Appendix 1: Multipole Moments

The quadrupole moment tensor is a three by three matrix that describes the relative orientation of the four poles making up the quadrupole. As a quadrupole is effectively two antiparallel dipoles, it is possible to describe any quadrupole in two ways. For example, Figure A1.1 shows a quadrupole split into two antiparallel dipoles. It is apparent from this that there are two ways to describe the quadrupole moment tensor for a single quadrupole.

![Quadrupole Moment Tensor](image)

(a) $m_x=1$, $d_y=1$  
(b) $m_y=1$, $d_x=1$

$$
\begin{bmatrix}
Q &=& 0 & 1 & 0 \\
    & 0 & 0 & 0 \\
    & 0 & 0 & 0 
\end{bmatrix} \quad \begin{bmatrix}
Q &=& 0 & 0 & 0 \\
    & 1 & 0 & 0 \\
    & 0 & 0 & 0 
\end{bmatrix}
$$

Figure A1.1. The quadrupole moment tensor is equal to its transpose.

A simple algorithm has been constructed to generate the magnetic field response around a magnetic quadrupole, given a quadrupole moment tensor. By running two forward models around quadrupoles with the quadrupole moments given in Figure A1.1, there is no difference between the plots. This suggests that the quadrupole moment tensor matrix is equal to its transpose. A proof of this follows.

Repeated here are equations (8-23) to (8-27), denoted equation (A-1) to (A-5), where the constants $A$ to $S$ represent combinations of $x$, $y$, $z$ and $B_{ij}$ as defined in Chapter 8. The $m_xd_i$ (or $m_yd_j$) terms represent the components of the quadrupole moment tensor. The equations govern the magnetic gradient tensor field responses for a magnetic quadrupole. Note that the coefficients of the non-diagonal terms are similar. For example, in equation (A-1), the coefficient for the $m_xd_y$ term and the $m_yd_x$ term is $B$. The coefficient for $m_xd_z$ and $m_yd_z$ is $C$. 


and for $m_dz$ and $m_yz$, it is $E$. This is apparent in all the equations (A-1) to (A-5). Therefore, if the quadrupole moment tensor is replaced with its transpose, the coefficients will remain the same, and the same magnetic field response will be produced.

\[ A_m d_x + B_m d_y + C_m d_z = O \]  
\[ D_m d_x + G_m d_y + H_m d_z = P \]  
\[ B_m d_x + D_m d_y + G_m d_z = Q \]  
\[ E_m d_x + H_m d_y + L_m d_z = R \]  
\[ C_m d_x + E_m d_y + F_m d_z = S \]

This is an important concept for Chapter 8, where the number of independent components of the quadrupole moment tensor plays an important part when attempting to invert to a quadrupole source.

In a similar vein, the static magnetic octupole theoretically has 27 components comprising its moment. This can be simplified to just 10 components through a method similar to that shown above.
Appendix 2: Publications arising from the Thesis

The following journal articles, extended abstracts, conference presentations and posters were produced as part of this thesis research.

*Journal Articles:*


*Extended Abstracts and Conference Presentations:*


A summary of magnetic gradient tensor techniques entitled “Magnetic Gradiometry: Forward modelling, filters and inversion” was presented in Adelaide at the Mineral Exploration Through Cover conference 2006.

Posters:


“Determining near-surface objects with the magnetic tensor.” Prepared for the Department of Physics annual poster competition (won the Bob Crompton prize in Physics for best poster)) in 2005.

Also, the extended abstracts for the three CRC LEME symposia are available for free from the Internet, via the CRC LEME website (http://www.crcleme.org.au/index.html). An extended abstract of the work (Heath et al., 2006) can also be found on the Internet (http://www.earth2006.org.au/).
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