

**PART C: HISTORICAL
ASPECTS OF THE
TEACHING OF
PROBABILITY, WITH
SPECIAL REFERENCE
TO SOUTH AUSTRALIA**

In this Part of the thesis the BSEM outlined in Chapter 9 is used as a lens to examine some aspects of the teaching of probability since 1959. The chapters contain precise delimiting dates, but these are only a general guide and have been interpreted flexibly when deciding where to insert which material. While much of the focus is on SA, for reasons summarised in Chapter 1, events in other parts of the world will also be discussed when they are able to elucidate the value of the model. Only the times of significant change are examined in detail, partly for reasons of space, but principally because it is at such times that the model is of most value. So that the thread of local history is not broken these more detailed chapters are linked by a briefer chapters covering the intervening periods.

CHAPTER 11: PREPARATIONS (1959–1964)

King Arthur invented Conferences because he was secretly a Weak King and liked to know what his memorable thousand and one Knights wanted to do next.¹

Much of this chapter addresses outcomes of various conferences which provided South Australian mathematics educators with a rich background on which to base their decisions about New Mathematics and probability in particular. They will give us a basis to assess whether the decisions made were merely a Colonial Echo, mature decisions based on evaluation of the available data, ecological responses in the sense of the BSEM, or a mixture of these and other explanations.

NEW MATHEMATICS

The change is more a change in emphasis than in content. In algebra, for example, where the main emphasis has been on the manipulative skills, the modern approach consists of the development of a proper understanding and command of the properties of a number field; it is concerned with skill in manipulation in so far as that skill can exemplify important principles or lead to common applications. Criticism on the basis that the new algebra underemphasizes techniques is baseless; the techniques now have a logical place as part of a unified system and will in fact receive a much wider application than previously.²

Nineteenth century mathematics textbooks³ show that probability and combinatorics were seen then as part of some anglophone mathematics curricula. As early as 1892. Sydney Lupton, formerly a teacher of secondary chemistry, was arguing for the that probability should form part of every student's general education because of its intrinsic interest, its value as a mental discipline and its uses in adult life.⁴ However, for reasons I do not yet understand, the subject fell out of favour in schools, probably in the early twentieth century, until the 1960s.*

¹ Sellar & Yeatman (1930, p. 10)

² P. Hughes (1960, p. 167)

³ E.g., Nicholson (1823); Todhunter (1875)

⁴ Lupton (1892). I must thank Dr Bob Petersen, University of Sydney, for drawing my attention to this interesting reference.

* Bibby (1986, p. 491) points out that teaching of probability and statistics in European and Japanese schools goes back to the second half of the nineteenth century, in some cases as a result of pressure from geographers. This practice had little influence on practice in English-speaking schools.

But we do well to remember that some aspects of the New Mathematics, including probability, were not new, and were influenced by earlier experiences.

The term *New Mathematics* has been used with many different meanings. The most general, least emotional, one refers to new material and approaches introduced into western culture schools (especially secondary ones) mainly from the early 1960s. It dealt with new mathematical topics (of which probability was one), increased precision of language, increased emphasis on mathematical structure, and non-Euclidean ways of approaching geometry. It was influenced by psychologists like Piaget, who was concerned with *children's* understanding of mathematics, and Bruner, who saw child-centred activities as necessary for the maximisation of learning.⁵ For some, especially applied mathematicians, the term soon acquired strong pejorative overtones: it represented a change for the worse.⁶ The more neutral meaning will be used here.

The New Mathematics reflected a dramatic broadening and deepening of tertiary mathematics. A nineteenth century mathematician would have understood most of the mathematics being taught and discovered in Cambridge in 1938, but would have found more than half of the papers reviewed in *Mathematical Reviews* in 1958 unintelligible.⁷ The broadening came from a resurgence of interest in applied mathematics,⁸ and from the development of the methods of statistical inference which were based on probabilistic ideas. At its best, the New Mathematics was an innovation of the highest quality:

Mathematics, always an exacting and exiguous subject, was suddenly released into disorientingly new dissolutions of its own premises by the last work of Turing, and then by Barnach and Ziman, as well as Gödel in the 1960s.[†] For once, most unusually and beneficially, what intellectuals thought changed what happened in school classrooms. In a rare case, the line from genius to the 10-year-old was quite direct.⁹

Many of the New Mathematics changes involved new ways of teaching old topics. But probability was a totally new topic requiring different approaches, rather than an old topic needing to be approached in a different way, so it was often not

⁵ Bruner (1963)

⁶ E.g., Hammersley (1968) in the UK, Kline (1973) in the USA, Potts (1974) in Australia. This issue is discussed further in Chapter 13.

⁷ Room (1960)

⁸ Bullen (1960)

[†] A.M. Turing, B. Barnach, J.M. Ziman, K. Gödel. I have been unable to locate any works of B. Barnach in standard libraries.

⁹ Inglis (1985, p. 77)

viewed as part of the New Mathematics movement. So it was not associated with the most trenchant criticisms of the movement, and is indeed the only one of the new topics to retain any sort of place in the South Australian curriculum.

The forces for change, therefore, were principally Intellectual ones, some from the discipline of mathematics, some from psychology and social science. Social forces were minimal, and we shall see that this was to prove very important for the fortunes of the New Mathematics as it was introduced into schools. For simplicity only, the summaries which follow trace developments in different regions separately, but draw out links where relevant.

New Mathematics in the USA

In the USA

[a]bout the time of World War I a conscious decision was made to allow the small number of research mathematicians to put their energy into the discovery of new theorems and the training of Ph.D.'s, and to leave the problems of school and undergraduate mathematics to others. This policy was highly successful on the research side At the same time school mathematics suffered a serious decline for lack of contact with the exciting changes in the world of mathematical discovery.¹⁰

The restructuring necessary after the 1939–1945 World War highlighted the need for mathematics and science graduates. American industrial and academic mathematicians had a profitable working relationship¹¹ and set out to help fill this gap by establishing new mathematics projects for schools.¹² Although Social benefits were believed to flow from these projects, they were all essentially the result of Intellectual forces for change: they were arguments for new material to be taught and sometimes new ways of teaching it.

The earliest,¹³ in 1952, was overseen by the University of Illinois Committee on School Mathematics (UICSM). Others were assisted by the Carnegie Corporation in 1955. At least four had been established before the Russians launched Sputnik, the first artificial earth satellite, in 1957.¹⁴ Several more were established just after,¹⁵ but the foundations of the New Mathematics movement, and also the

¹⁰ Allendoerfer (1965, pp. 16–17)

¹¹ Whyte (1956/1969)

¹² Price (1961, p. 10)

¹³ Howson (1979, p. 141)

¹⁴ McAloon (1975, p. 177)

¹⁵ McAloon (1975, p. 177)

Physical Science Movement,¹⁶ definitely antedated post-Sputnik paranoia.¹⁷ Some Australian visitors to the USA at the time understood this,¹⁸ but others¹⁹ saw mainly the public demand for action which followed the launch.²⁰

Sputnik's influence on the American public was, of course, a significant Social force, but an enabling one, not a directed one. It helped to ensure that the public would accept change to improve their country's technological standing, but it did not help to develop understanding about the most appropriate forms of change. In the long term its influence may have been greatest within educational psychology. It called into question the value of the neo-behaviourist tradition which had been dominant for some 30 years.²¹ A conference at Woods Hole, USA, in 1957 brought together developmental psychologists like Bruner and Inhelder and exerted a profound influence on popular theories of education and thus on the New Mathematics Movement.²²

For Australians two projects were particularly influential. The UICSM, based on a behaviourist philosophy, and concerned with pure, rather than applied, mathematics,²³ was regularly visited from about 1958. The other was the School Mathematics Study Group (SMSG), founded at Yale in 1958 by Dr E.G. Begle and moved to Stanford in 1961.²⁴ Begle was a main speaker at the 1965 Sydney conference discussed below. While there were differences between the mathematics projects, all emphasised sets, structure and logic,²⁵ and insisted that classroom practice should move towards the discovery approach recommended by Bruner and others.²⁶ Some claimed that statistical inference and probability also formed a unifying theme,²⁷ but this claim was optimistic.

Not all modern courses taught probability, and if they did, they treated it as a disjoint unit. For example, it formed one part of the Second Book of the University of Maryland Mathematics Project and was taught only in Grade 8 of the SMSG

¹⁶ Hunwick (1970)

¹⁷ Allendoerfer (1965, p. 17); Keeves (1965, p. 1); Waring (1979, p. 63)

¹⁸ E.g., McMullen (1958, p. 1)

¹⁹ E.g., Thompson (1963, pp. 192, 195)

²⁰ Allendoerfer (1965, p. 17)

²¹ Phillips (1972, p. 531)

²² Phillips (1972, pp. 557–558)

²³ UICSM (1960)

²⁴ Begle (1979, p. 160)

²⁵ NCTM (1961, pp. 21–27)

²⁶ NCTM (1961, p. 79)

²⁷ NCTM (1961, p. 22)

course, but, as far as I have been able to tell, was not included in the courses from the UICSM,²⁸ Ball State Teachers' College or Boston College.²⁹

As early as 1954 statistical inference based on ideas of chance was being prominently advocated by some writers.³¹ About 1956 the College Entrance Examination Board (CEEB), under the chairmanship of the influential³² Albert W. Tucker from Princeton University, encouraged the development of an experimental course in stochastics by a small group of influential, respected professional mathematicians and mathematics teachers.* This was released in 1959 and is the first strongly argued, influential case for the inclusion of stochastics in secondary syllabuses.³³ The Board recommended a new structure for Year 12 college-capable students, including a new alternative course called "Introductory Probability with Statistical Applications".³⁴ Its arguments for including stochastics were based on:

- the then recent growth of the subject;
- the importance of understanding uncertainty in science, technology, and everyday life;
- the mathematical value of probability as a small axiomatic system which can provide a natural use for the idea of sets and can help to keep alive algebraic skills.

Such an approach saw probability as appropriate for study in Year 12, though it did not exclude the study of descriptive statistics at an earlier age.³⁵ These arguments, quoted at length in Appendix II, were based on a scientific way of looking at the world and on an awareness that mathematics and society were changing in ways which required a sound understanding of probability. Some of the ideas were not new, but the collaboration between teachers and influential academics helped them to be heard more clearly.³⁶

²⁸ Examination of UICSM holdings from 1959 to c. 1964 at University of Bielefeld, Germany.

²⁹ NCTM (1961, pp. 65–68)

³⁰ NCTM (1961, pp. 65–68)

³¹ Kramer (1954)

³² Blakers (1976, p. 148)

* CEEB (1959a). The book is a minor revision of a preliminary draft (CEEB, 1957) which was used by some teachers and examined carefully by a number of mathematicians. However, comparison between the two books does not reveal any textual differences in the two chapters on probability, although the later book does have some additional exercises. Interestingly, two sources of exercises for this text are American textbooks written respectively in 1924 and 1933.

³³ CEEB (1959a, 1959b)

³⁴ CEEB (1959b, p. 34)

³⁵ CEEB (1959b pp. 1–2, 29–32)

³⁶ Moore (1993, para. 12), reporting an interview with Frederick Mosteller, a key author.

Although its style is chatty and its examples interesting and potentially relevant, the text is a construction of pure mathematicians. It starts with chapters on statistical graphs and simple parameters before presenting two chapters on probability, the first using intuitive ideas, the second formal ones. From then on the approach is algebraic and set-theoretic. In particular, all events were seen as having theoretical probabilities, even if their exact value could not be found by current knowledge.³⁷ Practising statisticians would usually put more emphasis on experimental probabilities, and some made their concerns known:

One cannot escape the applications of probability in any walk of life, for they are with us at the race track, in public opinion polls, in industrial quality control, in all our economic data collection, and in many modern forms of decision making. The suggestion, then, is that students should start the study of probability as early in their lives as possible. A very real problem with such a course, however, is the availability of properly trained teachers. Probability is a special kind of mathematics to which many good mathematicians have never been exposed. Moreover, it can be best taught only by someone who has got his hands dirty by using it in the field. Unless your school has a teacher with these qualifications, it would be wise to discard this as a possible twelfth grade course.³⁸

Nevertheless, the book sold so well that the CEEB “decided to stop selling it because it was getting them into all sorts of problems about taxation”;³⁹ it was replaced by a commercially produced book written by some of the CEEB authors.⁴⁰

So the New Mathematics in the USA was primarily a product of Intellectual forces, enabled to some extent by the Social forces released by Sputnik. With the production of the CEEB text some of the Physical problems involved in introducing a new syllabus were overcome with reasonable success.

While the New Mathematics was also an Intellectual force in Europe, there were some significant differences. We start by discussing a seminar held under the auspices of the Organisation for European Economic Co-operation (OEEC) at Royaumont, France, in late 1959 “for the purpose of improving mathematical education”.⁴¹

³⁷ CEEB (1959a, p. 57)

³⁸ Allendoerfer (1965, p. 14)

³⁹ Moore (1993, para. 14)

⁴⁰ Mosteller et al. (1961)

⁴¹ OEEC (1961, p. 7)

1959 Royaumont Seminar

The seminar attracted representatives from many European countries as well as Canada and the USA, including Tucker and Begle, and also R.E.K. Rourke, a co-author of the CEEB text. Delegates observed that over the preceding fifty years the volume of mathematics had increased and its language had changed but this had not, in general, found its way into secondary school syllabuses. They argued that if “the difference between algebra and geometry [were] made gradually to disappear”⁴² then the difficulties in use of language could be overcome and suggested the removal of most of traditional Euclidean geometry. The report was similar to that of the CEEB, as may be seen by examining the extensive quotations in Appendix II. There was a strong desire to bring the new forms of mathematics into the secondary school curriculum and to deepen the knowledge of mathematics in the general community. This could be done by emphasis on rigour and efficient, general procedures. Both probability and statistical inference were seen as essential parts of the secondary school curriculum, partly because they reflected current understandings of mathematics more accurately, and partly because they could be founded on combinatorial analysis, a topic which could be taught using the ideas of sets. Such an approach reflected the prevailing practice of tertiary teaching and, as we have seen in Chapter 8, probably influenced Piaget’s work in this field. As useful extensions of a basic course, the topics of correlation, normal distributions, and stochastic processes were recommended.⁴³

As with the CEEB, science was seen as saviour, with probability an important disciple. But the Royaumont representatives went further. Not only did they argue for understanding of structure and for a unified approach to the traditional mathematical topics, but they strongly emphasised the need for considering what psychological research into learning had to say about how primary and secondary mathematics should be taught. They further argued for the re-education of teachers, for an adequate supply of good teachers, for the provision of improved teaching materials and for an increase in research into the pedagogy of the new topics. Their proposals were comprehensive and forward-looking.

In later days the phrase “Do sets and be saved”⁴⁴ was used to belittle the New Mathematics movement. But “Do science and we shall all be saved” or “Understand structure and ye shall have tasted of the tree of knowledge” might have

⁴² OEEC (1961, p. 106)

⁴³ OEEC (1961, pp. 37, 117–118)

⁴⁴ Used by Professor Geoffrey Matthews at an AAMT Conference in Brisbane in May 1972. Matthews was a leading member of the Nuffield Mathematics Project.

been fairer catch-cries. The ideas from Royaumont were much richer than mere set theory. They incorporated both pure and applied approaches, while remaining rigorously academic. In terms of Howden & How's curriculum parameters mentioned in Chapter 7 the ideas encompassed the views of the Academic Rationalists, the Humanists, and to some extent the Technologists.

But the seminar did not produce material which could be transferred easily to the classroom. It is probably this almost total concentration on Intellectual matters which meant that its findings had little direct influence in Australia,* though they certainly underpinned other writing considered by Australians. But even so, the statements provide a useful framework for examining the developments in Europe before looking at the Australian situation.

New Mathematics in Britain

A conference at Southampton University in 1961 was an important moment in the development of New Mathematics syllabuses in Britain.⁴⁵ It was convened and chaired by Bryan Thwaites, formerly a mathematics master at Winchester College, but at that time Professor of Mathematics at Southampton University. Thwaites also was to address the 1965 Sydney conference discussed below. His charm and access to resources ensured that the meeting was exciting and productive.⁴⁶ Of the 150 participants, two-thirds were secondary teachers, predominantly from independent boys' schools. The remainder came from universities, research establishments and industry. The presence of the latter ensured a significant emphasis on applied mathematics in the discussions. Many were authors of books on secondary mathematics, more would join them in the near future, six or so were to form, under Thwaites, the core of the writing teams for the influential School Mathematics Project discussed below. There were speakers from the USA and Belgium, and many addresses on applications of mathematics in industry.⁴⁷ Discussions followed from the premise that the teaching of mathematics in schools needed significant improvement and that a critical need was to encourage the most able people to enter the profession. It was felt that unless mathematics was seen as lively and exciting the teaching of mathematics would not be seen as an option for the best mathematicians. For this reason the conference did not address industrial or pedagogic issues—its concern was

* The earliest published Australian reference I have found was Kempster (1982, p. 2).

⁴⁵ For a more detailed discussion see Cooper (1985, Ch. 8)

⁴⁶ Thwaites (1961, p. 104)

⁴⁷ Thwaites (1961)

principally content.⁴⁸ The failure to address pedagogy might be seen today as surprising. Groups with alternative teaching approaches were developing at that time, notably the *Association of Teachers of Mathematics*, but the academic ethos which permeated British Grammar and Public schools meant that for the majority of secondary teachers in Britain, the issue of a radical reform of pedagogy was not one that was seriously considered.

Stochastics received a little attention. It had been available for examination at A level for many years, but had been taken by very few.⁴⁹ Participants felt that in any general secondary school course “some idea should be given of the importance of statistics, of the limitations of statistical method, and of some of the inferences which may and may not be drawn from a given set of data”.⁵⁰ Unlike the changes advocated for other topics no reasons for this position were provided. Probability was not mentioned.

The subcommittee concerned with syllabuses for A Level examinations felt that it was desirable to teach statistics, but could not see room for it at that time. They felt that a better articulation between physics and mathematics courses at this level might produce sufficient time for the subject to be studied adequately and they did concede that the topic might be seen as an alternative to applied mathematics or to differential equations.⁵¹ The subcommittee concerned with syllabuses for first year university students saw mathematical statistics (with applications in science and industry) as a possible additional topic in courses designed for able mathematicians, but saw no need for the topic in less specialist courses. It was the university mathematicians who were the strongest advocates for change, the classroom teachers were generally happy with the *status quo*.⁵² Social forces were not particularly significant.

The English wanted to attract the most able students to mathematics and to teach their current senior courses better. They saw content as critical. The Liberal Humanist arguments propounded in the USA and at Royaumont were less important. Social forces had greater influence, but they were Social forces from the Intellectual part of society. Pedagogic issues were not discussed, but designing appropriate examinations was. These differences in part reflected differences between the English and American systems and meant that there was

⁴⁸ Thwaites (1961, pp. x–xiv)

⁴⁹ Cooper (1985, p. 113)

⁵⁰ Thwaites (1961, p. 32)

⁵¹ Thwaites (1961, pp. 41–42)

⁵² Cooper (1985, p. 113)

no single “New Mathematics” in the English speaking world where Australians were most likely to seek advice and inspiration. Indeed, there were also differences within countries. Some American differences have already been mentioned, and some British ones will now be discussed, starting with the SMP which was to have special influence in Australia.

SCHOOL MATHEMATICS PROJECT (SMP)

The Southampton conference, seen by some as a “Nine Days Wonder”,⁵³ developed into the SMP, which became the largest of the British New Mathematics projects. The first book published, Book T,⁵⁴ designed for Year 10, started with a chapter on sets and had no probability, although the corresponding Teacher’s Guide did discuss the teaching of probability.

So far most of the examples on sets have been of an artificial nature and the reader may have gained the impression that set theory is a nice academic pastime but that its practical applications are few and not of the type that can be studied with profit at school level.

To counter this impression we shall now look [at probability] to show how, given a little elementary set theory, one can establish a theory of probability in which conditional probabilities appear naturally and which is an acceptable basis of work at a post-school level.⁵⁵

Book T was followed by Book T4, which presented a didactic introduction to probability using set theory.⁵⁶ The accompanying Teachers’ Guide discussed some philosophical aspects of teaching of probability and gave references for further reading.⁵⁷ Linking these with the text’s didactic style would not have been easy for teachers; my own copy of Book T4 text contains my handwritten notes of frustration like “The idea of giving rules seems to me to be out of keeping with modern maths”, and “What do these boys know about proof?”

The authors responded quickly to such frustrations. In the main SMP sequence, Books 1–5, the approach became much more experimental.⁵⁸

Probability is a topic that always goes down well in the classroom. Starting as it does with games of chance, it makes an immediate appeal to students. Moreover, probability is an important branch of applied

⁵³ Thwaites (1961, p. 104)

⁵⁴ SMP (1964)

⁵⁵ SMP (1965b, p. 82)

⁵⁶ SMP (1965a, ch. 7)

⁵⁷ SMP (1966, pp. 84–87, 97–98)

⁵⁸ SMP (1967a, ch. 1, 1968b, ch. 13)

mathematics which is of increasing use in science, economics and sociology. Its inclusion at this stage [Year 10] of the school course can be further justified by the way in which it encourages the pupil to think clearly and logically.⁵⁹

The chapter for Year 11 started with a number of questions about indeterminate events, pointed out that probability is a good way of describing these, and showed how probabilities could be useful for insurance.⁶⁰ A more advanced text for able Year 11 students also used an applied, intuitive approach.⁶¹

So, six years after the Southampton Conference, probability found a place in the SMP syllabus because of its relevance, not to pure mathematics, but to applied, and because it was popular with students. It was included in an essentially pure syllabus for pragmatic, applied reasons. In such ways may a movement driven by Intellectual considerations be forced to consider Social ones like teaching material which is likely to be well received by the students.

OTHER BRITISH PROJECTS

This pragmatism fitted the general pattern of course development observed by one Australian visitor to Britain:

New topics such as Probability Theory and Statistics have been acknowledged as important but the need is felt to relate them to meaningful situations so that the treatment does not degenerate into process of “teaching by recipe”.⁶²

Of course, pragmatic considerations have to include the capabilities of the teachers. So Scotland, which has its own educational system, could introduce “Probability and Statistics” as a distinct university entrance subject more easily than England because its changes were less radical and its secondary teachers were all graduates, and so better prepared to understand the changes.⁶³

Probability was also advocated for less able students on pragmatic grounds:

[M]any problems of *choice and chance* or *probability* are susceptible of treatment from first principles and demand no mathematical techniques beyond arithmetical multiplication and division.⁶⁴

⁵⁹ SMP (1967b, p. 1)

⁶⁰ SMP (1968b, pp. 249–250)

⁶¹ SMP (1968a, ch. 17; 1968c, p. 175)

⁶² March (1970, p. 106)

⁶³ March (1970, p. 106). *Vide etiam* the discussion of Ruthven’s findings in ch. 12.

⁶⁴ Mathematical Association (1959, p. 205)

However, not all of the new movements were concerned with stochastics. One influential book⁶⁵ concerned with developing new pedagogies did not mention probability at all—its prime concern was to show that ideas of structure could be well taught without using the axiomatic method.

So by 1965 different sets of forces had led in the English-speaking world to many different forms of New Mathematics, and to the teaching of probability. The developments in continental Europe were less readily available to Australians, but need to be mentioned briefly for completeness.

New Mathematics in Europe

Francophone Europe was substantially influenced by an Intellectual force—the writings of a group using the *nom de plume* Nicholas Bourbaki who emphasised understanding of structure. They were supported by some creative people who were at once mathematicians, educators and psychologists—Gattegno,[†] Dienes,⁶⁶ Georges and Frédérique Papy,⁶⁷—who, influenced by the work of Jean Piaget, found ways of communicating formal ideas to quite small children. Their influence was not, of course, restricted to francophone countries—indeed both Dienes and Gattegno spent time in Australia—but it was strongest there. Their success meant that Bourbakesque ideas predominated in this region and that no probability was taught in France in Years 7 to 10 and was very formal in Year 11 and 12.⁶⁸

One example will suffice. Félix, a Bourbakesque collaborator, outlined a formal approach in a book seen as sufficiently important to be translated into English six years after its publication.* Probability appears at the end of the chapter on topology. The chapter starts with neighbourhoods, limits, and numerical approximations before moving to averages and the interpretation of a scatter diagram. Statistics and probability are seen as the sciences which complete algebra and topology. Not surprisingly, this formal European approach, which was also far removed from the Royaumont statements, had little influence in Australia.

⁶⁵ Fletcher (1964)

[†] Gattegno (1971; 1974) are later summaries of his approach. Although Caleb Gattegno taught for many years in England, he was born into a francophone community in Egypt, and his wide linguistic and personal skills meant that he was able to transcend the linguistic and cultural barriers which were so strong at that time.

⁶⁶ Dienes (1964)

⁶⁷ Papy (1964) is an early English translation of one of their works.

⁶⁸ Commission Française pour l'Enseignement des Mathématiques (1996)

* Félix (1966). The title of her original work was *Mathématiques modernes ↔ enseignement élémentaire*, probably the first book title to incorporate the symbolism of sets.

BEGINNINGS OF THE NEW MATHEMATICS IN AUSTRALIA

Professor Davis concludes that in spite of all the attention it has been getting, mathematics education is a culturally deprived area—containing an occasional genius but largely populated by unsettled immigrants.⁶⁹

The ideas discussed above were known within Australia very quickly. Tucker visited Australia about 1959.⁷⁰ Keeves reported to South Australians in 1959 on moves to introduce statistics in England.⁷¹ Dr P.W. Hughes, who was to be a leader of experiment in Tasmania, wrote an article for the *Australian Journal of Education* in 1960⁷² which showed an awareness of American, British and French thinking, and which argued, mainly on Intellectual grounds, the reasons which justified change. Hughes was equally well aware of the need for teachers to change as well, and for them to be given adequate support. “Support”, for Hughes and many others, meant “the provision of texts and study guides ... and in-service training courses ... to acquaint teachers with the fields and methods”.⁷³

Some Australian states implemented trials of some aspects of the New Mathematics in the early 1960s. Cuisenaire[†] materials⁷⁴ and adapted versions of SMSG⁷⁵ were trialed in WA; Cuisenaire materials⁷⁶ and the UICSM⁷⁷ were trialed in NSW; changes were planned as early as 1960⁷⁸ in Tasmania, and CEEB courses were eventually trialed.⁷⁹ All these trials developed similarly; one example will indicate the important features and show that these trials were not mere replications, but serious attempts to modify overseas ideas to Australian conditions.

⁶⁹ Blakers (1970, p. 97)

⁷⁰ P. Hughes (1960, p. 167)

⁷¹ Keeves (1959)

⁷² P. Hughes (1960)

⁷³ P. Hughes (1960, p. 168)

[†] Cuisenaire materials, developed by Gattegno, are sets of coloured rods which may be used to provide a concrete embodiment of the number system.

⁷⁴ ACER (1964a, Annexure 2, p. 2)

⁷⁵ Sumner (1969)

⁷⁶ ACER (1964a, Annexure 2, p. 2)

⁷⁷ Sumner (1969)

⁷⁸ Hughes (1960, pp. 168–169)

⁷⁹ ACER (1964a, Annexure 2, p. 2)

Mr A. McMullen of Sydney Teachers' College was sent⁸⁰ as a teacher and planning consultant to the UICSM Project.⁸¹ His reports were made available to interested readers by the ACER in 1958–59 and influenced aspects of the changes implemented by Dr H.S. Wyndham, Director of Education for NSW in 1962.⁸² The UICSM course was a formal one based on the unifying idea of a set⁸³ but did not include probability,* although it did include statistics.⁸⁴ Nor did the courses McMullen developed on his return to Australia.⁸⁵ These texts were not lavish copies of American material. Indeed, McMullen wrote

There is a real advantage in making judicious and increasing use of the language of sets in school mathematics, not because this idea is a new and fashionable one in America, but because so many situations can be expressed very simply in the terminology of sets and the terminology is thus a very versatile and unifying one.⁸⁶

In practice, the set theoretic approach was soon found to be unsuitable in many places. McMullen might be criticised for failing to see its weaknesses in advance, but then so might many other leading educators in many countries. The historical question we are addressing here is the way in which ideas reached Australia and developed within it. Many of the forces for curriculum change operating in Australia were found in other countries at that time, and all educators were subject to them. Convergence may be a better explanation for the similarities than copying.[§]

This is just one example of the sorts of changes which were implemented in Australia in the 1960s. It reflects Australia's strongly centralist educational systems which relied on a very small number of individual leaders whose vision had been expanded by their overseas experiences. In at least three Australian States senior educationists were allowed to respond rapidly to the forces for change by trialing and evaluating overseas ideas in similar ways. This was not to be the case in SA.

⁸⁰ Keeves, pers. comm., Aug 1993

⁸¹ McMullen (1958)

⁸² Keeves, pers. comm., Aug 1993

⁸³ McMullen (1958, p. 3)

* Examination of the large but not complete UICSM holdings in the University of Bielefeld, Germany, has not revealed any material on probability at all. The material covers publications from 1959 to c. 1964, but has no overall index.

⁸⁴ McMullen (1958, p. 2)

⁸⁵ McMullen (1959, p. 1); McMullen & Williams (c. 1963); NSW. Department of Education for the Secondary Schools Board (1962)

⁸⁶ McMullen (1960, p. 35)

§ It is just possible that the Carnegie Corporation's connection with both the ACER (Connell, 1980, ch. 1) and the UICSM encouraged the strengthening of this particular link, but I have found no direct evidence for this.

BEGINNINGS OF THE NEW MATHEMATICS IN SOUTH AUSTRALIA

[T]he changes that occurred in the period from 1958 say to 1972 ... demonstrate a gradual change in philosophy of the Education Department. From the insular, chauvinistic self-satisfaction of the later 1950's (where all that was not parochial was virtually ignorable) to a balanced, adventurous and progressive maturity, mathematics education in South Australia has progressed perhaps 30 years in ten.

It would seem then that we are now but a few years behind the leading educationally minded countries of the world!⁸⁷

We saw in Chapter 10 that in the 1950s the education system in SA was a conservative, centralist structure, under considerable pressure from rapidly expanding school populations and an inadequate supply of teachers. These conditions would continue to obtain throughout the 1960s. But this did not mean that the its leaders kept their heads in the sand. Those in authority in SA knew of the overseas changes, but did not consider that they had much to learn from them:

In other parts of the world ... there have been criticisms that mathematics, as taught in schools, is out of date, that the teaching is poor and out of touch with modern developments, and that there is insufficient emphasis on mathematics in schools.

Many of those statements are sweeping and are to be roundly condemned. The critics do not say what part is out of date, what parts of the course should be omitted, what new material should be included. ...

Fortunately, the Director of Education, ... and others ... who have visited the United States of America, assure me that some of the criticisms levelled at the teaching of mathematics in the United States are not applicable to South Australia.

Mr Inspector Jones ... said that, on the average, we give double the time to mathematics that America gives, and our methods are designed to give greater power to the student of mathematics than the American student achieves.⁸⁸

A.W. Jones, later Director-General of Education,[†] had been a secondary mathematics teacher, was a joint author of several secondary textbooks commonly used

⁸⁷ J. Baxter (1972, p. 30)

⁸⁸ Statement by the Minister of Education, the Hon. Baden Pattinson, *Education Gazette. South Australia* 16 Feb 1959, p. 98

[†] Jones' role in changing the culture of South Australian schools is discussed in ch. 13.

in SA,* and a departmental representative on the Arithmetic and Mathematics Subject Committee of the PEB.⁸⁹ He had visited the USA in 1956 on a Fulbright Scholarship⁹⁰ and again in 1959.⁹¹ He, and other visitors, failed to hear the rumblings of change at that time, just a few months before the Royaumont conference.⁹² At that time inspectors “were travelling overseas more than their predecessors”⁹³ and they went with well-defined purposes on behalf of a heavily pressured system. Baxter claimed that they liked most the rigid, exam-dominated parts of the US system.⁹⁴ Why did they fail to hear the new ideas? What did they hear?

Some examined not curricula, but pressing social and administrative issues.⁹⁸ The Physical forces operating on schools which have been described in Chapter 10 were still very much present. Jones went to look at administration and supervision, but he also saw ordinary classrooms, which were largely uninfluenced by progressive educational movements, and he judged the pedagogy inside Australian classrooms to be much better.⁹⁹ Others, like Mr W. Thompson, Inspector of High Schools in 1962, did examine curricula and observed that:

[t]he courses in Science and Mathematics were rewritten with a view to teaching concepts as much as possible ...

The new Mathematics seem [sic] strikingly novel at first glance, but they teach much of the same material as before. The essential change results from basing the subject on a clearer grasp of numbers and using Algebra to provide training in deduction with a consequent modification of the Geometry which follows.¹⁰⁰

* H.M. Searle & A.W. Jones *A Three-Year Arithmetic, A Three-Year Geometry Part II, Intermediate Trigonometry, A Simpler Algebra II* are all listed as “Books for First Year and Second Year High School Classes” in *Education Gazette. South Australia* 1 Jun 1964, p. 200.

⁸⁹ *Education Gazette. South Australia* 15 May 1959, p. 187

⁹⁰ Australian–American Educational Foundation (1990). *Contra South Australian Teachers’ Journal* March 1959, p. 10, he was neither the first South Australian nor the first South Australian educationist to obtain a Fulbright Travelling Scholarship, but the first to obtain a Fulbright Scholarship in Educational Development. *Vide Berndt infra*.

⁹¹ *South Australian Teachers’ Journal* March 1959, p. 10

⁹² A.W. Jones (1958)

⁹³ A.W. Jones (1985, p. 298)

⁹⁴ J. Baxter (1972)

⁹⁵ A.W. Jones (1958)

⁹⁶ A.W. Jones (1985, p. 298)

⁹⁷ J. Baxter (1972)

⁹⁸ J. Barter (1961)

⁹⁹ A.W. Jones (1958)

¹⁰⁰ W. Thompson (1963, pp. 195–196)

The reports do not say which mathematics projects he saw, but his comments and itinerary suggest that the UICSM project was one of them. Thompson correctly saw that it was the change in approach which was different, rather than the content, but he was more interested in the content and in how this was changed.

When the review of some high school courses was undertaken after 1957, a number of men at the highest university levels became keenly interested and led committees which were created to write new courses. It has been noted earlier that these committees largely retained the traditional nature of the subjects as single year courses and much of the previous subject matter was retained of course. But by virtue of their non-involvement in secondary schools, these men were able to exercise [sic] out-of-date material and they did not hesitate to do so.¹⁰¹

What is missing from Thompson's analysis is the provision of adequate funding to develop change, and the carefully articulated reasons for change presented at Royaumont. Thompson's view of New Mathematics, and hence to some extent the Department's view, seems to have been dominated by one, fairly rigid, American approach. In SA, upper secondary syllabuses were dominated by university, rather than Departmental, interests, so it is not surprising that the CEEB proposals were not reported. But other Australians were also making visits¹⁰² or using the extensive collection of material being purchased by ACER.¹⁰³ Perhaps the issue is more complex.

More is needed than having *access* to ideas. They must also be *assimilated*. The Royaumont Proceedings, published only in 1961, were on the library shelves of the University of Adelaide on 19 June, 1962 but not borrowed until 20 August, 1963 and only about ten times in the following five years.* For my part, I wrote on the New Mathematics in 1964¹⁰⁴ and referred to several projects which included the teaching of probability, but did not mention probability at all, although I did observe that some projects included statistics. Statistics had formed part of my tertiary studies, but I had no frame of reference for probability. The same must have been true for most teachers and administrators, even the most experienced.

¹⁰¹ W. Thompson (1963, p. 195)

¹⁰² Blakers (1976, p. 150–151)

¹⁰³ Keeves, pers. comm., Aug 1993

* OEEC (1961, p. 7) The stamping sheet for one of the two copies is hard to read, so greater precision is not possible. The fact that the original stamping sheets are still (August 1993) attached to both copies is, of course, another indication that the books were not widely consulted. But equally, someone in 1961 thought that it was sensible to purchase two copies. Both copies were purchased for the University library; there is no evidence that a copy was bought for the Teachers' College Library which has now amalgamated with the University Library.

¹⁰⁴ J. Truran (1964)

The new ideas were available to them, but they had no convincing way of evaluating them, and hence were extremely cautious about applying them.

Change also came in other ways. In 1957 the Murray Committee recommended an expansion of the university system. So when H.W. Saunders retired as Professor of Mathematics at the University of Adelaide, he was replaced in 1958 by E.S. Barnes as Professor of Pure Mathematics and in 1959 by R.B. Potts as Professor of Applied Mathematics.¹⁰⁵ Both were young and vibrant, and had already had distinguished careers.¹⁰⁶ In 1960 E.A. Cornish, who had been teaching statistics to from second year up since at least 1949, was made Professor of Mathematical Statistics.¹⁰⁷ Mathematics teaching was expanding, the need for mathematics graduates was increasing, the universities were gaining new blood and new life.

Barnes would play a leading role in developing school mathematics curricula. On his arrival, he responded to teachers' requests for more knowledge about modern mathematical developments and gave a course of lectures for teachers on Linear Inequalities and Linear Programming.¹⁰⁹ He worked with Jones to hold discussions in May 1959 with about 160 teachers on the possibility of undertaking a review of the mathematics curriculum in schools.¹¹⁰ His views appeared radical.

He startled us by saying that he would not need heights and distances, the sine and cosine rule for Leaving mathematics (then matriculation mathematics). He was prepared to eliminate Euclidean geometry from the syllabus. He bemoaned the fact that our courses contained no work on sets, no linear programming, no number theory and that Coordinate Geometry was introduced too late.¹¹¹

Barnes was a pure mathematician, the CEEB text had not been released: not surprisingly, the teaching of probability was not discussed.¹¹²

Soon after, under Barnes' encouragement,¹¹³ an SA branch of the Mathematical Association was established,¹¹⁴ with Barnes as Chairman and a committee repre-

¹⁰⁵ *Calendar of the University of Adelaide* 1960

¹⁰⁶ *Who's Who in Australia* 1965

¹⁰⁷ *Calendar of the University of Adelaide* 1949; 1960

¹⁰⁸ *Calendar of the University of Adelaide* 1949; 1960

¹⁰⁹ *Education Gazette. South Australia* 15 Apr 1959. p. 156

¹¹⁰ A.W. Jones (1959, p. 346)

¹¹¹ A.W. Jones (1970, pp. 85–86)

¹¹² *Education Gazette. South Australia* 15 Jul 1959. p. 237

¹¹³ A.W. Jones (1970, p. 83)

¹¹⁴ A.W. Jones (1959)

sentative of what we now call “major stakeholders”.[†] This became known as the Mathematical¹¹⁵ Association of South Australia (MASA). Barnes’ Intellectual leadership ensured that SA had a formal forum for discussing mathematics education.* In 1960, Gattegno visited Adelaide to demonstrate his Cuisenaire materials and methods,¹¹⁶ one of the first of many to come and present their ideas directly to South Australian teachers and administrators. However, as we shall see in Chapter 12, it would be some years before this new committee began to bear fruit.

Clements has seen the fact that “Barnes wanted to introduce sets, linear programming, etc in the 1960s at exactly the same time as people in equivalent positions in other states” as a confirmation of the CEM.¹¹⁷ Similarly, he sees the various visitors like Gattegno as evidence for the CEM. There is no question that the ideas of others were of influence on the decision makers in SA. It would have been very difficult to have avoided being aware of them. But it will be shown in this and the succeeding chapter that South Australian educators were practising what I have described as “critical development”¹¹⁸ rather than what Clements sees as being “copying”.¹¹⁹ “Critical development” conforms much better with an approach which sees ideas as forming parts of various memes, memes which are subject to natural selection in the same way that genes are. The concept of a meme acknowledges that there is a form of selection for ideas, as much as there is for genes. We shall see throughout this thesis that mathematics teaching ideas have received quite different receptions in the different states of Australia.

In particular, the pressures for change in SA were of a quite different form from overseas. The official position was extremely cautious. After one demonstration of Cuisenaire material, teachers were warned that “... as the system needed more trial and was quite revolutionary, it would be unwise to introduce the Cuisenaire

[†] Secretary and Treasurer: I.P. Lang, BSc Dip Ed., probably a teacher of mathematics at Adelaide Boys’ Technical High School and from 1962 lecturer at Adelaide Teachers’ College (*Adelaide Teachers’ College Handbook 1962*). Departmental Schools: Mr R. Goldsworthy; Independent Schools: Mr J. Keeves; University of Adelaide: Mr M.C. Gray; Adelaide Teachers’ College: Mr R.W. Close; Catholic Schools: Revd Br Miller (*South Australian Teachers’ Journal* December 1959)

¹¹⁵ The term “Mathematics Association of South Australia” is used in Vol 1 No 1 of *The S.A. Mathematics Teacher* Mar 1967 but the term “Mathematical Association of South Australia” is used in Vol 1 No 1 of *School Mathematics Journal* Jul 1963

* By “Mathematical Association” is implied the English body whose history dates back to 1870. For a brief discussion on the links between Australian States and this body, *vide* ch. 12.

¹¹⁶ *Education Gazette. South Australia* 15 Nov 1960, pp. 306–307; 15 Dec 1960, pp. 234–238

¹¹⁷ Clements to J. Truran, 20 Jan 1994

¹¹⁸ J. Truran (1993anzhes)

¹¹⁹ Clements to J. Truran, 20 Jan 1994

Method without prior consultation with the District Inspector.”¹²⁰ In fact, only two official experiments in mathematics education were permitted at that time; a brief summary of their trials and tribulations will illustrate the forces operating on curriculum development in SA in the early 1960s and highlight the remarkable-ness of the changes which did happen in 1965.

Before doing so we may note that this absence of official experiment did not mean that others were not thinking about change. The mathematics methods section of my Diploma of Education studies in 1962 was taken by Mr Ron Close, who had ensured that a large amount of modern material from other countries was available in the Teachers’ College library, and was discussed in lectures. My memory is that he provided a clear summary of what was done in the various courses, and then gave his own views on the material. Presumably he shared his ideas (and perhaps even some of ours) with the other members of MASA.

An Experiment in Secondary Schools: Berndt at Enfield High School

The first experiments in SA with teaching the New Mathematics occurred within the state system but at the initiative of an individual.

... [T]he Education Department failed to initiate experimentation with the new courses in high schools. Perhaps this was due to the absence of a Curriculum Planning and Development Branch, which, as one of its functions, would have concerned itself with evaluation and consequent implementation of experimental courses. Thus with the initiative not forthcoming from the Department, it was left to teachers, outside of the official administrative area, to innovate the new mathematics course.*

In 1956 Mr K.L. Berndt, an experienced senior teacher of mathematics at Enfield High School, received a Fulbright Teacher Exchange Scholarship to visit the USA.¹²¹ While there he had experience of the UICSM. On his return he argued that traffic flow, interplanetary travel, communication theory and cybernetics had arisen within mathematics but were not being dealt with in mathematics classes.¹²² Berndt’s views were an Intellectual force, reflecting largely an Academic Rationalist and Technological position common in the USA. He argued his case

¹²⁰ *Education Gazette. South Australia* 15 Nov 1960, p. 305

* Sumner (1969, p. 5). This work is a detailed study based on personal interviews and documents, some of which are probably no longer extant. It forms the basis for this section.

¹²¹ Australian-American Educational Foundation (1990). *Contra* 1959 (J. Baxter, 1972).

¹²² P. 10 of an undated document *Why Change School Mathematics Syllabuses?* quoted by Sumner (1969). I have not been able to trace this document.

extensively and enthusiastically,¹²³ using the *South Australian Science Teachers' Journal* because many mathematic teachers taught science, the South Australian Science Teachers' Association was particularly active,¹²⁴ and there was no local mathematics teachers' journal. In particular, he questioned the nature of what children were being asked to learn in the light of overseas developments which emphasised structure, sets, abstractions and greater precision.¹²⁵

Permission was given to introduce an experimental course for all first year classes at Enfield High School in 1961. The algebra and arithmetic courses were influenced by both SMSG and UICSM, but the traditional Euclidean geometry was left virtually unchanged. Six out of the nine teachers involved in the experiment were untrained. One of the trained teachers was Keith Hamann, who would play a significant part in South Australian mathematics education for many years to come. Very soon the Department unexpectedly restricted the experiment to the top two first-year classes from the beginning of Term 2, and insisted that the terminology used was conformable with that used in the traditional classes.¹²⁶ This decision was taken, without publicly stated reasons, by Mr J.O.G. Glastonbury, the Department's first Curriculum Development Officer, who had initially authorised the experiment.¹²⁷

Sumner's analysis of the experiment is that Berndt's "... case for course change was strong, and the apparent lack of support from Departmental officials [was] somewhat surprising".¹²⁸ He thought that "[p]robably the greatest disadvantage under which it laboured in 1961 was that of its teachers' poor mathematics and professional qualifications"¹²⁹ and that "[q]uite possibly it arose from the Department's lack of understanding of existing modern mathematics movements throughout the world".¹³⁰ He felt that there was "no justifiable reason to stint the experiment, other than conservatism, or lack of adequate knowledge of recent mathematics developments, on the part of the Education Department".¹³¹

Such an assessment confirms the arguments presented so far in this thesis, but raises an important new issue. The forces for change in the secondary curriculum

¹²³ Keeves, pers. comm., Aug 1993

¹²⁴ Keeves, pers. comm., Aug 1993

¹²⁵ Berndt (1960)

¹²⁶ Sumner (1969)

¹²⁷ Sumner (1969); Berndt, pers. comm., May 1993

¹²⁸ Sumner (1969, p. 6)

¹²⁹ Sumner (1969, p. 7)

¹³⁰ Sumner (1969, p. 8). Keeves, pers. comm., Aug 1993, agrees.

¹³¹ Sumner (1969, p. 10)

which were led by Berndt were essentially Intellectual ones. The Department's concern with uniform terminology suggests that it too was driven by Intellectual considerations. But there is little evidence that the Department really understood what Berndt was arguing for. We have already met in Chapter 3 the concept of a debased philosophy. Here we are meeting an example of a debased understanding, and we shall do so many times more. In other words, Intellectual forces can be emasculated if they are not understood by those whom they are seeking to influence. They cease to be an important part of the educational ecosystem—more an irritant than anything deeper. The Department was concerned to provide a common course for all children entering the increasingly comprehensive high schools, and did not consider that the new approaches were compatible with the old, even though their content might be similar. It saw no need for change. An inservice conference in 1962 encouraged the continuation of Berndt's work,¹³² but it ceased in 1964. "Berndt had gone overseas and came back as a prophet, but was considered suspect."¹³³ The similarities with Plato's prisoner who leaves the cave and returns with new ways of seeing are marked.¹³⁴ Berndt moved to ATC, partly from frustration, partly to influence new teachers,¹³⁵ and partly to avoid a promotion ladder which would have led him out of the classroom.¹³⁶

While Berndt's work was not concerned with probability, the change in approach which his work required was at least as great as that needed for teaching probability. A similar tale of limited support for experimentation occurred in primary schools over the same period. Taken together the two stories highlight the difficulties which were likely to be encountered in the teaching of probability.

An Experiment in Primary Schools: Dienes and the Adelaide Mathematics Project

In late 1961 Dr Z.P. Dienes took up a position as Reader in Psychology at the University of Adelaide. He had studied mathematics and psychology, had worked at

132 J. Baxter (1972, p. 15)

133 Keeves, pers. comm., Aug 1993

134 Plato *The Republic* Book VII, para. 517

135 J. Baxter (1972, p. 12)

136 Berndt, pers. comm., May 1993

137 J. Baxter (1972, p. 15)

138 Keeves, pers. comm., Aug 1993

139 Plato *The Republic* Book VII, para. 517

140 J. Baxter (1972, p. 12)

141 Berndt, pers. comm., May 1993

Harvard with Bruner and had developed an interest in theories of learning.¹⁴² His approach reflected Academic Rationalist aims, but he brought to his work so much creativity and enthusiasm that it is reasonable to claim that it also contained an implicit Humanist component.* He soon established the Adelaide Mathematics Project (AMP) which undertook experimental work on teaching structure in mathematics using concrete aids at several primary schools, most importantly with about 1000 children at Cowandilla Demonstration School,¹⁴³ whose Principal, Mr F.W. Golding, became an influential supporter of his work. Philosophically the Project was firmly within the traditions of the Woods Hole Conference: it was based on using psychological understandings to provide children with a structured, active, and interesting learning environment in order to achieve more stable learning of mathematical ideas. But the activities used were mainly those developed by Dienes and were based on seeing many aspects of elementary mathematics as exemplars of the more advanced ideas of groups and fields.

Dienes was a charismatic figure. Lokan, who formally investigated some of his work at the time, has recalled that “he was a real missionary, and few others seemed to be able to emulate what he could do.” For her “the impact of the Dienes program was largely Dienes himself”.¹⁴⁵

After a careful analysis of the Project, Brinkworth concluded:

However, Dienes quickly discovered that the rigid arithmetic curriculum and the traditional methods of teaching used by teachers provided a gigantic stumbling block to progress. ... If any fruitful work was to be done changes in both curriculum and methods would have to be made, although the Department had stipulated that the requirements of the existing curriculum still had to be fulfilled, despite the “freedom” given to the headmasters of the schools to introduce new ideas. Dienes achieved these changes only very slowly through personal contact with teachers, to whom he lectured several times a week, and by working in the classroom.¹⁴⁶

¹⁴² *Who's Who in Australia* 1965

* I was a subject of some of his experiments, and had one friend who worked with him at Cowandilla and who never ceased to wax lyrical about Dienes' charismatic teaching.

¹⁴³ GRG 18/142

* I was a subject of some of his experiments, and had one friend who worked with him at Cowandilla and who never ceased to wax lyrical about Dienes' charismatic teaching.

¹⁴⁴ GRG 18/142

¹⁴⁵ Lokan, pers. comm., Nov 1994

¹⁴⁶ Brinkworth (1970, pp. 15–16) Brinkworth did not have access to many documents from the Education Department of South Australia, especially the Minutes of the Primary Schools Advisory Curriculum Board.

Dienes believed that probability could and should be taught to young children, because of his experiences with teaching them logic and because of what he knew of the work done in this field by his fellow Hungarian, Tamas Varga.¹⁴⁷ However, his slow progress meant that these ideas were not developed. The AMP was affiliated with the International Study Group for Mathematics Learning,[†] which produced first a quarterly bulletin, and later the *Journal for Structural Learning* whose advisory panel contained many prominent international educators.¹⁴⁸ By 1965 Dienes had been given a Personal Chair in Education, he continued his work enthusiastically, but had no secretarial support.¹⁴⁹ In December 1965 he left for a new position in Canada, revisiting Adelaide in 1967 and 1969. Like Berndt, eventually he gave up and moved on.

The Need for Change Becomes More Apparent

It is important to assess why both these men were not really successful even though Dienes' ideas found some favour in independent schools from 1963.¹⁵⁰ The Department found it hard to forego its traditional close supervision of classroom practice. Even after Jones' "Freedom and Authority" Memorandum in 1970 gave more independence to individual schools, the Department retained control over what was taught. But in 1964 its official view was that success with the new methods was determined by good teachers and close supervision.¹⁵¹

Certainly it did not have enough good teachers to support innovative change. "[E]ven the simplest and most basic aspects of [Dienes'] course were unfamiliar territory for teachers".¹⁵² Support material provided was of poor quality. After Dienes' departure, Golding and Glastonbury "formed a writing team which constructed the new courses for grades III to VII and which prepared teachers' handbooks for those courses".¹⁵³

In later years the official view was that the introduction of modern mathematics in South Australian primary schools was

a model for innovation of this kind. The basis for the change was the research of Dr Z.P. Dienes. Its validity was checked against opinions

¹⁴⁷ Golding (1975, p. 62)

[†] also known as the Dienes' Club (Keeves, pers. comm., Aug 1993)

¹⁴⁸ *Journal for Structural Learning* 1 (1)

¹⁴⁹ Dr J.A. Rowell, pers. comm., July 1993

¹⁵⁰ Brinkworth (1970, p. 24)

¹⁵¹ ACER (1964a, Annexure 2, p. 3)

¹⁵² Brinkworth (1970, p. 31)

¹⁵³ Brinkworth (1970, p. 24)

and judgements of mathematicians the world over. The development was carried out in two demonstration schools where its feasibility, viability and impact on children were tested. It was diffused through the whole education system by the snowball effect of inservice training in which consultants were trained to prepare other consultants. They put the work into intelligible booklet form for teachers and children.¹⁵⁴

This was curriculum evaluation by expostulation and presents the Department as merely a *factotum* acting without having any reason to act beyond that of obeying its political masters. This is insulting to the Departmental officials and to the politicians. It does seem to be that case that at that time the Department had no well developed criteria for assessing innovations and no staff skilled in doing research in mathematics education. Reporting of what was known was haphazard and unco-ordinated. In 1964 there was “some evidence to suggest that Dienes’ material did not lead to any noticeable loss in accuracy, but resulted in increased understanding and liveliness in classrooms”.¹⁵⁵ By 1966 the story was what there was “not yet sufficient evidence for us to be completely satisfied”¹⁵⁶ with the work of the AMP. What this evidence was is not now clear, although Lokan did conduct one investigation and published it informally, much to the Department’s chagrin.¹⁵⁷

All this said, the Department acted as an important and pro-active component in the natural selection of the New Mathematics ideas. What forces influenced it at that time? I have already suggested that the Intellectual forces were weak because they were not understood. They may not have been much wanted either. The courses of Golding and Glastonbury “were framed in the absence of consultation with university mathematicians or educational psychologists (as done overseas) ...”.¹⁵⁸ The principal pressure on the Department was the need to keep an overloaded system going. This was largely a Physical force, one so pressing that Social and Intellectual pressures for change could gain little purchase. Those Social pressures which were strongest sought a Technocratic curriculum to ensure that children could find suitable work in a rapidly developing country. It so happened that very soon a new, powerful Physical force would make change inevitable, but meanwhile the debates about change continued to remain largely in the Intellectual environment.

¹⁵⁴ A.W. Jones (1970, p. 84)

¹⁵⁵ ACER (1964a, Annexure 1)

¹⁵⁶ “Primary School Mathematics” Supplement to the *Education Gazette*. South Australia Dec, 1966, p. 4

¹⁵⁷ Lokan, pers. comm., Nov 1994

¹⁵⁸ Brinkworth (1970, pp. 41–42)

There was little departmental leadership in this area. From 1963¹⁵⁹ Glastonbury had responsibilities for all subjects in both primary and secondary curriculum development, but initially for the development of courses which would be suitable for the wider range of children staying in secondary schools.¹⁶⁰ SA was the second last Australian state to appoint a curriculum officer.¹⁶¹ Glastonbury had done major studies in Pure and Applied mathematics but he was not a mathematics innovator;¹⁶² he seems to have vacillated in his support for Berndt, and although he wrote Dienes-based materials, his views of Dienes were tinged with scepticism.¹⁶³

After travelling interstate he reported on changes being introduced for Years 9, 10 and 11* in NSW under the Wyndham scheme, which had been introduced in haste in 1962.¹⁶⁴ This contains the earliest reference I have found from SA to the teaching of probability.

The section on statistics includes work on probability, based on the notions of a set. This quotation from the notes shows the treatment required, and it also shows notation unfamiliar to many of our teachers.

If A is an event of the sample space S of equally likely weighted points, then for $n(A)$ (the number of points in A), the probability of A (in the sample space S) has been defined above as

$$\frac{n(A)}{n(S)}$$

and it is conveniently written as $P_S(A)$, or, where the sample space is not in doubt, as $P(A)$.¹⁶⁵

In this brief, uncritical, comment may be seen Glastonbury's concerns with what teachers will make of the idea, some feeling of awe about the symbolism, and little concern about how probability fits into either a mathematics or an educational curriculum. The NSW syllabus was based on the thinking expressed in CEEB (1957); Glastonbury brought back only a few technical difficulties, not

¹⁵⁹ GRS 1049. Primary Schools Advisory Curriculum Board Resolutions, 29 May 63

¹⁶⁰ Glastonbury (1963a)

¹⁶¹ A duplicated document *The Growth of Primary School Education in Australia with Comments on Arithmetic*. p. 8. Archives of ACER, Radford Papers Item 357 (Primary School Mathematics Conference)

¹⁶² Brinkworth (1970, p. 42)

¹⁶³ Brinkworth (1970, p. 24)

* More precisely, Forms II, III and IV in NSW terminology

¹⁶⁴ Wallent (1964)

¹⁶⁵ Glastonbury (1963b). This is a quotation from NSW. Department of Education for the Secondary Schools Board (1963b)

the educational vision. Curriculum officer he may have been, but he still had a lot to learn. Indeed, even in 1966 when he was explaining changes about to be implemented in primary schools, he restricted his comments on overseas projects to the UICSM.¹⁶⁶ Furthermore, he seems to have seen SA as a hot-bed of experiment, and wrote in August, 1965 that

[s]o many experiments in mathematics are now being conducted and variations to the Official Course of Instruction in Primary Schools have been so frequent since 1963 that an article on what is taking place in this subject is necessary ... to let teachers know all that is taking place and, at the same time, enable every teacher to know just where he stands.¹⁶⁷

But the article is in fact administrative, not educational.

Glastonbury's ambivalent involvement with the New Mathematics experiments is fascinating. He was formally well educated, but inexperienced in his role, and held responsibilities too large for any individual. Was he instructed to treat Berndt harshly? Was he constrained by a Department unwilling to get out of step with other States as a result of decimalisation, and wanting to wait until the planned national meeting in 1964 had developed general recommendations?¹⁶⁸ We cannot tell. The best that can be said is that SA lacked vibrant, informed leadership, and that Glastonbury was sufficiently near the top of the ladder to be identified as at least the administrator of a conservative, inconsistent, ultra-cautious set of policies, working within an environment of inadequate Physical resources, and little Social pressure for change.

Nevertheless, the Intellectual pressures to improve school mathematics continued to increase. By themselves, they were not yet enough to ensure that change did happen, but they did ensure that when the time was right the change would be significant. New Mathematics was increasingly discussed throughout Australia. For example, the influential *Current Affairs Bulletin* devoted one edition to the subject in 1961.¹⁶⁹ The author, an anonymous university mathematician, put forward arguments similar to those from Royaumont, provided a simplistic analysis of the American scene, and lamented "a certain dreariness in the emphasis in schools on routine drills rather than exciting new ideas".¹⁷⁰ He believed there were three groups of mathematicians—creators, professionals and technicians—and saw teachers as technicians. He showed limited awareness of the

¹⁶⁶ Glastonbury (1966, p. 3)

¹⁶⁷ *Education Gazette. South Australia* 2 Aug 1965, p. 267

¹⁶⁸ Brinkworth (1970, p. 27)

¹⁶⁹ "Mathematics in Australia"

¹⁷⁰ "Mathematics in Australia", p. 58

nature of schools and the forces acting on them, but arguments like these made it clear that even a decision to do nothing would be a highly significant decision.

However, people started to see that change was necessary. Early in 1962, Close, lecturer in Mathematics Methods at ATC, argued publicly that SA should prepare a new mathematics curriculum because of the changing clientele within schools, and the mechanical, limited approach to much SA mathematics teaching. He also argued that it would be foolish to design a local course, and preferable to choose an appropriate one from overseas.¹⁷¹ Close was clearly aware that the changes in school clientele, discussed in Chapter 10, constituted a Social force operating on the system and that the limited Physical resources of South Australian education would make the use of work produced elsewhere a rational economic alternative. In the language of the CEM he was advocating “echoing”, but it was a *conscious* choice to do so, and not one made because echoing was seen as the thing to do. An opportunity to make an appropriate choice would come when several South Australians went to a conference in Sydney later in the year with the intention of deciding on possible changes for SA and conducting an experiment in 1963.¹⁷²

1962: AUSTRALIAN MATHEMATICAL SOCIETY CONFERENCE IN SYDNEY

More extreme language is used by some others who castigate the standard American high school course as the “dead and frozen product of antiquity” or “the mummified mangled corpse of Euclid: or even as “worthless nonsense”. In similar vein, Howard Levi ... protests at what he calls a principle in American mathematical education that a topic must be learned in a wrong version first before a right one is presented.¹⁷³

In August 1962 the Australian Mathematical Society, at that time only six years old,¹⁷⁴ and the Mathematical Association (NSW Branch) convened a Seminar on Mathematical Education at the University of Sydney to discuss the teaching of secondary mathematics in Australia and overseas. More than 100 people from all Australian States, representing secondary, tertiary and training college teachers, administrators, and industry were present, including 15 from SA.¹⁷⁵

¹⁷¹ Close (1962)

¹⁷² Gray (1962)

¹⁷³ Chong (1962, p. 51) in his opening address to the conference

¹⁷⁴ McQualter (1980a, p. 53)

¹⁷⁵ The proceedings are summarised in *Australian Mathematics Teacher* 18 (3). November 1962.

In the opening address Professor F. Chong, University of Auckland, summarised his experiences of new approaches during nine months overseas, particularly of SMSG and UICSM.¹⁷⁶ He did not mention probability, even though he had met Tucker and at least one other Royaumont participant. He did mention the CEEB's 1957 work, but not their 1959 publication. He told of the American backlash against modern mathematics spearheaded by Kline,¹⁷⁷ and argued against the excessively rigid approach to algebra and geometry taken by many in the USA and concluded with a Biblical form of the Heisenberg Uncertainty Principle:*

Precision, rigour, abstraction generality may be worthy goals, but a programme which pursues them in a self-conscious, unnatural manner can be self-defeating. *Whosoever shall seek to save his life shall lose it.*¹⁷⁸

In the discussions Larry Blakers, Professor of Mathematics at the University of WA, who had studied at Princeton and had been involved with trials of the New Mathematics in WA, argued that Australians had to find their own solutions to the problems and that it was desirable that different solutions should appear in different parts of the country.¹⁷⁹ Another speaker presented a closely reasoned argument against the use of sets terminology in junior secondary schools.¹⁸⁰

The 1962 conference was the first ever major meeting of Australian secondary and tertiary mathematics teachers. There is no evidence that its members were uncritically copying overseas practices. They saw the American approach as "pure",¹⁸¹ the English as "applied",¹⁸² and made no comment on the European. They were more concerned with pedagogy than content, and advocated no changes in the curriculum.¹⁸³ Even though statistics had been mentioned by several teachers as useful for less academic students or prospective social sciences students, no recommendations about teaching any stochastics were made,

¹⁷⁶ Chong (1962)

¹⁷⁷ One summary of the American position may be found in *American Mathematical Monthly* 69 (3), March 1962, reprinted in *Vinculum* 24 (1), 1987.

* It is impossible to know both the position and velocity of a particle at the same time.

¹⁷⁸ Luke 17: 33; Chong (1962, p. 57)

¹⁷⁹ *Australian Mathematics Teacher* 18 (3): 67

¹⁸⁰ Kelly (1962)

¹⁸¹ In Moore (1993, paras 29–32) Mosteller confirms that because statistics was usually taught in mathematics departments the Americans were concerned to have a sound theoretical base for what was being taught. He goes on to explain how he came to the other approach of Data Analysis through the pressures of working with applied statisticians who needed urgent answers to their questions. This led to his writing *Biostatistics in Clinical Medicine*.

