



Interference of Intensity Noise in a Multimode Nd:YAG Laser

by

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Statement Of Originality

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To those of you about to embark on writing a thesis, may I humbly suggest you consult the wisdom of Joe Wolfe from the University of New South Wales.

<http://www.phys.unsw.edu.au/~jw/thesis.html>

The University of Queensland also has some valuable suggestions regarding the journey that is a Ph.D.

http://www2.ems.uq.edu.au/phdweb/fr_phsta.html

If it’s all getting too much, then for some light relief pickup the book by Cham [J. Cham, *Piled Higher and Deeper* (Piled Higher and Deeper Publishing, Palo Alto, California, 2002)] or go to:

<http://www.phdcomics.com>

Don’t stay too long or you’ll be procrastinating!

Abstract

We investigate the behaviour of the intensity noise in a multi-longitudinal mode Nd:YAG laser. This type of laser is a nonlinear system which exhibits complicated dynamics within the intensity noise. For example, antiphase dynamics is where there is cancellation of one or more collective modes of oscillation, which are distinct from the longitudinal modes, in the total output.

Commonly lasers are studied experimentally to discriminate between models used to describe them. They are convenient since many external influences can be controlled and the oscillations of interest are low frequency (in the kHz regime) making their direct measurement relatively simple.

In our laser, the collective modes of oscillation are excited by broadband ambient noise. Because the phase of the excitation is unknown, we develop the cross spectral technique to measure the antiphase dynamics directly and form a picture of the intensity noise interference for two to five mode operation. For three mode operation, we measure the contributions of the longitudinal modes to the collective modes.

We also calculate power spectral densities of the individual longitudinal modes and the total intensity. We test relationships between these quantities, at the collective mode frequencies, which are derived from modal rate

equation theory. For two mode operation, the theoretical relations are satisfied. For three mode operation, the relations are satisfied when the picture of interferences is taken into account. The cross spectral technique is therefore shown to be a sensitive test of the model developed by Pieroux and Mandel [T. Hill et al., Phys. Rev. A **66**, 063803 (2002)].

The behaviour of the multimode laser operating near the threshold of a longitudinal mode is measured. Transitions in the cross spectrum are noted in some pairs of longitudinal modes, for an arbitrary but small pump rate above threshold of a longitudinal mode. It has been shown that longitudinal modes with a high threshold pump power may become more intense than those with a lower threshold [K. Otsuka et al., Opt. Lett. **23**, 201 (1998), L. Stamatescu and M.W. Hamilton, (unpublished) (1999), N.B. Abraham et al., Phys. Rev. A **62**, 013810 (2000), P.A. Khandokhin, E.A. Ovchinnikov and E.Yu. Shirokov, Phys. Rev. A **61**, 053807 (2000)]. The AC noise component of the first two longitudinal modes to reach threshold, is found to exhibit similar properties to their intensity. The implications of the results of this thesis, on models used to describe the behaviour of solid state lasers, are also discussed.

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Chapter 1

Introduction

One reason for interest in arrays of coupled nonlinear oscillators is the somewhat frequent occurrence of instabilities in their output. Some practical examples of such arrays include Josephson Junction arrays [1] and multimode lasers [2] whilst neural networks [3, 4] and phase oscillators [5] are studied theoretically. Studies have shown that instabilities in the output are generally the result of a fundamental nonlinearity in the physical makeup of an array. Nonlinear oscillator arrays with a small number of oscillators ($\lesssim 20$) are also studied because they may be able to be described by fundamental theory. However, more complex systems, including arrays with a large number of nonlinear oscillators, may require the introduction of concepts that have no direct connection to fundamental ideas. Thus arrays with a small number of oscillators may also be a bridge between simple and more complex systems.

Typically the constituent oscillators of an array are coupled to their nearest neighbour or to every other oscillator, i.e. globally. Because the oscillators are coupled, they do not act independently, rather the array acts with

collective modes of oscillation. Of particular interest to this thesis is the phenomenon known as antiphase dynamics, which occurs at the collective mode frequencies [1, 6, 2]. A Fourier analysis of the instabilities of each of the constituent oscillators of an array would show peaks corresponding to each of the collective modes. For each constituent oscillator, these frequency components (i.e. the collective modes) can interfere either constructively (in phase) or destructively (out of phase) with the same frequency components of the other constituent oscillators. However, a Fourier analysis of the instability in the combined oscillation of the array may show only one main peak, the others being reduced or cancelled out due to the interferences. We illustrate antiphase dynamics using a Fourier analysis of the fluctuations in intensity, of a laser operating with two longitudinal modes,¹ see Fig. 1.1. The interference of interest occurs in the field intensity fluctuations, at the collective mode frequencies, and not in the complex field amplitude. The power spectrum (i.e. the modulus squared of the Fourier transform) of the intensity fluctuations of a longitudinal mode shows two frequency components corresponding to the collective modes, see Fig. 1.1(a). The power spectrum of the other longitudinal mode would show similar resonances. However, the power spectrum of the fluctuations of the total intensity shows only one peak, see Fig. 1.1(b), the other collective mode being reduced or destroyed through destructive interference [7, 8]. The oscillations of the total intensity of a multimode laser are therefore akin to that of a single mode laser [9].

Another example where antiphase dynamics has been observed is zero bias Josephson junctions. These particular devices operate near the region

¹A longitudinal mode is an integer number of standing waves of the electric field, between the mirrors forming the laser cavity.

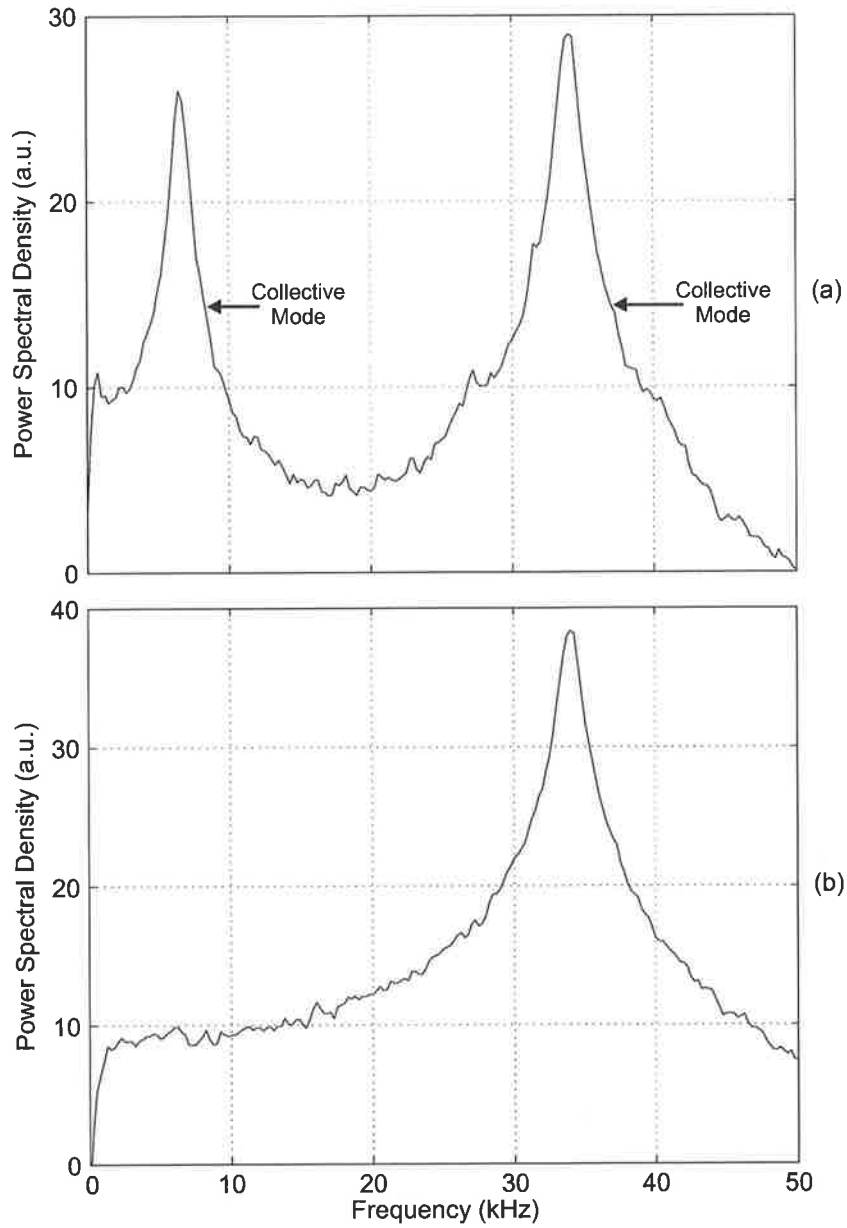


Figure 1.1: The collective mode frequencies are; 6.5kHz and 34kHz. (a) The power spectrum of a longitudinal mode. (b) The power spectrum of the total intensity fluctuations.

of chaos and are used for voltage standards [10]. In larger arrays, of order 10^4 oscillators as in certain fibre lasers, the antiphased collective oscillations of the array have been shown to be analogous to acoustical and optical phonons in a solid [11].

An understanding of the co-operative behaviour of oscillator arrays, their instabilities and their control is therefore of fundamental experimental and theoretical interest. In this context we note that nonlinear oscillator arrays and the instabilities exhibited, are many and varied. For example, coupled oscillators are used to model neural networks which control human body movement [3] or process visual data [4]. Coupled oscillators have also been used to model pacemaker cells which control the regularity of the human heartbeat [12, 13]. The efficient use of arrays in applications such as radio frequency transmission, has been proven with an array of Gunn diodes. When locked to a single resonant frequency of the circuit, the output power of the circuit scaled linearly with the number of elements of the array, i.e. power scaling [14]. York and Compton [14] also showed that when locked to a number of equally spaced frequencies, the array could generate large energy pulses, analogous to that of a mode-locked laser [15]. This phenomenon is an example, not of antiphasing, but the coherent behaviour of the collective modes of oscillation.

With lasers being of particular interest to this thesis, other optical examples of arrays include a soliton pulse [16, 17] and multimode gas ring lasers. The latter have been shown to exhibit antiphase dynamics in the chaotic fluctuations of the laser intensity [18, 19]. An understanding of semiconductor lasers as arrays is also important since they are in widespread

use, either on their own or as a source for another laser. For example, a multi-longitudinal mode single diode emitter, where the injection current is modulated with a large amplitude as in optical communication systems, can show routes to chaotic behaviour [20]. One recent development of diode lasers is vertical cavity surface emitting lasers (VCSELs). In this type of laser, the optical fields of the nearest neighbour emitters can be coupled, which gives rise to high power and beam steering applications [21, 22]. An array of high power diode bars, which are coupled in the same way, has similar applications [23, 24, 25]. It has also been shown that it is possible to couple separate solid state lasers, operating with a single longitudinal mode, via their optical fields and thereby control or synchronise the fluctuations [26, 27, 28]. Multiple longitudinal mode lasers form arrays through sharing the same gain medium. In this particular type of laser, antiphase dynamics has been observed in the fluctuations of the output of both fibre lasers [11, 29, 30] and a solid state crystal laser [31]. The influence of noise on the chaotic fluctuations of a multimode solid state crystal laser, that does not exhibit antiphase dynamics, has also been studied [32].

As we have indicated, antiphase dynamics occurs in a number of arrays. Historically the first array shown to exhibit antiphase dynamics was an array of Josephson junctions (JJ's) [1, 6]. The frequency of the collective oscillations in this array is typically 10GHz - 100GHz. This means that the oscillations are difficult to follow in real time and most research with these devices is therefore performed with simulations. Whilst the optical frequencies of a laser are much greater than that of JJ's, an appropriately designed laser also has frequencies of oscillation much less than that of JJ's, these are called

Class	Property
A	$\gamma_c \ll \gamma_p, \gamma_D$
B	$\gamma_p \gg \gamma_c \gg \gamma_D$
C	$\gamma_c \approx \gamma_p \approx \gamma_D$

Table 1.1: Class of lasers by decay rates.

the relaxation oscillations. Therefore lasers with these lower frequencies are more convenient for experimental investigation and can be readily compared to models used to describe them. Indeed, being used in many facets of life today, the behaviour of lasers is of interest in their own right. For example, Haken showed that the equations used to describe laser instabilities were isomorphic to the well known Lorenz equations for chaotic fluid dynamics [33]. This stimulated studies into the Lorenz like chaotic dynamics of Ammonia lasers [34, 35, 36]. However, it was just over a decade ago that experimental observation of antiphase dynamics in a laser was first reported [2]. Antiphase dynamics has since been shown to exist in lasers where the total intensity oscillates about a stable steady state [29, 37, 38, 39], with a long term limit which is periodic (limit cycle) [2, 31], or chaotically [2, 40, 41].

The oscillation of the total intensity of a laser can be divided into three classes based upon three decay rates: those of the laser cavity (γ_c), the dipole moment of the polarisation (γ_p) and the population inversion (γ_D), see monograph [42] and references contained therein. The relationships between the decay rates are given in Table 1.1. In Class C lasers (e.g. Ammonia), the decay rates are of similar magnitude and therefore each variable more or less equally influences the other. Intrinsically, the laser intensity oscillates

chaotically [34, 35, 41]. The Maxwell-Bloch equations, which were derived for homogeneously broadened laser, are a set of three coupled ordinary differential equations which describe the evolution with time, of the electric field, the polarisation and the population inversion density. The equations have been shown to qualitatively and quantitatively agree with the experimental results of a Class C laser [34].

In Class B lasers, the polarisation decay rate is very fast and it follows the dynamics of the electric field of the light. The laser cavity decay rate is faster than that of the population inversion, which can oscillate due to external influences and therefore, the total intensity likewise. Some examples of this class of laser are: Nd:YAG, LiNdP₄O₁₂, Ruby, CO₂ and semiconductor lasers. Beginning with the Maxwell-Bloch equations, the polarisation is eliminated such that the system is described by two coupled rate equations for the electric field and the population inversion. When perturbed, commonly the intensity of this class of laser fluctuates and returns to a stable steady state via damped relaxation oscillations, which are a nonlinear interaction between the electric field and the population inversion [9, 43, 44]. The model can be simplified further by linearising the equations about the steady state such that the equations are still coupled but the solution is simpler. However, depending on the number of oscillators, the pumping rate, the coupling strength between the oscillators, the strength or type of excitation of the collective modes, solutions are possible whereby the intensity oscillates with a limit cycle [2, 31] or chaotically [2, 40, 45].

In Class A lasers (e.g. organic dye, Argon-ion and HeNe), the decay rate of the laser cavity is much less than either of the other decay rates. Therefore

the population inversion and the polarisation follow the electric field such that they are eliminated from the equations of motion and there are no oscillations of the intensity [42].

Though we restrict ourselves in this thesis to homogeneously broadened lasers, we note that the intensity of He-Xe laser, which is inhomogeneously broadened, can also pulsate chaotically. Theoretical models of its behaviour have proven to be successful [35, 46]. For an extensive treatment of the instabilities in lasers based upon different gain media, types of cavities and so on, refer to monographs by Khanin [42] and Mandel [47] which describe in detail the equations required and the assumptions involved.

We choose a multi-longitudinal mode Nd:YAG (Neodymium doped Yttrium Aluminium Garnet) laser as our nonlinear oscillator array. For detailed studies of the interaction of an array of nonlinear oscillators, our laser is a good example, being able to be studied both experimentally and theoretically. It is also a relatively clean system for which external influences can be controlled. As stated previously, a laser has two characteristic frequency domains which apply whether operating in the single or multimode regime. There are optical frequencies, of order hundreds of THz, characterising the longitudinal eigenmodes, referred to herein as cavity modes. Also there are the relaxation oscillations, which are in the kHz to MHz range for common solid state lasers making their direct measurement relatively simple. Due to the standing wave nature of the electric field in a Fabry-Perot cavity laser, the population inversion (gain) is modified by spatial hole burning [15, 48]. The relaxation oscillations are therefore coupled due to the cross saturation of the laser gain [49] and correspond to the collective modes of oscillation of

the array. In this thesis, we focus on interferences of the field intensity fluctuations and not interferences of the complex field amplitude, at the collective mode frequencies.

For Nd lasers like ours, it takes little effort to excite the relaxation oscillations. There are several common methods: some examples are the use of ambient noise [50, 51], modulation of the pump, either continuously [31, 52, 53, 54] or by step function [55], and modulation of the resonator losses [56, 57, 58, 59]. In the work contained in this thesis we rely on broadband ambient noise to excite the relaxation oscillations. The noise originates from spontaneous emission (quantum noise) and from environmental perturbations. In a laser, there can be as many collective mode frequencies as there are cavity modes, although this is not an absolute rule [60, 61]. When there is more than one collective mode, the collective mode originating from single mode operation is usually the one with the highest frequency [9, 62], the others typically having about half that value. For a multimode laser, the signature of antiphase dynamics is defined as: each of the N cavity mode intensities relax with N frequencies, whilst the total intensity relaxes with just one [7, 8]. We describe the experimental setup used to study antiphase dynamics in Chap. 2.

In their studies of an array of JJ's, Hadley et al. [6] reported that the phases of the interferences of the individual oscillators, form a cluster at each collective mode frequency. This information is useful for determining which collective modes are antiphased and those which are not. For a laser operating with N cavity modes, a cluster at each collective mode frequency is divided into two groups, N' modes which fluctuate in phase, and $N - N'$

modes which fluctuate in antiphase to N' . Clusters similar to those found in JJ's have been predicted [55, 63] and observed [37, 51, 53, 54, 64, 65] in other lasers and in other arrays [5]. For studying antiphase dynamics within a laser excited by noise, a common method is to measure the power spectrum of the intensity fluctuations [38, 49, 66]. Because there is no phase information, the phase clusters cannot be determined without resorting to analytic theoretical methods [50, 51, 67].

One approach which determined the phase clustering directly is the transfer function technique [53]. The relaxation oscillations of a Nd:YAG laser were excited by modulation of the pump power with a swept frequency. Knowing the magnitude and phase of the excitation, and the complex response of the array, the authors calculated the phase clusters and also extracted the contributions of the cavity modes to the collective modes, i.e. the quantitative response of the array to excitation. In Chap. 3, we develop the cross spectrum technique to measure the antiphase dynamics of a laser excited by ambient noise. The cross spectrum is the Fourier transform of one variable, multiplied by the complex conjugate of the Fourier transform of a second variable. We determine the phase clusters, at the collective mode frequencies, for the laser operating with two or three cavity modes. We also calculate the contributions of the cavity modes to the collective modes for three mode operation and discuss how our technique might be applied to other oscillator arrays, for example high power diode lasers. The cross spectrum is also applied to four and five mode operation, the phase clusters being given in Chap. 5.

Whilst the power spectrum of intensity fluctuations cannot be used to

determine phase clusters directly, they are nevertheless a useful test of theoretical models used for lasers. Simple quantitative expressions describing the relationship between the power spectral densities of the modal intensities and the power spectral density of the total intensity, at the collective mode frequencies, have been derived analytically from modal rate equation theory for two [68] and three [69] mode operation of the laser. One necessary condition in the derivation of these relationships is that the lifetime of the laser cavity (inverse of the decay rate) is much smaller than the lifetime of the population inversion, i.e. a Class B laser. Experiments with a $\text{LiNdP}_4\text{O}_{12}$ laser (Lithium Neodymium TetraPhosphate - known as LNP) where these lifetimes differ by a factor of about 10^6 have confirmed these relations [68, 69] for two [50] and three mode operation [51]. In Chap. 4, we show that these relationships are satisfied in our laser, operating with two or three cavity modes, which has a smaller ratio of lifetimes than that of a LNP laser. Using a model similar to that outlined in this chapter, we also develop relationships to describe the phase clustering we measure.

Some recent studies of multimode Class B lasers concentrate on their behaviour when operated near the threshold of a cavity mode [70, 71, 54, 72, 73, 74, 75]. The authors independently showed that competition for gain between the laser cavity modes, profoundly affects their relative intensity and the relative height of the collective mode resonances in their intensity power spectra. Little or no effect was found on the total intensity or its power spectra however [71, 54, 72, 73]. It has been shown in both a Nd:YAG laser [71, 54, 72] and a LNP laser [70, 73], that the intensity of cavity modes with a high threshold pump power could eclipse the intensity of those with a lower

threshold pump power. In Chap. 6, we show that statistical measures of the intensity fluctuations of a cavity mode appear to display similar characteristics. We also apply the technique we developed in Chap. 3, to the laser operating at the threshold of a particular cavity mode. In so doing, we gain a better understanding of the antiphase dynamics at the operating conditions. We interpret our results in terms of both coupled nonlinear oscillators, and the competition for gain between cavity modes. Implications on the models used to describe the behaviour of solid state lasers are also discussed.

Chapter 2

Experimental Setup

This chapter describes the generic setup of the experiment. Any deviations from this are described within the individual chapters in which they occur.

We used Nd:YAG (Nd doped $\approx 1\%$, Yttrium Aluminium Garnet) as the gain medium in our Fabry-Perot cavity laser. The Nd:YAG crystal is a rod of 10mm length and 3mm diameter. It is perpendicularly cut at one end and Brewster cut at the other to force linear polarisation, see Fig. 2.1. This ensures that the laser cannot oscillate between two polarisation states as in other lasers [55, 76]. We therefore only consider intensity fluctuations. The perpendicular face of the rod forms one end of the cavity being highly reflective ($R \approx 99.9\%$) at 1064nm and partially transmitting ($T \approx 70\%$) at 808nm. The light from a 40mW diode laser (Sharp LT017MB) operating at 808nm, was focussed into the rod at this face, i.e. end pumping.

One way of controlling laser diode instabilities is to extend the cavity length by feeding back some of the light with an external mirror [77]. Another is to minimise reflections to the laser diode. We isolated the diode by placing a polarising beam splitter and a Fresnel rhomb between it and the

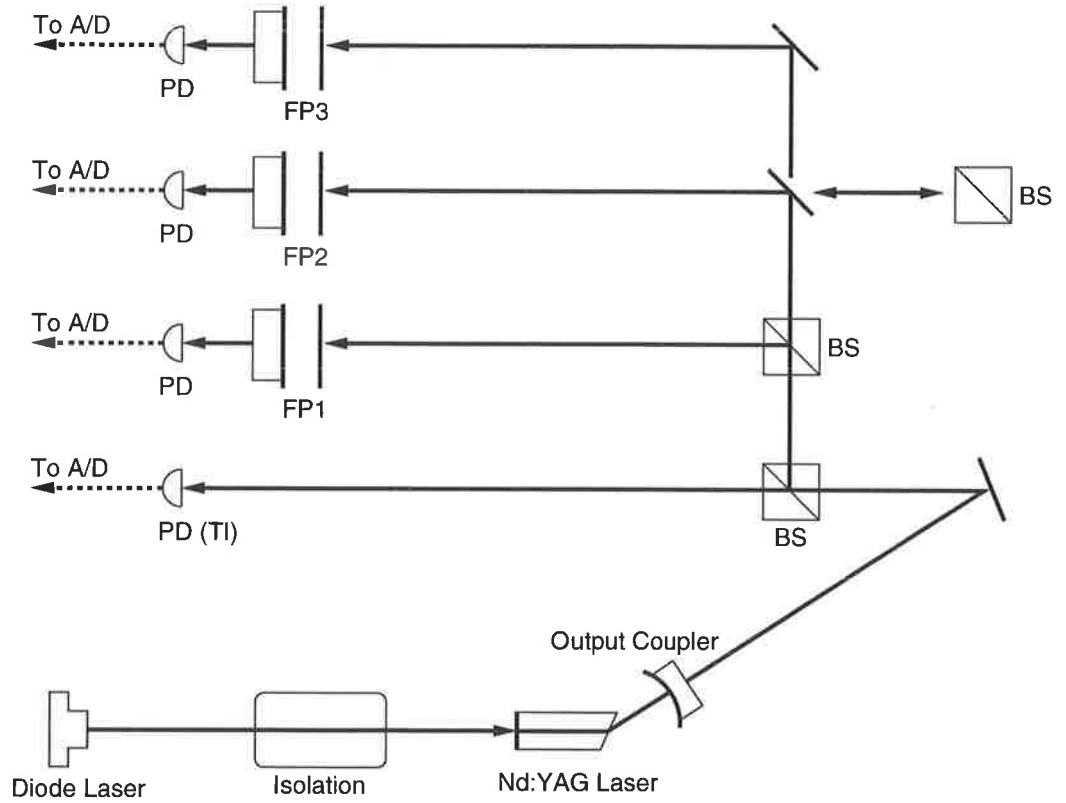


Figure 2.1: Schematic of the optical part of the experimental setup for two mode operation. A/D, Analog to Digital converter; PD, Photodiode; TI, total intensity; FP1, Fabry-Perot 1; FP2, Fabry-Perot 2; FP3, Fabry-Perot 3; BS, Beam Splitter. For three mode operation, one mirror is replaced by a BS as indicated.

YAG rod. We measured the isolation to be approximately 35dB and found this was sufficient to minimise reflections to, and therefore instabilities in the diode. We measured the isolation by reflecting a small amount of light back towards the diode laser, through the isolation optics, which was chopped at a fixed frequency. The light, which upon reflection was reversed in polarisation, was detected at the diode and at the output from the polarising beam splitter. Though some of the equipment shown in Fig. 2.1 was pre-existing, e.g. the diode laser, the isolation, the YAG laser, FP1 and a photodiode for it and the total intensity; I completely rebuilt the experiment and performed all measurements discussed in this thesis. I also manufactured four new photodetectors, added two more Fabry-Perots and beam splitters, and added the Analog to Digital converter.

The other end of the YAG laser cavity is a spherical mirror ($R \approx 98\%$ at 1064nm) with radius of curvature 2.5cm. The reflectance of the output coupling mirror was chosen for relatively low frequency relaxation oscillations rather than high output power, see for example [43, 44]. The output power of the YAG laser is of order 1mW and the beam had a good TEM₀₀ transverse intensity profile. The laser cavity lifetime (τ_c) of 2.0ns was calculated from the measured relaxation oscillation frequency of the laser in single mode operation and knowledge of the pump power [43]. The laser cavity lifetime was constant throughout the work contained in this thesis.

The optical length of the cavity is approximately 2.5cm. The laser operates on the 1064nm transition with several cavity modes, depending on the exact cavity length and pump power. We measured the optical spacing of the cavity modes, with a scanning Fabry-Perot interferometer, to be

approximately 6.15GHz. The output coupler was mounted on a piezoelectric transducer (PZT) to allow fine control of the cavity length, so that we could offset the cavity mode spectrum with respect to the gain spectrum and actively control its length with a servoloop.

By offsetting the cavity mode spectrum with respect to the gain spectrum, we could choose the relative gains of the cavity modes, see Figs. 2.2 & 2.3. We did this by increasing the pump power and viewing the optical spectrum with a scanning Fabry-Perot. We define two cavity modes as having equal relative gain in the case that they both reach threshold simultaneously. They must have the same threshold pump power, be spaced by 6.15GHz, and have the same area under their respective Fabry-Perot resonances. The latter is an important condition because it removes the effect of Fabry-Perot alignment change when operated as an optical spectrum analyser.

In three mode operation, the second and third modes to reach threshold may share the same gain. This is highlighted in Fig. 2.2, where the cavity mode spectrum is symmetric with respect to the gain spectrum. Due to the nature of the gain curve, it is impossible for the first three cavity modes to reach threshold to have equal gain. To achieve equal relative gain between the first two laser cavity modes to reach threshold, we moved the the cavity mode spectrum, with the PZT mounted on the output coupler, to the right or left of the centre of the gain curve by approximately 3.07GHz.

For the simplicity of the experiments and comparisons with other work, in this thesis we only operated the laser where the gains of the cavity modes were unequal, i.e. $1 > 2 > 3$ and so on. This is highlighted in Fig. 2.3 where the cavity mode spectrum is asymmetrically disposed about the gain

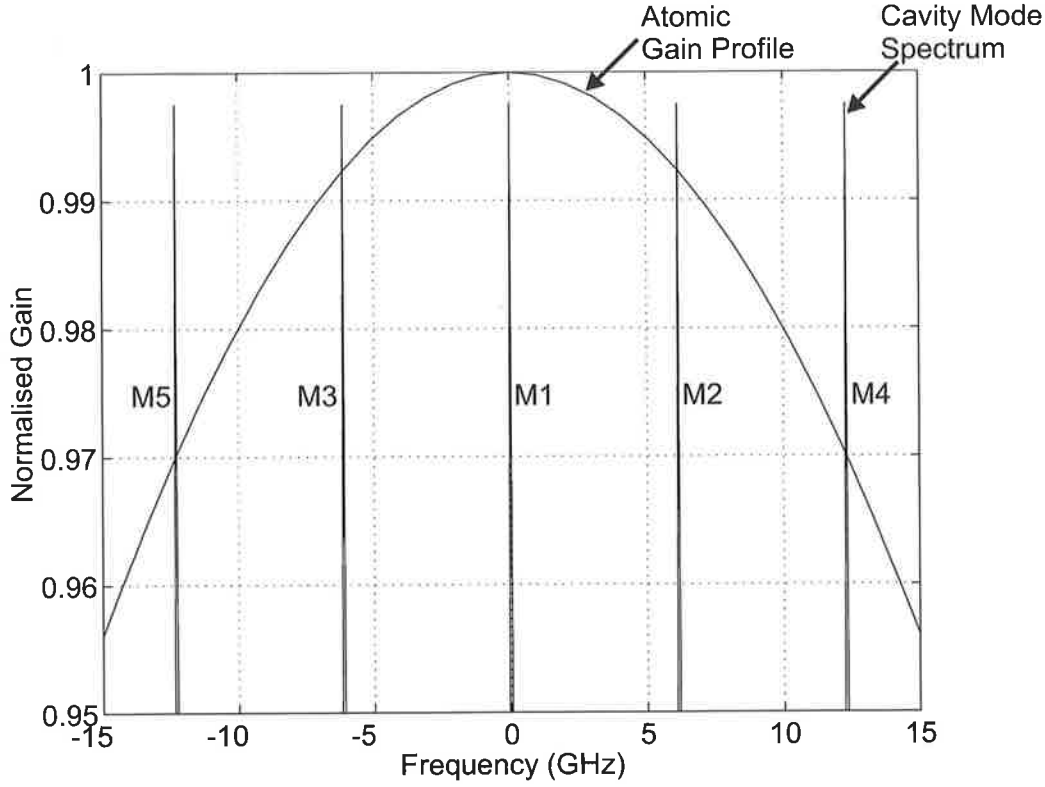


Figure 2.2: Commonly the atomic gain profile of Nd:YAG is assumed to be a symmetrical Lorentzian [15, 43, 53, 54, 72, 78]. Its full width at half maximum is commonly assumed to be in the range 120GHz [43] to 160GHz [78]. In this and the following example (Fig 2.3), we have chosen the FWHM to be approximately 140GHz.

The cavity mode spectrum is symmetrically disposed about the gain spectrum. The cavity mode with the largest gain is situated at 0GHz and is denoted M1, i.e. mode one. The other cavity modes are ordered according to their gain. In this example, cavity modes two and three share the same gain, and so on.

spectrum.

Given the separation of the cavity modes in frequency space, they were optically separated using Fabry-Perot interferometers and the total intensity detected directly, see Fig. 2.1. This enabled the individual cavity modes to be studied separately, whilst maintaining and not influencing the coupling of the array within the laser. Optical isolation with quarter wave plates and polarising beam splitters, measured to be approximately 28dB with a setup similar to that discussed previously, was used between the YAG laser and the Fabry-Perots, so that the oscillations of the collective modes were not driven by optical feedback. We found that removing the isolation between the Fabry-Perots and the YAG laser had no noticeable effect on the Fourier analysis of the intensity fluctuations. However, we left the isolation in situ. Though being outside the scope of this thesis, we note that other authors use optical feedback to drive [79] or control [59] the relaxation oscillations. For example, in the same way as one may sample part of the laser output and stabilise the laser frequency or intensity, i.e. injection locking, Khandokhin et al. have driven the relaxation oscillations of the laser [38].

The modal intensities transmitted by the Fabry-Perot interferometers and the total intensity were detected with high gain and bandwidth, low noise photodiodes. With a low noise, trans-impedance gain amplifier, the photocurrents were converted to voltages. The gain and bandwidth of each of the photodiode-amplifiers, which I constructed, were identical which ensured the same response of each detector, see Appendix C for the circuit diagram.

With the YAG rod and output coupler spaced by aluminium, the laser cavity length could change due to thermal drift and therefore the frequencies

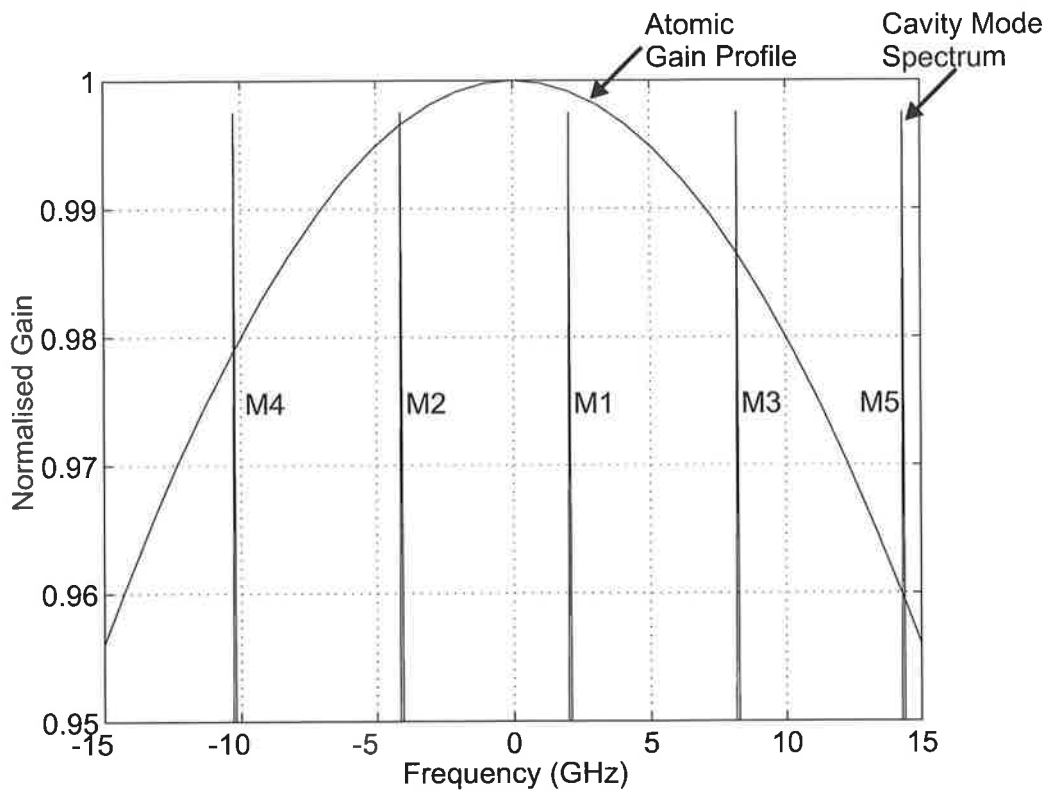


Figure 2.3: The cavity mode spectrum is asymmetrically disposed about the gain spectrum. The cavity mode with the largest gain is situated at $\approx 2.05\text{GHz}$, this is denoted as M1, i.e. mode one. The other cavity modes are ordered according to their gain, e.g. M2 (mode two) being the second largest gain, and so on.

and gains of the cavity modes. I added temperature stabilisation to the laser cavity to minimise this effect, see Appendix C for the circuit diagram. One Fabry-Perot (FP1 - TecOptics Ltd. SA-10, with a cavity lifetime of approximately 4.25ns and free spectral range 10GHz) was spaced with Invar which has a much lower coefficient of expansion than aluminium. We used it as a reference cavity to which we actively stabilised one of the laser cavity modes (the first to reach threshold) and therefore the laser cavity length. This fixed the position of the cavity modes with respect to the peak of the gain spectrum, and thus the relative gains of the modes. We did this by applying a very low level dither to the piezoelectric transducer (PZT) of FP1 at 1251Hz, see Fig. 2.4. When the dither frequency appears in the frequency domain, it is far enough away from the collective modes of the laser to be easily distinguishable, this is further discussed in Appendix A.

