Multichannel Synthetic Aperture Radar

by

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Abstract

Synthetic Aperture Radar (SAR) is a coherent imaging technique capable of generating fine resolution images from long stand off ranges, either day or night. The use of multiple antennas for SAR is referred to as multichannel SAR and offers the additional abilities to perform height estimation of the ground, improve the resolution and suppress backscatter and/or undesired interferences.

The first part of this thesis looks at the problem of image formation for a multichannel SAR. Three wavefront reconstruction algorithms are presented based on the multichannel matched filter imaging equation. They demonstrate differing levels of performance and accuracy which will determine their practical use. A fourth algorithm known as multichannel backprojection is then presented to provide comparative quality with the best wavefront reconstruction algorithm with a reduced computational load.

The remainder of the thesis is then focussed on image formation while suppressing undesired interferences. This topic has not received a lot of coverage in the literature as the application is primarily for defence. If present in the mainbeam of a SAR, a jammer can potentially destroy a large region of the SAR image. In addition to this, multipath reflections from the ground, known as hot-clutter or terrain scattered interference will add a non-stationary interference component to the image. The goal of interference suppression for SAR is to suppress these interferences while not significantly effecting the image quality by blurring, reducing the resolution or raising the sidelobe level.

A study into the effect of hot-clutter interference on a SAR image is provided with a comparison of suppression techniques using multichannel imaging, optimal slow and fast-time Space Time Adaptive Processing (STAP). Simulation results indicate that fast-time STAP offers the best potential for suppressing hot-clutter while maintaining a coherent SAR image. Further investigations using a sub-optimal fast-time STAP filter are then presented to determine the best location of the adaptive filter in the SAR processing chain and how to best implement it in the real world. A number of suitable filters are introduced with reduced computational load and rank. They are based on a constrained optimisation criteria and include data-independent methods such as fast-time element and beamspace STAP as well as data-dependent eigen-based methods and the multistage Wiener filter.
Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

Signed

Date
Acknowledgments

I’d like to firstly thank my principle supervisor, Professor Doug Gray who offered me great support, guidance and encouragement throughout the thesis. You have given me the skills I need to succeed as a researcher and I look forward to passing those on to others in the future. Thanks also goes to my external supervisor, Dr. Nick Stacy whose knowledge and insight into SAR was a great benefit. I know my thesis is now much better thanks to your advice.

I would also like to thank the institutions which have supported my research. Firstly, to the Defence Science and Technology Organisation who allowed me time to pursue this work and provided my scholarship over the past four years. Also, to the Cooperative Research Centre for Sensor, Signal and Information Processing and the University of Adelaide who provided a great work environment with friendly postgrads and staff to interact with. I’m going to miss spending time with you each day, the weekly noodles lunch and especially the conversations at coffee time!

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Luke Rosenberg
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Publications


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## Geometry

- $h$: Mean height of SAR and jammer platforms above ground (m)
- $h_p$: Height of SAR platform above ground (m)
- $h_J$: Height of jammer platform above ground (m)
- $\phi_{3dB}$: 3 dB antenna beamwidth (rad)
- $\varphi$: SAR integration angle (rad)
- $\psi$: Grazing angle (rad)
- $\psi_J$: Jammer grazing angle (rad)
- $R$: Radial distance (m)
- $R_{min}, R_{max}$: Minimum and maximum radial distances (m)
- $\bar{R}_J$: Radial distance along ground between SAR and jammer (m)
- $R_{rot}$: Slant plane rotation matrix
- $R_{rot3}$: Ground plane rotation matrix
- $\bar{\theta}_a$: Ground-plane aspect angle (rad)
- $\theta_a, \theta_a(x, y)$: Slant-plane aspect angle (rad)
- $\theta_{a,k}(u)$: Slant-plane aspect angle (rad)
- $\theta, \theta(u)$: Slant-plane aspect angle relative to centre of patch (rad)
- $\theta_J$: Jammer offset angle (rad)
- $\vartheta$: Elevation angle (rad)
- $x, x'$: Slant range and slant range at broadside (m)
- $\bar{x}, \bar{x}'$: Ground range and ground range at broadside (m)
- $\bar{x}_J$: Ground range between SAR and jammer (m)
- $X_c$: Offset range (m)
- $2X_0$: Range swath width (m)
- $y, \bar{y}$: Azimuth (m)
- $y', \bar{y}'$: Azimuth at broadside (m)
- $2Y_L$: SAR integration length (m)
- $2Y_S$: Synthetic aperture length (m)
List of Symbols

\[2Y_0\] Azimuth along track extent (m)
\[2Y_W\] Antenna along track extent (m)
\[2Y_{W,\text{min}}\] Minimum antenna along track extent (m)
\[2Y_{W,\text{max}}\] Maximum antenna along track extent (m)
\[\bar{z}, z'\] Ground plane height and ground plane height at broadside (m)

Signal Model - Main Variables

\[d_n\] Antenna offset position (m)
\[k, k_c\] Wavenumber and wavenumber defined at carrier frequency (rad/m)
\[k_u\] Slow-time frequency (rad/m)
\[k_x\] Range spatial frequency (rad/m)
\[k_{xc}\] Centre range spatial frequency (rad/m)
\[k_{x,\text{min}}, k_{x,\text{max}}\] Minimum and maximum range spatial frequencies (rad/m)
\[k_y\] Azimuth spatial frequency (rad/m)
\[k_{y,\text{min}}, k_{y,\text{max}}\] Minimum and maximum azimuth spatial frequencies (rad/m)
\[\lambda, \lambda_c\] Wavelength and wavelength defined at carrier frequency (m)
\[\omega\] Fast-time frequency defined at carrier frequency (rad/s)
\[\tilde{\omega}\] Fast-time frequency defined at baseband (rad/s)
\[\omega_c\] Fast-time carrier frequency (rad/s)
\[\omega_{up}\] Upsampled fast-time frequency (rad/s)
\[s_m\] The \(m^{th}\) slow-time sample (s)
\[t_l\] The \(l^{th}\) fast-time sample (s)
\[\tilde{t}\] Absolute time (s)
\[t_{up}\] Upsampled fast-time (s)
\[u_m\] The \(m^{th}\) slow-time spatial sample (m)
\[\bar{u}_m\] The \(m^{th}\) slow-time spatial sample over user defined imaging area (m)
\[\bar{u}\] The slow-time spatial vector over user defined imaging area (m)
\[z\] Hot-clutter scattering height autocorrelation variable (m)
\[\zeta\] Correlation fast-time variable (s)
\[\tilde{\zeta}\] Generic correlation variable (s)
List of Symbols

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\( f(t,u) \) Ground return at baseband
\( f(\theta, t, u) \) Ground return at baseband
\( \hat{f}(\omega, u) \) Ground return at baseband
\( \hat{f}(\Theta, F_d, \omega) \) Ground return at baseband
\( \tilde{F}(\omega, k_u) \) Ground return at baseband
\( F(k_x, k_y) \) Ground return at baseband
\( f_n(x, y) \) Ground return at baseband for the \( n^{th} \) channel
\( f_{sp}, f_{sp}(\theta, t, u) \) Ground return at baseband with spatial only filtering
\( f_{sp}(t, u) \) Ground return at baseband with spatial only filtering
\( \hat{f}_{ss}(\omega, u) \) Ground return at baseband with slow-time STAP filtering
\( \tilde{F}_{ss}(\omega, k_u) \) Ground return at baseband with slow-time STAP filtering
\( \hat{f}_{ss}(\Theta, F_d, \omega) \) Ground return at baseband with slow-time STAP filtering
\( \hat{f}(x, y) \) Estimated ground return at baseband
\( \hat{F}(k_x, k_y) \) Estimated ground return at baseband
\( \hat{f}_{dB}(x, y) \) Estimated ground return at baseband in dB
\( \hat{f}_{RF}(x, y) \) Estimated ground return at the carrier frequency
\( \hat{f}_{fs}(x, y) \) Estimated ground return at baseband with fast-time STAP filtering
\( \hat{f}_{ss}(x, y) \) Estimated ground return at baseband with slow-time STAP filtering
\( \tilde{F}_{ss}(k_x, k_y) \) Estimated ground return at baseband with slow-time STAP filtering
\( g(t, u) \) Temporal component of target signal
\( g_{\text{pre, } k}(t) \) Temporal steering signal for the \( k^{th} \) fast-time tap, pre RP
\( g_{\text{post, } k}(t) \) Temporal steering signal for the \( k^{th} \) fast-time tap, post RP
\( g \) Temporal steering vector
\( g_{\text{pre}} \) Temporal steering vector, pre RP
\( g_{\text{post}} \) Temporal steering vector, post RP
\( \tilde{G}(\Theta, F_d) \) Space/slow-time steering vector, stacked space/slow-time
\( \tilde{G}_J(\Theta_J) \) Jammer signal vector, stacked space/slow-time
\( \tilde{G}_T \) Target signal vector, stacked space/slow-time
\( \tilde{G}(\omega, u) \) Space/slow-time steering vector, post RP, stacked space/slow-time
\( \gamma(t, u) \) Total ground return at baseband
\( \gamma_n(t, u) \) Total ground return at baseband for the \( n^{th} \) channel
\( \Gamma(\omega) \) Space/time ground return at baseband
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<tr>
<td>(P_c(\tilde{\omega}))</td>
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<td>(\phi_n(u))</td>
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**List of Symbols**

$P, P(\theta)$ Mean output power

$P(\Theta, F_d)$ Mean output power

$P_{\text{MVDR}}$ MVDR mean output power

$P_{\text{SINR}}$ Maximum SINR mean output power

$P_b(\theta)$ Beampattern

$P_b(\Theta, F_d)$ Beampattern

$P_J(\omega)$ Jammer power spectral density

$P_z$ Hung Turner projection matrix

$r, r_1, r_2$ Random Gaussian vectors used for jammer waveform realisation

$r_J(\zeta)$ Jammer autocorrelation signal

$r_J$ Jammer autocorrelation vector

$r_{x_q,e_q}(u)$ Reference beam interference plus noise spatial cross covariance

$r_{i,x_q,e_q}(u)$ Reference beam interference plus noise space/fast-time cross covariance

$R_{x_q}(u)$ Reference beam interference plus noise spatial covariance matrix

$R_{i,x_q}(u)$ Reference beam interference plus noise space/fast-time covariance matrix

$R_x(u)$ Total received spatial covariance matrix

$R_z(u)$ Interference plus noise spatial covariance matrix

$\hat{R}_z(u)$ Estimated interference plus noise spatial covariance matrix

$\tilde{R}_z(u)$ Normalised estimated interference plus noise spatial covariance matrix

$\tilde{R}_{z,DL}(u)$ Normalised estimated interference plus noise spatial covariance matrix with diagonal loading

$R_{\tilde{Z}}$ Interference plus noise space/slow-time covariance matrix

$R_{\tilde{Z}_c}$ Interference plus ground clutter plus noise space/slow-time covariance matrix

$R_{Z_t,\text{pre}}(t, \zeta, \cdot)$ Interference space/fast-time instantaneous covariance element, pre RP

$\tilde{R}_{Z_t,\text{pre}}(\zeta, \cdot)$ Interference space/fast-time mean estimated covariance element, pre RP

$R_{Z_t,\text{post}}(t, \zeta, \cdot)$ Interference space/fast-time instantaneous covariance element, post RP

$\tilde{R}_{Z_t,\text{post}}(\zeta, \cdot)$ Interference space/fast-time mean estimated covariance element, post RP

$\hat{R}_{Z_t}(u)$ Estimated interference plus noise space/fast-time covariance matrix

$\hat{R}'_{Z_t}(u)$ Normalised estimated interference plus noise space/fast-time covariance matrix

$\hat{R}_{Z_t,DL}(u)$ Normalised estimated interference plus noise space/fast-time covariance matrix with diagonal loading

$s_n(u)$ Spatial signal model for the $n^{th}$ channel
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<td>Signal model, post RP</td>
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<td>$S_{\text{ref}, n}(\omega, k_u)$</td>
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<td>Principle component reduced rank transform</td>
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<td>Symbol</td>
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<td>-----------------------------------------------------------------------------</td>
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<td>$w_{SINR}(\theta)$</td>
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<td>$w_{GSC}(u)$</td>
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<td>$W_{f,LCMV}(u)$</td>
<td>LCMV weight vector, stacked space/fast-time</td>
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<tr>
<td>$w_{MWF}(u)$</td>
<td>MWF weight vector, stacked spatially</td>
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<tr>
<td>$W_{f,MWF}(u)$</td>
<td>MWF weight vector, stacked space/fast-time</td>
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<tr>
<td>$x(t, u)$</td>
<td>Total received SAR signal at baseband</td>
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<tr>
<td>$x_n(t, u)$</td>
<td>Total received SAR signal at baseband for the $n^{th}$ channel</td>
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<td>$x(t, u)$</td>
<td>Total received SAR vector at baseband, stacked spatially</td>
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<td>$\tilde{x}_n(\omega, u)$</td>
<td>Total received SAR signal at baseband</td>
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<td>$\tilde{x}(\omega, u)$</td>
<td>Total received SAR vector at baseband, stacked spatially</td>
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<td>$X_n(\omega, k_n)$</td>
<td>Total received SAR signal at baseband for the $n^{th}$ channel</td>
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<td>$X(\omega, k_n)$</td>
<td>Total received SAR vector at baseband, stacked spatially</td>
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<td>Total received SAR vector at baseband, stacked space/slow-time</td>
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List of Symbols

$\mathbf{X}_f(u), \mathbf{X}_f(t, u)$ Total received SAR vector at baseband, stacked space/fast-time

$x_{\text{pre}}(t, u)$ Total received SAR signal at baseband, pre RP

$x_{\text{pre}, n}(t, u)$ Total received SAR signal at baseband for the $n^{th}$ channel, pre RP

$x_{\text{pre}, \text{RF}}(t, u)$ Total received SAR signal at baseband at the carrier frequency

$x_{\text{ideal}}(t, u)$ Total received SAR signal at baseband with no interference present

$x_{n}(t, u)$ Total received up-sampled SAR signal at baseband for the $n^{th}$ channel

$x_{\text{RF}, n}(t, u)$ Total received up-sampled SAR signal at the carrier frequency for the $n^{th}$ channel

$x_q(t, u)$ Total received SAR vector at baseband for GSC, MWF algorithms, stacked spatially

$y_b(\Theta, F_d)$ Output beampattern signal

$y_s$ Output target signal post adaptive filtering

$y_z$ Output interference signal post adaptive filtering

$Y(x, y)$ Degraded image to be measured in the SDR

$z(t, u)$ Interference plus noise signal

$z_{n}(t, u)$ Interference plus noise signal for the $n^{th}$ channel

$z(t, u)$ Interference plus noise vector, stacked spatially

$\tilde{\mathbf{Z}}_s(\omega)$ Interference plus noise vector, stacked space/slow-time

$\mathbf{Z}_f(u), \mathbf{Z}_f(t, u)$ Interference plus noise vector, stacked space/fast-time

$z_{\text{pre}, n}(t, u)$ Interference plus noise signal for the $n^{th}$ channel, pre RP

$z_{\text{post}, n}(t, u)$ Interference plus noise signal for the $n^{th}$ channel, post RP

$z_{\text{dp}, n}(t, u)$ Direct-path jamming signal for the $n^{th}$ channel

$z_{\text{hc}, n}(t, u)$ Hot-clutter jamming signal for the $n^{th}$ channel

$z_{\text{spat}, n}(t, u)$ Spatial component of interference plus noise signal for the $n^{th}$ channel

$z_{\text{spat}}(t, u)$ Spatial component of interference plus noise vector, stacked spatially

$z_{\text{temp}}(t, u)$ Temporal component of interference plus noise signal

$z_{\text{TRAN}}(\tilde{t})$ Transmitted jamming signal

$z_{c, n}(t, u)$ Interference plus ground clutter plus noise signal

$\tilde{\mathbf{Z}}_c(\omega)$ Interference plus ground clutter plus noise vector, stacked space/slow-time
### List of Symbols

#### Signal Model - Parameters and Other Variables

2\(\alpha\)  
Chirprate (rad/s)

\(\alpha_{\text{norm}}\)  
Normalising level for spatial interference plus noise covariance

\(\alpha_{\text{f,norm}}\)  
Normalising level for space/fast-time interference plus noise covariance

\(\alpha_s\)  
Scale factor to constrain scatterer amplitudes in diffuse scattering model (dB)

\(\alpha_T\)  
Output target signal power (dB)

\(b_k\)  
Relative magnitude between the direct-path and \(k^{th}\) hot-clutter scatterer (dB)

\(\tilde{b}_k\)  
Antenna amplitude voltage for the \(k^{th}\) scatterer (V)

\(B\)  
Bandwidth (Hz)

\(B_C\)  
Pulse compressed bandwidth (Hz)

\(B_l\)  
Fractional bandwidth

\(B_x, B_y\)  
Range and azimuth mainlobe 3dB level in spatial frequency domain (rad/m)

\(\beta\)  
Angle made by the bisector of the hot-clutter incident and scattered rays with the \(\bar{z}\)-axis (rad)

\(\beta_0\)  
Upper \(\beta\) limit which defines hot-clutter scattering contour boundary (rad)

\(\tilde{\beta}\)  
Relationship between normalised slow-time Doppler rate and steering angles. This parameter determines the slope of the clutter ridge in the \(\Theta – F_d\) domain.

\(c\)  
Speed of light (m/s)

\(C\)  
Number of eigen-pairs used in reduced rank GSC

\(\delta\)  
Antenna spacing (m)

\(\delta_{\text{amp}}\)  
Amplitude constraint offset angle (rad)

\(D_y\)  
Antenna length (m)

\(\Delta\)  
Small number used in Taylor series

\(\Delta f_{k,k'}\)  
Differential Doppler shift between two scatterers (Hz)

\(\Delta t\)  
Fast-time sampling rate (s)

\(\Delta_x\)  
Spatial (range) sampling rate (m)

\(\Delta X\)  
3dB range resolution (m)

\(\Delta y\)  
Distance travelled between pulses / slow-time (azimuth) sampling rate (m)

\(\Delta Y\)  
3dB azimuth resolution (m)

\(\Delta\omega_{k,k'}\)  
Differential Doppler shift between two scatterers (rad/s)

\(e_q, \hat{e}_q\)  
Indicates paths in the GSC and MWF algorithms

\(\eta\)  
Diagonal loading level (dB)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>Dielectric constant (Fm$^{-1}$)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Random variable used for CDF in diffuse scattering model (rad)</td>
</tr>
<tr>
<td>$f_d$</td>
<td>Slow-time Doppler rate (Hz)</td>
</tr>
<tr>
<td>$f_{PRF}$</td>
<td>Pulse Repetition Frequency (Hz)</td>
</tr>
<tr>
<td>$F_d$</td>
<td>Normalised slow-time Doppler</td>
</tr>
<tr>
<td>$F_{d,T}$</td>
<td>Normalised target Doppler</td>
</tr>
<tr>
<td>$G_r$</td>
<td>Receiver gain (dB)</td>
</tr>
<tr>
<td>$G_t$</td>
<td>Transmitter gain (dB)</td>
</tr>
<tr>
<td>$\tilde{\gamma}$</td>
<td>Normalising coefficient for the maximum SINR weight vector</td>
</tr>
<tr>
<td>$\tilde{\Gamma}$</td>
<td>Fresnel reflection coefficient (dB)</td>
</tr>
<tr>
<td>$\tilde{\Gamma}_h, \tilde{\Gamma}_v$</td>
<td>Fresnel reflection coefficient for horizontal and vertical polarisations (dB)</td>
</tr>
<tr>
<td>$I_x, I_y$</td>
<td>Number of pixels in imported image</td>
</tr>
<tr>
<td>$\kappa_k$</td>
<td>The $k^{th}$ eigenvalue</td>
</tr>
<tr>
<td>$K_\beta$</td>
<td>Surface roughness parameter</td>
</tr>
<tr>
<td>$K_{av}$</td>
<td>Number of averages used to test jammer autocorrelation</td>
</tr>
<tr>
<td>$K_{gc}$</td>
<td>Number of ground patches included in ground clutter simulation model</td>
</tr>
<tr>
<td>$K_{hc}$</td>
<td>Number of hot-clutter ground patches included in jammer model</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of fast-time samples</td>
</tr>
<tr>
<td>$\hat{L}$</td>
<td>Number of fast-time taps</td>
</tr>
<tr>
<td>$L_P$</td>
<td>Number of range bins used in transmitted pulse, $TP$</td>
</tr>
<tr>
<td>$L_t$</td>
<td>Number of range bins used for covariance estimation</td>
</tr>
<tr>
<td>$L_{up}$</td>
<td>Number of upsampled fast-time samples</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Ideal number of fast-time taps</td>
</tr>
<tr>
<td>$L_J$</td>
<td>The total length of the jammer signal realisation</td>
</tr>
<tr>
<td>$L_Z$</td>
<td>Number of zeros used in upsampling</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>Singular value/eigenvector decomposition component vector</td>
</tr>
<tr>
<td>$M$</td>
<td>Number of pulses or slow time samples</td>
</tr>
<tr>
<td>$M'$</td>
<td>Number of pulses used for estimating covariance in optimal fast-time STAP</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Number of focus positions used in ground clutter model</td>
</tr>
<tr>
<td>$M_{\text{max}}$</td>
<td>Maximum number of pulses chosen for simulation cases</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Angle used to express differences in height for the diffuse scattering model (rad)</td>
</tr>
<tr>
<td>$n_x, n_y$</td>
<td>Number of range and azimuth pixels in final image</td>
</tr>
</tbody>
</table>
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of antenna channels or spatial samples</td>
</tr>
<tr>
<td>$N_{\text{con}}$</td>
<td>Number of constraints</td>
</tr>
<tr>
<td>$\omega_{d}, \omega_{d,k}$</td>
<td>General Doppler shift and for the $k^{th}$ scatterer (rad/s)</td>
</tr>
<tr>
<td>$P_{\text{over}}$</td>
<td>Azimuth over-sampling rate for simulation scenarios 2 and 3</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Received power (dB)</td>
</tr>
<tr>
<td>$P_t$</td>
<td>Transmitted power (dB)</td>
</tr>
<tr>
<td>$Q$</td>
<td>MWF maximum order</td>
</tr>
<tr>
<td>$Q_{\text{sub}}$</td>
<td>Azimuth subsampling rate for simulation scenario 1</td>
</tr>
<tr>
<td>$q_k$</td>
<td>The $k^{th}$ eigenvector</td>
</tr>
<tr>
<td>$\tilde{Q}$</td>
<td>Eigenvector decomposition matrix</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Relative power between direct-path and hot-clutter signal components (dB)</td>
</tr>
<tr>
<td>$\varsigma_k$</td>
<td>CSM ranking criteria</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>Hot-clutter scattering height standard deviation (m)</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>Hot-clutter ground return for the $k^{th}$ scatterer (dB)</td>
</tr>
<tr>
<td>$\sigma_r^2$</td>
<td>Thermal noise power/variance (dB)</td>
</tr>
<tr>
<td>$\sigma_q^2$</td>
<td>Quantisation noise power/variance (dB)</td>
</tr>
<tr>
<td>$\sigma_G^2$</td>
<td>Ground clutter power/variance (dB)</td>
</tr>
<tr>
<td>$\sigma_J^2$</td>
<td>Jammer power/variance (dB)</td>
</tr>
<tr>
<td>$\sigma_T^2$</td>
<td>Target signal power/variance (dB)</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>Scattering coefficient (m²)</td>
</tr>
<tr>
<td>$S_{\text{NR, opt}}$</td>
<td>Optimum signal to noise ratio (dB)</td>
</tr>
<tr>
<td>$S_{\text{NR,T}}$</td>
<td>Target signal to noise ratio (dB)</td>
</tr>
<tr>
<td>$T$</td>
<td>Fast-time sampling period (s)</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Fast-time sampling finish time (s)</td>
</tr>
<tr>
<td>$T_l$</td>
<td>Maximum temporal delay between hot-clutter scatterers (s)</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Total SAR integration time (s)</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Pulse width (s)</td>
</tr>
<tr>
<td>$T_{\text{PRI}}$</td>
<td>Pulse Repetition Interval (s)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Fast-time sampling start time (s)</td>
</tr>
<tr>
<td>$T_x$</td>
<td>Range swath echo time period (s)</td>
</tr>
<tr>
<td>$\theta_J$</td>
<td>Jammer signal DOA relative to broadside (rad)</td>
</tr>
<tr>
<td>$\Theta_J$</td>
<td>Normalised jammer signal DOA relative to broadside</td>
</tr>
</tbody>
</table>
\(\theta_{\text{lim}}\)  Angular slow-time limit (rad)
\(\theta_T\)  Target signal DOA relative to broadside (rad)
\(\Theta_T\)  Normalised target signal DOA relative to broadside
\(\Theta\)  Normalised slow-time steering angle
\(\tilde{U}\)  Singular value decomposition component vector
\(v_p\)  SAR platform speed along azimuth direction (m/s)
\(v_p\)  SAR platform velocity (m/s)
\(v_J\)  Jammer platform velocity (m/s)
\(\tilde{V}\)  Singular value decomposition component vector
\(x_{\text{ML}}\)  Half mainlobe width of point spread function (m)
\(\xi\)  Angle used to define contour in diffuse scattering model (rad)
\(\xi_A, \xi_B\)  Limits of \(\xi\) used to define the contour limits in the diffuse scattering model (rad)
\(z_h\)  Correlation distance for hot-clutter scattering autocorrelation function (m)
\(Z_r\)  Impedance at receiver (\(\Omega\))
\(Z_{\text{rat}}\)  Upsampling rate

**Other Functions and Operators**

\(b(x)\)  Rectangle function defined over \(0 \leq x \leq 1\)
\(\text{rect}(x)\)  Rectangle function defined over \(-0.5 \leq x \leq 0.5\)
\(\text{sinc}(x)\)  Sinc function defined as \(\sin[\pi x]/\pi x\)
\(0_N\)  Zero function of size \(N \times N\)
\(I_N\)  Identity function of size \(N \times N\)
\(\hat{\cdot}\)  Unit vector
\(\hat{\cdot}\)  Estimate
\(\top\)  Transpose
\(H\)  ‘Hermitian’ complex transpose
\(\dagger\)  Moore Penrose psuedo inverse
\(C_{N,M}\)  Matrix is complex with size \(N \times M\)
\(E\{\}\)  Expected value
\(\mathcal{F}\{\}\)  Fourier transform
\(\mathcal{F}^{-1}\{\}\)  Inverse Fourier transform
\(\ast\)  Convolution operator
\(\otimes\)  Kronecker product
# Glossary

## List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D, 3D</td>
<td>Two or Three Dimensional</td>
</tr>
<tr>
<td>ATI</td>
<td>Along Track Interferometry</td>
</tr>
<tr>
<td>CPI</td>
<td>Coherent Processing Interval</td>
</tr>
<tr>
<td>CFAR</td>
<td>Constant False Alarm Rate</td>
</tr>
<tr>
<td>CSM</td>
<td>Cross Spectral Metric</td>
</tr>
<tr>
<td>DOA</td>
<td>Direction of Arrival</td>
</tr>
<tr>
<td>DPCA</td>
<td>Displaced Phase Centre Array</td>
</tr>
<tr>
<td>DRFM</td>
<td>Digital Radio Frequency Memory</td>
</tr>
<tr>
<td>ERP</td>
<td>Effective Radiated Power</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Position Satellite</td>
</tr>
<tr>
<td>GSC</td>
<td>Generalised Sidelobe Canceller</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FM</td>
<td>Frequency Modulation</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field Programmable Gate Array</td>
</tr>
<tr>
<td>GLRT</td>
<td>Generalised Likelihood Ratio Test</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>INU</td>
<td>Inertial Navigation Unit</td>
</tr>
<tr>
<td>ISLR</td>
<td>Integrated Sidelobe Ratio</td>
</tr>
<tr>
<td>JNR</td>
<td>Jammer to Noise Ratio</td>
</tr>
<tr>
<td>LCMV</td>
<td>Linearly Constrained Minimum Variance</td>
</tr>
<tr>
<td>MF</td>
<td>Matched Filter</td>
</tr>
<tr>
<td>MSAR</td>
<td>Multichannel Synthetic Aperture Radar</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>MTI</td>
<td>Moving Target Indication</td>
</tr>
<tr>
<td>MVDR</td>
<td>Minimum Variance Distortionless Response</td>
</tr>
<tr>
<td>MWF</td>
<td>Multistage Wiener Filter</td>
</tr>
<tr>
<td>PC</td>
<td>Principle Components</td>
</tr>
</tbody>
</table>
RCS  Radar Cross Section
PRF  Pulse Repetition Frequency
PRI  Pulse Repetition Interval
PSD  Power Spectral Density
PSF  Point Spread Function
PSR  Peak Sidelobe Ratio
RF   Radio Frequency
RINR Residual Interference plus Noise Ratio
SAR  Synthetic Aperture Radar
SDR  Signal Distortion Ratio
SINR Signal to Interference plus Noise Ratio
SNR  Signal to Noise Ratio
STAP Space Time Adaptive Processing
TDC  Time Domain Correlation
TSI  Terrain Scattered Interference
WVD  Wigner-Ville Distribution

**Nominal Radio Frequency Bands**

<table>
<thead>
<tr>
<th>Band</th>
<th>Frequency Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>HF</td>
<td>3 MHz - 30 MHz</td>
</tr>
<tr>
<td>VHF</td>
<td>30 MHz - 300 MHz</td>
</tr>
<tr>
<td>UHF</td>
<td>300 MHz - 1 GHz</td>
</tr>
<tr>
<td>P-band</td>
<td>300 MHz - 350 MHz</td>
</tr>
<tr>
<td>L-band</td>
<td>1 GHz - 2 GHz</td>
</tr>
<tr>
<td>S-band</td>
<td>2 GHz - 4 GHz</td>
</tr>
<tr>
<td>C-band</td>
<td>4 GHz - 8 GHz</td>
</tr>
<tr>
<td>X-band</td>
<td>8 GHz - 12 GHz</td>
</tr>
<tr>
<td>Ku-band</td>
<td>12 GHz - 18 GHz</td>
</tr>
<tr>
<td>K-band</td>
<td>18 GHz - 27 GHz</td>
</tr>
<tr>
<td>Ka-band</td>
<td>27 GHz - 40 GHz</td>
</tr>
<tr>
<td>V-band</td>
<td>40 GHz - 75 GHz</td>
</tr>
<tr>
<td>W-band</td>
<td>75 GHz - 110 GHz</td>
</tr>
<tr>
<td>mm-band</td>
<td>110 GHz - 300 GHz</td>
</tr>
</tbody>
</table>

thetic Aperture Radar (SAR) is a well established radar imaging technology dating back to the early fifties. It is an active sensor system typically operating at microwave frequencies and can obtain real time, high resolution images of the ground from a long stand off range. One of its main features is the ability to operate in almost any weather, day or night, thereby overcoming many of the limitations of other passive imaging technologies such as optical and infra-red.

However, obtaining a resolution of a metre or less from a range of tens of kilometres would typically require an antenna hundreds of metres long. Since this is impractical for air or space-borne platforms, an antenna may be ‘synthesised’ by combining the radar returns from a single patch as the SAR platform moves along. Typical applications of SAR include ground mapping, remote sensing, search and rescue, environmental monitoring and detection and identification of moving ground targets, [Curlander and McDonough, 1991].

Current research areas for SAR include the combination of polarimetry with interferometry, [Stacy et al., 2003], improved range and azimuth resolution, [Ender and Brenner, 2002], moving target compensation, [Ender, 1998c] and bistatic image formation, [Horne and Yates, 2002]. All of these research areas utilise multiple sources of data, such as different polarisations, frequencies or antennas. The use of multiple antennas for SAR is referred to as Multichannel Synthetic Aperture Radar (MSAR) and offers a number of new and improved applications. The most common include cross-track interferometry for height estimation, [Graham, 1974] and Moving Target Identification (MTI) for detecting and compensating for moving targets.

The aim of MTI is to isolate moving targets while removing both unwanted reflections from the ground (ground clutter) and any other interferences from radio frequency signals in the same frequency band. There are however, additional problems with applying MTI to SAR. These are due to the longer integration time in SAR and image formation being designed to focus stationary ground clutter only. Moving targets will appear offset and blurred and techniques are required to not only detect the moving targets, but also to compensate for the effect of motion on the SAR image.

A very similar problem to MTI is that of image formation while suppressing unwanted interferences. This topic has not received the same coverage in the open literature as the application is primarily for defence. The interfering sources from a jammer include both a direct-path signal
1.1 Motivation

and multipath reflections from the ground, known as terrain scattered interference or hot-clutter. Using multiple antennas on a SAR provides spatial degrees of freedom and allows for beamforming to reject the interfering signals. Figure 1.1 shows the scenario of an airborne jammer with the different signal components.

Why then are there very few radar systems in the world which contain more than one or two antennas? The answer is due to the high cost of the hardware required and the immense data rates produced by a multichannel system. However, given the rate of technology increase for computer processing power and storage, MSAR’s will soon be the standard in surveillance radar. This thesis looks at the problem of image formation for a multichannel SAR, the effect of interference from an airborne jammer and how image formation can be modified to suppress unwanted interferences.

1.1 Motivation

The motivation for this work came after an extensive study of MSAR imaging, MTI in airborne radars and its application to SAR. MTI with non-stationary interferences, such as hot-clutter is a tough problem which has been addressed by a number of authors in the airborne radar, [Ward, 1994]-[Griffiths et al., 2000] and HF communities, [Abramovich et al., 1998]-[Fabrizio et al., 2004]. However, there has been little in the literature on the effect of non-stationary hot-clutter on SAR and multichannel SAR image formation under jamming conditions.

Multichannel SAR image formation comprises the first research area and is briefly summarised in Section 1.1.1. A study into suitable methods of hot-clutter suppression for SAR then comprises the second and main research area. It is summarised briefly in Section 1.1.2 to motivate the specific research undertaken in this thesis. These contributions aim to fill in the gaps in the literature and provide a foundation for future experimental work in this area.

Figure 1.1. SAR imaging scenario with airborne jammer
Chapter 1  

1.1.1 Multichannel SAR Image Formation

SAR imaging was originally designed with analogue/optical techniques including polar format and range-Doppler based imaging. Polar format imaging uses a plane wave approximation with range curvature correction, while the range-Doppler technique uses the Fresnel approximation for SAR inversion. With the introduction of powerful computers, more precise imaging algorithms are possible. These are based on wavefront reconstruction with a Matched Filter (MF) [Goodman, 1968], and offer the potential for imaging with greater accuracy. [Soumekh, 1999], has demonstrated a number of different algorithms based on this technique.

Literature on MSAR imaging includes a brief description of MF processing [Ender, 1998c] and how sidelobe suppression vectors can be used to reduce sidelobe leakage ambiguities for long range, low pulse repetition frequency or non-ideal antenna patterns [Goodman et al., 1999]- [Ender, 2000]. There has been no comprehensive study on how the many different imaging algorithms can be extended for multichannel systems.

1.1.2 Interference Suppression for Multichannel SAR

An airborne jammer can transmit in a number of different modes, such as narrowband ‘spot’, broadband ‘barrage’, random pulse or deceptive jamming, [Goj, 1992]. It can also combine different waveforms dynamically to try and stop or the SAR from imaging the ground or detecting moving targets. Nearly all the open literature on jammer suppression is concerned only with the case of a broadband continuous noise jammer, as the nature of different waveforms is clearly a sensitive topic. The two signal paths of an airborne jammer, the direct-path and terrain reflected path will cause different problems for the SAR. The direct path, while being wide in bandwidth, is narrow in angle and can be nulled quite easily. However, as the jammer signal reflects from the terrain, it is scattered diffusely over a range of azimuth angles. This leads to large angular spread and a non-stationarity as the scattering statistics change with the terrain aspect. This is important for MSAR as it means that slow-time Space Time Adaptive Processing (STAP) will not be effective at interference rejection as it relies on averaging over pulses.

The aim of MTI for both airborne radar and SAR is to isolate the ‘desired’ moving targets while removing both unwanted reflections from the ground and any other undesired interferences. The problem of image formation while suppressing interferences however, considers the ground clutter to be the desired signal. This is fundamental difference in philosophy between the two techniques and there has only been a small amount of literature on the latter topic.

The first publication to look at jamming for SAR was by [Condley, 1991]. He looked at the Sea Satellite (SEASAT) mission in 1978, [SEASAT, 1978] and calculated that if a typical spot noise jammer was to be used against SEASAT, it was possible to jam a signal from the mainbeam of the SAR, but if the jamming signal was aimed at the sidelobes, then more advanced sub-systems would be required. A book on electronic warfare for SAR was then published by [Goj, 1992] looking at how different methods of jamming will effect a SAR image and how the SAR could potentially defeat them. Since then a number of authors have looked at different methods for
1.2 Thesis Outline and Contributions

suppressing the direct-path signal. Most significantly is the work on STAP by [Ender, 1998a] and [Klemm, 2002].

It is unclear when hot-clutter was first identified as a problem in airborne radar, but mainstream publications on hot-clutter suppression techniques have been available since the mid-nineties, specifically at the Adaptive Sensor Array Processing conferences, 1995-1997. There are currently many different methods for suppressing hot-clutter, [Fante and Torres, 1995]-[Gabel et al., 1999], with fast-time STAP offering one of the best solutions. This technique involves adapting to the changing interference within each pulse and offers the advantage of exploiting the coherency between the direct-path jammer and other hot-clutter signals to provide improved interference rejection. The majority of publications are primarily focused on airborne radar and hot-clutter suppression for SAR is a problem that has not yet been addressed. The goal of interference suppression for SAR and hence this thesis, is to successfully suppress these interferences while not significantly affecting the image quality by blurring, reducing the resolution or raising the sidelobe level.

1.2 Thesis Outline and Contributions

This thesis looks at the problem of image formation for a MSAR and interference suppression for SAR using different STAP techniques.

MSAR imaging is the focus for the first part of the thesis where three wavefront reconstruction algorithms are presented. An analysis of their performance and accuracy is given as a guide to determine their practical use. A fourth algorithm known as multichannel backprojection is then presented as an alternative with comparable quality to best of the wavefront reconstruction algorithms and a reduced computational load.

An introduction to STAP in then presented for airborne radar and SAR followed by a detailed description of the jammer model. This model is used with the MSAR imaging algorithm to provide an analysis of the degradation resulting from hot-clutter, the limited restoration that multichannel imaging and optimal slow-time STAP can provide and how optimal fast-time STAP can improve the final image quality. Results from an ideal scenario indicate that fast-time STAP offers the best potential for suppressing hot-clutter while maintaining a coherent SAR image.

A description of how the optimal fast-time STAP filter can be approximated by a sub-optimal filter is then presented along with models for both the pre and post range processing space/fast-time steering vectors and the estimated interference covariance matrices. A number of comparisons are performed to determine the relative adaptive performance of both adaptive methods.

The final part of the thesis looks at a more realistic scenario using fast-time STAP algorithms. They are based on constrained optimisation criteria to suppress the hot-clutter interference while maintaining a good quality SAR image. The algorithms studied include data-independent methods such as fast-time element and beamspace STAP as well as data-dependent eigen-based methods and the multistage Wiener filter.
The main contributions of this thesis are:

- To derive the multichannel MF imaging equation and extend a number of SAR imaging algorithms to use multiple channels. An analysis of their complexity and Point Spread Functions (PSF) are used to compare their relative performance.

- To construct a realistic jamming model to provide an analysis of the degradation of a SAR image resulting from hot-clutter. Using this model, a study is performed of the limited restoration that multichannel imaging and optimal slow-time STAP can provide and how optimal fast-time STAP can improve the final image quality.

- To formulate a sub-optimal fast-time STAP filter and apply it both pre and post range processing to determine their adaptive performance. Constrained fast-time element and beamspace STAP algorithms are then applied to the problem of suppressing hot-clutter, while maintaining a good quality SAR image. Two reduced rank algorithms have been extended to use fast-time taps with constraints to achieve adaptive performance comparable to the full rank versions.

1.2.1 Chapter Summaries

Chapter 2 - Multichannel SAR Background
This chapter is focussed on understanding the characteristics and benefits of a MSAR, how the processing stages can be represented with a mathematical model and then simulated on a computer. The benefits of a MSAR include improved gain, ambiguity reduction, the ability for ground height estimation and the use of spatial degrees of freedom to suppress undesired signals. Details of the MSAR signal processing models are presented and include the imaging geometry, the SAR signal model, resolution and sampling, range compression and the extension to multiple channels. Due to lack of real data, simulation is the only means of testing and measuring the performance of different algorithms. A MATLAB simulation has been designed to match the signal models and allow the user to easily change parameters.

The main contribution in this chapter is to combine a number of mathematical models for MSAR and blend them into a useful simulation.

Chapter 3 - Multichannel SAR Imaging
This chapter uses the multichannel signal model to derive the MF imaging equation. Using this equation, three different wavefront reconstruction algorithms are described and extended to use multiple channels. Two of these algorithms, spatial MF interpolation and range stacking are based on the frequency domain to speed up the computation time. The simulated Point Spread Functions (PSF) however reveal some problems with this approach and the third algorithm, Time Domain Correlation (TDC) overcomes these by trading off accuracy for computation time. A mul-
1.2 Thesis Outline and Contributions

tichannel backprojection algorithm is then presented to both overcome the problems of 
the frequency domain algorithms and avoid the large computational cost of TDC.

The main contributions in this chapter are to derive a multichannel MF imaging 
equation, extend three SAR wavefront reconstruction algorithms to include 
multiple channels and provide an analysis of their PSF's. A multichannel back-
projection algorithm is described and its relative performance compared to the 
previous algorithms.

Chapter 4 - STAP Background This chapter presents a number of practical MTI tech-
niques for SAR as they form the basis of many interference suppression methods. An 
introduction to optimal adaptive array processing for airborne radar is then described and 
extended to slow-time STAP. The extension of STAP from airborne radar to SAR is then 
presented.

The main contributions in this chapter are to provide relevant background on 
adaptive filtering as applied to STAP for airborne radar and the extension 
required for SAR.

Chapter 5 - Jammer Background and Model The first part of this chapter presents a 
comprehensive literature review of jamming and anti-jamming techniques for SAR and 
methods of suppressing hot-clutter in airborne and high frequency over the horizon radar. 
The second part then describes mathematical models for the received jammer signal, the 
diffuse scattering of the hot-clutter and the jammer and noise waveforms.

The main contributions in this chapter are to provide a comprehensive liter-
ature review of SAR jamming and anti-jamming techniques and hot-clutter 
suppression algorithms in the open literature. A number of mathematical 
models are combined to form a physically based scattering and jammer model 
for use in the MATLAB simulation.

Chapter 6 - Optimal STAP for SAR This chapter uses the models presented in previous 
chapters to demonstrate the effect of hot-clutter on SAR imaging. Three different ap-
proaches for suppressing hot-clutter are introduced including multichannel imaging and 
optimal slow-time and fast-time STAP. Simulation results using ideal training data are 
then presented and quantified to measure the effectiveness of interference rejection for 
these three algorithms. A look at the effect of non-ideal training data is also presented to 
demonstrate the adaptive performance likely to be achieved in a real system.

The main contributions in this chapter are to provide an analysis of the degra-
dation in a SAR image resulting from hot-clutter, the limited restoration that 
multichannel imaging and optimal slow-time STAP can provide and how opti-
timal fast-time STAP can improve the final image quality. Realistic training 
data is used to demonstrate the adaptive performance likely to be achieved in 
a real system.
Chapter 7 - Fast-time STAP Performance This chapter describes how the optimal fast-time STAP filter can be approximated by a sub-optimal filter. Models for both the pre and post range processing space/fast-time steering vectors are introduced along with details of the estimated interference covariance matrix. Derivations of the exact instantaneous and mean estimated covariances for both cases are derived using a common signal model and different scattering scenarios are tested to determine the expected level of correlation with increasing fast-time taps. An analysis of the eigen-distribution is used to determine the rank of the interference and the Signal to Interference plus Noise Ratio (SINR) loss is used to measure their relative adaptive performance.

The main contributions in this chapter are to derive a sub-optimal fast-time STAP filter and pre and post range processing models for the space/fast-time steering vectors. Exact instantaneous and mean estimated models for the interference covariances are derived and tested with a number of simulated scenarios. The structure and relative performance of each model is compared using the space/fast-time steering vectors to determine their relative adaptive performance.

Chapter 8 - Constrained Fast-time STAP This chapter describes the development of constrained adaptive algorithms as applied to spatial and space/fast-time adaptive processing with image formation. Constrained fast-time adaptive filtering is presented for element space and beamspace algorithms via the Generalised Sidelobe Canceller (GSC). Two reduced rank formulations of the GSC are then presented which achieve similar adaptive performance to the full rank versions with reduced computational load. The first is based on an eigen-decomposition of the interference plus noise covariance matrix, while the latter uses a multistage Wiener filter.

The main contributions in this chapter are to apply constrained fast-time element and beamspace STAP algorithms to the problem of suppressing hot-clutter, while forming a good quality SAR image. Two reduced rank algorithms have been extended to use fast-time taps with constraints to achieve adaptive performance comparable to the full rank version.

Chapter 9 - Conclusions This chapter summarises the main results from the thesis and describes areas for future research.

Appendix A - $(\omega, k_u)$ Signal Model Formulation This appendix evaluates the $(\omega, k_u)$ post range processing system model used in Chapter 3.

Appendix B - Jammer Model Implementation This appendix describes the implementation of the diffuse scattering and jammer models presented in Chapter 5.

Appendix C - Sample Matrix Estimate using Ideal Training Data This appendix derives the Hung-Turner projection used in Chapter 6.
1.2 Thesis Outline and Contributions

Appendix D - Post Range Processing Covariance Derivation  This appendix contains the derivation of the post range processing covariance model from Chapter 7.
Chapter 2

Multichannel SAR Background

2.1 Introduction

This chapter is focused on understanding the benefits of a Multichannel Synthetic Aperture Radar (MSAR), how its processing can be represented with a mathematical model and then simulated on a computer. The most fundamental mode of operation in SAR is stripmap, where the antenna is fixed normal to the direction of flight and collects data from a series of pulses. The models in this chapter and the algorithms in later chapters are all based on this mode, but could be modified for other modes such as spotlight or scanSAR. There are numerous books on SAR detailing aspects such as system hardware, modes of operation, signal processing and imaging techniques. One very thorough book detailing the system and signal processing is [Curlander and McDonough, 1991]. Many of the technical models used in this thesis, such as Doppler, geometry and pulse compression are taken from this source. A book more focussed on signal processing and simulation is by [Soumekh, 1999]. His models for the slant-plane geometry, signal models and imaging algorithms have likewise been adapted for this thesis.

Section 2.2 provides a brief literature review on the benefits of a MSAR. These include improved gain, ambiguity reduction, height estimation and the use of spatial degrees of freedom. A number of operational MSAR’s are then listed with information on how they’ve exploited these benefits. Section 2.3 details the MSAR signal processing models which are used in the following chapters. They include imaging geometry, the SAR signal model, resolution and sampling, range compression and the extension to multiple channels. The final Section 2.4, describes the SAR simulation setup. Due to lack of real data, simulation is the only means of testing and measuring the performance of different algorithms. The simulation has been implemented in MATLAB to match the signal models and allow the user to easily change parameters.
2.2 Benefits of Multichannel SAR

Compared to a single channel SAR, there are many benefits in using multiple channels such as improved gain and ambiguity suppression for imaging large swaths, the use of across-track interferometry to estimate height and form Three Dimensional (3D) elevation or topographic maps and the potential to suppress undesired interferences using techniques such as Space Time Adaptive Processing (STAP). The following sections cover a literature review of these benefits and describe some operational MSAR’s which exploit them.

2.2.1 Improved Gain

By looking at the radar range equation, [Younis et al., 2002a]-[Younis et al., 2003] have analysed many of the tradeoffs available to a MSAR. If there are $N$ antennas receiving, the total antenna area has increased, and consequently, the gain in the radar range equation will increase by $N$. If there are also $N$ antennas transmitting, the gain will increase by $N^2$. This improvement can be used to:

- **Improve the Signal to Noise Ratio (SNR)** This follows directly from the radar range equation.
- **Improve the azimuth resolution** If the antenna area for a single channel and a linear array were the same, then each element on the array would be smaller. Since the azimuth resolution limit is determined by the transmit antenna size, a smaller antenna results in a finer azimuth resolution.

2.2.2 Change the Pulse Repetition Frequency

The Pulse Repetition Frequency (PRF), $f_{PRF}$, is the key parameter in keeping range and azimuth ambiguities from affecting a SAR image. Range ambiguities occur when simultaneous returns from two pulses in flight occur. To avoid this, an upper limit for the PRF is determined with the assumption that the antenna illuminates only the range swath $2X_0$,

$$f_{PRF} \leq \frac{c}{4X_0}. \quad (2.1)$$

where $c$ is the speed of light. This limit is more commonly a problem in space-borne SAR due to the larger range swath size.

To avoid aliasing problems in the azimuth domain, the radar’s PRF must exceed the frequency which ensures that a stationary target’s two-way distance never changes by more than half a wavelength between samples. The PRF lower limit is given by

$$f_{PRF} \geq 2 \left( \frac{v_p \sin \left[ \frac{\phi_{3dB}}{2} \right]}{\lambda_c/2} \right) \approx \frac{2v_p \phi_{3dB}}{\lambda_c} \quad (2.2)$$
where $v_p$ is the along track platform speed, $\lambda_c$ is the wavelength at the carrier frequency and $\phi_{3\text{dB}}$ is the 3 dB beamwidth of the real antenna and is typically quite small.

If a single transmitter illuminates a wide swath and $N$ smaller antennas aligned in the along track direction record simultaneously the scattered signal from the illuminated footprint, the effective PRF is $N$ times greater than the actual PRF. This allows two modes of operation as described in Table 2.1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Actual PRF (1 channel)</th>
<th>Effective PRF ($N$ channels)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>$f_{\text{PRF}}$</td>
<td>$N f_{\text{PRF}}$</td>
</tr>
<tr>
<td>Mode 2</td>
<td>$\frac{1}{N} f_{\text{PRF}}$</td>
<td>$f_{\text{PRF}}$</td>
</tr>
</tbody>
</table>

Mode 1 implies that a greater along track extent can be imaged with reduced azimuth ambiguities (see Equation 2.2). However, the azimuth attenuation imposed by the antenna beampattern will limit the useable along track extent that may be generated from a block of range lines. More importantly, this mode can be exploited for Moving Target Indication (MTI) and interference rejection as the system moves in the along track direction and essentially makes $N$ measurements of the same static information.

Mode 2 allows for a reduction of the actual PRF by a factor of $N$ without increasing azimuth ambiguities. This reduction of the azimuth sampling rate becomes possible by a coherent combination of the individual receiver signals where the ambiguous parts of the Doppler spectra cancel each other. Note that such an ambiguity suppression can also be regarded as digital beamforming on receive where nulls in the joint antenna pattern are steered to the ambiguous zones, [Krieger et al., 2004].

This principle has been applied by [Ender, 2000] who looked at dividing up a linear phased array into azimuthal sub-apertures. Since a single sub-aperture has a broader receive beam compared to the whole antenna, a longer synthetic aperture is possible. The azimuth resolution of the array can then be decreased to half the sub-aperture length and since combinations of the sub-aperture signals can be used to form narrow beams, the PRF can be reduced without introducing azimuth ambiguities.

Since the 3 dB antenna beamwidth determines the finest azimuth resolution, different combinations of receive and transmit antennas will also affect the PRF. For a linear array, both [Ender, 2000] and [Younis et al., 2002b] have analysed the tradeoffs. In the latter study, it was found that to avoid azimuth ambiguities, the transmit antenna should be longer than the aperture of a single receiving antenna. [Callaghan and Longstaff, 1999] have done a similar analysis for a quad-element spaceborne array and [Goodman et al., 2002]-[Goodman and Stiles, 2003] for a constellation of spaceborne receivers.
2.2 Benefits of Multichannel SAR

2.2.3 Cross-track Interferometry

Since Graham’s seminal work on cross-track interferometry, [Graham, 1974], there has been a large amount of literature published. Interferometry is based on the measurement of the phase difference between two complex SAR images on a pixel by pixel basis. Estimating height is related to the absolute phase and is ambiguous due to the $2\pi$-ambiguity of the difference phase. Therefore, phase-unwrapping is very important in accurately estimating the height.

The relationship between the phase is usually explained by means of geometric approaches where the system bandwidth is so small that the system is considered monochromatic, [Gatelli et al., 1994]. This assumption is linked to the centre frequency and no longer holds if the reflectivity of a scatterer depends strongly on the frequency. There are many alternative methods for performing phase-unwrapping such as direct residue removing, [Kagawa and Hanaizumi, 2000], neural networks, [Schwartzkopf et al., 2000] and binary mixture modelisation of coherent images, [Abdelfattah et al., 2001].

If the number of antennas is increased, [Rössing and Ender, 2002] have developed a technique by combining the returns from more then one channel to make the problem of phase unwrapping obsolete. The height estimation problem has been formulated as a combination of SAR image generation and interferometry. The technique works by looking at the frequency and look-angle dependent differential phase to measure the height for each pixel independently. An earlier technique by the same authors, [Rössing and Ender, 2000] looked at the case where scatterers in a single azimuth-range bin contain two or more distinct elevation angles. A parametric technique is derived to reduce the ambiguity by estimating the angles simultaneously.

There are now many SARs which apply cross-track interferometry for height estimation. One well known experiment was NASA’s1 shuttle topography mission, [Werner, 2000]. They collected data in both the C-band and X-band to form digital tomographic maps of most of the world.