¹⁸² Cherry (1962)

¹⁸³ *Australian Mathematics Teacher* 18 (3): 99–100

probably because it was seen as peripheral by the predominantly academic mathematicians, secondary and tertiary, who were present.

Jeff Baxter has described the small group from SA who attended this Conference as “arch-conservatives”, secretly frightened that mathematics in SA would not stand up well to international or interstate comparisons.¹⁸⁴ Indeed, he has claimed that SA did not take part in the 1964 IEA study which was being planned at this time because of a fear that the State would not show up well, especially in secondary schools.¹⁸⁵ As we shall see, the situation was more complex than these claims suggest. The group’s spokesperson observed that they had been “ruminating”¹⁸⁶ on the New Mathematics for several years. Two subsequent South Australian conferences for teachers from all systems, including the technical high schools, were held where university staff spoke on aspects of mathematics and Berndt spoke on his experimental work.¹⁸⁷ Predictably, probability was not mentioned. Yet nothing of immediate significance developed from these conferences.

Indeed, the Sydney conference did not in general act as a catalyst for any significant change anywhere in Australia. Its failure to do so is strong evidence that Intellectual forces by themselves were not sufficient to effect change within Australian schools at that time. Social forces for change were still increasing, and in fact reducing the Physical resources available for change as they did so. But the Sydney discussions were largely Intellectual ones and were marked by significant differences of opinion between pure and applied mathematicians and between secondary and tertiary mathematics teachers.¹⁸⁸ These were amicable enough, but meant that nobody saw a clear way forward. So Close’s appeal for change discussed above had no effect. The delegates did express a desire to meet again within two years¹⁸⁹ and planning for such a meeting did lead to the establishment of the Australian Association of Mathematics Teachers (AAMT),¹⁹⁰ but not until 1966.¹⁹¹ Two significant meetings did occur in 1964 and 1965, but under the auspices of the ACER, not tertiary mathematicians. Before discussing these meetings it is necessary to describe an important American meeting held in 1963.

184 J. Baxter (1972)

185 J. Baxter (1972)

186 *The Australian Mathematics Teacher* 18(3): 87

187 GRG 18/98

188 *Australian Mathematics Teacher* 18 (3): 86

189 *Australian Mathematics Teacher* 18 (3): 100

190 Pitman, pers. comm., Nov 1996

191 Horwood (1992)

1963: CAMBRIDGE (USA) CONFERENCE

The question of what is teachable and what is not depends largely on the organization of that subject matter. Only the very top level of expertise is likely to be sufficient to make the necessary determinations, and to set the stage for broader discussions ...¹⁹²

At this conference of 25 professional mathematicians and mathematics users, convened at relatively short notice¹⁹³ at Cambridge, MA, recommendations were made about syllabuses, but not pedagogy, for both primary and secondary schools. Most participants were from Ivy League Universities; three from large industrial undertakings. None of the participants taught in schools. This exclusion was deliberate.

The group felt that the American New Mathematics projects (in particular the UICSM and the SMSG) were not radical enough, because they were too constrained by the limitations of the traditional curriculum and the scarcity of adequately trained teachers. The recommendations were certainly radical.

Calculus was introduced in Year 9 for academically advanced students. Probability was seen as a branch of applied mathematics, with an emphasis on statistical inference and decision-making¹⁹⁴ which could provide “an antidote to the erroneous idea that ‘in mathematics there is always one *exactly* right answer’”¹⁹⁵ and that an understanding of probability and statistics provided an individual with additional ways of interpreting the world, one far more likely to be useable than the deterministic approaches of much traditional school mathematics. Probability and statistics, treated empirically, were seen as appropriate applications of mathematics for primary children which might be treated through the use of independent projects done by individuals. It was assumed that there would be such a rich set of experiences in the primary schools that in secondary schools probability could be taught extensively in Years 7 and 8, with discussions of sampling, conditional probability, independence, expectation and variance, Chebychev’s inequality, joint distributions, Poisson Distribution and hypothesis testing. No more probability would be taught until year 12 when it would be discussed for countable sample spaces and then for continuous distributions. An

¹⁹² Cambridge Conference on School Mathematics (1963, p. 3)

¹⁹³ Cambridge Conference on School Mathematics (1963, p. 3)

¹⁹⁴ Cambridge Conference on School Mathematics (1963, p. 70)

¹⁹⁵ Cambridge Conference on School Mathematics (1963, p. 72)

even more radical alternative proposal suggested teaching some of this Year 12 work in Year 10.¹⁹⁶

Very strong reasons were articulated for making probability such an important part of the Year 7 and 8 curriculum.

In every grade after elementary school a certain number of students are going to stop studying mathematics. For this reason, other things being equal, topics which have high value in liberal education deserve priority. In this case, other things are approximately equal: probability theory is not falsified by exposition in elementary terms; and its insertion does not disrupt the order of the curriculum, because it uses enough of the methods of the preceding courses to keep alive the skills that the student has already acquired.¹⁹⁷

This statement represents the zenith of formal advocacy of probability in western education. It was essentially Intellectual, and strongly within the Liberal Humanist tradition. But it represented what more and more teachers would come to see as an élitist approach to education and to the role of universities in influencing school practice. For as long as secondary schools were seen as being predominantly concerned with university preparation, this position could be maintained, but such days were numbered.

In fact, the changes which were soon to come in Australia, which were heralded by the two conferences next to be discussed, did not occur as a result of any of these forces but as a result of a much more mundane Physical force: substantial time would soon become available in the mathematics curriculum because of the government's decision to decimalise the currency.

¹⁹⁶ Cambridge Conference on School Mathematics (1963, pp. 43–46)

¹⁹⁷ Cambridge Conference on School Mathematics (1963, pp. 70–71)

1964: AUSTRALIAN COUNCIL FOR EDUCATIONAL RESEARCH CONFERENCE IN MELBOURNE

The contributors saw their task not solely as that of explaining some modern mathematics to teachers or of introducing them to older ideas which were to be presented as part of the mathematics in primary schools, but as a combination of that need and the need to indicate where necessary how such ideas might be presented to children. Sometimes this meant that a longer and more detailed presentation of a particular topic was needed than might have been thought necessary for the average teacher. At times it may even seem that the obvious has been labored, but the writers have all had experience in explaining these ideas to teachers and to children. ... Its fundamental intention is to make those teachers in the primary school who still feel hesitant and uncertain about their approach to the new course of study in mathematics more assured and more confident that they understand what they are now asked to teach.¹⁹⁸

This conference is often seen as a turning point in Australian mathematics curriculum development. The ACER, under the leadership of Dr W.C. Radford, was attempting to act as a catalytic leader of the fiercely autonomous conservative State systems in the quest for excellent and viable curriculum development. Radford believed that these systems, dominated in some states by leaders lacking in ability and imagination,¹⁹⁹ set too high a price of the cost of a mistake and that this led to curriculum development being imitative rather than original.²⁰⁰ The States, on the other hand, were mindful of their differing needs and wanted to be responsible for their own development, while utilising the experiences of others.²⁰¹

The driving force for change in the primary mathematics curriculum was not to be educational vision, but the large amount of free time released by the decision, announced in late 1961, to decimalise the Australian currency. More than one year of instruction time was expected to be saved by the abolition of pounds, shillings and pence.²⁰² The ACER commissioned a balanced and “state of the art” document on primary mathematics education.²⁰³ But probability was not mentioned. In April 1963 it was announced that the change would occur early in

¹⁹⁸ Radford (1966, p. 8) in his introduction to the book which arose out of the 1964 conference.

¹⁹⁹ Letter from Radford to Wyndham, 29 Jan 64. Archives of ACER, Vol 224

²⁰⁰ Archives of ACER, Radford Papers, Item 357

²⁰¹ Letters from Wyndham to Radford 6 Feb 64, 4 May 64. Letter from Mander-Jones to Radford 16 Apr 64. Archives of ACER, Vol 224

²⁰² Keeves, pers. comm., Aug 1993

²⁰³ Dodson (1962)

1966²⁰⁴ and the Directors of Education asked the ACER to organise a conference to help their planning.²⁰⁵ The ACER asked Radford to chair the meeting,²⁰⁶ and relevant Australian and overseas material was collected together. Mr J.P. Keeves, a Senior Research Officer at ACER, and previously a secondary mathematics and science teacher who had worked in independent schools in SA and England,²⁰⁷ was sent to NSW to examine their changes under the Wyndham scheme. At the ACER there was a sense of urgency²⁰⁸ and a desire to bring sound critical thought to the discussions.²⁰⁹

In November 1963, while Radford was overseas, a meeting of Australian curriculum officers and also Keeves was convened in Sydney by the Directors of Education²¹⁰ at the promptings of Wyndham, who was not willing to wait until Radford's return.²¹¹ This was probably the first Australian meeting of middle management education officers. Wyndham inspired²¹² the officers to use the opportunity to effect a more radical revision of the mathematics programmes for both primary and secondary schools.

The ACER meeting, held in Melbourne in March, 1964, was attended by one or two curriculum officers from each state and a representative from New Zealand. SA sent Glastonbury and Mr W. Westgarth, Master of Method at Lockleys Demonstration School, a member of the Primary Schools Advisory Board,²¹³ and the only practising teacher present. As well as issues arising from decimalisation, the conference was concerned with "new approaches to mathematics in primary school arising out of recent movements in the teaching of mathematics."²¹⁴ A large collection of information about work being done in the USA, Britain, and all Australian states was distributed beforehand and an extensive library of

²⁰⁴ Brinkworth (1970)

²⁰⁵ Letter from H.S. Osmond Secretary Directors' Conference to W.C Radford, Director ACER, 13 Jun 63. Archives of ACER, Vol 224

²⁰⁶ Letter from Radford to Directors of Education, 11 Sep 63. Archives of ACER, Vol 224

²⁰⁷ Keeves, pers. comm., Aug 1993

²⁰⁸ Keeves, pers. comm., Aug 1993

²⁰⁹ E.g., annotations on letter from Stephen White, Cambridge Conference on School Mathematics to S. Dunn, Acting Director of ACER, 24 Oct 63. Archives of ACER, Vol 224

²¹⁰ Letter from Mander-Jones to Radford [absent from the country at that time], 13 Nov 63. Archives of ACER, Vol 224

²¹¹ Letter from Wyndham to Radford 15 Jan 64. Archives of ACER, Vol 224

²¹² Keeves, pers. comm., Aug 1993

²¹³ Brinkworth (1970) describes him as a teachers' representative on the Advisory Curriculum Board.

²¹⁴ ACER (1964, p. 34)

materials was available during the conference. The findings of the Cambridge Conference received special attention.²¹⁵

Many of the Conference statements were visionary. They emphasised the need for teaching with understanding, acknowledged individual differences, and de-emphasised speed and accuracy in learning. They recommended that the term *Arithmetic* be replaced by *Mathematics* “since the infant and primary school should be concerned with the beginnings of the mathematical education of the child” and “the ... problem in considering the relationship between primary and secondary courses was to see the mathematical education of each child as a continuous process from the time he enters the school until the time his formal education ceases”.* As we shall see, none of these statements was to be well received in practice by all parties involved in education, save the change in terminology.

Principles of syllabus construction were drawn up by a subgroup²¹⁶ comprising Mr E. King, a District Inspector, and Mr G.McK. Brown, in his first week as a Curriculum Officer.²¹⁷ The principles were pragmatic, and showed little of the vision presented in overseas statements. Mathematics was seen as “... a study of the relationships between selected sets. It is a study of structure.”²¹⁸ Desirable reasons for selecting content were:

- (a) it is useful in contributing to successful daily living;
- (b) it has a cultural value;
- (c) it assists to develop desirable attitudes in the pupil (e.g., appeal to imagination);
- (d) it provides experience with fundamental patterns of thinking.²¹⁹

There is certainly some evidence of ideas taken from other places, especially those about structure and mathematics, and it is quite possible that these were uncritically accepted and not well understood by all the participants. But it is also the case that the structure argument is Intellectually an appealing one for providing deeper meaning in a topic known to be unpopular and in need of reform.

²¹⁵ Letter from S. White to Dunn (Acting Director, ACER), 24 Oct 63. Archives of ACER, Vol 224.

* ACER (1964a, p. 3). The change from arithmetic to mathematics had already been agreed to in principle in SA. (GRS 1049. Primary Advisory Curriculum Board Minutes, 19 Feb 64)

²¹⁶ *Education Gazette. South Australia* 1 Jun 1964, p. 187

²¹⁷ Keeves, pers. comm., Aug 1993

²¹⁸ ACER (1964a, Annexure 3, p. 1)

²¹⁹ ACER (1964a, Annexure 3)

In its statement of expected outcomes, the subgroup recommended, probably for the first time in Australia, that probability was a suitable topic for primary schools. How this happened is not clear. The draft version prepared on 19 March does not mention probability, but the revised version prepared on the next day lists “Development of an intuitive idea of probability.”²²⁰ as a desirable outcome. What caused this change? Which of the desirable criteria listed above were seen as relevant to probability?

I have not found a convincing answer to these questions. Hughes, then Principal of Hobart Teachers’ College, had had recent experience overseas and in Australia of teaching statistics courses, and also of the UICSM,²²¹ and this experience had led him to believe that statistical ideas were suitable and desirable for teaching in schools.²²² At the conference he was involved with the subgroup which constructed a Suggested Outline of Topics and Topic-Sequence²²³ which contains no mention of probability in spite of the recommendation mentioned above.²²⁴ The conference summary circulated soon after in SA also did not mention probability.²²⁵ In an official analysis ten years later of what happened to the 1964 recommendations, probability was not examined.²²⁶ It is all very strange, and a significant indication that the topic’s status was, at the very least, not a matter of pressing concern.

The conference documents were seen as working ones²²⁷ and distributed only to those with “a close concern with new curricula in mathematics and who have made a contribution to recent thinking.”²²⁸ In any case the stencils were soon to wear out.²²⁹ The possibility of producing topic textbooks to disseminate ideas was rejected because “this could lead to artificial divisions and treatments”²³⁰ and because the States were unwilling to have imposed on them a centralised

²²⁰ ACER (1964a, Annexure 3)

²²¹ Blakers (1976, p. 151)

²²² Keeves, pers. comm., Aug 1993

²²³ *Education Gazette. South Australia* 1 Jun 1964, p. 187

²²⁴ ACER (1964a, Annexure 10)

²²⁵ *Education Gazette. South Australia* 1 Jun 1964, pp. 187–188

²²⁶ Kee (1975)

²²⁷ Letter from Radford to Downs, Education Department of South Australia, 26 Jun 64. Archives of ACER, Vol 224

²²⁸ ACER (1965, p. 42)

²²⁹ Letter from Radford to Glestonbury, Archives of ACER, Vol 224

²³⁰ ACER (1964a, Annexure 2, p. 2)

curriculum which might date quickly.²³¹ But plans were made to produce “An Elementary Guide to Modern Mathematics”²³² which explained “in terms appropriate for primary teachers ... important ideas and topics that are being considered as the result of recent developments in mathematics teaching”.²³³ Two separate documents were available by December; one mentions probability and statistics, the other does not.²³⁴ Both were influenced mainly by American thinking.²³⁵ The guide, entitled *Background in Mathematics*, was produced in 1966²³⁶ and does contain a chapter on probability, drafted by Hughes & Berkeley²³⁷ but revised and extended by Keeves.²³⁸

This chapter is the first major published Australian argument for the teaching of probability, one developed *after* the topic had been proposed for inclusion in the syllabus. Drawing on the Cambridge Conference, it argued that “many of the ideas that are important in probability theory can be grasped clearly, in an intuitive way, by children”²³⁹ because of their experience in everyday life and in games. It made the neat point that “statistics has been termed the technological brother of probability;”²⁴⁰ this point would be consistently neglected in Australia for many years. Possible classroom approaches, such as the use of tacks and spinners, were outlined, drawn to some extent from Keeves’ experience with SMP.²⁴¹ Keeves’ provision of a competent, albeit brief, summary of the Intellectual arguments mentioned above is probably due more to his enthusiasm for the topic than to any authority it received at the Conference.

The recommendations were adopted in many states, albeit with significant modifications,²⁴² particularly in Tas and Q;²⁴³ those on statistics & probability

231 Letter from Wyndham to Radford, 4 May 64. Letter from Mander-Jones to Radford, 16 Apr 64. Archives of ACER, Vol 224

232 Letter from Osmond to Radford 26 Jun 64. Archives of ACER, Vol 224

233 ACER (1964a, p. 6)

234 Archives of ACER Volume 224

235 Archives of ACER Volume 224

236 ACER (1966) and lightly revised in 1972. Page references are to the 1972 edition.

237 Hughes, pers. comm., May 1995

238 Keeves, pers. comm., Aug 1993

239 ACER (1972, p. 185)

240 ACER (1972, p. 185)

241 Keeves, pers. comm., Aug 1993. Interesting Mosteller claims (Moore, 1993 para. 6) that he was the person, in 1948, to think of using the thumbtack “as a device for giving a fixed but unknown probability”.

242 Kee (1975)

243 Hughes, pers. comm., May 1995

were mainly not implemented in NSW & SA.* At least in Q the book formed a major resource for the inservice training of teachers preparing to teach the new curriculum.²⁴⁴ In SA, 800 copies were distributed, but “teachers found it difficult to read”,²⁴⁵ even though much of the book had been checked by the eleven-year old niece of the author who had found it sufficiently understandable to be able to point out some errors.²⁴⁶ Since the contents of the book as a whole were not well understood in SA, it is most unlikely that the unfamiliar section on probability would have been. The Physical force for change—decimalisation—remained, so change was going to happen, but the Intellectual forces for probability were not well enough understood, either mathematically or educationally, and probably pedagogically as well, for them to have any immediate influence. Radford’s hope for the book, quoted at the head of this section, was over-optimistic.

While the 1964 conference had little immediate impact on probability teaching in SA, the 1965 conference would have important consequences.

1965: UNESCO CONFERENCE IN SYDNEY

On the afternoon of Wednesday, 13th January, a number of excursions and sporting events were organized. A tour of the northern beaches, an inspection of the Art Gallery and the Opera House, visits to surf beaches and bowling greens were provided as diversions. Again, members were free to choose the activity or excursion which appealed to them.²⁴⁷

At Radford’s urging, and under the auspices of the Australian National Advisory Committee for Education, the NSW Department of Education organised a conference in Sydney in January 1965 for about 100 people from Australia and overseas concerned with secondary mathematics education. Its purposes were to “compare modern developments in mathematics teaching in America and England with those in the Australian States and New Zealand”,²⁴⁸ with the special intention of examining both modern syllabuses and also their teaching. A representative cross-section of 13 South Australians concerned with secondary mathematics education attended, including Barnes, Berndt and Hamann.

* Kee (1975, p. 257). NSW. Department of Education (1967) lists Rawlinson’s summary of *Background in Mathematics*, but contains no probability.

²⁴⁴ Nisbett (1978, p. 122)

²⁴⁵ Brinkworth (1970, p. 39, text and footnote)

²⁴⁶ Keeves, pers. comm., Aug 1993

²⁴⁷ UNESCO (1965, p. 5)

²⁴⁸ UNESCO (1965, p. 1)

Begle and Thwaites were invited with UNESCO support because it was felt that the mathematics which they were encouraging was rich and varied.²⁴⁹ Their addresses provided a significant focus for the thinking of members.

Begle had been a member of the Cambridge conference, and discussed probability in the context of the SMSG, which had undertaken extensive trialing prior to publication of a course in probability for the Junior High School between 1959 and 1961.²⁵⁰ The Group had also developed material in typescript form for the primary years in response to the recommendations of the 1963 Cambridge Conference.²⁵¹ Begle claimed that probability and statistics were becoming of increasing importance and were being introduced into the later stages of high school courses in the USA. Begle was described as using “quiet, calm, overpowering logic”²⁵² but the published summary of his arguments is somewhat superficial:

Teaching methods are constantly changing and advantage might be taken of improved outlook and methods. There has developed recently, for example, wide appreciation of such topics as ... statistics and probability so these could be given more important places.

Thwaites, on the other hand, scarcely discussed probability, saying only that “Probability and Statistics would occur after binary operations, and then recur throughout the course.”²⁵³ SMP texts and even trial texts were already available in Australia.²⁵⁴ What Thwaites brought to the conference were more SMP texts, convincing oratory, “boyishness, exuberance and infectious enthusiasm”.²⁵⁵ This was to prove critical for South Australian education.

One participant reported that both speakers emphasised the importance of substantial co-operation between university mathematicians, educational experts and classroom teachers, and the need for a flexible approach to change based on controlled clinical testing. Textbooks, they said, should be prepared by teams not subject to the blandishments of a publisher, and it would require many years

²⁴⁹ Keeves, pers. comm., Aug 1993

²⁵⁰ SMSG (1959; 1961). Kempster’s (1982, p. 4) statement that SMSG’s first probability course was published in 1966 is inaccurate, but their major work does seem to have been published after 1965 (Swinson, 1978, p. 310).

²⁵¹ SMSG (1966a–f)

²⁵² UNESCO (1965, p. 63)

²⁵³ UNESCO (1965, p. 9)

²⁵⁴ Cowban (1964)

²⁵⁵ UNESCO (1965, p. 63)

work to develop a full programme.²⁵⁶ Another participant was Mr M.J.G. Hearly, an English Inspector of Schools. He had not been officially invited to attend,²⁵⁷ but since he was present, he was invited to speak and “stated the ‘official’ English viewpoint”.²⁵⁸ He argued that the potential freedom of the English structure was being exploited by many small experiments and several other larger experiments.

The reports suggest that all speakers had a limited breadth of vision, but they certainly provided a focus for participants’ thoughts about change. “There was general agreement that courses for Australian schools should not remain static but, at the same time, the premature introduction of topics that only allowed a very restricted immediate development should be guarded against.”²⁵⁹ Most states were changing secondary syllabuses, but very slowly, and any change was in junior secondary classes not constrained by the demands of external syllabuses. Such developments as there had been in SA were in line with the main stream of developments in Australia.²⁶⁰

There were calls for Federal and industry money to support change, trialing of new courses, teachers’ guides,²⁶¹ “interesting and provocative”²⁶² students’ texts, and in-service training. Some thought that new syllabuses could provide a tactful way of retraining many current teachers who were not really competent. There was some debate about whether governments or mathematical associations were more fitted to receive financial aid and to co-ordinate the development, implementation and maintenance of new syllabuses—a debate which would continue until the present, and see an increasing strength of the associations at the expense of the government.

Blakers later recalled that the main purpose of the conference was to inform members on overseas developments and claimed that “there was little attempt at critical evaluation”.²⁶³ We shall see in the next chapter that adequate critical evaluation probably was missing, but the teachers came with problems to which they were seeking solutions and with strong views about what they wanted in their classrooms. And they were conscious of the danger of making a foolish

²⁵⁶ “A ‘New’ Syllabus” (1965)

²⁵⁷ Keeves, pers. comm., Aug 1993

²⁵⁸ UNESCO (1965, p. 49)

²⁵⁹ UNESCO (1965, p. 10)

²⁶⁰ Sumner (1969)

²⁶¹ UNESCO (1965, p. 33)

²⁶² UNESCO (1965, p. 34)

²⁶³ Blakers (1976, p. 147)

decision.²⁶⁴ They also wanted classroom teachers to be the leaders both in conducting experiments²⁶⁵ and also in deciding on change for all mathematics teaching except matriculation examinations.²⁶⁶

... while some rigour is necessary for experimentation, there is much value in informal experimentation, even to the extent of collecting 'data free' opinion. In other words, if the general impression of teachers to syllabus change is favourable, then such a change should be acceptable.²⁶⁷

The teachers were starting to see themselves as more than technicians. There is little doubt that this arose in part because they were having to face the teaching of quite different types of classes from those in the past. Not only did they want power over syllabuses, they wanted syllabuses which would be suitable for successful presentation by "all teachers to all children".* They discussed the needs of the bottom three quartiles of the school population at length and felt that all should develop the "simplest notions of statistical influences [sic]".²⁶⁸ In the light of this concern it is not surprising that they expressed "general support for the inclusion of [probability and statistics] and felt that work could be given at all levels".²⁶⁹

In terms of the BSEM some important changes may be seen in the report of the UNESCO conference. The traditional Intellectual force of university mathematicians was seen as relevant only to a limited part of the school population. The Social changes discussed previously were starting to have an effect. The university's place was to be taken by the teachers, even though it was freely admitted that many teachers were not particularly good at their job, and many had a limited understanding of their subject matter. We shall see the consequences of this significant and lasting change being worked out over the succeeding thirty years. One consequence would be an increased importance being attached to pragmatic solutions. If teachers were to have more influence, Physical forces were likely to be more important, and with them would come a greater concern for a Technocratic curriculum than a Liberal-Humanist one.

²⁶⁴ Cowban (1964)

²⁶⁵ UNESCO (1965, p. 25)

²⁶⁶ UNESCO (1965, p. 32)

²⁶⁷ UNESCO (1965, p. 24)

* UNESCO (1965, p. 10). Arguments for the teaching of statistics circulating just before the Conference may be found in McMullen & Williams (c. 1963).

²⁶⁸ UNESCO (1965, p. 16);

²⁶⁹ UNESCO (1965, p. 21)

It is important to note that this was a particularly Australian solution. For example, in Scotland, Ruthven has seen the New Mathematics changes as occurring because “the growing distance between the content and approach of the school curriculum and that of the university generated pressure for change in the school curriculum”.²⁷⁰ He does not describe what had caused this movement, but emphasises that the reconciliation was effected from within a Social structure where the universities were still seen as the legitimisers of appropriate forms of learning.²⁷¹ This was becoming much less the case in Australia.

THE WAY FORWARD

For Australia, the 1965 conference may be seen as the last of the grand exploratory conferences. At these conferences many Intellectual ideas were put forward and discussed. Those States which had not already started to change their mathematics teaching would soon have to do so because of decimal currency. So the memes which we group together under the term “New Mathematics” would be tested and subjected to natural selection. We shall see that the changes would be quite different from those developing overseas and quite different too in the different states. It is the existence of so many differences in a country with a remarkably uniform set of educational structures and population that shows the value of having a multi-faceted model of curriculum development rather than a narrow model like the CEM. The relative strengths of universities, industry, independent and government schools were quite different from those in the northern hemisphere. There was, for example, little hint of the radical ideas proclaimed by those who eventually formed the British Association of Teachers of Mathematics in an endeavour to construct a totally new understanding of classroom mathematics.²⁷²

Some of the overseas approaches, particularly the more rigid pure mathematics ones, were better known in Australia than others, but most were available for those who wished to look, particularly at the ACER in Melbourne. It is important to emphasise the great variety of ideas which were circulating. Decimalisation was providing time which everyone knew would have to be filled. It was simultaneously both an Ultimate and a Proximate force: one which was well defined and known about. Rarely has the need for change been so well defined in an educational environment. A great variety of ideas came forward as candidates for filling the niche. The Variation we should expect in a physical environment

²⁷⁰ Ruthven (1980, p. 143)

²⁷¹ Ruthven (1980, p. 182)

²⁷² Cooper (1985, Ch. 8)

was also present here. It would have been quite impossible for any Education Department merely to copy an overseas project; at the very least a suitable project would have to be selected and justified. The ACER, the only national organisation involved, was proposing two possible solutions. From an Intellectual perspective this was arguably the best ones available at the time. But other forces were operating.

One force was simply that change had already started to happen. Harold Wyndham, a visionary and charismatic leader, had already introduced major structural changes in NSW in 1962. But the New Mathematics introduced there, like other aspects of the curriculum,²⁷³ was rigid and uninspiring.[§] The broad visions which have been discussed above made little impact. Most States, in particular SA, had neither the resources nor the political vision at that time to look wide: pragmatic solutions were being looked for. Physical and Social forces, as we shall see in the next chapter, would also be brought to bear on the construction of a new curriculum

On his return from Sydney, Barnes convened a meeting of SA delegates to the AMS Conference. They told the Public Examinations Board they believed the time had come for a New Mathematics course in SA.²⁷⁴ Soon after, as we shall see in the next chapter, a major trial was established, and full scale changes were implemented within a year.

Mere changes of subject matter are not sufficient. A poor curriculum well taught is better than a good curriculum badly taught. A good curriculum well taught is the only acceptable goal.²⁷⁵

²⁷³ Connell (1993, p. 89)

[§] Interestingly, in 1998 NSW was the only state in Australia not to have officially introduced probability into the primary curriculum, probably because of the influence of university mathematicians on the Minister of Education.

²⁷⁴ Sumner (1969)

²⁷⁵ Blakers (1970, p. 101), citing CEEB (1959b)

CHAPTER 12: ESTABLISHMENT (1965–1970)

Australian history is almost always picturesque; indeed, it is so curious and strange, that it is itself the chief novelty the country has to offer and so it pushes the other novelties into second and third place. It does not read like history, but like the most beautiful lies; and all of a fresh new sort, no mouldy old stale ones. It is full of surprises and adventures, and incongruities, and contradictions, and incredibilities, but they are all true, they all happened.¹

McQualter saw the period 1958–1967 as one of optimistic innovation involving both acts of faith and trial & error,² followed by a time of replanning where reality overcame rhetoric.³ Connell, agreeing to some extent with the CEM, saw the changes as “inspired largely by ideas and programs currently operating in England or the United States”⁴ but “with a somewhat stronger social emphasis”.⁵ Here we shall certainly see acts of faith and we have already seen that “inspired” is an appropriate term. But we shall also find subsequent developments to be influenced by more than Social forces, not least the demands of classroom reality.

To provide evidence for the multi-faceted BSEM, it is necessary to provide a substantial amount of small detail, which does tend to make this chapter, like its predecessor, rather long. But only in the small details may the limitations of other, superficially attractive, models be seen. Precision comes at a cost of length.

We look first at junior secondary schools, followed by upper secondary and primary schools, and conclude with some interstate comparisons and an examination of professional development practices. The foundations for probability teaching which were laid down at this time were to last almost unchanged for some 25 years and also constitute an early stage in a shift in curriculum power away from universities into the hands of teachers, a theme which will be seen to be of critical importance for probability in the final chapters of this Part. This shift of power was accompanied by a change of emphasis towards *who* is learning and away from *what* is learned. Topics like probability, which were new to a relatively under-trained teaching force were likely to fare badly in the transition, and this is just what we shall find.

1 Twain (1897, p. 59)

2 McQualter (1982, p. 2)

3 McQualter (1982, p. 4)

4 Connell (1993, p. 135)

5 Connell (1993, p. 144)

JUNIOR SECONDARY SCHOOLS

[The teachers] ... expressed their appreciation of the interest evoked and the knowledge gained from an enthusiast such as Mr Hamann (rather than from books and correspondence). All members appreciated Mr Hamann's mastery of his subject, his comprehensive coverage of the subject, his lucidity and fluency, and the notes and models displayed.⁶

Junior secondary schools first felt the effects of the 1965 Conference. Probability's place in these changes was small, but understanding how it entered the syllabus helps to show how its pedagogy developed.

Deciding to Use SMP

After the Sydney Conference, the Education Department, probably under the influence of Frank Close, an Inspector of Schools, former mathematics teacher and brother of Ron, still at ATC, wanted some schools to start using modern material at once.⁷ At a meeting of the Mathematics Syllabus Committee in early 1965

the South Australian representatives at the U.N.E.S.C.O. Seminar recommend[ed] that there be an alternative exam held in Intermediate Mathematics in 1967, the syllabus as set out in the School Mathematics Project Director's report. [8] In 1968 the examination to be alternative and 1969 to be the sole examination.⁹

These plans, predicated on existing examination structures, unrealistically tried to change a three year course less than three years before the relevant examination. The first trial did not occur until 1966, in preparation for what became the last Intermediate Examination in 1968. In the meantime SMP Book 1¹⁰ was officially "recommended for enrichment" for all Year 8 classes.¹¹ It was seen to be relevant to every-day needs,¹² lively, sound in content though perhaps a little too hard,¹³

⁶ *Education Gazette. South Australia* 1 Feb 1967, p. 19, reporting on an in-service conference on the new syllabuses at Kadina in November, 1966

⁷ J. Baxter (1972, p. 20). Baxter's comments are based on *Minutes of a Meeting of Sydney Seminar Delegates* written by Hamann. The current provenance of the document is not known, but may perhaps be still held by Hamann.

⁸ Thwaites (1972 [sic], pp. 103–106).

⁹ J. Baxter (1972, p. 21) from p. 3 of the Committee's minutes

¹⁰ SMP (1965c)

¹¹ *Education Gazette. South Australia* LXXXI (941): 200, 1 Jun 65

¹² Maynard (1966) [Maynard was an Inspector of High Schools in SA.]

¹³ J. Baxter (1972), based on *Minutes of a Meeting of Sydney Seminar Delegates*

but “there [was] no evidence to suggest that the new material, which is mathematically important and has stimulated able students, is not acceptable to those of average or below average ability”.¹⁴

It involved 40% of the 1966 Year 8 cohort, mainly upper streams of metropolitan high schools,¹⁵ which were easier to staff with experienced teachers.¹⁶ Most changes were sufficiently accepted to survive the changes to the examination system described below. Officially, this trial was sponsored by MASA,¹⁷ perhaps to provide political protection for the Department, but, given the small number of key players and MASA’s limited resources, the idea of having a trial public examination sponsored externally is strange, but probably unimportant.

Although SMP was chosen as the syllabus basis, there were three other sets of recommended texts—two English and one McMullen’s adaptation of UICSM.* SMP had replaced Euclidean geometry by transformation (motion) geometry which was concretely based and so more suitable for the early secondary years. This radical change to one-third of the course, ostensibly replacing logic and proof by intuition, experiment and algorithmic approaches, meant that most re-training resources addressed geometry,¹⁸ and much of the rest the infamous and unpopular “curly bracket” set notation[†]. SMP happened to contain both probability and statistics; it was promoted as already being studied overseas and interstate, and often “natural extensions of the new material in the primary school curriculum”.

Some of the new material has already been tried as enrichment of the present courses. ... [T]he new topics have stimulated interest of students and in some schools, of parents. ... [T]he language of sets, the use of co-ordinates, and the concept of bases of numeration have been successful innovations and one senior is a strong advocate of motion geometry.¹⁹

14 GRG 18/98

15 Sumner (1969) states 21 metropolitan and four country high schools.

16 Maynard (1966)

17 PEB (1966, p. 32)

* Mansfield & Thompson (various dates), SMP (various dates), *Contemporary School Mathematics*, McMullen & Williams (various dates). *Contemporary School Mathematics* was a sequence called the “St Dunstan’s College Booklets” which were co-ordinated by Geoffrey Matthews, later of the Nuffield Project. The project was less grand than SMP, but within the same *genre*. The booklet relevant to probability was Sherlock (1964).

18 E.g., *Education Gazette. South Australia* 1 Feb 1967, p. 19

† A Year 9 student, for example, would no longer be asked to “sketch the graph of $y = 2x + 3$,” but to “sketch $\{(x,y): y = 2x + 3\}$ ”, which would be spoken as “sketch the set of ordered pairs x comma y , such that y is equal to $2x$ plus 3 ”.

19 PEB (1967, pp.63-64)

That senior was Hamann, a member of Berndt's Enfield project described in Chapter 11, and now Special Senior Master at Enfield, and secretary of MASA.²⁰ He was "released"[§] in 1966 to a new position, recommended by Inspector Doug Anders after a tour of the USA and Canada,²¹ as a half-time Schools Consultant in Mathematics. This was an important step in the move for Intellectual leadership to be more firmly based in the classroom.[†] Hamann, "a strong and informed advocate of the new Mathematics",²² helped to draft the new syllabus and gave lectures on motion geometry to large numbers of practising teachers, supported by copies of SMP publications provided by the publisher.²³

The in-service time devoted to motion geometry and set notation meant that developing a pedagogy for probability received little or no attention. Perhaps the very simplicity of the initial probability syllabus disguised the need:

Specific knowledge of the sum and product laws will not be required but simple problems on the combination of probabilities are included.²⁴

But this local syllabus left out the critical word "intuitive" which was included in the SMP syllabus on which the SA syllabus was claimed to be based, viz.:

Simple probability; problems involving the intuitive application of the sum and product laws may be set, but general statements of the laws will not be required.²⁵

Just which cognitive processes were needed to solve the "simple problems" was not specified. Keeves had seen SMP as "exciting but appearing to require a great deal of direct teaching and classroom discussion and relatively little individual study".²⁶ Child-centred learning was then unusual in SA high schools and SMP's forward-looking intuitive approaches* were soon replaced by more formal ones:

A practical, experimental approach to a definition of an event A (e.g. as a subset of the set of N equally likely possible outcomes of an experi-

²⁰ Maynard (1966)

[§] This term has unfortunate connotations given its usage within the prison system.

²¹ GRG 18/98. ED 32/9/28. Memo from Anders to Acting Director of Education

[†] Many MASA secretaries from now on would also be Departmental Consultants.

²² GRG 18/98. ED 32/9/28. Memo: Superintendent of High Schools to Director of Education

²³ Miss Nancy McConnan, Georgian House, Vic, distributors for CUP, pers. comm., c. 1976

²⁴ PEB (1967, p. 64)

²⁵ Thwaites (1972, p. 106)

²⁶ Keeves (1965, p. 14)

* These antedated Fischbein's (1975) work on probabilistic intuition, discussed in ch. 8.

ment); the probability $P(A)$ of an event A (e.g. $P(A) = n(A)/N$; $0 \leq P(A) \leq 1$; the graphical representation of the rules for the combination of probabilities of

(a) independent events: $P(A \text{ and } B) = P(A \leftrightarrow B) = P(A) \cdot P(B)$

(b) mutually exclusive events: $P(A \text{ or } B) = P(A \approx B) = P(A) + P(B)$ ²⁷

The objections to symbolism so recently raised by Glastonbury were no longer valid! Perhaps it was hoped that the “experimental approach” would develop intuitions, but introducing the formulae must have indicated to teachers what was seen as of most importance. The syllabus did discuss both experimental and theoretical approaches, but made no links between them and, almost unbelievably, applied the practical work to events but not to probabilities of events. This dominance of a “pure”, set-theoretic approach would lead to later difficulties.

So, at least for probability, SMP formed only an operational launching pad from which SA could develop a course most suited to its needs—a development quite different from that of other Australian States where

[l]ittle formal work on probability is introduced, the most common tendency being to think of probabilities as being long run relative frequencies (following the graphical work). The only alternative work on probability deals exclusively with the “equally likely” case and direct enumeration.²⁸

What were the origins of SA’s decision to include probability in the syllabus, and to include it in a deterministic way? I have found no specific documentary evidence. We have seen in Chapter 11 that Australian teachers were supportive of teaching statistics in secondary schools and we shall see below that statistics had already been introduced into some SA courses. Since SMP included statistics as well as probability, this was an additional reason for choosing it for use in SA, and its work was supplemented by two specialised textbooks, both English, which were recommended for background reading.²⁹ Hamann argued that

[t]he new topics and approaches in the new syllabus can provide a stimulus to the critical examination of data, to doubt and query rather than to blind acceptance, and to the highly motivating influence of interest.³⁰

²⁷ PEB (1968, p. 67)

²⁸ Douglas (1970, p. 305)

²⁹ Loveday (1958); P. Moore (1958)

³⁰ GRG 18/98 *Mathematics News Bulletin* Nov 66, p. 1

But I have found no evidence that Hamann or his colleagues realised that such “critical examination” should rest partly on probabilistic thinking. No textbook which they consulted took this approach. Moore included chapters on probability,³¹ but they were poorly integrated with other chapters. In both the “background” books the two topics were largely seen as disjoint. Probability arrived more as an adjunct of statistics than as a topic valued in its own right.

This lack of value had two consequences. Unlike other states, SA had retained a “core plus enrichment” structure which maintained strong links between grades and stages.³² This, coupled with strong Departmental control and a recommendation that schools use a standard text,³³ must have hindered experimentation and the development of a satisfactory pedagogy. It also provided an enrichment penumbra where teachers could place topics difficult for the students, or for the teachers. It is possible, but unproven, that probability was soon seen as peripheral, that is certainly where it would often be placed in the years to come.

Initially there was a general view that the new mathematics course was, as a whole, satisfactory,³⁴ but without specific reasons being given. Baxter has suggested that there were problems with textbook supply and inadequately trained teachers, and that the experiment only continued because it had received Departmental sanction.³⁵ But there were sound educational reasons behind the changes and some appreciation very early of the need to change “classroom culture” and to develop in-service training.³⁶ These were balanced by some pitiful weaknesses: Glastonbury had argued that the course attempted to provide Power for the students—the power of generalisation, symbolism, application and knowledge, but his concept of “power” seems to have addressed merely knowledge of how to do algebra and control over the use of brackets.³⁷ As for probability, it was simply there, loosely attached to the highly valued statistics, but the reasons for its attachment seem to have been poorly understood by the local innovators.

31 P. Moore (1958, chs 8–10)

32 Kee (1975, pp. 248–249)

33 Sumner (1969), citing Report of Mathematics Committee of High Schools Advisory Curriculum Board 9 Nov 66

34 Sumner (1969), citing Mathematics Committee of High Schools Advisory Curriculum Board. *Advice on Mathematics Courses for 1968 and 1969*

35 J. Baxter (1972, p. 17)

36 E.g., M. Watson (1968)

37 Glastonbury (1966)

Let us consider an analogy. If a family's house is crumbling away, it has several options. It may stay on, contenting itself with piece-meal repairs. It may move to rented accommodation while major repairs or total rebuilding are done to the property. Or it may purchase a different house. In all cases it will try to ensure that the best features of the old home are retained in the new, and that the worst features of the old are substantially improved. Cost and convenience will be the major influences on the decision. We may see SA mathematics education in 1965 as having made a decisive step to move to a new, architect-designed home. Many features of this home were familiar; others were radically new approaches to old needs. What is of special interest is that some rooms, notably the probability room, were new solutions to very new needs. We must now ask whether this decision to purchase an architect-designed house was a Colonial Echo, as Clements has claimed and Connell has hinted at, or a more considered decision?

Was This Change a Colonial Echo?

In Chapter 11 we saw that many early overseas movements for teaching probability in schools had made little impact in SA. Here we have seen that probability arrived in SA schools more on the shoulders of statistics and as part of the New Mathematics than as a topic in its own right, but in a form which was very theoretical and rather divorced from useful statistical applications. We have seen in Chapter 9 that Clements has talked about

the naive willingness of Australian educators to accept untested English ideas ... [and] to commit schools to large-scale, but ill-fated, reconstructions of their ... secondary mathematics curricula, through ... 'new Maths' movements.³⁸

For SA at this time Clements' claim cannot be justified either in general, or for probability in particular. First, for completeness, it is necessary to deal with his claim, also mentioned in Chapter 9, that the Nuffield Mathematics Project was a significant influence on Australian schools. This Project, for ages 5–13, started in 1964³⁹ and did not produce its first widely available texts until 1967⁴⁰ and its first text on probability until 1969.⁴¹ So for SA, Nuffield simply came too late, and its remit was such that it would almost certainly not have appealed to people looking for a viable way of improving a Year 8–10 syllabus reasonably quickly. We

³⁸ Clements et al. (1989, p. 68)

³⁹ Mansfield (1966, p. v)

⁴⁰ Nuffield Mathematics Project (1967a, 1967b, 1967c)

⁴¹ Nuffield Mathematics Project (1969)

shall see in Chapter 13 that Nuffield also had little influence on SA primary probability syllabuses in the changes of the early 1970s. It would be some years before the visionary Nuffield ideas had any influence in Australia;⁴² by then the groundbreaking changes forced by decimalisation were well in place.

Within Clements' main CEM argument, his claim that in 1966 SMP was an untested text is quite untenable. The texts had been trialed for two years before publication. Even the curly brackets had been trialed and found to be usable. This extensive trialing was known about in Australia at least as early as 1964 and acknowledged as being substantially greater than in many Australian and American projects,⁴³ even those which had received substantial funding.⁴⁴ As we have seen in Chapter 11, such trial texts were available in Australia. SMP had received much criticism in England,^{*} some of which led to an alternative course for less able students being developed.[†] At the same time the Project developed much faster than had been planned because of pressure from many practising teachers to have access to the ideas as quickly as possible.[§]

Clements may well be correct in claiming that many Australian innovations were "ill-fated", certainly when compared with long-running projects like SMP,[∞] though whether this was because Australian educators were naïve is another matter. We can be sure that they knew quite a lot about what was happening in other parts of the world. For example, Alan Patching was a Lecturer in Mathematics at Wattle Park Teachers' College, Adelaide. His 1965 speech⁴⁵ to an in-service Conference on "Children's Thinking" shows just how much one person could find out at that time about what was happening in the mathematics education world without travelling outside SA. Patching stated that his knowledge was based on reading and correspondence with overseas mathematics educators, on the report of the ACER 1964 Conference, and on his personal experience with the work of Dienes and Golding at Cowandilla. His brief article shows an awareness

⁴² Blakers (1976, p. 154). Matthews, by then its leader, came to address the 1972 AAMT Conference in Brisbane; the influence of his contribution there is not known.

⁴³ Cowban (1964)

⁴⁴ Keeves (1965, p. 5)

^{*} For example, in 1965, when I was working at Edmonton County Grammar School, we made limited use of SMP and some other less radical New Mathematics texts. The balance of opinion at that time in that school was against SMP.

[†] The SMP A–H sequence starting with SMP (1968d)

[§] My unverified memory is that some of the in-service courses given by SMP at this time worked from trial texts because of such pressure from potential users of the material.

[∞] A.G. Howson (1987) commemorates the 25th anniversary of the Project.

⁴⁵ Patching (1967)

of six American projects and several European ones. While Patching's interest was in primary education, there is no reason to believe that those concerned with secondary education knew any less. I have already observed in Chapter 11 that I was able to use many overseas secondary publications located in Adelaide libraries as early as 1962, and they were often referred to in Close's lectures.

Of course it may well be true that the SA leaders did not appreciate all the implications of SMP's new ideas. We have seen in Chapter 11 that their concerns tended to be with content than with method. They had little *regular* contact with mathematics education groups in other parts of the world compared with their peers in NSW and Victoria,* so the practical implications of the intuitive British approach to learning⁴⁶ may not have been fully understood in SA. But they were not fully understood in Britain either, even by some of us who wrote the texts themselves. We have seen how probability appeared in the SMP syllabus almost by default. We shall see other limitations on authors' understanding in Chapter 23. A little naïve the South Australians may have been, but no more so than their contemporaries elsewhere, even those whom they were supposed to be echoing.

In 1965 change became essential. Cheap hand-held calculators were still tools of the future. Pounds, shillings and pence would soon go, but Imperial units of measure would remain. No drastic changes were needed in the arithmetic course. Algebra remained difficult, but was still seen by all as an integral part of mathematics. Euclidean geometry, however, was notoriously difficult, unpopular and poorly understood.[†] There were good reasons to find a course with more accessible geometry, and if statistics were also in the course, then even better still because it would support changes already decided upon. The SMP books so ably purveyed by Thwaites in Sydney were extremely attractive: well bound, well laid out, and with many good illustrations, some unashamedly amusing. All books had been trialed for two years in England. Their cost was acceptable, and would in any case be borne by the parents. Plans were in hand to produce a complete sequence of books to stretch beyond Year 12 through a publisher of high stand-

* In 1937 there were but three SA members of the British Mathematical Association; in 1961 only 1, and he from an independent school. In 1937 the Sydney Branch of The Mathematical Association had 21 members and 129 associates, while the Victorian Branch had 9 members and 16 associates (Archives of The Mathematical Association, Leicester, UK, Box 544/12).

⁴⁶ Schools Council Research Studies (1973, p. 4)

[†] My pre-service training and experience had in no way prepared me for the levels of incomprehension which the students demonstrated in my first years of teaching, especially in geometry. Since they, and I, were judged by their results in weekly tests which were designed also to assess the understanding of the top stream, I saw no option but to resort to rote processes which would ensure that at least a few marks for geometry would be earned by at least some of the poor unfortunate boys being sacrificed at the altar of uniformity.

ing. The Department would have to pay the cost of retraining, but part of this could be done with the Teachers' Guides which accompanied the course. Euclidean geometry was replaced with transformation geometry, an approach which was far more concrete and intuitive, and more likely to go down well in the classroom. The approach to arithmetic and algebra was different but not radically so, especially when compared with the new American ways of approaching algebra. Thirty years later one senior educator observed that SMP represented the peak of British text-book production⁴⁷ and another senior curriculum writer

described [SMP] as being 'way ahead of its time when it was written and still way ahead of its time. Its writers were perceptive and wrote mathematics books, not time fillers.' This view was endorsed by six other professionals at the same time.⁴⁸

On many counts SMP appeared to be the ideal new home for SA to purchase and occupy. A similar view was also held in other States.⁴⁹ The ideas were purchased from England, so "colonial" might seem to have some validity, but historical reasons meant that it was easier then to purchase British books than American ones. In any case, SA had looked much wider than just England, and had made a decision which could be defended as the best possible one in the circumstances. Indeed, the NZ decision to follow the fragmented, less familiar American approach has been seen as a reason for the limited success of the New Mathematics there.⁵⁰

Furthermore, the changes cannot be seen as a lower-class copying of practice in elite schooling systems, as Cooper has seen the spread of SMP in Britain:

This increasing emulation of selective schooling within the non-selective sector tended to ensure that the reform of selective school mathematics initiated by groups like SMP would eventually affect pupils outside of selective schools themselves—at least those taking O-levels and CSEs modelled on O-levels. This development might be seen as a part of the general tendency ... for the curriculum of elite groups to be imitated by lower status actors. Differentiated curricula in stratified societies, where the desire for ... upward mobility exist, might be seen ... as inherently likely to result in attempts at imitation from below.⁵¹

While South Australians at that time certainly looked with respect towards what was done in Britain, we have seen that the need for change arose from much

⁴⁷ Quadling (1996, p. 124)

⁴⁸ Author unknown, cited by K. Truran (1993)

⁴⁹ E.g., "A 'New' Syllabus" (1965); Cowban (1964)

⁵⁰ Openshaw (1992, p. 152)

⁵¹ Cooper (1985rd, p. 100)

deeper causes than a desire to emulate the practice of a far-off élite. The variety of approaches examined by school systems around Australia, together with the variety of solutions obtained, make it clear that educational reasons took much higher precedence than reasons based on cultural ties or on any feeling that standards could not be established and monitored within Australia.

Finally, Clements overlooks the fact that changes to the SA mathematics curriculum *antedated* SMP. In a response to the broader high school intake, pilot alternative courses for less able students from Year 9 on had been developed as early as 1963.⁵² Furthermore, statistics, including relative frequency but not probability, had been part of the Leaving Arithmetic examination, taken mainly by weaker students, since at least 1961⁵³ and was retained when “mathematics” replaced “arithmetic” in later years.⁵⁴ It followed some descriptive statistics in the Year 10 Arithmetic course. Two British textbooks were recommended—one containing only descriptive statistics,⁵⁵ the other with some probability as well.⁵⁶ The use of British texts shows that teaching statistics to commerce-bound students was not an radical innovation by SA, and the use of supplementary notes indicates that it was not a Colonial Echo either. Since the course seems to have been successful, we may deduce that by 1964 SA had had a positive experience with purchasing outside material, albeit with the provision of some supplementary material.

It is likely that these well-established changes formed a basis for the little known decision to include elementary statistics in the *academic, mainstream* Intermediate Mathematics 1 and 2 examination in 1966.⁵⁷ Two textbooks were recommended—this time one British and one from the USA⁵⁸—and again notes “for the guidance of teachers” were in time made available as supplementary help.⁵⁹ The decision to implement this radical change, one which would have affected all able mathematics students, could not have been taken later than the end of 1964, and was probably made earlier. In other words, South Australian leaders were quite willing to change if they saw good reason, and they did so. Unfortunately, this well-documented change has been historically overshadowed by the depth of the

52 J. Baxter (1972); *South Australia. Education Gazette* LXXX (926): 17, 1964

53 PEB (1961)

54 Gaffney (1969)

55 Loveday (1958); (PEB, 1966, p. 110)

56 P. Moore (1958); (PEB, 1966, p. 110)

57 PEB (1966)

58 Loveday (1958); Johnson & Glenn (1961); (PEB, 1966, pp. 25–28)

59 PEB (1968, p. 37)

post-1964 changes which were accelerated by decimalisation. Clements' claim of "Colonial Echo" is untenable; so is Baxter's claim that the SA academic course was "the last bastion of conservatism".[§] What actually happened fits much more closely with Horwood's claim that "... the evolution of mathematics education in Australia took place alongside, if not before, its emergence overseas."⁶⁰

So until 1964 SA mathematics education leaders had looked at changes round the world, had thought critically about them, but had seen nothing apart from statistics which impressed them sufficiently to justify significant change, even in spite of the increasing pressures on the school system. They did not feel that in modern mathematics they had found that "philosopher's stone" which McQualter has claimed drove the changes in NSW;⁶¹ nevertheless, they were prepared to make changes when change seemed appropriate. Of course, decimalisation forced the issue, making further delay educationally irresponsible and politically indefensible.⁶² But the way in which a solution was found cannot be described as either naïve or irresponsible, even though it proved not to be totally successful.

As we have seen, probability really came as part of the housing package, rather like a jacuzzi for which a prospective new home buyer may have no special desire. But it merged comfortably enough with statistics, had some potentially interesting applications, and was a topic that the South Australians were starting to think about, so it was not seen to be a problem. There is certainly an element of chance in probability's arrival in the South Australian syllabus, but "echo" is not really the right word—"compliance" maybe closer, or perhaps "opportunism". It is important to draw this distinction. Statistics was not a stranger to SA schools, probability's arrival on the shoulders of statistics and SMP was not the result of any decision, conscious or unconscious, to echo what was being done elsewhere, but a logical consequence of a prior commitment to teaching statistics to all students and a moderately good managerial decision that SMP was the best way of effecting necessary major changes quickly.

After such a detailed examination of the CEM, the simplicity of the argument for the value of the BSEM may come as an anti-climax. But it is this simplicity which is its strength. We have seen that decimalisation demanded change. Some small changes had already been effected satisfactorily, but both the Physical and Intel-

[§] J. Baxter (1972, p. 17). My antipathy to aspects of SA education at this time (mentioned in ch. 1) means that I would prefer to support Baxter here, but the data demand otherwise.

⁶⁰ Horwood (1992)

⁶¹ McQualter (1980ocs, p. 69)

⁶² Maynard (1966)

lectual resources were probably not adequate for effecting a major change. In 1965 there had been “a further falling off in [the] general quality” of teachers⁶³ and the resignation rate of younger male staff, many of whom had good potential, was worrying.⁶⁴ The purchase of SMP was seen as being the most cost-effective way of providing a high-quality mathematics course in the short time available, because it had “access to greater resources of finance, status, time and connections with which to publicize and legitimize its materials in the early phase of reform”.⁶⁵ SMP was the best way of achieving necessary and desirable change with the least amount of disruption.

As we shall see, SMP had important weaknesses. But its adoption in SA was a reasonable response to a number of forces to ensure that available resources were not stretched beyond capacity. While Intellectual, Social and Physical forces all influenced the decision, it was the Physical ones which had the greatest influence because they had to be satisfied for the change which decimalisation demanded. Connell’s “stronger social emphasis”, mentioned above, was indirect at this stage. Once the changes were in place, the Intellectual and Social forces became stronger, and the weaknesses of SMP became more apparent.

Local Adaptation

Although SMP had many positive attributes, there were some important negative ones which soon made essential some revision of the SA course. These revisions constitute the first stages in the internalisation of probability teaching in SA among teachers and constitute a critical part of our study of influential forces.

Although currency changes forced the revision, it was not known whether Britain would also change its currency, and even if it did, it was unlikely to choose the same units of currency. So using SMP committed schools to text-books using an obsolete and/or different currency. Curiously, this issue does not seem to have been seen as important ; it is not mentioned in the documents which I have consulted, even those written locally soon after the changes had been made.⁶⁶

Another problem was that SMP was written for the top 25% of students, but SA high schools were becoming more comprehensive. So by 1967 Hamann, together with Lang and E.K.H. Clapp, a senior teacher at Adelaide Boys’ High School,

⁶³ GRG 18/98. Annual Report High Schools Branch 1965

⁶⁴ GRG 18/98. Analysis of Resignations from High Schools 1965

⁶⁵ Cooper (1985rd, p. 100), discussed in more detail in Cooper (1985, Ch. 10)

⁶⁶ Sumner (1969); J. Baxter (1972)

produced four textbooks in the *Secondary Mathematics Series* which were designed for the new intermediate syllabus and published by Rigby. The two on Transformation Geometry are not relevant here; of the two on Algebra,⁶⁷ *Algebra Book 1* contained work on statistics and statistical graphs, while *Algebra Book 2* contained a unit on probability.⁶⁸ The authors claimed that

[i]n South Australia, recent changes to syllabuses and teaching methods have been inspired and influenced by the work of the School Mathematics Project and its dynamic director, Professor Bryan Thwaites⁶⁹

This claim may have been too strong. While adaptation of overseas texts was common in Australia,⁷⁰ for probability at least, there is little evidence that SMP's approach was a dominant influence on the SA authors, and indeed the acknowledgement of its influence is missing from *Algebra 2*. Different technical terms and notation were used, and the change to a more rigid syllabus form noted above was balanced by a more didactic pedagogic approach based on definitions, worked examples and exercises.⁷¹ Compound probabilities were approached using the concept of independence, which was what was done in the first SMP approach,⁷² and later discarded,⁷³ but the SA style was quite different, not using trees at all.⁷⁴ The sheer speed of the SA production suggests that it was not based on substantial amounts of original thought about content, but it does not seem to have been strongly dependent on SMP. We shall see below how the rapid production of textbooks in France also led to a very rigid, didactic approach. Of course, such an approach fitted closely the didactic climate of SA schools of the time, so there was little opposition to it, and it was, at least on the surface, an efficient use of time. As Keeves observed:

It may be true that extensive use of discovery methods will produce greater understanding and the greater power to apply a mathematical idea or relationship, but compromises must be made in the interests of efficiency. To require every child to discover everything he learns is futile simply because it will take too long.⁷⁵

⁶⁷ Clapp et al. (1966; 1967)

⁶⁸ Advertisement, inside front cover, *South Australian Science Teachers' Journal* Oct 1966

⁶⁹ Clapp et al. (1966, Foreword)

⁷⁰ Rosier (1980, pp. 13–14)

⁷¹ Clapp et al. (1967, pp. 55–64)

⁷² SMP (1965a, ch. 7)

⁷³ SMP (1967a, ch. 1)

⁷⁴ Clapp et al. (1967, pp. 62–64)

⁷⁵ Keeves (1965, p. 17)

“Efficiency” would soon become a real problem for classroom teachers who tried to use discovery methods⁷⁶ and must have been one of the reasons why teachers were not fully implementing even the limited vision of the *Secondary Mathematics Series* authors, who were very soon writing in their Forewords:

It is important to preserve the order both in relation to treatment of topics and of exercises in order to maximise the benefit from the book.⁷⁷

It is envisaged that each student will gain maximum benefit from attempting every question in the book. As the course is seen as a whole sequence of examples, the random selection of these is to be deplored.⁷⁸

Given, for example, that *Algebra 2* makes no links between its chapters on Venn Diagrams and probability, the limitations of such a dictatorial approach are obvious. But limitations like this, as well as the differences from SMP, show that the books were not clones, but a real attempt to provide a text which overcame the perceived weaknesses of SMP for SA while retaining some of its new content.

Even though the authors saw the book as a whole course, the teachers, apparently, saw it more as a set of exercises, much as earlier textbooks had been. They lacked the broad vision. Clapp observed:

There can be little doubt that traditional mathematics was almost too well taught and that examiners had to be very careful to ensure that they were examining the student and not his teacher. ...

A teacher of the new mathematics is required to have a better knowledge in his subjects than was considered adequate for the teaching of traditional mathematics. Teachers are required to read, to study, to enquire, and to evaluate if the new courses are to be successfully taught.⁷⁹

Similar responses were found elsewhere in Australia. For example:

Fundamentally teachers of the late 60's [in Tasmania] were not yet ready to recognise the potential of any guideline type of course. ... Overnight a clamour arose for a suitable source of material—a text book of examples to cover the course.”⁸⁰

So it is not surprising that the authors' call for flexibility sat uneasily with their prescriptive approach, which highlights the need for ensuring that pedagogical

⁷⁶ Connell (1993, p. 148)

⁷⁷ Clapp et al. (1967tg, Foreword)

⁷⁸ Clapp et al. (1966, Foreword)

⁷⁹ Clapp (1967)

⁸⁰ Milbourne (1983, p. 56)

skills are adequate for implementing any curriculum change. *Secondary Mathematics Series* may be seen as trying to construct a new pedagogy with detailed provision of exercises, but it did not encourage study, enquiry and evaluation in the way that SMP did. For probability, the pedagogy was didactic and deterministic and unsupported by visual aids. It was part of a pure mathematics style which would be introduced, as we shall see below, into both upper secondary and primary schools and prove to be long-lasting.

The SA books had little market outside of the State; other authors were developing texts closely married to their local syllabuses. A Victorian critic saw it as “a workable text without any outstanding features. ... Victorians are better served by their own textbooks including those from the same publisher.”⁸¹

Apple, the radical deconstructionist, has pointed to the critical role of textbooks in presenting external cultures to schools. He has seen the rise of the textbook, under the auspices of powerful publishers, as important for developing central power and as an antidote to the perceived incompetence of many teachers.⁸² Certainly the new SA books were seen as agents for the dissemination of ideas legitimised by central authority and also as a mechanism for in-service education. Could they also be seen as a mechanism for obtaining power, and, if so, was it power for the system or power for individuals within the system? To return to the housing analogy, there is no doubt that the initial decisions were equivalent to moving to a new architect-designed house, but in practice this house was used only as temporary accommodation and a source of inspiration for the local tradespeople rebuilding the old house. Given that these decisions were made by people who were both the local tradesmen and the local designers, the question must be asked as to whether the decision-makers of 1965 were really as responsible as has been argued above, or very shrewd in being able to see a way ahead which they could lead by controlling the text-book market? Was the writing of these books an exercise in obtaining power? Was there a conflict of interest which was abused, as was being suggested for the primary curriculum at the same time?

It was inevitable that SMP, as a ground-breaking project, would need some revision. We have already mentioned that it established a new *A–H* sequence for those not in the top 25% of the school population;⁸³ much later, these books were

⁸¹ Review of *SMS Algebra 1, Algebra 2, Transformation Geometry 2*; author unknown, published in *Vinculum* 4 (3): 8, 1967

⁸² Apple (1993, ch. 5)

⁸³ Starting with SMP (1968d)

adapted for Australian conditions⁸⁴ although they never captured the enthusiasm of teachers in Australia as they did in England. But the academic SMP course was also revised very early, partly because the writers came to see more implications of the ideas they were proposing,⁸⁵ partly to simplify the approach, and partly to be available to different types of school. However, what remained in these revisions was the blend of academic rigour, mathematical vision and exploratory pedagogy which made SMP innovative and distinctive. There is no evidence that the SA leaders had realised that some revision would be necessary, even though they had had some experience of modifying imported texts. The charitable interpretation is to say that the SMP introduction was quite openly described as a trial and the trial was judged not to be successful. I know of no evidence to suggest any other interpretation.