A lock-in amplifier generated a correction signal from the intensity of the first laser mode to reach threshold which in turn was fed back to the output coupler of the YAG laser cavity, see Fig. 2.4. Because the thermal perturbation of the laser cavity length was continuous, proportional and integral gain was applied to the correction signal to negate the perturbation. The reasons for this are given in Appendix A. The other Fabry-Perot (FP2, cavity lifetime of approximately 230ps, borrowed from within our research group) was also dithered and, with a secondary feedback loop, locked to the cavity mode it transmitted (i.e. the second to reach threshold). The Fabry-Perots had a lifetime shorter than the characteristic timescale of the highest collective mode frequency ($\sim 10\mu\text{s}$), i.e. they act as a low pass filter. We therefore conclude that that Fabry-Perots do not add to, nor alter the fluctuations we

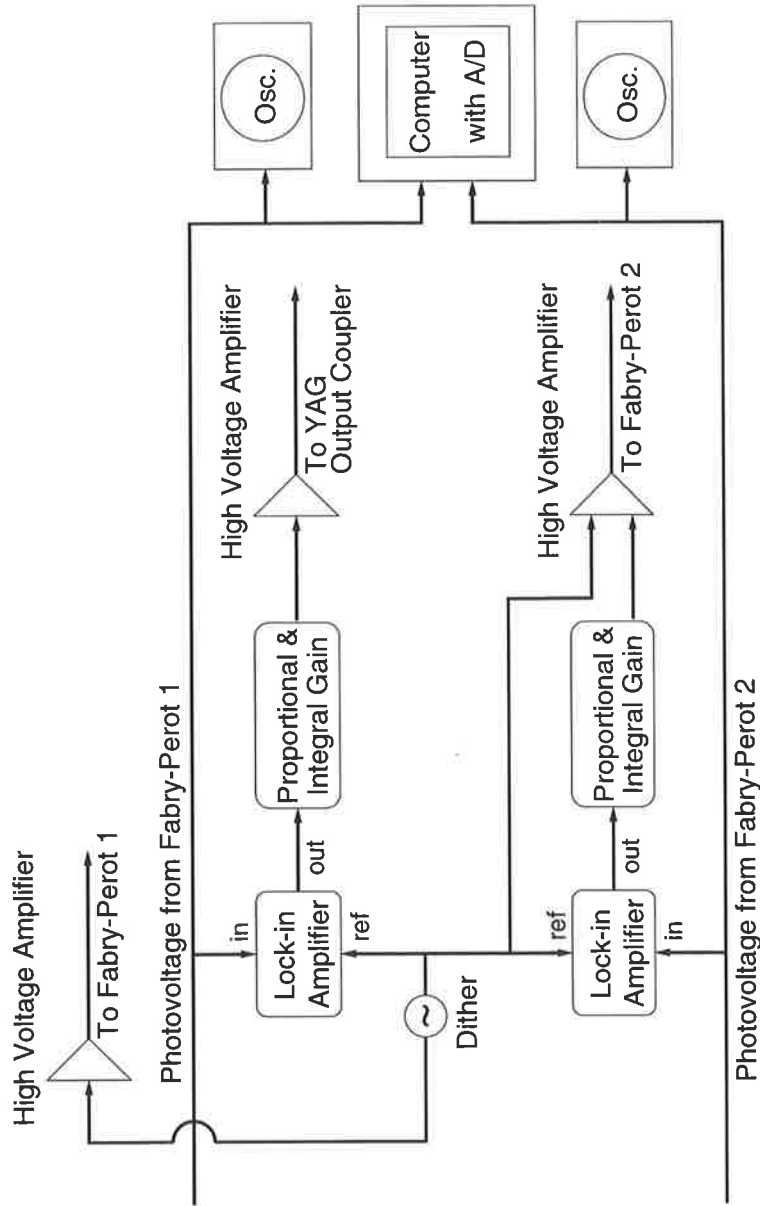


Figure 2.4: Schematic of the electrical part of the experimental setup. Osc., oscilloscope; For the lock-in amplifier: in refers to the input; out the output; and ref, the reference frequency. The oscilloscopes were used to monitor the intensities passed by each Fabry-Perot and to ensure each servoloop remained in lock.

measure.

The total and modal intensity fluctuations were recorded with two, 12 bit, 2 channel analog to digital (A/D) converters (Gage Applied Sciences Inc., CompuScope 512) mounted in separate PC's.¹ Each of the input channels recorded 2^{19} points at a sample rate of 1MS/s (megasample/second). The number of recorded points gave an adequate amount of data for averaging the background noise level in the frequency domain. The sampling interval was much shorter than the characteristic timescale of the collective modes which ensured that their frequencies were recorded accurately. Once the appropriate input range of the A/D was selected, we applied extra gain to the signal from the photodiode-amplifiers. This maximised the signal to background noise ratio (SNR) and resolution (i.e. significant number of bits) of the data. Appendix B discusses this process in more detail.

With up to four detection channels, each modal intensity and the total intensity could be sampled simultaneously, for operation with up to three cavity modes. The combination of active stabilisation of the laser cavity and simultaneous sampling of all the intensities minimised the effect of the thermal drift of the Nd:YAG laser cavity by keeping the time scale of the measurement short, approximately 525ms. These factors ensured that the relative gains seen by each of the cavity modes remained constant during the experiment such that any subsequent quantitative analysis was meaningful.

To make accurate calculations of the spectra and for comparing our data quantitatively with other results [50, 51], it was important to determine the scaling of each optical channel because each photodiode-amplifier had a different power of light impinging upon it. Each time an experiment was per-

¹The A/D's were triggered using an external trigger.

formed, the laser was started in single mode operation, and the intensity fluctuations recorded, i.e. all the channels were receiving the same information. The pump power was increased to the desired level for multimode operation, and the modal and total intensities recorded.

In the post acquisition processing, the standard deviation of the intensity noise was calculated from the single mode operation data for each detection channel. Using this information, the intensity fluctuations at the multimode operating point of interest were normalised to one of the channels. The last step involved transforming the normalised intensity noise data into the frequency domain, e.g. the power spectrum, using the modified periodogram method [80]. The benefits of this method compared with the standard Fast Fourier transform (FFt) are outlined in Appendix B.

When measuring the contributions of the cavity modes to the collective modes or testing relationships between the power spectral densities at the collective mode frequencies for three mode operation, the system was extended by replacing one mirror with a beam splitter and adding a third Fabry-Perot, see Fig. 2.1. No isolation was used between this Fabry-Perot and the YAG laser as we had found that it was not necessary. However, having four channels ensured that each of the modal intensities and the total intensity were sampled simultaneously. FP3 (cavity lifetime of approximately 480ps, borrowed from within our research group) transmitted the third laser cavity mode to reach threshold. In two mode operation, FP2 was locked to the laser mode that it transmitted, i.e. the second to reach threshold. The drift of FP2 without stabilisation during the time required to acquire the data ($\approx 525\text{ms}$) proved to be negligible. Therefore, for three mode operation

we did not lock FP2 or FP3 to the cavity modes that they transmitted. The photodiode-amplifier circuit used to detect the intensity transmitted by FP3 was identical to those used previously. After further gain, this signal was fed into the fourth available channel of the A/D's. Post processing of the intensity fluctuations was performed in the same manner as the two mode case.

Chapter 3

The Cross Spectral Technique

3.1 Development Of The Technique

In their studies of the antiphase dynamics in an array of JJ's, Hadley et al. [6] reported that the phases of the interferences of the individual oscillators, form clusters at each collective mode frequency. In a laser operating with N cavity modes, the clustering of the phases, at each collective mode frequency, is divided into two groups: N' modes which fluctuate in phase, and $N - N'$ modes which fluctuate in antiphase to N' . Phase clusters have been predicted [55, 63] and observed [37, 51, 53, 54, 64, 65] in lasers and have shown to be similar to those found in JJ's.

In a laser excited by noise, commonly the power spectrum of intensity fluctuations is used for studying antiphase dynamics. Because the power spectrum is an autocorrelation function, it has no phase information, and therefore cannot verify phase clustering directly. Some studies reporting clustering have resorted to analytic theoretical methods [67, 68, 69], or using the intensity traces to obtain such information [37, 49, 63, 79, 81]. Formulae

which calculate the contribution of the cavity modes to the collective modes, using the power spectrum, have also been derived analytically [68, 69] and confirmed experimentally [50, 51].

The response of a solid state laser to swept frequency has also been measured [53, 54]. The transfer function technique in its full form is a complex valued quantity. Knowing the magnitude and phase of the excitation and the complex response of the array, Stamatescu and Hamilton [53] measured the phase clustering and extracted the contributions of the cavity modes to the collective modes.

We desire a method to determine the clustering of phases, at the collective mode frequencies, and to measure the contributions of the constituent oscillators to the collective modes, for a laser excited by noise. Because we do not know the input excitation to the array, we choose the cross spectrum technique. The cross spectrum is the Fourier transform of one variable, multiplied by the complex conjugate of the Fourier transform of a second variable, see Eq. 4.7. It is a complex valued quantity, therefore a magnitude and phase can be calculated from the real and imaginary parts.

The intensity fluctuations of pairs of cavity modes and cavity modes paired with the total intensity were sampled simultaneously, see Fig. 2.1. We calculate the cross spectrum of these pairings, for two and three mode operation, to study the antiphase dynamics of the laser. Resonances at the collective mode frequencies are seen in the magnitude of the cross spectrum, zeros or antiresonances correspond to jumps in the phase. The relative phase at the collective mode frequencies is used to deduce whether the fluctuations of the intensity, for the pairing under consideration, are in, or out of phase.

By calculating the cross spectrum for all possible pairings, we build up a picture of the interference of intensity fluctuations of the array. For three mode operation we quantitatively measure the response of the array to ambient noise, i.e. the contributions of the cavity modes to the collective modes. A preliminary version of this work has been published [82] and is included in Appendix D.

3.2 Two Mode Operation

Figure 3.1 shows the cross spectrum between the pair of cavity modes for two mode operation. We find two collective mode resonances, in accordance with Khandokhin et al. [60, 61], at 7.5kHz and 36.25kHz in the magnitude of the cross spectrum. Given that the highest frequency collective mode is similar to that found in single mode operation [9, 62], we label this frequency Ω_1 . The lower collective mode frequency is labeled Ω_2 . There also appears a resonance at 72.5kHz, this is the second harmonic of Ω_1 . This frequency occurs in each of the spectra calculated in this thesis and is generally 10-15dB above the surrounding noise. We do not show this resonance in any subsequent spectra. Zeros occur at 4.75kHz and 13.5kHz and are associated with phase jumps of 180° . There also appears a peak at ≈ 0.75 kHz. This low frequency noise could be from a variety of sources, including for example mains harmonics.

In the phase of the cross spectrum, each collective mode frequency has a definite value; that is, it does not jump. Signals from switch mode power supplies are evident at approximately 32kHz, 44kHz and 64kHz.¹ We know

¹We discuss the low frequency noise peak and the noise from the switch mode power

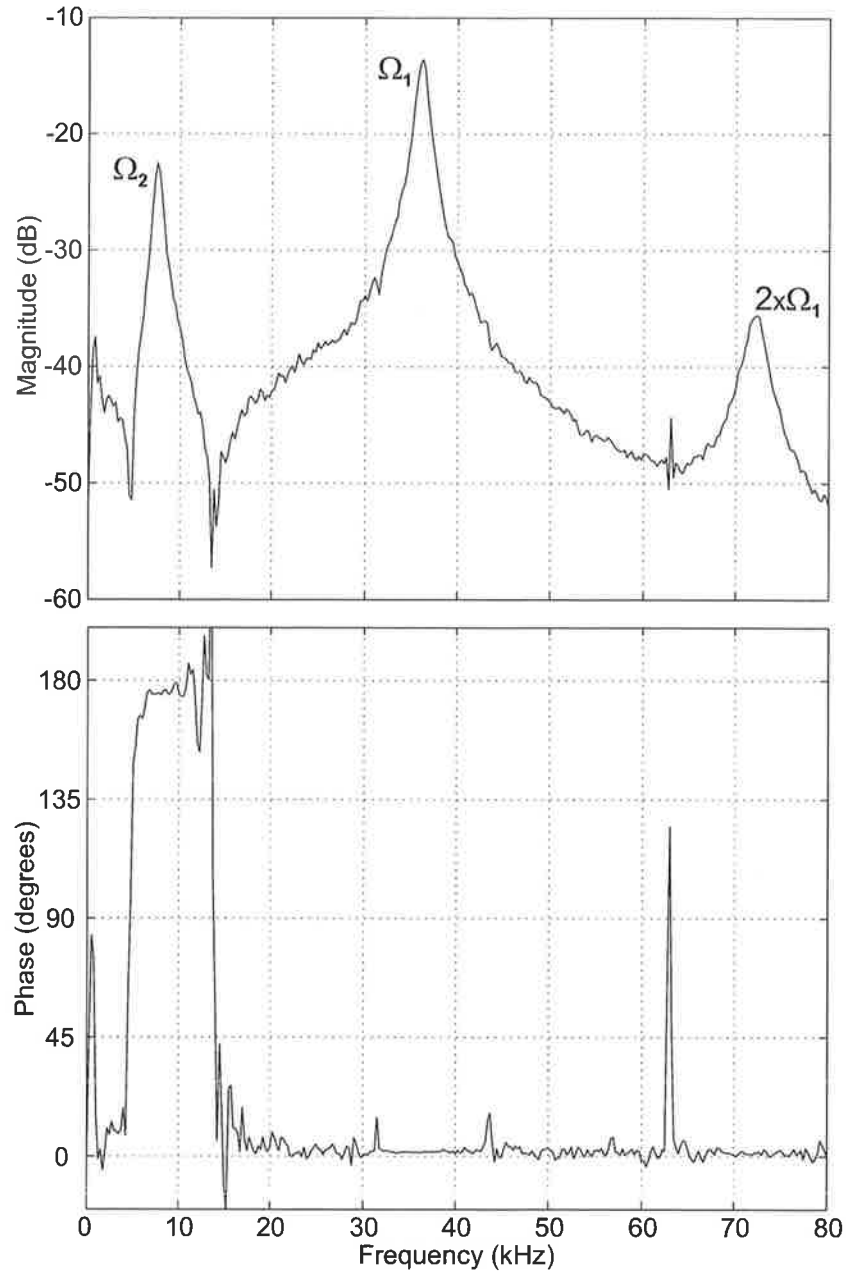


Figure 3.1: The cross spectrum of the intensity noise fluctuations between cavity modes one and two. The collective mode frequencies are; 7.5kHz and 36.25kHz. In this thesis we use a logarithmic scale, relative to unity, to highlight all frequency components in the magnitude of the cross spectrum or the power spectrum. Reasons for this are given at the end of Appendix B.

that the intensity fluctuations of all the laser modes contribute in phase (constructive interference) to the highest frequency collective mode because this is the mode which survives in the total intensity [7, 8, 50]. The phase of the cross spectrum is set so that it is approximately zero degrees at the highest frequency collective mode, Ω_1 , this is defined as the “in phase” condition. We checked this condition independently by making each of the Fabry-Perots resonant with the same laser cavity mode. The resultant cross spectral phase was flat across the frequency spectrum at zero degrees.

The number of 180° phase jumps between the highest frequency collective mode and any other resonance, tells us whether the intensity fluctuations, for the pairing under consideration, constructively or destructively interfere at the other resonances. By definition, the intensity noise fluctuations are in phase at Ω_1 . In this example, the phase of the intensity fluctuations at Ω_2 , are diametrically opposed to one another, i.e. antiphase. That the size of the phase jumps is a multiple of 180° , implies that we are seeing simple antiphase states. The simple antiphase states are consistent with other theoretical results, e.g. [50, 51, 55, 63, 67, 68, 69, 83] and experimental data for lasers, e.g. [37, 38, 49, 51, 53, 54, 64, 65, 66].

Figure 3.2 shows the cross spectrum of cavity mode one and the total intensity. In the magnitude, there appears a weak resonance at Ω_2 , approximately 2-3dB above the surrounding noise. There appears to be a zero to the left of the resonance however, it is associated with a sharp positive slope. The phase at Ω_2 is approximately 100° which is not a simple antiphase state, i.e. it does not appear to be directly in or out of phase. Although there

supplies in Appendix A. We found that neither of these pertained to the laser intensity fluctuations.

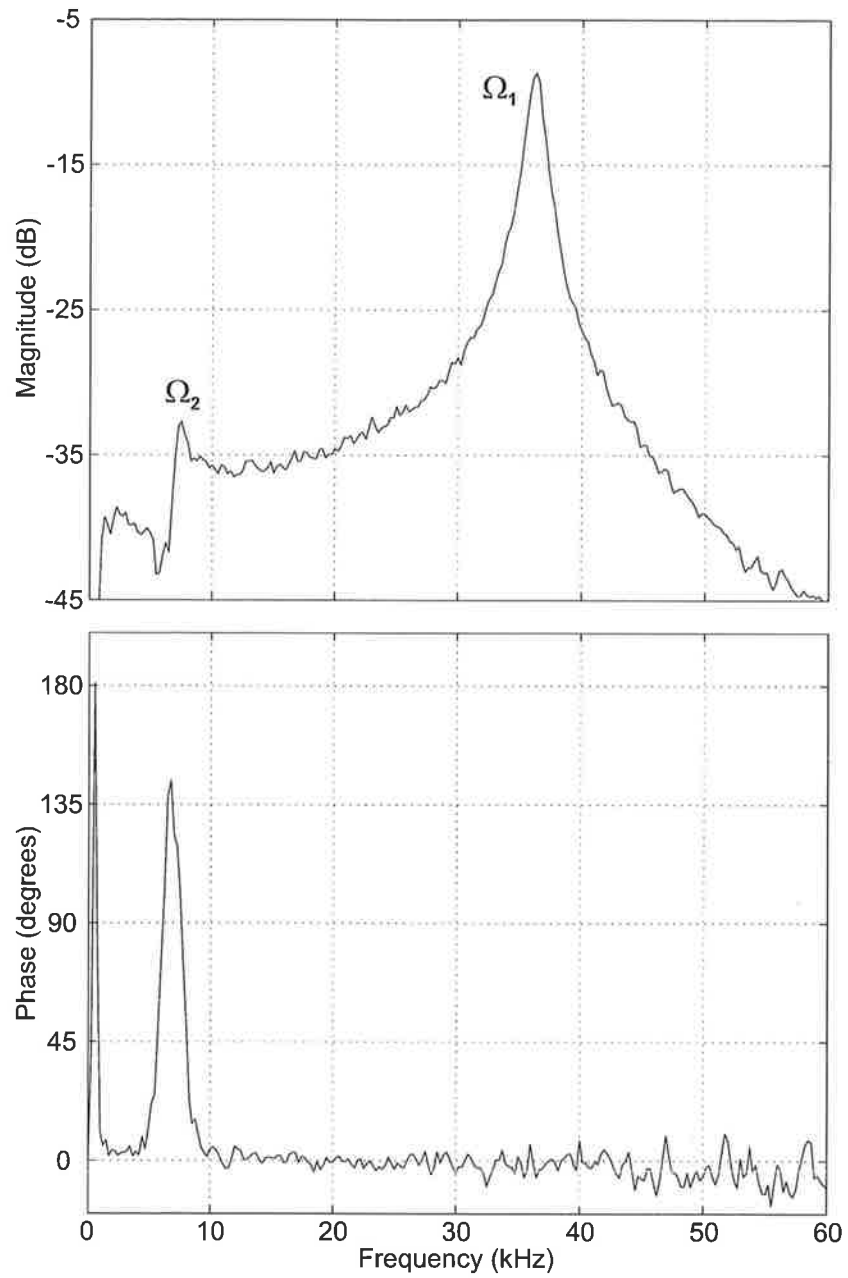


Figure 3.2: The cross spectrum of intensity noise fluctuations between cavity mode one and the total intensity, for two mode operation. The collective mode frequencies are; 7.5kHz and 36.25kHz.

is a superposed phase shift of $\approx 20^\circ/100\text{kHz}$, due to a slight timing difference² between the capture of the cavity modes and the total intensity, the fluctuations at Ω_1 are in phase.

The cross spectrum between cavity mode two and the total intensity is shown in Fig. 3.3. The intensity noise fluctuations for this pairing, at Ω_2 , appear to be opposite to the previous pairing. On the right hand side of Ω_2 , there appears to be a zero at 10.75kHz, although it is not associated with a 180° phase jump. The phase appears to “roll off” at Ω_2 such that the fluctuations are approximately -45° out of phase. Whilst the superposed phase shift is also evident in this spectra, the intensity noise fluctuations at Ω_1 are in phase. We found the phase of the fluctuations for each of the spectra to be repeatable to within $\pm 5^\circ$ for different pump parameters and positions of the cavity mode spectrum with respect to the gain spectrum.

We have shown that the interference of the intensity fluctuations between the cavity modes at Ω_2 is destructive. This is not altogether surprising given the limited range of possibilities for the phase of the fluctuations. The relative difference between the fluctuations of the two pairings of cavity modes and the total intensity (at Ω_2) appear to be $\approx 145^\circ$ out of phase. This is close to antiphase. We analyse this result using the complex plane. Given that we do not have a reference phase for the excitation of the relaxation oscillations, we assign the following complex variables to the intensity fluctuations of the cavity modes. We choose the phase of the fluctuations to be real in a first approximation, the reasons for this become evident when we discuss the

²Whilst the two A/D's were triggered simultaneously, they were housed in separate PC's. Therefore the CPU clock signals (to start recording) were out of synchronisation by a small margin, of order 100ns.

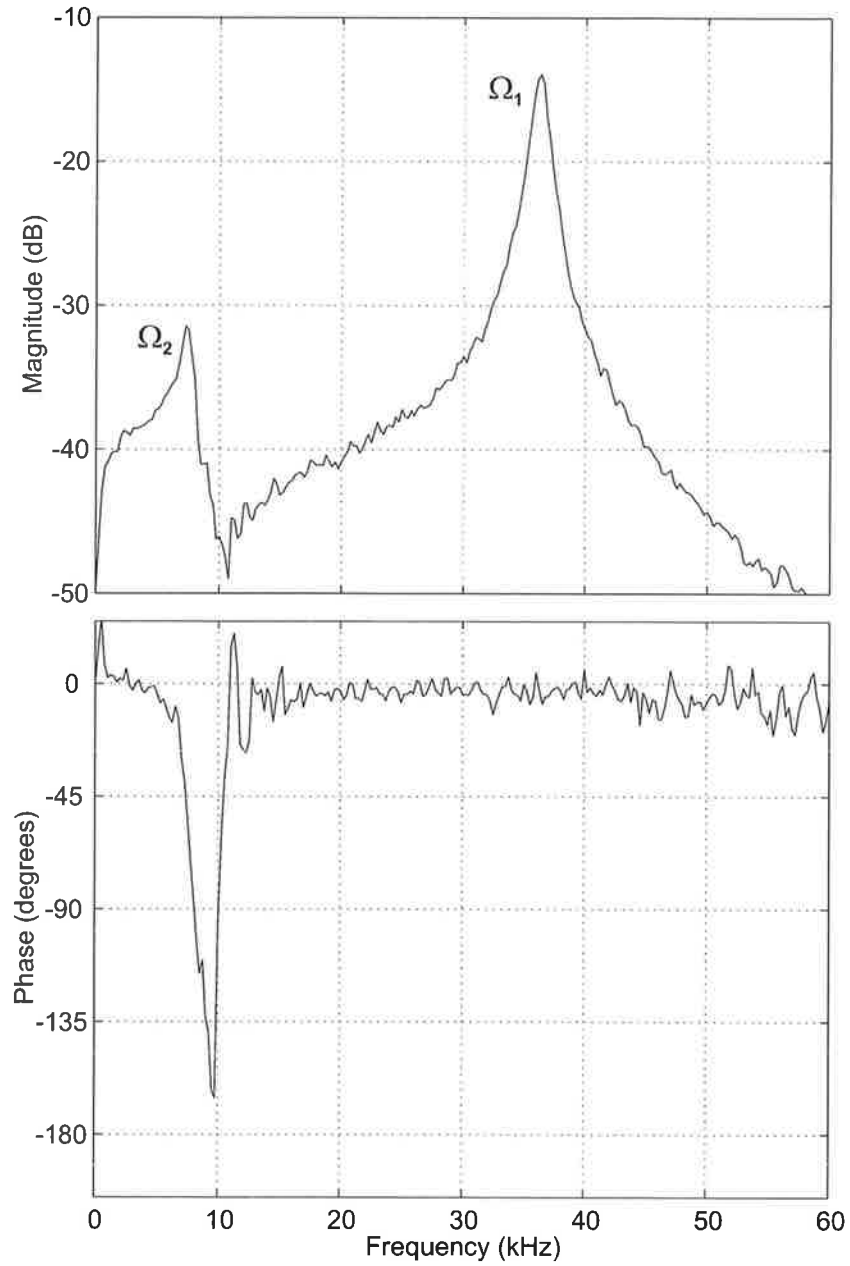


Figure 3.3: The cross spectrum of intensity noise fluctuations between cavity mode two and the total intensity, for two mode operation. The collective mode frequencies are; 7.5kHz and 36.25kHz.

model used to describe the laser in Chap. 4. We also choose the phase of the fluctuations to be approximately in antiphase, i.e. 180° out of phase at Ω_2 , as in Fig. 3.1.

$$\text{Mode One : } +1 \quad (3.1)$$

$$\text{Mode Two : } -1 + \psi * (i)$$

where $i = \sqrt{-1}$ and $\psi \ll 1$. The latter is used to represent a small uncertainty of the phase of the fluctuations between the cavity modes. The oscillation of the total intensity fluctuations is the summation of the fluctuations for cavity modes one and two, i.e.

$$+1 + (-1 + \psi * i) = \psi * i. \quad (3.2)$$

The fluctuations of the total intensity at Ω_2 is therefore very small. The fluctuations are approximately 90° out of phase with either of the modal intensity fluctuations, the latter being demonstrated in Figs. 3.2 & 3.3. This means that antiphase dynamics destroys nearly all of the fluctuations at Ω_2 . Therefore the cross spectral method shows the destructive interference explicitly at Ω_2 for two mode operation of our laser. Conversely, at Ω_1 in the complex plane, the fact that the fluctuations in each pairing are in phase means constructive interference occurs at this frequency. Therefore a resonance in the power spectrum of the total intensity should occur at this frequency. We confirm this, and the preceding discussion about the fluctuations at Ω_2 , in Fig. 3.4. The observed peak located at Ω_1 has a full width at half maximum (FWHM) of approximately 1.2kHz. There does not appear to be a peak at Ω_2 . This indicates that this collective mode is completely antiphased or is

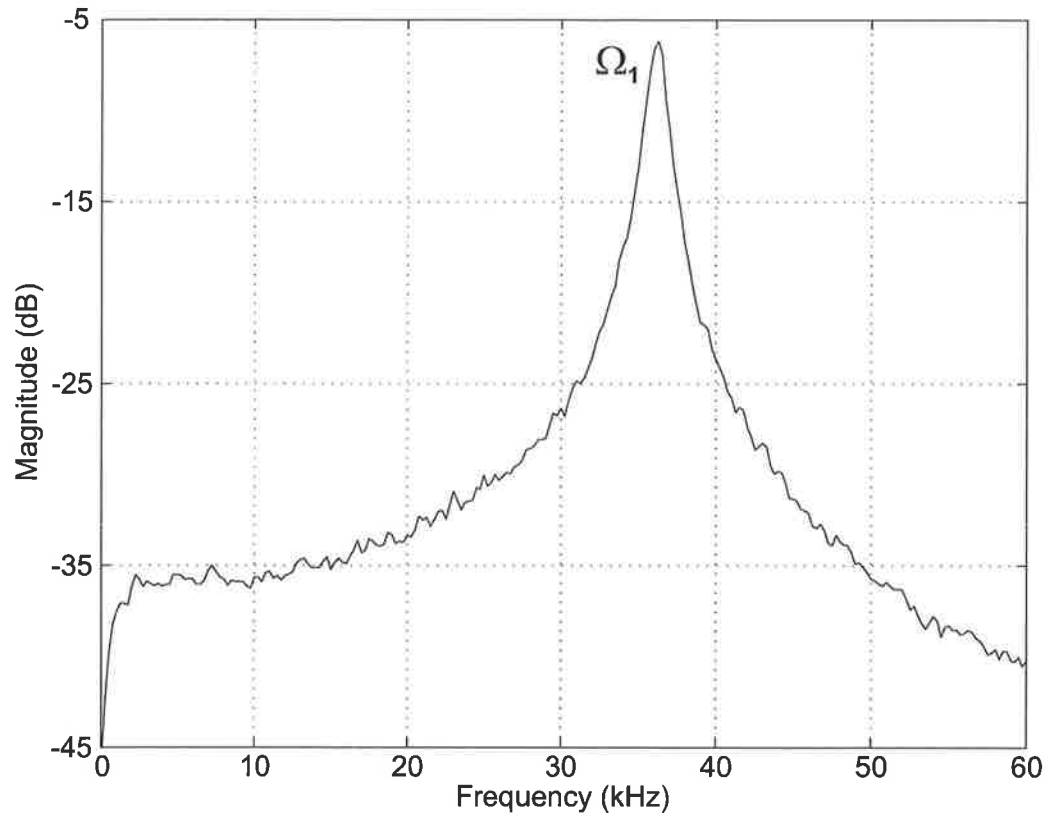


Figure 3.4: The power spectrum of the total intensity noise fluctuations for two mode operation. The collective mode frequencies are; 7.5kHz and 36.25kHz.

covered by the superposition of background noise, which we have minimised in our experimental setup, and the low frequency wing of the Ω_1 peak. For two modes receiving equal gain, it has been predicted that the destructive interference at the low frequency collective mode should be complete [47]. Therefore, in the power spectrum of the total intensity noise fluctuations, this resonance should not be visible. Conversely, when two cavity modes receive unequal gain, as in the case of our experiment, it has been predicted that the antiphasing at the low frequency collective mode should be incomplete and a vestige of Ω_2 survive in the total intensity noise fluctuations [47]. A similar prediction has been made and confirmed for three mode operation of a LNP laser [51]. This shows that a strong antiphase dynamics kills the oscillation of the low frequency collective mode.

When a measured value which is close to zero or the background noise, as is the case at Ω_2 in the total intensity fluctuations, is convolved with a small oscillation at the same frequency, for example the modal intensity fluctuations, the result is a very small signal to noise ratio. This explains the small resonance and the intermediate phase states found in Figs. 3.2 & 3.3. For two mode operation, we discuss the strength of the destructive interference quantitatively in Chap. 4 using relations between the power spectrum of the modal and total intensity fluctuations, at the collective mode frequencies. Reasons for not doing this using the cross spectrum are given in Sec. 3.3.3.

The existence of a resonance at $2 \times \Omega_1$, consistently in the spectra, is also interesting. In Fig. 3.1, this resonance was approximately 100 times (20dB) less than the peak of Ω_1 . It has been predicted [67] from a nonlinear

analysis of the Tang, Statz and deMars equations (TSD) [84] that Fabry-Perot cavity lasers like ours, may exhibit harmonic and mixing (sum and difference) frequency resonances of the collective modes. Given that we only found one such resonance, and that Ω_2 was almost completely antiphased in the power spectrum of the total intensity fluctuations, we conclude that nearly all the energy of the array is concentrated in Ω_1 . These findings are indeed signatures that the excitation of the array is close to linear [7]. This becomes important in Chapters 4 & 5 where we compare our results quantitatively with modal rate equation theory and qualitatively with data obtained from other solid state lasers, respectively.

3.3 Three Mode Operation

We apply the cross spectral technique to three mode operation of the laser and build up a picture of the interference of the intensity fluctuations of the array. We also measure the contributions of the constituent oscillators of the array to the collective modes.

3.3.1 Cavity Mode Pairs

We calculate the cross spectra for the three pairs of cavity modes. As an example, we show the pairing of cavity modes two and three in Fig. 3.5. In the cross spectrum magnitude, we find three collective mode resonances labeled Ω_3 , Ω_2 and Ω_1 , in order of increasing frequency. Zeros associated with 180° phase jumps occur at 10kHz and 21.5kHz. The low frequency noise peak at ≈ 0.5 kHz could be due to a variety sources including mains harmonics. As

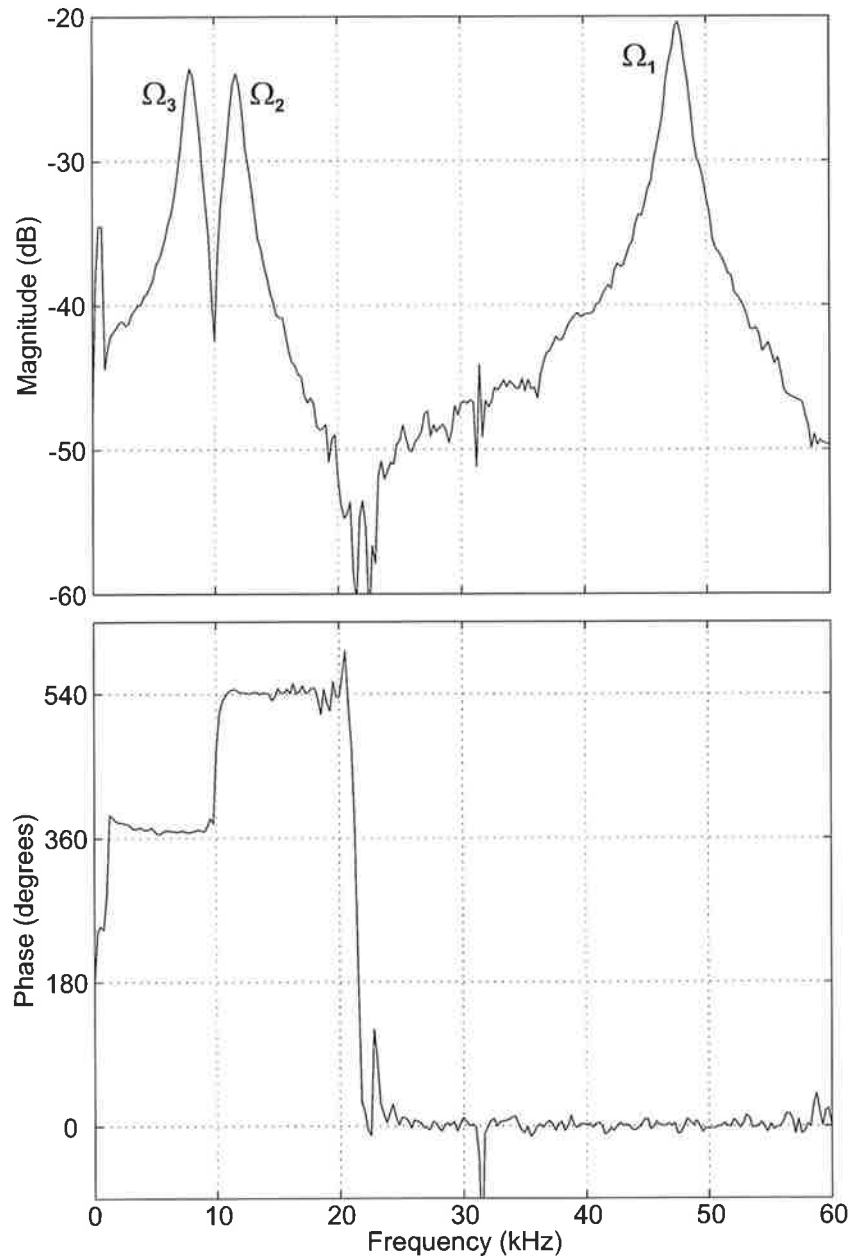


Figure 3.5: The cross spectrum of intensity noise fluctuations between cavity modes two and three, for three mode operation. The collective mode frequencies are; 8kHz, 11.75kHz and 47.75kHz. Electronic pickup from switch mode power supplies is visible at 31.5kHz.

shown in Appendix A, the low frequency components of the spectra, e.g. mains harmonics, the dither frequency (1251Hz) or its harmonics, do not pertain to the intensity fluctuations of interest. Rather, they are due to the equipment used in the experiment.

