

**The Teaching and
Learning of Probability,
with Special Reference to
South Australian Schools
from 1959–1994**

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June, 2001

Presented for the Degree of
Doctor of Philosophy
of the University of Adelaide

Faculty of Arts—Graduate School of Education
Faculty of Mathematical Sciences—Department of Pure Mathematics

DEDICATION

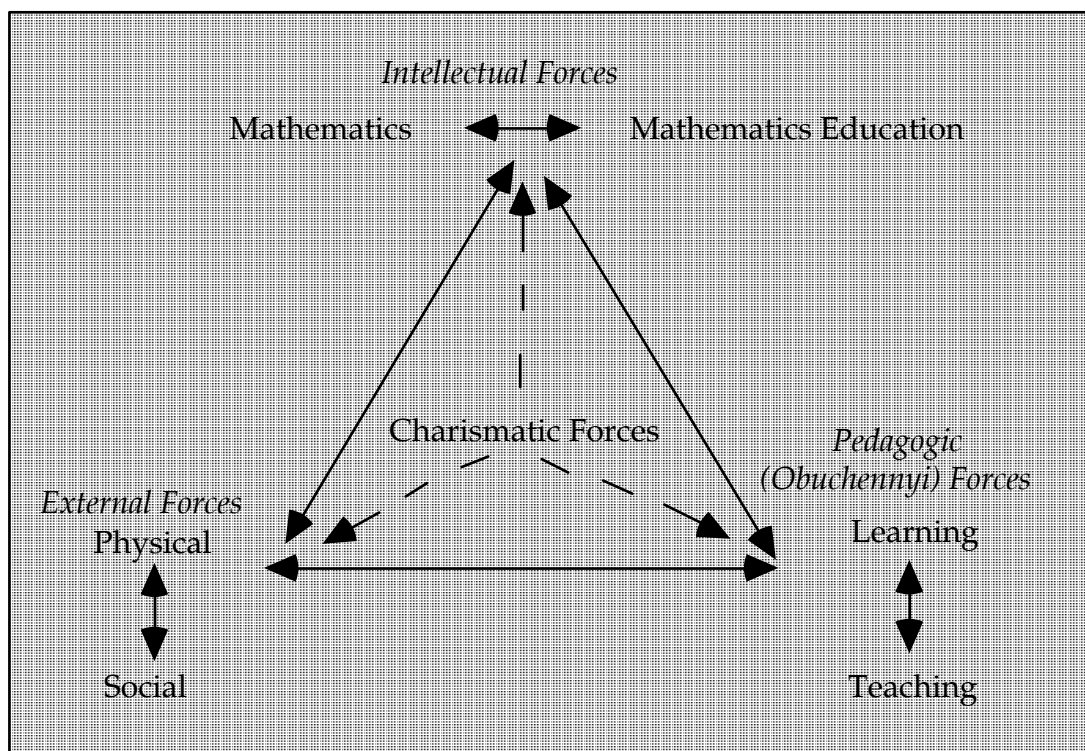
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1909 - 1941

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FRONTISPIECE



BROAD-SPECTRUM ECOLOGICAL MODEL FOR MATHEMATICS EDUCATION

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Brevity is the soul of wit¹

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¹ Shakespeare *Hamlet* II (2) l. 90

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As some day it may happen that a victim must be found
I've got a little list—I've got a little list.²

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² *Mikado*, Gilbert (1885, Act I)

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³ Barnard (1927), cited in Partington (1996)

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And speak when he is spoken to,
And behave mannerly at table,
At least as far as he is able.⁴

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⁴ R.L. Stevenson (1885) "Whole Duty of Children", cited in Partington (1996)

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ABSTRACT

For us, and for our tragedy,
Here stooping to your clemency,
We beg your hearing patiently.

Is this a prologue, or the posy of a ring?

'Tis brief, my lord.⁵

The reader who wishes to obtain an overview of this thesis is advised to read Chapter 1 (second half) Chapter 9 (second half), Chapter 24 (first half), Chapter 25 and Chapter 26.

The teaching of probability in schools provides a good opportunity for examining how a new topic is integrated into a school curriculum. Furthermore, because probabilistic thinking is quite different from the deterministic thinking traditionally found in mathematics classrooms, such an examination is particularly able to highlight significant forces operating within educational practice.

After six chapters which describe relevant aspects of the philosophical, cultural, and intellectual environment within which probability has been taught, a “Broad-Spectrum Ecological Model” is developed to examine the forces which operate on a school system. The Model sees school systems and their various participants as operating according to general ecological principles, where and interprets actions as responses to situations in ways which minimise energy expenditure and maximise chances of survival. The Model posits three principal forces—Physical, Social and Intellectual—as providing an adequate structure.

