Deformation Capacity and Moment Redistribution of Partially Prestressed Concrete Beams

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ABSTRACT

Ductility is a measure of the ability of a material, section, structural element or structural system to sustain deformations prior to collapse without substantial loss of resistance. The Australian design standard, AS 3600, imposes minimum ductility requirements on structural concrete members to try to prevent premature non-ductile failure and hence to ensure adequate strength and ductile-type collapse with large deflections. The requirements also enable members to resist imposed deformation due to differential settlement, time effects on the concrete and temperature effects, whilst ensuring sufficient carrying capacity and a safe design.

Current AS 3600 requirements allow a limited increase or reduction in elastically determined bending moments in critical regions of indeterminate beams, accommodating their ability to redistribute moment from highly stressed regions to other parts of the beam. Design moment redistribution limits and ductility requirements in AS 3600 for bonded partially prestressed beams are a simple extension of the requirements for reinforced members. The possibility of premature non-ductile failure occurring by fracture of the reinforcement or prestressing steel in partially prestressed members has not adequately addressed.

The aim of this research is to investigate the overload behaviour and deformation capacity of bonded post-tensioned beams. The current ductility requirements and design moment redistribution limits according to AS 3600 are tested to ensure designs are both safe and economical.

A local flexural deformation model based on the discrete cracked block approach is developed to predict the deformation capacity of high moment regions. The model predicts behaviour from an initial uncracked state through progressive crack development into yielding and collapse. Local deformations are considered in the model using non-linear material laws and local slip behaviour between steel and concrete interfaces, with rigorous definition of compatibility in the compression and tension zones. The model overcomes limitations of past discrete cracked block models by ensuring compatibility of deformation, rather than strain compatibility. This improvement allows the modeling of members with multiple layers of tensile reinforcement and variable depth prestressing tendons having separate material and bond properties.
An analysis method for simple and indeterminate reinforced and partially prestressed members was developed, based on the proposed deformation model. To account for the effect of shear in regions of high moment and shear present over the interior supports of a continuous beam, a modification to the treatment of local steel deformation in the flexural model, based on the truss analogy, was undertaken. Secondary reactions and moments due to prestress and continuity are also accounted for in the analysis.

A comparison of past beam test data and predictions by the analysis shows the cracking pattern and deformation capacity at ultimate of flexural regions in reinforced and partially prestressed members to be predicted with high accuracy. The analysis method accurately predicts local steel behaviour over a cracked region and deformation capacity for a wide range of beams which fail either by fracture of steel or crushing of the concrete.

A parametric study is used to investigate the influence of different parameters on the deformation capacity of a typical negative moment region in a continuous beam. The structural system consists of a bonded post-tensioned, partially prestressed band beam. The primary parameters investigated are the member height and span-to-depth ratio; relative quantity of reinforcing and prestressing steel; material properties and bond capacity of the steels; and lastly the compression zone properties.

Results show that the effects of the various parameters on the overload behaviour of partially prestressed beams follow the same trends as reinforced beams. A new insight into the local steel behaviour between cracks is attained. The deformation behaviour displays different trends for parametric variations of the local bond capacity, bar diameter and crack spacing, when compared to past analytical predictions from comparable studies. The discrepancy in findings is traced back to the definition of the plastic rotation capacity and the sequencing of the yielding of the steels. Compared to the other local deformation models, the current model does not assume a linear distribution of strain at a crack. The current findings highlight an important difference between predicted behaviours from different deformation compatibility requirements in local deformation models which has not yet been discussed in the literature.

The local deformation model evaluates the relationship between maximum steel strain at a crack and average steel deformation over a crack spacing for the entire loading history. The total steel percentage, hardening properties of the steel and concrete strength are shown by the model to have the greatest effect on these steel strain localisation factors. Section analysis, as currently used in design, can be improved with the proposed simplification of the relationships to identify and quantify the effects of steel fracture on deformation capacity and strength.
The numerical effort required to simulate the overload behaviour of practical beam designs with multiple reinforcement elements and a prestressing tendon are currently too great to be used in an extensive numerical study. The numerically more efficient smeared block approach is shown to accurately predict the ultimate carrying capacity of prestressed beams failing by crushing of the concrete. Consequently, this method is adopted to study the allowable limits of moment redistribution in the present investigation. Simplified relationships of the steel strain localisation factors evaluated in the parametric study of deformation capacity is used to predict maximum steel strains and premature failure.