The phase of the fluctuations at Ω_2 is 540° . That it is 540° and not 180° is due to the algorithm used to calculate the direction of the phase jumps rather than any physical significance. Therefore, the intensity noise fluctuations are out of phase at Ω_2 and in phase at Ω_3 & Ω_1 . The cross spectra for each of the other pairs show peaks at the same collective mode frequencies as in Fig. 3.5; however, the heights of the peaks and their relative phases differ. Table 3.1 shows the pattern of interferences thus determined. The cavity modes are labeled with Arabic numerals in order of increasing pump power threshold. The interferences are labeled “+” for in phase dynamics and “-” for antiphase dynamics. That the size of the phase jumps is a multiple of 180° , implies that we are seeing simple antiphase states and not splay phase states [6]. For example, this may mean 120° between the intensity fluctuations of each of the cavity modes. The simple antiphase states are consistent with other theoretical results, e.g. [50, 51, 55, 63, 67, 68, 69, 83] and experimental data for lasers, e.g. [37, 38, 49, 51, 53, 54, 64, 65, 66].

3.3.2 Cavity Modes Paired With The Total Intensity

We calculate the cross spectra for the three pairs of cavity modes and the total intensity. As an example, Fig. 3.6 shows cavity mode two paired with the total intensity. In the magnitude, we find peaks at Ω_3 and Ω_1 . There also appears to be a weak resonance at Ω_2 . There appears to be

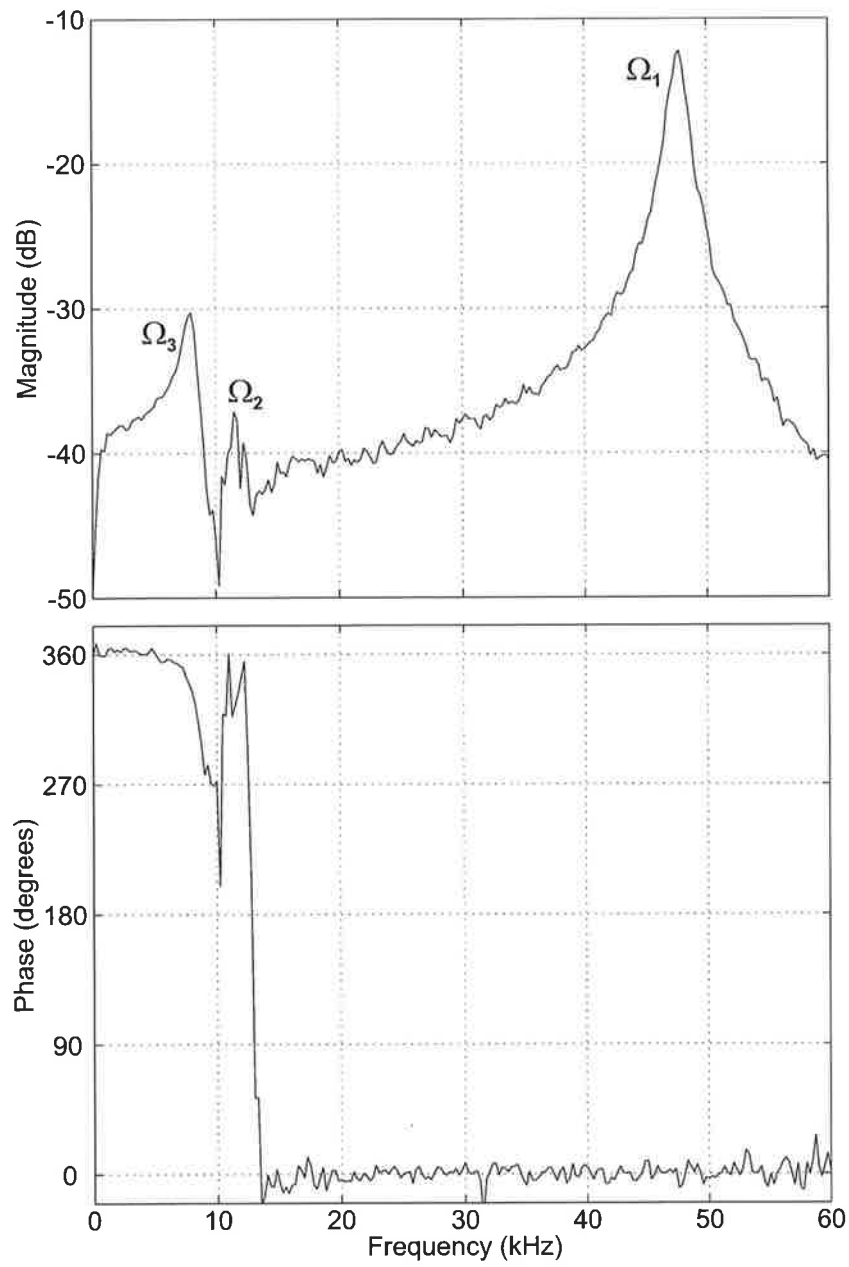


Figure 3.6: The cross spectrum of intensity noise fluctuations between cavity mode two and total intensity, for three mode operation. The collective mode frequencies are; 8kHz, 11.75kHz and 47.75kHz.

	1,2	1,3	2,3
Ω_3	-	-	+
Ω_2	-	+	-
Ω_1	+	+	+

Table 3.1: The phase pattern, expressed in terms of cavity mode pairs, for three mode operation. The columns correspond to the laser cavity mode pairs and the rows to the collective modes, where Ω_1 is the highest frequency.

zeros at approximately 10.25kHz and 13kHz. The phase of the intensity noise fluctuations at Ω_1 are in phase. The fluctuations at Ω_3 and Ω_2 are approximately 335° (-25° in relative terms), which is close to in phase. The cross spectra of the other pairs show similar features to Fig. 3.6; however, the heights of the peaks and their relative phases differ.

At the low frequency collective modes, the signal to noise ratio is low in each of the cross spectra because the total intensity oscillation is small at these frequencies, see Fig. 3.7. Given the result for the two mode case and that the phase of the fluctuations in these spectra were commonly $<45^\circ$ from a distinct phase state, we assumed that if the fluctuations were $> \pm 90^\circ$ (in relative terms), they were in antiphase. Conversely, if the fluctuations were $< \pm 90^\circ$ (in relative terms), they were deemed to be in phase. Table 3.2 shows the pattern of interferences thus determined, we use the same labeling conventions as Table 3.1.

	1,TI	2,TI	3,TI
Ω_3	-	+	+
Ω_2	+	+	-
Ω_1	+	+	+

Table 3.2: The phase pattern, expressed in terms of individual cavity modes paired with the total intensity (TI), for three mode operation. The columns correspond to the pairings and the rows to the collective modes.

3.3.3 Contribution Of The Cavity Modes To The Collective Modes

We use the phase patterns of Tables 3.1 & 3.2, to form phase clusters at the collective mode frequencies, i.e. the phases of the contributions of the cavity modes to the collective modes.

Unlike a transfer function which has a known excitation applied to the nonlinear oscillator array [53], we do not have a reference signal and therefore a reference phase. However, for two mode operation of our laser, we know that Ω_1 survives in the total intensity fluctuations, as it does in other lasers [8, 29, 38, 50, 53, 85]. Therefore we label the contributions of the cavity modes to be in phase (“+”) at this frequency. At the other collective mode frequencies, we define the contributions of cavity mode one to be in phase, the other phases being relative to this. For example, the phase of the contribution of cavity mode two at a given collective mode frequency, can be determined from the phase of the cross spectrum between modes one and two. Similarly the phase of the contributions of cavity mode three can be determined from

	1	2	3
Ω_3	+	-	-
Ω_2	+	-	+
Ω_1	+	+	+

Table 3.3: The group of clusters for three mode operation, where Ω_1 is the highest frequency collective mode.

the cross spectrum phase between modes one and three. We present the phase clusters in Table 3.3. The extra pairing of cavity modes, i.e. two and three, allows the consistency of the derivation of the phase clusters to be checked. In this case, the relative phases at the collective mode frequencies were found to be consistent.

As an independent check of the consistency of the phase clusters, we compare them with the phase pattern obtained for cavity modes paired with the total intensity, see Table 3.2. For example, at Ω_2 the fluctuations of cavity mode one and the total intensity are approximately in phase, similarly for cavity mode two and the total intensity. However, we have shown in the modal intensity fluctuations that they are in antiphase at this frequency, this is inconsistent. This is likely due to the low signal to noise ratio at the low frequency collective modes in the cross spectra between cavity modes and the total intensity. Therefore we cannot use this pairing as an independent check of the consistency of the phase clusters. Nevertheless, the phase clusters do not show a simple ordering of the phases at the collective mode frequencies as predicted in Josephson junctions [6] or other lasers [51, 55], or measured in a LNP laser [51]. If our laser were to follow a simple ordering, the relative

	1,2	1,3	2,3	1	2	3
Ω_3	$x = a.b$	$y = a.c$	$z = b.c$	$a = x/b$	$b^2 = z.x/y$	$c = b.y/x$
Ω_2	$t = d.e$	$u = d.f$	$v = e.f$	$d = t/e$	$e^2 = t.v/u$	$f = e.u/t$
Ω_1	$q = g.h$	$r = g.i$	$s = h.i$	$g = q/h$	$h^2 = q.s/r$	$i = h.r/q$

Table 3.4: On the left half of the table, the magnitude of the cross spectrum at each collective mode frequency, in terms of the contribution from cavity mode pairs. On the right half, the simultaneous equations used to calculate the contributions of the cavity modes to the collective modes.

phases at Ω_2 might be + + -.

We dis-entangle the magnitudes of the cross spectra, between pairs of cavity modes, to determine the contribution of the cavity modes with respect to the collective modes. In the cross spectrum magnitude between pairs of cavity modes, the height of an individual collective mode resonance is the product of the contributions from the two cavity modes concerned. This is shown in the left half of Table 3.4, at each collective mode frequency. Three simultaneous equations with three unknowns are used to solve for the magnitude of the contribution, these are shown on the right half of the Table. It is important to note that these contributions cannot be measured in the two mode case because there are two unknowns, whilst there is only one known value at a particular collective mode frequency.

In Table 3.5 we show the contributions, with their signs from the phase clusters, that we obtained for this set of three cross spectra. The column labeled “Total” represents the overall fluctuations of the array. This analysis is similar to the complex plane analysis used for two mode operation. Given

	1	2	3	Total
Ω_3	+0.12897	-0.05993	-0.07747	-0.00843
Ω_2	+0.03600	-0.10063	+0.04026	-0.02437
Ω_1	+0.34076	+0.17199	+0.05327	+0.56601

Table 3.5: The contributions of the cavity modes to the collective modes for three mode operation. The column labeled “Total” shows the sum of the contributions at the respective collective mode frequencies.

the arguments in the preceding paragraphs about the phase inconsistencies at the low frequency collective modes, we do not take account into the phase of the total oscillation here. At Ω_3 and Ω_2 , the relative size of the “Total” highlights that the overall oscillation of the array is very weak at these frequencies. This manifests itself in the power spectrum of the directly detected total intensity which was calculated separately, see Fig. 3.7. There does not appear to be a peak at Ω_2 . There appears to be a vestige of Ω_3 , approximately 1-2dB above the surrounding noise. The fact that the power spectral density of the total intensity is weak at these frequencies likely explains the inconsistency between the phase clusters and the cross spectrum between cavity modes and the total intensity and the phases of the overall oscillation in the subsequent calculation of the contributions of the cavity modes to the collective modes. We also discuss the strength of the destructive interference quantitatively in Chap. 4 using relations between the power spectrum of the modal and total intensity fluctuations, derived from modal rate equation theory, at the collective mode frequencies.

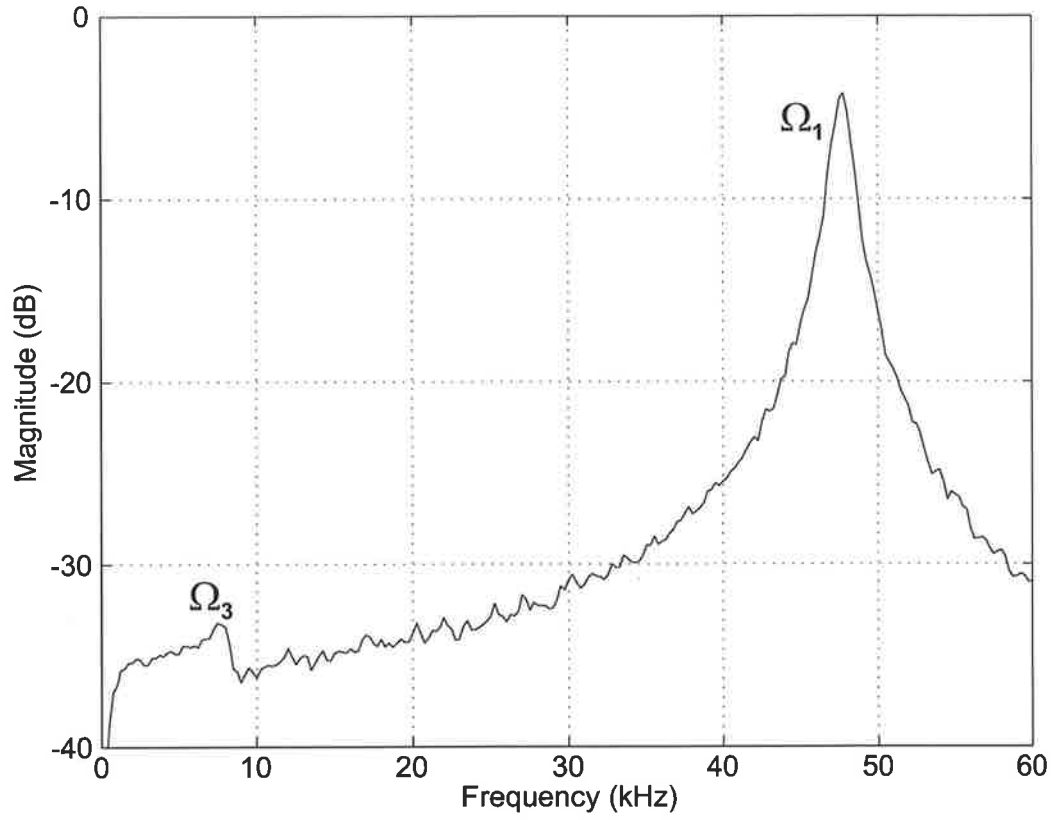


Figure 3.7: The power spectrum of the total intensity noise fluctuations for three mode operation. The collective mode frequencies are; 8kHz, 11.75kHz and 47.75kHz.

3.4 Conclusion

We have developed a method to measure the antiphasing directly in a nonlinear oscillator array excited by noise. We have applied this to two mode operation of the laser and showed that, although the range of possibilities for the phase of the intensity fluctuations at the collective modes was limited, the antiphase dynamics was determined explicitly. For three mode operation, the phases of the contributions of the cavity modes to the collective modes did not show simple ordering as in other arrays. This discrepancy is shown in the data presented in Chap. 4 and is discussed in detail in Chap. 6. We also measured quantitatively the response of the array to noisy excitation. In both regimes of operation, the power spectrum of the total intensity fluctuations showed that the destructive interference at the low frequency collective mode(s) was close to perfect.

It may be possible to use the cross spectral technique to compare different excitation methods of the array. For example, if the measured modal fluctuations are sufficiently small, the contributions of the cavity modes to collective modes using the cross spectral technique should be the approximately the same as those obtained using the transfer function technique [53], i.e. excitation with a swept frequency. We discuss this further in Chap. 5.

Our method may well be applied to other nonlinear oscillator arrays provided the individual oscillators can be separated from the overall fluctuations, whilst maintaining their coupling within the array. We have demonstrated the multimode laser, operating with a small number of cavity modes, to be a convenient example of an array because the laser cavity modes have different optical frequencies which makes them separable in the output beam. Some

other optical examples that might be considered for the application of this technique are: multimode fiber lasers [11, 29, 30], a soliton pulse [16, 17], mode-locked lasers [15], and semiconductor lasers [20, 23, 24, 25]. In this context we note that the cross correlation coefficients between the fluctuation of the spectral components of an intensity squeezed soliton pulse have been measured [16].

Our technique ought to be applicable in a fairly straightforward manner to the first example where the modes also have different optical frequencies. However, the last two examples are more complicated as the cross spectrum at optical frequencies ($\sim 10^{14}$ Hz) could be as important as that near the relaxation oscillation frequency (10^5 Hz - 10^{10} Hz). One aspect of diode lasers that makes them interesting as nonlinear oscillator arrays is the technique of making high power devices by placing several low power devices side by side [86]. The low power devices are coupled by the transverse light leakage from one diode junction to a neighbouring junction. These devices could be separated from the total output by “fibre-pigtailing” where an individual fibre is associated with an individual junction [86]. In other fields, the cross spectrum might be a means of probing theoretically the dynamics of JJ arrays [6] or phase oscillators [5].

Chapter 4

Power Spectra Relations

We developed the cross spectral technique because the power spectrum of the intensity fluctuations could not directly determine the clustering of the phases, at the collective mode frequencies. Nevertheless, there is much interest in power spectra because they are a useful test of the theoretical models used to describe the behaviour of multimode solid state lasers.

We measure the power spectral densities of the modal and total intensity noise fluctuations for the laser operating at low pump rate with two and three cavity modes. We test relationships between these quantities, at the collective mode frequencies, that are derived from modal rate equation theory. We also derive relationships that describe the phase of the intensity fluctuations, at the collective mode frequencies, for cross spectra between pairs of cavity modes and cavity modes paired with the total intensity. This work has been published [87] and is included in Appendix D.

4.1 Introduction

Many studies have shown that a rate equation approximation of the Maxwell-Bloch equations, adequately describe the evolution of the intensity and population inversion in a Class B lasers, refer monographs [42, 47] and references contained therein. Of these models, one of the earliest, well known and successful models for a Fabry-Perot cavity Class B laser, where the gain is modified by spatial hole burning (as is the case for our laser), is that due to Tang, Statz and deMars (TSD) [84]. In this model, a Fourier expansion of the population inversion density (also known as gain) rate equation is employed, with a rather arbitrary truncation (of the Fourier expansion) being needed to arrive at the set of ordinary differential equations that constitute the TSD model. In this context we note that a model which also includes the low frequency components of the expansion, known as the TSD⁺ equations, has recently been proposed [88] and analysed [89]. Two other basic assumptions that go into the TSD model are that the gain medium fills the cavity and the pump is distributed uniformly along the axis of the laser. Models which take into account a partially filled cavity and/or an inhomogeneous pump mechanism have been proposed [54, 71, 72, 74, 75, 85, 88, 89, 90, 91, 92, 93] however, as a general rule they employ the same modal expansion and truncation as TSD [54, 72, 85, 90, 91]. A more general approach that eschews the modal expansion, using the so-called global rate equations, was also developed [92, 93].

In the case of Nd:YAG lasers similar to ours [54, 71, 72], where one or both of the pump nonuniformities were accounted for, the model qualitatively described the data. Further, a recent comparison of a model incorporating

nonuniform pumping with one that assumes uniform pumping shows that this aspect hardly affects the threshold properties of a laser [75].

The TSD^+ equations account for the variation of the pump, along the axis of propagation, as a function of space. For homogeneous pumping, the TSD^+ equations closely approximate the TSD equations. For inhomogeneous pumping, as is the case in our laser, the TSD^+ equations predict that the intensity can oscillate with a limit cycle or chaotically. We operate our laser with a low pump rate, a small number of oscillators, and the excitation of the relaxation oscillations is close to linear such that the total intensity oscillates about a stable steady state. Whilst the TSD model has limitations, we therefore persist with it and because relations between the power spectral densities of the modal and total intensity fluctuations, at the collective mode frequencies, can easily be derived analytically as opposed to the TSD^+ equations. In this context we note that relations have been analytically derived for the two mode case with a rate equation model which takes into account the large scale nonuniformity of the pump [92], as one would find in an end pumped laser. So far this does not appear to be possible for three or more modes, which is of particular interest here. As the inhomogeneity of the population inversion acts mainly on the strength of the cavity mode competition [75], we expect the TSD model to give a qualitative description of the dynamics of inhomogeneously pumped lasers by using an effective pump value and effective mode gains.

Whilst these relations, referred to herein as the power spectra relations, are predicated on a particular laser model, they have a certain universality in the sense that they are independent of the detailed parameters of the

laser, for example the pump rate, insofar as operation with a given number of modes is maintained. The power spectra relations were derived for a Class B laser, i.e. the lifetime of the laser cavity is much smaller than that of the population inversion. Experiments with a LNP laser, where the lifetimes differ by a factor of about 10^6 , have confirmed these relations for two [50] and three mode operation [51].

In this thesis, we put forward another case where these power spectra relations are verified, but where the lifetimes differ by a smaller factor. In Nd:YAG, the fluorescence lifetime (τ_f) of the lasing transition, i.e. population inversion, is $230\mu\text{s}$ [78]. As stated in Chap. 2, we measured the laser cavity lifetime to be 2.0ns . The ratio of lifetimes (τ_f/τ_c) is therefore 1.15×10^5 which is typical for Nd:YAG lasers.

For two and three mode operation, we use the same intensity noise data as in Chap. 3 respectively. The power spectral densities of each of the modal intensity fluctuations and the total intensity fluctuations were calculated and the relations applied at the collective mode frequencies. We also computed the power spectral density of the sum of the modal intensities as a check of the consistency and the precision of our measurements. Using TSD the model, we also develop relations for the cross spectral phase, at the collective mode frequencies, and compare them with our experimental data.

4.2 Theoretical Results

The model discussed in this section was developed by D. Pieroux and P. Mandel from Optique Nonlinéaire Théorique, Université Libre de Bruxelles, Belgium.

4.2.1 The TSD Model

In 1963, Tang, Statz, and deMars proposed a simple model to describe the dynamics of multimode Class B Fabry-Perot lasers [84]. Starting from a noiseless semi-classical laser description, i.e. the Maxwell-Bloch equations, the derivation of the model proceeds in three stages. Firstly, the medium polarisation is adiabatically eliminated.¹ This simplification holds if the polarisation lifetime is much smaller than both the laser cavity and population inversion lifetimes, as is the case for LNP, Nd:YAG, and semiconductor lasers. Secondly, the population inversion is spatially Fourier expanded. The last stage consists of truncating the expansion by discarding all but the first order Fourier components. For a N mode laser, this results in $2N + 1$ nonlinear ordinary differential equations:

$$\begin{aligned}\frac{dI_m}{dt} &= \kappa I_m [\gamma_m (D_0 - D_m/2) - 1], \\ \frac{dD_m}{dt} &= \gamma_m D_0 I_m - D_m \left(1 + \sum_{k=1}^N \gamma_k I_k \right), \\ \frac{dD_0}{dt} &= w - D_0 - \sum_{k=1}^N \gamma_k (D_0 - D_k/2) I_k,\end{aligned}\tag{4.1}$$

with $m = 1, \dots, N$. In these equations, I_m is the intensity of mode m , D_0 and D_m are the zero and first order spatial Fourier components of the population inversion density, i.e.

$$D_0 = \frac{1}{L} \int_0^L D(x, t) dx \text{ and } D_m = \frac{2}{L} \int_0^L D(x, t) \cos(2k_m x) dx, \tag{4.2}$$

¹The rate equations for the fast variables are eliminated by integrating their equations of motion, whilst assuming the other variables are constant.

where $D(x, t)$ is the population inversion density, k_m the optical wavenumber of mode m and L the cavity length. Time t is measured in units of the population inversion lifetime, w is the pump parameter, normalised to the threshold of the YAG laser and averaged over the cavity length, and $\gamma_m \leq 1$ is the gain of mode m relative to the mode with maximum gain, $m = 1$. In what follows, we order the modes according to the pump threshold value, or equivalently in the case of the TSD model, such that $1 = \gamma_1 > \dots > \gamma_N$. We only consider this case since it corresponds to our experiments.

The ratio of lifetimes (κ), which in our laser is 1.15×10^5 , is assumed to be equal for all modes. For a large κ , and in the absence of modulation, e.g. the pump parameter, the TSD model displays fairly simple dynamics. For a normalised pump parameter less than unity, no modes oscillate within the Fabry-Perot cavity. At $w = 1$, the first mode ($m = 1$) starts lasing, followed by the other cavity modes as w increases. Whatever the pump value, the laser always possesses a single stable steady state such that modes whose threshold is below the current pump value are lasing, while the others are not.

If weakly perturbed, the laser returns to its stable steady state via damped relaxation oscillations. For a small enough perturbation ($\sim 1/\sqrt{\kappa}$), Eqs. (4.1) can then be linearised. Indeed in Chap. 3, we showed that this assumption is justified. Solving the linear equations using a Taylor expansion, the relaxation transient is given by:

$$I_m(t) = I_{m,\text{st}} + \sum_{k=1}^N (\epsilon_k a_{m,k} e^{(\gamma_k + i\Omega_k)t} + c.c.) + \mathcal{O}(\epsilon^2), \quad (4.3)$$

with $I_{m,\text{st}}$ being the steady state intensity of mode m , $\gamma_k + i\Omega_k$ the k^{th} eigenvalue of the linear problem and $a_{m,k}$ the amplitude of the m^{th} component

of the k^{th} eigenvector. As all the γ_k are negative, the transient is damped. The $a_{m,k}$ coefficients are independent of the perturbation, in contrast to the complex parameters ϵ_k which are uniquely determined by it. Being linear, Eq. (4.3) also applies to the total intensity $I_{tot} = \sum_{m=1}^N I_m$ if the $a_{m,k}$ are replaced by $a_{tot,k} = \sum_{m=1}^N a_{m,k}$. The eigenvectors being defined up to a complex scaling factor, we choose $a_{tot,k}$ real and not negative for all k .

4.2.2 Fourier Component Relations

In the limit of large κ , the collective mode frequencies Ω_k scale as $\kappa^{1/2}$ while the damping rates γ_k are of order one, i.e. they remain practically unaffected [7]. These two time scales lead to the existence of a range of durations T such that $\Omega_k^{-1} \ll T \ll \gamma_k^{-1}$. That is, during a time T , the laser displays many oscillations without being significantly damped. This fact simplifies the computation of the Ω_k components of the Fourier transformed intensities over a limited duration T :

$$\mathcal{F}_T(I_m, \Omega_k) = \int_0^T I_m(t) e^{-i\Omega_k t} dt \approx \epsilon_k a_{m,k} T. \quad (4.4)$$

This equation also holds for $\mathcal{F}_T(I_{tot}, \Omega_k)$ if $a_{m,k}$ is replaced by $a_{tot,k}$. It also allows one to recover the eigenvectors of the linearised TSD system from the Fourier transform of the laser intensity time traces, since ϵ_k and T can be eliminated to give:

$$a_{m_1,k} \mathcal{F}_T(I_{m_2}, \Omega_k) \approx a_{m_2,k} \mathcal{F}_T(I_{m_1}, \Omega_k). \quad (4.5)$$

4.2.3 Power And Cross Spectra Relations

Checking the validity of equation (4.5) is difficult as every $a_{m,k}$ depends on the pump w and the modal gain parameters γ_m . However, universal relations independent of the pump parameter and the gains can be established using the power spectral densities (PSD) and cross spectral densities (CSD), defined respectively by:

$$\mathcal{P}_T(I_m, \Omega_k) = \frac{1}{T} |\mathcal{F}_T(I_m, \Omega_k)|^2, \quad (4.6)$$

$$\mathcal{C}_T(I_m, I_n, \Omega_k) = \frac{1}{T} \mathcal{F}_T(I_m, \Omega_k) \mathcal{F}_T^*(I_n, \Omega_k) \quad (4.7)$$

with the asterisk denoting the complex conjugate. To derive these relations, it is necessary to refer to two properties of the $a_{m,k}$ coefficients that hold in the limit $\kappa \gg 1$. The first property is that all $a_{m,k}$ can be chosen real in a first approximation, as we did for the analysis of the cross spectra for two mode operation, see Sec. 3.2. This comes from the fact that $\text{Re}(a_{m,k}) \sim \kappa^{1/2} \text{Im}(a_{m,k})$. This fundamental property is at the origin of antiphase dynamics, i.e. the absence or the strong reduction of the low frequency collective modes in the total intensity transient, which we showed experimentally in Chap. 3. The second property of $a_{m,k}$ is that above the threshold of mode m , all $\text{sgn}(a_{m,k})$ (i.e. phase of $a_{m,k}$) are constant, insofar as operation with a given number of modes is maintained. These are given in Table 4.1, for three mode operation. These two properties have been demonstrated for any N and $\gamma_m \approx 1$ [94], for $N = 2$ and any γ_m [50] and for $N = 3$ and any γ_m [51], although they might be valid for any N and any γ_m .

Equations (4.6) & (4.7) become:

$$\sqrt{\mathcal{P}_T(I_{tot}, \Omega_k)} = \sum_m \text{sgn}(a_{m,k}) \sqrt{\mathcal{P}_T(I_m, \Omega_k)}, \quad (4.8)$$

and

$$\arg(\mathcal{C}_T(I_m, I_n, \Omega_k)) = \begin{cases} 0^\circ & \text{if } \text{sgn}(a_{m,k}) = \text{sgn}(a_{n,k}), \\ 180^\circ & \text{otherwise.} \end{cases} \quad (4.9)$$

For operation with a constant number of cavity modes, these relations are universal since they hold whatever the perturbation, the pump or the gain parameters. Table 4.2 illustrates Eq. (4.9) for all m , n and k , in the case $N = 3$. The same labeling convention is used as in Chap. 3. These results are consistent with the relative $\text{sgn}(a_{m,k})$ as shown in Table 4.1.

4.2.4 Application To Noisy Lasers

In practice, the laser can be a noisy device. The noise in our laser originates from spontaneous emission (quantum noise) and from environmental perturbations. Modeling the noise properties of our laser is a formidable task, outside the scope of this thesis. Therefore, we choose a qualitative approach by modeling the noise effect as a series of instantaneous, independent, random kicks that perturb the laser. Right after a kick, the laser relaxes towards its steady state, until it is pushed again by another kick. If the noise level is weak, the laser remains in the vicinity of its stable steady state and the linear theory we developed applies. The dynamics of the laser between two kicks is then described by the Eq. (4.3), with each ϵ_k being a random constant that is updated after each kick. If the average duration T between two kicks is such that $\Omega_k^{-1} \ll T \ll \gamma_k^{-1}$, the relations (4.8) and (4.9) still hold since they are independent of the perturbation. Moreover, the laser does not

	$a_{1,k}$	$a_{2,k}$	$a_{3,k}$	$a_{\text{tot},k}$
Ω_3	-	-	+	+
Ω_2	-	+	+	+
Ω_1	+	+	+	+

Table 4.1: Signs of the $a_{m,k}$ coefficients as predicted by the TSD model for three mode operation, where Ω_1 is the highest frequency collective mode. The two mode case is obtained by removing the Ω_3 row and the $a_{3,k}$ column. We measured the $\text{sgn}(a_{m,k})$, without the $a_{\text{tot},k}$ column, for three mode operation, i.e. the phase clusters, in Table 3.3.

	1,2	1,3	2,3	1,TI	2,TI	3,TI
Ω_3	$+\dagger$	-	$-\dagger$	-	$-\dagger$	+
Ω_2	-	$-\dagger$	$+\dagger$	$-\dagger$	+	$+\dagger$
Ω_1	+	+	+	+	+	+

Table 4.2: Theoretical prediction of the cross spectra complex phase $\arg(\mathcal{C}(I_m, I_n, \Omega_k))$ for three mode operation, expressed in terms of the cavity mode pairs and cavity modes paired with the total intensity. The two mode case is obtained by removing the Ω_3 row and the columns containing cavity mode three. Terms in opposition with our experimental data, for three mode operation, are highlighted by a “ $\dagger$ ”, see Tables 3.1 & 3.2.

have enough time to relax to its steady state before being perturbed again. Therefore, the upper integration bound of Eq. (4.4) can be taken much larger than T . Therefore long time traces, compared with the timescale of the dynamics, are required to compute the PSD's and the CSD's. For example, the recorded time traces in our experiment were $\approx 525\text{ms}$ whilst the oscillation of the collective modes was $\sim 10\mu\text{s}$.

4.3 Two Mode Operation

Using the experimental setup as described in Chap. 2, we check the validity of the relations (4.8) and (4.9). The same intensity noise data was used as in Chap. 3 where the collective mode frequencies were 7.5kHz and 36.25kHz . The YAG lasing threshold was 73.8mA of diode current, the threshold of the second cavity mode was 74.5mA and the operating current 75.4mA . The latter corresponds to a pump parameter of $w = 1.022$.

The power spectral density (PSD) of the directly detected total intensity was shown in Fig. 3.4. The observed peak located at Ω_1 had a full width at half maximum (FWHM) of $\approx 1.2\text{kHz}$. There did not appear to be a peak at Ω_2 , the PSD (at this frequency) being made up of a superposition of noise and the low frequency wing of the Ω_1 peak. The PSD values computed from the experimental time traces of the cavity modes and the total intensity, at Ω_2 and Ω_1 , are given in Table 4.3. The PSD of the sum of the modal intensities was also computed, shown in Fig. 4.1, to check the consistency and the precision of our measurement. The last column of Table 4.3 is obtained by

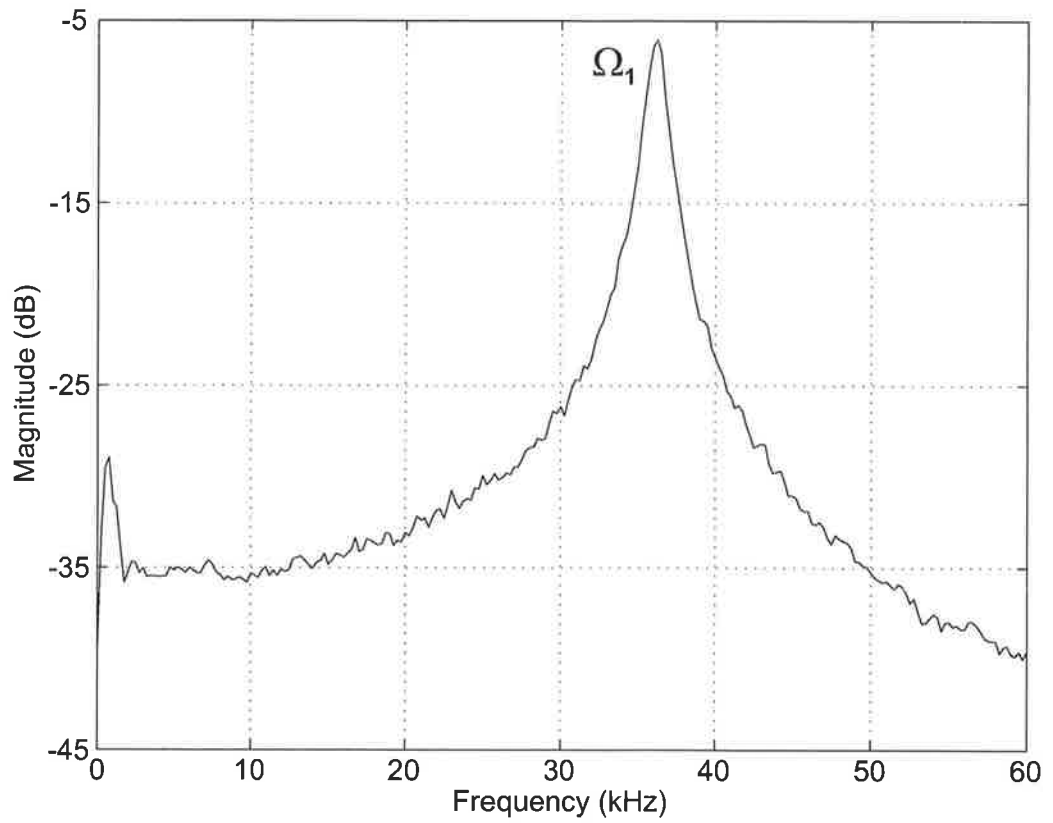


Figure 4.1: The power spectrum of the summed modal intensity fluctuations ($I_1 + I_2$) for two mode operation. The collective mode frequencies are; 7.5kHz and 36.25kHz. The low frequency noise peak could be from a variety of sources, including for example mains harmonics which were present in either of, or both modal intensity fluctuations.