The value of the Model as an interpretative framework is then assessed by examining three separate aspects of the teaching of probability. The first is a general survey of the history of the teaching of the topic from 1959 to 1994, paying particular attention to South Australia, but making some comparisons with other countries and other states of Australia. The second examines in detail attempts which have been made throughout the world to assess the under-

⁵ Shakespeare *Hamlet* III (2) ll. 159–163

standing of probabilistic ideas. The third addresses the influence on classroom practice of research into the teaching and learning of probabilistic ideas.

In all three situations the Model is shown to be a helpful way of interpreting the data, but to need some refinements. This involves the uniting of the Social and Physical forces, the division of the Intellectual force into Mathematics and Mathematics Education forces, and the addition of Pedagogical and Charismatic forces. A diagrammatic form of the Model is constructed which provides a way of indicating the relative strengths of these forces.

The initial form is used throughout the thesis for interpreting the events described. The revised form is then defined and assessed, particularly against alternative explanations of the events described, and also used for drawing some comparisons with medical education. The Model appears to be effective in highlighting uneven forces and in predicting outcomes which are likely to arise from such asymmetries, and this potential predictive power is assessed for one small case study. All Models have limitations, but this one seems to explain far more than the other models used for mathematics curriculum development in Australia which have tended to see our practice as an imitation of that in other countries.

STATEMENT

He that hath used no deceit in his tongue, nor done evil to his neighbour:
and hath not slandered his neighbour.

...

He that sweareth unto his neighbour, and disappointeth him not: though it
were to his own hindrance.

...

Whoso doeth these things: shall never fall.⁶

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by any other person, except where due reference is made in the text.

I give consent to this copy of my thesis, when deposited in the University Library being available for photocopying and loan.

.....

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⁶ Psalm 15, vv. 3, 5, 7

ACKNOWLEDGEMENTS

While they were on their way Jesus came to a village where a woman named Martha made him welcome in her home. She had a sister, Mary, who seated herself at the Lord's feet and stayed there listening to his words. Now Martha was distracted by her many tasks, so she came to him and said, 'Lord, do you not care that my sister has left me to get on with the work by myself? Tell her to come and lend a hand.' But the Lord answered, 'Martha, Martha, you are fretting and fussing about so many things; but one thing is necessary. The part that Mary has chosen is best; and it shall not be taken away from her.'⁷

For some of the last nine years I have been privileged to have the opportunity to sit and think about mathematics education. It is indeed an excellent way and I have been very sad when it was taken away from me. I was supported for three and one half years by an Australian Post-Graduate Research Award from the Government of Australia which was administered by the Graduate Studies Branch of the University of Adelaide. Further financial assistance came from my sponsoring departments and from the Faculty of Arts. I express my gratitude to all of these organisations and especially to the staff and Board of the former Graduate Studies Branch, whose concern for students is of the highest calibre.

Thanks are due to my supervisors, Mr Ian Brice, and Associate Professor Dr Paul Scott, for their extensive advice, patience and tolerance. The width of this thesis meant that both were to some extent flying blind, and I very much appreciate their willingness to support me in what they must have known was going to be an abnormally difficult task. I owe special thanks to Paul because it was one of his suggestions which led to the development of model which underlies this thesis. I also thank Dr Chris Dawson for acting as *locum tenens* and midwife after Ian moved to Sydney. Thanks are also due to the Departments of Education and Pure Mathematics for hosting my studies for much of this time, and especially to the Department of Education for providing me with the facilities for building up the networks which have made this thesis so much richer. These thanks extend also to the various fellow students who were asked by the Department to share a room with me and tolerate my energies and vagaries.

It is unfortunately necessary to record that the increasing financial restrictions on universities which have occurred during the 1990s, and the moves to manage-

⁷ Luke 10: 38–42

ment models which are insensitive to the ways in which academics need to work have meant that this thesis has been written in a period of declining resources, increased pressures on staff, and dangerous library cuts. It ought not to have been necessary for me to go to Bielefeld in Germany to read books which had been in my own university's library 35 years ago. These problems have been exacerbated by a substantial culling of the library of the Education Department of SA. Departmental officials have assured the relevant Minister that the process is a responsible one.⁸ For administrative records this advice may or may not be good; for curriculum and social records it is seriously flawed; those concerned do not seem to be able to conceive of the width of material which is of historical value.

In my own university, it ought not to have been necessary to have to fight so often for adequate equipment and, even more importantly, for rapid repairs to equipment which was usually satisfactory. It ought to have been possible to have provided each doctoral student with a computer which could not be interfered with or trashed by others. It ought also to have been possible to have had access to information about how best to use the expensive technology which we did have available, so that it was not necessary to spend long hours working things out for oneself, and that not always successfully. It ought to have been possible to provide us with good software, and not force us to work with obsolete and unfriendly material. It ought not to have been necessary to have spent so much time trying to boost my morale when all around me seemed to be crumbling. If universities are to be taken really seriously as educational institutions, they need to learn how to establish the best environments for learning and research to take place. So the thanks which I express here are particularly heart-felt: all of the people listed have been working in the same difficult circumstances.

