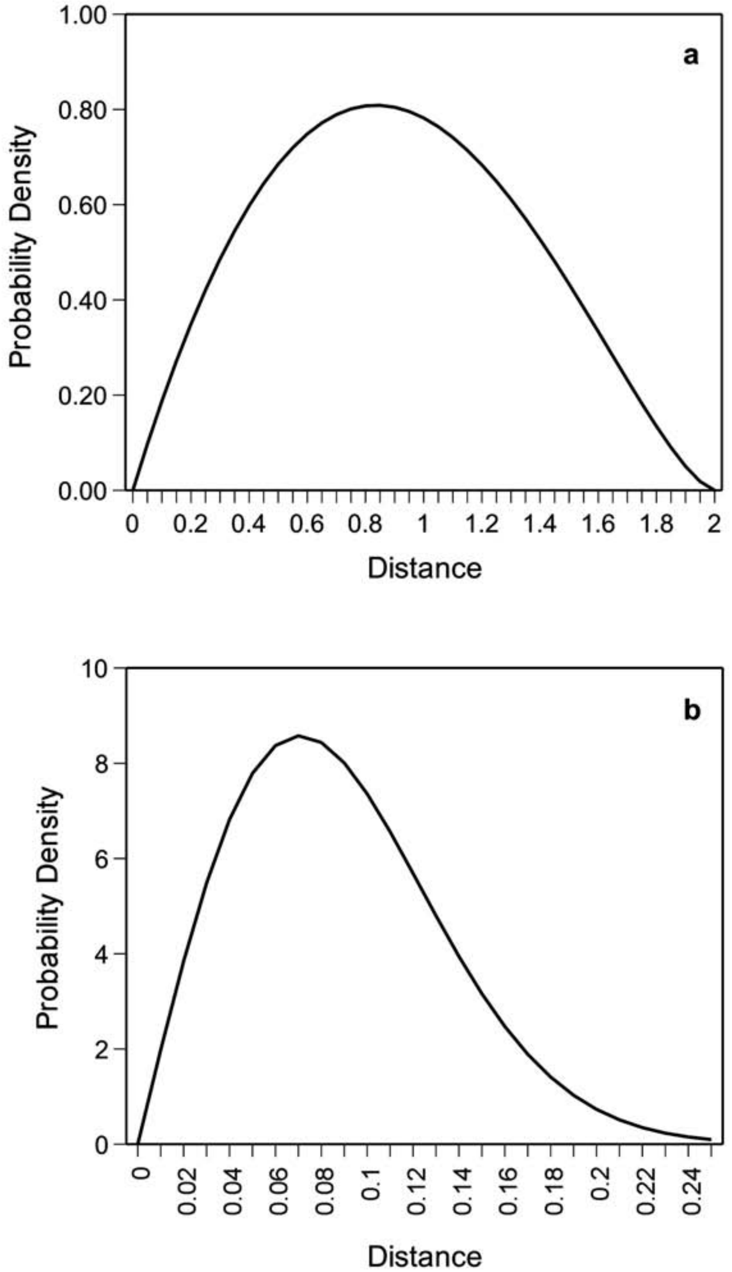
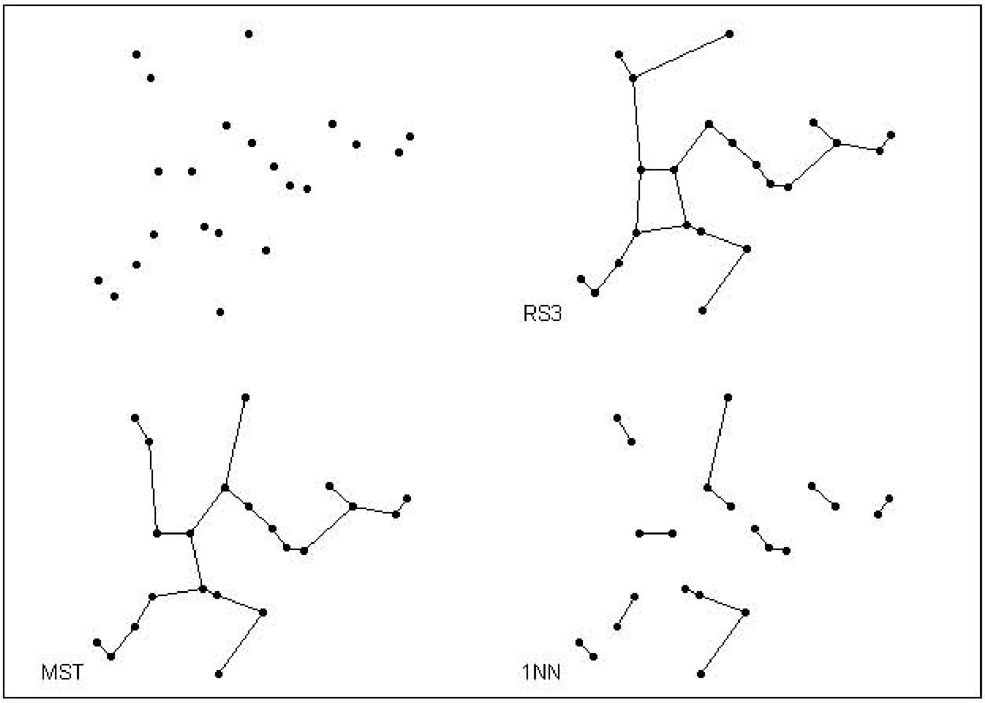