	$P(I_1)$	$P(I_2)$	$P(TI)$	$P(I_1 + I_2)$	$P_{\text{TSD}}(TI)$
Ω_2	0.00538	0.00619	0.00029	0.00033	2.85×10^{-5}
Ω_1	0.14774	0.01301	0.24136	0.24837	0.24844

Table 4.3: Power spectral densities (dimensionless) for two mode operation: $P(I_1)$, $P(I_2)$, $P(TI)$ correspond respectively to the measured modal and total intensity fluctuations, $P(I_1 + I_2)$ is computed by summing the normalised fluctuation time traces of the two modal intensities, and $P_{\text{TSD}}(TI)$ is obtained using Eq. (4.10). Ω_1 is the highest frequency collective mode.

application of the relations (4.8) which, in the two mode case, reduce to [50]:

$$\begin{aligned}
 P_{\text{TSD}}(TI, \Omega_2) &= \left(-\sqrt{P(I_1, \Omega_2)} + \sqrt{P(I_2, \Omega_2)} \right)^2, \\
 P_{\text{TSD}}(TI, \Omega_1) &= \left(\sqrt{P(I_1, \Omega_1)} + \sqrt{P(I_2, \Omega_1)} \right)^2.
 \end{aligned}
 \tag{4.10}$$

In Table 4.3, it is seen that the PSD's of the three different total intensities are in very good agreement at Ω_1 , their relative differences being smaller than 3%. At Ω_2 , $P(TI)$ and $P(I_1 + I_2)$ differ by 13%, however $P_{\text{TSD}}(TI)$ is ten times smaller. This apparent problem can be rather well explained. Indeed, the size of $P_{\text{TSD}}(TI)$, suggests that a strong antiphase dynamics cancels most of the Ω_2 oscillation in the total intensity. Therefore, $P(TI)$ and $P(I_1 + I_2)$ mainly measure the background noise intensity, leading to $P_{\text{TSD}}(TI) < P(TI) \approx P(I_1 + I_2) \ll 1$ which is illustrated in Table 4.3. Similar measurements in a LNP laser indicate the PSD, at this collective mode frequency, to be of a similar size, i.e. much less than the background noise [50].

	1,2	1,TI	2,TI
Ω_2	-	-	+
Ω_1	+	+	+

Table 4.4: Experimentally determined cross spectral phases, i.e. $\arg(\mathcal{C}(I_m, I_n))$, for two mode operation.

The cross spectral method, as described in Chap. 3, showed explicitly that the fluctuations of the two cavity modes to be in antiphase at the low frequency collective mode (Ω_2) whilst they were in phase at Ω_1 , see Fig. 3.1. This result is consistent with the relations (4.10). We present the experimentally determined cross spectral phase, at the collective mode frequencies, for each pairing in Table 4.4. For cross spectra between cavity modes and the total intensity, we used the same assumptions as in Sec. 3.3.2 to determine the phase at Ω_2 . These results confirm the theoretical predications given in Table 4.2, limited to two modes.

This experiment and analysis was repeated with success for different pump levels and positions of the cavity mode spectrum with respect to the gain spectrum. We therefore conclude that the relations (4.8) and (4.9) hold for Nd:YAG lasers operating with two cavity modes.

4.4 Three Mode Operation

Using the experimental setup described in Chap. 2 for three mode operation, we check the validity of the relations (4.8) and (4.9). We use the same intensity noise data as in Chap. 3 where the collective mode frequencies

were 8kHz, 11.75kHz and 47.75kHz. The threshold of the YAG laser was 76.4mA of diode current, the third mode threshold was 78.2mA and the operating point was 79.2mA, i.e. a pump parameter of $w = 1.037$.

For cavity modes with unequal gain, the theoretical values of $\text{sgn}(a_{m,k})$ are given in Table 4.1, such that Eqs. (4.8) reduce to [51]:

$$\begin{aligned} P_{\text{TSD}}(TI, \Omega_3) &= \left(-\sqrt{P(I_1, \Omega_3)} - \sqrt{P(I_2, \Omega_3)} + \sqrt{P(I_3, \Omega_3)} \right)^2, \\ P_{\text{TSD}}(TI, \Omega_2) &= \left(-\sqrt{P(I_1, \Omega_2)} + \sqrt{P(I_2, \Omega_2)} + \sqrt{P(I_3, \Omega_2)} \right)^2 \quad (4.11) \\ P_{\text{TSD}}(TI, \Omega_1) &= \left(+\sqrt{P(I_1, \Omega_1)} + \sqrt{P(I_2, \Omega_1)} + \sqrt{P(I_3, \Omega_1)} \right)^2. \end{aligned}$$

The PSD of the directly detected total intensity was shown in Fig. 3.7. The observed peak located at Ω_1 had a FWHM of $\approx 1.4\text{kHz}$. There appeared to be a vestige of Ω_3 , approximately 1-2dB above the background noise which is very weak compared with the Ω_1 peak. No peak was distinguishable at Ω_2 , the PSD (at this frequency) being made up of a superposition of noise and the low frequency wing of the Ω_1 peak. The experimental PSD's computed from the laser intensity noise time traces and the theoretical predictions of the TSD model are reported in Table 4.5. The PSD of the summed modal intensities is shown in Fig. 4.2.

The agreement between the PSD's of the directly measured total intensity and the sum of modal intensity time traces, columns four and five of Table 4.5, is not as good as in the two mode case, despite careful efforts in measuring the sensitivity of each optical channel. The relative differences between these two estimates of the total intensity PSD's being 56%, 150% and 16% at Ω_3 , Ω_2 and Ω_1 respectively. The large discrepancies at Ω_3 and Ω_2 can be partially explained by the small signal to background noise ratio. Nevertheless, there is an agreement on the relative size of the two estimates of the total intensity

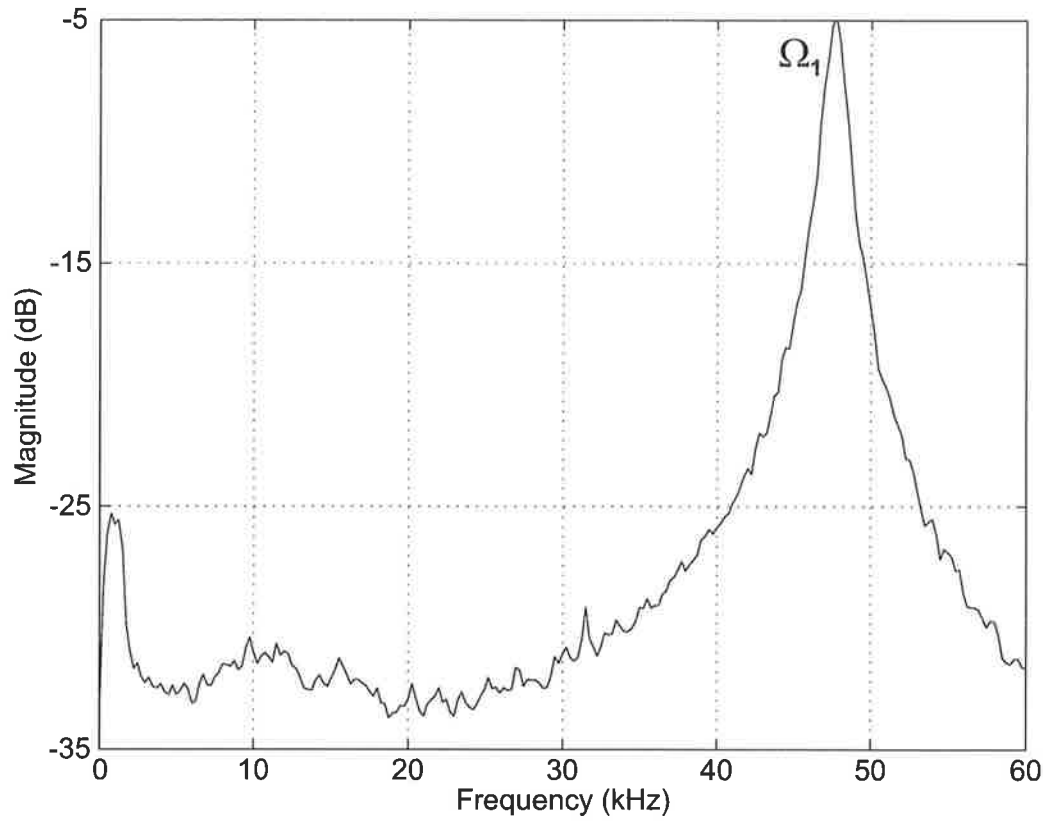


Figure 4.2: The power spectrum of the summed modal intensity noise fluctuations ($I_1 + I_2 + I_3$) for three mode operation. The collective mode frequencies are; 8kHz, 11.75kHz and 47.75kHz. The low frequency noise peak could be due to a variety sources, including the dither frequency (1251Hz) and mains harmonics. These frequencies occurred in one or more of the modal intensity fluctuations which make up this power spectrum.

	$P(I_1)$	$P(I_2)$	$P(I_3)$	$P(TI)$	$P(I_1 + I_2 + I_3)$	$P_{\text{TSD}}(TI)$
Ω_3	0.01687	0.00327	0.00628	0.00045	0.00071	0.01163
Ω_2	0.00225	0.00879	0.00217	0.00031	0.00077	0.00863
Ω_1	0.11626	0.02957	0.00886	0.37732	0.32585	0.36851

Table 4.5: Power spectral densities (dimensionless) for three mode operation: $P(I_1)$, $P(I_2)$, $P(I_3)$, $P(TI)$ correspond respectively to the measured modal and total intensity fluctuations. $P(I_1 + I_2 + I_3)$ is computed from the sum of the normalised modal intensity fluctuations. $P_{\text{TSD}}(TI)$ is calculated from Eq. (4.11). Ω_1 is the highest frequency collective mode.

PSD's, at each collective mode frequency. At Ω_1 , the theoretical estimation of $P_{\text{TSD}}(TI)$ matches well with the experimental data, $P(TI)$ (a 2% difference) and $P(I_1 + I_2 + I_3)$ (a 13% difference). However at Ω_3 and Ω_2 , $P_{\text{TSD}}(TI)$ is much larger than its corresponding experimental values. Therefore, the TSD model fails to verify the power spectra relations for our three mode laser setup.

Because the cross spectral method determined the dynamics implicit in the relations for the two mode case (Eqs. 4.8 & 4.9), we refer to our calculations of the cross spectra for three mode operation in Sec. 3.3. By analysing the experimental cross spectra, see Figs. 3.5 & 3.6, it appears that the intensity components of the eigenvectors can still be chosen real, as predicted by the TSD model in the asymptotic limit of large κ .

We reported the experimental evaluation of all the signs of $\arg(\mathcal{C}(I_m, I_n, \Omega_k))$ in Tables 3.1 & 3.2 and explained the inconsistency between the modal fluctuations and the total intensity, at Ω_2 , in Sec. 3.3.3. This is most probably

caused by the small amplitude of the total intensity at Ω_2 relative to the background noise. Therefore, we do not take into consideration the Ω_2 entries of the cross spectrum between cavity modes and the total intensity in Table 3.2. To remain consistent, we also do not consider the Ω_3 entries since they could be similarly affected. Even with these entries removed, the modal fluctuations are different to the theoretical predictions, highlighted by a “†” in Table 4.2.

Knowing that the entries of Table 4.2 are not all verified by the experiment, we re-write the signs of the terms in the R.H.S. of Eqs. (4.11) using the phase clusters in Table 3.3. The measured clusters do not show the simple ordering of phases predicted by our model, or predicted in other lasers [51, 55] or measured in a LNP laser [51]. Therefore the relations become:

$$\begin{aligned} P_{\text{TSD}}^{\text{corr.}}(TI, \Omega_3) &= \left(+\sqrt{P(I_1, \Omega_3)} - \sqrt{P(I_2, \Omega_3)} - \sqrt{P(I_3, \Omega_3)} \right)^2, \\ P_{\text{TSD}}^{\text{corr.}}(TI, \Omega_2) &= \left(+\sqrt{P(I_1, \Omega_2)} - \sqrt{P(I_2, \Omega_2)} + \sqrt{P(I_3, \Omega_2)} \right)^2, \\ P_{\text{TSD}}^{\text{corr.}}(TI, \Omega_1) &= \left(+\sqrt{P(I_1, \Omega_1)} + \sqrt{P(I_2, \Omega_1)} + \sqrt{P(I_3, \Omega_1)} \right)^2. \end{aligned} \quad (4.12)$$

The redefined total PSD's are shown in Table 4.6 column one. Note that at Ω_1 , these relations are the same as in Eqs. 4.11 and therefore the $P_{\text{TSD}}^{\text{corr.}}(TI)$ matches the value of the directly detected total intensity, $P(TI)$ in column four of Table 4.5, well. On the other hand, Table 4.6 shows that at both Ω_3 and Ω_2 , the PSD's are smaller than the corresponding experimental values of $P(TI)$. This can be explained by the arguments put forward in the two mode case, i.e. that $P(TI)$ and $P(I_1 + I_2 + I_3)$ mainly measure background noise and give the upper bound for the power spectra relations at these frequencies, and the fact that a similar analytic approach was used in a LNP laser operating with three cavity modes [51].

	$P_{\text{TSD}}^{\text{corr.}}(TI)$	P_{CS}
Ω_3	4.33×10^{-5}	1.96×10^{-5}
Ω_2	6.53×10^{-8}	0.00059
Ω_1	0.36851	0.32037

Table 4.6: The power spectral density $P_{\text{TSD}}^{\text{corr.}}(TI)$ is calculated from Eq. (4.12). The column labeled “ P_{CS} ” is calculated from the square of the sums of the contributions of the cavity modes to the collective modes, obtained using the cross spectrum, see Table 3.5.

In comparing the cross spectral technique with the power spectra relations, we show “ P_{CS} ” in column two of Table 4.6. This was calculated from the square of the sums of the contributions of the cavity modes to the collective modes, obtained using the cross spectrum, see Table 3.5 column four. These values are similar to the PSD of the summed modal intensities. It appears therefore, that the two measurement techniques give similar quantitative results.

For three mode operation, intensity noise data was recorded for different pump levels and positions of the cavity mode spectrum with respect to the gain spectrum. We checked the cross spectra between cavity mode pairs and cavity modes paired with the total intensity for each case. The phase of the fluctuations of the former were found to be repeatable whilst the latter were found to be repeatable to within $\pm 10^\circ$. Again, the phases of the cross spectra between cavity modes and the total intensity were found to be inconsistent with the phases of the cavity mode pairs. Because of the phase inconsistencies between the two types of cross spectra at the low frequency collective modes,

we only performed quantitative analysis using the cross spectrum once, see Sec. 3.3.3. The power spectra relations (Eq. (4.12)) analysis was repeated in each case with results being similar to those presented. These results also suggest that the cross spectrum between cavity modes and the total intensity is more sensitive to antiphase dynamics than the power spectrum of the total intensity and are indeed a signature of a strong antiphase dynamics.

4.5 Conclusion

We have applied the power spectra relations to a Nd:YAG laser for both two and three mode operation. The good agreement between the experimental data and the theory based on the TSD equations in the two mode case, is not altogether surprising given the restricted range of possibilities for the relative phase of the modal intensity fluctuations. Thus the two mode case is a check of the experimental technique rather than a check of a theoretical model.

With the wider range of possibilities for the relative phase of the modal intensity fluctuations in three mode operation, we have shown that the cross spectral measurements serve as a sensitive test of theoretical models. We conclude that the TSD rate equations fail to give the correct answer because some essential information is not used. We discuss this in more detail in Chap. 6 where we consider data presented in the rest of the thesis which further highlights deviations from the TSD model.

When two or more cavity modes receive different gains, the antiphasing of the low frequency collective modes should be incomplete and their vestige visible in the power spectrum of the measured total intensity fluctuations [47]. In both regimes of operation, the fluctuations of the total intensity, at

the low frequency collective mode(s), was very small. Therefore the relations were compared analytically with power spectrum of the directly detected total intensity, similar to measurements of a LNP laser [50, 51].

In principle, if predictions about the contribution of the wings of Ω_1 to the low frequency collective modes were included, together with knowledge of the background noise, it might be possible to be more quantitative at the low frequency collective modes. Alternatively, the oscillation of the low frequency collective modes should be greater than the superposition of background noise and the low frequency wing of the Ω_1 resonance, such that they are clearly visible in the power spectrum of the measured total intensity fluctuations. Given that we have optimised our experimental setup to maximise the signal to noise ratio and resolution of the data, we would require a laser where the FWHM of the main relaxation oscillation frequency is smaller. This would mean a shorter cavity lifetime of our Nd:YAG laser. The main resonance at Ω_1 would also be moved further in frequency space, from the low frequency collective modes [43].

Chapter 5

Four and Five Mode Operation

We have applied the cross spectral technique to the laser operating with a small number of modes, i.e. two or three. This has allowed us to compare the behaviour of the array with first principles theory and other experiments [50, 51]. Indeed, most publications deal with two or three modes because it is the simplest form of array oscillations. However, in practice a nonlinear oscillator array is likely to have more than three constituent oscillators. For example, a vertical cavity surface emitting laser (VCSEL) is made up of many emitters, the fields of which are coupled to their nearest neighbour [21, 22]. We have shown that the phase clusters, at the low frequency collective modes, for three mode operation were more complex than the two mode case. As the number of oscillators increases, we expect the complexity of the phase clusters to follow. However, the signature of antiphase dynamics should remain the same, i.e. each of the N modal intensities relax with N frequencies whilst the total intensity relaxes with just one [7, 8].

In this chapter, we apply the cross spectral technique to form a picture of the phase clusters and check the signature of antiphase dynamics for the laser

operating with four cavity modes. The same measurements and analysis are also performed for five mode operation.

It is well known that noise has important effects on nonlinear oscillator arrays. We compare our results with those obtained with Nd:YAG lasers, operating with five cavity modes and similar in design to ours, but excited by swept frequency [53, 54]. The laser relaxation oscillations being excited differently, we hope to gain a better understanding of the mechanisms of noisy excitation of an oscillator array.

For four and five mode operation, the intensity noise fluctuations were measured using the same experimental setup as for the two mode case, i.e. three detection channels which measure two modal intensity fluctuations and the total intensity fluctuations, see Fig. 2.1. There being more than one pair of cavity modes, the time to switch between pairs was kept very short, i.e. of order a couple of seconds. This maintained the length of the laser cavity and therefore minimised drift of the cavity mode spectrum with respect to the laser gain spectrum.

5.1 Four Mode Operation

For pairs of cavity modes, we show the cross spectrum between the intensity noise fluctuations of cavity modes one and two as an example, see Fig. 5.1. Four collective mode resonances are visible in the cross spectrum magnitude and are labeled; Ω_4 , Ω_3 , Ω_2 and Ω_1 in order of increasing frequency, where the highest frequency (Ω_1) is similar to that found in single mode operation [9, 62]. There are peaks at frequencies less than Ω_4 . These are from a variety of sources, including for example mains harmonics, the dither frequency

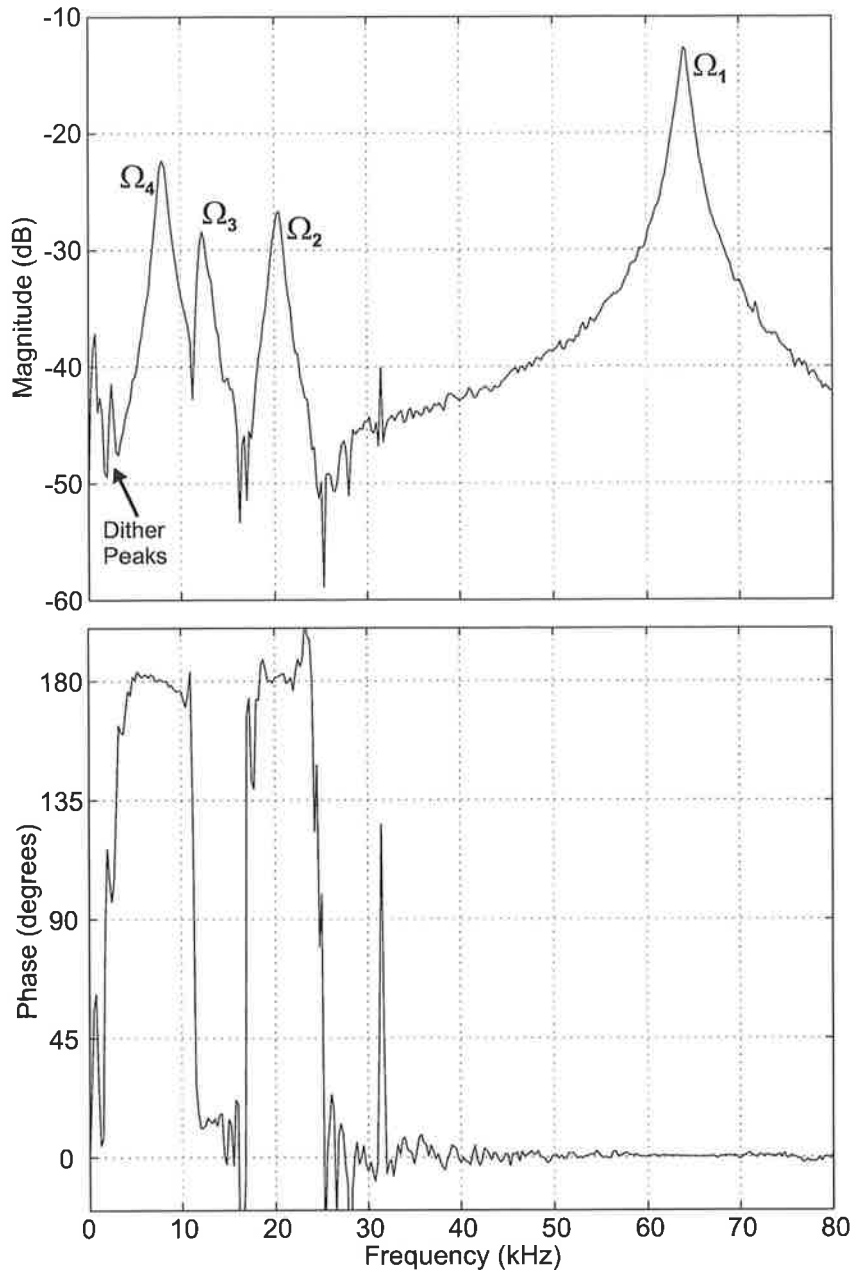


Figure 5.1: The cross spectrum of intensity noise fluctuations between cavity mode one and two, for four mode operation. The collective mode frequencies are; 8kHz, 12.25kHz, 20.5kHz and 64kHz.

	1,2	1,3	1,4	2,3	2,4	3,4
Ω_4	-	-	+	+	-	-
Ω_3	+	-	-	-	-	+
Ω_2	-	+	-	-	+	-
Ω_1	+	+	+	+	+	+

Table 5.1: The phase pattern, expressed in terms of cavity mode pairs, for four mode operation, where Ω_1 is the highest frequency collective mode.

(1251Hz) and its second harmonic. We label them “dither peaks”. The peak at 31.5kHz in both the magnitude and phase is due to electronic pickup from switch mode power supplies. As shown in Appendix A, the low frequency noise and the electronic pickup do not pertain to the intensity fluctuations of interest, rather they are due to the equipment used in the experiment.

Zeros and their corresponding phase jumps are found at 2kHz, 11.25kHz, 16.25kHz and 25.5kHz. The intensity fluctuations are in antiphase at Ω_4 & Ω_2 , in phase at Ω_1 and to within experimental error, in phase at Ω_3 . Table 5.1 shows the pattern of interferences for the cavity mode pairs.

Using the same labeling conventions and derivation method as outlined in Sec. 3.3.3, we present the phase clusters for four mode operation in Table 5.2. We check the consistency of the phase pattern by comparing the relative phase between each of the pairs of cavity modes, at a particular collective mode frequency. For example, at Ω_4 , the relative intensity noise fluctuations of cavity modes two and three are in phase which concurs with the phase pattern for that pair. Therefore the phase pattern was found to be self consistent. By definition, the contributions of cavity mode one to the intensity

	1	2	3	4
Ω_4	+	-	-	+
Ω_3	+	+	-	-
Ω_2	+	-	+	-
Ω_1	+	+	+	+

Table 5.2: The phase clusters for four mode operation.

	$a_{1,k}$	$a_{2,k}$	$a_{3,k}$	$a_{4,k}$
Ω_4	-	-	-	+
Ω_3	-	-	+	+
Ω_2	-	+	+	+
Ω_1	+	+	+	+

Table 5.3: Expected signs of the $a_{m,k}$ coefficients predicted by our TSD model for four mode operation, where Ω_1 is the highest frequency collective mode.

noise fluctuations of each of the collective modes are constructive. The other cavity modes then contribute both constructively and destructively. For example, the second cavity mode to reach threshold contributes destructively to Ω_4 & Ω_2 and constructively to Ω_3 , Ω_1 and so on. The phase clusters do not show the simple ordering as predicted in Josephson junctions [6] or other lasers [51, 55], or measured in a LNP laser [51]. We would expect the model developed in Chap. 4 (by Pieroux and Mandel) to show a simple ordering of the $\text{sgn}(a_{m,k})$ for four mode operation, see Table 5.3.

The cross spectra between cavity modes paired with directly detected total intensity were also calculated. As an example, Fig. 5.2 shows cavity mode

one paired with the total intensity. In the magnitude, there are resonances at Ω_4 , Ω_3 and Ω_1 . No peak at Ω_2 is visible, neither are any zeros. In the phase of the cross spectrum, there appears to be roll offs, i.e. not a jump but a sharp negative slope, such that the fluctuations are approximately 300 degrees (-60° in relative terms) out of phase at Ω_4 , 150 degrees out of phase at Ω_3 and in phase at Ω_2 & Ω_1 . The cross spectra of the other pairings show similar behaviour however, the heights of the resonances and their relative phases differ. The small signal to noise ratio at the low frequency collective modes is similar to the three mode case where we could not use this measurement to independently check the phase clusters. To remain consistent, we do likewise for four mode operation.

The power spectrum of the total intensity noise is shown in Fig. 5.3. No resonances are visible at Ω_4 or Ω_2 . There appears to be a vestige of Ω_3 , 1-2dB above the surrounding noise. This indicates that the antiphasing of the low frequency collective modes is almost complete as shown quantitatively in Chap. 4 for two and three mode operation. We therefore surmise that most of the energy in the array is concentrated in Ω_1 . This is the likely cause of the very small signal to noise ratio at the low frequency collective modes in the cross spectrum between the cavity modes and the total intensity, and therefore any inconsistencies between the latter and the phase clusters.

5.2 Five Mode Operation

We now compare the phase clusters determined from the cross spectra between cavity mode pairs with those obtained from similar Nd:YAG lasers, operating with five cavity modes and excited by swept frequency.

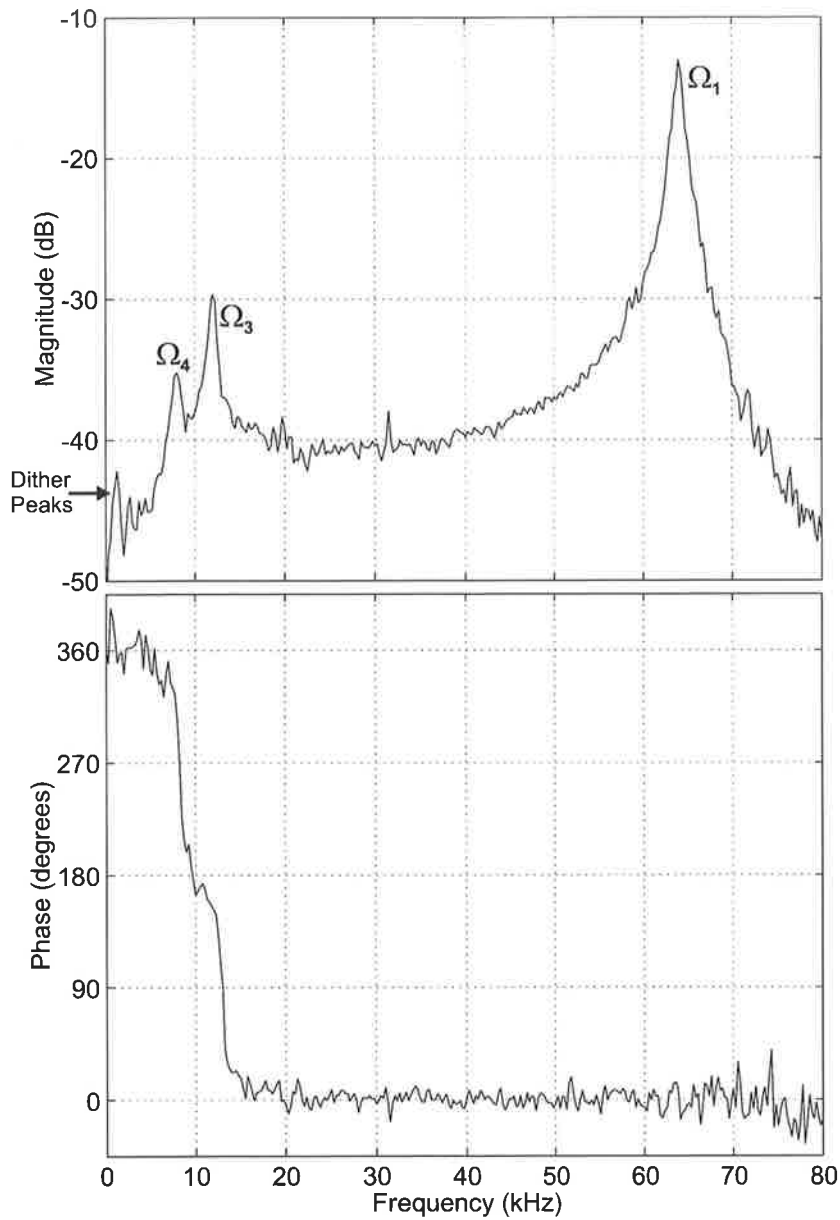


Figure 5.2: The cross spectrum of intensity noise fluctuations between cavity mode one and the total intensity, for four mode operation. The collective mode frequencies are; 8kHz, 12.25kHz, 20.5kHz and 64kHz. Weak electronic pickup is visible at 31.5kHz in the magnitude.

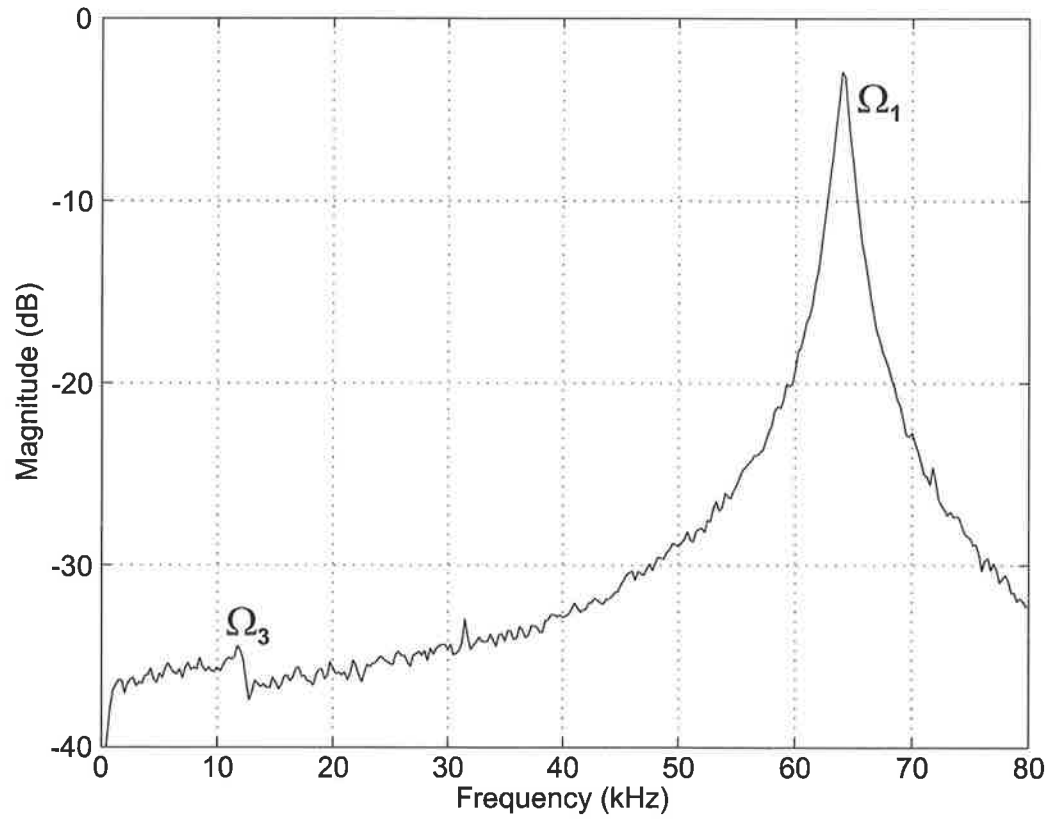


Figure 5.3: The power spectrum of intensity noise fluctuations of the total intensity for four mode operation, the collective mode frequencies are; 8kHz, 12.25kHz, 20.5kHz and 64kHz. Weak electronic pickup is visible at 31.5kHz.

	1,2	1,3	1,4	1,5	2,3	2,4	2,5	3,4	3,5	4,5
Ω_5	-	-	+	+	+	-	-	-	-	+
Ω_4	-	+	+	-	-	-	+	+	-	-
Ω_3	+	-	-	-	-	-	-	+	+	+
Ω_2	-	+	-	+	-	+	-	-	+	-
Ω_1	+	+	+	+	+	+	+	+	+	+

Table 5.4: The phase pattern for five mode operation, where Ω_1 is the highest frequency collective mode.

The cross spectrum between the intensity noise fluctuations of cavity modes one and two is shown in Fig. 5.4 as an example. Five collective mode resonances were found in the magnitude corresponding to the five oscillators. The collective modes are labeled; Ω_5 , Ω_4 , Ω_3 , Ω_2 and Ω_1 in order of increasing frequency. Zeros and their corresponding phase jumps are found at 1.75kHz, 14.75kHz, 22kHz and 29.75kHz. In this example, the intensity noise fluctuations are in antiphase at Ω_5 , Ω_4 & Ω_2 whilst they are in phase at Ω_3 & Ω_1 . Table 5.4 shows the pattern of interferences for the cavity mode pairs.

Using the same labeling conventions and derivation method as outlined in Sec. 3.3.3, the phases of the contributions of the cavity modes to the collective modes are presented in Table 5.5. We check the consistency of the phase pattern by comparing the relative phase between each of the pairs of cavity modes, at a particular collective mode frequency. For example at Ω_4 , the relative intensity noise fluctuations of cavity modes two and three are in antiphase which concurs with the phase pattern for that cavity mode pair.

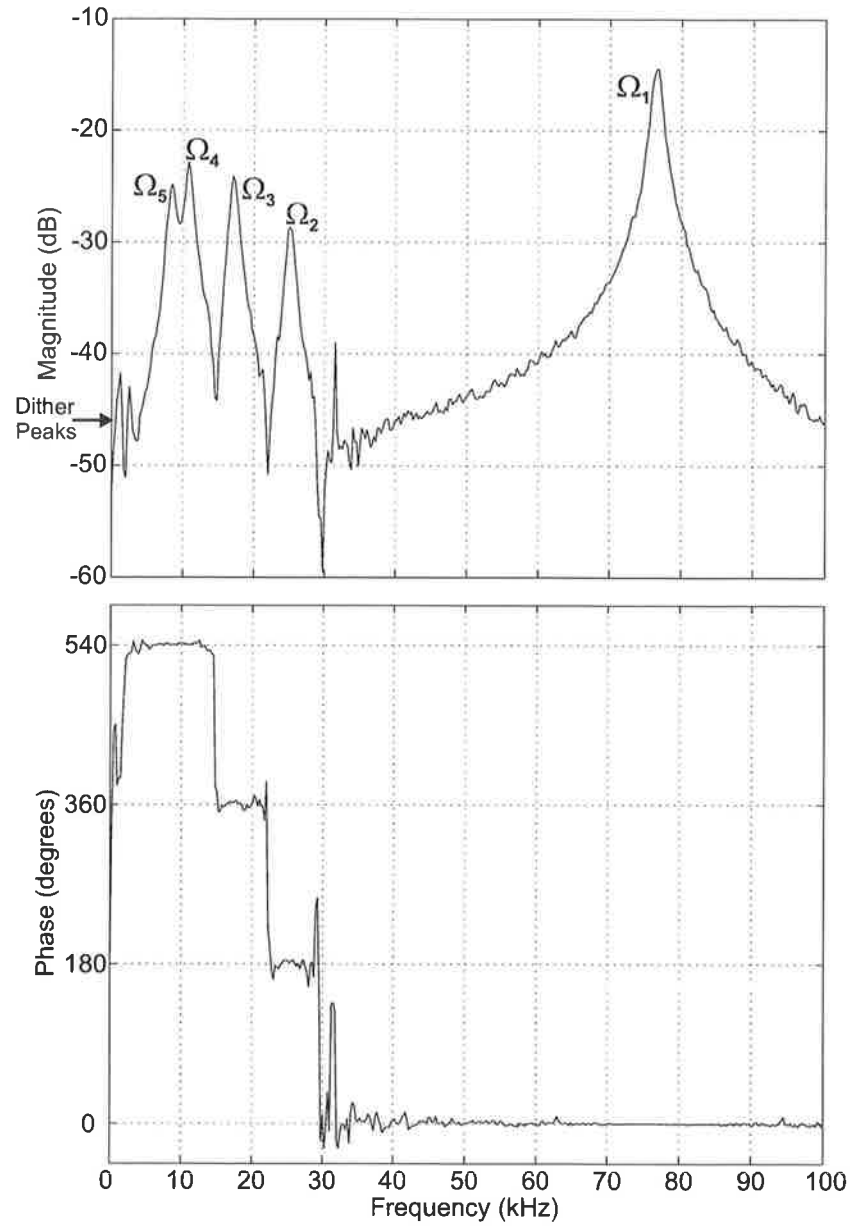


Figure 5.4: The cross spectrum of intensity noise fluctuations between cavity modes one and two, for five mode operation. The collective mode frequencies are; 8.5kHz, 10.75kHz, 17kHz, 25kHz and 76.75kHz. Electronic pickup is visible at 31.5kHz.

