PART E: LESSONS FROM HISTORY—DEVELOPING SOUND PEDAGOGICAL PRACTICE
Part B concluded with Henry Ford’s well known claim that “history is more or less bunk” and his less well known claim that “the only history that’s worth a damn is the history we make today”. Parts C and D have shown that the teaching and assessment of probability since its introduction into the curriculum in the 1960s have been fraught with difficulties. The BSEM has provided some reasons for this relative lack of success in a manner which hopefully has done something to refute Ford’s first claim. But the desire for relevance which is implicit in Ford’s second claim is probably a reasonable one, particularly within education, and especially so if Dewey’s claim, cited in both Chapter 6 and Chapter 9, that “theory is … the most practical of all things” is true.

So in this Part we examine some links between research into the obuchennyi of probability and classroom practice itself. If educational research is essentially an activity with useful practical consequences, then we should find that probability research has had some influence on classroom practice. However, given the difficulties outlined in the earlier Parts, it will come as no surprise to find that the links have been tenuous. Some of the evidence presented here can be interpreted by the original form of the BSEM, but not all. As we suggested in Part C there is a need for two further forces to be incorporated into the ecological model and this will be done in Chapter 24.
CHAPTER 21: RESEARCH, CLASSROOM PRACTICE AND CHANGE

Suppose 3 white balls, 4 black balls, and 5 red balls to be thrown pro-miscuously into a bag; required the probability that in two successive trials two red balls will be drawn, the first ball being replaced before the second trial.¹

The decimal parts of the logarithms of two numbers are taken at random from a table to 7 places: find the probability that the second term can be subtracted from the first without borrowing at all.²

These two questions from a popular nineteenth century algebra textbook suggest the nature of probability teaching at that time and contrast starkly with the material described in Chapter 16. But they are little different (changes in the meaning of certain words aside) from many questions found in textbooks today, as described in Chapter 4 and Part C. The classrooms of tomorrow will not be the same as those of today, or of yesterday. So in this Part we look at some attempts to improve the quality of probability teaching in classrooms. After summarising research into didactics in general, in subsequent chapters we look at research into probability teaching and case studies of some projects with which I have been personally involved. Finally, the BSEM is used to suggest why the discipline of Mathematics Education has not found a firm footing within probability classroom practice. We start by considering Mathematics Education as a discipline.

IS THERE A DISCIPLINE OF MATHEMATICS EDUCATION?

Even though its history is not a long history, research in mathematics education is a conversation that began well before today’s researchers appeared and that will continue long after they have gone. It is a conversation with thousands of voices speaking on hundreds of topics. Important issues are discussed, but the commentary is often difficult to understand. Listeners can get impatient and discouraged. It is easier to walk away and dismiss what is being said than to stay and listen and respond.³

I mentioned in Chapter 1 that Mathematics Education was seen by some as sterile and unfocussed, but expressed my own belief that is was potentially all-embracing. Disciplined study of mathematics teaching has extended over at most

¹ Todhunter (1875, p. 452)
² Todhunter (1875, pp. 478–479)
³ Kilpatrick (1992, p. 31)
two hundred years, and mainly in the last fifty. Now we may find all the features of an academic discipline—competing paradigms, professorial chairs, doctoral students seeking fame and glory, learned societies, journals, books, sequences of books, conferences, handbooks, and specialised subgroups in the process of developing their own structures. Many of these features have already been encountered at many places in this thesis.

So at the present there are two types of practitioners concerned with teaching mathematics: people who teach mathematics, and those who reflect in a disciplined way on its obuchennyi. Relatively few seem to do both, though there are notable exceptions. In general, the latter have a higher status in society than the former, although those who teach mathematics in universities probably have a higher status still, which can lead to special difficulties if they consider that there is little need for those who reflect on their practice as we saw in Chapter 15.

There has been a number of attempts to develop an overarching theory of Mathematics Education. One of the earliest was the establishment of the CIEAEM (International Commission for the Study and Improvement of Mathematics Teaching) in the period 1950–1952, but this has been a predominantly European and Francophone organisation which had relatively little direct influence on Anglophone thinking in its early years in spite of having early leaders like Piaget, Papy, and Gattegno. Perhaps the most influential in recent years, at least for Francophones, has been that of Brousseau, whose emphasis is always on what is learned by the child, and on the nature and effectiveness of didactical contracts between teachers and students. The careful approach of theorists like Brousseau, and their concern to expose cant and confusion contrast strongly with the pedagogic and assessment practices driven principally by “what works” which we have met in Parts C and D. Without suggesting that all work has been of this form, or that such work is not of value, it is true that much of what has been done in schools has had a limited perspective—perhaps “teaching”, perhaps “learn-
The overall vision encapsulated in the word *obuchennyi* has frequently been missing. It is the linking of these various perspectives into an overarching theoretical perspective which constitutes the discipline of Mathematics Education. The structure of this thesis, with its Sections on curriculum, assessment and pedagogy, is an attempt to demonstrate that this interlinking is necessary on practical grounds, and can produce further very broad insights. Its total is greater than the sum of its parts. Demonstrating this property alone is sufficient ground for claiming that a discipline of Mathematics Education exists.

Kilpatrick, a present-day doyen of Mathematics Education, has seen the failure of most researchers to set their work within a theoretical perspective as a key reason for the limited success of Mathematics Education research. He has suggested some reasons for this—the complexity of the topic, the desire to mimic the numerical social sciences, or the need to produce papers which attract university funding—and argued that Mathematics Education researchers should be working more as members of inter-disciplinary teams which have teachers as their central focus. He has argued for both theory and practice, though one might wish that he had argued for the centrality of the learner, rather than the teacher.

Certainly, theory is not enough. At one stage it was thought that Piaget’s theories could provide an adequate guide to what could or could not be learned by children at different ages. So some researchers tried to use his theory to develop syllabuses for topics like probability which had not been taught in schools before. This view proved to be simplistic; some of the reasons for this have been discussed when looking at Stage-related theories in Chapter 8 and in Part D.

There are several other broad theoretical perspectives in existence. That from Hans-Georg Steiner at the University of Bielefeld, has been perhaps the most comprehensive, but is very abstract, and has had relatively little influence outside

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§ Commission Internationale pour l’Etude et l’Amélioration de l’Enseignement des Mathématiques
8 Bernet & Jacquet (1998)
† The abundant material produced by CIEAEM over 50 years has only come to my attention in very recent years. This gap in my reading is a good example of the linguistic isolation discussed in Chapter 8.
10 Kilpatrick (1981)
11 Kilpatrick (1992, p. 31)
12 Kilpatrick (1992, p. 31)
13 E.g., Lovell (1971)
14 Vide, e.g., H.-G. Steiner (1987), Biehler et al. (1994).
of Germany. French research, including Brousseau’s, has shown great interest in “the complexity of the links between teaching and learning”,\(^\text{15}\) and may become more widely known now that Brousseau’s principal works have been translated into English.\(^\text{16}\) Research from the Netherlands, under the charismatic influence of Hans Freudenthal,\(^\text{17}\) has been perhaps the most influential, with its strong emphasis on both mathematics and children, and its setting within a Liberal-Humanistic/Constructivist framework. One example will be discussed in the next chapter. To analyse all these theories is not possible here, but their existence and diversity show that there are able practitioners who do believe that Mathematics Education does exist, and can produce findings not obtainable in other ways.

Many of these practitioners, and certainly most of the public, would presume that the enterprise would produce findings of special value to teachers of mathematics, just as medical research produces findings of value for medical practitioners. Theorising is not wrong, and not every finding has to be immediately applicable, but some findings must be demonstrably useful. Further, it is assumed that demonstrably useful findings will be used by teachers. But this is not always the case, as we shall see. So we may accept that the discipline of Mathematics Education is worth study, but that its tenuous links with teachers make its value difficult to see. Following Kilpatrick’s advice at the head of this section we shall try to “listen and respond”—in this case to evidence about the quality of these links.

THE QUALITY OF THE LINKS BETWEEN TEACHERS AND ACADEMICS

The doubts that we, as educational researchers, might have about the bearing of our research on educational policy and practice are, I believe, unfounded. In addition, we must recognize that in Australia ... there has been a “quiet revolution” in educational research over the past decade. While much of this more recent research has not been fully assessed and incorporated into the accumulated body of knowledge about education, the evidence of the past ... has been that research in education and related disciplines does have a very significant influence on educational policy and practice.\(^\text{18}\)

These words are from a presidential address to the Australian Association for Research in Education by John Keeves, then a senior member of ACER. Keeves defined three main foci of educational research:

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\(^{15}\) Laborde (1989, p. 31)


\(^{17}\) E.g., Freudenthal (1978, 1983)

\(^{18}\) Keeves (1982, p. 5)
• understanding the nature of the educational process;
• findings with direct applications to educational policy and practice:
• preparing material for direct use in schools and classrooms.19

He emphasised the importance of broad-based studies which tend to receive less funding than more focused studies, but even so often contribute significantly towards the development of a consensus in educational policy-making. This agrees with some of Kilpatrick’s points made above, but Keeves also insisted that the three types must not be confused with each other in general discussions. In this study, our concern is principally with the third focus, but it is important to see all three approaches as having different but equal validities.

In picking up his third point, Keeves addressed “the diffusion of useful knowledge”—a Victorian-era term deliberately chosen because he saw a need for the Victorian optimism of the 1840s to be restored to the 1980s. He argued that the “Linkage Model” of Havelock20 was an appropriate dissemination model. It links several of the approaches discussed in Chapter 7, analyses the flow of information between individuals and groups, and so has some of the elements of an ecological approach. What it lacks is analysis of the forces which cause the system to break down. Keeves described the sorts of desirable structures which would implement Havelock’s model, and showed that many were well in place within Australia at the time of his address. He concluded that

it will not be by the flooding of schools with detailed reports of research that significant effects will be achieved. Rather the greatest gains will probably be made by the identification of linkage agents and linkage institutions with the linkage medium and the establishing of networks around these agents and institutions.21

Here Keeves conceded that mere transmission of information is not enough, and noted the importance of Social forces, but he gave little evidence for the impact of research on classrooms, and did not address the complexity of teaching or the energy needed for stable change. His claim must be judged “not proven”.

Indeed, the weakness of the links between research and practice have been well documented by many researchers, and we have quoted some of these in Chapter 16. Don Hogben has used an argument based on Social forces to argue that

[u]nlike physicians and engineers (to take the two professional groups with whom teachers seem most often to be compared), teachers tend not

19 Keeves (1982, pp. 6–7)
20 Havelock (1969)
21 Keeves (1982, pp. 17–18)
to belong to strong academic/professional associations and tend not to subscribe to research journals.\textsuperscript{22} Hogben blamed this on poor pre-service training and a failure to show teachers how to interpret research in a world where neither the pressured school environment nor the reward structure encourage the reading or application of research. On the other hand, a recent British report saw the issue as being more complex:

Research is only one of the influences on policy formation and practice, and any impact is likely to be indirect rather than direct through a variety of transmission mechanisms and intermediaries. There is no simple model, and as a result the impact of research is difficult to isolate and measure. ... The actions and decisions of policy-makers and practitioners are insufficiently informed by research.\textsuperscript{23}

This report suggests that the Intellectual forces are too weak to be of influence, but does not suggest why. But for Musgrave the problem lies less in the practice of the teachers, as Hogben argued, but in how findings are disseminated.

The flow of information to serving teachers has repeatedly in many societies been found to be attenuated in the extreme. One difficulty is that much of the relevant information, especially in the form of research findings, is not made available to them in a suitable way. It is usually disseminated in expensive and scarce journals or texts often written in what teachers take to be jargon.\textsuperscript{24}

In this thesis we have seen supporting evidence for all of these assertions both from Mathematics Education in general and also from probability in particular. But we have also seen many examples of good linkages being established which were not actually working very well.\textsuperscript{*} The issue is clearly complex.

We saw in Chapter 7 that “buzz words” are often used by curriculum developers, but are interpreted in different ways by different teachers. Hargreaves has claimed that successful curriculum developments need slogans to carry workers through the inevitable troughs.\textsuperscript{25} Apple has seen \textit{Standards} documents as slogan systems enshrining a penumbra of vagueness to make them potentially compre-

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\textsuperscript{22} D. Hogben (1980, p. 56)

\textsuperscript{23} Hillage et al. (1998), cited by Keith Jones, general e-mail message, 6 Oct 1998

\textsuperscript{24} Musgrave (1985, p. 9)

\textsuperscript{*} As this chapter is being written two national conferences are scheduled to be held in Adelaide during 1999—one for mathematics teachers, one for mathematics educators. They are being held six months apart, and the links between the two planning committees are tenuous. A third, local, organisation—the Adelaide Consortium for Mathematics Education—held one of its rare meetings during the mathematics education conference at a time when the conference delegates would not easily have been able to attend.

\textsuperscript{25} Hargreaves (1994)
hensive and enchanting. Successful slogan systems, Apple claims, leave their unresolved tensions enshrined within them.\textsuperscript{26} The apparent neutrality of curriculum documents easily hides their underlying philosophy and inbuilt tensions.\textsuperscript{27} How much more so do slogans like “activity methods”, “Piagetian theory”, “relevance” and “numeracy”! How easily Orwell’s animals—weak readers and poor interpreters—were persuaded to change their chant from “four legs good, two legs bad” to “four legs good, two legs better” when their leaders started to emulate human practices.\textsuperscript{28} Anything for a quiet life.

Slogans and buzz words provide a good example of one aspect of linkages which is rarely discussed, but is implied in many writings. It is the assumption that the linkages are \textit{from} the researchers \textit{to} the teachers: they are “one way”. By contrast, as we saw in Chapter 7, Eisner has emphasised the complexity of teaching.

I start with the assumption that the improvement of education will result not so much from attempting to discover scientific methods that can be applied universally to classrooms throughout the land, or to individuals possessing particular personality characteristics, or to students coming from particular class backgrounds, but rather from enabling teachers and others engaged in education to improve their ability to see and think about what they do.\textsuperscript{29}

Similar ideas have been developed by Doyle in the form of three linkage paradigms—“process-product”, “mediating process”, and “classroom ecology”.\textsuperscript{30} It is “the richness and complexity of classroom settings”\textsuperscript{31} which makes the third paradigm so important. By turning aside from simplistic, deterministic “input-output” models, Doyle and Eisner have provided a basis for “a more complete understanding of the student competencies necessary in order to learn from classrooms”\textsuperscript{32} and a different way explaining how instructional effects occur.\textsuperscript{33} Doyle has concluded:

\begin{quote}
A\textsuperscript{t}tempts to attribute differences in student achievement to a few generalizable dimensions of teacher behavior or instructional materials may well be futile. The implication is that teacher effectiveness formulations should include both contextual variables and the meanings teach-
\end{quote}

\begin{itemize}
\item \textsuperscript{26} Apple (1992, pp. 413–415)
\item \textsuperscript{27} N. Gough (1989)
\item \textsuperscript{28} \textit{Animal Farm} Orwell (1945/1951)
\item \textsuperscript{29} Eisner (1985, p. 104)
\item \textsuperscript{30} Doyle (1977)
\item \textsuperscript{31} Doyle (1977, p. 182)
\item \textsuperscript{32} Doyle (1977, p. 183)
\item \textsuperscript{33} Doyle (1977, p. 185)
\end{itemize}
ers and students assign to the events and processes that occur in classrooms. One is even inclined to speculate, on the basis of an ecological analysis, that the teacher effectiveness question itself might be changed from “Which instructional conditions are most effective?” to “How do the instructional effects occur?”

Doyle’s perceptive insights are based on many research findings. In the 20 years since his work was written there has been a move to locate educational research more within the classroom than without. This has been a move away from the quantitative methods of the hard science to the qualitative ones of the social sciences. It may be argued that qualitative approaches are better suited to analyse complex practices like teaching, but they are open to the charge of being excessively subjective, and are often outside the experience of many who may make judgements about educational practice, as for example, we saw when discussing the influence of the Technocrats in Chapter 16. To understand the links between those involved in education, we shall now look at findings from both types of research, firstly with respect to teachers, and then to administrative structures and researchers.

THE GOOD TEACHER

Keating slammed his hand on the wall behind him, and the sound reverberated like a drum. The entire class jumped and turned to the rear. “Well,” Keating whispered defiantly. “I say—drivel! One reads poetry because he is a member of the human race, and the human race is filled with passion! Medicine, law, banking—these are necessary to sustain life. But poetry, romance, love, beauty? These are what we stay alive for.”

Not all teachers are bad, and not all are pilloried in reminiscences. John Keating is a fictional representative of those many teachers who are remembered for challenging their students. Certainly he is an extreme and a fictional example,† but we have already met Osborn’s memories of Dickie Holtham in Chapter 3 and Ferrers, in Alec Waugh’s youthful autobiographical novel The Loom of Youth, is

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34 Doyle (1977, p. 188)
35 Dead Poets’ Society Kleinbaum (1989, p. 41)
† It is an interesting comment on the status of education studies in universities that only one copy of this extraordinarily successful popularisation of important educational principles was held in the three South Australian universities, all of whom have Schools of Education. Perhaps this was because it was seen by them, as it was by the State Library, as “teenage fiction” and therefore irrelevant to a university. Certainly the only university copy was in a collection of light reading kept specifically for agricultural students at a rural campus, and could not possibly be made available for consultation by a researcher.
another example from the early part of this century, set in a time when the culture of teaching was particularly mouldy, based as it was to a large extent on the use of cribs which enabled answers to be provided without cognitive processing. Another, more conventional example comes a late Victorian-era girls’ school.

At Ellerslie the classes were small. ... We had a great deal of individual superintendence. We used textbooks, and a certain portion would be given to us to prepare, not to learn parrot-fashion, as at The Turrets, but merely to get the gist of the thing. Then the mistress would begin her lecture; using the textbook as her basis she would question us on the facts we had learnt from it, and would very much amplify and enlarge the subject herself, adding all kinds of illustrations and explanations, and encouraging us to ask about what we did not understand. ... Every girl who passed up the school would have had this tuition every other day for a period of six years, so she would have been very dense indeed if she did not derive some culture from it.

To complement such memoirs, researchers have examined the concept of an ideal teacher in some detail. A few examples, some of which were referred to in Chapter 1, will show that some agreement can been reached about good teaching, without denying that there will still be substantial variation, and that there will still be ec-centrics (used here again in a non-pejorative sense) who succeed brilliantly in the classroom, but who do not conform with the criteria. In general, for mathematics teachers, a person’s ability to communicate is critical, as is the nature of that person’s beliefs about the mathematics being taught. This comes close to the importance of passion for one’s subject which the literary examples emphasise, and shows the importance of the ideas summarised in Chapters 4–5.

Another, extraordinarily detailed, study in junior primary schools of six highly regarded teachers has shown the extent to which good teachers are required to be superb managers. Even so, these excellent teachers often found that management constraints prevented them from stretching some children to their limits. In a wider survey, it was found that exemplary mathematics teachers:

- have a thorough and comprehensive knowledge of the content they were to teach;
- have a range of teaching strategies that could be used without a great deal of conscious thought;
- use management strategies which facilitated sustained student involvement;

36 A. Waugh (1917/1955)
37 My Own Schooldays Brazil (1925), cited in Craig (1994, pp. 392–393). Brazil was a writer of popular teenage fiction set in girls’ schools.
38 Herrington et al. (1992)
39 Desforges & Cockburn (1987, p. 136)
• use strategies which encouraged students to participate in learning activities;
• use strategies designed to increase student understanding of mathematics;
• maintains a favourable classroom learning environment.40

Most of these results are not surprising, but they do come from extensive studies by experienced researchers, and so provide some secure data from which to work. They show that good teachers have valuable insights to bring to linkages between teachers and researchers because they:

• are enthusiastic;
• know their subject;
• are good classroom managers;
• have high pedagogical skills.

It is, of course, true that bad teachers will also bring insights to the linkages which will be ignored at peril. However, at this stage in the discussion it is easier to examine a world which, if not perfect, is at least working quite well.

Although most of the studies have not researched this feature, it is likely that such good teachers are well aware of their skills. So if they are called on to change their practices or develop new ones, they are most unlikely to cast away what is already working very well for them. There is an extensive literature on change processes in schools, some of which has been summarised in Chapter 7. Each of the perspectives described there has some value in providing a framework for examining different situations, but it is not easy to suggest a unifying structure for them in the way that Figure 3-1 was of help in integrating many different philosophies of education. A further difficulty is that few of the studies have examined the actual process of change, as opposed to its outcome. But it is not only important to design a bridge which will stand up very well once completed, it is also important to design one which is strong and stable during construction as well. So we also need to see how teachers work in times of significant change.

40 Tobin & Fraser (1988)
Examining the dynamics of change involves addressing tensions within schools, and it is here that an ecological model, which looks at how stability is achieved in the face of many conflicting forces, is of value. We live in a world where simplicity is seen as a virtue, and where tension is discouraged. Yet the simplicity is difficult to obtain, and tension is all around us. In looking at change, then, we need to take both aspects very seriously and try to understand how good teachers work within them. It is very important to look at change from a broad perspective because “the adoption of a particular perspective and its concomitant emphases and goals can lead to the advocacy of a particular form of professional development program, and to the undertaking of a specific type of research.”

**Some Examples of Change**

We shall look first at some theoretical models specifically concerned with change, as opposed to curriculum in general, and then look at some concrete examples. Such an approach seems an appropriate way to see if there really is a discipline of Mathematics Education which is securely linked with the chalk-face.

**THEORETICAL MODELS OF CHANGE**

Larkin has proposed a model to improve the quality of teaching and learning. It involves replacing the “Teach ∅ Illustrate ∅ Apply” sequence by an “Activity ∅ Learning ∅ Reinforcement” sequence and is introduced to teachers through an extensive sequence of activity-based workshops. His emphasis is very definitely on process and any products which are developed come as bonuses. They, like other artifacts [sic], are of greatest value to their creators. ... Laymen rarely become expert craftsmen by observing the finished products of the masters.

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41 French proverb
43 Larkin (1978, pp. 212–213)
44 Larkin (1978, pp. 213–215)
45 Larkin (1978, p. 218)
Larkin does not provide evidence for the efficacy of his model, which is very much a “one way” model, but it can be seen to have a Constructivist basis. On the other hand, Alan Bell has started the description of his much more “two way” model by listing psychological principles which he believes should underpin any teaching design. These are:

- Connectedness
- Structure and Context
- Feedback
- Reflection and Review
- Intensity

With the exception of intensity, which we shall discuss below, these are all fairly predictable and Bell is able to substantiate them by reference to his own experiments:

> Our own diagnostic teaching experiments ... have shown that intensity is related to successful learning. Games provide such experience in that many trials with limited variation are involved and a discussion focussed on one or a few points is itself an intensive experience. Our results suggest additionally that the most vigorous and intensive discussions resulted in the greatest amount of learning. However, it is clearly not extensive repetition alone which has the effect: this property was possessed also by the contrasting teaching materials which we used. The presence of feedback and a high level of personal engagement are important.

Bell uses these principles to present a theory for designing teaching and gives several examples of how his schema may be applied in practice:

- Situation, Task and Intervention
- Feedback
- Changes of Structure and Context
- Differentiation by Individualisation or Flexible Tasks.

Hunting & Davis have discussed the idea of a teaching experiment, a model which emphasises a teacher’s internal development. It is not a “one way” model which starts from the teacher, but one which emphasises that teachers’ beliefs and understanding are critical features of curriculum change. They argue:

> The professional development of teachers competent to lay the groundwork for children’s success in mathematics cannot be done through the

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46 A. Bell (1993, pp. 9–17)
47 A. Bell (1993, p. 17)
48 A. Bell (1993, pp. 18–20)
provision of information alone. Teachers’ views of mathematics learning processes need to be reconstructed, as an essential prerequisite to the improvement of teaching practices. Teachers whose use of numerical concepts and procedures has become routine over the passage of time need to understand the constructive processes of children as they attempt solutions to mathematical problems. They need to observe first-hand the behaviour of children under particular conditions and in particular contexts. Personal encounters with children, together with opportunities to reflect upon and compare with their peers their observations, were considered to be fundamental to the project’s success.

