

**ANALYSIS OF POTENTIAL FIELD GRADIENT  
TENSOR DATA: FORWARD MODELLING,  
INVERSION AND NEAR-SURFACE  
EXPLORATION**

by

Philip Heath

B.Sc. (Hons)

Discipline of Geology and Geophysics  
School of Earth and Environmental Sciences  
&  
Department of Physics  
School of Chemistry and Physics  
The University of Adelaide  
SA 5005, Adelaide

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## Abstract

In this thesis the mathematics of the potential field gradient tensor is thoroughly reviewed and the inter-relationships between the various components and quantities are established. Numerical forward modelling of 3D regolith scenarios is undertaken, showing that current instrumentation for measuring magnetic gradients is useful for near-surface exploration. Gravity gradiometry is only able to detect relatively large contrasts in density, and does not offer the same small-scale resolution as magnetic gradiometry. I examine the magnetic case further by incorporating surface measurements of magnetic susceptibility. These data are highly variable, and forward modelling shows while gradiometry is useful for regolith exploration, interpretation is difficult due to the high level of geological noise.

I have tested new and standard filters as tools for extracting information from gradient tensor data. This includes boundaries between geological units and possible mineralisation positions. New techniques developed involve calculating an Analytic Signal for the entire gradient tensor, and forming multiplicative combinations of the individual gradient tensor components. The determinant and inverse of the gradient tensor  $3 \times 3$  matrix can also yield useful information pertaining to source position and geological boundaries. These are illustrated by means of synthetic data examples.

Examination of inversion techniques on gradient tensor data illustrates that single component inversion is comparable in the quality of the final result to a multi-component inversion, albeit much faster. Eigenvalues and eigenvectors of the full tensor produce information about depth and direction, although interpretation must be taken with care. I have developed an inversion routine to locate magnetic dipole sources, and illustrate how it produces multiple solutions to a problem, and how it is not possible to unambiguously determine which of these is correct. A new inversion routine was therefore developed.

This new hybrid global inversion routine is an automated technique to locate a generalised magnetic multipole. It can locate dipole and quadrupole (and some octupole) sources successfully, and be used either in real time as data is being collected, or as post-processing on an entire data set.

## Statement

This work contains no material that has been accepted for the award of any degree or diploma in any university or college and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, or the School of Earth and Environmental Sciences, or the School of Chemistry and Physics, being available for loan and photocopying in the case where it is cited as a reference.

Philip John Heath  
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