	1	2	3	4	5
Ω_5	+	-	-	+	+
Ω_4	+	-	+	+	-
Ω_3	+	+	-	-	-
Ω_2	+	-	+	-	+
Ω_1	+	+	+	+	+

Table 5.5: The phase clusters for five mode operation.

Therefore the phase pattern was found to be self consistent. By definition, the contributions of cavity mode one to the intensity noise fluctuations of each collective mode are constructive. The other cavity modes contribute both constructively and destructively to the fluctuations, for example, cavity mode three contributes destructively to Ω_5 & Ω_3 and constructively to Ω_4 , Ω_2 and Ω_1 and so on. Therefore, for $N = 3 \dots 5$ cavity modes, we found that the phases of the contributions of the cavity modes to the collective modes, did not show a simple ordering.

The cross spectrum between cavity modes and the total intensity exhibited similar features to those discussed in this and the preceding chapters. For example, each of the low frequency collective modes did not appear consistently in each of the cross spectra. Where small resonances occurred in the cross spectrum magnitude, they were associated with roll offs in the phase. Therefore we do not show results from this pairing.

The very small oscillation of the total intensity, at the low frequency collective modes, see Fig. 5.5, is the cause of the weak resonances and phase roll offs observed in the cross spectrum between cavity modes and the total

intensity. No resonances at Ω_5 , Ω_4 and Ω_2 are visible in the power spectrum of the total intensity noise fluctuations shown in Fig. 5.5. However, there appears a vestige of Ω_3 approximately 2-3dB above the surrounding noise. At this frequency, Table 5.5 suggests that the constructive contributions of cavity modes one and two outweigh the destructive contributions from the other cavity modes.

Our array of globally coupled nonlinear oscillators showed that for $N = 2 \dots 5$ cavity modes, antiphase dynamics nearly destroyed all of the low frequency collective modes. For each N , the only harmonic or sum and difference frequency of the collective modes observed was $2 \times \Omega_1$. As an example, we showed this in Fig. 3.1 where it was approximately 100 times smaller than Ω_1 . The experimental results of this thesis show that nearly all of the energy in the array is concentrated in Ω_1 and are indeed signatures that the response of the array to perturbations is very close to linear [7]. This also confirms our assumption in our model, Eq. (4.3). Other authors excite similar Nd:YAG lasers by swept frequency [53, 54]. For meaningful comparisons of excitation by ambient noise and swept frequency, the response of the array to perturbations must be of the same order, for example linear.

In these articles, the antiphase dynamics nearly killed all of the low frequency collective modes in the total intensity fluctuations. The response of the arrays were measured to be very close to linear, indeed it was stated that linear excitation was a requirement of their experiment so they could compare the results with their model.

The laser used by Stamatescu and Hamilton [53] was almost identical to ours apart from the optical length of the laser cavity, the pump rate and the

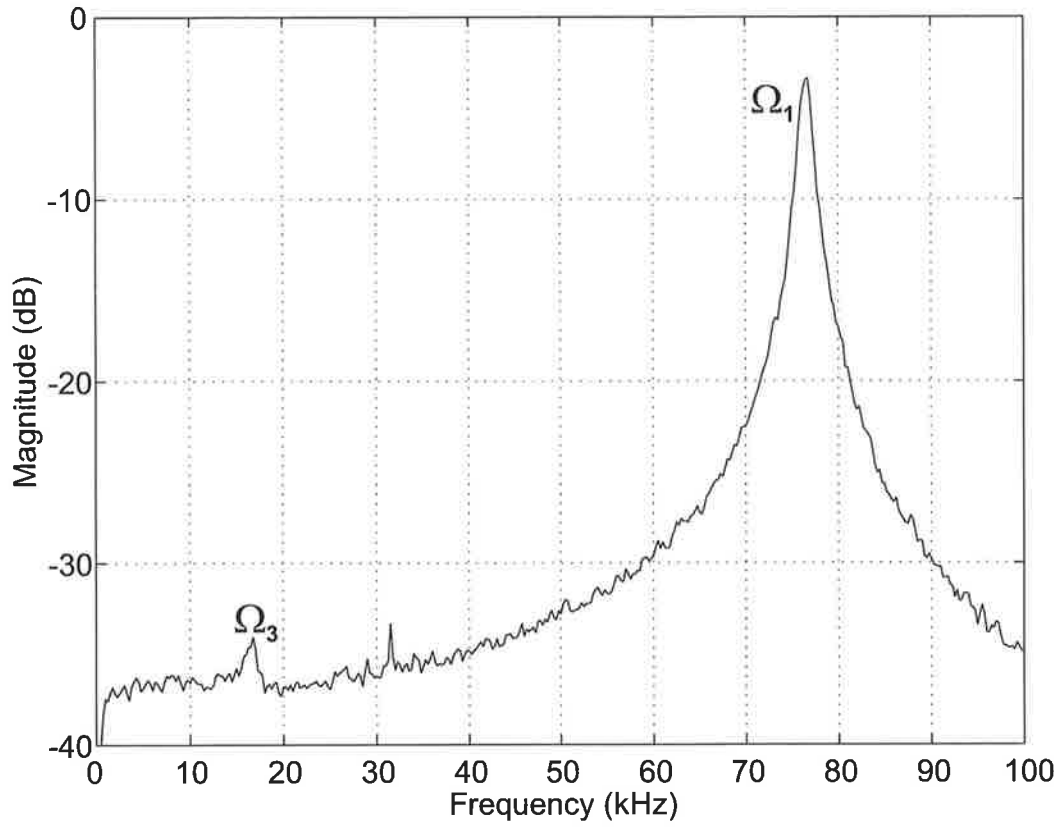


Figure 5.5: The power spectrum of total intensity noise fluctuations for five mode operation. The collective mode frequencies are; 8.5kHz, 10.75kHz, 17kHz, 25kHz and 76.75kHz. Electronic pickup is visible at 31.5kHz.

relative gains seen by each of the five cavity modes. Abraham et al. [54] used a configuration where the ratio of gain medium to cavity length (fill factor) was smaller than ours or Stamatescu and Hamilton [53]. Similarly, the pump rate and the relative gains of the cavity modes were different.

Stamatescu and Hamilton [53] used a similar model to ours described in Chap. 4, also based on the TSD equations but with a modulated pump parameter and found that the measured phase clusters did not match the simple ordering predicted. This is highlighted in Tables 5.6 & 5.7 which have been adapted from their data to suit my labeling conventions. In these tables, the phases were absolute measurements, whereas by definition ours are relative, see Table 5.5. This explains the difference at Ω_1 , which in both cases survived in the total intensity noise fluctuations. If we consider only the relative phases between each of the cavity modes, at each collective mode separately, we see that the only agreement between the techniques is that simple ordering of the phase clusters does not exist. The lack of simple ordering is not altogether surprising given that our lasers are almost identical.

Abraham et al. [54] measured the magnitude of the response of the total and modal intensities. They found that a change in the position of the cavity mode spectrum with respect to the gain spectrum, or changes in the pump parameter profoundly affected the response of low frequency collective modes in the modal intensities, i.e. the relative heights of the peaks. However, the highest frequency collective mode was not noticeably affected. In the total intensity transient, the low frequency collective modes remained antiphased whilst the highest frequency remained unaffected. Interestingly, the authors found kinks in the magnitude of the transfer function of the modal intensities,

	1	2	3	4	5
Ω_5	+	-	-	-	+
Ω_4	+	-	-	+	+
Ω_3	+	+	-	-	+
Ω_2	+	-	+	-	-
Ω_1	-	-	-	-	-

Table 5.6: The measured phase clusters for five mode operation using swept frequency excitation [53], where Ω_1 is the highest frequency collective mode.

	1	2	3	4	5
Ω_5	+	+	+	+	-
Ω_4	+	+	+	-	-
Ω_3	+	+	-	-	-
Ω_2	+	-	-	-	-
Ω_1	-	-	-	-	-

Table 5.7: The phase clusters for five mode operation obtained from the TSD equations [53].

similar to those found in our cross spectra between cavity modes and the total intensity. These were due to the combined responses of the collective modes which have different phases and strengths of response to the modulation. In their theoretical model, they took into account that the gain medium did not fill the cavity. The calculated phase clusters did not show simple ordering, nor did they match the experimental results of Stamatescu and Hamilton [53] or ours.

The measured differences in phase clusters could be attributed to the different type of excitation of the array. Indeed it was noted by Stamatescu [86] that in some cases, the relative magnitude of the peaks at the low frequency collective modes, with respect to each other, in the power spectrum of the intensity noise of an individual cavity mode were different in the transfer function of the same cavity mode [86]. This implies that excitation with broadband noise is a frequency mixing effect whereby all frequency components of the noise contribute to the excitation of the array, whereas excitation with a swept frequency occurs at discrete frequencies [86]. Another possible explanation for the different phase clusters could be the differences in laser parameters, for example, the ratio of gain medium to cavity length, the pump parameter, the length of the laser cavity or the relative gains of the cavity modes. Determining the exact source of these differences is outside the scope of this thesis. However, suggestions as to how one may pursue this and quantitatively measure the contributions of the cavity modes to the collective modes, for different excitations, are given in Chap. 7.

5.3 Conclusion

We have applied the cross spectral technique to the laser operating with four and five cavity modes. The results for the cross spectral phases of the cavity mode pairs were found to be repeatable, similar to the two and three mode cases. The phase of the fluctuations of the cross spectra between cavity modes and the total intensity were found to vary more than the two or three mode cases, this being further evidence of a strong antiphase dynamics. Importantly we found that the phase clusters, at the collective mode frequencies, became less ordered as the number of cavity modes increased. We discuss the implication of this result, on the TSD model, in Chap. 6. In comparing our five mode operation data with similar Nd:YAG lasers excited by swept frequency, we found that the phase clusters at the collective modes were different. This could be attributed to the different excitation of the array or to the different laser parameters.

Chapter 6

Threshold Behaviour

6.1 Introduction

It is well known that the behaviour of a single mode laser above threshold differs dramatically from that below threshold. For example, the linewidth of the emission is wide below threshold, whilst above it narrows sharply. As such, theoretical studies of, for example the intensity, treat the small region near lasing threshold as a special case [43, 44, 78, 95]. Therefore, we perform some detailed experiments on our multimode laser operating in the region of N modes and $N + 1$ modes. These measurements also provide information about the competition for gain between the cavity modes which has implications on the models used to describe solid state lasers.

It has been noted in a laser almost identical to ours, that the intensity of a cavity mode which has a low gain, may eclipse one which has a higher gain, even though the former has a higher threshold pump power [71]. Similar results have been demonstrated in other solid state lasers [54, 70, 72]. In performing the experiments discussed in the preceding chapters, we noted

that this behaviour sometimes occurred in our laser. We also noted on the oscilloscopes, which monitored the modal intensities and the servoloop lock, that the AC noise of a cavity mode might display similar characteristics to its intensity. We investigate this in detail by measuring statistical properties of the AC component of the intensity noise, of the first two cavity modes to reach threshold, as a function of pump parameter.

In Chaps. 3 & 4 we showed that two mode operation is the simplest form of antiphase dynamics. For three mode operation, the phase clusters were slightly more complex and did not show the simple ordering predicted or observed in other lasers [51, 55]. In Chap. 5 we showed that the clustering of phases of the individual oscillators, at the collective mode frequencies, became less ordered for increasing number of cavity modes. As the laser passes between N mode and $N + 1$ mode operation with an increase in the pump power, a new collective mode of oscillation is added to the array. We therefore expect the cross spectra of any pairing to change at the threshold of a cavity mode, thereby maintaining the assumption made in Chap. 4 that, above threshold of a cavity mode, all $\text{sgn}(a_{m,k})$ are constant, insofar as operation with a given number of cavity modes is maintained.

We test this hypothesis by measuring the interferences of the array for the transition from two to three cavity modes and three to four cavity modes. At the transition from N mode to $N + 1$ mode operation, we expect to find a resonance corresponding to the new collective mode in the cross spectrum magnitude and the phase of the fluctuations may change at one or more of the collective mode frequencies. As shown in the preceding chapters, the low frequency collective modes were difficult to observe in the cross spectrum

between cavity modes and the total intensity. Therefore, we only present results for pairs of cavity modes.

The optical layout used in these experiments was the same as for two mode operation, see Chap. 2. Similarly, the gain of each of the cavity modes was chosen to be different.

6.2 AC Noise

In Appendix B, we explain that the root mean square (rms) of the noise measured with a voltmeter is equivalent to the standard deviation of the noise we calculate from the data recorded by the A/D. The AC noise component of a cavity mode was measured with a HP400E Voltmeter. This device was used because it averaged the noise over a longer period of time than the A/D, of order 2-3s for each measurement point, thus it negates the effect of any large or small, instantaneous fluctuations. Using a voltmeter also means we do not need to process large volumes, of order hundreds of megabytes, of data.

The YAG laser was locked to FP1 which transmitted the first cavity mode to reach threshold. FP2 was used as an optical spectrum analyser to monitor the threshold position of the cavity modes with respect to the pump. The pump was increased in very small steps, 0.1mA of the diode current, near the threshold position of cavity modes one and two. Once these modes began to oscillate, the pump was increased in slightly larger steps, 0.2mA. The process was repeated for cavity mode two beginning at its threshold, i.e. the YAG laser cavity was locked to FP1 which was resonant with the second cavity mode to reach threshold.

Because the acquisition time for this data was of the order of minutes compared with a few seconds for the power or cross spectra measurements discussed in the preceding chapters, there was potential for drift of the cavity mode spectrum with respect to the gain spectrum. The measurement is therefore limited to the first two modes to reach threshold.

The measured noise is shown in Fig. 6.1. The threshold points of the first two cavity modes are 74.8mA and 75.2mA. The graph shows that the noise in each of the two cavity modes increases rapidly as they begin to oscillate. The noise in each mode peaks and then decays to a plateau, with the noise in the second cavity mode slightly eclipsing the first. The point at which the noise peaks and then decays appears to be approximately as the adjacent mode in the cavity mode spectrum, with respect to the gain spectrum, begins to oscillate, see Fig. 2.3. For example, in the case of the second mode to oscillate, it occurs as the fourth mode ($N + 2$) begins to oscillate.

The measured behaviour is similar to that of Nd:YAG lasers [54, 71, 72] of a similar design and a LNP laser [70] where the intensity noise of a mode which has a lower gain eclipses that of a mode which has a higher gain. Stamatescu and Hamilton [71] found that a local maxima of the intensity of a particular mode, was related to a new mode which began to oscillate. For example, the maximum intensity of the first mode occurred at the threshold of the third, which is adjacent to the former in the cavity mode spectrum. Khandokhin et al. [72] observed that modes with a higher gain did not necessarily enter the lasing process, rather adjacent modes in the cavity mode spectrum with a lower gain. As some of the modes expected to oscillate did

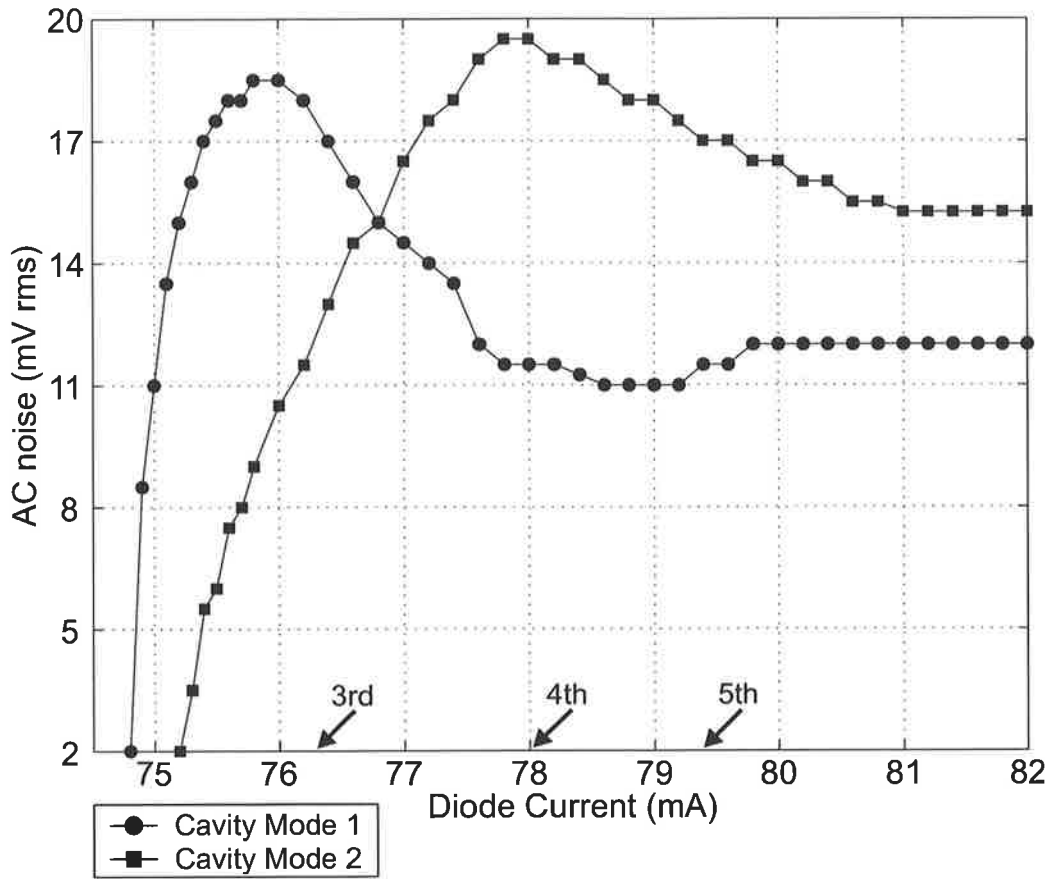


Figure 6.1: The measured AC noise in the first two cavity modes to reach threshold. The threshold pump parameter of the third, fourth and fifth modes are marked at 76.3mA, 78mA and 79.4mA respectively.

not, the optical spectrum became rarefied. Otsuka et al. [70] also observed that the intensity of a mode, on a separate lasing transition but with a lower gain, eclipsed those on a different transition with a higher gain. However, the intensity of some modes displayed a negative slope efficiency after a peak, which ultimately lead to their destruction.

6.3 Transition In The Cross Spectrum

A preliminary experiment was performed to establish any changes in the cross spectra. The laser was moved from single mode operation to three mode operation with large steps of the pump (0.3mA of the diode current) and all possible pairings of cavity modes recorded simultaneously with the total intensity. This was repeated as the laser was moved from two to four mode operation. The volume of data recorded was large compared with that acquired in Chaps. 3 & 4. Therefore an effort was made to keep the time to switch between pairs very short (of order a couple of seconds), thus maintaining lock of the laser cavity length and preventing drift of the cavity mode spectrum with respect to the gain spectrum.

In a more detailed study, we focused on specific pairings of cavity modes, namely 1,2 & 1,3 & 2,3. To capture the exact nature of any transition in the cross spectrum, the laser was moved from two to three mode operation with fine steps of the pump parameter, i.e. 0.1mA, and only one pairing measured. This was repeated at the transition from three to four mode operation and for the other pairings.

In analysing the data, we re-confirmed the phase patterns of the aforementioned modal pairings for three and four mode operation, when the pump

parameter was well above threshold of the third and fourth cavity modes respectively. The fluctuations of the intensity noise at the highest frequency collective mode (Ω_1), in any pairing, were found to be in phase. As shown in the preceding chapters, this is the mode which survives in the power spectrum of the total intensity. This discussion is therefore limited to the low frequency collective modes.

We first consider the transition between two and three mode operation. As an example of a transition in the cross spectrum, we show the pairing of cavity modes one and two in Figs. 6.2-6.4. Although, intensity fluctuations were recorded for 0.1mA increments of the pump parameter, for brevity we show figures where the pump parameter is separated by 0.2mA.

Figure 6.2 shows the laser operating with two cavity modes. The collective modes are labeled as in the preceding chapters. Zeros and their corresponding phase jumps occur at approximately 5.5kHz & 15.75kHz, the fluctuations of the intensity noise at Ω_2 are in antiphase. The intensity fluctuations are in phase at frequencies $<5\text{kHz}$ which is the expected position of a new resonance (Ω_3) for three mode operation.

The pump parameter in Fig. 6.3 is 0.1mA greater than threshold of the third cavity mode. Therefore the magnitude of the cross spectrum shows resonances at two low frequency collective modes. The “dither peaks” are visible in this spectra. As shown in Appendix A, these frequencies do not pertain to the intensity fluctuations of interest. Zeros occur at approximately 1.75kHz, 6.25kHz & 16.5kHz. The phase of the fluctuations at Ω_3 show a sharp negative slope and are $\approx 140^\circ$ out of phase whilst the fluctuations at Ω_2 are in antiphase.

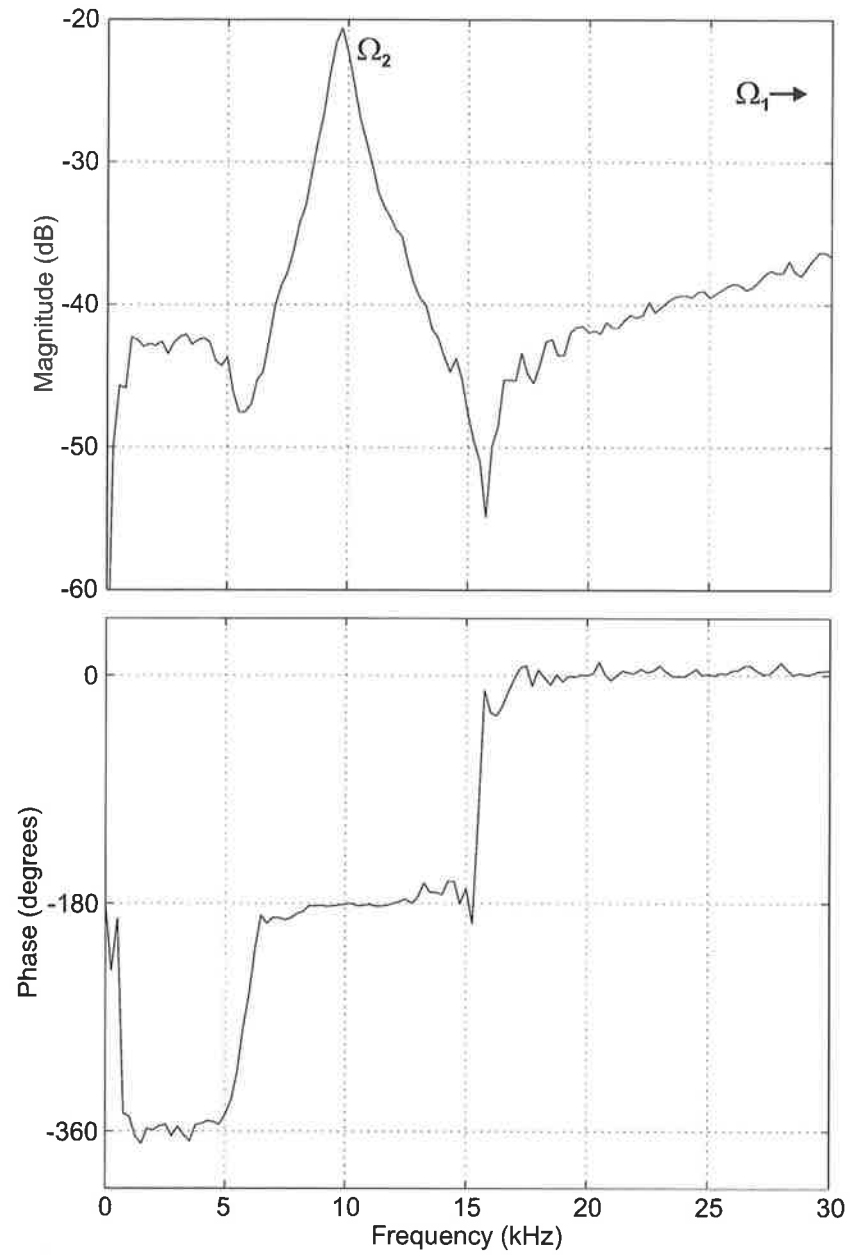


Figure 6.2: The cross spectrum of intensity noise fluctuations between cavity modes one and two, for two mode operation. The lowest collective mode frequency (Ω_2) is 9.75kHz. The pump parameter is 0.1mA less than the threshold of the third cavity mode.

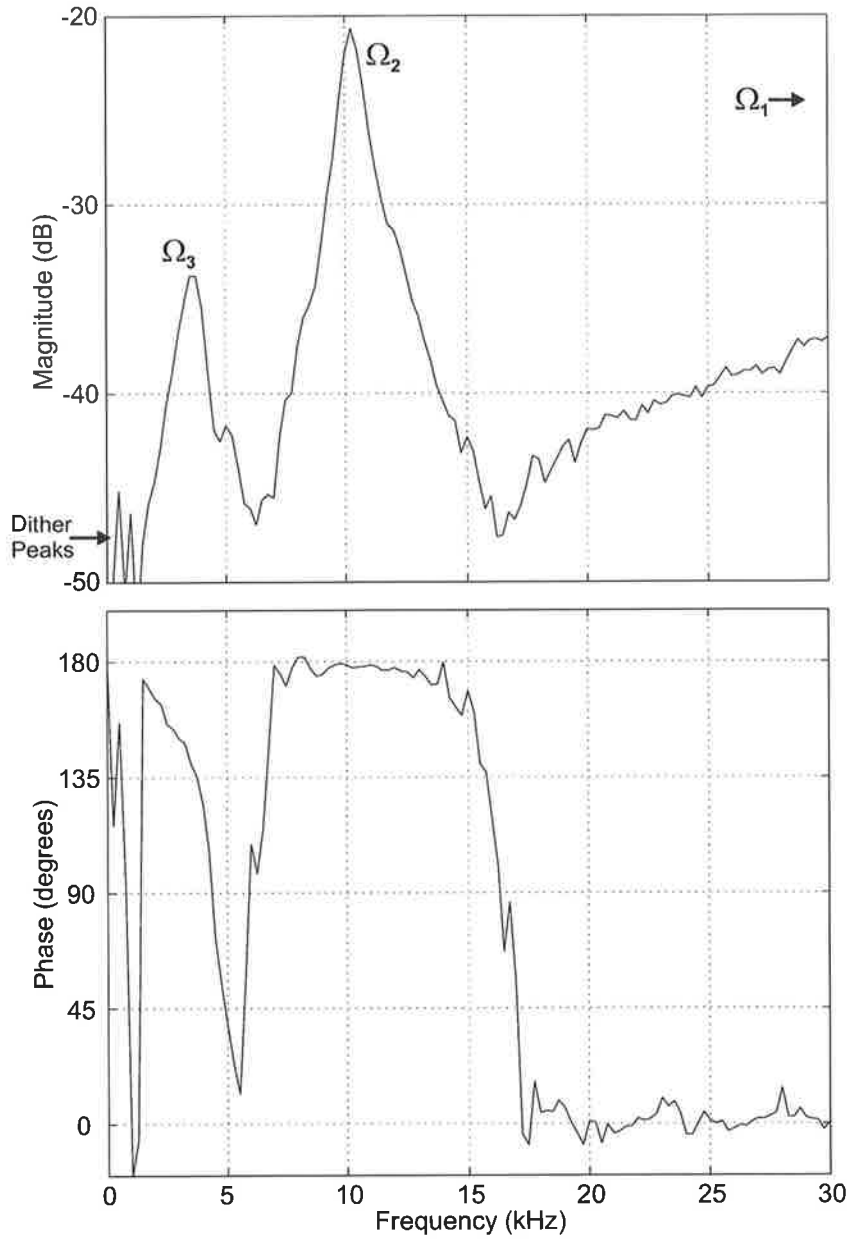


Figure 6.3: The cross spectrum of intensity noise fluctuations between cavity modes one and two, slightly above threshold of the third cavity mode. The low frequency collective modes (Ω_3 , Ω_2) are; 3.5kHz and 10.25kHz respectively. The pump parameter is 0.2mA greater than Fig. 6.2.

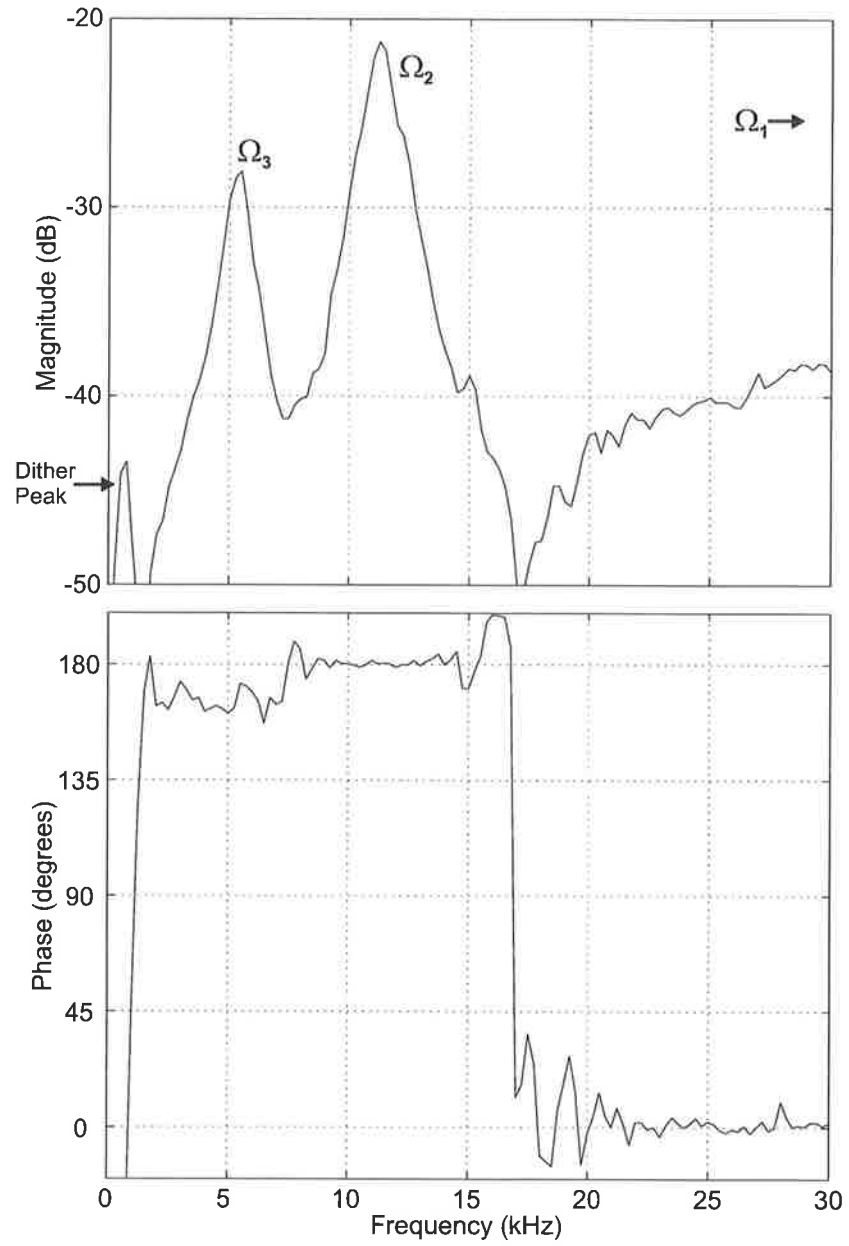


Figure 6.4: The cross spectrum of intensity noise fluctuations between cavity modes one and two, above threshold of the third cavity mode. The low frequency collective modes are; 5.5kHz and 11.25kHz. The pump parameter is 0.2mA greater than Fig. 6.3.

Pump Parameter (mA)	Figure	Ω_3 (kHz)	Phase (degrees)
threshold - 0.1mA	6.2	—	—
threshold of third mode	—	2.25	25
threshold + 0.1mA	6.3	3.5	140
threshold + 0.2mA	—	4.75	155
threshold + 0.3mA	6.4	5.5	175

Table 6.1: Phase of the intensity fluctuations between cavity modes one and two, at Ω_3 , for the transition from two to three mode operation.

In Fig. 6.4, zeros occur at approximately 1.5kHz & 17.25kHz in the magnitude. The zero is on the left hand side of Ω_3 whereas it was on the right hand side of this collective mode in Fig. 6.3. The fluctuations of the intensity noise, within experimental error, are in antiphase at Ω_3 , Ω_2 . Note that in each case presented, the phase of the fluctuations at Ω_2 do not change, that is they remain in antiphase.

The phase of the intensity fluctuations at Ω_3 , for 0.1mA increments of pump parameter, are given in Table 6.1. Figures 6.2-6.4 and Table 6.1 indicate a transition in the behaviour of the cross spectrum. We interpolate from them that the phase of the fluctuations at Ω_3 , began approximately in phase and moved to antiphase fluctuations, whilst one of the zeros decreased in frequency.

For the pairing of cavity modes two and three, we found a similar transition. The phase of the fluctuations at Ω_2 remained in phase, whilst the phase of the fluctuations at Ω_3 began approximately in antiphase and moved to in phase, with a zero being added in the magnitude. In analysing the pairing of

	1,2	1,3	2,3
Ω_3	+	-	-
Ω_2	-	+	-
Ω_1	+	+	+

Table 6.2: The interpolated phase pattern, expressed in terms of cavity mode pairs, at threshold of the third cavity mode. The columns correspond to the cavity mode pairs and the rows to the collective modes, where Ω_1 is the highest frequency collective mode.

cavity modes one and three, we did not find any transition in the behaviour of the cross spectrum at the low frequency collective modes.

The interpolated phase pattern at threshold of the third cavity mode is given in Table 6.2. The phase clusters, Table 6.3, were determined using the arguments set out in Chap. 3. The phase pattern was found to be self consistent by comparing the relative phases of the contributions of the cavity modes, at the collective mode frequencies. These tables (6.2 & 6.3) are different from those presented in Chap. 3, Tables 3.1 & 3.3, where the laser was operated further ($\approx 0.8\text{mA}$) above threshold of the third cavity mode. We found the transition between the behaviour represented by Tables 6.2 & 6.3 to that by Tables 3.1 & 3.3 to be repeatable.

As an example of a transition in the cross spectrum near threshold of the fourth mode, we show the pairing of cavity modes two and three in Figs. 6.5-6.7. The pump parameter in each of the figures is separated by 0.2mA with the pump parameter in the first figure 0.4mA higher than the threshold of the fourth cavity mode. Therefore three collective mode resonances are

	1	2	3
Ω_3	+	+	-
Ω_2	+	-	+
Ω_1	+	+	+

Table 6.3: The group of clusters at threshold of the third cavity mode.

visible in the low frequency component of the magnitude of the spectrum.

Zeros corresponding to phase jumps occur at 9.5kHz and ≈ 19 kHz in Fig. 6.5. The fluctuations of the intensity noise are in phase at Ω_4 & Ω_3 whilst they are in antiphase at Ω_2 .

