PART F: SUMMARY AND EVALUATION OF THE MODEL

The reader who wishes to obtain an overview of this thesis is advised to read Chapter 1 (second half) Chapter 9 (second half), Chapter 24 (first half), Chapter 25 and Chapter 26.

CHAPTER 25: SUMMARY

> Control of curriculum change in other words had reverted from its traditional locus in the professional education community to specialists in the academic disciplines. Secondly, as would be expected, the effort to replace the academic subjects as the basic building blocks of the curriculum, going back about half a century, was brought to an abrupt end.[1]

This was the interpretation made by an SA educational reformer of what had happened in SA in first half of the period under consideration here. This is partly supported by Connell's early view that curriculum construction in Australia was a "slow extension of the influence of the practising teacher over the determination of the content of the materials which he is called upon to teach"[2] But later Connell saw the period from the 1960s as having four major trends:

- a conscious and continual direction of educational policy and practice by economic, social and political interest;
- a reaffirmation of the relationship between education and culture;
- an attempted conversion of instruction into education;
- a rise of participatory decision-making for all levels of educational organisation.[3]

Neither of these pictures exactly matches the story we have been considering. We have certainly seen the control of SA mathematics education, at least for probability, moving from the Academic community to influential Pedagogues with wide authority, but with little influence from the developing discipline of Mathematics Education. Professional control has been further circumscribed by increased business and economic pressures on education, leading to a narrowing of educational purpose. But, although this is not often publicly admitted, these changes have had relatively little influence on the way in which probability has been taught.

Divergent interpretations are common in educational analysis, not least because the subject matter is so complex that it is easy for an analyst to focus too closely on just some of the important features. So this thesis has developed a model sufficiently comprehensive to discourage narrow focussing yet sufficiently simple to be manageable when examining *minutiae.* My purpose in assembling the many small details of Chapter 11–24 has been

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1 Hannaford (1986, p. 268)

2 Connell (1957, p. 69), cited by P. Hughes (1966, p. 258)

3 Connell (1993, p. 3)

> the single one of assessing the value of the ecological model to be described in Chapter 9 as a tool for increasing understanding of change processes in the teaching and learning of probability since about 1960.[4]

So in this chapter I shall use the BSEM to summarise and interpret the findings of Parts C–E, and in Chapter 26 I shall evaluate the effectiveness of this ecological metaphor, not only for the data examined here, but for a study of educational history in general.

INTRODUCTION

> With its shifting fashions, the world of ambitious educational reformers is not unlike the Hollywood most affect to despise. You might be able to recognise some Jackie Collins character types in your own backyard. Here the young ambitious beginning teacher, looking for the inspector's approval; there the established classroom practitioner, regularly attending in-service courses, 'dropping names' and gaining paper qualifications through the recitation of topical educational theories (which everyone who *is* anyone insists are really going to change things).[5]

Both Crombie and Hacking have described the development of probabilistic ideas in western culture.[6] One task of Part B has been to link some of their points with educational practice. For my touchstone I have made use of the Vygotskyan concept of *obuchennyi*—learning and teaching are seen as a unity in which both are deeply inter-related in complex ways. Chapter 3 summarised some educational philosophies as a background for Chapters 4–6, which linked probabilistic ideas with education and mathematics. Chapter 7 discussed general principles of curriculum development as a background for assessing probability's place in curricula. Chapter 8 examined the state of current research into the learning of probability—work which is quite recent and not widely known, but provides a useful background for understanding curriculum changes within a topic. Chapter 9 proposed a Broad-Spectrum Ecological Model to use for analysis and interpretation in Parts C–E, and Chapter 10 described the South Australian scene just before the period covered by this thesis.

Probability is a topic which easily passes a modified form of Bowen's "toilet test" for histories of education. This requires that the word "schools" should be replaced everywhere in a text by the word "toilets". If the text still reads coherently,

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4 Chapter 1

5 Openshaw (ndp., c. 1988, p. 4)

6 Crombie, (1994); Hacking (1975, 1990)

then it is not a history of education.[7] It is a history of administration. Such a test may be applied here by replacing the word "probability" by "arithmetic" or "algebra" or "calculus"—the three staples of primary, lower secondary and upper secondary mathematics syllabuses respectively. It quickly becomes clear that probability teaching really has many differences from the teaching of these staples, although there are, of course, also similarities. A history of probability teaching is not a history of mathematics teaching, and conversely.

To keep this study manageable it has been necessary to concentrate on happenings in SA, but to use data from other places where relevant. Three major themes were chosen—the history of probability in the SA school curriculum, the development of skills in assessing probabilistic understanding, and the development and implementation of a sound pedagogy for teaching probability. These themes provide sufficient depth and breadth to permit the making of sustainable claims about the value of the BSEM. I shall argue in Chapter 26 that the BSEM has been moderately effective for interpreting the restricted subject matter of this thesis. Who knows, it may perhaps one day be able to explain Hollywood itself!

SPECIFIC FINDINGS

> … in spite of more than two centuries that have passed since [Newton's] death, our vision of the world is still by and large Newtonian. … The two most important branches of modern physics, relativity and quantum mechanics, have not so far been integrated into a new universal synthesis; and the cosmological implications of Einstein's theory are still fluid and controversial.[8]
>
> What we have developed is a fragmented, fractured view of the cosmos and of our part in maintaining its health. We have encouraged people so much in the "bits" approach that they do not identify with the whole.[9]

In this Section each of the three major themes will be summarised separately, before concluding with a synthesis of the findings.

Historical Analysis

The largest part of this thesis has been an extended narrative describing the teaching of probability in schools in SA. It has been written within the *genre* of traditional historical writing about a specific set of events, as discussed in Chapter 9.

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7 Bowen (1973, p. 11)

8 Koestler (1959, p. 505)

9 Beare (1989, p. 18)

As secondary schooling after 1945 became more comprehensive, there was some enthusiasm for teaching statistics because of its practical value, but much less for its sister topic of probability. Indeed, probability's educational value and its exact relationship with statistics would be overlooked for many years on. Nevertheless, because of its theoretical relationship with the applied, increasingly important, topic of statistics, probability entered school curricula as part of the 1960s New Mathematics movement at a time when the Australian mathematics curriculum was Physically freed up because of the decimalisation of the currency.

New Mathematics brought several other new topics, such as matrices and difference equations, into secondary academic mathematics courses in SA as well. Some survived for a while, often grossly emasculated; all fell out eventually. Probability was the last to go. Now it is returning.

The pure Mathematicians who filled a leadership vacuum in the 1960s had the greatest influence on probability's introduction into SA schools. Statistics was seen as more suitable for weaker students; only probability and combinatorics were taught in senior academic courses. Few teachers had any formal training in the topic and its Pedagogy was poorly developed, so a pure mathematics philosophy and style prevailed—formal, deterministic, estranged from statistics, experimentation, and any deep attention to its underpinning ideas of randomness, uncertainty and variation. But this approach fitted the prevailing classroom ethos, and remained dominant throughout the period under discussion.

Various Intellectual arguments for probability's value in schools, such as those listed in Appendix II, had been proclaimed by various people since about 1960, and were rarely opposed. But they had little influence on early curricula, and were not widely espoused by schools or society. So there were no strong Social forces operating for or against the topic at this time, even though it seemed to be interesting, amenable to practical classroom activities, and directly related to the phenomenon of chance which permeated society. But it was also seen by many as peripheral to the real business of schooling—learning basic skills and facts. Probability entered the curriculum under the influence of Intellectual forces, largely because it was coming to be seen as relevant to the Ultimate force of the increasingly data driven nature of our society, but its intrinsic value and its links with statistics were dimly appreciated by most.

There were, however, strong Social and Physical forces developing which would demand further changes in school practice. The Physical forces arose from the Ultimate, inevitable, increasing industrialisation of Australian society, which led

to increases in immigration, demand for skilled labour, and school retention rates. The pressures these put on schools were later exacerbated by difficult economic times which led to large-scale unemployment and to schools being seen as holding paddocks for those students unable to find work. At the same time as schools were under increasing Social pressure to provide applied training for technical workplaces, the nature of authority within society was being challenged and traditional structures were being transformed.