A more serious issue is that the SA changes did not capture the vision of the new courses and retained the old pedagogy, which was disastrous for a non-deterministic topic like probability. We need to understand why this vision was lost.

COMPARISON WITH CHANGES IN BIOLOGY TEACHING

Some possible answers may be found in the experience of the biologists in making radical changes to the year 12 Biology syllabus in SA at about the same time. These were based on the Biological Science Curriculum Study (BSCS) developed in the USA from 1959. The USA developers “preferred to develop courses around the inquiry processes and the conceptual structure of the biology discipline because these attributes of science are more stable than technological applications. It was felt too that the traditional science courses were largely oriented towards the learning of facts and lacked conceptual unity.”⁸⁶ The similarities with the New Mathematics and its emphasis on structure may be easily seen here. After trials in SA from 1962, the new syllabus written for Australian conditions by Australians⁸⁷ was introduced in 1966 but encountered some unexpected pedagogic problems.

[I]n the absence of any direct guidance the teachers in using the BSCS texts made two general mistakes. The first was that the new texts emphasised concepts rather than remembering facts. When faced with a chapter which in comparison with more conventional texts appeared to be very superficial it was difficult for the teacher to restrain from

⁸⁴ *Books Ap-Hp*, starting with SMP (1974). The “p” stood for “Pacific”.

⁸⁵ Griffiths & Howson (1974, p. 186)

⁸⁶ Hunwick (1970, p. 5)

⁸⁷ Morgan et al. (1966)

enlarging that section from his or her own vast store of knowledge of facts. The result was an overloaded course. The second type of mistake revolved around the teacher's status in the classroom. No longer was he in a position of authority as one who knows, but he now had to admit the existence of ambiguities as well as that he didn't know the answer to many questions. This position was highlighted in the running of discussion groups, Neither the teachers nor the students were prepared or experienced in this aspect of teaching and it was not long before many fell back on the tried and tested approach of "chalk and talk".⁸⁸

The BSCS had been tested both in the USA and Australia before its full adoption in SA. Its Year 12 teachers were probably competent and its students reasonably well suited to the course. If problems of pedagogy and culture occurred under these almost ideal circumstances, it is not surprising that they also occurred with the introduction of SMP to poorly staffed junior secondary grades, especially with the new type of thinking which probability required.

COMPARISON WITH CHANGES IN SCOTLAND

Comparison with changes in another traditional, autonomous system can also clarify the forces operating in SA. In 1964 the Scottish Mathematics Group completed a conservative syllabus revision, presenting old ideas in the language of set, relation and function, and including some probability. Classroom teachers had a strong influence on this curriculum, which Ruthven has argued contributed both to its conservatism and to its success.⁸⁹ Even so, the modern mathematics proved too difficult for many students, and a 1972 revision extended the statistics, more because it had been received in the classroom than from any external pressures.⁹⁰ The revisions were well-funded and led by educational specialists working in the all-purpose Scottish Office and so relatively removed from political pressure. Even though the changes were supported by teachers, subsequent revisions were all "[retreats] back to traditional notions of pedagogy and [had] greater emphasis [on] traditional content and approaches to the subject".⁹¹

In Scotland the universities had the major influence on the direction of change, but the Scottish Education Department decided when and if the change would take place, and the teachers controlled to a large degree the extent and speed of

⁸⁸ Hunwick (1970, pp. 24–25)

⁸⁹ Ruthven (1980, pp. 154–159)

⁹⁰ Ruthven (1980, pp. 183–184)

⁹¹ Ruthven (1980, p. 194)

change.⁹² We can see here that the Scottish changes were influenced by a balance of Intellectual, Social and Physical forces. We may also see the existence of a Pedagogical force which does not yet exist within the proposed BSEM. The changes were effected within a country some five times more populous than SA with a traditionally very high respect for learning, and within a system staffed entirely by graduates.⁹³ Its special position in the United Kingdom made it a little easier to find adequate funds to effect well-considered changes. There was no need to purchase a new house from outside: there were architects and designers a-plenty available to do an in-house job.

THE FORCES OPERATING IN SOUTH AUSTRALIA

The SA writers primarily addressed content—the interest of most local educational globe-trotters at that time, as we have seen. Broader issues like culture, the nature of schooling and changes to pedagogical practice were not addressed. Were the writers trying to usurp Intellectual leadership from the universities?

Because of the influence and standing of the authors, their work might be seen as a “legitimation” of knowledge, to use Apple’s term.⁹⁴ So if the teachers and administrators could be shown to have taken over the role of the university at this point, then the claim that they were seeking power would be a strong one. But this does not seem to have been the case. SMP was always seen as a mathematically respectable course—too mathematically so for many. The SA revisions of SMP were conservative, the approach to probability particularly so. This fitted the approach developed in the upper secondary schools which was a deterministic “pure” approach. Given that the most radical aspect of the course—motion geometry—did not attract any serious criticism from universities for about 15 years, it must have been acceptable to the academics. Hamann certainly consulted them in some detail before he prepared his texts.⁹⁵ In other words, the principle Intellectual force remained at the tertiary level. The textbook was a means of defining classroom practice: it was Pedagogic, rather Academic, leadership.

For many years for SA students had used text-books matching the local syllabuses very closely and written by authors who were members of the relevant Subject Committee and involved with setting and marking the public examin-

⁹² Ruthven (1980, pp. 197–201)

⁹³ March (1970)

⁹⁴ Apple (1993, chs 3, 4)

⁹⁵ P. Scott, pers. comm, c. 1997.

ions. Not surprisingly, such books[∞] tended to have a near monopoly of use in schools, especially government schools.[†] It is probably the case that these works were a significant supplement to their authors' incomes and an encouragement to write them. But I do not recall ever hearing comments which suggested that an author was "only in it for the money". In a small state like SA, any author who did not also have some serious educational purpose would be unlikely to be well received. In those cases known to me personally from that time,[§] the authors did not live in a style out of kilter with their non-writing colleagues in similar positions though they certainly acquired some local status.

In my view we may see the *Secondary Mathematics Series* as being within this *genre* of locally produced texts prepared by influential local teachers. While teachers were free to use any book they liked, imaginative teachers would find it much easier to modify a purpose-built text to their own requirements than to modify any other course.* The course probably acquired an authority, and perhaps power, which it did not merit, but this was less because the authors were seeking power and authority but because it provided the most efficient solution available at the time. The concept of "efficiency", mentioned above, is equivalent to the concept of Optimisation within ecology, and is a good way for explaining similarities and differences between different systems.

The problems faced in SA were different from those in Scotland and from those of the biologists in SA. The content problems were more like those in France where

[l]es premiers manuels furent bâclés en deux mois, avant la rentrée scolaire, ... : cette course de vitesse, pour conquérir un marché fut menée par des auteurs peu compétents: plus précisément, ils ne connaissent

[∞] *Vide* the list of works by Searle & Jones in ch. 11.

[†] My own Headmaster at St Peter's College had written a Latin textbook which of course we had to use. We moved from this to another book which was not the standard local work, and I recall being aware, even at the age of 13, that we had a hard time covering the syllabus because neither of the books meshed easily with the local syllabus.

[§] Albie Jones (Mathematics), Ray Smith (Physics), Jim Giles (Latin).

[§] Albie Jones (Mathematics), Ray Smith (Physics), Jim Giles (Latin).

* My experience at Melbourne Grammar School is relevant here. This school decided that SMP was the most appropriate course for Years 7–10, but realised that it required significant modification to prepare students for the external Year 12 examination, which used a quite different theoretical approach. This led to my being involved in writing the series of booklets used in Year 11, of which Truran (1973a, 1973b) were part, which were designed to assist the transition. These remained in use for nearly 20 years, long after I had left the school. However, parallel texts designed for Year 12 soon fell into disuse because suitable commercially produced texts were readily and cheaply available. It was only in effecting the transition that it was rational for the school to spend a large amounts of money in developing and maintaining unique material.

pas simultanément les mathématiques et la psychopédagogie indispensable. ... Il fallut attendre une dizaine d'années pour qu'on dispose d'un stock suffisant d'activités enrichissantes pour les élèves, traitant de questions qui *font problèmes* pour les enfants, et qui se résolvent naturellement dans le cadre ensembliste. ...

Piaget avait beaucoup insisté sur le fait que le développement de l'intelligence de l'enfant s'appuie sur la conquête des activités ensemblistes, indépendamment des formulations verbales. Au début, au contraire les activités proposées aux enfants se réduisent uniquement à une mémorisation de mots. ... La réforme avait été conçue dans ses aspects administratifs: on avait négligé les aspects didactiques.⁹⁶

The pedagogical problems were more like those in the USA, where

[a]ll groups developing new programs have recognized that most teachers, through no fault of their own, are not properly equipped to present these programs successfully. The extent of the supplementary education required depends upon the teacher's mathematical competence, teaching skill, and in some measure, the program selected.⁹⁷

The need for change in SA was primarily Physical—increasing school populations and decimalisation. The Social forces calling for more appropriate courses were muted by a shortage of Physical resources. But the changes effected were mainly driven by Intellectual forces, with inspiring ideas dampened in practice because Physical demands dictated classroom practice. It would be many years before significant Pedagogic changes came to SA, or indeed to other conservative countries like France, which showed a similar regression of technique in the face of change, and the USA, which realised early that only massive retraining of teachers would permit effective radical change. Only in places like Scotland, where the teachers had a strong input was change really successful. The BSCS experiences show that the problem was not poor *teachers* or poor content knowledge, but inadequate *pedagogy*. The distinction is important: teachers in general *reflect* a society's understanding of teaching (i.e., Pedagogy) rather than *construct*

⁹⁶ Glaeser (1978) p. 94

The first texts were rushed out in two months, before the new academic year : this haste to conquer a market was led by incompetent authors who did not know both mathematics and psychopedagogical principles. It was to be a dozen years before a sufficient collection of enriching activities had been developed which addressed problems which *were problems* for the children and which fitted naturally into the study of set theory. ... Piaget had insisted on the fact that the development of a child's intelligence rests on his conquering set ideas independently of describing them verbally. But at the beginning the activities suggested for the children required only the memorisation of words. The reforms were conceived as administrative ones; pedagogical principles were neglected.

⁹⁷ NCTM (1961, p. 75)

it. Construction is too time-consuming for most busy teachers. On top of all these difficulties, probability fared particularly badly because there was no existing body of Pedagogical practice, and the many other changes and the general school pressures meant that there was little time or incentive to develop one.

So at the end of this first round of changes in mathematics education in SA, Intellectual authority and leadership remained with the universities. But they were also undergoing significant changes, which we now need to summarise to assist our understanding of the upper secondary changes.

ADMINISTRATIVE CHANGES

The Queen had only one way of settling all difficulties, great or small. "Off with his head!" she said without even looking round.

"I'll fetch the executioner myself," said the king eagerly, and he hurried off.⁹⁸

After 1964 SA education moved from a simple, tightly focused structure dominated by one university to a more diverse one where practising teachers had more influence. But initially the university influence was still strong, which led to a rigid way of teaching probability which would remain dominant for many years.*

In 1964 there were three public examinations run by the Public Examinations Board of the University of Adelaide (PEB): the Intermediate after three years of secondary study at the end of Year 10, the Leaving after Year 11, and the Leaving Honours after Year 12. University admission was based on the Leaving examination, but students were encouraged also to sit Leaving Honours before going to University. This ambivalent situation existed because the Education Department could not staff enough country Leaving Honours classes and the university and politicians did not wish to appear to discriminate against country people.

The Board comprised about 25 members with approximately equal representation from the University of Adelaide, the Department and the independent schools, both Catholic and non-Catholic.[†] One representative each also came from the Institute of Technology and the Commercial schools and the Chairman from

⁹⁸ Carroll/Gardner (1960, pp. 114–115)

* Leff's (1968) description of changes in the medieval universities of Paris and Oxford has helped me to accept and understand changes to the system within which I had grown up, and which had been presented to me then as being immutable.

[†] PEB (1969a, pp. 10-11). But there was no place for independent co-educational schools, even though there were two successful co-educational Lutheran schools in Adelaide at the time.

the university. The Board appointed Chief Examiners⁹⁹ who were answerable to the Board¹⁰⁰ and who informally gathered others to assist them on the various Subject Committees. For mathematics there were three Subject Committees—Arithmetic, Mathematics I and Mathematics II—which were the legitimators of school knowledge and practice, the arbiters of Intellectual pressures for change.

This simple structure could not last. The burgeoning population led to the establishment in 1966 of The Flinders University of South Australia* on the southern outskirts of the Adelaide suburbs. It had its own matriculation requirements and a different philosophy about how to structure university courses.¹⁰¹ So it was inevitable that Public examinations would need to be restructured.

The PEB could not remain an organ of the University of Adelaide and by 1971 it comprised 16 members from the two universities, 16 members from schools (of whom 10 were from the Department) and two from the Institute of Technology. Similar rearrangement took place in other states for similar reasons.¹⁰² The structure of Subject Committees was formalised so that most contained 24 members, mainly practising teachers. For the first time representatives of Subject Associations like MASA, were entitled to one representative on each committee. Later, in 1977, although the committee size was reduced to 21 members, the number to come from the relevant Subject Association was increased to three.¹⁰³ From 1969 the Intermediate examination was abolished. The Year 11 examination was retained to provide a qualification for the many students leaving school at that stage. The Year 12 examination became the Matriculation standard, normally of five subjects.¹⁰⁴ Results were no longer classified as pass, credit or fail, but were now graded by letters from A to G, with D or above constituting a pass. This simple change reflected the needs of teachers concerned with their students as people and was an early step towards more sophisticated assessment procedures. Some of the consequences of these changes will be discussed in Chapter 16 and in Part D.

⁹⁹ PEB (1969a, p. 11)

¹⁰⁰ A. Jones (1978, p. 36)

* Usually known just as Flinders University, and not to be confused with the later University of South Australia, an amalgam principally of former primary teaching training institutions and the former Institute of Technology.

¹⁰¹ Hilliard (1991, pp. 21–34)

¹⁰² E.g. Horwood (1994, pp. 13–14)

¹⁰³ A. Jones (1978)

¹⁰⁴ Statutes of the U of Adelaide Chapter IX, allowed 9 Jan 1969, enforced from 1 Apr 1969

So while the influence of the universities was still substantial, there was now room for more than one university opinion, and increasing room for the views of teachers and Subject Associations. The Intellectual forces were diversifying and becoming closer to those of the 1965 Sydney Conference. There were no revolutionary changes, the leaders were still “old amateurs”, rather than “new professionals” (in the language of Chapter 8), but it was now *possible* for teachers’ experiences to be a critical influence on change. This was a move away from Howden & How’s Humanist parameter and a strengthening of their Social Reconstruction parameter. But we shall see in later chapters that this shift in influence did not lead to a greater understanding by Academics of school-based Pedagogy—there were shifts in position, but not of understanding. This would lead to the totally new set of problems described in Chapter 16, but at this stage, at least for probability, teachers’ knowledge of the topic was still limited, so it tended to be Academics, principally pure mathematicians who had the greatest influence, as we shall now see in our discussion of the changes in the upper secondary schools.

UPPER SECONDARY SCHOOLS

According to the syllabus the treatment of probability is to be an intuitive one based on the discussion of equally likely events. Whether the tool used in discussion is the Venn diagram, set theory or straightforward enumeration, it is axiomatic that the probability must lie between 0 (impossible) and 1 (certain). This axiom was not grasped by at least half the candidates and so any calculations that they performed were worthless. But once this axiom is firmly established, any problem can be solved by a rigorous search for all the possible events and all the events which comply with the given solution. Only practice will help in choosing the best method of conducting the search. The extension to the probability of a combination of independent events is quite simple and logical.¹⁰⁵

In this section we shall illustrate the type of work in probability which was introduced during the changes to the upper secondary syllabuses in the late 1960s. As the example above suggests, this was principally “pure” work, and our primary interest will be particularly in the nature of the questions asked, and the ways in which the new topic was received in schools. but our secondary interest will be in how those responsible for developing the course understood students’ thinking, and how this influenced the way in which the course was presented. The links between the junior and secondary syllabuses were not obvious at this early stage, but will be discussed more in Chapter 13. The material for Years 11 and 12 was

¹⁰⁵ McNally (1967)

very similar, so we shall first summarise the material for each year, and then move to a general discussion of the two years taken together.

Year 11 Academic

For many years academic mathematics students had studied Mathematics I and II in Year 11, courses with Euclidean geometry but no stochastics. Almost all mathematics students studied both of them concurrently,¹⁰⁶ and together they occupied one-third of a full workload. An Alternative Syllabus was developed and the changeover, as in junior secondary schools, was rapid. In 1969 about half the Year 11 candidates sat for the new course;¹⁰⁷ by 1970 almost all students were sitting the new course¹⁰⁸ which by then matched the only available Year 12 courses.

The new Year 11 Mathematics I contained algebra, trigonometry and co-ordinate geometry, and Mathematics II contained further algebra, probability & statistics, and transformation geometry & vectors.¹⁰⁹ This “further algebra” included permutations and combinations and their application to probability, including Pascal's triangle and the binomial theorem.¹¹⁰ The probability & statistics syllabus was:

Simple extensions of the sum and product laws of probability. Binomial probability distribution. Frequency graphs; mean; median, quartiles, percentiles. Measures of dispersion: range, interquartile range.¹¹¹

Even though the syllabus for the traditional course had contained a list of helpful books and some guidance on appropriate teaching methods, no such advice was provided for this new course.¹¹²

One example of the types of questions asked on probability will suffice:

A rifleman is able, on the average, to hit the bull's-eye 80% of the time when shooting from a certain distance.

Find the probability that he will score

- (a) two bull's eyes with his first two shots;
- (b) exactly five bull's-eyes with his first eight shots.¹¹³

¹⁰⁶ PEB (1969b, p. 92). There are several p. 92s in this volume: this is the last one.

¹⁰⁷ PEB (1970, p. 480)

¹⁰⁸ PEB (1970, p. 530)

¹⁰⁹ PEB (1968, pp. 40–41)

¹¹⁰ PEB (1968, p. 41)

¹¹¹ PEB (1968, p. 41)

¹¹² PEB (1968, p. 40)

¹¹³ PEB (1970, p. 112)

We can see the formal tertiary approach here, as also in the Year 12 questions below. The examiners found this question to be satisfactorily done with the main error being the omission of $\binom{8}{5}$ in part (b).¹¹⁴ But they suggest no reasons for this omission; it is almost as though they did not realise that there might be reasons.

Year 12 (Matriculation)

In 1970 the experimental Year 12 course which had been introduced in 1967¹¹⁶ became the only Year 12 course available, leaving the examiners with

the Herculean labor of merging the syllabus of traditional and 'modern' courses and of providing a single, fair and adequate examination.¹¹⁷

The 1967 trial syllabus contained new topics like groups, and some element of choice was permitted in the topics taught.¹¹⁸ The evidence that these changes were driven by Intellectual forces from the tertiary sector is very strong. Even Baxter, a strong supporter of leadership by teachers, has commented:

The task of merging 'old' and new' mathematics courses had generally been the province of the Universities where students were deluged with new terms, symbolism and concepts early in their courses, and generally found the change from traditional school mathematics rather bewildering. By beginning the changing process in the final year of school it was felt that a less startling and more closely guided transition could occur, and in fact, it was generally agreed that this did happen.¹¹⁹

Some evidence that the difficulties were greater than suggested here can be found in the examiner's report quoted at the head of this section. This report also emphasises the tertiary Intellectual influence, and further evidence will be found as we look at the syllabuses and questions asked.

The Matriculation examination had two mathematics subjects: Mathematics I and II. Probability was included in Mathematics I in the following way, but without any specific textbooks being recommended:

¹¹⁴ PEB (1970, p. 483)

¹¹⁵ PEB (1970, p. 483)

¹¹⁶ PEB (1966, pp. 192–193)

¹¹⁷ J. Baxter (1972, p. 26)

¹¹⁸ PEB (1966, pp. 192–193)

¹¹⁹ J. Baxter (1972, p. 26)

Elementary theory of choice and chance. The product rule for independent choices; arrangement of n distinct objects; the notations $\binom{n}{r}$, $n!$; the binomial theorem for a positive integral exponent. Probability based on an intuitive discussion of equally likely events.¹²⁰

In 1970 a further Matriculation mathematics subject was offered: Mathematics IS,[†] a subset of Mathematics I and II for students needing mathematics at university but not intending to do major studies in mathematics, and which contained similar work on probability.¹²¹ By this time some optional topics were available: both Matrices and Geometry & Vectors were included in Mathematics II, each with its own question in the examination.¹²²

In Mathematics IS the examiners set what they believed to be straightforward questions, but they doubted that more than half the candidates had shown "any real appreciation of the mathematics in the course and could be said to have a chance of coping with tertiary mathematics".¹²³ They saw the single question on "probability" as a question which distinguished the competent from the incompetent even though this question was principally on Venn diagrams and sets!

Suppose that the probability of a man living to the age of 60 years is $\frac{2}{7}$ [sic], that for his wife this probability is $\frac{3}{8}$, and for them both together it is $\frac{3}{28}$. Find the probability that

- (a) the man will die before he is 60 years of age,
- (b) at least one of them will live to be 60 years of age.¹²⁴

The comment on probability as good predictor of overall success would be made many times in the future. A similar remark had been made in the previous year about the combinatorics question.¹²⁵ But the reasons for this were not addressed. Indeed, the examiners' comments reveal a lack of understanding of what the students were thinking as they answered the question above:

¹²⁰ PEB (1968, p. 106)

[†] The derivation of this name is instructive. I had always thought that the "S" stood for "single subject", but apparently the name arose because the single subject terminal first year course at Adelaide University had been called "Mathematics IS" because it included some statistics (Pitman, pers. comm., Nov 96). This is an unexpected piece of evidence for the power which universities exerted over the public examinations.

¹²¹ PEB (1970, pp. 350–352)

¹²² PEB (1970, p. 359)

¹²³ PEB (1970, p. 484)

¹²⁴ PEB (1970, p. 351)

¹²⁵ PEB (1969a, p. 299)

Only good candidates successfully explained part (b). Most candidates just combined the 3 fractions in some way, hoping they were doing the right thing!¹²⁶

Comments on whether and how the candidates had used trees and/or complements would have been helpful, but they are notably missing. Interestingly, the use of tree was referred to in an article for SA mathematics teachers in 1968,¹²⁷ but, like many articles at that time in *South Australian Mathematics Teacher* from tertiary academics, addressed only content and not also pedagogic implications. A similar lack of understanding of the needs of students and teachers is apparent in the comment on the probability question in Mathematics I.* The question was

- (i) A committee of 4 is chosen from a class of 20 students by drawing 4 names at random from a hat containing the 20 names. Show that the probability that a particular student, X, is chosen is $\frac{1}{5}$.
- (ii) The procedure described in (i) is carried out N times.
 - (a) Find an expression for the probability that a particular student, X, is chosen at most twice.
 - (b) Find also how large N must be to ensure that the probability that X is never chosen is less than $\frac{1}{10}$.¹²⁸

The examiners commented:

Most students were out of their depth with this one. Very few of them realised that part (ii) (a) needs a binomial distribution. Even fewer appreciated the meaning of "at most twice".¹²⁹

As we saw in Chapter 4 sound language skills are essential for probabilistic understanding, but the examiners seemed only to appreciate students' language failures. They show little awareness of the need for syllabuses and examinations to address weaknesses. One examiner did observe that the probability question was "somewhat controversial!" and "did not give the majority of candidates sufficient opportunity to display an understanding of the principles of probability".¹³⁰ We have no record of what students thought of these questions, but their teachers did make public comments. D.F. Sawley argued that, particularly in

¹²⁶ PEB (1970, p. 484)

¹²⁷ Duncan (1968)

* The examiners for each paper were probably different. The practice of appointing the chief examiner for each subject from different tertiary institutions seems to have been in place from at least 1966 (*South Australian Mathematics Teacher* 1 (1), March 1967, no pagination).

¹²⁸ PEB (1970, p. 355)

¹²⁹ PEB (1970, p. 487)

¹³⁰ Tamlin (1968)

Mathematics I, the actual questions asked were quite different from the type questions sent to schools for guidance before the examination.¹³¹ It is unknown whether the poor answers were seen as being partly a problem of language, but Sawley's complaints suggest strongly that this was the case. At the December 1967 meeting of MASA the following motion was carried unanimously:

We deplore the fact that again this year too many difficult questions were set in the Matriculation mathematics papers. This can only suppress enthusiasm for the student to continue with mathematics, whatever the grading of his results. Further, both teachers and students are perplexed by the present state of affairs. The papers set appear to be right away from the spirit of the course.¹³²

At this period of change in upper secondary schools we can see several important features for our story of probability. It is clear that the changes in general were not without pain in the classroom. The required approach to probability required was a formal, pure one which reflected little of the reasons for its importance which had been argued in the preceding decade. The examiners seem not to have understood why the students had difficulty with the topic; perhaps this is why they suggested that marks on the probability questions might be positively correlated with marks on the total examination. As mentioned above, Academics did not seem to understand the nature of school-based Pedagogy. The confidence of the comment at the head of this section which saw an understanding of axioms as the only pre-requisite for examination success is reflected in the confidence of the minimalist comments from the examiners. But the *angst* expressed by the teachers makes it equally clear that such confidence was ill-founded. We shall look in detail at the *genre* of the Examiners' comments in Chapter 14, but for the moment we can say that Intellectual forces were dominating the syllabuses, but the teachers were making their feelings more strongly known.

Neither teachers or examiners seem to have appreciated that there is no algorithm available for solving traditional probability questions, unlike, for example, calculus, vectors, or much of transformation geometry, but like Euclidean geometry which had been removed from the syllabus because it was too hard. Teachers at that time sought for algorithms, but our understanding of content was not good enough for us to appreciate that our quest was fruitless. The Section quotations in Chapter 22 show one teacher's attempts to solve this alchemistic conundrum.

¹³¹ Sawley (1967)

¹³² *South Australian Mathematics Teacher* 2 (1), Mar 1968

As well as these pedagogical difficulties, or perhaps because of them, the relevance of the content tended to be overlooked, as we have already seen for junior secondary schools, and shall soon see in primary schools. This failure to come to a deep understanding of the importance of probability represents a failure of the Intellectual environment to influence the Social environment which thus reduced the potential support for probability which might have been expected after some years of successful teaching.

CHANGES IN PRIMARY SCHOOLS

These texts were adopted as part of the free book list for all primary schools, along with a second set of texts, the Pacemaker Series written by some other members of the Mathematics Subcommittee.^[133] Because the South Australian course was different from interstate courses in both content and style, these two sets of texts were the only ones which the Committee considered recommending for adoption and thus both had a wide distribution among primary schools.¹³⁴

The primary school changes may be described more briefly. Compared with those in secondary schools there was more Intellectual influence from the ACER Conference and psychologists than from universities, and much less Social influence. But this influence did not extend to probability. There was the same production of home-grown courses with wide influence throughout the state, this time accompanied by some suggestions that key authors in official positions were unduly motivated by personal financial considerations. This unedifying tale is of value in illustrating the limited range of forces operating on SA primary schools at the time and the concomitantly limited quality of the outcomes.

Changes were promulgated for primary schools in SA soon after the ACER Conference, but did not include probability.¹³⁵ Suitable support material, claimed to be based on Dienes' experimental work,¹³⁶ was not developed for trialing until 1966, with plans for full implementation by 1968.¹³⁷ Dienes had advocated sets, but not probability,¹³⁸ so it is not surprising that probability was not mentioned,

¹³³ Henderson et al. (1968–1970)

¹³⁴ Brinkworth (1970, pp. 25–26), on *Primary Mathematics Series* (Golding et al., 1969–1972)

¹³⁵ *Education Gazette. South Australia* Feb 1965; Jul 1965; Aug 1965

¹³⁶ Maynard (1966)

¹³⁷ GRS 1049/2. Primary Schools Advisory Curriculum Board Resolutions, 13 Apr 65

¹³⁸ Dienes (1964)

although statistics and graphs were.¹³⁹ The changes were made when most classroom experiment was discouraged,¹⁴⁰ a situation which lasted until after 1968.¹⁴¹ These changes would not have concerned us at all, were it not for the rather surprising way in which a small amount of probability crept, largely unannounced, into classroom practice near the end of the introduction phase of these changes.

The new courses were developed in an atmosphere of some tension. For reasons I have not been able to locate, Glastonbury had not initially been involved with the Primary Advisory Curriculum Board.¹⁴² Many others were involved in the writing of the new primary school material, but they were strongly divided between the conservatives and those who believed that “[t]he course should be for the benefit of the children, and not for timid, uninspiring teachers”.¹⁴³ What is missing from the historical record, however, is a clear indication of the nature of the arguments being proposed by each group, and also by the two groups who were working to produce commercial material to match the new course—*Primary Mathematics Series* and *Pace-maker Maths*. Both of these are discussed further below.

Further problems arose developing a policy for working with commercial publishers anxious to capture new markets. At that time Rigby Limited was an Adelaide-based publishing house with a significant share of the national educational market. In primary mathematics it published the widely used but uninspiring *Pathfinder* sequence by F.G.N. Cawte, a former primary school principal,* which had been scathingly reviewed by Dienes in 1962-63.¹⁴⁴ The Advisory Curriculum Board quite reasonably decided that all publishers which expressed interest in producing material be given a copy of the courses together with assignment material.¹⁴⁵ But some people believed that Rigby was paying teachers for ideas obtained as the result of other people’s work¹⁴⁶ and others were concerned that Golding, whose work would also be published by Rigby, was

¹³⁹ Maynard (1966). GRS 1049/2. Primary Schools Advisory Curriculum Board Resolutions, 13 Apr 65

¹⁴⁰ GRS 1049/2. Primary Schools Advisory Curriculum Board Resolutions, 24 Aug 64

¹⁴¹ GRG 18/142

¹⁴² GRS 1049/2. Primary Advisory Curriculum Board Minutes, 29 May 63

¹⁴³ GRS 1049/2. Primary Advisory Curriculum Board Minutes, 29 Jun 66.

* Cawte had been my own primary school principal, and of course we used his books,

¹⁴⁴ Keeves, pers. comm., Aug 1993

¹⁴⁵ GRS 1049/2. Primary Schools Advisory Curriculum Board Resolutions, 2 Aug 67

¹⁴⁶ GRS 1049/2. Primary Advisory Curriculum Board Minutes, 2 Aug 67

offering financial remuneration to teachers for producing card material.¹⁴⁷ Here, as with the secondary curriculum, where Rigby was also deeply involved, those in governmental positions of authority stood to make financial gain, and it is possible that this interfered with administration of the best possible professional procedures. There is not enough evidence to be sure, but we may note, for example, that the Advisory Curriculum Committee did not incorporate into its planning any procedures for assessing the quality of the new material.¹⁴⁸

In due course Golding and his collaborators produced *Primary Mathematics Series*, a complete course for Years 1 to 7 between 1969 and 1972 which was based mainly on cards bound into a book.¹⁴⁹ The quotation at the head of this section shows that the work received an official *imprimatur* which came close to being a license to print money. Golding's work contained no probability, except in Year 7, where it was treated as part of "Measuring, Graphing, Geometry". Here both theoretical and experimental probabilities formed the basis of a total of a mere eleven questions.¹⁵⁰ A similar short section also appeared in the final book of the rival *Pace-maker Maths* sequence,¹⁵¹ and this commonality suggests that some structural change had occurred in SA. This was the case. By the time the final books in these sequences were being prepared another review of the syllabus was in train, and some probability was being recommended for Year 7. This change will be more appropriately discussed in Chapter 13, but we need to note here that the presence of probability in the first course and publications which developed out of the 1964 Conference is a later interpolation which did not form part of the original plans of those developing the New Mathematics primary course.

This is just one indication that the 1964 Conference was less crucial for probability in primary schools than Keeves' suggestions for teaching the topic in *Background in Mathematics*¹⁵² might imply. For example, in John Izard's early and very successful¹⁵³ *Individual Mathematics Programme*, a card-based system written to convey to classrooms the spirit of the 1964 Conference, prepared at ACER and also published by Rigby,¹⁵⁴ there is no probability, which is quite surprising, given its

147 Brinkworth (1970, p. 26)

148 GRG 18/160

149 Golding et al. (1969–1972)

150 Golding et al. (1972, pp. 169, 179, 189)

151 Henderson et al. (1971, pp. 127–128)

152 ACER (1966)

153 Connell (1993, pp. 151–152)

154 Izard et al. (1965, 1970)

sponsorship by ACER. But Izard's memory, many years later, was that it had done so.¹⁵⁵ Equally surprisingly, the same lapse of memory was committed by Hughes, who also believed in later years¹⁵⁶ that the texts he had produced in the 1960s¹⁵⁷ had included probability. Furthermore, Tasmanian work produced in 1966 over which Hughes must have had some influence made no mention of the 1964 Conference and did not include probability, even though its course outline¹⁵⁸ is very similar to that published in the Conference Report¹⁵⁹ in 1964. This identical failure of memory from two senior and still active people who were deeply involved in the reforms of the 1960s is unlikely to be mere coincidence. Their strong association of the Conference with the implementation of probability teaching suggests that the Conference was an important influence in raising the profile of probability teaching for primary schools. However, their inaccurate recollection of the stage at which their own work actually included probability suggests, along with the SA evidence mentioned here, that the principal forces leading to its introduction into primary schools lie in the 1970s, not the 1960s.

We shall return to this matter in Chapter 13. Meanwhile we need to describe some interstate and overseas systems, especially secondary ones, in order to see the SA developments in relation to other contemporaneous thinking.

SOME COMPARISONS WITH OTHER SCHOOL SYSTEMS

This does not mean that you will find any 1965 syllabus that closely resembled SMSG, UICSM, or SMP. In fact overseas ideas were introduced into the Australian States with, in most cases, considerable caution and considerable concern for the fact that many of the new ideas would be strange to Australian teachers A substantial infusion of old-fashioned Australian conservatism has resulted in syllabuses and teaching approaches which deviate considerably less from earlier norms than do many of the overseas counterparts. This has made it relatively easy to accommodate to recent Australian criticisms, some of which have merely been echoes of overseas critics and not always relevant to the Australian situation.¹⁶⁰

If the Colonial Echo model really were valid, we should expect to find similar approaches to those in SA happening across Australia. In this section we shall see

¹⁵⁵ Izard, pers. comm., Dec 1993

¹⁵⁶ Hughes, pers. comm., May 1995

¹⁵⁷ Hughes & Wilson (1966–1968)

¹⁵⁸ Tasmania. Education Department. Curriculum Centre (1966, pp. 72–73)

¹⁵⁹ ACER (1964a)

¹⁶⁰ Blakers (1976, p. 17)

just how much this was not so, and see how the BSEM, especially its concept of Convergence, provides a more parsimonious explanation. As Hamann observed later, the 1965 Conference had little uniform effect “because of the states wanting to ‘do their own thing’, and consequently eventuated in no direct action”.¹⁶¹

Even in the 1960s Australian states were remarkably isolated from each other, Letters sent by post were the most effective means of communication. Jet planes first flew on domestic routes only in 1964,¹⁶² and although they were not prohibitively expensive, there were few cut-rate fares. Trains were cheap, but the links between states were tenuous.* The standardisation of main interstate rail lines did not begin until 1962.¹⁶³ It was only in 1966 that the biennial meetings of the Australian Association of Mathematics Teachers (AAMT) were inaugurated.¹⁶⁴ By about then the major Australian urban centres were connected with cheap, automatic long distance telephone calls, but the whole country was not so connected until 1985.¹⁶⁵ Even with all these improvements, the traditional independence of Australian States in educational matters was sustained by political structures which could and did mean that different Education Departments were led by men of quite different personalities working under governments of quite different political complexions. Divergence was more to be expected than conformity.

In 1970 J.B. Douglas from the University of New South Wales prepared a careful and detailed summary of Australian stochastics teaching. He wrote:

As courses of study stand at present, there is a rather uneasy compromise in the treatment of probability, between the strict limitation to finite sample spaces with equally weighted points (rarely, in this case, using this kind of language), and the introduction of theory, more in line with modern views of probability models. This flows partly from the conservatism of many (school and university) teachers of mathematics, and one of the consequences of the restricted approach is that courses containing it tend also to restrict their treatment of statistics to descriptive aspects. The notion of random variable is little more than implicitly included, and statistical inference of any kind is almost entirely omitted. Such courses as these are in the minority, and do not reflect the general tendency of course revision. In the more common

¹⁶¹ Letter from Hamann to J. Baxter, cited in J. Baxter (1972, p. 20)

¹⁶² *Australian Encyclopaedia* (1988, vol. 1, p. 335)

* My rail journey from Adelaide to Armidale NSW in 1962 required three overnight trains, but seats for the second and third journeys could only be obtained after each prior leg had been completed.

¹⁶³ *Australian Encyclopaedia* (1988, vol 7, p. 2452)

¹⁶⁴ Horwood (1992)

¹⁶⁵ *Australian Encyclopaedia* (1988, vol 8, p. 2823)

case, however, it is possible to see rather frequently a good deal of confusion in the treatment of conditional probability (independence), and the existence of numerous textbooks sometimes rather hurriedly written after the revision of syllabuses suggests that the confusion will be present for some years.¹⁶⁶

The following summary will illustrate this diversity and confusion. In particular we shall be able to see how the different relative strengths of the academics and the teachers produced different outcomes in different states. Our first example will demonstrate what happened when the academics were dominant.

New South Wales

There had already been a significant revision of senior mathematics syllabuses in NSW in the 1950s under pressure from industrialists who wanted Technocrats with extensive knowledge of calculus.¹⁶⁷ This revision was not paralleled in the less industrialised states, so it is not surprising that NSW's development in the 1960s differed from the rest of the country. Furthermore, because probability was included in the NSW Year 12 syllabus in the 1950s as a minor extension of work in combinatorics, without any calculus of probabilities at all,¹⁶⁸ we would expect different outcomes there from those in states where probability was a totally new school topic. There were actually three markedly different approaches developed.

Firstly, a new syllabus for Years 10 and 11 students not proceeding to tertiary studies was drawn up in 1960 for the Leaving Certificate from 1962.¹⁶⁹ "Probability and Statistical Regularity" formed part of the statistics section and is so different from most other courses discussed here that it needs to be quoted in detail.

[P]upils should realise that the idea of probability derives from a frequency distribution. They should learn that the statistical calculations carried out on samples may be used to provide information about the populations from which the samples were drawn, that this information is not exact, and that the relation between a population and a sample can only properly be expressed in terms of probability. Relative frequency calculated from a frequency table is a measure of probability, the reliability of the measure increasing with sample size. This notion leads to the concept of statistical regularity, which can easily be related

¹⁶⁶ Douglas (1970, p. 306). *Vide supra* for another extract from his summary.

¹⁶⁷ Abraham (1975, pp. 72–76)

¹⁶⁸ *Education Gazette. New South Wales* Jan 1956. Specimen Papers for Revised Mathematics I Syllabus for Leaving Certificate. I thank Lindsay Grimison for showing me this example.

¹⁶⁹ NSW. Department of Education (1960)

to common experience. ... It is *not* expected that any probability calculus ... should be developed.¹⁷⁰

This remarkable linking of probability and statistics by emphasising understanding rather than calculation is a fine compromise between precision and pedagogic realities. It rested on the English text by Loveday (1958), which has no probability calculus, but the attached notes¹⁷¹ makes it clear that the NSW designers knew what they wanted, and would not be constrained by what happened to be in a particular textbook, no matter how suitable it was seen to be in other ways. Seven statistics textbooks from Britain and the USA were listed for background reading and may be seen as the basis for the Syllabus Committee's thinking.

On the other hand, the upper secondary changes of 1965 resemble a standard introduction to tertiary statistics.¹⁷² T.G. Room and John Mack, two pure mathematicians at the University of Sydney, lectured teachers on sets to help them with their teaching¹⁷³ and show them how set theory could open doors to a wide range of mathematics. They presented binary and ternary algebra in some detail, including a brief discussion of the probabilistic calculus as part of ternary algebra. The work was formal and difficult; it matched the formal approach of the Year 12 syllabuses, but would have been accessible only to the most academic teachers.

Finally, a totally different approach was proposed for junior secondary schools by the Wyndham reforms, starting with Year 7 in 1962, and with detailed syllabuses for Years 8–12 drawn up soon after.¹⁷⁴ The main mathematics course¹⁷⁵ contained probability and statistics in a form based on CEEB (1957), with its emphasis on theory, experiment, and reconciling the two. References to American textbooks were provided for teachers and also extensive, rather dogmatic notes¹⁷⁶ to bridge the gap between the Year 12 CEEB course and the needs of teachers of Years 8–10.

What the notes proposed is clear enough: what actually happened less so. I have only been able to examine books by McMullen & Williams,¹⁷⁷ which, because of

¹⁷⁰ NSW. Department of Education (1960, pp. 15–17)

¹⁷¹ "An informal study of Sections 107 to 111 provides some of the probability background of the course." NSW. Department of Education (1960, pp. 15–17)

¹⁷² NSW. Department of Education for the Board of Senior School Studies (1965a, 1965b, 1965c); NSW. Board of Senior School Studies (1965)

¹⁷³ Room & Mack (1966, pp. v–vi)

¹⁷⁴ Wallent (1964)

¹⁷⁵ NSW. Department of Education for the Secondary Schools Board (1962)

¹⁷⁶ NSW. Department of Education for the Secondary Schools Board (1963a, 1963b)

¹⁷⁷ McMullen & Williams (various dates)

McMullen's early involvement in the New Mathematics, might be seen to have had particular authority. Probability was introduced first into the Form 4 (Year 11) Advanced Course using a set terminology. The authors claimed to be using an intuitive approach, and did use tree diagrams, but the approach is formal and axiomatic with experimental probability left until the end of the chapter.¹⁷⁸ Both authors, in atypical career moves, went from Sydney Teachers' College to senior tertiary mathematics positions and their work on probability for schools reflects the tertiary pure mathematics style in just the same way that we have met in SA.

So in NSW three quite different approaches to stochastics teaching were introduced into different courses at about the same time. This cannot have been easy for teachers who were already struggling to cope with all the changes introduced by Wyndham. As one commentator observed:

For those who remember the early years of the "Wyndham Math", it would be difficult to forget the general confusion that surrounded the teaching of the subject when many teachers, like Christopher Columbus, started out not knowing where they were going and upon arrival did not know where they were and on returning did not know where they had been.¹⁷⁹

Connell has described the Wyndham Report as "unexciting", echoing ideas which had been around for decades.¹⁸⁰ Nevertheless the diversity described here makes it difficult to sustain any claim that the teaching of stochastics itself was either unexciting or a mere echo of ideas which had been around for decades.

Australian Capital Territory

Our second example comes from the ACT where in the 1960s students sat for NSW examinations. In 1966 the Canberra Mathematical Association released a small roneoed work entitled *Probability* written by the popular and influential Hanna Neumann as one of a sequence of booklets to help teachers teach the new topics.¹⁸¹ It echoed the very formal approach of Room and Mack, but it was more tightly focussed on probability as a subject in its own right. It sold better than any of the other booklets in the Canberra sequence, was used for first-year teaching at Latrobe University, and has been seen by her nephew as the best available at the

¹⁷⁸ McMullen & Williams (1965, pp. 246–248)

¹⁷⁹ Abraham (1975, p. 86)

¹⁸⁰ Connell (1993, p. 89)

¹⁸¹ Neumann (1966)

time.¹⁸² An advertisement claimed that “a number of people had found that [the approach of starting with arbitrary probability measure rather than the relative frequency measure] is readily accepted by pupils,¹⁸³ but in my view its approach was far too academic for ordinary students and teachers.

Victoria

In contrast with the dominance of pure mathematicians in NSW and the ACT the Victorian development was dominated by teachers. The Mathematical Association of Victoria (MAV), working in co-operation with the Education Department, gained significant control over the curriculum.¹⁸⁴ This was not easily won, but there is only space here to sketch some of the most relevant issues.

A formal study of probability and combinatorics was included in Victorian Year 12 courses as early as at least 1960, and was seen as one of the harder parts of the course.¹⁸⁵ Probability was retained in the 1965 alternative Year 12 syllabuses for implementation by 1967¹⁸⁶ but not, surprisingly, in the Victorian Universities Schools and Examination Board (VUSEB) Course of Study for Years 7–10.¹⁸⁷

By 1965 the MAV had established the School Mathematics Research Foundation (SMRF) which was based at the newer and generally more radical Monash University and which produced textbooks to help teachers implement the changed philosophy underlying the New Mathematics. It was felt that having an independent foundation would clear the MAV of any charge that it was trying to promote a monopoly textbook.¹⁸⁸ It is a reflection of the power of the teachers in Victoria that it was the MAV which was seen as needing to be neutral, not, as in SA, the government advisory bodies. SMRF started with Years 11 and 12, rather than with Year 7, and used UICSM as its basis, deliberately rejected the Royau-mont approach, and started too early to be significantly influenced by SMP.¹⁸⁹

182 Newman & Wall (1974, p. 76)

183 Advertisement by Canberra Mathematical Association in *South Australian Mathematics Teacher* 2 (1), Jul 1967

184 Horwood (1994)

185 Dr Brian Sherman, University of Adelaide, pers. comm., Nov 1993, reflecting on his own study of the topic in year 12

186 *Vinculum* 2 (1): 12, 1965

187 Byrt (1973)

188 Watterson, pers. comm, 15 Nov 1993

189 Cribb (1986)

The principal SMRF stochastics text was *Probability and Statistics*,¹⁹⁰ written by two Monash academics—Gordon Preston, a pure mathematician, and Geoff Watterson, a statistician—but the book was not published until 1972, when Preston was President of the SMRF. Watterson had been critical of an earlier text by Clements, who had no formal training in stochastics, but substantial experience and expertise in classroom teaching,¹⁹¹ and this may have been an attempt to formalise his point of view. The book is discussed in my Masters thesis;¹⁹² it was found to be so formal and unsuitable that one leading teacher took up textbook writing.* It was not a suitable marriage between mathematical precision and pedagogical expertise and came too late to be of value for teachers of the new syllabuses. The delay suggests some of the tensions present at the time.

More effective solutions came from within the schools.† One led to strong controversy. C.W. Lucas and Roy James from Wesley College prepared a trial Year 12 text, which was published by Nelson in 1966.§ Some Victorians believed it contained many important errors of fact (in one case it was claimed that there were 115 errors).# In SA, A.K. Duncan, Institute of Technology, damned the probability chapter because its definitions were inaccurate, quoting as an example "if A and B are independent events of non-zero probability then $A \leftrightarrow B = \emptyset$ ".¹⁹³ I have been told that VUSEB refused to arbitrate on whether there were errors of fact for fear of showing bias, that a potential legal action forced the publishers to withdraw the book, that Lucas & James published the book themselves, and that the book was not a success.∞ I can only observe that by 1970 my own school happily used two books by Lucas & James and published by Nelson¹⁹⁴ and that its traditional

190 Preston & Watterson (1972)

191 Clements, pers. comm., July 1993

192 J. Truran (1992, pp. 11, 103, 110–112)

* For obvious reasons I cannot reveal the source of this comment, which in any case is not chronologically consistent with my informant's publication list!

† One not discussed here was written by a group led by Bernie Fitzpatrick, Xavier College.

§ It has not been possible to identify which book or which edition. The bibliographic record of publications by Lucas & James at this period has been difficult to trace, even in the National Library of Australia, and I did not keep those of their books which I taught from.

Vide ∞ infra.

193 Duncan (1967lj)

∞ I have deliberately chosen not to reveal the source of this information. I have not been able to verify all of the claims, but simply because such claims were made is good evidence that in Victoria there were real difficulties establishing a course which was mathematically accurate and pedagogically sound.

194 Lucas & James (1968am, 1968pm) and subsequent editions.

style made it more popular with many schools, some of whom preferred it to the more mathematically correct SMRF text.¹⁹⁵

As one of those involved at the time has put it to me, the teachers were more concerned with style than with content. This had the double effect of removing pedantry but retaining formalism. The textbooks which came to be accepted softened the approaches of the academics, but they did not oppose them. Not only did the academics have a strong influence on teachers' thinking,¹⁹⁶ but they also remained influential on subject committees. And probability and statistics continued to be difficult for Victorian students, just as they would for students in SA.¹⁹⁷ I shall examine some of the reasons for these difficulties in Part E.

Finally, we may note two other paradoxical features of the Victorian situation. Firstly, for stochastics it was the older Melbourne University which was the more radical in its teaching approach. A former Englishman, G.H. Jowett, used a practical approach to statistics teaching during his five years there, and also took a significant interest in mathematics education in schools.¹⁹⁸ His innovative approaches, however, had little influence on the debate or on school practice. Secondly, we must note that one of those closely involved in the debates and in the work of the SMRF was Ken Clements. As I see the evidence, the Victorian development is a very good example of some Australians' trying to find a peculiarly Australian solution to new challenges, and Clements' contribution to this debate was an important and very positive feature of the developmental process. This makes his espousment of the CEM particularly difficult to understand.

Western Australia

In the two systems we have examined so far we have seen quite different solutions arising from the different relative strengths of those involved. The Western Australian experience is particularly interesting because it arose under much more evenly balanced leadership, but still encountered important difficulties.

Hume had developed a statistics course for Year 12 in WA by 1966.¹⁹⁹ She had the support of Nathan (Norm) Hoffman, at that time in the Research and Curriculum Branch, and later Superintendent of Mathematics for the Education Department

¹⁹⁵ Cribb (1986)

¹⁹⁶ E.g., Finch (1970); Watterson (1971)

¹⁹⁷ Powers (1972)

¹⁹⁸ Watterson, pers. comm., 15 Nov 1973; Petersen (1971)

¹⁹⁹ Hume (1966)

of WA, and also of Larry Blakers at the University of WA. Hoffman was a leading and early Australian advocate of stochastics teaching whose early work provided a basis for the work of Kempster discussed in Chapter 8 and Part D.²⁰⁰ It was Blakers who suggested that Hume might modify the CEEB course²⁰¹ to make it suitable for WA. It turned out that the modifications required were far more than had been anticipated.²⁰²

The book contained many experiments for students to do, even though its approach was a pure one and the style somewhat didactic. It had a leading educational publisher and by the standards of the time was attractively presented. In my view it is a good text for any Year 12 course going as far as the binomial and normal distributions, and also presents the material in a helpful way for a teacher with limited background in stochastics. I would have valued it as I struggled to learn stochastics and its teaching at that time on the other side of the world.

But Hume later judged the book to be not as successful as she had hoped. It did not prove practicable to take the course as far as simple statistical inference which she would have liked, and she believed that any improvements would rest primarily on ensuring a higher level of teacher knowledge.²⁰³

This early WA experience is particularly instructive. Unlike the situation in other States, the University and the Education Department were both moving together, led by competent and powerful men. A publicly examined syllabus was in place for which a good text had been prepared, a text sympathetic to the local environment. From this distance the balance between academia, government, and didactics seems to have been about right. But the project foundered on the ability of the teachers, and we may reasonably deduce that the experimental approach required more flexibility and background knowledge than the teachers felt comfortable with. This will be a key point in our discussion of the meaning of the variability which we have met here. But first it will be useful to look at an historical analysis of some New Zealand experiences at the same period of time.

200 Kempster (1982)

201 CEEB (1959a)

202 Hume (1970)

203 Hume (1970)

New Zealand

In NZ, Roger Openshaw has examined a number of curriculum developments from an historical perspective. Although he did not specifically examine the teaching of probability, three of his findings are relevant here.

In looking at the introduction of the New Mathematics²⁰⁴ Openshaw has shown how the NZ leadership came from several highly skilled mathematics teachers living in Christchurch at the same time. While NZ society was changing in ways which were supportive of more mathematics teaching in schools, its educational bureaucrats were as traditionally conservative as those in SA at the same time. But the Christchurch group had a leadership advantage because of

their monopolisation of expertise both as standard-bearers for the new approaches and as successful practitioners within an increasingly high-status academic discipline. ... In the face of Departmental Officers who often possessed only a limited background in traditional mathematics, the innovators commanded considerable respect.²⁰⁵

Although the potential reformers in SA had the advantage of living in a City-State with closer and easier access to administrators than the South Island New Zealanders, they simply did not have the New Zealanders' charismatic and authoritative advantage. The NZ teachers were clearly people of very high ability; in SA the leaders were competent but pedestrian.

Openshaw has also compared two developmental projects of this time—mathematics and social studies—and explained why one succeeded and one failed, even though both were equally well grounded.²⁰⁶ As well as the reasons cited above, he attributed the success of mathematics partly to the influence of academic mathematicians, partly to the perceived importance of mathematics for attaining national prosperity and partly to its dominance by male teachers with different and better defined career paths from those in social studies.

Finally, with respect to mathematics teaching, Openshaw has commented on “the tendency of New Zealand teachers to pragmatically borrow from a variety of sources whilst disregarding the underlying theoretical variations”.²⁰⁷ This, of

²⁰⁴ Openshaw (1992)

²⁰⁵ Openshaw (1992, p. 146)

²⁰⁶ Openshaw (1992); Openshaw et al. (1993)

²⁰⁷ Openshaw et al. (1993, p. 182)

course, parallels the “what works well in the classroom” criterion which we have already met among Australian teachers.

Openshaw’s observation about teachers’ pragmatic approach to classroom practice emphasises the practical limitations of Intellectual forces. His emphasis on the Charismatic influence of the Christchurch group adds a new dimension to our analysis of forces, which will be taken up in Part E.

Summary

Although these summaries of developments in other systems have been brief, they make it clear that the outcomes have been diverse and we shall meet a similar diversity in Chapter 16. Such diversity is a strong argument against the CEM, and the point does not need to be laboured further.

A striking feature of these summaries is the absence of Physical and Social forces on the new syllabuses. People, both from universities and schools, did provide leadership and develop texts, albeit under some pressure, so Physical limitations were not critical. Similarly, Society as a whole was not strongly for or against the developments, apart from some support for mathematics as an aid to increasing prosperity. But we have seen another force not included in the BSEM: a *Pedagogical force*. We have already met this in our discussion of Scottish developments, and in Victoria we have found practising teachers taking an active role in opposing some of the Intellectual forces, at least in so far as they affected classroom practice. But in general these Pedagogical forces were not strong enough to overcome the very formal approaches advocated by the academics, except in NZ where some teachers had a strong Charismatic influence. The well-planned WA course which might have been seen as an Intellectual and Pedagogical force on classroom practice foundered to some extent because the Intellectual force was not strong enough to influence teachers’ understanding of their subject-matter.

But even more interesting is that in spite of the various approaches and philosophies which drove the changes, including even the teacher-dominated ones in Victoria, the approach to secondary school probability in most parts of Australia ended up being remarkably uniform—a definitional, pure mathematics approach in which, after a short practical introduction, a simple calculus of probabilities formed the bulk of the course. We shall discuss this more in Chapter 14, but at this point it is sufficient to note that this is a classic case of Convergence and we shall meet another form of Convergence in Chapter 16. Here it may be useful simply to provide an small illustration of just how effortlessly Convergence can happen. Melbourne Grammar in 1969 was using its own internally written

textbook whose approach to probability was formal, abstract, and unpopular with staff and students. I had had experience when working with SMP with a more concrete approach using trees, and it was very easy to persuade my colleagues to make changes. Given two approaches, both of which are academically acceptable, it is natural to choose the one which goes down better in the classroom. We may reasonably deduce that similar choices led to a deterministic approach to probability becoming dominant across Australia because it fitted best with the Social and Physical forces in the classrooms of the time without breaching Intellectual rigour.

Pedagogical forces will be specially examined in Part E, where we shall also define a *Charismatic force* as part of the BSEM. But here we shall consider the relationship between in-service education and practical Pedagogy.

PREPARING TEACHERS TO TEACH PROBABILITY

However good a curriculum may look on paper, it is worthless if, by the time it is translated into real live lessons, its whole spirit is lost, and the pupils are turned into unthinking automatons. Thus, it must always be kept in mind that teaching may cause it to degenerate; its message may be garbled.²⁰⁸

We have already seen that many schools had poorly qualified staff. In primary schools, about 30% of teachers in 1973 did not “feel competent to teach all aspects of the mathematics course at present level or grade”.²⁰⁹ Gender differences were marked: males held four times as many academic qualifications than females in primary schools, but comprised only 32% of the teachers. The disparity was less in secondary schools, but still substantial.[†] Given these problems, what arrangements were made to assist the development of a sound pedagogy for probability?

Universities

Prospective secondary teachers were mainly bonded to the Education Department. Although their academic subjects were usually done at university, they were subject to tight control over their daily movements, and subjects studied. In 1967 probability was not taught to first-year students at Flinders,²¹⁰ the Institute

²⁰⁸ Griffiths & Howson (1974, p. 204)

²⁰⁹ EDSA(1974a, p. 49)

[†] Karmel (1971, pp. 108, 112). Table 6.33 (p. 112) on qualifications refers to 1968 and may contain unspecified double-counting.

²¹⁰ Abrahamson (1967)

of Technology,²¹¹ or in Adelaide's academic first year course, "Mathematics I".²¹² A new first year course containing both probability and statistics—"Mathematics IS"—was available for students at Adelaide who did not wish to proceed with mathematics beyond first year, and Adelaide's second year terminal course, "Mathematics II", also contained statistics.²¹³ Only in 1969 was probability ephemerally introduced into Adelaide's expanded Mathematics I as part of "some additional elementary topics",²¹⁴ but without any recommended textbook.²¹⁵ The first examination included the following probability question:

- 6 (i) What is meant by saying that two events are statistically independent? Give an example of two such events.
- 6 (ii) From a hand of 3 red cards and 2 black cards, two cards are drawn at random, without replacement. Let E be the event that at least one black card is drawn and F the event that at least one red card is drawn. Find the conditional probability $p(E/F)$.²¹⁶

Similar questions were set in 1970 and 1971.²¹⁷ Those who were taught some tertiary probability met a very formal topic, with obvious similarity to what was being required in Years 11 and 12. To make matters worse, from 1971 at the University of Adelaide, potential statisticians were advised to study Mathematics I and a statistics course called Mathematics IH in their first year,²¹⁸ so from 1972 probability was no longer included in the Mathematics I curriculum.²¹⁹ Potential teachers would not have been allowed by the Department to study Mathematics IH, or the full statistics major which had been available at Adelaide from at least 1966,²²⁰ because statistics would have no applicability to the classroom. So for most of the introductory period many, probably most, potential secondary mathematics teachers did not study any university statistics, and hence did not encounter formal probability as part of their academic training. They would have had only the model of school textbooks and examinations to guide them.

211 Duncan (1967)

212 E. Barnes (1967)

213 U of Adelaide (1967cal, pp. 978–979)

214 U of Adelaide (1969cal, pp. 976–977)

215 U of Adelaide (1969cal)

216 U of Adelaide (1969) Examination for Mathematics I SMO1 First Paper

217 U of Adelaide (1970, 1971)

218 U of Adelaide (1971cal, vol. II, pp. 873, 881)

219 U of Adelaide (1972cal)

220 U of Adelaide (1966cal, p. 833)

Some statistical training was available as part of other courses like Psychology which was taken by many teaching students.* They emphasised statistics rather than probability and were not directly relevant to classroom teaching.

Teachers' Colleges

Increasing school numbers led to more teacher training institutions. Secondary training remained at ATC, but Bedford Park Teachers' College was built as a part of the Education Department on the new Flinders University site with the aim of integrating the academic and professional development of student teachers in a way not possible at Adelaide. Professor Jim Richardson was appointed jointly to both institutions to lead the integration, but was unsuccessful, a victim of rigid Departmental control, cultural differences, and the geographical difficulties of the site.²²¹ Other suburban colleges were established for prospective primary teachers and offered internal certification rather than university degrees. Their significant tail of poor quality students was caused by paying teaching scholarships to secondary school students with ability but desultory work habits, by paying bonded tertiary scholarships which encouraged those uninterested in teaching to enrol for it, and by accepting applicants who had passed only internal examinations in Year 11.²²² The mathematics staff at these colleges tended to be graduates with a Diploma in Education and successful classroom careers, but who were not specialist mathematicians.²²³ Indeed, Western Teachers' College initially had no mathematics specialist, a deficiency not seen as such by the Principal.²²⁴

The limitations of the students influenced the lecturers' practice. M. Watson taught classical mathematics in her first year at Western Teachers' College, but this was unsuccessful.²²⁵ A simpler replacement course²²⁷ still generated poor attitudes and achievements.²²⁸ She included statistics without probability in this

* This was my introduction to the topic.

²²¹ Hilliard (1991, pp. 21–23)

²²² Esselbach (1967, pp. 12, 137, 188–189)

²²³ *Adelaide Teachers' College Handbook 1965–1966; Annual Reports of Western Teachers' College 1963–1969*

²²⁴ *First Annual Report of Western Teachers' College 1962* p. 5

²²⁵ *Second Annual Report of Western Teachers' College 1963*

²²⁶ *Second Annual Report of Western Teachers' College 1963*

²²⁷ *Third Annual Report of Western Teachers' College 1964*

²²⁸ E.g., *Third Annual Report of Western Teachers' College 1964; Fourth Annual Report of Western Teachers' College 1965*

²²⁹ E.g., *Third Annual Report of Western Teachers' College 1964; Fourth Annual Report of Western Teachers' College 1965*

course,[†] but the statistics was soon removed because it was being taught in Educational Psychology.²³⁰ Richardson has observed:

The 'average' teacher in South Australia is just not a professional. He lacks the foundation of an integrated, coherent body of personal knowledge (or theory) about primary education that he can use as a reference point for decision making. Professional expertise is not something that can be defined in precise terms and then handed out to teachers. It is associated with the whole nature of his training, and with the subsequent outlook and attitude he has to the task of being a teacher.²³¹

Clearly, changing the Intellectual skills of teachers would be difficult. The leaders in Teachers' Colleges, usually graduates with a commitment to change,^{*} were limited by their students, their employer, and their predominantly practical expertise. So when probability was being introduced into primary and secondary schools very few teachers, lecturers or student teachers had any formal academic experience of the topic. The lecturers could learn it easily enough, and some of the secondary teachers, but many found it confusing. Indeed:

Teachers themselves have very little feeling for probability. ... [T]he course will not succeed unless we can convince the teacher.²³²

The absence of a *proven* pedagogy was another problem. Not only could it not be taught in Teachers' Colleges, but suggestions which might be made on sound educational principles were likely to be rejected by those in authority without relevant experience, as the following description of difficulties in NSW shows:

The N.S.W. 1967 Syllabus Committee rejected some suggestions of the Melbourne Conference, for example ... experimental investigation of random events. This rejection is thought to have occurred because the teachers of senior classes on the Committee did not understand how these topics might be taught to primary children. They did not understand perhaps because there had not been sufficient trialing of new content in the senior grades. ...