Figure 6.6 shows a zero in the magnitude at approximately 20kHz. A resonance is visible at 9.25kHz (Ω_3) however, the phase has a sharp positive slope at this frequency. The fluctuations are approximately 220° out of phase, or -140° in relative terms. The phase of the fluctuations at Ω_4 are in phase whilst they are in antiphase at Ω_2 . In Fig. 6.7, zeros are visible at 8.5kHz and 21.25kHz. The fluctuations are in phase at Ω_4 whilst they are in antiphase at Ω_3 & Ω_2 . The phase of the intensity fluctuations at Ω_3 , for 0.1mA increments of pump parameter, are given in Table 6.4.

Figures 6.5-6.7 and Table 6.4 indicate a transition in the phase of the cross spectrum at Ω_3 from in phase fluctuations to antiphase. Correspondingly in the magnitude, a zero decreased in frequency.

The pairing of cavity modes one and two showed a similar transition in the cross spectrum. A transition began approximately 0.4mA above threshold of the fourth cavity mode. The phase pattern for this pairing, very close to but finitely above threshold of the fourth cavity mode was: - - - +, with

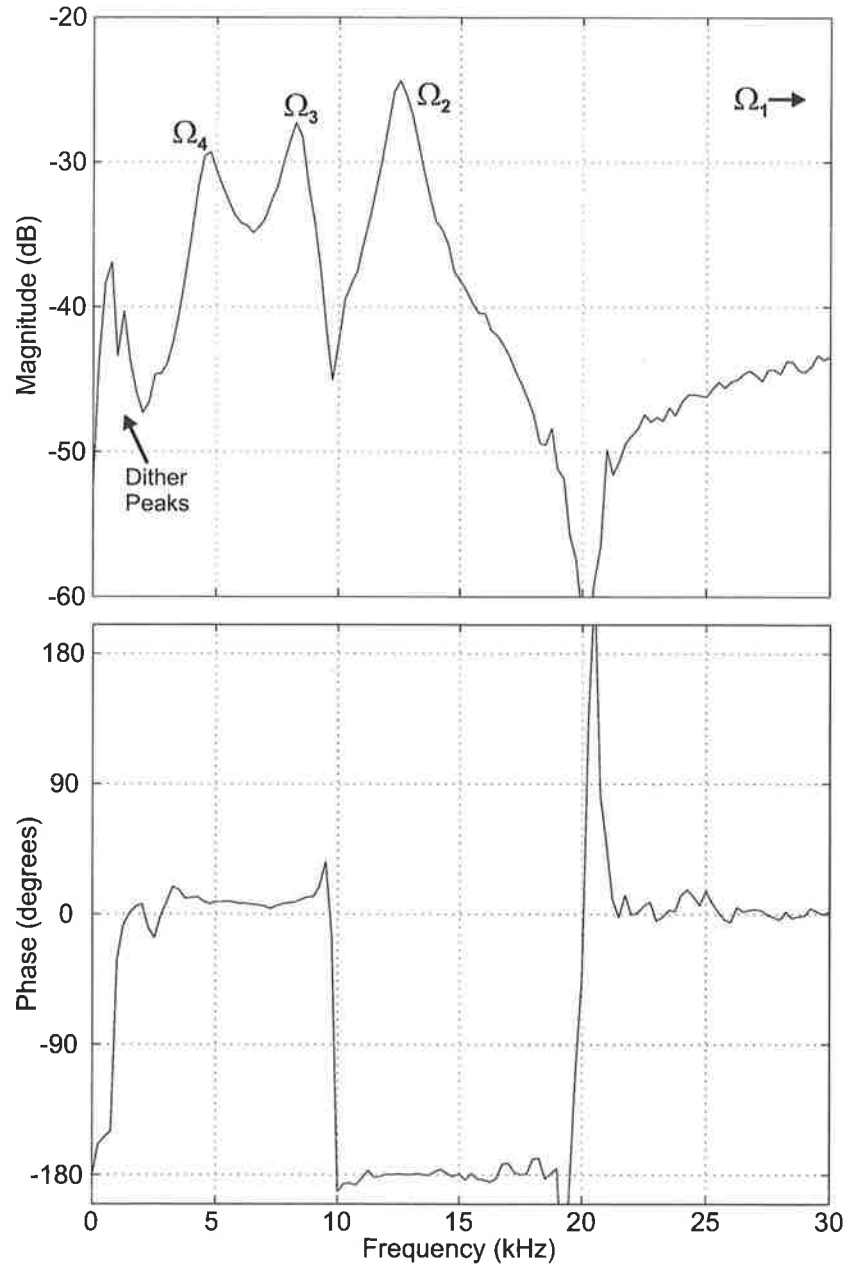


Figure 6.5: The cross spectrum of intensity noise fluctuations between cavity modes two and three, slightly above ($\approx 0.4\text{mA}$) threshold of the fourth mode. The low frequency collective modes (Ω_4 , Ω_3 & Ω_2) are; 4.75kHz, 8.25kHz and 12.5kHz respectively.

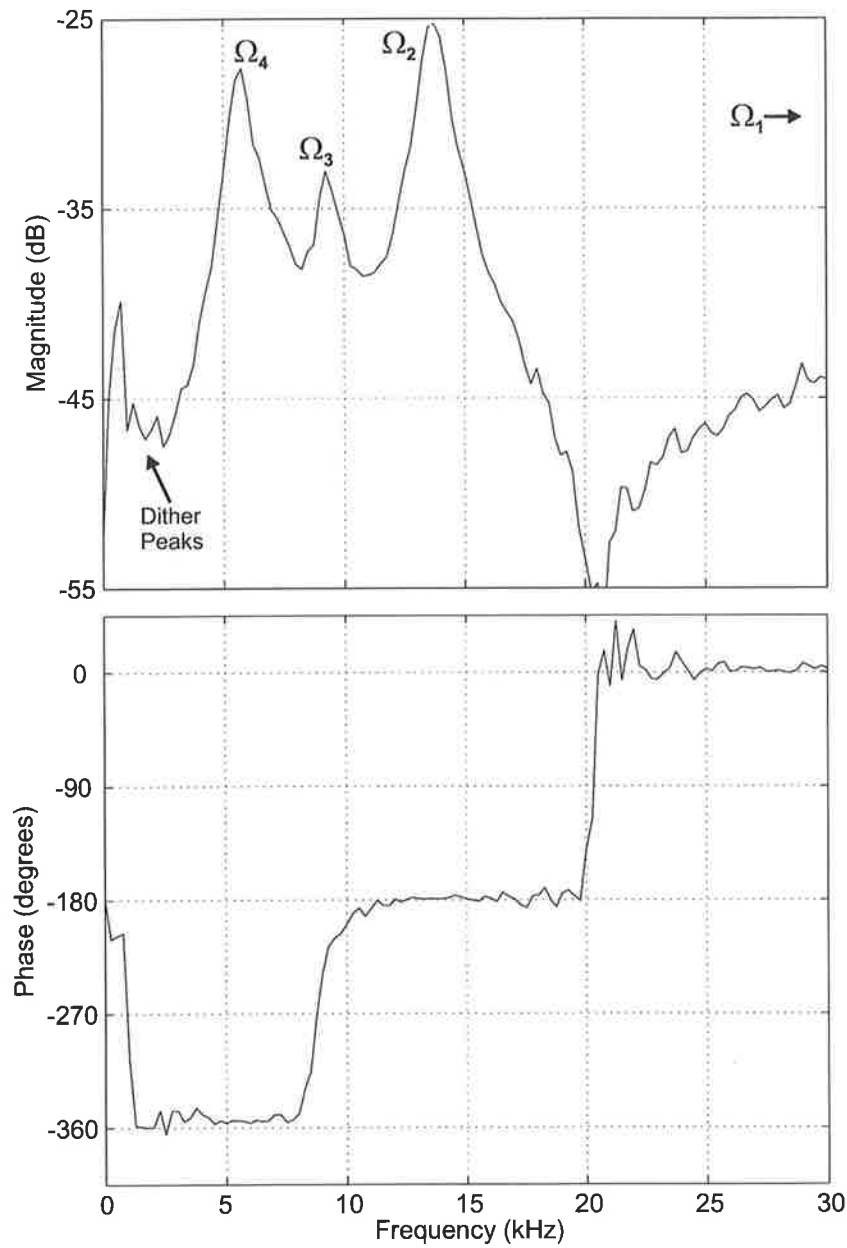


Figure 6.6: The cross spectrum of intensity noise fluctuations between cavity modes two and three, slightly above threshold of the fourth mode. The pump parameter is 0.2mA greater than Fig. 6.5. The low frequency collective modes are; 5.75kHz, 9.25kHz and 13.75kHz.

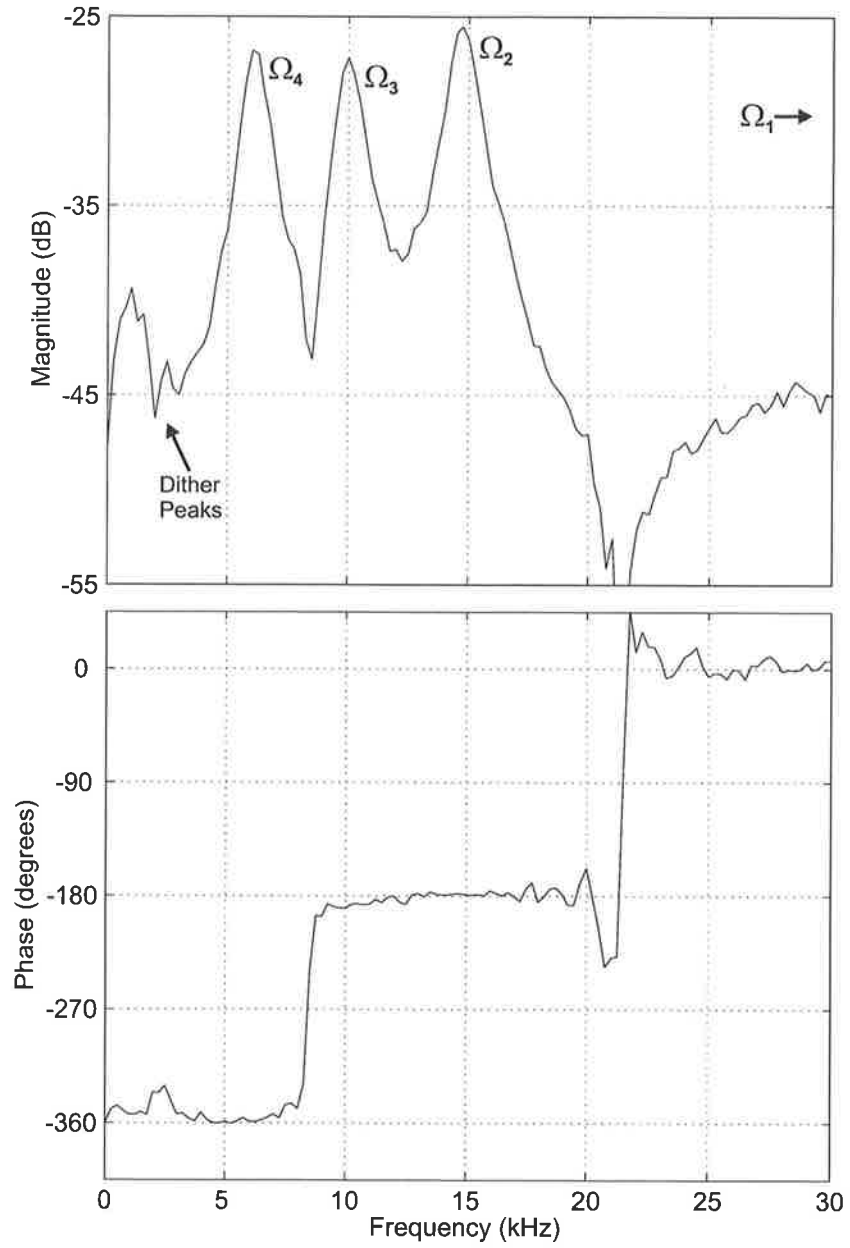


Figure 6.7: The cross spectrum of intensity noise fluctuations between cavity modes two and three, above threshold of the fourth mode. The pump parameter is 0.2mA greater than Fig. 6.6. The low frequency collective modes are; 6kHz, 10kHz and 14.75kHz.

Pump Parameter (mA)	Figure	Ω_3 (kHz)	Phase (degrees)
threshold of fourth mode + 0.4mA	6.5	8.25	5
threshold + 0.5mA	—	8.5	-15
threshold + 0.6mA	6.6	9.25	-140
threshold + 0.7mA	—	9.75	-170
threshold + 0.8mA	6.7	10	-185

Table 6.4: Phase of the intensity fluctuations between cavity modes two and three, at Ω_3 , slightly above threshold of the fourth cavity mode.

increasing collective mode frequency. Originally there was one zero in the cross spectrum magnitude. At the end of the transition, the phase pattern for this pairing was: $- + - +$, and correspondingly there were three zeros in the magnitude. No transition in the cross spectrum was found for the pairing of cavity modes one and three.

In summary, we found a transition in the behaviour of the cross spectrum of cavity mode pairs 1,2 & 2,3 for the laser operating near threshold of the third and fourth cavity modes. The transitions were repeatable and did not occur at the threshold pump parameter of these modes, rather a small but somewhat arbitrary pump level above them.¹ Therefore, the prediction that the $\text{sgn}(a_{m,k})$ (i.e. phase of $a_{m,k}$) are constant, insofar as operation with a given number of modes was maintained (see Sec. 4.2.3), does not hold for our

¹In the experiments presented in the preceding chapters, the laser was operated far above threshold of the nearest cavity mode ($\approx 0.8\text{mA}$) to try and prevent any ambiguity as to the identity of the cavity modes. As shown in this chapter, the AC noise was somewhat stable (at the plateaus) and any transition in cross spectrum was also avoided.

laser. This means that the relations, Eqs. 4.8 & 4.9, are not “universal”. For our laser, the “universality” of the relations is limited in the sense that, for operation with a given number of modes, they become valid once the pump parameter reaches a small but somewhat arbitrary pump parameter above threshold of a particular cavity mode.

We try to explain these transitions in the cross spectra in the framework of the global behaviour of the array, i.e. the effect of a new collective mode of oscillation on the total intensity fluctuations. The latter were recorded simultaneously with each pairing of cavity modes for each pump parameter. In this example, we use data from the pairing of cavity modes two and three. For brevity we show the transition from two to three mode operation although the transition from three to four mode operation showed similar results. To highlight the collective mode frequencies for each pump parameter, we show a power spectrum of a modal intensity fluctuation, conjointly with the power spectrum of the total intensity fluctuations.

Figure 6.8 shows the power spectrum of the second mode to reach threshold and the total intensity fluctuations, for two mode operation. The pump parameter is 0.1mA below the threshold of the third cavity mode. We use the label “dither peak” here also to indicate low frequency noise which does not pertain to the intensity fluctuations of interest. The collective mode frequencies (Ω_2 & Ω_1) are: 6.5kHz and 34kHz respectively. It can be seen from the figure that Ω_2 is antiphased in the total intensity fluctuations, similar to Figs. 3.4 & 4.1.

Figure 6.9 shows the power spectrum of the intensity fluctuations of mode three and the total intensity fluctuations, at the threshold of the third cavity

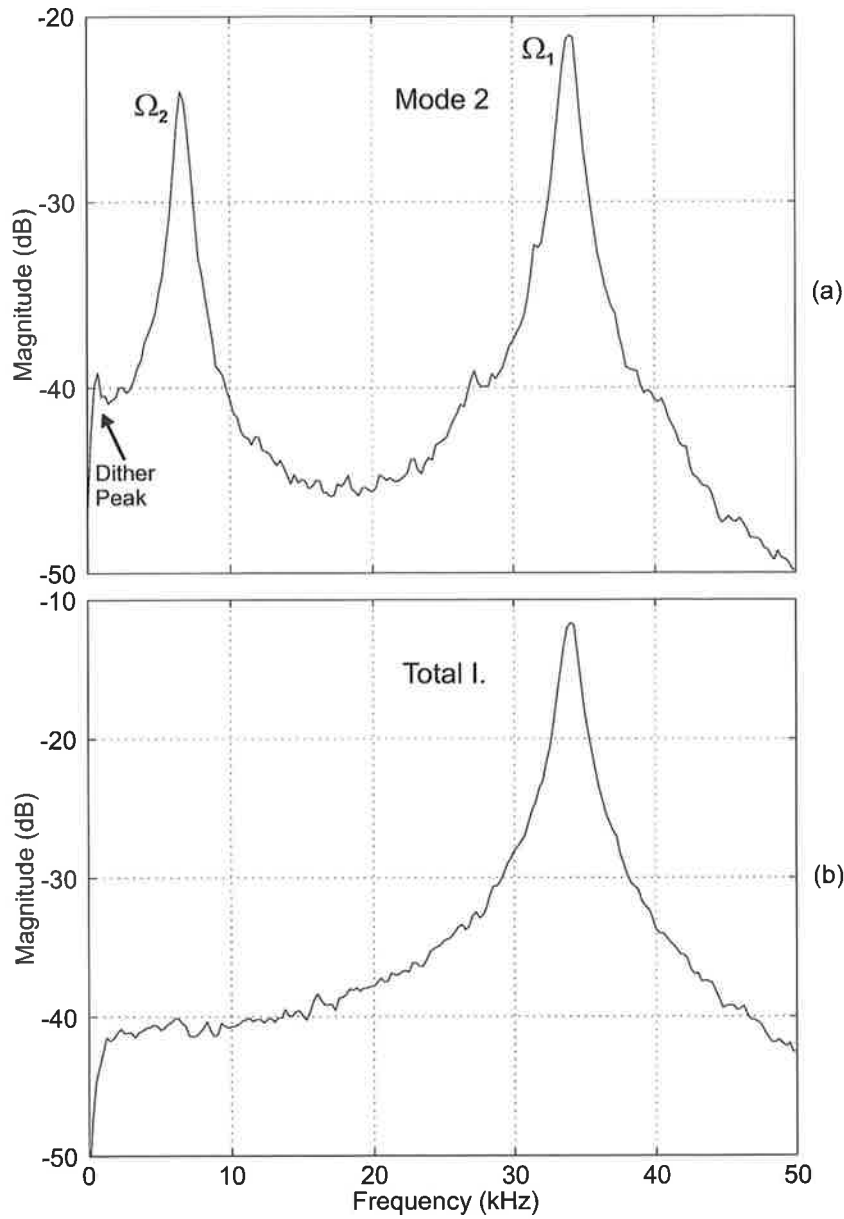


Figure 6.8: (a) The power spectrum of mode two. (b) The power spectrum of the total intensity fluctuations. In this figure, the pump parameter is 0.1mA less than the threshold of the third mode.

mode. Electronic pickup at $\approx 32\text{kHz}$ is visible in the modal intensity fluctuations because this mode has just begun to oscillate and the fluctuations are therefore small. The middle collective mode frequency Ω_2 , is obscured by the wing of Ω_3 . Its frequency was determined from the power spectrum of cavity mode two and the cross spectra between cavity modes two and three. The collective mode frequencies (Ω_3 , Ω_2 & Ω_1) are: 1.25kHz, 7kHz and 35kHz respectively. The total intensity fluctuations (Fig. 6.9(b)) do not appear to show any vestiges of the low frequency collective modes.

The power spectrum of cavity mode three is shown in Fig. 6.10(a) where the pump parameter is 0.1mA greater than the preceding figure. The collective mode frequencies (Ω_3 , Ω_2 & Ω_1) are: 3.25kHz, 7.5kHz and 36kHz respectively. The measured noise level in this cavity mode is low because it has just begun to oscillate, therefore a “dither peak” and electronic pickup are visible in this spectra. The resonance at 7.5kHz (Ω_2) is weak compared with the surrounding noise and is partially obscured by the wing of Ω_3 . The power spectrum of the total intensity fluctuations show that Ω_2 is antiphased whilst there appears to be a vestige of Ω_3 approximately 1-2dB above the surrounding noise. The vestige is similar to that presented in Chap. 3, see Fig. 3.7, where the laser was operated further above threshold of the third cavity mode.

These figures (6.8-6.10) indicate that a new collective mode is antiphased in the total intensity fluctuations as soon as it begins to oscillate, i.e. it has no effect on the global behaviour of the array. We found a similar result for the transition from three to four mode operation. Given the antiphase theorem which states that the N cavity mode intensities relax with N frequencies,

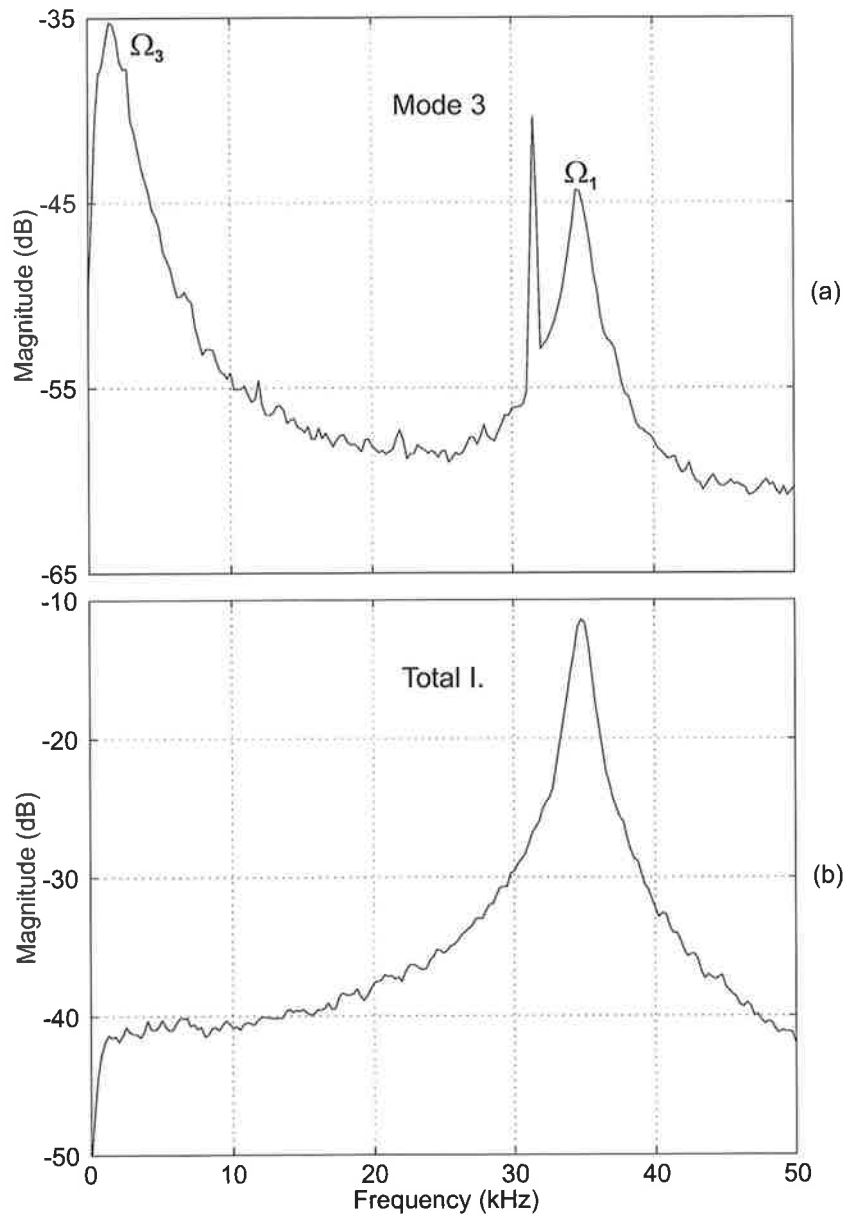


Figure 6.9: (a) The power spectrum of mode three at the threshold of the third cavity mode. (b) The power spectrum of the total intensity fluctuations at the threshold of the third cavity mode. The pump parameter is 0.1mA greater than Fig. 6.8.

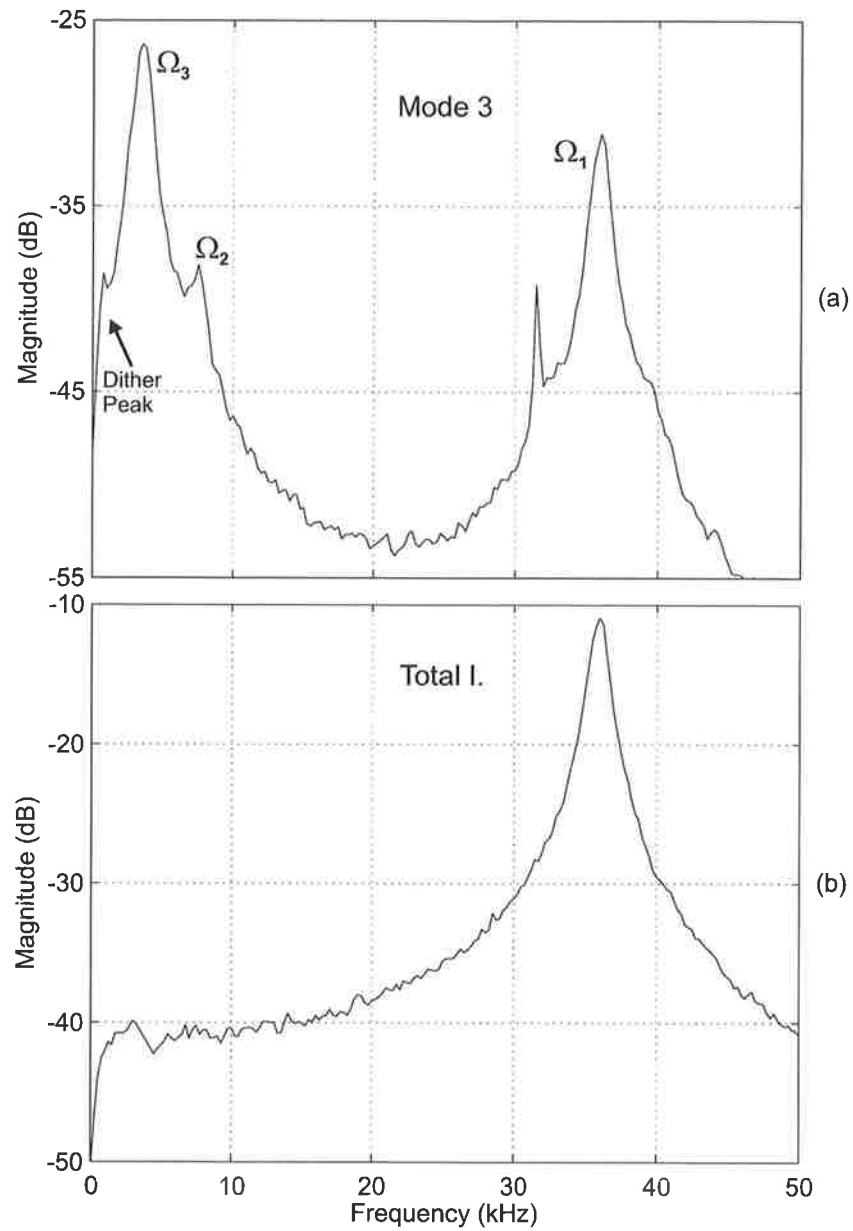


Figure 6.10: (a) The power spectrum of cavity mode three slightly above threshold of the third mode. (b) The power spectrum of the total intensity noise slightly above threshold of the third mode. The pump parameter is 0.1mA greater than Fig. 6.9.

whilst the total intensity relaxes with just one [7, 8], these results are not altogether surprising.

6.4 Deviations From The TSD Model

In this thesis, we have shown that for operation with $N = 3 \dots 5$ cavity modes, the phase clusters do not show a simple ordering of the phases at the collective mode frequencies, as predicted by the TSD equations. This manifested itself when we applied the power spectra relations to our laser operating with three cavity modes and used the phase clusters to redefine the relations.

In this chapter, we noted that the AC noise of a particular cavity mode peaks and then decays to a plateau as the pump is increased. The peaks of the noise occur approximately at the threshold of the neighbouring mode in the cavity mode spectrum. We also noted whilst performing our experiments that the intensity of a cavity mode with high threshold pump power could eclipse those with a lower threshold. The latter was not altogether surprising given that our laser is almost identical to Stamatescu and Hamilton [71]. In this context we note that the TSD equations predict that the intensities of the cavity modes increase monotonically with the pump and follow the order of threshold (or gains). In this chapter we also showed that a transition occurred in the cross spectrum between some of the pairs a cavity modes at a small but arbitrary pump parameter above threshold of the nearest cavity mode. Therefore, for a given number of cavity modes, the $\text{sgn}(a_{m,k})$ were not constant. These results indicate that our laser is outside the validity of the TSD equations.

The TSD equations fail to give correct answer because some essential information is not used. As stated in Chap. 4, we expected the TSD⁺ equations to give similar results to the TSD equations given the pump rate of, and the perturbations to our laser. We also know that spatial holeburning effects the gain of the cavity modes. Due to the standing wave nature of the cavity, the competition for gain between the cavity modes is largest at the ends of the cavity where the nodes of one mode overlies another [15, 43, 48, 71]. Since spatial hole burning gives rise to a small scale variation of the pump, where the gain length $\gg \lambda$ [75], a large scale variation is the likely origin of observed behaviour and the divergence from the TSD predictions.

We discuss some recent models for Fabry-Perot cavity Class B lasers, similar to the TSD equations, which have shown to successfully account for some of the behaviours we observed [54, 70, 71, 72, 74, 75, 88, 89]. In the cases where the authors compared their theoretical and experimental results, it was always qualitative [54, 70, 71, 72]. This is because it is relatively simple to discriminate against a model, for example one may use the phase clusters at the collective mode frequencies. It is however, difficult to quantitatively match both theoretical and experimental data. For example, the TSD equations are difficult to solve due to their hierarchical nature, also noise is inherently difficult to model. The latter is one of the reasons why the theoretical and experimental results of the power spectra relations for a LNP laser [50, 51] and a Nd:YAG laser [87] are compared analytically. Generally the difference between the TSD equations and the models I summarise, are that they take into account that the gain medium does not fill the cavity and/or the pump is not distributed uniformly along the axis of

the laser. However, the exact origin and nature of the competition between cavity modes remains a point for debate [54, 70, 71, 72, 74, 75, 88, 89]. For example, Stamatescu and Hamilton [71] showed that a modification to the equations using an exponential decay of the pump was able to qualitatively explain the intensity of modes with a low gain eclipsing those with a higher gain.

Abraham et al. [54] took into account that the gain medium did not fill the cavity. In their case, the variation in the the ratio of gain medium to overall cavity length (fill factor) was 0.2-0.4. They found the modification had a pronounced effect on the intensities of the cavity modes, similar to Stamatescu and Hamilton [71], yet showed little or no effect on the total intensity.

Khandokhin et al. [72] performed a study of both modifications. They found that the exponential absorption of the pump gave rise to hopping between adjacent modes on the cavity mode spectrum and sometimes to a rarefied optical spectrum, i.e. some of the cavity modes expected to oscillate did not. Their fill factor was varied between 0.2-0.3. They found that since it was not equal to unity, it led to non-monotonic changes in the modal intensities similar to those previously discussed [54, 71]. If both types of behaviours were observed, it was inferred that both non-uniformities were present.

Koryukin and Mandel [75] analysed theoretically the threshold position of the first two cavity modes in a solid state Fabry-Perot cavity laser. They found that the threshold position predicted by the TSD equations was modified only slightly by either the exponential decay of the pump or the fill factor.

This implied that the pump non-uniformity increased mode-mode competition slightly, which further increased with a smaller fill factor or stronger exponential decay.

The fill factor in our laser is ≈ 0.725 . Koryukin and Mandel [75] showed that the non-uniformity of the gain, for this configuration, is less than either Abraham et al. [54] or Khandokhin et al. [72]. Koryukin and Shirokov [74] showed that for a change in fill factor between unity and a half, the steady state intensities of the cavity modes barely changed in either the TSD⁺, or the so called global rate equations. Indeed these results were similar to the basic TSD equations [74]. These arguments lead us to suspect that since our laser was end pumped with a focused diode beam, the gain is concentrated near the input mirror. Therefore, the mode-mode competition is dominated by the non-uniformity induced by the exponential decay of the pump.

We therefore conjecture that the model we developed should be modified with an exponential decay of the pump. For completeness, we also suggest that a fill factor be included and a Fourier expansion of the population inversion, as used in the TSD⁺ equations [88], be included. These tasks are outside the scope of this thesis.

Chapter 7

Conclusion

7.1 Summary Of Results

Measurements of the intensity noise interference in a multi-longitudinal mode Nd:YAG laser were presented in thesis.

We developed the cross spectral technique to measure the antiphase dynamics directly. A picture of the interferences of the array was determined for two to five mode operation and the contributions of the cavity modes to the collective modes calculated for three mode operation. The sums of the contributions showed that the two low frequency collective modes were almost completely antiphased whilst the highest frequency collective mode was not. This manifested itself as a large resonance in the power spectrum of the total intensity noise.

Power spectral densities of the individual cavity modes and the total intensity were calculated for two and three mode operation. Relationships between these quantities derived from modal rate equation theory, at the collective mode frequencies [50], were satisfied in the two mode case. In

the three mode case, the power spectra relations were satisfied once the cross spectral measurements were taken into account. The cross spectral technique was therefore shown to be a sensitive test of the model developed by Pieroux and Mandel, Hill et al. [87].

The behaviour of the laser operating near threshold of a cavity mode was measured. The AC noise component of the individual cavity modes appeared to display similar characteristics to their intensity, i.e. modes with a high threshold pump power could become more intense than those with a threshold lower pump power [70, 71, 54, 72]. Transitions in the behaviour of the cross spectrum occurred at a small, but arbitrary pump rate above threshold of the third and fourth cavity modes, for some pairs of cavity modes. The observed behaviours were able to be explained qualitatively by the competition for gain between the cavity modes.

7.2 Future Directions

The results in this thesis have opened further avenues for study. They are:

- Comparison of excitation effects on an array. The laser could be excited by modulation of the pump and the total and modal intensity fluctuations recorded. Under the same experimental conditions but with the laser excited by ambient noise, the measurements could be repeated.
- Measuring the direct effect of noise on the array. This might be achieved by modulating the pump with noise and varying the modulation depth and statistics. Further questions might also be raised. For example, to which variable should the noise be applied, are the best measurement

techniques the transfer function or the cross spectrum? [86].

- Rigorously testing the AC noise relationships discussed in Chap. 6. For example, the noise of the the third and fourth cavity modes and the total intensity should also be measured. We expect the latter to increase monotonically with the pump, similar its intensity [70, 71, 54].
- Development and testing of a more rigorous model to describe the behaviour observed in our laser.

Appendix A

Ancillary Servoloop Information

A.1 Proportional And Integral Gain In Servoloops

First attempts at locking the laser cavity length to FP1, and FP2 to the cavity mode with which it was resonant, proved difficult. The problem was overcome with the application of proportional and integral gain (PI) to the correction signal from the lock-in amplifier, see Fig. 2.4. The article by Hamilton [96] forms the basis for the discussion as to why this was required.

We have a YAG laser cavity that is spaced with aluminium. Due to thermal fluctuations, the laser cavity length and therefore the frequencies and gains of the cavity modes could change. For active stabilisation of these fluctuations, a reference frequency is required. Commonly a Fabry-Perot resonance or a molecular spectral line is used [96]. The relevant resonance

must be detected and an error signal derived which changes sign as the reference frequency passes through the desired point. Commonly in laser systems, a dither frequency (a small AC signal) is applied to one end of the Fabry-Perot. This modulates the output power of the cavity, the phase of which is governed by the slope of the reference resonance.

With a photodiode used to detect the output power, an error signal appears at the lock-in amplifier which generates a correction signal that is applied to the laser cavity length. This removes both the error and correction signal and the system is driven back to its starting point. However, the thermal fluctuation remains. Increasing the gain of the servoloop brings the system closer to the desired point but not completely [96].

An integrator provides infinite gain at DC and therefore provides a signal to continually remove the perturbation. However, the integrator is not without problems. It suffers phase loss which means that the bandwidth of the servoloop is decreased, therefore the loop may not be able to cope with faster fluctuations. If the phase loss is large enough, the servoloop will itself oscillate. To offset the phase loss and increase the bandwidth of the servoloop, a combination of proportional and integral gain is used. A schematic of the circuit we used is shown in Fig. A.1. A response to higher frequency fluctuations, using differential gain, was not required because the thermal fluctuation was the dominant perturbation.