If, as we assumed, the mathematics of children and the mathematics of adults are different, we are obligated to consider the processes of mathematics learning from two different frames of reference: from the point of view of the child, and from the point of view of the teacher with whom the child has sustained communications.49

This important but often-forgotten feature of teaching is probably part of the classification mentioned above—“uses strategies designed to increase student understanding of mathematics”—but it is good to have it clearly spelled out.

Finally, a major difficulty in developing a model of change is in finding an appropriate place for evaluation. It is much harder to evaluate the change process than the outcome of the change, particularly because the wide range of people involved will make it the result of a much wider set of philosophies than its designers had envisioned. As we saw in Chapter 7, Howson has emphasised that all assessment is philosophically driven and that different forms are more suited to different teaching styles. It is much easier to evaluate facts than ideas. If the easy way is taken when evaluating a complex process like teaching, then the evaluation becomes banal.50 Indeed, the type of assessment typically required by curriculum funding agencies can lead to change being promoted principally by using a curriculum package with which teachers are expected to conform.51 Yet Bruner has argued:

Evaluation can only be of use when there is a full company on board, a full curriculum-building team consisting of the scholar, the curriculum maker, the teacher, the evaluator and the students. Its effectiveness is drastically reduced when it is used for a single purpose, say, of editing a chapter, making a film, devising a text.52

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49 Hunting & Davis (1991, p. 172)
50 Dale (1977)
51 Dale (1977)
52 Bruner (1966, p. 164)
So here, just as in Chapters 3 and 7, we have a variety of models being presented, none of which seems to be totally satisfactory. Yet there is still a strong need for a unifying structure to act as a balanced guide for practical behaviour. So let us now look at some practical examples of efforts to effect change to see whether they can give us deeper insights.

**PRACTICAL EXAMPLES OF CHANGE**

Links between research and classroom practice have been examined by the Children’s Mathematics Frameworks Project at Chelsea College, London. It has summarised children’s mathematical thinking in the middle school years, with a special concern on how to lead children from intuitive experiential thinking to more formal “mathematical thinking”. The researchers found that the process of formalisation was difficult. One reason is of special importance here:

Some teachers felt it was not always appropriate to teach for the formalization and preferred to place more emphasis on pre-formalization work—it may well be that for some children this only reinforced the notion that their more naive (e.g. counting or ‘intuitive’) strategies were appropriate for the tasks (problems) they were being asked to do.55

In other words, teachers’ perceptions may well dominate in classrooms, not because teachers are perverse, but because their perceptions may be relatively narrow. Similarly, in a smaller project, Long prepared a teaching booklet to illustrate the principles of a problem solving approach to classroom algebra to be used in classrooms by practising teachers. It was accompanied by a summary of the theory behind its construction, which teachers were encouraged to read. In general they found it of some value. However, in practice they did not always follow the precepts of the model, especially when working with children of markedly different levels of achievement in the same class. In particular, they were loathe to involve children in the mathematical discussion which Long saw as crucial to the experience.56

Similar tensions between teachers and researchers concerned with curriculum change have been observed by others. One set of researchers worked with senior educators in Colombia at a time when mathematics curricula were being changed as a result of governmental decree. The principal techniques used were those of action-research combined with many opportunities for reflection on what was

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53 D. Johnson (1989)
54 D. Johnson (1989)
55 D. Johnson (1989, p. 220)
56 Long (1989)
being done. Their report emphasised the tensions between beliefs and practice which participants felt throughout the exercise, and they saw this as a necessary part of the developmental process.\textsuperscript{57} A similar emphasis on the importance of systematic reflection, as well as the use of teachers’ own experiences has been presented by two Norwegian workers.\textsuperscript{58} Another researcher found substantial difficulty in persuading tutors in a tertiary genetics course to use educational practices like concept mapping and collaborative learning. While the minority of tutors who were successful in making the change strongly supported the practices, the majority abandoned their attempts to use the techniques, partly because of limitations in their pedagogic and content knowledge, and partly because of student resistance toward being involved in the learning process.\textsuperscript{59}

A recent comprehensive attempt to encourage change within a carefully monitored research environment has been the Calculator-Aware Number Project in the UK. Although the project was well funded and ably led it was found that it needed more thorough analysis still of both content and progression, and that researchers “should aim to develop and evaluate both a systematic design for a calculator-aware curriculum and an appropriate pedagogy of calculator use”.\textsuperscript{60} They also found that even a well-designed curriculum was only likely to be successfully implemented if it were “treated as part of a coherent and committed process of school development—and ultimately of systematic reform—rather than as the isolated responsibility of individual teachers”.\textsuperscript{61}

In most of these projects reported here researchers and teachers were working very closely together in a formal way, so that there were strong social pressures to maintain harmony between them. So whenever tension is noted in a research report it is reasonable to conclude that it was a significant feature of the enterprise: researchers are human and would prefer to report no more negative findings than is absolutely necessary. Of course, students have also generated some of the tensions by reacting against what they saw as activities not appropriate for their schooling. As suggested above, the student body can be the most conservative of all groups, especially when some proposed change calls for a different, usually more active, type of student involvement in classroom activities.

\textsuperscript{57} Valero et al. (1997)
\textsuperscript{58} Askerøi & Høie (1996)
\textsuperscript{59} Santhanan (1996, p. 229)
\textsuperscript{60} Ruthven (1999, p. 1-69)
\textsuperscript{61} Ruthven (1999 p. 1-70)
Partly to overcome all of these difficulties, other researchers have encouraged incremental change. Ransley taught student teachers how to make minor modifications to traditional teaching approaches so that they could experience the process of change before starting their work. The approach was carefully evaluated and found to be successful. The MCTP curriculum material for classroom teachers, mentioned several times before, was specifically developed to be introduced into a traditional classroom without major disruption.

But small steps are not always possible, particularly when change is driven by deeply held convictions. These convictions may be Intellectual, as with the British Nuffield Science Project in the 1960s and 1970s. Its leaders had a grand vision of science which meant that their approach would be “quite revolutionary because they wanted truly exploratory work with which teachers would be unfamiliar.”

The idea of science as a ‘magnificent human achievement’, motivated by a disinterested search for truth and requiring bold use of the imagination and, subsequently, severely critical testing, has given science a strong claim to be recognized as a humane study. If, however, the claim is to be used as a justification for science education, then science teaching must reflect it in some way.

Others have convictions based on a high vision of pedagogy rather than of content. A good example is the British Association of Teachers of Mathematics, which is strongly Constructivist and child-centred. The Association’s journal *Mathematics Teaching* contains examples of such approaches in every issue. The sheer size and persistence of this movement, which goes well beyond Britain, is good evidence that some teachers can become very effective in developing an alternative approach to teaching while working within a mainstream structure. But in spite of the movement’s importance, I know of no systematic analysis of how teachers involved with the Association actually change their practice from more traditional ones to more clinical ones. My own experience of the Association (personal contact with some of its leaders, regular reading of its journal over more than 30 years, and attendance at one of its conferences) suggests that change is effected and sustained by the creation of a Social environment which admits no other philosophy. Many of its leaders have a dogmatism about the rightness of their approach which, when coupled with the highly creative work which they succeed in doing with children, sustains the philosophy. The Association’s activities tend

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62 Ransley (1994)
63 Lovitt & Lowe (1993a, 1993b); Finlay & Lowe (1994)
64 Waring (1979, p. 124)
65 Waring (1979, p. 45)
to emphasise how change might be sustained, rather than effected. But they reiterate the findings of the calculator programme that change needs commitment from teachers and an environment supportive of the approach being taken.

We find a similar situation when we consider convictions based on social reasons. such as in the “hippy schools” of the long-haired seventies, one of whose concerns was to elevate relevance and denigrate examinations, just as is the case in the nineties. These schools were underpinned by a Phenomenological approach which claimed that when adolescents were involved structurally with the adult world, their resistance to rapid change was likely to be strong and so changes which did occur were more likely to be stable. They saw teachers and students as collaborators in a search for understanding, as philosophers whose job it is to question. Can such an approach work?

The answer ... is “yes”, provided that teachers are committed to such an approach, that they can work in teams, that they are not constrained by a rigid examination system, and provided that parents can be convinced that such an experience is worthwhile for their children.

Once again it is the commitment of teachers which comes first. This is something which goes beyond most of the curriculum analyses we have discussed. These tend to provide merely a framework for describing the skills and environment needed. This may well include the special skills needed by teachers, the nature of the society which is supporting the schools, and ways of describing the philosophical underpinnings of successful projects. But they tend to be static categorisations, paying little attention to the dynamics which are in place when effective change is occurring. Certainly, they may point out the need for dynamic leadership, but they do not describe the dynamics which lead a traditional classroom to develop a richer practice of teaching and learning. They may be seen as an educational analogue of tools used by a music critic to assess the performance of a violin concerto—technical skills, faithfulness to the score, rapport with the orchestra and conductor, consistency of interpretation, tempo. A good criticism will discuss many of these disparate aspects, but it will only become a really good criticism if it is able to bring all the aspects together into one holistic summary and show us what it is which has made a particular performance so inspiring.

The pot-pourri of examples cited above has often reported only partial successful changes. They have all emphasised the importance of the teacher for success, but they have rarely addressed the whole educational enterprise with its many
conflicting forces. The calculator programme probably came closest to doing so, but in its advocacy of more detailed advice for administrators and teachers it has run the risk of becoming reductionist. The examples have not really found a place for skilled connoisseurship and criticism within the change process, a place analogous to that occupied by the critic or master-teacher within the music education system.

Teaching is at least as complex an activity as playing a violin concerto. To understand teaching, and how it may change, demands awareness of this complexity. The examples cited here are diverse and not easily generalisable. But if the discipline of Mathematics Education really does exist, then it is necessary for a generalisation to be drawn from such disparate examples. So we come now to the focus of this Part—explication of the reasons why change has been so difficult here. The next section will provide some preliminary, general findings, and the issue will be taken up for the teaching of probability in Chapter 24.

**WHY HAVE CHANGE MOVEMENTS SO RARELY HAD LASTING EFFECTS?**

In England we have come to rely upon a comfortable time lag of fifty years or a century intervening between the perception that something ought to be done and a serious attempt to do it.\(^{68}\)

It is true that each author of a research report tries to present some theoretical rationale for his research, but these rationales rarely go beyond references to previous studies or to narrow hypotheses which someone has proposed. … Unless [information about mathematics education] is extracted from the scattered literature, organized and made widely available, I predict that future International Congresses will be much like past ones: new faces, new opinions, but very little new knowledge.\(^{69}\)

If the nature of good teaching is reasonably well understood, why are schools not able to effect useful changes? Why cannot research be used to improve practice? Is its failure to do so why some share Begle’s view that it is on a treadmill? Or is society as lethargic as Wells has suggested? Let us look at two possible reasons.

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\(^{68}\) Wells (1931, ch. 11), cited in Bartlett (1855/1980)

\(^{69}\) Begle (1979, pp. 156–157)
Weaknesses in Educational Structures

McKinlay, a practising teacher doing further research, has cited four reasons for the general non-implementation of critical educational science in the classroom:

- the questionable knowledge base of teachers;
- the elusive spark of enlightenment;
- the search for the Socratic facilitator;
- the democratic myth and the nature of institutions.\(^{70}\)

Some of these reasons arise from weakness in the educational structure of the schools, while others arise from weaknesses in the training or administrative structures. All may be found in earlier writings as well, such as those by Freeman Butts summarised in Chapter 10. From the BSEM perspective it is only the second and third which might be seen as forces, and these are relatively weak. What we find in analyses like these is that failures are attributed to the failure of a force, rather than to any mis-directed forces. Thus questionable knowledge-bases arise from the failure of Intellectual forces, and weaknesses in institutions arise from a failure to focus Physical forces appropriately.

McKinlay’s fourth reason is an administrative weakness: schools are not the democratic institutions which they are often claimed to be, because the locus of ultimate responsibility for what goes on in schools does not rest in the teachers.

His second and third reasons are related and are weaknesses of schools’ structures. He does not claim that teachers cannot demonstrate enlightenment, but rather that their lives are so busy with mundane matters that staffroom intercourse is almost inevitably pedestrian. He picks up the messages of Dead Poets’ Society, of Bell’s emphasis on intensity mentioned above, and of Keeves’ phrase mentioned in Chapter 7—“efficient and enthusiastic disseminator[s]”.\(^{71}\)

While he believes that change within schools is most likely to occur when each school has within it a charismatic challenger of received wisdom—a Socratic facilitator—he also believes that such a person is too challenging to be welcomed either by teachers, who prefer stability in their classroom, or by school administrations, who prefer to control than to encourage democratic decision making. The school environment is antagonistic to the natural internal development of such people.

\(^{70}\) McKinlay (1993)
\(^{71}\) Keeves (1982, pp. 8–9)
There has already been a great deal of evidence in this thesis that McKinlay’s first reason for minimal classroom change—a training reason and one rarely openly admitted—is almost certainly true. This Part will argue that perhaps his “questionable knowledge base” might be divided into a knowledge base about “content” and another about “practice”, remembering always that content knowledge is not necessarily synonymous with mathematical courses studied. With respect to practice, one experienced mathematics curriculum adviser has written of the teachers in the late 1970s:

Close scrutiny of many schools in country and metropolitan areas of Victoria and South Australia has revealed that decreasing numbers of teachers are employing activity-centred approaches to the teaching-learning of mathematics in schools. Discussion with advisory teachers and curriculum consultants from other Australian states confirms that this is in fact a tendency throughout the nation. Further, significant numbers of ‘teachers’, in some areas, have never been fully committed to such methods nor have they received adequate pre- or in-service assistance to allow them to gain full insight into the potential teaching power of those methods and approaches.

Supporting this, in an extensive survey of junior primary schools in Eastern Australia, Bob Wright found remarkably few changes in practice over the preceding 30 years, in spite of many changes in curriculum specifications. Teachers, he claimed, saw themselves as constrained by conventions and prescriptions about what is appropriate practice, and also lacked a repertoire of alternative skills. Among junior secondary teachers in WA, Haimes & Malone found both reluctance and resistance to change. They proposed several strategies for encouraging change, viz., giving teachers experiences of alternative teaching strategies and providing use-friendly teachers’ notes and appropriate teaching aids, but these proposals were not research-based, and although they sound reasonable enough, it is the argument of this thesis that such suggestions are jejune because they are based on a theory of “one-way” linkages, which have been shown to be relatively ineffective.

Hargreaves confirms McKinlay’s claim that teacher’s discussions in staff rooms are often at a very low level, and suggests that for a teacher to try to talk higher

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72 Neyland (1996)
73 Cooney (1994, p. 617)
74 Larkin (1978, p. 209)
75 B. Wright (1994)
76 Haimes & Malone (1993)
would risk conflict or exposure.\textsuperscript{77} He quotes Lortie’s distinction between talk based on “tricks of the trade” and talk based on underlying principles,\textsuperscript{78} and argues that it is the former which dominates discussions. If he is right, then some findings about teachers’ preferences about change modes need to be called into question. For example, some teachers of environmental science in SA were found to prefer group decision-making processes to change imposed from above, because they wanted to feel in control of what was happening.\textsuperscript{79} But if the analysis of McKinlay and the others reported here is correct, then the teachers may simply not have the resources amongst themselves to develop effective change, even with the provision of some outside support in the way suggested by Haimes & Malone.

There is some good evidence about the stochastics knowledge of teachers and pre-service teachers. After a detailed survey of 57 Spanish pre-service teachers one investigator concluded that “[c]on respecto a su concepción de la aleatoriedad, son muy pocos los sujetos que reflejan una idea clara de las características de los fenómenos aleatorios” and went on to describe some of the ways in which this lack of understanding was manifested.\textsuperscript{80} Similar results of have been found among South Australian pre-service teachers.\textsuperscript{81} In a study of Tasmanian teachers’ attitudes towards teaching the idea of a mean, some mismatch was found between their level of confidence in teaching the topic and their actual skills in doing so.\textsuperscript{82} It is likely that such a result will also hold for probability.

Ransley has also observed that when teaching student teachers new ways of presenting old material:

\begin{quote}

[i]f a group is to successfully reorganise the teaching material it needs at least one person who understands the mathematics beyond the level of rules to be learned and exercises to be done.\textsuperscript{83}
\end{quote}

We may also note that this concern with content knowledge is definitely not the same as the concern with “an emphasis on understanding what teachers know

\begin{itemize}
\item \textsuperscript{77} Hargreaves (1992)
\item \textsuperscript{78} Lortie (1975)
\item \textsuperscript{79} Millard (1988)
\item * With respect to the idea of chance, there were very few students who demonstrated a clear understanding of the characteristics of chance events.
\item \textsuperscript{80} Azcárate Goded (1996, p. 286)
\item \textsuperscript{81} K. Truran (1997mer)
\item \textsuperscript{82} Callingham et al. (1995)
\item \textsuperscript{83} Ransley (1994, p. 39)
\end{itemize}
about mathematics”, which has been how some Constructivists have addressed content knowledge. It may well be the case that we need better ways of preparing our teachers for classroom teaching, but the idea of content cannot be allowed to become determined purely by personal constructions.

While McKinlay’s four reasons provide a valuable summary of the problem from an administrative perspective, I would argue that there is a fifth reason which is rarely addressed—teachers’ knowledge about mathematics pedagogy.

**Weaknesses in Pedagogical Theory**

It is relatively rare in Mathematics Education to find strong suggestions like the following about how to tackle the teaching of a topic:

> Since naïve theories are inevitable, teachers will probably have to confront them directly. Students may have to be forced to pit their theories against the ones they are being asked to learn, to deal with conflict between theories in much the way that scientists do.

It is even rarer to find comments which put the blame for poor learning on the shoulders of the teacher’s pedagogy, as in this example:

> The scientific theories that children are taught in school often cannot compete as reference points for new learning because they are presented quickly and abstractly and so remain unorganized and unconnected to past experience.

I have not been able to find in the literature extensive research into pedagogy as an instrument of change. Many practising teachers do undertake “action research” when they do post-graduate studies, but these reports do not reach the standard research literature in great numbers and their reports are usually either not made public or are difficult to access. Within probability teaching, action research findings are more likely to be published by tertiary teachers because they have some financial inducement to do so. These results are very valuable,

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84 Cooney (1994, p. 617)
85 Resnick (1983, p. 478)
86 Resnick (1983, p. 478)
* The bibliography provided by Cooney (1994) has some examples, but they do not really form a significant corpus. The topic is not explicitly discussed in Grouws (1992) and Boero et al. (1996) is very pessimistic about pedagogy’s influence.
† In recent years there has been a tendency for honours theses not to be deposited in university libraries, and universities offering masters degrees on the course work plus minor dissertation model do not always ensure that the dissertations reach library catalogues, and tend merely to store them on over-flowing departmental shelves, where they are accessible only to those who know where to look.
and could provide a basis both for further research and for curriculum restructuring. For example, there is some evidence that the theoretical section on probability which is included near the beginning of many statistics courses is a barrier to students’ understanding⁸⁷ and some evidence that simulation is a more appropriate way for developing probability concepts than the learning of probability theory.⁸⁸ We shall meet many examples of research into probability learning in Chapter 22, but very few of these will address issues of change.

Some do not see pedagogy as a major interest of universities. For example, Peter Musgrave, writing from a chair of sociology in a faculty of education, has suggested that pedagogy is principally the function of professional associations.

Very rarely is the knowledge base of a subject to be found in the school, though innovatory pedagogy may be created by teachers. The agent of respectability for the academic knowledge from which the curriculum is constructed is usually the university and this can be filtered through a professional association, but a professional association can much more readily be a source of new teaching styles or materials and methods, since it consists of practising teachers whose pedagogical credentials are seen as respectable and who may have, at least locally, some charismatic authority.⁸⁹

While Musgrave may well be correctly interpreting current practice, his viewpoint implies that pedagogy is less important than content, and seems to accept that the authenticating authority for practice does not need as high a status in society as that for content. Given, as we have seen, that limited pedagogical skills and narrow classroom practices have been important reasons for the failure of many attempted changes, his vision for professional associations probably needs enlarging.

The problem is exacerbated because teachers’ practice tends to be heavily dependent on textbooks. Not only do textbooks act as filters of academic knowledge, but they also tend to dictate teaching style. By their very nature, they must act as one-way linkages without being able to address students’ misconceptions directly. They may explain well and they may suggest good exercises for the student, but teaching is more than this. It is inevitable that a teacher who is unsure of a topic like probability will initially follow the textbook style closely. Because content and presentation are so closely interwoven it is not surprising

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⁸⁷ Pfannkuch & Brown (1994)
⁸⁸ Lipson (1994)
⁸⁹ Musgrave (1985, p. 41)
that such teachers emulate, not only the content which they know they need, but also the style, which may not be appropriate.

But textbooks are often poor filters of academic knowledge as well. For example, two Spanish studies\(^\text{90}\) of commonly used textbooks, one in schools and one in tertiary courses, have both found inaccuracies, inconsistencies and incompleteness in the presentation of stochastic ideas. One has also claimed that some misconceptions of students may be attributable to poor textbook presentation.\(^\text{91}\) I have described elsewhere the process by which Fisher’s concept of the null hypothesis was poorly transmitted to students through textbooks even during Fisher’s lifetime.\(^\text{92}\) We saw in Chapter 4 some linguistic weaknesses used by textbooks when discussing probability. Sowey has argued that teachers need to ask fundamental questions about the textbooks they use, and is undertaking a survey of the way tertiary teachers actually use them.\(^\text{93}\) All of the studies mentioned here have examined tertiary statistical ideas, but they illustrate a process which is rarely described, and probably extends to other levels, especially when we remember the content weakness of many teachers of this subject.

Finally, it is interesting to note an unpublished report\(^\text{94}\) of a teacher who had a teaching problem and examined the literature for an answer. He understood statistical methods and could interpret and apply the finding successfully. This would come close to what would seem to be desirable, and fits the medical model which will be discussed in Chapter 24. However, Hogben has commented that such a situation must have been atypical at that time, especially in Australia, because few research endeavours directly answered teachers’ questions, and few teachers would have had the skills to find and interpret such answers as were available. I know of no evidence to indicate the present situation, but Hogben does suggest that a high level of skills is needed if a one-way linkage structure is likely to effective.

We have been able to cite evidence which confirms the views of Wells and Begle quoted at the head of this section, and we have been able to isolate some of the reasons for this. In other words, by following Kilpatrick’s advice to “listen and respond” we seem to be making progress. In trying to codify these reasons we shall see the need to make modifications to the BSEM. Such modification is the

\(^{90}\) Ortiz de Haro (1996); Estepa Castro & Sánchez Cobo (1998)

\(^{91}\) Estepa Castro & Sánchez Cobo (1998)

\(^{92}\) J. Truran (1998iconh)

\(^{93}\) Sowey (1998)

\(^{94}\) Cited by D. Hogben (1980, pp. 62–63)
essence of practising an academic discipline. They are proposed here to provide a background for Chapters 22 and 23, and will be evaluated in Chapter 24;

THE PLACE OF PEDAGOGY

I mentioned in Chapter 1 that I see both education and mathematics as all-encompassing disciplines, either of which would be adequate for the development of a rounded person. How much more so could be their union. So I started this chapter by asking if there were a discipline of Mathematics Education, and have argued that the evidence supports a “yes” reply, but that there is a need to generalise the disparate results which the discipline is currently producing, so that they may be more effectively implemented in the classroom. I see the addition of two forces to the BSEM as one way of effecting some generalisation, particularly with respect to examining change processes.

The first quotation above summarises a commonly held view providing one reason for this neglect of pedagogical theory. The other, diametrically opposed to the first, summarises the skills which we have seen in this chapter as being necessary attributes of a good teacher and strongly implies that such skills can be obtained and can be used. While there is little doubt that having an intuitive gift for teaching is a great asset (as it is for the practice of music and art), there is also little doubt that there are principles which can help any teacher to become better (as there also are in music and art).

