Chapter 6

Six Axis Isolator

6.1 Introduction

In this thesis so far, it has been shown that rotational moments can cause undesirable results in active vibration isolation experiments, if their contribution is ignored. In this chapter an active vibration isolator is described which is used to control the vibration transmission along all 6 axes of vibration, that is the 3 translational axes and 3 rotational axes, so that the transmission of all translational forces and rotational moments can be counter-acted by the active system.

6.2 Actuator Construction

The six axis isolator uses Ling Dynamics V201 electromagnetic shakers, which are placed in parallel with a passive vibration isolator, which is an elastic tube made from polyurethane. The control actuators and the passive isolator act against a large aluminium base plate. The base plate is attached to a six axis force transducer which is connected to the receiving structure. The 6 axis force transducer is described in chapter 7. The control actuators and the passive isolator are attached to a 4-arm cross bar. A mass, excited by a shaker, is attached to the top of the cross bar to simulate the mass and excitation forces of a rotating machine.
6.3 Finite Element Modelling of the Isolator

A finite element modelling program (Ansys) was used to calculate the resonance frequencies of the isolator system. It is important that the resonance frequencies of the isolator do not lie within the 0-200Hz frequency range, which is the frequency range of the active control systems investigated in this thesis. The finite element model of the isolator is shown in figure 6.1.

The results from the finite element analysis showed that the thickness of the base plate was important in minimizing the bending of the plate. The initial design used a 10mm thick base plate which resulted in several resonance frequencies within the 0-200Hz frequency band. The plate thickness was increased to 20mm which made the structure stiffer and shifted the resonance frequencies out of the 0-200Hz band. This design was built by the Department’s workshop and is shown in figure 6.2.

6.4 Coupling Elements

This active isolator is capable of reducing vibrations which cause small displacements of the top cross bar. Each end of the cross bar is attached to one or two control actuators. The control actuators provide a restoring force along a single translational axis. If the cross bar moves transverse to the axis of the control actuator, as shown in figure 6.3, then it is possible that the voice coil inside the shaker will be twisted and rub on the permanent magnet inside the shaker, which will eventually cause an expensive problem.

To relieve the shaker from these transverse loads, a connecting element is needed which is capable of transmitting only a translational force along the axis of the shaker, to the cross bar end and will not transmit shear forces or moments.

Three designs were tested as shown in figures 6.4, 6.5 and 6.6.
Figure 6.1: Finite Element Model of the 6 axis vibration isolator.

Figure 6.2: Picture of the 6 axis vibration isolator.

Figure 6.3: Transverse motion of the top cross bar will cause the voice coil inside the shaker to be twisted.
Figure 6.4: Rare earth magnets used to hold a ball bearing.

Figure 6.5: Rolling element bearing used to decoupled moments.

Figure 6.6: Thin wire used to decouple moments.
The first design, shown in figure 6.4, uses a hardened ball bearing held between two high intensity rare Earth magnets. Steel caps were placed on the ends of the magnet to prevent wearing of the magnet’s brittle surface. This design is theoretically the best of all three designs, but is limited by the strength of the magnet. When the control actuator is pushing on the element, there is a rigid connection between the shaker and the cross bar. However, when the control actuator is pulling on the element, the magnetic force of the rare Earth magnets limits the amount of force the control actuator can apply before the ball bearing loses contact with the steel end cap on the magnet and generates unwanted vibrations when the ball re-attaches to the steel end cap.

The second design shown in figure 6.5, uses a self-aligning rolling element bearing which is press fitted into an aluminium housing. This design is capable of transmitting translational forces and decouples all rotations, but does not decouple the shearing motions of the cross bar from the shaker. This design works well, but care must be taken when selecting the bearings to ensure that there is minimal looseness between the inner and outer race. The looseness can be reduced by a suitable press fit of the bearing into the bearing housing, but this is a delicate process. There must be sufficient clearance so that the bearing is free to rotate, but not an excessive amount so that the ball bearings lose contact with the inner and outer races, causing the assembly to rattle.

The third design, shown in figure 6.6, uses a thin spring steel wire soldered into a threaded stud which is screwed into the shaker and a non-threaded stud which is attached to the cross bar. This element is capable of transmitting all axes of vibration, but because the wire is thin, only a small amount of shear force and bending moment are transmitted.

The first two designs could eliminate transmission of several axes of vibration, but were limited by some mechanical aspect. The last design using the wire, does not entirely eliminate motion along any axis of vibration, but proved to be the most practical.
6.5 Conclusions

A 6 axis vibration isolator has been described which can be used to cancel vibration along 6 axes. Finite element modelling was used to check the resonance frequencies of the initial design and it was found that several torsional modes occurred in the operating frequency range. The thickness of the base plate was increased to help stiffen the plate and shift the resonance frequencies of the torsional modes outside the operating frequency range.

Three coupling elements were presented for attaching the control shakers to the cross bar on the active isolator. The most practical design was a thin metal wire which was soldered into two studs.

To minimize the vibration along several axes in an active vibration isolator, a transducer is needed which can measure the forces which are transmitted from the isolator into the receiving structure, such as a beam or a cylinder. In the next chapter a 6 axis force transducer is described which can be used with the 6 axis vibration isolator to perform active control experiments.
Chapter 7

Force Transducers

7.1 Introduction

The theoretical results from section 5.4 demonstrate that negative values of power transmission can be measured along a translational axis if small rotational moments are applied to the receiving structure. This has implications on active control experiments which can cause the value of total power transmission to be greater under active control than for passive isolation, when force or acceleration is minimized along a single translational axis. To fix this problem, it was suggested that power transmission along all 6 axes of vibration should be measured and controlled to ensure control of the total vibrational power transmission.

The measurement of power requires the measurement of velocity and force. Velocity can be measured using an array of 6 accelerometers and an integrating charge amplifier, which converts the acceleration output of the accelerometer into a velocity signal. The use of accelerometer arrays to measure acceleration along several axes is used frequently and needn’t be reviewed here.

The measurement of forces along several axes has not been considered in active vibration isolation experiments presented in the literature. In this chapter, 2 novel force transducers are described which were developed as part of the work described in this thesis to measure force along several axes.
The first transducer uses piezo-electric crystals, which have been shear polarised, sandwiched between two aluminium plates. The arrangement of the crystals permits the measurement of shear forces and the drilling moment about the vertical axes.

The second transducer uses 24 strain gauges to measure forces and moments along all 6 axes.

### 7.2 Shear Force Transducer

Piezo-electric crystals generate a small electrical charge when an alternating pressure is applied to the crystal. A charge amplifier is used to convert the small charge signal into a usable voltage signal which can be measured by instruments, such as a spectrum analyser. The amount of charge developed across the crystal is dependent on the direction of polarisation and the direction of the load on the crystal.

Piezo-electric crystals are polarised during the manufacturing stage when the crystals are being grown, by applying a large voltage between the ends of the crystal. This causes the crystal structure to align itself along the direction of the electric field generated by the applied voltage. Most piezo-electric crystals are "through" polarised, which means that the greatest charge is developed when a load is applied on the top and bottom faces of the crystal, as shown in figure 7.1.

The shear polarised crystals used in this sensor have the most sensitive direction in the plane of the crystal, so that the greatest charge is developed along the opposite faces of the crystal when a shear force is applied to the crystal, as shown in figure 7.2.

![Figure 7.1: A "through" polarised crystal with a load applied to the top and bottom faces of the crystal.](image-url)
The shear crystals were orientated as shown in figure 7.3, to measure shear forces \((F_x \text{ and } F_y)\) and the twisting drilling moment about the vertical axis \((M_z)\). To measure the force along the remaining three axes \((F_z, M_x \text{ and } M_y)\), an additional force transducer is needed. This additional force transducer could be similar in design to the shear force transducer, but using “through” polarised crystals instead of shear polarised crystals. Figure 7.4 shows a picture of the shear force transducer viewed from the side. The outer dimensions of the sensor are 80mm square by 30mm high. The sensor was designed so that the outer body could be used as the electrical ground. This was achieved by using two layers of crystals with a centre electrode used as the positive signal.
centre electrode is shown in figure 7.4 as the aluminium body separating the crystals. Figure 7.5 shows the disassembled view of the sensor.

Trials using this sensor showed that as expected, the sensor was very sensitive along the shear axes, however the sensor was also sensitive to compressive loads in the vertical direction.

Sensors such as accelerometers and force transducers are designed to measure acceleration or force along one translational axis and be insensitive to acceleration or force along the five axes. The transverse sensitivity ratio (Beranek 1988) (or cross axis sensitivity) is the term which describes the ratio of the output signal when the transducer is excited along a non-sensitive axes, to when it is excited along its most sensitive axis. The cross axis sensitivity is expressed as a percentage or as a negative decibel value.

Cross axis sensitivity is an important consideration for sensors which measure forces along multiple axes. Although the crystals used in this sensor were shear polarised, they generate a small charge when a compressive load is applied along the vertical axis.
The design of the sensor uses a cancellation technique to minimize the charge generated when a vertical load is applied. The top and bottom crystals were flipped so that when a vertical load was applied, the top crystal would generate a positive charge and the bottom crystal would generate a negative charge, thereby cancelling the effect of the vertical load. It was found that the cross axis sensitivity was about -10dB. This value could be further improved by using pairs of shear crystals which were matched in their sensitivity to an applied vertical force.

The shear force transducer is compact and has a reasonably high value of cross axis sensitivity. It was thus decided to developed another force transducer which has less cross axis sensitivity, which is described in section 7.3.

One disadvantage of using piezo-electric crystals in force transducers is that the transducer can only be calibrated using a dynamic load. Piezo-electric crystals generate a charge across their electrodes to an applied static load, but an alternating charge is required to generate a voltage and this requires an alternating load. Piezo-electric force transducers are typically calibrated by mounting the force transducer to a rigid mass which has an accelerometer attached to the back of the mass. A vibrating force is applied to the force transducer which will vibrate the mass and the accelerometer. The force transducer can be calibrated by using Newton’s second law, which says that the acceleration of the mass will be proportional to the applied force.