Increasing industrialisation also affected mathematics classrooms as calculators and computers became more readily available. These had the potential to free up classroom time and to develop more effective ways of teaching both old and new topics. Statistics could at last be freed from tedious pencil and paper calculations. Most teachers were slow to respond to these challenges. Change places great Physical demands on a teacher's time, and teachers, not unreasonably, tend to hear the many clarion calls for change as yet another cry of "Wolf!", especially if the calls are not accompanied by adequate funding. They needed simple, well established, classroom Pedagogies more than ever in their increasingly difficult classrooms, so there were good Physical reasons for them to be conservative.

Society too was ambivalent about providing electronic support for teaching. It wanted its children to be up to date, but it feared that they would not learn the old "Basics" well enough if they were provided with easy alternatives. The "Basics" won, supported by the Physical force of difficult economic times, and funding for the equipment required was severely limited.

Increasing population also led to the growth of Teachers' Colleges, later to become Colleges of Advanced Education, and later still parts of universities. Initially, they tended to be staffed by skilled Pedagogues, who emphasised the process of learning rather than the product, and who supported society's increasing concern for schools to be instruments of Social formation. Other skilled Pedagogues moved into government bureaucracies at a time when previously rigid central hierarchies were seeking to spread responsibility for education more widely across the community. This growing influence of skilled Pedagogues led to a reduction in the influence of Academic mathematicians over all but the Year 12 syllabuses. In time, and contrary to Hannaford's view at the head of this chapter, this led to a reconstruction of the school meaning of "mathematics" to something quite different from the Academic meaning, and also to an increased emphasis on an applied approach to school mathematics. Coupled with Technocratic forces, it

also led to that modularisation of the curriculum described above by Beare, even though it was accompanied by rhetoric advocating a more holistic approach.

The period also saw the development of a new Intellectual environment—Mathematics Education—which by 1990 had constructed a significant and usable *corpus* of understanding, albeit markedly incomplete. But the discipline tended to be rejected, alike by Pedagogues, Academic Mathematicians and Society in general. Society, which had lost faith in teachers during the long-haired seventies, seemed to share with Academics a view that "anybody can teach".

All these changes applied to mathematics teaching as a whole. But probability teaching differed in several important ways. Teachers' subject knowledge was poor because the topic was totally new to most of them and elementary probability seemed to rest on such disarmingly simple ideas like tossing dice that they did not appreciate its sophisticated nature. The links between probability and statistics were poorly made because they all rest on some form of numerical inference, and this was eschewed as being too difficult. Even when they were taught, usually at tertiary level, they tended to be taught deterministically. But few real-life examples of probability are amenable to pre-determined algorithmic analysis.

Indeed, the deterministic nature of most classrooms was more at odds with stochastics than with any other topic. This led to little classroom attention being paid to developing a deep understanding of the nature of chance, and to excessive attention being paid to routine calculations, even in courses with a specific emphasis on applications and/or modelling. These pedagogic problems were exacerbated because traditional classroom practice relied on refuting poor thinking by providing concrete counter-examples, which are not available for stochastic misconceptions. This further encouraged teachers who were not confident with the topic to teach by rule. The emphasis was on teaching, rather than learning: the unity encapsulated in the idea of *obuchennyi* was lost.

Throughout the period under discussion the teaching of probability saw little change, even though it was seen as difficult to teach, was often unpopular, and was leading to learning errors which were particularly difficult to interpret or correct. No specific effort was made by anyone in authority to address these known difficulties. Indeed the slow osmotic spread of probability into primary schools saw no change in practice other than more encouragement for classroom activities, but because experimental stochastic data is harder to interpret probabilistically than any other type, this only made for more difficulties.

The movements for change which were effective were those for making school mathematics more relevant to real life, which meant that statistics was increasingly seen as important, so probability was too. This led to their incorporation into the upper primary syllabus in SA, and to the provision of model lessons for primary and junior primary classes, which, however, had relatively little influence. In upper secondary schools, moves to introduce more applied courses based on modelling were not very successful. Teachers frequently could not work with open-ended material, and plans for a modern, applied Year 12 subject were strongly resisted by many tertiary Academics. Since probability had been placed in the new course because it was applicable to statistics, it lost its place in SA's mainstream academic curriculum just at the time when Australia-wide plans were developed for a national curriculum which included a significant place for stochastics from Reception upwards.

Indeed, by the 1990s probability was officially seen as a necessary part of the general education of every primary and junior secondary child. This time it was the Pedagogues, not the Intellectuals, who had seen the Ultimate force associated with increasing dependence on data in our society and had the power to write stochastics into curriculum documents. But, just as in the 1960s, Social forces were responding mainly to the Proximate needs of gaining jobs and earning good money, and Academic forces were more concerned with content than general educational issues. Little interest was shown in what Mathematics Education had to offer, either in terms of teaching or learning. So classroom practice was left to teachers who lacked adequate background and were working in situations where Physical forces were dominant. The three-phase pattern of interaction between groups and political systems—confrontation, consultation and co-operation—which Pouw-Bray[10] has seen as critical for obtaining political consensus at times of change did not occur for probability. So it is unlikely that Social or Intellectual forces in its favour will strengthen in the immediate future.

We can now see how much the introduction of probability did not fit Connell's four major trends listed above, even though we have seen a rise in the influence of skilled Pedagogues on the curriculum. Although probability is essential for understanding the very practical topic of statistics, its value was not appreciated, and its importance has not been adequately debated right across the community's decision-making structures. While it could form a rich way of making links between education and culture, such development has been hindered by its

10 Pouw-Bray (1979)

Pedagogic difficulties which have led to a classroom emphasis on instruction rather than education. By using the BSEM we have seen how well-defined, interacting forces can influence final outcomes, and such an approach seems to provide a richer interpretation than one based on trends, such as Connell's, or ones emphasising just a small part of the BSEM, such as Clements' or Horwood's. The special value of the ecological model has also been shown by illustrating Convergence of practice among economically similar countries.

Understanding of the circulation of blood and Newton's gravitational model is now widespread. Similar widespread understanding of the nature of chance has not yet been achieved. We cannot assume that it will be. After all, there is some evidence that strong Social forces have meant that half of the citizens of the USA do not believe in evolution.[11]

Assessment of Probabilistic Understanding

Part C provided a general survey of probability's fortunes within the SA educational and Social setting. It highlighted the difficulties experienced in developing a suitable Pedagogy and suggested that some of these difficulties arose from the failure of curriculum leaders to understand the full Intellectual implications of the topic. So Part D has looked in detail at one aspect of probability's Pedagogy—its assessment—in order to isolate some of these implications. Assessment was chosen partly because it tends to leave more written records than other educational practices, partly because it often provides fuel for political fires, and partly because it emphasises the often neglected "learning" part of *obuchennyi.* Assessment is also well linked with the BSEM's nodes. It is often driven by Social forces and can yield valuable Pedagogical implications, but the design and interpretation of questions is essentially an Intellectual activity.

The micro-analysis, which, *contra* Hannaford above, is based on an assumption that subject matter is an appropriate building block for the curriculum, looks at Intellectual aspects of probability, both in Mathematics and Mathematics Education. It rests on a clear understanding of underlying Mathematical concepts and structures, but also requires reliable ways of elucidating children's understanding—a task within the province of Mathematics Education. Questions from research questionnaires were chosen as its basis because it was believed that they would be diverse in approach, historiographically well-defined, carefully constructed, tightly focused, and also provide detailed discussion of results and their

11 Weiner (1994, pp. 295–297)

significance. They were also likely to be sought out by curriculum developers, so their influence on curriculum and Pedagogic practice was potentially greater than that of much research. For simplicity, the analysis was largely restricted to questions about single random generators with discrete outcomes.

The general summaries of research in Chapter 8 and of assessment projects in Chapter 17 had revealed many difficulties: results were dependent on question form, some questions were neither well-defined nor well constructed, discussion of results was often superficial, and one was left with some doubt about what useful information so much assessment energy had produced. Some projects had developed good questions which provided Pedagogically useful results, but there had been less progress in developing a *corpus* of questions which were effective for assessing the width and depth of a student's probabilistic understanding.