2.2.4 Spatial Degrees of Freedom

Exploiting the spatial degrees of freedom available in mode 1 of Table 2.1 provides three major benefits for a MSAR. It allows MTI for target tracking, compensation for moving targets in a SAR image and the ability to reject interferences while forming a SAR image. These applications are very tightly tied together with MTI only forming one of the three stages in moving target motion compensation for SAR.

The first stage involves the received SAR data going through two simultaneous stages. One of these is MTI and the other is image formation. The image formation step must reject the interferences in the data while preserving the ground clutter, while the MTI stage must reject the interferences and the ground clutter, while preserving the moving targets in the scene. The second stage takes the moving target information from the MTI block, estimates their position

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1 National Aeronautics and Space Administration
and velocity and then applies moving target compensation to the SAR image. The final SAR image will then have the moving targets at the correct location. An overview of this process is given in Figure 2.1 and as the techniques for both MTI and interference rejection during image formation are so closely related, a literature review of relevant MTI techniques for SAR is included in Chapter 4.

**Figure 2.1. Moving target compensation for SAR**

### 2.2.5 Operational Multichannel SAR Systems

This section provides details on a few operational MSAR’s which have exploited the benefits described in the previous sections. For a more complete list, see [Klemm, 2002].

**INGARA** This ‘Integrated Airborne Imaging Radar’ system is DSTO’s Australia’s dual channel polarimetric SAR capable of interferometry (see Figure 2.2). It operates at X-band with up to 600MHz bandwidth, [Stacy et al., 2003].

**AMSAR** This ‘Advanced Multichannel Synthetic Aperture Radar’ being developed by France, Germany and England in an aim to improve the ‘Euroradar’ which is being developed for the ‘Eurofighter’ and ‘Rafale’, [Eurofighter Typhoon Information Website, 2005].

**AER-II** The ‘Airborne Experimental Radar’ is FGAN’s first experimental MSAR, equipped with an X-band system with 1m x 1m resolution (see Figure 2.3). It has a fully polarimetric phased array allowing electronic azimuth beam-steering and four receiving channels. Its principle aim is the detection and positioning of slowly moving targets via multichannel algorithms such as STAP, [Ender, 1996]- [Ender, 1998b].

**PAMIR** The ‘Phased Array Multifunctional Imaging Radar’ is FGAN’s follow up system to the AER-II. It is designed to be an experimental X-band system which can image at very high resolutions, 10cm at a range of 30km in spotlight mode and 1m at a range of 100km. It has 256 receiver antennas which are formed into five channels to give good STAP performance for MTI, [Ender and Brenner, 2002].

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2. Defence Science and Technology Organisation
3. Forschungsgesellschaft für Angewandte Naturwissenschaften e.V. (Research Establishment for Applied Science)
2.2 Benefits of Multichannel SAR

**RAMSES** The ‘Regional earth observation Application for Mediterranean Sea Emergency Surveillance’ SAR was built by ONERA\(^4\) as an experimental test bench for radar imaging with high modularity and flexibility, [Dubois et al., 2002]. It can be configured with three bands picked among eight possible choices ranging from P-band to W-band. Each of these bands supports a four channel configuration using monopulse along-track interferometry for P to Ku-bands.

**DRA C-band** The Defence Research Agency in the UK have built an experimental C-Band MSAR. They use three antennas in an along track arrangement to test different MTI techniques, [Coe and White, 1996].

**CARABAS** The ‘Coherent All-Radio Band Sensor’ is the Swedish Defence Research Agency’s first MSAR system and uses interferometry to monitor forests. It operates in the VHF band between 20-90 MHz with HH-polarisation, [Franssoni et al., 2000].

**LORA** The ‘Low Frequency Airborne Synthetic Aperture Radar’ system is the Swedish Defence Research Agency’s second MSAR system. It works in the UHF band between 200-900 MHz and performs space/slow-time filtering with five channels. The aim is to detect moving objects on the ground, even when concealed by vegetation, [Ulander et al., 2003].

**AN/APG-76** Northrop-Grummen Aerospace have produced an operational Ku-band multi-mode radar system which has been used in Israel’s F-4 fighters, [Kopp, 1997]. This MSAR system is used for ground mapping, air search and track and air-to-ground targeting. Presently it is the only production system in the world to simultaneously produce high resolution radar imagery and perform MTI.

**APY-6** This X-band MSAR from Northrop-Grummen Aerospace is being put forward as a possibility to replace the F-111’s surveillance and possible attack radars, [Kopp, 1997]. The antenna is subdivided into a large panel for transmit and SAR receive, and three smaller panels for MTI receive. It can also be configured for either forward-looking or side-looking usage.

**RADARSAT-2** This is Canada’s latest fully polarimetric space-borne SAR. It is planned to launch in early 2006 and used primarily for scientific purposes such as interferometry and ground mapping from 3m resolution, [RADARSAT-2 Information Website, 2004].

**TERRASAR-X** Germany is also planning to launch an X-band fully polarimetric space-borne SAR in the last quarter of 2006 (see Figure 2.4). Its primary purpose is to create digital elevation maps down to a resolution of 1m, although it will also be used for thematic applications, [TERRASAR-X Information Website, 2005].

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\(^4\) Office National d’Etudes et de Recherches Aerospatiales (French Aeronautics and Space Research Center)
Figure 2.2. INGARA polarimetric SAR, image courtesy CSIRO Australia

Figure 2.3. AER-II side-looking MSAR, image courtesy FGAN Germany

Figure 2.4. TERRASAR-X satellite, image courtesy DLR Germany
2.3 Multichannel SAR Signal Model

The total MSAR signal model combines a number of different aspects which are explained in this section. Firstly, the 3D and projected 2D slant plane geometry is described with a number of definitions required for the signal model. The received signal model is then described in Section 2.3.2 with an absolute time variable before a pulse train is used to define slow-time and fast-time variables. Pulse compression using a chirp and the antenna beampattern are also introduced in this section.

In Section 2.3.3, both fast-time and slow-time sampling conditions are defined in terms of the respective range and azimuth resolutions. A general equation is derived for the slow-time sampling which can be applied to three different cases. These include an infinite SAR integration, an antenna beampattern limited case and an imaging area limited case. Range processing is then described in Section 2.3.4, followed in Section 2.3.5 by the multichannel extensions required for a MSAR simulation.

The block diagram in Figure 2.5 provides an overview of where the sections in this chapter fit into the overall SAR transmit and receive scheme. Note that the imaging algorithms are left to the following chapter.

![Figure 2.5. SAR transmit and receive block diagram](image-url)
2.3.1 Imaging Geometry

The imaging geometry is based on a 3D spherical coordinate system with a flat earth approximation. The SAR platform is travelling along the azimuth direction at a speed of $v_p$ m/s while transmitting radar pulses. The radar returns are reflected from the ground with a grazing angle, $\psi$ and measured in a 2D slant-plane relative to the true ground coordinates. Typically, the final image is formed by projecting onto the ground-plane, however to simplify the signal processing, the slant plane has used for the imaging geometry. This section introduces both geometries and shows how they are related using a radial unit vector.

3D Geometry

The 3D SAR geometry is based on the following figure:

Figure 2.6. 3D imaging geometry for stripmap SAR

where $(\bar{x}, \bar{y}, \bar{z})$ represent the ground-plane cartesian coordinate system for a flat earth. Relative to a point on the ground, $P$, $\bar{\theta}_a$ represents the ground-plane aspect angle, $\psi$ is the depression angle and $\vartheta$ is the elevation angle defined from the $\bar{z}$ axis. The SAR platform is located at a height $h_p$ above the ground-plane origin, travelling at $v_p$ m/s parallel to the $\bar{y}$ axis and $R(\cdot)$ is the radial distance from the SAR to the point $P$. The unit vector for the radial direction in terms of the cartesian ground unit vectors, $(\bar{x}, \bar{y}, \bar{z})$ is described by,
2.3 Multichannel SAR Signal Model

\[ \vec{R} = \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} = \begin{bmatrix} \cos \theta_a \sin \vartheta \\ \sin \theta_a \sin \vartheta \\ \cos \vartheta \end{bmatrix} \] (2.3)

and is commonly written in terms of a grazing angle \( \psi = \vartheta - \pi/2 \). In this case, the radial unit vector becomes,

\[ \vec{R} = \begin{bmatrix} \cos \theta_a \cos \psi \\ \sin \theta_a \cos \psi \\ -\sin \psi \end{bmatrix} \] (2.4)

As the backscatter is received at a grazing angle \( \psi \) relative to the ground, the three dimensional geometry can be projected to a two dimensional representation, known as the ‘slant-plane’,

\[ \begin{bmatrix} \vec{x} \\ \vec{y} \\ \vec{z} \end{bmatrix} \rightarrow \begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = \begin{bmatrix} \sqrt{\vec{x}^2 + \vec{z}^2} \\ \vec{y} \end{bmatrix} \] (2.5)

where the new coordinates \((x, y)\) are known as the slant-range and cross-range or azimuth. To avoid confusion, the slant-range will be known as range and the cross-range by azimuth during this thesis. The slant-plane radial unit vector is given by,

\[ \vec{R} = \begin{bmatrix} \cos \theta_a(x, y) \\ \sin \theta_a(x, y) \end{bmatrix} = \begin{bmatrix} \sqrt{\cos^2 \theta_a \cos^2 \psi + \sin^2 \psi} \\ \sin \theta_a \cos \psi \end{bmatrix} \] (2.6)

with the slant-plane aspect angle is defined by,

\[ \theta_a(x, y) = \arctan \left( \frac{y}{x} \right) \] (2.7)

and the radial distance by,

\[ R(x, y) = \sqrt{x^2 + y^2}. \] (2.8)

The origin of the slant-plane is related to the ground-plane by a translation of the SAR platform height, \( h_p \), in the \( \vec{z} \) axis. This ensures that the slant-plane origin is at the SAR position, while the ground-plane origin is below the SAR on the ground.

**Slant-plane Geometry**

If the 3D geometry is projected onto the slant-plane, the imaging scenario for a stripmap mode can be visualised from a top-down 2D view as shown in Figure 2.7.

The imaging area is defined with range \( x \in [X_c - X_0, X_c + X_0] \) and azimuth \( y \in [-Y_0, Y_0] \) with the SAR platform position offset at \((0, u)\). The physical aperture of the radar antenna, \( D_y \) determines the 3 dB beamwidth of the antenna \( \phi_{3dB} \), the radar ‘field of view’ and hence the maximum antenna along track extent is given by \( 2Y_{W,max} \). Other important parameters are the SAR integration length, \( 2Y_L \), which is defined by the distance travelled as the radar beam
crosses the along track extent, $2Y_0$ and the angle $\phi$ is the SAR integration angle defined relative to the far offset range, $X_c + X_0$. The synthetic aperture length is a function of range defined to maintain a fixed azimuth resolution. At the far range, it is defined by $2Y_S$ and is related to the SAR integration length by,

$$Y_L = Y_S + Y_0.$$  \hspace{1cm} (2.9)

### 2.3.2 Signal Model

This section describes the model of the transmit and receive radar signal. For a stationary SAR platform, the isotropic transmitted signal is given by,

$$p_t(\tilde{t}) = \sqrt{P_t} \mathcal{R} \{ p_c(\tilde{t}) \exp[j\omega_c \tilde{t}] \}$$  \hspace{1cm} (2.10)

where $\mathcal{R}\{\cdot\}$ indicates the real part of a signal, $p_c(\cdot)$ is the SAR signal waveform, $\tilde{t}$ is the absolute time, $\omega_c$ (rad/s) is the carrier frequency and $P_t$ is the transmitted power. In the direction of the scatterer located at $(x, y)$, the transmitted SAR signal becomes

$$p_t(\tilde{t}, x, y) = \sqrt{P_t a(x, y)} \mathcal{R} \{ p_c(\tilde{t}) \exp[j\omega_c \tilde{t}] \}$$  \hspace{1cm} (2.11)

where $a(x, y)$ is the two-way antenna beampattern at the carrier frequency. As the SAR moves along, it transmits a series of pulses called a ‘pulse train’ at the PRF. There are $M$ pulses in total and are separated in time by a Pulse Repetition Interval (PRI), $T_{PRI} = 1/f_{PRF}$. As shown in Section 2.2.2, the PRF must be chosen carefully so ambiguities in range (fast-time) and aliasing problems in the azimuth domain do not occur.

The received SAR signal model is defined with the SAR platform at the slant-plane origin, and ignoring curvature of the earth. Using the ‘stop-start’ approximation, the SAR platform travels
2.3 Multichannel SAR Signal Model

a distance \( \Delta_y = v_y T_{PRI} \) between pulses, giving it a relative ‘slow-time’ position \( u_m = m \Delta_y \) for the \( m^{th} \) pulse. Within each pulse the SAR platform is assumed to be stationary with the received pulse sampled \( L \) times, giving a relative ‘fast-time’, \( t_l \) for the \( l^{th} \) sample. With these definitions, the received SAR signal model after quadrature demodulation can be written in terms of the fast-time and SAR position,

\[
x_{pre}(t_l, u_m) = \gamma_{pre}(t_l, u_m) + \nu(t_l, u_m)
\]

where \( \nu(\cdot) \) represents the Gaussian system noise and the total ground return, \( \gamma_{pre}(\cdot) \) is defined as the integral over the SAR imaging area,

\[
\gamma_{pre}(t_l, u_m) = \int_y \int_x a(x, y - u_m)f(x, y)p_c(t_l - \tau(x, y - u_m)) \exp[-j\omega_c \tau(x, y - u_m)] dxdy
\]

where \( f(x, y) \) is the ground return or Radar Cross Section (RCS) of a scatterer and is scaled according to the radar range equation, [Curlander and McDonough, 1991]. The two-way delay for a scatterer at position \((x, y)\) is given by,

\[
\tau(x, y) = \frac{2R(x, y)}{c}
\]

with the received signal model defined as,

\[
s_{pre}(t_l, u_m, x, y) = p_c(t_l - \tau(x, y - u_m)) \exp[-j\omega_c \tau(x, y - u_m)]
\]

For ease of notation, the subscripts \( m \) and \( l \) will often be dropped during this thesis.

**Pulse Compression**

To obtain a fine range resolution, coded ‘pulse compression’ is used to increase the pulse-time/bandwidth product, \( BT_p \). Three common implementations include linear Frequency Modulation (FM), non-linear FM and phase-coding. Linear FM is typically used in SAR, as it is suitable for wide bandwidths and is easy to generate. It is also known as a ‘chirp’,

\[
p_c(t) = b\left(\frac{t}{T_p}\right) \exp[-jB\pi t + j2\alpha t^2]
\]

where \( 2\alpha \) (rad/s) is the chirp rate with duration defined by \( b(t) \) which is unity for \( 0 \leq t \leq 1 \) and zero otherwise. It is offset by half the bandwidth to ensure it is centred around the origin in the time/frequency domain. The observable bandwidth then becomes \( B = \alpha T_p / \pi \) (Hz) instead of just \( 1/T_p \). With this form of pulse compression, the received signal model in Equation 2.15 becomes,

\[
s_{pre}(t, u, x, y) = b\left(\frac{t - \tau(x, y - u)}{T_p}\right) \exp[-j\omega_c \tau(x, y - u) - jB\pi(t - \tau(x, y - u)) + j\alpha(t - \tau(x, y - u))^2].
\]
Chapter 2

Multichannel SAR Background

Antenna Beampattern

The SAR platform is travelling along the azimuth direction with its antenna in the side-looking orientation. It is also assumed that the transmit and receive antennas are the same. For an aperture planar antenna of length $D_y$, located at the centre of the synthetic aperture ($u = 0$), the two-way antenna beampattern in the azimuth spatial frequency domain is [Soumekh, 1999],

$$A(k_y) = D_y^2 \text{sinc}^2 \left( \frac{D_y k_y}{4\pi} \right)$$

(2.18)

with the two-way antenna beampattern for a scatterer at $(x, y)$ determined by the following relationship\(^{5}\) between the azimuth spatial frequency and fast-time frequency domains,

$$\tilde{a}(\omega, x, y) = \frac{1}{R^2(x, y)} A \left[ 2k \sin \left[ \theta_a(x, y) \right] \right]$$

(2.19)

where the range scaling is due to the two-way distance of the beampattern and the wavenumber, $k = \omega/c$. With this definition, the two-way antenna beampattern at fast-time frequency $\omega$ is

$$\tilde{a}(\omega, x, y) = \begin{cases} \frac{D_y^2}{R^2(x, y)} \text{sinc}^2 \left( \frac{D_y k}{2\pi} \sin \left[ \theta_a(x, y) \right] \right), & \theta_a(x, y) \leq \frac{\phi_{3dB}}{2} \\ 0, & \text{otherwise} \end{cases}$$

(2.20)

where the 3 dB beamwidth, $\phi_{3dB}$ determines the ‘radar field of view’ and the sidelobes are set to zero as shown in Figure 2.7. For an aperture planar radar, the 3 dB beamwidth is approximated by its one way beampattern,

$$\phi_{3dB} \approx \arcsin \left( \frac{\lambda}{D_y} \right)$$

(2.21)

with the along track extent of the antenna beam, $2Y_W(\cdot)$ determined at the antenna 3 dB positions by,

$$Y_W(x) = x \tan \left[ \frac{\phi_{3dB}}{2} \right].$$

(2.22)

For the signal model in Equation 2.13, the two-way antenna beampattern is defined at the carrier frequency, $a(x, y) = \tilde{a}(\omega_c, x, y)$.

2.3.3 Resolution and Sampling

The method of sampling will play an important role in the different STAP algorithms described in later chapters. After basebanding the received SAR signal for each pulse, it is sampled at a rate determined by the bandwidth. In Section 2.3.2, the absolute time variable was represented as the sum of slow-time and fast-time, where slow-time is concerned with samples from pulse to pulse, and fast-time by the samples within a pulse.

When sampling a return, it important to know when the SAR should expect an echo. This is calculated by the minimum and maximum radial distances expected from the imaging area.

\(^{5}\)It should be noted that these relationships are not Fourier Transform pairs.
2.3 Multichannel SAR Signal Model

Using the definitions in Figure 2.7:

\[ R_{\text{min}} = X_c - X_0 \]
\[ R_{\text{max}} = \sqrt{(X_c + X_0)^2 + Y_{W,\text{max}}^2} \]  \hspace{1cm} (2.23)

where the maximum antenna along track extent, \( 2Y_{W,\text{max}} \) is determined by

\[ Y_{W,\text{max}} = (X_c + X_0) \tan \left[ \frac{\phi_2 dB}{2} \right] . \]  \hspace{1cm} (2.24)

Using these definitions, the return from the closest scatterer occurs at

\[ T_s = \frac{2R_{\text{min}}}{c} \]  \hspace{1cm} (2.25)

and from the farthest reflector at

\[ T_f = \frac{2R_{\text{max}}}{c} + T_p \]  \hspace{1cm} (2.26)

giving the total length of time required for fast-time sampling as,

\[ T = T_f - T_s . \]  \hspace{1cm} (2.27)

Figure 2.8. Pulse timing

Fast-time

Range and azimuth resolutions are defined by the equivalent data extent in the spatial frequency domains, \((k_x, k_y)\) which are in turn related to measured frequency domain \((\omega, k_u)\) by,

\[ k_x = \frac{4k^2 - k_u^2}{2} \]
\[ k_y = k_u . \]  \hspace{1cm} (2.28)

The relationship in \(k_x\) is due to the two-way return of a curved wavefront and has been studied extensively by [Rocca et al., 1989] and [Soumekh, 1994]. The range resolution is dependent on the two-sided system bandwidth, \(B\) and can be approximated by twice the distance between the minimum and maximum values of the spatial frequency range domain, \(k_{x,\text{min}}\) to \(k_{x,\text{max}}\). These
define the range mainlobe width of the power spectrum and are found by setting $k_u = 0$ in Equation 2.28 giving,

$$B_x = k_{x,\text{max}} - k_{x,\text{min}}$$

$$= 2 \left( \frac{w_c + B\pi}{c} - \frac{w_c - B\pi}{c} \right)$$

$$= \frac{4B\pi}{c}. \quad (2.29)$$

In the range domain, the resolution is typically measured at the 3 dB points,

$$\Delta X \approx \frac{2\pi}{B_x} = \frac{c}{2B}. \quad (2.30)$$

The range sampling rate can be determined using the Nyquist sampling criteria for complex samples\(^6\) which ensures there are no overlapping components from aliased copies of the signal,

$$\Delta_x \leq \Delta X = \frac{c}{2B}. \quad (2.31)$$

Correspondingly, the sampling rate in the fast-time domain can be determined by the relationship $\Delta_t = 2\Delta_x/c$, giving

$$\Delta_t \leq \frac{1}{B}. \quad (2.32)$$

From Figure 2.8, the received fast-time signal needs to be sampled between the nearest and farthest expected return,

$$t \in [T_s, T_f] \quad (2.33)$$

with the $l^{th}$ fast-time sample located at,

$$t_l = T_s + (l - 1)\Delta_t, \quad l = 1 \ldots L \quad (2.34)$$

and the required number of samples, $L$ is determined by the fast-time sample rate, $\Delta_t$ and the total time required for fast-time processing, $T$,

$$L = 2^\lceil \frac{T}{2\Delta_t} \rceil \quad (2.35)$$

where $\lceil \cdot \rceil$ is the ceiling function. This variable is made an even number for future processing with Fast Fourier Transforms (FFT).

### Slow-time

For slow-time, the azimuth resolution is determined by the Doppler bandwidth in the $k_y$ domain. A general expression for the spatial frequency azimuth domain can then be given in terms of an angular limit $\theta_{\text{lim}}$, which will depend on either the antenna beamwidth or the synthetic aperture.

\(^6\)Over-sampling in the fast-time domain is used for adaptive processing in later chapters. This does not have an effect on the spatial resolution.
2.3 Multichannel SAR Signal Model

as described below. The azimuth spatial frequency mainlobe is determined by the distance between the limits $k_{y,min}$ and $k_{y,max}$ of the power spectrum giving,

$$B_y = k_{y,max} - k_{y,min} = 4k \sin \left[ \frac{\theta_{lim}}{2} \right]$$

(2.36)

and is a function of frequency. The ‘mean’ azimuth resolution is then determined by setting the frequency to the carrier frequency and measuring the 3 dB points,

$$\Delta_Y \approx \frac{2\pi}{B_y} = \frac{\lambda_c}{4 \sin \left[ \frac{\theta_{lim}}{2} \right]}$$

(2.37)

where $\lambda_c$ is the wavelength at the carrier frequency. Nyquist’s sampling criteria can then be applied to determine the slow-time sampling rate,

$$\Delta_y \leq \Delta_Y$$

(2.38)

with the required number of pulses determined by the sampling rate and the length of the SAR integration, $2Y_L$. Similar to the fast-time, the number of pulses is rounded to the greatest even number,

$$M = 2^r \left( \frac{Y_L}{\Delta_Y} \right)$$

(2.39)

The ‘ideal’ azimuth resolution is given by an infinite SAR synthetic aperture which corresponds to a radar field of view of $\pi$ (rad). In practice, the best azimuth resolution achievable for stripmap SAR is determined by the 3 dB beamwidth of the antenna beam covering the imaging area, $\phi_{3dB}$. However if the imaging area is small, it may be inside the antenna pattern for the duration of the CPI and the resolution will instead depend on the SAR integration angle, $\varphi < \phi_{3dB}$. The following section provides a comparison of these three different cases and the resultant synthetic aperture, angular limit and azimuth resolutions.

**CASE 1 - Infinite synthetic aperture** In this unachievable case, the synthetic aperture is infinite and has $\theta_{lim} = \pi$. The corresponding azimuth resolution becomes,

$$\Delta_Y = \frac{\lambda_c}{4}$$

(2.40)

**CASE 2 - Beampattern Limited** If the antenna beamwidth from Figure 2.7 is treated as a cone shaped region moving over the imaging area, the synthetic aperture, $2Y_S$ can be determined by the antenna beamwidth at the closest range, $X_c - X_0$ as it moves across a point in the along track extent,

$$Y_S = Y_{W,min}$$

(2.41)

where the minimum antenna along track extent, $2Y_{W,min}$ was determined by

$$Y_{W,min} = (X_c - X_0) \tan \left[ \frac{\phi_{3dB}}{2} \right].$$

(2.42)
To determine the azimuth resolution, the angular limit is twice the 3 dB beamwidth, $\theta_{\text{lim}} = \phi_{3dB}$. Substituting this into Equation 2.37 gives,

$$\Delta Y = \frac{\lambda_c}{4 \sin \left[ \frac{\phi_{3dB}}{2} \right]}$$

$$= \frac{D_y}{2} \quad (2.43)$$

where the resolution is independent of range and wavelength and only determined by the real antenna length $D_y$.

**CASE 3 - Imaging Area Limited** In this case, the synthetic aperture can be chosen to match a desired azimuth resolution and the large computational burden of case 2 can be avoided. If the imaging area is less than half of the antenna field of view,

$$Y_{W,\text{min}} \geq 2Y_0 \quad (2.44)$$

then the SAR integration angle is less than the 3 dB antenna beamwidth,

$$\varphi < \phi_{3dB} \quad (2.45)$$

and is determined by the synthetic aperture at the far range,

$$\varphi = 2 \arctan \left[ \frac{Y_S}{X_c + X_0} \right]. \quad (2.46)$$

Figure 2.9 shows a top view of this scenario.

![Figure 2.9. Imaging geometry for the imaging area limited case](image)

To establish bounds on the size of the imaging area, substitute this equation into the constraint condition of Equation 2.45.

$$2 \arctan \left[ \frac{Y_S}{X_c + X_0} \right] < \phi_{3dB}$$

$$\Rightarrow \quad \frac{Y_S}{X_c + X_0} < \tan \left[ \frac{\phi_{3dB}}{2} \right]. \quad (2.47)$$
Then using the definition of the maximum azimuth along track extent in Equation 2.24,\[ Y_S < Y_{W,\text{max}}. \] (2.48)

Finally, by using Equations 2.24 and 2.42, the relationship between the minimum and maximum azimuth along track extent can be shown to be,\[ Y_{W,\text{max}} = \left( \frac{X_c + X_0}{X_c - X_0} \right) Y_{W,\text{min}} \] (2.49)
and substituting this relationship into Equation 2.48, gives the constraint with the tightest limit from Equation 2.44 and entirely in terms of the imaging geometry,\[ Y_S < \left( \frac{X_c + X_0}{X_c - X_0} \right) 2Y_0. \] (2.50)

The azimuth resolution is determined by setting the angular limit \( \theta_{\text{lim}} = \varphi \) in Equation 2.37,\[ \Delta Y = \frac{\lambda_c}{4 \sin \left[ \frac{\varphi}{2} \right]} \approx \frac{\lambda_c}{2 \varphi} \approx \frac{\lambda_c (X_c + X_0)}{4Y_0}. \] (2.51)

Thus providing a direct link between the azimuth resolution and the synthetic aperture.

Table 2.2 shows a comparison of the synthetic aperture, angular limit and azimuth resolution for these three cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>( 2Y_S )</th>
<th>( \theta_{\text{lim}} )</th>
<th>( \Delta Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>( \infty )</td>
<td>( \pi )</td>
<td>( \frac{\lambda_c}{4} )</td>
</tr>
<tr>
<td>Case 2</td>
<td>( 2Y_{W,\text{min}} )</td>
<td>( \phi_{\text{dB}} )</td>
<td>( \frac{D_y}{2} )</td>
</tr>
<tr>
<td>Case 3</td>
<td>( \frac{(X_c + X_0)}{(X_c - X_0)} 4Y_0 )</td>
<td>( \varphi )</td>
<td>( \frac{\lambda_c (X_c + X_0)}{2\varphi} \approx \frac{\lambda_c}{4Y_S} )</td>
</tr>
</tbody>
</table>

### 2.3.4 Range Compression

After the SAR signal has been sampled, range compression must be applied to removed the coding in the SAR signal waveform defined in Section 2.3.2. This will also focus the target signal in range with the observable bandwidth defined by the chirp. There are a number of approaches to this problem, such as the deramp-FFT, deconvolution or Matched Filter (MF), [Munson-Jr., 1987].
The MF is formed as a convolution between the received signal and a reference signal defined by, \( s_{\text{MF}}(t) = p_c^*(-t) \),

\[
s_{\text{post}}(t,u,x,y) = \int_{-\infty}^{\infty} s_{\text{pre}}(t',u,x,y) s_{\text{MF}}(t-t') \, dt'.
\]

(2.52)

Using the pre range processing signal model in Equation 2.17, this becomes

\[
s_{\text{post}}(t,u,x,y) = \exp \left[ -j\omega_c\tau(x,y-u) \right] \int_{-\infty}^{\infty} p_c(t'-\tau(x,y-u)) p_c^*(t'-t) \, dt'.
\]

(2.53)

To evaluate this convolution in the frequency domain, Parseval’s Theorem can be used. At baseband, the fast-time frequency is defined as \( \tilde{\omega} \),

\[
s_{\text{post}}(t,u,x,y) = \exp \left[ -j\omega_c\tau(x,y-u) \right] \int_{-\infty}^{\infty} |P_c(\tilde{\omega})|^2 \exp [j\tilde{\omega}(t-\tau(x,y-u))] \, d\tilde{\omega}.
\]

(2.54)

Then integrating over the bandwidth, \(-B\pi \leq \tilde{\omega} \leq B\pi\) and assuming \( |P_c(\tilde{\omega})| = 1 \),

\[
s_{\text{post}}(t,u,x,y) = \pi B \exp \left[ -j\omega_c\tau(x,y-u) \right] \int_{-\pi B}^{\pi B} \exp [j\tilde{\omega}(t-\tau(x,y-u))] \, d\tilde{\omega}.
\]

(2.55)

where \( \text{sinc}[x] \equiv \sin[\pi x]/\pi x \). This signal model implies that each target response will be a sinc function modulated by an exponential with phase related to the round trip propagation delay.

After range compression, the fast-time domain becomes compressed, with the the total number of available samples \( n_x \), determined by the length of the transmitted pulse, \( L_p = \left\lceil T_p/\Delta t \right\rceil \),

\[
n_x = L - L_p + 1.
\]

(2.56)

The practical implementation of this filter is described in Section 2.4.4.

### 2.3.5 Multiple Channels

Both across track and along track interferometry require a large separation or baseline between antenna channels to provide a sufficiently different view of the imaged area. This enables them to measure height differences or detect moving targets as desired. From an array processing perspective however, a large baseline will provide a very small central beam at the expense of grating lobes and adaptive beamforming techniques will not be very successful.

To address this problem, both the AER-II and PAMIR MSAR’s use a phased array design, with a large number of array elements or antennas with half wavelength separation. To reduce the difficulties involved in sampling and processing each array element, these are summed together into a smaller number of sub-arrays or channels for further processing. By themselves, each channel will have a very tight beampattern with no grating lobes, while the array of channels will have a much larger separation with grating lobes. When combined however, the nulls of each channel will overlap and the effect of the grating lobes will be minimal.
2.3 Multichannel SAR Signal Model

A linear array is typically used over other array configurations to enable the narrowest mainbeam in the along track direction. This will result in the best clutter suppression without compromising slow moving targets. Also, when two or more elements are closely spaced in an array, mutual coupling can be a problem. This stems from mismatch between the antenna element and free space. While ideal matching can easily be accomplished for a non-steering array, phased arrays with steering will always have some degree of mismatch or mutual coupling. This mismatch can cause spurious lobes at some steering angles and ultimately limit the extent of steering. Due to the focus on stripmap SAR, it is assumed that the effect of mutual coupling is negligible throughout this thesis. Similarly, to simplify the interactions between the different array elements and sub-array’s, a single linear array is modelled with elements separated by a distance, $\delta$. Each array element is modelled as a horizontal dipole of physical length $D_y$, with the offset for the $n$th antenna defined as $d_n = n\delta$, with the phase centre in the middle of the array and $n \in \left[-(N-1)/2, (N-1)/2\right]$ for $N$ (odd) channels.

![Figure 2.10. Antenna placement](image)

From an array processing perspective, the backscattered signal is assumed narrowband, $c/B \gg N\delta$ [Mailloux, 1994], and the propagation delay across the array can be modelled as a phase shift. Therefore, if either the reference or all the antennas transmit simultaneously, the reflected signal incident on the array will contain an extra one-way spatial phase delay calculated as the time difference between the $n$th and antenna phase centre,

$$\tilde{\tau}_n(x, y - u) = \frac{1}{c} \left[ R(x, y - u - d_n) - R(x, y - u) \right]. \quad (2.57)$$

Then for the case of a transmitter located at the phase centre of the array, the existing temporal delay can be combined forming the space-time delay,

$$\tau_n(x, y - u) = \frac{2}{c} R(x, y - u) + \frac{1}{c} \left[ R(x, y - u - d_n) - R(x, y - u) \right]$$

$$= \frac{1}{c} \left[ R(x, y - u - d_n) + R(x, y - u) \right]. \quad (2.58)$$

The total ground return model for the $n$th channel then becomes,

$$\gamma_{\text{pre},n}(t, u) = \int_y \int_x a(x, y - u) f_n(x, y) s_{\text{pre},n}(t, u, x, y) dxdy \quad (2.59)$$

where $f_n(x, y)$ is the equivalent ground return for the $n$th channel and the pre range processing signal model is given by,

$$s_{\text{pre},n}(t, u, x, y) = p_c(t - \tau_n(x, y - u)) \exp \left[-j\omega_c \tau_n(x, y - u) \right]. \quad (2.60)$$
After range processing the ground return signal model becomes,

\[
\gamma_{\text{post}, n}(t, u) = \int_y \int_x a(x, y - u) f_n(x, y) s_{\text{post}, n}(t, u, x, y) \, dx \, dy
\]  

(2.61)

with the normalised post range processing signal model given by,

\[
s_{\text{post}, n}(t, u, x, y) = \text{sinc} \left[ B(t - \tau_n(x, y - u)) \right] \exp \left[ -j\omega_c \tau_n(x, y - u) \right].
\]  

(2.62)

Antenna Beampattern

For a dipole, the two-way antenna beampattern at the centre of the synthetic aperture \((u = 0)\), is defined by [Balanis, 1997],

\[
\tilde{a}(\omega, x, y) = \eta \frac{|I_0|^2}{8\pi^2} \left[ \frac{\cos \left[ \frac{k c}{2} \cos \left[ \theta_\alpha(x, y) \right] \right] - \cos \left[ \frac{k c}{2} \right]}{\sin \left[ \theta_\alpha(x, y) \right]} \right]^2
\]  

(2.63)

where \(I_0\) and \(\eta\) are constants. The 3 dB beamwidth is given by

\[
\phi_{3dB} = \pi - 2\theta'_\alpha(x, y)
\]  

(2.64)

where \(\theta'_\alpha(x, y)\) is determined when the normalised beampattern is equated to 0.5.

\[
\frac{\tilde{a}(\omega, x, y)}{\max(\tilde{a}(\omega, x, y))} = 0.5.
\]  

(2.65)

Within the received signal models in Equations 2.59 and 2.61, the two-way antenna beampattern is defined at the carrier frequency, \(a(x, y) = \tilde{a}(\omega_c, x, y)\) and it is also assumed that the two-way antenna beampattern for each element will be approximately identical due to their close proximity relative to the offset range, \(X_c\).

Spatial Steering Vector

The spatial steering vector is defined by the phase difference between the \(n^{th}\) antenna and the antenna phase centre for a scatterer at \((x, y)\). However, due to the large offset centre range, \(X_c \gg X_0\), the spatial steering can be approximated by a plane wave using the scatterer position at the centre of the imaging area, \((X_c, 0)\). The signal model is then formed using the spatial time delay from Equation 2.57,

\[
s_n(u) = \exp \left[ -j k_c (R(X_c, u + d_n) - R(X_c, u)) \right]
\]  

(2.66)

where \(k_c\) is the wavenumber at the carrier frequency and the phase term can be broken down using the second order binomial expression,

\[
\phi_n(u) = -k_c \left( \sqrt{X_c^2 + (u + d_n)^2} - \sqrt{X_c^2 + u^2} \right)
\approx -k_c \left( X_c + \frac{(u + d_n)^2}{2X_c} - X_c - \frac{u^2}{2X_c} \right)
= -k_c \left( \frac{u^2 + 2d_n u + d_n^2}{2X_c} - \frac{u^2}{2X_c} \right)
\approx k_c d_n \left( \frac{-u}{X_c} \right)
\]  

(2.67)
where it is assumed that \( \frac{d_n^2}{2X_c} \approx 0 \). Now, the aspect angle relative to the centre of the patch is given by,

\[
\theta(u) \equiv \theta_a(X_c, -u) = \arctan \left( \frac{-u}{X_c} \right)
\]  
(2.68)

and if \( \sin[\theta(u)] \approx -u/X_c \), the phase from Equation 2.67 can be written as,

\[
\phi_n(u) \approx k_c d_n \sin[\theta(u)]
\]  
(2.69)

giving the spatial reference signal,

\[
s_n(u) = \exp[jk_c d_n \sin[\theta(u)]]
\]  
(2.70)

This signal model can then be stacked to form the spatial steering vector,

\[
s(u) = [s_{-\lfloor(N-1)/2\rfloor}(u), \ldots, s_{\lfloor(N-1)/2\rfloor}(u)]^T \in \mathbb{C}^{N \times 1}
\]  
(2.71)

and used to approximate Equations 2.60 and 2.62 by separating the spatial and temporal components. This also requires the assumption that the carrier frequency is much larger than the bandwidth,

\[
\begin{align*}
    s_{\text{pre},n}(t, u, x, y) &\approx s_n(u)s_{\text{pre}}(t, u, x, y) \\
    s_{\text{post},n}(t, u, x, y) &\approx s_n(u)s_{\text{post}}(t, u, x, y).
\end{align*}
\]  
(2.72)

## 2.4 Multichannel SAR Simulation

The multichannel SAR simulation is performed using MATLAB and is the only method for evaluating the research in this thesis as real data was unavailable. A block diagram demonstrating how the simulation was performed is shown in Figure 2.11.

The first two blocks are responsible for setting up the SAR parameters and the environment for the simulation. The MSAR simulation determines the received SAR signal from both point scatterers and receiver noise. The total received signal is then range processed for each individual channel before being combined in the image formation algorithm.

### 2.4.1 Parameter Choices

The first simulation block sets up the main SAR parameters which were explained in the previous section. Since many of these are related, the user only needs to set a few parameters to define a simulation method. Table 2.3 provides a list of common simulation values.

The user defined parameters relate to the imaging area limited case described in Section 2.3.3. They include the azimuth resolution, \( \Delta Y \), number of range bins, \( n_x \) and the centre range, \( X_c \). The imaging area is determined by an imported image with size, \( (I_x, I_y) \) and is given by \( 2X_0 = \Delta Y I_x \) and \( 2Y_0 = \Delta I_y \). Often these choices contain too few samples to produce...
Table 2.3. Common simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>$f_c$</td>
<td>10 GHz</td>
</tr>
<tr>
<td>System bandwidth</td>
<td>$B$</td>
<td>300 MHz</td>
</tr>
<tr>
<td>Pulse Repetition Frequency</td>
<td>$f_{PRF}$</td>
<td>3 KHz</td>
</tr>
<tr>
<td>SAR platform velocity</td>
<td>$v_p$</td>
<td>200 m/s</td>
</tr>
<tr>
<td>Number of antenna elements</td>
<td>N</td>
<td>5</td>
</tr>
<tr>
<td>Element spacing</td>
<td>$\delta$</td>
<td>$\lambda_c/2$</td>
</tr>
<tr>
<td>Antenna length</td>
<td>$D_y$</td>
<td>0.99$\delta$</td>
</tr>
</tbody>
</table>

A reasonable image, so an option to increase azimuth sampling rate before the simulation is included to improve the final image quality. The parameter, $M_{\text{max}}$, determines how many slow-time samples are required in the imaging area and the azimuth sample rate, $\Delta_y$, is adjusted by the amount, $P_{\text{over}} = M_{\text{max}} / M$ without effecting the azimuth resolution. These choices are explained below in Figure 2.12.

2.4.2 Environment

The environment block defines the positions of the point scatterers on the ground, the jamming signals and receiver noise. The point scatterer positions can be defined manually or they can be read from an imported image as described in the previous section. The jamming signals are created from an airborne jammer model with multipath reflections using a diffuse scattering model described in Chapter 5.
2.4 Multichannel SAR Simulation

Choose:
- Number of range bins, \( n_x \)
- Centre range, \( X_c \)
- Azimuth resolution, \( \Delta_y \)

Imported image, size: \( (I_x, I_y) \)

\[
2X_0 = \Delta_x I_x, 2Y_0 = \Delta_y I_y
\]

Increase no. of pulses, \( M \rightarrow M_{\text{max}} \)

Increase sampling rate, \( \uparrow P_{\text{over}} \)

\[
\Delta_y = \frac{\Delta_y}{2P_{\text{over}}} \quad \Delta_y = \frac{\Delta_y}{2}
\]

**Figure 2.12. Parameter choice for SAR simulation**

For the simulations in this thesis, two different point scatterer scenarios are considered. They include a single scatterer used to determine the Point Spread Function (PSF) of an imaging algorithm, and an imported ‘S’ image for comparisons between algorithms. The location of the point scatterers for scenarios are shown in Figure 2.13.

**Figure 2.13. Point scatterer scenarios**

2.4.3 Simulation

The MSAR simulation simulates the returns from both the point scatterers and any jamming signals. It is performed at baseband and the SAR platform position is assumed to be perfectly known, making any autofocus techniques unnecessary. If there are \( K_{gc} \) patches on the ground representing the point scatterers, the total ground return signal is represented by a discrete version of Equation 2.59.

\[
\gamma_{pre,n}(t, u) = \sum_{k=1}^{K_{gc}} a(x_k, y_k)f_n(x_k, y_k)s_{pre,n}(t, u, x_k, y_k)
\]  

(2.73)
where \(f_n(\cdot)\) is the ground clutter magnitude. The jammer components are made up of the direct-path \(z_{dp,n}(\cdot)\) and ground reflected path (hot-clutter) \(z_{hc,n}(\cdot)\). These signals with the addition of receiver system noise \(\nu(\cdot)\), form the components of the received SAR data waveform.\(^7\)

\[
x_{pre,n}(t, u) = \gamma_{pre,n}(t, u) + z_{dp,n}(t, u) + z_{hc,n}(t, u) + \nu_n(t, u).
\] (2.74)

### 2.4.4 Range Processing

Range processing occurs directly following the MSAR simulation. It is implemented at base-band for each channel in the frequency domain as explained in Section 2.3.4. Prior to Fourier transforming, the baseband reference signal, \(s_{MF}(\cdot)\) is created with length \(L_p\). It is then zero padded to be the same length as the input signal so the convolution can be implemented in the frequency domain. The post range processing data signal, \(x_{post,n}(\cdot)\) is then obtained by Inverse Fourier transforming the result and trimming the output signal to be length \(n_x\). This removes the \(L_p - 1\) partially correlated samples from the convolution where the replica slides off the end of the input vector. Figure 2.14 shows a block diagram of this implementation.

![Range processing block diagram](image)

**Figure 2.14.** Range processing block diagram

### 2.4.5 Image Formation

Image formation is implemented to match the quadratic return from the moving SAR platform. Its goal is to extract the ground returns \(f_n(x, y)\) from the point scatterers for each channel.

\(^7\)Note: If there were moving targets in the scene, these would be additional components.
2.5 Conclusion

Multichannel implementations of several different imaging algorithms are explained in the following chapter. Images of both point scatterer scenarios are shown in Figure 2.15. Note that these images are the magnitude squared of the processed data and have a dB scale.

![Point scatterer images](image)

**Figure 2.15.** Point scatterer images

2.5 Conclusion

This chapter has described how combining multiple antennas with SAR can provide a number of benefits including improved gain, ambiguity reduction, the ability for height estimation and the use of spatial degrees of freedom. A detailed signal processing model was introduced for the imaging geometry, the SAR signal model, resolution and sampling, range compression and the extension to multiple channels. Each antenna element was modelled as a dipole with a linear array configuration chosen for superior mainbeam clutter suppression. These models were then combined into a multichannel SAR simulation to allow the user to easily test different parameters.
3.1 Introduction

Image formation comprises the final stage in the Synthetic Aperture Radar (SAR) processing chain. This step takes the range processed data and performs coherent processing to recover the azimuth focussed image of a scene. For an array of receivers in a Multichannel SAR (MSAR), coherent processing must also take into account the phase delays of each element. MSAR’s are typically formed with phased arrays and combine a large number of elements for both transmit and receive. Groups of these elements are then summed to form a smaller number of channels for further processing. Depending on the required usage, a MSAR can be configured to use either the entire array for transmit/receive or a single transmit with multiple receive channels. In the former case, each of the antenna elements will transmit in phase and assuming the receive signal is in the far field, the delay at the received antenna array is approximately identical to the single transmit channel case.

There are a number of potentially different schemes for combining the returns from multiple receivers. Some of these include: forming an image from each receiver and combining them together, compensating for each receiver delay within the imaging algorithm or a combination of these. This chapter is focussed on the second scheme where the spatial delays are incorporated into a Matched Filter (MF) imaging formulation.

While MSAR imaging has been briefly addressed by [Ender, 1998c], [Ender, 2003], there has been no comprehensive study on how different imaging algorithms can be implemented with multiple channels. Section 3.2 provides a background to the relevant work in single and multichannel imaging. Then using the multichannel signal model from the previous chapter, Section 3.3 derives the MF imaging equation and Section 3.4 describes how three different multichannel wavefront reconstruction algorithms can be implemented. Two of these algorithms, spatial MF interpolation and range stacking are based on the frequency domain to speed up the computation time. An analysis of the Point Spread Function (PSF) in Section 3.5 however reveals some problems with this approach. The third algorithm, Time Domain Correlation (TDC) overcomes these problems and trades off accuracy for computation time. To overcome the problems of the frequency domain algorithms and avoid the large computational cost of TDC, a multichannel backprojection algorithm is presented in Section 3.6. Finally, the conclusion in Section 3.7 summarises the pros and cons of each of these algorithms.
3.2 Background

Classical SAR imaging from the 1950’s and 60’s was based on analogue/optical techniques, [Cutrona et al., 1966] and used the Fresnel approximation. This early method of imaging is now known as ‘range-Doppler processing’, [Brown and Fredricks, 1969] and was first implemented in its digital form in the late 1970’s.

Shortly after, an imaging method known as ‘polar format processing’ was introduced for high resolution spotlight mode SAR, [Munson et al., 1983], [Ausherman et al., 1984]. Polar format processing uses a plane wave approximation with range curvature correction, and has been used successfully for a number of years. However, with the introduction of powerful computers, more precise imaging algorithms are now possible. These are based on wavefront reconstruction with a MF [Goodman, 1968], and offer the potential for imaging with greater accuracy. [Soumekh, 1999], has summarised a number of these techniques including spatial MF interpolation, range stacking and TDC.

The first two of these algorithms are formulated in the fast-time/slow-time frequency domains to speed up computation of the MF imaging equation. Spatial MF interpolation offers the least computation time but requires an interpolation to map from the measured data domain to the spatial frequency domain. It was originally used for seismic imaging and later applied to SAR by [Rocca et al., 1989]. It is also known in the literature as the \( (\omega, k_u) \), [Cumming and Wong, 2005] or range migration imaging algorithm, [Carrara et al., 1995]. Range stacking however, avoids the interpolation by explicitly calculating the MF for each range and offers a trade-off for accuracy to computation time. The third algorithm, TDC is the most computationally intensive as it relies on a separate reference vector for each range and azimuth position. As a result, it produces images with very high accuracy.

The application of conventional single channel imaging algorithms to the multichannel case has not received a lot of attention. The literature includes methods based on the MF processing [Ender, 1998c], orthogonal reference vectors for Moving Target Indication (MTI) [Ender, 2000] and sidelobe suppression vectors to reduce azimuth ambiguities for long-range low Pulse Repetition Frequency (PRF) or non-ideal antenna patterns [Ender, 2000], [Goodman et al., 1999], [Krieger et al., 2004]. Out of these, the only paper relevant to this thesis is by [Ender, 1998c], who has presented an algorithm resembling spatial MF interpolation. It is however presented at a single range and the extension in Section 3.4 shows how it can be extended to incorporate range curvature effects.

Unfortunately, the use of the spatial MF interpolation algorithm can present some unwanted problems such as artefacts from the interpolation which become spread over the entire image, the need to acquire a full aperture of pulses before processing can begin and its sensitivity to motion errors in the flight path. These result in poor focus for large integration angles and/or large motion errors, [Yegulalp, 1999].
Chapter 3 Multichannel SAR Imaging

Backprojection offers an alternative to the frequency domain based algorithms without the processing overhead of the TDC algorithm. The algorithm has been presented as convolution backprojection in [Desai and Kenneth Jenkins, 1992] and can be formulated as a modification to computer aided tomography. Recent work includes faster implementation methods, [Yegulalp, 1999], multi-resolution imaging with quadtree backprojection [Seung-mok Oh and McClellan, 2001] and enhancement using digital spotlight preprocessing [Nguyen et al., 2004]. However, there has not yet been a backprojection formulation suitable for MSAR.

3.3 Multichannel Image Formation

The first part of this Section 3.3.1 summarises the SAR signal model from the previous chapter and derives multichannel signal models in both the \((\omega, u)\) and \((\omega, k_u)\) domains. The second part 3.3.2 then goes through a derivation of the multichannel MF imaging equation which has been extended for the multichannel case from [Soumekh, 1994] and is used as the basis for the wavefront curvature algorithms. The final Section 3.3.3 provides details on how the algorithms in this chapter can be implemented.

3.3.1 SAR Signal Model

The SAR platform introduced in the previous chapter was travelling along the azimuth direction and imaging an area on the ground described by, \(x \in [X_c - X_0, X_c + X_0]\), \(y \in [-Y_0, Y_0]\). An \(N\) channel linear antenna array is used in the azimuth direction with half-wavelength spacing as shown in Figure 3.1.

![Figure 3.1. Multichannel SAR imaging](image)

The received signal model was derived in Chapter 2. After shifting to baseband and post range processing, the received signal at the \(n^{th}\) antenna is given by,

\[
x_n(t, u) = \gamma_n(t, u) + \nu_n(t, u)
\]  

(3.1)
3.3 Multichannel Image Formation

where the total ground return is given by,

\[ \gamma_n(t, u) = \int_y \int_x a(x, y - u) f_n(x, y) s_{\text{post}, n}(t, u, x, y) dx dy \] (3.2)

with \( a(\cdot) \) the two-way antenna beampattern, \( f_n(x, y) \) the ground return from the \( n^{th} \) channel for a scatterer located at point \((x, y)\) and the post range processing signal model is given by,

\[ s_{\text{post}, n}(t, u, x, y) = \exp[-j\omega c \tau_n(x, y - u)] \text{sinc}[B(t - \tau_n(x, y - u))] \] (3.3)

with carrier frequency \( \omega_c \) (rad/s), bandwidth \( B \) (Hz) and the variables \((t, u)\) representing ‘fast-time’ within a pulse and the SAR platform ‘slow-time’ position respectively. The temporal delay at the reference receiver is given as twice the distance to the scatterer,

\[ \tau(x, y - u) = \frac{2}{c} R(x, y - u) \] (3.4)

where \( R(x, y) = \sqrt{x^2 + y^2} \) is the radial distance to the focus point. The spatial delay is determined by the difference between the \( n^{th} \) and the antenna phase centre,

\[ \tilde{\tau}_n(x, y - u) = \frac{1}{c} [R(x, y - u - d_n) - R(x, y - u)] \] (3.5)

with the antenna offset \( d_n = n\delta \) for antenna spacing \( \delta \) with \( n \in [-\frac{N-1}{2}, \frac{N-1}{2}] \) for \( N \) (odd) antenna elements. By using the following Taylor expansion about the point \( x + \Delta / 2, \)

\[ E(x) + E(x + \Delta) \approx 2E(x + \Delta/2) + \frac{\Delta^2}{4} E''(x + \Delta/2) \] (3.6)

where \( \Delta \) is small and the received signal is from the far field, the total delay to the \( n^{th} \) channel can be given by the sum of the temporal and spatial components,

\[ \tau_n(x, y - u) = \frac{1}{c} [R(x, y - u) + R(x, y - u - d_n)] \]

\[ \approx \frac{2}{c} R(x, y - u - 0.5d_n), \quad \text{for } X_c \gg d_n. \] (3.7)

Note that higher order terms must be considered when the azimuth swath size approaches the offset range.

The following imaging algorithms also require signal models in the \((\omega, u)\) and \((\omega, k_u)\) domains. The \((\omega, u)\) domain model is found by taking the Fourier Transform of the fast-time domain expression given by Equation 3.3,

\[ \tilde{s}_{\text{post}, n}(\omega, u, x, y) = \text{rect} \left[ \frac{\omega - \omega_c}{\pi B} \right] \exp[-j\omega \tau_n(x, y - u)]. \] (3.8)

where the fast-time frequency \( \omega \) is related to the range compressed time with \( n_x \) range bins instead of the entire fast-time containing \( L \) samples.

To convert this model into the \((\omega, k_u)\) domain, the slow-time Fourier Transform must be calculated. This derivation involves the principle of stationary phase and is given in Appendix A. The final signal model is given by,

\[ S_{\text{post}, n}(\omega, k_u, x, y) = \exp \left[ -j \sqrt{4k^2 - k_u^2} x - jk_u (y - 0.5d_n) \right] \] (3.9)
where the wavenumber, $k = \omega / c$ and the slowly fluctuating amplitude has been normalised. One important result for this signal model is the linearity of the range and azimuth components in the phase, which implies they may be separated during image formation.

