



Active Isolation of Machinery Vibration from Flexible Structures

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Abstract

Vibrating machinery must be isolated from a supporting structure if the vibration is likely to cause fatigue of components or annoyance to people due to direct vibration exposure or from the noise radiated by the vibrating structure. Active vibration isolation can be applied in these situations to extend the low frequency performance of passive vibration isolators.

In this thesis, theoretical and experimental investigations are described for a vibrating rigid body that is passively and actively isolated from a beam and a cylinder, respectively. The focus of the work is to investigate the vibrational power transmitted by translational forces and rotational moments into the support structure.

For the investigation of the simply supported beam, a classical mathematical model is examined and finite element modelling is used to predict the power transmission into the beam when active vibration control is used. The results show that power transmission by moments is significant and cannot be ignored when the vibrational power transmission into the support structure is actively controlled. To control the power transmission by translational forces and rotational moments, a novel six axis active vibration isolator and a novel six axis force transducer were constructed to be used in the experimental investigations.

Using vibrational power transmission as a cost function to be minimized in active control experiments presents unique problems because negative values of translational power transmission are possible when power transmission from rotational moments is ignored or when phase errors occur in the transducer outputs. Active control attempts which converge the cost function to a negative value of power transmission along a

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particular axis can result in overall vibration levels in the structure which are greater than without active control. To prevent the increase in vibration levels, minimization of the squared value of power transmission is investigated as a potential cost function. A method is described to combine force and velocity signals into a signal which is proportional to the vibrational power transmission and is suitable for use with an existing filtered- x Least Mean Squares controller, so that the squared vibrational power transmission can be minimized.

Experimental trials were performed to actively minimize the power transmission into a simply supported beam from a vibrating rigid body using a single axis and a six axis active vibration isolator. The purpose of the experimental work was to confirm the theoretical findings and to find a practical method to measure power transmitted by rotational moments.

The vibrational power transmission from a vibrating rigid body that is passively and actively isolated from a cylinder was also investigated. The theoretical model of the cylinder was similar to the beam model, although the dynamics of the cylinder makes the solution more complicated. Two experimental trials were conducted to verify the theoretical model and involved the use of the single axis and the six axis active vibration isolators, respectively.

Statement of Originality

To the best of my knowledge and belief, all the material presented in this thesis, except where otherwise referenced, is my own original work, and has not been published previously for the award of any other degree or diploma in any university. If accepted for the award of the degree of Doctor of Philosophy, I consent that this thesis be made available for loan and photocopying.

Carl Howard

Date:

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