Since describing understanding is a desirable end-point of any assessment, Chapter 18 described a framework developed by the author to categorise the wide range of probabilistic situations and questions about them in order to provide a way of comparing questions and results from diverse sources and of evaluating the comprehensiveness of sets of questions. This seemed to be particularly necessary because probability's newness meant the potential difficulties were less widely appreciated than they were for more traditional topics. In Chapter 19 a representative set of questions and projects was assembled, and for each question the results were summarised, a categorisation constructed, and some comments made on its strengths and weaknesses. This material formed a data base for the discussion which took place in Chapter 20.

"Correct" answers to questions may be given for quite "incorrect" reasons or because of linguistic weaknesses, so good testing should seek reasons for responses, thus greatly increasing cost, especially if clinical interviews are used. Some attempts have been made to find inexpensive ways around this problem, but none has proved totally satisfactory. The issue of cost is important, because much testing has been initiated by government bodies, and the analysis made it clear that their questions were often poorly constructed and poorly reported, with an emphasis on statistical analysis at the expense of adequate understanding about what information the questions might reasonably have provided.

Given the constraints of cost it is not surprising that governments seem to have been seeking small question sets. But only one researcher, Green, has actually set out to establish a general test of probabilistic understanding. His work was well-designed and imaginatively constructed, but statistical analysis reduced his test

to a limited subset lacking sufficient generality to be a test of probabilistic understanding. A shorter, less general, test was developed by Fischbein & Gazit to identify some well-known misconceptions, and Godino et al. found two statistical factors in Fischbein & Gazit's test and fifteen in Green's as well as some correlation between them. But they concluded that a good test of probabilistic understanding required a much better bank of questions. Given that many researchers in the field work on a very small scale, this may prove difficult to achieve. But the framework of Chapter 18 has been able to provide an indication of the breadth and width of sets of questions which need to be asked, while making it clear that practical constraints will prevent anything even near a full coverage from being achieved in any one test. Once again, the situation for probability is quite different for that from other topics.

This difference is also found with assessment precision, because children often give markedly and unpredictably different responses to very similar questions. At least two major reasons for this instability have been identified. The first has been found in the very popular "comparison of probability" questions, where it is clear that quite different heuristics are used for different sets of numbers, probably to achieve computational simplicity. The second has been seen in the differing interpretations by respondents of what is being asked, and seems to be dependent on the precise verbal form used in a specific question.

Some analyses, notably Green and Watson's, have tried to interpret their results in terms of a developmental model. The theoretical structures proposed have been sensible, but the evidence gathered to date has been unconvincing, partly because of the limitations of their questions, and partly because of the difficulties of measuring and analysing sufficient changes of a large number of individuals.

Although there has been quite a lot of use by later researchers of the questions of earlier ones, there has been much less integrative analysis of different sets of results and a relative neglect of findings made within the psychological tradition. It may be that a lack of focus on what is really meant by "probabilistic understanding" has led researchers to focus more on the statistical distributions of their results than on what their results are telling us about the thinking and development of individual respondents.

While Part D focussed on the Mathematics Education node of the BSEM it did not seen it in isolation from the other nodes. There have been clear Social pressures from governments and Technocrats for questions which can measure the efficacy of teaching, and sometimes they have provided reasonable funding for the re-

search as well. However, teachers have been relatively little interested in the results, although they have not shown the strong resistance to this part of Mathematics Education which they have to other parts. Part D concluded with a close examination of Mathematics Education itself, using a modified version of the BSEM, and this helped to explain the both the strengths and weaknesses of the research findings so far published.

Research and Pedagogy

The micro-analysis of the assessment of probabilistic understanding in Part D was directed to one small part of the Pedagogy of probability. In Part E Pedagogy was examined from a much wider perspective, and with a special awareness that it had been slow to develop. My principle purpose was to see whether, for probability at least, the Mathematics Education node of the BSEM had disciplined knowledge of value to offer teachers practising at the Pedagogic node.

Chapter 21 examined general findings about Pedagogic change, to try to identify the forces which have caused many change movements to have limited and short-lived effects on classroom practice. The role of teachers in implementing change was seen as crucial, and it was argued that many proposed changes were ineffective because teachers lacked sound Intellectual and Pedagogic knowledge. These general conclusions were examined with respect to the teaching of probability in Chapter 22, where it was found that, although there was a need for much more work to be done, some effective Pedagogic strategies had been identified by researchers, but they were rarely used by most classroom teachers. Similar work was done for many mathematics topics, but the need for probability has been greater because it has been so poorly learned, and even here the work done has been largely disregarded, in spite of moderately good dissemination. In Chapter 23 I presented six case studies of stochastics curriculum change with which I have been involved in order to present a different form of evidence from that which is normally available in curriculum studies, as I traced my own development from a Pedagogue to a Mathematics Educator in an unfamiliar topic.

The earlier work was done from a Pedagogical perspective, and with very limited consideration of Intellectual issues. Mathematical difficulties which were encountered tended merely to be brushed over. Later work, which was based on an increasing understanding of Mathematics Education, attempted to blend Mathematics and Pedagogy, and later work still attempted to integrate research findings as well. One planned project foundered because of Physical constrains,

which had not been a serious issue in the other cases. The most successful project provided adequate Physical support to allow me to work beside a practising teacher in order to help her understand how the discipline of Mathematics was able to have a positive effect on Pedagogical practice.

The evidence from the whole thesis was used in Chapter 24 to construct a diagrammatic model of the interacting forces and to link this with the BSEM proposed in Chapter 9. Some modifications were made to the BSEM so that it could be linked with the Diagrammatic Form to constitute the Model to be assessed in Chapter 26. The Diagrammatic Form was then redrawn to provide a summary illustration of the relative strengths of the interacting forces which had been described in all three Parts with respect to the teaching of probability in SA for the period under consideration. It was argued that this illustration could provide a way of comparing systems at different times or in different places.

Part E concluded by examining the nature of a profession and showing how the structure of the BSEM could be applied isomorphically to Medical Practice. Diagrammatic models of the forces for Medicine and Mathematics Education could then be compared, and the differences helped to explain differences in status and success between the two professions as well as providing some evidence for the general validity of the BSEM as an interpretative tool.

OVERVIEW

> The colleges had it in their power to command good building. … The material was there. The genius of William Morris the Second in motor manufacture could hardly have been expected to extend to town planning. But there were all those books in the Bodleian, all those 'first-class brains' churning away in panelled senior common rooms.
>
> You would have thought some concerted action had been possible. You would have imagined that architecture and town-planning had been heard of in the home of culture.
>
> As it is, Oxford remains an unplanned muddle. Motopolis, Christminster and the University are jostled together in hopeless disorder.[12]

Betjeman's analysis of the reasons for the unplanned recent growth of Oxford shows a remarkable similarity with the reasons adduced here for the unsatisfactory construction of probability teaching in SA schools and beyond—little reflect-

[12] Betjeman/Guest (1978, p. 136), written in the 1930s.

ion, Intellectual neglect of both pure and applied theory, and the dominance of Proximate Physical forces.

Such tensions may well be general, at least in complex societies, and this thesis has summarised some of the ways in which they have been manifested within education. The BSEM and especially its diagrammatic model have acted as a synthesis of the many complex interacting forces. It is this interactive structure which is its special strength. This thesis has also highlighted some of the divisions within the principal components of the model—pure and applied approaches to mathematics, differing emphases on product and process in classrooms, differing emphases on the humanising and vocational aspects of education in society, to name just a few, but these can all be fitted into a model which is built on describing tensions and interactions.

So in this thesis we have seen how the Optimal position for probability in the school curriculum has been found because the Physical demands of classrooms have had far greater influence than any Intellectual reasons, or even of Social reasons. Over the last 35 years any vision that has existed of the importance for both individuals and society of a widely held understanding of the nature of chance has been lost. Lost in ignorance and isolationism, and obfuscated by anathema to theory and by entropic debasement of practice. The BSEM helps to show where the underlying problems have principally been located—in the relative weakness of the Intellectual and Charismatic nodes. This is a quite different interpretation from the more common one of blaming the teachers or, perhaps even more common, of blaming the students. Our educational practice, unlike our medical practice, has been based less on evidence than on feelings. But our culture is prepared to accept this neglect of evidence for education.