### 3.3.2 Matched Filter Formulation

This section derives the multichannel MF imaging model used for the wavefront reconstruction algorithms. It has been adapted from [Soumekh, 1994] and extended for multiple channels. The goal is to form an expression for the estimated ground return of a scatterer in terms of the measured signal, $x_n(\cdot)$ given in Equation 2.12. Due to the linear superposition between the ground reflected signal and the noise, the following derivation can be simplified by considering only the total ground return, $\gamma_n(\cdot)$ and not the receiver noise, $\nu_n(\cdot)$. The two-way antenna beampattern, $a(\cdot)$ is also assumed to be uniform over the imaging area and set to unity. The form of this signal for the $n^{th}$ channel is given post range processing as,

$$x_n(t, u) = \int y \int x f_n(x, y) \gamma_{\text{post}, n}(t, u, x, y) dxdy. \quad (3.10)$$

By taking Fourier Transforms in each dimension, this can be written in the $(\omega, ku)$ domain as

$$X_n(\omega, ku) = \int y \int x f_n(x, y) S_{\text{post}, n}(\omega, ku, x, y) dxdy \quad (3.11)$$

where $S_{\text{post}, n}(\omega, ku, x, y)$ is defined in Equation 3.9. Substituting it into this equation gives,

$$X_n(\omega, ku) = \int y \int x f_n(x, y) \exp \left[-j \sqrt{4k^2 - ku^2} x - j ku y \right] \exp [j ku 0.5d_n] dxdy. \quad (3.12)$$

Then using the relationship between the measured and spatial frequency domain,

$$k_x = \sqrt{4k^2 - ku^2}, \quad k_y = ku, \quad \text{(3.13)}$$

this equation can be written as,

$$X_n(\omega, ku) = \left[ \int x f_n(x, y) \exp \left[-jkx x - jky y \right] dxdy \right] \exp [j ky 0.5d_n]$$

$$= F_n(k_x, k_y) \exp [j ky 0.5d_n] \quad (3.14)$$

where $F_n(k_x, k_y)$ is the the two-dimensional Fourier Transform of $f_n(x, y)$. The next step is to multiply both sides by the complex conjugate of the exponential to give,

$$F_n(k_x, k_y) = \exp [-j ky 0.5d_n] X_n(\omega, ku). \quad (3.15)$$

Then to obtain an image in the $(x, y)$ domain, take the two-dimensional inverse Fourier Transform in the spatial domain to give,

$$f_n(x, y) = \int_{k_y} \int_{k_x} F_n(k_x, k_y) \exp[jkx x + jky y]dk_x dk_y$$

$$= \int_{k_y} \int_{k_x} \exp [j kx x + j ky (y - 0.5d_n)] X_n(\omega, ku) dk_x dk_y. \quad (3.16)$$
Then using the relationships in Equation 3.9 and 3.13,

\[
fn(x, y) = \int_{ky} \int_{kx} \exp \left[ j \sqrt{4k^2 - k_u^2} x + jk_u(y - 0.5d_n) \right] X_n(\omega, k_u) dk_x dk_y
\]

\[
= \int_{ky} \int_{kx} S_{\text{post}, n}(\omega, k_u, x, y) X_n(\omega, k_u) dk_x dk_y.
\]

As the received signal is represented in (\(\omega, k_u\)) domain, the integration variables must be changed from (\(k_x, k_y\)) to (\(\omega, k_u\)) in order to solve the integral,

\[
f_n(x, y) = \int_{k_u} \int_{\omega} S^*_{\text{post}, n}(\omega, k_u, x, y) X_n(\omega, k_u) \mathcal{J}(\omega, k_u) d\omega dk_u
\]

where the Jacobian,

\[
\mathcal{J}(\omega, k_u) = \frac{4k}{c\sqrt{4k^2 - k_u^2}}
\]

has a slowly fluctuating amplitude and can be neglected [Soumekh, 1999], giving

\[
f_n(x, y) = \int_{k_u} \int_{\omega} S^*_{\text{post}, n}(\omega, k_u, x, y) X_n(\omega, k_u) d\omega dk_u.
\]

This is the MF equation for a single channel with \(S^*_{\text{post}, n}(\cdot)\) defined as the MF reference signal. To focus multiple channels in the broadside direction, the focussed outputs from each channel are summed,

\[
f(x, y) = \int_{k_u} \int_{\omega} \frac{1}{N} \sum_n S^*_{\text{post}, n}(\omega, k_u, x, y) X_n(\omega, k_u) d\omega dk_u
\]

where \(f(x, y)\) is the ground return from the focussed real array. A convenient form for multiple channels is to stack the reference and data signals each channel to give the normalised signal vectors,

\[
S_{\text{post}}(\cdot) = \frac{1}{\sqrt{N}} [S_{\text{post}, -(N-1)/2}(\cdot), \ldots, S_{\text{post}, (N-1)/2}(\cdot)]^T,
\]

\[
X(\cdot) = \frac{1}{\sqrt{N}} [X_{-(N-1)/2}(\cdot), \ldots, X_{(N-1)/2}(\cdot)]^T
\]

and then Equation 3.21 can be written as an inner product,

\[
f(x, y) = \int_{k_u} \int_{\omega} S_{\text{post}}^H(\omega, k_u, x, y) X(\omega, k_u) d\omega dk_u
\]

where \(^H\) is the complex transpose or Hermitian operator. Due to the linear relationship mentioned at the start of this section, the data vector \(X(\cdot)\) can now be written in terms of both the ground reflected signal and the noise. The bounds for both \(\omega\) and \(k_u\) are determined by the bandwidth and the azimuth angular limit, \(\theta_{\text{lim}}\) described in Section 2.3.3. They will result in an estimated version of \(f(x, y)\), which is denoted \(\hat{f}(x, y)\),

\[
\hat{f}(x, y) = \int_{-2k_c \sin(\theta_{\text{lim}})}^{2k_c \sin(\theta_{\text{lim}})} \int_{\omega_c - B\pi}^{\omega_c + B\pi} S_{\text{post}}^H(\omega, k_u, x, y) X(\omega, k_u) d\omega dk_u.
\]
3.3.3 Implementation

The algorithms presented in this chapter are demonstrated for a stripmap SAR mode and are implemented discretely using the sampling definitions in the previous chapter. However, there are two different domains that need to be considered carefully. These are the measured data domain which is slow-time/fast-time and the image domain which is azimuth/range.

The synthetic aperture will always be greater than the desired imaging area and consequently, slow-time samples will cover a wider area than the azimuth swath size, $2Y_0$. The actual number of samples in the final image is determined by the imaging geometry and is defined by $n_y < M$.

For the fast-time, the number of range bins, $n_x$ will be less than the number of fast-time samples, $L$ due to the range processing stage described in Section 2.3.4.

3.4 Wavefront Reconstruction Algorithms

The theory of wavefront reconstruction is a method of imaging using a MF which does not approximate the received signal as a plane wave. Instead, signal models based on the MF imaging Equation 3.24 can be derived with different tradeoffs for accuracy and computational expense. There are three direct implementations of this equation:

(a) The first is detailed in Section 3.4.1 and is known as spatial MF interpolation. It uses a single reference vector over the whole SAR field of view to evaluate the MF imaging equation and as a result offers the least computation time. This approach also requires a Stolt interpolation when mapping from the measured into the spatial frequency domain and account for the range migration relative to the focus point. An alternative space/slow-time implementation of the first algorithm is given in Section 3.4.1 as it forms a basis for the slow-time Space Time Adaptive Processing (STAP) algorithm in Chapter 6.

(b) The second implementation of Equation 3.24 is known as range stacking and is described in Section 3.4.2. It offers a trade-off for accuracy to computation time by explicitly calculating the MF at each range.

(c) TDC is the final algorithm presented in Section 3.4.3. It is evaluated in the time domain and is the most computationally intensive of the three algorithms. It relies on a separate reference vector for each range and azimuth to produce images with high accuracy. For this reason, the MF process is now different to the previous two implementations and changes to the shape of the PSF will result.

Figure 3.2 shows the location of the reference vectors for these three algorithms. Following the algorithm descriptions, a comparison of the relative performance is presented in Section 3.5. PSF’s are created with the second method described in the previous chapter and measures of the Peak Sidelobe Ratio (PSR) and Integrated Sidelobe Ratio (ISLR) are used to quantify performance.
3.4 Wavefront Reconstruction Algorithms

3.4.1 Spatial Matched Filter Interpolation

This algorithm was originally formulated for seismic imaging and later applied to SAR by [Rocca et al., 1989]. It is also known in the literature as the \((\omega, k_u)\) algorithm, [Cumming and Wong, 2005], range migration imaging algorithm, [Carrara et al., 1995] or the spatial MF interpolation, [Soumekh, 1999]. It has also been proposed for multichannel SAR by [Ender, 1998c], though his implementation was only for a single range. The implementation in this section combines these two approaches by using spatial vectors combined with the Stolt interpolation.

The spatial MF interpolation algorithm evaluates the MF imaging Equation 3.24 with a fixed focus position at the centre of the imaging area, \((X_c, 0)\). To implement this algorithm for multichannel SAR, a reference vector is chosen, \(S_{ref}(\omega, k_u) = S_{post}(\omega, k_u, X_c, 0)\) and the following inner product calculated,

\[
\tilde{F}(\omega, k_u) = S_{ref}^H(\omega, k_u)X(\omega, k_u). \tag{3.25}
\]

The final SAR image however is represented in the \((x, y)\) domain and a change of variables is required. The spreading of the target signal over a number of range cells is known as range migration, and the change of variables is a method of range migration compensation relative to the reference or focus point. It is implemented using the relationship in Equation 3.13 and is known as a Stolt Interpolation, [Carrara et al., 1995]. To improve the result, a sinc function smoothed by a hamming window is used as the interpolation kernel.

\[
\tilde{F}(\omega, k_u) \xrightarrow{\text{interp.}} \hat{F}(k_x, k_y). \tag{3.26}
\]

The interpolation can be seen graphically in Figure 3.3, where the curved wavefront is sampled onto a rectangular grid. The final step is to use a two-dimensional inverse Fourier Transform to obtain the estimated ground return, \(\hat{f}(x, y)\).

\[
\hat{f}(x, y) = \int_{k_y} \int_{k_x} \hat{F}(k_x, k_y) \exp[jk_x x + jk_y y] \, dk_x \, dk_y. \tag{3.27}
\]

A block diagram summarising this algorithm is shown in Figure 3.4.
This algorithm is very quick to run as it only relies on a single MF reference signal to focus over the entire SAR field of view, giving a complexity of $O(n_x MN)$. This will also cause a degradation in the final image quality as artefacts from the Stolt interpolation become spread over the entire image. Other real world problems include the need to acquire a full aperture of pulses before processing can begin and its sensitivity to motion errors in the flight path. These result in poor focus for large integration angles and/or large motion errors, [Yegulalp, 1999].

**Space/Slow-time Matched Filter Interpolation**

[Ender, 1998c] first suggested this space/slow-time implementation as it formed the basis of the slow-time STAP algorithms he used for MTI. It is similarly used for this purpose in Chapter 6, but can also however be used to interpolate a different set of azimuth points to achieve an image with much greater detail. It is performed with space/slow-time vectors and requires the slow-time inverse Fourier Transform of Equation 3.25 to be rewritten as a discrete convolution in the azimuth domain.
3.4 Wavefront Reconstruction Algorithms

\[ \tilde{f}(\omega, u_m) = \frac{1}{M} \mathcal{H}_{\text{ref}}(\omega, u_m) * \tilde{x}(\omega, u_m) \]
\[ = \frac{1}{M} \sum_{k=1}^{M} \mathcal{H}_{\text{ref}}(\omega, u_m - u_k) \tilde{x}(\omega, u_k) \] (3.28)

where the extra summation from the convolution now requires normalising by the total number of pulses, \( M \). This equation can be rewritten by forming an inner product over space/slow-time vectors stacked over the entire range of \( u \). To maintain the phase centre at the centre of the synthetic array, the centre of the imaging area occurs at \( u_M/2 \) and hence both the slow-time steering and data vectors can be stacked over pulse delays where \( u \) varies over the set \( u_1, u_2, \ldots, u_M \),

\[ \tilde{G}(\omega, u_m) = \frac{1}{\sqrt{M}} [\tilde{x}^T(\omega, u_m - u_1), \ldots, \tilde{x}^T(\omega, u_m - u_M)]^T \in \mathbb{C}^{MN \times 1}, \]
\[ \tilde{X}_s(\omega) = \frac{1}{\sqrt{M}} [\tilde{x}^T(\omega, u_1), \ldots, \tilde{x}^T(\omega, u_M)]^T \in \mathbb{C}^{MN \times 1} \] (3.29)

so that Equation 3.28 can be written as,

\[ \tilde{f}(\omega, u_m) = \tilde{G}^H(\omega, u_m) \tilde{X}_s(\omega). \] (3.30)

To form the estimated ground return, the result must be Fourier Transformed into the \((\omega, k_u)\) domain and then mapped from the measured to the image domain as in the previous section where a two-dimensional inverse Fourier transform maps the spatial frequencies into \((x, y)\) image coordinates. A block diagram of this algorithm is presented in Figure 3.5.

**Figure 3.5. Space/slow-time MF interpolation block diagram**

To improve the final image quality, the convolution operation in Equation 3.28 can be designed to focus over a different set of azimuth points, thereby upsampling the final image,

\[ \tilde{f}(\omega, \bar{u}_m) = \sum_{k=1}^{n_y} \mathcal{H}_{\text{ref}}(\omega, \bar{u}_m - u_k) \tilde{x}(\omega, u_k) \] (3.31)

where \( \bar{u} \) defines the new sampling region with \( n_y > M \) points. Using this technique, the complexity of the algorithm will increase to \( O(n_x M n_y N) \) and depends on the desired number of points in the interpolated image. Note, that upsampling the final image is not unique to this algorithm and can implemented in a number of ways.
3.4.2 Range Stacking

A more exact frequency domain implementation of Equation 3.24 is referred to as range stacking [Soumekh, 1999]. This algorithm uses a different MF reference signal for each of the \( n_x \) range bins, integrates over the fast-time frequency \( \omega \), and then performs an inverse Fourier Transform to get the final result. By using these extra reference signals, the Stolt interpolation from the previous algorithm is not required and the final image does not suffer from the interpolation artefacts. The tradeoff for this approach however is the added computation time of the algorithm.

Consider the \((\omega, k_u)\) domain signal model for the \( n^{th} \) channel in Equation 3.9. If it is broken into two parts,

\[
S_{\text{post},n}(\omega, k_u, x, y) = \exp \left[ -j \sqrt{4k^2 - k_u^2} x + jk_u 0.5d_n \right] \exp \left[ -jk_u y \right] \\
= S_{\text{RS},n}(\omega, k_u, x) \exp[-jk_u y] \quad (3.32)
\]

and stacked as in Equation 3.22 to form the reference vector \( S_{\text{RS}}(\omega, k_u, x) \), the MF imaging Equation 3.24 can be written as,

\[
\hat{f}(x, y) = \int_{k_u} \left[ \int_{\omega} S_{\text{RS}}^H(\omega, k_u, x)X(\omega, k_u) d\omega \right] \exp[jk_u y] dk_u \quad (3.33)
\]

where \( k_u = k_y \). This can be interpreted as an inner product for the MF at range \( x \), followed by an inverse Fourier Transform in the azimuth domain. Figure 3.6 represents this algorithm in a block diagram.

![Figure 3.6. Range stacking block diagram](image)

Range stacking increases the computational cost to \( O(n_x^2 MN) \) due to the extra \((n_x - 1)\) MF reference signals required to form an image. The advantage of calculating the MF at each range is to avoid the artefacts produced by the Stolt interpolation. However, if this algorithm was implemented in the real world, it would still need to acquire a full aperture of pulses before processing can begin.
3.4 Wavefront Reconstruction Algorithms

3.4.3 Time Domain Correlation

Time Domain Correlation is the most exact implementation as it solves the MF imaging equation precisely for each range and azimuth focus point. As a result, the MF process is now different to the previous two algorithms and more closely resembles a spotlight imaging mode. The consequences of this can be seen by closer spaced azimuth nulls in its PSF.

The integration is now performed explicitly over both time variables to obtain the estimated ground return, $\hat{f}(\cdot)$ and results in a high computational expense, but also highly accurate results. For a single channel, the estimated ground return is obtained by the convolution of the received SAR data with the matched filter, $s_{TDC}(\cdot)$,

$$\hat{f}(x, y) = \int_u \int_t s_{TDC}(t - \tau_n(x, u - y), y - u)x(t, u)dtdu$$

(3.34)

where the time delays are functions of the focus point $(x, y)$. To extend this algorithm for multiple channels, the reference vector must also include the delay to the respective antenna element. The spatially stacked MF vector is related to post range processing signal vector in Equation 3.3, by $s_{TDC}(t - \tau_n(x, u - y), y - u) = s_{post,n}^*(t, -u, x, y)$. Then, if the MF imaging Equation 3.24 is transformed into the $(t, u)$ domain using Parseval’s theorem, it can be written as,

$$\hat{f}(x, y) = \int_u \int_t s_{post}^H(-t, -u, x, y)x(t, u)dtdu$$

(3.35)

To speed up the overall implementation, this equation can avoid one convolution by calculating the return in the $(\omega, u)$ domain instead,

$$\hat{f}(x, y) = \int_u \int_\omega \tilde{s}_{post}^H(\omega, u, x, y)\tilde{x}(\omega, u)d\omega du$$

(3.36)

with $\tilde{s}_{post}(\omega, u, x, y)$ defined in Equation 3.8. A block diagram of the algorithm is shown in Figure 3.7.

![Time domain correlation block diagram](image)

Figure 3.7. Time domain correlation block diagram

Due to the extra number of MF reference signals, the TDC algorithm is a very computational intensive algorithm with a complexity of $O(n_x^2M^2N)$. It also produces the most accurate results by using a different MF reference signal for each range and azimuth again avoiding the Stolt interpolation. Unfortunately, it is also too computationally expensive to implement in real time.
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Multichannel SAR Imaging

3.5 Comparative Results

To compare the relative performance of the three wavefront reconstruction algorithms, the MATLAB simulation from Chapter 2 has been used. The simulation parameters have been oversampled by increasing the number of azimuth samples $M$ and doubling the fast-time sampling rate, $\Delta t$. This allows the final image to be formed with greater detail.

The PSF for each algorithm is determined by forming the image of a single scatterer at the centre of the imaging area. This also minimises the effect of signal mismatch compared to imaging a scatterer in the far corner. To quantify the relative computational load of each algorithm, Section 3.5.1 summaries the complexity using some typical MSAR parameters and to access the accuracy, two different measures of performance are explained in Section 3.5.2.

The first of two scenarios is then explained in Section 3.5.3, demonstrating a typical stripmap operation where the offset range is very large and the SAR integration angle is very small. The alternative in Section 3.5.4 then uses a large SAR integration angle. This is more commonly the case for a spotlight mode of operation and can be used to get a much finer azimuth resolution and potentially show more clearly any differences between the imaging algorithms. From the definition of $\varphi$ in Equation 2.46, this is typically achieved with a large synthetic aperture or a small range offset. As the former would require a very large number of pulses to form an acceptable image, the range offset for the simulation is reduced to a small value. Consequently, this simulation scenario is now unrealistic, although the results are comparable to the case for a longer synthetic aperture and offset range. Parameters for both simulations are shown in Table 3.1. The final Section 3.5.5 then provides a summary of the results including a discussion of which algorithm offers the best tradeoff for computational cost and accuracy.

3.5.1 Complexity

The three wavefront reconstruction algorithms each tradeoff accuracy for an increased computational load. A summary of the complexity of the three algorithms are shown in Table 3.2, with both the simulated parameters and some typical parameters taken from DSTO’s Ingara SAR. For the simulation, there are $N = 5$ channels, $M = 250$ pulses and $n_x = 150$ range bins. The Ingara SAR typically processes stripmap imagery with $M = 4246$ pulses and $n_x = 4096$ range bins and results are based on a $N = 5$ channel system. Table 3.2 indicates three distinct levels of computational load equating to a relative timeliness, with the spatial MF interpolation algorithm classed as short, range stacking as medium and TDC as large.
3.5 Comparative Results

<table>
<thead>
<tr>
<th>Table 3.1. Simulation parameters</th>
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</thead>
<tbody>
<tr>
<td><strong>Parameters</strong></td>
</tr>
<tr>
<td>Carrier frequency</td>
</tr>
<tr>
<td>Bandwidth</td>
</tr>
<tr>
<td>Number of fast-time samples</td>
</tr>
<tr>
<td>Number of range bins</td>
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<td>Number of pulses</td>
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<td>Number of elements</td>
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<td>Platform velocity</td>
</tr>
<tr>
<td>Pulse repetition</td>
</tr>
<tr>
<td>Range resolution</td>
</tr>
</tbody>
</table>

**Long range scenario**
- Range centre: $X_c = 10$ km
- Azimuth resolution: $\Delta_Y = 2.5$ m
- SAR integration angle: $\varphi = 0.34^\circ$
- Swath size: $2X_0, 2Y_0 = 6, 30$ m
- Image size: $n_x, n_y = 150, 211$

**Short range scenario**
- Range centre: $X_c = 20$ m
- Azimuth resolution: $\Delta_Y = 0.075$ m
- SAR integration angle: $\varphi = 11.48^\circ$
- Swath size: $2X_0, 2Y_0 = 6, 0.9$ m
- Image size: $n_x, n_y = 150, 95$

<table>
<thead>
<tr>
<th>Table 3.2. Algorithm complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial MF Interpolation, $O(n_x M N)$</td>
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<tr>
<td>Simulation</td>
</tr>
<tr>
<td>Typical</td>
</tr>
</tbody>
</table>

3.5.2 Measures of Performance

To determine the difference in quality between the different algorithms, an analysis of different slices through the centre of the PSF image can be used as a comparison. An example slice is shown below in Figure 3.8 where the PSF slice is represented by $\hat{f}_{dB}(x) = 10 \log_{10} \left| \hat{f}(x) \right|^2$ and $x_{ML}$ indicates the first null of the PSF.

The first measure of performance is the Peak Sidelobe Ratio (PSR) which determines the difference between the mainlobe and greatest sidelobe. Better performance is indicated by a larger PSR.

$$PSR = \max \left[ \hat{f}_{dB}(x) \right] - \max \left[ \hat{f}_{dB}(x < -x_{ML}), \hat{f}_{dB}(x > x_{ML}) \right].$$  (3.37)
The second performance metric is the Integrated Sidelobe Ratio (ISLR) which measures the ratio of all energy in the first three sidelobes to the energy in the mainlobe. Lower sidelobes will result in a lower ISLR which indicates better performance.

\[
ISLR = 10 \log_{10} \left[ \frac{\sum_x \left( |\hat{f}(x < -x_{ML})|^2 + |\hat{f}(x > x_{ML})|^2 \right)}{\sum_x |\hat{f}(-x_{ML} \leq x \leq x_{ML})|^2} \right].
\]  

(3.38)

### 3.5.3 Small Field of View

A small SAR integration angle can be achieved by using a large offset range. This is the case for the first comparison scenario using the parameters in Table 3.1. The PSF’s for the three different algorithms are shown below in Figure 3.9, with range along the horizontal x-axis and azimuth along the vertical y-axis.

Visually, the PSF’s looks very similar with a slight ripple present in the sidelobes of the spatial MF interpolation and range stacking algorithms. There is also a noticeable difference in the azimuth sidelobes between these and the TDC algorithms. This is due to the more exact TDC algorithm using a different MF for each azimuth pixel.
3.5 Comparative Results

To look more carefully at these results, Figure 3.10 shows range, azimuth and diagonal slices taken through the centre of the PSF’s. The diagonal slice is taken from the top left to bottom right and is projected onto the range axis.

![Figure 3.10](image)

Figure 3.10. PSF slices for small field of view scenario: (—) spatial MF interpolation, (- -) range stacking, (---) TDC

The range slices indicate very similar results for all three algorithms with the mainlobe and first three sidelobe peaks being nearly identical. There are however, slightly some minor deviations at the edges of the spatial MF interpolation. This is where the Stolt interpolation is not as accurate as using the exact MF reference vectors for each range. The azimuth and diagonal slices however show near identical results for first two algorithms, while the TDC shows significantly different sidelobes. This difference is due to the extra \( y \)-varying azimuth term present in the MF (see Equation 3.8) and produces lower and slightly offset nulls between the sidelobes.

An analysis of the PSR and ISLR is shown in Table 3.3. The PSR is nearly identical for the three range slices and shows only a minor improvement for the TDC azimuth slice. When considering only the first three sidelobes, the ISLR of the range and azimuth slices are nearly identical for all three algorithms. The diagonal slice has an ISLR which is lower due the energy being located primarily in the range/azimuth directions. For this slice, the TDC is slightly larger by 0.12 dB than the other two algorithms.
### Chapter 3 Multichannel SAR Imaging

#### Table 3.3. PSR and ISLR comparisons for small field of view scenario

<table>
<thead>
<tr>
<th></th>
<th>Spatial MF interpolation</th>
<th>Range stacking</th>
<th>TDC</th>
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</thead>
<tbody>
<tr>
<td>Range PSR (dB)</td>
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<td>12.93</td>
<td>12.93</td>
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<td>Azimuth PSR (dB)</td>
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<tr>
<td>Azimuth ISLR (dB)</td>
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<td>Diagonal ISLR (dB)</td>
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<td>-25.30</td>
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</tbody>
</table>

#### 3.5.4 Large Field of View

The second comparison scenario is for a large SAR integration angle. The simulation parameters are given in Table 3.1 where an unrealistic offset range has been chosen to reduce the simulation size. The PSF’s for the three different algorithms are shown below in Figure 3.11.

![Figure 3.11. PSF comparison for large field of view scenario (horizontal - range, vertical - azimuth)](image)

Each of these results show a warping or ‘butterfly’ like effect due to the large SAR integration angle. Visually there is very little difference between the first two images, while the TDC image indicates deeper nulls and better defined azimuth sidelobes. Figure 3.12 shows range, azimuth and diagonal slices through the centre of the PSF.

With this scenario, there are very similar results for the range and diagonal slices with minor differences at the edges between the first two and TDC algorithms. The azimuth slices however are quite different with lower sidelobes and nulls on the TDC than the other two algorithms. These differences again are due to extra MF’s used in the image formation.

Table 3.4 provides an analysis of the PSR and ISLR for this scenario. There is now a difference in both TDC range and azimuth PSR’s of 0.25 dB and 0.10 dB respectively. The range PSR is above the ‘ideal’ 13.26 dB\(^8\) due to the warping caused by the large SAR integration angle. These results are reflected in the ISLR measurements for the TDC algorithm with improvements of 0.18 dB and 0.31 for the range and azimuth slices. The improvement for the diagonal slice is also significant at 0.14 dB.

---

\(^8\)This is the value obtained by analysis of the sinc function sidelobe levels.
3.5 Comparative Results

Figure 3.12. PSF slices for large field of view scenario: (—) spatial MF interpolation, (- -) range stacking, (---) TDC

Table 3.4. PSR and ISLR comparisons for large field of view scenario

<table>
<thead>
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<th>TDC</th>
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</thead>
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</tr>
<tr>
<td>Diagonal ISLR (dB)</td>
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<td>-22.82</td>
<td>-22.96</td>
</tr>
</tbody>
</table>

3.5.5 Summary

These results indicate that out of these three algorithms, the spatial MF interpolation offers the best tradeoff between accuracy and computational expense. There is very little difference in the results for this and the range stacking algorithms, indicating that the Stolt interpolation has worked well.
The improvement gained from the TDC algorithm has been demonstrated in the second scenario with a larger field of view. In this case, both the PSR and ISLR are improved over the other two algorithms. For practical use however, the TDC algorithm has an extremely large complexity and the tradeoff mentioned above should determine its use.

Another important real world problem is that of motion compensation. [Soumekh, 1999] has described a number of methods which can be applied to the MF formulations in this section. They assume that the error position has been estimated using either the Global Position Satellites (GPS), Inertial Navigation Unit (INU) or a combination of both. If the beamwidth is small, a phase shift in the \((\omega, u)\) domain can be used to correct the motion, with further filtering required as the beamwidth gets larger. More advanced motion compensation can be achieved by exploiting the phase response of in-scene targets.

### 3.6 Backprojection

As explained in Section 3.4.1 there are a number of known problems with the spatial MF interpolation algorithm. These include artefacts from the Stolt interpolation and sensitivity to motion errors in the flight path that result in poor focus for large integration angles and/or large motion errors, [Yegulalp, 1999].

Backprojection offers an alternative to the wavefront reconstruction algorithms without the processing overhead of TDC or the problems of the frequency domain implementations, [Yegulalp, 1999]. This method is also more closely related to a spotlight mode of imaging and will produce PSF’s which match the TDC algorithm. It has been presented as convolution backprojection in [Desai and Kenneth Jenkins, 1992] and can be formulated as a modification to computer aided tomography. The implementation here is based on Soumekh, [Soumekh, 1999] and extended for multiple channels.

Backprojection works by tracing back the \((t, u)\) domain data to obtain that component of the return for each point \((x, y)\). This value however contains contributions from a number of points at the same range. Therefore, when the delayed returns from a number of SAR locations are integrated, contributions from the point \((x, y)\) add in phase, while returns from other points add out of phase. This technique also inherently addresses the problem of range migration by selecting the appropriate points over adjacent range bins.

To achieve good accuracy, the range compressed fast-time samples are interpolated from the measured data to obtain the return for each point. For ease of implementation this is done by Fourier Transforming the data into the \((w, u)\) domain, zero-padding and then Inverse Fourier Transforming from the upsampled frequency domain, \(\omega_{\text{up}}\) to the corresponding time domain, \(t_{\text{up}}\). The amount of zero-padding is a trade-off between the accuracy of the interpolation and the extra computational load. If there are \(n_x\) range compressed fast-time samples, the total number of zero-padded samples is given by \(L_{\text{up}} = Z_{\text{rat}}n_x\) with the total number of zeros determined by \(L_Z = n_x(Z_{\text{rat}} - 1)\).
If the range compressed fast-time slice at SAR location \( u \) is defined as \( x(:, u) \), then the baseband interpolated signal is given by,

\[
\bar{x}(t, u) = F^{-1} \left\{ \left[ 0_{L_z/2} F \{ x(:, u) \} 0_{L_z/2} \right] \right\} \in \mathbb{C}^{L_{\text{up}} \times 1}.
\] (3.39)

However, for the interpolation to work, the measured data must reverse the baseband conversion performed by the receiver hardware. This is in preparation for the interpolation stage which requires the upsampled data to have a similar phase to the original received data. It must be implemented with the upsampled time \( t_{\text{up}} \), to bring the SAR signal back to the bandpass fast-time frequency.

\[
\bar{x}_{\text{RF}}(t_{\text{up}}, u) = \bar{x}(t_{\text{up}}, u) \exp \left[ j\omega_c t_{\text{up}} \right].
\] (3.40)

The interpolation step then finds the closest upsampled point in \( t_{\text{up}} \) to \( \tau(x, y - u) \).

\[
\bar{x}_{\text{RF}}(t_{\text{up}}, u) \xrightarrow{\text{interp.}} \bar{x}_{\text{RF}}(\tau(x, y - u), u).
\] (3.41)

The target function can then be determined by integrating over all the pulses.

\[
\hat{f}_{\text{RF}}(x, y) = \int_{u} \bar{x}_{\text{RF}}(\tau(x, y - u), u) du.
\] (3.42)

The final step is to correct for the upsampled phase by coherently basebanding the signal to bring the two-dimensional spectrum to the baseband \((k_x, k_y)\) domain. This phase shift will not effect the final image however which depends only on the magnitude of \( \hat{f}(\cdot) \). The estimated ground return is then given by,

\[
\hat{f}(x, y) = \hat{f}_{\text{RF}}(x, y) \exp \left[ -jk_{xc}x \right]
\] (3.43)

where the centre range spatial frequency, \( k_{xc} = (k_{x,\text{min}} + k_{x,\text{max}})/2 \). The multichannel extension for this algorithm involves compensating for the spatial phase delay at each interpolated point. As explained in Section 2.3.5, the spatial reference signal is formed by focussing at the centre of the imaging area,

\[
s_n(u) = \exp \left[ jk_c d_n \sin \left( \theta(u) \right) \right]
\] (3.44)

where \( k_c \) is the wavenumber at the carrier frequency and

\[
\theta(u) = \arctan \left( \frac{-u}{X_c} \right).
\] (3.45)

Once the interpolation has occurred for each channel, both the data vector and the spatial steering vector are stacked as in Equation 3.22. Equation 3.42 then becomes,

\[
\hat{f}_{\text{RF}}(x, y) = \int_{u} s^H(u) \bar{x}_{\text{RF}}(\tau(x, y - u), u) du
\] (3.46)

and as this algorithm is dependent on the chosen quality of the interpolator, it generally performs better then the Stolt interpolation from the spatial MF interpolation algorithm, [Soumekh, 1999]. Figure 3.13 presents a block diagram of this algorithm.
Chapter 3 Multichannel SAR Imaging

The longest path through the algorithm will be determined by either the chosen upsampling rate or the final image size. As a result, the complexity is given by $O((n_x n_y + L_{up} \log_2(L_{up})) MN)$ and will become $O(L_{up} \log_2(L_{up}) MN)$ if the upsampling rate is high enough.

The next section looks at the backprojection performance as the upsampling rate is varied. This is essential to determine the correct tradeoff for quality and computation time. The final Section 3.6.2 then compares the backprojection performance with the three wavefront reconstruction algorithms.

### 3.6.1 Backprojection Performance

The performance of the backprojection algorithm is very closely tied to the level of upsampling prior to interpolation. A level of at least 100 has been proposed for good reconstruction, [Soumekh, 1999] and this section looks at two methods of checking this proposition and determining an appropriate upsample rate to tradeoff computation time and image quality. The simulation parameters used are based on the small field of view scenario from Section 3.5.3.

The first measure is the ISLR of the range, azimuth and diagonal slices of the PSF. For comparison, the mainlobe width is fixed at the TDC level to ensure that a reasonable result will still occur if the image is poorly focussed. After plotting the ISLR for a number of upsampling ratios, a reasonable threshold level can then be chosen.

Using the small field of view scenario, an ‘S’ image is used to represent scatterers on the ground. To obtain a quantitative measure of image quality, the Signal Distortion Ratio (SDR) can be determined by viewing the backprojection image as a distorted version of the ideal TDC reference.
3.6 Backprojection

image. Let $Y(x_p, y_q)$ denote the degraded backprojection image and $D(x_p, y_q)$ the TDC reference image with range and azimuth pixels $p = 1 \ldots n_x, q = 1 \ldots n_y$. The SDR is then defined by,

$$ SDR = 10 \log_{10} \left[ \frac{\sum_{p,q} |D(x_p, y_q)|^2}{\sum_{p,q} |Y(x_p, y_q) - D(x_p, y_q)|^2} \right]. \quad (3.47) $$

Point Spread Function

The quality of the backprojection PSF depends on the upsampling rate. If it is set too low, the interpolation will perform poorly and if it is too high, computations are wasted. Using the small field of view scenario, a comparison of the PSF for three upsample rates are shown in Figure 3.14. A more detailed representation of range, azimuth and diagonal slices through the centre of the PSF are then shown in Figure 3.15.

$$ Z_{rat} = 20 \quad Z_{rat} = 50 \quad Z_{rat} = 100 $$

Figure 3.14. Backprojection PSF for varying upsample rates (horizontal - range, vertical - azimuth)

From these figures, the upsample rate of 20 appears very distorted, 50 is slightly distorted only in azimuth and 100 appears undistorted. It is clear that an appropriate upsample rate is somewhere between 50 and 100. A more analytic way of measuring the image quality is to measure the ISLR as the upsampling rate changes. Figure 3.16 shows a comparison of the ISLR for range, azimuth and diagonal slices of the PSF over the upsampling range 15 to 150.

This figure indicates that the range slice reaches its limit of -10.42 dB very quickly at around $Z_{rat} = 20$. The azimuth slice appears to increase in ISLR or decrease in performance with the low upsampling rate due to the very poor focussing of the backprojection. However, at around 20, the ISLR starts to decrease exponentially until it reaches $-13.65$ dB at the rate of 40. The diagonal ISLR behaves similarly to the azimuth slice and decreases with slight fluctuations to a mean level of $-25.20$ dB.

These final levels are either the same or very close to the TDC ISLR’s of $-10.42$ dB for range, $-14.00$ dB for azimuth and $-25.30$ dB for diagonal. To provide a clearer idea of performance however, further analysis with a simulated image is performed in the next section.
Figure 3.15. Backprojection PSF slices for $Z_{rat} = (\_\_\_\) 20, (\_\_\_) 50, (\_\_.\_) 100

Figure 3.16. Integrated sidelobe ratio comparison: (\_\_) range slice, (\_\_) azimuth slice, (\_.\_) diagonal slice
3.6 Backprojection

Imported Image

An imported ‘S’ image is used to represent scatterers on the ground and details of this implementation are explained in Section 2.4. To visualise the effect of the changing the upsampling rate, Figure 3.17 shows the backprojection results as the upsampling rate is set 20, 50 and 100 respectively. It is clear that an upsampling rate of 20 is insufficient to recover an image, 50 produces poor quality results and 100 is greatly improved, though not to the same standard of the TDC reference image.

![Backprojection image for varying upsample rates (horizontal - range, vertical - azimuth)](image)

Figure 3.17. Backprojection image for varying upsample rates (horizontal - range, vertical - azimuth)

Figure 3.18 shows a comparison of the SDR as the upsampling rate is increased from 15 until 200. The SDR results increase with small fluctuations until it plateaus at a level of 21.8 dB at the rate of 180. Further increases in the upsampling rate only fluctuate around this value.

Soumekh’s rate of 100 will still produce a reasonable quality image, but these results show that higher upsampling rates can offer further improvements. Based on these results and the PSF in the previous section, a rate of 150 is chosen for the algorithm comparison in the following section.

![Backprojection SDR with varying upsample rate](image)

Figure 3.18. Backprojection SDR with varying upsample rate
3.6.2 Algorithm Comparison

The main benefits of the backprojection algorithm are to improve the overall image quality while maintaining a reasonable computational load. This section compares the backprojection and TDC algorithms to demonstrate these points.

Image Quality - Small Field of View Scenario

To access the relative image quality, both the small and large SAR integration angle scenarios from Section 3.5 are used with the backprojection algorithm. For the small field of view scenario, Figure 3.19 shows a comparison of the PSF for the TDC and backprojection algorithms. Figure 3.20 then shows range, azimuth and diagonal slices through the centre of the PSF.

![Figure 3.19. PSF comparison with backprojection for small field of view scenario (horizontal - range, vertical - azimuth)](image)

There are very few differences between these two results with only slightly lower nulls present in the azimuth slice for the TDC algorithm. Table 3.5 compares the PSR and ISLR for backprojection with those listed earlier for the wavefront reconstruction algorithms. Compared to the first two algorithms, the backprojection azimuth results show a lower PSR with a slightly larger ISLR. When compared to the TDC algorithm, they are slightly worse by 0.29 dB in PSR and 0.24 dB in azimuth ISLR. The diagonal ISLR is also less than the TDC by 0.29 dB.

<table>
<thead>
<tr>
<th></th>
<th>Spatial MF interpolation</th>
<th>Range stacking</th>
<th>TDC</th>
<th>Backprojection</th>
</tr>
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<tbody>
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</table>
3.6 Backprojection

![Graphs of amplitude vs. range, azimuth, and diagonal slices through the center of the PSF for backprojection and time domain correlation.](image)

**Figure 3.20.** PSF slices with backprojection for small field of view scenario: (—) backprojection, (- -) time domain correlation

**Image Quality - Large Field of View Scenario**

The short range scenario from Section 3.5 showed a real improvement in the TDC algorithm. To compare this to the backprojection algorithm, Figures 3.21 shows a comparison of the PSF for the TDC and backprojection algorithms. A closer look at range, azimuth and diagonal slices through the centre of the PSF is then shown in Figure 3.22.

![Images of backprojection and time domain correlation for large field of view scenario.](image)

**Figure 3.21.** PSF comparison with backprojection for large field of view scenario (horizontal - range, vertical - azimuth)
The only visible differences are again in azimuth with slightly different sidelobes around the third and fourth null. Table 3.6 shows a comparison of the PSR and ISLR for each slice. The backprojection PSR is lower than the TDC by 0.03 dB in range and 0.09 dB in azimuth. For the ISLR, the backprojection results are the worst of all three algorithms by 0.24 dB, while the diagonal measurement is different by 0.08 dB.

Table 3.6. PSR and ISLR comparisons with backprojection for large field of view scenario

<table>
<thead>
<tr>
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<th>Backprojection</th>
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<tr>
<td>Diagonal ISLR (dB)</td>
<td>-22.82</td>
<td>-22.82</td>
<td>-22.96</td>
<td>-23.04</td>
</tr>
</tbody>
</table>
3.7 Conclusion

Computation Time

Similar to the comparison in Section 3.5.1, a summary of the complexity of the backprojection with the three wavefront reconstruction algorithms is shown in Table 3.7. The simulated parameters are $N = 5$ channels, $M = 300$ pulses and $n_x = 150$ range bins with an image size based on the small field of view scenario, $(n_x, n_y) = (150, 107)$. Typical parameters are taken from DSTO’s Ingara SAR with stripmap imagery processed with $M = 4246$ pulses and $n_x = 4096$ range bins. A typical image size is $(4096, 4096)$ and results are based on a $N = 5$ channel system.

An upsampling rate of $Z_{\text{rat}} = 150$ is chosen for backprojection with $L_{\text{up}} = Z_{\text{rat}} n_x = 22500$. The results indicate that the backprojection algorithm is slightly more computationally intensive than the range stacking algorithm and much less than the TDC. Based on the algorithm timeliness definitions, this puts backprojection into the medium computation time class.

<table>
<thead>
<tr>
<th></th>
<th>Spatial MF Interp., $O(n_x M N)$</th>
<th>Range Stacking, $O(n_x^2 M N)$</th>
<th>Backprojection, $O([n_x n_y + L_{\text{up}} \log_2(L_{\text{up}})] M N)$</th>
<th>TDC $O(n_x^2 M^2 N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>$1.9 \times 10^5$</td>
<td>$2.8 \times 10^7$</td>
<td>$5.2 \times 10^8$</td>
<td>$7.0 \times 10^9$</td>
</tr>
<tr>
<td>Typical</td>
<td>$8.7 \times 10^7$</td>
<td>$3.6 \times 10^{11}$</td>
<td>$3.6 \times 10^{11}$</td>
<td>$1.5 \times 10^{15}$</td>
</tr>
</tbody>
</table>

3.6.3 Summary

This section has demonstrated how the backprojection algorithm can be extended to cater for a SAR with multiple receive and transmit antennas. By selecting an appropriate upsampling rate for the smaller field of view scenario, the new algorithm can match the TDC performance with smaller computational requirements. This however is not the case for the large field of view scenario, which indicates worse performance even when compared to the spatial MF interpolation algorithm.

Motion compensation can be incorporated into the backprojection algorithm by modifying the delay used for the interpolation in Equation 3.41. As this method is different to the Fourier based motion compensation, the use of in-scene targets cannot be used to improve the result.

3.7 Conclusion

This chapter has presented four new multichannel imaging algorithms suitable for MSAR. These are summarised in Table 3.8 which also provides an overview of the pros and cons of each. The first three are based on wavefront reconstruction methods for solving the MF imaging equation. Simulation results of the PSF indicated that the spatial MF interpolation offers the best tradeoff between accuracy and computational expense. The Stolt interpolation appears to work very well as there was only a minimal improvement with the range stacking algorithm. The TDC on the
other hand uses the azimuth MF’s to produce improved visual results, especially when there is a large SAR integration angle. For practical use however, it has an extremely large complexity and the tradeoff between accuracy and computational expense should determine its use. The final algorithm was multichannel backprojection and by selecting an appropriate upsampling rate, can match the TDC performance for the small field of view scenario with smaller computational requirements. For the large field of view scenario however, it offers worse performance even when compared to the spatial MF interpolation algorithm.

### Table 3.8. Algorithm summary

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Pros</th>
<th>Cons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial MF interpolation</td>
<td>Short computation time</td>
<td>Stolt interpolation artefacts, Real world problems</td>
</tr>
<tr>
<td>Range stacking</td>
<td>Avoids Stolt interpolation</td>
<td>Medium computation time, Real world problems</td>
</tr>
<tr>
<td>Time domain correlation</td>
<td>Improved accuracy</td>
<td>Large computation time, Can not implement in real time</td>
</tr>
<tr>
<td>Backprojection</td>
<td>Small field of view quality comparable to TDC</td>
<td>Medium computation time, Poor results for large field of view</td>
</tr>
</tbody>
</table>
Chapter 4

STAP Background

4.1 Introduction

The original use of ‘RADAR’ was for radio detection and ranging. It soon became apparent that if the radar could adapt to different environments, target detection could be greatly improved. Adaptive beamforming applied to sonar and (theoretically) radar can be traced back early sixties, but it has only been in the last ten years or so that the development of phased array technology has allowed its practical application to airborne radar and more specifically, Synthetic Aperture Radar (SAR). The goal of Moving Target Identification (MTI) is to isolate moving targets while removing both unwanted reflections from the ground (ground clutter) and any other interferences from Radio Frequency (RF) signals in the same frequency band.

One of the earliest papers on adaptive radar came from [Brennan and Reed, 1973] who presented a space/slow-time adaptive processor which senses the Direction of Arrival (DOA)/Doppler distribution of the external noise field and adjusts a set of radar parameters for maximum Signal to Interference plus Noise Ratio (SINR) and optimum detection performance. Then in the early eighties, [Klemm, 1983] took up work on adaptive ground clutter suppression for airborne phased array radars and demonstrated how the ground clutter spectrum can be suppressed with a two dimensional space/slow-time filter.

In 1994, [Ward, 1994] wrote the first major summary of Space Time Adaptive Processing (STAP) techniques. He added to Klemm’s earlier work and proposed new suppression techniques for both ground clutter and wideband direct-path jammers. There was also an experimental program in 1995-96 performed by the Airforce Research Laboratory in the USA to test slow-time STAP algorithms on monostatic and bistatic data with no jammers present. This experiment is known as the Multi Channel Airborne Measurement (MCARM) program, [Sanyal, 1999].

The next major contribution was by [Klemm, 2002] and [Guerci, 2003] who each wrote a book on STAP, focussing on both optimal and sub-optimal techniques for airborne radar. Klemm has also written a more recent book, [Klemm-R., 2004], which is a collection of work focussing on different STAP applications including SAR, sonar, HF over the horizon radar, acoustics, communications and more.
4.2 Moving Target Indication Techniques for SAR

While the original application of slow-time STAP was focussed on airborne radar, the application of SAR for MTI has been a hot topic for the past ten or so years. There are however a number of additional problems due to the longer integration time in SAR and image formation being designed to focus stationary ground clutter only. Moving targets will appear offset and blurred and techniques are required to not only detect the moving targets, but also to compensate for their motion in the SAR image. Slow-time STAP has now proven to offer the best MTI results for SAR [Ender, 1998c], although there were many stepping stones. To build a greater appreciation for the application of slow-time STAP, the next Section 4.2 looks at a number of practical MTI techniques for SAR. Section 4.3 then presents an introduction to adaptive array processing and the optimal beamformers used in this thesis. This leads into Section 4.4 which extends this theory to slow-time STAP as applied in airborne radar. The final Section 4.5 then addresses some of the changes required to apply slow-time STAP to SAR.

4.2 Moving Target Indication Techniques for SAR

As presented in Chapter 2, the process of MTI for SAR is only one step in the total moving target motion compensation process. A generic overview of this process is given in Figure 4.1, where the received SAR data goes through an image formation stage in parallel to MTI. The image formation must reject the interferences in the data while preserving the ground clutter and the MTI stage must reject the interferences and the ground clutter, while preserving the moving targets in the scene. Following MTI, the position and velocity of each moving target is estimated, before the moving targets are compensated to form the final SAR image.

![Figure 4.1. Moving target compensation for SAR](image)

There have been a number of different MTI techniques applied to SAR. Not all of these have used multiple antennas to null the ground clutter and/or interference. A good literature review of single channel MTI techniques for SAR can be found in [Legg, 1997]. This section however, covers only those techniques using two or more antenna channels for MTI.

There have been some good summaries of space/time MTI techniques for SAR, [Sun et al., 2000] and [Klemm-R., 2004] being the most comprehensive. The best possible STAP processor would combine every spatial channel, pulse and range-bin to form a three dimensional (3D) adaptive
weight, [Lombardo, 1996]. Of course, the computational load of such a processor is prohibitive and sub-optimal processors must be used. Even optimal slow-time STAP is too computationally expensive and typically processors use only a few pulses for adaption, [Klemm, 1996b]. The following sections summarise five of the most popular sub-optimal MTI techniques being used for SAR.

4.2.1 Adaptive Displaced Phase Centre Array

Displaced Phase Centre Array (DPCA) for side-looking radar is one of the earliest techniques for MTI, [Skolnik, 1990]. It is typically formulated as in Figure 4.2 with two identical antennas displaced in the flight direction. The transmit antenna is switched from pulse to pulse in such a way that the position of the second antenna at the second pulse is the same as the position of first antenna at the first pulse. That is, the distance between the two channels is the SAR platform velocity times the pulse repetition interval, $v_p T_{PRI}$. Echoes received from stationary targets should be the same and if their signals are subtracted, the difference signal should show up both the noise contribution and the phase shift from any moving targets with radial velocity components.

![Figure 4.2. Principle of the DPCA technique](image)

With this design, the potential for channel mismatch is very high in a SAR MTI system. [Coe and White, 1996] have reformulated the DPCA algorithm into a two channel slow-time STAP framework, also known as Adaptive DPCA. This adaptive modification offers an improvement in channel mismatch and reduced computation over competing STAP algorithms under non-homogeneous ground clutter conditions, [Zhiqin and Shunji, 1996].

4.2.2 Time-Frequency Transforms

Time-Frequency distributions are extensions of traditional Fourier analysis used to cope with time-varying processes. [Cohen, 1989] has described a general class of bilinear time-frequency distributions, of which the continuous Wigner Ville Distribution (WVD) is a member. [Rieck, 1996]-[Rieck, 1997] demonstrates how the space/slow-time covariance matrix can be transformed using the WVD to be a real-valued time-frequency distribution incorporating the phase information.
4.2 Moving Target Indication Techniques for SAR

due to the cross-WVD terms. This new distribution is then used to estimate the target motion parameters. However, even by using FFT techniques, the computational load of this distribution is quite moderate.

An alternate implementation separates the clutter cancellation from the target estimation, [Lombardo, 1997]-[Barbarossa and Farina, 1992]. This approach reduces the computational load but has the side effect of mismatching fast moving targets with a trajectory parallel to the radar path.

4.2.3 Joint Domain Localised Processor

The Joint Domain Localised processor was formulated by [Wang and Cai, 1994] and later applied to MTI for SAR by [Lombardo, 1998]. It uses a time-varying transformation to compensate for the time-varying Doppler frequency of the clutter echoes. A two dimensional Fast Fourier Transform (FFT) then transforms the data into the DOA/Doppler plane where only the data localised around the centre is used for adaption. A Generalised Likelihood Ratio Test (GLRT) is then used to detect targets. This reduces the computational complexity and offers close to optimal detection performance, but requires a proper time-varying transformation and is less intuitive for targets with an along track velocity component, [Klemm-R., 2004]. This processor is shown below in Figure 4.3.

![Figure 4.3. Joint domain localised processor](image)

4.2.4 Along Track Interferometry

Along Track Interferometry (ATI) is a widely used and proven technique for MTI in SAR, [Schulz et al., 2002]- [Gierull, 2002]. It uses two displaced antennas connected to parallel receiving channels. SAR images are generated for each channel and the spatial time delay is compensated by two different reference signals. If the first image is multiplied by the complex conjugate of the second, the remaining phase is zero for stationary objects and non-zero otherwise.

The conventional ATI implementation however, suffers from a lack of sensitivity in detecting targets. To improve its performance, the antennas have to be widely separated which leads to a number of blind velocities, [Ender, 1999a]. One variation is to use monopulse techniques to measure the velocity and DOA of the moving targets, [Cantalloube, 2002]. However, by increasing the sensitivity of target detection, the alignment errors are also increased. Therefore the MTI signal processing primarily uses the radial velocity component of the target in the detection process and is often adaptive. By using combinations of channels, there are a number of different methods for cancelling the ground clutter. Cantalloube concludes the necessity of
having three channels or more for the MTI task, since using only two provides poor localisation of targets. Further detail on monopulse techniques can be found in [Soumekh, 1999].

### 4.2.5 Slow-time STAP

Ender and Klemm published the first papers on the application of space/slow-time Finite Impulse Response (FIR) filters for ground clutter suppression in SAR, [Ender and Klemm, 1989]-[Klemm and Ender, 1990a]. They showed that space/slow-time filters can achieve good rejection performance well in excess of conventional single channel MTI techniques, particularly with targets that move slower than the radar platform. A two-dimensional least squares FIR filter can be used to approximate the optimum solution and if the Pulse Repetition Frequency (PRF) is chosen so that the total bandwidth is sampled at the Nyquist rate, no degradation due to the signal bandwidth will occur.