Let us start by considering extracts from two reports of classroom activities. The first is from an experienced educational researcher in the UK, the second from a primary teacher in SA. Both, I think, would regard themselves as Constructivists,

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95 Folk saying.
96 Shulman (1987, p. 9)
though the researcher had been working in the same vein long before Constructivism came into fashion. The first author reported in this way:

However, the result shows clearly that the learning of group B has been well retained, while that of Group A has been substantially lost. ... The matter is of considerable importance since the scheme from which Method A was taken is in use in a large proportion of British secondary schools. It has some visible merit in terms of presentation and is generally well liked. However, a number of teachers have misgivings regarding the depth of learning which pupils achieve with it. This experiment shows not only the comparative ineffectiveness of the individual booklets, it shows also the high level of learning, retention, involvement, and enjoyment achieved with the method of conflict and investigation.97

By contrast, the second author reported:

I introduced the second task by looking at the results from one of the completed presentations. I asked them how they could test their predictions. They came up with a survey to other schools but they wanted to only ask other yr. 6/7 classes. So they came up with a list of topics to ask questions about. I limited it to seven topics so they could work in groups of three with a topic each. I showed a model of what a written survey looked like and the types of questions they might ask. Two children were given the task of coming up with the questions which were checked through and agreed by the class. The class discussed who they would send the survey to, it was agreed that it would be interesting to send it to one outer suburb school and one country school as well as complete it themselves. I arranged this with the other teachers but the children had to write a letter to the other teachers explaining what they were doing and what they wanted. I provided a recipe for this type of letter and they made sure the letter had all the parts included.98

We can see in these reports two totally different ways of viewing the education process. The first considers possible alternatives, and is able to weigh up the strengths and weakness of each. The principle criterion for success is academic learning, but enjoyment is also seen as important. The second is activity centred, where the source of ideas is the children, and the teacher is seen mainly as a manager and co-ordinator. Without denying the value of children’s activities, I would argue that the consensus of society today is that sound learning, preferably done with some enjoyment, is a principle objective of schooling. Just as doctors are expected to provide their patients with either cure and comfort wherever

97 A. Bell (1993b, p. 129)
possible, so are teachers expected to provide their students with understanding and skills. We have already met in Chapter 12 the importance of pedagogical influences on curriculum change, especially in New Zealand. In the remaining chapters of this Part we shall look at pedagogic practice using the criteria of learning, and argue that such an approach defines a discipline which has practical and educational value. Such a discipline does not fit well into the traditional understanding of an Intellectual force—it may be necessary to extend the structure of the BSEM to ensure that judgements about pedagogy are seen to be an important component of looking at educational practice. This confirms our decision in Chapter 12 to define a Pedagogical force which will be discussed in detail in Chapter 24.

This has practical implications. Commentators now observe that researchers and practitioners are being subsumed by a new group—managers. In this situation

the academic and theoretical workers find themselves being part of the resource, used up in contract work and projects, advisers on the development of cultural policies whether in curriculum or tourism, in selling art or education on the international market, adding their academic’s mite to value-added commerce, rather than attempting to understand the nature of the process of adding value.99

There is not space to examine this issue in detail, but the role of the Technocrats in effecting recent changes to curriculum, and the circumscribed vision they have encompassed has been discussed in Chapter 16 and provides one example of the process White is describing. If the reality of Pedagogical forces can become as much part of Technocratic decision-making as Physical (usually financial) forces are now, then there is some hope for more effective decisions in the future than we have had in the past.

**THE PLACE OF CHARISMA**

There is one more thing to say, because this chapter has confirmed what we have seen from time to time in Part C—the improvement of mathematics obuchennyi in schools requires more than the practice of the discipline of Mathematics Education—it also requires charisma coupled with enthusiasm for good teaching.

We have seen that both McKinlay & Keeves believed that enthusiasm was an important attribute of a good teacher, and we have also cited some real and fictional examples of this enthusiasm in practice. We have seen In Part C how much charismatic individuals like Wyndham and Thwaites were able to influence educ-

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99 D. White (1989, p. 21)
ational practice, and we have met Openshaw’s clear statement about the extent of the influence of the charismatic Christchurch group on the New Mathematics path taken by New Zealand. Weber has seen the charismatic approach to classroom practice as being predicated by an heroic/magical approach to content, in the same way that he has seen traditional control as matched with cultural content and rational bureaucratic control as match with a training content. This seems too narrow a view. All of the examples cited were of teachers working with very traditional content (cultural in Weber’s terms). Enthusiasm may well be an attribute with far more general effectiveness.

This is an important finding for the BSEM which we are examining because enthusiasm has the capacity to make more energy available than is believed to be present. Yet it is difficult to classify enthusiasm as an Intellectual, Social or Physical force. The discussion here suggests that we need to extend our model to include a Charismatic force as well, which will be done in Chapter 24.

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So Mr McChoakumchild began in his best manner. He and some one hundred and forty other schoolmasters had been lately turned out at the same time, in the same factory, on the same principles, like so many pianoforte legs. ... The sciences of compound proportion, algebra, land-surveying and levelling, vocal music, and drawing from models, were all at the ends of his ten chilled fingers. ... Ah, rather overdone, McChoakumchild. If he had only learned a little less, how infinitely better he might have taught much more!  

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100 Vaughan & Scotford-Archer (1971, p. 16); Gerth & Mills (1957, p. 243)
101 *Hard Times* Dickens (1854/1954, p. 17)
CHAPTER 22: RESEARCH INTO THE PEDAGOGY OF PROBABILITY

Until the counting techniques are mastered, you are wasting your time teaching probability.¹

In 1977, Swinson concluded, after an extensive survey, that adequate techniques were available for teaching probability to children.² In the light of the general lack of success since then in teaching probability, there is a clearly a need to establish whether Swinson’s judgement was incorrect, or whether suitable techniques have not been employed.

The bons mots in this chapter come from an article, also written in 1977, by a senior South Australian teacher suggesting how best to teach Year 12 probability. They represent a good example of the pedagogy which became established around this time, as discussed in Chapter 13, and which has remained more or less in place ever since. So they may serve as a backdrop against which to see the research findings.

We shall summarise some relevant papers under two main headings—reports of classroom practice and examples of systematic classroom experiments. The coverage can never be complete, but is sufficiently representative to indicate the principal findings. Words like “imaginative” are used to describe some of these papers. I make no secret of my belief, albeit untested, that it is these papers which are of the most value.

ANECDOTAL REPORTS OF CLASSROOM PRACTICE

It really makes me wonder why we have trouble with probability when there really are only a few situations to be learnt.³

As noted in Chapter 8, there are many published articles of the “this is how I do it” genre but which are not supported by formal research studies, though more recent documents may contain links to relevant curriculum statements. Some

¹ Brereton (1977, p. 45)
² Swinson (1977; 1978)
³ Brereton (1977, p. 43)
such articles are used here because they provide valuable insights into pedagogic practice and contemporary views of the learning environment.

**General Reports**

One of the earliest general anecdotal reports was a direct response to the 1959 Cambridge Conference, and described in detail a number of classroom activities and, more importantly, children’s responses to the activity.\(^4\) Papers like this can also give special insight into the intuitive approaches which naïve teachers are likely to bring to the topic and to how much can be learned about students’ thinking by a naïve teacher very quickly when teaching a new subject. The authors observed, *inter alia*, that the children

felt that experiments with a pair of dice should be performed with two dice of the same size. They had no objection to a red die and a green die together, but objected strongly to the use of a large die and a small die as a pair. In an attempt to determine the basis for this objection, the teacher asked whether they thought that a large die was more likely to have a large number on it. Their response indicated that they thought this was a foolish question, but they gave no other rationale for their objection.\(^5\)

By contrast, other reports, often from German-speaking countries, have placed less emphasis on cognitive features, but more on mathematics, and are based principally on a theory of didactics. They have discussed only basic structure\(^6\) or advanced mathematical analysis of interesting stochastic situations such as probability,\(^7\) randomness\(^8\) or the binomial theorem.\(^9\) Sometimes activities to illustrate these ideas have also been included.\(^10\) One creative publication of this type has come from Engel, who combined formal mathematics with imaginative activities\(^11\) to develop the detailed course discussed in Chapter 13.\(^12\)

Reports linking probability with other school subjects are all too rare. One interesting paper which has well developed links with both theory and other teaching

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\(^4\) Wilkinson & Nelson (1966)
\(^5\) Wilkinson & Nelson (1966, p. 103)
\(^6\) E.g., Schupp (1986)
\(^7\) Warmuth (1991)
\(^8\) Siet (1993)
\(^9\) Grünewald (1991)
\(^10\) Grünewald (1991)
\(^11\) Engel (1966)
\(^12\) Engel (1970)
aids has proposed ways of using primary children’s enjoyment of singing to construct links with chance activities. Another writer has argued for a totally “applied” approach, but he has not really shown how this aim might be achieved:

No amount of playing with dice and coins (unfortunately, because it has been interesting and enjoyable) or with developing probability set theory will help us in real life. ... We should start from everyday life and learn as much as possible from that, and only later discover those strange, artificial but interesting exceptions: coins, dice lotteries.

Some other writers have made what seem to be purely speculative recommendations, but they may still be of value, as long as their limitations are appreciated:

It seems to me that constructivism strongly suggests teaching probability and statistics using a Bayesian approach. Students get confused and phobic when they are confronted with traditional statistical methods, such as significance testing and confidence intervals, which violate their intuitions and which ignore their personal knowledge in the name of a spurious objectivity.

Other researchers have suggested activities based on what has been learned from research into misunderstandings, sometimes with evidence that such an approach does lead to increased understanding. Most have reported successful activities, but some researchers have argued that the diversity and unpredictability of children’s responses casts doubts on whether many primary school children have sufficient number sense to engage in activities in a probabilistic way. There is neither space nor need here to describe large numbers of the general anecdotal reports in detail, but some of the more interesting ones include:

- probabilistic paradoxes;
- probabilistic games;
- using non-standard dice;
- using real applications;
- using counter-examples in tertiary stochastics courses;

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13 R. Smith (1994)
14 Rogerson (1982, p. 88)
15 A.J. Bertie, Centre for Educational Software, Open U, e-mail comment 3 Nov 98
16 E.g., Shaughnessy & Bergman (1993)
17 E.g., Bright & Hoeffner (1993, pp. 83–87)
18 Taylor & Biddulph (1994); Taylor (1995)
19 Green (1993)
20 Bognár & Nemetz (1977); Bright & Harvey (1986)
21 Drake (1993)
22 Bungartz (1989)
• using subjective probability as a basis for teaching the other forms;\textsuperscript{24}
• looking at ways of applying the Australian Profiles.\textsuperscript{25}

So we may conclude that there have been many papers providing a rich pool of ideas for any teacher of probability, although some reports are unstructured and difficult to apply.\textsuperscript{26} These papers provide evidence of ideas which have worked well for one person in one classroom, but their eclecticism means that they cannot illustrate general principles of curriculum development without much deeper analysis than most classroom teachers have the time to undertake. We may note that very few of those whose work is reported here have also been reported in Chapter 8 (Green being a notable exception); in general the good ideas of teachers are not being evaluated by researchers. This pool of ideas is dominated by the “what works” syndrome, which is essentially a Physical force.

**Reports on Teaching Aids**

Teaching aids may be very simple, but their development may reasonably be seen as one valid form of research. How the equipment is used is of much more importance than its complexity, and the best material tends to have instructions which are facilitative, rather than prescriptive. In other words, all equipment needs to be interpreted within the context where it is used. One good example of facilitative use of simple equipment may be found in a paper\textsuperscript{27} which proposed six imaginatively different experiments using only paper clips. Some of these were specifically designed to avoid equiprobable outcomes and hence counteract the “equiprobability bias” which many children are known to exhibit.\textsuperscript{28}

A more complex aid which has been carefully developed is the DIME Probability Kit,\textsuperscript{29} which provides an imaginative set of RGs, all in sealed translucent cylinders.\textsuperscript{*} The kit also provides a brilliantly designed recording chart which encourages “guessing & checking” and rapid recording, although it can only be used for binary classifications of outcomes. Giles has provided a few suggestions for using

\begin{itemize}
\item Stoyanov (1986)
\item Orton (1988)
\item Mangan (1994)
\item Knott (1994)
\item Green (1995)
\item Lecoutre (1992)
\item G. Giles (1977)
\item A later version of the Kit did not provide sealed containers. While this has made for greater flexibility, it has also led to removal of some of the more unusual examples.
\end{itemize}
the Kit, but he has left its use largely to the teacher, and I am aware of only one discussion in print on appropriate teaching approaches for using the material.\footnote{Crawford (1997)}

One electronic kit\footnote{Olssen (1987)} containing both physical RGs and computer software has been judged by a reviewer in the following way.

This package is best used as a follow-up and natural extension to performing trials in class using the physical materials provided. ... It will be a valuable resource for teaching and learning about chance events, from the introductory stage right through to advanced modelling for senior students. An experimenter’s dream after probabilities have been assigned.\footnote{Sweeney (1990)}

This report assumes that the teacher will know what to do with the material, even though it is deliberately not set into any underlying structure. I first saw Olssen’s material during its preliminary trialing, and was unenthusiastic, but could not then explicate my reasons. It was the finding of K. Truran, reported in Chapter 8, that children do not see mathematically equivalent RGs as being truly equivalent which clarified my concern. Once such a generalisation has been made, then such an electronic aid may well be an “experimenter’s dream”, but much preliminary work seems to be necessary to attain this position.

**Anthologies of Activities**

There have been many collections of suitable activities. An early one was by Anderson, Kempster and Pegg,\footnote{Anderson et al. (1985), summarised in Anderson & Pegg (1988)} presumably based partly on Kempster’s thesis,\footnote{Kempspter (1982)} which had been supervised by Pegg. The activities were developed and trialed by students in the Advanced Curriculum Studies Course at Armidale CAE, NSW, in 1984. At about the same time Clarke proposed a set of examples based on MCTP thinking,\footnote{David Clarke (1985)} and later Watson & Reeves developed a set with a strong emphasis on language.\footnote{Watson & Reeves (1993)} The MCTP material\footnote{Lovitt & Lowe (1993a; 1993b); Finlay & Lowe (1993)} has already been mentioned several times, and *Australian Maths Works*\footnote{Watson (1994aamt)} was discussed in Chapter 16.
Anthologies tend to provide a rich set of ideas but within a limited philosophical framework. In any case

> [t]he choice of activities and the essential discussion are the responsibility of the teacher, and must embody an awareness of the most prevalent misconceptions and a commitment to learning as an activity which must wholly engage the pupil, and which is primarily concerned with the generation of meaning.³⁹

Furthermore, as Ahlgren & Garfield have observed

> [i]t seems unlikely … that ad hoc activities would have the coordination of theoretical and empirical viewpoints that Steinbring believes important to optimizing the understanding of probability.⁴⁰

In other words, we can see from these two sections that many useful ideas and aids are readily available, but that they may well require real pedagogical skill to implement effectively in the classroom. Many of the systematic teaching experiments described next assume the existence of high pedagogic skills. Finding a way of linking these will be the focus of Chapter 24.

**RESEARCH ANALYSES OF CLASSROOM PRACTICE**

Probability can be reasonable if the students are given a very comprehensive exposure via a multitude of exercises.⁴¹

Psychologists often use the term “probability learning” to mean a process quite different from the way teachers use it. So in order to avoid any confusion, it needs to be discussed briefly here before moving on to an analysis of more traditional forms of classroom practice.

**Analysis of Probability Matching Behaviour**

“Probability learning” in the technical sense used by psychologists refers to whether subjects can demonstrate from their responses that they are learning something about the structure of an unknown random generator. Typically, they are asked to make a very large number of predictions about which one of two or more outcomes will be the one delivered by the unknown random generator at its

³⁹ David Clarke (1985, p. 290)
⁴⁰ Ahlgren & Garfield (1991, pp. 125–126)
⁴¹ Brereton (1977, p. 47)
next operation. For ease and speed of operation and recording, the random generator is usually electronic or electrical; subjects make their predictions by selecting one switch, and outcomes are indicated by a specific light being switched on. I have described this type of learning as “implicit probability learning”\(^\text{42}\) and it is claimed to have occurred when subjects exhibit what is known as “probability matching behaviour”: i.e., when the number of successful predictions increases, presumably because the subjects are making valid deductions about the structure of the RG.

Although this form of research has been very common, and has been a basis for Fischbein’s claim about the existence of probabilistic intuition at a very early age, it has had little influence within schools, partly because the nature of the task is not amenable to most school environments. Even so, researchers have used the technique to establish some interesting findings. For example, Gruen & Weir showed that imposing penalties on students for incorrect responses improved their success rate, and that information about the nature of the RG were more likely to be effective with older students.\(^\text{43}\)

### Analysis of Short Instructional Courses

Occasionally instruction has taken place as part of a project designed to evaluate the effect of a specific teaching aid, particularly computers. For example, Bright used computers to ask students to choose which of two urns was more suitable for playing a certain game, but found no significant difference between the treatments.\(^\text{44}\) This work was really a computer extension of Bright’s earlier extensive work on concepts of fairness,\(^\text{45}\) and was not really research which made special use of the computer’s complex decision-making capacity. Similar experiments with computers and probability seem to have been quite rare in the past\(^\text{46}\) and frequently “long on didactical suggestions but short on research suggestions”\(^\text{47}\). This situation is changing rapidly. One of the richest has been that of Pratt,\(^\text{48}\) discussed in Chapter 8, but this was not research directly concerned with classroom practice.

\(^{42}\) J. Truran (1992, p. 89)  
\(^{43}\) Gruen & Weir (1964)  
\(^{44}\) Bright (1985)  
\(^{45}\) Bright et al. (1981)  
\(^{46}\) Shaughnessy (1992, p. 484)  
\(^{47}\) Shaughnessy (1992, p. 484)  
\(^{48}\) Pratt (1998)
A more typical example of a short course is that of Green, who developed a small learning package to broaden children's understanding of the relation between runs of heads and tails when tossing a coin and the idea of randomness.\textsuperscript{49} This used computer technology to minimise mechanical recording and collation by the students and to enable concentration on the relevant features of each distribution. The package

- introduced the concepts of: Heads/Tails; Pairs, Runs; and their distributions;
- provided data in an interesting, challenging and meaningful context;
- required pupils to undertake analysis of the distribution of Heads-Tails, Pairs, Runs and Maximum Run Length;
- required pupils to undertake comparisons and draw conclusions as to the randomness and bias of the data.

The students examined in a structured way the outcomes of tossing of a fair coin, and then used this information to investigate eight computer-based random generators—one fair, and seven unfair—each with a different systematic bias. Green found that the approach [required] of the pupils the willingness to make decisions for themselves and discuss their work, and [required] of the teacher the competence and confidence to allow pupils a free rein but to be able to provide support and guidance when the occasion demands it.\textsuperscript{50}

He went on to argue that the experience was worthwhile because the materials, when used imaginatively, could provide:

- a serious introduction to ideas of ‘bias’, ‘randomness’ and ‘fairness’;
- important concepts of distribution, dispersion, sampling and runs in a meaningful context;
- an opportunity to do things rather than just passively learn things;
- unique data for each pupil or group so that the conclusions are personal and yet can contribute to the overall class experience and encourage a move away from the right/wrong dichotomy prevalent in much mathematical work;
- an emphasis on the application of the scientific method which in practice is generally quite foreign to the mathematics classroom.\textsuperscript{51}

A more traditional approach was taken by Fischbein & Gazit, who developed a course of twelve lessons to introduce upper primary children to basic probabilist-\textsuperscript{49} Green (1987)
\textsuperscript{50} Green (1987, p. 13)
\textsuperscript{51} Green (1987, pp. 13–14)
ic ideas. They assessed effectiveness using a questionnaire which asked fairly traditional questions about these ideas, such as:

Limor rolls two dice and sums the obtained figures. Referring to the possible outcomes of that experiment:

- a: give two examples of certain events.
- b: give two examples of chance events.
- c: give two examples of impossible events.
- d: give two examples of simple events and two examples of compound events.

They also developed the questionnaire, which has been discussed in Chapters 19 and 20, to examine more general ideas about chance, particularly those identified as incorrect heuristics by other researchers. This was administered to students receiving probability instruction and also to a control group. The instructed group were less likely to exhibit intuitively based misconceptions than the control group. On the other hand, the instruction had a negative effect on children’s ability to reason proportionally. Although this work has been done by eminent researchers, we saw in Chapter 20 that it has important limitations. We may also observe that it is extremely structured and context specific, which probably reflects the nature of the instruction but limits the generality of the findings.

In contrast to these short instructional courses, some researchers have undertaken the more demanding task of presenting and assessing a full course in probability.

**Analysis of Full Instructional Courses**

In my Masters thesis I summarised a number of early experiments designed to assess whether probability could be taught in classrooms. Many of these tested the effectiveness of teaching a substantial unit of probability, and probability was specifically chosen in order to test a pedagogic theory with subject matter which the children would not at that time have encountered formally. I wrote:

Gipson (1971), using an instructional sequence on an individual basis, found that third grade is an appropriate grade to introduce concepts of a finite sample space and the probability of a simple event. Ojemann et al. (1965) found that third-graders could be taught to be more efficient in predicting the outcome of indeterminate situations.

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52 Fischbein & Gazit (1984)
53 Fischbein & Gazit (1984, p. 5)
54 J. Truran (1992, pp. 93–95)
Armstrong (1972) designed experiments to be used with a fifth grade and a sixth grade class. Using the same test as a pre-test and immediate post-test he concluded that both fifth and sixth graders possessed the ability to learn the concepts of outcome, event, probability of a finite event, and mutually exclusive events; sixth graders could also learn the concept of an outcome space. Yet it might be argued that the concept of an outcome space is a pre-requisite for understanding probability of a finite event.

Oguntebe & Lappan (1983) found that a course in probability was significantly effective in teaching probability to boys and girls in years 6, 7, and 8 “in any socio-cultural setting”, and that such a course succeeded in eliminating those sex-differences in knowledge level which were present before the course was given. They also considered that Grade 7 was the optimal level for the presentation of such a course. Lovell (1971) reports the work of Shepler (1969) which was very successful in teaching probability concepts to Year 6 students provided that the numbers used were fairly small. 55

Shepler’s work was tested further in collaboration with Romberg. 56 They continued to find successful probability teaching, but their comments highlight the difficulties arising from traditional methods of research.

Even though it seems unlikely that large item gains and high retention rates could be due to test-treatment interaction, the design used did not control this effect. 57

Although the high level of retention after four weeks is sufficient to account for any functional forgetting, the question remains: “Is four weeks long enough to be practically significant?” 58

What Romberg & Shepler did not address at all was whether their 72-item test distributed over 14 behavioural outcomes was actually a measure of probability concepts. While this test has not been analysed in Part D (the actual questions are not easily available), given the time when it was constructed, it probably contained a very limited range of questions. Certainly the behavioural outcomes (which are listed) suggest that the questions posed would be similar to those in the first questionnaire of Fischbein & Gazit discussed above.

I have never seen any of these reports referred to in my reading for this thesis. Shaughnessy has observed that many of the earlier investigations of this type

55 J. Truran (1992, pp. 93–94)
56 Romberg & Shepler (1973)
57 Romberg & Shepler (1973, p. 31)
58 Romberg & Shepler (1973, p. 31)
may have involved too short a duration for significant changes in attitudes and skills to actually show up. What was needed—at all age levels—was longer, more intensive exposure to stochastics, and greater use of clinical methodologies to investigate students’ thinking processes.59

This may well be true, but it is difficult to believe that so many reports found so little that they all now belong only in the dust-bin.