The limits of moment redistribution in bonded, post-tensioned partially prestressed band beams are explored by comparing the design load and predicted carrying capacity, for different section ductilities and design moment redistribution. In addition, the effects of different concrete strengths, up to 85 MPa, along with as three reinforcing and prestressing steel ductilities are quantified and compared to current Australian and international design requirements. Limitations in the carrying capacity are investigated for different reinforcement and prestress uniform elongation capacities.

More than one thousand beam simulations produce results showing that current design moment redistribution and ductility requirements in the Australian design code for concrete structures (AS 3600) are sufficient for normal strength concretes (<50 MPa). A suggestion for design moment redistribution limits, section ductility requirements and steel ductility limits is made for members constructed from higher strength concretes. A special high steel ductility class is proposed for both the reinforcement and prestressing steel to allow moment redistribution in higher strength concrete. No moment redistribution is proposed for members reinforced with low ductility (Class L) steel. An increase of the current elongation limit of Class L steel from 1.5 % to 2.5% is suggested to ensure strength and safety. An increase in the current ductility requirements from $f_{ud}/f_y=1.03$ and $\varepsilon_u=1.5 %$ to $f_{ud}/f_y=1.05$ and $\varepsilon_u=2.5 %$ for low ductility Class L steel is suggested to ensure strength and safety.
STATEMENT OF ORIGINALITY

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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Mark Rebentrost

Date
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NOTATION

Material constants

$E_{co}$  Tangent modulus of elasticity of unconfined concrete
$E_{cu}$  Secant modulus of elasticity of unconfined concrete
$E_p$  Modulus of elasticity of prestressing steel
$E_s$  Modulus of elasticity of reinforcing steel
$E_{sw}$  Modulus of elasticity of stirrup reinforcing steel (taken to be 200 GPa)
$k, k_1$  Material constants
$n, N$  Material constants

Strains

$\varepsilon_c$  Unconfined concrete strain
$\varepsilon_{c,\text{lim}}$  Concrete strain limiting application of CEB MC 90 (1993) ascending curve
$\varepsilon_{c0}$  Outermost maximum compressive concrete strain
$\varepsilon_{ct}$  Confined concrete strain
$\varepsilon_{cp}$  Concrete strain at the prestressing steel level
$\varepsilon_{cs}$  Concrete strain at the reinforcing steel level
$\varepsilon_{ct}$  Concrete tensile strain
$\varepsilon_{ct,u}$  Concrete tensile fracture strain
$\varepsilon_{ca}$  Concrete strain at maximum stress
$\varepsilon_y$  Maximum concrete strain in the concrete at the yield of the reinforcement
$\varepsilon_p$  Prestressing steel strain
$\varepsilon_{pd}$  Prestressing steel strain at decompression
$\varepsilon_{pu}$  Ultimate tensile prestress elongation
$\varepsilon_{py}$  Yield strain of prestress
$\varepsilon_s$  Reinforcing steel strain
$\varepsilon_{su}$  Ultimate reinforcement steel elongation
$\varepsilon_{sy}$  Yield strain of reinforcing steel

Stresses

$f_k$  Maximum grout strength (mean value)
$f_c$  Maximum unconfined concrete strength (mean value)
$f_{ct}$  Fracture stress of unconfined concrete
$\sigma_c$  Unconfined concrete stress
$\sigma_{ct}$  Outermost compressive concrete stress
\( \sigma_{cc} \) \quad Confined concrete stress
\( \sigma_{ci} \) \quad Stress in the concrete along an inclined strut
\( \sigma_{cp} \) \quad Concrete stress at the prestressing steel level
\( \sigma_{cs} \) \quad Concrete stress at the reinforcing steel level
\( \sigma_p \) \quad Prestressing steel stress
\( \sigma_{pd} \) \quad Prestressing steel stress at decompression
\( \sigma_s \) \quad Reinforcing steel stress
\( \sigma_{sw} \) \quad Stirrup reinforcement stress