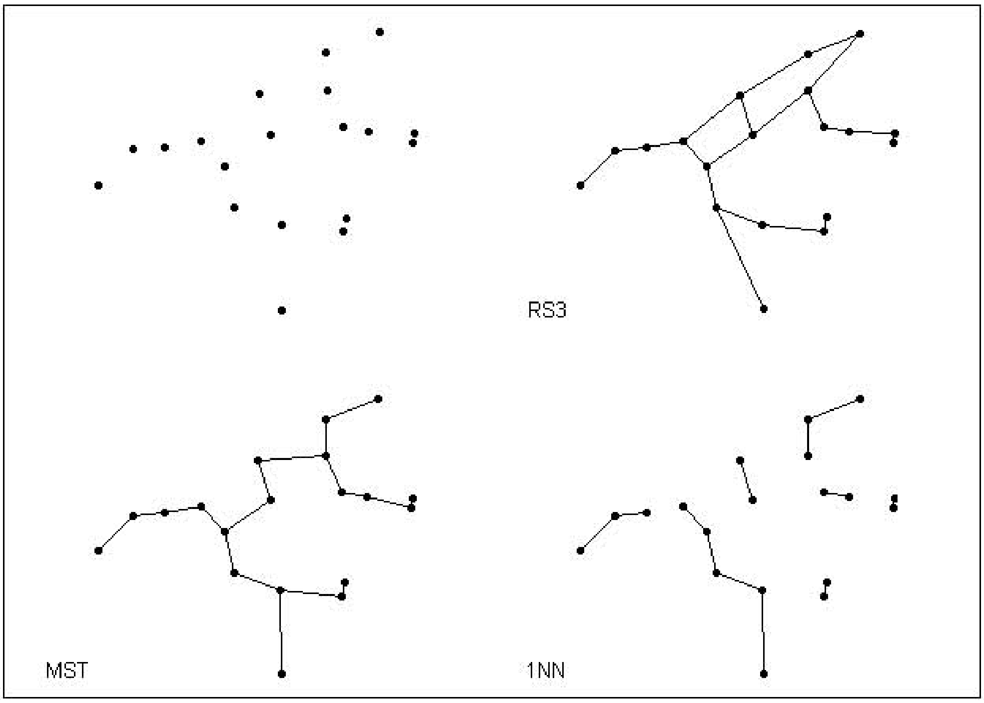