An alternative to using piezo-electric crystals in a force transducer is to use strain gauges mounted to an elastic body. When a stress is applied to the elastic body, the material will distort which can be measured by the strain gauges. An advantage of using strain gauges is that the force transducer can be calibrated using static loads. A force transducer using strain gauges is described in the next section.
7.3 Six Axis Force Transducer

Six axis force transducers are made by a few manufacturers for use on the arms of robots. These transducers are expensive and do not provide analog signals which can be used for active control experiments. This led to the development of a 6 axis force transducer which is shown as a schematic in figure 7.6. This force transducer uses 24 strain gauges mounted to a cylindrical tube to measure forces and moments along all six axes ($F_x$, $F_y$, $F_z$, $M_x$, $M_y$ and $M_z$). Figure 7.7 shows a close up view of the strain gauges mounted to the cylindrical tube. The outer dimensions of the sensor are 70mm outside diameter and 50mm high.

A group of four strain gauges are combined to form a full bridge Wheatstone circuit, which measures the force or moment along a single axis. The full bridge circuit has advantages in increased sensitivity (4 times greater compared to a quarter bridge), negligible thermal effects and decreased cross axis sensitivity.

Each group of four strain gauges is orientated to reject off axis loads (Hoffmann 1976). Consider the group of four strain gauges used to measure the bending moment from the applied vertical force as shown in figure 7.8. In this figure a fully clamped square bar has a pair of strain gauges attached to the top and bottom sides of the bar.
In the actual 6 axis force transducer shown in figure 7.7 the strain gauges are attached to a circular tube. The strain gauges are placed in pairs about the neutral axis of the bar. If a bending load is applied by the application of a vertical force then strain gauges 1 and 3 will be slightly elongated ($\epsilon_1 = \epsilon_3 = \delta$) and strain gauges 2 and 4 will be slightly shortened ($\epsilon_2 = \epsilon_4 = -\delta$). The voltage output of the full bridge circuit will
be

\[ V_0 = K \left( (\epsilon_1) - (\epsilon_2) + (\epsilon_3) - (\epsilon_4) \right) \]

\[ = K \left( (\delta) - (-\delta) + (\delta) - (-\delta) \right) \]  

(7.1)

\[ = 4K\delta \]

where \( V_0 \) is the output voltage, \( K \) is the strain constant in units of volts per unit strain, \( \epsilon_i \) is the strain measured at strain gauge \( i \) and \( \delta \) is an arbitrary small value of strain. Equation (7.1) shows that the output from the full bridge circuit is four times that which would be obtained using a single strain gauge.

If a cross axis load is applied such as a bending moment caused by the application of a horizontal force then strain gauges 3 and 4 will be slightly elongated \( (\epsilon_3 = \epsilon_4 = \delta) \) and strain gauges 1 and 2 will be slightly shortened \( (\epsilon_1 = \epsilon_2 = -\delta) \). The voltage output of the full bridge circuit will be

\[ V_0 = K \left( (\epsilon_1) - (\epsilon_2) + (\epsilon_3) - (\epsilon_4) \right) \]

\[ = K \left( (-\delta) - (-\delta) + (\delta) - (\delta) \right) \]  

(7.2)

\[ = 0 \]

Hence the cross axis load will be electrically cancelled.

The 5 other groups of four strain gauges exhibit these two properties of rejecting cross axis loads and also amplifying the voltage along the most sensitive axis. The orientation of the remaining 5 groups is described below, but similar derivations will not be discussed here.

A group of 4 strain gauges are used to measure the shear force across the transducer and are orientated as shown in figure 7.9. Two groups of 4 strain gauges at 90° apart are used to measure the shear forces \( F_x \) and \( F_y \).

A group of 4 strain gauges are used to measure the drilling moment \( (M_z) \) and are orientated as shown in figure 7.10. The difference in the orientation of the strain gauges used to measure the shear forces and the drilling moment is that strain gauges 3 and
7.3 Six Axis Force Transducer

Figure 7.9: The group of four strain gauges used to measure the shear forces $F_x$ and $F_y$.

Figure 7.10: The group of four strain gauges used to measure the drilling moment $M_z$.

The strain gauges used to measure the compressive axial load $F_z$ are orientated as shown in figure 7.11. When the bar is compressed axially then strain gauges 1 and 3 will become slightly shorter and strain gauges 2 and 4 will become slightly longer from the Poisson’s ratio effect. A similar derivation as the bending case can be made to show that strain gauges 2 and 4 will act to increase the voltage output of the full bridge circuit and to cancel cross axis loads.

An advantage of using a force transducer made with strain gauges compared to piezo-electric crystals is that the transducer can be calibrated using static loads. Force transducers that use piezo-electric crystals can only be calibrated with dynamic loads.
Figure 7.11: The group of four strain gauges used to measure the compressive load $F_z$.

Figure 7.12: Calibration of the 6 axis force transducer.

### 7.4 Calibration

Figure 7.12 shows how the 6 axis strain gauge force transducer was calibrated with static loads. The force transducer was mounted to an angle plate and secured to a table. A beam was bolted to the top flange of the transducer and a mass was hung from a hook on the end of the beam. Several masses and orientations of the transducer were used to determine the on-axis and cross axis sensitivities of the sensor. Table 7.1 lists the sensitivities of the sensor. The cross axis sensitivity of each axis is approximately -20dB relative to the axial sensitivity.

Power transmission is calculated by using the force measured in the cylindrical
Table 7.1: Sensitivities of the 6 axis force transducer

<table>
<thead>
<tr>
<th>Axis</th>
<th>Sensitivity</th>
<th>Cross Axis Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_x$</td>
<td>47 mV/N</td>
<td>-18 dB</td>
</tr>
<tr>
<td>$F_y$</td>
<td>46 mV/N</td>
<td>-19 dB</td>
</tr>
<tr>
<td>$F_z$</td>
<td>-10.5 mV/N</td>
<td>-17 dB</td>
</tr>
<tr>
<td>$M_x$</td>
<td>2.32 mV/(Nm)</td>
<td>-18 dB</td>
</tr>
<tr>
<td>$M_y$</td>
<td>2.35 mV/(Nm)</td>
<td>-18 dB</td>
</tr>
<tr>
<td>$M_z$</td>
<td>2.20 mV/(Nm)</td>
<td>-18 dB</td>
</tr>
</tbody>
</table>

body multiplied by the velocity at the point on the cylindrical body where the force is measured.

An experiment was performed to measure the phase accuracy of this 6 axis force transducer. An experiment was conducted which was based on Newton’s second law; the acceleration of a mass is proportional to the applied force. The experimental setup is shown in figure 7.13. For this test, the 6 axis force transducer was attached to a steel mass which was hanging vertically. A Brüel and Kjær Type 8200 force transducer was attached to the 6 axis force transducer and a Brüel and Kjær Type 4393 accelerometer was attached to the back of the hanging mass. The Brüel and Kjær transducers were electrically connected to Brüel and Kjær Type 2635 charge amplifiers. The system was vibrated horizontally with band limited random excitation.

Figures 7.14 and 7.15 shows the comparison of the amplitude and phase accuracy of the 6 axis force transducer compared with an accelerometer mounted to the rigid mass. The results show that there is a variation in amplitude of about $\pm 0.1$ dB and a variation in phase of about $\pm 1^\circ$, with a bias offset of about $+1^\circ$. The bias phase error is caused by the filters in the analog strain gauge amplifiers which can be corrected by digital filtering (Horner & White 1990). The experimental results presented in this thesis which use the force signals from this transducer have been corrected to take account of this bias phase error. The random phase errors cannot be corrected.

The phase accuracy of the transducers is important to the overall accuracy of the measured value of power transmission. When power transmitted into a lightly damped structure is measured, the phase angle between force and velocity signals are almost
Figure 7.13: Experimental setup to measure the phase accuracy of the 6 axis force transducer.

Figure 7.14: The amplitude accuracy of the 6 axis force transducer.
90° apart, hence the cosine of the angle is close to zero. Small phase errors will result in a large error of the measured power transmission. When the phase angle between force and velocity is relatively large (say 10°) in a heavily damped structure, the phase errors become less important.

Power transmission is calculated by using the force measured in the cylindrical body multiplied by the velocity at the point on the cylindrical body where the force is measured. An ideal transducer used to measure power transmission would be infinitely stiff, have zero inertia, perfect phase accuracy, extremely small and cheap. In this case, when an external force is applied to the body of the cylindrical tube of the force transducer, the tube elongates and is measured by the strain gauges to provide values of force and moments along the 6 axes of motion. The elongation of the tube is undesirable because it can introduce resonance problems and more importantly the velocity measured at the base of the force transducer may not necessarily be the same as the velocity of the material beneath the strain gauge. For this reason, piezo-electric impedance heads, which are a combined piezo-electric force transducer and accelerometer, are preferable because piezo-electric crystals are extremely stiff and consequently the measurement of velocity at the end of the force transducer is well correlated with
the velocity of the piezo-electric material which is measuring the force.

7.5 Measurement of Velocity

Preliminary investigations showed that the most important axes which need to be considered are the axial $Z$ and the two rotational axes $\theta_x$ and $\theta_y$. When horizontal forces are applied to the top of the force transducer, moments are generated on the cylindrical tube and are measured by strain gauges which measure bending moments. Therefore the measurement of the shear forces $F_x$ and $F_y$ and the corresponding accelerations were not required. The measurement of the torsional vibration was also unnecessary because the torsional vibration does not couple well with the transverse vibration of the beam and the cylinder systems investigated in this thesis.

The question arises: where should the accelerometers be placed to measure the velocity of the material beneath the strain gauges? To answer this question, a finite element model was constructed of one experimental setup used in this thesis, as shown in figure 7.16. A vibrating rigid body was attached to a visco-elastic spring which isolated the vibration from a simply supported beam. At the bottom of the vibration isolator is a lumped mass which is attached to the 6 axis force transducer. The force transducer sits on a 1mm thick washer and is attached to the simply supported beam. The 6 axis force transducer is modelled as a cylindrical tube with a plate attached to each end of the tube. The cylindrical tube had an outside diameter of 22mm, an inside diameter of 20mm and a length of 40mm. The top and bottom plates have an outside diameter of 60mm and a thickness of 15mm. The simply supported beam was 25mm wide, 25mm thick and 1.495m long. This finite element model can be used to compare the velocity of the material beneath the strain gauges on the 6 axis force transducer with the velocity measured at a practical location, such as on the beam or on the plates of the force transducer. The top mass of the finite element model was applied with a harmonic load of a vertical force of $F_z = 1$N and a rotational moment around the $\theta_y$ axis of $M_y = 0.002$Nm over a frequency range of 2-200Hz.
It was hypothesised that the axial velocity of the 6 axis force transducer could be measured on the base plate of the force transducer, as the axial stiffness of the 6 axis force transducer is much greater than the stiffness of the receiving structures examined in this thesis. Figure 7.17 shows a comparison of the velocity measured at the location of the strain gauges, the top plate and the bottom plate. The figure shows that the axial velocity beneath the strain gauge can be accurately approximated by measuring the velocity at either the top and bottom plate.