It was very early in the main thrust of research into probability learning, the charismatic Shaughnessy put his hand where his heart was and set up a classical long-term experiment60 conducted during an intensive ten-week* course for tertiary students. This started with a large number of experiments in both probability and statistics, accompanied by extensive discussion. Many of the activities were specifically designed to confront some of the well known probabilistic misconceptions which have been summarised in Chapter 8. Only after lots of experience with real data did Shaughnessy attempt to develop the mathematical theory. The similarity with Rogerson’s approach mentioned above is obvious, but Shaughnessy has provided hard findings, of which the main one was that many students clung to their misconceptions in spite of all their carefully structured experiences. Shaughnessy’s experiment was unusual in that he had a control class which was taught in the traditional lecture format manner, and was thus able to provide some strong evidence about the effectiveness of two methods in reaching the same, traditional, end-point. Shaughnessy attributes much of the success of his course to the instructional model employed, while conceding that such a model may not be sufficient for success. The model was:

- make a guess;
- gather and organise data;
- explicitly confront misconceptions with experimental evidence;
- build a theoretical model.61

A similar approach was undertaken with Spanish students aged 14–15 by Castro.62 One group received traditional teacher-centred teaching, the other used a set of structured activities as its basic learning approach. Castro found a signi-

59 Shaughnessy (1992, p. 481)
60 Summarised in Shaughnessy (1977)
* Shaughnessy is ambivalent on how long his course was—sometimes it is 9·5 weeks (Shaughnessy, 1977), sometimes ten weeks (Shaughnessy & Bergman, 1993, p. 169), sometimes twelve (Shaughnessy, 1992, p. 481). The value “9·5” seems most likely.
61 Shaughnessy (1992, pp. 481–483)
significant advantage for the activity method on all measured variables except attitudes towards mathematics. His research methodology was extremely thorough, well founded in the literature, used imaginative activities, and employed pre-tests and post-tests. His final objective was to teach pure probability concepts, which is what is required by the conservative Spanish curriculum, but he was able to set these into a remarkably creative environment. Undoubtedly his approach will need some modification before it can be transferred to other cultures, but it is one of the most significant pieces of research yet done which has been able to construct a sound blend of pure and applied mathematics.

A project which has put great emphasis on applied situations has been that developed in Utrecht during the 1970s by the Institute for the Development of Mathematical Education (IOWO*), now the Freudenthal Institute.† One of these was a book called *Look on Luck* which attempted to fit all aspects of stochastics into the upper primary school curriculum. Children were encouraged to discuss and analyse stochastic situations. Their feelings and emotions were seen as an important part of what they brought to their analyses, particularly because many of the activities raised issues of fairness. Some of the problems are quite difficult, and involve using simulation to find answers.63 Both of the reports which I have found of this project describe many of the proposed activities, which are much richer than most found in school textbooks. One example will suffice:

> Two children approach each other doing heel-toe pacing. The one who closes the gap wins. Is it fair if they have different shoe sizes?64

Personally, I would find it a real challenge to use this activity as a basis for extracting mathematics. Yet the project continues to today,65 has extended to secondary schools throughout the Netherlands, and is still using problems of the same genre.66 So we must assume that it has been successful, although I am unaware of formal or informal assessments of its effectiveness. Given that the approach represents a quantum change in classroom practice, one would presume that it is the result of a comprehensive and co-ordinated set of forces, but unfortunately I am unaware of any formal analysis of the process of change.

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* Institute voor de Ontwikkeling van het Wiskunde Onderwijs
† *Vide Educational Studies in Mathematics* 7 (3), 1976 for a complete edition devoted to the first five years of IOWO.
63 Collected (1976luck)
64 Morley (1975, p. 16)
65 de Lange (1994)
66 de Lange (1995)
Summary Papers

Finally, we must mention some material which has been written specifically to assist teachers to teach the topic well. This may have been a short presentation at a conference\(^{67}\) or in a journal,\(^{68}\) or it may have been a full-length chapter in a text designed for pre-service teacher education.\(^{69}\) Given the space constraints which authors are under when writing such material, such articles as I have read seem to be helpful summaries based on research findings and experience and which provide a useful list of further references. One ring-bound book on stochastics teaching from the USA bought for an SA tertiary library contained classroom activities based on research findings and substantial lists of suitable further reading.\(^{70}\) The list for probability teaching has been stolen, so someone seems to have found it useful The major weakness of these summaries is that they inevitably have to leave much unsaid. It is difficult to deal with a complex practice like teaching even just one topic in no more than twenty pages.

This why I have been working on the concept of a Handbook Model, to be discussed in Chapter 23. Such an idea was proposed at least as early as 1977 when Bognár & Nemetz identified the problem of children's false ideas arising as part of a game, and stated that teachers need detailed advice of what to do in such circumstances.\(^{71}\) They proposed that teachers “should be given a very carefully prepared manual”.\(^{72}\) To my knowledge, this was never produced by them.

We shall also meet another example of this summary genre in Chapter 23, one which was specifically developed to accompany a specific teaching course, and we shall see how difficult this proved to use in practice.

Comment on the Research Papers

The summary provided in this chapter is representative but not comprehensive. Some other relevant works, such as those by G. Jones\(^ {73}\) and Kempster,\(^ {74}\) have been mentioned in Chapter 8. Many others have been summarised by

\(^{67}\) E.g., M. Barnes (1994)
\(^{68}\) E.g., M. Barnes (1998); Shaughnessy (1993)
\(^{69}\) E.g., J. Watson (1995)
\(^{70}\) Hoffer (1978, pp. 311–321), purchased for the Underdale campus of the SACAE
\(^{71}\) Bognár & Nemetz (1977)
\(^{72}\) Bognár & Nemetz (1977, p. 403)
\(^{73}\) G. Jones (1974; 1977)
\(^{74}\) Kempster (1982)
Shaughnessy. By comparing post-test results with pre-test ones, researchers usually concluded that the topic of probability can be taught. Very few of the experiments used a testing structure which controlled variables like the learning effect of the test, the Hawthorne effect of the treatment, or the learning effect of time. Such processes are expensive, and difficult to administer in schools. So all the findings rest to some extent on a critical assessment by the experimenter of the validity of his or her findings. This is not necessarily bad. Indeed it may be quite efficient. But it does mean that the results are more subjective than their experimental design might suggest to the uninitiated.

One of the special value of these experiments is that they have identified some of the specific difficulties encountered, of which the principle one has been that of overcoming students’ prior misconceptions. These difficulties have arisen even in those experiments, such as Shaughnessy’s, which have paid special attention to a pedagogic structure of student involvement that the experimenter felt would help to overcome these difficulties. In other words, Shaughnessy, a skilled and knowledgeable teacher, found similar problems working with ordinary classroom teachers to those found in the Children’s Mathematical Framework Project reported in Chapter 21. One would predict then that more difficulties would be encountered by ordinary teachers when working with an atypical topic like probability.

But there is one report which suggests that these difficulties can be overcome for probability. Year 12 students were taught theoretical probability using a Constructivist perspective which involved group work and cognitive conflict. The teacher considered that the approach was successful because the students did very well in traditional assessment tasks and because they “arrived at an understanding that appeared the same as [his].” So, as an example of good pedagogy, this investigation must be seen as a success and point to a way of overcoming the problem. But the author has conceded that his understanding of probability is limited, so it may well not be the same understanding as that proposed in this thesis. Given that this understanding has been shown to be much richer than is commonly presented in many courses, it is an open question whether it may be as

75 Shaughnessy (1992, pp. 480–485)
76 E.g., by using a Solomon Four-group design (Campbell & Stanley, 1963)
78 D. Johnson (1989)
79 Borg (1998)
80 Borg (1998, p. 28)
effectively achieved using the same methods. This brings us to the critical question of this Part.

**CAN PROBABILITY BE TAUGHT?**

Just as we found in Chapter 8 for probability learning, so this summary chapter of probability teaching has shown that many research findings are diverse and poorly integrated. Even so, the consensus of opinion supports Swinson’s claim that probability can be taught, while leaving only partially answered the question of how best it can be taught.

Such a finding has deeper implications that mere classroom practice. The summary here has been concerned only with the teaching of elementary probabilistic ideas. As mentioned in Chapter 8, Tversky & Kahneman have suggested that when probabilistic ideas are encountered in more complex forms in real life, people do not use formal mathematical approaches and make significant errors in consistency. They have also reported that these errors in thinking are not easily able to be eradicated. 81

This view has recently been called into question. For example, others have shown that significant improvement can be obtained in people’s ability to avoid the conjunction fallacy (i.e. that \( \text{pr}(A \leftrightarrow B) \geq \text{pr}(A) \)), although they concede that the improvements may have been built on a shallow foundation. 82 We have already mentioned in Chapter 5 that some psychologists have argued that teaching about probabilistic ideas does have a general transfer benefit. This research team from Michigan have specifically taught statistical techniques to adults, 83 and have found that “[t]raining increases both the likelihood the people will take a statistical approach to a given problem and the quality of the statistical solutions”. These empirical findings have enormous practical implications. 84

In other words there is some evidence that developing good probability pedagogy may have long-term implications for the quality of stochastic thinking within our society—our level of stochastic understanding may well catch up with our level of understanding of the circulation of blood. The evidence of Chapter 21 suggests that there does exist a discipline of mathematics education, but that it is still developing and often unrecognised, even by practising teachers. The evid-

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81 Tversky & Kahneman (1974)
82 Agnoli & Krantz (1989)
83 Nisbett et al. (1983); Fong et al. (1986)
84 Nisbett et al. (1983, p. 339)
ence of this chapter is that mathematics educators have made some progress in applying the principles of mathematics education to probability \textit{obuchennyi}. However, as we have seen in Parts C and D, these findings have had little influence on actual pedagogic and assessment practice.

If we are to learn from history, then we need to be able to say why many good findings have had so little influence. After all, the situation is so different in medicine. In the next chapter we shall look at some specific examples of change, some successful, some not, in order to bring out those features which were helpful or unhelpful in implementing change. The final chapter in this Part will bring the earlier chapters together by pursuing the medical analogies hinted at here and interpreting the findings within the BSEM.

Let us consider a possible attack within the year 12 course. ... This is where you have to do some preparation. No text covers the needs of this topic adequately.

(1) Collect texts, reference books, revision guides, exam papers, etc that have relevant questions.

(2) Select questions of each type \textsuperscript{85}.

\textsuperscript{85} Brereton (1977, p. 45)
CHAPTER 23: SOME CASE STUDIES OF PROBABILITY AND PEDAGOGY

‘We’re supposed to be educating them for a working life and three-quarters of the time they’re bored stiff.’

‘I should have thought being bored stiff for three-quarters of the time was an excellent preparation for working life,’ was the flip reply.

‘Humphrey,’ I said firmly, ‘we raised the leaving age to sixteen to enable them to learn more. And they’re learning less.’

Suddenly he answered me seriously. ‘We didn’t raise the leaving age to enable them to learn more. We raised it to keep teenagers off the job market and hold down the unemployment figures.’

In Chapter 21 we examined some examples of attempts to make links between the classroom and mathematics education research. In Chapter 22 we looked in detail at these links for probability. Here I present a sequence of case studies with which I have been personally involved. These constitute a set of cameos which are based unashamedly on a view that schools are there in part to teach children useful and entertaining knowledge in the most skilled way possible. They provide a different type of evidence from that in Chapter 22, and hence a richer environment for testing the power of the BSEM, which will be the focus of Chapter 24.

OVERVIEW OF THE CAMEOS

Education will fulfil its traditional role as a destroyer of family and community.

Education is a risky business. As it seeks to encourage the development of individuals it challenges accepted norms. Ideas about probability, as we have seen in Chapter 5, have the potential to challenge family and cultural norms, and are perhaps one of the most challenging within the whole educational offering of western society. The cameos are not about cultural issues in the broadest sense, but they are very much about finding a place for stochastic ideas within traditional western classroom culture. As we shall see, this is a substantial challenge in itself, a challenge which raises important questions about the

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1  Yes Prime Minister Lynn & Jay (1987, pp. 220–221)
2  Murray-Smith (1986, p. 221)
3  Hannaford (1986)
relationships between the four parameters of classroom practice that were illustrated in Figure 3.1—the teacher’s perspective, the learner’s perspective, the real world, and abstract mathematics.

Because these cameos are personal stories they are well able to describe vividly the challenges involved. The examples chosen are presented roughly in chronological order and illustrate both successes and failures. The first three deal with curriculum construction and textbook writing, the second three with approaches to curriculum change. The two sets are linked by a brief reflection on dissemination practices for probability.

Using personal story-telling as a principal form of evidence is open to a number of forms of bias, but it has the great strength of providing insights which are unavailable from other sources, and is becoming a more commonly used tool of academic research. Some of the cameos are concerned to illustrate two pedagogical difficulties in teaching probability, the stages I went through in coming to resolve them, and the relationship of my partial solutions with prevailing practice. One difficulty concerns the nature of independence, for which the mathematical background may be found in Chapter 6. The other concerns the links between probability and statistics, in particular that of statistical inference. Others are concerned with aspects of encouraging pedagogical change with respect to the teaching of probability.

As Balacheff has observed

\begin{quote}
Psychology is only part of the relevant approaches to the problems raised by mathematics learning and teaching, and for example one must be able to take teaching processes as an object of study as such, as well as the epistemology of mathematics from a teaching/learning perspective.\footnote{Balacheff (1997)}
\end{quote}

It is the emphasis on pedagogical difficulties and practice which makes the use of personal anecdotes so important. As mentioned in Chapter 1, my academic studies have been the result of a dissatisfaction with the quality of what I was teaching. I remain to this day a teacher of mathematics, both with children and adults, and almost daily am called to reflect on the quality of how I have assisted my students’ learning. The story which will be told here is that of one person’s attempt to develop an effective pedagogy for probability. The story is disjointed, because it is part of the story of a rich and diverse life lived partly outside the

\footnote{A recent example is that of Chick (1998), who analyses within the SOLO Taxonomy her cognitive functioning during the preparation of a PhD dissertation in abstract algebra.}
mainstream of educational systems. For the same reasons even the successes have had relatively little influence. But it is only part of my life which has been outside the mainstream: in some ways it has been very much within the mainstream. This partial involvement/partial detachment is an ideal position for making broad reflections on the process of pedagogical development of a new topic.

THREE CAMEOS OF CURRICULUM DEVELOPMENT

[after having tossed 90 heads in a row, Rosencrantz comments:]
It must be indicative of something, besides the redistribution of wealth. List of possible explanations.

One. I’m willing it. Inside where nothing shows I am the essence of a man spinning double-headed coins, and betting against himself in private atonement for an unremembered past. …

Two: time has stopped dead, and the single experience of one coin being spun once has been repeated ninety times….

Three: divine intervention, that is to say, a good turn from above concerning him, cf. children of Israel, or retribution from above concerning me, cf. Lot’s wife.

Four: a spectacular vindication of the principle that each individual coin spun individually is as likely to come down heads as tails and therefore should cause no surprise each individual time its does.⁵

Rosencrantz has been well taught. He can see that a very unexpected situation can be interpreted within several different frameworks, including a formal mathematical one. We might see his response as an example of what we should hope all our students to be able to provide after a sound course in probability. The cameos which follow will indicate just how hard it is to develop such a course.

School Mathematics Project (1968)

As mentioned in Chapter 1, I spent three years working at one of the pilot SMP schools between 1965 and 1968. During that time I was teaching from draft texts prepared by other authors, and attended conferences which revised these texts in the light of our classroom experiences. We were not paid for this work, although authors did receive royalties in later years. But when we met for our conferences we were very well looked after,* which was certainly an inducement to be involved, and encouraged a purposeful attitude to our work.

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⁵ Rosencrantz and Guildenstern are Dead Stoppard (1967)
* My diary records “plenty of free booze”. (J. Truran, Diary, 27 Mar 1968)
In due course I was invited to prepare some chapters myself. In particular, I prepared Chapter 12 in Book 5, the final book in the main academic Years 7–11 sequence, together with the parallel section in the accompanying Teachers’ Guide. The chapter was a summary chapter of all that had been dealt with concerning Statistics and Probability in the five books of the sequence. I had no special training in the topic, much of what I knew had been learned from teaching the very chapters which I was asked to summarise. But by this time, I was teaching statistics at A Level (and attempting to learn it at the same time), and offering an enrichment course for O level students, particularly those who would take biology in Sixth Form, but who would not study any mathematics. I was available, skilled writing resources were scarce, so there was no good reason why I should not be asked to prepare the chapter.

Six pages of the chapter address probability. The section starts with a clear distinction between experimental and “expected” probability (“symmetric” in the language of this thesis) and describes the latter as “exact”. The style does not allow a lot of room for discussion or debate. This preliminary discussion concludes with the words “Most of our work deals with expected probability”, and the remainder of the text presents a formal definition of the probability rules using set notation, but supported with three types of illustration—Venn diagrams, lattices, and trees. The text might be described as that of a good lecture/classroom presentation—moving not too fast, taking one point at a time, trying to maintain some element of surprise and interest, and providing many concrete examples, which were in fact not specifically generalised. The revision meeting seemed satisfied: my diary records “[My chapter] was up for discussion against the clock, but was generally taken as being sound”. My memory of that session is that I was very disappointed at not receiving more criticism: I felt that the chapter could have benefited from it. As I re-read the chapter now, I can see

6 SMP (1969)
7 SMP (1970tg5m)
† This enrichment course was unencouragingly called a “Minor Option”, and contained a student whose father was an academic biologist. He informed the boy that the subject was quite worthless, and that it would be better to wait until university to learn any statistics. Both the biology teachers at the school also saw little need for their students to know any statistics. The A Level statistics course I was teaching had very few students, and was only offered because a very senior member of the town’s principal industry (who coincidentally sat next to me in the town choral society) had insisted that his son should be allowed to learn the subject while also studying biology.
8 SMP (1969, pp. 224–230)
9 SMP (1969, p. 225)
10 J. Truran, Diary, 29 Mar 1968
what the problems were, but I suspect that few of those present then would have been able to articulate the weaknesses.

The most important weakness was found in the discussion on trees. I wrote, “Notice that we multiply the probabilities.”11 Some justification for this had been provided in the previous paragraph, but the didactic style stands out as being contrary to the reasoned approach of the remainder of the exposition. There was no mention of independence or of conditional probabilities to support the multiplication: the concrete examples were seen to be sufficient for the time being. We shall see in the next cameos that this difficulty, seemingly of minor importance, was one which would not easily go away.

The other interesting feature of the chapter is that it concluded with a short section on significance.12 In this section I tried to address the fact that theoretical and experimental probabilities were rarely exactly the same, but that the differences between them tended to be small, and I tried to provide some easily understood meaning for “small”. The method used was simulation, and involved calculating means of increasingly large samples—a tedious process in those days of hand-cranked calculating machines. This work was additional to the material found in the earlier chapters of the SMP sequence, and represented an attempt to come to terms with the linking of probability and statistics. In the Teachers’ Guide I wrote

[I]t cannot be too strongly emphasized that it is the analytic side, of which significance is a key concept, that is the real justification for the subject.13

The exercises set were fairly straight-forward, though I did try to use interesting examples, and to link probability and statistics by using this data to provide the basis for calculating experimental probabilities. However, I used the question “what is the probability … ” which in later years I would denounce.14 One set of exercises provided some possible experiments which might be conducted to illustrate the stochastic principles covered in the chapter. I thought it necessary to include this because I believed then, as now, that “[i]t is not as easy to think of examples of practical statistical work as textbooks sometimes suggest”.15 These included selecting holly leaves at random (i.e., with eyes shut!) to see if there

11 SMP (1969, p. 228)
12 SMP (1969, pp. 230–231)
13 SMP (1970tg5m, p. 133)
14 J. Truran (1994amtprob)
15 SMP (1970tg5m, p. 139)
were any correlation between the number of spikes per leaf and the height of the leaves above the ground. There is no doubt that this approach reflected my special experiences in teaching a more advanced course of statistics for biologists.

So this chapter indicates where I started in what became my quest for understanding how best to teach independence and inference. On the subject of independence I had skated over the difficulties: not only was my Intellectual knowledge inadequate, but so was that of most of the people working with me. What I wrote was merely the rather imprecise received opinion. In terms of inference no received opinion had developed on what was possible from children. My applied experiences has given me some insight into what seemed to be appropriate for a school course, and I made use of the freedom provided by SMP to suggest one way in which it might be approached. I could have used a better model, but the general approach which I took was sound, concretely based, and appropriate for the age-range, so it was accepted by my colleagues as part of the course. I was fortunate in being asked to write the only chapter in the course which covered both probability and statistics; in many courses, as we shall see, the two topics tend to be kept separate, to the detriment of both.

One writer has observed that one of the critical differences between the British and USA approaches to the new mathematics was that the British did not draw a distinction between teachers and scholars; the British projects tended to be a genuine collaboration between tertiary and secondary mathematicians.16 This was certainly the case with SMP. But the human factor was also important. My diary records expressions like:

I came away from the place more & more convinced that the people writing these texts do not have a sufficient appreciation of the average teacher or of the average child.17

Some of the writers are very much in, & know far more than the rest, & it is difficult to argue back at them from one’s ignorance. And some, esp. XXX and YYY are just plain selfish in their behaviour.18

These were well educated men, who were seen by their schools as successful teachers. They brought strong Intellectual forces to our meetings, but their understanding of pedagogy and of mathematics education was more limited. What they know of these topics they knew well, but it was based mainly on their own experiences. Since these experiences tended to be limited to particular types

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16 Dale (1977, p. 72)
17 J. Truran, Diary, 31 Mar 1967
18 J. Truran, Diary, 27 Mar 1968
of schools, almost entirely privileged and élite, they had trouble in making generalities. A broader grounding in the discipline of Mathematics Education might have modified some of the more extreme outbursts.

Finally, the one aspect of my writing on which I have absolutely no evidence at all is how the work was received in the classroom. I had left the UK by the time the book was published, never taught from it myself, and I do not think that it was trialed in any classroom before publication. This was not to be the case in the second cameo which I shall now describe.

**Melbourne Grammar School (1973)**

On returning to Australia I worked at Melbourne Grammar School, a school with a strong academic bias which was using SMP Books 1–4 for Years 7–10. By that time the school was able to award its own Year 11 certificate for boys wishing to leave at that stage, so the only public examination which the boys sat was in Year 12. The differences in approach between SMP and the Victorian Year 12 syllabus were not great: both used a set theoretical approach, and both made use of models such as mapping diagrams for illustrating the concept of relations and functions, which were the underpinning concepts of most of the course.

Nevertheless, the school felt that a transitional Year 11 text was needed to bring the students from the SMP approach to the Victorian Year 12 approach. Because very few schools were using SMP,* there was no commercial market for such a book, so the staff decided to prepare their own. We worked from various drafts for several years, and then it was decided that in 1973 I would be granted time to codify our experiences and produce a more permanent version. One of the chapters and its accompanying Teachers’ Guide was on probability, statistics did not form part of the course, but combinatorics did.

This chapter represents a serious attempt on my part to explicate, both for students and staff, what was proving to be a difficult topic to teach well. I had the freedom to try anything I liked, my drafts were trialed in the classroom by three or four colleagues with a range of mathematical expertise and educational philosophies, and criticised helpfully at weekly faculty meetings. The text was then revised and printed in preparation for future years and is the version quoted

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* Only two or three schools had tried SMP by 1968, and one, a particularly “prestigious” school, had given it up because it was found to be “too hard for the staff”. Letter (no longer extant) from Miss Nancy McConnan, Cambridge University Press, Melbourne, to author, c. April 1968.

19 J. Truran (1973a, 1973b)
from here. In other words, the structure of this piece of curriculum development was remarkably good. It was classroom based, carefully trialed, adequately funded, and represented many years of diverse, accumulated wisdom. What it most lacked was any member of the group with deep knowledge of stochastics; once again, I was seen as the expert more by default than by qualification.

I started by pointing out that all the stochastic experiments which the students had done over the previous few years had failed to address the issue of “how near” when comparing symmetric and experimental probabilities. In other words, I picked up the inference issue which I had addressed in SMP Book 5. I also addressed the meaning of randomness, and stated:

> The essential point about random events is that the outcome of any event has no influence whatever on the outcome of any subsequent event. This can be summed up by the expression, ‘A coin has no memory.’

This, of course, is really a definition for the independence of random generators, but none of us picked this up at the time.

I also suggested a mapping diagram which linked the outcomes of an infinite number of tosses of a die to its six possible events, and this approach eventually led to the structure presented in Figures 6.1 and 6.2. Finally, I linked symmetric (still called “expected”) and experimental probabilities by making use of the concept of a model. This introduction was more sophisticated than the one provided for SMP 5, and made use of ideas which were becoming more common in schools, particularly that of a model. Nevertheless it covered much the same ground, and finished with the same limp excuse:

> In this chapter we shall be dealing almost entirely with expected probabilities. Their relation with experimental findings will be left to a later date.

