THE CLOUDY BAG MODEL

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CONTENTS

Preface

As a method of classification of the observed, strongly interacting particles the quark model was invented in the mid-sixties. However, the general belief that quarks are the fundamental constituents of hadronic matter (not simply a useful mathematical device) did not come for a further decade. The discovery of the J/Ψ, and the evidence for pointlike constituents inside hadrons from SLAC, were the major reasons for acceptance. However, the emergence of a local gauge theory for the strong interactions of coloured quarks (QCD) was also an important factor.

Amongst the attractive features of QCD were the proof that it is asymptotically free, and the strong possibility that its infrared behaviour could be confining. Thus QCD suggested the solution to two problems simultaneously. Quarks had not been observed because of confinement, however they could still behave as free particles inside hadrons (and hence yield structure functions that scale).

The mid-seventies saw the construction of many phenomenological models of hadron structure, motivated by QCD. The non-relativistic quark model (NRQM) is the most widely used, because of its simplicity and its phenomenological success. However it suffers from terrible inconsistencies through its non-relativistic nature, with typical quark momenta larger than the corresponding rest mass.

The MIT bag model had two major advantages. It was constructed in a completely covariant way and the introduction of massless quarks was no problem. Furthermore, because the quarks were free inside the bag, the phenomenon of scaling, at least for values of Bjorken-x greater than $(2mR)^{-1}$, was built in. Amongst the early successes of the MIT bag model were the agreement within (20-30)% for baryon magnetic moments, and the excellent value of $g_a/g_v = 1.09$ -

compared to 1.66 for non-relativistic models.

Almost immediately it was realized that the MIT bag model had one important flaw. Massless QCD has an exact chiral symmetry which is broken in the MIT bag model. Indeed, it seems that any model which confines quarks would also break chiral symmetry. Even though this problem was noticed in 1974 by Chodos and Thorn, and by Inoue and Maskawa, little more was done until 1979, when Brown and Rho raised the issue in a major way.

It is a fundamental belief in nuclear physics that the long-range N-N force is well described by pion exchange. In 1979 Gerry Brown and Mannque Rho realized that chiral symmetry could be restored in the bag model by coupling the pion field to the quarks at the bag surface. Their hope was that the pressure exerted by this external Bose field would compress the bag to a radius small enough that nuclear physicists could continue to ignore quark degrees of freedom, as they had done until that time.

Until 1979 my major work had been in intermediate energy physics, where the pion-nucleon interaction was usually treated as a phenomenological input. On a flight back from a conference in Houston in February 1979, Jerry Miller and I realized that the Brown-Rho approach offered a path to a much deeper understanding of the pion-nucleon interaction. Paper 1 was completed within a few months, but took quite a while to be published. The editor at Physical Review Letters said that the work was possibly more significant than the original MIT bag, but that it was a bit complicated for the general audience of P.R.L. Eventually it was published in Physics Letters, but only after the title was changed from "Pion-Nucleon Scattering in the Cloudy Bag Model" to "Pion Nucleon Scattering in the Brown-Rho Bag Model". (The editor of Physics Letters is in the same corridor at SUNY

Stony Brook as Gerry Brown.)

That first paper was confused as to how exactly the pion field should be treated. We could not decide whether it should be quantised or not, or whether a renormalisation program was appropriate. By the time paper 2 appeared in December 1980 all of this was resolved. The discussion of renormalisation of the CBM which appears there is mine, and represented the major foundation stone for all that followed. In that paper we gave a definitive answer concerning the nature of the $\Delta(1231)$ resonance, which is now universally accepted. This paper alone has been cited more than one hundred times since its publication, and has prompted an enormous amount of subsidiary theoretical work.

The later papers discuss the implications of our work for nuclear physics and for hadron structure. With respect to nuclear structure, we found that the bag radius (typically 1fm in the MIT bag model) was not compressed much by the pion field. This implied that quark degrees of freedom would play an important role in the short-distance N-N force, and modify the conventional picture of nuclear structure in a major way. This view was almost heresy in 1980, and led to heated arguments with the Stony Brook group at various international conferences. In 1985 it would be hard to find such an argument. Plans for new accelerators in Canada, the U.S., France and Germany are now largely based on the need to understand better the role of quarks in the nucleus.

Amongst the papers included here, I would mention only a few specifically. Paper 3 represents work done while on study leave at the University of Melbourne and during a short visit to this University. It was a crucial step for most of the recent developments involving KN and $\bar{K}N$ systems. In it I showed that instead of coupling the pion field to the confined quarks only at the bag surface, one could transform the theory to a form where the pions coupled throughout the bag volume. Remarkably, the second version gives the same $B'B\pi$ coupling constants and form-factors (as long as the quark radial orbit is unaltered). However it also gives the correct results for the s-wave pion-nucleon scattering lengths, and higher order corrections to it are much more convergent. Indeed, this new version of the CBM has largely superseded the original.

To conclude this introduction I should mention a major theme of the last two years. Deep-inelastic lepton scattering is the tool "par excellence" for investigating the internal structure of matter. Paper 19 puts a new limit on bag size using DIS data. Paper 26 suggests a new measurement which would establish the presence of an exotic, 6-quark component in the deuteron. Finally, papers 20, 26, 29, 31 and 33 deal with the natural extension of all the earlier CBM work towards an understanding of the EMC effect, which is the first clear evidence of a change in the quark structure of a nucleon when imbedded in a nuclear medium.

Note: The published work reproduced here represents in many cases collaborative work with students*, post-doctoral fellows and professional colleagues. In every one of the papers included here I have been at the very least an equal partner in the research. I feel that the international recognition of this, through more than 10 invitations to present talks on this work (in plenary session) at international meetings in the last five years, is sufficient testimony.

*Dr. S. Théberge was my Ph.D. student at the University of British Columbia, and the papers with him as co-author were submitted with his thesis in partial fulfilment of the requirements for the Ph.D. there in 1982.

DECLARATION

I certify that this thesis does not incorporate, without acknowledgement, any material previously submitted for a degree or diploma in any University and that, to the best of my knowledge and belief, does not contain any material previously published or written by another person, except where due reference is made in the text.

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PAPER 1

PION–NUCLEON SCATTERING IN THE BROWN–RHO BAG MODEL

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Pion–nucleus scattering in the (3,3) resonance region is described using a version of the Brown–Rho bag model. Terms involving iterations of the crossed Born graphs, as well as excitation of the three-quark delta, both give significant contributions to the scattering amplitude.

Recently Brown and Rho [1] introduced a new bag model of hadronic structure in which the radius can be much smaller ($\approx 1.5\ m_{\pi}^{-1}$) than the MIT bag [2]. In this model the nucleon consists of a small bag of three quarks surrounded by a cloud of pions, which exist only outside the bag. The pion cloud may be thought of as a crude representation of $q\bar{q}$ admixtures in the hadronic wave function. This bag model is useful because the pions couple to the bag surface with the familiar Yukawa coupling. Thus hadrons can interact with each other through conventional pion exchange mechanisms.

In this note we extend the Brown–Rho (BR) model to describe pion–nucleon scattering in the (3,3) resonance region. The new feature of our model is that two distinct mechanisms combine to produce the observed resonance. First there is a series of terms involving the cloud of pions about the nucleon (see fig. 1a). With the Yukawa coupling (eq. (2) below), and a suitable choice of cutoff function [3], this series of crossed graphs alone can reproduce the P_{33} resonance. However, in the bag model there is an additional pion–nucleon coupling in which the pion field changes the three-quark nucleon bag into a three-quark delta state. This coupling give rise to the series of graphs shown in figs. 1b and c. In what follows we sum the graphs of

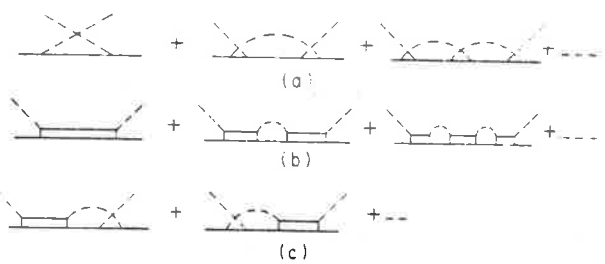

Fig. 1. Mechanisms for πN scattering in the P_{33} channel involving: (a) the nucleon's pion cloud, (b) the elementary (three-quark) delta and (c) interference terms.

fig. 1 in the static limit and obtain a good representation of the experimental P_{33} phase shifts. Our result is a completely new model of the off-shell pion–nucleon T-matrix.