The measurement of the rotational velocity at the location of the strain gauges on the force transducer is made difficult by the low bending stiffness of the cylindrical tube compared to the bending stiffness of the simply supported beam. A practical location to measure the rotational velocity would have been to mount accelerometers to the beam along the axial direction on both sides of the attachment point of the force transducer to the beam. Finite element modelling showed that this was a poor approximation to the measurement of rotational velocities. A better approximation of the rotational velocity at the strain gauge can be made by measuring the angular velocities of the top and bottom plates and interpolating between the two measurements. The angular velocity of each plate can be measured by using two accelerometers placed on opposite edges of plate and by calculating the difference between the two translational velocities.

Figure 7.16: Schematic of the finite element model used to determine suitable locations for the accelerometers to measure the velocity of the 6 axis force transducer.
and dividing by the distance which separates them. Figure 7.18 shows a comparison of the angular velocity measured at the location of the strain gauge, the top plate, the bottom plate and the difference in angular velocities between the top and bottom plates. The results show that the angular velocity at the strain gauge can be reasonably approximated by the difference in the angular velocities of the top and bottom plates.
7.6 Experimental Verification

An experiment was performed to verify that the 6 axis force transducer could be used with an accelerometer array to measure power transmission into a simply supported beam. Figure 7.19 shows the experimental setup. The six axis force transducer was bolted at 0.75m along a simply supported beam of dimension 1.5m length, 25mm square and a steel washer placed in between the force transducer and the beam. An aluminium bar was sandwiched between the top of the force transducer and an aluminium plate. This arrangement simulates a cantilever which is fully clamped to the force transducer. The axis of the bar was in parallel with the axis of the simply supported beam. Five accelerometers were attached to the beam to measure an approximation of the kinetic energy of the beam. The accelerometers were located at 0.30m, 0.35m, 0.40m, 0.45m and 0.50m from the end of the beam. The frequency range of interest is between 0-200Hz, which corresponds to the first 3 vibration modes of the beam. A shaker was attached to the end of the cantilevered beam through a Brüel and Kjær Type 8200 force transducer which was used to measure the force applied by the shaker. Accelerometers were attached to the top and bottom plates on the force transducer as shown in figure 7.16.

A Finite Element Model was constructed using the software package ANSYS of the experimental setup described above. The model was constructed so that predictions

Figure 7.19: The experimental setup of a cantilevered beam attached to the 6 axis force transducer to measure the power.
could be made of the forces, displacements, power transmission and kinetic energy in the beam which could be compared with the experimentally measured values to verify the accuracy of the measurement transducers.

Figure 7.20 shows the predicted and measured accelerations on the top and bottom plates of the force transducer.

Using the acceleration measurements on the top and bottom plates of the force transducer, the vertical and angular displacements of the top and bottom plates can be calculated, which can be used to predict the vertical displacement and the angular displacement of the material beneath the strain gauges used to measure the axial force and bending moments. Figures 7.21 and 7.22 show the vertical and rotational displacements of the force transducer, respectively. The results presented in figures 7.20 to

Figure 7.20: Accelerations on the top and bottom plates of the force transducer compared with theory. (a) Bottom plate accelerometer 1, (b) Bottom plate accelerometer 2, (c) Top plate accelerometer 3, (d) Top plate accelerometer 4.
Figure 7.21: Comparison of the theoretically predicted and experimentally measured displacement along the vertical $Z$ axis.

Figure 7.22: Comparison of the theoretically predicted and experimentally measured angular displacement of the material beneath the strain gauge used to measure bending moment.
Figure 7.23: Comparison of the theoretically predicted and experimentally measured force along the vertical $Z$ axis.

7.22 compare favourably. The resonance peaks in the graphs do not exactly match but this is due to the non-ideal boundary conditions in the experimental setup. The FEM assumes that the cantilevered beam is fully clamped to the top of the force transducer. In reality, the clamping arrangement permits some rotational motion at the bolted connection and is therefore less stiff than the FEM would predict. The lower stiffness results in a lower resonance frequency of the cantilevered beam.

The force along the vertical $Z$ axis and the bending moment along the $\theta_y$ axis are presented in figures 7.23 and 7.24 respectively. The experimental results compare favourably with the theoretical predictions.

The kinetic energy in the simply supported beam is theoretically calculated by the integral of the squared translational and squared rotational velocities over the length of the beam. In practice, only an approximate measure of the kinetic energy is ever obtained by using a finite number of accelerometers attached to the beam and in this case, 5 accelerometers were used. Figure 7.25 shows the theoretically predicted and experimentally measured sum of the squared accelerations at the 5 accelerometer locations. This measure will be called the kinetic energy in the beam. This result shows there is good agreement between the theoretically predicted and experimentally
The experimental results presented so far in figures 7.20 to 7.25 are for amplitude measurements. The phase information in the signals has only been required to calculate the angular displacement of the force transducer. The calculation of power transmission into the simply supported beam requires a high degree of phase accuracy because the
beam is very lightly damped. Figures 7.26 and 7.27 shows the predicted and measured power transmission into the beam along the vertical $Z$ axis and around the $\theta_y$ axis respectively.

The power transmission by rotational moments shown in figure 7.27 has poor agreement between theory and experiment, which is due to the unaccounted resonances in
Figure 7.28: Comparison of the theoretically predicted and experimentally measured phase angle for the rotational displacement along the $\theta_y$ axis.

the theoretical modelling. Figure 7.28 shows the phase angle of the rotational displacement of the material beneath the strain gauges used to measure bending moments. It can be seen that between 0-50Hz, the phase angle passes through $90^\circ$ phase shifts which corresponds to a rotational resonance and is not taken into account by the theoretical model.
7.7 Conclusions

Two force transducers were built to measure forces along several axes. The first transducer used piezo-electric crystals that were shear polarized to measure shear forces in the plane of the transducer. The transducer worked well but was limited by the cross axis sensitivity. The second transducer used 24 strain gauges mounted to a cylindrical tube. Sets of four strain gauges were combined into a single full bridge Wheatstone circuit which could be used to determine the force or moment along an axis. The orientation of the strain gauges was important in minimising cross axis loads.

In the following chapter, the six axis vibration isolator described in chapter 6 and the 6 axis force transducer described in this chapter are used in active vibration isolation experiments to verify the theoretical results from chapter 5 which analysed the power transmission into a simply supported beam from an actively isolated vibrating rigid mass.
Chapter 8

Experimental Investigation of Active Isolation of a Rigid Mass From a Simply Supported Beam

8.1 Introduction

This chapter presents the experimental investigation of the active vibration isolation of a vibrating rigid mass from a simply supported beam. Two types of active vibration isolators are used in the experiments. The first active vibration isolator has a single control actuator which is orientated vertically. The second active vibration isolator is the 6 axis active vibration isolator which was described in chapter 6. The 6 axis strain gauge force transducer described in chapter 7 is used in both experiments to measure the force transmitted into the simply supported beam.

The experimental measurements of the vibration isolation performance are derived from measured transfer function data. The analysis for this method is similar to that described in section 5.4.2, where theoretical transfer functions were obtained between the primary forces and the error sensors and the control actuators and the error sensors, to calculate the power transmission into the beam assuming that a perfect active vibration controller had been used. The transfer function method is described in the
The vibration isolation performance is determined by the change in approximate kinetic energy (KE) of the beam measured by summing the squared accelerations of 5 accelerometers mounted at 0.30m, 0.35m, 0.40m, 0.45m and 0.50m from the end of the beam. This measurement is not affected by phase errors and provides a reasonable approximation of the global KE of the beam. It also provides an independent measure of the isolation performance. Comparisons of the isolation performance using a single sensor, for example the acceleration at the base of the isolator, does not provide a good measure because it is possible to minimize the vibration at the sensor and increase the vibration elsewhere on the supporting structure. The true value of the KE requires the summation of an infinite number of acceleration measurements over the length of the beam to measure the translational and rotational accelerations. Some of the theoretical predictions and experimental results to follow are limited in validity as only 5 accelerometers were used to measure the KE. This limitation is addressed when it is apparent that it affects the results.

Another active vibration control experiment was performed to demonstrate that the heterodyning technique, which was described in section 3.2, can be used to minimize the squared vibrational power transmission into the beam. The experiment was not designed to compare the isolation performance of various cost functions; The efficacy of the various cost functions is examined in the experiments which use the transfer function method.

Initially, active vibration isolation experiments were conducted using the same rig as used by Pan et al. (1993). Pan used a vibrating rigid mass which was mounted on a steel helical spring. An aluminium beam, of dimensions 10mm thick, 1.5m long, 160mm wide, was held between two flat bars which was meant to simulate a simply supported end condition. Modal analysis of the system showed that the beam was not simply supported and that severe rotational resonances existed in the active vibration isolator which would corrupt any active control experiments. Considering this information,
Table 8.1: Parameters used in the new active isolator and beam system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam length</td>
<td>1.500m</td>
</tr>
<tr>
<td>Beam thickness</td>
<td>0.025m</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>207 GPa</td>
</tr>
<tr>
<td>Beam density</td>
<td>7800 kg/m³</td>
</tr>
<tr>
<td>Isolator stiffness $k_z$</td>
<td>45870 N/m</td>
</tr>
<tr>
<td>Isolator stiffness $k_{θ_y}$</td>
<td>216 N/rad</td>
</tr>
<tr>
<td>Isolator location</td>
<td>0.760 m</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>$1.6 \times 10^{-5}$ m⁴</td>
</tr>
<tr>
<td>Beam damping</td>
<td>$7.48 \times 10^{-6}$ sN/m</td>
</tr>
<tr>
<td>Isolator damping $c_z$</td>
<td>140 sN/m</td>
</tr>
<tr>
<td>Isolator damping $c_{θ_y}$</td>
<td>140 sN/rad</td>
</tr>
<tr>
<td>Top mass</td>
<td>7.4 kg</td>
</tr>
<tr>
<td>Bottom mass</td>
<td>8.2 kg</td>
</tr>
</tbody>
</table>

it was necessary to construct a new experimental rig. The new rig used knife edge supports to constrain a new beam which was a steel bar, with dimension 25mm square and 1.55m long. This bar was substantially stiffer than the bar used by Pan and was capable of supporting the mass of the 6 axis vibration isolator. A new single axis active vibration isolator was constructed which used a polyurethane cylindrical tube as the passive vibration isolation element and inside the tube a Ling Dynamics V203 shaker that was used as a control actuator.