Ender later published two more papers on STAP MTI for SAR, [Ender, 1998c]-[Ender, 1999a]. He looked at implementing slow-time STAP in both the time and frequency domains and compares the optimal solution with a number of reduced rank filters. These approaches are also known as pre and post-Doppler STAP. In the pre-Doppler domain, filtering suffers from unequal sub-aperture characteristics and the filter lengths have to be extended more and more to achieve acceptable clutter suppression. While in the post-Doppler domain, interference suppression is performed for each Doppler cell and these variations cancel out. Also in this domain, the clutter energy is contained in a one-dimensional subspace and can be cancelled without sensitivity to any amplitude variations. This is the reason why post-Doppler STAP is recommended for SAR as there are often high dynamics and strong target returns. Using this domain, a subspace based transformation has been presented by [Ender, 1999b] which offers stabilisation against rapid flight manoeuvres, coverage of uncertain frequency regions and avoids the suppression of moving target signals. These algorithms have shown good results with both the AER-II and PAMIR SAR systems presented in Section 2.2.5, [Ender, 1996]- [Meyer-Hilberg et al., 2002].

A more complete summary of different slow-time STAP algorithms is presented by [Ward, 1994]. Apart from the optimal formalisation, he categorised four classes of sub-optimal slow-time STAP algorithms. These include the pre and post Doppler element space algorithms already mentioned, as well as formulations in the spatial frequency domain, known as beamspace. In SAR however, there are typically a large number of pulses and only a small number of spatial channels. This implies that post-Doppler element space techniques are most suitable for SAR. The relationship between these four classes are shown in Figure 4.4.

Nearly all the literature on slow-time STAP for SAR has used it as a means to apply MTI. It can however, be used equally well to reject stationary interferences, as shown by [Ender, 1998a]. This chapter provides an overview on how slow-time STAP can be applied to either case. Later chapters will explore the application to interference rejection in more detail.
4.3 Adaptive Array Processing

For an $N$ channel uniform linear array, the Coherent Processing Interval (CPI) is used to represent the interval when the received raw data is coherent. It has three dimensions, slow-time, fast-time and spatial, where slow-time represents the $M$ samples from pulse to pulse and fast-time by the $L$ samples within a pulse. It is commonly represented as a CPI datacube, as shown in Figure 4.5.

This section considers the problem of detecting moving targets using spatial only beamforming. These concepts will then be applied to STAP and also modified for use in SAR. Consider the effect of a unit-amplitude, narrowband plane wave incident on the linear array with antenna location relative to the phase centre, $d_n = n\delta$. This ensures that the spatial steering vector is given as the phase difference between the $n^{th}$ antenna and the antenna phase centre for a
scatterer at an angle $\theta$ to broadside.

$$s_n(\theta) = \exp \left[j k_c d_n \sin \theta \right]$$  \hspace{1cm} (4.1)

where $k_c$ is the wavenumber at the carrier frequency. To focus an array of receivers, the spatial steering vector is used to determine the response for each focusing direction, $\theta$. If a vector is formed from the $N$ channels of the received data signal for the $m^{th}$ pulse and $l^{th}$ range bin, $x_n(t_l, u_m)$, when no range compression is being used, the data and spatial steering vector can be written as\(^9\),

$$\mathbf{x}(t_l, u_m) = \frac{1}{\sqrt{N}} \left[ x_1(t_l, u_m), \ldots, x_N(t_l, u_m) \right]^T \in \mathbb{C}^{N \times 1},$$

$$\mathbf{s}(\theta) = \frac{1}{\sqrt{N}} \left[ s_1(\theta), \ldots, s_N(\theta) \right]^T \in \mathbb{C}^{N \times 1}$$  \hspace{1cm} (4.2)

where both vectors have been normalised. With these definitions, the focussed or beamformed conventional output is given by,

$$f(\theta, t_l, u_m) = \mathbf{s}^H(\theta) \mathbf{x}(t_l, u_m).$$  \hspace{1cm} (4.3)

A more general form of Equation 4.3 replaces the steering vector with an adaptive weight vector. A block diagram of this adaptive process is shown in Figure 4.6.

$$f_{sp}(\theta, t_l, u_m) = \mathbf{w}^H(\theta) \mathbf{x}(t_l, u_m).$$  \hspace{1cm} (4.4)

---

\(^9\)The reference channel does not have to be the first element. As shown in Section 2.3.5 to maintain the phase centre for an odd number of elements, it is often chosen to be at the middle of the array.
4.3 Adaptive Array Processing

4.3.1 Optimal Processing

The optimal weight vector can be selected with different criteria. The two used in this thesis are the maximum SINR and the minimum mean output power of the interference plus noise. The latter criteria, for narrowband signals, also satisfies the maximum SINR constraint but offers the additional benefit of adding constraints to the optimisation. For ease of notation, the dependence on \((t_l, u_m)\) will be dropped during this section.

**Maximum Signal to Interference plus Noise Ratio**

The first optimal criteria is the maximum SINR which is a measure of the total signal power to mean interference plus noise power. Consider an output signal with a deterministic target, \(s_T = s(\theta_T)\) of strength \(\alpha_T\) and an interference plus noise vector, \(z\), which is random with zero-mean and finite variance,

\[
x = \alpha_T s_T + z \in \mathbb{C}^{N \times 1}.
\]

If this is used in Equation 4.4,

\[
f_{sp} = w^H (\alpha_T s_T + z) = y_s + y_z
\]

then the SINR can be defined as

\[
S_{\text{INR}} \equiv \frac{|y_s|^2}{E\{|y_z|^2\}}
\]

where the target signal power is given by

\[
|y_s|^2 = \sigma_T^2 |w^H s_T|^2 = \sigma_T^2 S_{\text{NR},T} |w^H s_T|^2
\]

with \(\sigma_T^2 = |\alpha_T|^2\) is the target power, \(\sigma_T^2\) is the noise power and \(S_{\text{NR},T}\) is the target SNR. The expected value of the interference plus noise power is determined by,

\[
P = E\{|y_z|^2\} = E\{|w^H z|^2\}
\]

\[
= w^H E\{zz^H\} w
\]

\[
= w^H R_z w
\]

where \(R_z = E\{zz^H\} \in \mathbb{C}^{N \times N}\) is the covariance matrix of the noise plus interference. The SINR can then be written as

\[
S_{\text{INR}} = \frac{\sigma_T^2 S_{\text{NR},T} |w^H s_T|^2}{w^H R_z w}
\]

and the corresponding maximum SINR criteria stated as:

\[
\max_w \left\{ \frac{|w^H s_T|^2}{w^H R_z w} \right\}.
\]

The solution is found using Schwartz’s inequality and is given by,

\[
w_{\text{SINR}} = \gamma R_z^{-1} s_T \in \mathbb{C}^{N \times 1}
\]
where $\tilde{\gamma}$ is a scalar that does not affect the SINR. With this weight vector, the corresponding SINR is given by,

$$S_{\text{INR}} = \sigma^2 \nu S_{\text{NR}, T} |(\tilde{\gamma} R_z^{-1} s_T)^H s_T|^2 \left(\tilde{\gamma} R_z^{-1} s_T\right) R_z \left(\tilde{\gamma} R_z^{-1} s_T\right)$$

$$= \sigma^2 S_{\text{INR}, T} (s_T^H R_z^{-1} s_T)^2$$

$$= \sigma^2 S_{\text{INR}, T} s_T^H R_z^{-1} s_T.$$

Similarly, the interference plus noise mean output power using the maximum SINR constraint is found by substituting the weight from Equation 4.12 into Equation 4.9,

$$P_{\text{SINR}}(\theta_T) = |\tilde{\gamma}|^2 s_T^H R_z^{-1} s_T.$$

### Minimum Output Power with Linear Constraints

An alternate optimisation criteria involves minimising the mean output power of the interference plus noise subject to a set of linear constraints, $C^H w = d$. The constraint matrix is defined by $C$ with the corresponding constraining values defined by $d$. This is also known as the Linearly Constrained Minimum Variance (LCMV) beamformer, [Van-Trees, 2002] and can be formally stated as

$$\min_w \left\{ w^H R_z w \right\} \text{ subject to } C^H w = d. \quad (4.15)$$

The constrained optimisation problem is solved using Lagrange multipliers to find the weight vector [Johnson and Dudgeon, 1993],

$$w_{\text{LCMV}} = R_z^{-1} C \left[ C^H R_z^{-1} C \right]^{-1} d \in \mathbb{C}^{N \times 1}. \quad (4.16)$$

This processor is often presented using the total covariance, which implies the constraint condition is minimising the total power instead of just the interference plus noise power. It then has the same solution with $R_z$ being replaced with $R_x$. [Van-Trees, 2002] refers to the latter processor as the optimal linearly constrained minimum power processor.

One of the most common constraints is the Minimum Variance Distortionless Response (MVDR) and constrains the beampattern to be unity in the steering direction. It is formed by setting $C = s_T$ and $d = 1$ in Equation 4.16. The weight vector then has the form,

$$w_{\text{MVDR}} = \frac{R_z^{-1} s_T}{s_T^H R_z^{-1} s_T} \in \mathbb{C}^{N \times 1}. \quad (4.17)$$

which equals Equation 4.12 when $\tilde{\gamma} = 1/(s_T^H R_z^{-1} s_T)$. With this choice of optimisation constraint, the maximum SINR constraint is satisfied and the corresponding SINR is the same. The output power will also be the same in the absence of any jammer signals. When they are present.
4.3 Adaptive Array Processing

however, the processor will form a tight beam on the jammer direction, as the beamformer meets its constraint of unity in the beamsteered direction. The form of the MVDR mean output power is found by substituting the weight from Equation 4.17 into Equation 4.9,

$$P_{\text{MVDR}}(\theta_T) = \frac{1}{{s_T^H R_z^{-1} s_T}}$$  \hspace{1cm} (4.18)

Other constraints used later in Chapter 8 include amplitude and first and second order derivative constraints. These can be used to broaden the adaptive peak and make the adaptation more robust to steering errors or non-stationary interference.

4.3.2 Beamforming Example

Consider an example with an $N = 15$ element uniform linear array with $\delta = \lambda_c/2$ spacing between elements. A target signal is incident at broadside, $\theta_T = 0^\circ$ with power $\sigma_T^2 = 0$ dB and a jammer signal, $\theta_J = 40^\circ$ with power $\sigma_J^2 = 20$ dB. Both signals are modelled as plane waves, with thermal noise also present from the receiver with power $\sigma_n^2 = 0$ dB. This gives the total received signal as,

$$x_n(t,u) = g(t,u)s_n(\theta_T) + z_n(t,u)$$  \hspace{1cm} (4.19)

with the interference plus noise,

$$z_n(t,u) = J(t,u)s_n(\theta_J) + \nu(t,u)$$  \hspace{1cm} (4.20)

and $g(t,u)$ and $J(t,u)$ are the time varying components of the target and jammer signals. The ideal interference plus noise covariance is modelled using the spatially stacked version of the interference, with the mean square jammer and noise signals approximated by the jammer and noise powers respectively. The cross terms are assumed to be zero as the signal components are uncorrelated.

$$R_z = E\{z(t,u)z^H(t,u)\}$$
$$= E\{|J(t,u)|^2 s(\theta_J)s^H(\theta_J)\} + E\{\nu(t,u)|^2 I_N\}$$
$$= \sigma_J^2 s(\theta_J)s^H(\theta_J) + \sigma_n^2 I_N \in C^{N \times N}$$  \hspace{1cm} (4.21)

Beampattern

From the previous section, the weights for a beam steered in direction $\theta_T$ are given by,

$$w_{\text{conv}} = s_T \in C^{N \times 1}$$
$$w_{\text{SINR}} = R_z^{-1}s_T \in C^{N \times 1}$$
$$w_{\text{MVDR}} = \frac{R_z^{-1}s_T}{s_T^H R_z^{-1} s_T} \in C^{N \times 1}$$  \hspace{1cm} (4.22)

where the SINR weight has $\bar{\gamma} \equiv 1$. The beampattern is formed by measuring how much of the power from a signal $s(\theta)$ will leak into the adaptive beam. When $\theta$ is varied over a range of angles, $-90^\circ \leq \theta \leq 90^\circ$,

$$y_b(\theta) = w^H s(\theta)$$  \hspace{1cm} (4.23)
which forms the beampattern,

\[ P_b(\theta) = |y_b(\theta)|^2. \] (4.24)

The beampattern of the conventional, maximum SINR and MVDR beamformers are shown in Figure 4.7 for \( \theta_T = 0^\circ \). The two adaptive beamformers produce identical results as they focus on the target direction and null the jammer at 40°.

\[ \text{Figure 4.7. Array beampattern example for } N = 15, \text{ (—) conventional, (--) max. SINR constraint, (- -) MVDR constraint} \]

**Mean Output Power**

The mean output power is formed by varying the steering direction \( \theta_T \) and taking a snapshot of the data at a given time, \((t_l, u_m)\). The angle varying weights are therefore given as,

\[
\begin{align*}
\mathbf{w}_{\text{conv}}(\theta_T) &= \mathbf{s}(\theta_T) \in \mathbb{C}^{N \times 1} \\
\mathbf{w}_{\text{SINR}}(\theta_T) &= \mathbf{R}_z^{-1}\mathbf{s}(\theta_T) \in \mathbb{C}^{N \times 1} \\
\mathbf{w}_{\text{MVDR}}(\theta_T) &= \mathbf{R}_z^{-1}\mathbf{s}(\theta_T)\mathbf{s}^H(\theta_T) \in \mathbb{C}^{N \times 1}
\end{align*}
\] (4.25)

which are then used to filter the data vector,

\[ f_{sp}(\theta_T, t_l, u_m) = \mathbf{w}^H(\theta_T)\mathbf{x}(t, u) \] (4.26)

and form the power spectrum,

\[ P(\theta_T) = E \left\{ |f_{sp}(\theta_T, t_l, u_m)|^2 \right\}. \] (4.27)

Figure 4.8 shows the output power of the conventional, maximum SINR and MVDR beamformers. The conventional power spectrum follows both the target signal and the jammer as the steering vector sweeps in angle. The SINR power spectrum maintains the target peak, but nulls the jammer signal. The MVDR power spectrum however, keeps the target peak but forms a sharp peak in the jammer direction. This is consistent with the optimisation criteria to minimise the mean output power of the interference.
4.4 Airborne STAP Overview

Airborne STAP is a technique that combines not only spatial information, but also that from the temporal slow-time domain. It offers the ability to null ground clutter and any broadband direct-path interferences while detecting and estimating the position (DOA) and velocity (Doppler) of any moving targets. The relationship between the Doppler and the DOA is a common way of determining the performance of a multichannel radar, where \( v_p \) is the platform velocity in the along track direction and \( \lambda_c \) is the wavelength.

\[
f_d = \frac{2v_p}{\lambda_c} \sin \theta
\]  

or normalised with respect to the PRF and spacing between antennas as,

\[
F_d = f_d T_{PRI} = \frac{2v_p T_{PRI}}{\delta} \left( \frac{\delta}{\lambda_c} \sin \theta \right)
= \tilde{\beta} \Theta
\]  

where

\[
\tilde{\beta} = \frac{2v_p T_{PRI}}{\delta}
\]  

is the relationship between normalised Doppler and steering angles as shown in Figure 4.9. This famous figure is a representation of the two-dimensional power spectrum of the received signal at a single range bin and includes the ground clutter, a direct-path jammer and a slow and fast moving target. It was first created by [Klemm and Ender, 1990b] and later refined by [Klemm, 2002].
Figure 4.9. Principle of space/time filtering

The ground clutter signal was determined by the relationship in Equation 4.29. Its position is determined by the slope of the clutter ridge, $\tilde{\beta}$, which in turn relates the spatial separation between pulses and elements in the linear array. The shape of the ground clutter signal will also be modulated by the two-way antenna pattern, which gives it a sinc-squared appearance. The other major signal components are the slow and fast moving targets which are assumed to remain fixed in DOA and Doppler over the CPI and the direct-path jammer signal which covers the entire Doppler band at a fixed DOA angle.

To suppress the ground-clutter using only temporal processing, an inverse temporal filter on the $F_d$ axis could be used. This is depicted at the back of the plot, where the ground-clutter notch is determined by the mainlobe of the projected ground clutter signal. Likewise, if only spatial processing were used to cancel the ground clutter or direct-path jammer, an inverse spatial filter on the $\Theta$ axis could be used. Both of these inverse filters however form a broad stop-band and blind the radar in these intervals.

A space/slow-time filter which operates over the whole $\Theta - F_d$ plane can form a very narrow ridge along the trajectory of the interference spectrum so that even slow targets may fall into the pass band and be detected. Both the ground clutter and direct-path jammer signals can be cancelled with the same filter or separately if preferred.
Signal Model

For a linear array of equi-spaced receivers, the form of the $n^{th}$ spatial channel signal model in terms of the normalised angle, $\Theta$ is given by

$$s_n(\Theta) = \exp[jk_c d_n \sin \theta] = \exp[j2\pi(n-1)\Theta]$$  (4.31)

with corresponding steering vector,

$$s(\Theta) = \frac{1}{\sqrt{N}} [1, \exp[j2\pi\Theta], \ldots, \exp[j2\pi(N-1)\Theta]]^T \in \mathbb{C}^{N \times 1}. \tag{4.32}$$

Now, since each ground clutter patch has a normalised Doppler shift associated with it, each vector of array outputs will have a temporal linear phase progression. Over the $M$ pulses of the CPI, the temporal steering vector is given by,

$$g(F_d) = \frac{1}{\sqrt{M}} [1, \exp[j2\pi F_d], \ldots, \exp[j2\pi(M-1)F_d]]^T \in \mathbb{C}^{M \times 1}. \tag{4.33}$$

If both of these steering vectors are combined using the Kronecker product, the total space/slow-time steering vector can be obtained,

$$\tilde{G}(\Theta, F_d) = s(\Theta) \otimes g(F_d) \in \mathbb{C}^{MN \times 1}. \tag{4.34}$$

To apply a space/slow-time adaptive filter, the CPI data must also be stacked accordingly. If a spatial snapshot is formed for the $l^{th}$ range bin and $m^{th}$ pulse from Equation 4.1, then a $MN \times 1$ space/slow-time vector is formed in the fast-time frequency domain by stacking data from consecutive pulses,

$$\tilde{X}_s(\omega_l) = \frac{1}{\sqrt{M}} [\tilde{x}^T(\omega_l, u_1), \ldots, \tilde{x}^T(\omega_l, u_M)]^T \in \mathbb{C}^{MN \times 1} \tag{4.35}$$

where $\omega_l$ is the $l^{th}$ fast-time frequency sample. With these definitions, the focussed or beam-formed conventional output for a given frequency bin is given by,

$$\tilde{f}(\Theta, F_d, \omega_l) = \tilde{G}^H(\Theta, F_d)\tilde{X}_s(\omega_l) \tag{4.36}$$

or more generally, the steering vector could be replaced with an adaptive weight vector as described in Section 4.3.1. This filtering process is shown in Figure 4.10.

$$\tilde{f}_{ss}(\Theta, F_d, \omega_l) = W^H(\Theta, F_d)\tilde{X}_s(\omega_l). \tag{4.37}$$

The general form of the maximum SINR weight vector is similar to Equation 4.12, but with space/slow-time vectors used instead,

$$W_{SINR}(\Theta, F_d) = \tilde{\gamma}R_{\tilde{Z}_c}^{-1}\tilde{G}(\Theta, F_d) \in \mathbb{C}^{MN \times 1} \tag{4.38}$$

where $R_{\tilde{Z}_c} = E\{\tilde{Z}_c(\omega_l)\tilde{Z}_{c}^H(\omega_l)\} \in \mathbb{C}^{MN \times MN}$ is the space/slow-time covariance of the clutter plus interference plus noise vector, $\tilde{Z}_c(\cdot)$. 


The method of obtaining the data without targets present is an estimation problem called ‘training’ and the method of obtaining the training data is called the training strategy. The final step in airborne STAP is to detect the moving targets and estimate their DOA and Doppler. The detection technique is generally based on a specified Constant False Alarm Rate (CFAR) to threshold the targets from the background noise. This entire scheme is presented in Figure 4.11.
4.4 Airborne STAP Overview

4.4.1 STAP Example

Consider the previous example with $N=15$ spatial channels and $M=15$ pulses. There is a broadside target and jammer at $\theta_J = 40^\circ$ both modelled as plane waves, with the target signal having a normalised steering angle of $\Theta_T = 0$ and normalised Doppler, $F_{d,T} = 0.3$. The jammer signal has a normalised steering angle of $\Theta_J = 0.32$, but is broadband in Doppler. The ground clutter is modelled as a $\sigma^2 = 20$ dB signal coming from each steering direction with Doppler determined by Equation 4.29. This gives the total received signal as,

$$x_n(t, u) = g(t, u)s_n(\theta_T) + z_{c,n}(t, u) \quad (4.39)$$

with the interference components, $z_{c,n}(\cdot)$ now comprising the $M_f$ ground clutter focus positions and the interference plus noise,

$$z_{c,n}(t, u) = \gamma(t, u) \sum_{k=1}^{M_f} s_n(\theta_k) + J(t, u)s_n(\theta_J) + \nu(t, u). \quad (4.40)$$

The ideal interference plus clutter covariance matrix is modelled using the space/slow-time stacked version of the interference plus clutter in the fast-time frequency domain, with the mean square jammer, ground clutter and noise signals approximated by the jammer, ground clutter and noise powers respectively. Again, the cross terms are zero as the signal components are uncorrelated.

$$R_{\tilde{Z}_c} = E \left\{ \tilde{Z}_c(\omega_l)\tilde{Z}_c^H(\omega_l) \right\} = E \left\{ \Gamma(\omega_l)^2 \sum_{k=1}^{M_f} \hat{G}(\Theta_k, F_{d,k}) \right\} + E \left\{ |J_s(\omega_l)|^2 \hat{G}_J(\Theta_J)\hat{G}_J^H(\Theta_J) \right\} + E \left\{ |\Xi(\omega_l)|^2 I_{MN} \right\}$$

$$= \sigma^2 \sum_{k=1}^{M_f} \hat{G}(\Theta_k, F_{d,k}) + \sigma^2 J_\gamma \hat{G}_J(\Theta_J)\hat{G}_J^H(\Theta_J) + \sigma^2 \nu I_{MN} \in \mathbb{C}^{MN \times MN} \quad (4.41)$$

where $\hat{G}_J(\cdot)$ is the space/slow-time broadband jammer vector and $\Gamma(\cdot), J_\gamma(\cdot)$ and $\Xi(\cdot)$ are the space/slow-time frequency varying ground-clutter, jammer and noise components.
Beampattern

From the previous section, the weights for a beam steered in the direction $T$ are formed by using the target signal, $\tilde{G}_T = \tilde{G}(\Theta, F_d, T)$,

$$
W_{\text{conv}} = \tilde{G}_T \in \mathbb{C}^{MN \times 1}
$$

$$
W_{\text{SINR}} = R_{Z_c}^{-1} \tilde{G}_T \in \mathbb{C}^{MN \times 1}
$$

$$
W_{\text{MVDR}} = \frac{R_{Z_c}^{-1} \tilde{G}_T \tilde{G}_T^H R_{Z_c}^{-1}}{\tilde{G}_T \tilde{G}_T^H} \in \mathbb{C}^{MN \times 1}
$$

(4.42)

where the SINR weight has $\bar{\gamma} \equiv 1$. The beampattern is then formed by measuring how much power from $\tilde{G}(\Theta, F_d)$ will leak into the adaptive beam. Both the normalised steering angle and Doppler are varied over $-0.5 \leq \Theta, F_d \leq 0.5$, to give,

$$
y_b(\Theta, F_d) = W^H \tilde{G}(\Theta, F_d)
$$

(4.43)

which produce the two dimensional beampatterns by,

$$
P_b(\Theta, F_d) = |y_b(\Theta, F_d)|^2.
$$

(4.44)

It should be noted that the optimal beampattern can be formed with either of the optimal weights in Equation 4.42. Examples of both the conventional and optimal beampatterns are shown in Figure 4.12. Its clear from this figure that the optimum processor is using very high resolution nulls to remove the ground clutter and jammer signals which are not in the steering direction.

Figure 4.12. STAP beampattern example
4.4 Airborne STAP Overview

Mean Output Power

Similarly to the spatial case, the mean output power is formed by varying the steering direction and Doppler, \((\Theta_T, F_{d,T})\) and fixing the data vector at a frequency \(\omega_l\). The weights now have the form,

\[
\begin{align*}
W_{\text{conv}}(\Theta_T, F_{d,T}) &= \hat{G}(\Theta_T, F_{d,T}) \in \mathbb{C}^{MN \times 1} \\
W_{\text{SINR}}(\Theta_T, F_{d,T}) &= R_{Z_c}^{-1} \hat{G}(\Theta_T, F_{d,T}) \in \mathbb{C}^{MN \times 1} \\
W_{\text{MVDR}}(\Theta_T, F_{d,T}) &= \frac{R_{Z_c}^{-1} \hat{G}(\Theta_T, F_{d,T}) \hat{G}^H(\Theta_T, F_{d,T})}{R_{Z_c}^{-1} \hat{G}(\Theta_T, F_{d,T})} \in \mathbb{C}^{MN \times 1}.
\end{align*}
\]

These are used to filter the data vector \(\hat{X}_s(\cdot)\),

\[
\tilde{f}_{ss}(\Theta_T, F_{d,T}, \omega_l) = W^H(\Theta_T, F_{d,T})\hat{X}_s(\omega_l)
\]

and form the power spectrum,

\[
P(\Theta_T, F_{d,T}) = E \left\{ |\tilde{f}_{ss}(\Theta_T, F_{d,T}, \omega_l)|^2 \right\}
\]

which is shown for the three algorithms in Figure 4.13. The conventional power spectrum shows only the jammer signal and very little detail of anything else. The max SINR spectrum of the other hand, has muled the jammer and ground clutter signals and preserved the target of interest. The MVDR power spectrum has minimised the interference and clutter signals and just reveals the target signal. Obviously, the intended application will determine which approach to use in a real system. The max SINR approach should be used for preserving the signal of interest, while the MVDR is designed to minimise the clutter and/or interference and will not guarantee the target’s detection.

![Figure 4.13. STAP power spectrum example](image-url)
4.5 From Airborne Radar to SAR

SAR is a high resolution imaging system interested only in a small patch of ground a long way from the radar platform. This is in contrast to the airborne radars which scan a wide region to detect moving targets. The biggest difference between SAR and MTI is the required resolution, which in turn relates to the integration time required to achieve a fine azimuth resolution and the bandwidth required for fine range resolution.

4.5.1 Integration Time

The longer integration time will cause both the target signal and ground-clutter signals to change in DOA and Doppler. Therefore the Doppler/DOA representation from the previous section will not be accurate and a more exact representation of the space/slow-time steering vectors is required. [Farina and Lombardo, 2004] present a new method of representing a SAR echo using a three dimensional representation in the slow-time/frequency/DOA domain to illustrate both the non-linear phase modulation and the angular position at the same time.

A more accurate signal representation is determined by the SAR signal model post range processing and was derived in Section 2.3.5. If the fast-time Doppler is considered negligible and \((t, u)\) represents the fast-time within a pulse and the SAR position respectively,

\[
s_{\text{post},n}(t, u, x, y) = \exp \left[jk_{c}d_{n}\sin \theta(u)\right] \text{sinc} \left[B(t - \tau_{n}(x, y - u))\right] \tag{4.48}
\]

where \(B\) is the SAR bandwidth, \(\theta(u) = \arctan \left(-\frac{u}{X_{c}}\right)\) is the aspect or steering angle relative to the offset range, \(X_{c}\) and the two-way space/time delay is given by,

\[
\tau_{n}(x, y) = \frac{2R(x, y - d_{n})}{c} \tag{4.49}
\]

with radial distance \(R(\cdot)\).

4.5.2 Bandwidth

Higher range resolution will generate effects which can drastically reduce STAP performance, [Cerutti-Maori, 2002]. The limiting factor is the range migration will be present over a number of neighbouring range bins. This is common to both single and multiple channels and can cause a reduction of the signal to ground clutter power ratio, seriously impairing target detection capabilities, especially for objects moving with low radial velocities. To avoid these problems, the STAP formulation must be modified to take this effect into account.
4.6 Conclusion

This chapter has covered a brief literature review of MTI techniques for SAR, including adaptive DPCA, ATI and slow-time STAP. The latter technique has shown the best results in a real system and is now the benchmark for any MTI scheme. The second section introduced adaptive array processing and described a number of optimal processors commonly used in beamforming. This led into a description of slow-time STAP for airborne radar. This approach is not directly suitable for SAR however and the final section looked at the main differences with applying STAP for SAR.
Chapter 5

Jammer Background and Model

5.1 Introduction

Airborne surveillance radars need to be able to image the ground and/or detect targets in environments that include a variety of interference sources. These include both terrestrial communications and hostile airborne or ground based jammers. A typical barrage noise jammer transmits a wideband direct-path signal and a reflected multipath signal known as hot-clutter or Terrain Scattered Interference (TSI) as shown in Figure 5.1.

As the jammer signal reflects from the terrain, it is scattered diffusely over a range of azimuth angles which leads to non-stationarity in the received data as the scattering statistics change with the terrain. A secondary cause of non-stationarity occurs from the changing motion between the Synthetic Aperture Radar (SAR) and jammer platforms which induces a bistatic Doppler shift for each reflection. Hot-clutter will greatly limit a radar’s ability to observe targets and the non-stationarity requires adaption within a pulse for effective suppression.

Figure 5.1. Interference Sources in SAR
5.2 SAR Jamming Literature Review

The first part of this chapter contains a comprehensive literature review of jamming and anti-jamming techniques for SAR and methods for suppressing hot-clutter in airborne and High Frequency Over The Horizon Radar (HF OTHR). The second part from Section 5.4 describes mathematical models for the transmitted jammer signal, the diffuse scattering used to describe the hot-clutter reflections and the jammer and noise waveforms.

5.2 SAR Jamming Literature Review

As there has not been any published results on the effect of hot-clutter for SAR imaging, the first part of this literature review looks at the main jamming techniques proposed for SAR. This is followed by information on some real world jammers in Section 5.2.2 and anti-jamming techniques for SAR in Section 5.2.3.

5.2.1 Jamming Techniques for SAR

There are only a small number of publications addressing jamming for SAR. Two summaries include a book on electronic warfare for SAR by [Goj, 1992] and a paper by [Yongfu and Shuyuan, 1996]. These look at how different methods of jamming will effect a SAR image and Goj suggests methods to potentially defeat them. The four main jamming techniques summarised in this section include barrage jamming, spot jamming, random pulse jamming and deceptive jamming.

**Barrage Jamming**

Barrage jamming radiates random noise over a wide band of frequencies, typically larger than the SAR bandwidth. This ensures that some of the jammer noise will be in the passband of the radar, raising the noise floor and thereby obscuring moving targets or features of interest. This technique however requires the Effective Radiated Power (ERP) to be thinly spread over the its total bandwidth reducing the amount of jammer noise power within the SAR’s passband.

[Dumper et al., 1997] derived a simple expression for the jammer to ground clutter ratio. The expression shows that the only system level parameters which effect the SAR’s susceptibility to jamming are the average transmitted power, the jammers ERP and the imaging geometry. The effect of a barrage jammer on a SAR image is to add random speckle noise of uniform intensity over the entire swath in the same manner as thermal noise. Figure 5.2 is an image taken from DSTO’s INGARA SAR with and without a simulated barrage jammer. Only a small amount of noise power has been used for the demonstration as too much results in a completely noisy image. Degradation can be seen at the two highlighted regions.
Spot Jamming

Spot jamming is the most effective jamming method when the centre frequency of a SAR is known as it allows the ERP and hence the noise level to be completely within the passband of the radar.

[Condley, 1991] looked at the possibility of spot jamming the SEASAT\textsuperscript{10} SAR. He calculated that it was possible to jam a signal from the mainbeam of the SAR, but if the jamming signal was aimed at the sidelobes, more advanced sub-systems would be required. The effect of spot jamming on a SAR image will be a speckle noise of uniform intensity covering the entire swath. However, since the spot jamming signal is narrower in frequency than the thermal noise, frequency domain processing will result in blurring of the speckle noise along the range dimension. This increases the overall effectiveness of the jammer as more of the image will be distorted, [Goj, 1992].

Random Pulse Jamming

Random pulse jamming implies the jamming pulses are transmitted at random intervals. This technique was originally designed to create a large number of false targets for a conventional pulsed radar and has the effect of raising the noise floor in a SAR proportionally to the average ERP.

The jamming noise will extend over the entire range swath in the SAR image and will appear as speckle noise stretched in range. The random pulse jammer speckle however will exhibit more pronounced brightness variations as fewer noise samples are added non-coherently and the smoothing effect of multiple looks is reduced.

\footnote{\textsuperscript{10}The 1978 SEASAT mission was a spaced based single channel SAR, [SEASAT, 1978].}
5.2 SAR Jamming Literature Review

Deceptive Jamming

Deceptive or coherent jamming can be classed into ‘repeater’ and ‘response’ jamming. Repeater jamming aims to intercept the transmitted SAR pulse, estimate the operating parameters and directly retransmit false targets into the SAR return. Response jamming on the other hand, can create false targets at any desired pulse. It uses prior knowledge of the operating parameters before receiving a SAR pulse and can adapt its parameters after receiving one. For both of these techniques, the deceptive jamming signal will appear as a barrage jammer if the parameters are not correctly estimated.

[Hyberg, 1998] has looked at the effect of coherent jamming on SAR. He used a simulation model verified with several flight trials and showed that the effectiveness of coherent jamming for SAR is limited by geometrical defocusing, extended range migration and pulse compression mismatch.

[Wei et al., 2004] has looked at jamming a cluster of spaceborne SAR satellites using deceptive jamming techniques. Their technique is a ‘parameter swiftly changing’ method which modifies the chirp rate or centre frequency to modify the received ground clutter of the scene. A second paper by [Wei et al., 2005a] looks at emulating moving targets using multiple jammers at specific ground locations.

5.2.2 Real World Jammers

There is only a small amount of information available in the public domain on real world airborne jammers. Nearly all the literature dealing with jamming considers only barrage noise jammers and more complex waveforms are generally left to the classified domain. A good source of information is [Jane’s Defence Weekly, 2005]. Basic details on a number of real world airborne jammers are available, including the radar band, physical description, power, dimensions and the contractor who designed it. While not intended for use against SAR specifically, these jamming systems can be used in all the modes from the previous section except deceptive jamming.

One example is the ALQ-88K radar jammer from LG Innotek Co. Ltd. which is a pod-mounted self-protection jammer for use on tactical aircraft such as the F-4 and the F-16 and is designed to counter surface-to-air, anti-aircraft artillery and airborne intercept radar emitters. It operates between 2 and 20GHz with 8.5kVA of power and can produce a range of noise, (airborne target) deception and complex jamming modes.

An example of a deceptive jammer for SAR is described by [Kristoffersen and Thingsrud, 2004]. It is typically referred to as a Digital Radio Frequency Memory (DFRM) and uses Field Programmable Gate Array (FPGA) circuits to do the signal processing.
5.2.3 Jammer Suppression for SAR

The majority of publications on jammer suppression for SAR have looked at STAP techniques for removing a barrage jammer incident in the SAR mainbeam. One of the earliest papers by [Klemm, 1996a] looked at suppression of both ground clutter and a single mainbeam jammer using a forward looking SAR in the space/slow-time domain. After extensive simulations, he concluded that due to every Doppler frequency containing interference signals from two DOA’s (ground clutter and direct-path jammer), performance was sensitive to the number of spatial channels. The greater the number of channels, the more degrees of freedom are available and the closer performance approaches the optimum.

[Ender, 1998a] has provided the most comprehensive paper on SAR anti-jamming against a direct-path jammer. A number of techniques were analysed and tested with simulation parameters mirroring the AER-II SAR. He first looks at spatial only methods, using the ideal and estimated covariance, with and without constraints. Better performance is then demonstrated using a space/slow-time anti-jamming filter with image reconstruction using a reduced rank Wiener Filter.

[Bidigare, 2001] also presented a space/slow-time technique for direct-path jammer suppression of mainbeam jamming signals. His paper looks at the improvement over spatial only beamforming and suggests an alternate scheme for when the interference source is not white noise or when it is desirable to estimate the location of the interfering signal.

[Klemm, 2002] later looked at a direct-path jammer suppression technique using fast-time STAP. His models were take from [Compton Jr., 1988a] and applied pre range processing to simulated data. The paper concluded that firstly, fast-time filtering will degrade SAR resolution by broadening the point spread function mainlobe and increasing its sidelobes and secondly, as range resolution improved, sensitivity to filtering increased. Due to the filter weights changing over azimuth, these effects were worse when focusing near to the azimuth filter notch. It was found that a larger array can keep the notches narrower and hence reduce the influence of jammer notches.

A technique for preventing spaceborne SAR jamming is presented by [Wei et al., 2005b]. They propose a method to transform the non-uniform spacing of the satellite constellation into a uniform sequence and apply slow-time STAP to remove the direct-path jammer. This technique will raise the computation cost however, which is very expensive for a space platform.

Protection against deceptive repeat jamming in SAR is a very new area of publication. [Soumekh, 2005] suggests a pulse diversity scheme to detect against false targets. It uses a modulated chirp signal which varies for each pulse and is known only to the SAR. The merits of this scheme are shown using the autocorrelation and cross correlation of these signals.
The hot-clutter signal will limit a radar’s ability to observe targets and spreading over azimuth during the coherent processing interval means that space/slow-time suppression techniques will only be partially effective. Also, a finite bandwidth implies multipath reflections are partially coherent with the direct-path jamming signal and as the interference varies in power from one range bin to the next, hot-clutter suppression should be undertaken by employing adaptive processing in both space and fast-time, [Griffiths et al., 2000].

It is unclear when hot-clutter was identified as a problem in airborne radar, but mainstream publications on hot-clutter suppression techniques have been available since the mid-nineties. There is now a great number of papers addressing this problem, many of which were presented at the Adaptive Sensor Array Processing workshops held each year at Lincoln Labs since 1993.

The first part of this section describes two experimental programs performed for researchers to test their hot-clutter suppression algorithms. Three Dimensional (3D) STAP is then introduced in Section 5.3.2 as the optimal method of suppressing both hot-clutter and ground clutter simultaneously. Of course, the computational requirement of this approach is unachievable in a real system, and the majority of work has been focussed on reduced rank schemes. Section 5.3.3 looks at element-space techniques and Section 5.3.4 addresses frequency domain and sub-band STAP implementations. Beamspace processing and sidelobe cancellers are addressed in Section 5.3.5 with subspace methods and the Multistage Wiener Filter (MWF) in Section 5.3.6. Constrained techniques for implementing many of the previous algorithms are then presented in Section 5.3.7 followed by the summary of a number of non-mainstream contributions in Section 5.3.8. The final Section 5.3.9 is not concerned with STAP, but rather addresses waveform diversity techniques for suppressing hot-clutter.

5.3.1 Experimental Work

There has only been two experimental programs to test and evaluate hot-clutter suppression algorithms on real airborne radar data. The first was performed by the Defense Advanced Research Projects Agency (DARPA) and is known as the ‘MOUNTAINTOP’ program, [Titi and Marshall, 1996]. It started in the early 1990’s and was designed to study advanced processing techniques and technologies required to support the mission requirements of next generation airborne early warning platforms. The experiment was performed with the radar receiver positioned on top of a mountain to simulate an airborne environment. The antenna array was receiving only in these trials, so there is no ground clutter in any of the data sets.

A second experimental trial was conducted by the Hughes Aircraft Company and the Wright Patterson Air Force Laboratory in the USA, [Brovko et al., 1997]. Their goal was to collect joint ground and hot-clutter flight test data, verify simulation models and demonstrate post Doppler STAP suppression algorithms. The flights were conducted in 1996 over the California coast and desert area near Edwards Airforce base.
5.3.2 Three Dimensional STAP

One of the most helpful papers on hot-clutter suppression is by [Rabideau, 2000], who introduces space/slow-time/fast-time or 3D STAP as well as alternative techniques for partitioning the processing to reduce the computational load. The most common method is to cancel the hot-clutter first and then the ground clutter in a cascaded arrangement. There are however two side effects with this approach. The first is a ‘training modulation’ which arises due to averaging over a finite number of different realisations and the second is a ‘coherence modulation’ which results from non-stationarity in the data being used for the covariance estimate. These will act to blur the ground-clutter and reduce the effectiveness of MTI. The goal therefore of reduced rank algorithms is to suppress the hot-clutter while reducing the impact of these modulations.

Three other good summary papers comparing 3D STAP to reduced rank alternatives are by [Gabel et al., 1999] and [Guerci et al., 1999]-[Guerci et al., 2000]. They conclude that although 3D STAP achieves optimal results, it is computationally unfeasible and reduced rank techniques are essential.

Another relevant paper is by [Techau et al., 1999a] who developed a method to calculate the interference covariance matrix under specific topographical conditions and determine performance bounds for 3D STAP based on different terrain models. [Parker and Swindlehurst, 2001], also presented a cancellation technique using a STAP model to form a 3D autoregressive filter. Their results show improved target detection performance, primarily around the ground clutter notch.

5.3.3 Element-Space Processing

Element space processing is the traditional method using an FIR filter with fast-time taps behind each element. The block diagram presented in Figure 4.10 applies to this case except now the delay between samples is the fast-time sampling period. The use of fast-time taps enables the coherency from hot-clutter reflections to aid in the total interference suppression. [Fante and Torres, 1995] and [Kogon et al., 1998], [Kogon et al., 1997] have implemented this technique and shown an improvement in the overall Signal to Interference plus Noise Ratio (SINR). For more detail of this technique, see Chapter 8.

5.3.4 Fast-time Frequency and Subband Processing

[Compton Jr., 1988a] has demonstrated how adding fast-time taps can improve the adaptive SINR performance of a STAP processor. A second paper by [Compton Jr., 1988b] then studied the equivalence of filtering in the space/fast-time versus the space/fast-time frequency domain. He found that the adaptive SINR was identical for filtering in either domain.

A practical technique to reduce the computational load known as ‘subbanding’, [Fante, 1991], [Weiss et al., 1999], [Steinhardt and Pulsone, 2000]. This splits the total bandwidth into a number of subbands and instead of performing a 2D STAP operation for each frequency bin, its now performed for each subband. The final step is to then recombine the output from each subband after adaptive filtering.
5.3 Hot-clutter Suppression Literature Review

An early study by [Fante, 1991] looked at the tradeoff with subbanding and the number of fast-time taps. For the case of two channels, he used a simulation model and an analytic expression for the cancellation ratio to determine the number of fast-time taps and subbands required to achieve a specified cancellation level.

One problem with subbanding is found when targets become spread across subbands and errors in the subband covariance matrix estimation causes the range sidelobe levels to increase following subband recombination. [Hoffman and Kogon, 2000] has addressed these issues by tuning the STAP filters to use a constant velocity rather than a constant Doppler and using linear constraints to control the subband STAP beampatterns at multiple points across the beamwidth.

A comparison of two different subbanding techniques is presented by [Jouny and Culpepper, 1994] and [Jouny and Amin, 1996]. They compare uniform subbanding with the discrete Fourier transform with non-uniform subbanding using the wavelet transform. Their results show that the best performance for correlated arrivals is achieved with the Fourier transform.

5.3.5 Beamspace Processing and Sidelobe Cancellers

Beamspace processing is implemented in the spatial frequency domain and under ideal conditions can be shown to be equivalent to element space approaches, [Gray, 1982]-[Godara, 1987]. If the spatial signals are transformed into a series of beams steered in desired directions, the number of beams may be less than the number of channels and reduced rank STAP processors can be formulated.

By first filtering the hot-clutter prior to the ground clutter (cascaded filtering), modulations will be introduced and can effect the detection of moving targets. A technique to reduce this effect is to use an auxiliary reference beam pointed at the direct-path signal. This technique is known as the ‘single reference beam canceller’, [Kogon, 1996] or the ‘selected auxiliary TSI mitigation processor’, [Gabel et al., 1999] and has also been used by [Fante et al., 1996] for mainbeam jammer suppression. It uses the reference beam to remove the hot-clutter in the mainbeam and since the reference beam is strong, the magnitude of the corresponding adaptive weights will be reduced, thereby reducing the effect of the modulations. In practice however, the desired signal may leak into the reference beam, causing unwanted target mitigation and there can be a long convergence time before the adaption works.

To overcome these problems, the generalised sidelobe canceller, [White, 1983]-[Johnson and Dudgeon, 1993] has been applied to this problem by [Kogon et al., 1996a], [Kogon, 1996]. His implementation uses fast-time taps to form a beamspace/fast-time STAP processor. The mainbeam is now steered in the target direction and the reference beams go through a blocking matrix to remove the desired signal from the data. This signal then goes through an adaptive filter before being subtracted from the mainbeam. The beamspace formulation also provides a much smaller adaptive convergence time due to a reduction of the temporal filter length. This algorithm is explained in more detail in Chapter 8.
Kogon has also looked at a number of reduced rank techniques for applying beamspace/ fast-time STAP, [Kogon, 1996], [Kogon et al., 1996b]. These include eigen-decomposition methods and normalised cross-covariance thresholding. He demonstrated good hot-clutter cancellation results with the MOUNTAINTOP data set.

An extension to Kogon’s work is presented by [Seliktar et al., 2000] who added the slow-time dimension to the beamspace formulation allowing joint ground and hot-clutter suppression. The technique is called ‘beam-augmented’ STAP and has shown improved interference cancellation.

An alternative study by [Goktun and Oruc, 2004] implemented the sidelobe canceller based on the analog ‘Howells-Applebaum’ control loop. They used the main antenna signal directly and adapted the reference signals without use of a blocking matrix. They showed that increasing the number of reference antennas improves the final cancellation level of the hot-clutter interference. However, only a small number of hot-clutter scatterers were used in their simulations and no fast-time taps were used to take advantage of the temporal correlation between scatterers. The other problem with this approach is the potential for the target signal to leak into the reference antennas due to the lack of a blocking matrix.

5.3.6 Subspace Techniques and the Multistage Wiener Filter

To reduce the computational load of the processors above, subspace techniques such as eigen-decomposition can be performed on the interference plus noise covariance matrix. By selecting the dominant eigenvectors or the ‘principle components’ associated with the largest eigenvalues, a lower rank covariance matrix can be formed, [Gabriel, 1986]. To improve the robustness in the subspace selection and achieve a reduction in the target detection dimensionality, a cross spectral metric ranking for the eigenvectors based on a minimal mean square error metric, can be used to further compress the interference subspace, [Goldstein and Reed, 1997b], [Goldstein and Reed, 1997a], [Goldstein and Reed, 1997c].

These subspace techniques have been applied to the problem of hot-clutter suppression by [Goldstein et al., 1999]. They contrast them with a newer technique known as the MWF. The MWF provides a faster rank reduction by using a nested chain of traditional Wiener filter stages. This is implemented by a sequence of orthogonal subspace projections to decompose the Wiener solution in terms of the cross-correlation observed at each stage. The process requires no knowledge of the eigen-structure of the covariance matrix and does not require any matrix inversions. It is demonstrated that this approach provides excellent performance while reducing the rank below any previously known eigen-based methods. More information on these three techniques can be found in Chapter 8.

Two recent modifications of this algorithm include adding target motion to the formulation, [Ricks et al., 2000] and applying the method of conjugate gradients for an order of magnitude speed improvement, [Weippert et al., 2003].
5.3 Hot-clutter Suppression Literature Review

5.3.7 Constrained Processing

If the filter weights of an adaptive filter are chosen with constraints, a number of different formulations are possible for STAP. This includes the Minimum Variance Distortionless Response from Chapter 4 which constrains the adaption to be unity in the steering direction. Other constraints such as the ‘derivative constraint’ allows a more robust adaptive processor in the presence of steering vector mismatch or a non-stationary environment as will be shown in Chapter 8.

Many of the previous algorithms have been modified to use constraints. For example, [Buckley and Griffiths, 1986] has implemented constrained beamforming using the Generalised Sidelobe Canceller, and later using principle components, [Buckley, 1987], while [Nguyen, 2002] has shown how the MWF can been modified to use constraints.

A different technique by [Hughes and Hayward, 1997] constrains the adaptive beamformer outside the mainbeam which allows mainbeam suppression of the jamming signals. They demonstrate superior performance to other algorithms utilising diagonal loading.

There is only one paper by [Griffiths, 1996] who has applied constraints to the problem of hot-clutter suppression. He has chosen constraints to preserve the characteristics of the desired signal while suppressing hot-clutter and shown good results with the MOUNTAINTOP data set.

5.3.8 Other STAP Techniques

TSI Finding

A technique called ‘TSI finding’ is presented by [Madurasinge and Shaw, 2005]. Their technique looks for the time delay associated with the highest point of the hot-clutter spectrum corresponding to the most powerful hot-clutter signal component available. This directs the STAP processor to select two desired range returns to form a reduced rank space/fast-time snapshot. The result is an efficient STAP processor with greatly reduced computational load.

Wold Decomposition

A parametric approach for modelling, estimation and target detection for STAP data has been derived in a paper by [Francos and Nehorai, 2003]. Their procedure uses information in only a single range bin and thus achieves high performance when the data in fast-time cannot be assumed stationary. The model is based on the 2D Wold-like decomposition to estimate the noise and interference components of the data which can then be used to form a parametric estimation of the interference plus noise covariance matrix.

Monopulse Processing

An alternative method of obtaining the filter coefficients is presented by [Seliktar et al., 1998]. Simultaneous optimisation of the two channels with appropriate constraints yields the desired
sum and difference components of the monopulse processor. Their performance results are tested using MOUNTAINTOP data and showed good results.

**Stochastic Constraints and Time-Varying Spatial Adaptive Processing**

The method of ‘stochastic constraints’ by [Abramovich et al., 1998]-[Abramovich et al., 2000] was developed to overcome the lack of good training data in HF OTHR. It assumes an autoregressive model for the ground clutter and sets up data dependent constraints to determine space/fast-time filter weights that provide high hot-clutter rejectability while maintaining acceptable ground clutter rejection. This technique however requires a large amount of computational complexity and is not suitable for practical implementation.

A different approach by [Fabrizio et al., 2004] addressed this problem and came up with a method called ‘time-varying spatial adaptive processing’. Their approach divides the Coherent Processing Interval (CPI) into a number of non-overlapping sub-CPI’s and the weight associated with each one is constrained to be orthogonal to the ground clutter subspace. It is also shown that this technique is robust to weight estimation errors that restrict the performance of a practical system.

**Pre-Filtering**

Another method to remove the effect of the modulations induced by cascaded filters is to use a pre-filter designed to cancel the ‘clutter of interest’ prior to the hot-clutter cancellation stage. This is chosen so that when the remainder is modulated, it lies below the noise floor. Using this approach, [Rabideau, 2000] has shown results with a much greater SINR and hence better target detection performance.

**Deconvolution**

[Nelander, 2002] presents an algorithm that requires an estimate of the complex multipath impulse response from a short time interval of the received signals. By convolving this signal with the direct-path reference signal, the interference estimate can be subtracted from the main beam jammer signal to suppress the hot-clutter. Experimental work in an anechoic chamber demonstrated the feasibility of the approach.
5.4 Jamming Scenario

5.3.9 Waveform Diversity Techniques

The only non STAP based hot-clutter suppression techniques are instead concerned with modifying the transmit waveform. By using polyphase coding for pulse compression, [Doherty, 1995] has demonstrated technique which can separate ground clutter from hot-clutter. It works on the principle that polyphase codes are uncorrelated in fast-time and as the correlated portions of the returned signal are due to hot-clutter, a fast-time FIR filter prior to pulse compression can be used to separate the uncorrelated ground clutter. Once separated, the hot-clutter can then be easily subtracted from the received data.

Since typical jammer signals are modelled as broadband white noise, [Techau, 1999] has shown that if the correlation function of this waveform have a finite time extent, then the ability to suppress hot-clutter is significantly reduced. These results may influence the choice of pulse compression used in a radar system, as pulse compression prior to adaptive filtering will impact the resulting correlation function.

A recent study by [Bergin et al., 2005] has looked at waveform optimisation which can adjust any or all of the radar’s transmitted degrees of freedom including the pulse shape, pulse repetition frequency, bandwidth and operating frequency. They find as the eigenvector subspace used for the optimised waveform shrinks, poorer pulse compression and an unsteady amplitude result. This presents a trade-off between the SINR improvement and the ‘quality’ of the radar waveform.

5.4 Jamming Scenario

From Chapter 2, the received SAR data waveform for the \( n^{th} \) channel comprised the desired ground return, \( \gamma_n(\cdot) \) and the receiver system noise \( \nu_n(\cdot) \). However, if the SAR is being jammed by an airborne platform, there will be two extra components required in the data model, the direct-path \( z_{dp,n}(\cdot) \) and hot-clutter \( z_{hc,n}(\cdot) \).

\[
x_n(t,u) = \gamma_n(t,u) + z_{dp,n}(t,u) + z_{hc,n}(t,u) + \nu_n(t,u).
\] (5.1)

This section comprises the three models required to simulate the interference signals. They include the:

Received Jammer Signal Model Section 5.4.1 describes the overall model for the direct-path and hot-clutter components. It relates the jammer waveform, the diffuse scattering coefficients, the bistatic time-delay and Doppler shift between the jammer and SAR platforms.

Diffuse Scattering Model Sections 5.4.2 and 5.4.3 describe models for the position and magnitude of the ground reflections over a designated ground area. They are based on a statistical model based on the platform heights, their distance apart and the roughness of the ground.

Note: If there were moving targets in the scene, they would be additional components.
Jammer and Noise Waveform Models

The final Section 5.4.4 describe models for both the jammer and thermal noise waveforms based on the received power spectral density as seen by the SAR. Different jamming characteristics are then presented in Section 5.4.5, before the last Section 5.4.6 which describes three different levels of realism for the jammer simulation.