The inclusion of specific content in a primary school syllabus cannot be justified to N.S.W. teachers solely on the grounds that a child is able thereby to commence this mathematical topic at an earlier age. Rather

[†] M. Watson (1968). She seems to have been unaware that 1964 Conference had recommended the teaching of probability.

²³⁰ *Seventh Annual Report of Western Teachers' College 1968*

²³¹ EDSA (1974d, p. 95)

^{*} By 1968 the mathematics staff Adelaide Teachers' College included Berndt as well as Ron Close and Lang, who had both been influential in the 1965 changes.

²³² M. Watson (1968)

the test is, can the child appreciate and enjoy the experience? Perhaps the test really is, can the teacher appreciate and enjoy teaching the topic?²³³

The SA experience was not as extreme as this one, because there was some concern for experimental investigation,²³⁴ but there was little trialing and the topic remained unpopular and often unsuccessfully taught.

In the quotation at the head of this Section Griffiths & Howson suggested that the “garbling” of a topic was the fault of teachers. Eisner, on the other hand, has suggested that the education objectives which tend to dominate the thinking of curriculum changes may simply not be useful to practising teachers.

... when teachers plan curriculum guides, their efforts first to identify over-all educational aims, then specify school objectives, then identify educational objectives for specific subject matters, appear to be more like exercises to be gone through than serious efforts to build tools for curriculum planning. If educational objectives were really useful tools, teachers, I submit, would use them. If they do not, perhaps it is not because there is something wrong with the teachers but because there might be something wrong with the theory.²³⁵

The NSW experience quoted above would suggest that it was not so much the theory, but the failure to link it with practice which was the problem. We shall pick up the issue of “linkages” in Chapter 21; for the moment it is sufficient to observe that an effective educational environment is a complex place responding to many interacting forces. We shall conclude this Section by looking at the various forms of in-service training which were available at this time.

In-Service Training

In SA there was little inservice training in mathematics prior to 1965. After that, particularly under the dynamic influence of A.W. Jones, Superintendent of Recruiting and Training,²³⁶ that there were many courses, often one day, sometimes more.* But Brinkworth has argued that the inservice training was unsuccessful because it was *ad hoc*, unco-ordinated and did not use teachers’ college

²³³ E. King (1975, p. 147)

²³⁴ Clapp et al. (1966; 1967)

²³⁵ Eisner (1985, p. 31) *Vide* White’s (1992) similar claim in ch. 24.

²³⁶ *Education Gazette. South Australia* LXXIX: (915): 1, 1963

* The extent of the change may most easily be seen by comparing the in-service course offered in the early months of 1964 (*South Australia. Education Gazette* LXXIX (924): 366–370; 1963) and 1965 (*Education Gazette. South Australia* LXXX (935): 367–373; 1964).

staff as much as it might because the Education Department wanted to divorce theory (the job of teachers' colleges) from practice (the job of the Department).²³⁷

When we consider the simplistic "Snowball" model which was used by the Education Department for ensuring that teachers learned how to teach the new topics, it is not surprising that there were many failures. The method has been described by Jones in this way:

Diffusion of ideas was carried out basically through the snowball effect of inservice training, particularly of mathematics consultants who trained other teachers to be consultants. These consultants, with the co-operation of Inspectors and Heads who were also involved at an early stage, produced a constant flow of booklets and other printed material for the Curriculum Board to distribute for the assistance of teachers and children. ... Parents, too, could attend evening classes provided by the schools to give them an understanding of the new mathematics.²³⁸

Jones saw this process of trialing and phased introduction as "bureaucracy at its best"²³⁹ and in-service courses as an important component of the change process.²⁴⁰ Not everyone agreed:

The swiftness of its introduction caused many teachers to feel lost, they did not know what they were doing or where they were going ... If there had been any critics (and it probably took them some time to recover their voices, lost through astonishment) who had called for evidence to support the claims of the new course, they would have been lucky to see any owing to the scarcity of research done on the introduction of New Mathematics in Australia. Their queries might have included, what were the causes for the change and how was it to be beneficial?²⁴¹

Once, and if, the teachers recovered their voices, what resources were available for them to ensure that they were well equipped to teaching probability? In other States, notes to the syllabuses had frequently been used as a form of in-service training,²⁴² but such notes were not always available in SA, and, as we have seen above, examiners' comments were often simplistic. We shall show in Chapter 16 that most teachers rarely use published sources of support. Nevertheless, it is

²³⁷ Brinkworth (1970, p. 47)

²³⁸ A. Jones (1971)

²³⁹ A. Jones (1971)

²⁴⁰ A. Jones (1970, p. 86)

²⁴¹ Carolyn Dawson (1972, p. 147)

²⁴² Douglas (1970, pp. 303–304)

relevant to see whether potentially appropriate material about probability for teachers was available. The answer is a strong “yes”.

Popular books on statistics containing some probability had been available for some years. Moroney’s immensely popular *Facts from Figures* had been first published by Penguin in 1951 and had seen two revisions and five reprintings by 1962.²⁴³ Weaver’s *Lady Luck* was first published in 1963.²⁴⁴ Hogben’s *Mathematics for the Million* certainly contained a chapter on probability and statistics in its 1951 third edition, and perhaps earlier.²⁴⁵ These books about statistics written by experts for lay people were best-sellers, and could have provided some encouragement to teachers that the proposed changes were valuable ones. Furthermore, as we have seen in Chapter 11, the ACER’s *Background in Mathematics* had contained a section on probability with many practical suggestions for classroom use.²⁴⁶

Although MASA had been established in 1959 and began to produce material for students in 1963²⁴⁷ it did not establish a journal for its members until 1967, from which time formal links between mathematics teachers and the South Australian Science Teachers’ Association were dissolved. The editorial for the first edition of the *S.A. Mathematics Teacher* by Mr M.C. Gray from the University of Adelaide argued that the pending abolition of the Intermediate examination would force teachers to consider not “How to teach it”, but rather “What to teach, and how to teach it.”²⁴⁸ The prophecy was over-confident, as we shall see, especially for probability, even though teachers did come to acquire more autonomy.

The first edition also contained a “Reading List for Mathematics Teachers Recommended by the Mathematical Association of South Australia, March 1967”. Only one book specifically on probability was listed—Mosteller’s *Probability with Statistical Applications*²⁵⁰ but there was also reference to CEEB (1959b), the Royau-mont Seminar, and part of a National Council for the Teaching of Mathematics Yearbook,²⁵¹ all of which were relevant to probability. Neumann’s book was ad-

243 Moroney (1956)

244 Weaver (1963)

245 L. Hogben (1951)

246 ACER (1966)

247 *School Mathematics Journal* 1 (1), Jul 1963

248 Gray (1967)

249 Gray (1967)

250 Mosteller et al. (1961)

251 Page (1959)

vertised in the following edition,²⁵² Duncan made his critique of Lucas & James' book (discussed above) in the next. Later, Hume reported²⁵³ on an International Conference on the Teaching of Probability and Statistics in Schools held at the University of Southern Illinois in 1969 under the leadership of Lennart Råde from Sweden well before the proceedings²⁵⁴ were published, giving detailed descriptions of Engel's creative classroom work which will be described in Chapter 13.

There is little doubt that the Department's in-service mathematics courses were concerned with the more pressing issues of set theory and transformation geometry than with probability. But probability material was available and while not all of the material to be summarised in Chapter 13 was widely circularised, MASA was clearly doing a good job in bringing at least some of new thinking in probability teaching to the attention of SA teachers. But none of this was effective in developing well-established practice for sound probability teaching. We need to ask why. Answering this question will provide a foundation for understanding how the BSEM can provides a rich structure for interpreting change.

DISCUSSION

At the beginning of the chapter I argued that the weaknesses in the SA educational system augured badly for a new and very different topic like probability, and we have seen some examples of probability's fickle fortunes. Yet Hughes has argued that by the 1960s, although highly qualified specialists were not involved in Australian curriculum development as they were overseas,²⁵⁵ the main elements of curriculum reform were all well in place, viz.:

- Co-operation of practising teachers in the process.
- Provision of full-time curriculum specialists as writers and consultants.
- Use, without wholesale adoption, of a variety of sources.
- The practice of drafting, trialing and revising.
- Use of continuous revision rather than recurring revision.²⁵⁶

Hughes has also argued that some curriculum decisions may rest on logic or empirical evidence, while others rest on value judgements,²⁵⁷ and suggested that

²⁵² *SA Mathematics Teacher* 1 (2): no pagination, Jun 67

²⁵³ Hume (1969)

²⁵⁴ Råde (1970)

²⁵⁵ P. Hughes (1966, p. 270)

²⁵⁶ P. Hughes (1966, pp. 259–260)

²⁵⁷ P. Hughes (1966, p. 261)

[p]erhaps the major weakness in current curriculum design in Australia lies in the failure to provide appropriate organizational form for these two types of decision.²⁵⁸

The evidence here and in Chapter 13 makes it clear that for probability's introduction into SA schools, not one of these five elements was securely in place, and some were totally absent. Empirical evidence was skimpy—the discipline of Mathematics Education was very much in its infancy, and what was known was largely disregarded. The logical evidence was restricted to content, not Pedagogy. Furthermore, the hurried writing of textbooks acquired commercial overtones which may possibly not have been in the topic's best interests. Probability was a new topic, so it is easier to see the depth of the structures underlying curriculum change: they were not as deep as Hughes has suggested. In the late 1960s Physical and Social forces had priority. We shall see the consequences of this very limited support for probability in later chapters.

Other difficulties arose because there was a change in emphasis from vocational education to Social developmental education.²⁵⁹ The number of public examinations was reduced, most technical high schools were abolished, open space units and unstreamed classes became the norm.²⁶⁰ Changes were constrained by what could be done by the staff actually employed, and with little attention being paid to educational theory.²⁶¹ The educational historian, Alan Barcan has observed:

The new curriculum was thus antipathetic to the liberal-humanist curriculum ... and the development in pupils of an organized body of knowledge. ... The new education may be termed neo-progressive rather than progressive, for it lacked important elements for its progressive doctrines, notably a concern with educational theory and a strong emphasis on citizenship and on vocational training.²⁶²

Stochastics, as we have seen in Chapters 10 and 11, had often been advocated as important elements of a Liberal-Humanist education. But it was being introduced at a time when such a philosophy was being seen as less important in schools. In a sea of changes at this time, the ship of knowledge, lurching in high seas driven by McQualter's "acts of faith and trial & error" needed to find a safe port, and this safe port of what McQualter called "reality" would be Pure Mathematics.

²⁵⁸ P. Hughes (1966, p. 261)

²⁵⁹ Hamann (1975b)

²⁶⁰ Thiele (1975, pp. 208–234)

²⁶¹ Barter (1967)

²⁶² Barcan (1980, p. 349)

In a short time, the mathematics of the engineer [was] replaced by the mathematics of the pure mathematician. Strangely it happened first in technical high school courses where we had been at pains to make the course redolent with practical applications in industry and in daily life.²⁶³

Furthermore, the captains of the mathematical ships were losing faith in the potential of their vessels. Of course, many of them had thought little about where their vessels were sailing. For as long as mathematics was acting as a diagnostic sieve for tertiary entrance there were always passengers for mathematical cruises. But this situation was changing both with the increasing range of school clientele and the increasing range of jobs which were available. Latin was beginning to follow Greek into obscurity. While no-one believed that mathematics would go the same way, very few had thought about its educational value, and of convincing society at large that such value was real. After all, society at large had, in the main, only unhappy memories of the subject, memories they were only too willing to remind teachers about at parent evenings and all sorts of other social occasions. So it is not surprising that Hamann, in some ways the leader of the SA reform movement, and conscious that in the newly comprehensive schools increasing amounts of time were being taken up by unmotivated disruptive pupils, would argue in 1975 that “mathematics [could not] expect, (nor does it deserve), to maintain its past position in the curriculum”.²⁶⁴

Hamann went on to predict that improvement in SA would come less from attention to content and more from attention to teachers, methods of teaching and the development of a science of teaching. This was just how Keeves had argued ten years earlier when he wrote “if effective changes are to occur then the teacher needs to be trained and to be enthusiastic and informed about the changes”.²⁶⁵ But little had actually happened to change the Pedagogical forces in schools.

Given all the difficulties, it is surprising that any of the New Mathematics topics managed to gain any purchase at all in such a sea of change. What purchase they did gain was tenuous: in the future all but probability would eventually be cast off their ships, and even probability spent some time in the emergency dinghy being towed behind the main vessel. This is why an understanding of the influential forces acting on the curriculum is so important.

²⁶³ A. Jones (1970, p. 86)

²⁶⁴ Hamann (1975b, p. 224)

²⁶⁵ Keeves (1965, p. 16)

Both Physical and Social forces ensured that probability's introduction into the curriculum was not easy. As Openshaw has pointed out, understanding change requires examining more than ability and organisation: "wider factors such as resources, status and legitimacy" are also important.²⁶⁶ The cogent argument of the early reformers that understanding probability would be a Social asset to children was a new but ineffective Social force because it was often not well understood and so lacked legitimacy. Statistics, which was seen both as an applied tool and as easy to teach in a classroom, had more legitimacy. Status and legitimacy tend more to reflect Ultimate forces than Proximate ones. So statistics has benefited in status from the increasingly data-driven nature of our society.²⁶⁷ But the Intellectual links between probability and statistics have been poorly understood so probability has taken longer to acquire status.

This long chapter has described the laying of new ground which would last for some twenty years without, for probability at least, significant change. Subsequent chapters will need only to show how little probability changed in spite of wave after wave of change to schools' culture and practices. I shall argue that incorporation of both Pedagogical and Charismatic forces into the BSEM makes this easier to explain. But already we have found many examples of what we met in our opening *bon mot*—Twain's surprises and adventures, incongruities, contradictions and incredibilities—but they *are* all true and they *did* all happen. It is exactly these features which make McQualter's argument, quoted below, which is a variation on the CEM, much too simplistic an interpretation of all that happened.

... the fact noted by both overseas and Australian observers of the Australian educational scene that Australian educational practice is greatly influenced by and relies heavily upon what is happening in U.S.A. and U.K. Consequently, there is a time lag in Australian educational practice when conforming to overseas educational developments.²⁶⁸

²⁶⁶ Openshaw et al. (1993, p. 145)

²⁶⁷ Hacking (1990)

²⁶⁸ McQualter (1980mea, p. 53)

CHAPTER 13: ENTRENCHMENT (1970–1975)

There is a need to return to the old tried and tested methods of primary school teaching with a return of emphasis on ... straight old-fashioned arithmetic, learning of "time-tables" [sic] etc.¹

By 1978 probability was a component of almost every Australian Year 12 course,² was found to some extent in junior secondary schools,³ and was starting to be introduced into primary schools. Having described its introduction in Chapter 12, we shall here examine, somewhat more briefly, its consolidation during a time marked both by radical political changes and also by the rise of the "Back to Basics" movement, a set of Social and Intellectual forces seeking to reverse what was seen as the loss of sound knowledge engendered by the New Mathematics.

McQualter has seen this period as one of stabilisation and evaluation,⁴ but probability was still not well enough understood to have reached this stage. In any case, although the general freeing up of school systems saw significant changes in the locus of curriculum power, the actual changes in mathematics teaching styles were in most cases quite small.

POLITICAL AND STRUCTURAL CHANGES OF THE PERIOD

Since the early 1970s schools in Australia have been treated as key instruments for advancing various ideals of social equality. Official policies have reflected a curious mixture of egalitarian and meritocratic theory. The present emphasis on equal average educational attainment among groups regarded as socially significant is not primarily for the good we call education. Rather, it is for economic, social, and political benefits attached to types of occupation in the society, to which a certain level of formal education happens to give access.⁵

In 1976 an expanding population was still an important influence on schools, exacerbated by increasing retention rates: the percentage of Year 7 students still at school in Year 11 rose from 36% in 1967 to 67% in 1985,⁶ and also increased for

1 EDSA (1974d, p. 64), submission by a country primary school council

2 Rosier (1980, pp. 36–39)

3 Rosier (1980, pp. 21–26)

4 McQualter (1982)

5 Crittenden (1986, p. 120)

6 DEET (1991)

Year 12.⁷ Part of this increase came from a relative increase in girls' retention rates: they had been about 7% less than those of boys in 1967, but reached equality in about 1976, and would rise a further 4% by 1985 and 8% by 1990.⁸ Post-1945 migration had significantly changed the composition of Australian society so that in 1976 of the whole Australian population, 23.4% had been born overseas.⁹ In SA schools, 3% of the children had been born overseas, and 16.9% had been born in Australia to parents born overseas. The impact of this change was felt most strongly in Catholic schools where 3.7% of the children had been born overseas and a further 32.6% had parents born overseas¹⁰ and where the pupil-teacher ratio remained at least two pupils higher than in the government sector.* These pressures on the politically powerful Catholic Church hastened an increase in Federal support for schools, thereby further eroding some of the States' previously unfettered control over school education. Some of this support went into curriculum development emphasising Social needs and so is relevant here.

In SA, after a brief return to power in 1968, the Liberal Country League was ousted again in 1970, this time for a reformist Labor government led by Don Dunstan, with H.R. Hudson, an academic economist, as Minister for Education. J.S. Walker, who had followed Mander-Jones as Director-General of Education in 1967, was succeeded in 1970 by A.W. Jones, a former mathematics and science teacher who has already been mentioned in earlier chapters. Jones and Hudson worked well together and provided strong leadership for many years in a time of an increasing rate of change in many areas, especially technological ones.¹¹

On the surface there were radical changes. In 1970 Jones issued a "Freedom and Authority" Memorandum. This gave school principals increased control over their schools in matters of management, but not in matters of curriculum.¹² In 1972 school councils were given advisory powers about curriculum and "[s]chools were considered as the centres of 'expertise' for curriculum decision

⁷ Commonwealth Department of Education (1985, p. 3)

⁸ DEET (1991)

⁹ Stevens (1979, p. 27)

¹⁰ Stevens (1979, p. 74)

* For 1964 the ratios were 25.7 for government and 27.0 for non-government schools. For 1976 the figures were 17.4 and 19.8 respectively (*South Australian Year Book* No 1: 1966, pp. 132–133; No 12: 1976, pp. 222–223). Part-time teachers in 1964 were estimated to be doing a 0.5 load. These generalised figures do not show any asymmetries in the distribution, of which the most marked was almost certainly a higher pupil-teacher ratio in Catholic primary schools, but it has not been possible to locate appropriate figures.

¹¹ Thiele (1975, p. 226)

¹² Thiele (1975, pp. 226–227)

making”,¹³ although some centralised curriculum guidelines were retained. The Department wanted curriculum change to be achieved, not by a Research-Development-Dissemination model, but by extensive use of a committee structure.¹⁴ In the view of one member of senior management at that time:

the best implementation of curriculum modifications is to be achieved by minimal use of hierarchical authority, not so much by deserting authority as by ensuring that it is not required: that is, by ensuring a maximum consensus of contending [sic] parties.¹⁵

Jones’ change is often seen as a watershed in SA education,¹⁶ but for Connell it was “neither revolutionary nor controversial”,¹⁷ although this was not always the view of teachers at the time.¹⁸ As we have seen in Chapter 7 Howson has described the committee structure as a school-based personality model. However, while the SA developments were influenced by personalities, these personalities were rarely based in schools. Hamann was only the first of a number of skilled mathematics teachers to move from the classroom to a consultancy position. The extra freedom for teachers offered by school-based curriculum development was unpopular (mainly because of a shortage of time and resources), and eventually led to a return to more central decision making.¹⁹ Physical constraints proved critical. Committees based on consensus take time to achieve results, and teachers in schools are busy people. For curricula, the changes which arose from the Freedom and Authority Memorandum led to changes in leadership to “new professionals” *outside* the schools, not a transfer of leadership to *within* the schools.

One new form of leadership came from newly appointed primary and secondary mathematics consultants. According to Hamann, the consultants often replaced tertiary staff as authorities for classroom teachers,²⁰ but others have suggested that teachers relied on textbooks as their first source of printed help and preferred to receive support via easy access to advisory teachers within a school rather than from experienced former classroom teachers who had moved to desk-based work.²¹ The Primary Mathematics Teachers’ Association was also established at

¹³ Prideaux (1988, p. 8)

¹⁴ J. Giles (1970, p. 77)

¹⁵ J. Giles (1970, p. 51)

¹⁶ Thiele (1975, p. 227)

¹⁷ Connell (1993, p. 204)

¹⁸ Thiele (1975, p. 227)

¹⁹ Prideaux (1988, p. 14)

²⁰ Hamann (1975b, p. 227)

²¹ EDSA (1974a, p. 49)

this time. It was set up because MASA, which was dominated by tertiary and secondary teachers, was seen as failing to reach primary teachers.²²

Most importantly, the Teachers' Colleges were developing into autonomous degree-granting Colleges of Advanced Education (CAEs) which from 1974 were fully supported by Federal money.²³ The new CAE staff, "[f]reed from the shackles of Education department uniformity",²⁴ and working at a time when rapid promotion both in and out of classrooms was feasible,²⁵ sought to carve out leadership niches for themselves,* niches which Horwood has seen as being "with interests related not so much to content but to social concerns" and as being a direct result of the development of the social sciences as academic disciplines.²⁶ Until then what little mathematics education research there was had been done mainly by the Department, and concentrated on classroom implementation of syllabuses. The results were neither widely disseminated²⁷ nor influential,²⁸ and slowly more research was done by CAE staff. This had the unexpected and important side-effect that university mathematicians began to take less interest in being leaders of school mathematics education.²⁹

Not only were leadership roles changing, but authority in general was also coming under increasing challenge, particularly because of Australia's involvement in the Vietnam War. Much of the opposition to the War came from within tertiary and secondary educational institutions, not least because from 1964 all 20-year olds were liable to be balloted into National Service.³⁰ And because this was an Asian-American war, not a European one, Australia's involvement was a significant step towards widening its social and cultural ties.

In the aftermath of wars, countries tend to spend money on education to try to build a new and better order. The reformist Whitlam Labor Government held Federal office from 1972 until its dismissal in November, 1975. This time was

²² Hamann (1975b, p. 227)

²³ Connell (1993, pp. 374–382)

²⁴ Horwood (1992, p. 17)

²⁵ Connell (1993, p. 172)

* This thesis has drawn on four introductory pieces of work—J. Baxter (1972), Brinkworth (1970), Hunwick (1970) and Sumner (1969)—done by young CAE staff to provide evidence for University authorities that they had the capacity to undertake advanced studies.

²⁶ Horwood (1992, p. 17)

²⁷ Sumner & Lafleur (1972); Lafleur et al. (1973)

²⁸ Education Department. Research and Planning Division (1975)

²⁹ Blakers (1978)

³⁰ Hilliard (1991, p. 54)

marked by funding for education unknown previously or since, together with the encouragement of a massive expansion of the tertiary sector. The Schools Commission was established 1973 to fund innovative ventures, and the Curriculum Development Centre (CDC), established in 1975, would be in a position to provide well-funded leadership for curriculum change—a change, however, concerned “more with the processes of education”³¹ than with the content. This approach had already been experienced in SA in 1971 when a Commission into education,³² chaired by Professor Peter Karmel, recommended many changes, but “had little of significance to say on the curriculum or on the actual practice of education in the schools.”³³

This change in focus from Intellectual to Social issues was long-lasting, and contributed in part to a decline in the position of mathematics within school curricula as educators, including mathematics teachers themselves, responded to the needs of the changed school population.³⁴ The decline of traditional leadership positions gave teachers more opportunities to have their views heard. Karmel had recognised that these needed to be balanced by other perspectives when he argued that

wider professional representation in the bodies concerned with curriculum development. Neither the scholarship that the university disciplines could contribute nor the experience and interest of the non-government schools appear to be adequately used.³⁵

This challenge fell largely on deaf ears. Even the extension of representation on Subject Committees did not occur until 1977.[†] The Intellectual diversification of this period made Karmel’s broad consensus difficult to attain. Intellectual forces constituted less a pressure group than a “Speakers’ Corner” with divergent views stridently put forward, but not easily blended. Such debate was probably necessary, and the freeing up of educational administration was initially very popular. The old, rigid administrative and political systems were becoming increasingly unworkable, and the weaknesses exposed by the New Mathematics changes meant that the community had lost some of its faith in teachers. Long-powerful Intellectual forces were in decline while Social forces were in the ascendant. Phys-

³¹ Connell (1993, p. 204)

³² Karmel (1971)

³³ Connell (1993, p. 227)

³⁴ Hamann (1975b, p. 224)

³⁵ Karmel (1971, p. 512)

[†] *Vide* ch. 11.

ical forces tended to be less important as liberal Federal funding from Labor governments flowed into the system. The time was ripe for change.

But, in the midst of all these changes, mathematics teaching became more conservative, and probability teaching changed little. Other issues were seen as much more important: they led to the “Back to Basics” Movement—a strong and powerful backlash against the New Mathematics.

THE “BACK TO BASICS” MOVEMENT

No educator has yet convinced us that New mathematics can replace simple arithmetic and addition, subtraction and tables without the use of the fingers should be the norm not the exception.³⁶

We saw in Chapter 11 that objections to New Mathematics were being described as early as the 1962 Sydney conference.³⁷ More objections were raised in the 1970s. In 1973, Potts delivered a scathing attack on modern mathematics as part of his presidential address at the ANZAAS Congress in Perth. He argued that sets in school were not being used as they were by mathematicians, and that many statements made in schools were wrong. He proclaimed

I do not favour a return to the old maths. I believe Australia urgently needs new maths. My plea is for professional mathematical scientists to become involved with teachers and mathematical educators in ridding the present texts of what is useless and nonsense and helping to produce more useful, sensible new maths texts.”³⁸

It is important to contrast Potts’ Australian complaints with others in the world which appear to be similar, but in fact reflect quite different ideas. In the U.K., for example, J.M. Hammersley delivered his famous paper “On the Enfeeblement of Mathematical Skills by ‘Modern Mathematics’ and by Similar Soft Intellectual Trash in Schools and Universities” in 1967.³⁹ Soon after came some Black Papers in Education advocating a return to more basic ideas were published, and a little later, in 1976, the issue was brought to a head with a speech at Ruskin College by James Callaghan, the Prime Minister.⁴⁰ Hammersley and Potts were both applied

³⁶ EDSA (1974d, p. 71) from a regional association of rural school welfare clubs

³⁷ Chong (1962)

³⁸ Potts (1974)

³⁹ Later published as Hammersley (1968)

⁴⁰ Cooper (1993, pp. 9–10) [Documents in the Private Domain]

mathematicians, but Potts' concern here was with ensuring that pure mathematics was properly taught in schools while Hammersley was concerned with the dominance of pure mathematicians over school syllabuses. They represented two quite different Intellectual forces, which may be contrasted with Callaghan's Social concern that children should be able to use their mathematics in everyday life. We may note that the arguments put forward were diverse, and but each was largely monochromatic.

Another manifestation of similar disquiet may be found in employers' distrust of new certification procedures which had been set up in order to provide achievable and practical goals for students without a strong academic interest.

In 1971 the Annual Report of the Secondary Division of the Education Department revealed that, despite the efforts by staff and moderators, employers were generally not accepting the Years 11 and 12 S.S.C.[*] as an indication of ability and achievement of students. Many still preferred unsatisfactory grades in the P.E.B. to satisfactory S.S.C. grades.⁴¹

Finally, the same distrust of the New Mathematics was also felt among rank and file citizens, including teachers and parents. In 1974 L.W. Whalan, at that time Superintendent of Primary Education, co-ordinated a wide-ranging review of the primary school curriculum, which generated many conservative responses like those heading this Chapter and Section. Reading, writing, and arithmetic were still seen as the principle aims of primary schooling. Teachers and parents rejected almost all the ideas from the New Maths, none more vehemently than "Sets" with their associated visual aid of Venn Diagrams,⁴² on which the pure mathematics approach to the teaching of probability was based.

[t]here are a few of the old ideas that still apply. In our varying professions ... we all agree that in all the avenues of work, the old basic principle of maths applies, without them we could not carry out our duties. Very rarely are the new maths applied to solve problems.⁴³

We saw in Chapter 10 that statistics had been seen as way of helping children to understand their world since at least the 1930s, but this understanding had not been assimilated by the community as a whole. The vision of the NEF had faded. Primary school mathematics was seen as a "basics" course, especially for the

* Senior Secondary Certificate, an internally assessed certification discussed below.

⁴¹ A. Jones (1978, p. 44)

⁴² E.g., EDSA (1974d, pp. 63, 66, 93)

⁴³ EDSA (1974d, p. 9), from a parent

increasing number of students with language difficulties.⁴⁴ I show below that the stochastics changes in SA primary schools were led by probability, rather than by statistics, as had initially occurred ten years earlier in secondary schools. This meant that the Social value of stochastics was largely unappreciated by teachers in spite of a desire to relate mathematics to everyday living.⁴⁵ In any case, teachers were fully occupied developing new pedagogies for familiar topics: they did not feel competent to make a radical reassessment of what was important in mathematics education, let alone to justify it to their students' parents. They did not want freedom and authority for curriculum decisions of this magnitude.⁴⁶

Like an earlier evaluation in about 1971,⁴⁷ Whalan's questionnaire did not mention probability, even though by 1973 it had a small place in primary schools. So very few teachers or parents made any comment about it.⁴⁸ The Primary Division Inspectors' Association felt that simple probability could be omitted from Year 7,⁴⁹ and the Secondary Mathematics Curriculum Advisory Committee considered probability to be a low priority subject for primary schools.⁵⁰ No one specifically supported the topic. A few opposed it, but with poorly articulated reasons.

Whalan's conservative report listed many topics felt to be marginal to primary schools, including Venn Diagrams and probability,⁵¹ and urged that greater emphasis be placed on core subjects like routine arithmetic.⁵² So just as schools and teachers were gaining new freedoms and diversity in the 1970s, strong monochromatic Intellectual and Social pressures were forcing schools to undo the changes of the 1960s. Potts' argument is representative of the Intellectual pressures, and the quotations from Whalan's report are representative of the Social pressures. While probability was not specifically attacked, it was not seen as important, and its status as a bed-fellow of sets and Venn Diagrams probably did it a little harm. It is ironic that the Back to Basics Movement was strongest at a time when so much creative work was being done in probability pedagogy around the world—developments which might have provided inspiration for SA reformers.

44 EDSA (1974d, p. 54)

45 EDSA (1974a, p. 44, point 6.1.7)

46 EDSA (1974a, p. 44, point 6.1.2)

47 EDSA. Research and Planning Branch (?1971, before 1975)

48 EDSA (1974b)

49 EDSA (1974d, p. 125)

50 EDSA (1974d, p. 127)

51 EDSA (1974a, p. 47)

52 EDSA (1974a, p. 44, point 6.1.6)

OVERSEAS TRENDS IN PROBABILITY TEACHING

Les mathématiques “modernes” ont amené une rénovation nécessaire des enseignements. Mais les professeurs auraient désiré être largement consultés pour l’élaboration des nouveaux programmes, ce qui aurait certainement évité à ces derniers d’être étendus et aussi abstraits. C’est leur parution trop tardive qui est responsable en grande partie de la rédaction hâtive, donc insatisfaisante, des ouvrages scolaires.⁵³

This quotation from a working party of about 60 French lycée* teachers highlights some of the perceived difficulties of the New Mathematics, but also shows how teachers might attempt to overcome them. In the early 1970s good material was developed in Europe and America, and a small number of Australians attended international conferences and returned to disseminate their experiences via conferences and publications.⁵⁴ So the already significant dissemination of overseas ideas within Australia in the 1960s continued in the 1970s, and Australian changes may reasonably be assessed against contemporaneous overseas thinking.

In France a group of teachers, under the influence of P.L. Hennequin & M. Glaymann, published two brochures—*Pour apprendre à conjecturer*—as early as 1967⁵⁵ and further material about ten years later.⁵⁶ Between 1970 and 1972 Brousseau & Hennequin developed material to facilitate understanding of decimals based on predicting the contents of an urn from the results of several draws and gave advice on appropriate forms of teacher intervention,⁵⁷ but this has unfortunately not been included in the English collection of Brousseau’s work.⁵⁸ In Bordeaux, O. Eyssautier-Laborderie developed a careful analysis of the subtleties of probabilistic language,⁵⁹ showing how children use different words for “I” to indicate

⁵³ IREM de Montpellier (1971)

New Mathematics led to a necessary renewal of teaching practices. But the teachers should have been more widely consulted in the development of these new programmes, and this would certainly have avoided their becoming so elaborate and abstract. The late publication of the programmes has led in large part to hasty and therefore unsatisfactory editing of these scholarly works.

* Academic secondary school

⁵⁴ E.g., Hume (1969); J. Truran (1972). *Vide etiam* discussion in ch. 14 on ICOTS and ICME.

⁵⁵ *Learning to Surmise*. APMEP (1967, 1968)

⁵⁶ APMEP (n.d., ?1976)

⁵⁷ Brousseau (1970–1990/1997, p. 117), which does not indicate a source, but this is probably Brousseau et al. (?1972) .

⁵⁸ Brousseau (1970–1990/1997)

⁵⁹ Eyssautier-Laborderie (1973)

relative degrees of conviction—on, je, moi je.[§] She showed how words may have different meanings in different stochastic contexts, and contrasted the language typically used by a teacher with that of a child in similar situations. This valuable work, of course, has addressed the issues discussed here in Chapters 4 and 5, and is similar to the work reported in Ch 6 where track gamblers use different meanings for the single word “certainty”.⁶⁰

In Holland, Freudenthal, ever the leader, had published *Waarscijnlijkheid en Statistiek*⁶⁶ as early as 1957. A. Engel from Germany spent time at Carbondale IL in the late 1960s, where he worked with Lennart Råde on a large and well-tested project⁶⁷ with an emphasis less on content than on pedagogy.⁶⁸ His lively ideas, similar to those of Hennequin & Glaymann, were published in German in 1973,⁶⁹ translated into French soon after,⁷⁰ summarised by Hume for the *South Australian Mathematics Teacher* as early as 1969,⁷¹ and influenced Kempster’s pioneering study in NSW.⁷² Glaymann later worked with T. Varga from Hungary to produce *Les Probabilités à l’école*⁷³ where they argued that the interest elicited by popular books on statistics* had not been reflected in school curricula. This fine teachers’ book is based on games, but underlain with theory. Soon after, Varga & M. Dumont produced a stochastics book for primary school children,⁷⁴ and Engel &

§ “On” ascribes no source, and may be translated by the passive (e.g, “it is said” for “on dit”). “Je” is “I”, but “moi je” is the emphatic “I certainly”, often conveyed by italics in English.

60 M.B. Scott (1968, pp. 82–83)

61 APMEP (n.d., ?1976)

62 Brousseau (1970–1990/1997, p. 117), which does not indicate a source, but this is probably Brousseau et al. (?1972) .

63 Brousseau (1970–1990/1997)

64 Eyssautier-Laborderie (1973)

§ “On” ascribes no source, and may be translated by the passive (e.g, “it is said” for “on dit”). “Je” is “I”, but “moi je” is the emphatic “I certainly”, often conveyed by italics in English.

65 M.B. Scott (1968, pp. 82–83)

66 *Probability and Statistics*. Freudenthal (1957)

67 Engel (1970)

68 Blakers (1970, p. 75)

69 Engel (1973), later revised as Engel (1987)

70 Engel (1975)

71 Hume (1969)

72 Kempster (1982, p. 5)

73 *Teaching Probability in Schools*. Glaymann & Varga (1973)

* *Vide* discussion in ch. 12.

74 Varga & Dumont (1973)

Varga collaborated with W. Walser to produce *Zufall oder Strategie?*⁷⁵ also written for children, with lively illustrations and an emphasis on the playing of games. By 1975 one of I. Adler's American works had been translated into French,⁷⁶ F. Mosteller had led a team which produced books and teachers' manuals for a basic stochastics course,⁷⁷ and Råde had written *Statistics at the School Level*.⁷⁸ In Britain the Mathematics for the Majority Project had produced the rather formal *Luck and Judgement*,⁷⁹ the Scottish Mathematics Group's books had been adapted for Australian schools,⁸⁰ and the Mathematical Association had produced a small pamphlet designed to clarify statistical ideas for teachers with some formal training in mathematics.⁸¹ Soon after, a very interesting and still useful collection of example of practical applications of stochastics was produced in the USA.⁸²

So from the early 1970s SA teachers had increased opportunities for introducing changes, and for engaging with a wide variety of imaginative thinking around the world, most of which followed Bruner in emphasising a practical approach to probability teaching. The newly developing discipline of mathematics education was producing Intellectual ideas. Although these ideas were not yet Intellectual *forces*, they would have made it easier for Australian curriculum developers to respond effectively to pressures to change or develop the teaching of probability.

PROBABILITY IN PRIMARY SCHOOLS

What are $P(o)$, $P(\Delta)$, $P(x)$, when a single member is chosen from $\{oooo\Delta\Delta\Delta xxx\}$?*

But SA did not respond to these imaginative ideas. As soon as the new courses described in Chapter 12 had been introduced, further ones were planned, begin-

⁷⁵ *Chance or Strategy?* Engel et al. (1974)

⁷⁶ I. Adler (1973)

⁷⁷ Mosteller et al. (1973a–d)

⁷⁸ Råde (1975)

⁷⁹ Mathematics for the Majority (1971)

⁸⁰ Davidson (1972)

⁸¹ Mathematical Association (1975)

⁸² Tanur et al. (1978)

* EDSA (1974cm, p. 227). It is not usual to include identical items more than once in a set, and it is usual to separate items by commas. The use of subscripts for the probability notation is also non-standard. So this example provides a strong indication that the material was prepared with little influence from outside SA (and probably inside as well).

ning with a trial in 1972,⁸³ formal *Statements of Objectives* in 1973,⁸⁴ and then an interim revision.⁸⁵ None of Whalan's *bêtes noires* was expunged from the syllabus, nor were the relative weights of the topics more clearly defined. Probability formed one Objective out of 16 in the section on Statistics and Graphs for Years 5, 6 and 7,⁸⁶ with suggested exercises like that at the head of this section and also:

- If you choose any member of {5, 6, 7, 8, 9, 10, 11, 12} what is the probability that you will choose
- a. an even number?
 - b. a prime number?
 - c. a multiple of three?
 - d. a factor of 12?⁸⁷

Similar questions had been circulating for a few years. Some are found in the small sets of questions included in the final books of the *Primary Mathematics Series* and *Pacemaker Maths* sequences in c. 1971 and mentioned in Chapter 12. Why did probability appear at this time, and not earlier? Although the Nuffield Project had recently produced an imaginative, developed, and tested pedagogy for probability,⁸⁸ in spite of what Clements has claimed,[∞] this does not seem to have had any influence, as a look at just one typical Nuffield example can show:

- Take 4 pennies.
 Make a guess as to how many times in 16 tosses of the 4 pennies together 4 heads will appear.
 Then do the experiment.
 How near was your guess?
 Do the experiment a number of times. Each time try to improve on your previous guess.⁸⁹

The contrast between theoretical and experimental approaches is obvious. It is most likely that probability appeared osmotically in SA primary schools through familiarisation with its teaching in secondary schools. The SA syllabus was "pure", especially in its failure to address adequately the two types of probabilities, which must be distinguished, as we saw in Chapter 6, for the sake of good mathematics and, I shall argue in Chapter 23, for the sake of effective teaching.

⁸³ J. Baxter (1972)

⁸⁴ EDSA (1973c)

⁸⁵ EDSA (1974cm) covers Years 6 and 7 and is the only volume relevant here.

⁸⁶ EDSA (1973c, p. 46)

⁸⁷ EDSA (1974cm, p. 227)

⁸⁸ Nuffield Mathematics Project (1969)

[∞] As discussed in ch. 12

⁸⁹ Nuffield Mathematics Project (1969, p. 46)

Whalan's review occurred just after probability had crept into the primary curriculum. Although it did not recommend that probability should be included in the core work⁹⁰ which was bound to dominate classroom practice, the curriculum to which it gave rise did contain some careful thought about probability by an influential group of SA primary school leaders, perhaps for the first time. The curriculum notes explain briefly why probability is an important branch of mathematics, but do not address its Social value.⁹¹ Symmetric probability is formally defined, but other forms of probability are not mentioned, some classroom activity is strongly encouraged. Suggestions of appropriate activities, which are a mixture of Classical and Bayesian approaches,[†] are suggested, but the practical difficulties of relating symmetric and experimental probabilities are skated over. In particular, the Bayesian problem of inferring the contents of an urn from a number of draws is, in practice, not easy for beginners (or anyone else for that matter),^{*} and the suggestions assume that the experimental results will be very close to the actual ones: i.e., that the Law of Large Numbers applies to small numbers.

The syllabus was constructed mainly by teachers or former teachers, but the pure mathematicians' influence may still be seen, and the committee does not seem to have used others' experience or pre-tested the suggested classroom activities with children. Probability did not arrive in SA primary schools with any of the excitement of exploration to be found in many other contemporary approaches. This would have alienated the teachers, even teachers of young children, who saw concrete aids as confusing for children, often time-wasters and a "prop" for less adventurous pupils. For them discovery was important but had its limits.⁹² At best then, probability was an enrichment topic,⁹³ just in New York at the same time where it tended to be taught at the end of the school year.⁹⁴

What a contrast with the syllabus in WA! As well as Hume's moderately successful Year 12 course described in Chapter 12, stochastics was part of the number

⁹⁰ EDSA (1974a, pp. 45–47)

⁹¹ EDSA (1974cm, pp. 224–229)

[†] Bayesian inference is also discussed in Ch. 18.

^{*} In 1994, during an in-service activity which I was leading, a group of 17 New Zealand teachers unanimously, aggressively and wrongly agreed on the contents of an urn on the basis of 16 draws with replacement. Quite fortuitously, the 17th draw provided a convincing counter-example and a very lively class response.

⁹² GRS 1049. Primary and Secondary Advisory Curriculum Boards. Minutes of Meetings 1955–present. Box 3. Minutes Book. Report to the Advisory Curriculum Board on State of the Curriculum. Mathematics

⁹³ Kee (1975, p. 257); *Vide etiam* "wet Friday afternoons" topic in ch. 24.

⁹⁴ Mocchiola (1975)

strand of the primary syllabus, using informal exploration⁹⁵ such as investigating pre-numerically the effect on outcomes of variations in random generators by, for example, changing the order on a spinner or the size of the spinner sectors.⁹⁶ Ironically, much of this work was developed after Pender Pedler moved from Adelaide to Perth in c. 1975; SA did not seem to be able to use his creative skills.

Although a new syllabus was developed in 1974, Whalan's report was still seen as an interim one because the ACER, after delicate diplomatic footwork with the States,⁹⁷ was convening a conference in August, 1975 to examine the changes since its 1964 conference.⁹⁸ SA wanted to delay major changes until it had heard what insights this Conference would bring.⁹⁹

THE 1975 MELBOURNE CONFERENCE

[P]rimary teachers criticise, (with some justification), the secondary school and teachers who often appear unsympathetic and intolerant in their acceptance of students and categorise them instantly on the basis of their own (unrealistic) expectations.

The traditional role of the teacher and textbook [is] being usurped by 'card' material, individual progression programs, work sheets In many cases the new approaches lead to cardboard indigestion or communal correspondence courses.¹⁰⁰

This was a conference of plain speaking, but it had little influence on primary education in SA. The ACER was planning a national assessment of literacy and numeracy which did not include probability,¹⁰¹ so it is not surprising that probability received little attention. King, an advocate of probability at the 1964 Conference, argued that probability might well receive a better reception in schools in 1975 than in 1964,¹⁰² and the Conference recommended that "greater emphasis at the primary level [could be given to] introductory experiences in random processes and activities based on probability".¹⁰³ Hamann saw probability and stat-

⁹⁵ Education Department of WA (1979), Pedler (1977a, 1977b, 1980), Hoffman (1980)

⁹⁶ Hoffman (1980, p. 345)

⁹⁷ ACER Archives Vol 321. Conference on Primary Mathematics August 1975

⁹⁸ Jeffrey (1975)

⁹⁹ EDSA (1975)

¹⁰⁰ Hamann (1975b, pp. 227–228)

¹⁰¹ Bourke & Lewis (1976)

¹⁰² E. King (1975, p. 156)

¹⁰³ Jeffery (1975, pp. 12; *vide etiam* pp. 3–5)

istics as suitable for junior secondary syllabuses “not [because of] a wish to force students into quicker development but to enable students to work creatively with topics which are interesting for the world of today”.¹⁰⁴ This, of course, emphasised experimental probability; experience was seen as coming before theory.

Balancing theory and practice was the basis of the plain speaking which highlighted important differences between academics and teachers, differences which would be very important in the next twenty years of curriculum development in Australia, and which helped to make the Conference uninfluential. Speakers tended to present personal opinions or experiences, unsubstantiated by detailed evidence. The academics tended to speak as though their position gave them very broad authority on all matters educational; the teacher-administrators tended to speak as leaders often pressed to make rapid, difficult decisions.

As well as the tensions between teachers of different levels cited above, the Conference also revealed a distaste among the academics for teachers and psychologists. Blakers predicted that Piaget would prove to be substantially wrong in many areas, described many primary teachers as poor and “beyond retrieval”, and considered improving the quality of in-service training to be impossible.¹⁰⁵ Henry Finucan, a mathematician from University of Queensland, argued, probably with cutting humour, that academic streaming would be more egalitarian than the approaches then current.¹⁰⁶ T.H. MacDonald, Chairman and Head of the Mathematics Department of Melbourne State [Teachers’] College, attributed, probably unreasonably, the formalism of the New Mathematics to austere psychologists like Bruner.¹⁰⁷ Preston argued that Victoria had been spared the worst excesses of the new maths movement because in that state the teaching profession was well qualified and knew enough mathematics to ensure that “mathematics and not educational theory rule[d]”.¹⁰⁸ Clements might not have agreed. Scott, a pure mathematician from the University of Adelaide, at that time President of MASA and described as a man with “considerable interest in and knowledge of the curriculum”,¹⁰⁹ condemned aspects of the correspondence course provided in SA,

104 Hamman (1975a, p. 59)

105 Blakers (1975)

106 Finucan (1975)

107 MacDonald (1975)

108 Preston (1975)

109 Barnes to Keeves (ACER) 17 Mar 75. ACER Archives Vol 321. Conference on Primary Mathematics August 1975

both as an academic and as a parent whose children had had to use the course, and argued for a more balanced development of an appropriate syllabus.¹¹⁰

On the other side, those responsible for keeping the state systems going showed little interest in these criticisms from academia. Their approach was rather like that of a football coach at half-time:

The next decade is likely to be one of even greater change and one hopes that a school programme can be developed which will enable students to cope with the demands of the time in which they live.¹¹¹

Hoffman was content to use unsubstantiated phrases like “often inappropriate”, “has widespread acceptance”, or “is widely criticized”.¹¹² Hamann mirrored this approach with phrases like “I believe that ...”, “we must teach for ...”, and “experience would point to the fact that ...”.¹¹³

The two groups of rhetoricians failed to engage with each other at this Conference, and were to drift further apart in the years to come. Serious consequences of this rift will be described in Chapters 15 and 16, but in the mean time the secondary syllabuses were being modified at a time when the relative strengths of academics and teachers were being tested. But, as we shall see, the dominant, highly conservative, power remained the practising teacher.

PROBABILITY IN SECONDARY SCHOOLS

A person who has studied this book will not have been prepared for a specific examination, and yet he should find himself able to complete the syllabus for any statistics examination he is likely to meet at this level. Meanwhile, in carrying things further, he should read only those statistics books whose *mathematics* is as far advanced as he can cope with. The danger with anything less is that he may be fobbed off with recipes.¹¹⁴

In secondary schools this conservatism was marked by the retention of the pure deterministic cook-book approach to probability which had flowed down into the primary schools. The content and style of questions in the public examination syllabuses changed little. Only major changes will be discussed here: the develop-

110 P. Scott (1975)

111 Hoffman (1975, p. 168)

112 Hoffman (1975)

113 Hamann (1975a)

114 Durran (1970, p. xiii)

ment of courses, textbooks and teaching approaches are better discussed as part of the longer time span of Chapter 14, and also in Part E.

Two small changes, however, demonstrate that the old order was changing, albeit under protest. In 1971, Roman numerals were abolished in subject titles: Mathematics I, II and IS came to be seen as Mathematics 1, 2, and 1S.¹¹⁵ This seems to have met with some resistance: Roman numerals reappeared for a time¹¹⁶ but were eventually conquered. In 1972 the hierarchical ordering of subjects in official documents was replaced by an alphabetical one. No longer would English head the list, followed by the classical languages, with mathematics forming a bridge between the arts and science subjects.¹¹⁷ These changes are, of course, trivial in themselves, but they are poignant indicators that a new environment was being developed in which old assumptions and hierarchies would be challenged.

The Intermediate Examination had already been abolished, and there were moves towards internally assessed certification. The Education Department provided the Senior Secondary Certificate (SSC) as a Leaving Statement mainly for those who had not studied an academic course,¹¹⁸ but, as shown above, not all employers had confidence in it. When the external Leaving Examination was abolished from 1975, the SSC was the only form of Year 11 certification until School Leaver Statements were introduced. Its syllabuses are largely beyond the scope of this thesis because non-academic stochastics courses tended to be based more on statistics than on probability. For academic courses Year 12 requirements would continue to influence most Year 11 practice for the whole of the period covered by this thesis.

Year 12

Only one significant change occurred in Year 12 examination courses at this time. From 1969 a moderately advanced Statistics Option formed part of the incoming syllabus for Mathematics I, covering normal and probability distributions, Bayes' Theorem, inference and hypothesis testing¹¹⁹ and was examined in the end-of-year examinations. John Darroch, Professor of Statistics from Flinders, and Pedler provided some notes for teachers¹²⁰ which were mainly a reading list and a set of

115 PEB (1971s)

116 E.g., in the examination papers for 1978, PEB (1978)

117 PEB (1972s)

118 A. Jones (1978, p. 48)

119 PEB (1969a)

120 Darroch & Pedler (1971)

model questions. Rigby published a set of exercises¹²¹ prepared by Sawley, Hamann, Clapp, and also John Gaffney, an able, quietly charismatic teacher who would soon take over Hamann's leadership role as a Consultant. The exercises contained a very practical suggestion, probably due to Gaffney, that simple cases of Bayes' theorem should be presented using trees. Hartley Hyde* has recalled that from at least 1966 probability was approached only algebraically, probably under Hamann's influence, but that when Gaffney illustrated the ideas using trees at a MASA Conference in c. 1973 he made the subject live. The authors also provided a short list of texts¹²² which might be consulted and bravely wrote

This work is to be regarded as a set of exercises and not a substitute for a text. We would like to feel that it was being used in classroom tutorials, in individual preparations, and in conjunction with a number of texts ...¹²³

It is unlikely that their expectations were fulfilled.

Year 11

The abolition of the Leaving Certificate had little short term effect on school practice. But in 1972 two new mathematics examinations were offered.

Mathematics 3 was an academic course containing a selection of work from Mathematics 1 and 2, and was probably taken mainly by students who expected to study no more mathematics. This examination contained questions on combinatorics and permutations, and an optional question on statistics.¹²⁴ The inclusion of combinatorics reflects the pure mathematics influences on the syllabus. The examiners were disappointed with the answers, and suggested that many students had not studied parts of the course, including the newer topics of statistics and vectors.¹²⁵ But they were happy with the answers to the probability question.¹²⁶ Mathematics 4 replaced the previous Arithmetic examination¹²⁷ and included a

¹²¹ I have only seen Sawley et al. (1974), but the first edition was published in 1971.

* Hartley Hyde, pers. comm., Mar 1993. I mentioned in Chapter 12 how Melbourne Grammar School also moved from formal algebraic models to tree diagrams: the shackles of rigid formalism had to be broken if the ideas were to become available to ordinary students.

¹²² Arthurs (1965); Mode (1966); Backhouse (1967); Spiegel (1961)

¹²³ Sawley et al. (1974, Preface)

¹²⁴ PEB (1972, pp. 160–163)

¹²⁵ PEB (1972, p. 643)

¹²⁶ PEB (1972, p. 644)

¹²⁷ PEB (1972, pp. 643, 644)

significant amount of statistics and linear programming, but no probability. The examiners expressed general satisfaction with the answers to this paper.¹²⁸

These syllabuses reflect the two approaches to stochastics found within secondary schools at this time. Statistics was for the less able students, and probability was eschewed. But probability was important for the more able students, and it seems as if their teachers had less experience of teaching statistics, or perhaps less enthusiasm for doing so. We shall see in the Discussion below how poorly grounded the SA practices were with respect to the teaching of probability.

Junior Secondary

Probability was also included in the recommendations for junior secondary mathematics. The 1974 revision saw the mathematics curriculum as:

- (1) providing a basis for further mathematics by developing the ability to read, understand, and translate information into meaningful mathematical statements;
- (2) helping to fulfil vocational and general educational needs,
- (3) developing in students the ability to reason and apply mathematics to situations and problems in their lives;
- (4) revealing mathematics as a part of the culture of the community.¹²⁹

Unlike the situation in primary schools, probability was seen as a core part of the syllabus and contained:

Notions of experiment, simple event, sample space A , event A , probability $p(A)$, where

$$p(A) = \frac{n(A)}{n(S)}, 0 \leq p(A) \leq 1.$$

Display of sample spaces, e.g. listing, tree diagrams, graphs, grids; their use in solving simple problems including those involving—

- (a) independent events, and
- (b) mutually exclusive events.

Formal statements on independent events and mutually exclusive events are not recommended.¹³⁰

This syllabus was developed nearly ten years after probability was first taught in SA high schools, and would remain largely unchanged for another 15 years. It is a pared-down version of the upper secondary syllabus, couched in much the same language, but a little less didactic. However, the experimental probabilities of the

¹²⁸ PEB (1972, p. 644)

¹²⁹ EDSA (1974jsms, p. 3)

¹³⁰ EDSA (1974jsms, p. 8)

initial syllabus introduced under the influence of SMP have been removed and subjective probabilities are not mentioned. There are no links with three of the four aims of the mathematics course. It is still a pure mathematician's syllabus, modified just a little by a Pedagogue's awareness of the need for concrete presentation. The very formula which Glastonbury thought would be unfamiliar to SA teachers in 1963 is now standard fare for junior secondary students—only it is not as precisely expressed as it was then. Intellectual forces had lost so much ground that the syllabus had become academically “debased” just as we saw in Chapter 3 that educational philosophies could become debased.

DISCUSSION

By 1975, the general situation in SA was that less able secondary students studied practical statistics without probability and combinatorics, while more able students started with a formal probabilistic approach and could go on to study substantial amounts of probability, statistics and combinatorics. So in ten years stochastics had found a respectable place in secondary schools, and was starting to be seen as suitable for primary students. The pure mathematics style of these reformers influenced the approach taken to its teaching at all levels within both secondary and primary schools. The evidence that probability's introduction and development was driven by a narrow set of Intellectual forces is overwhelming, and is in sharp contrast to the developments in other parts of the world.

Because Chapters 14–16 will describe probability's partial fall from favour, here we shall look at its rise within a broader context and show that its development did not match the general trends of educational change identified by many commentators. Connell has claimed that by the 1970s curriculum construction had moved from being the task of those who “[knew] thoroughly the content of the subject they were dealing with and ...[had] some feel for the grade placement of content, items, ...”¹³¹ to being the task of a specialist body concerned principally with the processes of education¹³² and the social implications of hidden curricula. For probability this summary is too broad, just as is Hughes' claim from Chapter 12 that the principal elements of curriculum reform were all in place in Australia. So too is Blakers' claim that

[t]he process of adoption was extremely useful because it involved many teachers and administrators in thinking critically about the mathematics content and teaching methods involved.¹³³

131 Connell (1993, p. 172)

132 Connell (1993, p. 171)

133 Blakers (1978)

Yet even an American visitor to Australia in 1977 observed that

the value of probability and statistics is fairly clear, and I am glad to see that these topics are more firmly established within Australian syllabuses than seems to be the case in the United States.¹³⁴

The next three chapters will show that, at least for SA in the case of probability, all these claims are too strong, even those specifically addressing mathematics teaching. The probability of the secondary curriculum was deterministic, theoretical, and uninspiring, and was cloning itself in primary schools. We shall see in Chapter 15 just how poorly established it really was. The recommended approaches made little use of the more exciting approaches available in other states and countries, in contrast with the situation in science teaching in SA, where a book had been published to assist science teachers to use their new freedoms to develop their own courses, to evaluate them and to justify them.¹³⁵ For probability, the SA system also contrasted markedly with the findings of an overview of probability teaching around the world made by Colmez in 1979:

We have been able to distinguish various goals in the teaching of probability at the elementary level:

- (1) To provide children with the experience of chance
- (2) To introduce a useful and precise vocabulary
- (3) To construct models of probability
- (4) To start a more systematic study¹³⁶

Colmez also noted that some places restricted their activities to the first two levels and particularly to statistics, while others tried to deal with all four goals which tended to lead to

the risk of stressing the third goal too soon and of reducing the teaching of probability to a use of arithmetical and combinatorial [sic] techniques, and of making the simulation without the pupils having actually constructed the models which justify it.¹³⁷

By 1975, when there had been time to consolidate the New Mathematics changes, probability teaching in SA disregarded most of these approaches. Goal (1) was largely neglected, and most courses at all levels started with goal (4). Striking as probability's rise was, it is clear that in SA at least this rise was atypical of the

¹³⁴ A. Robertson (1977, p. 4)

¹³⁵ Secondary Science Curriculum Committee (1977)

¹³⁶ Colmez (1979, pp. 15–16)

¹³⁷ Colmez (1979, p. 16)

broad curriculum developments taking place at this time—developments subject to a much wider range of forces—Intellectual, Pedagogical and Social.

How can we explain the conflict between the many claims of advancement and stability cited above and probability's subsequent fall from favour? This question is crucial for understanding the value of the BSEM. It seems that the claims are often concerned with what is present at a given moment, or with what processes have been used to construct a given situation, and rather than with the stability of what is actually in place. If a system is in moderately unstable equilibrium, this may not become obvious until some forces change sufficiently to put it out of kilter. We have already seen the limited range of forces which led to probability's introduction. In the next three chapters the change of forces will make it clear that the foundations which had been laid were far less secure than they appeared in 1975.

Chapter 14 will describe a relatively long period during which only minor changes were made to the teaching of probability. We finish this chapter with a comment by Green, who as a teacher, historian, statistician, and psychologist, is well fitted to assess "the story so far", particularly because his work has brought him much closer to teachers and students than many educational theorists. He provides some clues about the sources of potential disequilibrium, which will help to provide a focus for the description of the "consolidation" period.

Over the past two decades the topic of 'Probability' has been brought into the mathematics curriculum but it may be that this is more an empty gesture rather than a sound strategy.

It seems very unlikely that the kind of mechanical probability calculations asked for in most texts are of much help. These are often based on an *a priori* approach with the experimental side played down (so as not to spoil the pure mathematics).

There is a strong tendency of adults to assume that immature children possess the probabilistic intuitions and concepts which they themselves find so 'natural'. Experience may educate the teacher otherwise if he takes the trouble to analyse the statements made by his pupils.¹³⁸

¹³⁸ Green (1979)

CHAPTER 14: OPTIMISATION AND CHALLENGE (1975–1985)

Comments on the Front Cover of *Australian Mathematics Teacher* for 1983

The 4th, 5th and 8th dice cannot be conventional because not all the opposite sides add up to 7.

Ken Ellis, Colac

I would have thought that anyone with any aesthetic sense would have found the 1983 cover of A.M.T. jars horribly, even if it not totally offensive. As seen, by any educated eye, none of the dots appears circular, as they should be.

... I believe that any attempt to "tart up" mathematics to make it more attractive will be disastrous [sic] in the long run.

John Barton, North Carlton

The panel in selecting the 1983 cover did indeed recognise the point [sic] made about the dice in the letters printed above. However we believed then and still believe now that the artistic merit of the cover overrides these other considerations.

Editorial Committee¹

In Chapter 13 we saw the formal approach to probability spreading through school mathematics at the same time as conservative forces were pushing for sound, usually unimaginative, teaching of basic skills. I argued that probability was less securely in place than many were claiming. Here I shall show why the position and teaching of probability changed very little in the period 1975–1985, in spite of much rhetoric for change.

OVERVIEW OF THE PERIOD

Great innovations should not be forced on slender minorities.²

Connell has described this period relatively negatively:

About 1975, there was a failure of nerve. The promise of renewal in education, the rejuvenation of the years 1965 to 1975, faltered and broke down. Criticism rather than creativity took over. An economic recession

¹ *Australian Mathematics Teacher* 39 (3): 21. The spelling error is almost certainly not Barton's.

² Attributed to Thomas Jefferson by Goodman (1997, p. 128)

began, conservatism re-emerged, financial support for schools became tighter, school support services were reduced, recently established Federal agencies were reorganised or eliminated, schools, teachers and students were criticised for poor discipline and lack of thoroughness in their work, and education became increasingly urged to direct its programs more closely towards ways of improving the nation's economic productivity.³

But for mathematics education, Connell's summary overlooks the strong pressures for radical change at this time. These pressures were moves away from formal rigour towards the alleged creativity of applied and applicable mathematics. They were not post-modernist rejections of mathematics as a discipline, rather moves for a significant change of emphasis. The response to valid criticism which is quoted above is symbolic of the new approach—the editors did not deny their technical errors, but merely saw other issues as being of more importance. These other issues were to sustain the pressure for change in school mathematics at a time when other disciplines were experiencing a “failure of nerve”.

In Chapter 13 we mentioned some forces for change, notably the changing nature of authority and the expansion of Teachers' Colleges. There were others. One was a need to improve the actual teaching of mathematics, rather than its content, and there was some co-operation between Australian States to achieve this aim.⁴ Another was the development of a pluralist, multicultural society which led to the rising up of many pressure groups who “sought to advance their ideological beliefs through the school curriculum”.⁵ A third was the increasing availability of computers and calculators which made it reasonable to question the type of mathematics needed in schools, and foreshadowed the possibility of teaching old ideas in radically new ways. The importance of calculators was formally stressed in a CDC/AAMT position paper in 1988.⁶ All of these forces meant that there were increasing pressures for a less rigid, more applied form of school mathematics. There was also a strong development of research in Mathematics Education, and a slow growth of research into probability learning, but this research had relatively little influence on classroom practice. The increasing potential of the research summarised in Chapter 8 to have assisted classroom practice must be seen as a background to all that is written in the remainder of this Part, and it will receive detailed attention in parts of Chapters 15 and 16.

