

A Unified Approach to Nuclear Matter and Quark Matter

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Abstract

The properties of hadronic and quark matter are studied as a function of density using a chiral model based on quark degrees of freedom. Nucleons are described as quark - diquark states in the Faddeev approach and this description is extended to infinite nuclear matter in the mean field approximation. We calculate the properties of two flavour quark matter, allowing for the possibility of colour superconductivity in the form of a spin zero condensate (i.e. the 2SC phase). These calculations are performed using the proper-time regularisation method. We find that the phase diagrams for asymmetric matter in this description can have charge neutral phase transitions from the hadronic phase to the deconfined phase, depending on the pairing strength for quarks in the 2SC phase. We study the evolution of the phase diagrams as a function of the pairing strength. The properties of nuclear matter are significantly improved once we take into account the self-energy of the nucleon. We also find that the structure of the nucleon has important consequences for the phase diagram. The charge neutral equations of state are used to produce compact star configurations by solving the Tolman-Oppenheimer-Volkoff (TOV) equations. We use these solutions to investigate the possibility of hybrid stars.

Statement of Originality

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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