In the BR model [1], the coupling of the pion field to the surface of the nucleon bag is obtained by assuming $\partial_\mu A_\mu = 0$, where A_μ is the axial vector current. Inside the bag this is carried entirely by the quarks, and outside by the pions. By considering the integral $\int d^4x\ \partial_\mu A_\mu = 0$ over a pill-box through the bag surface, one finds that at the surface S [1,4]:

$$\hat{n}\cdot\nabla\boldsymbol{\phi}_\pi|_S = -\mathrm{i}(f_\pi^{-1}/2)\bar{q}\gamma_5\boldsymbol{\tau}q|_S, \qquad (1)$$

where f_π is the pion decay constant.

In order to specify completely the meaning of eq.

are both the Θ-functions with a cutoff at momentum p_M. This is the second parameter in our model, but based on the uncertainty principle we expect p_M to be of order R^{-1} (R is the bag radius).

The pion–nucleon transition amplitude $t(E)$ corresponding to the sum of all the graphs of fig. 1 can be shown to have the form

$$t(E) = t_1(E) + t_2(E)\,, \tag{7}$$

where t_1 and t_2 satisfy the coupled equations

$$t_1(E) = \frac{v_N}{E} + \frac{v_N}{E}\,\frac{E}{h_0}\,\frac{1}{E^+ - h_0}\left[\frac{E}{h_0}t_1(E) + t_2(E)\right], \tag{8a}$$

$$t_2(E) = \frac{v_\Delta}{E - s_0} + \frac{v_\Delta}{E - s_0}\,\frac{1}{E^+ - h_0}\left[\frac{E}{h_0}t_1(E) + t_2(E)\right], \tag{8b}$$

and h_0 is the pion total energy operator. In the limit where g^2 is zero, eqs. (7) and (8) reduce to the Chew–Low [7] theory without crossing. When f^2 is zero we get a pure Δ-model.

There are a number of additional approximations (standard in earlier treatments of πN scattering) implicit in these equations. Terms associated with pion crossing are neglected, and we approximate the crossed Born graph so that there is no pion production cut. These neglected terms provide some attraction in the (3,3) channel [8], and their inclusion could modify the values of p_M and s_0 that we extract.

Using eqs. (7) and (8) we find that values of $p_M = 900$ MeV/c and $s_0 = 1400$ MeV (dotted line in fig. 2) give a very good fit to the total cross section (solid curve) [9] in the (3,3) channel. The value of p_M is consistent with the assumption of a small bag, but s_0 seems very large. The difficulty stems from the large mass of the bare little bag ($\approx 5.4 m_n$). In the BR paper it was argued that strong self-energy corrections from virtual pion emission and absorption (especially figs. 3a and 3b) would bring the bare nucleon mass down to the physical mass. Indeed we have assumed that this renormalization procedure is valid and have used the physical values of m_N and f^2 in evaluating the terms of fig. 1. On the other hand, while self-energy graphs like fig. 3c can be assumed to have been included in s_0 (i.e. $s_0 = 5.4 m_N + 3c + ...$), important self-energy terms for the Δ, such as fig. 3d, are included in the solution of eqs. (7) and (8). Since Brown and Rho found a contribution of $-2.9 m_N$ from figs. 3a and 3b in the nucleon case, our value of s_0 is not unreasonable.

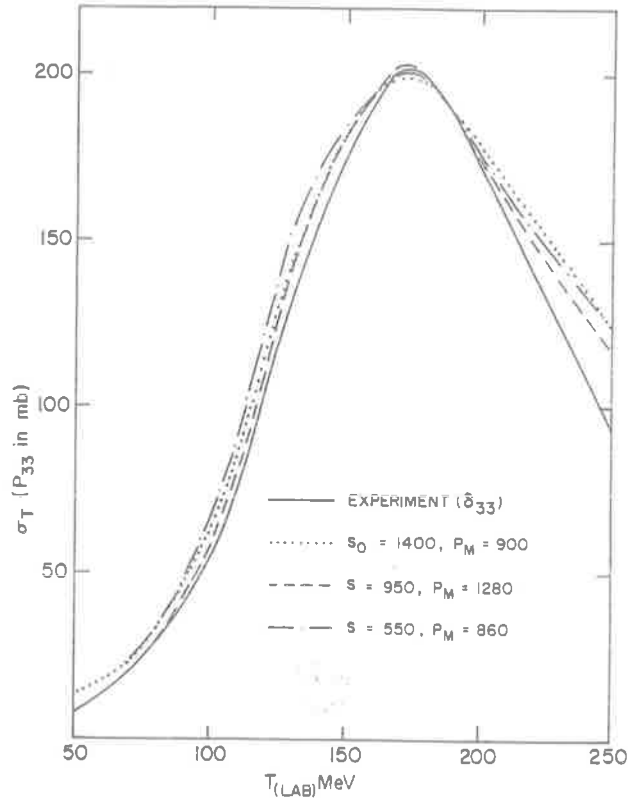

Fig. 2. Total cross section for πN scattering in the P_{33} channel (see text).

However, there is a question of consistency, because f^2 is a fully renormalized coupling constant, while some parts of the diagrams in fig. 1b may be regarded as renormalizations of the ΔNπ coupling. In particular, we expect that $(g/f)^2$ should be given by the quark model only if both f and g are fully renormalized. This has led us to apply standard nonrelativistic renormalization techniques [10] to the Δ Green's function. We first make a subtraction on the quantity $[E^+ - s_0 - g^2\Sigma(E)] \equiv G_\Delta^{-1}(E)$ (see fig. 3d) to guarantee that $\mathrm{Re}\,[G_\Delta^{-1}(E = s)] = 0$. Thus we have

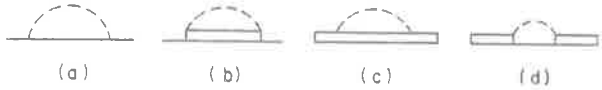

Fig. 3. Self-energy corrections considered by Brown and Rho [1] for the nucleon ((a) and (b)), and by us ((c) and (d)). Diagram (d) represents the term $g^2\Sigma(E)$ in the text (above eq. (9)).

$$G_\Delta^{-1}(E) = E - s - g^2\,[\mathrm{Re}\,\Sigma(E) - \mathrm{Re}\,\Sigma(s)] - \mathrm{i}g^2\mathrm{Im}\,\Sigma(E). \tag{9}$$

Furthermore, a renormalization of the $\pi N\Delta$ coupling constant is to be performed. To do this define the real quantities $I(E, s)$ and $F(E, s)$ by

$$\mathrm{Re}\,\Sigma(E) - \mathrm{Re}\,\Sigma(s) = (E-s)I(E,s) \tag{10a}$$

$$= (E-s)\,[I(s,s) + (E-s)F(E,s)]\,. \tag{10b}$$

The use of eq. (10) in eq. (9) leads to

$$G_\Delta^{-1}(E) = Z(E-s)\,[1 - g_\mathrm{R}^2 F(E,s)] - \mathrm{i}Zg_\mathrm{R}^2\,\mathrm{Im}\,\Sigma(E)\,, \tag{11}$$

where $Z = 1 - g^2 I(s,s)$ and g_R^2, the renormalized $\pi N\Delta$ coupling constant, given by $g_\mathrm{R}^2 = g^2/Z$. The use of $G_\Delta^{-1}(E)$ of eq. (11) in each Δ propagator of the series represented by fig. 1 enables us to eliminate g^2 in favour of g_R^2 in *all* terms of $t(E)$. Hence this renormalization scheme is a consistent one. The only change in eqs. (8) is that eq. (8b) is replaced by

$$t_2(E) = \frac{v_\Delta}{E-s} + \frac{v_\Delta}{E-s}\left\{\frac{1}{E^+ - h_0}\,\frac{E}{h_0}\,t_1(E) - \frac{(E-s)^2}{E^+ - h_0}\left[\frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{P}{s-h_0}\right)\right] t_2(E)\right\}, \tag{8b$'$}$$

where P denotes the principal value part of the integral. The potential v_Δ is now proportional to the square of the renormalized $\pi N\Delta$ coupling constant and $g_\mathrm{R}^2/f^2 = 72/25$.