That later date, if it were ever attained, would be in Year 12! In other words, inference was pushed to one side. In a course with no statistics, there was no room for it.

Most of the text followed a similar approach to that used in SMP 5. However, the way in which independence was discussed clearly indicated that I was trying to come to terms with what I saw as an illogicality in the traditional teaching

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20 J. Truran (1973a, p. 1)
21 J. Truran (1973a, p. 3)
approach. The problem boiled down to defining the circumstances under which it was appropriate to multiply probabilities. I had found that the didactic approach used in SMP 5 simply did not satisfy either me or my very able students: something had to be done to resolve this.

So I introduced a section headed “Simultaneous and Sequential Events”, where I considered two questions:

(a) A die is tossed. What is the probability that it will show an even number and a prime number?
(b) A coin and a die are tossed. What is the probability that they will show a head and a prime number?22

I observed that although these questions have the same verbal structure, the first refers to the outcomes of one RG, which might be illustrated by the intersection of two sets on a single Venn diagram, while the second refers to the outcomes of two RGs, which needs to be analysed using a set of ordered pairs of outcomes.† I did observe that the term “sequential” was a poor one and that “distinguishable” was probably better, but did not emphasise this point. The text then went on to deal with the second case using trees, and to observe that the probabilities on the branches of the trees after the first set of nodes were all conditional probabilities. So a question concerning the drawing of two balls without replacements from an urn containing 7 Apricot and 3 Blue balls could be represented as shown in Figure 22·1, where “A1” stands for “an Apricot ball is taken on the first draw”, etc.§

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22 J. Truran (1973a, p. 13)
† For brevity, I have mainly used the language of this thesis for this summary; the original language was not quite as sophisticated, but its development is not relevant here.
§ After J. Truran (1973a, p. 18). The colours have been changed to obtain capital letters of similar appearance in both Palatino and Symbol fonts, since “↔” demands the use of Symbol font in diagrams. In the original, “A1” was represented as “A₁”, etc. Fractions have been represented with sloping lines for ease of production here: they were better presented in the original.
The discussion then went on to argue that if in general

$$\text{pr}(A_2|B_1) = \text{pr}(A_1)$$ and $$\text{pr}(B_2|A_1) = \text{pr}(B_1)$$

then the events A and B are **independent**. This means that the approach taken in the tree, viz.:

$$\text{pr}(A_1 \leftrightarrow B_2) = \text{pr}(A_1) \times \text{pr}(B_2|A_1)$$

may be simplified, for independent sequential events, to

$$\text{pr}(A_1 \leftrightarrow B_2) = \text{pr}(A_1) \times \text{pr}(B_1)$$

and, without explanation, further simplified to

$$\text{pr}(A \leftrightarrow B) = \text{pr}(A) \times \text{pr}(B).$$

This enabled us to reach the classical form for the definition of independence, but without simultaneous events and with some unexpressed doubts about the meaning of the term “intersection” as used in the tree. I went on to discuss a simple simultaneous situation, and showed that it could be analysed using both a Venn diagram and a tree. I found this quite surprising, because I saw conditional probabilities as having a chronological component, but it seemed to me to be the clue towards understanding how the two situations could be integrated within the same formula, so I stated that
The concept of independence we looked at intuitively for sequential elements of the range of a random function has been extended to include corresponding elements in different ranges of the same random domain. Herein lies another example of the creative power of mathematics, though why this particular example has been created I am at a loss to say.23

Nor, might I add in my defence, could my learned colleagues say either. In the Teachers’ Guide,24 written after the text had been prepared, I pointed out that my use of “intersection” caused difficulties because it was symmetric, and therefore did not indicate order in the way implied in Figure 23.1. There, for the time being, I left the problem, because I resigned from the school at the end of 1973.

However, my text continued to be used for many years after I had left, and was revised.25 I am unaware of who did the revision, but the handwriting suggests a former colleague for whose personal, mathematical and pedagogical skills I have a high regard. It is interesting to examine his changes to the probability section.26

There were many more exercises added, but the basic argument and order of argument was retained. When discussing sequential events a guideline was inserted “If two events are physically independent, the events are independent in the probability sense”.27 When discussing simultaneous events the antiphon of confusion quoted just above was replaced by

Such situations may usually be easily recognized, because they are more conveniently illustrated on a Venn Diagram than on a Tree Diagram.28

This overlooked the point I had made about its being surprising to me that they could be illustrated in both fashions, and begged the questions of what is meant by “more conveniently”.

The revision had clearly been thoughtfully done, some of the more eccentric Truranisms had been removed, but the text remained essentially mine. There were far more alterations to my text in the Probability Section than in any of the other sections of the whole course. In other words, at least one person of ability

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23 J. Truran (1973a, p. 23)  
24 J. Truran (1973b)  
25 J. Truran (ndp, c. 1980)  
26 J. Truran (ndp, c. 1980, pp. 84–126)  
27 J. Truran (ndp, c. 1980, p. 94)  
28 J. Truran (ndp, c. 1980, p. 97)
had seriously tried to improve on the argument I had put forward in 1973, but was not able to see a significantly better way forward.

One other interesting change was made. My opening section on randomness and significance was removed. However, two new sections were added dealing with statistics, which included normal distribution functions and tests of significance for correlation coefficients. I am unaware of why this was done. While I was pleased to see that statistics and probability were finally being brought together, it seemed to me that this meant that there was a greater need than ever for an understanding of randomness.

The difficulties which both of us had in refining the probability section illustrate the problems inherent in constructing academically accurate texts. Our versions were both improvements on their predecessors. But although my colleagues were very supportive on pedagogical matters, and some were better trained mathematicians than I, none of us was a probabilist, and I was essentially working alone in terms of getting the Intellectual content of the text correct. This was not because our mathematics faculty was working in isolation. Many members were involved with the Mathematical Association of Victoria and attended local, national and international conferences, we had a well-stocked faculty library, and good relations with most of the university mathematicians. Indeed, the father of one of my students had written a book on probability. But we tried to solve the problem ourselves!

It might reasonably be asked whether we would have done better if we had sought more advice from outside our school. In my preparation for this thesis I have read many of the printed documents from this time written by others, and talked again to one of those who might have been seen as experts in probability at that time. My suspicion is that we might not have done better, because, to use the language of Figure 3-1, the abstract mathematicians did not have a clear appreciation of the views of either teachers or learners. Our need was pedagogic more than mathematical. The experiences reported here, ones firmly grounded in the classroom, are a major reason why I shall propose in Chapter 24 that the Intellectual force of the BSEM needs to be split in order to distinguish “traditional” content knowledge from academic understanding of pedagogy. As we have seen in Part C, those Intellectual forces concerned with subject matter often do not gel well with those concerned with pedagogy.
SOME COMMENTS FROM OTHERS CONCERNED WITH THIS WORK

In order to confirm my memories of my time at Melbourne Grammar School I consulted two of my former colleagues, Michael Arnold and Gordon Jones, about their memories of how change was effected at that time.29

Both acknowledged that the staff of the time knew little about many of the new ideas, especially of probability. Jones had met some in his tertiary studies of physics, but they agreed that no-one in the school had sufficient prior knowledge to compose a text-book. Arnold recalled a Third year University student complaining that he had not been taught about conditional probability at school. The staff admitted that they did not know what it was, but set about finding out and improving the course as a result. He also recalled a new teacher who had previously taught only pure mathematics, and who found the teaching of probability anathema. The clear memory of both men was that the staff at that time were willing to sit down and learn. Jones recalled after school teaching sessions for the mathematics faculty when the staff would do exercises themselves.* The staff talked to each other and Arnold observed that the mathematics faculty was large enough to form a critical mass for sustaining change in difficult circumstances. I would add that the academic expertise also formed a critical mass.

Neither gave reasons why the school chose to change its syllabus, but Arnold (who also taught physics and so was concerned with two sets of radical change) emphasised that they did look for a course which emphasised underlying principles. Jones put time into developing a pedagogy which used “swat cards” which were looked at for five minutes each lesson to emphasise the big picture. Arnold believed that SMP was chosen by the head of department (who had attended the 1965 UNESCO Conference) because he saw it as a genuine new course, and not a cut-and-paste construction. Very few texts were available at that time. American ones were not so well known, and, in Jones’ view, the school was so Anglophile† that American texts were unlikely to have been considered. Even as it was, Jones recalled many problems with obtaining texts on time for the school’s needs. In Arnold’s view the staff had little contact with academics, except

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* A little later, when computing became part of the curriculum, I recall that some of these classes were taken by senior students who knew far more about computing than any of us.

† Of the four staff who were mentioned in the discussions (including Arnold and Jones), all were Australian, but two had Oxford degrees and all four had taught at Public schools in England or Scotland. Jones had also taught in an English Grammar School. The principal author of the school’s initial internal text had been a Rhodes Scholar.
at occasional conferences, and were more likely to have contact with staff from other schools, particularly independent schools.

These comments confirm the argument of this thesis that effecting significant change in a school is time-consuming. Even in a school with very able staff, some material developed had serious gaps in it, and little time was available for communication with others. Both the culture and the size of the mathematics faculty made change easier than in a school with fewer staff and/or a more industrial model of work commitment. In terms of the BSEM, the Intellectual forces were internal, rather than external, Physical forces meant that the work tended to be done internally, and Social pressures were also mainly internal. Clearly, such circumstances are rare. Nevertheless the example presented here indicates one set of forces which are sufficient for effecting change and shows how the BSEM is able to locate what some would see as an aberrant story within a wider model.

**Technical and Further Education (1989)**

My third cameo discusses a much later book, *Probability*, prepared for a Year 11 distance education course for the Department of Technical and Further Education in South Australia. This work, which had a long gestation period, was prepared concurrently with my Masters thesis, constitutes Appendix VII of that thesis, and is analysed there in Chapter 4. It is only necessary to examine one point here.

The text first examined functions of one random variable, and illustrated these using Venn diagrams. From this position it was possible to present the classical definition of independence in a totally straight-forward and convincing way. Gone were all the technical details and pedantic circumlocutions. I wrote

> Imagine that it is found that wine drinkers are likely to develop some serious disease. If it is known that wine drinkers are also likely to drink beer, then it would be wise to look for the cause of the disease in both wine and beer. If, however, wine drinkers are no more likely to drink beer than anybody else, then it would be better to concentrate on looking for the cause in the wine.31

After some calculations using hypothetical figures. I concluded

> In this case the proportion of wine drinkers is the same, no matter whether we choose the whole population or just the beer drinkers.

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30 J. Truran (1989)

31 J. Truran (1989, p. 66)
When the two values are equal we say that the two events are independent.\(^{32}\)

Symbolically, using obvious abbreviations

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pr(W \mid B) = pr(W).
\]

Of course the usual form in terms of multiplication may easily be deduced from this, but it is the conditional probability form which most strongly illustrates the meaning of independence in this case. This may seem a small pedagogical step forward, but the fact is that the vast majority of textbooks do not approach the situation in what I would argue is the simplest fashion. Helpful pedagogy has been hidden by the propensity of texts to clone.

It is hard to convey in what must be a simplified summary just how much more satisfying this approach to independence is than what I had tried before, which had been based on what I had read from other people. I had identified the two possible approaches in 1973, but chose to start with the more confusing one. In 1989, I chose the simpler one, and felt at last that I had found an approach which would not need significant modification. Although I had examined many textbooks,\(^{33}\) I was not aware of having met this approach in any of them, in spite of its inherent simplicity and beauty.

Of course, I still had to deal with the alternative approach to independence which involved more than one random function. I did this by defining “independent” to have two meanings—one for random trials (RGs in the language of this thesis), and one for random outcomes. In this analysis I corrected the error made in 1973 of relating “a coin has no memory” to randomness rather than to independence, and provided an example of physically non-independent RGs which were still probabilistically independent, thus correcting the guideline mentioned above introduced into the revision of my 1973 work. Once the concept of independence of RGs had been established, it was then possible to introduce trees and to justify the multiplication of probabilities along the branches. The argument was simple and straight-forward.

There remain two difficulties with this approach. Students must learn to use the word “independent” with two different meanings, depending on context. This is awkward, but achievable. More seriously, the existence of two different meanings is not acknowledged by examiners, and the textbooks which try to conform with

\(^{32}\) J. Truran (1989, p. 67)  
\(^{33}\) J. Truran (1992, pp. 100–140)
examiners’ wants. It became clear that if I were to be able to advocate my approach, I would need to understand clearly the received view on independence.

**CLARIFYING MY INCREASING UNDERSTANDING**

Now we see through a glass, darkly, but then face to face:

So part of my Masters thesis examined the formal mathematics of probability, and proposed a mathematical structure for the two types of independence which has been summarised in Chapter 6. But it has been difficult to find ways of making this proposal heard, let alone having it subjected to formal criticism. Even my examiners, two well-respected members of the mathematics education community, did not examine it carefully. Pp. 46–47 are not logical, and arise from using a copy and paste instruction without subsequently making the planned alterations to the pasted version.* A paper summarising my approach was rejected by one learned journal but without any of the referees giving any indication that they had understood the argument. The editor promised to send the paper to referees with a stronger background in statistics, but did not do so. Eventually, the material formed part of a paper presented to an international conference, and was published in the proceedings of the Topic Group.36

Now it is in press, but I am unaware of anyone who has taken any notice of it. The mathematicians do not believe it is true, but cannot tell me what is wrong with it. The teachers do not read it, and would not go against the wishes of the examiners, who are mathematicians. At this stage I would happily have been prepared to rate the strength of Intellectual forces within academia, especially those enshrined in theses in isolated universities, as “Force 0”.

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34 I Corinthians 13: 12, *King James’ Version*

35 J. Truran (1992, pp. 31–48)

* I am grateful to Dr Brian Sherman, University of Adelaide, for pointing out this serious error out to me.

36 J. & K. Truran (1997ind)

† There is a culture in Mathematics Education of not reading theses: it is assumed that the author will later disseminate his or her work in more accessible journals and in more digestible lengths. Indeed, in preparing J. & K. Truran (1996) we had to argue strongly to be allowed to include theses in our data base. In practice, of those whose original theses I have read, Green is one of the few in probability research to have disseminated the bulk of their thesis findings, but not all, as we saw in Chapter 19, Green 6. Most of Peard’s (1994) findings have not published and he has not mentioned them to me in our meetings since then.
The rejection of my journal article led to my taking some interest in the process of dissemination within the mathematics education community.

**STANDARD DISSEMINATION PROCEDURES**

Go out on to the highways and along the hedgerows and make them come in; I want my house to be full.\(^37\)

I have already summarised the evidence I found about dissemination in Chapters 16 and 21, and shown that the linkages between teachers and researchers have often been tenuous, particularly the Pedagogical ones. I have been arguing in this chapter, and in Chapter 6, that there are weak Intellectual links as well. For all of these inter-related aspects of education, going out and “making them come in” has not proved an effective strategy.

If this topic is to be well taught, especially in primary schools where the problems are greater, then a richer model of curriculum development seemed to be called for. It therefore seemed appropriate to try to find ways which might have more effectiveness, and these will be discussed in the next three cameos.

**THREE CAMEOS OF CURRICULUM CHANGE**

Chance (referred to as probability in secondary schools) has usually been the domain of secondary school classrooms. I recall, all too [sic] well and without fondness, trying to learn probability in years 11 and 12. It was vastly different (or to me it seemed that way) to the earlier years that were related to dice, cards and counters. We no longer used materials I was familiar with—the cards and dice were gone. We no longer used vocabulary I was familiar with—instead we used words like permutations and symbols that reminded me of elements and their atomic weights from the periodic table. If it wasn’t for probability (and statistics) I would have been an “A” maths student.

To me, chance should be taught in the same way that the other strands of mathematics should be taught. ... In such an approach the teacher decides on the curriculum area and sets up activities that motivate children to pursue an investigation.\(^38\)

The plaint quoted here by a Constructivist primary educator raises important issues. It emphasises the need for a sound concrete base, and for language which enables meaningful use. But it also, as we would expect, avoids the issue of

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\(^{37}\) Luke: 14: 23

\(^{38}\) Becker (ndp., ?1994)
relevant knowledge, and gives no hint that the Constructivist position may be particularly difficult to sustain effectively in probability. Dare one say it, but it was perhaps the lack of pedagogical skill of any sort for stochastics among Becker’s upper secondary teachers which affected his grades rather than their lack of a Constructivist approach. This section summarises three attempts to find alternative ways of improving pedagogical skills—one is unproven and one was still-born, but one was a heartening and instructive success.

The Development of a Handbook Model

In Chapter 24 we shall discuss some analogies between the practice of medicine and the practice of pedagogy. I have already mentioned above Shaughnessy’s useful summary of research into stochastics learning, published in 1992. This, and the more recent but less easily readable summary by Borovcnik & Peard, have both been commented on in Chapter 8. Both summaries are written primarily for researchers, so teachers would need to work hard to interpret them in ways which could be quickly used in the classroom, even if they could afford the cost of the books and the time involved. It therefore seemed to me that it might be helpful to develop a “Handbook Model” which was strongly research based, but which presented findings in a well-structured and easily accessible form and which might help to overcome the communication problems noted in Chapter 22.

The model is called a “Handbook Model” because the style and structure conform with those in handbooks produced within medicine and the hard sciences, where each individual part of the text is directed specifically to a specific aspect of the subject under consideration. References are cited, but they are subordinate to the summary. The structure is directed specifically to those whose primary concern is teaching rather than research and to act as a complement to standard curriculum documents and text-books. The model presented here is based on the analysis of findings about tossing a die summarised in the section “What Does a Literature Review tell Us?” in Chapter 8. This small, simple area of stochastics research has been taken as a basis for building a structure which is a prose summary of the findings set within a hierarchy of contexts. It is argued that the hierarchy makes the material readily accessible to those with specific needs, and that the prose is an effective medium for expressing the complexities of the topic.

39 Shaughnessy (1992)
40 Borovcnik & Peard (1996)
41 E.g., Higgins & Davies (1996)
42 It was published as J. Truran (1998pmehab); further models were presented as J. & K. Truran (1999hab) for comparison of RGs and (2000pmehab) for independence.
It might well be argued that an interactive form such as is now available using CD-ROM or the Web might be more effective. It is my personal view that this is not the case because interactive forms tend to increase the level of complexity of presentation, but to my knowledge the relative benefits of the methods have yet to be evaluated. In any case, at this stage of our development of new technology, it is appropriate to develop alternative forms so that they can be compared and evaluated, rather than to assume that the newer is the better as a matter of course.

In this preliminary construction I have taken the view that the fragmented pattern formed by current research findings can be reduced by using the concept of an RG as a unifying feature, principally because it is a fundamental probabilistic concept, far more fundamental than stochastic outcomes. So the construction uses a hierarchical structure—RGs, Dice-Family RGs, and Symmetry of D-FRGs—to indicate how the hierarchy might be extended. It also brings together both the holistic and outcome approaches to RGs, thus clarifying the circumstances under which inconsistent reasoning appears.

In the model I have presented here I have deliberately omitted the references, which would normally be included as footnotes or within parentheses, in order to emphasise the innovative aspect of this work—its structure.

**AN EXAMPLE OF A HANDBOOK MODEL**

**Random Generators**

Children’s responses to RGs of all sorts depend on a number of features which are unrelated to the RGs themselves. The more closely a child is involved with the operation of the RG, the more likely it is that misconceptions will be revealed. Promising rewards for successful operation of an RG is particularly effective for highlighting misconceptions.

RGs may be viewed holistically or as a means of generating a specific outcome which is of immediate interest. Asking children to predict the next outcome from an RG (an "outcome approach") is mathematically unreasonable. However, such situations do arise in life and children’s predictions can reveal understandings about RGs which holistically oriented questions do not bring out. So it is desirable that both approaches are used when teaching or diagnosing.

**Dice-Family Random Generators**

D-FRGs are RGs with between 4 and 12 discrete equi-probable outcomes. RGs with 2 or 3 outcomes tend to produce different types of responses because of their simplicity. More than 12 outcomes are difficult for children to conceptualise, so it is likely that they will use different ways of interpreting them. Mature understanding about the behaviour of D-FRGs is probably positively correlated with thinking ability.
Symmetry of Dice-Family Random Generators

Many children do not believe that these RGs are symmetrical; this belief often seems to decline with age, for reasons which are not well understood. The misconception may be revealed by asking questions like “which number is easiest to get or are they all equally easy”, but minor changes in wording, such as replacing “easiest” by “hardest”, can lead to quite different responses. It is wise to ask several questions of different forms.

Many children and adults whose understanding of the holistic approach to dice is sound will still prefer a specific number when asked to use an “outcome approach”, and many of these tend to choose central numbers (a representativeness heuristic). It is not known how such people respond when the outcomes concerned are not able to be placed easily in order in the way that numbers are.

It cannot be assumed that children who seem to have a clear understanding of one embodiment of a D-FRG will necessarily have the same clear understanding of others which are mathematically identical. The larger the number of outcomes the more likely children are to reveal misconceptions. Children need experience with a variety of forms.

DISCUSSION OF THE MODEL

This model was presented to a PME Conference in 1998. It attracted favourable comments from the referees, and from those who attended the presentation, but, as is usually the case at conferences, there were no real deep comments about the value of the model as a long-term aid for improving teaching.

The major discussion arose with Andrew Ahlgren of the American Academy of Science, who is developing a map of ideas required by the numerate adult of 2060. Ahlgren’s approach is cartographic, making direct links between nodes which indicate fundamental concepts, whereas my approach is linear. While we could see some of the strengths and weaknesses of both, we could not see any reason why one approach had any significant advantages over the other. But it was valuable to meet someone else who also saw the need for well-constructed summaries of ideas as an aid towards teaching.

An Experiment within a School

The second attempt to influence classroom change was the development of a booklet to support teaching a unit of probability in schools. Because the teaching of probability was no longer part of the year 12 academic syllabus in SA, its importance at junior secondary level had tended to decline, in spite of the National Statement, and it proved difficult to find a suitable school. However, eventually a school was found, and two teachers agreed to teach the topic for two weeks to their Year 9 mathematics classes. The time lines for the exercise were
tight, there was little room for manoeuvre, and circumstances eventually led to calling off the project. However, since it was planned to be a principal part of the research for this thesis, some aspects of its preparation can indicate some useful features of the difficulties of effecting curriculum change. So it is worth recounting those parts of the story which shed some light on the arguments of this thesis.

GENERAL PROCEDURE

I am conscious in quoting from the booklet that it gives the impression of being a very formal, verbal document, and it may well be seen to be quite inappropriate for its purpose simply because of its style. This issue will be discussed further in Chapter 24, but at this stage I shall simply say that while some such criticism would be valid, at the time my intention was to gather together what I saw as useful background, and there simply was not any opportunity to soften its presentation. In any case, I was working in a sympathetic environment, among people who understood that this was really only the first of a number of trials.

The co-operating school was a boys’ school with a strong academic focus, whose Head of Mathematics I had known for many years. He allowed me access to classes in Years 8 and 9, but only at a time which did not disrupt their basic programme, which was largely controlled by the dates of formal topic testing. Two teachers agreed to help: one was a senior member of staff and an experienced mathematics teacher whom I had met previously on one or two occasions, the other a man I did not know, of about ten year’s experience whose principal subject was biology, but who taught some junior mathematics as well.

The basic plan was to produce a Booklet entitled “A Short Course in Probability for Middle School Students”, which would provide a reasonably self-contained background for any teacher of the topic. In the introduction I wrote:

It is my belief that curriculum change is only lasting and effective when its reasons are well understood by teachers. Top-down models are notoriously prone to producing difficulties at the chalk-face. The relatively conservative changes proposed in this booklet are presented in a form which assumes a bottom-up model of change.

The booklet first summarised the four aspects of teaching probability which are summarised below, and then I added:

Many textbooks do not address some of these issues or do not make them explicit. The reality of classroom practice is that teachers tend to follow very closely the approaches of the textbook. I argue that by providing a meta-cognitive overview of a course then a teacher is more easily able to decide how to use textbooks and other resources in pursuing the long-term aim of developing children’s understanding about a particular mathematical idea. At the same time it is essential that any such overview must be closely related to
current classroom practice, so that the changes required of teachers are achievable within the amount of time which they can reasonably give to preparation.