5.4.1 Received Jammer Signal Model

Consider the geometry of Figure 5.3 where the ground is discretely sampled and the three points of interest have a given range, azimuth and velocity. By defining $x_{J,k} = |x_J - x_k|$, $y_{J,k} = |y_J - y_k|$ and $y_{p,k} = |u - y_k|$, the radial distances $R_{p,k} = R(x_k, y_{p,k})$ and $R_{J,k} = R(x_{J,k}, y_{J,k})$ can be calculated. The direct-path length can be calculated similarly by defining $y_{p,J} = |u - y_J|$ and using $R_{d,0} = R(x_J, y_{p,J})$. Unit vectors in each direction are indicated by $\vec{R}_{d,0}$, $\vec{R}_{J,k}$ and $\vec{R}_{p,k}$.

![Figure 5.3. Jammer geometry](image)

The bistatic jammer model is adapted from [Fante and Torres, 1995] and assumes there are $K_{hc}$ hot-clutter patches within the area on the ground that is being irradiated by the jammer. It is assumed that the jammer platform is directing its transmit beams to achieve maximum interference power on the SAR platform for both the direct-path and hot-clutter components. These components and the receiver noise are combined into a single variable $z_n(\cdot) = z_{dp,n}(\cdot) + z_{hc,n}(\cdot) + \nu_n(\cdot)$ where the noise signal $\nu_n(\cdot)$ represents the receiver noise for each channel.

The transmitted jammer signal is transmitted continuously with absolute time, $\tilde{t}$ and received at the SAR according to the fast-time, $t$ and SAR platform position, $u$. It is assumed that the bandwidth of the jammer signal waveform will totally cover the SAR’s and therefore can be
modelled at the carrier frequency by,

\[ z_{\text{tran}}(\tilde{t}) = J(\tilde{t}) \exp \{ j\omega_c \tilde{t} \} \quad (5.2) \]

where \( J(\cdot) \) is the baseband signal at the jammer platform. The received jammer signal from the \( n^{th} \) receiver after base-banding is given by the superposition of the delayed narrowband reflections from each patch,

\[ z_n(t,u) = \sum_{k=0}^{K_{\text{he}}} b_k J(t - \bar{\tau}_n(x_k, y_k - u)) \exp \{ -j\omega_c \bar{\tau}_n(x_k, y_k - u) \} \exp[-j\omega_{d,k} t] + \nu_n(t,u) \quad (5.3) \]

where \( \bar{\tau}_n(\cdot) \) are the space-time bistatic delays and for the \( k^{th} \) scatterer, \( \omega_{d,k} \) is the Doppler frequency and \( b_k \) is the hot-clutter ground return. It is defined below as the relative magnitude between the direct-path and reflected jammer signal using the bistatic radar range equation with the zero index referring to the direct-path, \( b_0 \equiv 1 \). Using the geometry in Figure 5.3, the space-time bistatic delays for the \( k^{th} \) scatterer are given by,

\[ \bar{\tau}_n(x_k, y_k - u) = \begin{cases} \frac{1}{c} R(x_J, y_p, t - d_n), & k = 0 \\ \frac{1}{c} \left[ R_{J,k} + R(x_k, y_p, k - d_n) \right], & k > 0 \end{cases} \quad (5.4) \]

and the Doppler frequencies are given by the inner product of the following vectors (in bold)

\[ \omega_{d,k} = \begin{cases} \frac{2\pi}{\lambda_c} (\vec{v}_p - \vec{v}_J)^T \vec{R}_{d,0}, & k = 0 \\ \frac{2\pi}{\lambda_c} (\vec{v}_p^T \vec{R}_{p,k} - \vec{v}_J^T \vec{R}_{J,k}), & k > 0. \end{cases} \quad (5.5) \]

**Relative Magnitude**

The parameter \( b_k \) is the relative magnitude between the direct and hot-clutter signals. In terms of the radar range equation, the average received powers for the direct and hot-clutter signal components are,

\[ \tilde{P}_{r,0} = \frac{P_{t,0} G_{t,0} G_{r,0} \lambda_c^2}{(4\pi R_{d,0})^2} \]
\[ \tilde{P}_{r,k} = \frac{P_{t,k} G_{t,k} G_{r,k} \sigma_k A_k \lambda_c^2}{(4\pi)^{3/2} (R_{p,k} R_{J,k})^{3/2}} \quad (5.6) \]

where \( A_k \) is the effective area of the \( k^{th} \) patch and \( \sigma_k \) is a random variable describing the hot-clutter ground return. In the direction of the direct and hot-clutter paths from the jammer platform, \( P_{t,0} G_{t,0} \) and \( P_{t,k} G_{t,k} \) are the effective transmitted powers and \( G_{r,0} \) and \( G_{r,k} \) are the receive gains from the SAR platform. The received voltage is determined with the simple relationship,

\[ \tilde{b} = \sqrt{\tilde{P}_r Z_t} \quad (5.7) \]
where $Z_r$ the impedance at the receiver. The antenna amplitude voltages are then,

$$\tilde{b}_0 = \sqrt{\frac{P_{t,0}G_{t,0}G_{r,0}Z_r\lambda_c}{4\pi R_{0,d}}}$$
$$\tilde{b}_k = \sqrt{\frac{P_{t,k}G_{t,k}G_{r,k}\sigma_k A_k Z_r\lambda_c}{4\pi R_{p,k}R_{f,k}}}$$

(5.8)

with the relative magnitude for $k = 1 \ldots K_{hc}$,

$$b_k = \frac{\tilde{b}_k}{b_0} = \left( \frac{P_{t,k}G_{t,k}G_{r,k}\sigma_k A_k}{P_{t,0}G_{t,0}G_{r,0}4\pi} \right)^{1/2} \frac{R_{0,d}}{R_{p,k}R_{f,k}}$$

(5.9)

with $b_0 \equiv 1$. If the effective transmitted powers for both paths are assumed to be equal, this reduces to,

$$b_k = \left( \frac{G_{r,k} \sigma_k A_k}{G_{r,0}4\pi} \right)^{1/2} \frac{R_{d,0}}{R_{p,k}R_{f,k}}, \quad k > 0$$

$$= \sqrt{\rho \sigma_k}$$

(5.10)

where $\rho$ is defined as the relative hot-clutter power. A diffuse scattering model presented in the following section is used to determine the hot-clutter ground return, $\sigma_k$.

### 5.4.2 Diffuse Scattering Model

The physically based diffuse scattering model was originally presented by [Beckmann and Spizzichino, 1987] and implemented for hot-clutter modelling by [Fante, 1991]. It is a statistical based model describing the scattering distribution between the incident and reflected signals over a ‘rough’ or ‘glistening surface’.

This section is broken into a number of parts: the first describes the physical model for the rough surface, the second details the geometry of the glistening surface and the location of the hot-clutter scatterers and the third presents the statistical model for the hot-clutter ground return and how it relates to the scatterer positions. The final part shows a number of distributions corresponding to different rough surfaces.

#### Physical Model

This model considers the surface of the earth as rough with scatterer heights normally distributed with standard deviation, $\sigma_h$. The autocorrelation, $C_h(z)$ is used to characterise the scattering behaviour and determines the correlation (or lack of independence) between two random values of $\Sigma$ separated by a distance $z$,

$$C_h(z) = \exp \left[-\frac{z^2}{2\bar{h}}\right].$$

(5.11)
5.4 Jamming Scenario

This function will decrease from its maximum value of $C_h(0) = 1$ to $C_h(\infty) = 0$ with a ‘correlation distance’, $z_h$ which determines when the autocorrelation drops to $e^{-1}$. Physically, this parameter describes the nature of the rough surface; if $z_h = 0$, the model implies no correlation between the two random values of $\Sigma$, while a small $z_h$ implies a surface with densely packed irregularities such as that formed by tree-tops of a dense forest. Finally, if $z_h$ is large, points on the surface will be correlated even if they are far apart, such as a gently rolling surface.

The rough surface is modelled by a number of small elements, each of which acts as a small mirror reflecting the incident wave according to the law of geometrical optics. The distribution of slopes of these mirrors can then be used to completely define the model with two parameters; $\beta$ describing the scattering geometry and $\beta_0$, the rough surface. The first parameter, $\beta$ relates the angle made by the bisector of the incident and scattered rays, while $\beta_0$ is defined as a measure of the mean (absolute) slope of the irregularities forming the rough surface, [Beckmann and Spizzichino, 1987]. It can be interpreted as the mean value of the ratio of vertical and horizontal dimensions of the irregularities and is related to the standard deviation and correlation distance by,

$$\tan \beta_0 = \frac{2\sigma_h}{z_h}. \quad (5.12)$$

To use this model, a few assumptions must be made:

1. The earth is flat.
2. The heights of the transmitting and receiving antennae are small with respect to the distance between them.
3. The antenna gains are practically constant over the glistening surface.

Scatterer Location

Using ground plane coordinates defined in Section 2.3.1, the glistening surface can be defined by the region where $\beta \leq \beta_0$ and is represented by the shaded region in Figure 5.4.

In this figure, the SAR and jammer heights are given by $h_P$ and $h_J$ respectively, separated by a distance $\bar{x}_J$ in the ground-plane with the contour boundary $C$ representing the limit $\beta = \beta_0$. Hot-clutter scatterers such as the point $P$ can be represented with coordinates $(\bar{x}_P, \bar{y}_P) = (\bar{x}_2, \bar{y}(\beta))$, where the azimuth ground coordinate is a function of $\beta$ and $\bar{x}_1 = \bar{x}_J - \bar{x}_2$.

$$\bar{y}(\beta) = \pm \frac{\bar{x}_1 \bar{x}_2}{\bar{x}_J} \left( \frac{h_J}{\bar{x}_1} + \frac{h_P}{\bar{x}_2} \right) \sqrt{\tan^2 \beta - \frac{1}{4} \left( \frac{h_J}{\bar{x}_1} - \frac{h_P}{\bar{x}_2} \right)^2}. \quad (5.13)$$

Scattering Coefficient

The hot-clutter ground return for the $k^{th}$ scatterer is defined by,

$$\sigma_k = |\bar{\Gamma}| \Sigma_k \quad (5.14)$$
where \( \tilde{\Gamma} \) is the Fresnel reflection coefficient and \( \Sigma_k \) is the scattering coefficient. The ground return is modelled as a real random variable with zero phase. Once all the scatterers are combined in Equation 5.3, the sum of temporal delays will appear as a random phase which is consistent to [Fante, 1991] and [Techau et al., 1999a] who instead use a random phase term for \( \sigma_k \).

The scattering coefficient can be described by its mean absolute square, \( E \{ |\Sigma|^2 \} \), allowing statistically identical random samples to be drawn from a given distribution. This relationship can be described in terms of the scatterer positions and platform heights from Figure 5.4 or more simply, by two random variables \( \xi \) and \( \mu \) interpreted as angles varying from 0 to \( \pi \) radians, [Beckmann and Spizzichino, 1987]. Only the latter angle has any physical meaning and describes the asymmetry of the glistening surface. The first angle \( \xi \) is related to the ground range by the following relationship,

\[
\bar{x}_1 = \bar{x}_J \sin^2 \frac{\xi}{2}
\]

\[
\bar{x}_2 = \bar{x}_J \cos^2 \frac{\xi}{2}
\]  

while the second angle \( \mu \) allows differences in height to be represented by,

\[
\tan^2 \frac{\mu}{2} = \frac{h_J}{h_P}
\]  

and is related to the jammer and SAR heights by,

\[
h_J = 2h \sin^2 \frac{\mu}{2}
\]

\[
h_P = 2h \cos^2 \frac{\mu}{2}
\]  

where \( h \) represents the average height of the two platforms,

\[
h = \frac{h_J + h_P}{2}.
\]
5.4 Jamming Scenario

The mean absolute square of the scattering coefficient can then be determined by integrating over the contour boundary,

\[
E \{ |\Sigma|^2 \} = \frac{2}{\pi} K_\beta \int_{\xi_A}^{\xi_B} \left( \frac{1 - \cos \mu \cos \xi}{\sin^2 \xi} \right) \sqrt{1 - \left( \frac{K_\beta \cos \xi - \cos \mu}{\sin^2 \xi} \right)^2} d\xi
\]

where the limits of the integral \( \xi_A \) and \( \xi_B \) are determined by the validity of the expression under the square root. The function \( f_s(\xi) \) can be thought of as the probability distribution of the scatterers and \( K_\beta \) is the surface ‘roughness’ parameter. If the jammer grazing angle is defined as,

\[
\psi_J = \arctan \left( \frac{h}{\bar{x}_J/2} \right)
\]

for a ray that is specularly reflected to the maximum slope \( \beta_0 \) of scatterers in the terrain, then the surface roughness is defined by the ratio of tangents of the grazing angle and maximum slope and can be written in terms of the standard deviation of irregularities in the terrain, \( \sigma_h \),

\[
K_\beta = \frac{\tan \psi_J}{\tan \beta_0} = \frac{h z_h}{\bar{x}_J \sigma_h}.
\]

A rougher surface with diffuse reflections then corresponds to a smaller value of the surface roughness parameter as will be shown in the next section.

Distribution Analysis

To visualise the two distributions, \( f_s(\xi) \) and \( E \{ |\Sigma|^2 \} \), the roughness parameter is varied with platform heights set the same, giving \( \mu = \pi/2 \). Figure 5.5 is a plot of the scattering distribution as a function of \( \xi \), while Figure 5.6 is a plot of the mean absolute square of the scattering coefficient as a function of the roughness.

The level of diffuseness will greatly effect the final image quality. For example, a high \( K_\beta \) will cause the hot-clutter reflections to be specular and it will appear spatially that only one jamming source is present. However, when \( K_\beta \) is low, the diffuseness is large and the hot-clutter will spread in angle, giving the appearance of a number of different jammer sources incident on the SAR.

A step by step implementation for this model is presented in Appendix B, including the mapping from ground plane to slant plane coordinates required for the multichannel SAR simulation in Chapter 2.
5.4.3 Scenario Scatterer Distribution

The section demonstrates how the distribution of hot-clutter scatterers vary with the diffuseness of the ground. Three levels of diffuseness are considered, corresponding to very diffuse, moderately diffuse and specular scenarios. The heights of both platforms have been set to \( h_J = h_P = 3 \) km, the jammer platform is offset from the SAR in ground range by 50 km and the direct-path is incident at broadside. This will be approximately constant over the CPI due to the large offset range and as both platforms are travelling at a speed of 200 m/s.
5.4 Jamming Scenario

Two plots are shown in each of the following three parts. The first corresponds to a top down view of $K_{hc} = 10000$ scatterers with the SAR platform on the left and the jammer on the right, while the second shows a histogram of the scatterer incident angles as seen by the SAR. There is a border around each of the scattering regions indicating the maximum bisecting angle, $\beta_0$.

**Very Diffuse Scenario**

The very diffuse scenario ($K_{\beta} = 0.01$) is represented in Figure 5.7 and shows a wide spread of scatters out to 35km in azimuth. This leads to a very wide spread of $\pm 35^\circ$ in incidence angles as shown in Figure 5.8. Interestingly, the scattering spread doesn’t fill the entire area. This result is not pursued further, as it is not the main focus of this thesis.

![Figure 5.7. Random scatterer location for very diffuse scenario](image)

![Figure 5.8. Scatterer distribution as incident on SAR for very diffuse scenario](image)
Moderately Diffuse Scenario

The moderately diffuse scattering scenario \( (K_\beta = 0.7) \) shown in Figure 5.9 has an order of magnitude decrease in the azimuth locations. The distribution now resembles a cone shape and is within ±0.35° incidence with the SAR as shown in Figure 5.10.

![Figure 5.9. Random scatterer location for moderately diffuse scenario](image)

![Figure 5.10. Scatterer distribution as incident on SAR for moderately diffuse scenario](image)
5.4 Jamming Scenario

Specular Scenario

The specular case ($K_\beta = 10$) in Figure 5.11 shows a very small spread in azimuth. This is confirmed with the histogram in Figure 5.12 which shows a deviation of $\pm 0.0036^\circ$ incidence with the SAR.

![Figure 5.11. Random scatterer location for specular scenario](image)

![Figure 5.12. Scatterer distribution as incident on SAR for specular scenario](image)
5.4.4 Jammer and Noise Waveform Models

Both the jammer and thermal noise waveforms are wideband signals with their power spectrum covering the entire SAR bandwidth. The jammer signal will have a greater magnitude and is correlated across receivers, while the noise waveform will be uncorrelated. A temporal correlation can be introduced to both signals if the fast-time domain is oversampled.

Jammer Waveform Model

The jammer waveform, $J(\cdot)$ is a realisation of Gaussian random noise with correlation due to the finite SAR bandwidth. Although the jammer bandwidth is greater than the SAR bandwidth $B$, the received signal is filtered within the receiver to match the SAR bandwidth. The Power Spectral Density (PSD), $P_J(\omega)$ of the signal at baseband is a rectangle function with its autocorrelation determined by an inverse Fourier transform. If the PSD is

$$P_J(\omega) = \frac{\sigma^2_J}{B} \text{rect} \left( \frac{\omega}{\pi B} \right) \quad (5.22)$$

with power $\sigma^2_J$, then the inverse Fourier transform is

$$r_J(\tilde{\zeta}) = \sigma^2_J \text{sinc}(B\tilde{\zeta}) \quad (5.23)$$

and oversampling in the fast-time by $\tilde{\zeta} < \frac{1}{B}$ will result in samples that are no longer in the nulls of the sinc function. Details on the jammer waveform implementation are presented in Appendix B.

Noise Waveform Model

The receiver system noise waveform comprises two components representing the thermal noise and the quantisation noise. The thermal noise will have an autocorrelation model exactly the same as the jammer waveform, except the power is now given by $\sigma^2_{\nu}$. It is generated in the same manner as the jammer waveform with each realisation of the noise being independent. The quantisation noise on the other hand is introduced during the analog to digital conversion, is wideband and does not depend on the fast-time sampling. It is generally of less power than the thermal noise and is represented by $\sigma^2_q$.

5.4.5 Jammer Characteristics

The incidence of the jamming signal, $\theta_J$ will greatly impact the performance of the adaptive algorithm, especially as it approaches the SAR integration angle. However, this is typically very small due to the large offset range, $X_c$. The class of jamming is typically defined relative to the mainlobe and sidelobes of the real array beampattern. If the jammer incidence angle is within the SAR integration angle, it is known as inner mainlobe jamming, while the region between the
5.4 Jamming Scenario

SAR integration angle and first mainlobe null is outer mainlobe jamming. Beyond the mainlobe null, the jamming signals are known as sidelobe jamming.

As shown in Chapter 4, adaptive filtering can distort the beampattern where the SAR image is formed. The effect will be worse as the jammer incidence gets closer to the SAR integration angle and once inside, can cancel the ground clutter along with the interference. Figure 5.13 shows the three jamming regions.

![Figure 5.13. Real array beampattern showing: (—) SAR integration angle, (--) inner mainlobe jamming, \( \theta_J = 0^\circ \), (--) outer mainlobe jamming, \( \theta_J = 5^\circ \), (···) sidelobe jamming, \( \theta_J = 40^\circ \)]

5.4.6 Training Level of Realism

Estimating the interference plus noise covariance matrix plays a big role in the performance of STAP algorithms. Previously in Chapter 4, it was assumed that a measure of the interference plus noise data was available separately to the total received signal. This is not always the case however and different training methods have been devised to separate the two signals components. For simulation purposes, three different cases are presented below corresponding to different levels of realism.

Case 1 The ideal case is where the SAR receiver has exact knowledge of the interference plus noise signal and the training data contains no ground clutter. This case is used to test the performance of different optimal STAP algorithms in Chapter 6.

Case 2 This case is based on techniques described by [Ward, 1994] and [Ender, 1998a], where there is access to a training region separate from the received data. This can be outside the receiver bandwidth or at the end of a pulse where the received ground clutter signals have a low amplitude. The former technique assumes that the interference plus noise
signal is present outside the SAR bandwidth and the latter that statistics of the hot-clutter are consistent in both regions. This method is used in Chapter 8 with two different realisations of the interference plus noise signal are simulated with only the total received signal containing ground clutter.

**Case 3** The most realistic case is where there is no separate region and the training must be taken from the total received data signal. As the training data contains ground clutter, guard bands need to be used to prevent self-nulling. It is generally difficult to get a good estimate of interference plus noise statistics with this method and correspondingly, it is not used in this thesis.

### 5.5 Conclusion

This chapter has provided a detailed literature review of jamming and anti-jamming techniques for SAR and methods for suppressing hot-clutter in airborne and HF OTHR. These techniques provide important background material for this thesis and many will be adapted in later chapters for use in hot-clutter suppression algorithms for SAR.

The second part of this chapter combines a number of different physical models to form an accurate scattering and jammer model to use for simulation. These included models for the transmitted jammer signal, the diffuse scattering used to describe the hot-clutter reflections and jammer and noise waveforms.
6.1 Introduction

Coherent Synthetic Aperture Radar (SAR) imaging is very sensitive to additive noise and an airborne broadband jammer has the potential to render it useless. The two jammer components introduced in the previous chapter include the direct-path from the jammer platform to the SAR and multipath reflections from the ground. The direct-path of the jammer signal is defined by a narrow azimuth region and while the long integration times involved in SAR image formation can be used to ‘burn-through’ the interference [Skolnik, 1990], spatial degrees of freedom are required for effective cancellation. It has been shown that by combining the multichannel data from multiple pulses (slow-time) and performing slow-time Space Time Adaptive Processing (STAP), much greater suppression is possible [Ender, 1998a]. On the other hand, due to the diffuse reflection from the ground, the hot-clutter component is spread in azimuth and its properties can change rapidly with time, even over several adjacent pulses. This leads to a non-stationarity over slow-time and degrades the performance of slow-time adaptive filtering. Slow-time STAP works well for suppressing signals which are narrow in azimuth, though as the hot-clutter becomes more dominant, interference contributions spread in angle, resulting in images that are blurry and of poor quality. A secondary cause of non-stationarity comes from the changing motion between the SAR and jammer platforms which induces a bistatic Doppler shift for each scatterer. This effect is considered minimal in SAR as the jammer platform is typically a long distance away and the Doppler shift is relatively constant.

To effectively account for the effect of non-stationarity between pulses, cancellation of the interference should occur before image formation. Also the finite bandwidth of SAR means that multipath reflections are partially coherent with the direct-path jamming signal and adaptive filtering can exploit the temporal correlation within each pulse or over fast-time. Hot-clutter suppression can therefore be undertaken by employing adaptive processing in both space and fast-time, forming a space/fast-time adaptive processor for each pulse [Griffiths et al., 2000], [Fante and Torres, 1995].

The relevant background on interference suppression for multichannel SAR was covered in Section 5.2.3, while hot-clutter suppression techniques were described in Section 5.3. These were
first identified as a problem in airborne radar and the application of hot-clutter suppression for SAR is a problem that has not yet been addressed.

The first part of this chapter summarises the SAR and jammer signal models from chapters 2 and 5 and demonstrates the effect of hot-clutter on SAR imaging. The next Section 6.3 then describes three different approaches for suppressing hot-clutter including multichannel imaging, optimal slow-time STAP and optimal fast-time STAP. Optimal STAP refers to the formulation using all the pulses for the slow-time algorithm and likewise all the range compressed fast-time samples for the fast-time algorithm. It does not mean that the ideal covariance is used for these algorithms, as this would not demonstrate the effect of averaging over pulses. Simulation results are presented in Section 6.4 and quantified to measure the effectiveness of interference rejection for these three algorithms. To simplify the analysis, it is assumed that the interference waveform is known, resulting in an idealised scenario where the training data waveform is identical to that of the jamming component of the received data. This approach is used to determine the potential of the proposed algorithms. However, a look at the effect of non-ideal training data is included in the final Section 6.5 to demonstrate the performance likely to be achieved in a real system.

6.2 Simulation Models

The first part of this Section 6.2.1 is a brief summary of the multichannel SAR signal model presented in Chapter 2 while the second part 6.2.2 summarises the jammer model presented in Chapter 5. Section 6.2.3 then shows simulation results when these models are combined into a multichannel SAR simulation.

6.2.1 SAR Signal Model

Consider a SAR travelling along the y-axis, imaging a patch in the slant-plane $x \in [X_c - X_0, X_c + X_0]$, $y \in [-Y_0, Y_0]$. For each of the $M$ pulses, the radar transmits a broadband chirp and the received signal is base-banded, sampled and range processed for each of the $N$ channels of a linear antenna array. If the SAR is being jammed by an airborne platform, the signal components received by the $n^{th}$ channel of the SAR include the direct-path $z_{dp,n}(\cdot)$, ground reflected path (hot-clutter) $z_{hc,n}(\cdot)$, ground clutter return $\gamma_n(\cdot)$ and the receiver system noise $\nu_n(\cdot)$,

$$x_n(t,u) = z_{dp,n}(t,u) + z_{hc,n}(t,u) + \gamma_n(t,u) + \nu_n(t,u). \quad (6.1)$$

The total ground return for the $n^{th}$ channel is represented by the integral over all scatterers with ground return $f_n(x,y)$,

$$\gamma_n(t,u) = \int_y \int_x a(x,y - u)f_n(x,y)s_{post,n}(t,u,x,y)dxdy \quad (6.2)$$
where \( a(\cdot) \) is the two-way antenna beampattern and the form of the narrowband post range processing received signal model is given by,

\[
s_{\text{post},n}(t, u, x, y) = \exp\left[-j\omega_c\tau_n(x, y - u)\right] \text{sinc}\left[B\pi(t - \tau_n(x, y - u))\right]
\] (6.3)

where \( \tau_n(\cdot) \) is the temporal delay for the \( n^{\text{th}} \) channel, \( \omega_c \) (rad/s) is the carrier frequency, \( B \) (Hz) is the bandwidth and the variables \((t, u)\) represent fast-time within a pulse and the SAR platform position respectively. The signal model in the fast-time frequency domain was derived in Section 3.3.1 and is given by,

\[
\hat{s}_{\text{post},n}(\omega, u, x, y) = \text{rect}\left[\frac{\omega - \omega_c}{\pi B}\right] \exp\left[-j\omega\tau_n(x, y - u)\right].
\] (6.4)

where the fast-time frequency \( \omega \) is related to the range compressed time with \( n \) range bins instead of the entire fast-time containing \( L \) samples. The spatial component of the SAR signal model was derived in Section 2.3.5 as

\[
s_n(u) = \exp[jk_c d_n \sin[\theta(u)]]
\] (6.5)

where \( k_c \) is the wavenumber at the carrier frequency, \( \theta(u) = \arctan[-u/X_c] \) is the steering angle and \( d_n = n\delta \) is the antenna offset from the array phase centre with antenna spacing \( \delta \) and \( n \in [- (N - 1)/2, (N - 1)/2] \) for \( N \) (odd) antenna elements. By assuming that the carrier frequency is much larger than the bandwidth with a small array aperture, this signal model can be used to approximate Equation 6.3 by separating the spatial and temporal components,

\[
s_{\text{post},n}(t, u, x, y) \approx s_n(u)s_{\text{post},0}(t, u, x, y)
\] (6.6)

where the temporal component, \( s_{\text{post},0}(\cdot) \) is defined at the antenna phase centre.

### 6.2.2 Jammer and Noise Models

The bistatic jammer model assumes there are \( K_{hc} \) hot-clutter patches within the area on the ground that is being irradiated by the jammer and that the jammer platform is directing its transmit beams to achieve maximum interference power on the SAR platform for both the direct path and hot-clutter components. These components and the receiver noise are combined into a single variable \( z_n(\cdot) = z_{\text{dp},n}(\cdot) + z_{\text{hc},n}(\cdot) + \nu_n(\cdot) \). The noise component is defined in Section 5.4.4 as the sum of thermal and quantisation noise. Both are modelled as broadband Gaussian processes with zero mean and variance, \( \sigma^2_\nu \) and \( \sigma^2_q \) respectively.

The output of the \( n^{\text{th}} \) receiver due to the jammer \( z_n(\cdot) \) is then given as the superposition of the delayed reflections from each patch,

\[
z_n(t, u) = \sum_{k=0}^{K_{hc}} b_k J(t - \bar{\tau}_n(x_k, y_k - u)) \exp\left[-j\omega_c\bar{\tau}_n(x_k, y_k - u)\right] \exp\left[-j\omega_{d,k} t\right] + \nu_n(t, u)
\] (6.7)

where \( J(\cdot) \) is the signal at the jammer platform modelled as broadband Gaussian noise with zero mean and variance \( \sigma^2_J \). The bistatic delay \( \bar{\tau}_n(\cdot) \) and Doppler frequency, \( \omega_{d,k} \) for the \( k^{\text{th}} \) scatterer
6.3 Jammer Suppression for SAR

are defined in Section 5.4 according to the geometry of the jammer platform, the ground and the SAR platform. Likewise, the hot-clutter ground return $b_k$ is defined as the relative magnitude between the direct-path jammer signal and the reflection of the $k^{th}$ scatterer. It was derived in Section 5.4 using the bistatic radar range equation with the zero index referring to the direct-path signal, $b_0 \equiv 1$ and is represented for $k = 1 \ldots K_{hc}$ by,

$$b_k = \sqrt{\rho \sigma_k} \quad \text{(6.8)}$$

where $\rho$ as the relative hot-clutter power and $\sigma_k$ the hot-clutter ground return for the $k^{th}$ scatterer.

6.2.3 Simulation

To determine the effect of hot-clutter on a SAR image, these models can be combined into a simulation. Section 6.4 describes how this is achieved and presents a summary of the parameters used. To visualise the effect, Figure 6.1 shows three images including the point scatterers before simulation, a synthetic SAR image and the same image with hot-clutter added. This image shows so much blurring that the original image is indecipherable. To quantify this degradation, a number of image metrics are presented in Section 6.4.1 and applied to the SAR images with varying relative hot-clutter power levels.

![Figure 6.1. Image comparison](image)

6.3 Jammer Suppression for SAR

Three alternative algorithms are now presented which have the ability to both reject interferences and perform SAR imaging. The first is a space/slow-time multichannel imaging algorithm which was presented in Section 3.4.1 and relies on a large number of pulses to ‘burn-through’ the interference, [Skolnik, 1990]. The second algorithm is optimal slow-time STAP which has been shown to produce good jammer suppression for signals which are stationary over the coherent processing interval and are narrow in azimuth, [Ward, 1994]. The third algorithm is optimal fast-time STAP and has demonstrated good performance with suppressing non-stationary interference in airborne radar, [Fante and Torres, 1995]. This algorithm is also able to account for any non-stationarity due to changing geometry between pulses. These points are summarised in
Table 6.1 and a block diagram of the simulation with the three alternate processing schemes is presented in Figure 6.2.

<table>
<thead>
<tr>
<th>No adaption</th>
<th>Fast-time STAP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can suppress only with large number of pulses (burn-through).</td>
<td>Accounts for changing geometry between pulses.</td>
</tr>
<tr>
<td><strong>Slow-time STAP</strong></td>
<td>Necessary for non-stationary interference incident in the SAR mainbeam.</td>
</tr>
<tr>
<td>Shown to produce good jammer suppression for stationary interferences which are narrow in azimuth.</td>
<td></td>
</tr>
</tbody>
</table>

**Table 6.1. Jammer suppression techniques**

6.3.1 Multichannel SAR Imaging

High resolution SAR imaging requires a much longer integration time than conventional airborne radar. This is a desirable property for rejecting noise jammers as it enables the ground return to ‘burn-through’ the interference, [Skolnik, 1990]. One suitable method for coherent integration/imaging of a multichannel SAR is the space/time Matched Filter (MF) interpolation algorithm. This algorithm was presented in Chapter 3 and is now briefly summarised as it forms the basis of the slow-time STAP algorithm in the following section.
This algorithm is formed in the \((\omega, u)\) domain and requires a reference signal defined at the point \((X_c, 0)\) giving \(\tilde{s}_{\text{ref, } n}(\omega, u) = \tilde{s}_{\text{post, } n}(\omega, u, X_c, 0)\). Both this signal and the received SAR data are first stacked to the form spatial vectors,

\[
\tilde{s}_{\text{ref}}(\omega, u) = \frac{1}{\sqrt{N}}[\tilde{s}_{\text{ref, } -(N-1)/2}(\omega, u), \ldots, \tilde{s}_{\text{ref, } (N-1)/2}(\omega, u)]^T \in \mathbb{C}^{N \times 1},
\]

\[
\tilde{x}(\omega, u) = \frac{1}{\sqrt{N}}[\tilde{x}_{-(N-1)/2}(\omega, u), \ldots, \tilde{x}_{(N-1)/2}(\omega, u)]^T \in \mathbb{C}^{N \times 1},
\]

(6.9)

and then stacked again over the entire range of \(u\) to give space/slow-time vectors. To maintain the phase centre at the middle of the synthetic array, \(u_M/2\) occurs as the SAR platform passes the centre of the imaging area and hence both the slow-time steering and data vectors can be stacked over pulse delays where \(u\) varies over \(u_1, u_2, \ldots, u_M\),

\[
\tilde{G}(\omega, u) = \frac{1}{\sqrt{M}}[\tilde{s}_H(\omega, u - u_1), \ldots, \tilde{s}_H(\omega, u - u_M)]^T \in \mathbb{C}^{MN \times 1},
\]

\[
\tilde{X}_s(\omega) = \frac{1}{\sqrt{M}}[\tilde{x}_H(\omega, u_1), \ldots, \tilde{x}_H(\omega, u_M)]^T \in \mathbb{C}^{MN \times 1}.
\]

(6.10)

The MF imaging Equation can then be written as,

\[
\tilde{f}(\omega, u) = \tilde{G}^H(\omega, u)\tilde{X}_s(\omega)
\]

(6.11)

and is represented in Figure 6.3. To form the final image, the result must be Fourier transformed into the \((\omega, k_u)\) domain and then mapped from the measured to the image domain with a Stolt interpolation. A two-dimensional inverse Fourier transform then maps the spatial frequencies into \((x, y)\) image coordinates. A block diagram of this algorithm is presented in Figure 6.3 where the wavenumber \(k = \omega/c\). Note that the coherent averaging over pulses in the Fourier Transform \(u \to k_u\), can be interpreted as ‘burn-through’.

![Figure 6.3. Multichannel spatial/MF imaging block diagram](image)
6.3.2 Slow-time STAP

Principle causes of non-stationarity are due to the relative motion between the two platforms and the changing super-position of the direct-path and hot-clutter components of the interference. The degree of non-stationarity will depend on the relative power between these components as well as the geometrical and physical features of the ground which vary from pulse to pulse. If the relative power of the direct-path signal is much greater then the hot-clutter component, the total interference can be classed as ‘approximately stationary’ and less intensive filtering using slow-time STAP algorithms may be sufficient to remove the predominant interference.

Optimal slow-time STAP involves replacing the slow-time reference or steering vector with the optimal weight for each frequency to maximise the Signal to Interference plus Noise Ratio (SINR) for each \((\omega, u)\). The weight is a space/slow-time vector and includes the interference plus noise components that are to be cancelled.

\[
\hat{f}_{ss}(\omega, u) = W^{H}(\omega, u)\hat{X}_{s}(\omega) \tag{6.12}
\]

where

\[
W(\omega, u) = \hat{\gamma}R_{\hat{Z}_{s}}^{-1}\bar{G}(\omega, u) \in \mathbb{C}^{MN \times 1} \tag{6.13}
\]

and \(R_{\hat{Z}_{s}}\) is the ideal interference plus noise covariance evaluated at the carrier frequency. The optimisation criteria chosen is the maximum SINR which is suitable for the ideal training case used in this chapter. More realistic training conditions are used in Chapter 8 and will require the use of steering constraints as described in Section 4.3.1. The arbitrary scaling constant, \(\hat{\gamma}\) will not effect the SINR performance of this algorithm and is set to unity.

To measure the effect of averaging over pulses, the ideal interference plus noise covariance is replaced by the sample matrix estimate. This is found by averaging over \(n_x\) frequency bins with \(\eta\) dB of diagonal loading.

\[
\hat{R}_{\hat{Z}_{s}} = \frac{1}{n_x} \sum_{l=1}^{n_x} \hat{Z}_{s}(\omega_{l})\hat{Z}_{s}^{H}(\omega_{l}) + \eta I_{MN} \in \mathbb{C}^{MN \times MN} \tag{6.14}
\]

where \(\hat{Z}_{s}(\cdot)\) is the interference plus noise space/slow-time vector defined similarly to Equation 6.10. Diagonal loading has a number of benefits which are investigated in Chapter 8. The primary use of diagonal loading for this chapter is to regularise the covariance matrix to help with its inversion. Since this chapter is focussed on formulating a comparison between different anti-jamming techniques, an ideal training situation is considered where the actual interference plus noise waveform is used to form \(\hat{R}_{\hat{Z}_{s}}\). A more realistic training situation will also be used in Chapter 8.

The final step in forming a SAR image is to perform range migration compensation by using Stolt interpolation and inverse Fourier transforming as described in the previous section. This process is described by the block diagram in Figure 6.4.
6.3 Jammer Suppression for SAR

6.3.3 Fast-time STAP

Interferences which are non-stationary require fast-time processing for effective cancellation. This formulation combines both spatial and fast-time samples to form a space/fast-time processor with new weights for each pulse. Space/fast-time focussing is accomplished by a convolution between the reference vector, \( \mathbf{H}_{\text{post}}(\cdot) \) and the incoming data vector, \( \mathbf{x}(\cdot) \). The entire \( n_x \) range bins are used to form the optimal filter, which is non-causal due to the delay being in the reference function and not the data,

\[
x_1(t_l, u) = \frac{1}{n_x} \sum_{k=1}^{n_x} \mathbf{H}_{\text{post}}^H (t_l - t_k, u) \mathbf{x}(t_k, u)
= \mathbf{H}_{\text{post}}^H (t_l, u) \mathbf{X}_1(u).
\] (6.15)

The received SAR space/fast-time vector is formed by stacking spatially and then temporally to give,

\[
\mathbf{x}(t_l, u) = \frac{1}{\sqrt{N_x}} \left[ x_{-(N-1)/2}(t_l, u), \ldots, x_{(N-1)/2}(t_l, u) \right]^T \in \mathbb{C}^{N_x \times 1},
\]
\[
\mathbf{X}_1(u) = \frac{1}{\sqrt{n_x}} \left[ \mathbf{x}^T (t_1, u), \ldots, \mathbf{x}^T (t_{n_x}, u) \right]^T \in \mathbb{C}^{n_x \times N_x \times 1}
\] (6.16)

and the spatial signal model defined in Equation 6.5 is vectorised by stacking each element to give,

\[
\mathbf{s}(u) = \frac{1}{\sqrt{N}} \left[ s_{-(N-1)/2}(u), \ldots, s_{(N-1)/2}(u) \right]^T \in \mathbb{C}^{N \times 1}
\] (6.17)

where the antenna phase centre is at the centre of the array. For the temporal steering signal, the post range processing signal model in Equation 6.3 is used with the non-time dependent term in the exponential removed as it have no effect on the fast-time filter output,

\[
g_{\text{post}, k}(t_l) = \text{sinc} \left[ B(t_l - t_k) \right].
\] (6.18)

This signal model can similarly be stacked over fast-time delays to give,

\[
\mathbf{g}_{\text{post}}(t_l) = \frac{1}{\sqrt{n_x}} [g_{\text{post},1}(t_l), \ldots, g_{\text{post},n_x}(t_l)]^T \in \mathbb{C}^{n_x \times 1}.
\] (6.19)
The space/fast-time vector is then formed by taking the Kronecker product of both steering components,

\[ \tilde{H}_{\text{post}}(t_l, u) = \mathbf{g}_{\text{post}}(t_l) \otimes \mathbf{s}(u) \in \mathbb{C}^{n_xN \times 1} \]  

(6.20)

where this signal has the form for the \( n^{th} \) channel and \( k^{th} \) fast-time sample,

\[ H_{\text{post},n}(t_l - t_k, u) = \exp \left( jk_c d_n \sin \left( \theta(u) \right) \right) \text{sinc} \left( B(t_l - t_k) \right) \]  

(6.21)

or if stacked spatially, resembles the vector \( \mathbf{H}_{\text{post}}(t_l - t_k, u) \) from Equation 6.15.

Optimal fast-time STAP involves replacing the fast-time steering vector with the optimal weight for each pulse,

\[ x_{\text{fs}}(t, u) = \mathbf{W}_f^H(t, u) \mathbf{X}_f(u) \]  

(6.22)

where

\[ \mathbf{W}_f(t, u) = \tilde{\gamma} \mathbf{\hat{R}}_{Z_l}^{-1} \tilde{H}_{\text{post}}(t_l, u) \in \mathbb{C}^{n_xN \times 1}. \]  

(6.23)

Once again, the maximum SINR criteria with \( \tilde{\gamma} = 1 \) has been chosen to provide a comparison between the slow-time STAP formulation. The estimated space/fast-time interference plus noise covariance, \( \mathbf{\hat{R}}_{Z_l} \), can be determined by averaging the outer product of the space/fast-time interference plus noise vector \( \mathbf{Z}_l(u) \), over a number of pulses. Analogous to the slow-time case, this can be achieved by a sample matrix estimate over \( M \) pulses,

\[ \mathbf{\hat{R}}_{Z_l} = \frac{1}{M} \sum_{m=1}^{M} \mathbf{Z}_l(u_m) \mathbf{Z}^H_l(u_m) + \eta \mathbf{I}_{n_xN} \in \mathbb{C}^{n_xN \times n_xN}. \]  

(6.24)

However, in the presence of non-stationary hot-clutter, averaging over pulses produces unwanted modulations which will distort the covariance estimate, [Rabideau, 2000] and so this estimator cannot be used. The effect of these modulations is investigated further in Section 6.5 and a different estimate without averaging is used. Consequently, it must be reevaluated at each pulse, \( u \).

\[ \mathbf{\hat{R}}_{Z_l}(u) = \mathbf{Z}_l(u) \mathbf{Z}^H_l(u) + \eta \mathbf{I}_{n_xN} \in \mathbb{C}^{n_xN \times n_xN}. \]  

(6.25)

Without diagonal loading, this is a rank one matrix. Therefore \( \eta \) dB of diagonal loading has been included to enable its inversion. Also, with a known interference and small level of diagonal loading, the filter output is known as the Hung-Turner projection, [Hung and Turner, 1983]. This is shown in Appendix C.

The final step in forming a SAR image is to focus the modified single channel data \( x_{\text{fs}}(\cdot) \) in azimuth. This is done using the multichannel spatial Matched Filter interpolation algorithm from Section 6.3.1. Figure 6.5 represents an overview of this algorithm.
6.4 Simulated Results

An X-band simulation is used to compare these three algorithms while varying the relative level of hot-clutter power. To provide a comparison of the performance of the algorithms presented above, the moderately diffuse scenario is chosen from Section 5.4.2. The direct-path incidence angle chosen is $\theta_J = 1^\circ$, classing the scenario as outer mainlobe jamming with the spread of azimuth angles, $1^\circ \pm 0.35^\circ$. With the parameters described below in Table 6.2, this range of angles will fall outside the SAR integration angle, $\varphi = \pm 0.17^\circ$ and hence a STAP processor is able to partly cancel the interference while maintaining the look direction. The real array beampattern is shown below in Figure 6.6 to demonstrate this.

![Figure 6.5. Fast-time STAP block diagram](image)

![Figure 6.6. Real array beampattern showing: (—) SAR integration angle and (- -) hot-clutter angular spread around $\theta_J$](image)

In the following sections, both a sample image and Point Spread Function (PSF) are evaluated and their performance measured against an ideal image with no interference added. As the sample image contains more ground clutter scatterers than the PSF, it will have a greater total ground clutter to noise ratio and the jammer power is set to 95 dB for the sample image and 60 dB for the PSF.
dB for the PSF. These values are chosen to provide a good comparison using the performance metrics described in the following sections.

Also due to the computational load required to run the STAP algorithms, only $n_x = 200$ range bins are used, thereby providing $0.4NM$ samples for training. As a result, diagonal loading is used to regularise the covariance with a level proportional to the interference power. It was found from simulation results that a level of $\eta = 20$ dB is suitable to avoid a non-singular inverse and not significantly influence the adaptive performance.

### Table 6.2. Simulation parameters

<table>
<thead>
<tr>
<th>Main parameters</th>
<th>Symbol</th>
<th>Value</th>
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</thead>
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<td>Carrier frequency</td>
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</tr>
<tr>
<td>Bandwidth</td>
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</tr>
<tr>
<td>Number of fast-time samples</td>
<td>$L$</td>
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<tr>
<td>Number of range bins</td>
<td>$n_x$</td>
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</tr>
<tr>
<td>Number of pulses</td>
<td>$M$</td>
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</tr>
<tr>
<td>Number of elements</td>
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</tr>
<tr>
<td>Element spacing</td>
<td>$\delta$</td>
<td>$\frac{\lambda}{2}$ m</td>
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<tr>
<td>Pulse repetition frequency</td>
<td>$f_{PRF}$</td>
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<tr>
<td>SAR platform velocity</td>
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<tr>
<td>Pulse length</td>
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<tr>
<td>Ground clutter power</td>
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<td>Thermal noise power</td>
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</tr>
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</tr>
<tr>
<td>Range centre</td>
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</tr>
<tr>
<td>SAR height</td>
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<td>3 km</td>
</tr>
<tr>
<td>Jammer height</td>
<td>$h_J$</td>
<td>3 km</td>
</tr>
<tr>
<td>Jammer offset</td>
<td>$\bar{x}_J$</td>
<td>50 km</td>
</tr>
<tr>
<td>Interference</td>
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<td></td>
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<td>Number of hot-clutter scatterers</td>
<td>$K_{hc}$</td>
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<td>95 / 60 dB</td>
</tr>
<tr>
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<td>$v_J$</td>
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</tr>
<tr>
<td>Jamming offset angle</td>
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<td>$1^\circ$</td>
</tr>
<tr>
<td>Diagonal loading factor</td>
<td>$\eta$</td>
<td>20 dB</td>
</tr>
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</table>
6.4 Simulated Results

6.4.1 Image Comparisons

Example images of the algorithm results are shown in Figure 6.7 for the direct-path only signal and in Figure 6.8 for $K_{hc} = 100$ hot-clutter scatterers of relative power level, $\rho = 13$ dB. From a visual inspection of both figures, it appears that by using the multichannel imaging algorithm, the SAR image is just visible with the direct-path signal alone and becomes completely degraded with the hot-clutter interference. The slow-time STAP algorithm has rejected the interference quite well for the direct-path only case, but becomes blurry and unfocussed with the hot-clutter interference. The fast-time STAP algorithm however manages to suppress the interference and maintain good image quality for both cases.

![SAR image comparison with direct-path only](image1)

**Figure 6.7.** SAR image comparison with direct-path only, $\rho = -\infty$ dB, (horizontal - range, vertical - azimuth)

![SAR image comparison with direct-path and hot-clutter](image2)

**Figure 6.8.** SAR image comparison with direct-path and hot-clutter, $\rho = 13$ dB, (horizontal - range, vertical - azimuth)

To obtain a quantitative measure of image degradation, the filtered image can be compared to an ideal reference image such as in Figure 6.1. The Signal Distortion Ratio (SDR) was introduced in Chapter 3 with $Y(x_p, y_q)$ denoting the filtered image and $D(x_p, y_q)$ the reference image with range and azimuth pixels $p = 1 \ldots n_x, q = 1 \ldots n_y$. The SDR was defined by,

$$ SDR = 10\log_{10} \left[ \frac{\sum_{p,q} |D(x_p, y_q)|^2}{\sum_{p,q} |Y(x_p, y_q) - D(x_p, y_q)|^2} \right]. $$

Figure 6.9 shows the SDR when the relative power of the hot-clutter interference is varied from $\rho = -\infty$ dB (direct-path only) to $\rho = 13$ dB. The imaging only case demonstrates poor performance with the direct-path only signal and even worse as hot-clutter is included. Slow-time STAP offers some improvement for the direct-path only case and with small levels of
relative hot-clutter power. However, the best suppression is achieved by fast-time STAP which maintains good performance independent of the strength of the hot-clutter. Note that due to the different forms of sample matrix estimate being used for slow-time and fast time STAP algorithms, (see Equations 6.14 and 6.25), the slow-time SDR will not converge to the fast-time SDR as $\rho \to -\infty$.

![Figure 6.9. SDR comparisons for: (—) conventional imaging, (- -) slow-time STAP, (-.-) fast-time STAP](image)

### 6.4.2 Point Spread Function Comparisons

To further analyse the effect of the interferences and suppression algorithms, the PSF is formed from a single scatterer at the centre of the imaging area. This also minimises the effect of signal mismatch compared to a scatterer in the far corner. The algorithm results for the direct-path only case are shown in Figure 6.10. They show similar results to the previous image case, so slices through the centre of each image are used to more accurately visualise the algorithm performance. Figures 6.11 and 6.12 show range, azimuth and diagonal slices for the direct-path only case and also with a high level of hot-clutter power\(^{12}\). The diagonal slice is taken from the top left to bottom right and is projected onto the range axis.

The results for the direct-path only case show reasonable results for the slow and fast-time STAP, while multichannel imaging shows some distortion for each slice. With a strong level of hot-clutter power, the range slice shows good results for the slow and fast-time STAP and severe distortion for multichannel imaging. The azimuth and diagonal slices however maintain good fast-time STAP results, but show severe distortion for both the slow-time STAP and multichannel imaging algorithms. The remainder of this section uses the Integrated Sidelobe Ratio (ISLR) and Peak Sidelobe Ratio (PSR) from Chapter 3 to more accurately measure the change in image quality along the azimuth direction.

\(^{12}\)Note that a non-interfered image has not been included as a reference, as it would be indistinguishable from the fast-time STAP result.
6.4 Simulated Results

Figure 6.10. SAR PSF comparison with direct-path only, $\rho = -\infty \text{ dB}$, (horizontal - range, vertical - azimuth)

Figure 6.11. SAR PSF slice comparison with direct-path only, $\rho = -\infty \text{ dB}$ for:

(—) conventional imaging, (- -) slow-time STAP, (-.-) fast-time STAP

**Integrated Sidelobe Ratio**

The ISLR was presented in Section 3.5.2 as a measure of the ratio of all energy in the first three sidelobes to the energy in the mainlobe. Lower sidelobes will result in a lower ISLR which indicates better performance. It was defined as,

$$ISLR = 10 \log_{10} \left[ \frac{\sum_x (|\hat{f}(x < -x_{ML})|^2 + |\hat{f}(x > x_{ML})|^2)}{\sum_x |\hat{f}(-x_{ML} \leq x \leq x_{ML})|^2} \right]$$

(6.27)
where the PSF slice is represented by \( \hat{f}_{dB}(x) = 10 \log_{10} \left( |\hat{f}(x)|^2 \right) \) and \( x_{ML} \) indicates the edge of the mainlobe.

Figure 6.13 shows a comparison for the azimuthal slice of the ISLR for a single point scatterer as the relative hot-clutter power is varied. As the range ISLR remains almost constant for each of the three cases, only the azimuth slice of the PSF is analysed. The multichannel imaging results show poor performance for all but the direct-path only case and the slow-time STAP algorithm gets gradually worse as the hot-clutter level increases. The fast-time STAP algorithm on the other hand, remains constant at -10 dB, independent of the level of hot-clutter power.

**Peak Sidelobe Ratio**

The PSR determines the difference between the mainlobe and highest sidelobe, with better performance indicated by a larger PSR,

\[
PSR = \max \left[ \hat{f}_{dB}(x) \right] - \max \left[ \hat{f}_{dB}(x < -x_{ML}), \hat{f}_{dB}(x > x_{ML}) \right].
\]  

(6.28)

Similar results to the ISLR are seen in Figure 6.14 with the fast-time STAP algorithm performing...
6.4 Simulated Results

Figure 6.13. ISLR comparisons for: (—) conventional imaging, (- -) slow-time STAP, (---) fast-time STAP

well with a constant level of 13 dB and the conventional image showing poor performance for all but the direct-path only case. The slow-time STAP algorithm however shows strange behaviour, as it first increases before getting worse. This is due to the energy spreading away from the first sidelobe and into the others. It is demonstrated in Figure 6.15 with azimuth slices for three different levels of relative hot-clutter power.

Figure 6.14. PSR comparisons for: (—) conventional imaging, (- -) slow-time STAP, (---) fast-time STAP

Figure 6.15. Slow-time STAP azimuth slice for $\rho =$ (—) -inf dB, (- -) -7.4 dB, (---) 13 dB
6.5 Realistic Training

Only an ideal training situation has been considered in this chapter with excellent interference suppression demonstrated for the fast-time STAP algorithm using the Hung-Turner projection. The second stage of realism from Section 5.4.6 is achieved when there is different data for the received data signal and that used for training in the covariance matrix estimate.

To implement optimal fast-time STAP in this case, the interference plus noise covariance must be averaged over pulses to produce a reasonable estimate of the covariance. Unfortunately, any averaging over pulses in the presence of non-stationary hot-clutter will produce two side-effects as described by [Rabideau, 2000]. The first is ‘training modulation’ which arises due to averaging over a finite number of different realisations and the second is ‘coherence modulation’ which results from non-stationarity present in the data used for the interference plus noise covariance estimate. This modulation is very similar to the problem inherent in slow-time STAP and this analysis should show similar results.