Once again we adjust two parameters (s and p_M) so that the solution of eqs. (7), (8a) and (8b$'$) fits the pion–nucleon data. We find a best fit to the total cross section with p_M = 1280 MeV and s = 950 MeV (dashed curve in fig. 2). However, fits of comparable quality can be found with a range of values of these parameters. For example, we also show in fig. 2 (dash–dot curve) the case p_M = 860 MeV and s = 550 MeV. This value for the splitting between the nucleon and delta masses is small enough to be reasonably attributed to gluon exchange mechanisms. In each of these calculations both the Δ and pionic aspects are important. Setting g_R or f arbitrarily equal to zero results in dramatic changes of σ_T (sometimes even removing the resonance). For example, setting $g_\mathrm{R} = 0$ in the calculation represented by the dot–dash curve results in a resonance at $T_\mathrm{lab} \approx 350$ MeV. All of our solutions are consistent with the idea of a small bag.

A successful representation of the πN scattering data has been obtained, and the present model offers a novel description of the off-energy-shell behaviour of the πN transition matrix. Pion–nucleon scattering in the (3,3) resonance region is determined by two basic mechanisms. One involves the excitation of the bag-like core from a nucleon to a delta. This process alone has been considered by Brown et al. [11], who found Γ_Δ = 94 MeV, and earlier by Chodos and Thorne [12] (in the MIT bag model) who found Γ_Δ = 32 MeV. The other mechanism involves the pion cloud which surrounds the nucleon core. By itself the latter corresponds to Chew–Low theory [7].

The cloudy bag model is richer than either the old Chew–Low theory or the pure quark models. Furthermore it provides a calculable framework to study a number of unanswered problems involving widths of resonances [13].

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Pionic corrections to the MIT bag model: The (3,3) resonance

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By incorporating chiral invariance in the MIT bag model, we are led to a theory in which the pion field is coupled to the confined quarks only at the bag surface. An equivalent quantized theory of nucleons and Δ's interacting with pions is then obtained. The pion-nucleon scattering amplitude in this model is found to give a good fit to experimental data on the (3,3) resonance, with a bag radius of about 0.72 fm.

I. INTRODUCTION

The problem of understanding pion-nucleon (πN) scattering in the energy region of the (3,3) resonance has had a long and fascinating history. Chew[1,2] showed that a field theory which involves pions and nucleons interacting via a Yukawa coupling could be used to explain the appearance of this resonance in πN scattering. The Chew theory consisted of summing (within the static model) the series of graphs of Fig. 1. Chew and Low[3] showed that a resonant scattering amplitude could also be obtained by solving a nonlinear integral equation (the Low equation) that was the forerunner of dispersion relations (e.g., Refs. 4 and 5). An expansion of the Low equation in powers of the coupling constant is the same as summing the series of Fig. 1, but it was also pointed out that there are an infinite number of solutions of the Low equation.[6] There has been much recent interest in pion-nucleus scattering as a probe of nuclear structure. The consequent need to understand πN scattering in a very precise fashion has led to a recent series of very sophisticated applications and modifications of the original Chew-Low theory.[7,8]

Shortly after the work of Chew and Low a vast number of πN resonances and other new particles were discovered. In order to find some order among all the particles Gell-Mann and Ne'eman[9] introduced the eightfold way. In this model the πN P_{33} resonance is essentially a stable particle (the Δ), which consists of three quarks. The corresponding πN t matrix can be calculated by defining Fig. 2(a) to be a K matrix. In this way the t matrix includes all the self-energy graphs of Fig. 2 with an on-energy-shell pion. There have been several recent calculations[10-12] of πN scattering using models of this kind.

The observed πN resonances can therefore be "explained" either in terms of pions and nucleons (Fig. 1), or in terms of Δ's that consist of quarks (Fig. 2). In the present work we unify these apparently contradictory views of the (3,3) resonance.

In our model, as in the work of Chodos and Thorn,[13] the Stony Brook group,[14-16] and Jaffe,[17] the baryon is regarded as consisting of three quarks confined in a bag that is surrounded by a cloud of pions (hence the name cloudy bag). We use the MIT bag model,[18-21] which has been very successful in describing hadronic structure.

In its simplest form the MIT bag model gives a degenerate nucleon and Δ, consisting of three massless up or down quarks moving freely in a spherical region of space of radius R, called a bag. The confinement of the quarks is guaranteed by demanding that no color-electric or -magnetic fields penetrate the surface of this region, that the quark wave functions are zero outside the bag, and that the pressure exerted by the quarks on the bag surface is balanced by an external pressure. The radius of the MIT bag is typically of the order of 1.2 fm, which yields an average nucleon and Δ mass of about 1.1 GeV. This degeneracy is removed by including the color-magnetic interaction between the quarks—essentially a spin-spin force. For a summary of the many achievements of this model we refer to several recent review articles.[17,19,22-24]

The MIT bag model raises a number of fascinating problems when looked at in the context of nuclear physics. In particular, there has been little effort to include the coupling of the pion to the nucleon in the MIT model, even though it is well established that the long-range part of the N-N force is given by one-pion exchange.[25] Even given some $NN\pi$ coupling, it is rather difficult to see how two nucleon bags in a nucleus, which would be touching, could easily interact through pion exchange. There is also the controversial question of the stability of nuclear matter against

FIG. 1. The Chew series. Nucleons are represented by solid lines and pions by dashed ones.

percolation[25] if the nucleon bag has the MIT radius.

In an attempt to overcome these objections, Brown and Rho (BR) showed how the ideas of PCAC (partial conservation of axial-vector current) and the "Princeton bag"[26] could be used to derive an $NN\pi$ coupling. They obtained an equivalent Yukawa theory in which the parameters of the nucleon and the $NN\pi$ vertex could be related to the bag-model parameters. At large internucleon separation, this automatically yields the usual one-pion-exchange force. In an earlier report[27] we extended the BR model by observing that the equivalent Yukawa theory should include both nucleon and Δ bag states, and the appropriate interaction vertices.

In the present work we derive (Sec. II) the cloudy bag model in a much more rigorous way, by imposing chiral invariance on the MIT bag model. One advantage of the new derivation is that one obtains exact expressions for the $NN\pi$, $\Delta N\pi$, and $\Delta\Delta\pi$ vertex functions and coupling constants in a very straightforward manner.

In Sec. III formal expressions are obtained for the nucleon wave function and the πN scattering amplitude. The complete renormalization procedure is also discussed in some detail. An explicit expression for the πN scattering amplitude in the P_{33} channel, based on this formalism, is obtained in Sec. IV.

Numerical results are presented and discussed in Sec. V. There are two parameters in our model: R, the bag radius, and ω_Δ, the difference between the renormalized Δ and nucleon masses. The quantity ω_Δ is not necessarily the resonance energy (293 MeV) because the terms of Fig. 1 contribute to πN scattering. We find that the best fit to experimental data is obtained with $R = 0.72$ fm and $\omega_\Delta = 294$ MeV. With these parameters the effects of the pionic terms are relatively small: the Δ terms contain about 80% of the strength of the resonance. However, the pionic terms do contribute a non-negligible background. If they are completely neglected, but otherwise the same parameters are used, the position of the calculated resonance is shifted upward by 50 MeV.