**Bond stress-slip parameters**

- \( s_1 \): Slip at which mechanical interlock is fully activated
- \( s_2 \): Slip at which mechanical interlock capacity starting to decrease
- \( s_3 \): Slip at which friction acts only
- \( s_4 \): Adjusted slip \( s_2 \) for post-yield range
- \( s_5 \): Adjusted slip \( s_4 \) for post-yield range
- \( s_y \): Slip at steel yield
- \( s_{yR} \): Slip at which friction acts only for post-yield range
- \( \tau \): Bond stress
- \( \tau_{\text{max}} \): Maximum steel bond stress
- \( \tau_p \): Prestressing steel bond stress
- \( \tau_f \): Friction steel bond stress
- \( \tau_s \): Reinforcing steel bond stress
- \( \tau_{\text{unit}} \): Unit bond stress
- \( \tau_{\text{ymax}} \): Maximum steel bond stress for post-yield range
- \( \tau_{yR} \): Friction steel bond stress for post-yield range
- \( x_{cr} \): Distance to nearest crack along prestress or reinforcing element in bar diameters

**Factors**

- \( \alpha_s \): Compatibility factor relating peak to average reinforcing steel strain
- \( \alpha_p \): Compatibility factor relating peak to average prestressing tendon steel strain increment
- \( \alpha_{co} \): Compatibility factor relating peak to average compressive concrete strain
- \( \alpha_{dc} \): Compatibility factor relating neutral axis depth at the crack to the neutral axis of deformation

**Deformations**

- \( \delta_{c0} \): Outermost compressive concrete deformations
- \( \delta_p \): Concrete deformations at the prestressing steel layer
- \( \delta_s \): Concrete deformations at the reinforcing steel layer
- \( d_{lab} \): Neutral axis of deformations over a region
- \( \delta_p \): Prestressing steel deformations
- \( \delta_s \): Reinforcing steel deformations
- \( \kappa \): Curvature
Notation

- $s_p$: Prestressing tendon slip
- $s_i$: Reinforcing bar slip
- $w_p$: Crack opening at the prestressing tendon layer
- $w_i$: Crack opening at the reinforcement steel layer
- $w_{sp}$: Splitting crack opening

**Forces and moments**

- $C$: Compression force
- $C_c$: Compression force in the concrete
- $C_{sc}$: Compression force in the reinforcing
- $\phi$: Angle along which a force is directed
- $F_i$: Inclined strut force
- $F_x$: Horizontal force component
- $F_y$: Vertical force component
- $M$: Moment
- $M_1$: Primary moment due to prestress
- $M_2$: Secondary moment due to prestress
- $M_{cr}$: Cracking moment
- $M_{driver}$: Driver moment to set the bending moment level in the analysis
- $M_{py}$: Prestress yield moment
- $M_{sw}$: Self-weight moment
- $M_{ry}$: Reinforcement yield moment
- $M_y$: Yield moment
- $N$: Axial force
- $N_u$: Axial force capacity
- $P$: Effective prestressing force
- $P_{Gb, eq}$: Point load equivalent to $w_{Gb}$ for simulating interior support regions with $L_{eq}$
- $R$: Reaction force
- $S$: Stirrup force
- $\bar{s}$: Distance to resultant stirrup force
- $T$: Tensile force
- $T_c$: Tensile force in the concrete
- $T_p$: Tensile force in the prestressing
- $T_s$: Tensile force in the reinforcing
- $V$: Shear force
- $w$: Cracking load (first crack $w_{cr1}$, second crack $w_{cr2}$...)
- $w_{cr}$: Uniformly distributed load balanced by prestress
- $w_{Gb}$: Uniformly distributed self-weight load
- $w_{sw}$: Uniformly distributed load