The theoretical results in chapter 5 were based on the physical properties of the beam presented in Pan et al. (1993). A new beam and isolator is used in this experimental investigation and it exhibits the same general behaviour under active control as the beam described in chapter 5. The physical properties of the new beam and isolator system are shown in table 8.1 and are used with the general theoretical analysis of chapter 5 to produce the theoretical results presented in this chapter.

### 8.2 Transfer Function Method to Predict the Vibration Isolation using Active Control

The method of using transfer function data to predict the isolation performance of the system is similar to the method described in section 3.2. The only difference is that instead of using a software package to calculate the transfer functions on a finite element model, the transfer functions are experimentally measured on a real
structure. A similar method has been used by Dorling, Eatwell, Hutchins, Ross & Sutcliffe (1987) and Dorling, Eatwell, Hutchins, Ross & Sutcliffe (1989) where measured acoustic transfer function data were used to predict the sound pressure levels inside an aircraft cabin for active noise control.

Transfer functions are measured between the driving force on the structure and the response at the error sensors. The driving force is measured by placing a force transducer between a shaker and the structure. Response measurements are made at the 6 axis force transducer and the accelerometers which measure the acceleration of the structure. Transfer functions are measured between the primary shaker and the error sensors and between the control shakers and the error sensors.

The error signals from the error sensors can be written in matrix form as (Nelson & Elliott 1992, Appendix A.5)

\[
e = d + Cx
\]

where \( e \) is a \((n_e \times 1)\) vector of \( n_e \) error signals, \( x \) is a \((n_c \times 1)\) vector of control signals, \( d \) is a \((n_e \times 1)\) vector of the error signals resulting from passive control and \( C \) is a \((n_e \times n_c)\) matrix of the transfer functions between the control signals and the error signals when the primary disturbance is turned off. The usual goal of active control systems is to determine the amplitude and phase of the control signals which will cancel the primary disturbance, and is given by the re-arrangement of equation (8.1) as

\[
x_0 = -(C)^{-1}d
\]

Equation (8.2) can be solved when there are an equal number of control signals and error signals \((n_c = n_e)\). If there are more error signals than control signals \((n_e > n_c)\) then the problem is said to be over-determined. The matrix \( C \) is not square and cannot be inverted, and generally it is not possible to achieve complete cancellation at all of the error sensors. The problem can be transformed into a least-squares problem such that
the cost function $J$ which is minimized is the squared amplitude of the error signals $e$, which can be written as

$$
J = e^H e 
$$
(8.3)

$$
= x^H C^H C x + x^H C^H d + d^H C x + d^H d 
$$
(8.4)

Equation (8.4) is in the general Hermitian quadratic form, and has a minimum value when the control signals are given by

$$
x_0 = -(C^H C)^{-1} (C^H d) 
$$
(8.5)

When there are more control sources than error sensors ($n_c > n_e$), the minimization problem becomes un-determined and there are an infinite number of solutions for the control sources which will minimize the error signals. The problem can be redefined to include a control effort term, such as $x^H x$, so that the cost function $J$ is minimized with the least amount of control effort. The cost function $J$ is minimized when the control source is given by

$$
x_0 = -C^H (C C^H)^{-1} d 
$$
(8.6)

For example, consider the system shown in figure 5.1 (page 62) where the velocity along the vertical axis at the base of the isolator is to be minimized when the top rigid body is subjected to a harmonic vertical primary force. A transfer function measurement is taken over the frequency range of interest, between the primary driving force and the velocity along the vertical axis at the base of the isolator and this transfer function is called $Z_{vp}$. The primary driving force is then turned off and a transfer function measurement is taken between the force exerted by the control shaker and the velocity along the vertical axis at the base of the isolator and this transfer function is
called $Z_{vc}$. The terms $d$ and $C$ become

$$d = Z_{vp}f_p$$

(8.7)

$$C = Z_{vc}$$

(8.8)

where $f_p$ is the $(n_p \times 1)$ column vector of primary forces, which for this example is $f_p = 1$.

In the experiments that follow, the optimal control forces are calculated by using equation (8.5) or (8.6) depending on the number of error sensors and control forces. In the experiments where squared power transmission is minimized, the optimal control forces are calculated by a different method that is explained later in this chapter.

Gardonio et al. (1997a) has suggested minimizing the weighted sum of squared velocity and squared force along the vertical axis. They gave the vector of optimal control forces as

$$x_0 = -(A)^{-1}b$$

(8.9)

where

$$A = Z_{vc}^H Z_{vc} + \mu Z_{fc}^H Z_{fc}$$

(8.10)

$$b = Z_{vc}^H Z_{vp}f_p + \mu Z_{fc}^H Z_{vp}Z_{fp}f_p$$

(8.11)

$\mu$ is the weighting factor which is applied to the squared force signal so that the amplitudes of the squared velocity signals and squared force signals are similar, $Z_{ij}$ is a transfer function between velocity or force, $i$, and primary or control force, $j$. For example, $Z_{vc}$ is the transfer function matrix of dimensions $(n_e \times n_c)$ between the velocity measured at an error sensor and the driving control force.

When there are more error sensors than control forces, equations (8.9) to (8.11) presented in Gardonio et al. (1997a) cannot be solved and have to be re-written in
terms of the least squares problem formulation. The velocities and forces at the error sensors can be written as

\[
\begin{align*}
v &= Z_{vp} f_p + Z_{vc} f_c \
f &= Z_{fp} f_p + Z_{fc} f_c
\end{align*}
\] (8.12) (8.13)

The terms \(d\) and \(C\) become

\[
\begin{align*}
d &= \begin{bmatrix} Z_{vp} f_p \\ \sqrt{\mu} Z_{fp} f_p \end{bmatrix} \\
C &= \begin{bmatrix} Z_{vc} \\ \sqrt{\mu} Z_{fc} \end{bmatrix}
\end{align*}
\] (8.14) (8.15)

Equation (8.5) or (8.6) can now be used to calculate the optimal control forces depending on the number of error sensors and control forces.

### 8.3 Resonance Frequencies of the Beam

The resonance frequencies of the beam were measured and compared with theory (appendix C, equation (C.15)) to ensure that the knife edge supports provided simply supported boundary conditions. The beam was driven with a shaker at a non-symmetric location and the driving force was measured with a force transducer. An accelerometer was also mounted on the beam at a non-symmetric location to measure the acceleration of the beam to the applied force. A transfer function measurement was made between the driving force and the acceleration of the beam and the frequencies of the peaks in the transfer function were noted as the resonance frequencies of the simply supported beam. These measured resonance frequencies are compared with the theoretically predicted values using equation (C.15) as shown in table 8.2. The results show that there is good agreement between experimentally measured and theoretically
Table 8.2: Experimentally measured and theoretically predicted resonance frequencies of the simply supported beam.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Experiment (Hz)</th>
<th>Theory (Hz)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29</td>
<td>26</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>103</td>
<td>105</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>234</td>
<td>236</td>
<td>-1</td>
</tr>
</tbody>
</table>

predicted values for the 2nd and 3rd modes. The measured frequency of the 1st mode is 12% greater than predicted by theory, which seems large but is actually only a 3Hz difference. These results indicate that the experimental beam is behaving very nearly as a simply supported beam.
8.4 Vibration Isolation Using a Single Axis Active Isolator

8.4.1 Description of the Experimental Setup

Figure 8.1 shows a picture of the experimental rig and figure 8.2 shows how the instruments were connected. A steel beam, of dimensions 1.55m long by 25mm square, was mounted between two knife edges which provided a simply supported end condition. The 6 axis force transducer, described in section 7.3 was bolted to the beam at 0.75m from the end of the beam. Attached to the top of the force transducer was the lower mass that was used to support the end of the vibration isolator. The vibration isolator was a cylindrical polyurethane tube and inside the tube was a Ling Dynamics V203 shaker which provided a cancelling force to counter-act the vibrations which passed through the outer tube. On top of the vibration isolator was a solid steel cylindrical mass which weighed 7.4kg. Five accelerometers were attached to the beam to measure the residual vibration in the beam when active control was applied. The five accelerometers were used to measure the approximate KE of the beam. Four accelerometers were attached the 6 axis force transducer which were used to calculate the acceleration of

![Experimental setup diagram]

Figure 8.1: Experimental setup for the single axis isolator on the simply supported beam.
the material beneath the strain gauges, using the method described in section 7.5.

Figure 8.2 shows how the instrumentation was connected. All the transducers were connected to amplifiers which were connected to the Brüel and Kjær Pulse System which in turn measured the transfer functions. The primary shaker was connected to the top mass and applied a harmonic force which swept in frequency between 5Hz to 200Hz.

8.4.2 Transfer Function Method

In this section, the vibration isolation performance using passive isolation is compared with an active vibration isolation system when various cost functions are minimized. The isolation performance is measured by the approximate KE of the beam. The transfer function method, which was described in section 8.2 is used to calculate the cost functions and the approximate KE of the beam.
Passive Isolation

The displacement, force and power transmission into the beam along the vertical Z axis at the intersection of the force transducer and the beam for passive isolation is shown in figures 8.3 to 8.5 respectively. Figures 8.3 and 8.4 show good agreement between the theoretical and experimental values. Figure 8.5 shows that the experimentally measured values of power transmission into the beam do not match the theoretically predicted values. The difference is attributed to the phase errors in the transducers. The phase accuracy of the force transducer-accelerometer pair was measured to be within ±2°, as shown in figure 7.15. Two curves have been drawn on figure 8.5 to show the theoretical power transmission into the beam for a ±2° phase error in the measurement of the phase angle between the force and displacement. The experimentally measured values generally lie with these two theoretical bounds.