For the thesis research, then, the key ideas are presented personally to the teachers concerned at an introductory meeting, and the whole booklet is made available to them to reflect on before presenting the course. They are free to make suggestions about what they want in the course as well and I am available to discuss possible approaches with the teachers before and during the course presentation. The process is interactive, not dogmatic. After the course teachers’ assessments are collected both informally and by means of a simple, structured questionnaire.

The precise research question which the experiments sought to answer was:

To what extent and in what ways has the provision in this booklet of material addressing each of the above issues enhanced or hindered the learning of basic probabilistic ideas by the children?

A question like this was chosen rather than the traditional approach of comparing methods of teaching because of a belief that such approaches do not make adequate allowance for the complexity of the teaching task. The teachers involved were asked to familiarise themselves with the ideas in this book, and to make a conscious effort to incorporate the ideas into their own teaching. I did not expect them to incorporate all of the ideas in the book into one course. Individual classes vary in age, experience and culture, and I saw it as the role of the individual teachers to use their pedagogic skills to decide what was appropriate for their class and for individuals within that class. For my part I was willing to provide assistance in any way required, either during class time or at other times. It was not anticipated that I would normally attend the lessons, but the possibility was not excluded.

Of course, as soon as a researcher starts working with real live teachers living under pedastress he or she is forced to make sure that what is proposed is as teacher friendly as possible. So the booklet highlighted four major aspects of the teaching of probability:

- The Mathematics of Probability;
- Research Findings about Children’s Understanding of Probability;
- Some Suggestions about Pedagogy;
- Some Suggestions about Assessment.†

It then went on to define six basic mathematical ideas which I saw as a foundation for a course in basic probability.

† The assessment section was significantly incomplete when the project was due to start.
• The Concept of Randomness;
• Using Numbers as a Measure of the Likelihood of Random Events Happening;
• Interpreting the Meaning of Differences between Different Probability Measures;
• The Concept of Independence of Random Generators;
• Using Numbers to Analyse Compound Probability Functions;
• The Concept of Independence of Events.

It rapidly became clear that it would be too much for my colleagues to handle all this at once, so I also developed a heavily shortened summary which emphasised the main ideas and showed where more information in the body of the text might be found when needed. This seemed a reasonable compromise. In the following section, I present some extracts to indicate the depth and style of the approach.

EXAMPLES FROM THE PROPOSED TEXT

The main body of the text dealt with all of these issues, and was a selective summary of the material gathered for this thesis at that time. I quote one example for each major aspect as a basis for the discussion on mathematical didactics in Chapter 24. The reader will recognise the approach, examples, and even some of the phrases, as being well within the general approach of this thesis. Inference is seen as a key way into the topic, and independence is mentioned at a later stage.

Mathematics—Reconciling Three Different Probabilities

Once again this is a topic which is often overlooked in school textbooks. In particular, it is not uncommon to see books which mention subjective probability initially, but then disregard it. Many books also suggest that it is theoretical probability which is the true probability and are ambivalent about the place of experimental probability.

But reconciling the three probabilities is a critical reason for studying probability. Most people believe that the number of boys and girls born is approximately the same. They see conception as a random generator which has the same structure as a coin. In both cases two outcomes are possible and both are equally likely. The subjective probability of 0.50 matches a theoretical probability of 0.50 based on arguments about X and Y chromosomes. But government records make it clear that experimental probabilities of a male child being born are consistently more than 0.51. Is the difference between 0.50 and 0.51 so great as to suggest that the subjective and theoretical probabilities are poor descriptions of what is actually happening?

The short answer to this question is "almost certainly, yes". No probabilistic answer can be absolute, but this one is as close to being absolute as is practically possible. Establishing such a degree of certainty involves formal analysis beyond the skills of middle school students. But the principles behind such analysis are accessible to students.
Consider an urn containing 50 red balls and 50 green balls. Children could collect samples (with replacement) of 10, 20, 30, … balls from the urn and plot the experimental probabilities of obtaining a red ball. Once they have understood the general idea then this work could be assisted by computer simulation. Plotting their results will show them that the larger the sample the less their experimental probabilities are likely to vary from 0.5. This work could be repeated with an urn containing 55 red balls and 45 green balls. While an experimental probability for red of 0.5 is quite likely from this urn for a sample of 10, they will see that it is highly unlikely for large samples. It is not difficult for them to accept that for very large samples (such as are provided in government statistics) a difference between experimental and theoretical probabilities as low as 0.01 may well mean that the theoretical model being used is not the best one available.

Hopefully the experimental results will do even more than persuade the children to revise their theoretical probabilities. They may also persuade them to revise their subjective probabilities. This is harder to achieve, but it is a critical reason for teaching western scientific thought and probability in particular. No one can ever force a child to change his or her opinion that a die is loaded against six. Nor can anyone devise an experiment which will show absolutely that a die is not loaded absolutely against six. But we can teach our students that the western scientific approach provides ways of collecting data which make certain interpretations to be highly likely and others to be highly unlikely. Under such circumstances a person is well advised to revise his or her subjective opinions.

This is dangerous educational ground. The disagreements between the creationists and the evolutionists are well known. Legal authority is employed in some parts of the world to enforce the teaching of views which many regard as a very poor models of the available evidence. Children's cultural background may be so firmly entrenched and so different from western scientific culture that the two cannot be comfortably reconciled. I have personally seen Papuan students attribute the strong growth of some sweet potato plants and not others to friendly spirits rather than to the presence of superphosphate which they had personally added to these specific plants when planting the crop.

But education is a risky business. Western science believes that probabilistic thinking is a valuable way of thinking. This belief is based on a large amount of evidence. We have an obligation to our students from all cultures to ensure that they understand the process and its strengths and weaknesses. For middle school children the approach to statistical inference will be concrete and semi-numeric, but it cannot be omitted from a well-balanced course.

Children’s Understanding—Availability

Statistically naïve people assign probabilities according to the ease with which they can recall a similar event happening in their past experience. For example, some people argue that children believe that six is the least likely number to obtain from tossing of a die because of the ease with which they can recall situations in board games where they sat waiting for a six for what seemed a very long time.

Availability may also depend on the ability to conceive an idea about which one has had little previous experience. For example, many people believe that the number of committees of two which can be formed from ten people is far
more than the number of committees of eight which can be formed from ten people, because they can more easily conceive committees of two.

**Pedagogy—Understanding, Estimating and Measuring Chance Variation**

The listing here suggests a developmental sequence, and numerical representations of chance events are delayed until Level 5. There is no research evidence that such a developmental sequence has any validity. There is good research evidence that many children acquire some of the skills listed for the upper levels much earlier than is suggested in the Profiles. For example, children in upper primary schools (Levels 3 and 4) are well able to express both theoretical and experimental probabilities numerically and to place them on a number line. Similarly, they are well able to use ideas of complementarity to calculate probabilities. So the Profiles should not be treated as a guide to normative development. Children are capable of far more than the Profiles require, and they are capable of it much earlier.

Some of the statements made are mathematically unsatisfactory. The distinction between events arising from random generators and those arising from more complex situations is not made. The use of "things" as a synonym for "events" is unfortunate. The three forms of probability are not clearly distinguished. Probabilities cannot be assigned using complementarity and independence: they can only be calculated.

Some of the outcomes imply a specificity which cannot be justified. The splitting of impossible, certain, and uncertain events into two Levels (2 and 3) is unrealistic, and in the way presented could lead to a belief that "possible" and "uncertain" had different meanings in this context. Equal likelihood is omitted in Level 2, but included in Level 3. Events may be placed in order on subjective grounds as well as numerical grounds (Level 4). To start the teaching of numerical statements about probability (Level 5) with equally likely outcomes is likely to lead to under-generalisation. Similarly, there are sound techniques for reasoning about two-stage events which do not require the assumption of equiprobability.

Some of the sequence is mathematically invalid. Reasoning about two-stage events (Level 6) cannot take place without some assumptions of independence (Level 7).

We may note that this section in the Profiles on “Chance Variation” does not in fact discuss variation.

**Assessment—Questioning**

**Good Questions**

**Questions asking for one basic outcome to be selected**

*I am going to toss this die, and I am going to ask you to choose one number to bet on. I know that you cannot predict the outcome, but I still want you to make a bet. You may choose any of the possible basic outcomes, please write down the outcome of your choice.*

This is an effective question for finding out what children believe is going to happen at the next operation of a random generator. It is a realistic question because we all have to make choices about uncertain outcomes. These may
be trivial matters like filling in the numbers on a Cross-Lotto ticket, but they may be much more serious like deciding whether to insure a new bicycle against theft and damage, or to decide that the risk of losing the bicycle is not balanced by the cost of the premiums. Not buying insurance is as much a decision about future uncertain events as is buying the insurance.

**Questions which allow for the possibility that events or basic outcomes are equally likely**

*When I toss this die which number is most likely to come up, or are they all the same?*

*When I toss this die which number is most likely to come up, or are some of them the same?*

*When I toss this die which number is least likely to come up, or are they all the same?*

*When I toss this die which number is least likely to come up, or are some of them the same?*

It goes without saying that providing the option of equal likelihood should be included in situations when no outcomes are equally likely as well as when they are. Otherwise students will learn to answer by interpreting their teacher’s questioning style, rather than by examining the mathematics of the situation being discussed. Similarly, questions which use the expression "some of them" should not be used only when only some of the outcomes are equally likely, the question is equally applicable to situations when all of the outcomes are equally likely.

**Poor Questions**

**Predicting a single event or basic outcome from a random generator**

Common questions which are put to children about random generators include:

*When I toss this die what face do you expect to come up?*

*When the raffle is drawn what numbers do you think will not win?*

There is little doubt that children (and adults) do have views about the next basic outcome from a random generator. These views may be about what will happen, but they may also be about what will not happen. There is a true story of a football club raffle where no-one would buy ticket number 1. So the club secretary bought this ticket to complete the sales, the draw was made and number 1 was drawn. There were howls of protest from club members who claimed that the draw had been rigged.

Children will happily give you answers to these questions. After all, questions from teachers require answers, and definite ones at that. But the best answer to these questions is "It is impossible to tell", and in general children do not offer this as a possibility. In other words, the question does not give them sufficient opportunity to present a mature view of the situation; it forces them to answer in a manner which does not match the question, but which does match

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43 This story was told to me by my local butcher.
their limited expectations about the nature of classroom mathematical discourse.

If you want to know their feelings, as opposed to their mathematical understanding, then use questions like the one listed above which says "I am going to toss this die, and I am going to ask you to choose one number to bet on. I know that you cannot predict the outcome, but I still want you to make a bet. You may choose any of the possible basic outcomes, please write down the outcome of your choice."

**Asking for a Choice between Equally Likely Outcomes**

- *When I toss this die which number is most likely to come up?*
- *When I toss this die which number is least likely to come up?*

These questions do not give children the opportunity to state that they believe that all numbers are equally likely to come up, so they encourage the provision of a wrong answer and perhaps also a belief that there is a most/least likely number.

**Questions asking for "the probability"**

- *When I toss this die what is the probability that it will come down a "six"?*

We have seen that there are at least three different types of probability. So the term "the probability" has no meaning at all. Textbooks often use questions of this sort; usually they mean "the theoretical probability", but they rarely make this assumption explicit.

**REASONS FOR SUSPENDING THE EXPERIMENT**

My colleagues were not sure how best to introduce the ideas we had agreed to teach. Solving this problem proved to be the critical one in the experiment. After some discussion, we decided that I should suggest appropriate activities from the MCTP Chance & Data materials⁴⁴ as a basis for their classroom introductions. Once we had overcome this problem of getting the teaching off to an interesting start, we were confident that their teaching skills would be able to run with the fortnight’s work without serious difficulties. I presented some suggestions, and we focused on a small number of activities which involved computer simulations.

This proved our downfall. The MCTP comes with computer programs to support its activities. I had used the material, and believed they would engage the students’ interest. The school had well equipped laboratories, and wanted to use computers more in the classroom. The experiment was becoming mutually beneficial. However, I had only used the Macintosh version. We had trouble purchasing an MS-DOS version and it was the Friday before the week we planned to start before the right disks arrived. A trial in the school’s computer laboratory on the Friday

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⁴⁴ Finlay & Lowe (1993); Lovitt & Lowe (1993a, 1993b)
failed, as did a longer and less hurried one on the Saturday. We could go no further. The school’s hospitality could not extend to delaying the project for a few weeks; so the approach reported here remains untested. While I was particularly disappointed, the attempt still has a lot of lessons for aspects of curriculum change which is why it has been reported in some detail.

Fortunately, the final experiment with which I was involved was far more successful, and in ways which were totally unexpected.

**An Experiment with a University Statistics Class**

The teaching of introductory statistics “service” courses, which include probability, to undergraduates (usually in their first year at university) has greatly increased in recent years, has raised a number of pedagogical challenges, and has often generated antipathy to the subject and claims that the work was “irrelevant”. I had been working as a tutor in such a course in the Department of Economics at the University of Adelaide when a decision was taken by Anne Arnold, lecturer-in-charge for Semester 1, 1997, to assess the course with a view to making minor improvements as the semester developed, and major ones in later years. She obtained a Departmental grant to pay me to use my background in the discipline of mathematics education to help in deciding the best strategies to use.

Although we found a number of suggestions in standard journals like *Teaching Statistics* and *Journal of Statistics Education* about good ways of teaching such courses, almost all were anecdotal and lacked standard forms of research evidence. So we decided to ask a small, stratified random selection of students to keep a confidential journal on a weekly basis. In it they would evaluate the ways in which they responded to the course and fulfilled its requirements so that we could isolate the specific needs of the students and to plan appropriate modifications to the course. This decision was taken on the basis of a substantial mathematics education literature on the values of journal keeping.45

The full details of the project, which was developed further in 1998, are described elsewhere.46 Although we had problems in obtaining a stratified random sample, we were able to investigate students’ reactions in several independent ways and obtained remarkably consistent results. With a few exceptions, the students were satisfied with the course. It was not seen as terribly exciting, and it did require students to stay up to date with their work, but its demands were reasonable, and

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45 E.g., Bell & Bell (1985, pp. 219–220); Borasi & Rose (1989); Miller (1991)
46 Arnold & Truran (1999)
it was seen as relevant. It became clear that the silent majority had been drowned by a small and noisy minority. Since it proved easy to implement quickly a number of good suggestions for improvements to the course, we were confident that a fair assessment of the course would be better than “satisfactory”, but not outstanding. Obviously this finding was a great morale booster for the lecturer.

But one aspect of the exercise was of relevance for this thesis. While some of the questions put to the journalists covered fairly routine matters, we were also anxious to try to find out more about their understanding of the content of the course and also their response to it, using approaches which could not be used in traditional examination procedures. Some examples are given in the next section.

EXAMPLES OF QUESTIONS USED

In the original sheets handed out, sufficient space after each paragraph was left for journalists’ responses to be written in.

**Hypothesis Testing**

This is a critical part of the course. Please explain as carefully as you can how you feel about it, what its purpose is, and what further help you might need.

How would you explain to an intelligent person who has not studied statistics why it is not possible to accept $H_0$?

What criteria would you use to decide whether a one-tailed or a two-tailed test was more appropriate?

When a report says “We reject the null hypothesis ($p < 0.05$)”, what is the significance of the $p$-value?

**Other Statistical Issues**

Why is it important for samples used in analyses to be random? What sort of difficulties arise in ensuring that they are random?

How do practising statisticians weigh up the differences between Type I and Type II errors in Hypothesis testing? In your answer please make it clear that you know what each type is.

Of course, the students’ responses were of use in helping us to assess the effectiveness of the teaching. But they also generated another, quite unexpected, response. When asked to indicate what benefits they had gained in writing their journal, they produced comments like:

- It has made me realise that I need to learn some definitions better.
- Getting you to think about the overall relevance of the topics, not just plugging in the numbers.
It’s made me notice that I have trouble answering the simplest questions about things I ought to know by now.

It has helped me to realise what are the most concerning issues. It points out what we should focus on and therefore has an impact on what to study and revise.

If anything, it has increased my understanding of the topics because I am forced to think about them.

Clearly the approach we had taken was having learning benefits for the students which we had not anticipated, but which were very obvious to them. This was our most important finding.

For many years the course had been taught by several lecturers fairly rigidly. The students preferred to learn procedures rather than concepts and tended to use rote learning to tackle standard problems, and disregard anything else! During our experiment the course was offered twice a year and the other lecturer was strongly in favour of a procedural emphasis, while Anne had felt that something deeper could be tackled.* We now found that at least some students would express a preference for learning for understanding if they were given the opportunity to experience its benefits. Admittedly, they tended to be better, more committed students, but universities do have obligations to these students too.

Two features of this story are of relevance for this thesis. The first is that many of the more probing questions which were put to the journalists were constructed from with my own expertise in mathematics education. It might be difficult to convince an outsider of this, because there is no hard evidence, and because the questions really look to be disarmingly simple.∞ Either of us might have written the administrative questions equally well. But I was constructing the cognitively more demanding questions from a basis of 30 years of asking cognitively demanding questions in relatively relaxed ways† and I had structures of levels of difficulty§ which formed part of my way of thinking about mathematics teaching and which could be accessed quickly and efficiently. This was the special skill which I brought to the exercise, and which proved to be so important.

* This is, of course, essentially a debate between “instrumental” and “relational” understanding explicated by Skemp (1971/1986).

∞ Part of the pleasure involved in watching skilled television interviewers like Michael Parkinson is because they manage to ask penetrating questions while giving the impression that they are disarmingly simple people.

† Some documentary evidence for this may be found in J. Truran (1973a)

§ Such as that proposed in Bloom et al. (1956)
The second feature is the effect which it has had on Anne’s planning for future courses. While she had always had a lingering belief that there might be better ways of doing things, the experiment has shown her that students will accept deeper approaches and has given her some insights into why this is the case. She implemented some changes during first Semester, 1988, and then it happened that in second Semester she was asked to take the second year “economics statistics” course. So she has been able to compare students who have come from different first year courses being conducted in quite different ways. The benefits of a deeper approach in the foundation year have been obvious both to her and to her colleague, a lecturer who had not been involved in first year teaching at all, and knew very little about our work.

So our work showed that developing the ideal environment for being able to make substantial change involves spending some time thinking about what is currently being done, and in cleaning out any cobwebs which may have crept into one’s teaching. The role of the mathematics educator was to bring a different perspective to the course, and, very importantly, to be in and around, dropping in for discussions, and generally providing some assurance that the lecturer is on the right track.

In my view, we are now ready to make more important alterations to the course—tightening its approach to probability and independence, using more sophisticated statistical inference, and a richer variety of teaching aids, &c. But departmental funding has been frozen, so the changes will be much slower, and will not make as much use of my mathematics education expertise as they might!

**CONCLUSION**

It was fortunate that the final cameo was so successful, or this chapter would have presented a rather depressing tale. In my view it is significant that the experiments made use of holistic mathematics education skills (as would the school experiment), and it is this holism which I see as an important rebuttal to those who would argue for any one aspect of mathematics education as having primacy over another.

All of the cameos presented here have been principally related to Intellectual forces. Most of them describe situations where a need has been felt for change which has generated sufficient funding to provide an adequate Intellectual basis for effecting this change. The first three cameos demonstrate ways of supporting change by the provision of suitable new texts. But they also illustrate the difficulties of ensuring that the mathematics is sufficiently precise, even in a
supportive and skilled learning environment. The other three cameos address change by providing teachers with a deeper background than has usually been seen necessary. While they do not provide enough evidence for making detailed generalisations, they do strongly suggest that such an approach is very complex, and that the mere provision of printed material, no matter how good, may not be sufficient for stable change.

The cameos illustrate a variety of curriculum development models, ranging from the use of textbooks as agents of change to Howson’s school-based personality model. They are neither sufficient in number nor diverse enough in selection to constitute a basis for generalisations. But they do indicate how my experiences have influenced the arguments in this Part.

Finally, the final cameo also demonstrates the claim made above that education is a risky business. It was only Anne Arnold’s willingness publicly to try alternative approaches which enabled both of us to develop so much understanding of what was needed to improve the course and to effect changes. An interesting corollary to this project was the claim of one very senior member of the economics staff that work of this form was not research. A generous response to this comment might be to suggest that he had not really appreciated the nature of the discipline of mathematics education, and it is this matter which we shall address in the final chapter of this research report.

Young man, I have been thinking about this topic for more than thirty years, and have never discovered a clearer way of explaining it than what I have just repeated to you.

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*R.A. Fisher, from an apocryphal, but probably true, story told to me by Professor Alan James, University of Adelaide. Fisher had been interrupted during a lecture and asked to explain one of his points in more detail. He had repeated exactly the same words as he had used the first time. The student, nothing daunted, had asked for further elaboration. This led Fisher to walk right up to the student (Fisher was extremely short-sighted) in order to give the above reply.*
CHAPTER 24: WHY IS PROBABILITY POORLY TAUGHT?

Educational researchers have an important role to play in developing policies based on evidence rather than on hunch and anecdote. Too often analysis is divorced from solution. Many education papers I read point out the deficiencies in the system and detail at length the causes. I would welcome suggested courses of action to make the necessary structural changes.

Such policies could help bypass the rhetoric of the educational professionals. ... Actual reform of our education system is not only possible, it is needed. Such reform is too important to be left to the politicians and the professionals alone. Education is for the people, all the people, not just a self-selected few.¹

These words from a former NZ Prime Minister reflect the feelings of many who see academics as divorced from reality, and schools as failing to do their duty. Lange has found academics to be too equivocal to be of much use when change has been necessary. His view would not be universally shared, but it is practical and constructive and must be addressed by serious educational researchers.

AN OVERVIEW OF THE PROBLEM

The range of events that scientists can think about statistically has been increasing slowly, but in a decided, positively accelerated fashion since at least the 17th century. The work of Kahneman and Tversky may be regarded as one of the most dramatic inflection points on that curve.

We believe that, with a lag in time, lay people have been following a similar curve of ever-widening application of statistical reasoning, ... Piaget’s young subjects reasoned about the behavior or randomising devices with a sophistication that seems quite unlikely for people of earlier centuries. Will our own descendants differ as much from us as we do from Bernouilli’s contemporaries?²

There is little doubt that the need for probabilistic understanding to interpret our society meaningfully is now an Ultimate Social force on the curriculum: those who resist it may be classed as either ill-informed, Luddites or visionaries. Probabilistic understanding has been spreading through society for some centuries now, but its progress, as mentioned in Chapter 22, has been much slower than, for example, that for understanding about the nature of the circulation of blood. In Australian schools, its position is not yet totally assured: it still has to

¹ Lange (ndp, c. 1988, p. 5)
² Nisbett et al. (1983, pp. 361–363)
reach the NSW primary curriculum, and its status in the SA secondary curriculum remains tenuous.* In this Part we have seen that developing an effective and wide-spread pedagogy for probability has proved difficult, and in Part D we have seen how hard it has been to establish evidence about assessing probability learning. These are serious educational problems for our increasingly data driven society, so we need to examine why the problem continues to exist.

Some reasons have already been discussed:

- the conflict between axiomatic formal probability and the perceived need for applied approaches to its teaching;
- the conflict between the traditional deterministic classroom and the non-deterministic nature of probability;
- the failure to develop deep links between the teaching of probability and statistics;
- the fact that early teachers had no pedagogic model to follow, and later teachers often had only the poor pedagogic models of their predecessors.

We have described many experiments in Chapters 13, 14, 15, 22 and 23. Some have been pragmatic attempts by individuals with no special skills in pedagogy; some more systematic small-scale enquiries. Many have been successful for their developers within their individual institutions and cultures. Others have been well-funded productive projects which have produced teaching materials with good claims to be workable over a fairly broad range of schools and cultures. The assessment research summarised in Part D has found good ways of talking to children about probability, and good situations for encouraging the development of their conceptions. The assessment structure proposed in Chapter 18 also has obvious advantages for the preparation of sound Pedagogic presentations. While research has not yet found most of the answers, its findings have not been so equivocal as to be unusable, as the Handbook Model in Chapter 23 has shown.