The schematic shows a switch to short circuit the integrator. It was used for example, to reset the integrator before moving the PZT to the top of a Fabry-Perot resonance and closing the servoloop. This prevented an undesired correction signal, from the charge stored on the integrator, being

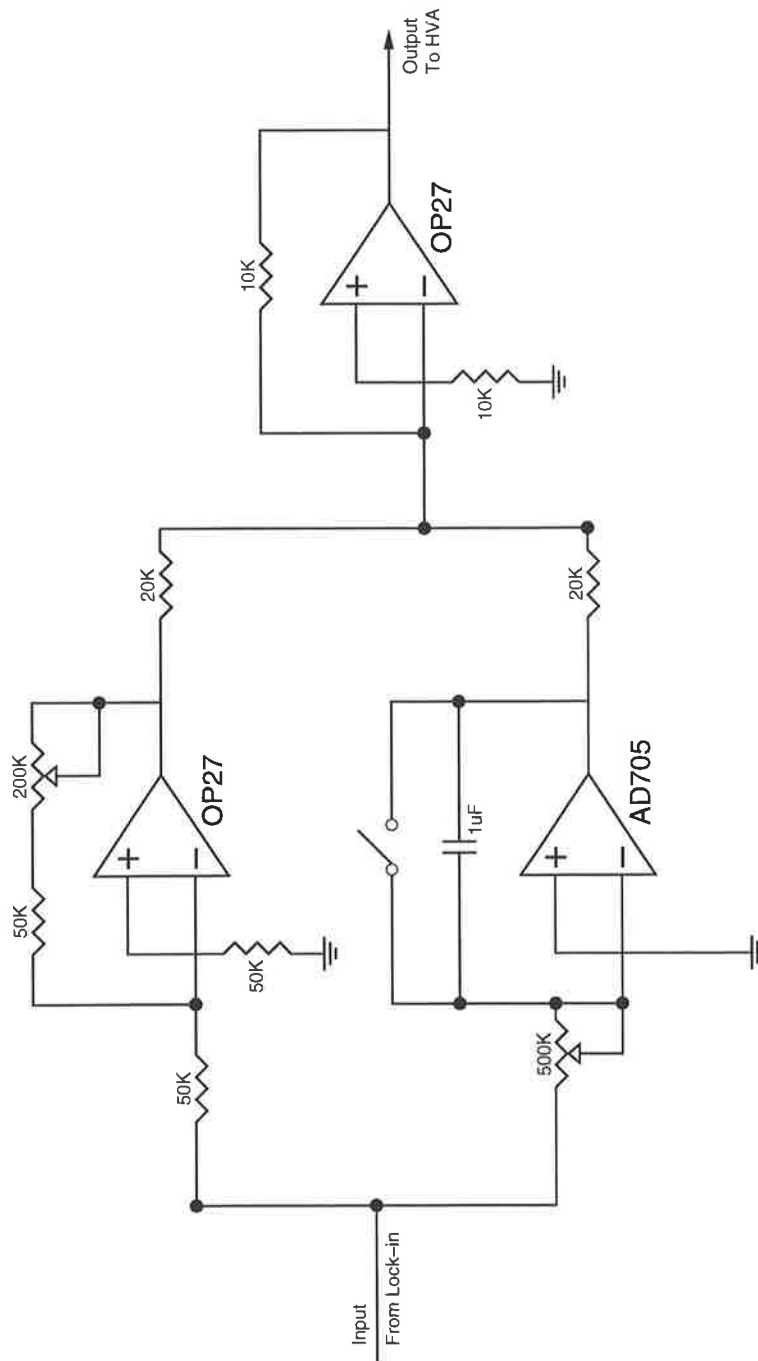


Figure A.1: The schematic of the PI circuit. All values of the resistors are in ohms. The circuit was operated from a $\pm 12\text{V}$ battery power supply.

applied to the PZT. The proportional and integral gain were summed before being passed to a high voltage amplifier and applied to a PZT.

A.2 Dither Frequency

Although I could find no published works on this matter, conventional wisdom suggested that using the same dither frequency in two adjacent servoloops was inviting disaster. The servoloops could potentially cross couple into each other with undesirable results. I performed a series of experiments to try and determine if the servoloops were cross coupling and if I could use the same dither frequency in both loops, see Fig. 2.4.

Six conditions were tested with the laser operating with two cavity modes. The pump power and position of cavity mode spectrum with respect to the gain spectrum was the same in each case. The dither frequencies were limited to $\leq 10\text{kHz}$ because a piezoelectric (PZT) was used as the element to control the laser or Fabry-Perot cavity length. They and their sum and differences were very close to, but not equal to mains harmonics, which in Australia is 50Hz. They were also far enough away from the collective modes to be easily distinguishable in the frequency domain. The modal and total intensity fluctuations were recorded with the A/D. Power spectra of the modal intensities were also calculated simultaneously with a signal analyser (HP35670A).

- a) Both servoloops were locked with different dither frequencies, namely 1251Hz & 1502Hz.
- b) Both loops were locked with the same dither frequency (1251Hz). From this point on in these tests, the same dither frequencies were used.

- c) A dither was applied to FP2 yet this servoloop was not locked. The laser cavity length remained locked to FP1.
- d) The laser cavity length remained locked to FP1 and no dither was applied to FP2.
- e) Both loops were unlocked and a dither applied to both Fabry-Perots.
- f) Both loops were unlocked and no dither was applied to the Fabry-Perots.

The same overall size and shape of the collective modes, in the power spectral densities of the intensity fluctuations, were found for each condition. The power spectra relations (see Sec. 4.3) were satisfied in each case. The only noticeable differences in the spectra were at low frequencies, e.g. the dither frequency(ies). We found that we could remove the dither frequencies from a spectra by not applying a dither to a Fabry-Perot. Similarly, electronic pickup from switch mode power supplies at approximately 30kHz could be removed by turning off some of the monitoring equipment, e.g. the oscilloscopes.¹ Feedback of the dither frequencies to the YAG laser, manifesting itself as peaks in the power spectrum of the total intensity noise fluctuations, was not found either.

These results appear to suggest that the only effect of the “dither peaks” on a spectrum, was in the region 0Hz to 3kHz, i.e. the second harmonic of 1502Hz. They also suggest that whilst these frequencies and the switch mode

¹In separate experiments, we found that if we connected the photodiode-amplifiers directly to the signal analyser (HP35670A), the switch mode power supply noise was not evident. This suggests that the spurious noise was being picked up by the extra gain amplifier preceding the A/D (see Appendix B), or the A/D itself.

power supply noise may affect a spectrum, they do not pertain to the modal or total intensity noise fluctuations of interest. No attempt was therefore made to prevent them effecting a spectrum. No cross coupling between the two servoloops was able to be detected either. Therefore, we used the same dither frequency in each servoloop in our experiments.

Appendix B

Data Manipulation

B.1 Intensity Noise Measurement

To maximise the signal to background noise ratio and resolution (significant number of bits) of the intensity noise data, its properties must first be understood.

The peak level of the intensity of the cavity modes fluctuations, i.e. AC noise. This can be directly measured with an AC voltmeter that has the appropriate bandwidth to measure all the frequency components of interest. The measured RMS (root mean square) of the noise corresponds to sigma (σ) [97], which is the standard deviation. For a Gaussian distribution, 3σ covers 99% of all possible values. However when measuring noise, common engineering practice utilises 5σ , this is known as the “crest factor” and ensures even rare values are covered [97].

We use an A/D to measure the intensity noise so that we can study the Fourier components of it. By AC coupling the board, the standard deviation can be calculated directly from the intensity noise data. Whilst

the data could be manipulated mathematically, for example multiplied by a scale factor, it does not change the signal to noise ratio and resolution inherent in the fluctuations.

The following is an example of the process we therefore followed. A $\pm 1\text{V}$ input range of the A/D was chosen and the standard deviation of the recorded noise calculated to be 10mV . Therefore the corresponding crest factor is 50mV . The idea being to match the crest factor to the input range, electronic gain in the form of a simple inverting operational amplifier of 20 times could be applied to the photovoltage before sampling. Alternatively, the input range of the A/D could be changed to 100mV and an electronic gain of 2 utilised. The bandwidth of the additional operational amplifier was sufficient such that the photodiode-amplifier was the limiting factor. It was also a low noise device so that it did not influence the intensity noise being measured.

B.2 Transformation Into The Frequency Domain

In discussing the optimum method to convert our N point ($\approx 2^{19}$) time series of the intensity fluctuations into the frequency domain, we summarise the relevant information contained in monographs [80, 98].

By definition, the discrete Fourier transform (FT) gives the best estimate of a spectrum because the frequency resolution is also N points [80, 98]. However, it takes N^2 computations to calculate the spectrum, which can be a long period of time. The Fast Fourier transform (FFt) takes $N\log_2(N)$

computations which is a significant decrease. The FFt is therefore used by many Scientists and Engineers because of its speed and efficiency.

The most common use of the FFt is the power spectrum, i.e. the modulus squared of the FFt. However, the estimate of the spectrum is commonly jagged in appearance as it is made up of N discrete frequencies. Increasing the number of recorded points further increases the number of discrete frequencies, however this does not change the variance (or difference) between adjacent Fourier components [80].

A compromise between the accuracy of the discrete Fourier transform and the speed and efficiency of the FFt is therefore required. The monograph entitled “Numerical Recipes” [80] gives two examples of ways to achieve this: the modified periodogram estimate (MPE), and the correlation method. The freeware program “Scilab”, developed at Inria France, has two routines which utilise these methods. I could not test the correlation routine as it caused the computer to crash.

The modified periodogram estimate uses three methods to reduce the variance: the N point time series is partitioned into small segments, the segments are overlapped to include data points from either side of a particular segment, and a weighted window is applied to each segment.

By transforming each segment separately into the frequency domain and averaging at each frequency component, the variance is reduced in direct proportion to the number of segments [80]. This is also computationally efficient because many short FFt’s are calculated and averaged instead of a long one [80, 98].

The application of the weighted window smoothes the spectrum and ex-

presses a better relationship between the estimate at a discrete frequency and the underlying continuous spectrum at nearby frequencies [80, 98].

In calculating spectra, the segment length should be chosen with a view to the correlation length of the time series data. For example, if an arbitrary point is chosen, how many surrounding points are influenced by this point. It has been shown numerically that a better estimate of the spectrum is produced when each segment includes half the data points of the preceding and following segments [98].

The procedure of the MPE is: segment the data, place the overlapped data each side, apply the window, calculate the FFT and repeat for each segment until the end of the time series. The segmented FFT's are then averaged at each Fourier component. For a quantitative treatment of the reduction in variance, see Rabiner & Gold [98].

The A/D's we used (Gage Applied Sciences Inc., CompuScope 512) had a maximum memory of 524k samples for each channel and a maximum sample rate of 5MS/s. We always captured the maximum number of data points to give better accuracy in the estimate of the spectrum. The maximum frequency measurable by this device is governed by the Nyquist theorem, i.e. half the sample rate. A frequency greater than is will be aliased, i.e. falsely translated in a spectrum. Therefore, the highest measurable frequency with this device is 2.5MHz.

In determining the balance between frequency resolution and variance in the spectrum, the following rule was applied [80]. For a constant number of data points, an increase in the number of segments decreases the segment length and therefore the frequency resolution and variance. For a constant

segment length, an increase in the sample rate gives rise to a decrease in frequency resolution and the variance.

We derived the following formula Eq. (B.1), to express this relationship for our measurement system:

$$\frac{\text{sample rate (kS/s)}}{\text{segment length (data points)}} = \text{frequency resolution (kHz)}. \quad (\text{B.1})$$

To obtain the best compromise between frequency resolution and variance between adjacent Fourier components, we compared spectra of the same type (for example the cross spectrum of a cavity mode pair), for a particular operating point of the YAG laser. The intensity fluctuations were recorded at different sample rates (namely 500kS/s, 1MS/s and 5MS/s) and spectra calculated with a different number of segments for the same sample rate. We decided on a sample rate of 1MS/s and a frequency resolution of 0.25kHz, these were kept constant throughout the work contained in this thesis.

The power spectrum and the magnitude of the cross spectrum are plotted using a logarithmic scale, relative to unity, to highlight all of the frequency components. Though we recorded photovoltages, we use a $10\log_{10}$ scale rather than $20\log_{10}$, because spectra at a discrete frequency represent a power function.

Appendix C

Circuit Designs

C.1 Photodiodes

For the detection of intensity noise it is important to keep all other noise sources small. Therefore, photodetectors were required to have an excellent signal to noise ratio (SNR) and give the same response relative to each other, i.e. gain and bandwidth (BW).

The photodiodes (Epitaxx, ETX 1000T) were made from InGaAs and were chosen because they have a very high responsivity at 1064nm. They also had a high bandwidth (of order hundreds of MHz), low dark current and high shunt (or junction) resistance. The latter two parameters give excellent noise performance. Their small active area can be a problem in some instances, for example detecting the light from a divergent beam exiting a Fabry-Perot. This was overcome with the use of a lens to focus the light onto the diode.

The photodiode was used in reverse bias configuration. To convert the photocurrents to voltages, a trans-impedance gain (TIG) amplifier configuration was used, see Fig. C.1.

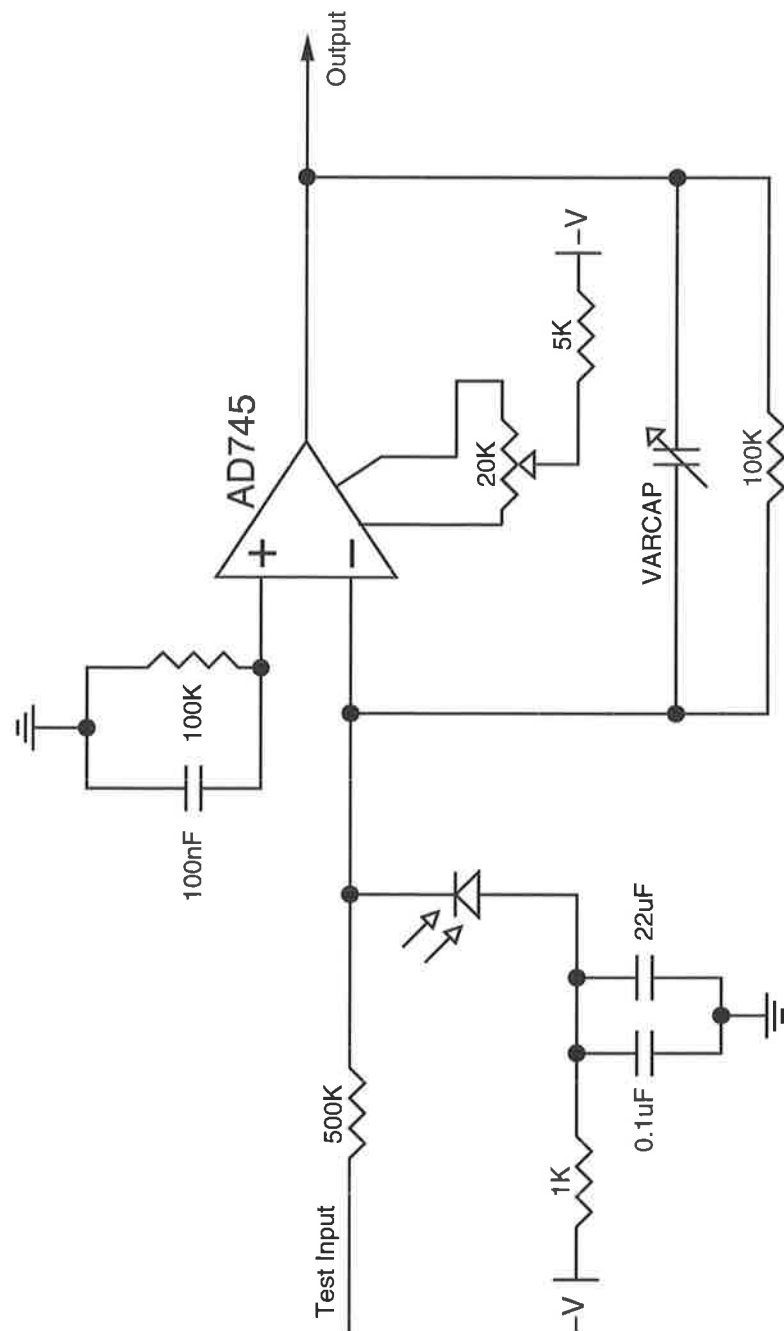


Figure C.1: The schematic of the photodiode-amplifier circuit. The feedback capacitor was variable between 2pF and 10pF. The negative rail was -12V. All values of the resistors are in ohms.

The requirements for the TIG amplifier circuit were: high gain, low BW and low noise to maximise the signal to background noise ratio. We chose an AD745 operational amplifier for this application. We used the definition for the 3dB point of the BW to be $1/\sqrt{2}$ of the maximum amplitude. We set the 3dB point to be 265kHz which was greater than the third harmonic of the highest frequency collective mode for five mode operation. We calculated the resistive and reactive feedback, R_f & C_f respectively, using Eq. (C.1).

$$f_{3dB} = \frac{1}{2\pi R_f C_f} \quad (\text{C.1})$$

It was found that the addition of resistive and reactive balance, R_B & C_B respectively, to the non-inverting input of the operational amplifier, improved the noise performance of the circuit. The latter were calculated from the parallel of their feedback and junction values respectively. By the appropriate choice of components, we were able to approach the limit of the overall noise of the circuit to be proportional to the Johnson noise of the feedback resistor [99]. The important parameters of the photodiode-amplifier are listed in Table C.1.

In further efforts to minimise background noise, the photodiode-amplifiers were operated from a $\pm 12\text{V}$ battery power supply and the tracks on the circuit board kept thin and short. The latter also prevented the circuit oscillating autonomously. Other features of the circuit are: power supply decoupling and filtering (not shown in Fig. C.1), a DC offset to the amplifier, and a variable capacitor (C_f , 2pF to 10pF) to accurately set the BW.

In many cases the bandwidth of a light detection circuit is measured with a modulated light source. In this application, the BW of the TIG amplifier

Parameter	Value
Active Area	1mm ²
Dark Current	20nA
Responsivity (R) @ 1064nm	.875A/W
Junction Resistance (R_j)	25M Ω
Junction Capacitance (C_j)	100pF
Gain	100k
Bandwidth	265kHz
Feedback Resistance (R_f)	100k Ω
Feedback Capacitance (C_f)	6pF
Balance Resistance (R_B)	100k Ω
Balance Capacitance (C_B)	100pF

Table C.1: Parameters for the photodiode and amplifier circuit.

is the limiting factor rather than the photodiode. Therefore a test input was included. An AC signal can be applied to this input and the BW measured with an oscilloscope.

C.2 Temperature Control

To control the temperature of the YAG laser cavity, we used a circuit based on a design from Bradley et al. [100]. This device has been optimised and standardised for our research group by Mudge [101].

The control circuit is based on a Wheatstone Bridge. An instrumentation amplifier is used to determine the difference between the actual and desired points. The error signal is given gain (PI & D) before application to the controlling element, in this case a Peltier cell (Thermo-electric cooler). The benefits of proportional and integral gain have been outlined in Appendix A.

The circuit by Mudge [101] was slightly modified to suit our particular application and availability of parts, the schematic is shown in Fig. C.2. Response at higher frequencies using differential gain was not required because thermal fluctuations are slow. A regulated power supply was custom made to suit this controller and was housed in a single box with the control circuit.

Figure C.2: The schematic of the temperature control circuit. All values of the resistors are in ohms. The voltage rails unless otherwise shown were $\pm 9\text{V}$. The variable resistor, R_G , sets the gain of the instrumentation amplifier.

Appendix D

Publications

D.1 Publications Resulting From This Work

T. Hill, L. Stamatescu and M.W. Hamilton, “*Method For Determining Antiphase Dynamics in a Multimode Laser*”, Phys. Rev. E, **61** (5), R4718-R4721, 2000

T. Hill, M.W. Hamilton, D. Pieroux and P. Mandel, “*Intensity Coherence of a Multimode Nd:YAG Laser*”, Phys. Rev. A, **66** (6), 063803, 2002

Other refereed article to which I contributed.

M.W. Hamilton, T. Hill and L. Stamatescu, “*Noisy Excitation of Nonlinear Oscillator Arrays*”, Unsolved Problems of Noise and Fluctuations. UPoN’99: Second International Conference. Adelaide, Australia. AIP Conference Proceedings: **511**, 295-300, 2000

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Method for determining antiphase dynamics in a multimode laser

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We measure the cross spectrum of the intensity fluctuations of pairs of modes for a multilongitudinal mode neodymium-doped yttrium aluminum garnet laser operating in the steady state regime. From the data we build up a picture of how the longitudinal mode fluctuations interfere and directly show the antiphase dynamics of the intensity fluctuations.

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It is well known that antiphase dynamics occurs in a range of nonlinearly coupled systems, for example, Josephson junction arrays [1,2], phase oscillators [3], and lasers (e.g., [4] and references contained therein). Antiphase dynamics is where the cooperative behavior of the system results in cancellation of one or more of the collective modes. The behavior of the total system is then somewhat simpler than might be expected.

Here we report on the use of the cross spectrum of the intensity fluctuations of pairs of cavity modes to study the antiphase dynamics of a multimode laser. The cross spectrum is the Fourier transform of the cross correlation function and is a complex valued quantity. Peaks at the collective mode frequencies are seen in the cross spectrum magnitude and the phase at these frequencies is used to deduce whether the two cavity modes contribute to the collective modes in or out of phase.

We use a multimode solid-state neodymium-doped yttrium aluminum garnet (Nd:YAG) laser in which the relaxation oscillations of the cavity modes comprise an oscillator array that is globally coupled by gain sharing. The laser operates in the steady state regime. For N cavity modes there are N collective mode resonances. These occur in a range of frequencies up to the single mode relaxation oscillation frequency [5]. The frequencies of these resonances are typically of the order of tens of kHz, making their direct study relatively simple. For Nd lasers it takes little effort to excite the collective modes. There are several common excitation methods: examples are the use of ambient noise [6], and modulation of the pump, either continuously [7–9] or by step function [10]. In the work considered in this Rapid Communication we rely on ambient noise to excite the collective modes.

A typical method for studying antiphase dynamics is the power spectrum of intensity fluctuations which for an individual cavity mode shows peaks corresponding to the collective modes, many of which are greatly diminished in the power spectrum of the total intensity [6]. This technique gives no phase information; therefore, this method cannot directly verify the phase clustering as predicted for Josephson junctions [2] and other lasers [10]. One approach that has successfully verified phase clustering is the transfer function technique [9] using modulation of the pump. The informa-

tion contained in the pole-residue representation of a transfer function describes how each of the cavity modes contributes to each collective mode.

Our Nd:YAG laser (Nd ~ 1% concentration) is end pumped by the combined beams of two 40 mW diode lasers operating at 808 nm; see Fig. 1. The YAG rod is 10 mm long and 3 mm in diameter. It is perpendicularly cut at the front face and Brewster cut at the other to force linear polarization. This ensures that the laser system does not exhibit polarization instability and switching [11]. The front face of the YAG rod forms one end of the cavity and is highly reflective at 1064 nm but partially reflecting ($R \sim 30\%$) at 808 nm. Optical isolation (measured to be approximately 35 dB) with a polarizing beam splitter and a Fresnel rhomb was therefore used between the YAG rod and the laser diodes to minimize instabilities in the diodes. The other end of the YAG laser cavity is a spherical mirror ($R \sim 98\%$ at 1064 nm) and radius of curvature 2.5 cm. The output coupling mirror was chosen for relatively low frequency relaxation oscillations rather than high output power. The output power is of the order of 1 mW. The optical length of the cavity is approximately 2.5 cm, giving rise to a measured longitudinal mode spacing of 6.15 GHz. The laser operates on the 1064 nm transition with up to six longitudinal modes, depending on the exact cavity length and pump power. Each longitudinal mode has a good TEM₀₀ transverse intensity profile. The output coupler was mounted on a piezoelectric transducer to allow fine control of the cavity length, so that we could offset the laser mode spectrum with respect to the gain spectrum.

To measure the cross spectrum, we recorded simultaneously the intensity of two different laser modes using Fabry-Perot filters to separate them out. Optical isolation

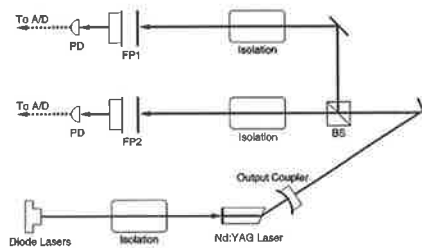


FIG. 1. Schematic of the optical part of the experimental setup. A/D, analog to digital converter; PD, photodiode; FP1, Fabry-Perot 1; FP2, Fabry-Perot 2; BS, 50/50 beam splitter.

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(measured to be approximately 28 dB) with quarter wave plates and polarizing beam splitters was used between the YAG laser and the Fabry-Perot interferometers so that oscillations of collective modes were not driven by feedback. The intensities of the modes transmitted through the Fabry-Perot interferometers were detected with low noise photodiode-amplifiers. The photocurrents were recorded with a 12 bit analog to digital (A/D) converter (Gage Applied Sciences Inc., CompuScope 512) mounted in a PC. Each of the two input channels recorded 2^{19} points at a sample rate of 5 MS/s. The sampling interval was much shorter than the shortest characteristic timescale of the dynamics ($\sim 5 \mu\text{s}$) and the number of recorded points gave an adequate amount of data for averaging in the frequency domain. Once the appropriate input range of the A/D was selected, extra gain was applied directly after the photodiode-amplifiers to maximize the signal-to-noise ratio and resolution of the data. The recorded intensity data was transformed into the frequency domain (i.e., power or cross spectrum) using the modified periodogram method [12].

An aluminum spacer was used in the laser cavity. Due to thermal expansion the frequencies of the longitudinal modes and their associated gains could change. One Fabry-Perot (FP1 - TecOptics Ltd., SA-10 with a cavity lifetime of approximately 4.25 ns) was spaced with Invar which has a much lower coefficient of expansion than aluminum. We used it as a reference cavity to which we stabilize one of the laser modes and therefore the laser cavity length. We stabilized by applying a very low level dither to the piezoelectric transducer of FP1 at 3 kHz, which is far enough away in frequency from the collective mode frequencies of the laser not to be a problem. A lock-in amplifier generated an error signal from the intensity of the transmitted laser mode which was fed back to the output coupler. This maintained a sufficiently constant position of the laser cavity modes with respect to the gain spectrum for the time needed to acquire all the data. The other Fabry-Perot (FP2-cavity lifetime of approximately 230 ps) was also dithered and, with a secondary feedback loop, locked to the laser mode it transmitted.

For any pair of cavity modes, the magnitude of the cross spectrum shows peaks at the collective mode frequencies. Zeros (or antiresonances) in the magnitude correspond to phase jumps. Figure 2 shows an example of this for three mode operation. We see three collective mode resonances; i.e., I, II, and III. The labeled feature IV is the second harmonic of the highest frequency collective mode III. The group of features labeled V includes zeros, the phase of the cross spectrum is set so that it is approximately 0° at the frequency of collective mode III. There is a superposed phase shift of approximately $20^\circ/100 \text{ kHz}$ due to the amplifier rolloff on the detector used for FP1. The cross spectra for each of the other pairs of modes show peaks at the same frequencies as in Fig. 2; however, the heights of the peaks and their relative phases differ.

At each collective mode frequency the phase has a definite value; that is, it does not jump. We know that the intensity fluctuations of all the laser modes contribute in phase (constructive interference) to the highest frequency collective mode resonance because this is the mode which survives in the total intensity [6,9]. The number of 180° phase jumps between the highest frequency collective mode and any

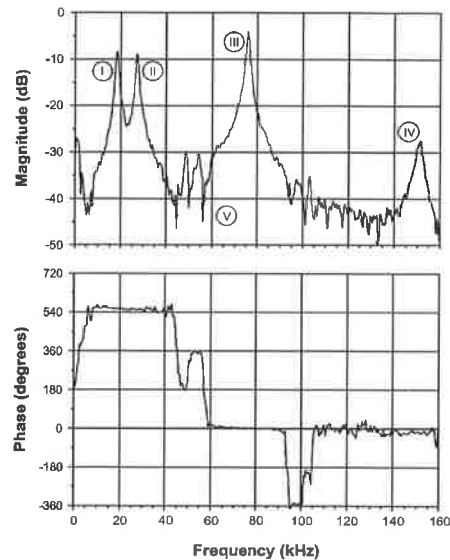


FIG. 2. Cross spectrum of intensity noise fluctuations of cavity modes 1 and 2, for three mode operation. The labeled features in the magnitude are collective modes I, II, and III, at frequencies 18, 26, and 76 kHz, respectively. Feature IV is the second harmonic of mode III, i.e., 152 kHz. The group of features labeled V includes zeros associated with phase jumps.

other, tells us whether the fluctuations of each of the two laser modes constructively or destructively interfere at the other collective mode frequencies. That the size of the phase jumps is always a multiple of 180° in this system, implies that we are seeing simple antiphase states and not splay phase states [2]. This is consistent with other experimental data [9] and theoretical work [10,13].

Table I shows the pattern of interferences thus determined for pairs of longitudinal modes. We call this a phase pattern. The collective modes are labeled with Roman numerals in order of increasing frequency, where III is equivalent to the single mode relaxation oscillation frequency [5]. Cavity modes are labeled with Arabic numerals in order of increasing pump threshold. Interference between cavity mode pairs, at a particular collective mode, is labeled “+” for in-phase dynamics and “-” for antiphase dynamics.

It has been predicted [14] from a nonlinear analysis of the Tang, Statz, and deMars equations (TSdM) [15] that Fabry-Perot lasers may exhibit harmonic and mixing frequency

TABLE I. Phase pattern, expressed in terms of cavity mode pairs, for three mode operation. The columns correspond to the laser cavity mode pairs and the rows to the collective modes.

Mode	1,2	1,3	2,3
I	-	-	+
II	-	+	-
III	+	+	+

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TABLE II. Contributions of the cavity modes to the collective modes. The labeling of the modes is the same as in Table I. The column labeled "Total" shows the sum of the contributions for the respective rows.

Mode	1	2	3	Total
I	+0.667	-0.221	-0.439	+0.007
II	+0.252	-0.530	+0.455	+0.177
III	+0.615	+0.643	+0.142	+1.400

resonances of the collective modes. For example, in Fig. 2 the labeled feature IV is the second harmonic of collective mode III. However, it is not clear if mixing frequency resonances exist. The two small peaks in the group of features labeled V in Fig. 2 might be such resonances but the signal to noise ratio (at those frequencies) is not good enough to be sure. Also they do not appear consistently in the cross spectra of all possible cavity mode pairs. Another interesting question is whether zeros (of the amplitude), for example, in the group of features labeled V in Fig. 2, are associated with mixing frequencies [16].

To build up a picture of how the cavity modes contribute to the collective modes, we use cavity mode 1, which is the first to reach threshold, as the reference phase. The phases of the contributions of cavity modes 2 and 3 at a given frequency can then be determined from the phase of the cross spectrum between modes 1 and 2, or between 1 and 3 as appropriate. The height of a collective mode peak is the product of the contributions from the two cavity modes concerned. For a given collective mode frequency the heights of that peak in each of the three cross spectra (for three mode operation) can be used to extract the magnitudes of the cavity mode contributions to that collective mode. In obtaining the data the gains of the two detection channels were kept constant between measurements of the time series data for the different pairs of modes, so that this analysis is meaningful. In Table II we show the contributions, with their signs, that we obtained from the set of three cross spectra. The spectrum in Fig. 2 is one of the three. Thus, for collective mode I the fluctuations of cavity modes 2 and 3 are in antiphase with the fluctuation of cavity mode 1. At collective mode II the fluctuations of cavity mode 2 are in antiphase to the fluctuations of cavity modes 1 and 3, while at collective mode III all cavity mode fluctuations are in phase.

The completeness of the destructive interference is another interesting issue. A conjecture has been made [17] that perfect antiphasing will only occur if the modal gains have a symmetric distribution; i.e., the laser gain spectrum is symmetric and the cavity modes are symmetrically disposed about its peak. In a Nd laser the gain spectrum of the 1064 nm line is slightly asymmetric [18]. Furthermore, in our experiment the cavity modes were not symmetrically distributed about the gain peak, so that, according to the conjecture,

remnants of the low frequency collective modes will still appear in the power spectrum of the total intensity fluctuations. This imperfect antiphasing was observed experimentally [9] using the transfer function technique, and we also observed this in the experiment reported here. In column 4 of Table II the sums of these contributions are shown. These sums represent the collective mode amplitudes extracted from the cross spectra. For collective mode I this amplitude is quite small, showing that only a vestige of this mode remains. We observed this vestige in the power spectrum of total intensity, which we measured separately.

In this paper we have presented the cross spectrum as a convenient method for detecting what type of antiphasing the dynamics of a nonlinear oscillator array show. If the measured modal fluctuations are sufficiently small, the cross spectral method should give the same information as the transfer function technique [9]. These two techniques would then make it possible to compare the excitation of a nonlinear oscillator array with a single frequency and with noise in the case where nonlinearities are important.

Our method may well be applied to other nonlinear oscillator arrays provided the individual oscillators can be separated. The relaxation oscillation dynamics of a multimode laser is only one manifestation of such an array, albeit a particularly convenient one because the laser cavity modes have different optical frequencies. This makes them separable in the output beam, and thus the cross spectrum at nonoptical frequencies can be measured. Some other optical examples that might be considered are multimode fiber lasers [19], mode-locked lasers, and an array of semiconductor lasers [20]. This technique ought to be applicable in a fairly straightforward manner to the first example. However, the latter two are more complicated, as the cross spectrum at optical frequencies ($\sim 10^{14}$ Hz) will likely be just as important as that near the relaxation oscillation frequency (10^5 – 10^{10} Hz). In the semiconductor laser array, one of the important observables is the "beam quality," which depends on the relative optical phases of the emitters. In this case the individual oscillators in the array might be separated by fiber-coupling of individual emitters in the near-field, or careful imaging of the emitters with a lens to enable separation in the far-field. The cross spectrum might also be a means of probing the dynamics of Josephson junction arrays. Solitons may also be viewed as a nonlinear oscillator array. In this context we note that the cross correlation coefficients between the fluctuation of the spectral components of an intensity squeezed soliton pulse have been measured [21].

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Intensity coherence of a multimode Nd-doped yttrium aluminum garnet laser

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We measure the power spectral densities, at relaxation oscillation frequencies, of the individual longitudinal modes and the total intensity of a Nd-doped yttrium aluminum garnet laser at low pumping rate. We then test relationships between these quantities that are derived from the modal rate equations theory. The theoretical relations for the two mode case are confirmed by the experiment. However, in the three-mode regime, theory and experiments do not agree well.

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I. INTRODUCTION

There has been much recent interest in the spectra of intensity fluctuations in multimode lasers because these lasers exemplify nonlinear oscillator arrays and because the spectra are a useful test of the theoretical models used for the lasers. One of the earliest and most well known models is that due to Tang, Statz, and deMars, referred to herein as TSD [1]. In this model a modal expansion of the inversion density (or the gain) is employed, with a rather arbitrary truncation being needed to arrive at the set of ordinary differential equations that constitutes the TSD model. Two other basic assumptions that go into the model are that the gain medium fills the cavity and that the gain is distributed uniformly along the axis of the laser. Other models which do not make these latter assumptions still, as a general rule, employ the same modal expansion as TSD [2–6]. A more general approach that eschews the modal expansion, using so-called global rate equations, was also developed [7]. These models do not incorporate the phase. At the level of the rate equations the phase variables obey an evolution equation that is decoupled from the equations that describe the intensity dynamics. To reconnect the phase and amplitude variable requires a deep revision of the adiabatic elimination of atomic polarization, which is performed in the rate equation derivation. This task is outside the scope of the present paper.