Figure 8.6 shows the phase angle between force and displacement for theory and experiment. There is a reasonable agreement between the two sets of results, but a closer inspection, shown in figure 8.7, reveals that there is a random ±2° phase error. The difference in phase angles between force and displacement is close to 180°.

![Figure 8.3: Theoretically predicted and experimentally measured displacement of the beam along the vertical Z axis for a vertical force of $F_z = 1N$.](image-url)
Figure 8.4: Theoretically predicted and experimentally measured force along the vertical $Z$ axis for a vertical force of $F_z = 1$N.

Figure 8.5: Theoretically predicted and experimentally measured power transmission along the vertical $Z$ axis for a vertical force of $F_z = 1$N. The theoretical values of power transmission are shown for $0^\circ$, $+2^\circ$ and $-2^\circ$ phase errors.
8.4 Vibration Isolation Using a Single Axis Active Isolator

Figure 8.6: Theoretically predicted and experimentally measured phase angle between force and displacement.

Figure 8.7: Theoretically predicted and experimentally measured phase angle between force and displacement.

which means that the difference between the force and velocity would be very close to 90° and hence the small errors in the phase measurements have led to the erroneous measurements of power transmission.
Active Isolation

Theoretical and experimental predictions are presented in this section of the approximate KE of the beam for the passive and active isolation of a vibrating rigid mass that is actively isolated from a simply supported beam. The theoretical results from section 5.3 show that it is possible to stop the vibration from the rigid mass from reaching the simply supported beam if the primary force is aligned with the control actuator. In reality, this is difficult to achieve as there is usually a small misalignment between the primary shaker and the centroid of the rigid mass. For the theoretical results presented in this section, it is assumed that there is 2mm of misalignment, so the primary load on the top mass is $F_z = 1$N and $M_y = 0.002$Nm.

A reasonable approach to the active vibration isolation of this system is to minimize the squared acceleration along the vertical axis at the base of the isolator. Figure 8.8 shows the theoretically predicted KE of the beam for passive isolation, minimization of squared acceleration $A_z^2$ along the vertical axis and the minimization of the sum of the squared accelerations $A_z^2 + A_{\theta y}^2$ along the vertical and rotational axes. The minimization

![Figure 8.8](image-url)
of the sum of the squared accelerations $A_z^2 + A_{\theta y}^2$ along the vertical and rotational axes has a higher KE in general than the minimization of squared acceleration $A_z^2$ along the vertical axis as is also shown in figure 5.11. At 108Hz in figure 8.8 and at 35Hz in figure 5.11, the KE at the rotational resonance is lower when controlling the sum of the squared accelerations along the vertical and rotational axes than when controlling the squared acceleration along the vertical axis. When minimizing the sum of the squared accelerations $A_z^2 + A_{\theta y}^2$ there are 2 error sensors and 1 control source so the cost function is over-determined ($n_e > n_c$). It is not possible to calculate a control force such that the amplitude of both error signals will equal zero. In this case, the sum of the squared accelerations $A_z^2 + A_{\theta y}^2$ along the translational and rotational axes is minimized by increasing the squared acceleration along the vertical axis compared to when minimizing only the squared acceleration $A_z^2$ along the vertical axis. In control systems which have more error signals than control sources ($n_e > n_c$), it is possible to weight the contributions of each error signal to the overall cost function, by multiplying each error signal by a weighting factor. A large weighting factor places greater emphasis on the corresponding error signal in the cost function. Cost functions which use a weighted sum of the error signals are discussed further in this section.

An experiment was conducted to verify the theoretical predictions shown in figure 8.8 and the results are shown in figure 8.9. These experimental results confirm the two theoretical predictions that: 1) in general the KE of the beam for minimization of the sum of the squared accelerations along the vertical and rotational axes is greater than that obtained by minimizing the squared acceleration along the vertical axis and 2) that the KE at the rotational resonance is lower when the sum of the squared accelerations along the vertical and rotational axes is minimized than when the squared acceleration along the vertical axis is minimized.

Figures 8.8 and 8.9 show the limitation of using only 5 accelerometers to measure the KE. Figure 8.8 shows that at 95Hz and 115Hz the KE for the minimization of the sum of the squared accelerations along the vertical and rotational axes is lower
than that obtained for the minimization of squared acceleration along the vertical axis. Figure 8.10 shows the same theoretical predictions as figure 8.8, but this time 10 accelerometers mounted along the beam and 4 on the force transducer were used to calculate the KE of the beam. The theoretical results in figure 8.10 show that by using 14 accelerometers to measure the KE of the beam, the isolation performance at 95Hz and 115Hz when minimizing the sum of the squared accelerations $A_z^2 + A_{\theta y}^2$ along the vertical and rotational axes is similar to minimizing the squared acceleration $A_z^2$ along the vertical axis. The corresponding theoretical total power transmission is shown in figure 8.11 and it is similar to the theoretically predicted KE using 14 accelerometers shown in figure 8.10. The power transmission spectrum can be related to the KE spectrum by a frequency dependent function as shown by Pavić (1993).

It has been suggested by Gardonio et al. (1997a) that the minimization of a weighted sum of the squared velocity and squared force along the vertical axis will have a similar result to the minimization of total power transmission. The purpose of using a weighted sum of squared velocity and squared force is to adjust the signal levels to be a similar order of magnitude. Figure 8.12 shows that when the theoretically predicted value of
8.4 Vibration Isolation Using a Single Axis Active Isolator

Figure 8.10: Theoretically predicted KE (using 14 accelerometers) for the passive isolation, minimization of squared acceleration $A_z^2$ along the vertical axis and the minimization of the sum of the squared accelerations $A_z^2 + A_{\theta y}^2$ along the vertical and rotational axes.

Figure 8.11: Theoretically predicted total power transmission for the passive isolation, minimization of squared acceleration $A_z^2$ along the vertical axis and the minimization of the combined squared accelerations $A_z^2 + A_{\theta y}^2$ along the vertical and rotational axes.
squared force is reduced in amplitude by multiplying by $10^{-8}$ it has a similar signal level to the theoretically predicted squared velocity. Figure 8.13 shows the theoretically predicted KE of the beam for passive isolation, minimization of squared velocity $V_z^2$, minimization of squared force $F_z^2$ and the minimization of the weighted sum of squared velocity and squared force $V_z^2 + \mu F_z^2$, where $\mu = 10^{-8}$.

An experiment was conducted to verify the results from figure 8.13 and the results are shown in figure 8.14. These results show that there is some improvement in the minimization of the weighted sum of squared velocity and squared force over the minimization of squared acceleration. Figure 8.15 shows the experimentally measured squared velocity signal and the experimentally measured squared force signal multiplied by $\mu = 10^{-8}$. It can be seen in figures 8.12 and 8.15 that the squared velocity signal level is greater than the weighted squared force signal, except in the frequency range between about 150Hz and 170Hz. It is then reasonable to expect that the predicted theoretical and experimental results of the minimization of the weighted sum of squared velocity and squared force should follow the response of the squared velocity except in the frequency range between 150Hz and 170Hz where it should follow the results for the minimization of the squared force, which can be seen in figure 8.14.
8.4 Vibration Isolation Using a Single Axis Active Isolator

Figure 8.13: Theoretically predicted KE for the passive isolation, minimization of squared velocity $V_z^2$ along the vertical axis, minimization of squared force along the vertical axis $F_z^2$ and the minimization of the weighted sum of $V_z^2 + \mu F_z^2$ the squared velocity and squared force where $\mu = 10^{-8}$.

Figure 8.14: Experimental results of the KE for the passive isolation, minimization of squared velocity along the vertical axis $V_z^2$, minimization of squared force $F_z^2$ along the vertical axis and the minimization of the weighted sum of $V_z^2 + \mu F_z^2$ the squared velocity and squared force.
Figure 8.15: Experimental results of scaling the squared force signal to be a similar order of magnitude as the squared velocity signal.

Gardonio et al. (1997a) suggested minimizing the weighted sum of squared velocity and squared force along the vertical axis. Another possibility is to minimize the weighted sum of the squared velocities along translational and rotational axes, squared forces and squared moments. Figure 8.16 shows the experimentally measured KE of beam for passive isolation, the minimization of squared velocity along the vertical axis and the minimization of the weighted sum of $V_t^2$ (sum of the squared velocities along the vertical axis $V_z^2$ and around the rotational axis $V_{\theta_y}^2$) and $F_t^2$ (the sum of the squared force along the vertical axis $F_z^2$ and the squared moment $M_y^2$). The minimization of $V_t^2 + \mu F_t^2$ the weighted sum of squared velocities, squared forces and squared moments along translational and rotational axes has slightly better vibration isolation performance than $V_z^2 + \mu F_z^2$ the weighted sum of the squared velocity and squared force along the vertical axis.
Figure 8.16: Experimental results of the KE for the passive isolation, minimization of the weighted sum of $V_z^2 + \mu F_z^2$ the squared velocity and squared force along the vertical axis and the minimization of the weighted sum of $V_t^2 + \mu F_t^2$ the squared velocities, squared forces and squared moments along translational and rotational axes.
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The results from section 5.4 have indicated that active control using signed power transmission as a cost function to be minimized will converge to a negative value if moments are present and could result in the overall vibration response of the receiving structure being greater than it was with only passive isolation. It has been of concern to researchers that small phase errors in the measurement of power transmission can corrupt the true measure of power transmission such that attempts to reduce vibration transmission using active vibration control, with signed power transmission as a cost function, will be unsatisfactory. Figure 8.17 shows the approximate KE of the beam for the minimization of the signed total power transmission when there is a $0^\circ$, $-2^\circ$ and $+2^\circ$ phase error in the measurement of force. The results show that phase errors only slightly affect the active control response, as was also shown in figure 5.15. This assumes that the phase error is constant with time. Real transducers have a random phase error which varies with time. Figure 8.18 shows a theoretical prediction of the approximated KE for passive isolation and minimization of signed power transmission and the minimization of signed power transmission when there is a random $\pm 2^\circ$ phase error. A phase error between $\pm 2^\circ$ was applied to the transfer function measurement.