One technique to provide a tradeoff between the accuracy of the estimate and the modulations is to form the sample matrix estimate over only the previous \( M' \) pulses. This will act to increase the training modulation and reduce the effect of the non-stationarity. On the \( m^{th} \) pulse,

\[
\hat{R}_{Z_i} = \frac{1}{M'} \sum_{\Omega_m} Z_i(u_m)Z_i^H(u_m) + \eta I_{n_xN} \in \mathbb{C}^{n_xN \times n_xN} \tag{6.29}
\]

where the summation bounds, \( \Omega_m = m - M' + 1 : m \) and the first \( M' \) pulses average from \( \Omega_m = 1 : m \). In his paper, Rabideau derived expressions for the coherency and training modulations using the statistical properties of the training data and an ideal interference plus noise covariance matrix. The training modulation was only present when non-ideal training data was used, while the coherency modulation appeared when the non-stationary hot-clutter was introduced. It is important to understand the effect of these modulations and how they affect a focused SAR image so that the ideal case in this chapter can be tested against a more realistic scenario.

6.5.1 Training Modulation

To isolate the training modulation, a simulation with only the direct-path signal is used as it represents a stationary environment with no coherent modulations. Realistic training data is used by simulating two different realisations of the interference plus noise signal, one for the received data signal and the other for the training data. This was described in more detail in Section 5.4.6.

In a stationary environment, the training modulation will have less effect as the amount of averaging increases. This is consistent with [Reed et al., 1974], who show that to achieve an estimate within 3 dB of optimal, the number of training samples used for the estimate must be \( M \geq 2n_xN \). However, this is very difficult to achieve for optimal fast-time STAP and diagonal loading is used to both improve the estimate and avoid a non-singular inverse. It is therefore
expected that the estimate using all the pulses in Equation 6.24 should provide a better estimate of the stationary interference.

Figure 6.16 shows a comparison of three PSF slices. The first is the optimal PSF using the ideal covariance, the second is formed using the estimate from Equation 6.29 averaged over the previous \( M^\prime = 20 \) pulses and the third uses the estimate from Equation 6.24 where all the pulses are used. It is not surprising that the third estimate provides the closest results to optimal, even though only \( M = 100 = 0.1n_xN \) pulses were used for averaging. It is the diagonal loading which acts to improve the estimate when there are insufficient pulses available for averaging, [Rabideau, 2000].

![Figure 6.16. PSF showing training modulation for: (—) optimal estimate, (- -) averaging over 20 pulses, (---) averaging over all pulses](image)

### 6.5.2 Coherency Modulation

Coherency modulation is a result of averaging over pulses in the presence of non-stationary hot-clutter and will occur for both the ideal and realistic training data. To isolate the coherency modulation, an ideal simulation with hot-clutter of relative power, \( \rho = -3 \) dB is used with both the covariance estimates in Equations 6.29 and 6.24. Figure 6.17 shows the same comparison as the training modulation case, with the optimal and two estimated PSF slices.
The effect of the non-stationarity is to distort the PSF along the range slice when using estimates averaged over pulses and is noticeably worse when averaging over all pulses. As adaption occurred over fast-time, the range is most effected and the azimuth slice does not distort as badly. These results are also similar to those seen with slow-time STAP as the inherent problem of averaging over pulses is the same.

![Graph showing coherency modulation for: (—) optimal estimate, (- -) averaging over 20 pulses, (---) averaging over all pulses](image)

**Figure 6.17.** PSF showing coherency modulation for: (—) optimal estimate, (- -) averaging over 20 pulses, (---) averaging over all pulses

### 6.5.3 Controlling Modulation

Both of these modulation effects severely reduce the quality of the PSF and any SAR image produced would be severely distorted and blurred. To aid in this problem, [Rabideau, 2000] has suggested a number of techniques to control these modulations:

**Training size** Increasing the training size may be used to control training modulation, but this is not always possible due to the number of training samples available. Also, as a side effect, the coherency modulation will increase as the training size increases. Therefore, the total effect of both modulations must be used to select the training size.
6.6 Conclusion

Reduced rank beamspace For controlling both modulations, a reduced rank beamspace approach can be used to reduce the size of the covariance and also the impact of mismatched signals. This is due to each beam emphasising hot-clutter from some directions, while attenuating it from others, thereby providing an overall reduction in the modulation.

Constrained adaption Constrained adaption can also be used to control modulation effects by shaping the beampattern over a desired region, reducing the impact of adapting in a non-stationary environment and prevent suppression of the desired signals.

Pre-filter The use of a pre-filter before adaption can be used to attenuate strong ground clutter returns and cause the weak clutter regions to fall below the noise floor after modulation. It is important to note that a pre-filter does not try to completely eliminate clutter, only attenuates selected regions of it. This more relaxed objective permits the use of non-adaptive clutter filtering techniques.

As well as the modulation effects, optimal fast-time STAP is extremely computationally intensive and could never be used in a real system. As a result, further work in this thesis is focussed on sub-optimal fast-time STAP, where a smaller number of samples are used to form the adaptive filter.

6.6 Conclusion

This chapter has demonstrated how hot-clutter can degrade a multichannel SAR image and cause it to be severely distorted. Three algorithms have been presented to show how both stationary and non-stationary interferences can be suppressed. The first algorithm was a space/slow-time multichannel imaging algorithm and demonstrated some interference reduction through the process of burn-through. The second algorithm was optimal slow-time STAP which has the ability to reject the direct-path interference with a small amount of hot-clutter, while the third algorithm, fast-time STAP has shown to be far more effective than the first two algorithms in the presence of strong hot-clutter. The drawback of this algorithm comes when using realistic training data.

In this case, averaging over pulses is required to achieve a reasonable estimate of the interference plus noise covariance matrix and modulations become a problem due to the number of averages and the non-stationarity of the hot-clutter. A number of techniques have been suggested in the literature to address these modulations, although the computational load of optimal fast-time STAP makes many of these prohibitive in a real system. Sub-optimal implementations are therefore required to obtain a more reasonable runtime and reduce the impact of the modulations. Chapter 7 looks at this formulation and contrasts the performance of adaption pre and post range processing, before the final Chapter 8 looks at using constrained adaption with sub-optimal fast-time STAP.
7.1 Introduction

Fast-time Space Time Adaptive Processing (STAP) was introduced in the previous chapter as having the best potential for suppressing the non-stationary hot-clutter interference. However, an optimal implementation is not feasible due to the computational requirements and the inability to get a good estimate of the covariance matrix. Also in the previous chapter, the entire received pulse was used to model the steering vector and the performance of the adaptive filter would have been identical if it was placed pre or post range processing (RP). The performance of sub-optimal adaptive filtering however will depend on how well the steering vector matches the target signal within a limited number of range bins. In the literature, there have been performance studies on each of these adaptive methods but not a direct comparison on which one offers the best adaptive results, particularly for SAR. This chapter therefore provides analytic models for both the pre and post RP interference covariances and steering models and then compares their relative performance as the number of fast-time taps are increased. Figure 7.1 shows the Synthetic Aperture Radar (SAR) processing chain including transmission of the chirp signal, formation of the received data signal, adaption, RP and image formation.

Relevant literature includes a performance study using three dimensional (3D) STAP pre RP by [Techau et al., 1999a], who derived an analytical model for the hot-clutter covariance and tested its performance under different simulated topographical conditions. A similar study for two dimensional (2D) fast-time STAP post RP has been performed by [Fante and Torres, 1995]. They derived an analytical model for the hot-clutter covariance and use a simulated scenario to compare element space and beam space approaches. The only study on fast-time STAP performance for SAR is by [Klemm, 2002]. He looked at simulated data applied pre RP and concluded that firstly, fast-time filtering will degrade SAR resolution by broadening the point spread function mainlobe and increasing its sidelobes and secondly, as range resolution improved, so did its sensitivity to filtering.

The first part of this chapter in Section 7.2 describes how the optimal fast-time STAP filter can be approximated by a sub-optimal filter. As the fast-time STAP filter now only covers a small
number of samples, its performance will depend on how well the fast-time steering vector matches the true representation of the target signal. Models for both the pre and post RP space/fast-time steering vectors are therefore introduced along with details of the estimated interference covariance matrix. The following two sections 7.3 and 7.4 then present a derivation of the exact instantaneous and estimated covariances for both cases using a common signal model. Although the derivations have been published previously, this allows a direct comparison to analyse the best correlation conditions. A number of scattering scenarios are then tested in Section 7.5 to determine the expected level of correlation with increasing fast-time taps. An analysis of the eigen-distribution is then used to determine the rank of the interference and see how it varies between models and the Signal to Interference plus Noise Ratio (SINR) loss is used to measure their relative adaptive performance.

7.2 Fast-time STAP

Optimal space/fast-time focussing was introduced in Section 6.3.3 as a convolution with the post RP reference vector, $\mathbf{H}_{\text{post}}(\cdot)$ and the incoming range processed data vector, $\mathbf{x}(\cdot)$ both stacked spatially over $N$ channels,

$$x_f(t, u) = \frac{1}{n_x} \sum_{k=1}^{n_x} \mathbf{H}_{\text{post}}^H(t - t_k, u)\mathbf{x}(t_k, u)$$

$$= \tilde{\mathbf{H}}_{\text{post}}^H(t, u)\mathbf{X}_f(u) \quad (7.1)$$
where \( n_x \) is the number of range bins and \((t_l, u)\) represents the \( l^{th} \) fast-time sample within a pulse and the SAR position respectively. The received SAR space/fast-time vector is formed by stacking spatially and then temporally to give,

\[
x(t_l, u) = \frac{1}{\sqrt{N}} \left[ x_{-(N-1)/2}(t_l, u), \ldots, x_{(N-1)/2}(t_l, u) \right]^T \in \mathbb{C}^{N \times 1},
\]

\[
X_f(u) = \frac{1}{\sqrt{n_x}} \left[ x^T(t_l, u), \ldots, x^T(t_d, u) \right]^T \in \mathbb{C}^{n_xN \times 1}.
\] (7.2)

Fast-time STAP then involves replacing the fast-time steering vector with the weight for each pulse,

\[
x_{fs}(t_l, u) = W^H_f(t_l, u) X_f(u) \quad \text{(7.3)}
\]

where the maximum SINR weight is given by,

\[
W_f(t_l, u) = \tilde{\gamma} \hat{R}^{-1}_{Z_f}(u) \tilde{H}_{\text{post}}(t_l, u) \in \mathbb{C}^{n_x N \times 1} \quad \text{(7.4)}
\]

where \( \tilde{\gamma} \) is an arbitrary scaling constant and \( \hat{\mathbf{R}}_{Z_f}(\cdot) \) is the estimated interference plus noise space/fast-time covariance matrix. To reduce the computational load and the inability to get a good estimate of the covariance matrix, the number of samples or taps used in the STAP filter can be reduced to \( \tilde{L} \), providing an approximation to the full convolution in Equation 7.1 and allowing a better estimate of the covariance matrix by reducing its variance when \( \tilde{L} \ll n_x \). It can be implemented by updating the range of \( k \) at each sample by \( k = l : l + \tilde{L} - 1 \) and reducing both the data and steering vectors in size. If \( \mathbf{H}(\cdot) \) and \( \tilde{\mathbf{H}}(\cdot) \) represent generic steering vectors, then the convolution from Equation 7.1 can be written as,

\[
x_f(t_l, u) = \frac{1}{L} \sum_{k=1}^{l+\tilde{L}-1} \mathbf{H}^H(t_l - t_k, u) x(t_k, u)
\]

\[
= \tilde{\mathbf{H}}^H(u) X_f(t_l, u) \quad \text{(7.5)}
\]

where the steering vector is now a function of the relative difference between time taps and the corresponding sub-optimal data vector is,

\[
X_f(t_l, u) = \frac{1}{\sqrt{\tilde{L}}} \left[ x^T(t_l, u), x^T(t_{l+1}, u), \ldots, x^T(t_{l+\tilde{L}-1}, u) \right]^T \in \mathbb{C}^{\tilde{L}N \times 1} \quad \text{(7.6)}
\]

with data components for the final \( \tilde{L} \) taps set to zero. Similarly, the fast-time STAP filter will have the form,

\[
x_{fs}(t_l, u) = W^H_f(u) X_f(t_l, u) \quad \text{(7.7)}
\]

with the maximum SINR weight given by,

\[
W_f(u) = \tilde{\gamma} \hat{R}^{-1}_{Z_f}(u) \tilde{\mathbf{H}}(u) \in \mathbb{C}^{\tilde{L}N \times 1}. \quad \text{(7.8)}
\]

Note that the covariance, \( \hat{\mathbf{R}}_{Z_f}(u) \) is now reduced in dimension to \( \tilde{L}N \times \tilde{L}N \). 

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**Chapter 7 Fast-time STAP Performance**
7.2 Fast-time STAP

7.2.1 Space/fast-time Steering Vectors

As the fast-time STAP filter now only covers $\tilde{L}$ fast-time taps, its performance will depend on how well the fast-time steering vector matches the true representation of a target signal. The forms of the pre and post RP space/fast-time steering vectors are based on the received signal models defined in Chapter 2. Due to the fact that the carrier frequency is much larger than the bandwidth and the scene is in the far field of the real array, both signal models can be written with separate spatial and temporal components,

$$s_{\text{pre},n}(t_l, u, x, y) \approx s_n(u)s_{\text{pre},0}(t_l, u, x, y)$$  \quad (7.9)

$$s_{\text{post},n}(t_l, u, x, y) \approx s_n(u)s_{\text{post},0}(t_l, u, x, y)$$  \quad (7.10)

where the temporal components, $s_{\text{pre},0}(\cdot)$ and $s_{\text{post},0}(\cdot)$ are defined at the antenna phase centre and the spatial steering model for the $n^{th}$ channel is given by,

$$s_n(u) = \exp[j k_c d_n \sin(\theta(u))]$$  \quad (7.11)

where $k_c$ is the wavenumber at the carrier frequency, $d_n = n \delta$ is the antenna offset from the array phase centre with antenna spacing $\delta$ and $\theta(u)$ is the steering angle relative to the centre of the imaging patch. If there are $N$ antenna elements, this signal can be stacked with the antenna phase centre at the centre of the array to form the spatial steering vector,

$$s(u) = \frac{1}{\sqrt{N}} [s_{-(N-1)/2}(u), \ldots, s_{(N-1)/2}(u)]^T \in \mathbb{C}^{N \times 1}. \quad (7.12)$$

The pre RP fast-time steering model is given by Equation 7.9 with the non-time dependent delay removed. For the $l^{th}$ sample and $k^{th}$ fast-time tap,

$$g_{\text{pre},k}(t_l) = \exp[-j B \pi (t_l - t_k) + j \alpha(t_l - t_k)^2]$$  \quad (7.13)

where $B$ is the bandwidth, $2\alpha$ represents the chirp rate and the samples occur at $t_l = T_s + (l-1) \Delta t$ where $T_s$ is the pulse collection starting time and $\Delta t$ is the fast-time sampling rate. As the steering vector is a chirp function, it can only provide a partially accurate representation of the target signal within $\tilde{L}$ taps. The extent of the mismatch can be seen by the pre RP temporal steering vector,

$$g_{\text{pre}} = \frac{1}{\sqrt{L}} \left[ 1, \exp[j B \pi \Delta t + j \alpha \Delta t^2], \ldots, \exp[j B \pi (\tilde{L} - 1) \Delta t + j \alpha((\tilde{L} - 1) \Delta t)^2] \right]^T \in \mathbb{C}^{\tilde{L} \times 1}.$$  \quad (7.14)

The post RP fast-time steering model has been defined in Section 6.3.3 as a sinc function,

$$g_{\text{post},k}(t_l) = \text{sinc}[B(t_l - t_k)]$$  \quad (7.14)

which is consistent with the received signal model in Equation 7.10 with the non-time dependent term in the exponential removed as it has no effect on a temporal filter’s output. As this model
Chapter 7 Fast-time STAP Performance

is a decaying function, it can be accurately represented by a small number of taps as shown by
the post RP temporal steering vector,

\[
g_{\text{post}} = \frac{1}{\sqrt{L}} \left[ 1, \text{sinc} [B \Delta t], \ldots, \text{sinc} \left[ B(\tilde{L} - 1)\Delta t \right] \right]^T \in \mathbb{C}^{\tilde{L} \times 1}. \tag{7.15}
\]

Also, if no oversampling is used, this model matches the delta function commonly used in
literature, [Kogon, 1996], [Techau et al., 1999b] where it is assumed that the target occupies a
single range bin. The combined space/fast-time steering vector for each case is then formed by
the Kronecker product of the spatial and temporal components to give,

\[
\tilde{H}_{\text{pre}}(u) = g_{\text{pre}} \otimes s(u) \in \mathbb{C}^{LN \times 1}
\]

\[
\tilde{H}_{\text{post}}(u) = g_{\text{post}} \otimes s(u) \in \mathbb{C}^{LN \times 1}. \tag{7.16}
\]

7.2.2 Estimated Sample Matrix Covariance

The estimated interference plus noise covariance matrix is found by estimating over \( L_t < n_x \)
samples. According to [Reed et al., 1974], to achieve an estimate within 3 dB of optimal when
there are \( LN \) independent blocks to average over, the number of samples used for the estimate
must be \( L_t \geq 2LN \). However, the formulation in this thesis uses overlapping fast-time taps and
this criteria only holds approximately. As the minimal number of samples will be less than this
value and the estimate quality improves with more averaging, this value is instead used as a
lower bound.

The pre and post RP received interference plus noise signals, \( z_{\text{pre, n}}(t, u) \), \( z_{\text{post, n}}(t, u) \) must also be
correctly stacked in the same manner as the data vector in Equation 7.6. If \( Z_{\ell}(\cdot) \) represents either
the pre or post RP interference plus noise vector, then the corresponding ‘sample covariance
matrix’ is given by,

\[
\hat{R}_{Z_{\ell}}(u) = \frac{1}{L_t} \sum_{l=1}^{L_t} Z_{\ell}(t_l, u)Z_{\ell}^H(t_l, u) \in \mathbb{C}^{LN \times LN}. \tag{7.17}
\]

7.3 Covariance Model Pre Range Processing

The pre RP covariance model is based on the derivation by [Techau et al., 1999a]. The main
difference in this approach is to use a baseband model for the jammer and assume the hot-clutter
scatterers are stationary within a pulse. The received jammer plus noise signal after basebanding
is given by Equation 5.3. Only the jammer component is included in this study and is given as,

\[
z_{\text{pre, n}}(t, u) = \sum_{k=0}^{K_{hc}} b_k J(t - \bar{\tau}_n(x_k, y_k - u)) \exp \left[ -j\omega_c \bar{\tau}_n(x_k, y_k - u) \right] \exp \left[ -j\omega_d k t \right]. \tag{7.18}
\]

The first part of this Section 7.3.1 uses this signal model to derive the exact instantaneous
covariance model for a given fast-time. To remove this dependence, the mean estimated sample
covariance is then derived in Section 7.3.2, before being split into spatial and temporal
components in Section 7.3.3 to help with analysis of the covariance matrix.
The exact instantaneous covariance model is given at time $t$ by,

$$R_{Z_i,\text{pre}}(t, \zeta, n, n') = E\{z_{\text{pre},n}(t, u)z_{\text{pre},n'}(t - \zeta, u)\}$$

$$= \sum_k \sum_{k'} b_k b_{k'} E\{J(t - \bar{\tau}_{n,k}) J^*(t - \zeta - \bar{\tau}_{n',k'})\} \exp[-j\omega_c(\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) - j\omega_{d,k'}\zeta] \exp[-j\Delta\omega_{k,k'}t]$$  (7.19)

where the dependence on $u$ is not included, $\bar{\tau}_{n,k} \equiv \bar{\tau}_n(x_k, y_k - u)$, $\Delta\omega_{k,k'} = \omega_{d,k} - \omega_{d,k'}$ and $\zeta$ is the delay between two fast-time taps. The form of the covariance lag in the expectation of Equation 7.19 is found by substituting $\chi = t - \bar{\tau}_{n,k}$, giving

$$E\{J(t - \bar{\tau}_{n,k}) J^*(t - \zeta - \bar{\tau}_{n',k'})\} = E\{J(\chi) J^*(\chi + \bar{\tau}_{n,k} - \bar{\tau}_{n',k'} - \zeta)\}$$  (7.20)

where the broadband jammer is a stationary random process within the pulse interval, $r_J(\tilde{\zeta}) = E\{J(t) J^*(t - \tilde{\zeta})\}$. The form of the covariance pre RP then reduces to,

$$R_{Z_i,\text{pre}}(t, \zeta, n, n') = \sum_k \sum_{k'} b_k b_{k'}^* r_J(\zeta - \bar{\tau}_{n,k} + \bar{\tau}_{n',k'}) \exp[-j\omega_c(\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) - j\omega_{d,k'}\zeta] \exp[-j\Delta\omega_{k,k'}t]$$  (7.21)

or by using the jammer covariance derived in Section 5.4.4,

$$r_J(\tilde{\zeta}) = \sigma_J^2 \text{sinc}[B\tilde{\zeta}]$$  (7.22)

can be written as,

$$R_{Z_i,\text{pre}}(t, \zeta, n, n') = \sigma_J^2 \sum_k \sum_{k'} b_k b_{k'}^* \text{sinc}[B(\zeta - \bar{\tau}_{n,k} + \bar{\tau}_{n',k'})] \exp[-j\omega_c(\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) - j\omega_{d,k'}\zeta] \exp[-j\Delta\omega_{k,k'}t].$$  (7.23)

With this covariance model, the only fast-time dependence is from the Doppler components of the hot-clutter scatterers. This implies that a number of fast-time taps must be used to accurately represent the exact covariance. In practice however, an estimate is calculated from a number of fast-time taps as is shown in the following section.

### 7.3.2 Mean Estimated Covariance Model

The sample covariance matrix estimate from Section 7.2.2 offers a good estimate of the exact covariance presented in the previous section. To determine its relative performance, an analysis of the mean estimated sample covariance model from Equation 7.17 can be determined. This will also allow a performance comparison between both pre and post RP models by removing the
dependence on fast-time, $t_l$. Note, that if the broadband jammer is assumed to be a stationary random process within the pulse interval, so will the received interference signal, $z_{\text{pre},n}(\cdot)$.

$$
\hat{R}_{Z_{\text{t,pre}}}(\zeta, n, n') = E \left\{ \frac{1}{L_t} \sum_{l=1}^{L_t} z_{\text{pre},n}(t_l, u) z_{\text{pre},n'}(t_l - \zeta, u) \right\} \\
= \sigma_j^2 \sum_{k} \sum_{k'} b_k b_{k'}^* \text{sinc} \left[ B(\zeta - \tau_{n,k} + \tau_{n',k'}) \exp \left[ -j \omega_c (\tau_{n,k} - \tau_{n',k'}) - j \omega_{d,k} \zeta \right] \right] \\
\frac{1}{L_t} \sum_{l=1}^{L_t} \exp \left[ -j \Delta \omega_{k,k'} (T_s + (l - 1) \Delta_t) \right] \\
= \sigma_j^2 \sum_{k} \sum_{k'} b_k b_{k'}^* \text{sinc} \left[ B(\zeta - \tau_{n,k} + \tau_{n',k'}) \exp \left[ -j \omega_c (\tau_{n,k} - \tau_{n',k'}) - j \omega_{d,k} \zeta \right] \right] \\
\exp \left[ -j \Delta \omega_{k,k'} T_s \right] \frac{1}{L_t} \sum_{l=1}^{L_t} \exp \left[ -j \Delta \omega_{k,k'} (l - 1) \Delta_t \right]. \quad (7.24)
$$

The final sum can then be written as a geometric series [Techau et al., 1999a],

$$
\frac{1}{L_t} \sum_{l=1}^{L_t} \exp \left[ -j \Delta \omega_{k,k'} (l - 1) \Delta_t \right] = \exp \left[ -j 0.5 \Delta \omega_{k,k'} (L_t - 1) \Delta_t \right] \frac{\sin \left[ 0.5 \Delta \omega_{k,k'} L_t \Delta_t \right]}{L_t \sin \left[ 0.5 \Delta \omega_{k,k'} \Delta_t \right]} \quad (7.25)
$$
giving the final result,

$$
\hat{R}_{Z_{\text{t,pre}}}(\zeta, n, n') = \sigma_j^2 \sum_{k} \sum_{k'} b_k b_{k'}^* \text{sinc} \left[ B(\zeta - \tau_{n,k} + \tau_{n',k'}) \exp \left[ -j \omega_c (\tau_{n,k} - \tau_{n',k'}) - j \omega_{d,k} \zeta \right] \right] \\
\exp \left[ -j \Delta \omega_{k,k'} (T_s + 0.5 (L_t - 1) \Delta_t) \right] \frac{\sin \left[ 0.5 \Delta \omega_{k,k'} L_t \Delta_t \right]}{L_t \sin \left[ 0.5 \Delta \omega_{k,k'} \Delta_t \right]} \frac{1}{L_t} \sum_{l=1}^{L_t} \exp \left[ -j \Delta \omega_{k,k'} (l - 1) \Delta_t \right]. \quad (7.26)
$$
When compared to the exact instantaneous covariance in Equation 7.23, there is now a minor multiplicative bias determined by the number of averaged fast-time taps, $L_t$.

### 7.3.3 Separable Covariance

To further analyse the covariance matrix, it is useful to separate the spatial and temporal components. This can be achieved by first separating these components in the reflected hot-clutter signal, $z_{\text{pre},n}(\cdot)$.

$$
z_{\text{pre},n}(t, u) = \sum_{k=0}^{K_{\text{fin}}} b_k J(t - \tau_n(x_k, y_k - u)) \exp \left[ -j \omega_c \tau_n(x_k, y_k - u) \right] \exp \left[ -j \omega_{d,k} t \right]
\\
\approx \sum_k b_k z_{\text{spat},n}(\theta_{a,k}(u)) z_{\text{temp}}(t, u, x_k, y_k). \quad (7.27)
$$

To define these signal components, the delay $\tau_n(\cdot)$ can be written as,

$$
\tau_{n,k} = \tau_n(x_k, y_k - u) = T_{d,n}(\theta_{a,k}(u)) + \tau_0(x_k, y_k - u) \quad (7.28)
$$
where the temporal component is defined at the real array phase centre and the spatial component is given by,

\[ T_{d,n}(\theta_{a,k}(u)) = \frac{d_n}{c} \sin [\theta_{a,k}(u)] \] (7.29)

with the scatterer incidence angle defined by,

\[ \theta_{a,k}(u) = \theta_a(x_k, y_k - u) = \arctan \left( \frac{y_k - u}{x_k} \right). \] (7.30)

Also, since the broadband jammer model is defined at baseband, the spatial component within \( J(\cdot) \) is very small compared to the temporal and can be set to zero. The signal components can then be written as,

\[ z_{spat,n}(\theta_{a,k}(u)) = \exp \left[ -j\omega_c T_{d,n}(\theta_{a,k}(u)) \right] \] (7.31)

\[ z_{temp}(t,u,x_k,y_k) = J(t - \bar{\tau}_0(x_k, y_k - u)) \exp \left[ -j\omega_c \bar{\tau}_0(x_k, y_k - u) \right] \exp \left[ -j\omega_{d,k} t \right]. \] (7.32)

Substituting these into the mean estimated covariance model with \( \bar{\tau}_{0,k} \equiv \bar{\tau}_0(x_k, y_k - u) \) gives,

\[ \tilde{R}_{Z_{i,pre}}(\zeta, n, n') = \sigma_f^2 \sum_k \sum_{k'} b_kb_{k'}^* \exp \left[ -j\omega_c T_{d,n}(\theta_{a,k}(u)) + j\omega_c T_{d,n'}(\theta_{a,k'}(u)) \right] \]

\[ \begin{align*}
\text{sinc} & \left[ B(\zeta - \bar{\tau}_{0,k} + \bar{\tau}_{0,k'}) \right] \exp \left[ -j\omega_c (\bar{\tau}_{0,k} - \bar{\tau}_{0,k'}) - j\omega_{d,k'} \zeta \right] \\
& \exp \left[ -j\Delta \omega_{k,k'}(T_s + 0.5(L_t - 1)\Delta_t) \right] \frac{\sin [0.5\Delta \omega_{k,k'} L_t \Delta t]}{L_t \sin [0.5\Delta \omega_{k,k'} L_t \Delta t]}.
\end{align*} \] (7.33)

Finally, to form the full space/fast-time covariance matrix, \( \tilde{R}_{Z_{i,pre}} \) the spatial components, \( n, n' \) and the temporal delay \( \zeta \) must be varied over the antenna elements and fast-time taps respectively.

### 7.4 Covariance Model Post Range Processing

The covariance model post RP is based on the derivation by [Fante and Torres, 1995]. The main difference in this approach is to baseband the signals before RP and assume the hot-clutter scatterers are stationary within a pulse. This derivation also provides a more straightforward solution and does not require a series of cases to interpret the results.

The post RP covariance requires the range compression to be included in the derivation. At baseband, the reference signal is given by \( s_{MF}(t) = p_c^*(-t) \), where the SAR waveform is a chirp defined in Section 2.3.2,

\[ p_c(t) = b \left( \frac{t}{T_p} \right) \exp \left[ -j B\pi t + j\alpha t^2 \right] \] (7.34)

with the chirp duration defined by \( b(t) \), which is unity for \( 0 \leq t \leq 1 \) and zero otherwise. This gives the post RP jammer signal model as,

\[ z_{post,n}(t, u) = \int_{-\infty}^{\infty} z_{pre,n}(t', u)p_c^*(t' - t)dt'. \] (7.35)
This section is analogous to the pre RP case with the first Section 7.4.1 now using the signal model above to derive the instantaneous covariance model for a given fast-time. The mean estimated sample covariance is then derived in Section 7.4.2, before being split into spatial and temporal components in Section 7.4.3 to help with analysis of the covariance matrix.

### 7.4.1 Exact Instantaneous Covariance Model

The exact instantaneous covariance is given by,

$$
R_{Z_{t} \text{post}}(t, \zeta, n, n') = E\{z_{\text{post}, n}(t, u)z_{\text{post}, n'}^*(t - \zeta, u)\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E\{z_{\text{pre, n}}(t', u)z_{\text{pre, n'}}^*(t'' - \zeta, u)\}
$$

$$
p_c^s(t' - t) p_c(t'' + \zeta - t) dt' dt''
$$

and with the jammer signal in Equation 7.18 substituted,

$$
R_{Z_{t} \text{post}}(t, \zeta, n, n') = \sum_k \sum_{k'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_k b_{k'}^* E\{J(t' - \bar{\tau}_{n,k}) J^* (t'' - \bar{\tau}_{n',k'})\}
$$

$$
\exp \left[-j \omega_c (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[-j \omega_d k' t' + j \omega_d k' t'' \right]
$$

$$
p_c^s(t' - t) p_c(t'' + \zeta - t) dt' dt''.
$$

This equation can then be simplified by defining $\chi = t' - \bar{\tau}_{n,k}$, modifying the expectation term as

$$
E\{J(t' - \bar{\tau}_{n,k}) J^* (t'' - \bar{\tau}_{n',k'})\} = E\{J(\chi) J^* (\chi + t'' - t' + \bar{\tau}_{n,k} - \bar{\tau}_{n',k'})\}
$$

where the broadband jammer is a stationary random process within the pulse interval. This term can then be written in terms of the jammer covariance with $r(\tilde{\zeta}) = r(-\tilde{\zeta})$, giving

$$
R_{Z_{t} \text{post}}(t, \zeta, n, n') = \sum_k \sum_{k'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_k b_{k'}^* r_{J} (t'' - t' + \bar{\tau}_{n,k} - \bar{\tau}_{n',k'})
$$

$$
\exp \left[-j \omega_c (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[-j \omega_d k' t' + j \omega_d k' t'' \right]
$$

$$
p_c^s(t' - t) p_c(t'' + \zeta - t) dt' dt''.
$$

This model is simplified in Appendix D for a chirp of duration $T_p$, giving the final form given as

$$
R_{Z_{t} \text{post}}(t, \zeta, n, n') = \sigma_j^2 \sum_k \sum_{k'} b_k b_{k'}^* \text{sinc} \left[ B \zeta' (1 - \zeta' / T_p) - \Delta f_{k,k'} (T_p - \zeta') \right]
$$

$$
\exp \left[-j (\omega_c + \omega_d k') (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right]
$$

$$
\exp \left[-j \Delta \omega_{k,k'} (t + 0.5 (T_p - \zeta')) \right].
$$

(7.40)

where $\zeta' = \zeta - \bar{\tau}_{n,k} + \bar{\tau}_{n',k'}$ and $\Delta f_{k,k'} = 0.5 \Delta \omega_{k,k'}/\pi$. This model is very similar to the pre RP case with the sinc function containing the relative temporal delays and the correlation variable. There are now however extra terms related to the pulse width and the differential Doppler. These differences are also present in the two exponential terms and will cause only a very small difference in the overall model.
7.5 Correlation Analysis

7.4.2 Mean Estimated Covariance Model

Similarly to Section 7.3.2, to compare the performance of both pre and post RP models, the dependence on fast-time, \( t_f \) must be removed. For the post RP case, this is achieved by finding the mean of the estimated sample covariance model given in Equation 7.40 with the same assumption that since the broadband jammer is a stationary random process within the pulse interval, so must be the post RP interference signal, \( z_{\text{post},n}(\cdot) \).

\[
\bar{R}_{Z_{t,\text{post}}} (\zeta, n, n') = \sigma_j^2 \sum_k \sum_{k'} b_kb_k^* \text{sinc} \left[ B \zeta' (1 - \zeta'/T_p) - \Delta f_{k,k'} (T_p - \zeta') \right] \\
\exp \left[ -j(\omega_c + \omega_{d,k'}) (\bar{r}_{n,k} - \bar{r}_{n',k'}) \right] \\
\left\{ \frac{1}{L_t} \sum_{l=1}^{L_t} \exp \left[ -j\Delta \omega_{k,k'} (T_s + (l - 1)\Delta_t + 0.5(T_p - \zeta')) \right] \right\} \\
= \sigma_j^2 \sum_k \sum_{k'} b_kb_k^* \text{sinc} \left[ B \zeta' (1 - \zeta'/T_p) - \Delta f_{k,k'} (T_p - \zeta') \right] \\
\exp \left[ -j(\omega_c + \omega_{d,k'}) (\bar{r}_{n,k} - \bar{r}_{n',k'}) \right] \\
\exp \left[ -j\Delta \omega_{k,k'} (T_s + 0.5(T_p - \zeta' + (L_t - 1)\Delta_t)) \right] \sin \left[ \frac{0.5 \Delta \omega_{k,k'} L_t \Delta_t}{L_t \sin \left[ 0.5 \Delta \omega_{k,k'} \Delta_t \right]} \right]. \quad (7.41)
\]

7.4.3 Separable Covariance

The separable form of the post RP mean estimated sample covariance is determined using the assumptions in Section 7.3.3,

\[
\bar{R}_{Z_{t,\text{post}}} (\zeta, n, n') = \sigma_j^2 \sum_k \sum_{k'} b_kb_k^* \exp \left[ -j \omega_c T_{d,n}(\theta_{a,k}(u)) + j \omega_c T_{d,n'}(\theta_{a,k'}(u)) \right] \\
\text{sinc} \left[ B \zeta' (1 - \zeta'/T_p) - \Delta f_{k,k'} (T_p - \zeta') \right] \exp \left[ -j(\omega_c + \omega_{d,k'}) (\bar{r}_{0,k} - \bar{r}_{0,k'}) \right] \\
\exp \left[ -j\Delta \omega_{k,k'} (T_s + 0.5(T_p - \zeta' + (L_t - 1)\Delta_t)) \right] \sin \left[ \frac{0.5 \Delta \omega_{k,k'} L_t \Delta_t}{L_t \sin \left[ 0.5 \Delta \omega_{k,k'} \Delta_t \right]} \right]. \quad (7.42)
\]

Then similarly to the pre RP case, the full space/fast-time covariance matrix, \( \bar{R}_{Z_{t,\text{post}}} \) can be formed by varying the spatial variables, \( n \) and \( n' \) and the temporal delay \( \zeta \) over the antenna elements and fast-time taps.

7.5 Correlation Analysis

To measure the effect of using fast-time taps and the relative performance of the pre and post RP models, the analytic models must be carefully analysed. This is done in the first Section 7.5.1 by looking at the required conditions for correlation between hot-clutter scatterers. A look at how fast-time taps can improve the correlation condition and hence the benefit of using fast-time taps for interference suppression is then presented and demonstrated for a number of scenarios in Section 7.5.2.
Chapter 7 Fast-time STAP Performance

The second part of this section looks at comparing the pre and post RP covariance models. Section 7.5.3 looks at the eigen-distribution of the different scenarios for both models to determine their relative interference rank. The final Section 7.5.4 then uses the steering models from Section 7.2.1 and the different covariance models to measure the relative adaptive performance for a real filter.

7.5.1 Scatterer Correlation Conditions

Conditions for correlation between scatterers will depend on the sinc function common to both pre and post expressions approaching unity. This will effect the rank of the covariance matrix and hence the ability of an adaptive filter to improve interference suppression with fast-time taps. For the pre RP case, the sinc function is given by,

\[
\text{sinc} \left[ B(\zeta - \bar{\tau}_0,k + \bar{\tau}_0,k') \right]
\]  

(7.43)

and for the post RP,

\[
\text{sinc} \left[ B\zeta(1 - \zeta'/T_p) - \Delta f_{k,k'}(T_p - \zeta') \right].
\]  

(7.44)

The two limiting cases for correlation can be seen by setting the bandwidth \( B \) to either narrowband or broadband. The best case is achieved by a narrowband jammer with bandwidth approximately zero. This results in a covariance matrix with a rank equal to one and an adaptive filter will easily be able to cancel the interference. Alternatively, the worst case is when the jammer is broadband with bandwidth approaching infinity. Providing no scattered path has the same delay, the sinc function approaches zero and the resulting matrix has high off diagonal elements with rank \( \max\{N\bar{L},K_{hc}\bar{L}\} \). The large bandwidth ensures that each scatterer is uncorrelated with any others and will look like an independent source. Consequently, there will be no benefit from using temporal taps to remove the interference.

For systems with a finite bandwidth, the level of correlation will depend on the relative bistatic delay of the scatterers. If \( T_p \gg \zeta' \) and the fast-time delay, \( \zeta \) and Doppler terms, \( \Delta f_{k,k'} \) are set to zero, this is achieved for both cases by the differential delay bandwidth product being smaller than unity,

\[
|B(\bar{\tau}_{0,k} - \bar{\tau}_{0,k'})| < 1.
\]  

(7.45)

This result is not good for SAR, where a high bandwidth is required for good range resolution. However, by storing the sampled returns at a number of fast-time taps, it is possible to improve the overall correlation.

If the fast-time delay is included, the correlation condition becomes,

\[
|B(\zeta + \bar{\tau}_{0,k} - \bar{\tau}_{0,k'})| < 1
\]  

(7.46)

and implies that as the number of fast-time taps is increased, this condition is better satisfied. More correlations between scatterers are picked up as the overall correlation now comprises contributions from all fast-time taps.
If the pulse width constraint is relaxed and the Doppler terms are included in the post RP correlation function, the post RP correlation condition becomes

$$|B\zeta'(1 - \zeta'/T_p) - \Delta f_{k,k'}(T_p - \zeta')| < 1$$  (7.47)

which indicates that the relative Doppler between two scatterers and the pulse width will also contribute to the overall correlation. The Doppler term is very small compared to the relative bistatic delay bandwidth product and its effect on the correlation is minimal. Similarly, the pulse width term is typically greater than $\zeta'$ and if not, it works to reduce the differential delay bandwidth product, thus providing slightly better correlation!

The optimal number of fast-time taps required to satisfy the correlation condition is determined by the largest delay bandwidth product as complete suppression of the hot-clutter requires that the fast-time interval, $T_I = (L_I - 1)\Delta_t$ span this delay, [Fante and Torres, 1995], [Kogon, 1996]. This gives the ideal number of fast-time taps as,

$$L_I = \frac{T_I}{\Delta_t} + 1$$  (7.48)

where $L_I$ is rounded to the closest integer. Examples of different optimal tap numbers are provided in the following section, however in many scenarios the fast-time interval may require an unrealistic number of taps greater than the number of range bins, $n_x$. This means that perfect correlation may not be achieved and this measure instead should be used as an upper bound. Also, if less than this number of taps is used, extra scatterer pairs will still be picked up, thereby improving the overall correlation and still providing partial hot-clutter suppression.

### 7.5.2 Fast-time Taps

The fast-time tap spacing is an important parameter and has been studied by a number of authors, [Compton Jr., 1988a], [Fante and Torres, 1995]. For the application in this chapter, the fast-time tap spacing is fixed to the fast-time sample rate and to gain any advantage from using fast-time taps, the correlation condition in Equation 7.46 shows that the sample rate must be of the same order as the relative bistatic delay of the scatterer pairs. Secondly, since the correlation condition is based on the sinc function, upsampling can be used to increase the number of non-zero points in the correlation function. However, this improvement will also depend on the relative bistatic delays which are unknown and hence a factor of two is used for upsampling, giving $\Delta_t = 1/2B$, [Fante and Torres, 1995].

The differential delay for $K_{hc} = 100$ hot-clutter scatterers, when the jammer incidence angle is at $\theta_J = 0^\circ$ is shown in Figure 7.2. Three scenarios are compared, representing very diffuse ($K_\beta = 0.01$), moderately diffuse ($K_\beta = 0.7$) and specular scattering ($K_\beta = 10$). It is clear that the differential delay increases with the level of diffuseness.

To measure the effect of using fast-time taps, the delay $\zeta = t_2 - t_1$ is varied by setting $t_1 = 0$ and varying $t_2$ from 0 to $(\tilde{L} - 1)\Delta_t$. The level of correlation between all scatterers, $k = 1 \ldots K_{hc}$
is then measured using Equation 7.46 and if the absolute differential delay bandwidth product falls below 1, it is considered to contribute to the correlation. The first plot in Figure 7.3 shows the number of correlated pairs with each fast-time tap given as a percentage out of $K^2$ possible correlations. The second plot in this figure represents the total number of scatterer pairs meeting the same criteria.

The results for the first plot show scatterers which are uncorrelated with the number of taps a huge 84047! In the moderately diffuse scenario however the ideal number of taps is a more reasonable 11, although there appears to be minimal improvement with taps above 9. This result is due to the level of correlation for later taps being very small and using taps above 9 is expected to provide only a minor improvement. The specular scenario contains 100% correlated scatterers, so adding fast-time taps will make no difference to the correlation. This is observed in the first plot as the delay pairs are very similar and all the scatterer pairs meet the correlation condition with no tap delays (1 fast-time tap) and also with one tap delay.
The second plot reveals a constant level of 1% correlation for the very diffuse scenario and there is no benefit in using fast-time taps. For the moderately diffuse case, the number of new scatterer pairs increases by 20% with increasing fast-time taps and plateaus at 81% with 5 taps. It never reaches 100% as there are no new scatterer pairs introduced beyond this point and \( n_x < L_1 \). This value does not mean that adding taps beyond 5 will limit performance, as it is the correlation per tap which contributes to the total correlation. This level does however provide an indication on the level of hot-clutter which would be cancelled using fast-time taps. For example, the specular scenario shows that 100% of the scatterers meet the correlation condition and adaptive filtering should remove it completely.

### 7.5.3 Eigen-distribution Analysis

To measure the differences between the covariance models, the eigen-distribution can be analysed. This method can be used to determine the dominant rank of the covariance matrix by measuring the number of eigenvalues above the noise floor. The three models compared include the estimated covariance using simulated data and the mean estimated pre and post RP covariance matrices from Equations 7.33 and 7.42. The thermal and quantisation noise are not included in the simulation model and a noise floor of 0 dB is imposed by adding an identity matrix to the covariance models.

To provide a more detailed view of the interference subspace, the number of fast-time taps is varied, thereby increasing the total number of eigenvalues to \( \tilde{L}N \). Although this analysis is to compare the different covariance models, an accurate measure of the dominant interference subspace allows for reduced rank interference suppression techniques as shown in Chapter 8. For these comparisons, simulation of a single pulse was performed using parameters shown below in Table 7.1.

The eigen-distribution is obtained by a eigen-decomposition to determine \( \tilde{Q} \) and \( \Lambda \) such that \( R_z = \tilde{Q}\Lambda\tilde{Q}^H \). The eigen-distribution then forms the diagonal of \( \Lambda \) and is sorted from largest to smallest. The following Figure 7.4 shows nine plots, corresponding to the three scattering scenarios and three different fast-time taps. Only the simulated pre RP has been included and uses \( L_1 = 3\tilde{L}N \) range bins for averaging.

For the very diffuse scenario, the first two plots show a large interference rank equal to the maximum number of eigenvalues, with the third only just reaching the noise floor. This is expected as each of the 100 hot-clutter scatterers represent statistically independent signals. Completely orthogonal signals would however have eigenvalues of equal magnitude and these plots show a decrease. The reason for this is the spatial component of the covariance, which provides some correlation due to the closeness in angle between many of the scatterers. The pre and post RP distributions match identically for the first two plots, but separate very slightly for the third. They both however, cross the noise floor at the same level, giving equal rank. The simulated covariance estimate on the other hand falls slightly short of the ideal eigen-distribution due to the difficulty of getting a good estimate with diffuse signals.
Table 7.1. Simulation parameters

<table>
<thead>
<tr>
<th>Main parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>Carrier frequency</td>
<td>$f_c$</td>
<td>10 GHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>$B$</td>
<td>0.3 GHz</td>
</tr>
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<td>Number of fast-time samples</td>
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<td>Number of range bins</td>
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<td>Number of elements</td>
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</tr>
<tr>
<td>Element spacing</td>
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<tr>
<td>Relative hot-clutter power</td>
<td>$\rho$</td>
<td>$-3$ dB</td>
</tr>
<tr>
<td>Jamming offset angle</td>
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<tr>
<td>Training size</td>
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<td>$3LN$</td>
</tr>
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</table>

Figure 7.4. Eigen-distribution for: (—) simulated pre RP, (- -) pre RP, (.-.) post RP
7.5 Correlation Analysis

For the moderately diffuse scenario, the interference rank is reduced for the three plots. There is now no noticeable difference between the pre and post RP distributions and they both cross the noise floor at approximately the same place giving an equal rank measure. The specular scenario has an interference rank equal to the number of fast-time taps and all three covariances match perfectly.

7.5.4 Adaptive Performance

The final comparison is to look at the adaptive performance between the two adaption methods. Using the spatial only case ($\tilde{L} = 1$), the SINR loss is defined using the SINR based on the maximum SINR weight in Section 4.3.1 divided by the optimal Signal to Noise Ratio (SNR), giving a range between 0 and 1. From this section, the optimal spatial SINR was given by,

$$S_{\text{SNR}}(\theta(u)) = \sigma^2 \nu S_{\text{NR},T} s^H(u) R_z^{-1}(u) s(u)$$  \hspace{1cm} (7.49)

where $\sigma^2$ is the noise power, $S_{\text{NR},T}$ is the target SNR and $R_z(u)$ is the spatial interference covariance matrix. The optimal SNR is determined by the target SNR in the absence of interference multiplied by the spatial gain for an arbitrary $u$,

$$S_{\text{NR, opt}} = s^H(u) s(u) S_{\text{NR}, T} = N S_{\text{NR}, T}.$$  \hspace{1cm} (7.50)

The SINR loss is then defined as,

$$L_{\text{SINR}}(\theta(u)) = \frac{S_{\text{SNR}}(\theta(u))}{S_{\text{NR, opt}}} = \frac{\sigma^2 s^H(u) R_z^{-1}(u) s(u)}{N}.$$  \hspace{1cm} (7.51)

which is related within a scale factor to the inverse of the mean output power defined in Equation 4.14. To extend the SINR loss to use space/fast-time vectors, the equivalent steering vectors for both pre and post RP must be used and will then be combined to form the equivalent space/fast-time steering vectors. The SINR loss for an arbitrary fast-time steering vector, $\tilde{H}(-)$ is then defined in the same manner as the spatial only case with the optimal SNR given by the inner product of the space/fast-time steering vector for an arbitrary $u$,

$$S_{\text{NR, opt}} = \tilde{H}^H(u) \tilde{H}(u)$$  \hspace{1cm} (7.52)

giving the SINR loss as,

$$L_{\text{SINR}}(\theta(u)) = \frac{\sigma^2 \tilde{H}^H(u) R_z^{-1}(u) \tilde{H}(u)}{\tilde{H}^H(u) \tilde{H}(u)}.$$  \hspace{1cm} (7.53)

To compare the SINR loss, simulation of a single pulse was used to model the covariance matrices, including the simulated estimated pre RP model from Equation 7.17 and the space/fast-time versions of the mean estimated pre and post RP models in Equations 7.33 and 7.42. Results are
given in Figure 7.5 for the three scattering scenarios and for three different fast-time taps as the steering angle, $\theta(u)$ is varied. The diffuse and specular scenarios show a good match between the pre and post RP cases, while the moderately diffuse scenario shows a slight mismatch between the two. The estimated SINR loss however does not match the sample covariances in the very diffuse scenarios due to the mismatch in the covariance estimation. Between fast-time taps, there is also a noticeable decrease in the SINR loss of approximately 20 dB for the moderately diffuse scenario. This is due to the covariance matrix being able to provide a measurement of the extra correlations between hot-clutter scatterers.

To analyse this scenario more closely, the minimum point of the SINR loss is measured as the number of fast-time taps is increased. Figure 7.6 shows plots for the three different cases. All three of the cases show an increase of close to 24 dB and it is clear that the SINR loss decreases as more taps are used. From Equation 7.48, the ideal number of taps is 9 and there is very little improvement with adding taps beyond this limit. Looking back at Figure 7.4, it is also apparent that for the moderately diffuse scenario, the interference rank is 9 for 6 fast-time taps and only increases to 11 when there are 12 taps. This again indicates that there is little improvement in using too many fast-time taps to cancel the interference.

![Figure 7.5. SINR loss with varying steering angle for: (---) simulated pre RP, (- -) pre RP, (---) post RP](image_url)
7.6 Conclusion

This chapter looked at how a sub-optimal space/fast-time filter can be used to approximate the optimal filter introduced in Chapter 6 and how its performance improves as more fast-time taps are used. Adaptive performance both pre and post RP were analysed with the corresponding steering vectors given as a chirp and sinc function respectively. The advantage of the post RP sinc function is that it can more accurately be represented by a smaller number of samples, while the pre RP chirp can not. To determine the adaptive performance, a pre RP simulated estimated covariance model was calculated and both the pre and post RP mean estimated sample covariances were derived analytically. The end result of the latter models showed them to be very similar with the form of an exponential multiplied by a sinc function.

A number of scattering scenarios were then tested to determine the expected level of correlation with increasing fast-time taps. The benefit of this approach however was only noticed when the tap-spacing was of the order of the relative bistatic delay and results showed a good match between the ideal number of taps and the correlation analysis of simulated hot-clutter scatterers. This number also indicates the point where the optimal level of interference suppression was reached and adding more taps would not improve the level of hot-clutter suppression. It was shown that the performance was limited by the amount of extra correlations between hot-clutter scatterers that could be represented by the covariance matrix. The overall correlation level was less than 100%, indicating that not all of the interference would be removed in an adaptive filter.

An analysis of the eigen-distribution was then used to determine the rank of the interference and measure any variation between the pre and post RP mean estimated models. This showed a good match between the two covariance models for all but the most diffuse scenario and even for this case both models still gave equal rank. Similarly for this scenario, the simulated estimated
covariance fell short of the ideal eigen-distribution due to the difficulty of getting a good estimate with diffuse scattering.

Finally, an analysis of the SINR loss was used to measure the expected performance of both models as they would be applied in an adaptive filter. There was a big increase in performance for the moderately diffuse scenario due to the extra correlation between hot-clutter scatterers. Results for the two mean estimated sample covariances showed little difference between scenarios, while the simulated estimated model had trouble matching the other cases for the very diffuse scenario.

In summary, both pre and post RP adaptive filters perform equally well at suppressing hot-clutter. However, the pre RP steering vector is not able to detect targets within the received data as well as the post RP case and the latter model is therefore the preferred choice for SAR.
Chapter 8

Constrained Fast-time STAP

8.1 Introduction

Typical SAR imaging is performed with a large offset range and small field of view. Jammer signals incident in the SAR sidelobes will typically be suppressed with minimal effect on the final SAR image. However, as the incident angle moves inside the mainbeam and approaches the SAR integration angle, the range profile of a target can be nullled and consequent image formation will lead to a blurry final image. The goal of interference suppression for SAR is to successfully suppress both the direct-path and hot-clutter interferences while not significantly effecting the image quality by blurring, reducing the resolution or raising the sidelobe level.

Chapter 6 demonstrated that non-stationary interference or hot-clutter will cause the training statistics to change from pulse to pulse and slow-time Space Time Adaptive Processing (STAP) will not be effective. Fast-time STAP was then demonstrated as an effective method to suppress non-stationary hot-clutter and offers the advantage of exploiting the coherency between the direct-path jammer and hot-clutter signals to provide improved interference rejection. The previous chapter looked at fast-time STAP implementations pre and post range processing using a small number of fast-time taps. Adaption post range processing was found to most accurately match the SAR signal model over the adaptive period.