Society and individuals will both survive if they know little about the nature of chance. They survive now. And equally, deep understanding will not close the casinos or save compulsive gamblers from poverty and shame. But the idea that chance events can have underlying regularities is one of our culture's great discoveries. It is beautiful, explanatory, and useful. And counter-intuitive, unlike most other mathematics topics. In our society we teach reading and writing to all so that all may be empowered to use their individual freedoms richly. How much more should we also ensure that the great insights of our culture are also available to all as enrichment and empowerment of individual freedoms. The ability to measure chance is one of these great insights.

When I started my formal studies in this field in about 1978 my aim was to understand why probability was difficult to teach in the classroom. This led to the identification of a weakness in the underlying theory and my Masters thesis (Volume 1 of my writings) was in part a clarification of the theory and a suggested modification of Pedagogical practice. It also looked at how children develop probabilistic concepts and introduced me to the world of educational research. This showed me how much more there was to know about the field. When I started this thesis my aim was to understand why probability had proved difficult to introduce into the curriculum. This has led to the identification of weaknesses in educational structures, and this thesis (Volume 2) has developed into the construction of a model which might help to explain why these weaknesses occur and remain.

The summary above suggests that this thesis has presented a fairly comprehensive description of the difficulties probability has encountered in being received into the curriculum. If the BSEM is found to be useful, then my question will become, "What can be done to make a mature understanding of chance as common in our society as is an understanding of the circulation of blood?" That will be a task for Volume 3. But first I need to conclude this Volume by establishing that the BSEM provides a convincing explanatory model. This will be done in Chapter 26.

> From bias free of every kind
> This trial must be tried.[13]

[13] Gilbert (1875, p. 43)

CHAPTER 26: EVALUATION OF THE MODEL

> BERNARD [NIGHTINGALE]: Yes. One of my colleagues believed he had found an unattributed short story by D.H. Lawrence, and he analysed it on his home computer, most interesting, perhaps you remember the paper?
>
> VALENTINE [COVERLY]: Not really. But I often sit with my eyes closed and it doesn't necessarily mean I'm awake.
>
> BERNARD: Well, by comparing sentence structures and so forth, this chap showed that there was a ninety percent chance that the story had indeed been written by the same person as *Women in Love*. To my inexpressible joy, one of your maths mob was able to show that on the same statistical basis there was a ninety per cent chance that Lawrence also wrote the *Just William* books and much of the previous day's *Brighton and Home Argus*.[1]

Qualitative hypotheses cannot have the luxury of being tested by mathematical methods. But their evaluation can still be disciplined and systematic. I shall use Lakatos' criteria for a research programme as my guide. Others are possible, but this seems to fit well the exploratory nature of my investigation. Chalmers has summarised the principal features of such a programme:

> A Lakatosian research programme is a structure that provides guidance for future research in both a positive and a negative way. The *negative heuristic* of the programme involves the stipulation that the basic assumptions underlying the programme, its *hard core*, must not be rejected or modified. It is protected from falsification by a *protective belt* of auxiliary hypotheses, initial conditions, etc. The *positive heuristic* is comprised of rough guidelines indicating how the research programme might be developed. Such development will involve supplementing the hard core with additional assumptions in an attempt to account for previously known phenomena and to predict novel phenomena. Research programmes will be *progressive* or *degenerating* depending on whether they succeed in leading to the discovery of novel phenomena.[2]
>
> Two ways in which the merit of a research programme is to be assessed have emerged from the foregoing outline. Firstly, a research programme should possess a degree of coherence that involves the mapping out of a definite programme for future research. Secondly, a research programme should lead to the discovery of novel phenomena at least occasionally. A research programme must satisfy both conditions if it is to qualify as a scientific one.[3]

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1 Stoppard (1993, p. 19)

2 Chalmers (1976, p. 76)

3 Chalmers (1976, p. 79)

I see the research programme within which I am working as "the use of ecological principles to interpret historical data". The use of principles from other disciplines to interpret history is not uncommon, though rarely popular, as noted in Chapter 9. The BSEM represents a development of the ideas of Crombie, an historian of science, not of education, so some of the changes made to his model have arisen because I have been looking at educational history.

I am aware of Windschuttle's criticisms of such an approach:

> [H]istorical explanations are based on the movements of events over time and their conclusions from the evidence the historian finds during research into the subject. This is the opposite of a theoretical approach in which large-scale generalisations about human society or human conduct are taken as given before either research or writing starts. … Any evidence that might be brought into play is used to confirm the theory that has already been chosen. … [T]o draw findings from large generalisations or from anything resembling scientific laws is not part of the historical approach.[4]

I find this view simplistic, and easily contradicted by reading several histories which cover the same time and place.[*] Windschuttle is a vociferous opponent of Post-modernism, Constructivism, and anything which denies the existence of absolute truth or the validity of inductive arguments.[5] Although I share many of his concerns, I cannot share his unwillingness to accept that some knowledge is both provisional and relative, though I concede that many thinkers have carried their relativistic arguments too far. Nor can I share his claim that theoretical models should not be used to inform historical interpretation. It is partly for these reasons that I have chosen to use Lakatos' approach: his auxiliary hypotheses allow for theory tempered with revision and modification, and we have already seen a need for these as the BSEM has been weighed against the evidence.

The purpose of this chapter, then, is to show that the research programme:

- has a well defined hard core;
- is protected from falsification;
- is able to generate a programme for future research;
- has predictive value;
- is able to generate novel results.

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4 Windschuttle (1994, p. 19). I am grateful to Dr Barry Jermonson, for drawing my attention to Windschuttle's views.

* My current reading of Irish history provides more than enough evidence for this claim.

5 Windschuttle (1994, p. 36)

THE HARD CORE OF THE MODEL

> … never forget what I believe was observed to you by Coleridge, that every great and original writer, in proportion as he is great or original, must himself create the taste by which he is to be relished; he must teach the art by which he is to be seen; this, in a certain degree, even to all persons, however wise and pure may be their lives, and however unvitiated their taste; but for those who dip into books in order to give an opinion of them, or talk about them to take up an opinion—for this multitude of unhappy, and misguided, and misguiding beings, an entire regeneration must be produced; and if this be possible, it must be a work of *time*.[6]

We must first show that "the use of ecological principles to interpret historical data" is a well defined hard core for interpreting the evidence gathered here in a systematic way. While this evidence is somewhat eclectic, both its locus and focus are sufficiently well defined to form an appropriate domain for testing the model.

The ecological model is obviously developed from Darwin's theory, which is often summarised by the rather simplistic slogan "survival of the fittest", overlooking the fact that the theory may refer either to individuals or to co-operative groups as small as families or as large as whole species, and may well lead to different outcomes in different environments. The in-built tension and diversity of the BSEM provides a systematic way of examining the workings of such co-operative behaviour in a variety of environments.[†]

But an educational model must go beyond Darwin's because it deals with the construction and preservation of ideas, which cannot be done genetically. It must go beyond a memetic model because the ideas are complex and require reflection for their preservation. This reflection has produced Intellectual environments containing many differing educational philosophies which have led to a variety of Social pressures within any particular Physical environment.

Because of this complexity, educational research must be qualitative, and its ethical guidelines are difficult to define. It is not enough to require that experiments be reported honestly and be capable of replication, though this is still important. Nor is it enough that sources be meticulously quoted and referenced. Something else is needed. I have used Eisner's concept of connoisseurship as a

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6 Wordsworth to Lady Beaumont, 21 May 1807, from de Selincourt/Moorman (1969, p. 150). I am grateful to Dr Margaret Scott, U of Adelaide, for drawing my attention to this letter.

† I am grateful to Dr Robin Sanderson, School of Mechanical Engineering, U of Adelaide for his comments on this matter.

guide. This demands disinterest, depth and breadth, but sees such technical skills as subservient to providing insight—the "aha" effect—for both the writer and the reader. This is to some extent testable, though it does require a willingness to take time to understand the model before making a judgement, as Wordsworth has suggested above. In my experience, not all historical writing does provide insight, though it may be both technically meticulous and firmly underpinned by theory.