³ Connell (1993, p. 683)

⁴ Keeves & Stacey (1999, p. 207)

⁵ Barcan (1996, p. 6)

⁶ CDC & AAMT (ndp, c. 1988)

It was this period which Clements⁷ saw as the time when Australian mathematics education came of age. His reasons, quite different from Connell's, were:

- the high level of co-operation between mathematics educators in the different States and territories;
- the high quality and distinctive character of recent Australian mathematics education projects;
- the improved preservice and inservice professional development of teachers;
- the greater awareness of, and planning for, cultural factors (including language backgrounds) which influence mathematics learning;
- the development of mathematics curriculum frameworks;
- moves to increase the access and success in mathematics of previously disadvantaged groups ... ;
- the development of new assessment procedures

For Clements this was a time where overseas shackles were thrown off and a Golden Age of Australian mathematics education emerged. But by 1985, for probability at least, very little would have changed in practice. Our challenge is to use the BSEM to show why so many pressures for change were so ineffectual in circumstances which the CEM would see as highly propitious. We shall also need to examine whether the attempts to make mathematics more accessible led to "tarting up" or merely to different, but still respectable, approaches. I shall start by summarising some important potential sources of overseas influences to provide a background for assessing how much Australian developments were independent of those elsewhere. The argument must concentrate on probability, which means that many interesting issues will of necessity receive little or no mention.

POTENTIAL OVERSEAS INFLUENCES

They spell it Vinci and pronounce it Vinchy; foreigners always spell better than they pronounce.⁸

One might have anticipated that overseas influences would have increased at this time because of an increasing Australian involvement in the international mathematics education scene. This was made possible by the introduction of jumbo jets on flights between Australia and Europe from 1971⁹ which led to a sharp decline in fares. In 1967 a full economy one-way fare to Europe was 9·5 weeks work by

⁷ Clements et al. (1989, p. 71); *vide* ch. 9.

⁸ Twain (1869, ch. 19), cited in Partington (1996)

⁹ *Australian Way* 54: 81, Dec 1997

the average Australian wage-earner; in 1979 the same ticket required only 4.7 weeks. Even more importantly, the cheapest available tourist return ticket took 11.9 weeks work in 1967, and only 4.9 weeks in 1979, having dipped briefly to 3.8 times in 1978.¹⁰ When we remember that teachers and academics earn more than the average wage, often receive support from their employers, and could claim the balance as a tax deductible expense, we can see how the jumbo jet almost overnight enabled many Australians to learn about the educational affairs of the world, and to play an active role in them, in ways never before possible.*

Four major events occurred during the period covered by this thesis which had the stature to influence developments in Australian mathematics education. Examining their actual influence will provide us with good evidence of the relationship between Australian practice and overseas influence.

The British Cockcroft Report

From 1978 a major review of mathematics teaching in England and Wales led to the publication in early 1982 of *Mathematics Counts*, usually known as the “Cockcroft Report”.¹¹ This Report argued against the “Back to Basics” movement without denying the need for routine skills, and emphasised the need for practical work, problem solving, the use of electronic tools, and the presentation of mathematics in applicable situations. It stated strongly that children should enjoy doing mathematics and be confident about their learning. Because some mathematical skills could not be assessed effectively in timed tests, it urged that teacher assessment should form part of formal, external, assessment procedures. The Committee members, none of whom was a statistician,¹² saw the ability to interpret simple statistical data as important,¹³ but did not specifically mention understanding of probability as important for all. It is unlikely that they fully appreciated the argument of one submission that “... the larger part of the mathematical know-how required by most school leavers not continuing in further education is arithmetical and ... a substantial proportion of its applications in ordinary working and social life is statistical”,¹⁴ but it must be conceded that this claim was not

¹⁰ Findlay (1983); *Official Yearbook of the Commonwealth of Australia* No 58, 1972 *et seq.*

* Just after the first drop in prices my return airfare from Melbourne to ICME2 in England in 1972 cost AUD1410. It is still possible (April 2001), even after substantial inflation, to find air fares to London at only AUD1500, so accessibility has continued to increase.

¹¹ Cockcroft (1982)

¹² Royal Statistical Society and the Institute of Statisticians (1979, p. 8)

¹³ Mann (1984, p. 2)

¹⁴ Royal Statistical Society and the Institute of Statisticians (1979, p. 8)

supported by hard evidence. Even for upper secondary schools, Committee members refrained from recommending teaching probability in all academic courses, in spite of seeing the topic as important at this level, because they did not believe there were enough teachers available to do so with confidence, and they were loathe to oust mechanics from its privileged position in British mathematical education.¹⁵

A bonus of the Investigation was the production of three commissioned summaries of the findings of mathematics education researchers. These were made public, and provided valuable “state of the art” overviews.¹⁶ The influence of these overviews was, of course, much wider than just Britain.

Reports like Cockcroft’s tend to generate large amounts of discussion, but are only as influential as politicians want them to be. As I recall it, Australians tended to see the report as being designed for a different country and not directly transferable, but all of the issues noted here had, or would, become important for Australian mathematics education. This is very important for our evaluation of the historical models which have been proposed because it suggests that the influences operating on the two countries were similar. In the Coda to Chapter 16 I shall propose that Convergence represents a parsimonious explanation of the fact that similar societies tend to experience similar problems at about the same time in their histories.

USA Reports 1980–1989

Another society similar to Australia is the USA. In 1980 its NCTM produced *An Agenda for Action*¹⁷ advocating problem-solving as the focus of school mathematics, and a wider view of basic skills in mathematics than mere computational facility. It also advocated an enrichment of assessment methods, together with a number of recommendations for a higher standard of mathematics teaching overall.¹⁸ It did not have a strong emphasis on applied mathematics, and made no specific comments about probability, but it did see some aspects of statistical thinking as being important basic skills, viz.:

- locating and presenting quantitative information;
- collecting data;
- organizing and presenting data;

¹⁵ Cockcroft (1982, p. 173, para 569; p. 174, para 574)

¹⁶ Bell et al. (1983); Bishop & Nickson (1983); A.G. Howson (1983)

¹⁷ NCTM (1980)

¹⁸ NCTM (1980, p. 1)

- interpreting data;
- drawing inferences and predicting from data.¹⁹

A survey of responses to the *Agenda* from groups loosely involved with schools was conducted soon after its publication in order to assess what changes would be acceptable to teachers and school systems in general. Probability and statistics did not receive strong support for inclusion in basic education²⁰ or in compulsory general education for all students²¹ although 60% of respondents supported their inclusion somewhere in school curricula.²² Strongest support came from supervisors of mathematics (73%), and weakest from presidents of school boards (43%).²³ Those respondents who supported introducing the topics emphasised that probability and statistics should not be separated, that they should be treated concretely, and that formal approaches to compound and conditional probabilities were inappropriate for primary schools. They did not consider that probability and statistics should replace work on fractions in primary schools.²⁴

Responses to questionnaires are partly determined by the questions asked. Nevertheless, these findings did point to some differences between teachers and society (as embodied by its representatives on school boards), and suggested that some teachers had had their fingers burned using poor pedagogical approaches to probability in primary schools. They uncovered no strong belief that stochastics is an essential tool for understanding the world. The 1963 Cambridge Conference had had little impact; probability was still seen as a bed-fellow of formalism.

These differences illustrate some of the ways in which the different forces of the BSEM were identified by this survey as being out of kilter with each other. Just as Whalan found in SA, American Social forces had a limited view of the general value of stochastics, and Intellectual forces were seen as having a poor influence on classroom practice. Also, as we have seen in Chapters 12 and 13, a Pedagogical force is clearly at work as teachers look for material which is not only useful but also teachable. And indeed, at just this time, as we shall see below, moves were afoot to develop an international structure specifically directed towards improving the teaching of statistics.

¹⁹ NCTM (1980, p. 7)

²⁰ NCTM (1980, p. 30)

²¹ NCTM (1980, p. 32)

²² NCTM (1980, p. 7)

²³ NCTM (1980, p. 7)

²⁴ NCTM (1980, pp. 11–12)

More reports were to follow *An Agenda for Action*. Two years after its publication the National Commission on Excellence in Education produced *A Nation at Risk* which saw both stochastics and applications of mathematics as forming parts of high school mathematics for all students.²⁵ Seven years later still two important statements were prepared. One formed the NCTM's *Standards* documents²⁶ which are discussed further in Chapter 16 and were principally a Social and Pedagogic force. The other was a report by the Mathematical Sciences Education Board of the National Research Council, principally an Intellectual force, with the short title of *Everybody Counts*.²⁷

This report argued that the New Mathematics had failed because it was not developed within schools,²⁸ over-emphasised formalism,²⁹ needed very good teachers, and also pointed to economic, racial,³⁰ and gender³¹ inequalities. Mathematics teaching, it claimed, had lost the confidence of parents and had become an object of public ridicule.³² It endorsed current trends towards a democratisation of mathematics, more experiential approaches, more use of technology, and a more creative view of the subject.³³ But most importantly it argued that

Mathematics today involves far more than calculation; clarification of the problem, deduction of consequences, formulation of alternatives, and development of appropriate tools are as much a part of the modern mathematician's craft as are solving equations or providing answers.

Statistics, the science of data, has blossomed from roots in agriculture and genetics into a rich mathematical science that provides essential tools both for analyses of uncertainty and for forecasts of future events.

...

Today's mathematics is a creative counterpoint of computation and deduction, rooted in data while unfolding in abstraction.³⁴

Although many advocates of change mentioned the changing nature of mathematics, few explained just how it had changed as succinctly as we find here. And

²⁵ National Commission on Excellence in Education (1982, p. 25)

²⁶ Initially and most importantly NCTM (1989)

²⁷ Mathematical Sciences Education Board of the National Research Council (1989)

²⁸ Mathematical Sciences Education Board of the National Research Council (1989, p. 79)

²⁹ Mathematical Sciences Education Board of the National Research Council (1989, pp. 78–79)

³⁰ Mathematical Sciences Education Board of the National Research Council (1989, p. 13)

³¹ Mathematical Sciences Education Board of the National Research Council (1989, p. 23)

³² Mathematical Sciences Education Board of the National Research Council (1989, pp. 78–79)

³³ Mathematical Sciences Education Board of the National Research Council (1989, pp. 81–84)

³⁴ Mathematical Sciences Education Board of the National Research Council (1989, p. 5)

while many proposed changes incorporated some stochastics, few treated the subject as being fundamental to all mathematics. Even stochastics' strongest advocates did not usually put forward the full strength and potential of the subject in the way that *Everybody Counts* did.

The 1982 ICOTS Conference in Sheffield, UK

In 1979, under the auspices of the Applied Probability Trust, the Institute of Statisticians, the International Statistical Institute and the Royal Statistical Society, the journal *Teaching Statistics* was established and run from the Department of Probability and Statistics at the University of Sheffield, England, under the editorship of Peter Holmes.³⁵ The journal is still running successfully in 2001.

The synergy which brought these five organisations together subsequently led to the holding of the first International Conference on Teaching Statistics (ICOTS1) at Sheffield, attended by some 400 participants, and at regular four-yearly intervals from then on. Unlike Råde's 1970 Conference to which we have referred several times³⁶ this conference was firmly set within established international structures for teachers and practitioners of statistics. Out of this meeting evolved the informal Probability and Statistics Study Group (now the Statistics Education Research Group) which aimed to provide a link with stochastics researchers around the world.³⁷ The link was less effective than one might have hoped for, simply because its maintenance required time from already very busy people.

However, the Conference was successful in crossing linguistic barriers. Of the 74 papers or reports 45% came from countries whose first language is not English. It was also successful in assembling a galaxy of stars from the statistics education constellation—Borovcnik, Falk, Fischbein, Green, Hennequin, Shaughnessy, Varga, to name a few. But although it had special sessions devoted to each of primary, junior secondary and upper secondary education, it was unsuccessful in attracting contributions from practising school teachers—66% of papers or reports were from tertiary staff, and the vast majority of the others from people working in curriculum bodies or government statistical institutions. Only two papers might have been from secondary teachers, and none was from a primary teacher. Furthermore, there were no papers at all from Australia or NZ.³⁸

³⁵ *Teaching Statistics* 1 (1): cover pages

³⁶ *Vide* Douglas (1970), Engel (1970a), Hume (1970).

³⁷ Garfield & Green (1988)

³⁸ Grey et al. (1982)

Non-presenting participants (about 300 in all) were not listed in the Proceedings, so I cannot tell how many Australians were present. Jeff Baxter did attend, and brought back copies of Minitab[®], a statistical analysis program, which formed the basis for an experimental course at Flinders University in 1983.³⁹ I know of no other work specifically influenced by the Conference even though, as we are seeing in this Part, many Australians were interested in teaching stochastics. Indeed, some Australian tertiary staff were already contributing to *Teaching Statistics* as early as 1981,⁴⁰ and two of the trustees of the Applied Probability Trust (one of the journal's sponsors) were academics at the Australian National University, and a third trustee had worked in Australia for many years.⁴¹ So, even though Australians were by then taking to the international conference trail in moderate numbers, those interested in stochastics teaching had little influence on this significant meeting, and only one State seems to have been directly influenced by its discussions. The contrast with Australians' response to ICME Congresses is marked.

The 1984 ICME Congress in Adelaide

International Congresses on Mathematics Education (ICME) have been held in 1969 and every four years from 1972. At the first three Congresses there were respectively 3 Australians out of 655 participants, 17 out of 1384 (1%), and 38 out of 1854 (2%).⁴² But a large Australian contingent attended ICME4 in 1980 and bid, successfully, to hold ICME5 in Adelaide in August 1984. Planning, led in Australia by Mack, Potts and Baxter, had taken some five years and involved large numbers of the SA mathematics education community. Mack lived for many months in Adelaide, there was a great coming and going of mathematics educational leaders, and for a time Adelaide seemed to be the centre of the mathematics education world. Numbers are not easily accessible, but about 40% (c. 800) of those attending were Australians,⁴³ many of them classroom teachers. Attendances at the biennial AAMT conferences have never reached such levels.[∞] It was a very successful conference, both academically and socially, and is still remembered with affection by many of the 2000 participants.*

³⁹ Whitford (1988, pp. 118–119)

⁴⁰ Croucher (1981); L. Johnson (1981)

⁴¹ Hitchcock (1980, p. 14)

⁴² J. Becker (1977)

⁴³ Carss (1986, pp. 382–399)

[∞] E.g., attendance at the fairly central Canberra Conference in January, 2001 was c. 300.

* Paradoxically, the train strike and torrential downpour which occurred on Excursion Day seem to have increased the level of pleasant memories.

An ICME Satellite meeting of PME held in Sydney before the main Congress contained no stochastics learning papers.⁴⁴ At ICME itself there were two poster presentations on probability and one on statistics,⁴⁵ but no general oral presentations on stochastics.⁴⁶ However, two formal Congress Sections were devoted to stochastics, and several of the presenters had attended the ICOTS1. Råde convened a section on statistics, but none of those deeply involved was Australian.[†] Bentz & Borovcnik convened a Section on “Theory, Research and Practice with respect to Probability and Statistics”, where one prepared paper was given by an Australian school-teacher,⁴⁷ and I was able to give an impromptu presentation of some of my taped interviews. There is no firm data on the influence of these two Sections on Australian stochastics education. About twenty attended the one run of Bentz & Borovcnik, who in fact presented most of the papers.⁴⁸ These were very abstract and difficult to apply in schools, so it is unlikely that they had much influence. The only direct influences of this Section of which I am aware were the personal links I made with Holmes and Borovcnik, the former of which led to my first publication in the field,⁴⁹ the latter to its subsequent translation for a German journal.⁵⁰

We shall see in the next section that at this time stochastics was increasingly being seen in Australia as an important part of school curricula. This was probably a response to the same Ultimate forces which were leading to similar conclusions in England and the USA. But the international stochastics teaching research community had little influence on early Australian moves for change, which would, by and large, be developed in isolation. For stochastics in general, there seems to be some truth in Clements’ claim that from about 1975 Australian thinkers began to question what was being taught in schools by looking at first principles, not the practices of other parts of the world. There was also an overseas influence from good new technology—for example, Minitab[®] and the video sequence⁵¹ *Against All Odds*—to be discussed in Chapter 15. But the argument of this thesis is that regardless of how much good material was used, for probability teaching many

44 Southwell et al. (1984)

45 Fifth International Congress on Mathematical Education (1984pp)

46 Fifth International Congress on Mathematical Education (1984op)

† The Proceedings of the Congress (Carss, 1986) give only very general summaries of what actually happened and who was involved where and I did not attend this session at all.

47 K, Evans (1985)

48 Seven out of nine! Bell et al., 1985, pp. 257–330)

49 J. Truran (1985)

50 J. Truran (1986), although it was c. four years before I found out that it had been translated!

51 Moore & McCabe (1989tv)

first principles were neglected, and there was no Golden Age. There were not enough readily available answers to the Social forces which were developing.

MATHEMATICS FOR LIFE

With the disappearance of certificates (except at year 12) and prescribed courses and syllabuses ... [c]urriculum development was seen as a process of determining the needs of students, and of using teaching methods, materials and content to meet their needs and aspirations.⁵²

Most efforts at reform in the 1980s failed for three major reasons. Firstly, the Departments of Education lacked sanctions to enforce their proposed reforms. They had little control over the education and training of future teachers, no effective system of inspection, no effective form of external examination. Secondly, the leadership of the teachers' unions and parents' associations usually opposed reform. They feared that school councils would undermine their centralised structures and power. Finally the various Departments of Education now contained many 'professional educationists', many of whom sympathised with and had a vested interest in progressive pedagogy. Torn between the pressures of conflicting interest groups, all the Education Departments could do was issue a flood of documents which had minimal impact.⁵³

The Back to Basics movement could not last. Educationally it was too limiting, and studies of people at work⁵⁴ had found that while a small core of mathematics could be defined as being essential for most jobs, there was an immense variety of other material which might be used, which led to the dilemma that

[i]f one reports only the core as being the basic skills then one understates the mathematics required and undervalues the sophistication. But to list all the skills which may appear in the optional section would be an almost endless task which would exaggerate the magnitude of mathematical learning required.⁵⁵

But even when using a very fine net, these studies found little place for probability. There was a small need for statistics, but the only skill identified remotely involving probability was "appreciation of accumulation of errors".⁵⁶ The studies' meticulous Academic findings had little influence on more powerful Social forces which continued to press for school mathematics to be "vocationally useful".

⁵² EDSA (1981ostp, p. 14)

⁵³ Barcan (1996, p. 16)

⁵⁴ E.g., A. Fitzgerald (1983a, 1983b)

⁵⁵ Foyster (1988, p. 31)

⁵⁶ A. Fitzgerald (1976)

Such pressures were exacerbated by a period of economic growth (as measured by Gross Domestic Product) which peaked in 1975/76 oscillated into and out of a recession in 1982/83.⁵⁷ This economic instability led to Utilitarian pressures for a more pragmatic approach to solving the country's economic ills than Liberal-Humanist philosophies would have advocated. While the election of a Liberal/National Party government led by Malcolm Fraser at the end of 1975 brought to a rapid end the visionary social agenda of the Whitlam Labor government, the return of Labor to power in 1982 under Bob Hawke did not lead to a significant swing back in educational policy: economic issues were far too pressing, and their educational consequences will be seen in the remainder of this Part.⁵⁸

And yet, as we shall see below, probability still found a place in the rhetoric of relevance which supported the emerging concept of "numeracy" as an equal partner for "literacy".[†] This was principally because she travelled along the curriculum highway in the shadow of her more obviously useful brother, statistics.* As we have seen, and shall continue to see, the closeness of the relationship has not always been obvious to curriculum planners whose frequent concern to parcel up knowledge into discrete units was often antipathetic to presenting stochastics as a way of thinking in the manner advocated so eloquently in *Everybody Counts*.

South Australia

At this time all States were producing detailed teaching guidelines to replace the externally imposed examination syllabuses of the past.⁵⁹ In SA the implications of Whalan's earlier report⁶⁰ were thought through, and the ensuing 1977 *R-12 Mathematics Report*⁶¹ distinguished two levels of basic mathematics content—Social and Prevocational. The Social Mathematics course defined *minimal competencies* which it was hoped all students would achieve during their years of schooling,

⁵⁷ McTaggart et al. (1996, pp. 508–512)

⁵⁸ Barcan (1996, p. 9)

[†] The first recorded use of this term is in 1959 (*Oxford English Dictionary*, 1989, vol .X, p. 595). It was coined for use in a British educational report (the Crowther Report), but with a far more learned emphasis than it had acquired by the 1980s (Cockcroft, 1982, p. 11, para. 37).

^{*} The gender metaphor follows Keeves, (ACER , 1972, p. 185), cited in ch. 11, Statistics may be seen as Adam's Task, probability as Eve's, though Keeves would probably not have used this reasoning at the time.

⁵⁹ Rosier (1980)

⁶⁰ *Vide* ch. 13.

⁶¹ EDSA (1977r)

and which it was expected that most students would not only achieve by Year 7, but maintain in later years. The stochastics requirements were:

- a. Determine averages for numerical data.
- b. Apply simple notions of probability in everyday situations.
- c. Recognise techniques for predicting and estimating from samples (e.g. Gallup polls, TV ratings).⁶²

Prevocational Mathematics concerned junior secondary schools and covered material later defined as “a set of skills and concepts ... upon which specific vocational training could be based”,⁶³ and was seen as being of sufficiently broad workplace application to warrant teaching to all. As well as simple statistical procedures and interpretation the following aspects of probability were listed:

Counting procedures (product rule)

Statistical probabilities

Prediction of outcomes based on data; experience of everyday situations involving probability (medicine, traffic, quality control, etc.)

A priori probabilities

Notion that p is a limit in the “long run”; sample space of outcomes, probability of single outcomes; tree diagrams, listing, array, graph (of sample spaces; probability of events containing more than one outcome, (using above methods).⁶⁴

Although the *R-12 Mathematics Report* accepted many of Whalan’s recommendations, such as the removal from the primary syllabus of logic and modular arithmetic,⁶⁵ it retained both Sets (albeit in a reduced and deformed form) and Probability. The *Report* marks an early Australian example of justifying probability on vocational grounds, and of making sound links between probability and statistics. However, it specifically excluded stochastics from courses leading to tertiary studies,⁶⁶ it continued the practice of seeing statistics as being more a useful topic than a mathematical one, and it failed to provide a clear model of a suitable classroom relationship between probability and statistics.

The ideas of the 1977 report were developed further in the major curriculum restructuring of the early 1980s headed “Our Schools and Their Purposes”.⁶⁷ This

⁶² EDSA (1977r, Appendix A), retained in EDSA (1980mcg, p. 53)

⁶³ EDSA (1984mcg, p. 2)

⁶⁴ EDSA (1977r, Appendix B), retained in EDSA (1980mcg, p. 56)

⁶⁵ EDSA (1977r, pp. 45–46)

⁶⁶ EDSA (1977r, p. 7)

⁶⁷ The basic document is EDSA (1981ostp); others followed later.

was developed contemporaneously with a Committee of Enquiry in the Education in SA chaired by Keeves,⁶⁸ one which recommended a strong emphasis on language and mathematics, probably reflecting to some extent the difficult economic times.⁶⁹ The restructuring included a resource paper on literacy and numeracy which admitted the limitations of curricula dominated by survival skills or minimal competencies, and presented a wider view of numeracy as “[helping] a person to participate actively and effectively in Australian society”.⁷⁰ In this paper both probability and statistics were seen as important: statistics for “reading, interpreting and constructing tables and charts”, probability for “using mathematics to predict”.⁷¹ The links between the two were not drawn, and the formal approach to probability which was all too evident in the prevocational mathematics syllabus was replaced by requirements that

... students should understand

- how elementary notions of probability are used to determine the likelihood of future events
- how the likelihood of chance events is measured in real life, where the most common units are “odds against” or percentage.

It is also important for students to learn to identify situations where immediate past experience does not affect the likelihood of future events.⁷²

This is, at least at first sight, a much more applied approach. But we shall see in Chapter 15 that when an attempt is made to write a probability course from an applied perspective, it is necessary to provide a significant amount of pure theory, which may easily become dry and didactic, and so negate any educational value gained from a more user-friendly applied introduction.

So the period of this chapter saw probability officially well established as important for all students. It seems to have been Social forces and the trend towards emphasising applications which led to this situation, but the reasons are only briefly touched on in the documents.⁷³ The underlying Intellectual arguments for stochastics teaching were little heeded. It was probably also felt that using an applied approach would remove some of the Pedagogic and classroom manage-

⁶⁸ Keeves, (1981sa; 1982sa)

⁶⁹ Prideaux (1988, p. 10)

⁷⁰ EDSA (1983ln, 21)

⁷¹ EDSA (1983ln, p. 23)

⁷² EDSA (1983ln, p. 23)

⁷³ EDSA (1981ostp, pp. 13, 23)

ment difficulties encountered with the more formal approach, but this was rarely stated explicitly. The failure of curriculum documents to address Pedagogic issues for probability is one of their greatest weaknesses and may well have contributed to our “swings and roundabouts” educational development, a development which an inter-active model like the BSEM is well suited to explain.

National Statements

We have seen earlier that developments within any one State were often paralleled in others, but rarely cloned. Since our principal interest is in SA, we shall only make comparisons here with one national statement, which had considerable input from SA. This is work from the CDC, whose establishment, discussed in Chapter 13, was leading towards a more visible federal influence over curriculum and a greater amount of interstate co-operation.

The CDC tried to turn a rudimentary Back to Basics movement into a visionary relating of education to real life.⁷⁴ As a federal body with no direct control over schools it had to develop and disseminate ideas and resources, for which the time was ripe. After the new Mathematics problems “principals, teachers and parents [were] beginning to object to teachers being withdrawn from their teaching commitments to attend in-service courses”.⁷⁵ Of course the CDC had no say in whether its materials were ever used. Its funding was drastically reduced in 1981, and only partially restored in 1984,⁷⁶ but even so some good material was produced. We shall look at some of its curriculum materials in the next section, here we shall examine part of its attempt to define a “Core Curriculum for Australian Schools”.⁷⁷ The CDC saw the curriculum not as a set of compulsory subjects but as something related to the “defined characteristic and major needs of contemporary society and of youth”.⁷⁸ It emphasised the *process* of learning⁷⁹ and a concern with wider ideas than those which might be easily tested, because “core learnings are varied and complex”.⁸⁰ It was opposed to national testing⁸¹ and saw problem solving, relevance and applications as the key ideas, and technology

⁷⁴ Archer et al. (1993)

⁷⁵ R. Phillips (1975, p. 120)

⁷⁶ L. Bartlett (1993)

⁷⁷ CDC (1980)

⁷⁸ CDC (1980, p. 13)

⁷⁹ CDC (1980, p. 13)

⁸⁰ CDC (1980, p. 4)

⁸¹ CDC (1980, p. 16)

as an increasingly important pedagogical tool.⁸² Such ideas were not dissimilar to those expressed in Britain and the USA, mentioned above, but they were not closely linked to the culture of the stochastics education community which emphasised learning research more than Social aspects of education. The CDC's ideas were not popular with conservative Social forces, even though it was aware that "our traditional way of packaging knowledge into required subjects no longer satisfies either society or students"⁸³ and was trying to rectify this.

As part of its emphasis on the process of learning,⁸⁴ the CDC nominated seven specific processes which should underpin teaching practice:

- learning and thinking techniques;
- ways of organising knowledge;
- dispositions and values;
- skills or abilities;
- forms of expression;
- practical performances;
- inter-personal and group relationships.*

CDC funding for curriculum projects stimulated non-governmental groups. AAMT was undergoing a resurgence, and, after some difficulties with funding,⁸⁵ it produced an A3 sized document rather grandly entitled "Australian Mathematics Education Project", but actually addressing "Basic Mathematical Skills and Concepts for Effective Participation in Australian Society".⁸⁶ The authors are not stated, but Marjorie Carss from Q and Baxter must both have been closely involved.† Section 7—"Using Mathematics to Predict"—argued strongly for the teaching of probability as part of a general education.

Students should understand how the likelihood of chance events occurring is measured in real life. They should learn how elementary notions of probability are used to determine the likelihood of future events occurring, and be able to identify situations where immediate past experience does not affect the likelihood of future events. They should be familiar with how mathematics is used to make decisions where a number of possible outcomes is likely, e.g. lotteries, weather forecasting, in-

⁸² CDC (1980, p. 19)

⁸³ CDC (1980, p. 7)

⁸⁴ CDC (1980, p. 13)

* CDC (1980, p. 16). This list was a precursor of the Finn and Mayer lists discussed in ch. 16.

⁸⁵ Carss (1979, 1982)

⁸⁶ CDC (ndp, ?1982). It must be said that the absence of bibliographic data and the inappropriate title give this document less authority that was presumably intended.

† Indeed, Olssen (1988, p. 106) attributes the Statement to Carss.

surance, superannuation, census. Students should be able to extrapolate and interpolate from data by making predictions based on trends.

Keeves made a similar comment in 1982:

The type of mathematics that is of importance in society at a particular time necessarily depends on the type of problems that the society wishes to solve. New problems have recently emerged in our society arising from a greater facility to calculate probabilities and to optimize conditions for success⁸⁷

To my knowledge these are the first formal Australian statements showing the same appreciation of the educational value of the topic found in much earlier overseas statements. The CDC statement had an unexpectedly rapid influence on SA primary schools, which will be described below. But in general the statement really formed just a small force in a world full of many competing forces. It was not received as a dramatic call to arms which could not go unheeded. Much more work had to be done to convince influential members of Social, Academic and Pedagogical environments. One important step in this process was the production of good textbook material which exemplified the principles being outlined.

APPLICATIONS TEXTBOOKS

According to one idea, there is no question at this stage of learning a theory, but essentially of acquainting the pupils at an early stage with probabilist thought in action, which is different from formal, "determinist" thought. ... According to opposing opinion, the theory of probability is a mathematical theory like other mathematical theories, and should be taught as such. It is therefore only suitable at the teaching level beyond the lower stage of secondary school.⁸⁸

During this period several sets of texts were produced which had a marked emphasis on applications. We shall discuss two in detail, and mention a number of others more briefly.

Mathematics at Work

One of the earliest to be produced was *Mathematics at Work* which was sponsored by the Australian Academy of Science and comprised six well-produced, extensively trialed, but awkwardly laid-out booklets of about fifty A4 pages each,

⁸⁷ Keeves (1982sa, p. 105)

⁸⁸ Krygowska (1979, p. 38)

designed for Year 11.⁸⁹ Its Supervising Committee was chaired initially by Barnes and then by Potts, and its Director from 1974 was Gaffney. Although there were representatives from most states on the Committee, its membership was strongly biased towards SA, but contained only two practising teachers, although several others had had extensive classroom experience in the past.⁹⁰

Gaffney, one of the former classroom teachers, would later become Superintendent of Schools (Mathematics).⁹¹ We saw in the last chapter that he had the skill of seeing and solving Pedagogic needs, and his experiences teaching academically weak Year 11 students had made him specially aware of a need to link probability and statistics,⁹² which was reflected in the work he helped to write for the project. He was followed as Director in 1980 by Vern Treilibs, formerly a high school teacher in SA, who had just returned from leave in Nottingham UK to investigate mathematical modelling in secondary schools.⁹³ Five of the six texts were prepared by South Australians, three by Treilibs. The close liaison between the Academy, senior academics, and senior teachers was undoubtedly an important factor in Adelaide's being invited to host ICME5, and is good evidence that mathematics curriculum thinking in SA at the time was widely seen as being of high quality by many. It means too that the output of the project should have been acceptable to Intellectual leaders, as well as to reformers concerned with applicability, problem solving and technology.

The probability booklet—*Taking Your Chances*—was written by Margaret Vaughton from the University of Adelaide, assisted by Gaffney.⁹⁴ The introductory chapters deal with the formal concepts of simple and compound probability, but have far more suggested activities than most other textbooks of this period. By encouraging the use of spinners by providing printed templates, students encounter symmetric and asymmetric random generators early in the course. Some questions were deliberately constructed to check whether students had developed a full understanding of new concepts.⁹⁵ Such a level of sophistication in assessment is still rare. On the other hand, the probability theory is presented

⁸⁹ Vaughton & Gaffney (1980, p. viii)

⁹⁰ Vaughton & Gaffney (1980, p. v)

⁹¹ Gaffney (1986)

⁹² Gaffney (1969, p. 18)

⁹³ Treilibs (1979)

⁹⁴ Vaughton & Gaffney (1980)

⁹⁵ E.g., Vaughton & Gaffney (1980, p. 7)

very didactically, the term “the probability” is frequently used, and, surprisingly, independence is wrongly defined as lack of influence.⁹⁶

I am unaware of any evaluation of how much or how the books were used in classrooms. They were sometimes used in academic Year 11 classes, but were more often used in the non-academic courses. I used them occasionally, particularly the work on optimisation,⁹⁷ and found them useful and usable, but many sections of work required more time for students’ to conquer than in traditional courses. They may be seen as the first major SA step towards making academic mathematics an applied, rather than a pure, study.

Miscellaneous Examples

By far the most refreshing aspect of curriculum development at this time was the way in which producers of school television tried to find interesting ways of presenting films on mathematics. Material on statistics and its applications was being screened in SA as early as 1963.⁹⁸ By 1973 there were four ABC films on Year 11 Probability being screened in Victoria.⁹⁹ In 1978 an ABC sequence entitled “Mathshow”, designed for ages 11–13 and screened across Australia, presented dramatic situations “to stimulate mathematical work in an entertaining way”.¹⁰⁰ The film “A Likely Story” illustrated both experimental and theoretical approaches to probability and analysed their strengths and weaknesses.¹⁰¹ In 1981 the BBC films “Maths Topics” were screened in SA; they included three films on statistics followed by two on probability.¹⁰²

I do not personally recall any of these films, but I did use a number of other films from the same stables with junior secondary classes and they were well received, so I presume that those on stochastics were also of good quality. Before the spread of video-cassette recorders, using television films in secondary classes was logistically difficult, and I am unaware of how widely the films were used or of how effective they were when used. Exhaustive searching in the archives of the ABC in Adelaide and Melbourne, personal contact with people working in the

⁹⁶ Vaughton & Gaffney (1980, p. 14)

⁹⁷ Treilibs & Gaffney (1980)

⁹⁸ ABC (1963, pp. 38–44). Relevant titles were: How Many People?, On the Average, Chance and Life [on insurance]. The other two films were on finance.

⁹⁹ *Vinculum* 10 (1), 1973

¹⁰⁰ *ABC Education Program Notes* Term 2 1978, p. 45

¹⁰¹ *ABC Education Program Notes* Term 2 1978, p. 45

¹⁰² *ABC Education Program Notes* Term 3 1981

ABC in the 1970s and with a small number of the older schools in Adelaide, has failed to locate any of the films themselves. It was suggested to me by staff at an ABC reception in June 2000 that the file copies were probably all over-written with other material at a time of financial crisis. Given their failure to survive it seems reasonable to presume that they had relatively little influence.

Television work started in the 1970s and was an early example of changes around 1980 which sought to make the public face of mathematics more attractive. Another was the production from 1979 of a much more physically interesting *Australian Mathematics Teacher* whose new format was probably partly responsible for Barton's rather unjust term "tart[ing] up" mentioned above. The applied movement was part of these efforts, but most of the work emphasising applications did not come until electronic aids became more readily accessible.

Four projects provide a good example of what was being produced. A useful booklet on practical applications of statistics was produced in NZ,¹⁰³ and was used at Flinders University in its early experiments with Exploratory Data Analysis.¹⁰⁴ A project sponsored by the Institution of Engineers—*Project CAM*—surprisingly did not include any applications of probability, even though at least two of the authors were statisticians.¹⁰⁵ Galbraith & Carr, after some experience in England, prepared a proposal for a project called "Practical Applications of Mathematics" which contained many interesting topics.¹⁰⁶ However, the project does not seem to have come to fruition, probably because funding sources became more scarce. Finally the *Mathematics at Work* material was restructured, revised and extended by Lowe at the end of the 1980s.¹⁰⁷

All these projects produced good work written by authors of status, but none had widespread success. We shall examine possible reasons for this in Chapter 15. The Education Department of SA would have purchased overseas materials for use if it had felt they would be suitable.¹⁰⁸ But it did not do so. It was the Australian Mathematics Curriculum and Teaching Program (MCTP)¹⁰⁹ and its pre-cursors which produced the most widely spread and long-lasting material.

¹⁰³ Gubbins et al. (1982)

¹⁰⁴ Whitford (1988)

¹⁰⁵ Costello et al. (1985, pp. 51–52)

¹⁰⁶ Galbraith & Carr (1987)

¹⁰⁷ Lowe (1988, 1991)

¹⁰⁸ Gaffney (1986, p. 2)

¹⁰⁹ CDC (1988)

Mathematics Curriculum and Teaching Program

While MCTP was a national project, its leadership came mainly from Victoria, and was the outcome of many years of development.* MAV's journal, *Vinculum*, was more lively than those from some other States, and it contained many ideas from the period.¹¹⁰ In the early 1980s Vic supported a project called "RIME"—Reality in Mathematics Education—which produced two sets of materials. One was a collection of activities called, for reasons I have never ascertained, *glima too*,¹¹¹ which included many ideas for teaching probability. The other, usually called just "RIME" was a set of model lessons for Years 7 and 8 (the first two years of Vic secondary schools) with photocopiable worksheets and extensive advice on teaching the material.¹¹² Each set contained a little stochastics. Although this material was not published until 1984, it was extensively trialed and publicised before then, so at least some of it would have been available from about 1980 to those who were interested. I have seen references to *glima too* in Victorian curriculum statements for 2001, so the material has proved to be of long-term value.[†]

MCTP drew on the wide experience of all of these projects and others¹¹³ to develop material for activities which could be used to supplement and enrich the presentation of a traditional mathematics course. It encouraged critical discussion of data so its philosophy and approach are worth explicating in some detail. I have chosen to use a unit of work¹¹⁴ prepared a little later in the USA by one of the authors, because it represents a culmination of the Project's aims, and I have heard Doug Clarke describe its development, and so have a deeper insight into his approach for this particular problem than for many of the printed examples.

Clarke saw a model of the world's tallest man at the Guinness Book of Records display in London and saw its mathematical potential. He developed a detective game using numerical data about a buried radius bone found by police. Children

* I first heard Charles Lovitt present his practical approach, using geodesic domes as his example, at an MAV Conference in 1972 or 1973.

¹¹⁰ Examples concerned with practical approaches to probability include J. Gough (1980); Lovitt (1980); Nicholls (1980); Seidel (1976)

¹¹¹ Cribb (1984)

¹¹² Lowe & Lovitt (1984a, 1984b)

[†] The reference is the CD-ROM "curriculum@work" demonstrated by Margaret Davies at the 2001 AAMT Conference. This is available on-line, but not on open Web access. This is one of the ways in which Web access is making more difficult the traditional scholarly process of citing documents which can be freely accessed. This is, of course, the very opposite of the hyperbole which surrounds the Web.

¹¹³ Vide Gaffney (1986, pp. 3–4) for a summary of SA involvement.

¹¹⁴ *Mathematics, Measurement and Me*, later published as Doug Clarke (1996)

had to decide what features, such as gender or height, they could deduce about the overall shape of the person concerned. This led to their estimating the ratio of a person's radius length to total height by collecting data and also considering normal variation in human radius bones. Once they had realised some of the difficulties of collecting adequately reliable information, they were given data from the Smithsonian Institution. They were given further information about the deceased at appropriate times, and asked to make more complicated measurements and deductions. At every stage the principle of "estimate and check" was employed to increase students' interest, and they were strongly encouraged to produce numerical relationships between sets of figures. The actual formula used by anthropologists for estimating height from bones was held in reserve to provide students with an indication of the level of precision reached by experts. Finally the project contained some work on relating skin area to height, and the relevance of these figures to the treatment of burns was emphasised.

The critical features of this approach are

- a dramatic and interesting context;
- substantial child involvement in collecting data;
- emphasis on making approximations;
- emphasis on looking for relationships;
- use of estimate and check strategy;
- making connections with real-life situations;
- providing an opportunity for discussing wider social issues in the context of mathematics.

The MCTP philosophy has created some imaginative classroom situations within a structure which could encourage teachers to try the material for themselves. The detective game was designed to cover several weeks of work, but most Australian material was developed to be completed in one or two lessons, and so not seriously disrupt the fairly rigid syllabuses of many Australian schools. The main books¹¹⁵ presented model classroom lessons known as vignettes, sometimes supported by video tapes or on computer disks. A briefer book¹¹⁶ provided guidelines for effecting curriculum change within schools or systems, and other brief books suggested alternative ways for assessing mathematics¹¹⁷ and how these alternative approaches might be integrated with the material in the main books.¹¹⁸

115 Lovitt & Clarke (1988a, 1988b)

116 Owen et al. (1988)

117 David Clarke (1988c)

118 David Clarke (1988a, 1988b)

In keeping with the theme of relevance, most of the probability vignettes in MCTP examined common forms of gambling, using structured games which, it was hoped, would lead children to appreciate the outcomes which might be expected if the gambling activity were pursued consistently for a long time. Some of the activities discussed would be seen as morally sensitive by some members of the community. The TAB is dealt with in detail, and is supported by videos of complete trotting and greyhound racing meetings, accompanied by form guides, starting prices and tips from journalists. The authors do warn that such lessons need to be handled sensitively, but they argue that if parents understand that the lessons are designed to show children the mathematical structure of such activities and not to make judgements about their rightness or wrongness, then there is likely to be considerable support from most parents.

So by the end of the period discussed here Australian teachers had a wide variety of carefully prepared examples of alternative models of teaching available for them to follow. These models supported the first of the two views in this Section's *bon mot* and in SA were supported by Keeves' thoughtful 1982 report:

In the past not many students could envisage using mathematical ideas at the level of sophistication necessary for success in problem solving. Now, with the rapidly increasing availability of computers of various sizes and power, the need for mathematical skills to facilitate the solving of an increasingly wide range of problems in daily life has meant that mathematics is no longer a subject to be studied only by the most able, but a system for solving problems that substantial numbers would benefit from if they had the necessary skills.¹¹⁹

But change to SA practice would come very slowly.

CHANGES IN SOUTH AUSTRALIAN STRUCTURES

What's the good of a home if you are never in it?¹²⁰

Because the most important changes were in primary schools, we shall start there. We shall see moderate changes in curriculum prescriptions which matched the developing ideas described above. However we shall see at the end of this chapter that many of the structural changes had little real impact.

¹¹⁹ Keeves (1982sa, p. 106)

¹²⁰ Grossmith (1894, ch. 1) cited in Partington (1996)

Primary Schools

In 1980 *Mathematics Curriculum Guidelines* were published for primary schools using three basic strands—number, space and measurement.¹²¹ The modular approach assumed that teachers would draw links between the strands, but instead it “tended to direct attention towards structuring learning in small steps ...”.¹²² But holistic approaches are difficult and the approach remained, as we shall see in Chapter 16. Teacher support material for primary schools, matched to the structure of the *Guidelines* and known as “the Modules”, was also prepared at this time and is discussed below in the section on pedagogy.

In Chapter 13 we saw that probability was recommended only as an extension activity in primary schools. In the 1980 *Guidelines* it was not mentioned at all, so it is reasonable to conclude that the recommendation had had little impact. But when the *Guidelines* were revised and extended to Year 10 in 1984,¹²³ probability was included extensively in the Measurement strand,¹²⁴ starting at Module 19 (about Year 4/5), and appearing at frequent intervals from then on.

This dramatic change in policy was unheralded, and only justified in the *Guidelines* by the attachment of the CDC Statement as an appendix,¹²⁵ so we may assume that it was introduced for the reasons in Section 7 quoted above. This seems reasonable when we remember that Baxter was involved with the CDC statement, that he and Gaffney shared similar views about mathematics, and that Gaffney held considerable authority within the Department. Furthermore, given the earlier lack of enthusiasm for probability in primary schools, it is reasonable to assume that this was primarily a change imposed from above, with little pressure from teachers.

So often probability had come stealthily into the syllabus as a travelling companion of statistics. Here there is still stealth, but it enters as a full equal with its sibling.¹²⁶ And, to the Department’s credit, its introduction was accompanied by extensive advice on its pedagogy and an emphasis on experimental work. Nevertheless, as we shall see below, the advice retained many of the old formalisms.

121 EDSA (1980mcg)

122 Gaffney (1986, p. 1)

123 EDSA (1984mcg)

124 EDSA (1984mcg, pp. 55–56)

125 CDC (ndp, ?1982)

126 EDSA (1984mcg, pp. 33. Module 27), discussed below

What is important for this thesis is knowing which forces were dominant in effecting this dramatic change. The CDC statement was in part influenced by the Ultimate force of the increasingly data driven nature of our society. Its arguments are far more practical than those outlined by Keeves after the 1964 Conference¹²⁷ and far more clear than the statements from the Conference itself. They reflect the applied mathematics movement found in the curriculum documents described above. The 1984 primary school introduction may be seen as reflecting the same pressures for change as were operating in secondary schools and they were introduced under the influence of people whose principal concern was secondary mathematics, so there will usually be no need in this analysis to distinguish primary and secondary changes.

Some of these pressures for change were Social and Physical. The increasingly difficult economic circumstances meant that students stayed at school longer because there were no jobs, and that schooling was expected to provide far more practical and employable skills than it had offered in the past. Pedagogical pressures from classrooms were minimal. As we shall see in Part E, Pedagogical practice frequently did not reflect Social aims. Although some academics, mainly from the Institute of Technology and its structural successors, were involved in developing applied approaches in schools, Academic forces were not strong. The main impetus came from people like Gaffney, Treilibs and Baxter who were essentially educational administrators and curriculum leaders.* Essentially these leaders were skilled teachers who had moved out of schools to positions of curriculum leadership and authority. They were one form of the “new professionals”, mentioned at the beginning of Chapter 8, who were seeking to usurp authority from the “old amateurs”—the professional mathematicians, some of whom wished to maintain their involvement in school mathematics.

In the 1960s people like Hamann, Ron Close and Frank Close were all in similar positions. Does this mean that the BSEM needs to be expanded further to include a new environment and force? I do not think so. Intellectual forces have some form of theoretical base, and it is a concern with educational theory as well as practice which distinguishes the discipline of Mathematics Education from its practice. I see the “new professionals” as being good teachers who have needed to move out of the classroom in order to have time to exert their influence

¹²⁷ ACER (1972, p. 185). *Vide* ch. 11.

* Baxter did teach in a CAE, which became part of a university, but his interests were not strongly directed to academic enquiry or research. To my knowledge he never completed any post-graduate qualification. Treilibs had undertaken academic reflection, but his principle interests were not in academia. Gaffney had moved directly from the classroom to various positions in Head Office.

adequately. Some of them did their best work when they had only just left the classroom, others returned to the classroom at a later stage, some were closely involved with teachers in training, all worked closely with classroom teachers. Their principal concern was a pragmatic improvement of classroom content and practice, rather than the betterment of society or the development of educational theory, although of course many had concern for these issues as well. They stand out from most teachers, and indeed sometimes differed from them, because of their skill, not their basic approach. But then Academic and Social environments are also divided, so for all parts of the BSEM it is necessary to remember that no force is likely to reflect the position of all members of its environment. There is no need to see these curriculum leaders as being more than a particularly influential part of the Pedagogical environment.

So we may say that it was a Pedagogical force which was the Proximate reason for probability's being securely introduced in the SA primary schools. But earlier, unsuccessful, moves for its introduction were also Pedagogical. Why was this one successful? One partial reason is that Social forces were stronger in 1984 than in 1964. It was easier to see how much society was becoming data-driven, but this is more an argument for the introduction of statistics than for probability. There is little published justification, but the most likely explanation is that the curriculum leaders understood the importance of probability for interpreting models, and they had sufficient authority to secure its introduction on these grounds, even though most teachers would not have been aware of the deep reasons. Although some Academics, particularly from the Institute of Technology, were supportive of the approach, it was essentially a Pedagogical force, not an Intellectual one.

Now that we have seen probability finally occupy a significant part of the official primary syllabus, other changes in SA may be described more briefly.

Junior Secondary Schools

While the 1984 *Guidelines* extended as far as year 10, the junior secondary support material (Modules 37–48)¹²⁸ never progressed beyond a loose-leaf draft form, but were widely distributed. The details were much briefer than for Modules 1–36, but they contain a rich set of recommended further resources. The three modules on probability referred to most of the newer material discussed above, as well as to the Haese & Haese texts mentioned below. One Module dealt with basic ideas and was seen as part of Social Mathematics;¹²⁹ the other two were seen as extens-

¹²⁸ EDSA (1985)

¹²⁹ EDSA (1985, 40.M.186)

ions and covered simulation¹³⁰ and modelling.¹³¹ Teachers were strongly advised to use an experimental approach when teaching the topic. The influence of these Modules is not known, but classroom practice seems to have changed little.

Upper Secondary Schools

Over the period covered by this chapter there were substantial changes in school enrolments in SA. Retention rates increased dramatically. In 1987, 66% of the cohort were still at school in Year 11 (an increase of 29% over the 1967 figures) and 49% reached Year 12 (an increase of 26%). Furthermore the tendency for girls to leave school earlier than boys, which had been quite marked in the 1960s, had been replaced by quite the opposite situation in 1987, where retention rates for girls were 6–8% higher than for boys.¹³² But at the same time there was a decline in upper secondary mathematics enrolments in SA¹³³ and a sharp decline in the time allocated to the teaching of mathematics.¹³⁴ In junior secondary classes the recommended time was 240 minutes per week, almost one half of that obtaining in some schools in the 1960s.¹³⁵ The reduced time was a strong Physical force encouraging more rigid teaching: quite antipathetic to the more reflective approaches to mathematics which were being advocated, and which probability teaching, in particular, required. The declining enrolments may have been influenced by the reduced time, and they certainly provided a further Social impetus to make the mathematics course more obviously relevant and attractive. These pressures extended to all tertiary institutions who were also experiencing a decline in mathematics and science enrolments, and the pressures were greatest at the less prestigious campuses.

The Leaving examination was abolished from 1975, but the PEB continued to provide an “advisory”¹³⁶ syllabus for Year 11 which was unchanged from the preceding examination syllabus.¹³⁷ This, not surprisingly, carried considerable weight within schools, because Year 11 studies were still precursors of the Year 12 examinable subjects taken by many students.

¹³⁰ EDSA (1985, 43.M.196)

¹³¹ EDSA (1985, 47.M.210)

¹³² Connell (1983, p. 81)

¹³³ Dekkers et al. (1986, p. 41)

¹³⁴ AAMT (1981, p. 2)

¹³⁵ EDSA (undated, c. 1984, p. 11) and personal memories

¹³⁶ PEB (1974s)

¹³⁷ PEB (1973s, pp. 77–78); PEB (1974s, pp. 79–81)

The SSC mentioned in Chapter 13 as an internally assessed, moderated course with continuous assessment had been extended to Year 12 by 1979¹³⁸ with a mathematics syllabus requiring the study of five seven-week options out of ten, of which probability was one. This option covered counting techniques and the binomial probability distribution. Applications suggested were X-lotto, poker, betting odds, runs, expectations, and simulation such as traffic light models.¹³⁹ The applied emphasis is clear, but the formal pure roots have been retained.

However, the SSC never attained the same status or popularity as publicly examined subjects did,¹⁴⁰ and it was decided to establish a single assessment authority to cover all types of courses. Eventually, in 1984 the Senior Secondary Assessment Board of South Australia (SSABSA) was established with a brief to maintain the status quo for 1984 and 1985 while developing new policies for implementation for a five-year period commencing in 1986.¹⁴¹ Curriculum Area Committees and Syllabuses Working Groups were established, although initially no Curriculum Officer (Mathematics) was appointed.¹⁴² But SSABSA's first task was to administer the new "Year 12 Certificate of Achievement"¹⁴³ where subjects of quite different depths of difficulties were presented as being equal. They were, of course, not equal, and were not seen as equal, and Chapter 15 will describe some of the difficulties arising from the tensions which were inherent in such a system.

Meanwhile, school-based assessment of public examinations, with a weighting of 25%, had been introduced in 1977, and the required publicly examined option was replaced by a scheme where teachers taught and examined internally a option of their choice. Heads of schools were expected to certify that their teachers had taught appropriate optional material, even though some had already argued that teachers and students seemed to be neglecting the required thorough attention to non-examinable English skills.¹⁴⁴ These changes were related both to each other and to the need to provide a manageable way of persuading teachers to incorporate the use of calculators into mathematics practice and assessment. It was argued that many good optional topics were unsuitable for assessment questions which could take no more than 30 minutes and the variations between

¹³⁸ *Education Gazette. South Australia* New Series 7 (27): Supplement 79/11 "Secondary School Certificate. Enrolment and Assessment. Procedures 1979" 22 Aug 1979

¹³⁹ Nolan (1980, pp. 72, 75)

¹⁴⁰ A.W. Jones (1978)

¹⁴¹ SSABSA (1984ar, pp. 2, 6)

¹⁴² SSABSA (1985ar, 1984ar)

¹⁴³ SSABSA (1985ar)

¹⁴⁴ Casse (1977, p. 23)

options produced some unfairness.¹⁴⁵ Extended, internally assessed options could make better use of modern technology than could uniform external testing. A deliberate decision was also made not to provide official support materials or advice to teachers, but to rely on their interest and enthusiasm to generate material appropriate for their students.¹⁴⁶ One suggested course showing links between two of the New Mathematics topics was that on Markov processes, which use matrix theory to calculate long-term probabilities. Although an approach very suitable for Year 10 had been developed by SMP,¹⁴⁷ the recommended approach was a very formal, tertiary approach, and the further reading comprised four tertiary books on finite mathematics.¹⁴⁸ I have not seen any analysis of how teachers used their freedom. and the “core plus option” structure disappeared in 1984¹⁴⁹ when some university staff secured a reintroduction of Euclidean geometry.¹⁵⁰

We may note that five years after these changes, the Cockcroft Committee recommended an internally assessed component of external examinations for the same reasons as were put forward in SA. This is good evidence that on at least one issue SA was not following overseas practice but leading it. However, given that it was a response to an Ultimate force of increasingly accessible technological support for mathematics, it is perhaps less an indication of Clements’ Golden Age than of the greater ease with which small States can respond to outside forces.*

Through all of these structural changes, the place of standard probability and combinatorics in the Mathematics 1S and 1 syllabuses remained virtually unchanged and, consequentially, so did their place in the syllabuses for the lower years. So too did their Pedagogical difficulties. Earlier chapters have shown that probability tended to be taught in a fairly rigid fashion, strongly influenced by its deterministic, pure mathematics, pedagogical origins. We now need to examine the Pedagogical practice of this period to see how much it was influenced by these new curriculum developments. Of course, determining actual teaching practice is difficult. As with plays, it is only the text which survives, performances

145 B. Baxter (1977, p. 19)

146 B. Baxter (1977, p. 21)

147 SMP (1967a, ch. 6)

148 Colgan (1977)

149 PEB (1983)

150 PEB (1982, 1983)

* In the late nineteenth century reforms to mathematics teaching in SA antedated those in the UK by some twenty years because SA had a still forming university and a young Professor of Mathematics (J. Truran, 1991)

are necessarily ephemeral. However, some information is available which allows a moderately comprehensive picture to be sketched.

HOW WAS PROBABILITY TAUGHT?

The 1975 syllabus for the lower secondary area are [sic] certainly considerably different from those of 1960, but (as compared with SMSG, UISCAM, and SMP) they could still be described as “trad”—certainly not as “trendy”. A substantial infusion of old-fashioned Australian conservatism has resulted in syllabuses and teaching approaches which deviate considerably less from the earlier norms than do many of the overseas counterparts. This has made it relatively easy to accommodate to recent Australian criticisms, some of which have been merely echoes of overseas critics and not always relevant to the Australian situation.¹⁵¹

In SA, the construction and use of the Department’s curriculum materials was carefully evaluated in 1986.¹⁵² Differences in approach between junior primary, primary and secondary schools were found to lead to transition difficulties for children and teachers.¹⁵³ There was a general lack of curriculum support facilities and some reluctance to undertake out of school professional development, especially if it cost money.¹⁵⁴ But, as we have already seen, since support materials were available for probability and the approaches to the topic at all levels were remarkably similar, for probability at least the situation was probably more complex than identified by this evaluation. It was a weakness of the investigation that the teaching of specific topics was not investigated, but this does enable us to argue from silence that the special difficulties of probability teaching were not seen by senior management as important enough to attract special attention.

Clearly textbooks are important, especially for inexperienced teachers, and most teachers were inexperienced in teaching probability. We have already seen that the texts used in SA tended to be didactic and deterministic, for example:

If you toss a penny in the open air it is absolutely certain to come down again. We call this a probability of 1. ... If you toss a penny in the open air it is absolutely certain that it will never come down a half-crown, no matter how often you try. We call this a probability of 0.¹⁵⁵

¹⁵¹ Blakers (1976, p. 17)

¹⁵² Starr (1987)

¹⁵³ Starr (1987, pp. 28, 44–45)

¹⁵⁴ Starr (1987, pp. 35, 51)

¹⁵⁵ *Probability Theory*, p. 15 [documents in the private domain]

What is significant about this example is that it comes from a privately prepared roneoed text developed at a fairly academic Adelaide high school at some time in the late 1960s and *was still in use* in 1985. It is a work of tertiary pure mathematics but with language modified for children, and reflects nothing of the more modern trends we have been describing. If this level of conservatism could occur at a successful school with committed teachers, it is likely to occur elsewhere as well.

We have already see how much imaginative work was available throughout the world, and pointed out that at least some of these ideas reached SA. The Variation which one might expect in a new environment certainly occurred. Later work continued to be disseminated in SA, sometimes accompanied by critical reviews, albeit somewhat superficial.¹⁵⁶ But, in spite of the many projects described above, with which some South Australians were involved, one model alone soon dominated, viz., that developed by traditional text-book writers. Teachers often adopted textbook approaches in preference to official guidelines.¹⁵⁷ In Chapter 21 we shall show how much reliance teachers place on textbooks for developing their teaching style. It is sufficient to say here that most SA secondary schools used the sequence of terse, didactic textbooks developed during this period by R.C. & S.H. Haese and their collaborators, and self-published with increasing quality as the books gained in popularity. They followed the pure mathematics approach for probability which mirrored the revised syllabuses described in Chapter 13, but, as we saw in Chapter 4, some of their probability questions had important linguistic weaknesses. Nevertheless, most chapters provided a large number of exercises, and so the books remained in favour with teachers. Other textbooks used in SA maintained the same didactic style. This is not surprising, because commercial publishers have to make profits and so tend to act as marked Physical influence against change.*

Material from a collection of papers used in the late 1970s at an Adelaide metropolitan high school can also give some idea of some aspects of the pedagogy used.¹⁵⁸ At this school students were encouraged to think widely, and supplementary questions were prepared such as "What is the probability of choosing a point at random inside a circle that is nearer the centre than the circumference?" But the answer was provided immediately beside the question, so diminishing

¹⁵⁶ E.g., EDSA (undated, c. 1978)

¹⁵⁷ Starr (1987, p. 39)

* The material of which J. Truran (1973a, 1973b) formed part was not accepted for general publication on the grounds that it was too revolutionary to sell well. This was almost certainly a correct business decision.

¹⁵⁸ High School Papers (c. 1975) (documents in the private domain)

much of the exploratory joy. Suitable questions had been sought from much older books—the collection contained photocopies from a text on probability which used the factorial notation “ \underline{n} ”, rather than the more modern “ $n!$ ”. A worked solution to a Year 12 test question shows a formal, compact approach being presented as a model. This is quite appropriate for someone who has a sound grasp of the ideas, but, as we shall see below, many students at this level did not.

Question

The probabilities that three men hit a target are respectively $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$.
Each shoots once at the target.

- (i) Find the probability that exactly one of them hits the target.
(ii) If only one of them hits the target, what is the probability that it was the first man?

Model answer.

Let A, B, C be events that each man hits target.

$$P(A) = \frac{1}{6} \quad P(B) = \frac{1}{4} \quad P(C) = \frac{1}{3}$$

$$\begin{aligned} \text{(i) } P(\text{exactly one hit}) &= P(AB'C') + P(A'BC') + P(A'B'C) \\ &= \frac{1}{6} \cdot \frac{3}{4} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{1}{4} \cdot \frac{2}{3} + \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{1}{3} \\ &= \frac{31}{72} \end{aligned}$$

$$\begin{aligned} \text{(ii) } P(A \mid \text{exactly one hit}) &= \frac{P(A)}{P(\text{exactly one hit})} \\ &= \frac{\frac{1}{6}}{\frac{31}{72}} \approx \frac{12}{31} \\ &= \frac{6}{31} \end{aligned}$$

Another source of evidence is reports from Australians at ICOTS conferences. ICOTS2, held in Vancouver in 1986, attracted nine Australian presentations,¹⁵⁹ of which one summarised the state of statistics teaching in Q,¹⁶⁰ and one the development of materials for schools education,¹⁶¹ but neither was published in the Proceedings. At the 1990 NZ ICOTS there were many Australian contributions,¹⁶²

¹⁵⁹ Davidson & Swift (1986)

¹⁶⁰ Davidson & Swift (1986, p. 503), unpublished paper by Peard

¹⁶¹ Davidson & Swift (1986, p. 507), unpublished paper by Phillips on Project CAM, *vide infra*.

¹⁶² Vere-Jones (1990)

but none in the sections on either primary teaching or psychological research into learning. Most addressed tertiary teaching, but three related directly to schools. One described the new practices being introduced for Year 12 in Victoria¹⁶³ but made no comment on either pedagogy or stochastic learning research. Another provided a good collection of activities to illustrate sampling principles, some of which were demonstrated in detail.¹⁶⁴ All the references were to statistics textbooks; the paper described examples of good teaching in action but with no underlying theoretical principles. The third, from a practising, but ec-centric,* classroom teacher, proposed approaches which were more likely to arouse students' interests than traditional ones.¹⁶⁵ This paper did address pedagogy, but the author's fervent advocacy and the abundant self-citation strongly suggest that his sensible ideas were not being practised in many classrooms. Taken together, these three papers strongly suggest that curriculum changes were not usually producing pedagogical changes, and that theoretical studies of pedagogy were having little influence, even among committed teachers.

Further evidence on pedagogy comes from the way SA primary teachers used official teaching support materials prepared from 1980–1984. The 36 booklets covering the 36 modules of the primary Guidelines were informally known as "The Modules".¹⁶⁶ They were unattractively produced, but did contain helpful background, teaching ideas, extension material, links to commonly used textbooks, and suggestions about assessment. Apart from their omission of research findings and references, they were a useful *pot-pourri* for any teacher who wanted support for improving his or her teaching skills in a particular topic.

Section 27.M.129 on Probability can serve as a representative example; it aimed to introduce children to:

Probability statements from graphs.
Predict the likelihood of events from graphs.¹⁶⁷

The teacher is encouraged to let the children make estimates of expected outcomes and then to carry out their own experiments before graphing the results using a variety of forms and observing all the standard graphing conventions. Teachers are advised that

¹⁶³ FitzSimons & Money (1990)

¹⁶⁴ Petocz (1990)

* The ec-centric spelling emphasises that the term is not being used pejoratively.