Our results are summarized and plans for future work are discussed in Sec. VI.

FIG. 2. The Δ model. The wiggly line is the bare Δ.

II. THEORETICAL FOUNDATION

As demonstrated by Chodos and Thorn,[13] it is possible to incorporate both the Dirac equation for massless quarks and the two-boundary conditions of the MIT bag model in a single Lagrangian density

$$\mathcal{L}(x) = \left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/} q_a(x) - B\right]\theta_v - \tfrac{1}{2}\sum_a \bar{q}_a(x) q_a(x)\Delta_s . \tag{2.1}$$

In this equation $q_a(x)$ is the usual Dirac field (color a), B a phenomenological energy density, θ_v a function which is one inside the confinement volume and zero outside [$\theta_v \equiv \theta(R-r)$ in the static case], and finally Δ_s is a surface δ function. By demanding that the action

$$S = \int d^4x\, \mathcal{L}(x) \tag{2.2}$$

be invariant under the variations of the fields and bag surface

$$q_a(x) \to q_a(x) + \delta q_a(x)\,, \tag{2.3a}$$

$$\bar{q}_a(x) \to \bar{q}_a(x) + \delta\bar{q}_a(x)\,, \tag{2.3b}$$

$$\theta_v \to \theta_v + \epsilon\Delta_s\,, \tag{2.3c}$$

$$\Delta_s \to \Delta_s - \epsilon n\cdot\partial\Delta_s\,, \tag{2.3d}$$

(where n^μ is an outward normal to the bag surface), we find

$$i\partial\!\!\!/ q_a(x) = 0, \quad x \in V \tag{2.4a}$$

$$i\gamma\cdot n q_a(x) = q_a(x), \quad x \in S \tag{2.4b}$$

$$B = -\tfrac{1}{2}\, n\cdot\partial\left(\sum_a \bar{q}_a(x) q_a(x)\right) = P_D\,, \quad x \in S \tag{2.4c}$$

(where P_D is the Dirac pressure exerted on the bag surface).

The first boundary condition (2.4b) guarantees that there is no current flow through the bag surface, and the nonlinear relation (2.4c) expresses conservation of momentum at the bag boundary. Taking the static limit [$n = (0, \hat{r})$], we find that Eq. (2.4b) leads in the familiar way[19,22] to a set of quantized energy levels for the quarks, and (2.4c) provides a relation between B and R.

A. Chiral symmetry

Thus far we have been able to confine the quarks and guarantee energy and momentum conservation. Unfortunately, the necessary reflection of the quarks at the bag boundary violates chiral invariance, and the axial current associated with (2.1) is far from being conserved. Formally, this is equivalent to the observation that under the global chiral transformation

$$q_a(x) \to q_a(x) + \frac{i}{2}\epsilon\gamma_5 q_a(x) \quad (2.5)$$

the third term is not invariant, viz.,

$$\mathcal{L}(x) \to \mathcal{L}(x) - \tfrac{1}{2}\sum_a \bar{q}_a(x) i\epsilon\gamma_5 q_a(x)\Delta_s\,. \quad (2.6)$$

Indeed, as Jaffe[17] has observed, the linear boundary condition (2.4b) is not unique in guaranteeing vector current conservation. The most general condition which guarantees this is

$$i\gamma\cdot n q_a(x) = e^{i\alpha\gamma_5} q_a(x), \quad x \in S \quad (2.7)$$

and our solution above corresponds to the choice $\alpha = 0$.

A very natural way to make up for this lack of invariance is to introduce a compensating pointlike pseudoscalar field ϕ. Of course, this will eventually be identified as the pion, and we must therefore exclude the pion from those states described by $\mathcal{L}(x)$. Since our main interest is nuclear and intermediate-energy physics, we shall consider only two quark flavors, up and down. The new Lagrangian density is

$$\mathcal{L}_{\text{CBM}}(x) = \left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/} q_a(x) - B\right]\theta_v - \tfrac{1}{2}\sum_a \bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)\Delta_s + \tfrac{1}{2}[\partial_\mu\vec{\phi}(x)][\partial^\mu\vec{\phi}(x)] \quad (2.8)$$

(where the subscript CBM means "cloudy bag model"). Notice that for the moment the isovector pseudoscalar field $\vec{\phi}(x)$ is massless and that Eq. (2.8) reduces to (2.1) when $\vec{\phi}$ is zero. If one now performs a variation on the $\vec{\phi}$ field as well as the variations (2.3), one obtains the field equations

$$i\partial\!\!\!/ q_a(x) = 0, \quad x \in V\,, \quad (2.9a)$$

$$i\gamma\cdot n q_a(x) = e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x), \quad x \in S\,, \quad (2.9b)$$

$$B = -\tfrac{1}{2} n\cdot\partial \sum_a [\bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)], \quad x \in S\,, \quad (2.9c)$$

$$\partial^2\vec{\phi}(x) = -\frac{i}{2f}\sum_a \bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f}\vec{\tau}\gamma_5 q_a(x)\Delta_s, \quad \forall x\,. \quad (2.9d)$$

Once again it is easy to show that the linear boundary condition (2.9b) implies current conservation at the surface, viz.,

$$\bar{q}_a(x) i\gamma\cdot n q_a(x) = n^\mu J^a_\mu(x) = 0, \quad x \in S\,. \quad (2.10)$$

The new equation (2.9d) shows explicitly that the $\vec{\phi}$ field is free except for a source term at the bag surface. Of course, the major reason for introducing $\vec{\phi}(x)$ is that the new Lagrangian density $\mathcal{L}_{\text{CBM}}(x)$ is invariant under the global chiral transformation

$$q_a(x) \to q_a(x) + \frac{i}{2}\vec{\tau}\cdot\vec{\epsilon}\gamma_5 q_a(x)\,, \quad (2.11a)$$

$$\vec{\phi}(x) \to \vec{\phi}(x) - \vec{\epsilon} f\,. \quad (2.11b)$$

Associated with this invariance of the Lagrangian there is, of course, a conserved axial current. This can be shown in the standard way,[28] to have the explicit form

$$\vec{A}^\mu = \tfrac{1}{2}\sum_a \bar{q}_a\gamma^\mu\gamma_5\vec{\tau} q_a\,\theta_v + f\partial^\mu\vec{\phi}\,. \quad (2.12)$$

Of course, in the real world we want to identify $\vec{\phi}$ as the pion field. If we add a mass term $[-\frac{1}{2}m_\pi^2\vec{\phi}^2(x)]$ to the Lagrangian density (2.8), instead of the current (2.12) being exactly conserved ($\partial_\mu A^\mu = 0$), we find (since $\partial_\mu\partial^\mu\vec{\phi} = m_\pi^2\vec{\phi}$)

$$\partial_\mu A^\mu = f m_\pi^2\vec{\phi}\,. \quad (2.13)$$

This is exactly the form required for PCAC but derived at a somewhat deeper level than in the original work of Gell-Mann and Levy.[28]

Since we shall eventually deal with a Hamiltonian formulation of pion scattering, we now construct the Hamiltonian as

$$\hat{H} = \int d^3x\, T^{00}(x)\,, \quad (2.14)$$

where

$$T^{00}(x) = \sum_r \frac{\partial_0\mathcal{L}}{\partial(\partial_0\psi_r)}\partial^0\psi_r - \mathcal{L}g^{00}\,. \quad (2.15)$$

If we define $\vec{\pi}$ as the usual field conjugate to $\vec{\phi}$ ($\vec{\pi} = \partial^0\vec{\phi}$), Eq. (2.14) (with a pion-mass term) becomes

$$\hat{H} = \int d^3x\left[\left(\frac{i}{2}\sum_a q_a^\dagger\overleftrightarrow{\partial}_0 q_a + B\right)\theta_v + \tfrac{1}{2}\sum_a \bar{q}_a e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/f} q_a\Delta_s + \tfrac{1}{2}(\vec{\pi}\cdot\vec{\pi} + \vec{\nabla}\vec{\phi}\cdot\vec{\nabla}\vec{\phi} + m_\pi^2\vec{\phi}^2)\right]. \quad (2.16)$$