The BSEM in Practice

The BSEM was developed out of Crombie's three-faceted model of the growth of scientific thought, and modified as evidence about educational systems accumulated. The challenge was to balance the complexity of the data with the simplicity needed for an interpretative metaphor. In the end, seven nodes were proposed, which could be seen either as environments or forces, with one viewpoint being more helpful than the other in different circumstances.* The model was set in a background symbolising each individual experiencing educational growth.

All operating forces influence all the other environments, and a thorough investigation must pay attention to all forces and all environments, even though some forces may seem to be ineffectual, and some environments may appear to be very simple. This thesis has made use of some basic ecological principles to underpin its interpretation; these principles, which are highlighted below, provide the coherence looked for in a good scientific research programme.

A feature of the evidence presented in Parts C–E has been the great **Variation** in possible practice which has developed as probability has entered into school curricula, but a marked lack of variation in classroom practice. The BSEM provides a mechanism for explaining why so much potentially useful material is generally not used. Because it sees the educational environment in terms of equilibrium, stable or unstable, between forces, it has a greater place for tension than in some other models discussed,[7] and as a result is better able to use the ecological principle of **Optimisation** to explain the origins and continued hegemony of any particular situation. It does this by looking for a minimisation of *total* energy expended, rather than viewing energy expenditure from the narrow perspective of just one node. For example, we saw in Chapter 12 that the way in which the New Mathematics was introduced into SA seemed to represent the best available compromise given the forces operating at the time. Using this principle makes it

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* Rather as one can describe light by a particle or a wave model

7 E.g., Keeves' Linkage model (ch. 21); Popkewitz (ch. 9)

much harder to rely on unimodal explanations, such as saying that the SA introduction failed simply because the books were too academic for the students. Other constraints like the urgent need for change and the limited resources available with the State were also relevant. Optimisation is also useful in explaining the wide-spread debasement of educational practice occurring when a system has insufficient energy to reduce entropy and maintain complexity. Because "debasement" involves subjective judgement, it is not an ecological principle as such, but it is certainly relevant to educational discourse.

The BSEM can also set an environment into historic and teleological perspectives, because it considers, and distinguishes, **Ultimate** and **Proximate** aspects of operating **forces**. Many developments described here may be directly attributed to two separate, but more and more inter-related, Ultimate, global forces—the increasingly data driven nature of our society and the increasing availability of electronic technology. Others are directly attributable in Australia to the Proximate forces arising from rapid population expansion. Educational hyperbole often does not distinguish these aspects;[∞] the BSEM does so as a matter of course.

The Coda to Chapter 16 showed how the ecological principle of **Convergence** can provide a better explanation of similarities found in different environments than models based on straight-forward copying. As Gibbon has pointed out:

> Much learned trifling might be spared, if our antiquarians would condescend to reflect, that similar manners will naturally be produced by similar situations.[8]

Finally, a scientific model should be **Parsimonious**—the simplest which can account for all the data. Historical explanations using the BSEM are not as succinct as scientific laws, of course, but the Diagrammatic Model allows complex forces to be interpreted within a simple model which is easily accessible as a whole.

In this thesis I argue that ecological principles do provide a well defined, coherent hard core which has allowed meaningful and insightful interpretation within three quite separate educational domains.[†]

[∞] E.g., the Mathematical Sciences Education Board (1989, pp. 81–84) saw "changes in public attitudes about mathematics" and "increasing use of calculators and computers" as two equally important historical trends. But the first is an unexplained and possibly fickle change in the Social environment, the second a result of an irresistible Ultimate force.

[8] Gibbon (1776–1788, ch. 9, footnote 71), cited by Grafton (1997, p. 184)

[†] On a personal note, I have been struck by how often it has forced me to make judgements—both positive and negative—which were contrary to my *a priori* beliefs, and has also proved capable of providing insights into areas well beyond my immediate field of study.

PROTECTION FROM FALSIFICATION

> A miracle, my friend, is an event which creates faith. … Miracles are not frauds because they are often—I do not say always—very simple and innocent contrivances by which the priest fortifies the faith of his flock.[9]

If new evidence seems to contradict the hard core of the model, two responses are possible. One is to reject the model on the grounds that it has been falsified, which means, of course, the end of the research programme. The other is to construct auxiliary hypotheses—modifications of the model which can accommodate the new evidence while preserving the underlying hard core of the programme.

We have seen in several places that each of the individual nodes of the BSEM also constitutes a complex environment. There is no one Social force, just as there is no one Pedagogical method. We examined one such situation in detail in Part E, which was unusual in examining a very small aspect of the research process from a very broad perspective. This produced some results which have been rarely commented on. So Chapter 20 attempted to accommodate this weakness by proposing a "zoom model" similar to the BSEM but just for the Mathematics Education node itself. This was of some value in explaining the nature and strength of the links within Mathematics Education and also between Mathematics Education and the other nodes. It provided a better explanation of the whole picture than the BSEM did by itself. Since it was constructed very firmly within the hard core of the research programme—a model based on ecological principles—it constitutes an auxiliary hypothesis.

It would clearly be possible to construct similar auxiliary hypotheses to explain other discrepancies. For example, we have noted that teacher leaders were often developers of new classroom approaches. One auxiliary hypothesis could have been to define a new node for teacher leaders. But I saw them as similar to classroom practitioners because they were more concerned with practice than with theory, and decided that a new node was not appropriate. Although it has not been tested here, a "zoom model" for the "Teaching" node would seem to be a satisfactory alternative auxiliary hypothesis. There are possibly other auxiliary hypotheses which might allow for the differences as well. All that it is necessary to do here is to show that when evidence is found which is not a perfect fit for the model, then alternative hypotheses can be constructed to accommodate them.

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9 The Archbishop of Rheims in *Saint Joan* (Shaw, 1923/1953, pp. 94–95)

THE VALUE OF THE MODEL FOR FUTURE RESEARCH

> [T]he pioneer scholars of education placed a great emphasis on quantification as well as on identifying what were taken to be invariable certainties—laws of learning, formulas for administrative efficiency, and the like. Convinced that philosophy led to conflict … they believed the findings of science could and would guarantee consensus.[10]

This criterion of Lakatos may be addressed quite briefly. There is obviously potential for wide testing of the model within both Mathematics Education and education as a whole. This would examine whether the model is a better, or at least an equally good, fit for events which have been examined in the past. It would also be possible to make prognoses based on the Model, and later to test them. Finally, it would be possible to use the Model to make recommendations about appropriate future practice and to evaluate the outcomes in the same way that predictions were tested.

In my opinion, the most urgent need for Mathematics teaching is to find ways of integrating Mathematics Education Research with Pedagogical practice. The findings of Parts D and E have made clear how the BSEM might help in identifying the forces which need to be overcome if such a project were to succeed. Looking at the training structures of other professions may help to define a suitable structure for education which could facilitate such integration. Such a project could utilise all three of the testing approaches suggested above. If it could succeed it would constitute a significant improvement in the links between theory and practice.[11]

Another focus for future research will follow from the claim made below that an ecological approach is a research programme which is able to generate novel results. If this claim is correct, then the implications of these novel results will also provide a fruitful source of future research.

None of this research is likely to lead education back to the deterministic certainties of its pioneer scholars. But because the BSEM is able to accommodate creative tension, it may lead to a richer synthesis of qualitative and quantitative research than has been possible in the past.

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10 Lagemann (2000, p. 21)

11 *Vide* Helme & Stacey (2000, pp. 115–116) for one statement of the exasperation which many skilled researchers feel with any failure to break through the research/practice divide.

THE PREDICTIVE VALUE OF THE MODEL

> I will tomorrow,
> And betimes I will, to the weird sisters:
> More shall they speak; for now I am bent to know,
> By the worst means, the worst.[12]

Next, we need to examine the BSEM's predictive power, which will hopefully provide partial confirmation of our oft-repeated claim that theory is the most practical of all things. Just as this thesis is being completed, the first significant SA post-1994 curriculum revision has been released—the *South Australian Curriculum, Standards and Accountability Framework*—usually known as the "SACSA Framework" or "SACSA".[13] So we can examine how much the Framework is a predictable development of the forces summarised in Figure 24·6. This is not quite "predictive power"—predictions cannot be made *a posteriori*—but it is the best that can be done now. We shall find that the changes were largely foreseeable, but there were two exceptions whose omission will need to be explained.