The first Section 8.2 of this chapter describes the development of constrained adaptive algorithms as applied to spatial and space/time adaptive processing. The next Section 8.3 then contains a brief summary of the SAR and jammer signal models presented in chapters 2 and 5, as well as information on the simulation and performance metrics. An introduction to constrained adaptive filtering is then presented for element space algorithms in Section 8.4 and for beamspace algorithms in Section 8.5 via the Generalised Sidelobe Canceller (GSC). Reduced rank formulations of the GSC are then presented in the final two sections 8.6 and 8.7. These algorithms use fast-time taps with constraints to both minimise the effect of modulations due to the non-stationary hot-clutter and potential signal suppression in the mainbeam. The first approach is based on an eigen-decomposition of the interference plus noise matrix, while the second uses a Multistage Wiener Filter (MWF). The goal of both of these algorithms is to achieve
8.2 Background

Constrained adaptive beamforming is a technique used to shape the beampattern over a desired region, reduce the impact of adapting in a non-stationary environment, prevent suppression of the desired signals and minimise the modulation effects. The most common use of constraints is the Minimum Variance Distortionless Response (MVDR), which was first presented by [Frost III, 1972] in a constrained least mean square space-time power minimisation algorithm. The goal of his approach was to respond to signals in a chosen look direction while rejecting signals incident from other directions. Derivative constraints were later introduced by [Buckley and Griffiths, 1986] for a beamspace GSC and by [Er and Cantoni, 1993] for an element space formulation. These constraints force the first and second order spatial derivatives of the array power response in the look direction to zero and allow direct control of the sensitivity of the beampattern to mismatch between the steering and arrival angle of the desired signal. This provides robustness against steering mismatch and allows a tradeoff between the spatial resolution capability of the beamformer and in the case of a jammer, the degree to which it can be eliminated.

[Thng and Cantoni, 1993] have shown however that ‘sufficient’ conditions for the spatial derivatives to be zero are in general quadratic, and the resulting weight vector solution space is non-convex. [Tuthill et al., 1995] suggests using quadratic constraints to resolve this problem and still provide the desired performance. Another problem shown by [Buckley and Griffiths, 1986] occurs if the phase centre is not at the middle of the real array and results in the beamformer response varying greatly with undesirable sidelobes. An alternative is presented by [Tseng, 1992] and uses phase independent derivative constraints to avoid this problem.

A similar technique involves placing a number of unity amplitude constraints around the steering direction, [Johnson and Dudgeon, 1993]. This also provides an approximation to derivative constraints and can be varied to change the effect of the constraint. The only application of constrained adaptive filtering for hot-clutter suppression is by [Griffiths, 1996]. He used constraints to preserve the characteristics of the desired signal while suppressing hot-clutter and has shown good results with the MOUNTAINTOP data set. Further background material on element and beamspace STAP implementations as applied to hot-clutter suppression can be found in Section 5.3.

Fully adaptive processing can be very computationally intensive and not suitable for real time operation. Moreover, if the interference is non-stationary over the coherent processing interval, the eigenvalues of the estimated covariance matrix will spread, increasing the interference rank and therefore the degrees of freedom required to effectively cancel it. This problem is also analogous to the Moving Target Indication application where the ground clutter returns may
not be stationary due to real world effects, such as aircraft crabbing, non-linear array geometry, intrinsic clutter motion, and scattering from near-field obstacles, [Guerci and Bergin, 2002].

Reduced rank techniques work to reduce the rank associated with the interference plus noise covariance matrix. Many of the methods in the literature promise performance near or better than their full rank counterparts but with reduced sample support and computational load. The most common technique is known as an ‘eigen-canceller’ which uses a Principle Component (PC) decomposition of the estimated interference covariance matrix, [Gabriel, 1986]. By selecting the dominant eigenvectors associated with the largest eigenvalues, a lower rank covariance matrix can be formed. An alternative method is the Cross Spectral Metric (CSM) which utilises knowledge of the signal steering vector to further compress the interference subspace by selecting only those eigenvectors which minimise the Mean Square Error (MSE), [Goldstein and Reed, 1997a]. Applying derivative constraints to this technique was originally shown by [Buckley, 1987] and later applied to the GSC by [Van Veen, 1988], [Er and Ng, 1992] and [Haimovich et al., 1997].

An alternative technique is known as the MWF and was first formulated by [Goldstein et al., 1998] and later applied to hot-clutter suppression by [Goldstein et al., 1999]. It provides a faster rank reduction by using a nested chain of traditional Wiener filter stages. Weights are estimated at each stage to maximise the cross covariance energy between the main and reference beams, thereby requiring no knowledge of the eigen-structure of the covariance matrix and avoiding any matrix inversions. It was demonstrated that this approach provides excellent performance while reducing the rank below any previously known eigen-based methods. It was later modified by [Nguyen, 2002] to improve robustness to signal mismatch by using derivative constraints.

### 8.3 Simulation Models

The first part of this Section 8.3.1 is a brief summary of the multichannel SAR signal model presented in Chapter 2 while the second part 8.3.2 summarises the jammer model presented in Chapter 5. Details on the simulation used in this chapter is then presented in Section 8.3.3 and the metrics used to measure the algorithm performance in Section 8.3.4.

#### 8.3.1 SAR Signal Model

Consider a SAR travelling along the y-axis, imaging a point in the slant-plane $x \in [X_c - X_0, X_c + X_0]$, $y \in [-Y_0, Y_0]$. The radar transmits a broadband chirp and the received signal is base-banded, sampled and range processed for each of the $N$ channels of a linear antenna array. If the SAR is being jammed by an airborne platform, the signal components received by the $n^{th}$ channel of the SAR include the direct-path $z_{dp,n}(\cdot)$, ground reflected path (hot-clutter) $z_{hc,n}(\cdot)$, ground clutter return $\gamma_n(\cdot)$ and the receiver system noise $\nu_n(\cdot)$,

$$x_n(t, u) = z_{dp,n}(t, u) + z_{hc,n}(t, u) + \gamma_n(t, u) + \nu_n(t, u). \quad (8.1)$$
8.3 Simulation Models

Using the fact that the carrier frequency is much larger than the bandwidth, the form of the narrowband post range processing received signal model can be approximated by separating the spatial and temporal components,

\[ s_{\text{post},n}(t, u, x, y) = \exp[jk_c d_n \sin(\theta(u))] \text{sinc}[B\pi(t - \tau_n(x, y - u))] \approx s_n(u)s_{\text{post},0}(t, u, x, y) \quad (8.2) \]

where \( \tau_n(\cdot) \) is the temporal delay for the \( n^{th} \) channel, \( k_c \) is the wavenumber at the carrier frequency, \( B \) is the bandwidth and the variables \((t, u)\) represent fast-time within a pulse and the SAR platform position respectively. While there are \( L \) fast-time samples within each pulse, post range processing the available number of range bins becomes \( n_x \). The spatial component of the SAR signal model was derived in Section 2.3.5 with \( \theta(u) = \arctan(-u/X_c) \) as the steering angle and \( d_n = n\delta \), the antenna offset from the array phase centre with antenna spacing \( \delta \) and \( n \in [-\frac{N-1}{2}, \frac{N-1}{2}] \) for \( N \) (odd) antenna elements.

### 8.3.2 Jammer and Noise Model

The bistatic jammer model assumes there are \( K_{hc} \) hot-clutter patches within the area on the ground that is being irradiated by the jammer and that the jammer platform is directing its transmit beams to achieve maximum interference power on the SAR platform for both the direct path and hot-clutter components. These components and the receiver noise are combined into a single variable \( z_n(\cdot) = z_{\text{dp},n}(\cdot) + z_{hc,n}(\cdot) + \nu_n(\cdot) \). The noise component is defined in Section 5.4.4 as the sum of thermal and quantisation noise. Both are modelled as broadband Gaussian processes with zero mean and variance, \( \sigma^2_{\nu} \) and \( \sigma^2_q \) respectively.

The output of the \( n^{th} \) receiver due to the jammer \( z_n(\cdot) \) is then given as the superposition of the delayed reflections from each patch,

\[ z_n(t, u) = \sum_{k=0}^{K_{hc}} b_k J(t - \bar{\tau}_n(x_k, y_k - u)) \exp[-j\omega_c \bar{\tau}_n(x_k, y_k - u)] \exp[-j\omega_{d,k} t] + \nu_n(t, u) \quad (8.3) \]

where \( J(\cdot) \) is the signal at the jammer platform modelled as broadband Gaussian noise with zero mean and variance \( \sigma^2_J \). The bistatic delay \( \bar{\tau}_n(\cdot) \) and Doppler frequency, \( \omega_{d,k} \) for the \( k^{th} \) scatterer are defined in Section 5.4 according to the geometry of the jammer platform, the ground and the SAR platform. Likewise, the hot-clutter ground return \( b_k \) is defined as the relative magnitude between the direct-path jammer signal and the reflection of the \( k^{th} \) scatterer. It was derived in Section 5.4 using the bistatic radar range equation with the zero index referring to the direct-path signal, \( b_0 \equiv 1 \) and is represented for \( k = 1 \ldots K_{hc} \) by,

\[ b_k = \sqrt{\rho\sigma_k} \quad (8.4) \]

where \( \rho \) as the relative hot-clutter power and \( \sigma_k \) the hot-clutter ground return for the \( k^{th} \) scatterer.
8.3.3 Simulation

The entire processing chain for the MATLAB simulation is shown in Figure 8.1. It is based on the multichannel SAR simulation described in Chapter 2 with a set of parameters given in Table 8.1. A moderately diffuse scenario has been chosen from Section 5.4.2 to measure the effect of increasing the number of fast-time taps. Also, to demonstrate the worst case scenario, both the direct and hot-clutter paths are incident in the SAR mainbeam, with the hot-clutter spread over the range of angles, $-0.7^\circ$ to $0.7^\circ$.

After the chirp and the jammer signals are received, sampled and range processed for each spatial channel, the data is filtered for each pulse. The final step in the processing chain is image formation, which is performed using the multichannel spatial Matched Filter interpolation algorithm from Section 6.3.1. The sample image used for simulation is shown in Figure 8.2 along with an image with hot-clutter interference and an image which has been filtered to remove the interference. Note, that the degraded SAR image with hot-clutter is now visible as opposed to the image from Chapter 6, which had a stronger jammer signal present.

![Figure 8.1. SAR processing diagram](image-url)
### 8.3 Simulation Models

#### Table 8.1. Simulation parameters

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<th>Symbol</th>
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<td>Direct-path jammer power</td>
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<td>$L_t$</td>
<td>$3\bar{L}N$</td>
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</table>

#### 8.3.4 Performance Measures

To measure the performance of the algorithms presented in this chapter, two different metrics will be used. The first is the Residual Interference plus Noise Ratio (RINR), which measures adaptive performance of the received SAR signal before image formation. It is determined by measuring the residual interference plus noise power after cancellation relative to an ideal received data signal with no interference present. If $x_{fs}(\cdot)$ is the adapted signal and $x_{\text{ideal}}(\cdot)$ is the output signal with no interference present, then the RINR is given by,

$$RINR = 10\log_{10} \left[ \frac{1}{LM} \sum_{l,m} \frac{|x_{fs}(t_l, u_m)|^2}{|x_{\text{ideal}}(t_l, u_m)|^2} \right]. \quad (8.5)$$
A second measure of performance looks at the quality of the final image by measuring the Signal
Distortion Ratio (SDR) between an ideal and the filtered image. It was defined in Chapter 3
by letting \( Y(x_p, y_q) \) denote the adapted images for pixels \( p = 1 \ldots n_x, q = 1 \ldots n_y \) and \( D(x_p, y_q) \)
denote the ideal image with no jammer added. The SDR can then be defined as,
\[
SDR = 10 \log_{10} \left[ \frac{\sum_{p,q} |D(x_p, y_q)|^2}{\sum_{p,q} |Y(x_p, y_q) - D(x_p, y_q)|^2} \right].
\] (8.6)

### 8.4 Constrained filtering

This section introduces constrained fast-time STAP using an element space implementation. The constraints are used to minimise the effect of adaptive modulations and prevent potential
signal suppression while preserving the final image quality.

The first Section 8.4.1 describes the spatial only signal model using the constrained optimisation
criteria. The covariance estimation is then described in Section 8.4.2 along with information
on diagonal loading. The following sections then describe a number of constraint techniques
including the MVDR in Section 8.4.3, derivative constraints in Section 8.4.4 and amplitude
constraints in Section 8.4.5. The extension to fast-time STAP is then described in Section 8.4.6
and the entire section summarised in Section 8.4.7.

#### 8.4.1 Spatial Signal Model

To focus an array of receivers, a spatial steering vector is required to correctly evaluate the
response for each focussing position. If vectors are formed from the \( N \) channels of the received
signal \( x_n(t, u) \) and the spatial steering vector \( s_n(u) \) from Equation 8.2,
\[
\begin{align*}
\mathbf{x}(t, u) &= \frac{1}{\sqrt{N}} \left[ x_{-(N-1)/2}(t, u), \ldots, x_{(N-1)/2}(t, u) \right]^T \in \mathbb{C}^{N \times 1}, \\
\mathbf{s}(u) &= \frac{1}{\sqrt{N}} \left[ s_{-(N-1)/2}(u), \ldots, s_{(N-1)/2}(u) \right]^T \in \mathbb{C}^{N \times 1}
\end{align*}
\] (8.7)
then the focussed or beamformed conventional output is given by,

\[ f(t, u) = s^H(u)x(t, u). \]  

(8.8)

The adaptive beamformer replaces the steering vector with a weight vector \( w(u) \),

\[ f_{sp}(t, u) = w^H(u)x(t, u). \]  

(8.9)

It is designed to minimise the mean square value of the undesired signal component at each pulse subject to a set of constraints. The chosen method of adaption was described in Section 4.3.1 and is known as the Linearly Constrained Minimum Variance (LCMV) method, [Van-Trees, 2002]. Formally, the output power of the general optimisation problem is stated as

\[
\min_{w(u)} \left\{ w^H(u)R_z(u)w(u) \right\} \text{ subject to } C^H(u)w(u) = d
\]

(8.10)

with \( R_z(u) \) is the ideal interference plus noise covariance matrix, \( C(u) \) is the constraint matrix of size \( N \times N_{\text{con}} \) and \( d \) is a column matrix of constraining values, size \( N_{\text{con}} \times 1 \). The constrained optimisation problem is solved using Lagrange multipliers to find the weight vector [Johnson and Dudgeon, 1993],

\[
w_{\text{LCMV}}(u) = R_z^{-1}(u)C(u) \left[ C^H(u)R_z^{-1}(u)C(u) \right]^{-1} d \in C^{N \times 1}.
\]

(8.11)

As explained in Section 4.3.1, this processor is often presented using the total covariance, which implies the constraint condition is minimising the total power instead of just the interference plus noise power. It then has the same solution with \( R_z(u) \) being replaced with \( R_z(u) \). [Van-Trees, 2002] refers to the latter processor as the optimal linearly constrained minimum power processor.

### 8.4.2 Covariance Estimation and Diagonal Loading

For the algorithms in this chapter, the normalised estimated spatial interference plus noise covariance matrix, \( \hat{R}_z(u) = \alpha_{\text{norm}}^{-1}\hat{R}_z(u) \) replaces the ideal covariance matrix \( R_z(u) \). \( \hat{R}_z(u) \) is known as the sample matrix estimate and is determined by averaging over \( L_t \) range bins,

\[
\hat{R}_z(u) = \frac{1}{L_t} \sum_{l=1}^{L_t} z(t_l, u)z^H(t_l, u) \in C^{N \times N}
\]

(8.12)

and the normalising value, \( \alpha_{\text{norm}} = \text{Tr}\left\{ \hat{R}_z(u) \right\} / N \) provides a relative measure of the effect of diagonal loading. It is assumed that techniques described in Section 5.4.6 can be used to get different realisations of the interference plus noise signal without any targets present and the interference plus noise vector, \( z(\cdot) \) is formed similarly to the data vector \( x(\cdot) \).

Diagonal loading can be included by adding a scaled identity matrix to the estimated covariance matrix,

\[
\hat{R}_{z,DL}(u) = \hat{R}_z(u) + \eta I_N \in C^{N \times N}.
\]

(8.13)
It was used in Chapter 6 to regularise the covariance matrix and help with its inversion. For this chapter, the number of training samples, $L_t \geq 2N$ and the diagonal loading instead acts to improve the robustness by smoothing the adaption via compression of the eigenvalues of the covariance matrix [Ward et al., 2003]. In addition it acts to minimise the norm of the weight vector, thereby reducing adaptive sidelobes, regularising the estimated covariance inverse and improving the overall performance of the adaption, [Carlson, 1998]. [Cheremisin, 1982] has shown that a diagonal loading level of approximately three times the sensor noise yields optimum results in most cases. Other more exact techniques for determining the optimal loading factor are given by [Li et al., 2003] and [Ma and Goh, 2003]. Interestingly, the former paper formulates diagonal loading as an additional constraint on the adaption, which can be solved using a combination of Lagrange multipliers and Newton’s method. For this chapter however, the optimal level of diagonal loading is hard to estimate due to the varying level of temporal correlation between components of the hot-clutter and is instead found by searching over a range of possible levels.

### 8.4.3 Minimum Variance Distortionless Response

The most common use of the weight vector in Equation 8.11 is to constrain the steering direction to be unity. This is commonly referred to as the Minimum Variance Distortionless Response (MVDR), [Frost III, 1972] and is formed by substituting

$$C_{\text{MVDR}}(u) = s(u) \in C^{N \times 1} ; \quad d_{\text{MVDR}} = 1$$

into Equation 8.11. This technique provides good interference cancellation with sharp nulls in each interference direction. Unfortunately, as the hot-clutter scatterers change from pulse to pulse, so does the interference direction for each corresponding patch. As range profiles are built up with each pulse, coherent modulations are formed over the entire processing interval. Then after image formation, the SDR of the MVDR adaption can end up being no better than the conventional beamformer! These secondary effects have been studied previously for target detection in airborne radar [Rabideau, 2000] and require modifications to the single constraint optimisation.

### 8.4.4 Derivative Constraints

A second method to reduce potential target signal suppression requires first and/or second order derivatives to be zero in the steering direction, [Buckley and Griffiths, 1986]. The form of the derivatives is based on differentiating the spatial steering vector, $s_n(u)$ in Equation 8.2,

$$\frac{\partial s_n(u)}{\partial \theta(u)} = s_n(u) [j d_n k_c \cos[\theta(u)]] ,$$

$$\frac{\partial^2 s_n(u)}{\partial \theta^2(u)} = j d_n k_c \left[ -s_n(u) \sin[\theta(u)] + \frac{\partial s_n(u)}{\partial \theta(u)} \cos[\theta(u)] \right]$$
allowing the first order constraint to be written as a combination of the MVDR unity steering vector and first order derivative constraints,

\[
C_{\text{deriv1}}(u) = \begin{bmatrix} s(u), \frac{\partial s(u)}{\partial \theta(u)} \end{bmatrix} \in \mathbb{C}^{N \times 2} ; d_{\text{deriv1}} = [1, 0]^T
\]

and the second order by a combination of the MVDR unity steering vector with first and second order derivative constraints,

\[
C_{\text{deriv2}}(u) = \begin{bmatrix} s(u), \frac{\partial s(u)}{\partial \theta(u)}, \frac{\partial^2 s(u)}{\partial \theta^2(u)} \end{bmatrix} \in \mathbb{C}^{N \times 3} ; d_{\text{deriv2}} = [1, 0, 0]^T.
\]

This implementation of derivative constraints is based on the array phase centre being at the centre of the array, thereby avoiding the undesirable sidelobes described in [Buckley and Griffiths, 1986].

**Mean Output Power with Derivative Constraints**

Consider an example similar to that in Section 4.3.2 with an \( N = 5 \) element uniform linear array with \( \delta = \lambda_c / 2 \) spacing between elements. Both the target and jammer signals are incident at broadside, \( \theta_T = 0^\circ, \theta_J = 0^\circ \) with powers \( \sigma^2_T = 0 \text{ dB} \) and \( \sigma^2_J = 50 \text{ dB} \) respectively. They are modelled as plane waves, with thermal noise also present from the receiver with power \( \sigma^2_\nu = 0 \text{ dB} \). This gives the total received signal as \( x_n(t,u) = g(t,u)s_n(\theta_T) + z_n(t,u) \) with the interference plus noise given by \( z_n(t,u) = J(t,u)s_n(\theta_J) + \nu(t,u) \) and \( g(t,u) \) and \( J(t,u) \) are the time varying components of the target and jammer signals. The ideal interference plus noise covariance can then be formed as,

\[
R_z \approx \sigma^2_J s(\theta_J) s^H(\theta_J) + \sigma^2_\nu I_N \in \mathbb{C}^{N \times N}.
\]

The constrained adaptive weights for the MVDR and first and second order derivative constraints are found by substituting Equations 8.14, 8.17 and 8.18 into Equation 8.11. These are then used to filter the data vector,

\[
f_{sp}(\theta(u),t,u) = w^H(u)x(t,u)
\]

and form the power spectrum,

\[
P(\theta(u)) = E \{|f_{sp}(\theta(u),t,u)|^2\}.
\]

Figure 8.3 shows the power spectrum with the conventional, MVDR and first and second order constraints. Its clear that as each derivative is included, the adaptive beam becomes wider, thereby rejecting slightly less interference power while improving its performance against potential signal mismatch.
Simulation Results with Derivative Constraints

Greater interference suppression is achieved by a good match between the constraint vector and the interference being minimised. Consequently, the spread of hot-clutter signal components around the direct-path incidence angle $\theta_J$ will reduce the interference suppression expected from a narrow constraint such as the MVDR. The desired null will need to be broader and shallower such as that achieved by use of the derivative constraint.

To measure the relative effect of both the MVDR and derivative constraints as the diagonal loading level increases, a number of simulation runs are performed as described in Section 8.3.3. Figure 8.4 shows a plot of the resulting RINR and SDR for this case. As expected, when the diagonal loading increases, the RINR decreases and the SDR increases for all three cases. Then as the level is further increased, both of the metrics approach their conventional levels. For this simulation case, the conventional RINR is 10.1 dB and the SDR is 3.9 dB.

For the RINR plot, the MVDR case reaches a minimum of 8 dB at $\eta = -70$ dB before increasing again and approaching the conventional level. The first order case behaves similarly and reaches a slightly lower level of 7.6 dB at the same value of $\eta$, while the second order case doesn’t perform as well as the other two, reaching the conventional level with a relatively low level of diagonal loading.

The SDR results are almost analogous to the RINR except for the MVDR behaviour. For this case, the SDR improves to a level of 5.3 dB at $\eta = -80$ dB, then drops below the conventional level between -60 and -20 dB! This effect is not seen in the RINR results and is due to mismatch between the hot-clutter scatterers and the MVDR constraint as explained above. The first order derivative case surpasses the MVDR case with a peak of 6 dB at $\eta = -70$ dB, while the
8.4 Constrained filtering

second order case only improves the SDR slightly. For this reason, the second order derivative constraints are not used further in this chapter.

![Figure 8.4. Simulation results with derivative constraints, varying $\eta$: (---) MVDR, (- -) first order, (---) second order](image)

8.4.5 Amplitude Constraints

A third constraint option is to set amplitude constraints around the steering direction, [Johnson and Dudgeon, 1993]. A three point constraint is used here and is described by,

$$
C_{\text{amp}}(u) = [s(u, -\delta_{\text{amp}}), s(u), s(u, \delta_{\text{amp}})] \in \mathbb{C}^{N \times 3} ; \quad d_{\text{amp}} = [1, 1, 1]^T
$$

(8.22)

where the $n^{th}$ element of $s(u, \delta_{\text{amp}})$ is given by,

$$
s_n(u, \delta_{\text{amp}}) = \exp[jk_c d_n \sin[\theta(u) + \delta_{\text{amp}}]].
$$

(8.23)

The main drawback of this technique is uncertainty of where $\delta_{\text{amp}}$ should be placed as the optimal position will depend on the scenario.

Mean Output Power with Amplitude Constraints

The example from Section 8.4.4 is now repeated with the amplitude constraint described in Equation 8.22. A small angle of $\delta_{\text{amp}} = 0.5^o$ and a larger angle of $\delta_{\text{amp}} = 15^o$ is used for comparison in Figure 8.5. The smaller angle gives a wider response than the MVDR and first derivative, but is slightly smaller than the second derivative, while the larger angle shrinks the centre beam of the amplitude constraint so it is only slightly wider than the MVDR constraint.

Simulation Results with Amplitude Constraints

A number of simulation runs are now performed to measure the effect of amplitude constraints as both the angle $\delta_{\text{amp}}$ and the diagonal loading level, $\eta$ are varied. Figure 8.6 shows a plot of the resulting RINR and SDR. There is no apparent improvement with adding diagonal loading and the best result is achieved with $\eta = 0$ dB and a very small angle of $\delta_{\text{amp}} = 0.4^o$, giving an RINR of 8.2 dB and an SDR of 6 dB. From Figure 8.5, this is the region which closely resembles the second order derivative result. Then as the angle is increased, the RINR increases and the SDR
Figure 8.5. Mean output power with amplitude constraints for direct-path jammer only: (–) conventional, (– -) MVDR, (– -) amplitude, $\delta_{\text{amp}} = 0.5^\circ$, (– - -) amplitude, $\delta_{\text{amp}} = 15^\circ$.

decreases below the conventional level indicating mismatch between the hot-clutter scatterers and the amplitude constraints. Finally, as the angle increases to around $\delta_{\text{amp}} = 20^\circ$, the RINR again drops to 8.2 dB with the SDR at a level of 4.8 dB. This result is not unexpected as it is the region where the amplitude constraint approaches the narrow MVDR result. Although there are regions which show improvement over the conventional case, they are not as good as the first derivative case and due to the uncertainty of the angle $\delta_{\text{amp}}$, this constraint will not be used further in this thesis.

Figure 8.6. Simulation results with amplitude constraints, varying: $\eta$, $\delta_{\text{amp}}$. 
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8.4.6 Fast-time Extension

Optimal fast-time STAP was introduced in Section 6.3.3 as a convolution with the reference vector, \( H_{\text{post}}(\cdot) \) and the incoming data vector, \( X_f(\cdot) \). Then in Chapter 7, it was modified to adapt over only the future \( \tilde{L} \ll n_x \) fast-time samples. This provides an approximation to the full convolution and allows a better estimate of the covariance matrix. This section summaries these results and shows how the constrained adaptive filter can be extended to use fast-time taps.

As shown in Section 7.2, the spatial data vector can be stacked over the future \( \tilde{L} \) taps to create space/fast-time vectors,

\[
X_f(t_l, u) = \frac{1}{\sqrt{\tilde{L}}} \left[ x^T(t_l, u), x^T(t_{l+1}, u), \ldots, x^T(t_{l+\tilde{L}-1}, u) \right]^T \in \mathbb{C}^{\tilde{L}N \times 1} \quad (8.24)
\]

with data components for the final \( \tilde{L} \) taps set to zero. The \( k^{th} \) fast-time component of the post range processing steering vector was given in Section 7.2.1 as,

\[
g_{\text{post},k} = \text{sinc} \left[ B(k - 1)\Delta_t \right] \quad (8.25)
\]

and can be stacked to give the fast-time steering vector,

\[
g_{\text{post}} = \frac{1}{\sqrt{\tilde{L}}} \left[ 1, \text{sinc} [B\Delta_t], \ldots, \text{sinc} [B(\tilde{L} - 1)\Delta_t] \right]^T \in \mathbb{C}^{\tilde{L} \times 1}. \quad (8.26)
\]

The fast-time filter is then represented by the convolution,

\[
x_f(t_l, u) = \tilde{H}_{\text{post}}^H(u)X_f(t_l, u) \quad (8.27)
\]

with the space/fast-time steering vector formed by the Kronecker product of the spatial and temporal steering vectors,

\[
\tilde{H}_{\text{post}}(u) = g_{\text{post}} \otimes s(u) \in \mathbb{C}^{\tilde{L}N \times 1}. \quad (8.28)
\]

The fast-time equivalent of Equation 8.9 is similarly given by,

\[
x_{fs}(t_l, u) = W_f^H(u)X_f(t_l, u) \quad (8.29)
\]

where the constrained optimisation problem and solution from Section 8.4.1 is extended to use fast-time vectors instead of spatial ones,

\[
W_{f,\text{LCMV}}(u) = R_{Z_l}^{-1}(u)C_l(u) \left[ C_l^H(u)R_{Z_l}^{-1}(u)C_l(u) \right]^{-1} d_f \in \mathbb{C}^{\tilde{L}N \times 1}. \quad (8.30)
\]

The corresponding fast-time constraint matrix for \( N_{\text{con}} \) constraints is then given by,

\[
C_l(u) = I_L \otimes C(u) \in \mathbb{C}^{\tilde{L}N \times \tilde{L}N_{\text{con}}} \quad (8.31)
\]

with corresponding desired response vector,

\[
d_f = g_{\text{post}} \otimes d \in \mathbb{C}^{\tilde{L}N_{\text{con}} \times 1}. \quad (8.32)
\]
For the fast-time implementation, the normalised estimated space/fast-time interference plus noise covariance matrix, \( \hat{\mathbf{R}}_{Z_t}^\prime(u) = \alpha_{f,norm}^{-1} \hat{\mathbf{R}}_{Z_t}(u) \) is also similar to the spatial case, except \( \hat{\mathbf{R}}_{Z_t}(u) \) is now determined by averaging over the space/fast-time interference plus noise vector, \( \mathbf{Z}_{t}(\cdot) \),

\[
\hat{\mathbf{R}}_{Z_t}(u) = \frac{1}{L_t} \sum_{l=1}^{L_t} \mathbf{Z}_{t}(t_l, u) \mathbf{Z}_{t}^H(t_l, u) \in \mathbb{C}^{LN \times LN}
\]  

(8.33)

and the normalising value is now given by, \( \alpha_{f,norm}^{-1} = \text{Tr}\left\{ \hat{\mathbf{R}}_{Z_t}(u) \right\} / (\hat{L}N) \). Diagonal loading is included similarly to Equation 8.13,

\[
\hat{\mathbf{R}}_{Z_t,DL}(u) = \hat{\mathbf{R}}_{Z_t}(u) + \eta \mathbf{I}_{LN} \in \mathbb{C}^{LN \times LN}.
\]

(8.34)

Constrained Fast-time Results

To measure the effect of including fast-time taps, the simulation was run for the MVDR and first derivative cases as \( \hat{L} = 1 \) to 15. The diagonal loading level is also varied to measure how the interaction between fast-time taps can improve both the RINR and SDR. As shown previously, diagonal loading can improve the results to a point, before which further increases result in both metrics approaching the conventional level.

The MVDR results are shown in Figure 8.7. As the number of taps is increased with no diagonal loading, the RINR decreases from 9 dB to 6.2 dB, while the SDR increases from 4 dB to 5.2 dB at \( \hat{L} = 7 \) before dropping down to the conventional level of 3.9 dB when \( \hat{L} = 15 \). This again demonstrates mismatch between the hot-clutter scatterers and the narrow MVDR constraint. In this case, the large number of fast-time taps reduces the benefit of the extra correlation in the covariance estimate. With the diagonal loading set to \(-60 \text{ dB}\) however, the RINR reaches 5.3 dB and the SDR continues upwards till it reaches a peak of 6.5 dB.

The first derivative results in Figure 8.8 show a decrease in RINR from 9.2 dB to 6.2 dB when there is no diagonal loading. At this point the SDR increases from 4.8 dB to 6.2 dB and only increases slightly to 7.1 dB with \(-90 \text{ dB}\) to \(-60 \text{ dB}\) of diagonal loading. While the best SDR results for both constraints are within 0.6 dB, the exact level of diagonal loading is hard to determine in practice. For this reason, the first derivative constraint would be the preferred algorithm with a small level of diagonal loading.
8.4 Constrained filtering

![Figure 8.7. Fast-time simulation results with MVDR constraint, varying: \( \bar{L}, \eta \)](image)

![Figure 8.8. Fast-time simulation results with derivative constraints, varying: \( \bar{L}, \eta \)](image)

8.4.7 Summary

This section presented constrained fast-time STAP using an element space implementation. A number of constraints were introduced as a method to minimise the effect of adaptive modulations and prevent potential signal suppression while preserving the final image quality.

Both the MVDR, first and second order derivative constraints were tested with the simulation and the RINR and SDR were measured. With spatial only processing, the first order derivative offered the best results, with the MVDR slightly behind and the second order results similar to the conventional beamformer. Amplitude constraints were also shown to offer good results when the angle \( \delta_{\text{amp}} \) is small. However this optimal level is hard to find in practice and this constraint will not be used further in this thesis.

As fast-time taps were included, the MVDR achieved the greatest results with \(-80\) dB of diagonal loading. Again this level is hard to find in practice and the first order derivative results would be the safer algorithm with a small level of diagonal loading.
Chapter 8

Constrained Fast-time STAP

8.5 Generalised Sidelobe Canceller

Beamspace processing was introduced in Section 4.2.5 as a method of implementing STAP in the spatial frequency domain. It is commonly implemented as a GSC, which under ideal conditions can be shown to be equivalent to element space approaches, [Gray, 1982]-[Godara, 1987]. Fast-time taps were originally used by [White, 1983] and applied to hot-clutter interference rejection by [Kogon et al., 1996a]-[Kogon, 1996]. Derivative constraints have also been used in the GSC by [Buckley and Griffiths, 1986], but there have been limited publications combining the use of fast-time STAP with a constrained GSC. Further background on the GSC is presented in Section 5.3.5. This section presents a general method of applying constraints to the fast-time GSC with a general blocking matrix design method. It is also important as a basis for the reduced rank GSC algorithms in sections 8.6 and 8.7.

The first Section 8.5.1 describes the GSC spatial signal model and the corresponding weight vectors. Design of the blocking matrix to make the GSC equivalent to the element space formulation is then described in Section 8.5.2 before the extension to use fast-time taps in Section 8.5.3. No new simulation results are shown for this section, as they would be identical to those in the previous section.

8.5.1 Spatial Signal Model

The GSC shown in Figure 8.9 forms a set of ‘beams’ with the main beam in the ‘desired’ target direction and the other ‘reference’ beams going through a blocking matrix $B(u)$ to remove the desired signal from the data. This signal then goes through an adaptive filter to minimise the output power, before being subtracted from the main beam. While the element space adaptive method relies on constraints to let the target signal through, the GSC is formulated to keep its main beam fixed on the target signal. A reference beam is also formed with only the undesired terms present, thereby allowing reduced rank schemes to be more easily implemented, [Guerci, 2003]. Using $N_{\text{con}}$ constraints also results in a loss of $N_{\text{con}}$ degrees of freedom in the adaption and less training data is required for the same adaptive performance. The most important difference however is its behaviour with steering vector mismatch, which is superior to the element space adaptive processor, [Godara, 1987].

Figure 8.9. Generalised sidelobe canceller
8.5 Generalised Sidelobe Canceller

The canceller’s output is given by
\[ x_d(t, u) = w_d^H(u)x(t, u) - w_a^H(u)w_a(u)R_x(u)x(t, u) \]
\[ = [w_d(u) - B(u)w_a(u)]^H x(t, u) \]
\[ = w_{GSC}^H(u)x(t, u) \] (8.35)

where the desired weight \( w_d(u) \), is given by
\[ w_d(u) = C(u)\left[C^H(u)C(u)\right]^{-1}d \in \mathbb{C}^{N \times 1}. \] (8.36)

The adaptive weight vector \( w_a(u) \), is designed to minimise the scalar MSE between the two paths, \( e_0 \) and \( \hat{e}_0 \) by solving the unconstrained optimisation [Johnson and Dudgeon, 1993],
\[ \min_{w_a(u)} \left\{ E \{ |e_0 - \hat{e}_0|^2 \} \right\} \]
\[ \Rightarrow \min_{w_a(u)} \left\{ [w_d(u) - B(u)w_a(u)]^H R_x(u) [w_d(u) - B(u)w_a(u)] \right\} \] (8.37)
giving the adaptive weight,
\[ w_a(u) = [B^H(u)R_x(u)B(u)]^{-1}B^H(u)R_x(u)w_d(u) \in \mathbb{C}^{(N-N_{con}) \times 1}. \] (8.38)

As the reference beam is orthogonal to the mainbeam and providing there is no mismatch between the input signal and reference beam, the following holds for both the total received covariance and the interference plus noise only covariance [Van-Trees, 2002],
\[ B^H(u)R_x(u) = B^H(u)R_z(u). \] (8.39)

Equation 8.38 then reduces to,
\[ w_a(u) = [B^H(u)R_z(u)B(u)]^{-1}B^H(u)R_z(u)w_d(u) \]
\[ = R_{x_1}^{-1}(u)r_{x_1,e_0}(u) \in \mathbb{C}^{(N-N_{con}) \times 1} \] (8.40)

where \( R_{x_1}(u) \) is the ‘reference’ covariance matrix at \( x_1(\cdot) \) and \( r_{x_1,e_0}(u) \) is the cross covariance between \( x_1(\cdot) \) and \( e_0 \). If this is substituted into into Equation 8.35, the overall weight is,
\[ w_{GSC}(u) = w_d(u) - B(u)[B^H(u)R_z(u)B(u)]^{-1}B^H(u)R_z(u)w_d(u) \]
\[ = [I_N - B(u)[B^H(u)R_z(u)B(u)]^{-1}B^H(u)R_z(u)]w_d(u) \in \mathbb{C}^{N \times 1}. \] (8.41)

As explained in Section 8.4.2, the ideal interference plus noise covariance, \( R_z(u) \) is replaced by the normalised sample matrix estimate with diagonal loading \( \hat{R}_{z,DL}(u) \) for the GSC implementation.

8.5.2 Blocking Matrix Design

To make the GSC result equivalent to the element space form, the GSC weight vector, \( w_{GSC}(u) \) must be equal to the constrained solution in Equation 8.11. This equality can then be used to
find a general form for the blocking matrix $B(u)$, [Johnson and Dudgeon, 1993].

$$w_{GSC}(u) = \begin{bmatrix} I_N - B(u) \left[ B^H(u)R_z(u)B(u) \right]^{-1} B^H(u)R_z(u) \end{bmatrix} w_d(u) = R_z^{-1}(u)C(u) \left[ C^H(u)R_z(u)C(u) \right]^{-1} d \in C^{N \times 1}. \quad (8.42)$$

Pre multiplying both sides by $B^H(u)R_z(u)$ gives

$$B^H(u)C(u) \left[ C^H(u)R_z(u)C(u) \right]^{-1} d = 0 \quad (8.43)$$

where $B^H(u)$ cannot be invertible or this equation would be meaningless. Instead, the null space of $B^H(u)$ is forced to contain $C(u) \left[ C(u)R_z(u)C(u) \right]^{-1} d$ and be independent of the data characteristics being filtered, [Johnson and Dudgeon, 1993]. To satisfy this, the blocking matrix must satisfy,

$$B^H(u)C(u) = 0 \quad (8.44)$$

with each column being orthogonal to the desired weight vector, $B^H(u)w_d(u) = 0$. To meet this design criteria, the overall size of the blocking matrix is $N \times (N - N_{\text{con}})$ with each of the $N - N_{\text{con}}$ columns formed as shifted versions of a single orthogonal vector. The vector is designed to have $N_{\text{con}} + 1$ elements corresponding to the orthogonal constraint and $N - N_{\text{con}} - 1$ zero elements to improve the orthogonality.

Using the Moore-Penrose Pseudo Inverse, [Golub and Van Loan, 1989] a modified constraint matrix $A(u)$ can be formed as the concatenation of $N - N_{\text{con}} - 1$ unit vectors with the constraint matrix $C(u)$. The unit vectors are used to define the position of zeros within the blocking matrix as shown below. The matrix $A(u)$ is given by,

$$A(u) = [e_{N_{\text{con}}+1} \ldots e_N]C(u) \in C^{N \times (N - N_{\text{con}})} \quad (8.45)$$

where the $N \times 1$ unit vector $e_q$ is defined with zeros in all positions except for the $q^{th}$ element, which is unity. The Moore-Penrose Pseudo Inverse is then given by,

$$A^\dagger(u) = [A^H(u)A(u)]^{-1}A^H(u) \in C^{(N - N_{\text{con}}) \times N} \quad (8.46)$$

with the first orthogonal column of the blocking matrix formed as,

$$b(u) = I_N - A(u)A^\dagger(u) = [\tilde{b}^T(u)0_{N - N_{\text{con}} - 1}]^T \in C^{N \times 1} \quad (8.47)$$

where $\tilde{b}(u)$ contains $N_{\text{con}} + 1$ non-zero elements and $0_{N - N_{\text{con}} - 1}$ is an $(N - N_{\text{con}} - 1) \times 1$ vector of zeros. The blocking matrix then has the form,

$$B(u) = \begin{bmatrix} \tilde{b}(u) & 0 & 0 & 0_{N - N_{\text{con}} - 2} & 0 \ \\ \tilde{b}(u) & \ddots & \tilde{b}(u) & \vdots & \tilde{b}(u) \ \\ 0_{N - N_{\text{con}} - 1} & 0_{N - N_{\text{con}} - 2} & 0_{N - N_{\text{con}} - 1} & \tilde{b}(u) \end{bmatrix} \in C^{N \times N - N_{\text{con}}}, \quad (8.48)$$
8.5.3 Fast-time Extension

To extend this algorithm to use fast-time taps, the desired fast-time weights are formed by the Kronecker product with the fast-time steering vector,

$$W_{f,d}(u) = g_{\text{post}} \otimes w_d(u) \in \mathbb{C}^{LN \times 1}$$

and the fast-time blocking matrix expanded similarly to the constraint matrix in Equation 8.31,

$$B_f(u) = I_L \otimes B(u) \in \mathbb{C}^{LN \times L(N-N_{\text{con}})}.$$  

The fast-time adaptive weight can then be formed using Equation 8.41 with the fast-time vectors,

$$W_{f,a}(u) = \left[ B_f^H(u) R_{Z_f}(u) B_f(u) \right]^{-1} B_f^H(u) R_{Z_f}(u) W_{f,d}(u) \in \mathbb{C}^{LN \times 1}$$

and the overall fast-time weight can then be written as,

$$W_{f,GSC}(u) = W_{f,d}(u) - B_f(u) W_{f,a}(u) \in \mathbb{C}^{LN \times 1}.$$  

As in the spatial case, the ideal interference plus noise covariance, $R_{Z_f}(u)$ is replaced by the normalised sample matrix with diagonal loading, $\hat{R}_{Z_f,DL}(u)$.

8.6 Reduced Rank Generalised Sidelobe Canceller

The goal of reduced rank techniques is to reduce the rank of the interference plus noise covariance matrix and the overall computational load of an adaptive filter. They can be applied to any adaptive filter, but are better suited with the GSC implementation, [Guerci, 2003]. For this section, the reduction in rank is achieved by decomposing the estimated interference plus noise covariance matrix into eigenvector/eigenvalue pairs and selecting a reduced number of eigenvectors to form a reduced rank transform. Both Principal Components (PC) and the Cross Spectral Metric (CSM) are used as ranking criteria, with the adaption subject to the MVDR and first derivative constraints.

The new contribution in this section is to generalise the constrained reduced rank GSC formulation in [Haimovich et al., 1997] and extend it to use fast-time taps similar to [Kogon, 1996]. The first Section 8.6.1 describes the reduced rank GSC model and the two transformation methods are described in Section 8.6.2. Simulation results are then presented in Section 8.6.4 and the entire section summarised in Section 8.6.5.

8.6.1 Signal Model

The spatial reduced rank GSC implementation is shown in Figure 8.10. The reduced rank transform is contained in the matrix $U(u)$ which acts to reduce the size and rank of the reference beam after going through the blocking matrix. The output of the reduced rank GSC is given
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Figure 8.10. Reduced rank generalised sidelobe canceller

\[ x_{fb}(t, u) = w_d^H(u)x(t, u) - w_a^H(u)U^H(u)B^H(u)x(t, u) \]

where the transform \( U(u) \) has size \( N \times C \) with \( C < N \) for rank reduction. With the reduced rank, the optimisation criteria from Equation 8.54 is modified to,

\[
\begin{align*}
\min_{w_a(u)} & \left\{ w_d(u) - B(u)U(u)w_a(u) \right\}^H R_{x}(u) \left[ w_d(u) - B(u)U(u)w_a(u) \right] \\
& \text{(8.54)}
\end{align*}
\]

giving the rank reduced adaptive weight, [Haimovich et al., 1997],

\[
w_a(u) = \left[ U^H(u)B^H(u)R_x(u)B(u)U(u) \right]^{-1} U^H(u)B^H(u)R_x(u)w_d(u) \quad \text{(8.55)}
\]

Using the substitution from Equation 8.39, this can then be written in terms of the interference plus noise only covariance matrix, \( R_z(u) \),

\[
\begin{align*}
w_a(u) &= \left[ U^H(u)B^H(u)R_z(u)B(u)U(u) \right]^{-1} U^H(u)B^H(u)R_z(u)w_d(u) \\
&= \left[ U^H(u)R_{x1}(u)U(u) \right]^{-1} U^H(u)r_{x1,0}(u) \\
&= R_{z1}^{-1}(u)r_{x1,0}(u) \in \mathbb{C}^{C \times 1}. \quad \text{(8.56)}
\end{align*}
\]

Similar to the past two sections, the ideal interference plus noise covariance, \( R_z(u) \) is replaced by the normalised sample matrix, \( \hat{R}_z'(u) \). Diagonal loading is now included by adding a scaled identity matrix to the reduced rank covariance matrix,

\[
w_a(u) = \left[ R_{x}(u) + \eta \mathbf{I}_C \right]^{-1} r_{x_1,0}(u) \in \mathbb{C}^{C \times 1} \quad \text{(8.57)}
\]

and the extension to fast-time is exactly the same as the full rank GSC presented in Section 8.5.3, except that the normalised covariance must be used instead of the diagonal loaded version. The transform now has size \( \tilde{L}N \times C \) with good rank reduction achieved when \( C \ll \tilde{L}N \).

### 8.6.2 Reduced Rank Transforms

Eigen-decomposition of the reference interference plus noise covariance matrix involves separating the interference plus noise space into a number of orthogonal basis vectors. They are related
where \((\kappa_k, q_k)\) represents the \(k^{th}\) eigenvalue/vector pair. Using this decomposition, the matrix inversion within the adaptive weight simplifies to,

\[
R_{x_r}^{-1}(u) = \sum_{k=1}^{\tilde{L}N} \frac{q_k q_k^H}{\kappa_k} \in \mathbb{C}^{\tilde{L}(N-N_{con}) \times \tilde{L}(N-N_{con})}
\]  

(8.59)

and can be approximated by ordering \(C \ll \tilde{L}N\) eigen-pairs according to a ranking criteria. The two most common are PC and the CSM, which are both presented below.

**Principle Components**

The first criteria is also known as an ‘eigen-canceller’ and selects eigenvectors associated with the largest eigenvalues or ‘principle components’ to form a lower rank covariance matrix. If the eigenvalues are ranked with \(\kappa_1 \geq \kappa_2 \geq \ldots \geq \kappa_{\tilde{L}(N-N_{con})}\) with corresponding eigenvectors, \(q_1, \ldots, q_{\tilde{L}(N-N_{con})}\), then the PC are taken by selecting the \(C\) greatest eigen-pairs to form a reduced rank interference canceller. The corresponding equivalent non-transformed reduced rank weight vector is given by,

\[
W_{eq}(u) = \sum_{k=1}^{C} \frac{q_k^H r_{x_r,e_0}(u) q_k}{\kappa_k} \in \mathbb{C}^{\tilde{L}(N-N_{con}) \times 1}
\]  

(8.60)

and the PC transformation, also known as a truncated Karhunen Loeve transformation is defined by,

\[
U_{PC}(u) = [q_1, \ldots, q_C] \in \mathbb{C}^{\tilde{L}(N-N_{con}) \times C}.
\]  

(8.61)

The transform can also be used to represent the reduced rank interference plus noise covariance matrix as,

\[
R_{x_r}(u) = U_{PC}^H(u) R_{x_r}(u) U_{PC}(u) = \text{diag} [\kappa_1, \kappa_2, \ldots, \kappa_C] \in \mathbb{C}^{C \times C}
\]  

(8.62)

where \(\text{diag}(\cdot)\) represents a square matrix with the contained elements along the diagonal and the remainder set to zero. This transform will also reduce the computational complexity of the matrix inverse from \(O(\tilde{L}N)^3\) to \(O(C)^3\).

**Cross Spectral Metric**

The PC criteria does not directly consider the maximum output SINR performance measure, which is also a function of the cross covariance vector. The CSM ranking criteria first proposed by [Goldstein and Reed, 1997a] uses this knowledge to prioritise the eigen-pairs of the covariance
matrix associated with the reference channels. Its form is found by expressing the SINR in terms of its eigenvalues,

\[ S_{INR}(u) = \frac{|w_d^H(u)w_d(u)|^2}{E \{|e_0 - \hat{e}_0|^2\}} \]

\[ = \frac{1}{E \{|e_0 - w_a^H(u)x_e(t,u)|^2\}} \]

\[ = \frac{\sigma_e^2 - \sigma_a^2}{\sigma_e^2 - \sigma_a^2} \frac{1}{R_x(u)} + \frac{w_a^H(u)R_x(u)w_a(u)}{R_x(u)} \]

where \( w_d^H(u)w_d(u) = 1 \), \( \sigma_e^2 = E \{|e_0|^2\} \) is the mainbeam output power, \( r_{x_e,e_0}(u) = E \{x_e(u)\bar{e}_0\} \) is the cross correlation between \( x_e(u) \) and \( \bar{e}_0 \) and the reduced rank covariance, \( R_x = E \{x_e(u)x_e^H(u)\}. \)

This expression can be simplified by substituting in the adaptive weight from Equation 8.56, giving

\[ S_{INR}(u) = \frac{1}{\sigma_e^2 - \sigma_a^2} \frac{1}{R_x(u)} \frac{1}{\sigma_e^2 - \sigma_a^2} \]

\[ = \frac{L_N - 1}{\sum_{k=1}^{N-1} |q_k^H r_{x,e_0}(u)|^2} \]

The CSM criteria is based on maximising the SINR and hence minimising the denominator of Equation 8.64. This can be accomplished by selecting those eigenvectors corresponding to maximum values of

\[ \varsigma_k = \frac{|q_k^H r_{x_e,e_0}(u)|^2}{\kappa_k} \]

and yields the following equivalent reduced rank adaptive weight vector,

\[ W_{eq}(u) = \sum_{k=1}^{C} \frac{q_k^H r_{x,e_0}(u)}{\kappa_k} q_k \in \mathbb{C}^{L(N-N_{con}) \times 1} \]

where \((\kappa_k, q_k)\) denotes the \(k\text{th}\) eigen-pair ranked according to the CSM, with \( \kappa_{k1} \) the eigenvalue producing the largest value of \( \varsigma_k \), \( \kappa_{k2} \) the second largest, and so on. In terms of the CSM subspace transformation,

\[ U_{CSM}(u) = [q_{k1}, q_{k2}, \ldots, q_{kC}] \in \mathbb{C}^{L(N-N_{con}) \times C}. \]

### 8.6.3 Eigen-distributions

Based on the size of \( R_{x_1} \) when \( L = 15 \) taps are used, the maximum interference rank for the MVDR constraint is 60 and for the derivative constraint is 45. The eigen-distributions are shown in Figure 8.11 with the normalised noise floor at \( 1/\alpha_{f,\text{norm}} = -117 \) dB. Both are nearly full rank as they cross the noise floor before reaching the maximum rank.
8.6 Reduced Rank Generalised Sidelobe Canceller

![Figure 8.11. Reference beam eigen-distribution for: (- -) MVDR constraint, (-.-) derivative constraint](image)

8.6.4 Simulation Results

This section presents simulation results using both the MVDR and first derivative constraints with the PC and CSM transforms. The rank of the covariance matrix is determined by the chosen number of eigenvalues. Both the rank and the diagonal loading level is varied with $\hat{L} = 15$ fast-time taps. As the rank increases, the results will match the full rank case shown in Figure 8.7 and as the diagonal loading becomes large, both the RINR and SDR results will approach the conventional beamformer.

The first two Figures 8.12-8.13 show simulation results using the MVDR constraint. With no diagonal loading the CSM performs better with a low rank, achieving an RINR of 6.8 dB and SDR of 5.3 when the rank is 10. The PC then reaches an RINR of 5.6 dB with an SDR of 6.4 dB, nearly equal to the highest value in the full rank SDR. Beyond this level however, both metrics get worse as the results start matching those of the full rank filter. As the diagonal loading is increased, the CSM still performs better with a low rank, although when the rank increases beyond 20 and $\eta = 60$ dB, both metrics achieve the best RINR of 5.3 dB and SDR of 6.5 dB.

The next two Figures, 8.14-8.15 show the simulation results using the first derivative constraint. Again the CSM performs better with the rank less than 10. At this point however, both filters reach their peak with an RINR of 6.2 dB and SDR of 7.1 dB. With a small amount of diagonal loading and a rank of only 10, this filter choice can safely achieve the same RINR and SDR level as the full rank case!

8.6.5 Summary

This section has shown how the constrained fast-time GSC can be formulated with reduced rank and computational complexity using a transform based on the eigen-decomposition. This approach uses the adaptive degrees of freedom more effectively to remove the interference and the inclusion of constraints aids to minimise distortions in the final SAR image.
Both the PC and CSM ranking criteria were used to determine which eigenvectors to include in the reduced rank transform. From the simulation results, the CSM ranking criteria shows improved performance over PC only when the rank is low. Beyond this, there was little difference between the results for the two ranking criteria.