But most teachers have been little influenced by research and pedagogic developments for probability. This is difficult to substantiate, but there are many pieces of evidence which, when taken together, make the claim reasonable. The first is the reasons given in Chapter 15 for withdrawing the topic from the SA academic syllabus—teachers found it hard to teach, and most children did not enjoy it. The second comes from comments made by teachers to student teachers or visiting

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* As this thesis is being completed in 2000, moves are afoot to reintroduce probability to the Year 12 academic mathematics curriculum, but as an adjunct to statistics rather than as a foundation of statistics (Anthony Harradine, address to Statistical Society of SA, 16 Aug 00). Changes to the syllabuses for other years are discussed briefly in Ch. 26.
supervisors and reported back to academic researchers. Sometimes supervising teachers even ask their student teacher to teach probability because they are not confident in doing so.\(^3\) The third is the large research grants to the Tasmanian SOLO group 30 years after the topic was first taught in Tasmanian schools. Presumably our parsimonious funding agencies believed that the results would have some generality. The fourth is the evidence from Chapter 16 that many poor curriculum materials have been based on classroom experiences supported more by philosophic approaches than by research findings. The fifth is the unstructured and diverse nature of the pedagogical suggestions summarised in Chapter 22. The sixth, discussed in Chapter 21 and in Chapter 2 of my Masters thesis, is the slow speed at which clear mathematical ideas become embedded in text-book presentation. Finally, Chapter 20 has shown that even within the quality control of the research world there has been some debasement of question quality, and in Chapter 8 has shown that even findings about common probabilistic situations are not yet well codified in researchers’ minds, let alone in those of teachers.

It is true that this failure to adopt new methods in classrooms has been paralleled in other topics. For example, extensive practical work done by several groups of researchers into understanding how better to teach algebra has had very little influence in schools.\(^4\) So a failure to develop a sound probability pedagogy might be seen as due to a general failure of communication between researchers and teachers. But, unlike algebra, understanding probability is now being seen as an essential skill of any numerate person, and not a topic from which a student can be withdrawn because it is “too demanding”. Good probability pedagogy practices have to be found and implemented because chance is now recognised as a critical mathematical feature of our society in the same way that counting and measurement are. It is a feature which will not go away.

So the task in this chapter is to interpret this failure to implement sound research and pedagogic experience. We shall develop a model for analysing change which incorporates the findings of this Part and also tests the validity of the BSEM. This will lead to some formal changes in the BSEM as has already been mentioned, and it seems best to indicate the proposed changes as they arise here, in order to make clear what circumstances have made them necessary.

\(^3\) Kath Truran, pers. comm.

\(^4\) E.g., Küchemann (1981); MacGregor & Stacey (1995)
It was argued in Chapter 21 that the curriculum development models of Chapter 7 provide static snapshots of processes but do not address the dynamics of change. They are useful as snapshots, but still need integration. The BSEM suggests that this must incorporate some element of tension because optimal solutions under changing conditions usually arise out of competing claims. So we shall use the tetrahedral model of Figure 3·1, reproduced below as Figure 24·1 with a slightly expanded heading, to summarise some tensions between various legitimate interests. Because this form shows the Real World at the centre of the diagram the model might be seen as emphasising Social forces on the curriculum.

**Figure 24·1** Perspectives of Mathematics Education with Social Forces Central

However, it would be possible to emphasise Intellectual forces on the curriculum by representing the tetrahedral model in the form of Figure 24·2.

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5 *Emile* Rousseau (1762, p. 184)

I really do not like verbal explanations. The young retain very little of them. Chatter, chatter, chatter! I can never repeat too often that we ascribe too much power to words: with our babbling way of teaching we produce only babblers.
Two other transformations are also possible: placing either the teacher or the learner at the centre of the model. At different times in history educators have seen the subject, the child, or the real world as being the principal concern of education. But they have rarely seen the teacher as a principal concern. This contrasts with the views we have met that see teachers as both agents and resisters of change, and sometimes Charismatic leaders. It also contrasts with the evidence of Chapter 22 that some problems of probability effectively solved by mathematics education have often not been utilised by classroom teachers.

One might argue that the teacher is merely the servant of the child, and should never be the centre of the educational process. This was the position taken by Rousseau in *Emile*, and in the quotation at the head of this section Rousseau warns against dangers which sometimes arise when the teacher is placed at the centre of the stage. This is a legitimate comment, but it is here that the idea of *obuchennyi* shows its value because it emphasises the close relationship between teaching and learning—between the teacher’s and the child’s perspectives. It also provides a specific place in the model—at the interface of teaching and learning—for the assessment structure used in Part D. How can *obuchennyi* be incorporated into these diagrammatic structures?

I shall present a sequence of diagrams to indicate the development of my thinking. “Freudenthal has observed that many a text-book presents only a final version of an author’s thoughts, and gives few clues about the difficulties encountered
by the author when composing a text.”\(^6\) So I start with Figure 24·3, which is a two dimensional re-drawing of Figure 24·1 incorporating obuchennyi.

![Figure 24·3 Perspectives of Mathematics Education Using Idea of Obuchennyi](image)

This model retains the tensions of the earlier forms, but provides a specific node to represent the important tension between teaching and learning. Within obuchennyi Teaching is placed below Learning to emphasise that the learner’s needs are paramount in education. The depth of tensions in learning may easily be overlooked. The following example from coin tossing shows the teacher’s role in mediating some of the deep tensions generated.

Intuitive ideas … indicate that it is impossible to predict the next outcome with absolute certainty, and yet the individual feels the need to master the chaos in the environment. At this point, mathematics enters the stage and promises to calculate randomness. However, while reflection yields only abstract weights which are not intuitively accessible, action would necessitate a procedure for accurate prediction. This primary intuitive conflict is exacerbated by the lack of direct feedback from reality, leading to a re-interpretation of any statement from theory into a recipe to solve the prediction problem. Consequently, mathematical statements are either over-interpreted (e.g. the law of large numbers being misused to derive the prediction of Tails after five consecutive Heads) or abandoned, e.g., re-establishing causal schemes for prediction.\(^7\)

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\(^6\) J. Truran (1992, p. 6), taken from Freudenthal (1974)

\(^7\) Borovcnik (1993)
Mathematics is at the top because it is the subject of discussion, even if different philosophies may construct different meanings for it. Physical and Social Forces are together as the External Forces under which education takes place, but Physical Forces are above Social because they are seen as more fundamental. The shading represents the individual learner who is subject to all these interacting forces. This model is more complex than earlier ones, but complexity in inherent in education and any effective model must reflect it. At the same time I claim that Figure 24·3 provides space for obuchennyi without sacrificing parsimony.

Once the integrated concept of obuchennyi has been incorporated into the model it becomes clear that further modification is required. Part of the argument for the importance of obuchennyi has been the failure of many teachers to take notice of relevant findings from mathematics education research. We also saw in Chapter 16 how the curricula of the early 1990s were not as good as they might have been because many mathematics education research findings were not considered, and we have seen in Part D and in earlier chapters of this Part similar failures from time to time among researchers to pay adequate attention to the findings of their colleagues. In short, we have shown that there has been a significant body of Academic knowledge about obuchennyi (taken in its broadest sense) which has had less influence than its quality deserved. This is tantamount to saying that there is another form of Intellectual force which is separate from that of Mathematics. We showed in Chapter 21 that judgements about pedagogy are an important component of looking at educational practice and this seems to be best included in the model by placing the disciplines of Mathematics and Mathematics Education together to constitute two distinct Intellectual Forces. This is done in Figure 24·4 and also reflects the comment in Chapter 21 that it is useful to distinguish knowledge of “content” from knowledge of “practice”.

The two forces are placed at the same level, because they are seen to be equally important and the discussions in Part C provide more than enough evidence to justify the arrow between the two disciplines to indicate the presence of tension, a tension which has been neatly explained in the following way:

My reading of the Maths Wars debates is that professional mathematicians have no conception of evidence that falls short of the kind of logical proof required in mathematics. They seem to believe that any assertion that falls short of this level of proof is a matter of opinion, and their opinion counts for as much as anyone else’s. The flaw in this kind of arguing was pointed out (I think by Gary Cizek) as being equivalent to stating that because completely aseptic conditions are impossible to

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8 Neyland (1996)
achieve in a hospital operating theatre, one might as well do surgery in a sewer ....\(^9\)

We are clearly working towards a physical representation of the BSEM. The \textit{obuchennyi} node does not at the moment have an analogue within the BSEM, and the Intellectual node now has two distinct parts. The BSEM was proposed as a structure for interpreting history, and mathematics education history in particular, which could fit the data better than the CEM or the MTM. It filled this role quite well in Parts C and D, although from time to time some weaknesses have been commented on. In this Part, these weaknesses have become more apparent. So the structure of Figure 24-4 suggests the following revised form of the BSEM:

- **External Environment**
  - Physical
  - Social
- **Intellectual Environment**
  - Mathematics
  - Mathematics Education
- **Pedagogic (Obuchennyi) Environment**

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9 Dylan William, Kings College, London, e-mail posting, 10 Jan 00
The term “Ecology” used in Chapter 9 has been replaced by “Environment” here, seeing “Environment” as a broad system which ecological principles try to explain. The Pedagogic Environment includes, as defined in Chapter 14, those “new professionals” who work in positions of curriculum leadership but whose concern is more with practice than with theory. We shall now use this model to analyse reasons for the classroom neglect of research and pedagogic experience discussed at the end of Chapter 22. This will show the value of the modifications, but will also show the need for adding a fourth environment—a Charismatic one—as has already been suggested in earlier chapters. This will provide the final structure of the BSEM in both a verbal and a diagrammatic form: it is this form which will be evaluated in Part F.

SOME POSSIBLE REASONS FOR THE NEGLECT OF RESEARCH AND EXPERIENCE

No little Gradgrind had ever seen a face in the moon; it was up in the moon before it could speak distinctly. No little Gradgrind had ever learned the silly jingle, Twinkle twinkle little star; how I wonder what you are! No little Gradgrind had ever known wonder on the subject, each little Gradgrind having dissected the Great Bear like a professor Owen. ... No little Gradgrind had ever associated a cow in a field with that famous cow with the crumpled horn who tossed the dog who worried the cat who killed the rat who ate the malt: ... [it] had only been introduced to a cow as a graminivorous ruminating quadruped with several stomachs.10

We start by showing that many commonly presented reasons for the neglect of research and experience are not particularly convincing. However, if they are considered in terms of the strengths of links between the elements of the BSEM, then a more convincing rationale may be proposed.

The Research Corpus is Not Well Enough Developed

Ahlgren & Garfield have argued that we simply do not yet know enough to be able to give helpful advice to teachers. Their case needs to be quoted at length.

While bright ideas for improvements in instruction should surely be tried, it may be too early for research to provide a sound and extensive basis for improvement. First there must be cross-sectional studies of how students think—not just whether they get the right answers—and

10  *Hard Times* Dickens (1854/1954, p. 19)
from those studies estimates made of how the understanding of probability develops. These estimates should be corroborated and refined by longitudinal studies .... As a reliable research base accumulates ... instructional methods should be tried through a sequence of alternating revisions and investigations.

What is needed are extensive, cross-disciplinary research projects, ... studies that draw on all of the different sources of probability curriculum (statistics, mathematics, psychology, etc) ...

This is not to imply, however, that research alone can determine efficient educational practice. Research provides information about connections and constraints which the curriculum maker can ponder, and provides feedback about the efficacy of the design. But the design itself requires mathematical and pedagogic creativity that draws on other sources of art and wisdom.\footnote{Ahlgren & Garfield (1991, pp. 131–132)}

Few would quarrel with what Ahlgren & Garfield have written when it is viewed as a recommendation for a research programme. They are really amplifying and supporting Romberg’s judgement about the relationship between mathematics education and “Normal Science” which we have discussed in Chapter 8.

But Lange and other power-brokers would have little patience with this position. Since they pay the piper, they quite reasonably want to choose at least some of the tunes. Below we compare medical and pedagogical professionalism, and we may note here that while our political masters do not expect every medical problem to be solved quickly, they do expect some progress some of the time. In particular, they do expect that if a well-tested treatment has been found to be therapeutic and without obvious disadvantages then it will immediately be used when appropriate, even if the reasons why it works are not well understood. The need for patients to be healed is immediate—so too is the need for students to learn.

**The Research Done has been Irrelevant to Schools**

Richard White claims that research is not used because it is irrelevant to schools:

It is easy to think of reasons why teachers and curriculum developers reject or remain ignorant of research. Teachers work in a complex social context, a tough world in which they must from moment to moment take a wide range of decisions. For most of this century research has tried to find answers to this complex world of the classroom by creating a simpler, artificial one. This worked in physical science .... . Complex and elegant designs for experiments inevitably interfere with the normal functioning of a school, so such experiments tend to be brief and
carried out in the artificial setting of a psychological laboratory rather than in normal classrooms. To use a phrase made well known by [Bronfenbrenner:(1979)], the experiments lacked ecological validity.\(^\text{12}\)

It is true that much still needs to be done, but is all of what has been done not usable in any way? School research cannot replicate the school environment exactly, but its conclusions may still be transferable. We saw in Chapter 22 that many probability teaching experiments have been done in schools, some of high technical quality and by researchers with strong backgrounds in both teaching and research. All this experience, as shown in Chapter 23, can have immediate application in classroom and assessment discourse. As mentioned above, we have enough experience now to know that probability can be taught, and, although there is a need for more understanding of how this may best be done, there have already been generated many model lesson plans and readily accessible support materials, which busy teachers are willing to use.

In other words, for probability research at least, White’s claim is too strong. It is true that some probability research is abstruse and/or difficult to understand and that recent mathematics education research has tended to focus on paradigms\(^\text{13}\) rather than on “innovations”. What innovations have been explored have often been presented as posters at conferences\(^\text{14}\) and so been less accessible to teachers. But on the other hand, Chapters 8 and 21 make it clear that much probability research has come from present or former teachers working within schools, and much has been specifically directed to pedagogical issues. There is a corpus of relevant results which has the ecological validity which White seeks.

**Inadequate Dissemination of Ideas**

Perhaps these useful results are not being used because they are not being disseminated. But I showed in Chapter 16 that the ideas circulating in the early 1990s, many of which were very soundly based, had been quite well disseminated within SA thanks to a moderate infusion of federal funding. Rather, the ideas were rarely taken on board by teachers. Similarly, we saw in Chapter 12 that for the changes of the 1960s dissemination was also quite good. It is just not reasonable to argue that new ideas have not been reasonably accessible to any teacher who cared to look, and in recent years they have been very well disseminated.


\(^{13}\) Boero et al. (1996, p. 1101)

\(^{14}\) Boero et al. (1996, p. 1108)
Inadequate Reception of Ideas

These three reasons are not strong ones, so it does seem that the problem must lie to some extent with what teachers have done with research findings. In saying this I do not want to give any impression that I am “teacher bashing”. After their excellent study of six highly skilled teachers Desforges & Cockburn observed:

We have demonstrated in detail how several constraining forces operate in concert and how teachers’ necessary management strategies exacerbate the problems of developing children’s thinking.

The teachers in our study held elaborate views of children’s learning and correlated views of appropriated teaching. They subscribed to the same aspirations for their children as those pronounced by mathematics experts. We showed that holding these aspirations and working with consummate industry were not enough to overcome the constraining forces of the classroom.

… We [suggest] that current approaches to enhancing mathematics do not take into account the complexity of the teachers’ job.15

I agree entirely. But the BSEM is not a deficit model, rather a model about tensions and complexity. So discussion about what teachers do not do must focus on why they behave as they do, and must start from the assumption that the vast majority of teachers do want to do what they see as being best for their students.

In a careful ACER survey of teachers’ behaviour during the “National Curriculum” changes, Lokan listed four factors as being essential for successful implementation of changes:

- congruence of the state or territory’s curriculum to the version of the profiles with which teachers have been issued;
- teachers’ belief that the profiles approach is ‘there to stay’ in their system and school;
- the presence of a ‘teacher leader’ within the school;
- sufficient time for teachers individually and in collegial groups to talk through the necessary changes.16

Making these positive factors negative provides four convincing reasons why committed teachers may choose not to implement well-recommended ideas.

15 Desforges & Cockburn (1987, p. 155)
16 Lokan (1997, p. x)
LACK OF BELIEF IN THE VALUE OF THE PROPOSED CHANGES

Lokan’s first point specifically addresses content (the Mathematics node) and she talks about the extent to which new ideas match the current curriculum. However, she only addresses change in an instrumental (procedural) way, and I would argue that this is part of a wider, relational, category\(^{17}\) which reflects teachers’ underlying beliefs in the value of any proposed change.

But evidence on teachers’ beliefs about the value of the content which they teach is relatively scarce, and there is no reason to believe that it matches those of the curriculum innovators or of the minority of teachers who are seeking to better themselves by further study, which are the groups whose views are most likely to be preserved in printed form. Teachers have a high degree of autonomy and privacy within their classrooms; they can quietly disregard some “requirements” about content, and many about method because even in today’s open society most of what they do is not systematically observed or recorded.

In particular, discussion among ordinary teachers about the value of teaching probability has rarely been formally recorded, and constitutes a serious gap in our research knowledge. Chapter 7 reported some disagreement among teachers about what knowledge is of most worth,\(^{18}\) and Chapter 15 revealed a range of views about teaching probability in upper secondary school, and it is often seen as peripheral in primary schools and treated as a “wet Friday afternoons” topic.\(^{§}\) Because probability is related to gambling and recent curriculum development in Australia\(^{19}\) has regarded study of common forms of gambling as a necessary part of a mathematics curriculum on social grounds, this may have led some teachers to see the topic as undesirable on ethical grounds.\(^{†}\) And, at least for the concept of independence, many teachers prefer inaccurate textbooks to the received views of mathematicians.\(^{20}\) So it is quite possible that a wide range of views exists among teachers about the teaching of probability at all levels, but there is very little hard evidence.

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\(^{17}\) Using the instrumental-relational dichotomy of Skemp (1976)

\(^{18}\) McGaw et al. (1992, p. 54)

\(^{§}\) The term is K. Truran’s. The evidence for this claim is hard to obtain, and rests largely on the nods which have greeted her use of this description when presenting workshops.

\(^{19}\) E.g., CDC (1988)

\(^{†}\) In 1984 (and possibly still) playing cards were illegal in Papua New Guinea for social reasons, and they are socially unacceptable in some sub-groups of modern Australian society.

\(^{20}\) J. Truran (1992, p. 99)
On the other hand, while there is little evidence about the views of “ordinary teachers” about a new and different topic like probability, there is some on their views of mathematics itself. For example, it has been found that pre-service primary school teachers see mathematics as “a brightly wrapped but empty gift box”, a view which reflects their dislike of and/or lack of confidence in the subject, coupled with an awareness that they will have to teach the content to children whom they wish above all else to protect from the anguish they have experienced when learning mathematics. If teachers have no special commitment to teaching a topic, it is reasonable to expect that they will be unenthusiastic in internalising relevant research or pedagogic findings.

CAUTIOUSNESS ABOUT CHANGE

While Lokan’s first reason highlights the relations between Mathematicians and teachers, her second matches the BSEM’s Social force. There have been so many changes in curricula and methodology in the last 30 years that teachers have become understandably cautious about any new approach on board unless they are convinced that it has come to stay. Society is not well agreed about what it wants from its schools, and until there is reasonable agreement, the influence of pressure groups is likely to be transitory, because teachers will not invest large quantities of energy in preparing for a change which is not likely to endure.

INADEQUATE TIME

Taking Lokan’s fourth point next, we can see that it matches some of the BSEM’s Physical forces. Effective change demands time and energy, and Lokan noted that in places where exemplary work has been done, vast quantities of time have been invested; this time has typically been supplied by teachers staying at school for long hours after classes are finished, often several nights a week, for several terms on end.

This finding is important because it suggests that, as at Melbourne Grammar, time can be found if the circumstances are right, and that it may well be the teacher’s own time, not the employer’s. Taking new ideas on board requires extra time and energy, and Lokan provides some systematic evidence that in the absence of officially provided time, teachers will make their own available.

* Papers like Callingham et al. (1995) have examined teachers’ confidence in teaching the topic, but this is a different, albeit related, issue. Edwards (ndp) lists many papers on attitudes to aspects of statistics teaching, but not towards the value of the subject itself.

21 Schuck (1999)
22 Lokan (1997, p. x)
provided that they are convinced that they can create lasting, stable benefits. I am unaware of any research which looks at whether innovation with generous time allowance is more or less successful than innovation which relies more on teachers’ own generosity.

But one aspect of “time” which is rarely mentioned is the complexity of teaching. Solutions can rarely be found by a “quick fix”. In my reading this has rarely been explicitly discussed in detail by educationists. As mentioned in Chapter 21, it has been a special interest of French researchers, but the language barrier and the very theoretical models used by the French seem to have kept their work from the rest of the world. But complexity is also implicit in the findings of Tobin & Fraser (1988) and Desforges & Cockburn (1987), mentioned here and in Chapter 1, and in many of the models of curriculum development discussed in Chapter 7. In this thesis it has been an important part of the discussion of National Curricula in Chapter 16, and in the analysis of assessment in Part D. Even though the cameos of Chapter 23 illustrated changes effected in relatively stable environments they demonstrated that change was quite complex, and required considerable time.

As a young teacher I frequently returned from conferences full of inspiring ideas to try in the classroom. Most of those I tried failed. I did not then understand why, but suspected my inexperience: my colleagues seemed to know so much more about teaching than I. Now I would place more emphasis on the complexity of what I was trying to do and on the need to devote substantial time to understanding this complexity. This thesis is in part an attempt to do just this.

We first met complexity in Chapter 7 as part of Eisner’s advocacy of connoisseurship—Eve’s task. Here we meet it while examining how to implement change—Adam’s task. If it is important for both Eve and Adam then it should a key element in any analysis of educational practice. As a complex model with an emphasis on tensions, the BSEM seems to be a useful model for illustrating at least some aspects of pedagogical complexity, and there is a place within its Physical environment for the specific influence of Time over change processes.

LACK OF ADEQUATE LEADERSHIP

Finally, Lokan’s third point—the importance of a “teacher leader”—presents one way of addressing complexity in successful change situations. A “teacher leader” embodies the Charismatic force mentioned at various places in Part C and discussed in detail in Chapter 21—a charismatic leader who sustains the final vision
as the project members trudge through the muddy slough of preparing new lessons, new approaches, new material, and who helps others to see how all their little achievements will contribute to the final success. Charisma may also, as Dickens reminds us in his disparagement of Gradgrind’s pedagogy, contribute much to learning and hence to the joy of teaching. Lokan’s claim is well supported by a variety of evidence, so it is appropriate to add Charisma to the formal structure of the BSEM. It seems best to present it as a fourth environment, because it may influence all the other environments. The BSEM thus becomes:

- **External Environment**
  → Physical
  → Social

- **Intellectual Environment**
  → Mathematics
  → Mathematics Education

- **Pedagogic (Obuchennyi) Environment**
  → Teaching
  → Learning

- **Charismatic Environment.**

How can the Charismatic force be added to Figure 24:3? This is difficult. “The wind blows where it wills; you hear the sound of it, but you do not know where it comes from, or where it is going.” Still, a decision must be made. Charisma is relevant to all three Environments so in Figure 24:5 it is placed in the centre of the triangle of forces, with one-way arrows, to suggest that it can alleviate at least some tension, and broken lines, to indicate that it is not always present.

This diagram, unlike Figure 24:1, is intended to be viewed basically as a two-dimensional diagram with the exception of the shaded section representing an individual. This illustrates the point made in Chapter 9 that the BSEM sees individuals as acting under the influence of a complex environment, rather than as acting autonomously within it. The diagram does not show the place of the Ultimate and Proximate forces in the system. This would require a time scale which would unduly complicate the structure. We shall now use this diagram to illustrate the forces acting on SA probability teaching in the period under discussion and to draw some comparisons between pedagogy and medicine.

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24 John 3: 8
CAN THE BSEM HELP US TO UNDERSTAND THIS NEGLECT OF RESEARCH AND EXPERIENCE?