¹⁶⁵ Rogerson (1990)

¹⁶⁶ EDSA (various dates, c. 1980), individually referred to in this text as *Module n*

¹⁶⁷ *Module 27*, pp. 97–101

A probability statement compares the number of times a specific result occurs with the number of times the action took place. For example: an even number was thrown on three out of six throws of a dice [sic].¹⁶⁸

A comment is made about how any prediction made on collected data need not be accurate, but without any explanation. Two experiments are discussed—eye colour and the relative frequency of double letters in English text—with many specific teaching tips. Further experiments are referred to, as well as games and computer dice-throwing packages. The “text references” show that less than half the texts commonly used in SA primary schools dealt with this aspect of probability (including *Primary Mathematics Series*¹⁶⁹ and *Pacemaker Maths*¹⁷⁰). Finally, the Assessment section suggests making a record of the number of children per family and provides some structured statements which require the children to fill in just the numbers which they have read from their graphs, and to make some comment on the constancy of their results with different groups of children.

There are obvious weaknesses in this section, not least the requirement to fill in gaps in sentences. A formal approach to probability is dominant, as seen in the use of “a priori” where “theoretical” or “symmetric” would have done just as well.* “Counting procedures” means combinatorics here, and are not the simplest way of introducing probability. As we shall see in Chapter 18 “prediction of outcome” is an illogical approach—Konold’s “Outcome approach” discussed in Chapter 8—which tries to adapt deterministic thinking to stochastic situations. Finally, no detailed pedagogy is suggested. Nevertheless the section does provide a substantial amount of support for the teacher, and does try to breathe some life into the topic; the same may be said of most parts of the Modules. So it is not surprising that some teachers saw the Modules as defining the curriculum, rather than supporting it,¹⁷¹ and many felt there was a strong need for such material.¹⁷²

If the story stopped here it would describe some gradual improvement. But soon after the Modules were prepared, a set of books was prepared commercially with supplementary material for classroom use in a form closely linked to the official structure, but which was largely just cognitively undemanding worksheets. Here

¹⁶⁸ *Module 27*, p. 97

¹⁶⁹ Golding et al. (1969–72)

¹⁷⁰ Henderson et al. (1968–70)

* After all, a small minority of pre-service primary student teachers whose work I marked in 1999 believed that the “a” in “a priori” was the indefinite article!

¹⁷¹ Gaffney (1986)

¹⁷² Starr (1987, p. 25)

is an example from Module 27.[†] After asking children to toss two dice with certain numbers on them, and then to multiply the results and graph them on a grid with the scales already marked, the book asks the children to

complete the probability [sic] statement ‘ _____ out of a total of _____ results were between 20 and 29’.

Later they had to “predict from the graph” by filling in the gaps in the statement

It is _____ likely that a result of 30–39 will be obtained than a result of _____.

The worst part of the *Modules* was the part which was emphasised! Both Kath and I at this time encountered student teachers who referred to these collections of worksheets as “the Modules” and were also unaware that the teacher support material was what we meant when we used the term. The worksheets remain in schools two to three curriculum revisions later; the *Modules* are gathering dust.¹⁷³

Many teachers in both primary and secondary schools at this time were not soundly equipped to teach mathematics,¹⁷⁴ and this may well have contributed to the debasement of practice described here. Taken with the commercial pressures mentioned above which tend to make textbooks quite conservative, it is reasonable to conclude that most teaching at all levels, including that of probability, was relatively formal and deterministic, in spite of the availability of more imaginative models. In 1986 Gaffney wrote of

...the need to forge more closely the links between the use of language and the learning of mathematics; the need to promote active investigations by students; the appropriate use of calculators and computers; ensuring that applicability is maintained throughout the R–10 continuum; identifying practices which are deliberately inclusive of groups at risk, and exemplifying observational and assessment practices to respond to the diversity of student learning.¹⁷⁵

When we read between these diplomatic lines we can reasonably surmise that the “old-fashioned conservatism” of Blakers, mentioned at the head of this section, remained dominant. In particular, probability was rarely being well taught to most students. Some evidence for this claim will be deferred to Chapter 15; here

[†] Machin & Tilsley (1985, p. 31). “Module 27”, without italics, indicates material related to Module 27 in the *Guidelines*, and not to the resource booklet forming part of EDSA (various dates, c. 1980).

¹⁷³ Kath Truran, pers. comm., March 2001

¹⁷⁴ Starr (1987, pp. 30, 51)

¹⁷⁵ Gaffney (1986, p. 4)

we shall look at comments by the Year 12 examiners. They were able to identify poor learning, but had little idea why it was happening, and very limited views of what could be done about the problem.

EXAMINERS' COMMENTS

It appears that, over the past few years, the questions on probability have become a little more straightforward. More hints are included to overcome conceptual problems. However, many students still seem reluctant to “think”. Perhaps the concept of a “probability distribution” (i.e. how the total probability of 1 is distributed among the various outcomes) ought to be considered more deeply. Certainly there are too many candidates who hope that vaguely understood formulae will come to their assistance. This is definitely a mistaken idea, particularly with respect to conditional probability. Confidence breeds confidence. This could hardly be more true than in the teaching of probability.*

Public examination practice at Year 12 continued to be dominated by tertiary academics. Comments by examiners provide one of the few sources for forthright statements about school practices. The above quotation is a typically depressing example of their response to students' work, and seems to have neglected the following points:

- “Hints” usually replace the making of conceptual decisions, and hence lower the cognitive level at which the question will be answered by all students—good and bad. But this does not necessarily make them more straight-forward.
- The concept of a probability distribution is most likely to be well learned if several different types of distribution are studied,¹⁷⁶ but the simplified syllabus of Mathematics 1S discouraged teachers from doing this.
- A standard principle of questioning students is to assume that they will try to give the best answer they can: it may be true that many students have not thought well, but this is no evidence for saying that they have not tried to think as well as they can. It is unnecessarily demeaning to assume otherwise.
- Since the marking scheme employed gave credit for any signs of usable knowledge, it is not surprising that students wrote down poorly understood formulae. They were probably advised by their teachers to write down anything which seemed relevant.

* Colgan (1978, pp. 40–41) commenting on a Mathematics 1S paper. This article was not a formal “Examiners' Comment”, subject to modification by the teachers who were also involved with the setting and examining: it represents the personal, public view of the Chief Examiner, and is in fact softer than the formal Comment where he reported that “the concept of conditional probability ... is still not understood at all” (PEB, 1977, p. 2).

¹⁷⁶ Skemp (1971/1986, ch. 2)

- “Nothing succeeds like success” and “start from where the child is” must be the two most strongly emphasised maxims in all courses of teacher training, and guiding principles for many successful teachers. Colgan seems to imply that “confidence breeds confidence” will be a new idea for most teachers. This is as demeaning of teachers as the earlier comment was of students.

This example has not been chosen for the sake of “lecturer-bashing”.[†] This section is concerned to show the gulf which exists between tertiary and secondary teachers—between Intellectual and Pedagogical forces. Examiners’ comments provide one of the few available sources for demonstrating this gulf.

The underlying theme of the examiners’ reports on probability questions (many of which also included combinatorics) during this period is one of confusion about why students did poorly. They believed they could understand “inadequate mathematical skills, and careless presentation”¹⁷⁷ or the overlooking of “many opportunities for self-checking by internal consistency”.¹⁷⁸ They could even understand what they believed to be the practice of some students, and perhaps their teachers, of simply not bothering to study the topic in any detail.¹⁷⁹

But they could not understand why in some years that the quality of students’ answers to the probability question was inversely related to the quality of their other answers. Usually they reported this in a straight-forward way,¹⁸⁰ but one year they cuttingly observed that “[s]ome of the best answers came from candidates who, showing more wit than skill, had performed poorly on the rest of the paper”.¹⁸¹ Simplistic, sometimes acerbic, remarks like this are frequently found in their comments. The following examples refer specifically to stochastics and indicate clearly some of the examiners’ confusions:

But the answers to probability questions simply confused them. The values $\frac{1}{4}$, $\frac{1}{5}$, $\binom{4}{1}$, $\binom{5}{1}$ were muddled up with gay abandon.¹⁸²

[†] SA is a small state: I need to say that I have always found Len Colgan to be pleasant and approachable, and that all our dealings have been without any form of antagonism. The trenchant criticism here is of his statements, and in no way of the man himself.

¹⁷⁷ PEB (1978, p. 1)

¹⁷⁸ PEB (1978, p. 2)

¹⁷⁹ E.g., PEB (1982exm1, pp. 1–2)

¹⁸⁰ E.g., SSABSA (1983exm1, p. 1)

¹⁸¹ PEB (1978, p. 4) *Vide* ch. 17 for similar findings by Kempster (1982).

¹⁸² PEB (1976, p. 2); one of the last times the word was used with this meaning in a public document

In past years, the probability questions has been placed last, and suffered, we thought as a result. The question was poorly done again, and attempted last by some candidates in spite of its placing this year.[†]

This section was generally well done, particularly the question relating to independence of events. However, inexplicably the conditional probability was virtually never recognised, even by the very best candidates.¹⁸³

Fairly well attempted by comparison with previous years, which was not surprising since a similar question has appeared twice in the past ten years. ... Quite often, the idea of conditional probability was not detected in b (3).¹⁸⁴

Very poorly done by most. Many answers could only be described as wild lunges at the problem. ... [Some] used (implicitly) the “formula” $P(A|B) = P(A)$, arrived at by assuming that all pairs of events are independent.¹⁸⁵

This question was extremely poorly done. Many candidates could not list the events in set ‘A’, and did not appear to know the meaning of ‘sample’ space. ... Without, apparently, considering the underlying sample space, they attempted to write down probabilities. Not surprisingly, these were usually wrong. ... Many candidates were unable to explain their answer coherently and might have scored better had they been able to communicate their findings to the examiners.¹⁸⁶

Again, this list is not presented to indulge in examiner-bashing. It is presented to show which issues concerning examiners could have been helpfully explained from within Mathematics Education. Most of the comments address the desire of students to use formulae without understanding. We know why this happens, and how to address the problem, but changes in teaching methods are needed. If the examiners had been prepared to accept that the problem might be able to be solved, and had tried to find out how, improved learning might have resulted. It was certainly unlikely to have happened as a result of their unsubtle put-downs.

To be fair, examiners did try to suggest more productive approaches, especially the use of Venn diagrams.¹⁸⁷ For example, in 1985 they made a serious effort to provide constructive advice:

[†] PEB (1978, p. 4). The question had been moved from seventh to sixth out of seven!

¹⁸³ PEB (1980, p. 2)

¹⁸⁴ PEB (1982ex, p. 3)

¹⁸⁵ (PEB (1983exm1, p. 3)

¹⁸⁶ SSABSA (1986, p. 4)

¹⁸⁷ E.g., PEB (1980, p. 3); SSABSA (1983, p. 3)

Most students seem to attempt questions on probability by asking the wrong question, viz., “What is the probability that the described event occurs?”. The lead-in to question 13 was supposed to make them think one step back and ask the simpler question, “In how many ways can the described event occur?”. This is helpful only if the student clearly sees the sample space. Without this they are again left to grope for numbers. It is, of course, not possible to list every element in a sample space of reasonable size, but it is possible to think of some elements that are in the space. For example, (3, 8, 7) is in the space for part (a), and so is (8, 3, 7). These two elements are different: in the first, number ‘3’ wins the first prize, whereas, in the other, number ‘8’ wins the first prize. (4, 6, 4) is not in the sample space of part (a) as tickets, once drawn, are not replaced, but it is an element of the sample space of part (b). By asking questions about which elements are in the sample space, and which are not, candidates might have some chance to lead themselves to a sensible answer. ... There is no guarantee that this will lead students to the correct answer. ... The examiners, however, believe that the above might be a more fruitful approach than scribbling numbers on a page and then crossing them out again.¹⁸⁸

Putting aside the cynical peroration, this clearly presented approach still does not make explicit the underlying assumption of equiprobability. It also avoids the important pedagogical issue of deciding between Venn diagrams, trees, or other ways for a given question. This issue has never been addressed in any SA examiner’s comments which I have read, and it was never addressed in the textbooks most commonly used in SA schools at the time.[†] In my experience it is a critical issue. It was addressed in some Victorian texts,¹⁸⁹ and these were used by a few schools,¹⁹⁰ but they do not seem to have had any wide influence in SA.

There were other issues which the examiners failed to address. One was the poor reputation which Venn diagrams had acquired in the days of New Mathematics. While the textbooks did use them, the teachers may not have, and certainly many students did not. Another was their assumption that the term “the probability question” could be reasonably applied to quite a wide range of questions which may or may not have included combinatorics. The examiners seem to have assumed that the whole content of the syllabus would have been learned as a whole, all to the same level of understanding.

¹⁸⁸ SSABSA (1985ex1s, pp. 4–5)

[†] In Year 12 most schools used Haese et al. (1980), a hand-written text, later marginally expanded to the better produced Haese et al. (1982m1, 1982m1s)—books which the authors themselves describe in their introduction as a only “semi-texts”.

¹⁸⁹ E.g., Lynch et al. (1977, pp. 227–228)

¹⁹⁰ Usually Lynch et al. (1977) and later versions

The problems identified here were present ever since probability had been introduced into the secondary syllabus. While the problems of students' lack of understanding were well recognised, examiners' lack of understanding does not seem to have been. This section has shown that Intellectual forces may have limitations, made the more serious by a general lack of awareness of their existence. Green pointed out in the *bon mot* at the end of Chapter 13, "Experience may educate the teacher ... if he takes the trouble to analyse the statements made by his pupils."¹⁹¹ The examiners had not even realised that they too needed to take some trouble.

SUMMARY

This chapter has strongly suggested that amidst all the turmoil of Social and curriculum change little was changing in classrooms with respect to the teaching of probability. The "Optimisation" of the Chapter's title was an Optimisation of the forces, not of *obuchennyi* quality. At the same time as Adelaide was enjoying the excitement of ICME5, and also acting as a centre for some creative new work, the teaching of probability in SA was largely uninspiring and unsuccessful.

Most of Clements' claims, cited above, about mathematics education for this period do not hold for probability teaching in SA. There was good Australian work, and improved professional development and interstate co-operation but it was not having any marked effect in the classrooms. Furthermore, issues of culture, language and disadvantage had not been part of the discourse, many children were continuing to fail, and assessment procedures had changed little. This was a time of some good change, but it was scarcely a Golden Age and it would not improve markedly in the near future.

If probability teaching was being unsuccessful, why was it not changing or being removed, as Q teachers were advocating?¹⁹² Was it merely that teaching expertise takes about ten years to develop and differs from the logical rational model of planning which student teachers get taught?¹⁹³ Was it that many teachers do not reach this stage, but remain in Nolder & Johnson's "coping" phase reported in Chapter 7?¹⁹⁴ SA teachers were probably coping, and the *bons mots* which head the sections in Chapter 22 suggest that this was at the expense of the topic itself.

¹⁹¹ Green (1979)

¹⁹² G. Jones (1979)

¹⁹³ Condon (ndp, c. 1991)

¹⁹⁴ Nolder & Johnson (1995)

But the extensive broad reflection which Condon has suggested is one mark of an expert teacher does not seem to have been happening.¹⁹⁵

Reflection was certainly happening among a small group of curriculum leaders, and their work was extensively trialed in many schools. Many of these leaders were influential in SA, so it is not surprising that the new approaches received enough support to overcome teacher opposition and some teacher lethargy. These leaders were sufficiently strong that mathematics curriculum development was to some extent running against the trend of most States, which were seeking to regain the curriculum initiative they had lost to the radical progressive educators in the period 1967–1974.¹⁹⁶

But leadership of this small group was largely only a Pedagogic force, weakly supported by Social and Intellectual forces, and it came from only one part of the teaching profession, largely those who were not in classrooms. Some of the curriculum leaders, Lovitt and Doug Clarke in particular, were very good at introducing material to teachers, and they did exert some Charismatic leadership. But inevitably this leadership could not extend to supporting classroom teachers across Australia having difficulties introducing the new approaches. The leaders do not seem to have been concerned with theoretical issues within the developing discipline of Mathematics Education, nor with making specific links with those closely concerned with statistical education, such as, for example, by contributing to the section on modelling at the NZ ICOTS Conference.¹⁹⁷

The curriculum changes of this period were driven principally by one force—a Pedagogical one of a very specific kind, and only weakly supported by others. These were certainly different forces from those operating when probability came into the syllabus, and the reasons for their change may be reasonably explained by the substantial Social changes of the ensuing twenty years.

But a change driven by one force is likely to encounter more than its fair share of opposing forces. One which is always present is the Physical force of the time required to implement effective change, and some debasement of practice is therefore inevitable. Another which is inherent in the applied approach is the difficulty of producing valid and reliable assessment. Practical aspects of this will be discussed in Chapter 15 and theoretical aspects in Part D. Here we may note that assessing learning requires the skills of Mathematicians, Mathematics Educators,

¹⁹⁵ Condon (ndp, c. 1991)

¹⁹⁶ Barcan (1996, p. 1)

¹⁹⁷ Vere-Jones (1990, pp. 121–143)

and Pedagogues. The old amateurs, the new professionals and the uncommitted practitioners are all part of the same system. We have seen what can happen if Mathematicians have too much influence: Chapters 15 and 16 will show respectively what can happen when Pedagogues and Social forces are dominant.

Community confidence in teachers and their assessments is vital for ensuring their influence on educational practice. Recently, complaints like the following became common in Victoria, which has experienced the most widespread implementation of and applied approach:

There is widespread copying of work requirements by less organised students. Less work (content) is learned than ever before, but more hours are put into school requirements. At year 12 level, there is a growing use of outside assistance for Project [Common Assessment Tasks].¹⁹⁸

Significant changes in Victorian certification procedures had to be effected to restore public confidence in educational leaders.¹⁹⁹

This chapter has described a situation where the forces for change were strong, competent, and well organised, but had only marginal impact. The BSEM suggests that this was because the reformers did not address all aspects of curriculum change, especially the Physical forces acting on classroom teachers, and ensuring that the changes led to improved learning. One school of thought holds that classroom change in general can only be effective by changes in examinations.²⁰⁰ In Chapter 15 we shall describe attempts to advance the cause of modeling by just this method. The attempt would prove almost fatal for probability.

... in the words of Chuck Colson, President Nixon's henchman, when you've got them by the balls, their hearts and minds will surely follow.²⁰¹

¹⁹⁸ Roberts (1994, p. 39)

¹⁹⁹ Michell & Stephens (1995, p. 13)

²⁰⁰ E.g., Grouws & Meier (1992, pp. 89–90); Sherrington et al. (1987, pp. 113–114)

²⁰¹ *Yes Minister* Lynn & Jay (1981, p. 121)

CHAPTER 15: DECLINE (1985–1990)

VALENTINE: Yes. There was someone, forget his name, 1820s, who pointed out that from Newton's laws you could predict everything to come—I mean, you'd need a computer as big as the universe, but the formula would exist.

CHLOË: But it doesn't work, does it?

VALENTINE: No. It turns out the maths is different.

CHLOË: No, it's all because of sex.

VALENTINE: Really?

CHLOË: That's what I think. The universe is deterministic all right, just like Newton said, I mean it's trying to be, but the only thing going wrong is people fancying people who aren't supposed to be in that part of the plan.

VALENTINE: Ah. The attraction that Newton left out. All the way back to the apple in the garden. Yes. *(pause.)* Yes, I think you're the first person to think of this.¹

For probability in SA the important changes in this period were its removal from Year 12 academic mathematics courses and the partial blackballing of Quantitative Methods by some tertiary institutions. SA was unexpectedly running against the stream of both national and international trends. In education as a whole, governments were regaining control over schools and curricula through a corporate management model, in which outcomes were seen as the critical measure of success.² The inspirational movements of the early 1980s had come too late. So it is only necessary here to describe the general background of this period and the major changes: most aspects of probability teaching remained unchanged.*

GENERAL BACKGROUND

There is a certain relief in change, even though it be from bad to worse; as I have found in travelling in a stage coach, that it is often a comfort to shift one's position and be bruised in a new place.³

In Chapter 14 we showed how upper secondary school populations were increasing and hence diversifying: new approaches became necessary, and this chapter concentrates on the most radical of the proposed mathematics changes—the need to be applicable.

¹ Stoppard (1993, pp. 73–74)

² Bessant (1989)

* I am grateful to Michael Wheal for his comments on an earlier draft of this chapter.

³ Irving (1824), taken from Goodman (1997, p. 128) and Partington (1996)

Such pragmatism was also behind the Labor party's return to national power under the leadership of Bob Hawke during a period of economic downturn in 1982. This was a right-wing Labor, quite different from Whitlam's Labor just eleven years earlier, and it oversaw a period of intense economic rationalism. The dominant political ideologies of the 70s—equality and empowerment—had to compete with those of productivity, efficiency and international competitiveness.⁴ The corporate federalists played down the Humanising benefits of education and lauded training for Technical skills.⁵ Two Federal Ministers, John Dawkins and Clive Holding, set education on a path whose destination would be the National Curriculum documents of the early 1990s,⁶ to be discussed in the next chapter. But in spite of all these managerial decisions, the high economic growth which had occurred in 1984 had slumped by 1987, and, after a moderate rise, slumped again in 1990/91.⁷

We saw in Chapter 14 that mathematics teaching at this time was often uninspired. As the economy lurched up and down, salaries increased more slowly than prices* and school discipline became more difficult. Teaching was becoming less and less attractive. There were few mathematics specialists, and some teachers with an antipathy towards the subject. Mathematics teachers tended to teach in several faculties, so it was difficult to develop strong mathematics departments. The Modules approach in primary schools had led to a greater diversity of knowledge held by children entering secondary schools, which led Year 8 teachers to adopt very rigid approaches.⁸ The times were not ripe for creative change.

Yet at the same time as governments and industrial leaders were advocating an increased training role for schools, others, such as SA's charismatic Garth Boomer, were arguing for a greater relevance and liveliness in school mathematics. In an address to the 1986 AAMT Conference, Boomer said that his experience of learning and teaching mathematics had been that it was "cut and dried", "frozen", in marked contrast to his more creative experiences teaching English. He argued that in the 1980s mathematics, alone of all school subjects, had remained un-

⁴ Dudley & Vidovich (1995, p. 102)

⁵ Lingard (1993, p. 27)

⁶ L. Bartlett (1993)

⁷ McTaggart et al. (1996 pp. 508–512)

* The "Index of Weekly Award Rates of Pay for Wage and Salary Earners" for SA rose from 100 to 205 for men and from 100 to 212 for women between 1976 and 1984, but the Consumer Price Index from 1974/75 to 1984/85 rose from 100 to 255 (*South Australian Year Book and Year Book Australia*).

⁸ Starr (1987)

transformed: its teachers were rigid and didactic, rather than flexible and challenging. Boomer wanted mathematics classrooms to become centres of mathematical production and communication, using a Constructivist, applied approach based on well-developed theories of learning and increased mathematical knowledge of the teachers. He also claimed that

the present state of mathematics is a threat to democracy. ... Human dignity is undermined by the submerged guilt about inadequacy that resides with so many of our citizens.

In addition to calling for a major overhaul of content, curriculum and textbooks Boomer argued that there was a “need for mathematics teachers to wrest back responsibility for what happens in the classrooms”.⁹

Less Charismatic views were expressed by Newton, from the Victorian Ministry of Education, at the 1988 MAV Conference. He saw current practices as merely “an accumulation of mathematical details”¹⁰ and claimed that “[i]n fact students have got it right: education *should* be useful, not later, but now”.¹¹ and that “[we need] to reduce the amount of content and come up with a reduced number of mathematical ideas—the core, essential ideas”.¹²

Both these views may be compared with the following view which we met in Chapter 6:

Hungarian children are not told that mathematics is useful. ... They are led to believe it represents a pinnacle of mankind’s intellectual achievement. That not knowing and understanding mathematics is something about which one should be ashamed. ... That their teachers are teachers of mathematics and not some over-rated aspect of social education.¹³

The serious attempts described in Chapter 14 to develop more creative and inspiring models for teachers were consonant with the views of Boomer and Newton, but stood in marked contrast to the desire of governments as a whole to regain control of schools and their curricula. And none came anywhere near the Hungarian view that mathematics is as much a significant part of the world’s cultural heritage as are Music and Art. They all retained a strong Utilitarian streak.

⁹ Boomer (1986)

¹⁰ B. Newton (1988, p. 72)

¹¹ B. Newton (1988, p. 71)

¹² B. Newton (1988, p. 72)

¹³ Andrews (1998, p. 4)

The prognosis for successful radical change, at least for probability, was not hopeful. A comprehensive international survey undertaken just before this period had found that probability had become established in many countries, but with many different emphases.¹⁴ Most importantly, while probability calculus had received some degree of emphasis in most countries, there were no countries at all which saw comprehension, application and analysis as being important at age 13+. Indeed, the researchers found that it had

proved easier to introduce new techniques that [could] readily be put to use on collections of exercises, than abstract concepts: a fact that should cause no surprise, but which should not be overlooked when planning for the 1990s.¹⁵

This is just what Freudenthal, who shared the Hungarian, Liberal-Humanist view of mathematics, had predicted in 1973 when he wrote:

There is little chance left that incorporating probability into school programmes can be prevented. Must this lead to spoiling the last oasis of reality-related mathematics? I am afraid this is inevitable if probability is considered as a subject that is worth devoting two hours a week in some of the higher grades.¹⁶

Australia would take the same path as many other countries. In Chapter 14 we saw an increasing rhetoric about the value of probability for a general education. There were variations between States, but in most places, including SA, its position had remained part of traditional, academic, pure mathematics courses and their precursors lower down the school system. In spite of the CDC statement which included stochastics in its list of essential mathematical skills for all,¹⁷ many radical reformers of the late 1980s do not seem to have seen probability as being a necessary part of a useful curriculum, let alone as essential for all in the deep way advocated by the 1963 Cambridge Conference.¹⁸ For example, in the revision of the syllabus which Newton was developing at that time, probability did not appear until Year 7.¹⁹ The MCTP material contained relatively little stochastics, even though there was a considerable emphasis in Victoria on this topic, and this deficiency was only rectified²⁰ after the publication of the National Curr-

14 Howson & Wilson (1986)

15 Howson & Wilson (1986, p. 47)

16 Freudenthal (1973, p. 613)

17 *Vide* ch. 14.

18 *Vide* ch. 11.

19 Cribb et al. (1988, pp. 96–101)

20 Lovitt & Lowe (1993a, 1993b); Finlay & Lowe (1993)

riculum. An Australian questionnaire prepared about 1990 to establish what various groups of people considered to be desirable skills possessed by a mathematically literate person did not even include probability at all, though it did mention tables and graphs.²¹ The rhetoric often lacked supporting commitment.

As the opposing factions sought to stretch educational practice to fit their own perceptions of what society and its members needed, the teachers remained at their black-boards, working harder and in much more difficult conditions. The rhetoric of the reformers was full of praise for the teachers, but the majority of teachers—uncommitted practitioners—would act as a markedly conservative influence against all moves for change, whether from their political masters or their Charismatic leaders. Pedagogy is discussed in detail in Part E, and we have already seen that a Pedagogical force will need to be built into the BSEM, so it needs to be kept in mind throughout the stories told here and in Chapter 16.

Similarly, the background material in Part B also needs to be kept in mind. The educational philosophies being proposed at this time were very different from each other, and their advocates and curriculum implementers seemed to have had little idea that other philosophies were even defensible, let alone possibly desirable. There seems to have been little room for Eisner's connoisseurship. Using the model of Howden & How described in Chapter 7 we may see the principal tension as being between the Technologists and the Humanists, but the Academic Rationalists also had a part to play. The influence of the Social reconstructionists had diminished markedly with the fall of the Whitlam government and the consequential reduction of funding for the creative centres it had tried to establish.

For me the most striking aspect of the situation is the lack of evidence that many people really understood the place which probability might fill in a general education, using the term in a Liberal-Humanist sense rather than a Utilitarian one. Because it was an applicable topic, its identity became lost in the push for applicability. Paradoxically, the very obviousness of the need for members of our society to be able to read statistics caused the non-deterministic nature of the underlying probability theory, and also its implications for school and educational discourse, to be overlooked. It is the material discussed in Chapter 5—The Meaning of Probability—which seems most to have been neglected. In that chapter I quoted the following:

²¹ Galbraith et al. (1992)

Thus before we can carry out an empirical investigation of teaching methods we are faced with the difficult task of getting clear what is involved in, say, thinking historically, and thus in learning to think in this way.²²

The issue of the clarifying what was involved in thinking stochastically was rarely addressed at this time. Statistics was seen more as a tool than as a significant contribution to our culture and way of thinking.[†]

So at a time when some Social forces were becoming quite conservative, and world-wide experience suggested that a conservative, rather rigid, approach to teaching probability was the default one, other Charismatic, Social forces within education were becoming more radical. Society's emphasis on instrumental, rather than relational, understanding was accompanied by an Intellectual failure to appreciate the real nature of statistical thinking. This chapter will examine one attempt by teachers to inspire, and gain control of, the curriculum; the next will examine an attempt by governments to do so. Both will demonstrate the unsatisfactory consequences of responding to poorly thought out forces.

NEW PEDAGOGICAL MODELS

If birds of a feather flock together, they don't learn enough.²³

In Chapter 14 I provided a variety of evidence to support my claim that most teaching of probability was uninspired and deterministic and did not reflect the rich set of pedagogical models which were then being developed. Three other models need to be described here. The first is a reasonably good attempt to link research and practice; the second is the development of non-analytic approaches to statistics commonly called "Exploratory Data Analysis" (EDA); the third is a locally produced text which formed a basis for some of the major changes which will be described in subsequent sections. In these sections we shall see how the failure of teachers to develop a sound pedagogy was one of the reasons for probability's demise.*

²² Hirst (1974, p. 117)

[†] Papers like Wild & Pfannkuch (1999) were not available at the time.: perhaps it was too early for the questions they address even to have been conceived.

²³ Attributed to Robert Half by Goodman (1997, p. 258)

* Describing recent and controversial events which occurred in a small city and with which I had a very minor involvement makes it difficult to balance objectivity, confidentiality and sensitivity. While I believe that I have retained my objectivity in what I am about to

The Open University

An Open University distance education teacher preparation course from the end of this period provided pedagogical ideas based on a rich set of experiences and was in a form amenable to wide dissemination. It was reasonably attractively produced, and included some mathematical background and also research findings.²⁴ While it could be criticised on several grounds (for example, it repeats²⁵ the belief about children's antipathy to "6" being due to experience of board games, discussed in Chapter 8), it was a marked step forward in probability pedagogy which emphasised that good teaching needed to integrate:

- language patterns;
- different contexts;
- standard misconceptions;
- imagery/sense of topic;
- root questions;
- techniques and methods.

In other words, its approach was holistic and supported many of the points which are being made in this thesis. I am not aware of what evaluation was done on this material, but it represented a quantum leap forward in the development of a soundly based pedagogy, and showed what was achievable at that time. The material included EDA, but had a much wider educational background and was recommended as background reading²⁶ for the new SA Year 12 course discussed below. Another book from the Open University was also being recommended for SA teachers with an interest in statistics at that time, but this only just touched on

describe, perhaps I do need to state my position. Michael Wheal has been both a colleague and friend with whom I have maintained amicable relations over more than thirty years. Jeff Baxter and I had one serious and unresolved disagreement about administrative matters for ICME, but in all other ways our relationships have been pleasant enough. My relationships with all the others involved have been friendly, although not without some professional disagreements. I voted against MASA's taking legal action, but my sympathies were with neither side. Both held to their positions without what I saw as clearly explicated reasons, and I did not find individuals particularly willing to debate alternative solutions. I did have to provide student teachers who had done no Euclidean geometry with a crash course in the topic, which was certainly not easy in the time available. At the time my Masters studies were not far enough advanced for me to have come to a full appreciation of the nature of Mathematics Education, but I did know that there was a wealth of research evidence which was not being utilised. In places I know of no written documentation for my claims, so I have been forced to cite my own experiences.

²⁴ Gates (1989)

²⁵ Gates (1989, p. 12)

²⁶ SAIT(2) (1989) [documents in the private domain]

numerical estimates of significance, and we shall see below the limitations of this approach.²⁷ The full influence of these two books in SA is not known.

Exploratory Data Analysis

At the end of the 1980s teaching statistics using EDA became increasingly common around the world. EDA was developed by John Tukey at Princeton, and sought to provide an introduction to statistical inference and sampling distributions based almost entirely on the analysis of data sets. It required little algebra, was amenable to computer support, and was seen to be particularly valuable for potential users of statistical results, many of whom had weak mathematical backgrounds. The approach was experimentally based and mathematically rigorous, and several textbooks were produced by highly regarded statistics teachers.²⁸ Tukey saw his approach as complementing traditional statistics, not replacing it.

Once upon a time, statisticians only explored. Then they learned to confirm exactly—to confirm a few things exactly, each under very specific circumstances. As they emphasized exact confirmation, their techniques inevitably became less flexible. ... Anything to which a confirmatory procedure was not explicitly attached was decried as “mere descriptive statistics”, no matter how much we had learned from it. ... **Today, exploratory and confirmatory can—and should—proceed side by side.**²⁹

Not all statisticians were as enthusiastic as Tukey, and division in the statistical ranks influenced the events to be described below. EDA was first taught in a tertiary course in Australia at Flinders University in 1983.³⁰ Bob Hall, at the SA Institute of Technology, also took it up, both in his Institute teaching and in classes which were offered to practising teachers undertaking a Graduate Certificate in Mathematics Education.[†] His teaching was later enhanced by a set of teaching videos, *Against all Odds*.³¹ Although I was on the Graduate Certificate committee

²⁷ Graham (1987, pp. 49–51)

²⁸ E.g., a very chatty section by D. Moore in Garfunkel & Steen (1988); Moore & McCabe (1989); Rouncefield & Holmes (1989)

²⁹ Tukey (1977, p. vii), bold as in original.

³⁰ Whitford (1988, pp. 118–119). *Vide* section on ICOTS in ch. 14. My memory is of hearing about EDA from a statistician friend some five years earlier than this, so it had been known about in some Australian universities well before 1983.

[†] The Graduate Certificate in Mathematics Education was a joint project of all SA tertiary institutions, which provided in-service education for teachers under the auspices of the Adelaide Consortium for Mathematics Education. It had a strong focus on modelling and statistics, and attracted small numbers of teachers for as long as they received release-time funding, but went into recess soon after the funding dried up.

³¹ Moore & McCabe (1989tv)

for some years, I never heard any reasons expressed as to why EDA was to be preferred to other ways of teaching statistics, or what it was which distinguished it from the other ways. But it was clear that those proposing the course believed that it was *the* way to teach statistics in secondary schools and basic tertiary courses, and they were working towards achieving that aim.

Wheal's South Australian Text

In c. 1986 Wheal wrote a probability text³² to show how the topic could be integrated into a mathematics course with a strong applied emphasis. The book reflects Wheal's later description of his educational philosophy:

The aim of having students develop their capacities to operate independently has been common in mathematics curricula for a long time. Until recently, in most education, this was to be achieved after the students had acquired the fundamental mathematical knowledge of the time. Unfortunately, in the process the vast majority of the population was gradually weeded out and in the end only a few survivors learned enough to be allowed to try to exercise the freedom to take risks in mathematical thinking.

In recent times there has been a change proposed in how this ideal should be achieved. Instead of beginning by rattling the bones of the past, the first step was to encounter situations where particular knowledge was valuable in developing understanding of the situations and problems which arose within them. Trial and error, experimentation, modelling and exploration were promoted as the way to go.³³

The book shows no evidence of using research findings, rather it was a simplification of some ideas in the AAS's *Taking Yours Chances*.³⁴ Wheal wrote specifically for correspondence students, but, given his position as Chief Moderator for Year 12 Applied Mathematics from 1987–1990, it probably had wider influence. The discussion here is based on a later revision of the original text³⁵ because this probably contains what Wheal would see as the clearest example of his approach.[†]

The text claims to be an example of mathematical modelling. It starts with a simulation of a cricket game using dice, which forms a basis for some fairly tradition-

³² Wheal (ndp., c. 1986)

³³ Wheal (1991mob, p. 19)

³⁴ Vaughton & Gaffney (1980)

³⁵ Wheal (1994text)

[†] The text was in fact little revised. It represented a compromise between what Wheal wanted to write and what he thought would be received by his potential audience (Wheal, pers. comm. Apr 2001).

al teaching about, and exercises on, probability. This is followed by non-traditional material showing simple ways of using relative frequency and length of runs to decide whether an RG really is fair. Some interesting practical activities (including baking chocolate chip biscuits) come next to illustrate the value of simulations as opposed to actual experiments. After some assignment work for formal assessment Wheal presents more traditional teaching, this time on permutations and combinations, to develop the binomial theorem for probabilities and to introduce the hypergeometric distribution for sampling without replacement. Extensive use is made of clearly drawn trees for this work. The exercises and subsequent assignment contain traditional problems. Finally the opportunity is provided for students to design and carry out a project of their own which uses a modelling approach and theory taught in the book to compare real with modelled data. Detailed, handwritten model answers are provided for the exercises, so it is possible to decide what Wheal hoped his students would be able to do as a result of studying his text and, presumably, what other teachers would do with their students.

Wheal has certainly managed to show how probability theory may enable the construction of mathematical models. His worked answers show how he has utilised the availability of calculators to work with awkward numbers, and to encourage the practice of sensible rounding of results. On the other hand, the teaching sections of his text look remarkably traditional. This is not necessarily bad, but Wheal repeats the weaknesses of traditional texts in, for example, failing to distinguish theoretical, experimental and subjective probabilities. Given that this distinction is critical to his modelling aim, this is a serious omission. He is clearly aiming for statistical inference, but does not reach a sufficient level of numericisation to do so effectively, which means that his interpretations are fairly general:

Because of the small number of observations, fifty each, there is not sufficient evidence for suspicion about the randomness of either student's results.³⁶

The average run length is shorter but the poker set distribution is more like that predicted in 31 (iv). Even if these results had been exactly as predicted by the fair coins theory the small number of results would have made it necessary to be very cautious about accepting the fair coins assumption. More observations are needed.*

³⁶ Wheal (1994text, Solutions, p. 15)

* Wheal (1994text, Solutions, p. 19) This answer comes perilously close to the fallacious approach which advocates continued resampling until the desired results are obtained, overlooking the fact that increasing the number of samples increases the probability of obtaining a statistically significant, but erroneous, result.

This text, which, unexpectedly, is not pedagogically revolutionary, suggests the level of change sought by the “applied” reformers. We now describe some changes of this period and the structure within which they were effected.

NEW SSABSA STRUCTURES

... an emphasis on subject achievement related to syllabus objectives.³⁷

By 1986 permutations, combinations and the binomial theorem formed part of all Australian Year 12 academic mathematics syllabuses, except in Q, where it was optional, but there was much less unanimity across the country about more advanced probability and statistics. Only WA saw both as a compulsory part of a standard course; some other States provided substantial optional units.³⁸ SA held to a middle path, but this would soon change.

As mentioned in Chapter 14, SSABSA was established in 1984 as an organisation controlled by a much wider range of interests than the former examination boards, and with a structure to encourage greater flexibility and diversity in accreditation for students in Years 11 and 12. This flexibility and diversity was marked by some extraordinary terminology, of which the *bon mot* above is a good example. Two changes will be of special interest here—the status of probability and statistics in academic courses and the difficulties of accrediting with integrity courses formally certified entirely by school-based assessment. Some of the events described here occurred after the period technically covered by this chapter, but they belong more with the changes resulting from the introduction of SSABSA than with the National Curriculum changes of the early 1990s. As discussed below, the changes of this period had some influence on how much probability was taught below Year 11, but they did not influence how it was taught, so there is no need to discuss primary or junior secondary teaching in this chapter.

In 1986 the SSABSA Mathematics syllabuses were accredited for the ensuing five years. These syllabuses were conservative modifications of previous practice, and included the return of Euclidean geometry which university pressures had effected in 1984. There was an expectation that the five year period would be used to think through further revisions which would bring the courses more closely into line with SSABSA’s more inclusive remit. Interestingly, the assumed

³⁷ SSABSA (1986)

³⁸ Mack (c. 1986)

knowledge for Mathematics 1 and 1S, both of which contained probability, made no reference to probability at all, only to the binomial expansion and Pascal's triangle.³⁹ We shall see below other examples of the belief that the less students are taught early, the better they are likely to learn later.

For many years SA mathematics education had been led by the competent and popular John Gaffney who died prematurely in the middle of 1987, aged 45. After his death, the impetus for change came principally from three people all about the same age as Gaffney—Jeff Baxter, Vern Treilibs and Michael Wheal. Baxter, from Sturt CAE, had been the financial wizard behind the Adelaide ICME, a key player in establishing the national AAMT office in Adelaide, President of AAMT for some of this period, and Chair of SSABSA's mathematics Curriculum Area Committee (CAC) from late 1985 until 1990.⁴⁰ His special interests were in making resources from around the world readily available, and the AAMT's mail-order book shop expanded substantially under his leadership. Wheal and Treilibs had spent some years together as the Education Department's two Mathematics Consultants. Treilibs, already mentioned in Chapter 14, had earlier implemented radical changes to mathematics teaching structures at a challenging suburban High School, had been involved in the *Mathematics at Work* project, and was SSABSA's Mathematics Curriculum Officer, from 1987 to 1990. At about the same time as Baxter left the CAC, he returned to a classroom, this time in Indonesia, and was replaced by the much less evangelical Graham Benger. Wheal had been a Consultant prior to 1987, when he began three years as a writer at the SA Correspondence School. After that he spent most of his time in various positions at SSABSA apart from one year in 1991 as a Senior teacher in a High School.

Year 11

SSABSA's flexible approach allowed schools to design their own "Registered Subjects", and at least one such course in Probability was constructed.⁴¹ But most schools followed the courses constructed by SSABSA, where five mathematics courses were available in Year 11.⁴² Business and Technical Mathematics contained no probability. General Mathematics was based on the *Mathematics at Work* sequence, and included both statistics and simple probability. Mathematics

³⁹ SSABSA (ndp. c. 1986, pp. 3, 13–14)

⁴⁰ SSABSA (1986ar) *et seq.*

⁴¹ Goodwood High School, 1987, Course R586

⁴² EDSA. Mathematics Curriculum Committee (ndp, c. 1987)

Applied, which required several “case studies”, e.g., of supermarket queues, or of the relationship between times for winning the Melbourne Cup and weight carried, had a substantial amount of probability, statistics, combinatorics and simulation. It was based on texts written by The Spode Group⁴³ and also a part of Jacobs’ *Mathematics. A Human Endeavour*.⁴⁴ The latter was a lively and attractive presentation of traditional material, the former emphasised applications, such as simulation. Pure Mathematics (a double subject leading to academic Year 12 mathematics) contained about three weeks’ work on traditional probability and combinatorics to provide a basis for the Binomial Theorem, permitted a project on statistics, optimisation or mathematical modelling, and recommended that “an open-ended problem solving approach should be used wherever possible”.⁴⁵

Year 12

From 1986 the weighting for school-based assessment of Publicly Examined Subjects (PES) was increased to 50%.⁴⁶ The assessment itself was not moderated, but the marks were statistically scaled to try to make allowances for otherwise uncontrolled differences between schools. This procedure was generally acceptable to tertiary institutions. Other subjects were known as School Assessed Subjects (SAS) and were awarded entirely on a school-based assessment which was moderated by a group of experienced teachers, many of whom were also teaching the subject. Students were permitted to study a small number of SAS subjects without forfeiting the possibility of university entrance. All subjects were acceptable for inclusion on a student’s Certificate of Achievement, but individual tertiary institutions retained the right to decide whether a particular PES subject should be acceptable as a “Higher Education Selection Subject” (HESS), i.e., as part of an application for tertiary entrance, and whether a “Subject Achievement Score” of an SAS subject which was also accepted as a HESS subject should have its “Higher Education Entrance Score (HEES) lowered by two marks out of twenty to make allowances for the assumed easiness of the subject.”[§]

⁴³ E.g., Spode Group (1986)

⁴⁴ Jacobs (1977), an Australian adaptation of his 1970 book

⁴⁵ EDSA. Mathematics Curriculum Committee (ndp, c. 1987, p. 20)

⁴⁶ SSABSA (1987ar)

[§] E.g., *The SATAC Guide* (1993, p. 10). The tautological use of HEES, HESS, PES and SAS as adjectives is common, and seems to be necessary for clarity. The term “Subject Achievement Score” does not seem ever to have been abbreviated, to avoid confusion with “School Assessed Subjects” which were! None of these abbreviations should be confused with “HECS”—the Higher Education Contribution Scheme—which was the principal method used by the Federal government to pay for the universities who were going to such trouble to refine their admission procedures.

Two SAS subjects, Applied and Business Mathematics, included probability. Applied Mathematics was the big brother of the Year 11 course. One unit covered probability, and contained two sections, “Concepts and Techniques” and “Applications”. The concepts listed were simulation, randomness, probabilities, odds & expectations, counting techniques, and the binomial probability distribution. The formal statement indicated some of the difficulties which were encountered:

It is intended that the study of probability should demonstrate its application to chance processes in real life. Consequently, priority need to be given to its role as a predictor, with some estimate of error, from both the a-posteriori and the a-priori perspectives (although these words need not be used). The language, techniques and concepts should arise from the applications rather than as subject matter to be mastered before applications can be undertaken. ... The separation of the syllabus material into two sections is to nominate the routine skills and tasks and to emphasise the need for applications.⁴⁷

Implementation of the New Approaches

How successful were teachers in working with such new ideas in such a flexible examination environment? Could they successfully walk the tightropes in the curriculum documents? Could they conform with the Board’s precept that “[a] major test of whether learning has occurred is a demonstration by the student of the particular concept or skill in an applied situation”?⁴⁸ The Year 12 Applied Mathematics course provides us with the best answers, because of its special focus and greater flexibility, and its use of internal moderation which has been openly reported on by Wheal. It shows that the changes were difficult to implement, and that some teachers did not conform with the spirit of the course, but:

presented courses dominated by routine skills work in which the students [were] told what to do and given few opportunities to use their own initiative in investigating problems. Courses such as these gave students no opportunity to achieve grades higher than C.⁴⁹

An evaluation conducted in 1990 reported

... there has not been enough time available for the modelling emphasis to be addressed well enough to meet the subject's aims adequately.

A feature of this subject is criterion based assessment. The determination of grades from the nature of the work students have successfully

⁴⁷ SSABSA (1992am, p. 28)

⁴⁸ SSABSA (ndp, p. 8)

⁴⁹ Wheal (1990)

undertaken, rather than from their scoring of given percentages of marks has in practice led to some misunderstanding and undue apprehension in teachers. These problems have been magnified by the high rate of teacher turnover.⁵⁰

In spite of the development of resources for teachers looking to develop projects and/or questions, it proved especially difficult to find suitable projects for probability and most used tended to be too complex.⁵¹ Even after about seven years of the course the Moderator could report

Many students were given only two opportunities to undertake project work. The project tasks were often unsuitable from the outset, allowing little scope for mathematics to be used and causing students to draw unclear or inappropriate conclusions. There was no clear basis for the allocation of marks for many projects; on other occasions the allocation of marks for components of projects was inappropriate.⁵²

The course was labour-intensive and required a lot of administrative work from teachers. As time went on staffing pressures increased, and funding cuts reduced the richness of the moderation which could be provided.⁵³ Placing so much responsibility in the hands of teachers had not proved very successful, even though SSABSA had stated:

The maintenance of appropriate standards, and the resultant public credibility of the individual courses, will be determined largely by the professionalism of the teachers conducting them in the schools.⁵⁴

SSABSA's requirements from teachers of the academically weak Year 11 General Mathematics students were even more demanding:

The major component of the assessment should be the assessment of the problem solving strategies and skills required by the students. This implies less emphasis on formal testing and more on the teachers' judgement of student work in class and assignment work. ... Students will, however, find it hard to record their mathematical thinking and teachers should be realistic in their expectations.⁵⁵

⁵⁰ SSABSA (1985–1990) [documents in the private domain] *Draft Evaluation Report Applied Mathematics (4unit) Applied Mathematics (2unit)* presented to Mathematics CAC 19 Jun 90

⁵¹ Wheal (1991)

⁵² Wheal (1993)

⁵³ Wheal (1994)

⁵⁴ SSABSA (ndp, p. 8)

⁵⁵ EDSA. Mathematics Curriculum Committee (ndp, c. 1987, p. 12)

Few teachers were likely to be able to live up to the laudable aims of the reformers. The evidence from Applied Mathematics is that few had the necessary levels of expertise or professionalism.* The evidence from this thesis in general and from Part E in particular is that it was unrealistic for a public assessment body to expect such a high level of Pedagogic expertise for probability because it had not been incorporated into the majority of classrooms. Social forces were running ahead of Pedagogic knowledge. Academically less demanding courses which did not have a high emphasis on content knowledge were often referred to by students at that time as “vegie maths”: there seems to have been some defensible reasons for their using the term.†

We shall now discuss two changes which are of critical importance for evaluating probability’s position with the curriculum. In doing so we need to remember that during this period, the Gilding report⁵⁶ was being constructed and this would lead to a new structure for upper secondary schools. Years 11 and 12 would be treated as two stages of a single course leading to the award of single “South Australian Certificate of Education” (SACE). A consequence of the new structure would be a further reduction in the amount of time allowed for mathematics in Year 11.⁵⁷ Stage 1 of the Certificate was introduced in 1992,⁵⁸ followed by Stage 2 in 1993.⁵⁹ We also need to remember that on 1 Jan 1991 the University of South Australia was established, an unlikely marriage of the Institute of Technology with three of the CAE campuses, which had themselves grown out of the former Teachers’ Colleges. At the same time both the existing universities were amalgamated with one each of the remaining CAE campuses.

* As a parent of an Applied Mathematics student in 1987, I was consulted by an experienced teacher on the mathematics underlying spherical geometry. This was certainly a highly professional approach, but she was a remarkable person, with a degree of self-confidence which could support seeking advice from her students’ parents. It is unlikely that she was typical of Applied Mathematics teachers at this time, but I do not have firm evidence.

† My observation on the principal project done by Tim in 1987 was that it was competently and carefully done and on those criteria deserved the good mark he obtained. However, it utilised no mathematical knowledge taught after Year 7 and contained no element of proof or generalisation. In my opinion he learned that he could apply primary school mathematics to a practical situation, but he did not learn anything about mathematics as such, and had in fact done similar types of thinking on his own behalf at least two years earlier. I would not use the term “vegie”, but I would not use the term “maths” either—he was *using* the arithmetic and geometry which he already knew quite well, but he was not *learning* new things about them, and it is the integrating of new ideas which I would see as of the essence of mathematical thinking.

56 Gilding (1988)

57 Bullock et al. (1990)

58 SSABSA (1991mem42)

59 SSABSA (1991mem86)

TWO MAJOR CHANGES

What will I learn? In Year 12 you will have the opportunity to ... gain more insight into probability, a topic essential in interpreting the results of scientific research and handling much media information.

How will it help me? You will be able to ... apply probability to problems involving chance events.⁶⁰

And all men kill the thing they love,
 By all let this be heard,
 Some do it with a bitter look,
 Some with a flattering word,
 The coward does it with a kiss,
 The brave man with a sword.⁶¹

In this official SSABSA advice to students of Mathematics 1S we see two major themes—probability's relevance to general education, and the importance of mathematical applications. Both views would be challenged by 1993. We consider first probability's removal from Year 12 academic mathematics courses.

The Removal of Probability from PES Courses

From 1985 to 1990, SSABSA's Mathematics CAC developed major changes to the Year 12 courses. An extensive evaluation of Mathematics 1, 2 and 1S in 1986 and 1987 included a survey of teachers' views about the Year 12 mathematics courses. This obtained a good response rate of 63% of schools and 140 individual returns.

For Mathematics 1, 68% of respondents saw no need for changes to the probability section, 3% wanted it deleted, 14% reduced, 5% enlarged and 9% modified but without changing size.⁶² ⁶³Of the thirteen topics listed, probability was fourth of those seen as most requiring change, but it did not generate large pressure for deletion.⁶⁴ It was certainly not seen as a topic which most teachers enjoyed teaching (differential and integral calculus were the outstandingly popular ones) but there were only 13% of teachers who expressed a strong or moderate lack of enjoyment in teaching the topic, about the same percentage as for geometry, inequalities and conics.⁶⁵ On the other hand, the teachers believed that the students saw it as

⁶⁰ SSABSA (ndpa), layout compressed

⁶¹ Wilde *The Ballad of Reading Gaol*, part 6, final stanza, from Wilde (1994, p. 899)

⁶² SSABSA (ndp, ? 1987, p. 3)

⁶³ SSABSA (ndpa), layout compressed

⁶⁴ SSABSA (ndp, ? 1987, p. 3)

⁶⁵ SSABSA (ndp, ? 1987, p. 4)

being a difficult topic: 37% as very difficult, and 41% as moderately difficult. Only geometry, which was new to the syllabus, received a similar judgement. A proposal that probability distributions (presumably normal and binomial, and perhaps Poisson) might be added to the syllabus met with a variety of responses, with 7% considering it to be highly desirable, 18% very undesirable, and the remainder spread evenly over the middle three points of the Likert scale.⁶⁶

In 1988 the CAC debated a recommendation that probability be omitted from the Mathematics 1 and 1S courses. It was argued that while a minority of Mathematics 1S teachers were in favour of its removal, this did not apply to Mathematics 1 teachers, and 10% of Maths 1 and 2 students had found probability the most useful subject they had studied. It was felt that for probability not to be studied by many beyond Year 10 was a serious loss in educational terms, and the NCTM report⁶⁷ was cited as support for this position. Relocation to Mathematics 2 was put forward as a possible compromise, and the argument was put that the planned new course would contain probability, so it would not really be lost.⁶⁸

This last argument was the kiss of death for probability. In 1989 a decision was taken to offer three new subjects.

- a 'New Mathematics 1' which approximates the present Mathematics 1S, and which may be taken as a 'stand-alone' subject
- a 'New Mathematics 2' which approximates the remainder of the present Mathematics 1 and 2, and which has been designed as an extension of the 'New Mathematics 1'
- a 'New PES Mathematics' with a strong emphasis on contemporary applications of mathematics, which will normally be taken as a 'stand-alone' subject but which may be taken in conjunction with the 'New Mathematics 1'.⁶⁹

The names for these subjects varied over time. For the sake of simplicity and convenience they will be referred to here as *Mathematics 1*, *2* and *3* respectively, and will be printed in italics to avoid any confusion with older courses.

The suggestion that *Mathematics 1* would "approximate the present Mathematics 1S" disguised an intention to recommend that probability should not be re-

⁶⁶ SSABSA (ndp, ? 1987, p. 9)

⁶⁷ NCTM (1980)

⁶⁸ SSABSA (1985–1990) [documents in the private domain]

⁶⁹ SSABSA (1989pesm, p. 1)

tained,⁷⁰ and probability was not included in the draft *Mathematics 1* syllabus.⁷¹ This was an ignominious end for the most resilient of the new topics from the New Mathematics movement some 20 years earlier. It flew in the face of the official statements, such as those quoted at the head of this section. It had an immediate influence on the teaching of probability in junior secondary schools, where it was far more often neglected.⁷² It was a move, in Australia almost a lone move, against a world-wide surge of support for probability. What force was strong enough to overcome such dominant Social and Pedagogical forces?

Half the answer to this question is the Intellectual force exerted by some tertiary mathematicians, mainly pure ones, to reintroduce Euclidean geometry in 1984 on the grounds that the type of thinking promoted by studying Euclidean proof was necessary for mathematics students. This claim may or may not have been valid, but if it really were believed to have been valid, then one would have expected the universities to have taught the topic to all of their students during the years of its exile. In fact they had disregarded its study, which meant that about fifteen years of worth of teachers had entered their profession with no experience of the topic. The universities did not have a good case. But they had the power to enforce major changes on the curriculum.[†]

The inclusion of a large, unfamiliar, and difficult topic had added to what many saw as the time pressures of the Year 12 curriculum, and the intention was that the first SSABSA revision should cover less work, but at more depth.⁷³ Given that *Mathematics 3* was being planned as a potential sibling for *Mathematics 1* and that probability could be a good applied topic, it was proposed to move probability to *Mathematics 3*.⁷⁴ This associated probability with a course which would inevitably have low status for some years, would not encourage its teaching in junior secondary classes and would not ensure that future specialist mathematics teachers would experience its teaching while at school. Such a decision flew in the face of all the pious statements which had emphasised the fundamental importance of probability. But, in any case, even the good reasons for the decision were predic-

⁷⁰ SSABSA (1989pesm, p. 4, recommendation 16)

⁷¹ SSABSA (1991mss)

⁷² Personal experience arising from discussions with teachers in schools.

[†] I do not know for sure which mathematicians were involved, though I can guess at some. This is one case where my evidence has to be personal experience arising from living through the events. The only corroborative evidence I can offer is the fact that the change took place, and this is quite strong in itself.

⁷³ Personal recollections of public meetings explaining the changes.

⁷⁴ SSABSA (1989pesm, p. 4, recommendation 19)

ated on a general acceptance of *Mathematics 3* and that would take many years to achieve. We shall now examine the forces opposing this general acceptance.

Mathematics 3—A New Applied Course

Mathematics 3 was seen as being of value for the social and biological sciences, and, when taken with *Mathematics 1*, as being very suitable either for those with a general interest in mathematics and/or for a wide range of future studies except engineering and specialised mathematics. The 1991 Draft Syllabus for “Contemporary Mathematical Applications” (changed to “Quantitative Methods” later in the year) covered statistics (c. 40%), operations research (c. 35%), models of growth (c. 15%) and a project (c. 10%). SSABSA announced that the new course was ready for examination, but this claim was premature.⁷⁵

The 1991 Draft was revised in 1992.⁷⁶ In both forms probability was not seen as a topic in its own right, but as a support for the applications. The bibliography for statistics and the extensive list of available teacher resources indicated familiarity by the authors with a wide range of recent work. The emphasis was on classroom material which had been carefully developed, often by well-funded groups,^{*} and which made use of modern technology and teaching approaches. Evaluation material and reference to standard research were little mentioned, although some of the projects mentioned were certainly based on significant basic research. The emphasis for statistical work was to be

on interpretation and the use of statistics to solve problems rather than the mechanics of drawing graphs and making calculations and consequently the use of electronic calculators and computer packages is essential.⁷⁷

This emphasis on technological support matches the view that mathematics subjects should change to reflect changes in society. The differences between the two documents provide some useful indicators of differing points of view about what constituted suitable content. In 1991 it was assumed that students would

understand the concept of probability and have a working knowledge of the addition rule for mutually exclusive events and the multiplication rule for independent events [and] be familiar with random numbers,

⁷⁵ SSABSA (1992ar)

⁷⁶ SSABSA (1992qm)

^{*} E.g., NCTM, MCTP, Shell Centre (where Treilibs had studied), Open University (UK)

⁷⁷ SSABSA (1991cma, p. 8; 1992qm, p. 5)

some simple techniques for selecting simple random samples and have had some experience with elementary simulation techniques.⁷⁸

This is a little vague, but in 1992 the words were modified to require merely that students should “be familiar with ... the basic concepts of probability [and] simulations with dice or other randomisers”, as well as some elementary statistical techniques, including “correlation from inspection of a scatter plot”.⁷⁹ This new form skates over even more potential difficulties. The term “basic concepts of probability” is vague, and does not indicate whether all three types of probability should be specifically discussed. The replacement of “have had some experience with” by “be familiar with” is scarcely consistent with any theory of learning which argues that children learn best by doing. Finally, correlation by visual inspection of scatter plots is extraordinarily difficult to do successfully and lessons based purely on a visual approach are difficult to teach, which is why formal correlation analysis is usually needed.* So there is some evidence that the changes made were strongly influenced by those who had little understanding of the mathematics of probability, learning theory, or the teaching of children. Yet the principal advocates for *Mathematics 3* were “skilled teachers in positions of curriculum leadership”, to use the language of Chapter 14. The precise reasons for the Intellectual weakening of the syllabus are not known, but the influence of the “uncommitted practitioners” seems to be the most likely explanation.

There were two main parts to the statistics section. The first was based on EDA and involved designing a simple survey, and analysing the data. The second examined the normal and binomial distributions, and the standard statistical procedures of sampling, confidence intervals and tests of significance. We saw that Wheal’s text had difficulties in dealing with statistical significance without using formal numerical procedures. For this course the only formal test of significance used was the two-sided sign test.⁸⁰ This simple, non-parametric test was chosen

because it can be applied in a range of situations (even though it is not often the most efficient test) [and] involves a sampling distribution

⁷⁸ SSABSA (1991cma, p. 9)

⁷⁹ SSABSA (1992qm, p. 4)

* This is not to deny the value of visual inspection as one analytic tool among several, but in practice students offer such a wide variety of lines of best fit, especially for the small data sets which need to be used for introductory teaching, that they can easily lose confidence in the value of visual methods. Going the other way, the need for some visual inspection is illustrated by a well-known collection of data sets which presents four markedly different scatter plots all of which have the same regression line. [The collection is well known, but locating its source has proved difficult!]

⁸⁰ Presumably as in Siegel (1956, pp. 68–75), but the specification is not really clear

which is accessible to students. For simplicity, the complication of deciding between one or two sided alternative hypotheses is avoided by only considering the two sided case. The prime intention is that the structure and philosophy of the testing procedure be understood.⁸¹

It was courageous⁸² to aim for understanding rather than technical competency. The sign test requires minimal calculation and should, in theory, allow students to concentrate on principles rather than details. But teaching the null hypothesis is difficult and has received little attention. Both researchers and textbook authors have often misunderstood its meaning.⁸³ Choosing just one type of test contradicts Skemp's principle that many examples are needed in the development of a concept,* and this test limits hypothesis testing solely to paired data, an approach which is relatively uncommon because its data are expensive to collect. In Tas the tetrachoric coefficient had been taught at one stage⁸⁴ and this could have provided a more general measure of association without using paired data, as well as drawing on some proven experience of its teaching. In my experience the χ^2 test is fairly easy to teach, and very commonly used, and this too could have been used as well as, or in place of, the sign test. The balance of evidence seems to be against the wisdom of the decision, in spite of its good intentions.

It was planned that Quantitative Methods should be available for examination in 1993, the beginning of the new arrangements for Stage 2 (Year 12) of SACE, and that it should be a full PES subject with HESS status. There was some tertiary and specialist support for this proposal, which had developed from experience with Minitab[®] at Flinders in 1983 and from Hall's teaching of EDA, both mentioned above. There had also been early support for a subject involving significant statistics from the committee of the SA branch of the Statistical Society of Australia.⁸⁵

⁸¹ SSABSA (1991cma, p. 11)

⁸² Sir Humphrey Appleby (Lynn & Jay, 1981, pp. 131–2) uses "courageous" as a synonym for "certain to lose an election" so the word seems peculiarly appropriate here.