Results from using the MVDR constraint were very sensitive to the level of diagonal loading. This is seen when the rank approaches full rank and the results match those from Figure 8.7. However, when the first derivative constraint was used, the results were not as greatly affected by the diagonal loading level. With a rank of only 10 and a small amount of diagonal loading, this filter can safely achieve the same RINR and SDR level as the full rank case.
8.7 Multistage Wiener Filter

The last fast-time STAP algorithm is known as the MWF and provides a faster rank reduction than the GSC by using a nested chain of traditional Wiener filter stages. Weights are estimated at each stage to maximise the energy between the main and reference beams and the implementation requires much less sample support than the previous methods. This method also does not need an eigenvector decomposition or large covariance matrix inversion which makes it more suitable for real world implementation. Both the MVDR and derivative constraints are again used to improve the final image quality in the non-stationary hot-clutter environment.

The original formulation was by [Goldstein et al., 1998] and adapted slightly to be more efficient by [Guerci, 2003]. Derivative constraints have also been applied to this algorithm by [Nguyen, 2002], who presented a very basic derivation. The new contribution in this section is to present a simplified description of the MWF with modifications to use arbitrary constraints and fast-time taps. The first Section 8.7.1 presents the MWF derivation with the extension to fast-time taps.
in Section 8.7.2. Simulation results are then presented in Section 8.7.3 and the entire section is summarised in the Section 8.7.4.

8.7.1 Spatial Signal Model

The MWF is formed from a number of filter stages. A block diagram of the first stage is shown in Figure 8.16, where $\text{null}[w_d(u)]$ represents the nullspace of $w_d(u)$.

\begin{equation}
x_{b}(t,u) = w_d^H(u)x(t,u) - w_{a,1}(u)h_1^H(u)B_1^H(u)x(t,u)
\end{equation}

where the blocking matrix $B_1(u)$ has size $N \times (N - N_{\text{con}})$ and the received data vector after transformation, $x_1(t,u)$ has size $(N - N_{\text{con}}) \times 1$. The overall weight can then be written as,

\begin{equation}
w_{\text{MWF}}(u) = w_d(u) - L_1(u)w_{a,1}(u) \in \mathbb{C}^{N \times 1}
\end{equation}

with

\begin{equation}
L_1(u) = B_1(u)h_1(u) \in \mathbb{C}^{N \times 1}.
\end{equation}

The optimal choice for the weight $h_1(u)$, can be found from the minimal MSE solution of the GSC in Equation 8.40. However, this requires inversion of the reference covariance matrix, $R_{x_1}(u)$ and is contrary to the reduced rank solution desired here. Therefore, a sub-optimal basis vector is required to estimate $e_1$ without this information. The goal of the MWF is to choose a rank one basis vector $h_1(u)$ such that the cross-correlation energy between $e_1$ and $e_0$ is maximised,

\begin{equation}
\max_{h_1(u)} \left\{ E\{|e_1e_0^*|^2\} \right\}
\end{equation}

\begin{equation}
\Rightarrow \max_{h_1(u)} \left\{ |h_1^H(u)r_{x_1,e_0}(u)|^2 \right\}.
\end{equation}

By applying Schwartz’s inequality with a unity norm constraint, $||h_1(u)|| = [h_1^H(u)h_1(u)]^{1/2} = 1$, the optimum $h_1(u)$ is given as,

\begin{equation}
h_1(u) = \frac{r_{x_1,e_0}(u)}{||r_{x_1,e_0}(u)||} = \frac{r_{x_1,e_0}(u)}{\sqrt{r_{x_1,e_0}(u)r_{x_1,e_0}^H(u)}} \in \mathbb{C}^{(N-\text{N}_{\text{con}})\times 1}
\end{equation}
8.7 Multistage Wiener Filter

with the reference covariance and cross covariance given in Equation 8.40,

\[ R_{x_1}(u) = B_1^H(u)R_z(u)B_1(u) \in \mathbb{C}^{(N-N_{\text{con}}) \times (N-N_{\text{con}})}, \]  
(8.73)

\[ r_{x_1,e_0}(u) = B_1^H(u)R_z(u)w_d(u) \in \mathbb{C}^{(N-N_{\text{con}}) \times 1}. \]  
(8.74)

and the identity in Equation 8.39 has been used to substitute the total received covariance, \( R_x(u) \) with the interference plus noise only covariance, \( R_z(u) \). Using this choice of \( h_1(u) \), the optimum rank one scalar weight is given by,

\[ w_{a,1}(u) = R_{e_1}^{-1}(u)r_{e_1,e_0}(u) = \left[ L_1^H(u)R_1(u)L_1(u) \right]^{-1} L_1^H(u)R_z(u)h_1(u). \]  
(8.75)

Now since \( h_1(u) \) is not formed with the full covariance and cross correlation information, it will not span the entire space of the interference plus noise. Additional stages must therefore be added to the above process. Figure 8.17 shows the two-stage MWF, where the space spanned by \( x_2(t,u) \) is orthogonal to the span\{\( w_d(u), h_1(u) \)\} due to the blocking matrices, \( B_1(u) \) and \( B_2(u) \).

![Figure 8.17. Two stage Wiener filter](image)

As with the single stage case, the final weight can be written as a sequential decomposition,

\[ w_{\text{MWF}}(u) = w_d(u) - L_2(u)w_{a,2}(u) \in \mathbb{C}^{N \times 1} \]  
(8.76)

where

\[ L_2(u) = [B_1(u)h_1(u), B_1(u)B_2(u)h_2(u)] \in \mathbb{C}^{N \times 2} \]  
(8.77)

and the second stage blocking matrix is designed to reject the basis vector of the previous stage. To maintain the size of the blocking matrix, this is calculated by,

\[ B_2(u) = \mathbf{I}_{N-N_{\text{con}}} - h_1(u)h_1^H(u) \in \mathbb{C}^{(N-N_{\text{con}}) \times (N-N_{\text{con}})} \]  
(8.78)
with the basis vector for the second stage, $h_2(u)$ determined using the same optimisation criteria as the first stage,

$$
\begin{align*}
\max_{h_2(u)} \left\{ E\{|e_2 e_1^*|^2\} \right\} \\
\Rightarrow \max_{h_2(u)} \left\{ |h_2^H(u)r_{x_2,e_1}(u)|^2 \right\}
\end{align*}
$$

(8.79)

which is again found using Schwartz’s inequality,

$$
h_2(u) = \frac{r_{x_2,e_1}}{|r_{x_2,e_1}|} = \frac{r_{x_2,e_1}}{\sqrt{r_{x_2,e_1}^H(u)r_{x_2,e_1}(u)}} \in \mathbb{C}^{(N-N_{\text{con}}) \times 1}.
$$

(8.80)

with the second stage reference covariance and cross covariance given by,

$$
\begin{align*}
R_{x_2}(u) &= B_2^H(u)R_{x_1}(u)B_2(u) \in \mathbb{C}^{(N-N_{\text{con}}) \times (N-N_{\text{con}})}, \\
r_{x_2,e_1}(u) &= B_2^H(u)R_{x_1}(u)h_2(u) \in \mathbb{C}^{(N-N_{\text{con}}) \times 1}.
\end{align*}
$$

(8.81)

(8.82)

This gives the final two stage weight as,

$$
w_{\text{a,2}}(u) = R_{x_2}^{-1}(u)r_{x_2,e_0}(u) = \left[ L_2^H(u)R_z(u)L_2(u) \right]^{-1} L_2^H(u)R_z(u)w_d(u) \in \mathbb{C}^{2 \times 1}
$$

(8.83)

If this process is repeated $Q$ times, a cascaded filter bank is formed as in Figure 8.18.

The overall weight vector for the $q^{th}$ stage can be written as,

$$
w_{\text{MWF}}(u) = w_d(u) - L_q(u)w_{\text{a,q}}(u) \in \mathbb{C}^{N \times 1}
$$

(8.84)

with the sequential decomposition given by,

$$
L_q(u) = [B_1(u)h_1(u), B_1(u)B_2(u)h_2(u), \ldots, B_1(u)B_2(u) \cdots B_q(u)h_q(u)] \in \mathbb{C}^{N \times q}.
$$

(8.85)
A general form for the basis vector can be given by,

$$h_q(u) = \frac{r_{x_q,e_g-1}}{||r_{x_q,e_g-1}||} = \frac{r_{x_q,e_g-1}}{\sqrt{r_{x_q,e_g-1}(u)r_{x_q,e_g-1}(u)}} \in \mathbb{C}^{(N-N_{con}) \times 1} \quad (8.86)$$

where the reference covariance and cross covariance have the form,

$$R_{x_q}(u) = B_q^H(u)R_{x_{q-1}}(u)B_q(u) \in \mathbb{C}^{(N-N_{con}) \times (N-N_{con})}, \quad (8.87)$$

$$r_{x_q,e_g-1}(u) = B_q^H(u)R_{x_{q-1}}(u)h_{q-1}(u) \in \mathbb{C}^{(N-N_{con}) \times 1}. \quad (8.88)$$

with $h_0(u) = w_d(u)$, $R_{x_0}(u) = R_z(u)$ and the blocking matrices for $q > 1$,

$$B_q(u) = I_{N-N_{con}} - h_{q-1}(u)h_{q-1}^H(u) \in \mathbb{C}^{(N-N_{con}) \times (N-N_{con})}. \quad (8.89)$$

The $q^{th}$ order weight vector can then be written as,

$$w_{a,q}(u) = R_{e_g-1}^{-1}(u)r_{e_g,e_0}(u)$$

$$= [L_q^H(u)R_z(u)L_q(u)]^{-1}L_q^H(u)R_z(u)w_d(u) \in \mathbb{C}^{q \times 1}. \quad (8.90)$$

Similar to the previous algorithms, the ideal interference plus noise covariance, $R_z(u)$ is replaced by the normalised sample matrix, $\hat{R}_z(u)$. Diagonal loading is included by adding a scaled identity matrix to the modified covariance matrix,

$$w_{a,q}(u) = [L_q^H(u)R_z(u)L_q(u) + \eta I_q]^{-1}L_q^H(u)R_z(u)w_d(u) \in \mathbb{C}^{q \times 1}. \quad (8.91)$$

### 8.7.2 Fast-time Extension

The space/fast-time MWF weight vector is determined by an extended version of Equation 8.84,

$$W_{f,MWF}(u) = W_{f,d}(u) - L_{q,f}(u)W_{f,a,q}(u) \in \mathbb{C}^\hat{L}N \times 1 \quad (8.92)$$

where the extended sequential vector is given by,

$$L_{f,q}(u) = [B_{f,1}(u)h_{f,1}(u), B_{f,1}(u)B_{f,2}(u)h_{f,2}(u), \ldots, B_{f,1}(u)B_{f,2}(u) \cdots B_{f,q}(u)h_{f,q}(u)] \in \mathbb{C}^{\hat{L}N \times Q}$$

and the expressions for the desired signal vector, $W_{f,d}(u)$ and first stage blocking matrix, $B_{f,1}(u)$ are given in Section 8.5.3. The space/fast-time basis vectors are then defined by,

$$h_{f,q}(u) = \frac{r_{f,x_q,e_g-1}}{||r_{f,x_q,e_g-1}||} = \frac{r_{f,x_q,e_g-1}}{\sqrt{r_{f,x_q,e_g-1}(u)r_{f,x_q,e_g-1}(u)}} \in \mathbb{C}^{\hat{L}(N-N_{con}) \times 1} \quad (8.93)$$

with reference covariance and cross covariance,

$$R_{f,x_q}(u) = B_{f,q}^H(u)R_{f,x_{q-1}}(u)B_{f,q}(u) \in \mathbb{C}^{\hat{L}(N-N_{con}) \times (L-N-N_{con})}, \quad (8.94)$$

$$r_{f,x_q,e_g-1}(u) = B_{f,q}^H(u)R_{f,x_{q-1}}(u)h_{f,q-1}(u) \in \mathbb{C}^{\hat{L}(N-N_{con}) \times 1} \quad (8.95)$$
where $h_{f,0}(u) = W_d(u)$, $R_{f,x_0}(u) = R_{Z_f}(u)$ and the blocking matrices for $q > 1$,

$$B_{f,q}(u) = I_{L(N-N_{con})} - h_{f,q-1}(u)h_{f,q-1}(u)^H \in C^{L(N-N_{con}) \times L(N-N_{con})}. \quad (8.96)$$

The $q^{th}$ order space/fast-time weight vector can then be written as,

$$W_{f,a,q}(u) = R_{f,eq}^{-1}(u)r_{f,eq}(u) = \left[L_{f,q}^H(u)R_{Z_f}(u)L_{f,q}(u)\right]^{-1}L_{f,q}^H(u)R_{Z_f}(u)W_{f,d}(u) \in C^{q \times 1}. \quad (8.97)$$

As in the spatial case, the ideal interference plus noise covariance, $R_{Z_f}(u)$ is replaced by the normalised sample matrix, $\hat{R}_{Z_f}(u)$. Diagonal loading is also included in the same way as the spatial case,

$$W_{f,a,q}(u) = \left[L_{f,q}^H(u)R_{Z_f}(u)L_{f,q}(u) + \eta I_q\right]^{-1}L_{f,q}^H(u)R_{Z_f}(u)W_{f,d}(u) \in C^{q \times 1}. \quad (8.98)$$

Interestingly, the size of the space/fast-time weight vector is still determined by the MWF order, $Q$. To take advantage of the fast-time taps, this should be of the order of $\tilde{L}$. This will also reduce the computational complexity of the matrix inverse from $O(\tilde{L}N)^3$ to $O(Q)^3$ without requiring an eigen-decomposition as in the previous section.

### 8.7.3 Simulation Results

This section presents simulation results using both the MVDR and first derivative constraints with the MWF. Both the filter order and the diagonal loading level are varied with $\tilde{L} = 15$ fast-time taps. It is expected as the order approaches $\tilde{L}$, that the results should approach the full rank case shown in Figure 8.7 and as the diagonal loading level becomes large, both the RINR and SDR results will approach the conventional beamformer.

Figure 8.19 shows simulation results using the MVDR constraint. It takes an order of 14 before the filter matches the RINR and SDR and behaves like the full rank case. In contrast, the derivative constraint results in Figure 8.20 show that a small order of 5 can meet the full rank case. This is a huge difference of 11 filter orders and demonstrates the superiority of using the derivative constraints with the MWF. In addition, the SDR has increased to 7 dB which is 0.5 dB greater than the MVDR case.

### 8.7.4 Summary

This section has demonstrated how the MWF can be used for hot-clutter rejection using fast-time STAP with constraints. The main benefit of the MWF is a faster rank reduction that does not require an eigen-decomposition or large covariance matrix inversion.

Simulation results showed that the MVDR constraint requires an order of approximately the number of fast-time taps to achieve performance similar to the full rank case. With derivative constraints however, a much smaller order is sufficient to achieve results which are the same as the full-rank case.
8.8 Conclusion

This chapter has introduced a number of sub-optimal fast-time STAP algorithms to suppress the direct-path and hot-clutter interferences, while preserving the final image quality. The first algorithm was an element space implementation designed to minimise the interference power subject to a number of constraints. These included the MVDR, first and second order derivative and amplitude constraints. It was found that the second order derivative and amplitude constraints were not suitable and only the MVDR and first order derivative constraint were used for comparison. Simulation results with fast-time taps found that the first-order derivative constraint would be the preferred constraint choice as the exact level of diagonal loading is hard to determine in practice.

The next section introduced the GSC as a beamspace implementation of the constrained adaptive filter. With no signal mismatch, it will perform identically to the element space approach and no further simulation results were performed. This algorithm formed the basis of the reduced rank GSC, which utilised an eigen-decomposition of the interference plus noise covariance matrix to
form two different reduced rank transforms. These were based on the PC and CSM ranking criteria for the eigenvectors. Simulation results showed that the CSM performed better with a low rank, but quickly becomes equal to the PC results. With a small amount of diagonal loading and a filter with transformed rank of only 10, the first order derivative constraint choice was found to safely achieve the same performance as the full rank filter!

The final algorithm was the MWF which is able to provide a faster rank reduction and does not require an eigen-decomposition or large covariance matrix inversion. Simulation results showed the first order derivative constraint combined with fast-time taps can be exploited to provide performance on par with the full rank processor and a much smaller computation load.
Chapter 9

Conclusion

In this thesis, the two problems of image formation for a Multichannel Synthetic Aperture Radar (MSAR) and suppressing interferences while forming a good quality image have been addressed. For the first problem, three wavefront reconstruction algorithms were presented based on the multichannel Matched Filter (MF) imaging equation which demonstrated differing levels of performance and accuracy. A fourth algorithm known as multichannel backprojection was also presented to provide comparative quality with a reduced computational load.

To address the second problem, a detailed jammer model was described and tested with a multichannel imaging algorithm to demonstrate the effect of hot-clutter on a SAR image. Multichannel imaging and optimal slow-time Space Time Adaptive Processing (STAP) were shown to only partially suppress the hot-clutter interference, while optimal fast-time STAP demonstrated a much greater performance.

Implementing optimal fast-time STAP however is not feasible in a real system and a sub-optimal filter was presented as a substitute. To determine the location of the filter which will provide the best adaptive performance, models for both pre and post range processing (RP) space/fast-time steering vectors and the estimated interference covariance matrices were derived and their performance compared. It was found that they had equal potential for hot-clutter suppression as the number of fast-time taps increased, but the pre range processing temporal steering vector was not able to detect targets within the received data as well as the post RP case and this model is therefore the preferred choice for SAR.

The final part of the thesis looked at a more realistic scenario using post RP adaption and sub-optimal fast-time STAP algorithms. They were based on constrained optimisation criteria designed to suppress the hot-clutter interference while maintaining a coherent SAR image. The full rank algorithms presented included fast-time element and beamspace STAP using the Generalised Sidelobe C canceller (GSC). The derivative constrained filter with fast-time taps and a small amount of diagonal loading, demonstrated the best performance in terms of its final image quality. Data-dependent reduced rank filters were then introduced based on eigen-based methods and the Multistage Wiener Filter (MWF). Both of these demonstrated close or equal full rank performance with a much smaller computational load than the full rank filters.

The next part of this Chapter 9.1 summarises the main conclusions from each chapter and a number of future research areas are described in Section 9.2.
9.1 Chapter Summaries

Chapter 2 - Multichannel SAR Background  This chapter was focussed on understanding the characteristics and benefits of a MSAR, how the processing stages can be represented with a mathematical model and then simulated using MATLAB. By combining multiple antennas with SAR, a number of benefits can be achieved. These include improved gain, ambiguity reduction, the ability for height estimation and the use of spatial degrees of freedom. A number of signal processing models were then introduced for the imaging geometry, the SAR signal model, resolution and sampling, range compression and the extension to multiple channels. Each antenna element was modelled as a dipole with a linear array configuration chosen for superior mainbeam clutter suppression. These models were then combined into a multichannel SAR simulation with a number of scenarios to allow the user to easily test different parameters.

Chapter 3 - Multichannel SAR Imaging  This chapter presented four new multichannel imaging algorithms suitable for MSAR. The first three were based on wavefront reconstruction methods for solving the MF imaging equation and the fourth was a multichannel backprojection algorithm.

Each of the three wavefront reconstruction algorithms offer different levels of computational expense and accuracy, with the spatial MF interpolation only using a single MF and relying on the Stolt interpolation to correct for range migration. Range stacking then uses MF’s at each range to improve the accuracy, while the Time Domain Correlation (TDC) uses a MF at each range and azimuth location! Simulation of the point spread function was used to measure the accuracy of each algorithm and results indicated that the spatial MF interpolation offers the best tradeoff between accuracy and computational expense. The Stolt interpolation appears to work very well as there was only a minimal improvement with the range stacking algorithm. The TDC on the other hand avoids the stationary phase artefacts and was shown to produce superior results when there is a large SAR integration angle. For practical use however, it has an extremely large complexity and should only be used when there is a demand for high accuracy. The final algorithm was multichannel backprojection and by selecting an appropriate upsampling rate, it was able to match the TDC performance with smaller computational requirements.

Chapter 4 - STAP Background  This chapter presented relevant background material on adaptive filtering as applied to STAP for airborne radar and SAR. A number of practical MTI techniques suitable for SAR were presented as they provide a useful background to the adaptive algorithms used in this thesis. Adaptive array processing was then introduced with a number of optimal processors commonly used in beamforming. This led to a description of slow-time STAP and its application for airborne radar. However, this approach is not directly suitable for SAR and the final section looked at the main differences with applying STAP for SAR.
Chapter 5 - Jammer Background and Model  This chapter provided a detailed literature review of jamming and anti-jamming techniques for SAR and methods for suppressing hot-clutter in airborne and HF over the horizon radar. The second part of the chapter introduced a number of mathematical models for an airborne broadband jammer, including the transmitted jammer signal, the diffuse scattering of the hot-clutter reflections and the jammer and noise waveforms. These were then combined together and incorporated into the MATLAB simulation.

Chapter 6 - Optimal STAP for SAR  This chapter combined the simulation models from the previous chapters to analyse the degradation in a SAR image resulting from hot-clutter. Three algorithms were introduced and tested using ideal training data to show how both stationary and non-stationary interferences can be suppressed. The first algorithm was a space/slow-time multichannel imaging algorithm and demonstrated a small level of interference reduction through the process of burn-through. The second algorithm was optimal slow-time STAP which has the ability to reject the direct-path interference and a small amount of hot-clutter, while the third algorithm, optimal fast-time STAP has shown to be far more effective than the first two algorithms in the presence of strong hot-clutter. When realistic training data was used however, fast-time STAP suffered from two modulation effects. The first is a ‘training’ modulation which arises from averaging over a finite number of pulses, while the second is a ‘coherency’ modulation due to the non-stationarity in the hot-clutter. A number of techniques have been suggested in the literature to address these modulations, although the computational load of optimal fast-time STAP makes many of these prohibitive in a real system. Sub-optimal implementations are therefore required to obtain a more reasonable runtime and reduce the impact of the modulations.

Chapter 7 - Fast-time STAP Performance  This chapter looked at how a sub-optimal space/fast-time filter can be used to approximate the optimal filter and how its performance improves as more fast-time taps are used. Adaptive performance both pre and post RP was analysed with the corresponding steering vectors given as a chirp and sinc function respectively. As the fast-time STAP filter now only covers a small number of fast-time samples, its performance will depend on how well the fast-time steering vector matches the true representation of the target signal. The advantage of the post RP sinc function is that it can more accurately be represented by a smaller number of samples, while the pre RP chirp can not. To determine the adaptive performance, a pre RP simulated estimated covariance model was calculated and both the pre and post RP mean estimated sample covariances were derived analytically. The final result of the latter models showed them to be very similar with the form of an exponential multiplied by a sinc function.

A number of scattering scenarios were then tested to determine the expected level of correlation with increasing fast-time taps. The benefit of this approach however was only noticed when the tap-spacing was of the order of the relative bistatic delay and there was...
9.1 Chapter Summaries

A good match between the ideal number of taps and the correlation analysis of simulated hot-clutter scatterers. This number also indicates the point where the optimal level of interference suppression was reached and adding more taps would not improve the level of hot-clutter suppression.

An analysis of the eigen-distribution was then used to determine the rank of the interference and measure any variations between the pre and post RP mean estimated models. Results showed a good match between the two covariance models for all but the most diffuse scenario and even for this case both models still gave equal rank. An analysis of the SINR loss was then used to measure the expected performance of both models as they would be applied in an adaptive filter. The moderately diffuse scenario showed a big increase in performance due to the extra correlation with using fast-time taps and results for the two mean estimated covariances showed little difference. This implies that both pre and post RP adaptive filters would perform equally well at suppressing hot-clutter.

Chapter 8 - Constrained Fast-time STAP This chapter looked at applying a number of sub-optimal fast-time STAP algorithms to the problem of image formation while suppressing the direct-path and hot-clutter components of a mainbeam jammer. The adaptive filters were constrained to reduce the impact of the non-stationary environment, prevent suppression of the desired signals and reduce the impact of modulation effects.

The first algorithm was an element space implementation designed to minimise the interference power subject to a number of constraints. These included the Minimum Variance Distortionless Response (MVDR), first and second order derivative and amplitude constraints. Diagonal loading was also included to improve the robustness by smoothing the adaption via compression of the eigenvalues of the interference plus noise covariance matrix. It was found that the second order derivative and amplitude constraints were not suitable and only the MVDR and first order derivative constraint were used for comparison. Simulation results with increasing fast-time taps showed an improvement in the final image quality for both cases. The first-order derivative constraint was the preferred constraint choice however, as it was not as dependent on diagonal loading to produce a good quality final image, for which the exact level is hard to determine in practice.

The next section introduced the GSC as a beamspace implementation of the constrained adaptive filter. With no signal mismatch, it will perform identically to the element space approach and no further simulation results were performed. This algorithm formed the basis of the reduced rank GSC, which utilised an eigen-decomposition of the interference plus noise covariance matrix to form two different reduced rank transforms. These were based on the Principle Components (PC) and Cross Spectral Metric (CSM) ranking criteria for the eigenvectors. Simulation results showed that the CSM performed better with a low rank, but quickly become equal to the PC results. With a small amount of diagonal loading and a small rank, the first order derivative constraint choice was found to safely achieve the same performance as the full rank filter! The final algorithm was the MWF which is
able to provide a faster rank reduction and does not require an eigenvector decomposition or large covariance matrix inversion. Simulation results showed the first order derivative constraint combined with fast-time taps can be exploited to provide performance on par with the full rank processor and a much smaller computation load than both the full rank processors and the reduced rank GSC.

### 9.2 Future Work

Multichannel SAR is still a relatively new area in terms of research and development. This thesis has looked a number of areas, but there is still a lot of research left to be done. The first extension would be to use a realistic phased array model for the simulation, instead of just a linear array. This opens up a number of alternatives, such as the best way to combine sub-channels and how to use different waveforms along the array to achieve a wide bandwidth similar to PAMIR, [Ender and Brenner, 2002]. There are also a number of potentially different schemes for combining the returns from multiple channels and forming a multichannel image. Some of these include: forming an image from each channel and combining them together, compensating for each channel delay within the imaging algorithm or a combination of these. A comparison between the different methods would be useful to determine the ‘best’ method of image formation.

The diffuse scattering model in Chapter 5 produced some interesting results where the scattering spread didn’t fill the entire designated area. Although this didn’t affect the results in the thesis, it may have further significance and should be investigated further. Similarly, the performance of the sub-optimal fast-time STAP filters was investigated in Chapter 7 in terms of the signal to interference plus noise ratio loss and not actually tested with simulated data containing targets. This could be performed to confirm the result that the pre RP steering vector will not perform as accurately as the post RP vector for imaging and target detection purposes.

The final chapter looked at a number of constrained adaptive filters for suppressing the direct-path and hot-clutter interferences while forming a reasonable SAR image. Improved implementations of these filters in terms of the computational load is one topic which could be pursued, especially if real data was available for testing. However, there are many other methods which could be applied to this problem as described in Section 5.3.8. These include sub-band processing methods, [Fante, 1991], TSI finding, [Madurasinge and Shaw, 2005], pre-filtering, [Rabideau, 2000] and the many others.

A method which hasn’t yet been applied to hot-clutter suppression, but has demonstrated potential with signal mismatch is covariance matrix tapers, [Gueric, 1999]. This method adds a ‘taper’ to the interference plus noise covariance matrix to improve its robustness to signal mismatch and should improve the adaptive perform in a non-stationary environment. Recently, [Nguyen, 2002] has combined this method with the MWF and has shown superior results to the derivative constraint formulation.
9.2 Future Work

Finally, the problem of non-stationarity in adaptive filters is not just limited to hot-clutter. Effects such as aircraft crabbing, non-linear array geometry, intrinsic clutter motion, and scattering from near-field obstacles can cause a similar effect known as subspace leakage, [Guerci and Bergin, 2002]. The algorithms in this thesis could be applied to this problem quite easily with similar performance improvements. Also, the difference between moving target indication and interference suppression for image formation is very small and the algorithms in this thesis could be modified accordingly.
Appendix A

(\(\omega, k_u\)) Signal Model Formulation

This appendix evaluates the \((\omega, k_u)\) post range processing system model from Chapter 3 and is formed by taking the Fourier Transform of the \((\omega, u)\) system model in the slow-time domain. This derivation does not include the spatial phase offset due to multiple antennas and the fast-time frequency is defined at the carrier with \(\omega = \tilde{\omega} + \omega_c\),

\[
\tilde{s}_{\text{post}}(\omega, u, x, y) = \exp \left[-2j\omega \tau(x, y - u)\right] = \exp \left[-2jk\sqrt{x^2 + (y - u)^2}\right] \quad (A.1)
\]

and the Fourier Transform in the \(u\)-domain is given by,

\[
S_{\text{post}}(\omega, k_u, x, y) = \int_u \exp \left[-2jk\sqrt{x^2 + (y - u)^2}\right] \exp [-jk_u u] du
= \int_u \exp \left[-2jk\sqrt{x^2 + (y - u)^2} - jk_u u\right] du
= \int_u \exp [jk_u \phi(u)] du \quad (A.2)
\]

where

\[
\phi(u) = \frac{2k}{k_u} \sqrt{x^2 + (y - u)^2} - u. \quad (A.3)
\]

The method of stationary phase can be used to solve this equation by evaluating a Taylor Series at the point where the first derivative is zero, \([\text{Papoulis, 1968}]\), \([\text{Easton-Jr, 2005}]\),

\[
S_{\text{post}}(\omega, k_u, x, y) \approx \exp \left[\frac{j\pi}{4}\right] \sqrt{\frac{1}{k_u \phi''(u')}} \exp [jk_u \phi(u')] \quad (A.4)
\]

where the second derivative, \(\phi''(u') = \frac{\partial^2 \phi(u')}{\partial^2(u')}\). The stationary point, \(u'\) is found by solving,

\[
\left.\frac{\partial \phi(u)}{\partial u}\right|_{u=u'} = 0
\Rightarrow \left.\frac{\partial}{\partial u} \left(-\frac{2k}{k_u} \sqrt{x^2 + (y - u)^2} - u\right)\right|_{u=u'} = 0
\Rightarrow -2k(y - u') = k_u \sqrt{x^2 + (y - u')^2} - 1 = 0
\Rightarrow -2k(y - u') = k_u \sqrt{x^2 + (y - u')^2}. \quad (A.5)
\]
Then squaring both sides and rearranging gives,

$$\Rightarrow 4k^2(y - u')^2 = k_u^2(x^2 + (y - u')^2)$$
$$\Rightarrow (4k^2 - k_u^2)(y - u')^2 = k_u^2x^2$$
$$\Rightarrow (y - u')^2 = \frac{k_u^2x^2}{4k^2 - k_u^2}$$
$$\Rightarrow (y - u') = \frac{k_u^x}{\sqrt{4k^2 - k_u^2}}$$
$$\Rightarrow u' = -\frac{k_u^x}{\sqrt{4k^2 - k_u^2}} + y. \quad \text{(A.6)}$$

The next step is to substitute the stationary point into the phase of Equation A.3 giving,

$$\phi(u') = \frac{2k}{k_u}\sqrt{x^2 + (y - u')^2} - u'$$
$$= -\frac{2k}{k_u}\sqrt{x^2 + \left(y - \left(-\frac{k_u^x}{\sqrt{4k^2 - k_u^2}} + y\right)\right)^2} - \left(-\frac{k_u^x}{\sqrt{4k^2 - k_u^2}} + y\right)$$
$$= -\frac{2kx}{k_u}\sqrt{\frac{4k^2}{4k^2 - k_u^2}} + \frac{k_u^x}{\sqrt{4k^2 - k_u^2}} - y$$
$$= -\frac{4k^2x}{k_u}\sqrt{\frac{4k^2}{4k^2 - k_u^2}} + \frac{k_u^x}{\sqrt{4k^2 - k_u^2}} - y$$
$$= -\frac{x}{k_u}\frac{4k^2 - k_u^2}{\sqrt{4k^2 - k_u^2}} - y$$
$$= -\frac{x}{k_u}\sqrt{4k^2 - k_u^2} - y \quad \text{(A.7)}$$

and substituting this back into Equation A.4 gives

$$S_{\text{post}}(\omega, k_u, x, y) = \exp\left[j\pi \frac{1}{4k^2} \frac{1}{k_u\phi'(u')} \exp\left[-j\sqrt{4k^2 - k_u^2}x - jk_u y\right]\right]. \quad \text{(A.8)}$$

Since the amplitude of this signal model is slowly oscillating and essentially constant, [Soumekh, 1999], it is not important for the imaging algorithms in this thesis. This equation is therefore normalised, giving a new definition for the signal model,

$$S_{\text{post}}(\omega, k_u, x, y) \equiv \exp\left[-j\sqrt{4k^2 - k_u^2}x - jk_u y\right]. \quad \text{(A.9)}$$
This appendix describes the implementation of the diffuse scattering and jammer models presented in Chapter 5. The first two parts describe how the magnitudes and positions are determined for the hot-clutter ground returns, while the third part describes how the jammer waveform is implemented.

**B.1 Scatterer Magnitude**

The magnitude $b_k$ for $k = 1\ldots K_{hc}$ is determined by the following seven steps. These include formulation of the Probability Distribution Function (PDF), $f_s(\xi)$ and the Cumulative Distribution Function (CDF), $F_s(\xi)$ which allows random magnitudes to be drawn. They are then scaled according to mean square value, $E\{|\Sigma|^2\}$ to produce the correct output power before the Fresnel reflection coefficient is used to model the effect of different antenna polarisations and soil types.

1. Calculate the mean square scattering coefficient $E\{|\Sigma|^2\}$ based on Equation 5.19. The upper and lower limits, $\xi_A$ and $\xi_B$ are found numerically by determining the valid range of the square root in this equation. The numerical integration is implemented by MATLAB using the adaptive Lobatto quadrature method.

2. Calculate the CDF $F_s(\xi)$ for a range of $\xi$. This is found by integrating from the lower limit $\xi_A$ to the the variable of interest $\xi$,

$$F_s(\xi) = \int_{\xi_A}^{\xi} f_s(\xi')d\xi'. \quad (B.1)$$

3. Determine a random $\xi$ from the CDF. This is done by first choosing a random variable, $\varepsilon_k \in [0, \max\{F_s(\xi)\}]$ and then finding the closest matching value in the CDF, $\xi_k = F_s(\xi_k)$.

4. Determine the PDF magnitude of the random variable from Equation 5.19, $f_s(\xi_k)$. 
5. The scattering coefficient \( \Sigma_k = \alpha_s f_s(\xi_k) \), now needs to be correctly scaled so the amplitudes are constrained with total power,

\[
\sum_{k=1}^{K_{hc}} |\Sigma_k|^2 = E \{ |\Sigma|^2 \} 
\]

(B.2)

giving,

\[
\alpha_s = \sqrt{\frac{E \{ |\Sigma|^2 \}}{\sum_{k=1}^{K_{hc}} |f_s(\xi_k)|^2}}
\]

(B.3)

6. Multiply by the Fresnel reflection parameter, \( \sigma_k = |\tilde{\Gamma}|\Sigma_k \). If \( \psi \) is the grazing angle of the SAR at the offset range \( X_c \),

\[
\psi = \arctan \left[ \frac{h_P}{X_c} \right]
\]

then for a horizontally polarised antenna,

\[
\tilde{\Gamma}_h = \frac{\cos \psi - \sqrt{\epsilon_r - \sin^2 \psi}}{\cos \psi + \sqrt{\epsilon_r - \sin^2 \psi}}
\]

(B.5)

where \( \epsilon_r \) is the dielectric constant of the ground. Similarly, for vertical polarisation,

\[
\tilde{\Gamma}_v = \frac{\epsilon_r \cos \psi - \sqrt{\epsilon_r - \sin^2 \psi}}{\epsilon_r \cos \psi + \sqrt{\epsilon_r - \sin^2 \psi}}
\]

(B.6)

Note that only a single polarisation was used in the simulation with \( \tilde{\Gamma} \equiv \tilde{\Gamma}_h \). The dielectric constant was arbitrarily chosen for a ground with soil moisture of 12%.

7. Multiply by the relative hot-clutter power and square root the result to obtain the final hot-clutter ground return, \( b_k = \sqrt{\rho \sigma_k} \).

### B.2 Scatterer Location

The following four steps describe how the positions, \((x_k, y_k)\) for \( k = 1 \ldots K_{hc} \) of the hot-clutter ground returns are determined. This involves first determining the ground location \((\bar{x}_k, \bar{y}_k)\), projecting onto the slant-plane and then rotating to account for the jammer offset angle.

1. The ground range \( \bar{x}_k \) is found by substituting the random variable \( \xi_k \) from step 3 of the magnitude calculation into Equation 5.15. This means that the scatterer locations are also determined by the CDF in step 2 above.

2. The ground azimuth location \( \bar{y}_k \) was defined in Equation 5.13 and is a function of the angle \( \beta \). This is modelled as a uniform random variable with its upper bound \( \beta_0 \), defined by Equation 5.21 and lower bound by evaluating the real part of the square root in Equation 5.13.

\[
\beta \in \left[ \arctan \left( \frac{1}{4} \left( \frac{h_J}{x_1} - \frac{h_P}{x_2} \right) \right), \arctan \left( \frac{2h}{x_J} \right) \right]
\]

(B.7)

The final component required to calculate \( \bar{y}_k \) is the sign. This is randomly selected as either \( \pm 1 \).
3. The ground plane scatterers are then projected onto the slant plane to match the simulation geometry. The projection of any scatterer from the ground plane onto the slant-plane is along a circular arc where the centre of the circle is at the SAR location. This circular arc becomes a straight line when the SAR is sufficiently far away. With this approximation, projecting onto the slant-plane requires the broadside range, \( x'_{kJ} > \bar{x}_k \):

\[
\bar{x}_k = x'_k \cos \psi \rightarrow x'_k = \frac{\bar{x}_k}{\cos \psi}.
\]

The new geometry is shown in Figure B.1 where the scatterer height is calculated as \( \bar{z} = (\bar{x}_k - x_C) \sin \psi \). Note that the azimuth position is unchanged for both domains, \( \bar{y}_k = y'_k \).

4. If the jammer is not directly in line with the SAR look direction (\( \theta_J > 0^\circ \)), there will be a jammer offset angle and the slant-plane geometry must be rotated accordingly. Rotational matrices are used to adjust the scatterer positions on the slant-plane. If the scatterer location at broadside is given by \( (x'_k, y'_k) \), then the two-dimensional rotational matrix for an angle \( \theta_J \) is given by:

\[
R_{\text{rot}} = \begin{bmatrix}
\cos \theta_J & \sin \theta_J \\
-\sin \theta_J & \cos \theta_J
\end{bmatrix}
\]

implying that the correctly oriented scatter positions are given by:

\[
\begin{bmatrix}
x_k \\
y_k
\end{bmatrix} = R_{\text{rot}}^{-T} \begin{bmatrix}
x'_k \\
y'_k
\end{bmatrix}.
\]

The distance on the ground between the SAR and jammer platforms also needs to be modified. It is now defined as a radial distance,

\[
\bar{R}_J = \frac{\bar{x}_J}{\cos \theta_J}.
\]
B.3 Jammer Waveform

The final geometry is shown in Figure B.2. In terms of the original coordinate system, a three-dimensional rotation about the $z$-axis is required. If the broadside scatterer location is given by $(\bar{x}_k', \bar{y}_k', \bar{z}_k')$, with rotation matrix,

$$
R_{\text{rot3}} = \begin{bmatrix}
\cos \theta_J & \sin \theta_J & 0 \\
-\sin \theta_J & \cos \theta_J & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

then the new positions are

$$
\begin{bmatrix}
\bar{x}_k \\
\bar{y}_k \\
\bar{z}_k
\end{bmatrix} = R_{\text{rot3}}^T \begin{bmatrix}
\bar{x}_k' \\
\bar{y}_k' \\
\bar{z}_k'
\end{bmatrix}.
$$

---

B.3 Jammer Waveform

The transmitted jammer signal $J(\cdot)$ is modelled as continuous white Gaussian noise. When received at the SAR, its form is given in Equation 5.3 and must contain the correct delay for the direct-path and each hot-clutter scatterer. This requires a knowledge of the hot-clutter scatterer positions and the corresponding delay. To fit the jammer signal to the model, it must be created with enough points to encompass each delay component and have each sample separated by the fast-time sample rate, $\Delta t$. Each scatterer contribution can then use the signal as a lookup table to extract its contribution for each pulse.
To implement this model for a diffuse scenario, simulation of a very large jammer signal would be required. To reduce the computation time, intervals of the pulse repetition interval, $T_p$, are assumed to be uncorrelated with the one before it. The following implementation can then be used to reduce the size of the generated jammer signal without compromising its accuracy.

1. Calculate the delays from each scatterer to the antenna phase centre based on the ground location given in diffuse scattering model. The delays are determined using Equation 5.4 with $d_n = 0$.

2. Sort the delays and determine the amount of overlap between each, given the total length of each signal component is $(L - 1)\Delta t$.

3. Generate portions of the jammer signal for each overlapping group, given the total length of the jammer signal portion is $L_J > L$. The jammer signal realisation is based on the models in the previous section. To create a random variable with this autocorrelation, a singular value decomposition is used to determine $\tilde{U}$, $\Lambda$ and $\tilde{V}$ such that $r_J = \tilde{U}\Lambda\tilde{V}^H$.

To model the two components of the quadrature receiver, two random Gaussian vectors $r_1$ and $r_2$ of length $L_J$ with zero mean and variance 1 are combined,

$$r = \frac{r_1 + jr_2}{\sqrt{2}} \in \mathbb{C}^{L_J \times 1}$$

(B.14)

with length equal to the number of fast-time samples. This random vector is then transformed into one which fits the desired autocorrelation by

$$J = \tilde{U}\sqrt{\Lambda}r \in \mathbb{C}^{L_J \times 1}.$$  

(B.15)

4. Extract a jammer signal of size $L$ for each scatterer by looking up the jammer signal portion, $J$ to find the closest point matching the delay.

Using this technique, a correlated scenario would have very similar hot-clutter delays and correspondingly would use the same jammer realisation, while a very diffuse scenario would have no overlap between delays and would have independent jammer realisations due to the uncorrelated Gaussian white noise. To demonstrate this procedure, an example is used to comparing an estimated autocorrelation matrix with the exact version. The fast-time domain is oversampled by a factor two to demonstrate the correlation function. If $J_k$ represents the $k^{th}$ realisation of this process, then an estimate of the autocorrelation can be formed by averaging the outer-product $K_{av}$ times,

$$\hat{r}_J = \frac{1}{K_{av}}\sum_{k=1}^{K_{av}} J_kJ_k^H \in \mathbb{C}^{L \times L}.$$  

(B.16)

The ideal and averaged autocorrelation matrices are shown below in Figure B.3. It is clear that the averaged function is very similar to the ideal.
B.3 Jammer Waveform

Figure B.3. Jammer autocorrelation comparison (real part, $\sigma_J^2 = 1$)
This appendix derives the Hung-Turner projection used in Chapter 6. Consider the following rank one covariance estimate of the interference plus noise signal $\mathbf{z}$ with $\eta$ dB of diagonal loading,

$$
\hat{R}_z = \mathbf{z}\mathbf{z}^H + \eta \mathbf{I}.
$$

If the interference plus noise signal is known and this estimate is used in the maximum Signal to Interference plus Noise (SINR) algorithm with small diagonal loading, then the interference plus noise will be completely cancelled. The result is known as a Hung-Turner projection, [Hung and Turner, 1983], [Gierull, 1996] and is derived below.

Consider the following maximum SINR filter with the desired target represented by $\mathbf{s}_T$,

$$
x_{sp}(t, u) = (\mathbf{z}\mathbf{z}^H + \eta \mathbf{I})^{-1}\mathbf{s}_T^H (\mathbf{s}_T + \mathbf{z}) = \mathbf{s}_T^H (\mathbf{z}\mathbf{z}^H + \eta \mathbf{I})^{-1} (\mathbf{s}_T + \mathbf{z}).
$$

Using the matrix inversion lemma gives:

$$
x_{sp}(t, u) = \mathbf{s}_T^H (\eta^{-1} \mathbf{I} - \eta^{-2} \mathbf{z}(1 + \eta^{-1} \mathbf{z}^H \mathbf{z})^{-1} \mathbf{z}^H)(\mathbf{s}_T + \mathbf{z})
\approx \mathbf{s}_T^H (\mathbf{z}\mathbf{z}^H + \eta \mathbf{I})^{-1} (\mathbf{s}_T + \mathbf{z})
\approx \mathbf{s}_T^H \left[ \mathbf{I} - \frac{\eta^{-1} \mathbf{z}\mathbf{z}^H}{\eta + \mathbf{z}^H \mathbf{z}} \right] \mathbf{s}_T
$$

and if the diagonal loading is small compared to the interference plus noise power, this reduces to

$$
x_{sp}(t, u) \approx \mathbf{s}_T^H \left[ \mathbf{I} - \frac{\mathbf{z}\mathbf{z}^H}{\mathbf{z}^H \mathbf{z}} \right] \mathbf{s}_T
$$

where $\mathbf{P}_z$ is a (Hung-Turner) projection onto the space orthogonal to the interference plus noise.
Appendix D

Post Range Processing Covariance Derivation

This appendix contains the derivation of the post range processing covariance model in section 7.4. From this section, the hot-clutter covariance model post range processing is given by

\[
R_{Z_{f}, \text{post}}(t, \zeta, n, n') = \sum_{k} \sum_{k'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_{k} b_{k'}^{*} r_{J} \left( t'' - t' + \bar{\tau}_{n,k} - \bar{\tau}_{n',k'} \right) \exp \left[ -j \omega_{c} (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ - j \omega_{d,k} t' + j \omega_{d,k'} t'' \right] p_{c}^{*} (t' - t) p_{c} (t'' + \zeta - t) \ dt' dt''.
\]  
(D.1)

It can be simplified by making two substitutions,

\[
\begin{align*}
\chi_1 &= t' - t \quad \text{(D.2)} \\
\chi_2 &= t'' + \zeta - t \quad \text{(D.3)}
\end{align*}
\]

giving,

\[
R_{Z_{f}, \text{post}}(t, \zeta, n, n') = \sum_{k} \sum_{k'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} b_{k} b_{k'}^{*} r_{J} \left( \chi_2 - \chi_1 - \zeta + \bar{\tau}_{n,k} - \bar{\tau}_{n',k'} \right) \exp \left[ -j \omega_{c} (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ - j \Delta \omega_{k,k'} t \right] p_{c}^{*} (\chi_1) p_{c} (\chi_2) \ d\chi_1 d\chi_2
\]

\[
= \sum_{k} \sum_{k'} b_{k} b_{k'}^{*} \exp \left[ -j \omega_{c} (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ - j \Delta \omega_{k,k'} t \right]
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r_{J} \left( \chi_2 - \chi_1 - \zeta + \bar{\tau}_{n,k} - \bar{\tau}_{n',k'} \right) \exp \left[ - j \omega_{d,k} \chi_1 + j \omega_{d,k'} (\chi_2 - \zeta) \right] p_{c}^{*} (\chi_1) p_{c} (\chi_2) \ d\chi_1 d\chi_2
\]  
(D.4)

where \( \Delta \omega_{k,k'} = \omega_{d,k} - \omega_{d,k'} \). To reduce one dimension of the integral, the jammer covariance is assumed to be a delta function of power \( \sigma_{J}^{2} \) at the point, \( \chi_2 = \chi_1 + \zeta - \bar{\tau}_{n,k} + \bar{\tau}_{n',k'} \). The covariance then becomes,

\[
R_{Z_{f}, \text{post}}(t, \zeta, n, n') = \sigma_{J}^{2} \sum_{k} \sum_{k'} b_{k} b_{k'}^{*} \exp \left[ -j \omega_{c} (\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ - j \Delta \omega_{k,k'} t \right]
\int_{-\infty}^{\infty} p_{c}^{*} (\chi_1) p_{c} (\chi_1 + \zeta - \bar{\tau}_{n,k} + \bar{\tau}_{n',k'}) \exp \left[ - j \omega_{d,k} \chi_1 + j \omega_{d,k'} (\chi_1 - \bar{\tau}_{n,k} + \bar{\tau}_{n',k'}) \right] \ d\chi_1.
\]  
(D.5)
Then by grouping the temporal delays in the first exponential, making a third substitution
\( \zeta' = \zeta - \tau_{n,k} + \bar{\tau}_{n',k'} \)
and using the SAR signal waveform,

\[ p_c(t) = b \left( \frac{t}{T_p} \right) \exp \left[ -jB\pi t + j\alpha t^2 \right] \]  

(D.6)

where \( T_p \) is the pulse width and \( b(t) \) is the chirp duration which is unity for \( 0 \leq t \leq 1 \) and zero otherwise, the covariance becomes,

\[
R_{Z_t,\text{post}}(t, \zeta, n, n') = \sigma_j^2 \sum_k \sum_{k'} b_k b_{k'}^* \exp \left[ -j(\omega_c + \omega_{d,k'})(\tau_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ -j\Delta\omega_{k,k'}t \right] 
\]

\[
\int_{-\infty}^{\infty} b \left( \frac{\chi_1}{T_p} \right) b \left( \frac{\chi_1 + \zeta'}{T_p} \right) \exp \left[ -jB\pi\zeta' - j\alpha (\chi_1^2 - (\chi_1 + \zeta')^2) \right] 
\]

\[
\exp \left[ -j\Delta\omega_{k,k'}\chi_1 \right] d\chi_1 
\]

(D.7)

where the integration area is defined by the overlap between the two rectangle functions. It is only valid when \( T_p > |\zeta'| \), giving,

\[
R_{Z_t,\text{post}}(t, \zeta, n, n') = \sigma_j^2 \sum_k \sum_{k'} b_k b_{k'}^* \exp \left[ -j(\omega_c + \omega_{d,k'})(\tau_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ -j\Delta\omega_{k,k'}t \right] 
\]

\[
\exp \left[ -jB\pi\zeta' \right] \int_{-\infty}^{\infty} \frac{\chi_1}{T_p - \zeta'} \exp \left[ j\alpha (2\chi_1\zeta' + \zeta'^2) \right] \exp \left[ -j\Delta\omega_{k,k'}\chi_1 \right] d\chi_1 
\]

\[
= \sigma_j^2 \sum_k \sum_{k'} b_k b_{k'}^* \exp \left[ -j(\omega_c + \omega_{d,k'})(\tau_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ -j\Delta\omega_{k,k'}t \right] 
\]

\[
\exp \left[ -jB\pi\zeta' + j\alpha\zeta'^2 \right] \int_0^{T_p - \zeta'} \exp \left[ j\chi_1(2\alpha\zeta' - \Delta\omega_{k,k'}) \right] d\chi_1. 
\]

(D.8)

This integral can then be solved to give,

\[
\int_0^{T_p - \zeta'} \left( \cdot \right) d\chi_1 = \left. \frac{\exp \left[ j\chi_1(2\alpha\zeta' - \Delta\omega_{k,k'}) \right]}{j(2\alpha\zeta' - \Delta\omega_{k,k'})} \right|_0^{T_p - \zeta'} 
\]

\[
= \left( \frac{\exp \left[ j(T_p - \zeta')(2\alpha\zeta' - \Delta\omega_{k,k'}) \right] - 1}{j(2\alpha\zeta' - \Delta\omega_{k,k'})} \right) 
\]

\[
= \exp \left[ j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta') \right] 
\]

\[
\left( \frac{\exp \left[ j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta') \right] - \exp \left[ -j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta') \right]}{2j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})} \right) 
\]

\[
= \exp \left[ j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta') \right] \sin \left[ \frac{j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta')}{\omega_{d,k'}} \right] 
\]

\[
= (T_p - \zeta') \exp \left[ j(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta') \right] \sin \left[ \frac{(\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta')}{\pi} \right] 
\]

(D.9)

and normalising the result by \( (T_p - \zeta') \) and substituting it into the covariance gives,

\[
R_{Z_t,\text{post}}(t, \zeta, n, n') = \sigma_j^2 \sum_k \sum_{k'} b_k b_{k'}^* \exp \left[ -j(\omega_c + \omega_{d,k'})(\tau_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ -j\Delta\omega_{k,k'}t \right] 
\]

\[
\exp \left[ -jB\pi\zeta' + j\alpha(\zeta'^2 + \zeta'(T_p - \zeta')) \right] \exp \left[ -j0.5\Delta\omega_{k,k'}(T_p - \zeta') \right] 
\]

\[
\sin \left[ (\alpha\zeta' - 0.5\Delta\omega_{k,k'})(T_p - \zeta')/\pi \right]. 
\]

(D.10)
Finally if $\alpha = \pi B/T_p$, the covariance reduces to

$$R_{Z_t,\text{post}}(t, \zeta, n, n') = \sigma_f^2 \sum_k \sum_{k'} b_k b_{k'}^* \exp \left[ -j(\omega_c + \omega_{d,k'})(\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right] \exp \left[ -j\Delta\omega_{k,k'} t \right]$$

$$\exp \left[ -j0.5\Delta\omega_{k,k'}(T_p - \zeta') \right]$$

$$\text{sinc} \left[ B\zeta'(1 - \zeta'/T_p) - \Delta f_{k,k'}(T_p - \zeta') \right]$$

(D.11)

where $\Delta f_{k,k'} = 0.5\Delta\omega_{k,k'}/\pi$, and rearranged slightly the final form becomes,

$$R_{Z_t,\text{post}}(t, \zeta, n, n') = \sigma_f^2 \sum_k \sum_{k'} b_k b_{k'}^* \text{sinc} \left[ B\zeta'(1 - \zeta'/T_p) - \Delta f_{k,k'}(T_p - \zeta') \right]$$

$$\exp \left[ -j(\omega_c + \omega_{d,k'})(\bar{\tau}_{n,k} - \bar{\tau}_{n',k'}) \right]$$

$$\exp \left[ -j\Delta\omega_{k,k'}(t + 0.5(T_p - \zeta')) \right].$$

(D.12)
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