Whether in the single-mode or in the multimode regime, two frequency domains have to be distinguished. There are the optical frequencies, characterizing the cavity eigenmodes. They are shifted from the empty cavity frequencies as a result of the frequency pulling and pushing induced by the light-matter interactions. However, there are also the relaxation oscillation frequencies (ROF's) which are in the kHz to MHz range for common solid state lasers. In principle, there are as many ROF's as there are modes, although this is not an absolute rule [8]. When there is more than one ROF, the ROF originating from the single-mode regime is usually the one with the highest frequency, the others having

typically about half that value. A feature of both the experimental data and the aforementioned models is the phenomenon of antiphasing. This occurs at the ROF's when there is destructive interference of the contributions of the individual laser cavity modes to the fluctuations of the total intensity. Simple quantitative expressions that relate the power spectral densities of modal intensity fluctuations to total intensity fluctuations have been derived for two- [9] and three- [10,11] mode operation of the laser. The relations describe how these interferences occur at each ROF, specifying which mode pairs interfere constructively and which interfere destructively. For example, in two-mode operation where there is only one-mode pair, at the lower ROF the modes are in antiphase (destructive interference) and at the higher frequency they constructively interfere. Thus if the destructive interference is complete there will only be one resonance seen in the power spectrum of total intensity fluctuations, that at the higher system frequency. It is important to stress that these interferences at the ROF's correspond physically to the fluctuations of the field intensity and not the complex field amplitude.

While these expressions are predicated on a particular laser model (i.e., TSD), they have a certain universality in the sense that they are independent of the detailed parameters of the laser, such as pump rate (insofar as operation with a given number of modes is maintained). A necessary condition for the derivation of these simple relations is that the lifetime of the cavity modes is much smaller than that of the population inversion. For solid state lasers with the Nd^{3+} ion as the active species this is generally the case. Experiments with a LNP (lithium neodymium tetraphosphate) laser [12], where these rates differ by a factor of about 10^6 , have confirmed these relations. One purpose of this paper is to put forward another case where these relations are tested, but where the decay rates differ by a smaller factor. It will be seen that the relations are verified for two-mode operation but not for three mode. It is well known that the TSD model has limitations when applied to real lasers. In particular it assumes that the gain medium fills the cavity and that the pumping is uniform. Although neither of these apply to our laser, we persist with the TSD model because with this model the relations can be derived analytically. Relations

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have been analytically derived for the two-mode case with a rate equation model which takes into account the large scale nonuniformity of the pump [7], as one would find in an end-pumped laser, but do not so far appear to be possible for three modes or more, which is of particular interest here.

We use a multimode Nd:YAG (neodymium-doped yttrium aluminum garnet) laser, in which the upper state lifetime of the lasing transition is 230 μ s [13] and the cavity lifetime is 2.0 ns. This latter value is derived from the ROF of the laser in single-mode operation. The ratio of decay rates is thus $\sim 1.1 \times 10^5$. The laser is operated in steady state with either two or three longitudinal modes, in which the relaxation oscillations of the laser modes are globally coupled by gain sharing. For N laser modes there are N ROF's which occur up to the single-mode ROF [14], and are of the order of tens of kHz making their direct study relatively simple. The ROF's are excited only by ambient noise in this paper.

In Sec. II we review the theory behind antiphase dynamics of the multimode laser in the context of the TSD model, making explicit the connection between the experimentally obtained spectra and the variables of the theory. Then in Sec. III the experimental arrangement is described. Experimental results for the two-mode case are presented in Sec. IV and there it is demonstrated that they agree well with theoretical predictions of power spectral densities of the modal intensities at each of the ROF's. A confirmation of these results based on the cross-spectra between modal pairs is also presented. In contrast to Sec. IV, we present in Sec. V results from three-mode operation that show clear quantitative disagreement with the predictions of the theory based on the TSD model. The data suggest that in the correct model the relative phases of the modal intensity fluctuations will be different, but that power spectra relations similar to the ones introduced below will be obtained. Further, a recent comparison of a model incorporating nonuniform pumping with one that assumes uniform pumping shows that this aspect hardly affects the threshold properties of a laser [15]. This leads us to suspect that the assumption of pump uniformity is not the relevant limitation of the TSD model.

II. THEORETICAL RESULTS

A. The TSD model

In 1963, Tang, Statz, and deMars proposed a simple model to describe the dynamics of multilongitudinal mode Fabry-Perot class-B lasers [1]. Starting from a noiseless semiclassical laser description, the derivation of the model proceeds in three stages. Firstly, the medium polarization is adiabatically eliminated. This simplification holds if the polarization lifetime is much smaller than both the photon and population inversion lifetimes. This is in particular the case for the LNP, Nd:YAG, and semiconductor lasers. Then, the population inversion is spatially Fourier expanded. The last stage consists of discarding all but the first order Fourier components. For a N mode laser, this results in $2N+1$ nonlinear ordinary differential equations,

$$\begin{aligned} \frac{dI_m}{dt} &= \kappa I_m [\gamma_m (N_0 - N_m/2) - 1], \\ \frac{dN_m}{dt} &= \gamma_m N_0 I_m - N_m \left(1 + \sum_{k=1}^N \gamma_k I_k \right), \\ \frac{dN_0}{dt} &= w - N_0 - \sum_{k=1}^N \gamma_k (N_0 - N_k/2) I_k, \end{aligned} \quad (1)$$

with $m = 1, \dots, N$. In these equations, I_m is the intensity of mode m , N_0 and N_m are zero- and first-order spatial Fourier components of the population inversion, i.e.,

$$\begin{aligned} N_0 &= \frac{1}{L} \int_0^L N(x, t) dx, \\ N_m &= \frac{2}{L} \int_0^L N(x, t) \cos(2k_m x) dx \end{aligned} \quad (2)$$

with $N(x, t)$ the population inversion density, k_m the optical wave number of mode m , and L the cavity length. Time t is measured in units of the population inversion relaxation time, w is the pump parameter averaged over the cavity length, and $\gamma_m \leq 1$ is the gain of mode m relative to the maximum gain mode $m=1$. In what follows, we order the modes according to their lasing threshold value, or equivalently in the case of the TSD model such that $1 = \gamma_1 > \dots > \gamma_N$. Equality between two or more mode gains occurs in degenerate cases, but we do not consider this possibility here since it does not correspond to any of our experiments. The κ parameter is the inverse photon lifetime that we assume equal for all modes. One of the formal limitations of the TSD model is that it describes a homogeneously pumped medium filling the cavity over its whole length. More complex models have been recently proposed for a partially filled cavity or an inhomogeneous pump mechanism [6,7,16,17]. However, as the inhomogeneity of the inversion acts mainly on the strength of mode-mode competition, we expect the TSD model to give a qualitative description of the dynamics of inhomogeneously pumped lasers by using an effective pump value and effective mode gains.

As mentioned in the preceding section, κ for Nd:YAG lasers is about 10^5 . For such a large value, and in the absence of parameter modulation, the TSD model displays fairly simple dynamics. For $w < 1$, all modes are switched off. At $w = 1$ the $m=1$ mode starts lasing, followed by the other modes as w increases further. Whatever the pump value, the system always possesses a single stable steady state such that modes whose threshold is below the current pump value are lasing while the other modes are not. If weakly perturbed, the laser returns to its stable steady state via damped oscillations. For an N mode laser with all unequal gains, N different relaxation frequencies $\omega_1 > \dots > \omega_N$ are found in the power spectra of the relaxation transients of each mode.

For a small enough perturbation, Eqs. (1) can then be linearized. Solving the linear equations, the relaxation transient is given by

$$I_m(t) = I_{m,sl} + \sum_{k=1}^N (\epsilon_k a_{m,k} e^{(\gamma_k + i\omega_k)t} + \text{c.c.}) + O(\epsilon^2), \quad (3)$$

with $I_{m,sl}$ being the steady intensity of mode m , $\gamma_k + i\omega_k$ the k th eigenvalue of the linear problem, and $a_{m,k}$ the amplitude of the m th component of the k th eigenvector. As all the γ_k are negative, the transient is damped. It is important to note that the coefficients $a_{m,k}$ are independent of the perturbation, contrary to the complex parameters ϵ_k that are univocally determined by it. Being linear, Eq. (3) applies also to the total intensity $I_{tot} = \sum_{m=1}^N I_m$ if the $a_{m,k}$ are replaced by $a_{tot,k} = \sum_{m=1}^N a_{m,k}$. The eigenvectors being defined up to a complex scaling factor, we choose $a_{tot,k}$ real and not negative for all k .

B. Fourier component relations

In the limit of large κ , the frequencies ω_k scale as $\kappa^{1/2}$ while the damping rates γ_k are of $O(1)$ [18]. These two time scales lead to the existence of a range of durations T such that $\omega_k^{-1} \ll T \ll \gamma_k^{-1}$. That is, during a time T , the laser displays many oscillations without being significantly damped. This fact simplifies the computation of the ω_k components of the Fourier-transformed intensities over a limited duration T :

$$\mathcal{F}_T(I_m, \omega_k) = \int_0^T I_m(t) e^{-i\omega_k t} dt \approx \epsilon_k a_{m,k} T. \quad (4)$$

This relation holds also for $\mathcal{F}_T(I_{tot}, \omega_k)$ if $a_{m,k}$ is replaced by $a_{tot,k}$. The set of relations (4) allows one to recover the eigenvectors of the linearized TSD system from the Fourier transform of the laser intensity time traces, since the ϵ_k and T parameters are easily eliminated to give

$$a_{m_1,k} \mathcal{F}_T(I_{m_2}, \omega_k) \approx a_{m_2,k} \mathcal{F}_T(I_{m_1}, \omega_k). \quad (5)$$

C. Power and cross-spectra relations

Checking the validity of the set of relations (5) is a difficult task as every $a_{m,k}$ depends on the pump w and the modal gain parameters γ_m . However, universal relations independent of the pump and the gains can be established using the power spectral densities (PSD) and cross-spectral densities (CSD) defined, respectively, by

$$\mathcal{P}_T(I_m, \omega_k) = \frac{1}{T} |\mathcal{F}_T(I_m, \omega_k)|^2, \quad (6)$$

$$\mathcal{C}_T(I_m, I_n, \omega_k) = \frac{1}{T} \mathcal{F}_T(I_m, \omega_k) \mathcal{F}_T^*(I_n, \omega_k), \quad (7)$$

with the asterisk denoting the complex conjugate. To derive these relations, it is necessary to refer to two properties of the $a_{m,k}$ coefficients that hold in the limit $\kappa \gg 1$. The first property is that all $a_{m,k}$ can be chosen real in a first approximation. This comes from the fact that $\text{Re}(a_{m,k}) \sim \kappa^{1/2} \text{Im}(a_{m,k})$. This fundamental property is at the origin of the antiphase laser dynamics, i.e., the absence or the strong reduction of the $\omega_2, \dots, \omega_N$ frequency peaks in the total

TABLE I. Sign of the $a_{m,k}$ coefficients as predicted by the TSD model for three-mode operation. The two-mode case is obtained by removing the $a_{3,k}$ row and the ω_3 column.

	ω_1	ω_2	ω_3
$a_{1,k}$	+	−	−
$a_{2,k}$	+	+	−
$a_{3,k}$	+	+	+
$a_{tot,k}$	+	+	+

intensity transient. Indeed, the set of N modes is divided in two clusters for each ω_k : the N' modes whose $a_{m,k}$ are positive, and the $N - N'$ modes whose $a_{m,k}$ are negative. Modes belonging to the first cluster oscillate in phase, but they oscillate out of phase (i.e., phase shifted by π) with the modes of the other cluster. Therefore, the total intensity oscillates with an amplitude resulting from the contribution of the first cluster reduced by the contribution of the second one. The second property of $a_{m,k}$ is that above the threshold of mode m , all $\text{sgn}(a_{m,k})$ are constant. These are given in Table I for three-mode operation. These two properties have been demonstrated for any N and $\gamma_m \approx 1$ [19], for $N=2$ and any γ_m [12], and for $N=3$ and any γ_m [11], but we conjecture that they are valid for any N and any γ_m . It is now straightforward to show that

$$\sqrt{\mathcal{P}_T(I_{tot}, \omega_k)} = \sum_m \text{sgn}(a_{m,k}) \sqrt{\mathcal{P}_T(I_m, \omega_k)}, \quad (8)$$

and

$$\arg[\mathcal{C}_T(I_m, I_n, \omega_k)] = \begin{cases} 0 & \text{if } \text{sgn}(a_{m,k}) = \text{sgn}(a_{n,k}) \\ \pi & \text{otherwise.} \end{cases} \quad (9)$$

These relations are universal since they hold whatever the perturbation, the pump or the gain parameters. Table II illustrates Eq. (9) for all m, n , and k , in the case $N=3$. Equations (8) and (9) describe interference of the modal intensity fluctuations. If $\arg[\mathcal{C}_T(I_m, I_n, \omega_k)] = \pi$, modes m and n are in antiphase; that is, their fluctuations at ω_k interfere destructively.

TABLE II. Theoretical prediction of the cross-spectra complex phase $\arg[\mathcal{C}(I_m, I_n, \omega_k)]$ for three-mode operation. The two-mode case is obtained by removing the row and the column labeled I_3 , as well as the three columns corresponding to ω_3 . A null value indicates that modes m and n oscillate in phase, while π means that they oscillate with opposite phase. These results follow immediately from Eq. (9) and Table I.

	ω_1			ω_2			ω_3		
	I_1	I_2	I_3	I_1	I_2	I_3	I_1	I_2	I_3
I_1	0	0	0	0	π	π	0	0	π
I_2	0	0	0	π	0	0	0	0	π
I_3	0	0	0	π	0	0	π	π	0
I_{tot}	0	0	0	π	0	0	π	π	0

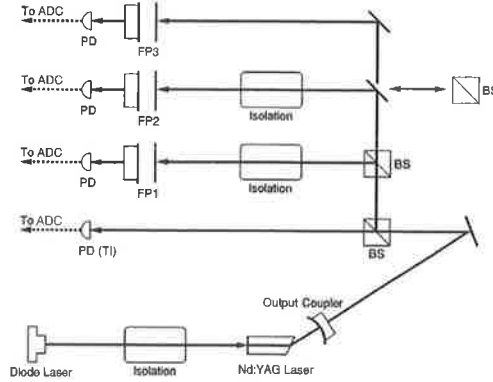
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FIG. 1. Schematic of the optical part of the experimental setup for two-mode operation. ADC, analog to digital converter; PD, photodiode; TI, total intensity; FP1, Fabry-Perot 1; FP2, Fabry-Perot 2; FP3, Fabry-Perot 3; BS, beam splitter. For three-mode operation, one mirror is replaced by a BS as indicated.

D. Application to noisy lasers

In practice, every laser is a noisy device. The noise originates from the spontaneous emission (quantum noise) and from environmental perturbations. Modeling the noise properties of an experimental Nd:YAG laser system is a formidable task, out of the scope of this paper. Therefore, we choose to use a qualitative approach by modeling the noise effect as a series of instantaneous, independent and random kicks that perturb the laser. Right after a kick, the laser relaxes towards its steady state, until it is pushed again by yet another kick. If the noise level is weak, the laser remains in the neighborhood of its stable steady state and the linear theory we have developed previously applies. The dynamics of the laser between two kicks is then described by Eq. (3), with each ϵ_k being a random constant that is updated after each kick. If the average duration T between two kicks is such that $\omega_k^{-1} \ll T \ll \gamma_k^{-1}$, the relations (8) and (9) still hold since they are independent of the perturbation. Moreover, the laser does not have enough time to relax to its steady state before being perturbed again. Therefore, the upper integration bound of Eq. (4) can be taken much larger than T . As a result, long time traces are appropriate to compute the PSD's and the CSD's.

III. EXPERIMENTAL SETUP

Our Nd:YAG laser (Nd~1% concentration) is end pumped by a 40 mW diode laser operating at 808 nm; see Fig. 1. The YAG rod is 10 mm long and 3 mm in diameter. It is perpendicularly cut at the front face and Brewster cut at the other to force linear polarization, ensuring that the laser system does not exhibit polarization instability and switching [20]. The front face of the YAG rod forms one end of the cavity; it is highly reflective at 1064 nm and partially reflecting ($R \sim 30\%$) at 808 nm. Optical isolation (measured to be approximately 35 dB) with a polarizing beam splitter and a

Fresnel rhomb was used between the YAG rod and the laser diode to minimize instabilities in the diode. At the other end of the YAG laser cavity is a spherical output coupling mirror ($R \sim 98\%$ at 1064 nm) with radius of curvature 2.5 cm. The output coupling mirror was chosen for relatively low frequency relaxation oscillations rather than high output power. The output power is of order 1 mW. The optical length of the cavity is approximately 2.5 cm giving rise to a measured longitudinal mode spacing of 6.15 GHz. The laser operates on the 1064 nm transition with up to six longitudinal modes, depending on the exact cavity length and pump power. The output beam had a good TEM₀₀ transverse intensity profile. The output coupler was mounted on a piezoelectric transducer to allow fine control of the cavity length, so that we could offset the laser mode spectrum with respect to the gain spectrum and actively control its length with a servo-loop.

Figure 1 shows the optical setup for two-mode operation. The laser cavity modes were separated using Fabry-Perot interferometers. Optical isolation with quarter wave plates and polarizing beam splitters (measured to be approximately 28 dB) was used between the YAG laser and the Fabry-Perots. The modal and total intensities were detected with identical high gain, low noise photodiode-amplifier circuits (Photodiodes - Epitaxx, ETX 1000T). With up to four detection channels, each modal intensity and the total intensity could be sampled simultaneously. Simultaneous sampling minimizes the effect of the thermal drift of the Nd:YAG laser cavity by keeping the time scale of the measurement short (525 ms), and allows a determination of the scaling of each optical channel. We can then compare the measured total intensity (TI) directly with the summed modal intensities, which is a useful test of our method.

The intensities were recorded with two, 12 bit, 2 channel analog to digital converters (ADC's) (Gage Applied Sciences Inc., CompuScope 512) mounted in separate PC's. Each of the input channels recorded 2^{19} points at a sample rate of 1 MS/s. The sampling interval was much shorter than the period of highest ROF ($\sim 20 \mu\text{s}$) and the number of recorded points gave an adequate amount of data for averaging in the frequency domain. Once the appropriate input range of the ADC was selected, extra gain was applied directly after the photodiode-amplifiers to maximize the signal-to-noise ratio (SNR) and resolution of the data. The recorded intensity data was transformed into the frequency domain (i.e., power or cross-spectrum) using the modified periodogram method [21].

An aluminum spacer was used in the Nd:YAG laser cavity. Due to thermal expansion, the frequencies of the laser modes and their associated gains could change. We temperature stabilized the laser cavity to minimize this effect. One Fabry-Perot (FP1) (TecOptics Ltd., SA-10 with a cavity life-time of approximately 4.25 ns) was spaced with Invar which has a much lower coefficient of expansion than aluminum. In the experiments we used it as a reference cavity to which we actively stabilized one of the laser cavity modes (the first to reach threshold) and therefore the laser cavity length. This fixed the position of the cavity modes with respect to the peak of the gain spectrum, and thus the relative gains of the modes. We did this by applying a very low level dither to the

piezoelectric transducer of FP1 at 1251 Hz which is far enough away in frequency from the ROF's of the laser to be easily distinguishable when it appears in the power and cross-spectra. A lock-in amplifier generated an error signal from the intensity of the first laser mode to reach threshold which in turn was fed back to the output coupler of the YAG laser cavity. The combination of active stabilization of the laser cavity and simultaneous sampling of all intensities maintained a constant position of the laser cavity modes with respect to the gain spectrum for the duration of the experiment. The other Fabry-Perot (FP2) (cavity lifetime of approximately 230 ps) was also dithered and, with a secondary feedback loop, locked to the laser cavity mode it transmitted (i.e., the second to reach threshold).

For three-mode operation, the system was extended with the addition of a third Fabry-Perot (FP3) (cavity lifetime of approximately 480 ps), see Fig. 1. This Fabry-Perot transmitted the third laser cavity mode to reach threshold. In two-mode operation FP2 was locked to the laser mode that it transmitted, using the same technique that was used for stabilizing the laser cavity to FP1. In fact the data was acquired in about 0.5 s and the drift of FP2, without stabilization, in this time proved to be negligible. Thus for three-mode operation we did not lock FP2 or FP3 to the modes that they transmitted. The photodiode-amplifier circuit used to detect the intensity from FP3 was identical to those used previously. After further gain, this signal was fed into the fourth available channel of the ADC's.

For both two- and three-mode operation, the relative gains of the laser cavity modes were always chosen to be unequal. To do this we offset the laser mode spectrum with respect to the gain spectrum while viewing the laser modes with a Fabry-Perot used as an optical spectrum analyzer.

In both operation regimes, each optical channel had a different amount of light impinging on the photodetector. To make accurate calculations (and for comparing our data with other results [9–12]), it is therefore important to determine the appropriate scaling for each optical channel. This was done by starting the laser in single-mode operation (each time an experiment was performed) and recording that mode with each channel. The standard deviation of the intensity noise was calculated from the data and the scalings found. Therefore in both two- and three-mode operation, the intensity series were normalized to one of the optical channels before transformation into the frequency domain.

For each operating point the total intensity and the modal intensities were recorded. The power spectral densities of the fluctuations of each of the total intensity, the sum of modal intensities, and the individual modal intensities were calculated. The sum of modal intensities, and its power spectrum,

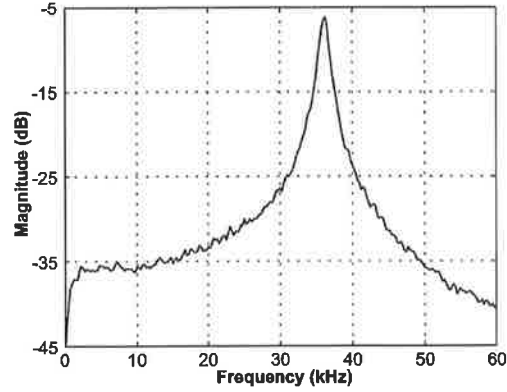


FIG. 2. Power spectrum of the total intensity noise fluctuations, two-mode operation. The system ROF's are 7.50 and 36.25 kHz.

were calculated as a check of the scaling of the different recording channels. Cross-spectra were also calculated for pairs of modal intensities.

IV. TWO-MODE OPERATION

In this section we use data recorded by our experimental setup working in two-mode operation to check the validity of the relations (8) and (9). The data set has been obtained for a pump value $w=1.022$, the first mode threshold being $w_1=1$ by definition and the second mode threshold being located at $w_2=1.010$. The two relaxation frequencies, $\omega_1 \approx 36.25$ kHz and $\omega_2 \approx 7.50$ kHz, have been determined from the location of the resonant peaks of a modal intensity power spectrum. The PSD of the directly measured total intensity is shown in Fig. 2. The observed peak is located at ω_1 and has a full width at half maximum of 1.22 kHz. No peak is apparent at ω_2 , the PSD being made of a superposition of noise and the low frequency wing of the ω_1 peak. The ω_1 and ω_2 PSD values computed from the experimental time traces of I_1 , I_2 , and I_{tot} are given in Table III. To check the consistency and the precision of our measurement, we also computed the PSD of I_1+I_2 . The last column of Table III is obtained by application of the relations (8) that, in the two-mode case, reduce to [12]

$$\begin{aligned}\sqrt{\mathcal{P}_{\text{TSD}}(I_{tot}, \omega_1)} &= \sqrt{\mathcal{P}(I_2, \omega_1)} + \sqrt{\mathcal{P}(I_1, \omega_1)}, \\ \sqrt{\mathcal{P}_{\text{TSD}}(I_{tot}, \omega_2)} &= \sqrt{\mathcal{P}(I_2, \omega_2)} - \sqrt{\mathcal{P}(I_1, \omega_2)}.\end{aligned}\quad (10)$$

TABLE III. Power spectral densities for two-mode operation: $\mathcal{P}(I_1)$, $\mathcal{P}(I_2)$, $\mathcal{P}(I_{tot})$ correspond, respectively, to the measured modal and total intensity fluctuations, $\mathcal{P}(I_1+I_2)$ is computed by summing the fluctuation time traces of the two modal intensities, and $\mathcal{P}_{\text{TSD}}(I_{tot})$ is obtained by application of Eq. (8).

	$\mathcal{P}(I_1)$	$\mathcal{P}(I_2)$	$\mathcal{P}(I_{tot})$	$\mathcal{P}(I_1+I_2)$	$\mathcal{P}_{\text{TSD}}(I_{tot})$
ω_1	0.14774	0.01301	0.24136	0.24837	0.24844
ω_2	0.00538	0.00619	0.00029	0.00033	2.85×10^{-5}

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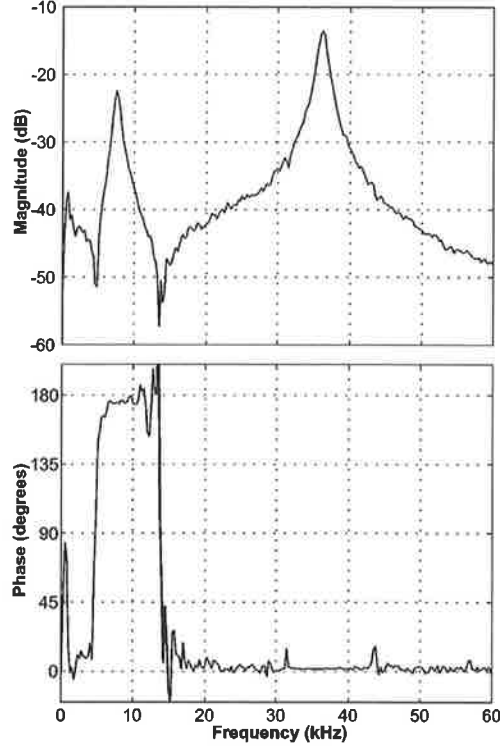


FIG. 3. Cross-spectrum of intensity noise fluctuations between laser modes 1 and 2, for two-mode operation. The ROF's are 7.50 and 36.25 kHz. Phase jumps occur at approximately 4.75 and 13.5 kHz. A weak diather signal (1251 Hz) can be seen in the magnitude. Electronic pickup from switch mode power supplies can be seen in the phase at approximately 32 and 44 kHz.

In Table III it is seen that $\mathcal{P}(I_{tot}, \omega_1)$, $\mathcal{P}(I_1 + I_2, \omega_1)$, and $\mathcal{P}_{TSD}(I_{tot}, \omega_1)$ are in a very good agreement, their relative differences being smaller than 3%. At first sight, the situation is far from being that good at ω_2 : while $\mathcal{P}(I_{tot}, \omega_2)$ and $\mathcal{P}(I_1 + I_2, \omega_2)$ differ by a mere 13%, $\mathcal{P}_{TSD}(I_{tot})$ is ten times smaller. However, this is only an apparent problem that can be rather well explained. Indeed, $\mathcal{P}_{TSD}(I_{tot}, \omega_2) \approx 3 \times 10^{-5}$ suggests that a very strong antiphase dynamics cancels most of the ω_2 oscillation in the total intensity. Therefore, $\mathcal{P}(I_{tot}, \omega_2)$ and $\mathcal{P}(I_1 + I_2, \omega_2)$ mainly measure the background noise intensity, leading to $\mathcal{P}_{TSD}(I_{tot}, \omega_2) < \mathcal{P}(I_{tot}, \omega_2) \approx \mathcal{P}(I_1 + I_2, \omega_2) \ll 1$ which is exactly the situation illustrated in Table III.

Using the cross-spectral method [22], the phase pattern between the two modes has been calculated, see Fig. 3. At ω_1 , the cross-spectral phase is approximately zero, meaning that the two modes are oscillating in phase. With decreasing ω , the phase of the cross-spectrum changes abruptly from 0 to π at about $\omega = 15$ kHz, leading to antiphase dynamics at

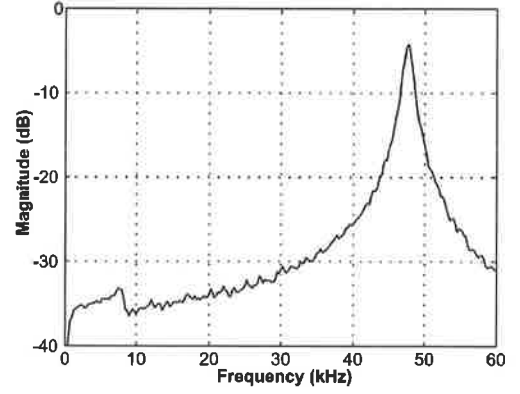


FIG. 4. Power spectrum of the total intensity noise fluctuations for three-mode operation. The ROF's are 8.00, 11.75, and 47.75 kHz.

ω_2 . This result is consistent with the relations (10) and confirms the content of Table II limited to 2 modes.

This experiment and analysis have been repeated with success for different pump levels and mode gains. We therefore conclude that the relations (8) and (9) hold for Nd:YAG lasers in two-mode operation.

V. THREE-MODE OPERATION

We consider now data obtained by operating our Nd:YAG laser system with three modes. The third mode threshold is located at $w = 1.024$ and we operate the laser at $w = 1.037$. The laser relaxation frequencies are $\omega_1 \approx 47.75$ kHz, $\omega_2 \approx 11.75$ kHz, and $\omega_3 \approx 8.00$ kHz. The total intensity power spectrum is shown in Fig. 4. The ω_1 peak is clearly visible, while a vestige of the ω_3 peak is observed 1 to 2 dB above the background noise. It is very weak in comparison with the ω_1 peak. No peak is distinguishable at ω_2 . The theoretical values of $\text{sgn}(a_{m,k})$ are given in Table I, so that Eqs. (8) reduce to [11]

$$\sqrt{\mathcal{P}_{TSD}(I_{tot}, \omega_1)} = \sqrt{\mathcal{P}(I_3, \omega_1)} + \sqrt{\mathcal{P}(I_2, \omega_1)} + \sqrt{\mathcal{P}(I_1, \omega_1)}, \quad (11a)$$

$$\sqrt{\mathcal{P}_{TSD}(I_{tot}, \omega_2)} = \sqrt{\mathcal{P}(I_3, \omega_2)} + \sqrt{\mathcal{P}(I_2, \omega_2)} - \sqrt{\mathcal{P}(I_1, \omega_2)}, \quad (11b)$$

$$\sqrt{\mathcal{P}_{TSD}(I_{tot}, \omega_3)} = \sqrt{\mathcal{P}(I_3, \omega_3)} - \sqrt{\mathcal{P}(I_2, \omega_3)} - \sqrt{\mathcal{P}(I_1, \omega_3)}. \quad (11c)$$

The experimental PSD's computed from the laser intensity time traces and the theoretical predictions of the TSD model are reported in Table IV. The agreement between the PSD's obtained by using the measured total intensity and by summing the modal intensity time traces (columns four and five) is not as good as in the two-mode case, despite careful efforts in measuring the sensitivity of each optical channel. The relative differences between these two estimates of the

TABLE IV. Power spectral densities for three-mode operation: $\mathcal{P}(I_1)$, $\mathcal{P}(I_2)$, $\mathcal{P}(I_3)$, $\mathcal{P}(I_{tot})$ correspond, respectively, to the measured modal and total intensity fluctuations. $\mathcal{P}(I_1+I_2+I_3)$ is computed by summing the fluctuation time traces of the three modal intensities. $\mathcal{P}_{\text{TSD}}(I_{tot})$ is computed from Eq. (8) and $\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot})$ from Eq. (12).

	$\mathcal{P}(I_1)$	$\mathcal{P}(I_2)$	$\mathcal{P}(I_3)$	$\mathcal{P}(I_{tot})$	$\mathcal{P}(I_1+I_2+I_3)$	$\mathcal{P}_{\text{TSD}}(I_{tot})$	$\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot})$
ω_1	0.11626	0.02957	0.00886	0.37732	0.32585	0.36850	0.36851
ω_2	0.00225	0.00879	0.00217	0.00031	0.00077	0.00863	6.536×10^{-8}
ω_3	0.01687	0.00327	0.00628	0.00045	0.00071	0.01163	4.33×10^{-5}

total intensity PSD's are 16%, 150%, and 56% at ω_1 , ω_2 , and ω_3 , respectively. The large discrepancies at ω_2 and ω_3 can be partially explained by the small ratio of the signal to background noise. Nevertheless, there is an agreement on the magnitude of the PSD's. The theoretical estimation $\mathcal{P}_{\text{TSD}}(I_{tot}, \omega_1)$ matches well with $\mathcal{P}(I_{tot}, \omega_1)$ (a 2% error) and $\mathcal{P}(I_1+I_2+I_3, \omega_1)$ (a 13% error). However, $\mathcal{P}_{\text{TSD}}(I_{tot}, \omega_2)$ and $\mathcal{P}_{\text{TSD}}(I_{tot}, \omega_3)$ are much larger than their corresponding experimental values. Thus the TSD model fails to verify the PSD relations for our three-mode laser setup.

Nevertheless, by analyzing the experimental cross-spectral figures, it appears that the intensity components of the eigenvectors can still be chosen real, as predicted by the TSD model in the asymptotic limit of κ large. We report the experimental evaluation of all $\text{sgn}(a_{m,k})$ in Table V. Before analyzing this table in depth, we have to explain the self-inconsistency found amongst its ω_2 -column entries. According to the table, I_1 and I_2 are oscillating out of phase (i.e., phase shifted by π in this case), while simultaneously oscillating in phase with the total intensity. This is most probably caused by the small amplitude of the total intensity at ω_2 relative to the background noise. Therefore, we decide not to take into consideration the ω_2 entries of the total intensity row. To remain consistent, we also do not consider the ω_3 entries of the total intensity row since they could also be similarly affected. Even with these entries removed, the remaining underlined entries of Table V are opposite to the theoretical predictions of Table II. Obviously, these differ-

TABLE V. Experimental measures of the cross-spectra complex phase $\arg[C(I_m, I_n, \omega_k)]$ for three modes. A null value indicates that modes m and n oscillate in phase, while π means that they oscillate with opposite phase. Terms in opposition with Table II are underlined.

	ω_1			ω_2			ω_3		
	I_1	I_2	I_3	I_1	I_2	I_3	I_1	I_2	I_3
I_1	0	0	0	0	π	0	0	π	π
I_2	0	0	0	π	0	π	π	0	0
I_3	0	0	0	0	π	0	π	0	0
I_{tot}	0	0	0	0	0	0	π	0	0

ences result from the inadequacy of the TSD model to describe the structure of our Nd:YAG laser system. Contrary to the hypotheses of the TSD model, (1) our laser active medium does not fill in the whole cavity, and (2) it is end-pumped and the pumping power decreases exponentially along the medium. These features affect the laser dynamics but they are not taken into account in the TSD model. This leads to clear discrepancies between the TSD model predictions and the experimental observations. For instance, we have observed in some experiments that the second mode becomes brighter than the first one once the pump is strong enough. Increasing the pump further can make the third mode to reach threshold the most intense. These observations cannot be explained in the frame of the TSD model and show that we are out of its range of validity.

Knowing that the entries of Table II are not all verified by the experiment, we rewrite Eqs. (8) using the experimental values of Table V. They become

$$\begin{aligned}\sqrt{\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_1)} &= \sqrt{\mathcal{P}(I_3, \omega_1)} + \sqrt{\mathcal{P}(I_2, \omega_1)} + \sqrt{\mathcal{P}(I_1, \omega_1)}, \\ \sqrt{\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_2)} &= \sqrt{\mathcal{P}(I_3, \omega_2)} - \sqrt{\mathcal{P}(I_2, \omega_2)} + \sqrt{\mathcal{P}(I_1, \omega_2)}, \\ \sqrt{\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_3)} &= -\sqrt{\mathcal{P}(I_3, \omega_3)} - \sqrt{\mathcal{P}(I_2, \omega_3)} + \sqrt{\mathcal{P}(I_1, \omega_3)}.\end{aligned}\quad (12)$$

These redefined total PSD's are shown in Table IV in column seven. Note that $\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_1)$ is unchanged relative to Eq. (11a). As already mentioned above, $\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_1)$ matches well with the experimental value. On the other hand, Table IV shows that $\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_2)$ and $\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_3)$ are now smaller than the corresponding experimental values; however, this can be explained with the arguments put forward in the two-mode case. Note that $\sqrt{\mathcal{P}_{\text{TSD}}^{\text{corr}}(I_{tot}, \omega_3)}$ is small but negative. This is due to the experimental noise that affects the measured functions and is of no significance here.

VI. CONCLUDING REMARKS

The good agreement in the two-mode case between the experimental data and the theory based on TSD is not too surprising because of the very restricted range of possibilities for the relative phase of the modal fluctuations. Thus the two-mode case is a check of the experimental technique

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rather than being a check of a theoretical model. It is in the case of three (or more) modes, with the wider range of possibilities for relative phase, that the cross-spectral measurements serve as a sensitive test of theoretical models as shown in Sec. V. The result is that the TSD rate equations fail to give the correct answer because some essential information is not used.

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