⁸³ J. Truran(1998iconh)

* Skemp (1971/1986, ch. 2), mentioned in ch. 8. An example may help because syllabus constructors often assume that simplifying a syllabus will make it easier, whereas the opposite may well be the case. Students tend to develop generalisations from simple experiences which are no longer sustainable when more complex, and usually more realistic, experiences are introduced, but are, by then, hard to eradicate. A simple example is the use of the superscript "-1". This usually refers to "inverse", as in " $\cos^{-1}x$ " or " $f^{-1}(x)$ ". However, students usually meet it first with examples like " a^{-1} ", where it refers to " $\frac{1}{a}$ ", a *multiplicative* inverse, and they wrongly generalise that all such superscripts refer to "one over something".

⁸⁴ Mathematical Association of Tasmania - Education Department (1972, p. 26) [documents in the private domain]. *Vide etiam* Guildford & Fruchter (1973, pp. 300–306), Leo et al. (1974, pp. 51–53).

⁸⁵ SSABSA (1985–1990) Correspondence 18 Oct 85 [documents in the private domain]

But there was also opposition. Some was based on practical issues like the availability of sufficient computer power for the recommended approach,⁸⁶ but this objection would eventually disappear. Some pure and applied mathematicians claimed that the proposed syllabus contained “wrong” mathematics,[#] and some may have shared the commonly-held view, already mentioned in Chapter 3, that statistics is not a branch of mathematics. An applied mathematician merely asserted that “there is no evidence that the [Year 12 publicly examined subjects] as now constructed are anything but ideal for their purpose”.⁸⁷

The term “applied mathematics” was probably being used with two rather different meanings at this time, though I have never read any documentary evidence for this claim. The reforming Pedagogues normally used the term “modelling” and the examples they constructed showed ways of constructing models which adequately fitted the data. This is certainly what the applied mathematicians did when solving real problems. But it is not quite what they taught. The majority of undergraduate applied mathematics being taught at this time presented ways of fitting the data to match the model. It was a very “pure” approach.

We saw above that Wheal’s text introduced rather abruptly some didactic material on probability theory just after collecting experimental data. Wheal wanted the students to be involved in their own mathematics, rather than to be soaking up the mathematics of others. But much of the practical work done by applied mathematicians rests on a deep, well internalised, “pure” base, and the inherent need for such a base, and for sufficient depth and security of understanding to use it in an applied situation does not seem to have been adequately appreciated.*

There were also divisions among the statisticians, some of whom had yet to come to terms with the full implications of EDA. We have mentioned the supporters above. One opponent saw EDA as useful but “not really mathematics” and felt that the available time was better spent on traditional mathematics.⁸⁸ He thought significance tests could be usefully taught to illustrate an important reason for

⁸⁶ Gaerntner-Jones (1993)

[#] I have not found documentary evidence of this claim, which I heard in private conversations twice, once from a pure and once from an applied mathematician. Neither explained what aspects of the syllabus were actually “wrong”.

⁸⁷ SSABSA (1985–1990). Correspondence 17 Feb 86. [documents in the private domain]

^{*} My own experience of teaching problem solving in traditional secondary classrooms and tertiary tutorials has been that problems which require post-primary mathematical skills usually founder because so much effort is spent concentrating on the algebra that the students lose sight of the need to look at the problem from different viewpoints.

⁸⁸ SSABSA (1985–1990). Correspondence 13 Jan 86. [documents in the private domain]

studying probability, but that their value would probably be negated by “the apparently wide-spread feeling of insecurity about probability among matriculation students. This insecurity is likely to be amplified in making the transition from the ordinary ‘forward reasoning’ of probability theory to the ‘backward reasoning’ of hypothesis testing”.⁸⁹ He also argued that because the underlying theory of t tests and χ^2 tests could not be taught in schools, they would need to be taught by a cook book approach, which was “totally undesirable in a mathematics course”.⁹⁰ Another statistician expressed even more deep-seated objections.

I am against any proposal to incorporate [statistics] into the year 12 Mathematics syllabus for at least two main reasons.

(i) A proper appreciation of the subtleties of a statistic argument requires some experience with real data, and with the problems generated by it. While this is true for someone studying statistics, it is much more true for someone trying to teach the subject. I very much doubt whether there are many teachers in the secondary schools with any real experience of statistical problems, and inexperienced teachers are likely to do more harm than good. Having Statistics as a section of, say, Mathematics I would encourage the view that the subject *is* just a branch of mathematics, even just a branch of probability theory, and thus miss the point entirely.

(ii) to be a good statistician a student needs first to be a capable and astute mathematician [*tertiary tail wagging societal dog again*]. I am sure the statisticians at tertiary institutions would prefer students to come forward with their mathematical skills as well developed as possible and not with some inevitable hazy notions of statistics at the expense of more basic mathematics. Mathematics can be taught more or less in isolation from other subjects; statistics really cannot [*Oh yeah*].⁹¹

The italicised interpolations are hand-written comments on the original letter which were presumably made by some unidentified person involved with the mathematics CAC. They summarise eloquently the rift which had grown between some Intellectuals and some Pedagogues advocating radical change. They provide a good example of Schön’s comment quoted in Chapter 3 that

[t]he protagonists of the various points of view do not reflect *on* their frames but act *from* them, seeking to defend their own positions and attack the positions of their opponents.⁹²

⁸⁹ SSABSA (1985–1990). Correspondence 13 Jan 86. [documents in the private domain]

⁹⁰ SSABSA (1985–1990). Correspondence 13 Jan 86. [documents in the private domain]

⁹¹ SSABSA (1985–1990). Correspondence 22 Nov 85. [documents in the private domain]

⁹² Schön (1983, p. 312)

The rift was an extension of the similar rift described in Chapter 13 which became evident at the 1975 ACER Conference. In both cases the underlying cause was not primarily caused by personality clashes, but by other factors—principally the effects of the rise of the “new professionals” from among the Pedagogues and the impoverished educational understanding of the Academics.

All these divisions came to a head at a Special General meeting of MASA held on 23 Jun 1993 to discuss and vote on three motions, all in Wheal’s name. Their purpose was to raise the formal status of Quantitative Methods. The first advocated seeking a broader base of community support for the subject. The second was:

That the Committee* take legal advice regarding the seeking of a court order requiring that the University of Adelaide and the Flinders University of South Australia grant immediately to Quantitative Methods the status of Higher Education Entrance Subject [sic].

The third motion called for MASA to initiate legal proceedings if support was obtained for their case, and if the legal advice was that such proceedings might be successful.⁹³ Altogether, either in person or by proxy, about 60 members were present, very large for MASA evening meetings at that time, and the first two motions were both lost by ratios of approximately 2 - 1 against. As a result, no seconder was found for the third motion. My recollections are that the meeting was happy to support an improved status for Quantitative Methods,⁹⁴ but that it saw legal proceedings as inappropriate for achieving this, not least because the estimated cost merely of obtaining a legal opinion was anything up to \$12 000, a figure which represented five to ten years worth of gross MASA subscriptions.

So when the course was first offered in 1993 it had both PES and HESS status at the University of SA, and SAS status (with a discounted HEES score) at Adelaide and Flinders.⁹⁵ Its initial enrolment was quite small—about 160 candidates from 11 schools in SA and NT.⁹⁶ The Chief Examiner, Hall, reported that “candidates were under-prepared in financial mathematics and probability” and, with respect to a question on hypothesis testing, “very few answers included clear statements of the hypotheses, test statistics, p. value, decision rule and conclusion”. On the other hand, “[n]ormal probability calculations were handled well by those at-

* Presumably the MASA Committee

⁹³ Andrew (1993)

⁹⁴ Confirmed by Andrew (1993)

⁹⁵ *The SATAC Guide* 1993

⁹⁶ SSABSA (1994ar)

tempting them".⁹⁷ This should not be surprising since such calculations had formed part of the junior secondary syllabus for many years, and are quite deterministic: they do not indicate very much about an understanding of stochastic thinking. The subject's partial black-balling, which was also experienced by the subject "Nutrition", remained until 1996.⁹⁸ In 1997 Flinders University granted HESS status to Quantitative Methods, and so did a small number of departments at Adelaide.⁹⁹ Some have suggested privately that these changes of heart arose, not from rational rethinking, but from falling enrolments, particularly in science.

The width and depth of feelings displayed by this whole debate provide us with particularly useful insights into the nature of the pressures on the teaching of probability during this period. We need to remember that there were about twenty members, many in positions of responsibility, who were prepared to initiate drastic measures to achieve their aims. Were they brave men, or cowards? Perhaps they were bluffing, but I have never heard this suggested, even in private. Certainly they felt that they had exhausted all other avenues. This chapter concludes with an attempt to see the issues in the broader perspective of the BSEM, which, had it been considered then, might have helped to assuage some of the unproductive tensions.

WHAT FORCES WERE ACTING?

At the head of this chapter Chloë argued that the reason that life did not always follow a deterministic Newtonian model was "people fancying people who aren't supposed to be in that part of the plan." Things did not go according to many people's plans in SA at this time, But this was nothing to do with sex, rather with a head-long clash between strong forces of limited vision.

Both Pedagogical and Intellectual forces acknowledged that changes would be needed because of the increasing Ultimate force of growing access to technology, though they probably differed about the rate at which change should be effected.

On most issues, the sides simply did not listen to each other. The Intellectuals regarded their subject matter as sacrosanct, at least for their most able potential recruits. They did not, somewhat inconsistently, seem to be much concerned with the enormous amount of cook-book teaching of statistics being presented in many

⁹⁷ Hall (ndp, c. 1994)

⁹⁸ *The SATAC Guide* 1996, p. 20

⁹⁹ *The SATAC Guide* 1997, p. 29. The Adelaide Departments were concerned with Music, Dance, Drama, Labour Studies and Agriculture.

disciplines. They also seemed to be little aware of how the Ultimate force of the increasingly data driven nature of our society might need to influence school practice. There was undoubtedly a fear by some Intellectuals that the subject was not academically rigorous and that, even if it were, there were not enough skilled teachers available to teach it. The latter point was certainly true.

On the other hand, the reformers, who were Pedagogues rather than Mathematics Educators, seemed to have some Social support for their aim of making mathematics more accessible, some general Pedagogical support, and a moderately well prepared programme for change, developed over many years, which did not overtax available Physical resources. It is true that some reformers were a little bellicose and some may have appeared arrogant, but not all the Academics radiated sweetness and light either.

It is ironic that these tensions arose between groups who had co-operated so well to run ICME less than ten years previously, and who had also established the Adelaide Consortium for Mathematics Education (ACME) as a formal structure for encouraging change and co-operation.* Indeed, this vehicle may well have increased the Intellectuals' caution about the Pedagogues and Mathematics Educators. Meetings at about this time were extremely difficult because some rampant Constructivists teaching pre-service teachers were proposing approaches quite beyond the comprehension of traditional tertiary mathematicians. Furthermore, former CAE staff now had the status of university lecturers, and there was some fear among those they were joining that they were of lower academic quality. Because many of the CAE staff were Pedagogues concerned with practice, rather than Academics concerned with theory, this view had some justification.

But there was also a failure by the Academics to understand the nature of teaching and the nature of teachers. For example, after a particularly difficult ACME meeting I invited one tertiary teacher with whom I had worked well in the past to one of my primary pre-service lectures so that he might see some of the difficulties faced in tertiary schools of education. All was going well: the students were slow to grasp new ideas, but that was to be expected and to be worked with. After some time, my friend asked for the chalk and went to the board to explain a number of things to the students. He had failed to see that I was working to get them to do the thinking, and believed that proper teaching merely involved giving people information. We were poles apart, even though neither of us was at the extremities of our disciplines. Recently, another Academic, formerly a Chief

* *Vide footnote supra.*

Examiner, declared to me that he would know more about teaching Year 12 mathematics than anyone else in the State *because* he had been a Chief Examiner.

If Dewey's claim that "theory is the most practical of all things"¹⁰⁰ is true the theory underlying Mathematics Education ought to have been of value. Its concern with both Mathematics and Education, with teaching and learning, as encapsulated in *obuchennyi*, should have been able to find a way of responding to the Ultimate forces in ways which preserved the legitimate interests of both Intellectuals and Pedagogues. This thesis is in part an argument that the discipline did have something to offer at that time. A micro-example of its value for curriculum development will be presented in Chapter 16. Cases for its broad value for assessment and Pedagogy will be presented in Part D, particularly Chapter 20, and Part E, particularly Chapters 21 and 24. Chapter 1 of my Masters thesis demonstrates how Pedagogic difficulties can identify weaknesses in received mathematical wisdom. Why was Mathematics Education not more influential?

Most of us had been exposed to a much wider vision at ICME. But those concerned with mathematics teaching in SA were sharply divided in philosophy. This led to some personality clashes, but these can often be overcome. More seriously, some Academic Mathematicians acquired an unearned status as Mathematics Educators. What I saw among most of those involved was a lack of awareness, in spite of ICME, that there were alternative points of view which were even worth listening to, let alone using as a model for change. Physical and Social forces cause most of us to stay within fairly narrow circles which, if we are moderately successful, we usually see no need to leave. Most individuals only consider leaving their circle if there is a catastrophe or if a Charismatic leader arises.

Although SA had problems, they were not catastrophic. So there was little reason for people to develop a wider perspective. Pedagogy had changed little from that described in Chapter 10, and was unlikely to change given the Social pressures from industry and a weakened economy. ICME's Charismatic influence did not last, SA's two most Charismatic leaders both died prematurely and Boomer was in any case not a member of the mathematics community.

It is possible that in a cross-disciplinary field like Mathematics Education, Charismatic leaders need to be respected within the individual disciplines as well as within the constructed one. This was certainly the case with Thwaites, and also with Freudenthal, perhaps the most successful of all Mathematics Education

¹⁰⁰ Dewey (1929, p. 17). *Vide* chs 3, 9.

leaders. We quoted above Freudenthal's fear that probability teaching would be degraded in most classrooms. He went on to say

I can, however, accept entirely A. Engel's ideas,[†] that is, to pervade all mathematics by probability at an early stage—as soon as the children get to know about fractions, not just because it is useful for future probability teaching but because this penetration brings mathematics nearer to reality. In Engel's approach probabilities are not the subject of arid theorems and formulae. They are acted out in the classroom, initially with dice and roulette-wheels, but soon stylized or simulated (that is the technical term) by the use of tables of random numbers.¹⁰¹

This language is very close to the language of the SA reformers. Freudenthal succeeded in changing the curriculum of a country; the SA reformers have continued to struggle for small changes which have been neither resoundingly successful nor abject disasters. They certainly did not fulfil Baxter's implied prophecy, quoted in Chapter 11, that "we are now but a few years behind the leading educationally minded countries of the world"¹⁰² or Clements' claim that this was a time when Mathematics Education in Australia came of age.¹⁰³

Rather, visions became debased, or not even seen. I claimed in Chapter 3 that debasement was common enough but rarely commented on. The BSEM would suggest that it is normal because it is usually a simplification and so is an example of increasing entropy, which is the default condition for our environment.¹⁰⁴ This is one reason why Charismatic forces are so important in times of change, because they generate energy which can reduce entropy. How could the following high-minded intentions not become debased in the reality of mundane classroom life?

It is intended that all of the objectives of mathematics learning (as listed in the Broad-field Framework) should be addressed during the study of each component, but not in an objective-by-objective way. The frameworks recommend that more holistic approaches (investigations, applications, practical work, project work and theoretical work) be used and it is assumed that when a balanced range of these is used adequate coverage of the objectives will occur (for both teaching and assessment purposes).¹⁰⁵

[†] Freudenthal is referring *inter alia* to Engel (1970a). *Vide etiam* ch. 13.

¹⁰¹ Freudenthal (1973, p. 613)

¹⁰² J. Baxter (1972, p. 30)

¹⁰³ Clements et al. (1989, p. 71). *Vide* chs 9, 14.

¹⁰⁴ *Vide* discussion in ch. 9.

¹⁰⁵ Bullock et al. (1990, p. 23)

The reformers do not seem to have acknowledged the absence of a sufficiently supportive Physical environment to support mature thought of this depth. Freudenthal saw the process of debasement in this way:

Probability has its own theory how it is applied—I mean mathematical statistics, which tells how probabilities can be estimated by samples. It is frightening to see this domain of mathematics, the paragon of free thinking activity, being turned into its opposite, into a system of rigid rules. It is, indeed, just a little step from freedom to slavery if people are not trusted to use and not misuse their freedom. Those who try to justify dogmatic didactics with the argument that pupils like the system and just wish to remain within the system, can draw on their own experience because they taught probability and statistics in this way.¹⁰⁶

The BSEM does seem to provide us with an explanation for what Freudenthal has observed. It emphasises that the forces operating are much wider than the contending parties will usually admit, and suggests that debasement is highly likely unless countering forces are strong. In SA at the time the forces from Mathematics Education and Charismatic leaders were both weak.

The events described here have been concerned with upper secondary schools and have retained some continuity with previous practice. In the next chapter we shall see how a different balance of forces influenced the structures of the National Curriculum for Years 0–10 in a way which led to a discontinuity in curriculum development. For such a drastic change the value of the discipline of Mathematics Education should have been obvious. But, for probability at least, this was not to be the case.

The Grand Old Duke of York,
He had ten thousand men,
He marched them to the top of the hill
And he marched them down again.¹⁰⁷

¹⁰⁶ Freudenthal (1973, p. 590)

¹⁰⁷ Traditional English song, so traditional I cannot locate a source

CHAPTER 16: RESTRUCTURING (1990–1994)

Attainment Levels—Fact, Fear or Fantasy

Attainment Level 3

Staff member is able to find the seven multi-coloured sections at the rear of the folder and is able to answer the following questions about them:

- The writers for which colour were the most concise?
- The writers for which colour had the least to say?
- Which Level of the "Health and Personal Development" is most relevant to you or your students?
- Which Level of the "Languages other than English" sections applies most to this school?

Attainment Level 4

Staff member is able to find the third section which contains the white Levels 1 to 6 Attainment Statements and is able to answer the following questions:

- Why did some idiot stick these near the end of the folder?
- In Level 1 which subject area uses the word "student" the most number of times? Why?
- Each of these statements is meant to be a rich description. Which is the richest?
- Name one student at this school who is working at Levels 5 or 6.
- Name a high school which will be able to easily adopt the Attainment Levels?

Attainment Level 5

Staff member able to find page 6.1 and is able to browse the mathematics section and is able to answer the following questions:

- In pages 6.1 to 6.3 find three bits of jargon which are totally unintelligible to a normal person.
- In the observable indicators, why are they called observable?
- In the observable indicators in Level 1 why are there bits written in italics and which of them makes the most sense or is the most beneficial?
- What is the meaning of Algebra and list your fond memories of it at school (please limit this to 10 items).¹

The final period for this historical discussion is a time when Attainment Levels and "National Curricula" were at the centre of the curriculum stage and Technological philosophies (to use the classification of Print from Chapter 7) were in the ascendant. State and federal governments to some extent pooled their resources to try to define what should be taught in schools, how it might be assessed, and, to a lesser extent, how it might be taught. Syllabuses emphasised contextualising

¹ *Attainment Levels—Fact, Fear or Fantasy*

mathematics, problem solving approaches, and linking mathematics with other school subjects. Many of the changes were revolutionary, not evolutionary, and formed an unconformity in the “geological” history of curriculum development. For probability the documents tried, for the first time in the country’s history, to set the topic systematically into the total mathematical and social curriculum and also recommended a seamless pedagogical join at the primary/secondary interface. So this is a good place to end the historical examination of this thesis, particularly since it will show that even changes as radical as these failed to break the teaching of probability out of the educational straight-jacket built around it in the 1960s and 1970s. It also raises deep questions about assessment, which will be addressed in Part D, and about teachers’ professional development, which will be addressed in Part E. It may also provide a spring-board for a third volume to complement this work and my Masters thesis and examine the development and implementation of an effective pedagogy for probability.

Because many developed countries in the late 1980s reacted to similar economic pressures in similar ways, the similarities and differences between their reactions help to clarify those forces which were most influential on curriculum change. This will provide a real bonus for our assessment of the BSEM. Furthermore, the professionalism and authority of the documents produced means that it is easy to assess them against the Intellectual climate and knowledge of the time in order to assess the influence on the curriculum of Intellectual forces. To a lesser extent it is also possible to assess the extent to which the documents were accepted in the classroom. Finally, the depauperate educational environment which developed in this period arose under the influence of a small number of groups with relatively little formal training in education and a limited view of mathematics and mathematics education. Because these forces were so different from those operating earlier, they are particularly useful for assessing the robustness of the BSEM.

So after setting the general economic and social scene we shall describe the main Australian National Curriculum documents which are relevant to SA. This needs to be detailed because the structures developed were complex and poorly integrated. We shall then examine those parts of the documents most concerned with probability to show how many potential Intellectual influences were largely excluded from their construction, especially those concerned with the pedagogy and mathematics of probability. We shall then examine how the material has been received in classrooms and conclude by comparing briefly the Australian documents with contemporaneous ones produced in economically and culturally similar countries in order to assess the relative merits of the CEM and the BSEM.

BACKGROUND TO THE CHANGES

By the Year 2000, Australian school systems [are] to be utilising national curriculum statements and frameworks in all major curriculum areas which will identify common learning tasks and agreed performance standards.²

In general, a standard is an answer to the question 'How much is good enough?' ... A standard must be defined on grounds external to the measurement process. An example of a standard would be the minimum height for entry into the armed services.³

The 1980s saw moderate economic decline and increasing unemployment within the Western world. Governments tended to become more conservative, and the traditional association of labour parties with the political Left and of conservative parties with the Right ceased to be so clear. Economic rationalism dominated political and business processes. Employers' groups took an increasing interest in education and were successful in influencing official policies to move educational practices closer to an industrial model. In Australia, the Industry Education Forum* was established in 1990 by senior executives from business and industry to "develop new structures and new relationships with educators to try to ensure that the outcomes of Australia's education system relate[d] closely to the economic realities of trade and commerce."⁴ They did this because "rapid changes in trade and commerce have resulted in differences between the expectations of the business community and the objectives of educators".⁵ The first quotation at the head of this section comes from their declared aims which also included:

- establishing a complete system of standards and competencies in Australian schools by 2000 (with mathematics, English and science in place by 1995);
- introducing performance and accountability assessments of student and teacher performance by 1995;
- enhancing the quality and status of teaching by 2000;
- establishing decentralised management structures for all schools by 2000;
- establishing a comprehensive set of links between education and industry by 2000.[†]

² *Declaration of Goals for Australia's Schools* (1991)

³ Masters (1993nief, p. 3)

* "Forum" is a misnomer: the organisation was essentially a pressure group.

⁴ *The Industry Education Forum* (c. 1991)

⁵ *The Industry Education Forum* (c. 1991)

[†] *Declaration of Goals for Australia's Schools* (1991). Not all of these aims were achieved by 2000.

Two influential reports commissioned by the Federal Government as part of its concern to make education an economically useful enterprise—the Finn and Mayer Reports—established a number of basic structures for post-compulsory education and training education in Australia. The Finn Report⁶—*Young People's Participation in Post-compulsory Education and Training*—proposed six *Key Areas of Competence* which all post-compulsory students should attain:

- Language & Communication;
- Mathematics;
- Scientific & Technological Understanding;
- Cultural Understanding;
- Problem Solving;
- Personal & Interpersonal.

The report recommended appropriate benchmarks to indicate attainment of steps within each competency, and wanted a system of national reporting in place by 1995. It also wanted schools to integrate its structures into their curricula.⁷ These recommendations all fitted into a national vocational education structure of competency-based accreditation to permit flexible yet standardised education pathways which could be easily linked to industrial awards.⁸

Soon after, the Mayer Committee in a document entitled *Employment-Related Key Competencies for Postcompulsory Education and Training* proposed eight *Key Competency Strands*:

- collecting, analysing and organising information;
- communicating ideas and information;
- planning and organising activities;
- working with others and in teams;
- using mathematical ideas and techniques;
- solving problems;
- using technology;
- using an understanding of cultures.⁹

Although these look similar to Finn's "Key Areas of Competence", the Mayer Committee saw them as different but related, and drew up a two-dimensional table to show the nature of the relationship.¹⁰ The four levels required for each competency were based on the Australian Standards Framework developed by

⁶ AEC Review Committee (1991)

⁷ *Finn Report* (1991, ch. 4)

⁸ *Finn Report* (1991, ch. 5)

⁹ Mayer Committee (ndp; 1992)

¹⁰ Mayer Committee (1992, p. 10)

the National Training Board, and reflected different levels of autonomy in a work-place situation.¹¹ The actual content of mathematics required was not specified: only the way in which it might be applied. The link with the quotation “how much is good enough?” from the beginning of this section is clear. Indeed, the committee took the view that

while the subject called Mathematics is an important anchor point for describing the area of competence, it is not synonymous with it. Competence in using mathematics is required, and may be developed and demonstrated, across a range of curriculum areas. Accordingly, it was proposed that the area be re-named ‘Using Mathematics’.¹²

Four strands were proposed for this area:

- evaluating (mainly in its literal sense);
- handling information;
- planning;
- designing.¹³

The value of some aspects of stochastics may easily be seen in this framework, but stochastic knowledge as such was not addressed. The limitations of the current penchant of industry for clear articulation of minute goals are not always appreciated. A useful example may be found in *Interim Australian Standards. General Conditions for Engagement of Consultants*,¹⁴ where the relationship between client and consultant is seen as an hierarchical one, even though there is substantial evidence that the relationship works much better when it is seen as an interactive, cybernetic one.¹⁵ Of course, most would agree that the listed aims are desirable outcomes of a truly educational process. But the exemplars provided for these aims are, like the desired relationships with consultants, so limited because they are heavily oriented to practical work-place situations. For example:

Solving problems—the capacity to identify when a problem exists and to achieve a solution. For example, handling a complaint from a customer or overcoming a personal conflict at work.¹⁶

Education has larger aims than these, as deep thinkers like Rousseau and Dewey have been so good at reminding us. But the industrial, product-oriented model

¹¹ Mayer Committee (1992, p. 12)

¹² Mayer Committee (1992, p. 64)

¹³ Mayer Committee (1992, p. 65)

¹⁴ Standards Association of Australia (1993)

¹⁵ E.g., Wild & Pfannkuch (1999); Zahn (1982)

¹⁶ *Key Competencies and How They Can Help Your Business*

emphasised presenting curricula in small, ostensibly discrete parts displayed on a spread-sheet. Each section of the structure was seen as discrete, and parts could be put together in a wide variety of ways without, as in the past, requiring rigorous pre-requisites. The 1980 *Guidelines* had also followed such a model and we saw in Chapter 14 that teachers had not make good links between the parts. Such an approach was becoming pervasive within the community, including the testing of applicants for a driver's licence. For example, in SA no longer did applicants need to undergo a single test during which all important skills were tested together; they could pass the whole test by showing, for example, their competence in parallel parking on one day and in changing lanes safely on another.¹⁷ The approach also made no allowance for the fact that skills might regress to a lower level. Yet, to choose another example from the field of driver training, discussions with classes of young adults have confirmed that a significant minority who have demonstrated their ability to perform a skill like parallel parking during a driving test make no attempt ever to do so again.

In a critical review of the National Curriculum, Piper has described two views of knowledge—one “molar” and one “molecular”.¹⁸ Molar knowledge is global and contextualised, with an emphasis on reactions between components. Molecular knowledge is analytic, incremental and sequential, with an emphasis on the components themselves. The industrialists' position clearly favours the molecular view. Piper sees such views are strongly influenced by one's view of learning, and suggests that emphasising dualisms of this type is highly likely to lead to a polarisation which will neglect much of value at either end of the spectrum.¹⁹ His approach is similar to Skemp's well-known analysis of instrumental and relational understanding,²⁰ but he is less concerned with cognition than with political pressures and the forces which operate on an educational environment.

In terms of the BSEM the period of “national curricula” may be seen as a period when Social forces based on economic imperatives, especially the need to produce young adults who would fit into current work-places, dominated over Intellectual forces. As we shall see, a structure was set up which required much more work from teachers at a time when the teaching population was aging and the physical demands of their work were increasing. While the Physical imperative of bringing the country out of its recession was real enough, the

17 SA. Department of Transport (1997, p. 10)

18 Piper (1997, p. 65)

19 Piper (1997, pp. 65–66)

20 Skemp (1976)

demands placed on schools to help in this process required more energy than was made available within schools at that time.

There is not space in this thesis for a full analysis of how an industrial model came to exert so much influence over the quite different enterprise of education. In industry, once something has been achieved, it has been achieved. A well run factory is highly likely to turn out well made products for customers who need them. Each production and sale represents an outcome which is a measurable success. Even attending to a repair under guarantee may be seen as a similar, measurable, successful outcome. Not so in education.²¹ A well taught lesson, an attractively written textbook, a pleasant, caring personality—even all of these together do not guarantee learning among the willing children, let alone among the unwilling. Even if something of value has been learned, it may not be apparent even to the learner for many years.

The competency approach has been the subject of considerable debate.²² Among those writers who have addressed issues of immediate relevance to this thesis, Ellerton & Clements have strongly denounced the administrative processes by which the documents were constructed,²³ and others have examined what the documents imply for our understanding of curriculum.²⁴ But there has been little attention paid to Intellectual issues about the content being taught or Pedagogic issues about how learning takes place, and even when this has been looked at, Chance & Data has received little or no attention.²⁵ In many ways the debate has moved far away from most of the issues which were discussed in Chapter 3–8; the emphasis on Technological priorities has left no room for the contributions of many educational thinkers. This is why the changes of the early 1990s need to be seen as a “geological unconformity”, and why it has been necessary to provide such a detailed background to the Social forces.

Yet the broad range of educational thought could have contributed much to the Technocrats’ approach, as is suggested in the following English comment:

When ... achievement relates to something as complex as learning mathematics the task of defining the performance criteria is equally complex. A particular issue is that individual statements of attainment

²¹ Soucek (1993, pp. 168–174) discusses this in detail.

²² *Vide* Collins (1993).

²³ Ellerton & Clements (1994mav; 1994)

²⁴ E.g., L. Bartlett (1993), Collins (1994)

²⁵ Stacey (1994)

require a good deal of exemplification if they are to be regarded as usable and not open to significant differences in interpretation.²⁶

The following sections will provide examples of how these contributions might have been made, particularly with respect to stochastics. I shall argue that the Australian curricula have significant Academic weaknesses, some of which may be directly attributed to the false industrial assumption that children's learning may be developed and assessed merely by undertaking certain fairly routine procedures which do not take into account the complexity of mathematics learning. Perhaps it is the failure to appreciate this complexity which is the biggest single hindrance to the acceptance of mathematics education as a worthwhile discipline.

AUSTRALIAN "NATIONAL CURRICULA", 1989–1994

Conservatives will tell you [that an Australian curriculum] should not be soft, and therefore not about women, Aborigines, peace or the environment. Liberals will utter reverse anathema. An Australian curriculum, they will say, should not be nationalistic or chauvinistic in tone or intent. ... [It] should give a critical view of Australian society and its myths.²⁷

The Technocratic view, among many others, sees mathematics as a-political, a-personal, and a-critical. The discussions in Part B, especially Chapter 5, have shown such a view to be simplistic. In order to assess the Mathematics National Curricula developed in the early 1990s we first need to describe in some detail the construction of this totally new way of approaching Australian curricula.

The Australian Education Council (AEC) is a meeting of Ministers and Chief Executive Officers of the nine government education systems in Australia. In 1989, after endorsing ten common and agreed goals for schooling in Australia,²⁸ its members agreed to collaborate to produce curriculum statements for Australian schools. These were produced by the AEC Curriculum and Assessment Committee (CURASS), published by the Curriculum Corporation (CC) and constitute what is informally known as the "Australian National Curriculum". But the term is a misnomer: it was always well understood that no statement had any formal authority in any State or Territory, and in any case many of the States and Territories chose to construct and use their own documents after or before the AEC's documents were produced. Nevertheless, the national documents provide a

²⁶ Keith Jones (1995, p. 40)

²⁷ Hannan (1989, p. 7)

²⁸ AEC (1989)

useful framework to examine thinking about probability and its teaching, especially its academic content. Others have discussed the larger picture of curriculum change in Australia;²⁹ our concern here is strictly with how the many forces operating within Australia influenced the production of a curriculum for probability.

For each subject it was intended to produce a *Statement*, a *Curriculum Profile*, and a set of *Work Samples*. In most cases two or all three of these documents were bound together. However, for mathematics the *National Statement on Mathematics for Australian Schools* was produced first, in 1991,³⁰ while *Mathematics—A Curriculum Profile for Australian Schools* and *Mathematics—Work Samples* came out much later, in 1994.³¹ These two documents were produced some six months after the AEC had decided that “the publication of statements and profiles should be the prerogative of each State and Territory”,³² officialese for the fact that the plans for a National Curriculum had foundered on ideological differences between states. Since then each State has gone its own way,³³ but these developments are beyond the scope of this thesis.

Although the *Mathematics Profile* and *Work Samples* use the same structure and are very closely inter-related, they diverge in a number of important ways from the *National Statement* so it is necessary to describe in some detail the structures of each type separately. The whole structure is so complicated that for the reader’s convenience copies of the most relevant pages from the curriculum documents are provided in Appendix III.

The National Statement

The *National Statement* proposes principles for school mathematics which reflect the importance of mathematics in the life and culture of Australians, the need for equity, and the increasing need in our society to be able to communicate mathematically. The learning principles enunciated have a Constructivist basis, and encourage an holistic approach to mathematics through active problem-solving, frequently in group situations, with assessment which reflects what is seen by its authors as important in mathematics learning. It claims to provide a framework around which schools can build their curriculum, and does not claim to be either

²⁹ E.g., Piper (1997); Ellerton & Clements (1994)

³⁰ AEC (1991)

³¹ AEC (1994a, b)

³² AEC (1994b, p. iv)

³³ Kennedy et al. (1995)

a syllabus or a curriculum.³⁴ Yet the main part of the text covers “mathematical understandings, skills, knowledge and processes which should typically be made available to students”³⁵ and is preceded by chapters which strongly advocate that attitudes, appreciations, and problem-solving must be seen as a part of the curriculum, as well as content knowledge. The width of its concerns matches the wide views of curricula we have discussed in Chapter 7, and make it difficult to accept the claim that the *National Statement* is not a curriculum.

The main framework is divided into four Bands covering the whole primary and secondary curricula. For Australia, this represents a highly innovative approach and is the logical culmination of the 1964 Conference decision to replace the term “arithmetic” in primary schools by “mathematics”. While the bands are cumulative and developmental, no precise age or grade levels are assigned. In this way it was hoped to balance a rigid structure with the need to allow for individual differences. Although four Bands were chosen quite deliberately so that they could not be mapped exactly onto Year levels,³⁶ it is suggested that Band A refer to the junior primary years, Band B to upper primary years, Band C to junior secondary years, and Band D to upper secondary years.³⁷ Within each Band there are seven main Strands—*Mathematical Inquiry, Choosing & Using Mathematics, Space, Number, Measurement, Chance & Data, and Algebra*.^{*} The first two overarching strands (if it is permitted to mix the metaphor) are linked with a quasi-strand on *Attitudes & Appreciations*. Within each Strand are several topics. For Chance & Data the topics are Chance, Data Handling, and, for Bands C and D, Statistical Inference. Several specific Outcomes are listed for each topic, together with brief sets of activities which might assist the attainment of these Outcomes.

One example will suffice: a part of the Chance section of Chance & Data in Band B. After a preamble which emphasises the need for activity by children and which encourages prediction, systematic analysis, awareness of variation and informal ordering, the *Statement* suggests that

[e]xperiences with chance should be provided which enable children to make statements about how likely are everyday experiences which

³⁴ AEC (1991, p. 1)

³⁵ AEC (1991, p. 2)

³⁶ McGaw (1997, p. 3)

³⁷ AEC (1991, p. 28)

^{*} As discussed in Chapter 2 “Chance & Data” is used here rather than the official “Chance and data” because it is visually more unified.

involve some elements of chance and understand the terms ‘chance’ and ‘probability’ in common usage.³⁸

The possible activities it suggests are:

- Make simple predictive statements about everyday events (e.g. in Perth, it is more likely to rain in July than in December) and understand and use appropriate words such as possible/impossible, likely/unlikely, certain/uncertain, biased/fair, a chance/no chance.
- Discuss situations in which one may wish to maximise or minimise the possible of certain events occurring (i.e. there is usually less traffic on the road I take to school than on the other road; I am less likely to have an accident than if I took the busy route).
- Interpret statements of numerical probability in common use (i.e. use the idea that very likely events have probabilities ‘close to 1’ and very unlikely events have probabilities ‘close to 0’ to interpret probability statements).
- Interpret alternative ways of expressing probabilities which are in everyday use (e.g. the probability of rain tomorrow is 30 per cent; there’s a fifty-fifty chance I will go to the concert).³⁹

The *Statement’s* applied philosophy may be clearly seen here. But there is little of an holistic approach, as we shall find when examining the even more fragmented *Profiles*. And there is no mention of the mathematical and pedagogical problems which the activity approach may engender. The *National Statement* was reviewed by the AAMT for the Commonwealth Government in 1993. I have seen only the draft report, which is not available for citing. Most of the criticisms were at the holistic level: comments on detailed aspects of the content were few—it is almost as though critics and authors had quite different agenda. To use the structure of Part B: the emphasis of the *Statement* and of its evaluation has been on different philosophical issues, with little attention to pedagogic issues as informed by research. In Part E we shall see some of the consequences of this oversight, which is found in many of the other National Curriculum documents.

The Profiles

The *Curriculum Profile* and *Work Samples* proclaim the same philosophies and use the same basic set of strands, but the three overarching ones have been com-

³⁸ AEC (1991, p. 170)

³⁹ AEC (1991, p. 170)

pressed into one strand, called *Working Mathematically*. The draft versions made available for public comment were closely linked to the *National Statement Bands*, but in the published version the Bands, covering twelve years, were replaced by eight Levels covering the ten years of compulsory schooling. Once again the number of Levels was deliberately chosen not to match the number of years of schooling covered. And since the eight Levels cover roughly the same development as the first three Bands, no simple mapping is available here either. Such a significant change makes a mockery of any claim that the documents were subject to a full consultation process. Worse still, there are no formal links made in these documents between the Levels and the Bands, though of course similarities may be found with careful reading. For some unknown reason Outcomes for *Working Mathematically* are provided only for Levels 1, 3, 5, 7 and 8. Given the holistic aims of the *Statement*, this seems to be indefensible.

At each Level in the *Profiles* some *Strand Organisers* or general outcomes are defined. There are five of these for Levels 1–7 of the Chance & Data Strand:

- Understanding, Estimating and Measuring Chance Variation
- Collecting Data
- Organising Data
- Displaying and Summarising Data
- Interpreting Data (from Level 2 onwards)

Level 8 is more general, and is essentially an attempt to link all the Strand Organisers together under the general theme of inference. Its generality seems to indicate some lack of confidence about the best way to approach the making of links with upper secondary curricula.

While the *Profiles* have expanded the “Data Handling” in the *National Statement*, the concepts involved are essentially the same. Here we shall address mainly the material from the first Strand Organiser which is the only one directly concerned with probability. This is numbered “n.23” in the sequence of Outcomes, where “n” represents the Level. For example, Outcome 4.23 refers to “Understanding, Estimating and Measuring Chance Variation” at Level 4. This structure provides a two-dimensional grid where each cell in the grid contains one Outcome. The *Profiles* provide a number of criteria for student behaviour which indicates success in each Outcome. For example, Level 1.23 is attained when students

[show] some recognition of the element of chance in familiar daily activities.

[This is] evident when students, for example:

- Respond appropriately to everyday language associated with uncertainty (will-won't-might, could-couldn't).
- Talk about events in ways which show they recognise their chance nature (say, 'Our new baby might be a girl or ...a boy').
- Use language such as 'won't happen, 'will happen' or 'might happen' appropriately (says, 'Tomorrow it might rain' or 'I need seven to win and that can't happen').
- Recognise and take into account the possibility of different results for repetitions of the same simple action, such as throwing a die for a game (say, 'I got 3 last time but I won't always').⁴⁰

Given the holistic approach of the general aims, this balkanised structure can cause difficulties. Indeed, during the compilation of the *Profiles* one contributor noted, "Whilst thinking along levels' lines it is easy to slip into thinking (or give the impression) that all of this is a linear progression; I would assume this to be the exception, rather than the rule in actual curriculum delivery."⁴¹ So at least some authors foresaw the problem, but were constrained by their brief. Experienced teachers might well be able to overcome such weaknesses, but, as we have seen in Chapter 14, many found it difficult, even with traditional materials.

Not only is linking between units a problem, so also are the standards for each Level. The approach taken was *a priori*: desired learning outcomes were formulated first,⁴² on the basis of what "experts" *believed* was possible, without checking that the outcomes were achievable. "Empirical considerations of what students can achieve [were] of secondary importance in the development of the curriculum profiles."⁴³ The Constructivist approach, loathe to define absolute knowledge, may be clearly seen here. Many other approaches were possible, which would each have gained their authority from a different group of "experts". Politically, it is easy to argue that the authority for any approach has come from consensus among skilled, interested parties. What defines "skill" and "interest" is of course problematic. The analysis of the *Work Samples* below strongly suggests that the skills and interests of those influential in its construction were, at best, narrow. Furthermore, in the same way that, as we shall observe in Chapter 26, it is important also for research workers to be disinterested, in the best meaning of the

⁴⁰ AEC (1994a, p. 32)

⁴¹ *Initial Thoughts on Levels 7 & 8 & Chance & Data* (Documents in the Private Domain)

⁴² McGaw (1995)

⁴³ McGaw (1995)

word, so it is important for curriculum planners to have a strong measure of disinterest, and one important aspect of attaining such neutrality is a width of vision. Of course, no decisions will ever be value-free, and all will have some measure of intelligent guesswork, but if there is a general expectation of breadth and neutrality, then decisions are much less likely to be damagingly lop-sided.

In any case, the *Profiles* were not always used as had been planned, which led to a further narrowing. In SA, for example

there was initially a tendency to ignore the Curriculum Statements and to use the Curriculum Profiles as a programming or curriculum development framework as well as a reporting framework. This, in its extreme form, led to a narrowing of curriculum provision and a use of the profile outcomes as curriculum determinants rather than as being illustrative of student achievement at particular levels.⁴⁴

The Work Samples

The basic structure of each item in the *Work Samples* is essentially the same, and some examples may be seen in Appendix III. For most Levels of most Strands tasks and responses are presented. For each task a child's work is provided which is usually written, but may be supplemented by notes from the teacher on relevant non-written aspects, such as verification of a written calculation by using a calculator. The work is discussed under the headings of Task, Background, Commentary, Relevant Outcomes, and Summary Content. The whole example usually occupies exactly one page for each task. In the commentary each piece of thinking that the child is believed to have done is carefully pointed out. The comments are almost entirely positive; there are very few statements of what the child is unable to do. The child's achievements are then interpreted in terms of the relevant Outcomes from the *Profiles* and the Summary Comment makes a value judgement about whether these Outcomes have been adequately achieved.

For example, Work Sample "Chance and Data 1" is the task relevant to outcome 1.23 quoted above⁴⁵ and will be discussed further below. Groups of students were presented with pictures of children in various activities and asked to comment on whether the activity might happen, would happen, or would not happen tomorrow with the teacher recording what each individual child said. The examples chosen were:

⁴⁴ Stehn (1997, p. 179)

⁴⁵ AEC (1994b, p. 116)

playing with a calculator;
 watching television;
 eating breakfast;
 playing in a paddling pool.

Many examples in the *Work Samples* were not tested or made available for public consultation in the draft version of the Profiles and Work Samples.⁴⁶ which, as we shall see below, resulted in some very poor exemplars.

But our tale of curriculum complexity does not end with these three documents. At least four other significant documents relevant to the teaching of probability in SA were produced at about the same time. Fortunately, they can be described more briefly. The seven documents taken together constitute what I shall call the “National Curriculum documents” and will provide the main evidence for the historical interpretation of curriculum forces operating during this period.

The Attainment Levels

While the *Profiles* were being constructed nationally the Education Department of South Australia, under the influence of Boomer,⁴⁷ was constructing its own outcomes-based R–10 curriculum under the title of *Attainment Levels*⁴⁸ in order to break, so it was claimed, a curriculum inertia associated with the devolution of curriculum decisions to schools.⁴⁹ Effective reporting was seen as the missing link in the “curriculum cycle” at that time, and hence the reforms emphasised procedures which were capable of being efficiently reported.

This document was produced in 1992 in loose-leaf form with poor cross-referencing, and with the planned set of exemplars missing. It was claimed to provide a framework which would enable comparisons between schools across the state and which would enable students and parents to know how well their students were progressing. It was further claimed to provide a structure which would assist teachers in assessing children’s work, and to enable schools and systems to gather data about students’ performance which could influence the distribution of resources especially towards under-achieving or minority groups.⁵⁰ In mathematics five mathematical Strands and three “affective” Strands were specified. These were the same as in the already published *National Statement* but there is

⁴⁶ *Mathematics Profiles Levels 1–8 Draft*

⁴⁷ Stehn (1997, p. 167)

⁴⁸ EDSA (1992)

⁴⁹ Stehn (1997, p. 169)

⁵⁰ EDSA (1992, p. 1.7)

nothing in the text to indicate this link. Six Levels were specified for each strand—at least these could be easily matched against the first three Bands of the *National Statement* though not against year levels.

The development of the Levels has been described by Stehn, a supporter and implementer of the system. Most of her description is procedural, but some issues are relevant to this discussion.⁵¹ Although she provides a statement about general influences on the *Attainment Levels*, it is not terribly precise.

Although it was not widely recognised at the time, in 1989 South Australian educators joined what was a global education trend in addressing these two questions.[*] ... The most common reference points for South Australians in addressing the two core questions have been the United Kingdom National Curriculum and more recently the United States New Standards movement. South Australia's work has influenced and been influenced by events and ideas in other educational arenas. Most influential however has been local notions of what is valued, what is practical, what is needed and what is wanted.⁵²

The project was based on “a high degree of confidence in teachers’ knowledge of their students’ learning needs and in teachers’ abilities to make judgements about their students’ progress is possible”,⁵³ and, although the decision to introduce reporting was imposed from above, it was developed within the local system.

Too few members of the profession realised, however, how much room for influence there was in the development of the detail of the content of the materials and how they could be used. The nature of the enterprise was very broadly framed by Boomer, drawing on some works by Royce Sadler in Queensland, and some practices from the Toronto Benchmarks, but mostly building on some very strong values within the South Australian teaching profession.

In other words, while the *Profiles* were being developed nationally, SA was also developing its own largely home-grown form to reflect Boomer’s specific aims. Watching two forms develop together would have been somewhat disturbing to teachers, especially since neither form was well received by the teachers’ union, who saw them as “rationalist economic checking up mechanisms”.⁵⁴ In due

⁵¹ Stehn (1997)

* “What should students learn?”; “What should teachers, parents, educational support groups, government and the public know about what children have learnt?”

⁵² Stehn (1997, p. 166)

⁵³ Stehn (1997, p. 172)

⁵⁴ Stehn (1997, p. 174)

course the *Attainment Levels* were replaced by the national material,⁵⁵ probably to the chagrin of the local curriculum developers. But the *Attainment Levels* are important for this thesis because they reflect specific South Australian thinking.

Other Support Material

In 1994, after the AEC had withdrawn from the National Curriculum project, the Curriculum Corporation published *Using the Mathematics Profile*⁵⁶ which was designed to provide concrete help for teachers in working within the new structure by discussing the approach and use of the *Profiles*, as well as aspects of assessing, recording and reporting. Examples came from all the Strands, so in places this document provides some advice on how to approach Chance & Data.

The Curriculum Corporation and its predecessors were also involved at the end of the 1980s with providing well-designed, heavily structured examples of good classroom practice as a new form of curriculum support materials for teachers. The MCTP material described in Chapter 14 was augmented in 1993 with three books devoted specifically to Chance & Data and written in a style similar to that used earlier.* Not only did they contain a very rich range of activities for all levels, but they also provided commentaries on the mathematics and on how the material might work best in classrooms. A computer disc enabled the examination of a much wider range of real data and stochastic situations than had previously been possible in a standard classroom. Although the material was tested in classrooms, my own limited use of it has found that not all lessons were as predictable as suggested,[†] and I am not aware of any detailed evaluation of its use or effectiveness. However, it may be seen as a constructive attempt to integrate Intellectual, Social, Physical and Pedagogical aspects of stochastics teaching.

Finally, official supplementary material to develop teachers' understandings was produced during 1992 and 1993 by AAMT with funding from the Commonwealth Department of Employment Education and Training (DEET). This was designed for workshop presentation to teachers and others, and formed the basis for some in-service courses eligible for tertiary credit. It was initially intended to

⁵⁵ Stehn (1997, pp. 173–179)

⁵⁶ Olssen et al. (1994)

* Lovitt & Lowe (1993a, b); Finlay & Lowe (1993). CDC (1988) is often seen as synonymous with MCTP, but the Program was wider than this, and included these stochastics books.

[†] E.g., able primary school children were not surprised by the contradictory results with duelling dice in the way predicted by Lovitt & Lowe (1993b, p. 411).

be for those who held positions of responsibility in mathematics education as a basis for designing programmes for teachers⁵⁷ but was later modified for use with classroom teachers.⁵⁸ The project was grossly underfunded, with only \$3500 being paid per unit for all writing and trialing costs.⁵⁹ The material could only be used by licensed presenters who paid an up-front fee of \$1600, and charged \$20 per course to be eligible to award certificates of completion to participants.⁶⁰ In effect, AAMT and the local subject associations were becoming the authenticating authorities for teachers' professional development.

In SA such courses were offered by MASA from at least 1994 with the possibility (which came to pass) that they would earn credit as part of the University of SA's Graduate Certificate of Education (Professional Practice). Each consisted of ten hours of work which closely followed the ten modules in the printed material. Several modules might be combined into, for example, one evening's work.⁶¹

One of the eight workshops is called *Teaching and Learning Chance and Data*.⁶² This collection of exemplary activities by an active stochastics researcher has a strong basis in some stochastics research, seeks to "develop teachers' own understandings and skills",⁶³ reviews some useful resources for teachers, has been tested in schools, and is augmented by an expensive video. It does not claim to be comprehensive⁶⁴ and because it was prepared before publication of the *Profiles* it only makes links to the *National Statement*. It emphasises statistics more than probability, and cautions against its use by those inexperienced in stochastics.⁶⁵ There are no published links with the standard research literature and none of the advisory committee had done significant research into stochastics learning.⁶⁶

The final module examines the *National Statement*. Participants—teachers, parents, or interested others—might have had only a few hours of contact with the subject, but were encouraged to answer questions about *Statement's* aims for

⁵⁷ MacGregor Papers 25 Jun 91 *Call for Expression of Interest* [Documents in the Private Domain]

⁵⁸ MacGregor Papers 18 Sep 1991, Kiley (executive officer) to MacGregor

⁵⁹ MacGregor Papers 18 Sep 1991, Kiley (executive officer) to MacGregor

⁶⁰ MacGregor Papers *Draft of Australian Maths Works* 11 Nov 1993

⁶¹ MASA Workshops (1994/95)

⁶² J. Watson (1994aamt)

⁶³ J. Watson (1994aamt, inside front cover)

⁶⁴ J. Watson (1994aamt, p. 1)

⁶⁵ J. Watson (1994aamt, p. 6)

⁶⁶ This paragraph is a revision of J. & K. Truran (1995, p. 533).

Chance & Data, such as “From your experience, do the levels suggested appear appropriate?” Other questions asked about what terms the participants did not know, and what activities they felt confident about using. Finally, they had to assess the value of Chance & Data in the school curriculum.⁶⁷ This module is the only section of the official material which specifically calls for comments from teachers and others and it asks only about feelings. Academic and/or pedagogic disciplines are not put on any agenda for discussion. This is one of the strongest indications that the National Curriculum documents were primarily driven by Social forces, rather than by Intellectual or Physical ones, even though the Chance & Data Workshop did have a significant research base.

The Use of the National Curriculum for Assessment

One effect of the Industrial influence was that schools in the 1990s were more and more called to be “accountable”, and the National Curriculum documents were constructed in a form which, it was hoped, would increase the precision of teachers’ reporting of children’s progress. So while the *Profiles* were being constructed, advice was sought from the ACER as to how the material could be used to infer students’ levels on a profile strand from records of their achievements. A discussion paper was produced⁶⁸ which claimed that either records of teacher judgements or student performances could be used to plot a student’s achievement level for each of the Strands on a continuum, together with an indication of the average achievement level of some chosen reference group and an indication of the range of achievement demonstrated for the middle 80% of the group.

The ACER’s plans were similar to those used by the National Assessment of Educational Progress in the USA⁶⁹ and by the International Assessment of Educational Progress movement.⁷⁰ In Chapter 19 we shall discuss some of the weaknesses of the approaches used by these two organisations. The plans were summarised for the Industry Education Forum in Australia and widely circulated among its members,⁷¹ but they have made limited impact on Australian schools as yet. The proposed model was “Rasch analysis”, discussed further in Chapter 17, which requires a computer program, and is based on a probabilistic argument and an assumption that the tasks can in some way be ordered. But as we have

⁶⁷ Watson (1994aamt, pp. 147–150)

⁶⁸ Masters (1993)

⁶⁹ Masters (1993nief, p. 3)

⁷⁰ Masters (1993nief, p. 7)

⁷¹ Masters (1993nief)

seen in Chapter 8, the existence of an order property for tasks in probability learning has increasingly been called into question. In Chapter 7 we first met Eisner's concept of "complexity": we shall show in Chapter 20 that the concept of "probabilistic understanding" has so many complex facets any valid measure of its assessment requires an almost impracticable richness and we shall see in Chapter 24 the effect of this complexity on the teacher's task. Here we only need to note that there seems to be some danger that the Technocratic approach may lead to the assessment tail trying to wag the pedagogical dog.

Summary

By 1994, almost entirely through government funding, Australian teachers of primary and secondary classes had been provided with a detailed set of aims for teaching stochastics, together with a variety of exemplary activities which might form a basis for developing a sound classroom pedagogy. Teachers saw many positive features in the *Profiles* but almost universally complained about unreasonable time pressures for implementing them, pressures exacerbated by poor central administration.⁷² Their comments were almost entirely about Physical aspects of the material, not of Intellectual or Pedagogical ones, or even of the Social philosophies which were underpinning the changes. The next section will examine Intellectual aspects, while Pedagogical issues will be discussed in Part E.

AN ASSESSMENT OF PROBABILITY IN THE NATIONAL CURRICULUM DOCUMENTS

The National Statements and Profiles project has the potential to be a centralised mechanism of conservative ideological control, or a liberating curriculum tool. ... [A]s currently conceived, it is more likely by default than by design, to be the former. But it doesn't have to be. Educators committed to social justice should be using the implementation phase of the project to argue the case for the project to adopt an education for social justice curriculum stance.⁷³

The quotation above is an example of a Social criticisms of the National Curriculum documents. There has been some Academic criticism of content,⁷⁴ but the field of probability learning has attracted little such attention. Yet, as we have seen in several places in Chapter 3, if the content of the curriculum is not

⁷² Lokan & Frigo (1995)

⁷³ A. Reid (1995, p. 17)

⁷⁴ McGaw (1997, p. 3)

academically reputable, then the potential for social injustice is very high. Without in any way denigrating the aims of those concerned with using school as an instrument for social justice, I would argue that academic rigour is a pre-requisite for achieving these aims. So this section seeks to assess whether the documents do have a sufficiently high level of academic rigour.

The complex structures outlined make it difficult to provide the reader with a clear and simple framework for the arguments now to be developed. The discussion will mainly follow the structure and approach of the *Profiles*, and refer to the other documents where appropriate. Given that the average teacher is likely to be looking for concrete examples to help understand what is required in this new and complex environment, particular importance will be paid to the *Work Samples*, bearing in mind that any example provided in an official text is likely to develop an authority far beyond that intended by its author. To keep the text simple, relevant parts of the documents are reproduced in Appendix III. This includes all sections of the *National Statement*, *Profiles* and *Attainment Levels* which are related to probability, together with those sections of the *Work Samples* and some other documents which are specifically referred to in the text. The original pagination and ordering is retained in Appendix III, so that items may be easily located using the formal reference quoted in the footnotes.

It is unnecessary to discuss all the Work Samples relevant to probability. A small number will be examined to make the point that the models provided in these documents have serious limitations. The reasons for this situation will then be examined and interpreted within the BSEM. I shall start with a brief examination of some general principles outlined in the *National Statement* to assist in assessing the material against the authors' intentions.

General Principles

Five important principles for teaching probability are proposed by the *National Statement* and may be summarised in the following form:

- the topic should be integrated with other school topics;
- [s]tudents should be helped to refine and extend their use of [colloquial] language so that they are more able to make sense of their everyday experiences;
- chance activities should be provided in schools from the earliest stages in order to help students develop more inclusive conceptions;

- children should learn about both experimental and theoretical probabilities;
- simulation, preferably with a computer, should be a common teaching technique.⁷⁵

The Constructivist philosophy underlying these principles may be clearly seen, particularly in the term “more inclusive conceptions”. As we have seen in Part B the importance which Constructivists attach to making meaning is a valuable counter-balance to the all too frequent tendency to regard teaching as telling. But if children are to develop understandings which are shared with the mathematical community, then it is essential that these shared meanings are well understood by teachers, and that teachers have some useful ways of helping children to develop them. Activity is not enough, as we saw in the discussion on Process Mathematics in Chapter 4. So although the principles propounded here appear to be rational ones, they fail to address important pedagogical issues. In the following discussion we shall see that:

- few examples are provided of integration with other school topics;
- the issue of learning formal language and distinguishing it from colloquial language is not addressed;
- little help is provided for the teacher to know in advance what are the commonly held misconceptions about probability;
- little help is provided for teachers in knowing how to ensure that activity really does lead to the elimination of misconceptions, and not the growth of more;
- subjective probability needs to be addressed as well as experimental and theoretical probabilities.

Critiques of Specific Examples

This section illustrates how and why the material was constructed. It looks first at how probabilistic language is addressed, and then examines three Outcome Statements—one from each of what would be Bands A to C in the *National Statement*. The examples have been chosen because each has a Work Sample available, and because they represent key steps in coming to an understanding of random processes.* It concludes with some general comments on relevant issues not picked up in the three examples.

⁷⁵ AEC (1991, pp. 163–164)

* Many of comments which follow were made in a letter to Mr Rob Nagy, of the South Australian Fulham Gardens Curriculum Centre on 22 Apr 94 in response to a limited review of the *Profiles* and *Work Samples* instituted by the Minister of Education to ascertain whether they contained serious errors of fact, omissions, or inappropriate emphases. To the best of my knowledge, none of the submissions made had any practical influence.

LANGUAGE

Aspects of probabilistic language have already been discussed in Chapter 4, including specific difficulties with some probabilistic terms, such as “fair” and “chance”. But in the instructions for the task in Work Sample “Chance and Data 8” at Level 2, the word “fair” is used as a synonym for “symmetrical”.⁷⁶ Similarly, the word “chance” is used at Level 3 in at least three different ways:⁷⁷

- as a synonym for “probability”, i.e., as a number;
- (in the title of the Strand “Chance & Data”) as a generic name for a set of situations;
- as something which can be influenced. (“[m]ake informal statements about how one might influence the chance of an event happening (... I am less likely to have an accident if I use the back road ...).”)

Using three quite different meanings does little to help children “refine and extend their use of [colloquial] language”.⁷⁸ The third meaning is the most worrying. Taking a back road does change the probabilities, but does not influence the factors leading to indeterminism: it engages different factors. This is not mere pedantry. Chapter 5 has shown the rich and diverse approaches to chance which many children and adults have. “Chance” is a vernacular word which is particularly difficult to “domesticate”⁷⁹ and has no place in the formal teaching of probability. This is an important example of how the documents have failed to find an effective way of linking Intellectual clarity with acknowledgement of the Social forces implicit in vernacular language.

Mathematical symbolism is another stumbling block. In Work Sample “Chance and Data 17”⁸⁰ the student writes “chance” values as “a in b”, and “probability” ones as “a/b”. If “chance” has any meaning here then it is as a synonym for “probability”, but it seems that the child has not understood this. And it would be easy for an inexperienced teacher reading this text to surmise that the words have different meanings which are reflected in how their values are written. A model like this without comment may make confusion worse confounded.

So in spite of strong emphasis in the *National Statement* on the need for precise language, we can see that the material provided for teachers is likely to lead to

⁷⁶ AEC (1994b, p. 123)

⁷⁷ AEC (1994a, p. 62)

⁷⁸ AEC (1991, p. 163)

⁷⁹ *Vide* ch. 5.

⁸⁰ AEC (1994b, pp. 134–135)

confusion. What does this tell us about the forces operating on those contracting the documents? The blame cannot lie with the industrialists because the responsibility for writing within the overall structure rested with the educationists.

The answer is probably that nobody thought about the issue. The research findings of which I am aware and which might have helped the writers did not come out until after 1992, and were not widely disseminated.⁸¹ But these findings often only formally confirmed what a small amount of thought about the issues might have predicted. On this issue, the official documents suggest that their authors were not experts in either stochastics or stochastics teaching, and were probably bravely working *de novo*, confident that their general classroom experience in other, more well-developed, fields would carry them through.* The writers were working, very much to a dead-line, with Social, rather than Intellectual aims.

EXAMPLE 1: THE NATURE OF CHANCE

The first example is that concerning young children's understanding of the concept of chance referred to above. We have seen in Chapter 8 that this is an area researchers have found hard to investigate. We discuss Outcome 1.23, illustrated in Work Sample "Chance and Data 1".⁸² In the Work Sample Suzi shows that she can use words like "might", "will" and "won't" sensibly, and the commentary emphasises that some children will have similar understanding, but be unable to articulate it. But we cannot say that Suzi has achieved Level 1.23 by showing "some recognition of the element of chance in familiar daily activities". The world of young children is dominated by adults. Things happen for reasons they neither understand nor question. Some, such as the consequences of heavy rain, may be seen by a child as outside the control of adults, but many are likely to be seen as within their control. This is not what mathematicians mean by "the element of chance". "Chance events" are more than events with unpredictable outcomes and more than events which are not subject to the control of an individual. They are events which are subject to random forces and are not, as shown in Chapter 5, reversible. But the examples here are very much reversible: for example, decisions

⁸¹ E.g., Peard (1993); J. Truran (1992)

* Such an approach is not uncommon. At the second meeting of a Discussion Group on the Teaching and Learning of Stochastics at the PME Conference in Lisbon, 1994, we were joined by a well-known mathematics educator who has three works cited positively in this thesis, but who, to my knowledge, had no particular experience in stochastics. Even though most participants did have substantial experience (including at least three recent research degrees), this "expert" proceeded to think his way through the problems in a way which took me back to my first musings before starting my Masters thesis. He made the same plausible leaps of logic which experience had eventually showed me to be unsatisfactory.