**Dimension**
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{sh}$</td>
<td>Cross-sectional area of stirrup steel</td>
<td></td>
</tr>
<tr>
<td>$B$</td>
<td>Width of member</td>
<td></td>
</tr>
<tr>
<td>$B_t$</td>
<td>Width of flange</td>
<td></td>
</tr>
<tr>
<td>$B_w$</td>
<td>Width of web</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>Concrete cover</td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td>Effective member depth</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>Height of the member</td>
<td></td>
</tr>
<tr>
<td>$d_f$</td>
<td>Depth of flange</td>
<td></td>
</tr>
<tr>
<td>$d_p$</td>
<td>Depth to the centroid of the prestressing tendon</td>
<td></td>
</tr>
<tr>
<td>$d_s$</td>
<td>Height of a slab</td>
<td></td>
</tr>
<tr>
<td>$D_s$</td>
<td>Depth to centroid of the steel layer</td>
<td></td>
</tr>
<tr>
<td>$d_w$</td>
<td>Height of web</td>
<td></td>
</tr>
<tr>
<td>$h$</td>
<td>Maximum tendon drape</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Span of the member</td>
<td></td>
</tr>
<tr>
<td>$b_{bp}$</td>
<td>Prestressing tendon slip distance</td>
<td></td>
</tr>
<tr>
<td>$b_{bpk}$</td>
<td>Deformation compatibility distance</td>
<td></td>
</tr>
<tr>
<td>$b_{bpkL}$</td>
<td>Prestressing tendon slip distance to the left of a reference point</td>
<td></td>
</tr>
<tr>
<td>$b_{bpkR}$</td>
<td>Prestressing tendon slip distance to the right of a reference point</td>
<td></td>
</tr>
<tr>
<td>$b_{rak}$</td>
<td>Reinforcing bar slip distance</td>
<td></td>
</tr>
<tr>
<td>$b_{rakL}$</td>
<td>Reinforcing bar slip distance to the left of a reference point</td>
<td></td>
</tr>
<tr>
<td>$b_{rakR}$</td>
<td>Reinforcing bar slip distance to the right of a reference point</td>
<td></td>
</tr>
<tr>
<td>$l_{D_{region}}$</td>
<td>D-region length</td>
<td></td>
</tr>
<tr>
<td>$L_{eq}$</td>
<td>Equivalent span of an interior support region (Chapter 6)</td>
<td></td>
</tr>
<tr>
<td>$l_{pad}$</td>
<td>Loading plate width</td>
<td></td>
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<tr>
<td>$l_{pad,eff}$</td>
<td>Effective loading plate width</td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>Stirrup spacing</td>
<td></td>
</tr>
<tr>
<td>$t_{slice}$</td>
<td>Thickness of a slice of concrete at a section</td>
<td></td>
</tr>
<tr>
<td>$U$</td>
<td>Circumference</td>
<td></td>
</tr>
<tr>
<td>$U_p$</td>
<td>Reinforcing bar circumference</td>
<td></td>
</tr>
<tr>
<td>$U_s$</td>
<td>Prestressing tendon circumference</td>
<td></td>
</tr>
</tbody>
</table>

**Section properties**

- $A_{ct,eff}$: Cross-sectional area of tension reinforcing steel
- $A_t$: Effective area of concrete in tension
- $A_p$: Cross-sectional area of compression reinforcing steel
- $A_s$: Cross-sectional area of prestressing steel
- $A_{sc}$: Gross section area
- $c$: Percentage of tensile steel
- $d_o$: Percentage of prestressing steel
- $k_e$: Neutral axis of strains at a section ($x$ is used in European design codes)
- $n_c$: Percentage of tension reinforcing steel
- $n_p$: Neutral axis parameter (AS 3600)
- $n_s$: Concrete cover
- $p$: Number of reinforcing layers at a section
- $p_p$: Number of prestressing layers at a section
\( p_s \)  
Number of concrete slices at a section

\( \omega \)  
Critical mechanical reinforcement ratio

\( \omega_t \)  
Mechanical ratio of the reinforcing steel in tension

\( \omega_p \)  
Mechanical ratio of the prestressing steel

\( \omega_c \)  
Mechanical ratio of the reinforcing steel in compression

\( \omega_s \)  
Mechanical reinforcing ratio

\( z \)  
Lever arm of internal forces at a section

**Common subscripts**

-avg  
An average value

.exp  
A value observed in experiments

.L  
To the left of a reference point

.max  
A maximum value

.min  
A minimum value

.R  
To the right of a reference point

.span  
Indicating a property in the span

.sup  
Indicating a property at an interior support
SI units are used throughout this thesis. Experimental values are quoted as published with SI conversions given as applicable.

Strain [mm/mm]
Stress [MPa]
Area [mm²]
Length [mm]
Angle [degrees]
Rotation [radians]
Force [kN]
Moment [kNm]

The following acronyms are used in this thesis:

ACI American Concrete Institute
AS Australian Standards
ASCE American Society of Civil Engineers
BAM Bundesanstahlt für Materialforschungen
BS British Standards
CEB Comité Européen du Béton
CEB MC 90 Commite-Euro-International du Beton Model Code 90
CSA Canadian Standards Association
DCB Discrete crack block (analysis)
DIN Deutsche Industrie Normen
EC Eurocode
EN English
FIP Fédération Internationale de la Précontrainte
GER German
PCI Prestressed/Precast Concrete Institute
PPC Partially prestressed concrete
RC Reinforced concrete
SLS Serviceability limit state
UDL Uniformly distributed load
ULS Ultimate limit state