Figure 1. Vickers et al.



Hercules



Ursa Major

Figure 2. Vickers et al.

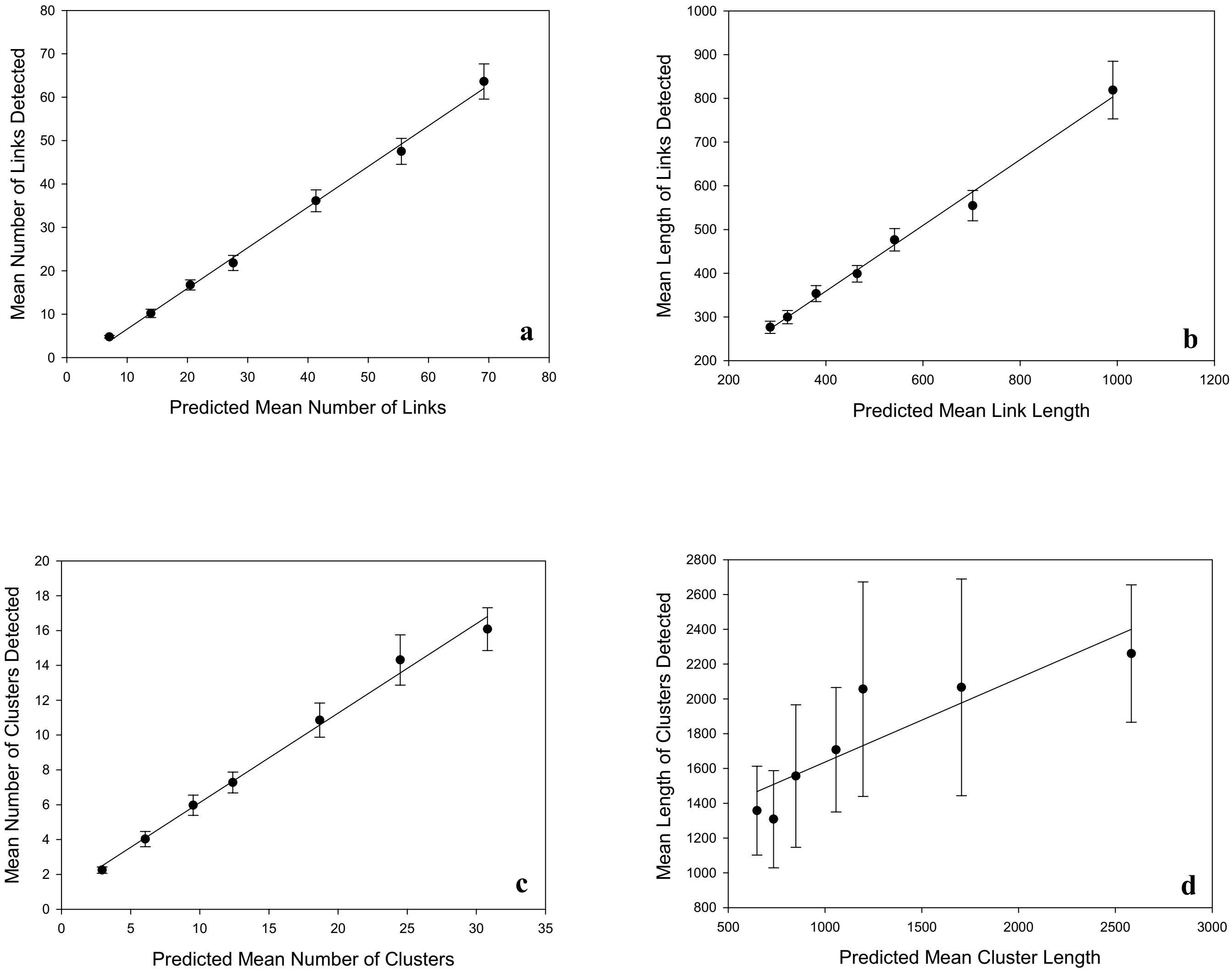


Figure 3. Vickers et al.