⁸² AEC (1994b, p. 116)

about watching television may well be reversed by adult *fiat*, as children well know. And even in countries whose weather forecasts provide a numerical estimate of “the probability of rain” such numbers may not be seen as estimates of chance in the mathematical sense.⁸³ The thinking underlying this Work Sample and its analysis is jejune. Does the support material provide clearer objectives?

Australian Maths Works presents this work first and uses a similar approach to the Work Sample, with a special emphasis on the use of chance words in the media. It also encourages the ordering of uncertain events and their representation on a scale from 0 to 1, and takes the work further to look at other issues like “odds” and conditional probabilities.⁸⁴ But, as we have seen, this does not address the specific issue of randomness and merely follows what has been defined by the basic documents: it has not addressed them critically.

The *Attainment Levels* do not use the approach based on common events. They restrict themselves to simple, common RGs, but say little about language or the use of a number line. The MCTP material sets the language of chance into a context of random draws and possibly computer simulation. The authors encourage the marking of estimates of chance on a line marked from “no chance” to “certainty”.⁸⁵ However, the use of appropriate words to describe different positions on the number line is not mentioned. Later, in the section on “Early Probability”, the authors concentrate on estimating experimental probabilities from the results of small experiments using spinners and urns. Children are encouraged to decide on the best strategy to employ to win games using these RGs, but they are not encouraged to use any form of the number line.⁸⁶ This is a very good example how a well-constructed work might have been greatly improved if more attention had been paid to available research findings, in this case, those of Acredolo et al. (1989) reported in the proportion section of Chapter 8.

Many of the MCTP activities are excellent, and probably help to develop intuitive ideas through games.* But we also need to decide whether these activities do link with the *Profiles* and *Work Samples*. For Outcome 1.23 the short answer is that they do not address *familiar* daily activities at all. They address structured activities

⁸³ Konold (1989)

⁸⁴ J. Watson (1994aamt, module 1)

⁸⁵ Lovitt & Lowe (1993a, p. 35)

⁸⁶ Lovitt & Lowe (1993a, pp. 48–58)

* The use of “probably” here is not meant to be dismissive. We do not know whether these activities live up to the claims made for them. Research on what children actually learn from such games has simply not been done. Nevertheless, the games are concrete, interesting and complex—basic criteria for educational success—and so are likely to be effective.

using specialised material. As such they are essentially sound, both mathematically and pedagogically. But they do not address Outcome 1.23.

Nor does the MCTP background material help because it defines “probability” as “long-run relative frequency”⁸⁷ which over-emphasises experimental probability. The brief comments about experimental and theoretical probability⁸⁸ are not deep enough because, although the MCTP work is based on RGs, the concept of an RG is neither mentioned nor generalised. Only when this idea is clearly understood can the fallacy of the *Work Samples* approach be clearly seen.

We may contrast this approach with a 1991 French text encouraging a frequentist approach in secondary classes.⁸⁹ Several pages of discussion intended for the teacher are devoted to the notion of probability and to the meaning of the Law of Large Numbers.⁹⁰ The discussion is formal and the circularity of some commonly used definitions is explicated. Strong emphasis is placed on accurate language.

In other words, some of the difficulties of using a frequentist approach to probability were known about when the National Curriculum documents were written. As we have seen in Chapter 8, by 1991 a well-developed network of stochastics researchers existed, so finding out what had been done in other language fields was possible though it might have taken time. However, such time was not made available to those writing the curriculum documents. So the existence of linguistic fences meant that Intellectual forces had little influence on the construction of this Outcome at least. Social forces with a strong concern for language development had a considerable part to play, but no other force is particularly obvious.

Framing appropriate criteria at this level is difficult. In 1999 a large assessment project discussed with me the possibility of writing suitable *brief* questions to assess probabilistic understanding in junior primary children, but the project was abandoned because its administration was seen as too hard. Similarly, the English National Curriculum eventually removed exemplars similar to the Australian ones, and probability was delayed until the upper primary school years.⁹¹

⁸⁷ Lovitt & Lowe (1993a, p. 38)

⁸⁸ Lovitt & Lowe (1993a, p. 46)

⁸⁹ Dupuis et al. (1992)

⁹⁰ Dupuis et al. (1992, pp. 69–74)

⁹¹ Keith Jones (1995)

EXAMPLE 2: THE NATURE OF A RANDOM GENERATOR

Our second example concerns the nature of an RG, and in particular, children's understanding of dice. It is covered, among other aspects, in Outcomes 3.23, 4.23 and 5.23,⁹² and in Work Sample "Chance and Data 8".⁹³ It is, of course, a fundamental part of an understanding of randomness and probability. It is true that the research summarised in Chapter 8 on this topic was all published after the *Profiles* were prepared, but the criticisms made below do not depend on the findings of this research at all, they are far more fundamental and obvious.

The child, in Year 6 or 7, is first asked to test whether or not a die is fair, then to make and test a prediction about the outcome of tossing a drawing pin. and then asked to investigate a non-standard die. Much of this task deals with the collection and presentation of data, but probabilistic outcomes are also involved.

Unfortunately, the analysis of this Work Sample has internal inconsistencies. It is seen as appropriate for Level 2, even though Debbie is seen as having reached aspects of Level 3; yet it is also described as generating outcomes at Levels 3, 4 and 5. The non-standard dice listed have four, eight or twelve faces, but the exemplar deals with one of ten. The three non-standard dice listed can be regular polyhedra, but the one actually used cannot be, although it presumably had sufficient symmetry for all faces to be equi-probable, but this is not clear from the text. The special problems of using a regular four-sided die which cannot land with just one face upward are also not mentioned.* Finally, there is no mention of the problematic meaning of "fair", as discussed above and in Chapter 4.

Putting these many issues aside, we can examine what Debbie and her partner did. They first conducted an experiment to estimate the experimental probability of a drawing pin landing point up. Their two samples produce slightly different results, so they took the mean as their best estimate. Debbie summarised her findings in several ways: she gave numerical results, she observed that the pin landed point up "the majority of the time" with a reason for this based on the heaviness of the head, and she rounded off her estimate to be able to use simple numbers to say that the tack landed point up two times out of three. Certainly she has achieved Level 3; she has been able to identify which outcome is the more likely. But her work is at a much higher level than this. She has made estimates of

⁹² AEC (1994a, pp. 62, 76, 92)

⁹³ AEC (1994b, p. 123)

* I can well remember encountering just this problem when I first started to experiment with non-standard dice in the classroom in c. 1970 and did not bother to test the tasks I was setting before presenting them to the class.

an experimental probability, and used them to make a better estimate. This is very nearly a Level 6 achievement. Furthermore, her use of reasons to support her findings represents Level 5.4 in Working Mathematically—“checks that answers fit specifications and make sense in the original situation”.⁹⁴

These points are important. Debbie is at least in Year 6, and her work is certainly at Level 4, if not higher. By using her work as an exemplar for Level 3 teachers may well be misled about reasonable expectations from a student just entering upper primary school. By failing to make links with other Strands, especially Working Mathematically, an opportunity has been missed to emphasise the holistic approach of the National Curriculum documents. There are Work Samples which do make links from Working Mathematically to Chance & Data,⁹⁵ but the linkage must be two-way and it is the harder way which has been overlooked.

Debbie then went on to toss the ten-sided die 23 times, and to report the outcomes. She knew that there was variation and observed that “this is probably just coincidence but maybe not”. This response reflects a critical dilemma in probability teaching—matching theoretical and experimental results. This is the basis of all statistical inference, as discussed in Chapter 6. Nowhere is this included in the *Profiles*. The concept of “significance” or “significant difference” in any form is missing. So a child might proceed right through this course and never come to a deeper understanding of how to resolve this dilemma other than by equivocation. We shall see this problem recur in Example 3. Discrepancies between theory and practice can be assessed subjectively. We do not need an expensive educational system to ensure that children attain this level of discrimination. Mathematics involves the numericisation of feelings—this is the special contribution which mathematics classes can add to a child’s development. Of course, this will not come immediately, but there is little suggestion in the Outcome Statements that this is the path along which we wish to lead our children. The idea of numericisation ought to be a *sine qua non* of mathematical curriculum development, but it seems not to have been for the National Curriculum documents. This is the great weakness of the Constructivist approach underlying these documents.

There are yet more reasons for being concerned about this Work Sample. Debbie has been asked to assess some RGs in a specific way and she has done so quite well. Does this mean that she has a deep understanding of these RGs? There is insufficient evidence to say. In Chapter 18 we shall see that there are many different situations in which a probabilistic understanding might be assessed:

⁹⁴ AEC (1994a, p. 5)

⁹⁵ E.g., AEC (1994b, p. 15)

- prediction of outcomes;
- comparison of random generators;
- comparison of outcomes;
- fair allocation of payout for bets;
- sequences of outcomes;
- questions of technical knowledge.

Debbie has only been asked to compare outcomes. It is certainly not the case that children who can answer one type of question well can necessarily answer the other types well. While several of these types are included in the Work Samples, they are dealt with in different situations, and there is no suggestion that the children need to be correct in a wide variety of situations before being judged to have attained a certain Level. Outcome statements have come into fashion because it is now appreciated that it is impossible to *know* what a student understands: it is only possible to *infer* what is understood from a student's statements, writings or actions. Each Outcome that a student produces contributes to the basis on which we make our inferences about his or her level of understanding. Inference is a "provisional" activity, to use Runcie's approach mentioned in Chapter 3. The more Outcomes which are available, the higher the probability that our inference will be correct. But this is not the same as the deterministic tenor of the *Work Samples* which posits that if a student has produced a certain outcome once, then that student has achieved the associated Level. The possibility for regression must be considered, as we have already mentioned.

This Work Sample does well to use unusual symmetric and asymmetric RGs, both of which are valuable for developing children's understanding of chance. But both raise pedagogic difficulties rarely encountered within the primary curriculum in the past, and neither is discussed in any detail in any of the principal National Curriculum documents. It is true that many of the difficulties have only been formally reported after the publication of the *Work Samples*, but the potential for difficulties is so obvious that it cries out for the provision of teacher support.

EXAMPLE 3: USING TWO RANDOM GENERATORS AT ONCE

Our final example deals with examining the outcomes arising from two RGs, such as from tossing two dice together. We consider Outcome 6.23⁹⁶ which is illustrated by Work Sample Chance and Data 17.⁹⁷ This Outcome is a complex

⁹⁶ AEC (1994a, p. 108)

⁹⁷ AEC (1994b, pp. 134–135)

one, covering several skills, of which allocation of probabilities and independence have not been discussed so far. Some technical issues need to be discussed first.

The claim that Jane had “generated a two-way table” is false. She has made entries in a table which had been generated for her—a much easier task than constructing a suitable table. The distinction between probability and proportion seems to have been glossed over in the commentary. Falk has observed that

[t]he ability to calculate proportions as such does not necessarily signify an understanding of probability. A realization of the impossibility neither of controlling nor of predicting the outcome of the immediate event is crucial.⁹⁸

This issue has been discussed by several workers. After considering their views and examining my own data from clinical interviews I am in agreement with Falk.⁹⁹ There is strong evidence in this Work Sample that Jane is behaving algorithmically, so it is simply not possible to tell from the evidence presented whether or not she understands the distinction between probability and proportion.

A similar judgement must be made about whether Jane has reasoned about equally likely events to assign probabilities. It is not obvious that the Cartesian product of two sets of equally likely events will produce equally likely events. In my experience this issue is poorly dealt with by both textbooks and teachers, and it is likely that Jane has simply manipulated numbers as taught[†] and not even formally decided whether the two RGs are independent or not. Because Level 6 discusses compound probability spaces, at least one concept of independence must be discussed at this stage, but the *Profiles* first mention independence at Level 7. I have discussed this issue in depth elsewhere;¹⁰⁰ all that needs to be said here is that independence has been assumed, but not made explicit.

This Work Sample also highlights the failure of the *Profiles* to include any calculus of probabilities in its requirements. It is possible that it might be required under “independence” in Level 7, but Work Sample 20¹⁰¹ does not suggest that it is. One important part of any course in probability should be to show that compound events can be analysed theoretically as well as experimentally. It is quite possible for children in Years 8–10 to do this sort of work with success.

⁹⁸ Falk et al. (1980, p. 183)

⁹⁹ *Vide* J. Truran (1992, pp. 200–206; 1994pme).

[†] *Vide* J. Truran (1992, pp. 34–35) for a deeper discussion of this issue.

¹⁰⁰ J. Truran (1992, pp. 31–47); J. & K. Truran (1997ind)

¹⁰¹ AEC (1994b, pp. 138–139)

It is important to note that this example was included in the draft form of the *Profiles* which was distributed for critical comment to the wider mathematics community.¹⁰² Although some of the exemplars in the draft were omitted from the final version for good reason,¹⁰³ this one, which is particularly poor, was not picked up by the vetting process, even though better examples were certainly available, because they are included in the Working Mathematically section.¹⁰⁴

In the support material, compound events are considered in Module 3 of *Teaching and Learning Chance and Data*.¹⁰⁵ Here they are presented in the context of a dice game whose rules favour those participants who have some understanding of the non-rectangular distribution of the sums of the outcomes of two standard dice. It is a good game, and children are encouraged to reflect on their experiences to try to find a systematic way of understanding the distribution. As one example of a constructive exploratory probability activity this Module is a valuable one.

This example raises important issues about the making of links between individual activities and modular nature of the *Profiles*. The task presents a rich, intuitive, holistic approach to an important probabilistic issue and for this reason responses will be difficult to match with the detailed outcome statements of the *Profiles*. Difficult, not impossible. Good matching will require a broad mathematical background from a teacher. The difficulty is that the teacher needs to have sufficient background to be able to interpret what mathematics is involved in the game, and to be able to see which aspects of the mathematics have been learned by the children. Undoubtedly, the activities were thoroughly trialed in the classroom, but one must ask whether the way in which teachers incorporated the activities into their wider knowledge, or lack of knowledge, was also trialed.

The MCTP material deals with work of this kind in some detail.¹⁰⁶ The examples start by considering the difference between the scores on two dice, presumably on the grounds that this will be less familiar and therefore more challenging to the students. Some computer simulation is also available. The work is extended to a number of examples which look at the mathematical meanings of “fairness” and “expectation”. A small amount of background material is provided for the teacher in the form of tables illustrating the Cartesian product for two dice according to different rules. While the issue of independence is not mentioned here,

¹⁰² *Mathematics Profiles Level 5 Draft pp. 24–25*

¹⁰³ E.g., a case where a child claimed that $10 + 4 = 6$ (*Mathematics Profiles Level 1 Draft p. 14*)

¹⁰⁴ E.g., AEC (1994b, p. 15)

¹⁰⁵ J. Watson (1994aamt)

¹⁰⁶ Lovitt & Lowe (1993a, pp. 59–71)

it is discussed briefly with reference to a more complex situation in a later chapter.¹⁰⁷ The material is lively and interesting, it is well constructed, it can be easily linked to the *Profiles*, and is a much better model than that provided in the *Work Samples*. We may say that, in this case, the support material is quite effective.

This particular moderately complex example illustrates the need for good activities to be supported by deep teacher knowledge to draw out the mathematics of a situation—a view which is not really contrary to the Constructivist view of learning. The support material has provided good ideas for activities, and some mathematical background, but not enough. And there is a danger, as we shall see below, that the very success of the support material in solving some of the Physical pressures on the classroom teacher may discourage teachers from delving deeper into the Intellectual issues. As we saw in Chapter 11, the rise of teacher control over the curriculum was partly motivated by a desire to find what goes down well in the classroom. This is a desirable aim, but not the only one.

SOME OTHER AREAS OF CONCERN

Finally, we need to look at five other aspects of the *Work Samples* and *Profiles* to illustrate some of their other limitations as models for stochastics teaching:

- links with other aspects of mathematics;
- links with the Language Learning Area;*
- links with other Learning Areas;
- links with pedagogical knowledge;
- links with research into stochastics *obuchennyi*.

Links with Other Aspects of Mathematics

Using Outcome statements for assessment is complex, but the official texts tend to overlook this. It would have been helpful to have been shown portfolios of students' work which, taken alone, suggest that a student has attained a Level, but which, when taken with other work, suggest that the student has not done so.

Similarly, there is a need to remind teachers that no one Strand is an isolated part of the mathematics curriculum. Probability rests heavily on Number, especially fractions and decimals, and to some extent on Space and Measurement. We have seen above that the links between Chance & Data and Working Mathematically

¹⁰⁷ Lovitt & Lowe (1993a, p. 78)

* "Learning Area" is the new name for "subject". The semantics may be significant. "Subject" implies that there is content to be conquered (thrown under); "Learning Area" implies that there is something which may be explored.

need to be spelled out, similarly the links with the other content Strands should also be addressed. They do seem to have been considered by the authors, because the knowledge about and skills with fractions, decimals and percentages required for Chance & Data at different Levels are included in the Number Strand at the same or earlier Levels. But it is important that they be emphasised in practice.

But one skill which does not seem to have been considered is the approach to arithmetical accuracy. Consider Work Sample “Chance and Data 20”¹⁰⁸ at Level 7. Tai is modelling a situation quite well, but makes a simple computational error when multiplying \$0.25 by 6 which has put his estimate of profit in error by 50%. So for this particular piece of work, but perhaps not in other cases, Tai has not achieved Level 4 of the Number Strand, which requires him to “[u]se a calculator efficiently for operating on decimal numbers ...”¹⁰⁹ or to “[use an] understood written [method] to ... multiply ... whole numbers, money and measures (two places, whole number multipliers ... to 10).”¹¹⁰ Nor has he attained Level 5 which requires him to “[estimate and calculate] mentally with whole numbers, money and simple fractions, including multiplying and dividing some two-digit numbers by one-digit numbers”.¹¹¹ Finally, Tai has also not achieved Level 5 in the Working Mathematically Strand which requires him to “[check] that answers fit specifications and make sense in the original situation”.¹¹² Without wishing to detract from Tai’s very real achievements in this problem I would argue that the *Profiles* make it clear that he ought to have found an error of this type and that his achievement in some Strands at lower Levels needs to be re-assessed. This is not easy for a teacher to do. The commentary on this Work Sample sees the error as “relatively minor”. In one sense it is: certainly less important than another error discussed in detail. But the Industrial model which has driven the construction of the *Profiles* will see it as very important because it requires that work is done within specified accuracy limits, which will certainly not be as high as 50%.* One can sympathise with a teacher who is aware of the significant intellectual work which Tai has done in this assignment, and who does not want to make too much noise about what may well be an atypical error made under pressure. The teacher is likely to concentrate on the more serious error and to commend the boy for his

¹⁰⁸ AEC (1994b, pp. 138–139)

¹⁰⁹ AEC (1994a, p. 9)

¹¹⁰ AEC (1994a, p. 9)

¹¹¹ AEC (1994a, p. 9)

¹¹² AEC (1994a, p. 5)

* The mathematical world will never live down its two top quality collaborating research teams who lost a space craft because they were not using the same system of units!

essentially logical, well-administered investigation. But good teaching does require that the error be addressed; a skilled teacher might let a child savour success for the time being, but will note that the issue needs to be returned to at a suitable time in the future. This is unlikely to happen if the error is labelled as “relatively minor” when it is a serious breach of the standards for another Strand.

Finally, some Work Samples which illustrate that a child has attained a certain Level still include examples of common weaknesses in children’s work which it is desirable to discourage. One example is the false distinction, already discussed above” between “1/6” and “1 in 6”. Another is the use of “the probability” instead of “the theoretical probability” as discussed in Chapters 4 and 6. Because there is no comment on these weaknesses, it is likely that teachers will come to take them as acceptable. This is not in children’s best interests.

Links with the Language Learning Area

Another common weakness is in spelling and grammar. Work Sample “Chance and Data 13”¹¹³ is a good piece of Level 4 work, but contains “watermetre” and “the average people using the wash taps”. These are not serious Level 4 errors, but Work Sample “Number 25”¹¹⁴ at Level 6 contains “accont”, “speacials”, “dependant” and “veiw”, which are serious, but allowed to pass without comment. The documents need to remind teachers that the spelling in a Work Sample may not be at the same Level as the mathematics, and that this mismatch needs to be recorded and acted upon. This is hard to carry out in a classroom, and the compartmentalisation of the *Profiles* does not encourage teachers to try.

Links with Other Learning Areas

Examples of links with probability and other non-scientific subjects are hard to find. This seems to be partly because the culture of such subjects is not mathematical. Yet, for example, examination of the Learning Area “Studies of Society and Environment”¹¹⁵ reveals a number of areas where probabilistic ideas are implicit:

- 3.6 Identifies issues about care of places arising from the different ways in which they are valued;
- 4.12 Shows how information is used as a resource to make and record decisions;

¹¹³ AEC (1994b, pp. 128–129)

¹¹⁴ AEC (1994b, p. 81)

¹¹⁵ AEC (1994d, pp. 4–9)

- 4.15 Identifies decisions that have to be made by groups and individuals about production and consumption;^w
- 5.12 describes and applies different ways of managing financial resources;
- 6.4 Explains and predicts variations in places over time by referring to processes that may affect natural and built features;
- 6.5 Explains consequences of human modifications of natural and built features of places;
- 6.13 Explains the ways in which various natural systems interact on a global scale and the ways people affect them;
- 8.5 Evaluates the role of planning and management in balancing or deciding among competing demands for the use of places.

All these Outcomes could be enriched by mathematical approaches such as modelling and simulation and support material is available. For example, a simulation game of the economics of running a cool drink stall, which would be an excellent part of a study of Outcome 4.15 and quite appropriate for the age level, has been available for 20 years. But the accompanying *Statement* for Studies of Society and Environment says very little about the making of links with other Learning Areas. It points out that the Learning Area is often approached as part of an integrated study and mentions potential links with the Learning Areas of Health and Language Other Than English, but does not mention of potential links with the Learning Areas of Mathematics, English or Science. Seven curriculum perspectives are suggested—none rest in discipline knowledge.¹¹⁶

In practice too, mathematical approaches are notably absent from many Work Samples provided. One Work Sample on budgeting¹¹⁷ is predominantly arithmetical, but does contain some simple optimisation principles. However, in almost all cases the analysis was non-numerical, even when numerical data would have been helpful. For example, one Work Sample at Level 4 reported political lobbying by students for a bicycle track to make journeys to school safer.¹¹⁸ The report did state the percentage of students who owned bicycles, but it also presented photographs of the proposed track with comments like, “We think there’s room for a track around this oval” and “There seems to be room without interfering with the soccer field”. This project was an important and eventually successful one. It would have been easy to have estimated the width required for a track and to have measured the width available beside the oval and the soccer field. This model provided for teachers of Society and Environment has omitted basic math-

¹¹⁶ AEC (1994c, pp. 1–10, 19, 35, 44)

¹¹⁷ AEC (1994d, pp. 132–133)

¹¹⁸ AEC (1994d, p. 71)

ematicisation. It is therefore unlikely that more sophisticated approaches like probability and simulation will easily find a place in the syllabus.

An interesting example of this with some pluses and some minuses is the following example from Level 4 of “Studies of Society and Environment”¹¹⁹ which touches on statistical variation, shows the importance of accurate technical language, and raises radical questions about the appropriateness of the general mathematics curriculum.¹²⁰ Two objectives identified at the Level are

- identifies the types of data and sources required by a task and decides how they will be used to gain information;
- translates information from one form to another.

These objectives are seen as being achieved when students *inter alia*

- choose between and design pro formas as a basis for data collection;
- explain the information in a graph, in speech or in writing;
- give a talk summarising data organised in a chart, timeline or map.¹²¹

The Work Sample uses a data base to find the best places in WA for dairy farming to illustrate “the factors that affect resource use and development—a suitable topic for the end of primary school. The Work Sample is well presented, and suggests that the student had understood most of the basic ideas. Not only were Chance & Data skills used, but several other mathematical skills as well, including simple subtraction, the use of suitable units and also “>”, as well as simple map reading. Two vernacular words were used where technical words would have been preferable—“difference” rather than “range” to measure spread, and “degrees” rather than “latitude” to measure location. The major omission from the model answer was any indication that the child appreciated the probabilistic nature of rainfall distributions. This is not a trivial point: Australia’s large climatic variation took European settlers a long time to appreciate and even longer to adapt to. The deterministic maxim “rain follows the plough” brought countless sufferings and financial collapses.¹²² So the probabilistic nature of rainfall is not a concept that children are likely to appreciate without some teaching.

The corresponding Level in the Mathematics Profile shows that the mathematical ideas which had been used correctly had all been covered before Level 4, with the

¹¹⁹ AEC (1994d, p. 69)

¹²⁰ The following section is a modification of part of J. Truran (1996)

¹²¹ AEC (1994d, p. 67)

¹²² Meinig (1962/1988, e.g., pp. 59–60)

possible exception of measures of rainfall. But those ideas expressed in vernacular language had not. Spherical geometry and the idea that a map is a transformation are both hinted at in the specifications for Mathematics Level 4, but neither is dealt with explicitly. The idea of extrema is mentioned in the Chance & Data specifications for Level 4, but the idea of a range is not specifically required until Level 6 and even then the term itself is not mentioned at all.

All comments on the example must of course be speculative, but they raise interesting issues. Did the mathematicians realise it would be useful to discuss and define “range” and probabilistic distributions rather earlier than is suggested in the *Profiles*? Had they thought about how spherical geometry could form a useful part of the upper primary syllabus? Did the environmentalists believe that technical terms are preferable or not? What concept of latitude did the child have—a two-dimensional one or a three-dimensional one? Most primary school teachers are in the privileged position of teaching all subjects to their students, so what did they do in practice to blend the demands of the two Learning Areas?

This example suggests that the curriculum documents for different subject areas were not prepared as a unified whole, and that where the areas are poorly integrated, then children’s understanding is less precise. Some degree of sensible integration is to be expected in many topics because experience will have led teachers to realise what is achievable at different ages. However, as we have already mentioned many times, and will do so again in Part E, there is not the same body of experience about how best to incorporate probability into the teaching of other subjects as there is with more traditional topics like number, space and measurement. If there are discrepancies even with these Strands, then it is not surprising that there are discrepancies and omissions with the Chance & Data Strand, some of which have been illustrated in this example.

Links with Pedagogic Knowledge

McGaw has claimed that the *Profiles* were developed using curriculum experience and “where available, research evidence on the developmental sequence of skill acquisition”.¹²³ This claim is difficult to substantiate for Chance & Data, and, even if it is true, his omission of both pedagogic and discipline knowledge as other bases suggests that the theoretical base of the *Profiles* is limited. For example, we have already mentioned that the *Profiles* make no suggestion about appropriate action if a student regresses to a lower Level, but all good pedagogues know the

¹²³ McGaw (1995)

importance of systematic practice for sustained learning. This seems to have no place in an outcomes-based assessment system.

We shall show below that Australian teachers often lack stochastics content knowledge and tend to be unwilling to acquire it. A discussion paper prepared for primary teachers¹²⁴ can give some indication of the situation in South Australian primary schools in 1992, a time when it was becoming clear to teachers that they would have to learn to teach stochastics well. Some teachers were asked to set certain tasks for children and to submit their responses for publication. The two tasks relevant to probability were:

- Throw 3 dice for 4 minutes and record what happens. Record your information on grid paper.
- Throw a die and record the numbers which are rolled.

The Process Mathematics approach may be clearly seen. The published responses from all Year levels show that the task was seen by the teachers, and hence also by the children, as a data collection and presentation task. The teachers' comments indicate the aspects of the work which they saw as important:

- the same task can be recorded in a variety of ways;
- all students can achieve success;
- students have organisational skills;
- there is a basis upon which students can develop their skills.

Although the comments also made suggestions about further work, they made no mention of the randomness inherent in the activity, and none of the published responses indicated that any of the children had observed this aspect of the situation. As we have seen in the environmental example discussed above, appreciating the randomness of a situation is an important real-life skill.

Presumably the teachers contributing to this discussion paper had a strong commitment to mathematics education, though not necessarily a strong knowledge of stochastics. The centre page spread of the discussion paper listed some key questions, including some on chance, but all concentrated on the outcomes and not the random nature of the generator. The centre page also listed several good activities as well as providing a list of possible resources. But it is not unreasonable to presume that because the emphasis in these activities was on what the child could discover unaided, it is likely that the work which would be produced

¹²⁴ *Primary Mathematics Notes from the [South Australian] Primary Mathematics Association. Discussion Paper no. 16—1992. Chance & Data*

would be similar to the data processing examples accompanying the article: competent, but uncritical, and without any focus on the concept of randomness.

By contrast, the opening section of the discussion paper from a university mathematics educator contained suggestions for classroom activities which were grouped according the level of skills required for organising data and making predictions and amplified by questions which encouraged the children to think about what was happening.¹²⁵ The contrast between the offerings of the academic and some committed but inexperienced teachers is marked, and shows that in 1992 there was a still need for the development of a pedagogy of stochastics teaching to make discipline knowledge accessible to children.

Links with Research into Stochastics *Obuchennyi*

Watson has observed that the inclusion of the Chance & Data strand in the Australian National Curriculum occurred

without the benefit of any previous educational research in [Australia] on the learning of probability and statistics and without any formal recommendations for future research on the learning and teaching of the topics or for evaluation of the efforts of preparing teachers or actually implementing the curriculum.¹²⁶

It was not true that there had not been any Australian research, as we have seen in Chapter 8, though it may well have been true that this was not taken notice of. In any case, non-Australian research was almost certainly of value for Australia and there was plenty of that. Watson's other comments are indisputable, and she has shown that it is possible to provide useful problems for in-service workshops which are closely related to National Curriculum requirements and based on known research findings.¹²⁷ We have seen above some examples of neglect of the research by the National Curriculum documents; these will be discussed further in Chapter 22 where we shall conclude that mathematics educators

have made some progress in applying the principles of mathematics education to probability *obuchennyi*, [but] these findings have had little influence on actual pedagogic and assessment practice.¹²⁸

¹²⁵ K. Truran (1992pma)

¹²⁶ J. Watson (1992, p. 556)

¹²⁷ J. Watson (1994aamt)

¹²⁸ J. Truran (2000phd, ch. 22)

This section on “An Educational Assessment of Probability in the National Curriculum Documents” has been a long one in order to illustrate the complexity of the pedagogic task involved in teaching superficially simple ideas. How were teachers assisted to respond to this task?

MAKING PROBABILITY MORE WIDELY KNOWN

Go out on to the highways and along the hedgerows and make them come in; I want my house to be full.¹²⁹

Pedagogy will be discussed in detail in Part E, but here we shall examine three aspects of the dissemination of probability *obuchennyi* as a background for the Chapter’s concluding discussion on the forces underlying the *National Statement* changes. We shall find that the spread of the new ideas has been slow and that there have been few changes since the 1970s. If modern doctors continued to use leeches as a principal means of treatment there would be a public outcry, as we shall point out in Chapter 20. But this is what has happened with probability.[†]

TEACHERS’ VIEWS ON PROBABILITY AND CURRICULUM CHANGE

There has been relatively little systematic work done to assess the impact of the National Curriculum process on teachers. The available findings are somewhat eclectic, so cannot lead to firm conclusions, but they are consistent both with the received wisdom and with what would be predicted by the BSEM, so it is likely that they are broadly applicable to Australia. Some results lie outside the period covered by this thesis, but they will be considered because they seem to be of value.

Many teachers tend not to have a high regard for mathematics content knowledge. A project which ran from 1994 to 1997 and was eventually called *LUD-DITE** was set up by AAMT with funding from DEET to see how teacher development might be enhanced using distance education through modern technology.¹³⁰ Chance & Data was chosen as the focus because of a perceived emerging need among teachers, parents and students for knowledge and skills in this

¹²⁹ Luke 14: 23

[†] Some parts of this section have been presented in J. Truran (1999ausico).

^{*} Learning the Unlikely at Distance Delivered as an Information Technology Enterprise

¹³⁰ J. Watson (1997amt)

area.¹³¹ Part of the project involved the making of a CD-ROM “to include as much information as possible of use to teachers of chance and data”¹³² and to supplement that material in the accompanying text and the videos. The project was carefully evaluated and the response of the teachers to the mathematical material is particularly interesting.

The inclusion of the mathematical material in the text and video was generally well received by teachers of older students, while teachers of primary children did not feel they had time to digest all of the content and preferred to concentrate on the lessons ready-prepared for classroom use. The belief of the author that a good understanding of the content is important before launching into a teaching sequence was not universally shared by teachers.¹³³

Not only do teachers tend not to be concerned with acquiring content knowledge, but they often have little stochastic content knowledge to start with. For example, K. Truran has discussed some aspects of the content weakness of pre-service primary student teachers in SA.¹³⁴ Her findings corroborate more extensive research done in Spain and suggest that the Spanish findings may be applicable to Australia. These findings include:

- [s]on muy pocos los sujetos que reflejan un idea clara de las características de los fenómenos aleatorios. ...
- Los niveles de comprensión y utilización de los modelos normativos son mínimos y, normalmente, restringida a contextos de juego. ...
- La mayoría de las argumentaciones expuestas por los sujetos reflejan características intuitivas de razonamiento, con un predominio de esquemas causales y de juicios heurísticos, entre sus estrategias de decisión.¹³⁵

Nor are many teachers concerned to fill in their knowledge gaps. A number of commercial mathematics schemes designed for the English National Curriculum were produced with accompanying teachers’ handbooks. The Qualifications Curriculum Authority investigated how teachers used these handbooks for Key

131 *AAMT/DEET Project in the Teaching and Learning of Chance and Data via Open Learning* p. 4

132 J. Watson (1997amt, p. 28)

133 J. Watson (1997amt, p. 28)

134 K. Truran (1997mer)

135 Azcárate Goded (1996, pp. 286–287)

Very few subjects convey a clear idea of the properties of chance phenomena. ... The levels of comprehension and the use of standard models are both minimal, and mainly restricted to the context of games. ... The majority of the reasons given by the subjects reflect a characteristic intuitive reasoning with a predominance of causal schema and heuristic judgements underlying their decision strategies.

Stage 2 work. It found that the handbooks were generally of high quality, but many teachers lacked the time to read them properly, so put them in a cupboard and attempted to operate the scheme without their assistance. If they used the material at all, it was to provide work for children and not to use its overall structure as a guide for their own creative teaching.¹³⁶ There has been a similar rapid commercial production of Australian material which claimed to conform with the new structures. For example, McGraw-Hill claimed in c. 1994 that their *Mathematics Today Series* “reflects aspects of the National Statement, Frameworks Course Advice and the Victorian Profiles Documents”.¹³⁷ It is unlikely that Australian teachers responded to such material differently from English ones. So we may conclude that the sheer complexity of the National Curriculum documents is likely to discourage the development of a deeper understanding among teachers of what is being taught and how it might best be learnt.

THE INFLUENCE OF EXTERNAL EXAMINATIONS

Some people claim that effective classroom change is best achieved by a top-down changing of the curriculum of external examinations,¹³⁸ though we have seen that this is not always effective. The National Curriculum was imposed upon schools which were already working within structures heavily dominated by the requirements of an examinations board at Year 12. The “Gilding Report”, mentioned in Chapter 15, was advised to place much more emphasis on applications in school mathematics curricula¹³⁹ and to include both probability and statistics in its concept of numeracy,¹⁴⁰ but its recommendations were more structural than specific.¹⁴¹ Until major changes were implemented by SSABSA in 1992, academic mathematics courses in SA still reflected the approaches of the 1970s. The SSABSA revision emphasised extensive flexibility in constructing courses and expected that learning and teaching would involve a balance of group and individual work, investigative and practical activities, consolidation, and the use of calculators, computers, and practical contexts.¹⁴² Such a remit is particularly applicable to statistical investigations, but the links between probability and statistics were poorly made. Only two statements referred to probability directly:

¹³⁶ Keith Jones, U of Southampton, UK, e-mail message, 28 Nov 97

¹³⁷ *Mathematics Today Series*

¹³⁸ Westbury (1980)

¹³⁹ Pitman (1990)

¹⁴⁰ Pitman (1990, p. 89)

¹⁴¹ Gilding (1988). Pitman (1990) was advice for Gilding (1988), but formally published after it.

¹⁴² SSABSA (1992, p. 70)

Expressing probabilities as decimals, fractions, ratios, and percentages, with examples from gambling activities, statistical data, and so on.

Representing chance processes by means of sample space graphs and diagrams, tree diagrams, probability tables, and so on.¹⁴³

In the suggested Assessment Plan probability was seen as suitable for a Skills Assessment Task, but not for Directed Investigation or Project Work.¹⁴⁴ It could have been applied to simulation, relative frequencies, and measures of spread, all of which were mentioned in the suggested syllabus. No doubt skilled teachers would make these links. But the suggested syllabus failed to do so.

Once again, we do not know the practical influence of this material within schools. But we are able to see that the intellectual thinking about probability among concerned and influential mathematics educators in South Australia continued to see it as a deterministic topic. It is therefore unlikely that the significant changes recommended by SSABSA had a major influence on classroom practice.*

THE EFFECTIVENESS OF ASSOCIATIONS AND JOURNALS

To see what support is available to teachers from publications and professional associations, let us look at what was readily available to teachers of probability in SA in 1994.¹⁴⁵

Soon after these changes were introduced some of the most recent editions of the Haese & Haese textbooks most commonly used in SA still presented probability only in a didactic way.¹⁴⁶ Others¹⁴⁷ did provide suggestions for simulations and investigations, but they gave little help for a teacher trying to develop appropriate new pedagogies for new topics or new approaches.

Three widely distributed journals are available in SA. These journals were examined from the years 1992–1994 to see what help they provided for a practising teacher. *Australian Mathematics Teacher* published five articles on stochastics in its twelve issues, two of which were on probability. One used a children's story to

¹⁴³ SSABSA (1992, p. 71)

¹⁴⁴ SSABSA (1992, p. 94)

* I have had some experience of working with individual students preparing for Year 11 and 12 examinations at conservative independent schools since 1992 and my experience suggests that the practical influence has been minimal.

¹⁴⁵ This section is a rewriting of J. Truran (in press).

¹⁴⁶ E.g., Haese et al. (1993, ch. 2)

¹⁴⁷ E.g., Haese et al. (1992, ch. 19), Bruce et al. (1992, ch. 4)

illustrate how the use of accurate probabilistic language might be encouraged.¹⁴⁸ The other was one of mine addressing the lack of meaning in the phrase “what is the probability of...?”¹⁴⁹ There were three reviews of statistical textbooks, and four of textbooks containing both probability and statistics for upper secondary classes, three by the same reviewer. There was almost no mention of stochastics in either the *Australian Senior Mathematics Journal* or the state journal, *Möbius*.

Ideas can also come from conferences and courses. MASA conferences were usually held for two days near the end of each year. In 1992 there were sessions specifically for teachers of the controversial “Quantitative Methods”, including two on statistics and one from two classroom teachers on new approaches to teaching probability. In 1993 Watson came to talk on “Intuitions and Misconceptions in Chance & Data”, and there were other sessions on statistics. The programme for 1994 contained no stochastics. Although MASA has run in-service courses for many years it is only since the publication of *Mathworks* that it has run courses on stochastics. Courses were first offered in Term 4 1994 with the claim that they would address *inter alia* “related research including the conceptual difficulties which children experience in Chance and Data”.¹⁵⁰

It is reasonable to expect overseas material to be seen as readily available to teachers if it is in book form. The NCTM’s 1981 Yearbook on teaching stochastics contained thirty chapters in all, as well as an extensive bibliography and a list of possible statistical projects.¹⁵¹ Nearly half of the chapters were relevant to probability teaching; many were written by acknowledged leaders in probability education. Yet the book seems to have little influence in Australia. One chapter of a 1992 Handbook provides a useful summary of research into the teaching and learning of stochastics which could be accessible to a busy teacher.¹⁵² The probability chapter in a later Handbook¹⁵³ is much too obscure for teachers. Furthermore, both books are expensive, and unlikely to be bought by many schools, especially primary schools without mathematics specialists.

So we may say that some material was readily available, although it was not comprehensive. But there is considerable evidence that teachers do not use even what is available. Peter Jeffery, at the time on the staff of ACER, evaluated the effect-

148 J. Watson (1993)

149 J. Truran (1994amtprob)

150 MASA Workshops (1994/95)

151 Shulte & Smart (1981)

152 Shaughnessy (1992)

153 Borovcnik & Peard (1996)

iveness of *Set: research information for teachers*, a periodical produced by ACER & NZCER which tries to bring research information into school staff rooms in a user-friendly way. He showed why those who accepted the R-D-D (agricultural) models of dissemination* would say that the publication was successful, but those who challenged the validity of the model would take the opposite view.¹⁵⁴ Jeffery pointed out the many logistical difficulties which caused the ideas being transmitted to take second place and argued that the publishers did not have the resources to do more at the time.¹⁵⁵

Few teachers have had direct experience with *Set* because the material seldom reaches them due to various factors in the delivery system. Prime among these is the problem of getting the material through the vagaries of within school dissemination systems. Even when transmission to end users is effected there remains a considerable problem in empowerment of the participants in implementation of ideas and knowledge.¹⁵⁶

Even more depressing in the long term was Jeffery's finding that the one school in his study which was actually using *Set* did so because the articles serendipitously matched school plans at the time, not because of any systematic attempt to use the type of material which *Set* was providing.¹⁵⁷

There have been some other Australian studies on this issue. It has been found that Q teachers do not regularly read professional journals¹⁵⁸ and that conferences "do not play a significant part in the professional life of mathematics teachers".¹⁵⁹ WA teachers have largely rejected efforts to suggest appropriate methodologies for teaching new material, and have resisted the use of excessively detailed resources.¹⁶⁰ For many teachers, textbooks are the main source of support.¹⁶¹ Queenslanders overwhelmingly cited "other teachers" as their main source of new information.¹⁶² Swinson has observed that many of these will be

* *Vide* ch. 7

154 Jeffery (1985, pp. 9–10)

155 Jeffery (1985, p. 10)

156 Jeffery (1985, p. 10)

157 Jeffery (1985, p. 17)

158 Swinson (1993, p. 537)

159 Swinson (1993, p. 539)

160 Haines & Malone (1993, p. 25)

161 Haines & Malone (1993, p. 24); Truran (1992, p. 99)

162 Swinson (1993, p. 539)

teachers not undertaking other forms of professional development activity.¹⁶³ The potential for Intellectual in-breeding is high.

We do not have systematic data for other States, but the situation is likely to be similar because the teaching cultures across Australia are remarkably similar. It is known that post-conference sales of the Proceedings of the annual and very large MAV Conferences are negligible.¹⁶⁴ In the UK Linda Haggarty conducted a small survey of ten secondary schools and 60 teachers in the UK¹⁶⁵ and found that only about 20% of the teachers subscribed to any mathematical association, and only two teachers had ever attended a full-scale conference. Their written comments did not suggest that teachers who had had experience with associations or conferences were dissatisfied: their reasons for not being further involved were mundane ones like shortage of money and time. Haggarty also found that four of her ten schools “appeared to be staffed by teachers who never or only rarely read either journal, never attended [conferences] and never attended any branch meeting”¹⁶⁶ This is partial confirmation of the Australian results. Recently the increasingly common practice of publishing conference papers on the web has increased accessibility and the AAMT 1997 Conference site received about 20 visits a day for some three months after publication, though it was suspected that more than half of these were from non-Australian visitors¹⁶⁷ and nothing is known about what the visitors did with what they saw.

Project-oriented approaches have been officially encouraged in SA for some ten years.¹⁶⁸ There is little published evidence about their influence on classroom practice but at the upper secondary level it is known that the probability has proved difficult to teach in any form and is less well understood than other topics, even in the new Year 12 “Quantitative Methods” which has a substantial component of stochastics and might be expected to be taught by specialists.¹⁶⁹

The teacher is at the white-board face. We have good evidence here, and will meet more in Part E, that teachers change very slowly. Yet curriculum developers tend to neglect teachers’ responses when they propose change. The symmetry of the forces represented by Figure 3·1 is neglected. One experienced English teacher

¹⁶³ Swinson (1993, p. 539)

¹⁶⁴ Tynan, pers. comm., 10 Nov 1999

¹⁶⁵ Haggarty (1992)

¹⁶⁶ Haggarty (1992, p. 29)

¹⁶⁷ Tynan, pers. comm., 10 Nov 1999

¹⁶⁸ Education Department of South Australia (1985)

¹⁶⁹ Hall (ndp, c. 1994); J. Truran (1994merhis, p. 637)

has argued that the National Statement approach is inherently bound to fail because it neglects the teachers.

The memorable textbooks are those which have been written by teachers with ambitions to reform, who believed that where they led, the curriculum (including the all-important examination syllabuses) would follow, supported by publishers whose confidence they enjoyed. They thrived on the freedom of individual teachers in the UK to teach how and (subject to loose constraints) what they wished. But the world of attainment targets, common cores and league tables is inimical to such initiatives. It seems unlikely that we shall see more textbooks to compare with the best of the 20th century until responsibility for curriculum development is returned to its proper guardian—the experienced teacher in the classroom.¹⁷⁰

Quadling wrote from within the English Public School System where both students and teachers tended to be better than average, so his view of a prior “golden age” is a little blinkered. Nevertheless, his plea for the importance of teachers is one which will form an important part of the analysis of change which will conclude this Chapter.

WHY HAVE THE NATIONAL CURRICULUM CHANGES BEEN SO POORLY CONSTRUCTED AND INTERPRETED?

That man can interrogate as well as observe nature, was a lesson slowly learned in his evolution.¹⁷¹

We might reasonably have expected that this concluding chapter of the historical Part of this thesis would have shown how the major changes of the 1990s would have drawn on the accumulated wisdom of thirty years of teaching experience to permit a large step forward in what is offered by schools to children. But the evidence presented here makes it clear that this has not been the case. So we need to conclude this Part by seeking reasons for such a situation, at the same time testing the value of the BSEM as a helpful interpretative tool.

Two forms of Intellectual forces have operated on the probability curriculum. One has been the traditional Academic force, which has declined markedly over the years. While we have seen in Chapter 15 that academics still held some power to define what constitutes academically respectable content, their understanding

¹⁷⁰ Quadling (1996, p. 126)

¹⁷¹ *Aphorisms from His Bedside Teachings* Osler (ndp), cited in Partington (1996)

of basic probability in 1994 would have been very similar to that held in 1969. Nor would many teachers (at least those who believed they understood the topic) have disagreed with the academics. We may say that the strictly Academic arguments for the teaching of probability and about the nature of probability had remained largely unchanged throughout the period under discussion, though they may well have received wider acceptance from some teachers.

During the last thirty years a new type of Intellectual force has developed—one concerned with developing a pedagogy based on disciplined findings about *obuchennyi*. For probability, researchers have marshalled large amounts of evidence, and although much of it is still poorly integrated, it is often of good quality, and is relevant to classroom practitioners. But, as we have seen, it has had little effect in the classroom. Text books have changed little.[§] The free market process which determines the content of journals and, to a large extent, conferences has produced uneven coverage and little critical debate. The best available summary of research findings on teaching stochastics¹⁷² is not in a form which is easily available to teachers.

By comparison with stagnant Intellectual and weak Pedagogical forces, Social forces have changed significantly. There is little doubt that Technocratic forces have been influential in establishing the modular approach to curriculum definition together with the insistent use of the “Moralistic Should”^{*} and the “Imperative Present”[†] in the curriculum documents. Such forces have often been well received within a society increasingly concerned for both relevance and accountability. In other words, the various forms of Social forces were tending to move in the same direction during the early 1990s.

There is little which needs to be said about Physical forces. Teachers’ work has become so much more obviously complex and demanding in the 1990s that it does not require detailed elaboration. In this Part we have seen how wave upon wave of changing pedagogical approaches has been required by those in authority so it is not surprising that an aging teaching force has become cautiously cynical about the messianic claims which inevitably accompany any new proposal.

§ A notable exception which helps to prove the assertion is work by David Moore, in the USE (e.g., Moore, 1991) first published in 1979. Moore is the much respected doyen of current United States statisticians, and yet the classroom influence of his books is very small as may be seen by a glance at the texts commonly used in tertiary courses.

¹⁷² Shaughnessy (1992, pp. 481–486)

^{*} E.g., “Experiences with chance should be provided which enable children to ... ” (AEC, 1991, p. 166)

[†] E.g., “Distinguishes possible from impossible events, ... ” (AEC, 1994a, p. 46)

We have already seen that the production of the support material for the *Profiles* was grossly under-funded. For the many teachers who can remember the 70s it is all *déjà vu*. The lack of integration between Learning Areas mentioned above and the weakness of the Level 8 material in the Mathematics Strand strongly suggest that the documents were produced in a hurry with inadequate resources, although I have only been able to obtain anecdotal evidence for this. We shall see in Chapter 23 just how much effort is required for a teacher to introduce a change effectively, but the concomitant extensive funding for the National Curriculum changes was just not available. In a school world of increasing complexity, regular changes of tack and limited resources, there can be little inducement for teachers to change more than absolutely necessary and who would blame them. In terms of the opening quotation, teachers have little enough time to observe their educational environment to try to interrogate it as well.

So when we look at the component forces of the BSEM, it would be easy to say that the Technocratic force was dominant, with the Physical forces acting for minimal change and the Intellectual forces being side-lined. This might explain the modular form of the curriculum and the disjoint Criterion Referenced form of assessment, but it does not explain why the curriculum documents did not respond to the Academic and Pedagogic Intellectual forces. These documents were prepared by a relatively small group of people, with good potential access to appropriate authorities and sufficient authority to incorporate appropriate findings. Yet, as mentioned above, the authors did not seem to use this material much. This failure to respond is a critical one and demands deeper investigation.

I have already suggested that some of the weaknesses of their work arose from a strident adherence to Constructivist principles. The CEM would suggest that it may be explained by copying. But neither answer may be sufficient. Fortunately we have data from a set of activities conducted in the early 1990s in Australia and three other countries with a similar economic structure. This will help us to see the most relevant forces. So we start on the Coda to this Chapter and this Part—a coda which will fulfil its late classical function¹⁷³ of providing insights of a quite different type from those previously encountered but which will help us to see all the previous themes into much better perspective.

¹⁷³ Scholes (1970, p. 200)

NATIONAL CURRICULA IN OTHER COUNTRIES— SIMILARITIES AND DIFFERENCES

Where order in variety we see,
And where, though all things differ, all agree.¹⁷⁴

Efforts to reconstruct the phylogeny ... on the basis of a single character or a series of interdependent characters are not very rewarding. Such attempts reveal not only a lack of understanding of convergence and divergence among birds, but also a naïve concept of basic morphology, including a failure to appreciate the dynamic interaction between developing bones and between bones and muscles.¹⁷⁵

When we compare national curriculum documents in Australia and three other economically developed English-speaking countries—England & Wales, the USA, and NZ—we see that while all countries have responded in similar ways to recent economic and social pressures there are so many differences in detail that it is highly unlikely that any one country merely copied from another. Rather the similarities are better explained by the principle of Convergence.[§]

In England & Wales, a national curriculum document was produced in 1989.¹⁷⁶ It was much briefer than the Australian ones, no doubt partly because it needed to be incorporated into an Order of Parliament¹⁷⁷ so that it could be used as a basis for a planned national testing programme. However, it has the same structure as the Australian one: the mathematics curriculum was divided into topics and levels and a “statement of attainment” was specified for each level as well as an example of what type of work would indicate that the level had been achieved.

Also in 1989, a similar document was produced in the USA.¹⁷⁸ While this document was also based on outcomes statements, it distinguished only three levels: K–4, 5–8 and 9–12. A separate section on evaluation was provided which emphasised the sorts of questions which might be used to evaluate achievement but without devoting much attention to the sorts of answers which would indicate that an outcome had been achieved. This document differed significantly from the Australian and English ones because it was produced independently by the

¹⁷⁴ *Windsor Forest* Pope (1711b, ll. 15–16), cited in Partington (1996)

¹⁷⁵ Van Tyne & Berger (1959/1971, p. 370)

[§] The following section is a revision of part of J. Truran (1997anzhes).

¹⁷⁶ GB. DES (1989)

¹⁷⁷ Great Britain. Parliament (1989)

¹⁷⁸ NCTM (1989)

National Council of Teachers of Mathematics (NCTM)—a professional body—and carried no form of governmental or semi-governmental authority.

Finally, the corresponding NZ document was produced in 1992.¹⁷⁹ As well as listing achievement objectives, suggested learning experiences and sample assessment activities, it also provided suggested activities for students learning more slowly or more quickly than the class norm.

There are undoubted similarities between these documents, of which three are particularly important because they represent significant changes from much previous curriculum practice:

- all provide a curriculum which crosses the traditional boundaries between primary and secondary schools;
- all express their aims in some form of outcome statement, followed by some way of assessing whether the outcome has been achieved;
- all include a section emphasising an holistic approach to mathematical thinking, setting up special sections like “Working Mathematically” (Australia), “Mathematics as Problem Solving”, “Mathematics as Communication (USA)”, “Using and Applying Mathematics” (England & Wales) and “Mathematical Processes” (NZ).

These similarities are striking, because they come from documents produced by four different types of organisations: a government ministry working towards a clear end-point in England & Wales, a government ministry in bi-cultural NZ, a loose federation of government ministries with restricted powers in Australia, and a professional teachers’ organisation in the USA. What are the antecedents of these ideas: does one or more of these documents represent uncritical copying of another or do they represent similar responses to similar situations?

Publication dates in themselves tell us little. There is a lead time of at least two years required to prepare document like these,¹⁸⁰ and the Australian ones at least took much longer.¹⁸¹ I argue that the documents reflect similar approaches to education arising particularly from two aspects of the industrial model:

- the belief of the business world that education is a way of making the young more employable, and thereby increasing their economic value, and reducing their socially disruptive potential;
- the belief that understanding can be measured by what is done, rather than by what is internalised.

¹⁷⁹ NZ. Ministry of Education (1992)

¹⁸⁰ NZ. Ministry of Education (1992, p. 5)

¹⁸¹ AEC (1991, p. i)

Such similarities in prosperous developed countries, subject to the same economic fluctuations which now spread so rapidly across the world, suggest that they may be most parsimoniously explained by a Convergence model. Of course, potential solutions being tried in one country are likely to be taken on board in other countries as well, quite possibly before they have been adequately evaluated, but this is not to suggest mere copying, rather thoughtful adaptation to similar environments.

A striking argument against the CEM may be found by examining how the various documents approach similar situations. Here the teaching of probability is of particular value because its pedagogy is much less well developed than for other subjects. So it should be a little easier to trace discrepancies in approach than it would be for a subject whose pedagogy has had more time to stabilise across and within countries. One example will suffice.

Consider the issue of the understanding of the symmetry of a die, discussed in Example 2 above, and later in Chapter 23. The findings of Green and other researchers had been well disseminated among English-speaking teachers and researchers, at least within their own countries. Given the excellent search facilities available today, this implies that any teachers and researchers drawing up national curricula, even in other countries, would have been able easily to locate these research findings. But the divergence in how the different countries have treated this issue is striking. The Australian curriculum does not mention a conventional die in Levels 4, 5 or 6, but assumes that children will be aware of equiprobability and happy to accept it.¹⁸² It totally neglects subjective probability. The English curriculum does discuss subjective probabilities at Level 4, and distinguishes theoretical and experimental probabilities at Level 5, but does not try to integrate the three forms.¹⁸³ The NZ curriculum discusses experimental probability in Levels 3–5, and introduces theoretical probabilities briefly at Level 5. It does not discuss subjective probability.¹⁸⁴ The USA curriculum is based on a modelling approach which looks for theoretical probabilities to match experimental outcomes. The inclusion of subjective probability in this syllabus is strongly implied, but not made explicit.¹⁸⁵

This is not the place to evaluate the pedagogy of these four approaches. All have strengths and weaknesses. But they show that in these four countries an

¹⁸² AEC (1994a, pp. 77, 92, 108)

¹⁸³ GB. DES (1989, pp. 43–44)

¹⁸⁴ NZ. Ministry of Education (1992, pp. 178–191)

¹⁸⁵ NCTM (1989, pp. 109–111)

important aspect of the curriculum has been treated remarkably differently, even though the underlying structures of the curriculum frameworks have been remarkably similar.

The most striking difference is the American approach with its intense concern for children's activity and for relevance. It might be argued that this reflects the interests of a professional organisation with strong roots in the classroom. But this is only a matter of degree. For example, the Australian curriculum was drawn up in consultation with many teachers, and all the *Work Samples* come from classroom trials, and it is certainly possible to read an emphasis on activity in the Australian documents, albeit not as strong as the American ones.

It would be legitimate to investigate differences in degree of emphasis like these, but our concern here is why the USA's *structure* is similar to the others' when it was drawn up, not under political patronage, but by a professional body of teachers with the avowed aim of changing, on educational grounds, both school practice and the requirements placed on schools by governments? Mathematics education in the USA has for many years been much more rigid and test-driven than in the other three countries. Technocratic forces were already strong there, and paradoxically we may see the *Standards* as to some extent a liberating force, albeit still quite structured, as indeed are aspects of at least the Australian documents.

Noss has argued that the British concern has been with how children learn, rather than what they learn. Education has been seen as a Social moulder far more than as a developer of the mind.¹⁸⁶ This is probably a fair summary of the position taken by influential mathematics educators in all four countries, and, again paradoxically, reflects more a Constructivist position rather than the increasingly dominant Technocratic one concerned more with "what" than with "how". It is this tension which provides a clue for understanding the similarities in the four structures.

I argue that the balkanised approach of the curriculum documents represents a potentially workable compromise between the Technocratic forces of Society, the creative educational philosophy of the Constructivists and the inherent conservatism of teachers constrained by increasingly demanding Physical forces. The modular structure which we now have may be seen as the best currently available way of increasing the probability that incremental change will be effective in real classrooms while allowing both flexibility in approach and clearly defined end-points. It might be seen as an efficient response to the wider ranges of press-

¹⁸⁶ Noss (1990)

ures on the educational system, which has arisen principally because of the increased influence of the Technocrats. The strength of this argument lies in the fact that the modular structure has been variously reached—sometimes with the Technocrats in the ascendant, sometimes with the Constructivists.

Corroborative evidence for this argument rests in the relative absence of Intellectual forces, both Academic and Pedagogic. This absence is evident in the lack of Intellectual commonality to be found in the four documents even in an area as basic as that of dice. We have already cited Pascal's delight that "truth is the same in Toulouse as in Paris"¹⁸⁷ and it is the absence of this commonality in four very similar countries which would suggest that either the Intellectual forces have been weak, or they have been seriously misguided. It is really not possible to argue that the Academic Intellectual forces are seriously misguided—this would destroy the whole discipline of stochastics—so it is reasonable to posit that they have been weak. The situation for the Pedagogic Intellectual forces is more difficult, and leads us back to the question of whether the discipline of Mathematics Education really exists. Parts D and E will provide two significant and focused answers in the affirmative, but I would argue that the evidence, for example, from Chapter 8 and the analysis of the National Curriculum documents undertaken above is also convincingly affirmative. I would argue, therefore, that because research into probability *obuchennyi* has produced many valuable results so far, it is not seriously misguided, and that the marked absence of its influence in the National Curriculum documents in three countries and its minimal mention in the fourth is good evidence that the Pedagogic Intellectual forces underlying the construction of the National Curriculum documents were weak.

The curriculum documents have all the marks of having been thought through *de novo* by writers with many pedagogic skills but without deep reference to the research knowledge which was available. For example, in Australia subjective probability was emphasised in the support material produced by the Curriculum Corporation, but it did not find a place in the contemporaneous *Profiles* and *Work Samples*,¹⁸⁸ even though, presumably by a different route, it did find a place in the English ones. In Australia the *Profiles* were trialed in schools, but they were not based on research evidence.¹⁸⁹ Indeed, only the American documents provide any references to other published material which might be helpful to a teacher, even though many suitable books and articles were readily available. The

¹⁸⁷ Je vois bien que la vérité est la même à Toulouse et à Paris. Pascal to Fermat 29 Jul 1654

¹⁸⁸ Lovitt & Lowe (1993a, b)

¹⁸⁹ Ellerton & Clements (1994, p. 188)

documents of the other countries tend to disempower teachers: they are so strongly self-contained that an inexperienced teacher could be forgiven for thinking that there was little more left to learn. Noss, more radically still, has argued that the de-skilling of labour and teachers has been a major aim of some of those trying to influence national curricula.¹⁹⁰ I would not go so far, because I would suggest that ignorance of the complexity of *obuchennyi* is a more parsimonious explanation for any de-skilling which may be observed.

This finding of similar outcomes interspersed with striking differences from four similar countries strongly suggests that some form of Convergence has occurred. It is Convergence which provides the theme for this coda, and which helps to make sense of all the data we have examined in the last six chapters. Because we have similar structures which have been developed in quite different ways, we are very close to just those situations which the biological model of Convergence has been developed to explain. It is Convergence which seems to have the power to allow for the great complexity of *obuchennyi* which we have observed throughout Parts B and C.

CONCLUSION

This is a suitable point to leave historical studies for the time being. I have produced evidence for the value of the BSEM as an interpretative model. Of course, the evidence (in spite of its great size) is still skimpy. I have already suggested, but without attempting any explanation, that developments in other Western European countries have been different. I have not looked at the development of the colonies of these countries. Nor have I examined the situations in developing countries with a small élite educated to the standards of the Western world. Nor have I taken up the challenge of Chapter 5 and looked at countries with strong ambient cultures which are outside the Western Christian tradition. The fact that so many fields for investigation may be suggested so easily is evidence that the BSEM provides a real “hard core” of a research programme (to use Lakatos’ model),¹⁹¹ one which is rich enough to justify the time spent in its refinement.

So the time has come to examine two other aspects of probability *obuchennyi*. The next Part will look at assessment, the following at Pedagogy. Both will have a double purpose—confirmation of my claim that a discipline of mathematics

¹⁹⁰ Noss (1990, pp. 24–25)

¹⁹¹ Chalmers (1976, pp. 76–80)

education does exist, and evidence of how the forces of the BSEM have influenced the observed outcomes.

A good paradigm will also have predictive value. This will be examined in Chapter 26. Our final *bon mot* for this Part was written in 1995, but it could equally have come from 1965. In its arguments for a sound mathematical base as well as its emphasis on modelling it is a useful homiletic reminder of what has been lost by the failure of Intellectual forces to have any influence in recent probability curriculum construction. Perhaps it is a seed encapsulating future changes, but not more than that.

In comparison with statistics, probability has taken something of a back seat in recent years. It has sometimes appeared to be concerned only with artificial situations involving dice, coins, and cards. In the simplest cases probabilities are easily predictable and examples hackneyed. ... However, probability has a strong claim for inclusion in A-Level work. It is in essence a modelling activity—urns, like perfectly smooth surfaces and point masses, are *models* which abstract the features of a situation. Many other areas of mathematics are used in work on probability—series, solution of equations, differentiation and integration, methods of counting, matrices. Also probability leads to predictions which can be tested experimentally by simulation methods¹⁹²

¹⁹² F.R. Watson (1995, p. 45)