There are so many people who have assisted. Those who made specific suggestions are acknowledged in the appropriate places. I am especially grateful to the staff of the school who agreed to help me with a teaching experiment, and am sorry that we were not able to carry it through to completion. Others who consented to be interviewed are listed in the Sources. Without their reminiscences this thesis would be much the poorer.

I must mention especially Dr Carmen Batanero, University of Granada. We first met by chance at a crowded morning tea in Marakesh, and since then have developed a high regard for each other's work, and have become collaborators in

⁸ Hon Robert Lucas, Minister for Education and Children's Service to author, 20 Aug 1994

so many activities. Our regular e-mail sessions have done much to sustain me in times of despair for mathematics education, and in those moments when I doubted my own abilities.

Many people have provided me with useful insights, advice, or information, and some have also sustained me more generally, often over long periods of time. These include:

Mrs Anne Arnold	University of Adelaide
Dr Nicolas Balacheff	Laboratoire Leibneitz, Grenoble France
Dr Andy Begg	University of Waikato. NZ
Mr Peter Brinkworth	Flinders University of South Australia
Miss Mary Brownlee	University of Adelaide
Professor Ken Clements	University of Brunei Darussalam
Dr Chris Dawson	University of Adelaide
Mr Jim Giles	Retired educational administrator
Dr Juan Godino	University of Granada, Spain
Dr David Green	Loughborough University, UK
Dr Brian Greer	San Diego State University, USA
Mr Lindsay Grimison	University of Sydney
Dr Dorothy Hudson	University of Adelaide
Professor Alan James	University of Adelaide
Dr Barry Jeromson	University of South Australia, Magill
Dr Mollie MacGregor	University of Melbourne
Mr Will Morony	Department for Education and Children's Services
Dr Andrea Peter-Koop	University of Münster
Dr Jon Opie	University of Adelaide
Dr Michael Price	University of Leicester, UK
Dr Rene Ritson	Queens University, Belfast
Dr Jack Rowell	University of Adelaide
Dr Margaret Scott	University of Adelaide
Dr Michael Shaughnessy	Portland State University, USA
Dr Brian Sherman	University of Adelaide
Dr Beth Southwell	University of Western Sydney, Nepean
Mrs Kath Truran	University of South Australia, Magill
Mrs Jenni Way	University of Western Sydney, Nepean
Dr Gregory Yates	University of South Australia, Magill

A few of these people have been involved with schools some of the time, but it is a sad reflection on both myself and mathematics education that I have had less to do with schools in constructing this thesis than would have been desirable. Some conferences of professional associations have helped to redress this imbalance, but most of those I have attended have been dominated by tertiary teachers and researchers. Nevertheless, they have provided a valuable way of meeting people

with like interests and an impetus to put one's ideas onto paper, although the level of critical debate at these conferences is rather less than it might be. The number of people involved is too great to list individually, but their contributions and friendship have been much appreciated. Thanks are due to the leaders of the organisations as well as the organisers of the individual conferences, and especially to MERGA for providing a base for so much of my writing. The organisations and conferences involved have been:

Australian and New Zealand History of Education Society (ANZHES)
Australian Association of Mathematics Teachers (AAMT)
International Conferences on Teaching of Statistics (ICOTS)
International Congress on Mathematics Education (ICME)
Mathematical Association of Victoria (MAV)
Mathematics Education Research Group of Australasia (MERGA)
New Zealand Association of Mathematics Teachers (NZAMT)
Psychology of Mathematics Education Group (PME)

I have been able to make use of many libraries as well as our own. In these days of increasing university cuts, it is especially important to acknowledge the help of the librarians and archivists working in the following institutions:

Australian Council for Educational Research
Flinders University of South Australia
Mathematical Association, Leicester, UK
Mathematical Association of Victoria
University of Adelaide
University of Bielefeld, Germany
University of Melbourne
University of Newcastle
University of South Australia
University of Sydney
University of Tasmania

The Department of Education Training and Employment in SA kindly allowed me to read a number of official records not on open access.

Special thanks are due to the inventors of the personal computer, and also to Mervyn Macintosh Truran II, without whose flexibility these thoughts would never have lost their entropy. Thanks are also due to TransAdelaide, whose trains provided the cheap, clean and comfortable seclusion which the preparation of this document required.

The Dedication is to three significant people in my life who are no longer living. Although I do not remember my father, he left me the legacy of a fine library, which was a wonderful support for me during childhood and adolescence and which, with my mother's determined encouragement in the face of many difficulties, set me well on the road towards my academic work. The dedication to Tim is an attempt to express, not only my sadness at what might have been but never happened, but also gratitude for the many good times we had together.

Finally, my thanks go to my wife and collaborator, Kath, for her support, both personal and professional, and especially for her patience and understanding. On top of all her trials and tribulations, she has seen me through so many of my own, and a large number of action replays as well. This work would not have been finished without her; words really cannot express my gratitude.

For this relief, much thanks; 'tis bitter cold
And I am sick at heart.⁹

⁹ Shakespeare *Hamlet* I (1) ll. 8–9