<table>
<thead>
<tr>
<th>Approx Kinetic Energy (dB re 1m/s$^2$)</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory Passive</td>
<td>0</td>
</tr>
<tr>
<td>Theory Min $Pz+\theta$ 0° error</td>
<td>-20</td>
</tr>
<tr>
<td>Theory Min $Pz+\theta$ -2° error</td>
<td>-40</td>
</tr>
<tr>
<td>Theory Min $Pz+\theta$ +2° error</td>
<td>-60</td>
</tr>
</tbody>
</table>

**Figure 8.17:** Theoretically predicted KE for the passive isolation, minimization of signed total power transmission when there is a $0^\circ$, $-2^\circ$ and $+2^\circ$ phase error in the measurement of force.
8.4 Vibration Isolation Using a Single Axis Active Isolator

Figure 8.18: Theoretically predicted KE for the passive isolation, minimization of signed power transmission $P_z$ along the vertical axis and when there is $\pm 2^\circ$ phase error.

Figure 8.19: The random $\pm 2^\circ$ phase error which was applied to the force transducer for the measurement of the (a) primary response and (b) response of the structure by the action of the control actuator.

between the force response of the structure and the primary load $Z_{fp}$, as shown in figure 8.19(a) and another phase error between $\pm 2^\circ$ was applied to the transfer function measurement between the force response of the structure and the force applied by the control actuator $Z_{fc}$, as shown in figure 8.19(b). The use of two different values of phase error for the primary and control actuator responses simulates a random phase error that varies with time. The theoretical result in figure 8.18 shows that the minimization of signed power transmission along a vertical axis with a small phase
error will produce unsatisfactory results. This was confirmed in an experiment as shown in figure 8.20. Similar results occur when the signed total power transmission is minimized. Figure 8.21 shows the theoretical approximate KE of the beam when the signed power transmission along both the vertical axis and the rotational axis are minimized for an accurate measurement of power and when there is a $\pm2^\circ$ phase error in the measurement of force. Figure 8.22 shows the corresponding experimental result.

The results presented in figures 8.18 to 8.22 verify that attempts to minimize signed power transmission along either a vertical axis or along the sum of the vertical and rotational axes will be limited by the phase errors of the transducers. This result agrees with the comments by Henriksen (1996) and Gardonio et al. (1997a).
8.4 Vibration Isolation Using a Single Axis Active Isolator

Figure 8.21: Theoretically predicted KE for the passive isolation, minimization of the sum of $P_z + P_{\theta y}$ the signed power transmission along the vertical axes and when there is $\pm2^\circ$ phase error.

Figure 8.22: Experimental results of the KE of the beam for passive isolation, minimization of $A_z^2$ and minimization of the sum of the signed power transmission $P_z$ and $P_{\theta y}$. 
The results from chapter 5 suggest that the minimization of squared power transmission would give results better than the minimization of signed power transmission when negative values of signed power transmission are possible. The random phase errors cause an error in the measurement of squared power transmission. The theoretical model can be used to show the effect of phase errors on the isolation performance. Figure 8.23 shows a contour plot of the squared total power transmission at 100Hz for the theoretical model when transducers have no phase errors. The shading indicates constant levels of squared power transmission and darker shading indicates values that are closer to zero. The axes are the real and imaginary parts of the control force and the white dot at the centre of the rings shows the value of the control force which minimizes the squared total power transmission. This result shows that if transducers had no phase errors, then the error surface would resemble a parabolic bowl. If the transducers have phase errors (the same random phase errors as shown in figure 8.19) then the error surface will not have a unique global minimum but will have an infinite number of solutions for the control force which will minimize the erroneous measure.

**Figure 8.23:** Contour plot of the theoretical squared power transmission $P^2_z + P^2_{\theta y}$ along the vertical and rotational axes with no phase error. The white dot shows the control force which minimizes the squared total power transmission. Power transmission is inversely proportional to the darkness of the contour.
Figure 8.24: Contour plot of the theoretical squared power transmission $P_z^2 + P_{\theta y}^2$ along the vertical and rotational axes with a $\pm 2^\circ$ phase error. The white dot shows the control force which minimizes the squared total power transmission with no phase error. Power transmission is inversely proportional to the darkness of the contour. Minimum power transmission is represented by the black ring.

of squared power transmission, as shown in figure 8.24 as the dark ring. Figure 8.24 shows a white dot which is at the same location as the white dot in figure 8.23. This is the control force which an adaptive controller should converge towards. The error surface shown in figure 8.24 resembles a parabolic bowl with an inverted bowl at the centre of the parabola. The same type of error surface was shown in figure 5.21 for the squared power transmission along the vertical axis. Figure 8.25 shows a close up of figure 8.24 around the control force which minimizes the total power transmission with no phase error. Figure 8.25 shows that the control force which minimizes the true value of squared total power transmission does not lie on the ring of solutions which minimizes the erroneous measure of squared total power transmission. Obviously an adaptive controller should converge towards the true value, but the controller could converge to any solution on the dark ring shown in figure 8.24. The controller needs to be guided towards the true value. Figure 8.25 also shows the control force which minimizes the squared acceleration along the vertical $Z$ axis, which is near the control
force which minimizes the true value of squared total power transmission. An adaptive controller could be guided towards minimizing the squared acceleration which will start the adaptation process in the correct direction towards minimizing the true value of squared power transmission. This technique was used here and the control force which was calculated is shown in figure 8.25 as a white dot which lies on the ring of solutions where the squared total power transmission (with phase errors) equals zero. These same three solutions for the control force are shown in figure 8.26 when the contours show the true value of total power transmission, that is with no phase error. The control force which is closest to the control force which minimizes the true value of the squared total power transmission is the better solution. In this case the control force which minimizes the squared acceleration along the vertical axis and the control force which minimizes the squared total power transmission with phase errors have about the same value of total power transmission as they are both on the same contour level.

It is not possible to experimentally demonstrate this phenomena as the force trans-
8.4 Vibration Isolation Using a Single Axis Active Isolator

Figure 8.26: Contour plot of the theoretical squared power transmission $P_z^2 + P_{\theta y}^2$ along the vertical and rotational axes with no phase error, showing the 3 different control forces which minimizes the squared acceleration along the vertical axis, the squared total power transmission with no phase error and the squared total power transmission with $\pm 2^\circ$ phase error. Power transmission is inversely proportional to the darkness of the contour.

...ducers and accelerometers used in the experiments have phase errors and cannot be compared with the an experiment without phase errors. It is possible to experimentally demonstrate the technique described above where the adaptation is guided towards the minimization of squared acceleration. Figure 8.27 shows the experimental results of the KE of the beam when the adaptive controller starts to minimize the squared power transmission along the vertical $Z$ axis from zero control force and when the controller starts from control force which minimizes the squared acceleration along the vertical $Z$ axis. This result confirms that the controller must be guided towards minimizing the true value of total power transmission (with no phase error).

The results which follow in which squared power transmission has been minimized uses this technique to initially guide the solution towards minimizing the squared acceleration.

Figure 8.28 shows the theoretically predicted KE for passive isolation, when the squared power transmission $P_z^2$ along the vertical axis is minimized and the sum of...
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Figure 8.27: Experimentally measured KE of the beam when the adaptive controller starts to minimize the squared power transmission along the vertical axis from zero control force and when the adaptation starts from the minimization of squared acceleration $A_z^2$.

The squared power transmission $P_z^2 + P_{\theta y}^2$ along the vertical and rotational axes when there is a random $\pm 2^\circ$ phase error. This result shows that phase errors associated with the measurement of power will not greatly affect the minimization of squared power transmission. This prediction was confirmed by experiment as shown in figure 8.29. The minimization of squared power transmission along the vertical and rotational axes results in a lower KE of the beam at the rotational resonance of 108Hz, which also confirms the predictions from section 5.4.9, figure 5.35.

Another experiment was conducted when the rigid mass was excited along both the vertical axis and the horizontal axis aligned with the beam. The results from these experiments are not presented as they show the same results described above, except the peak corresponding to the rotational resonance at 108Hz is larger.
Figure 8.28: Theoretical prediction of the KE of the beam for passive isolation, minimization of squared acceleration $A_z^2$ along the vertical axis, minimization of squared power transmission $P_z^2$ along the vertical axis with a random $\pm 2^\circ$ phase error and the minimization of the sum of the squared power transmissions $P_z^2 + P_{\theta y}^2$ along the vertical and rotational axes with a random $\pm 2^\circ$ phase error.

Figure 8.29: Experimental results of the KE of the beam for passive isolation, minimization of squared acceleration $A_z^2$ along the vertical axis, minimization of squared power transmission $P_z^2$ along the vertical axis and the minimization of the sum of the squared power transmissions $P_z^2 + P_{\theta y}^2$ along the vertical and rotational axes.
Figure 8.30 shows that the cost functions considered so far provide about the same level of vibration isolation. The greatest vibration isolation was obtained by the minimization of the weighted sum of squared velocity and force along translational and rotational axes.

Figure 8.30: Experimental results of the KE of the beam for passive isolation, minimization of squared acceleration $A_z^2$ along the vertical axis, minimization of the sum of the squared power transmissions $P_z^2 + P_{\theta y}^2$ along the vertical and rotational axes, minimization of the weighted sum of $V_z^2 + \mu F_z^2$ the squared velocity and force along the vertical axis and the minimization of $V_t^2 + \mu F_z^2$ the weighted sum of squared velocities, forces and moments along translational and rotational axes.
8.4.3 Adaptive Controller Method

An adaptive controller was used to actively minimize the vibration transmitted into the simply supported beam from a vibrating rigid mass. The purpose of this experiment was to confirm that the heterodyning technique, described in section 3.2 which is used to generate an error signal that is proportional to the squared power transmission, could be used in conjunction with a filtered-$x$ LMS adaptive controller to minimize the squared power transmission $P_z^2$ along the vertical axis.

The experimental rig used in this experiment was the same as shown in figure 8.1 and the instrumentation which was used is shown in figure 8.31. Velocity signals were obtained by integrating the acceleration signals from the accelerometers attached to the force transducer, using the analog integrator on the B&K 2635 charge amplifiers. The heterodyning technique was programmed in C and assembly language to run on a Sundance SMT326 Digital Signal Processing board. The board has one floating point 50MHz Texas Instruments TMS320C44 DSP chip, 32 analog input and 32 analog output channels. The force, velocity and reference signals were supplied to the input channels of the DSP board, which multiplied the force and velocity signals together, performed a low pass filter on the resulting signal using a 4 tap IIR filter with the 3dB point at 20Hz, then multiplied the low pass filtered signal by the reference signal and output the resulting signal. The output signal from the DSP board was connected to a conventional filtered-$x$ LMS controller, which in this case was an EZ-ANC active noise controller produced by Causal Systems.