Summary of the Period

The period 1994–2001 was economically difficult, and dominated by conservative political forces at both State and Federal levels, and a rise in new political parties espousing "old values". It also saw rapid changes as business became technologically driven and had less need of full-time employees, especially older ones. In schools, the rapid spread of personal computers and the Internet finally made new approaches to teaching and learning practically achievable. But the continued influence of the Technologists was a strong conservative force, more easily implemented as government control of educational practice increased, especially through an expanded use of comprehensive testing schedules at Years 3, 5 and 7.

Prognosis

Apart from the marked strengthening of these conservative forces, the late 1990s saw only minor changes in forces from other parts of the educational environment. The increased availability of electronic communication led to some increased pressure for change from both Physical and Social Environments, and saw a small, slow response from both Mathematics and Mathematics Education.

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12 Shakespeare *Macbeth* III (4) ll. 132–135

13 SA. DETE (2001)

The use of Constructivism by Mathematics Educators as a pedagogical framework continued to grow,[14] but its influence on school practices was small.[15] Schools continued to be concerned with teaching facts and preparing children for integration into the employment structures of the community.

Radical changes to stable structures are most likely to come from "wild cards": either natural disasters or Charismatic leaders, neither of which has had any recent influence in SA. We might therefore have predicted that the 2001 SACSA Framework would continue the Technological emphasis described in Chapter 16, together with an increased concern for electronics in teaching, but would neglect the findings of Mathematics Education. In general this is what happened, but with two unexpected features. We shall first summarise the Framework.

The SACSA Framework

The Framework aims to prepare children from Birth to Year 12 for a "knowledge and service-based society" which will respond to "new technologies and forms of communication".[16] It focuses on "Essential Learnings", "Coherence", "Enterprise and Vocational Education", "Equity" and "Standards".[17] It has been underpinned by Constructivism,[†] and emphasises holism and "learning across subject areas and between stages of education", but does not prescribe any specific "body of knowledge" or "way of going about teaching".[18]

The Framework has clearly grown out of both the Technological statements of the early 1990s and the National Statement documents of the same period. The principal sources were the first seven of Mayer's "Key Competencies",[*] and the *Profiles.*, which provided the material for the assessment "Outcomes" near the ends

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14 *Vide* T. Smith (2000), especially her neat summary of the critical underlying literature which sustains the social Constructivist perspective.

15 Frid (2000, pp. 29, 31)

16 SA. DETE (2001, General Introduction, p. 5)

17 SA. DETE (2001, General Introduction, pp. 7–8, elaborated on pp. 9–21)

† SA. DETE (2001, General Introduction, p. 10). The exact form of Constructivist philosophy embraced is not clearly stated. It is also important to realise that just as the SACSA Framework was for the first time in SA being deeply incorporated into a curriculum document, serious criticisms of some aspects of the philosophy were starting to surface within international Mathematics Education (e.g., Lesh, 2000).

18 SA. DETE (2001, General Introduction, p. 11)

* Collecting, analysing and organising information; communicating ideas and information; planning and organising activities; working with others and in teams; using mathematical ideas and techniques; solving problems; using technology. The one omitted is "using an understanding of cultures". *Vide* ch. 16.

of Years 2, 4, 6, 8, and 10, called "Standards 1–5".The one quite new component, five "Essential Learnings", could have been equally predictable:

- Futures
- Identity
- Interdependence
- Thinking
- Communication

The whole document resembles a two-dimensional hypertext with each statement code-linked to the relevant Key Competency, Essential Learning, and Standard. It is an evolutionary development and technical improvement[19] which could easily have been predicted in 1994 from the rhetoric of that time and the principles of the BSEM. Here we only need to discuss the two unexpected changes.

AN INCREASE IN THE IMPORTANCE OF STOCHASTICS

In mathematics, from as early as Reception, the Strand "Exploring Analysing and Modelling Data", which includes probability, has been given primacy over the Strands "Measurement", "Number", "Pattern and Algebraic Reasoning", and "Spatial Sense and Geometric Reasoning". This major change from the traditional ordering, especially in primary schools, fits with the increased emphasis on students' constructing their own mathematical meaning and on learning being set within children's "authentic experiences".[†] But it has not been accompanied by any change in content or approach. These have been based on the *Profiles:* and have retained the weaknesses identified in Chapter 16. There is also no evidence that the topic of probability or its links with data analysis have been rethought to ensure that they can be used to good effect in their new position.

It was predictable that Statistics would become more important once Technological developments made data analysis easier.[20] It was also highly likely that

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19 K. Truran reports that student teachers find interpreting the SACSA structure much easier than they did for the National Curriculum documents.

† SA. DETE (2000, Early Years Band, p. 226). It might be argued that the ordering is alphabetical. The ordering of the Learning Areas is definitely alphabetical (as can be seen from the thumb tabs), though sometimes this is over-ridden by space constraints. The Strands of some Learning Areas are arranged alphabetically, others are not. Given that the SACSA Framework is consciously modelled on the National Curriculum documents, it is reasonable to conclude that any change in ordering has been deliberate, especially because the new ordering conforms more closely with the ordering of the Key Competencies.

20 It was mentioned in Chapter 24 that statistics, but not probability, would in c. 2003 become a standard part of a more technologically driven academic Year 12 mathematics in SA.

probability would remain subservient to statistics, and that the ineffectual Mathematics Education node would not improve the *Profiles* or Pedagogic practice.

But it was not predictable from Figure 24·6 that Statistics would have become preeminent within the Mathematics Learning Area and that conservative Social forces urging greater emphasis on communication, greater integration of studies, and increased testing would have been associated with a rise in status of a radical, unstructured, and difficult aspect of basic Mathematics. Some forces at play must have been stronger than, or different from, those described in Figure 24·6. The Mathematical forces may also have been a little weaker, but the Model already shows them as being resisted, so this small error is probably not relevant.

It seems that the influential place of Mathematicians was filled by Pedagogues with a Social commitment of a quite different nature from that of the Technocrats. They claimed to draw Intellectual authority from the Constructivist movement, but went well beyond psychological Constructivism (a theory of learning) to social constructivism (a theory of knowledge, and hence of education). The evidence for this may be seen in an even more radical change, which involved a partial redefinition of the nature of Mathematics.

REDEFINING THE NATURE OF MATHEMATICS

This change is so radical that it is necessary to quote at considerable length from the introduction to the Mathematics Learning Area.

> Mathematics within the SACSA Framework involves an ongoing discussion and debate between two broad intellectual communities with contrasting points of view. … The dominant perspective represents a view that has been present in education for a long time: the second, emergent, one offers another set of understandings and associated practices.
>
> In summary form, the first position comprises a view based on a recognition that mathematics is a body of knowledge that involves certainty, consistency, and the capacity to bring order to random sets of information. Historically, this position has been important for producing methods and understandings that are required to make order out of various forms of information about the physical and social worlds.
>
> The second, emergent, view, suggests that mathematics is a body of knowledge that is fallible and a product of changing social circumstances. This position recognises the importance of human and social factors in the process of sifting, sorting, naming and applying numerical concepts. In this case mathematics is recognised as part of a chain of conversations concerning making and sharing meanings about physical,

> environmental and social processes and phenomena. This view demonstrates how, as new knowledge is generated about the physical and social worlds, mathematics contributes to and is adjusted to fit these new understandings.
>
> This second perspective stresses social values attached to mathematics, and the understandings that are deemed most socially appropriate for the issues, challenges and concerns of a given time/place. For example this perspective registers that in the technical/scientific world in which we live, most roads lead back to the wisdoms of the ancient Greeks and Egyptians and that the mathematical understandings derived from Aboriginal Dreaming are not given pre-eminent status in remaking mathematical understanding. Accordingly, this second perspective reminds us that there are many different forms of mathematical knowledge.
>
> Both of these major perspectives are important and have been, and are in the process of contributing to quality educational practices. The SACSA Framework aims to draw from both perspectives. Via a constructivist pedagogy, and through the use of the Essential Learnings and cross-curriculum perspectives (including equity and Enterprise and Vocational Education), educators and students develop sophisticated and contemporary understandings, capabilities and dispositions in mathematics. … The ongoing challenge is to produce forms of learning that unite these two approaches and, in the process, form the basis of new approaches to mathematics education.[21]

This debate is different from that described in Chapter 15, because the protagonists there were essentially in agreement about the nature of mathematics—their disagreement was about emphasis. Here the debate is on the nature of mathematics itself. One wonders just what sense teachers, parents, business people, and politicians will make of these statements. However, our concern here is to see why this remarkable statement was not easily predictable from Figure 24·6.