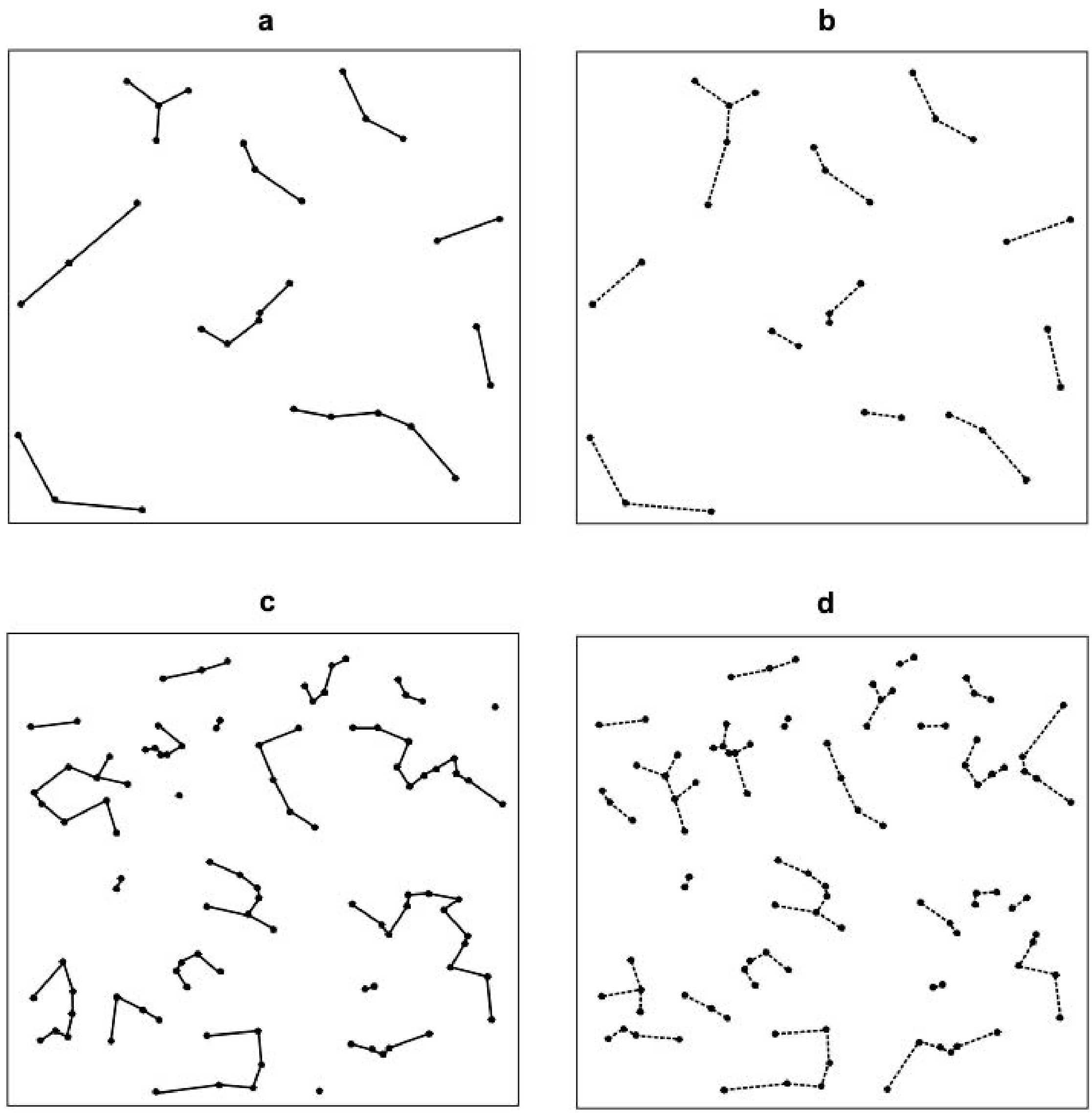


Figure 4. Vickers et al.