Figure 8.32 shows the approximate KE of the beam for the minimization of squared acceleration along the vertical $Z$ axis and the minimization of squared power transmission along the vertical $Z$ axis. The results illustrated in figure 8.29, which are based on the transfer function method, are also shown in figure 8.32 to verify that the transfer function method will give results which are similar to the results obtained using an adaptive controller. Figure 8.32 shows that minimizing squared power transmission along the vertical $Z$ axis performs as well as minimizing squared acceleration along the
vertical Z axis.

The x-LMS controller converged extremely slowly and was unstable when minimizing the squared power transmission along the vertical axis. The x-LMS algorithm uses an adaptive filter to model the transfer function between a signal that is injected at the control shaker and the resulting error signal. This model is called the cancellation path transfer function model. In this case, the error signal was the squared power transmission which was derived using the heterodyning technique and it was not possible to obtain a good model in this experiment, which is the cause of the poor convergence.

Figure 8.31: Instrumentation used in the adaptive control experiment which uses the heterodyning technique.
and instability of the controller. It can be seen from equation (3.13) on page 38, that the error signal $e(n)$ is a cubic function of the reference signal $x(n)$, which means that the formula for the error signal is a non-linear function. The cancellation path transfer function is assumed to be a linear transfer function and is therefore not able to model a non-linear transfer function accurately. Attempts were made to improve the cancellation path transfer function model by using a long filter length (100 taps), but this did not improve the stability of the system. Another attempt was made to improve the stability by replacing the $x$-LMS controller with an LMS controller which was programmed on the Sundance SMT326 DSP board. The LMS algorithm is almost the same as the $x$-LMS algorithm except the cancellation path transfer function is not used. The use of the LMS controller did not improve the stability of the system.

Further research would be required to improve the stability of the controller when using the heterodyning technique and this is beyond the intended scope of this thesis.
Chapter 8 Experiments on a Simply Supported Beam

8.5 Vibration Isolation Using a Six Axis Active Isolator

It was suggested in chapter 5 that it is theoretically possible to achieve complete cancellation of the vibration from the rigid mass reaching the simply supported beam if the primary load is aligned with the control actuators.

In this section, the 6 axis active vibration isolator, which was described in chapter 6, is used to cancel the vibration from a vibrating rigid mass from reaching a simply supported beam. The purpose of this experiment is to determine if the 6 axis active vibration isolator can improve vibration isolation over the use of a single axis active vibration isolator. Comparisons will be made between the results obtained when using the 4 shakers on the 6 axis isolator which are pointing vertically and driven with the same control force to simulate a single control actuator, to the results obtained when using a number of shakers that are driven independently.

8.5.1 Modal Analysis

To aid in the interpretation of the active control results which are to follow this section, a modal analysis was conducted on the 6 axis vibration isolator mounted to the simply supported beam, using the modal analysis software package called PcMODAL. Figure 8.33 shows the equipment used in the modal analysis. A shaker vibrated the rigid mass in a vertical direction and the applied force was measured using a B&K 8200 force transducer. A B&K 4393 accelerometer with a magnetic base was used to measure the vibration at various points on the 6 axis vibration isolator and the simply supported beam. The accelerometer output was connected to a B&K 2635 charge amplifier which in turn was attached to a B&K 2032 signal analyser. The signal analyser was connected to a personal computer via a GPIB interface. The personal computer was operating PcMODAL. The force transducer was connected to a B&K 2635 charge amplifier which in turn was connected to the B&K 2032 signal analyser. Additional modes shapes were
obtained by attaching the driving shaker to an arm on the 6 axis isolator which was aligned with the beam. The mode shapes of the isolator and beam are shown in figure 8.34. The results show that at 55 Hz the vibration isolator is compressed vertically and the beam is relatively motionless. At 92 Hz the beam is in its second mode of vibration and the bottom plate of the isolator is rotating. At 101 Hz the beam is in its second mode of vibration and the arms and the bottom plate of the isolator are rotating. At 126 Hz the beam is in its second mode of vibration and the isolator is relatively motionless. These results show that there is some relative rotational motion between the bottom plate of the isolator and the beam.

### 8.5.2 Experimental Setup

A picture of the experimental rig of the 6 axis vibration isolator and the beam is shown in figure 8.35. For this experiment, the transfer function method, described in section 8.2, was used to determine the isolation performance of the system. Eight control shakers were attached to the 6 axis vibration isolator and are numbered as shown in figure 8.36. By using the transfer function method, the isolation performance using any
combination of the 8 control shakers can be determined. In the experimental results that follow, 4 combinations of control shakers were used. The first combination used the 4 vertically orientated shakers numbered 1, 2, 3 and 4, and all shakers applied the same control force to simulate a single axis isolator. The second combination used the same 4 shakers but were driven independently to control vertical forces $F_z$ and
rotational moments $M_x$ and $M_y$. The third combination used the 6 shakers numbered 1, 3, 4, 6, 7 and 8, to control loads along all 6 axes of vibration. The forth combination used all 8 shakers to control loads along all 6 axes of vibration.

The primary load on the rigid mass was a vertical force of $F_z = 1\text{N}$ and a force along the axis of the beam of $F_x = 0.5\text{N}$.

8.5.3 Results

In the graphs which follow, the results below 30Hz and above 190Hz should be ignored as the transfer function measurements had poor coherence in these frequency ranges.

Figure 8.37 shows the KE of the beam for passive and active isolation for the simulated single control actuator, 4, 6 and 8 control actuators when the squared acceleration along the vertical axis $A_z^2$ was minimized. The figure shows that in the frequency range 80-90Hz, all the active control results are worse than achieved with only passive isolation. The modal analysis results show that at 92Hz the beam is resonant in its second mode and the onset of this beam mode would be noticeable at 80Hz. As the error sensor can only detect vibration along the vertical axis (which is zero for the second mode of vibration), the controller is unable to control the rotation of the beam (which characterises the beam motion at the base of the isolator for the second mode). In the
Figure 8.37: Experimentally measured KE of the beam using the 6 axis isolator for passive isolation, simulated single axis isolator, 4, 6 and 8 control actuators when the squared acceleration $A^2_z$ along the vertical axis was minimized.

In the frequency range 96-102Hz the results for using 6 and 8 control actuators are worse than passive isolation and using the simulated single axis or 4 control actuators is better than passive isolation. Using 6 or 8 control actuators applies translational forces $F_x$ and $F_y$ along the horizontal plane which must increase the rotational vibration of the isolator and therefore increases the KE of the beam. As the error sensors can only detect vibration along the vertical Z axis, the controller cannot take account of the increased rotational vibration. Figure 8.37 shows that the results for 6 and 8 control actuators are about the same which means that there is no advantage in using 8 control shakers. These results indicate that if control actuators are used to counteract moments, then error sensors should be used which can detect any increase in rotational vibration caused by the application of the control actuators.

Figure 8.38 shows the KE of the beam for passive isolation and active isolation for the simulated single control actuator, 4, 6 and 8 control actuators when the squared acceleration $A^2_z$ along the vertical axis and $A^2_{\theta x}+A^2_{\theta y}$ around the two rotational axes are minimized. The figure shows that the simulated single axis vibration isolator performs very poorly compared to the other 3 combinations of control actuators. The
controller has attempted to reduce the sum of the squared vibration along the vertical and rotational axes at the expense of increasing the squared vibration along the vertical axis. Similar levels of isolation are obtained when using 4, 6 or 8 control actuators. In these cases there are more control sources than error signals ($n_c > n_e$) and the control system is able to cancel the vibration along all axes. These results show the improvement in vibration isolation when error sensors are used which can measure the rotational vibration as compared to minimizing the squared acceleration along the vertical axis.

In theory, if the vibration along all axes in the force transducer can be reduced to zero, then no energy should be transmitted into the beam and the measured KE levels should be extremely low (about 320dB lower than passive isolation, which is the calculation noise floor of the software used to derive the results). The experimental results show that the active isolation system is not able to achieve this theoretical prediction and there are several plausible reasons to explain this difference. The modal analysis results, from figure 8.34, show that the force transducer is not rigidly attached to the beam along the rotational axes. If the force transducer were rigidly attached to
the beam, then the slope of the beam would be the same as the slope of the lower arm on the isolator which is aligned with the beam.

Consider the situation where the top and bottom plates of the force transducer have been rotated as shown in figure 8.39. Fictitious values for the translational displacement of the top and bottom plates have been assigned. The rotational displacements of the force transducer are calculated by the difference in translational displacements of the edges of the top and bottom plates of the force transducer. This situation would correspond to the mode shape at 126Hz shown in figure 8.34. The calculated rotational displacement of the force transducer will be zero \( \theta = 0 \) and yet the beam could be rotated regardless of the rotational rigidity between the lower plate and the beam. This could explain the poor isolation at 126Hz shown in figure 8.38.

Another experiment was conducted in which the 4 accelerometers were mounted to beam on each side of the attachment point as shown in figure 8.40. Two accelerometers were mounted on each side of the beam directly beneath the force transducer and another two accelerometers were mounted along the beam on each side of the attachment

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**Figure 8.39:** A possible distortion of the 6 axis force transducer.

**Figure 8.40:** The accelerometers mounted to the beam around the attachment of the 6 axis force transducer to the beam.
point of the force transducer to the beam. The minimization of acceleration at the location of these 4 accelerometers should have minimized the vertical and rotational accelerations at the intersection of the force transducer and the beam, which means that no power should be transmitted into the beam and therefore the isolator should have exhibited a large amount of vibration isolation. However this did not occur; the results showed the same level of vibration isolation as those presented in figure 8.38. The problem is caused by the small amplitude and phase errors in the measurement of the transfer functions for the primary and control cases. The technique used to predict the active isolation performance relies on accurate measurements of the transfer function between the force applied by a driving shaker and the response on the structure. An amplitude error of 0.1dB (= 1%) in the transfer functions can degrade the isolation performance from extremely high levels of vibration attenuation to only moderate levels of attenuation (20dB).