Who were these successful Constructivists? Their names are not listed in SACSA, so I must use informal knowledge. None was a regular practising member of the Australian Mathematics Education research community, i.e., an active member of MERGA.[22] Some were lecturers in Mathematics Education, some with research degrees. Others were Departmental subject specialists, classroom teachers, and tertiary mathematicians. Although many may well have been committed to Constructivist principles, it is likely that only a few had a deep understanding of the literature. But many from all of these groups would probably claim that they are Mathematics Educators, and perhaps position themselves in the Mathematics

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21 SA. DETE (2001, Senior Years Band—Mathematics, p. 216)

22 The only regular SA contributors are Kath Truran, Shirley Yates and myself.

Education node of the BSEM. They would certainly be seen that way by politicians and administrators, because teaching is usually seen as an holistic activity which is not split into specialised fields in the way that medicine is.

But my construction of the BSEM has seen Mathematics Education as an Intellectual environment which supports academic study subject to theoretical principles and traditional academic control mechanisms. This view is tempered but not invalidated by the control weaknesses identified in Chapter 20. Some of the SACSA writing team have had good academic training in Mathematics Education, and some were well aware that many Australian Mathematics Educators were working within a Constructivist paradigm and understood their arguments. However, for probability at least, the Framework was largely uninfluenced by the discipline, so it seems reasonable to deduce that the practical influence of Academic Constructivists was small. In my view, the contributors in the main were not at the Mathematics Education node of the BSEM, but at the Pedagogical node, just as were the innovators for the changes described in Chapters 14 and 15.

Why did Figure 24·6 fail to anticipate the strength of this Pedagogic force? In part, the influence of the Social reconstructivists after 1975 was underestimated. Their concern with equity, justice and empowerment, rather than skills and employment, has been an important part of educational debate for some thirty years. Although I had acknowledged the importance of multi-cultural pressure groups in Chapter 14, I argued in Chapter 15 that these and similar concerns had lost influence. This disregarded much evidence, of which the movement for Aboriginal reconciliation is one good example, that the Social forces for justice and equity within society and schools had remained strong at the same time as Technologically driven Social forces were exerting the strongest influence on Intellectual content. What I had particularly failed to note was that the justice and equity forces placed considerable emphasis on establishing personal identities, and so fitted neatly with Constructivist principles of creating personal knowledge.

This mixture of Constructivism and Social forces for equity and justice is almost sufficient to explain the elevation of stochastics to first place in the Mathematics Learning Strand. But not quite. It also requires a willingness of academic Mathematicians to allow the traditional subjects of arithmetic, algebra, and geometry to be down-graded. For example, one reason why NSW has still not introduced probability into the primary curriculum is because some Mathematicians have argued to the Minister that this would lead to a decline in the quality of children's

fraction knowledge.[23] As we have seen, academic Mathematicians have recently played a diminished role in influencing school mathematics teaching in SA, and in any case are usually ill-equipped to deal with the Constructivist arguments currently being presented. This reduced influence was clearly shown in Figure 24·6 and is able to explain why it was possible for the dual conceptions of mathematics to be inserted in SACSA.

Why did I fail to foresee the strength of this alliance between Social and Constructivist forces? Although Social issues had formed part of discourse about probability from as far back as the days of the New Education Fellowship, they had had very little influence on its teaching or on its place in the curriculum, even in the National Curriculum documents of the early 1990s, which were themselves largely developed by Constructivists. The influence of Constructivists presented in Chapters 15 and 16 seemed to me to be largely the wishy-washying of curriculum statements, and not of the taught curriculum itself. From my experience and discussion with others, the taught curriculum seemed to be changing very little. So I did not believe that Constructivist influence would change markedly in seven years of conservative government in difficult economic times, even with the relative freedom from Mathematical forces.[∞] The existence of the National Statements had, predictably, caused publishers to look for material which could be marketed as conforming with the latest official requirements, and at least some Constructivists had adapted their language to fit these commercial pressures.[§] But, after reflection, I stand by my judgement that the Constructivist influence on classrooms had been relatively small, though I may not have seen the forces sufficiently clearly. In my view, another factor was involved.

The writing of the Frameworks was put out to tender, and the contract was won by the University of South Australia. This university has a commitment to justice and equity, the largest School of Education in the State, and the only one preparing teachers for all levels of teaching from birth to Year 12. It clearly has strong paper qualifications for being awarded the contract, particularly by a government with a justifiable concern to improve the quality of the early years of schooling as

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23 Not surprisingly, no documentary support is available for this claim, which rests on personal discussions with people from NSW and those who know the Mathematician who is most strongly putting this claim forward.

∞ At some time in the 1990s AAMT withdrew from its membership of the Australian Mathematical Sciences Council, thus making another significant break in the links between Pedagogues and Academic Mathematicians, but I did not consider this as being of long-term concern for the Mathematical content of the curriculum either.

§ A good example is Bobis et al. (1999).

a matter of priority.† But it also had the highest concentration of Constructivist Mathematics Educators and the lowest concentration of Mathematicians, and these concerned mainly with Years 11 and 12 only, so this gave the Constructivists more strength than their numerical concentration would have predicted.* It seems that the tendering process for awarding the writing contract allowed an unbalanced set of forces to influence the construction of the SACSA Framework.

Governments rise and fall on their success in balancing conflicting pressures, so such an imbalance is very surprising. It is possible that the government was unaware that the University of SA team did not contain a representative set of views and that more care was needed to ensure that all stake-holders had a balanced input. It is also possible that the Government had an ideological commitment to Constructivism, or at least to its emphasis on establishing personal identification with learning. Given that the SACSA redefinition of mathematics is so unexpected, and so different from what many Social forces in the community would want, the former interpretation is more likely. Some confirmation of this view comes from the construction of SACSA's Science Learning Area. Although similar Constructivist proposals were made there, only the traditional view of science was included in the published Framework. This was at least partly due to substantial pressure from one person with advanced experience both as a Scientist and as a Science Educator at another University.[24] It shows that Governmental pressures on the construction of the Framework were not mandating the inclusion of a radical Constructivist theory of knowledge.

Evaluation of the BSEM's Predictive Value

It is reasonable to claim that many aspects of the SACSA Framework were predictable by the BSEM. The rise in importance of stochastics was probably also predictable, although I did not see it that way. But in my view the inclusion of dual views on the nature of mathematics was not predictable, even if the forces of Figure 24·6 had been drawn more precisely. This seems to have been a result of atypical government behaviour, which, within the ecological metaphor, might be seen as a mutation. It conforms with neither government policy nor the dominant

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† Part of the working out of this priority has been the appointment of Dr Philip Gammage to the joint role of Professor of Early Childhood Education at the University of SA and adviser on Early Childhood Education to the Minister of Education. This must have established a link which the government was unwilling to break lightly.

* They were not particularly anxious to broaden their base, either, in spite of the heavy workload required to fulfil the contract on time. Kath Truran was not invited to work on the Framework writing, even though she is on the staff of the U of SA!

24 Dr C.J. Dawson, U of Adelaide, pers. comm., April 2001

forces within society. Whether it proves to be a successful one remains to be seen. My prediction is that other Social pressures and the Physical demands of the classroom will mean that the radical SACSA changes will prove ineffective.§

In the light of these findings, is it reasonable to claim that the BSEM has predictive power? Some would see engaging the concept of a mutation as stretching the model too far. Others would see it as another example of an auxiliary hypothesis which effectively preserves the hard core of the research programme. In my view, an evaluation of these cases based on the model would show significantly better predictive power than other models in existence, while conceding that the prediction had a significant error. There is not space to examine this here, but I incline to the view that the model is remarkably good as a predictor, and that it also provides a context for interpreting the likely significance of unpredicted events.