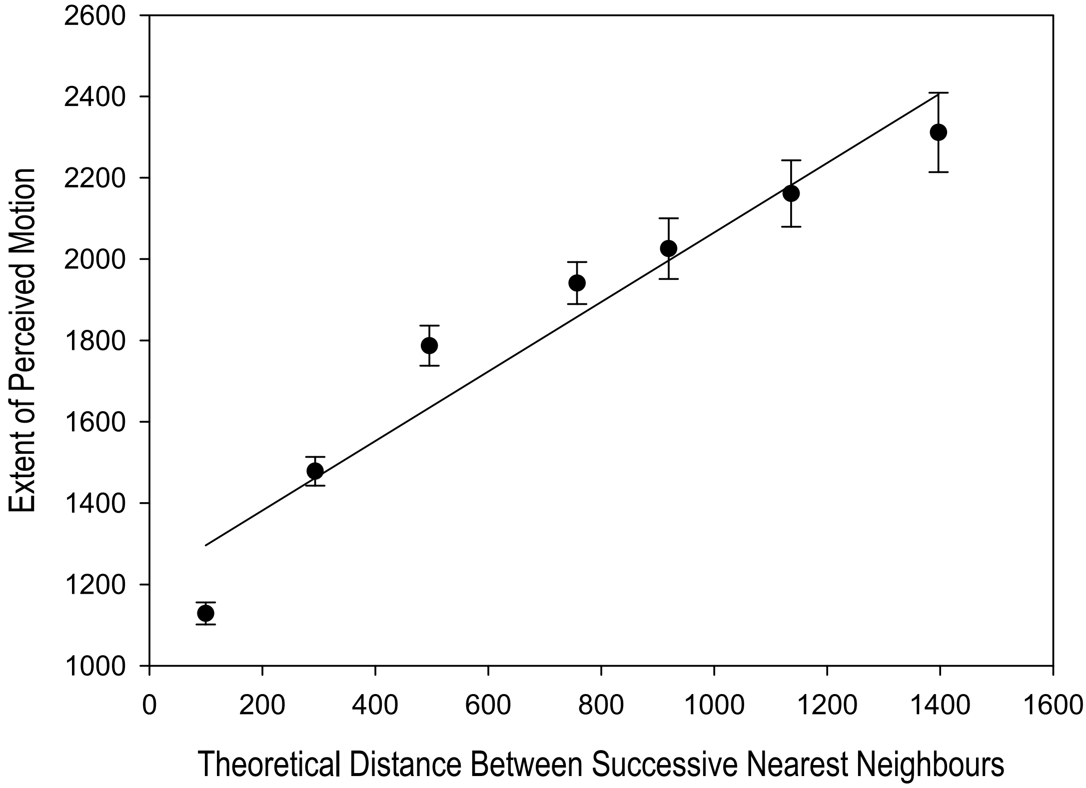


Figure 5. Vickers et al.

Appendix C

Transformational Analyses of Visual Perception

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Transformations and Symmetry Optimization in Visual Perception

Visual perception readily lends itself to conceptualization as an optimization process. A primary difference between perceptual theories concerns the nature of the optimized quantity. Most theories suggest that this is either economy of coding or some form of likelihood (Palmer, 1999).

A smaller number of theorists have sought to explain perception in terms of maximizing symmetry (e.g., Leyton, 1992). On our version of this view, the perceptual system subjects image elements to multiple transformations and represents structure by the parameters of those transformations that maximize correspondence with the current sensory input (Vickers, Navarro, and Lee, in press). This paper examines two applications of this approach.

Perception of Projections of the Platonic Solids

In an early experiment, Hochberg and Brooks (1960) showed that the tendency to see outline figures as two- or three-dimensional was a function of the number of angles and line segments required to specify them in two or three dimensions. According to a transformational approach, whichever perception is associated with more symmetry-preserving transformations will occur more readily than one associated with fewer such transformations.

To test this prediction with stimuli that are representative of major classes of geometrical objects, we asked 50 observers to rate printed examples of 16 and 18 orthographic projections, respectively, of the first two of the regular polyhedra (the Platonic solids): the cube and the tetrahedron. The projections were generated in Mathematica by systematically rotating the figures around the two axes orthogonal to the line of sight

The means and standard deviations in observers' preferences for a two- or a three-dimensional interpretation covaried in a continuous manner that was (weakly) predicted by both the discontinuous differences in the symmetries of the two- and three-dimensional figures and by a count of the number of distinguishable elements. Further analyses suggested that the data may be better accounted for in terms of subjectively perceived symmetry, either as rated by observers or as estimated by the symmetry maximizing program developed by Vickers, Navarro, and Lee (in press).

Memory and the Perception of Process History

Leyton (1992) has argued that visual perception consists of recovering the process-history undergone by an object. According to Leyton, this recovery proceeds by progressively removing asymmetries or "distinguishabilities", so as to infer an original object that is maximally symmetric. A similar evolution towards regularity is claimed for the successive reproductions of random arrays (Giraudo & Pailhous, 1999). However, there has been no quantitative investigation of either of these tendencies towards symmetry.