Up to this point our derivation has been exact. It may be possible to work directly with Eq. (2.16), and in the classical case we have made some progress which will be reported elsewhere. In the present work we intend to deal with a quantized pion field, and to make the calculations tractable we shall assume that the pion field is rather small. In that case, we can expand the exponential in Eq. (2.16) as

$$\tfrac{1}{2}\bar{q}_a e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a\Delta_s \simeq \tfrac{1}{2}\bar{q}_a q_a\Delta_s + \frac{i}{2f}\bar{q}_a\vec{\tau}\cdot\vec{\phi}(x)\gamma_5 q_a\Delta_s\,. \quad (2.17)$$

If we also neglect the second term in Eq. (2.17) in the linear boundary condition (2.9b), the quark fields will correspond to exactly the usual MIT model. Then we can write Eqs. (2.16) and (2.17) in the form

$$\hat{H}=\hat{H}_{\mathrm{MIT}}+H_\pi+\hat{H}_{\mathrm{int}}\,, \tag{2.18a}$$

where

$$\hat{H}_{\mathrm{MIT}}=\int d^3x\left(\frac{i}{2}\sum_a q_a^\dagger\overleftrightarrow{\partial}_0 q_a+B\right)\theta_v\,, \tag{2.18b}$$

$$H_\pi=\tfrac{1}{2}\int d^3x(\vec{\pi}\cdot\vec{\pi}+\nabla\vec{\phi}\cdot\nabla\vec{\phi}+m_\pi{}^2\vec{\phi}^2)\,, \tag{2.18c}$$

$$\hat{H}_{\mathrm{int}}=\frac{i}{2f}\int d^3x\sum_a\bar{q}_a\gamma_5\vec{\tau}\cdot\vec{\phi}q_a\Delta_s\,. \tag{2.18d}$$

Our procedure is to obtain the eigenvalues and other observables of $\hat{H}$. To do this we consider the sum ($\hat{H}_0$) of $\hat{H}_{\mathrm{MIT}}$ and H_π to be an unperturbed Hamiltonian, and work with matrix elements of $\hat{H}_{\mathrm{int}}$ in the representation of unperturbed direct product states. Let us examine the individual terms of Eq. (2.18a). The first, $\hat{H}_{\mathrm{MIT}}$, is simply the Hamiltonian describing the hadrons (excluding pions) of the original MIT bag model.[19,20] Consider the complete set of colorless baryonic bag states $|\alpha\rangle$. In this representation Eq. (2.18b) becomes

$$H_{\mathrm{MIT}}=\sum_\alpha m_\alpha|\alpha\rangle\langle\alpha|\,, \tag{2.19}$$

where m_α is the mass of the bare bag state.

Next we examine H_π which is simply the Hamiltonian for a quantized, free pion field. The eigenstates of H_π are described in terms of pion creation ($a_k^\dagger$) and destruction (a_k) operators. Then the free quantized field $\vec{\phi}$ is given by

$$\phi_j(\vec{x})=(2\pi)^{-3/2}\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(a_{j\vec{k}}e^{i\vec{k}\cdot\vec{x}}+a_{j\vec{k}}^\dagger e^{-i\vec{k}\cdot\vec{x}}) \tag{2.20}$$

and

$$H_\pi=\sum_j\int d\vec{k}\,\omega_{\vec{k}}a_{j\vec{k}}^\dagger a_{j\vec{k}}\,. \tag{2.21}$$

The interaction term in the Hamiltonian Eq. (2.18d) is particularly interesting in this representation, as nondiagonal matrix elements are not necessarily zero, viz,

$$H_{\mathrm{int}}=\sum_{\alpha,\beta}|\alpha\rangle\langle\beta|\left[\frac{i}{2f}\left\langle\alpha\right|\int d^3x\sum_a\bar{q}_a(x)\gamma_5\vec{\tau}q_a(x)\vec{\phi}(x)\Delta_s\left|\beta\right\rangle\right]. \tag{2.22}$$

To be more specific, not only will the matrix elements corresponding to $NN\pi$ and $\Delta\Delta\pi$ ($\alpha=\beta=N$ or Δ) vertices be defined by Eq. (2.22), but there will also be $\Delta N\pi$ and $N\Delta\pi$ vertices.

These interaction vertices can be calculated explicitly using the lowest-order bag-model wave functions. The latter are constructed in the usual way[22] in terms of the single-particle quark wave functions

$$q(\vec{r})=\frac{N}{\sqrt{4\pi}}\begin{pmatrix}j_0(\omega r/R)\\ ij_1(\omega r/R)\vec{\sigma}\cdot\hat{r}\end{pmatrix}v\,, \tag{2.23}$$

where v is a spin and isospin wave function, ω ($\equiv\omega_{1-1}$) = 2.04, and R is the bag radius. The normalization constant N is given by

$$N^2=R^{-3}\{\omega^2+\omega/[2(\omega-1)]\}\,. \tag{2.24}$$

If we now substitute the usual expression for the quantized pion field from Eq. (2.20), the $NN\pi$ term in H_{int} becomes

$$H_{\mathrm{int}}^{NN\pi}=|N\rangle\langle N|(2\pi)^{-3/2}\sum_j\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(v_{j\vec{k}}^{NN}a_{\vec{k}}+v_{j\vec{k}}^{NN\dagger}a_{\vec{k}}^\dagger)\,, \tag{2.25}$$

where

$$v_{j\vec{k}}^{NN}=\frac{i}{2f}\,\frac{\omega}{(\omega-1)}\,\frac{j_1(kR)}{kR}\left\langle N\right|\sum_a\vec{\sigma}_a\cdot\vec{k}\tau_{aj}\left|N\right\rangle. \tag{2.26}$$

Using the explicit nucleon wave functions, one finds that the quark spin and isospin operators can be eliminated in favor of the nucleon operators. That is,

$$\left\langle N\right|\sum_a\vec{\sigma}_a\cdot\vec{k}\tau_{aj}\left|N\right\rangle=\tfrac{5}{3}v_N^\dagger\vec{\sigma}\cdot\vec{k}\tau_jv_N\,, \tag{2.27}$$

where v_N is the nucleon spin-isospin wave function. At last we have an NN vertex of the usual form

$$v_{jk}^{NN}=i(4\pi)^{1/2}\left(\frac{f_{NN\pi}^{(0)}}{m_\pi}\right)u_N(k)v_N^\dagger\vec{\sigma}\cdot\vec{k}\tau_jv_N\,, \tag{2.28}$$

where the vertex function (normalized to unity at $k=0$) is

$$u_N(k)=j_0(kR)+j_2(kR)=3j_1(kR)/kR\,, \tag{2.29}$$

and the coupling constant is

$$(4\pi)^{1/2}\left(\frac{f_{NN\pi}^{(0)}}{m_\pi}\right)=\frac{5}{18}\left(\frac{\omega}{\omega-1}\right)\left(\frac{1}{f}\right). \tag{2.30}$$

It is interesting to notice that this value of $f_{NN\pi}^{(0)}$ is quite close to the observed $NN\pi$ coupling strength. Let us use the Goldberger-Treiman relation to replace f^{-1} on the right of (2.30). Then we obtain

$$\frac{f_{NN\pi}^{(0)}}{m_\pi}=\left(\frac{5}{9}\frac{\omega}{\omega-1}\frac{1}{g_A}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right)$$
$$=\left(\frac{1.09}{g_A}\right)\left(\frac{f_{NN\pi}}{m_\pi}\right), \tag{2.31}$$

and clearly the agreement is rather good. Note, however, that both $f_{NN\pi}^{(0)}$ and g_A will be affected by higher-order pion-quark interactions. For example, the πNN coupling constant will be renormalized (see Sec. III), and pion cloud contribution must be included in calculating g_A.