Classifications are more helpful for refining the quality of curriculum description than for curriculum criticism or prediction.25

In the previous Section we have shown that some of the common reasons proposed for teachers’ neglect of research and pedagogic findings about teaching probability have only limited validity, and that the most convincing reasons are teachers’ unwillingness to respond to available information because of a variety of External pressures, and possibly because of a lack of commitment to the topic. A full assessment of the value of the BSEM as a tool for understanding change processes will be done in Part F, but at this stage it will be useful to assess its value for explaining this neglect of research and experience. In particular, does it provide deeper insights than those provided in the following quotation, and which may be seen as typical of many commentaries on curriculum change?

25 Brady (1983, p. 73), also quoted in Ch. 7
As research on learning becomes better known among teachers and curriculum writers, tension will develop since much of the research is critical of current practice. It really does have implications for what happens in the classroom. The revolution in research should be followed by a revolution in practice. But because there are many more teachers than researchers the second revolution will take longer and may be resisted more strongly.26

The revised BSEM of Figure 24-5 has seen Pedagogic and Charismatic factors as being as important as Physical and Social ones. It has broadened the idea of Intellectual forces to include both Mathematics and Mathematics Education. Finally, it has emphasised the complexity of the tensions between all of these forces and the need to see any given situation as an equilibrium (not necessarily stable) attained under the influence of many interacting forces. How may this model be used to evaluate the critical features of the teaching of probability in SA—in particular, its unpopularity among teachers because it was difficult to teach, and its insecure position within the curriculum?

During the period under consideration there were initially Social forces arguing for the teaching of probability, but they were weak subsidiary ones reflecting Ultimate factors mainly oriented to statistical knowledge, had little influence on curriculum developers strongly influenced by two Proximate Physical factors—decimalisation and increasing school populations. Physical pressures stayed strong after numbers started to decline because of increased demands from a society dominated by Technological and Economic forces on those who still had jobs. Developments in the Pedagogy of probability in some parts of the world reached SA, but had little influence, partly because of the weakness of Mathematics Education at that time, but even when Mathematics Education was better developed, useful and useable Australian material also had little influence. Intellectual pressures came from pure mathematicians and were quite strong because of the universities’ influence over public examinations.

So, of the two External forces on the classroom, one was weak, and the other gave teachers little time to experiment or learn new techniques and ideas. Of the two Intellectual forces, the Mathematical one was moderately strong, but not well suited to young developing minds or to changes in the educational environment, and the Mathematics Education force was weak. This weakness was partly due to the small amount known about probability teaching, but there were still some very applicable findings available, some of which had been developed by educational leaders in SA. Some of the resistance from the Pedagogic environment may

26 R. White (1992, pp. 162–163)
have come from the low esteem which practising teachers have traditionally held for Academics not working in classrooms. Other resistance came from the poor subject knowledge of many teachers, arising from failure of the Mathematics Intellectual forces to make their message well understood. The Pedagogues were seeking to exert their own influence on the Mathematics environment, but were being strenuously resisted in part. Finally, there was little Charismatic leadership in SA, and its influence was short-lived and intermittent. In Figure 24.6 we illustrate this whole description by drawing the links of Figure 25.5 with different intensities.

The arrow lines have three thicknesses to illustrate the relative strengths of the forces. In cases where the two forces at a node have different relative strengths, or when the forces in each direction are unequal, then separate arrows are drawn. A cross-piece at the end of an arrow indicates resistance.

What Figure 24.6 does not show are the Ultimate and Proximate forces, as mentioned above, and the divisions within the individual environments. For our purposes the important divisions are those between pure and applied mathematics, and between the “new professionals” and “uncommitted practitioners” in the
Pedagogic environment. We need new dimensions to illustrate each of these, and this would sacrifice the simplicity which is also important. These forces remain as part of the BSEM, but they are not able to be portrayed in its diagrammatic form.

Not surprisingly, Figure 24.6 lacks the tidy symmetry of the original model—it represents real-life. Its details may well be debated, but my purpose here is simply to show that the diagram can model the forces we have seen in our historical survey, as well as providing a vivid summary of the principles underlying the BSEM. It shows Physical and Social forces exerting strong External pressures on the Pedagogic environment. Mathematics and Mathematics Education are poorly linked, reflecting the view of many that Mathematics Education is not a research discipline, and they exert quite different levels of Intellectual pressure on the Pedagogic environment. Indeed, that from Mathematics Education is being resisted. Teaching is seen as more significant than learning. Both teachers and Mathematics Education exert little effective pressure on the other environments. There is little Charismatic leadership encouraging quantum changes, which helps to show why there has been little use made of research and experience.

Indeed, the Figure provides a background for examining the potential value of particular administrative plans. For example, the Handbook Model discussed in Chapter 23 belongs within the link from Mathematics Education to Pedagogy which is both not strong and subject to some overt resistance. So an essential requirement for the use of the Handbook model is the removal or softening of the resistance. It also provides a partial explanation of Lange’s concerns cited above and indicates which links need to be strengthened to achieve his objectives.

A full evaluation of the BSEM will be delayed until Chapter 26, but, following the example of earlier Parts, it may be helpful to finish this Part too with a coda—one which examines the status of teaching as a profession, and draws comparisons between teaching and medicine. This will show that the BSEM is not only deeper than some of the critiques quote here, but may be also applied to other situations, and produce diagrams which may be constructively compared with Figure 24.6.
A COMPARISON WITH THE MEDICAL PROFESSION

Previous systems of performance appraisal for most teachers based upon regular inspections have all but disappeared. Modern appraisal systems are operational in many companies which serve a constructive purpose for both employees and their employers. These need to be adapted and applied within school settings.27

Many teachers would see the industrial model proposed in this quotation as a denial of their status as members of a profession. In Chapter 21, I cited Hogben’s claim that teaching was often compared with the medical and engineering professions. The comparison with medicine is worth examining in some detail because both professions are concerned with using special skills for the benefit of individuals, and have extended preparation requirements which include substantial “on the job” practical experience. So it may be possible to construct a comparison between teaching and medicine using the BSEM as its basis.*

Teaching as a Profession

The status of teaching as a profession is tenuous, as the bon mot cited above indicates. So we shall first consider briefly some of the criteria which define a profession. This will enable comparisons to be drawn with medical practice, in order to test the wider applicability of the BSEM. References will be relatively few: a full analysis would require a Part in itself, which is not appropriate for a coda.

Noddings, in a thoughtful and insightful article has argued that mathematics teachers—in fact, all teachers—fall short of professional status. At present, teacher-led organizations have little control over standard-setting for the profession. There is no consensus on the knowledge that teachers must have, and control over teacher-knowledge does not rest with teachers. Devotion to service, to the lives and well-being of students, has become a mark of semi-professional rather than true professional life. There is little prestige attached to teaching. Teachers still labour in isolation, lacking the collegiality necessary for rich professional life. Finally, external regulation has severely constrained individual teacher autonomy.28

27 NIEF (ndp., c. 1993, p. 9)

*I am grateful to Sarah Ketteridge, my long-suffering General Practitioner, for making me aware of the therapeutic potential available within our medical system. Some aspects of the comparison presented here have also been discussed in J. Truran (2000aamtjou).

28 Noddings (1992, p. 206)
This suggests several relevant criteria for assessing professionalism, and I shall add one or two more.

**CONSENSUS ON CONTENT KNOWLEDGE BASE**

People in the street as well as professional mathematicians would consider the knowledge base of mathematics to be well established, and not seriously in dispute. But Constructivism, with its emphasis on what individuals are thinking, has challenged this view for the practice of mathematics within school classrooms, and post-modernism may challenge it even further.

Here the contrast with medicine is most marked. While there are certainly alternative practitioners in our society and also some more general scepticism about western medicine, nevertheless mainstream medicine is well established and there is also a substantial body of agreement among both practitioners and the public about its underlying knowledge base and its efficacy.

**CONTROL OVER CONTENT KNOWLEDGE BASE**

Neither teachers nor most doctors have immediate control over their respective content knowledge bases: this is done within research institutions subject to the control mechanisms of academia, principally peer review and/or replicability.

Some medicos work both as researchers and practitioners and this must help in ensuring the applicability and acceptance of research findings. But only in tertiary institutions do people usually work both as mathematical teachers and researchers. Mathematics research is rarely immediately relevant to school mathematics, but it is possible that both groups would benefit from greater interaction.

**CONSENSUS ON PEDAGOGIC/CONSULTATION KNOWLEDGE BASE**

By contrast with the consensus on knowledge cited above, people in the street, and teachers too, have little belief in the validity of knowledge about mathematics teaching. Yet, as has been mentioned before, reminiscences of schooldays do distinguish good and bad teachers, and some well constructed studies have described important features of good teaching practice.29 Some have shown that it is possible for good teachers to assist less good teachers to develop their skills,30 and some have shown that good teaching is likely to be associated with further

29 Desforges & Cockburn (1987), cited above; Tobin & Fraser (1988); Askew et al. (1997)
30 E.g., Askew et al. (1997, p. 4)
study after the initial period of pre-service preparation. In other words, there is good research which strongly challenges the commonly held adages that “good teachers are born, not made”, “teaching is only common sense, anyway”, and Shaw’s view that “He who can, does. He who cannot, teaches.” While it is almost certainly true that some people do have a special gift for teaching, it is also true that there are disciplined ways of identifying the nature of their gifts and of improving them in an equally disciplined way. This is one reason for claiming that the discipline of mathematics education really does exist.

Since “dinner party gossip” on doctors’ consulting skills does seem to parallel that on teachers’ pedagogic skills, there may well be parallels between teaching and consulting.* I am told that doctors have in the past been taught very little about consulting skills in their pre-service training, but recently such skills have formed part of an holistic, problem-based approach to some entire medical training courses. Such courses may use a simulated consultation by an actor presenting as a “patient” as a basis for a directed unit of study pertaining to a certain area of medical knowledge and are supported by video cameras and post-consultation feed-back sessions. These approaches are obviously quite expensive, but there is evidence that, when combined with appropriate feedback on success achieved, they can produce better and more long-lasting skills than more conventional training. Furthermore, there are currently moves in some Australian universities to pay more attention to inter-personal skills when selecting candidates for admission to medical training. So we may say that there is an evolving discipline of medicine education which parallels that in mathematics education.

This thesis has made it clear that there is little consensus about the pedagogic knowledge base among teachers. I have not found evidence about doctors’ consulting knowledge, but given that they are human beings in a conservative and traditional discipline who are working under considerable pressure and often in their own small businesses, it is likely that little consensus exists here either. This is certainly what I have observed in my own experience.

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31 E.g., Askew et al. (1997, p. 4)
32 Shaw (1903, p. 230), also cited in ch. 1
* This paragraph forms part of Truran & Arnold (in preparation).
33 Moorhead (1992, p. 458)
34 E.g., S. Clarke (1997)
35 Maguire et al. (1986)
36 E.g., prospective medical students at the U of Adelaide are interviewed to assess their personal suitability for the profession prior to sitting their University Entrance examinations.
Noddings has argued that professionalism demands control by teachers over “teacher-knowledge” (presumably “pedagogic knowledge” in its broadest sense). Teachers should, of course, be one of the controlling agents for teacher-knowledge because they do have many important insights and skills. In recent years, for example, they have successfully responded to the Social pressures to make schools more humane and more supportive places. But, as we have seen in this Part, many teachers have a limited knowledge of appropriate content pedagogy (as opposed to social pedagogy) and this means that teachers cannot expect to be the sole controlling agent for “teacher-knowledge”.

The situation is probably similar for medical consulting because this is usually done in much greater privacy than classroom teaching. Medicine is often seen as a self-regulating profession, and may well have been a model for Noddings’ call for more collegiality in teaching. But recent events have shown that society has not been totally happy with some aspects of professional self-regulation. For example, doctors have been required to provide more information to patients before asking them to consent to treatment, and teachers have been much more limited in the forms of punishments which they may inflict. Collegiality is not an absolute professional model, even for well-established professions like medicine; it must still be subject to some extent to the needs of society as a whole.

Indeed, for just one group to have sole control is at odds with the complexity of the educational and medical environments. We have already seen that when curriculum development has rested largely in the hands of teachers, it has shown serious weaknesses in terms of content and also, paradoxically, of method. Of course, the lack of consensus about teacher-knowledge is a problem. This thesis has identified knowledge about probability, its pedagogy, and its assessment as three important areas which need consensus, and Figure 24·6 has shown very clearly where the links between groups of specialists have been weak, or even antagonistic, thereby impeding the development of consensus. But these present weaknesses negate neither the need for consensus nor the need for a wide range of interests to be involved in control processes.

It is here that comparisons with medicine are of special value. Although individual doctors may well differ in their treatment of the same case, there are underlying agreed principles which they breech at their peril. I refer not to well-agreed principles of social behaviour and legal obligations which are very similar for doctors and teachers. Rather I refer to principles based on received medical knowledge which dictate what a doctor should do if presented with certain
symptoms. If the doctor does not follow these and causes damage to a patient, he or she is legally liable for the consequences. Such litigation is becoming increasingly common. I do not support a culture of excessive litigation because it can lead to excessive caution, but we have already seen that within probability teaching serious errors of fact have crept into textbooks and classrooms, and that there is considerable disagreement about what probability means. It is at least arguable that if teachers constitute a profession, they should be legally liable for grossly incorrect teaching in the same way that medicos are for their treatment errors.

OBLIGATION TO MAINTAIN AND DEVELOP PROFESSIONAL SKILLS

A position which regards individual practitioners as being legally responsible for their actions demands that practitioners be up to date in both content and practice. Of course, people are human, time is finite, and no-one can know everything, but in my experience as a practising teacher and as an ever-increasing consumer of high-quality medical services, it is possible even for busy people to keep up with the main developments in their field. Two ways in which this might be done are worth special discussion.

The Use of Journals for Dissemination of Knowledge

Medical practitioners receive journals as a consequence of their professional registration, to say nothing of the material which they receive from drug companies. Every fortnight journals like Australian Medical Journal bring reports of the latest findings in pure research, as well as broader discussions of aspects of clinical practice. Of course, doctors do not read everything. But they regularly and frequently have new information put in front of them with the expectation that they will keep up to date with research as much as possible.

In recent years the issue of assimilating with the vast amount of data generated by research has been the development of “evidence-based medicine”, associated with organisations such as the Cochrane Library in the UK. Learned summaries of research findings and their significance for choosing the most appropriate treatment for each individual patient\(^{37}\) are now being constructed, and regularly updated. These summaries are patient-oriented, rather than disease oriented,\(^{38}\) and seek to overcome what some see as serious myths which have been taught to prospective doctors in medical schools.\(^{39}\) They also try to evaluate the probab-

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\(^{37}\) Geyman (2000, p. 5)
\(^{38}\) Geyman (2000, p. 7)
\(^{39}\) Paauw (200, p. 13)
ilities of making a correct diagnosis from the observable symptoms and to assess the value of an improved diagnosis against the cost of obtaining more detail about the symptoms. While the summaries are not particularly user-friendly at this stage, they are available on-line to subscribers, and their development reflects an appreciation that it is important, and good economic sense, to make research readily available to busy practitioners.†

Not only do teachers not have access to such sophisticated summaries, but they do not even have to subscribe to professional associations, and in any case, as we have already seen in Chapter 16, they are largely uninfluenced by writing in professional journals. The mechanisms which ensure that new medical treatments soon become widely used does not seem to exist in teaching. For example, the following editorial was published in a journal for secondary mathematics teachers to accompany a very thoughtful article which advocated a new notation for permutations and combinations.

My point here is not to try to justify the use of this new notation. But supposing for a moment that it is in fact a better notation, how would it be possible to bring it into general usage? Would the push come from enthusiastic teachers? Or from an alert curriculum designer? Or will this good idea, like so many past good ideas, be lost simply because there is no mechanism to take it up?40

We have seen throughout this thesis how radical educational changes regress towards the mean. The editor might well also have mentioned textbooks, which seem to have even greater influence over teachers than drug companies’ flow charts on diagnosis and treatment have over doctors. Yet the situations are the same: fairly straight-forward principles must be applied in similar situations but for highly variable individuals. If anything, the challenge for teachers is harder than for doctors because teachers lack precise measuring instruments.

Of course, publication and dissemination is not sufficient. Each medical practitioner is ultimately responsible for his or her own decisions. Even Harvey’s theory of the circulation of blood was subject to intense and bitter criticism for some thirty years after its formal publication at a time when scientific communication was in some ways more effective than now. His arguments did have limitations. For example, he could not explain how blood moved from the arterial system to the venal system because capillaries were only identified after his lifetime.41 Oth-

† I thank Sarah Ketteridge for drawing my attention to this movement.
40 P. Scott (1996)
41 A.G.R. Smith (1972, p. 148)
ers defended him, but he largely remained silent or, in later years, merely reiterated what he had done and observed. While his quantitative and anatomical arguments were more than adequate for his case, other practitioners needed time to develop the same faith. However, Harvey the researcher was also Harvey the Court physician, and Harvey the general practitioner. His practical background and personal links with other practitioners were also important for the acceptance of his theory. Mathematics education research is differently structured.

The Use of Referring Procedures for the Dissemination of Knowledge

Medicine, however, still has built-in mechanisms for in-service training. If a General practitioner (GP) is not sure about a condition, either because it is unusual, or because it requires more skills or equipment than he or she has, then there is a range of support services to call on. The principal one, of course, is that provided by specialists. Medical convention requires that the specialist reports back to the GP in writing, and frequently the management of the condition will be a joint undertaking between GP, specialist and patient. This structure provides a form of continuing education for GPs which can improve their own skills and efficiency.*

In my experience, putting aside GPs’ jokes about the social skills of orthopaedic surgeons, the two forms of medical practitioners usually operate as equals, where the narrower but deeper knowledge of “the expert” is not seen as a reason for despising the wider but shallower knowledge of the GP. And especially is it not seen as a reason for telling the specialist that he or she would not know what he or she was talking about, as teachers sometimes say to university academics.

There are other support services for the GP. These may be diagnostic services like pathological tests, X-rays, or therapeutic services from paramedical staff like physiotherapists. The diagnostic services will be useful both before and after treatment. They will often be able to provide information which could not be guessed at even by an experienced practitioner. At all stages of treatment both patients and doctors are working within complex and unpredictable situations. Each patient is a different organism who may respond in different ways to the same treatment for the same condition. Some outcomes cannot be predicted in

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42 Whitteridge (1971, pp. 149–200)

* An example may be instructive. I occasional suffer a mildly irritating condition which I diagnosed as being A, requiring treatment $\alpha$. My GP at the time said it was in fact B, requiring treatment $\beta$. I protested that this did not make sense. He agreed, and said that he had used to think it was A, but found that $\alpha$ was ineffective. So he sent his patients to a specialist, who explained why it was B, and $\beta$ has proved effective in curing the problem. Such an example demonstrates in-service education at its best, with benefits everywhere except in the specialist’s wallet.
advance, which is what makes the practice of good medicine such a challenge and why it relies so much on complex personal skills.

The non-deterministic nature of medical treatment parallels that of pedagogic practice, but with the luxury of dealing usually with only one patient at a time. Experts in learning, psychology, course development, and academic content are available in the community, but there are few direct linkages with teachers as individuals. Indeed, there is often a culture within schools which says that teachers should be able to handle all the problems which they encounter. Even so, many parents seek outside help for their children, and presumably more would do so if they could afford it. But there is little culture of communication between classroom teachers and outside tutors, and little place for Keeves’ “linkage agents” mentioned in Chapter 21. The cameos of Chapter 23 suggest that working beside teachers is probably the most effective way of establishing these links, but the evidence for this claim is still limited. We may note, as a comparison, the common practice in medicine of inviting GPs to assist specialist surgeons with operations on their own patients and so observe new techniques. Even formal written communication between academics and teachers is poorly structured as the work reported in Chapter 23 on developing and using Handbook Model has shown. Educational Handbooks do not have the same authority that medical ones have.

SUCCESS IN ACHIEVING PROFESSIONAL OBJECTIVES

The final criterion for professionalism in this brief discussion is that of professional success. Even though there are teachers who seem content to have a 70% failure rate, the fact remains that teaching exists so that students will learn, and medicine so that patients will be healed, or at least palliated. Noddings, surprisingly, does not include the effectiveness of teachers in allowing their students to learn as one of her criteria for professionalism. She thus neglects both the totality of the obuchennyi concept and the complexity of education itself. The seriousness of such an omission is highlighted when teaching and medicine are treated as comparable professions. But there are important differences in assessing success.

It is relatively easy in many cases for a lay person to decide whether a doctor has initiated an effective cure. Of course, criteria for repairing broken limbs will be more clear than for treating cancer. We know more about bones, the corrective procedures are less complex than they are for carcinogens, and the cures are more likely to be permanent. Nevertheless, patients’ beliefs that they are feeling better are usually valid indicators of their condition.

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43 J. Truran (1983, p. 326)
In education, on the other hand, while lay people do seem to be reasonably well agreed when making Social judgements about the personal skills of teachers, they find it much harder to judge teachers’ effectiveness in teaching content, even widely understood content like reading and writing. So it must be much harder for them to judge effectiveness in teaching advanced or unfamiliar ideas. Some problems can be identified because children show distress, or from test results, but, as we have seen, these may have serious limitations, which may only become obvious some years later. This is why so much attention has been paid in Parts C and D to understanding the efficacy of assessment procedures. There seems to be a prima facie need for mathematics education to provide parents with better ways of assessing the quality of children’s learning. This, of course, represents a refinement of the Pedagogical node in the BSEM, more precisely at the very interface of the teaching-learning dialectic. I would argue that it could play an important role in raising the professional status of teachers. Achieving such an aim would require a strengthening of the links between Mathematics Education and both the Pedagogical and the Social Environments.

Using the BSEM to Describe Medicine

The above discussion has shown that it is possible to discuss both teaching and medicine in terms of the same broad criteria. This suggests that it may be possible to describe medicine using the BSEM as well, and Figure 24.7 shows how this might be done quite easily. Following this, Figure 24.8 summarises the points made above about medical practice by illustrating relative strengths of the BSEM’s components. Charismatic forces are omitted because I am unaware of the extent to which they currently operate within medicine, but I would argue that the other links are approximately correct.

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44 J. Truran (1983, p. 326)
Figure 24.7  Diagrammatic Form of the Revised Broad-Spectrum Ecological Model for Medical Education

Figure 24.8  Broad-Spectrum Ecological Model Summarising Forces on Contemporary Medical Practice
As with Figure 24·6, it is not important here to debate small disagreements about the model. What is relevant is that the basic BSEM model for teaching can be transferred easily to medical practice and produces pictures substantially different from those constructed for mathematics education practice and which reflect the real differences we have observed in this coda. Examining medical practice has provided some confirmation of the value and generality of the BSEM as a whole. It has shown that the model has widely applicable discriminatory potential, and so seems to be suitable for using as an interpretative tool.

SUMMARY

At the beginning of this Part we considered Henry Ford’s complaint that history was irrelevant and in this chapter we considered Lange’s complaint that educational research was impractical. The whole of this Part has been a consideration of whether these complaints are valid. Can we learn from history? Has the Mathematics Education Enterprise been anything more than gossamer?

The BSEM seems to provide a discriminatory model of wide generality. When we look at Figure 26·6 we can see very clearly some of the weaknesses in mathematics education, weaknesses which, we have already seen, have been reasons for probability’s relative lack of success within the curriculum. This is not only a “lesson from history”, it is also a lesson in a form which shows what might be done to prevent history repeating itself. The weak linkages in the diagram are clear indications to politicians and other decision-makers about what needs strengthening. So the Figure can provide a useful response to Lange’s pragmatic, but reasonable plea for immediate guidance. And because it also describes relationships between individuals and many of the complex forces operating in society it can also provide a vision for educational development in the future.

Many so-called educators believe that it does not matter what a child learns as long as he is taught something. And, of course, with schools as they are—just mass production factories—what can a teacher do but teach something and come to believe that teaching, in itself, matters most of all?

For rigorous teachers purged my youth,
And purged its faith, and trimmed its fire,
Showed me the high, white star of Truth
There bade me gaze, and there aspire.

45 Neill (1962/1968, p. 40)