A triangular grid finite-difference model for wind-induced circulation in shallow lakes

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Thesis submitted for the degree of
Doctor of Philosophy
in
Applied Mathematics
at
The University of Adelaide
(Faculty of Engineering, Computer and Mathematical Sciences)

School of Mathematical Sciences

February 2005

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Abstract

In this study, the development and testing of a finite-difference model for wind-induced flow in shallow lakes, and, in particular, a new technique for improving the land–water boundary representation, are documented. The model solves nonlinear, as well as linear, versions of the two-dimensional depth-integrated shallow water equations.

Finite-difference methods on rectangular grids are widely used in numerical models of environmental flows. In these models, land–water boundaries are usually approximated by a series of perpendicular line segments, which enable the impermeability condition to be easily implemented. A disadvantage of this approach is that the actual boundary is often poorly approximated, particularly in regions which have complicated coastlines, and, as a result, currents in these regions cannot be accurately predicted.

A technique for improving the land–water boundary representation in finite-difference models is introduced. This technique permits the model boundary to contain diagonal line segments, in addition to the vertical and horizontal line segments used in traditional models. The new technique is based on a simple concept and can easily be included in existing finite-difference models.

In order to test the new method, the linearised shallow water equations are solved numerically for oscillatory wind-driven flow in lakes with simple geometry. Predictions obtained using the new approach are compared with predictions from the traditional stepped boundary and known analytic solutions. A significant improvement in the accuracy of results is noticed when the new approach is used, particularly in currents close to shore. The increased accuracy obtained using the improved boundary representation can lead to a significant computational saving, when compared with running the rectangular grid model with smaller grid spacings.

A second-order analytic solution to the nonlinear shallow water equations is developed for oscillatory wind-driven flow in a rectangular lake. Comparisons between this solution and numerical results, obtained using the traditional stepped boundary and the improved boundary, verify the finite-difference formulae used in these models, including the approximations used for the cross-advective terms close to shore. Once more, currents are predicted with greater accuracy when the new technique for representing the land–water boundary is implemented.

The lake circulation model is applied to the Lower Murray Lakes, South Australia, and predicted water levels at Tauwitchere Barrage are shown to agree very well with observations. The model is then used to examine the effectiveness of two schemes that have been proposed to increase wind-induced circulation, and therefore potentially decrease salinity, in Lake Albert, demonstrating the model's use as an efficient and effective tool for analysing flow behaviour in lakes.

Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

SIGNED: .... DATE: ....

Acknowledgements

I am indebted to my primary supervisor, Dr Michael Teubner, for his continued support, expert guidance, encouragement and patience throughout the development of this work. I am grateful for the many hours that he spent discussing and proof-reading my work, and appreciate the faith that he has shown in my ability.

I also wish to thank my secondary supervisor, Associate Professor John Noye, for encouraging me to commence this PhD and whose idea was the basis for this work. The support that he provided for me in the early stages of this research is much appreciated.

I express my sincere thanks to the Applied Mathematics staff at the University of Adelaide. In particular, I thank Dr Peter Gill, Dr Liz Cousins and Dianne Parish for their ongoing support.

I would like to thank the past and present members of the Adelaide University Computational Fluid Dynamics Group, as well as fellow mathematics postgraduate students, for the valuable discussions on my research, as well as their friendship and support over the years.

Special thanks go to my parents, Peter and Jan, for their encouragement and understanding. I am especially thankful to my mother who spent many hours editing my thesis. I also thank my brother, Ben, and sister, Kate, as well as many other family members and friends who have made this period of my life so enjoyable.

I wish to acknowledge the financial assistance from the Commonwealth Government of Australia, in the form of an Australian Post-graduate Award, that I received in the early years of this work.