By an analogous procedure one can also establish the form of the $\Delta N\pi$ interaction term

$$H_{\rm int}^{\Delta N\pi}=\sum_j (2\pi)^{-3/2}\int d\vec{k}(v_{j\vec{k}}^{\Delta N}a_{j\vec{k}}+v_{j\vec{k}}^{N\Delta\dagger}a_{j\vec{k}}^\dagger)|\,\Delta\rangle\langle N|+\text{H.c}\,. \tag{2.32}$$

The coefficients in Eq. (2.32) are related to the transition spin and isospin operators ($\vec{S}$ and $\vec{T}$—see Ref. 10) by the equation

$$v_{j\vec{k}}^{\Delta N}=i(4\pi)^{1/2}\left(\frac{f_{\Delta N\pi}^{(0)}}{m_\pi}\right)u_\Delta(k)v_\Delta^\dagger\vec{S}\cdot\vec{k}T_j v_N\,. \tag{2.33}$$

This coupling constant can also be expressed in terms of the parameters of the bag model. However, since the coupling occurs only at the surface of the bag the details of the wave functions are irrelevant, and $[f_{\Delta N\pi}^{(0)}/f_{NN\pi}^{(0)}]$ takes the SU(6) value. For the same reason, if the nucleon and Δ bag radii are the same, the form factors $u_N(k)$ and $u_\Delta(k)$ will be identical:

$$u_N(k)=u_\Delta(k)=j_0(kR)+j_2(kR)\,. \tag{2.34}$$

These form factors provide a very natural high-momentum cutoff for the theory $[u(k)\sim k^{-2}$ as $k\to\infty]$.

The practical problem with Eqs. (2.19) and (2.22) is that in principle there are an infinite number of terms in the expansion. In the present work we shall be concerned with the energy region where, at most, one real pion is allowed. Highly excited bag states should be suppressed by large energy denominators. Therefore, we shall truncate the expansion after α equals N or Δ.

B. Summary of the cloudy bag model

Given the bag model of baryon structure, we have shown that from considerations of chiral invariance one is led to include pion coupling to the quarks at the bag surface. In our model the system is in fact described by the Hamiltonian

$$H=H_0+H_I\,, \tag{2.35}$$

$$H_0=\sum_\alpha m_b^{(\alpha)}\alpha^\dagger\alpha+\sum_k \omega_k a_k^\dagger a_k\,, \tag{2.36}$$

$$H_I=\sum_{\alpha,\beta,k}[(v_k^{\alpha\beta})\alpha^\dagger\beta a_k+\text{H.c.}]\,. \tag{2.37}$$

Here α ($\alpha^\dagger$) and β ($\beta^\dagger$) are annihilation (creation) operators for static nucleon (N) or Δ bag states of bare mass $m_b^{(\alpha)}\equiv m_b^{(N)}$ or $m_b^{(\Delta)}$. The boson operators a_k and $a_k^\dagger$ obey the usual commutation rules, and the sum over k is a formal way to represent a sum over pion isospin labels and an integral over pion momenta

$$\sum_k\equiv\sum_j\int\frac{d\vec{k}}{(2\pi)^3}\,. \tag{2.38}$$

Finally we can write interactions $v_k^{\alpha\beta}$ in terms of the microscopic form factors $u_N(k)$, $u_\Delta(k)$ of Eq. (2.34) as

$$v_k^{NN}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\,\frac{f_{NN\pi}^{(0)}}{m_\pi}u_N(k)\tau_k\vec{\sigma}\cdot\vec{k} \tag{2.39}$$

and

$$v_k^{\Delta N}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\,\frac{f_{\Delta N\pi}^{(0)}}{m_\pi}u_\Delta(k)T_k\vec{S}\cdot\vec{k}\,. \tag{2.40}$$

The ratio of $(f_{\Delta N\pi}^{(0)}/f_{NN\pi}^{(0)})^2$ can be obtained from an evaluation of the appropriate bag-model matrix elements. Because the pion interacts with quarks at the bag surface the ratio is the same as for the SU(6) model,[10] i.e.,

$$\left(\frac{f_{\Delta N\pi}^{(0)}}{f_{NN\pi}^{(0)}}\right)^2=\frac{72}{25}\,. \tag{2.41}$$

It is convenient to group the hadronic creation and annihilation operators with the interaction strengths v, so that

$$V_k^{NN}=v_k^{NN}N^\dagger N\,, \tag{2.42}$$
$$V_k^{\Delta N}=v_k^{\Delta N}\Delta^\dagger N\,, \tag{2.43}$$

and so on. Then the interaction Hamiltonian becomes

$$H_I=\sum_{\alpha,\beta\in(N,\Delta)}\sum_k(V_k^{\alpha\beta}a_k+\text{H.c.})\,. \tag{2.44}$$

This model is a *combination* of the Lee model[30] and the Chew-Low model. Note that whereas the free Hamiltonian H_0 has two stable particles, since the observed P_{33} resonance is unstable, H has only one discrete eigenvalue.

In concluding this section we wish to add one caution. We are in no way attempting to solve the bag model with pion coupling self-consistently as has been done by Chodos and Thorn[13] and by Vento *et al.*[16] (This because we neglect the influence of the pion field on the quark wave functions.) We simply assume that a self-consistent solution exists, and then examine its properties in a somewhat phenomenological way. It is nevertheless very interesting that the bag radius which we find, namely $R=0.72$ fm, is within the range of solutions ($0.5 \leqslant R \leqslant 1.5$ fm) that the Stony Brook group has reported.

III. FORMAL DEVELOPMENTS

The effective Hamiltonian (2.35) is a combination of two textbook models, the Lee model and the Chew-Low model. In this section we extend standard treatments (see Refs. 31 and 32) of the Chew-Low model to include nucleon excitation. The key results are (i) an expression for the wave function of the physical (dressed) nucleon; (ii) an exact expression for the πN scattering amplitude, which is the basis for the developments of Sec. IV; and (iii) a proof that this scattering amplitude should obey the Low equation. (In Sec. IV we show that our solution does indeed obey the Low equation).[33] In view of point (iii), the CBM is an explicit (and we feel physically well motivated) example of the well known result that the Low equation does not have a unique solution.[6]

A. The physical nucleon

In developing perturbation expansions it is useful to use energy denominators involving physical nucleon masses. This is done [following Sec. XII(d) of Schweber[31]] by introducing a mass shift into H_0,

$$\delta m=\sum_{\alpha=N,\Delta}(m_\alpha-m_b^{(\alpha)})\alpha^\dagger\alpha\ , \tag{3.1}$$

where $m_b^{(\alpha)}$ is the bare (i.e., bag) mass, and m_α the mass of the physical particle. [The meaning of m_Δ is made clear in Sec. IV—see the discussion near Eq. (4.37).] Thus we find

$$H=\bar{H}_0+\bar{H}_I\ , \tag{3.2}$$

$$\bar{H}_0=H_0+\delta m\ , \tag{3.3}$$

$$\bar{H}_I=H_I-\delta m\ , \tag{3.4}$$

and $\bar{H}_0$ acting on the bag state ($|N\rangle$ or $|\Delta\rangle$) gives the physical mass

$$\bar{H}_0|\alpha\rangle=m_\alpha|\alpha\rangle\ . \tag{3.5}$$

Notice that the completeness relation for baryon number one is

$$1=\sum_\alpha|\alpha\rangle\langle\alpha|+\sum_{\alpha,k}|\alpha,k\rangle\langle\alpha,k| + \sum_{\alpha,k,k'}|\alpha,k,k'\rangle\langle\alpha,k,k'|+\cdots\ , \tag{3.6}$$

or in a shorthand form

$$1=\sum_n|n\rangle\langle n|\ , \tag{3.7}$$

where

$$\bar{H}_0|n\rangle=E_n|n\rangle\ . \tag{3.8}$$

For example, if $|n\rangle$ is $|\Delta,k\rangle$, the energy E_n is $(m_\Delta+\omega_k)$. (The kinetic energy of the Δ's and nucleons is neglected in our treatment.)