The next cost function which was investigated was the minimization of the power transmission into the beam. To measure the power transmission through the body of the force transducer, the array of accelerometers must be attached to the top and bottom plates of the force transducer, as shown in figure 8.39. Figure 8.41 shows the KE of the beam for passive isolation and active isolation for the simulated single control actuator, 4, 6 and 8 control actuators when the sum of $P_z^2 + P_{\theta x}^2 + P_{\theta y}^2$ the squared power transmission along the vertical axis and around the two rotational axes are minimized. The results show that using 8 actuators gives the greatest overall vibration attenuation.

Figures 8.42 and 8.43 show the KE of the beam for the minimization of the weighted sum of $V_z^2 + \mu F_z^2$ the squared velocity and squared force along the vertical axis and the minimization of the weighted sum of $V_t^2 + \mu F_t^2$ the squared velocities, squared forces and squared moments along translational and rotational axes using 4 and 8 shakers respectively. The results obtained for the minimization of $V_z^2 + \mu F_z^2$ are the same when using 4 or 8 shakers because the problem is undetermined as there are 4 control
Figure 8.41: Experimentally measured KE of the beam using the 6 axis isolator for passive isolation, simulated single axis isolator, 4, 6 and 8 control actuators when the sum of \( P_z^2 + P_{\theta_x}^2 + P_{\theta_y}^2 \) the squared power transmission along the vertical axis and around the two rotation axes are minimized.

Figure 8.42: Experimentally measured KE of the beam using the 6 axis isolator for passive isolation, for 4 control actuators when the weighted sum of \( V_z^2 + \mu F_z^2 \) the squared velocity and squared force along the vertical axis is minimized and when the weighted sum of \( V_t^2 + \mu F_t^2 \) the squared velocities, squared forces and squared moments along the translational and rotational axes are minimized.
8.5 Vibration Isolation Using a Six Axis Active Isolator

![Graph showing KE vs Frequency](image)

Figure 8.43: Experimentally measured KE of the beam using the 6 axis isolator for passive isolation, for 8 control actuators when the weighted sum of $V_z^2 + \mu F_z^2$ the squared velocity and squared force along the vertical axis is minimized and when the weighted sum of $V_t^2 + \mu F_t^2$ the squared velocities, squared forces and square moments along the translational and rotational axes are minimized.

sources and only 2 error sensors ($n_c < n_e$) and therefore the velocity and force along the vertical axis can be reduced to zero using either 4 or 8 shakers. Varying the weighting factors makes no difference to the result. The results for minimizing $V_t^2 + \mu F_t^2$ using 4 and 8 shakers are different because there are 4 error sensors. The weighting factors made a difference to the results when 4 shakers were used, but not when 8 shakers were used. When 8 shakers were used, the velocities and forces along the translational and rotational axes were reduced to zero. The minimization of the weighted sum of the squared velocities and squared forces provides a broad-band reduction in the KE of the beam compared to the single axis isolator experiment, which was unable to provide control at the rotational resonance. The result is also better than the minimization of squared accelerations along multiple axes. It has already been explained that the rotational vibration of the beam cannot be properly measured due to distortions of the 6-axis force transducer, as was shown in figure 8.39. When the bottom plate of the 6-axis force transducer rotates relative to the upright cylinder of the force transducer,
as shown in figure 8.39, a bending moment is generated on the upright cylinder and is measured by the strain gauges. Therefore, when the controller minimizes the squared velocities and squared forces along multiple axes, additional information is provided on the rotational vibration of the beam and can be counteracted by the control actuators. This results in improved performance compared to the minimization of squared accelerations along multiple axes.

A summary of the results is presented in figures 8.44 and 8.45 for using 4 and 8 shakers respectively, and shows that minimizing the weighted sum of the squared velocity and squared force along the vertical and rotational axes had the best vibration isolation performance.

![Graph](image)

**Figure 8.44**: Summary of the results for the minimization of the KE of the beam using the 6 axis vibration isolator using 4 shakers.
Figure 8.45: Summary of the results for the minimization of the KE of the beam using the 6 axis vibration isolator using 8 shakers.
8.6 Conclusions from the Beam Experiments

The passive and active isolation of a vibrating rigid mass from a simply supported beam has been experimentally investigated. Two types of active vibration isolators were used in the experiments. The isolation performance was measured by the approximate KE of the beam which was measured by summing the squared acceleration from 5 accelerometers mounted to the beam. The active vibration isolation performance was compared for the minimization of:

- the squared acceleration along the vertical axis,
- the unweighted and weighted sum of the squared acceleration along the vertical and rotational axis,
- the weighted sum of the squared velocity and squared force along the vertical axis,
- the weighted sum of the squared velocity and squared force along the vertical and a rotational axis, and
- the squared power transmission along the vertical and a rotational axis.

The first type of isolator that was used in the experiments was a single axis active vibration isolator. The experimentally measured passive vibration response of the system was compared with the theoretically predicted response for a vertical excitation force. The experimentally measured acceleration and force response at the intersection of the 6 axis force transducer and the beam compared well with the theoretically predicted response. The experimentally measured power transmission into the beam compared well with the theoretically predicted values assuming a ±2° phase error in the measurement transducers.

Active vibration isolation was investigated for the minimization of the cost functions itemized above, when a vertical force was applied to the rigid mass. The theoretical model showed that it is possible to stop the vibration from the rigid mass from reaching the beam, provided the control actuators can counteract all the forces and moments from the rigid mass. It was not possible to demonstrate this experimentally as small
misalignments in the system caused moments to be generated and consequently the vertically orientated control actuator was not able to counteract them.

The active minimization of the squared acceleration along the vertical axis resulted in about a 20dB reduction of the KE of the beam compared to passive isolation, except at rotational resonances, where active control slightly increased the KE of the beam compared to passive isolation. This behaviour was found to occur in the theoretical modelling and was verified by experimental testing. The theoretical modelling predicted that the KE of the beam would be generally greater for the minimization of the squared acceleration along the vertical and rotational axes than when minimizing the squared acceleration along only the vertical axis, except at the rotational resonance where better isolation was obtained from the minimization of the squared accelerations along vertical and rotational axes. This was confirmed in the experimental testing - the acceleration along the vertical axis increases in an attempt to reduce the sum of the squared acceleration along the vertical and rotational axes.

In control systems which have more error signals than control sources, it is possible to apply a weighting factor to each error signal so that greater control effort can be directed towards reducing a particular error signal. In this experimental setup, there was only 1 control actuator, so cost functions that were based on more than one error signal were candidates for the application of weighting factors. The isolation performance for active control is dictated by the error signal with the greatest amplitude. This was confirmed in an experiment in which the weighted sum of squared velocity and squared force along the vertical axis was minimized. When the weighted squared velocity signal was larger than the weighted squared force signal, the isolation performance was the same as if only the squared velocity signal had been minimized. It is possible to tailor the isolation performance by choosing appropriate weighting factors so that the cost function can be biased towards minimizing an error signal that will give the best isolation performance. For example, if the minimization of squared force in a particular frequency range gives better isolation performance than the minimization
of squared velocity, a large weighting factor can be applied to the force error signal.

Power transmission as a cost function does not require the selection of weighting factors. Velocity and force have different units and need to be scaled by applying weighting factors. Power transmission uses consistent units of Watts for vibration along vertical or rotational axes and hence the error signal is unbiased. Power transmission as a cost function is theoretically appealing but the practical measurement of power transmission is dogged by phase errors in the measurement transducers. Experimental results were presented that demonstrated the minimization of signed power transmission is useless because of phase errors in the transducers. The phase errors can cause negative values for the measurement of power transmission. The power transmission is minimized at the negative value that will often result in an increase of the KE of the beam.

Surprisingly, the minimization of squared power transmission was not entirely useless. The experimental results for the minimization of squared power transmission were almost the same as the minimization of squared acceleration along the vertical axis. Minimizing the squared power transmission along vertical and rotational axes was able to reduce the KE of the beam at the rotational resonance, and had similar performance to minimization of squared acceleration along the vertical axis over the frequency range examined in the experiment. Phase errors in the transducers still affect the cost function and the isolation performance but not to the same extent as the minimization of signed power transmission. The calculation of the control force that minimized the squared power transmission along the vertical axis involved initially guiding the searching process towards minimizing squared acceleration.

An experiment was conducted to demonstrate that an adaptive controller, which uses a filtered-\(x\) LMS algorithm, could be used to minimize the squared power transmission into the beam along the vertical axis. The heterodyning technique, which was described in section 3.2, was used to generate an error signal that was proportional to squared power transmission. The isolation performance when the squared power
transmission was minimized was about the same as when the squared acceleration or squared velocity was minimized. The adaptive controller was unstable when using the error signal generated by the heterodyning technique. The instability was caused by the inability of the cancellation path transfer function filter to model the non-linear error signal. Further research would be required to improve the stability of the controller. The experiment also verified that the same results could be obtained by using the transfer function method or by using an adaptive controller.

Another experiment was performed in which the vibration from the rigid mass was prevented from reaching the beam by using a 6-axis active vibration isolator. The rigid mass was excited along both the vertical axis and the horizontal axis aligned with the beam. It is theoretically possible to stop the vibration from the rigid mass from reaching the beam if control actuators can counteract all the forces and moments transmitted by the rigid mass. The six axis isolator was designed for this purpose; however it was not possible to demonstrate this experimentally.

The 6-axis isolator was able to provide broad-band reduction in the KE of the beam of around 20dB, whereas the single axis isolator was unable to provide a reduction at the rotational resonance. Various cost functions were minimized and the best isolation performance was obtained from the minimization of the weighted sum of squared velocity and force along multiple axes. It is theoretically possible to stop vibration reaching the beam by minimizing the acceleration along the vertical and rotational axes at the intersection of the force transducer and beam. This could not be demonstrated experimentally due to slight amplitude and phase errors in the measurement of the transfer functions.

The best isolation should have been obtained by minimizing the sum of the squared velocity along the vertical and rotational axes at the intersection of the force transducer and the beam, however the best performance was obtained by minimizing the weighted sum of squared velocities along the vertical and rotational axes and the squared forces and moments. The reason why the sum of the squared accelerations along the vertical
and rotational axes did not perform as expected was due to the inability of the accelerometers to measure the rotational vibration properly. The 6-axis force transducer was able to indirectly measure the rotational vibration by detecting the induced bending stresses in the body of the transducer as a result of rotational motion and therefore resulted in the best isolation performance.