An experiment, modeled on Bartlett's (1932) method of serial reproduction, was carried out, in which 44 observers, tested in five groups of 4 to 13, were asked to reproduce briefly presented, irregular heptagons, drawn randomly from an original pool of 168 figures. Observers were then presented with each other's (randomly allocated) reproductions and asked to reproduce them. This process was repeated until each observer had made 20 reproductions. In agreement with Leyton's hypothesis, analysis of the 57 (or more) figures that were reproduced at least 10 times showed that observers had a progressive tendency to reproduce figures with a smaller perimeter and with more nearly equidistant vertices, as measured by a reduction by a quarter and a third, respectively, in the mean and the standard deviation (normalized for perimeter size) of the lengths of the edges.

References for Appendix C

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Appendix D

Visual Basic 6 source code for a universal Glass pattern generator. (Referenced from Chapter 12, page 234.)

If trying out the program, copy the code into a New Project, Standard EXE, Code window for Visual Basic 6. To run, press F5. Note: text—shown in green—preceded by a single quotation mark is treated as a comment, not part of the program code.

```
'Universal Glass Pattern Generator

'Angles are expressed in degrees for ease of interpretation.
'Program setup produces a Glass screw pattern of 1000 pairs. Alternate setup, by lifting
'alternate choice comments, produces a Glass horizontal translation pattern of 1000 pairs.

'Insist on data type declarations
Option Explicit

'Declare user-defined data type for x and y coordinates
Private Type Points
  x As Single
  y As Single
End Type

'Declare other module level variables with corresponding data types
Dim i As Long, pi As Single, Ang As Single, xTemp As Single, yTemp As Single


Private Sub Form_Load()
  'Many form properties can be set with the properties window at design-time. However they are here set at
  'run-time so as to make the program directly work properly by executing upon copying.
  'Set form properties
  Width = 7995: Height = 7995
  ScaleWidth = 5000: ScaleHeight = 5000
  Caption = "Universal Glass Pattern Generator"
  'Declare procedure level constants with corresponding data types
  Const NumPairs As Long = 1000, Rad As Byte = 5
  'Declare procedure level arrays with corresponding data types
  Dim Buf1(1 To NumPairs) As Points, Buf2(1 To NumPairs) As Points
  Dim Buf3(1 To NumPairs) As Points, Buf4(1 To NumPairs) As Points

  pi = 4# * Atn(1#)   'Number of radians in 4 x 45 degrees (180 degrees) is pi

  'Show form
  Show

  For i = 1 To NumPairs

    'Generate random dots in two dimensional array
    Buf1(i).x = Rnd * ScaleWidth: Buf1(i).y = Rnd * ScaleHeight

    'Centre of rotation of Glass pattern (constant across pairs)
    xTemp = ScaleWidth / 2
    yTemp = ScaleHeight / 2
    'yTemp = 200 * ScaleHeight   'Alternate choice

    Ang = 3   'Choice for Glass pattern transformation angle
    'Ang = 3 / 1000   'Alternate choice
    'Invoke Rotate subroutine, passing appropriate arrays
    Rotate Buf1, Buf2

    'Centre of rotation of Glass pattern pair i (different for each pair)
    xTemp = (Buf1(i).x + Buf2(i).x) / 2: yTemp = (Buf1(i).y + Buf2(i).y) / 2
```

```
    Ang = 20   'Choice for angle of rotation of Glass pattern pair i
    'Ang = 0   'Alternate choice
    Rotate Buf1, Buf3
    Rotate Buf2, Buf4

    'Draw dots on form
    Circle (Buf3(i).x, Buf3(i).y), Rad
    Circle (Buf4(i).x, Buf4(i).y), Rad

  Next i

End Sub

Private Sub Rotate(BufA() As Points, BufB() As Points)

  BufB(i).x = Cos(Ang * pi / 180) * (BufA(i).x - xTemp) - Sin(Ang * pi / 180) * (BufA(i).y - yTemp) + xTemp
  BufB(i).y = Sin(Ang * pi / 180) * (BufA(i).x - xTemp) + Cos(Ang * pi / 180) * (BufA(i).y - yTemp) + yTemp

End Sub
```