The eigenstates of H correspond to the physical nucleon $|\tilde{N}\rangle$, and the set of scattering states $|\tilde{N},k\rangle$, $|\tilde{N},k,k'\rangle$, and so on, corresponding to an incident pion of momentum k scattering from a real nucleon [total energy $(m_N+\omega_k)$], and two incident pions of momenta k and k' scattering from a real nucleon [total energy $(m_N+\omega_k+\omega_{k'})$]. Notice that whereas $H_0(\bar{H}_0)$ has two discrete eigenstates, H has only one. The bare Δ becomes a resonance in the pion-nucleon system when $H_I(\bar{H}_I)$ is turned on.

The physical nucleon satisfies the equation

$$H|\tilde{N}\rangle=m_N|\tilde{N}\rangle\ . \tag{3.9}$$

To understand what are the components of $|\tilde{N}\rangle$ we rewrite $|\tilde{N}\rangle$ as

$$|\tilde{N}\rangle=Z^{1/2}|N\rangle+\Lambda|\chi\rangle\ , \tag{3.10}$$

where $\Lambda|\chi\rangle$ is to be determined—and the projection operator Λ is given by

$$\Lambda=1-|N\rangle\langle N|\ . \tag{3.11}$$

Thus $\Lambda|\chi\rangle$ includes the components other than the bare nucleon. To obtain $\Lambda|\chi\rangle$ use (3.2), (3.5), and (3.10) in (3.9) to find

$$\Lambda|\chi\rangle=(m_N-\bar{H}_0)^{-1}\bar{H}_I|\tilde{N}\rangle\ . \tag{3.12}$$

A useful integral equation for $|\tilde{N}\rangle$ may be obtained by using (3.12) in (3.10):

$$|\tilde{N}\rangle=Z^{1/2}|N\rangle+\Lambda(m_N-\bar{H}_0)^{-1}\bar{H}_I|\tilde{N}\rangle\ . \tag{3.13}$$

[In obtaining (3.13) the relationship $\Lambda^2=\Lambda$ has been used.]

To appreciate (3.13) let us iterate (3.13) and keep terms of first order in $\bar{H}_I$:

$$|\tilde{N}\rangle\simeq Z^{1/2}|N\rangle+Z^{1/2}\Lambda(m_N-\bar{H}_0)^{-1}\bar{H}_I|N\rangle\ . \tag{3.14}$$

However,

$$\begin{aligned}\bar{H}_I|N\rangle&=(H_I-\delta m)|N\rangle\\&=\sum_k(v_k^{NN*}|Nk\rangle+v_k^{N\Delta*}|\Delta k\rangle)-(m_N-m_b^{(N)})|N\rangle\ ,\end{aligned} \tag{3.15}$$

so that (3.14) may be written as

$$|\tilde{N}\rangle\simeq Z^{1/2}|N\rangle-Z^{1/2}\sum_k\left(\frac{v_k^{NN*}|N,k\rangle}{\omega_k}+\frac{v_k^{\Delta N*}|\Delta,k\rangle}{m_\Delta+\omega_k-m_N}\right). \tag{3.16}$$

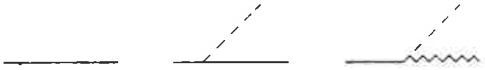

FIG. 3. The physical nucleon [from Eq. (3.16)].

Equation (3.16) is illustrated in Fig. 3. To the stated order, there is a probability Z,

$$Z^{-1}=1+\sum_k\left[\frac{v_k^{NN}v_k^{NN*}}{\omega_k^2}+\frac{v_k^{N\Delta}v_k^{\Delta N*}}{(m_\Delta+\omega_k-m_N)^2}\right], \quad (3.17)$$

that the physical nucleon is a bare three-quark state. In addition, there is some probability that the nucleon looks like either a nucleon or a Δ bag with a pion "in the air."

Finally we note that the mass shift can be obtained by considering the matrix element $\langle N|\tilde{H}_I|\tilde{N}\rangle$, which is zero because

$$\langle N|\tilde{H}_I|\tilde{N}\rangle=\langle N|H-\tilde{H}_0|\tilde{N}\rangle \quad (3.18a)$$

$$=0=\langle N|H_I-\delta m_N|\tilde{N}\rangle\,, \quad (3.18b)$$

where the last equation is obtained from (3.5). The use of the relation $\langle N|\tilde{N}\rangle=Z^{1/2}$ in (3.18a) then gives

$$\delta m_N=Z^{-1/2}\langle N|H_I|\tilde{N}\rangle\,. \quad (3.19)$$

To the lowest order in H_I, we find

$$\delta m_N^{(2)}=-\sum_k\left(\frac{v_k^{NN}v_k^{NN*}}{\omega_k}+\frac{v_k^{N\Delta}v_k^{\Delta N*}}{m_\Delta+\omega_k-m_N}\right), \quad (3.20)$$

which corresponds to the first two self-energy diagrams shown in Fig. 4(a).

B. Pion-nucleon scattering

Following Wick,[32] we suppose that the scattering wave function for a pion (k) incident on a nucleon leading to outgoing scattered waves is $|\tilde{N},k\rangle_+$. For this case the Schrödinger equation is

$$H|\tilde{N},k\rangle_+=(m_N+\omega_k)|\tilde{N},k\rangle_+\,. \quad (3.21)$$

FIG. 4. Nucleon and Δ self-energy terms.

The boundary condition is imposed by writing

$$|\tilde{N},k\rangle_+=a_k^\dagger|\tilde{N}\rangle+|\chi\rangle_+\,, \quad (3.22)$$

where $|\chi\rangle_+$ has only outgoing waves in the asymptotic region. As usual this amounts to letting E become $(E+i\epsilon)$ and taking the limit $\epsilon\to 0+$.

By following analogous steps to those in Sec. IIIA, one can find an integral equation for $|\tilde{N}k\rangle_+$:

$$|\tilde{N},k\rangle_+=a_k^\dagger|\tilde{N}\rangle+(m_N+\omega_k+i\epsilon-H)\sum_{\alpha,\beta\in(N,\Delta)}V_k^{\alpha\beta}|\tilde{N}\rangle\,. \quad (3.23)$$

For ingoing boundary conditions we simply replace $+i\epsilon$ by $-i\epsilon$, so that the S matrix is

$$S(\tilde{N}'k';Nk)={}_-\langle\tilde{N}'k'|\tilde{N}k\rangle_+$$

$$=\delta_{kk'}\delta_{\tilde{N}\tilde{N}'}-2\pi i\delta(\omega_k-\omega_{k'})\,{}_-\Big\langle\tilde{N}'k'\Big|\sum_{\alpha\beta}V_k^{\alpha\beta}\Big|\tilde{N}\Big\rangle. \quad (3.24)$$

Therefore the *exact* expression for the πN t matrix in the CBM is

$$t(\tilde{N}'k',\tilde{N}k)={}_-\Big\langle\tilde{N}'k',\Big|\sum_{\alpha\beta}V_k^{\alpha\beta}\Big|\tilde{N}\Big\rangle. \quad (3.25)$$

The operator $\sum_{\alpha\beta}V_k^{\alpha\beta}$ is simply related to the pion current, i.e.,

$$[H,a_k^\dagger]-\omega_k a_k^\dagger=\sum_{\alpha\beta}V_k^{\alpha\beta}a_k\equiv J_k\,. \quad (3.26)$$

To obtain the Low equation for any other model simply replace J_k by the corresponding operator for the other model. Equation (3.25) is used to obtain the πN phase shifts in Secs. IV and V.