For example, the dual views constitute an open sore which, if it festers, could do great damage to SA mathematics teaching. But they will have to be worked out in a society and an educational system which are intolerant of such dual views. There are good reasons for believing that this mutation will go the same way that most mutations go. As well as this, the analysis of why the sore was not predicted provides an effective way of identifying not only how it occurred, but also how it might be healed—i.e., by ensuring that the curriculum is constructed as a balanced response to all relevant forces. In other words, the BSEM is able to identify which forces were under-represented, and so it shows itself to be a *useful* model: it provides the type of information which Lange was calling out for.[25]

THE ABILITY TO GENERATE NOVEL RESULTS

> … there is nothing new under the sun.[26]

The prediction by astronomers in 1846 of the existence of the planet Neptune is a good example of a novel result arising from the application of Newton's gravitational model to planetary orbits. Neptune itself may not have been new, but

§ E.g., the Victorian reforms of the early 1990s introduced compulsory mathematical tasks which had to be completed, but did not have to be completed correctly. Not surprisingly, these "reforms" have since disappeared.

25 *Vide* quotation at the head of ch. 24.

26 Ecclesiastes 1: 9

knowledge about it certainly was. Here I shall re-examine one aspect of this thesis, and show that the BSEM illuminates it so differently from traditional interpretations, that the BSEM may be said to have generated a novel result.

In Chapters 20 and 24 I presented some criteria for establishing the existence of a profession to try to establish whether teaching really was a profession. There are many similar sets of criteria in existence, of which one brief example is:

> the two primary bases for specialization within a profession are (1) the substantive field of knowledge that the specialist professes to command and (2) the technique of production or application of knowledge over which the specialist claims mastery.[27]

For some people education is merely a "minor profession hopelessly non-rigorous [and] dependent on representatives of academic disciplines" as well as being subject to "shifting, ambiguous ends, … unstable institutional contexts of practice, and … *therefore* unable to develop a base of systematic, scientific, professional knowledge".[28] In this thesis I have argued strongly against such a view, while not denying many weaknesses in the practice of education. But I showed in Chapter 24 that educational and medical practice were both capable of being described by the same basic interactive structure, and hinted in Chapter 25 that the same model might well also apply to architectural practice. This suggests a quite different way of defining a profession, viz., as a system which is responsive to all of the forces from all of the environments defined in the BSEM. There is not space here to examine this proposal in detail. However, the structure clearly does allow for expert knowledge and its application in a responsible way within a society. It addresses the ambiguities over control and application of knowledge which were mentioned in Chapter 24, and it allows for shortcomings without denying the existence of a profession or requiring the definition of a new type of disciplined practice.

For the time being, this proposal must remain untested, but there is a *prima facie* case for arguing that an interactive approach more neatly encapsulates the complexities of professional practice than does a check-list approach. It is therefore possible at this stage to put forward a testable claim that the BSEM is able to generate novel results and therefore the ecological model does constitute a progressive research programme.

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27 W. Moore (1970, p. 141), cited in Schön (1983, p. 22), italics added

28 Glazer (1974, p. 346), cited in Schön (1983, p. 23)

CONCLUSION

O Wedding-Guest! this soul has been
Alone on a wide wide sea:
So lonely 'twas, that God himself
Scarce seeméd there to be.
…

Farewell, farewell! but this I tell
To thee, thou Wedding-Guest!
He prayeth well, who loveth well
Both man and bird and beast.[29]

Gage has summarised claims that social and educational research only yields obvious results, and showed that in practice quite contradictory positions may be agreed as being "obvious" when presented authoritatively. He suggested that this phenomenon was most common when people had limited background knowledge.[30] We have met many examples of educational decisions being made with limited background knowledge, and the interactive BSEM is designed to provide a structure to show where such gaps might exist, to predict their potential influence, and to help overcome them.

Sufficient evidence has been presented here to claim that the ecological metaphor fulfils all Lakatos' conditions for a progressive research programme. In particular, the BSEM is an embodiment of this metaphor which provides insightful interpretations that are able to be tested and have a high, though not absolute, degree of reliability. As a bonus, it is also able to be used as a model for future planning.

One commentator has claimed that "[t]here has been relatively little written about the history of curriculum development in Australia and few general principles have been articulated".[31] For Mathematics Education, the dearth has been even greater. I see Clements' Colonial Echo Model as related mainly to the Physical and Social nodes of the BSEM with an emphasis on the Physical node, and Horwood's Social model as emphasising the Social node. This is not to dismiss the insights these authors have presented, but to set them into a more comprehensive picture. I would also argue that use of the BSEM can lead to a more thorough and disciplined interpretation than what is often provided by curriculum history writers, even learned and distinguished ones. Three brief examples will suffice:

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29 Coleridge (1797–98, ll. 597–600, 610–614)

30 Gage (1991)

31 Seddon (1989, p. 1)

> In the end what became of the American curriculum was not the result of any decision, or victory by any one of the contending parties, but a loose, largely unarticulated, and not very tidy compromise.[32]

> Almost by a process of osmosis, the [Australian corporate] schools have done very much the same things at very much the same periods. … It is also possible to see that they have changed, for the most part, not because of the dynamic potential within this form of schooling, but rather in response to outside influences that have operated upon them.[33]

> About 1967 Australian education underwent changes in may respects more profound than any previously experienced, for they were part of a deep-seated alteration in the character of Western civilization.[34]

None of these interpretations is false, but they leave so much unsaid. Furthermore, they are not generalisable, explanatory, or testable. The BSEM seeks to be all three.

If my claims about the BSEM are sustainable, then, by implication, so must my belief in Mathematics Education as a discipline. This too goes beyond some learned opinions from distinguished workers in the field:

> Unless [information about mathematics education] is extracted from the scattered literature, organized and made widely available, I predict that future International Congresses will be much like the past ones: new faces, new opinions, but very little new knowledge.[35]

> Today, many mathematics education researchers believe that the very nature of educational activity—the complexity of the objects of study—means that educational research should not be expected to be a "science" in the traditional sense.[36]

To sum up. Some of the petitions in the Gorseth Prayer which introduced Chapter 1 have, for me at least, been granted. What I have learned about God through this thesis must remain within my own soul. But Knowledge, the centre-piece of the Prayer's list of petitions, is the most obvious gift I have received. Deeper insights about Justice, Power and Passion have also been granted, and hopefully some Wisdom as well.

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32 Kliebard (1986), cited by Musgrave (1988, p. 8)

33 Sherrington et al. (1987, p. 113)

34 Barcan (1980, p. 344)

35 Begle (1979) pp. 156–157

36 Lester & William (2000, p. 136)

But the force-driven nature of the Model does not seem to have said much about Protection and Goodness. The public image which educationists would like for themselves is that of kind, good, compassionate beings whose ultimate concern is for the enhancement of society and its children. This laudable aim can easily become debased through unbalanced practices like paternalism and iatrogeny. The ecological metaphor explains that such weaknesses require energy to overcome, and this energy is not automatically forthcoming. Perhaps this is why Lester & William have argued for the importance of including values as part of the research and dissemination enterprise.

> The relation between knowledge claims and evidence involves more than simply establishing a logical connection between the two. Instead, the relation is determined, in large part, by a set of beliefs, values, and perspectives operating in the context in which the empirical data are being assessed, How researchers go about convincing others of the claims they make and how they defend their claims on ethical and practical grounds are, only in part, matters of marshalling adequate contextualized evidence embedded in sets of beliefs and theories. Indeed, convincing others is also a matter of persuading them to accept the values the researchers holds about the objects and phenomena being studied, as well as about the very purpose of research itself.[37]

That must be a task for Volume 3.

> Now telleth me, eer that ye ferther went
> I ka namoore; my tale is at an end.[38]

37 Lester & William (2000, p. 136)

38 Chaucer *The Frankin's Tale* ll. 1623–4