C. The Low equation

If we now use the integral equation for ${}_-\langle\tilde{N}'k'|$ in Eq. (3.25) we find

$$t(\tilde{N}'k',Nk)=\Big\langle\tilde{N}'\Big|\sum_{\alpha\beta}V_k^{\alpha\beta}a_{k'}\Big|\tilde{N}\Big\rangle+\Big\langle\tilde{N}'\Big|\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^\dagger(m_N+\omega_{k'}-H+i\epsilon)^{-1}\sum_{\mu\nu}V_k^{\mu\nu}\Big|\tilde{N}\Big\rangle, \quad (3.27)$$

where the relation $[J_k,a_{k'}]=0$ has been used. To simplify the quantity $a_{k'}|\tilde{N}\rangle$ consider $Ha_{k'}|\tilde{N}\rangle$:

$$Ha_{k'}|\tilde{N}\rangle=a_{k'}m_N|\tilde{N}\rangle+[H,a_{k'}]|\tilde{N}\rangle\,. \quad (3.28)$$

Using the definitions of H (2.35) and $V_k^{\mu\nu}$ [(2.42) and (2.43)] we find

$$a_{k'}|\tilde{N}\rangle=(m_N-\omega_{k'}-H)^{-1}\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^\dagger|\tilde{N}\rangle\,. \quad (3.29)$$

Using (3.29) in Eq. (3.27) gives

$$t(\bar{N}'k',\bar{N}k)=\left\langle \bar{N}'\right|\sum_{\alpha\beta}V_k^{\alpha\beta}(m_N-\omega_{k'}-H)^{-1}\sum_{\mu\nu}(V_{k'}^{\mu\nu})^{\dagger}\left|\bar{N}\right\rangle$$
$$+\left\langle \bar{N}'\right|\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^{\dagger}(m_N+\omega_{k'}-H+i\epsilon)^{-1}\sum_{\mu\nu}V_k^{\mu\nu}\left|\bar{N}\right\rangle. \tag{3.30}$$

Equation (3.30) is the Low equation, as can be seen by inserting a complete set of eigenstates of H [c.f., Eq. (3.6)] with ingoing boundary conditions

$$1=\sum_{|\bar{n}\rangle_-}|\bar{n}\rangle_{-\,-}\langle\bar{n}|\,. \tag{3.31}$$

Using Eq. (3.31) in (3.30) we find

$$t(\bar{N}'k',\bar{N}k)=\sum_{|\bar{n}\rangle}\left[\frac{\langle\bar{N}'|\sum_{\alpha\beta}V_k^{\alpha\beta}|\bar{n}\rangle_{-\,-}\langle\bar{n}|\sum_{\mu\nu}V_{k'}^{\mu\nu}|\bar{N}\rangle}{m_N-\omega_{k'}-E_{\bar{n}}}+\frac{\langle\bar{N}'|\sum_{\alpha\beta}(V_{k'}^{\alpha\beta})^{\dagger}|\bar{n}\rangle_{-\,-}\langle\bar{n}|\sum_{\mu\nu}V_k^{\mu\nu}|\bar{N}\rangle}{m_N+\omega_{k'}-E_{\bar{n}}+i\epsilon}\right]. \tag{3.32}$$

However, from Eq. (3.25) (and its analogs for more incident mesons), this is simply

$$t(\bar{N}'k',\bar{N}k)=\sum_{|\bar{n}\rangle}\left[\frac{t^{\dagger}(\bar{N}'k,\bar{n})t(\bar{n},\bar{N}k')}{m_N-\omega_{k'}-E_{\bar{n}}}+\frac{t^{\dagger}(\bar{N}'k',\bar{n})t(\bar{n},\bar{N}k)}{m_N+\omega_{k'}-E_{\bar{n}}+i\epsilon}\right], \tag{3.33}$$

which is the familiar form of the Low equation.

In order to make (3.33) tractable some standard approximations are made. First, only the nucleon and one-meson–nucleon states are included in the sums over $\bar{n}$. Thus inelasticities in the πN amplitude are ignored. This should be a reasonable approximation for the (3.3) resonance region as the phase shift is real up to pion laboratory energies of about 500 MeV. We also keep only the nucleon-pole contribution in the first term on the right-hand side of Eq. (3.33). Since a solution of (3.33) that includes complete crossing symmetry has *never* been found, this seems to be reasonable for an initial study of our model. With these two approximations we find

$$t(\bar{N}'k',\bar{N}k)=\sum_{\bar{N}''}\frac{\langle\bar{N}'|\sum_{\mu\nu}V_{k'}^{\mu\nu}|\bar{N}''\rangle\langle\bar{N}''|\sum_{\mu\nu}V_k^{\mu\nu}|\bar{N}\rangle}{-\omega_{k'}}$$
$$+\sum_{\bar{N}''p}\frac{t^{*}(\bar{N}''p,\bar{N}'k')t(\bar{N}''p,\bar{N}k)}{\omega_{k'}-\omega_p+i\epsilon}. \tag{3.34}$$

The zero meson term arising from the second term of (3.33) has been ignored because it gives no contribution to scattering in the (3, 3) channel.

IV. THE P_{33} RESONANCE

With the theoretical basis described fully in Secs. II and III, it is relatively straightforward to derive equations for πN scattering in the (3, 3) channel. Our proof relies heavily on the renormalization techniques of Dyson[34] as applied by Chew[2] to the static model of the πN system. We briefly review Chew's arguments in Sec. IV A, before proceeding to the analogous treatment of the CBM in Sec. IV B. We show that with a small number of very reasonable assumptions a simple formula for the P_{33} scattering amplitude can be obtained.

A. The Chew model

This model is defined by our Eqs. (2.35)–(2.44), provided all mention of the Δ is omitted. That is,

$$H_{\text{Chew}}=\sum_k\omega_k a_k^{\dagger}a_k+m_0N^{\dagger}N+\sum_k(V_ka_k+V_k^{\dagger}a_k^{\dagger})\,, \tag{4.1}$$

where V_k $(\equiv V_k^{NN})$ includes a phenomenological (sharp) cutoff to eliminate ultraviolet divergences. Following Dyson, Chew grouped together all self-energy graphs (see e.g., Fig. 5) as $\Sigma(E)$. The full nucleon propagator $S(E)$ is therefore

$$S(E)=[E-m_0-\Sigma(E)]^{-1}\,. \tag{4.2}$$

At this stage it is customary to assume that the theory makes sense. That is, if $\Sigma(E)$ was evaluated exactly, to all orders, that $S(E)$ would have a pole at the nucleon mass. In practice, one can only evaluate the lowest-order terms, so that it is helpful to impose this pole on the approximate solution. Thus one expands $\Sigma(E)$ about the real nucleon mass m as

$$S(E)=\{E-[m_0+\Sigma(m)]+(E-m)\Sigma'(m)+\Sigma^R(E)\}^{-1}\,, \tag{4.3}$$

where $\Sigma^R(E)$ vanishes at least as fast as $(E-m)^2$ at $E=m$. Clearly we must now identify

$$m=m_0+\Sigma(m)\,. \tag{4.4}$$

FIG. 5. Some contributions to $\Sigma(E)$ in Chew's model (Refs. 1 and 2).

If $\Sigma(E)$ was very slowly varying, Eq. (4.3) would be simply $S(E)=(E-m)^{-1}$. Indeed the usual assumption, introduced by Chew, is that higher-order graphs such as Figs. 5(b) and 5(c) vary slowly with energy, and therefore $\Sigma'(E)$ and $\Sigma^R(E)$ get their major contribution from Fig. 5(a). That is,

$$\Sigma(E)=\Sigma(m)+[\Sigma_{N\pi}(E)-\Sigma_{N\pi}(m)]\,, \tag{4.5}$$

where $\Sigma_{N\pi}$ denotes the self-energy contribution of Fig. 5(a).

At this stage one has a choice. Since $\Sigma_{N\pi}(E)$ and its derivatives are all finite, one can work with the propagator of Eqs. (4.3), (4.4), and (4.5), viz.,

$$S(E)=\{(E-m)[1+\Sigma'_{N\pi}(m)]-\Sigma^R_{N\pi}(E)\}^{-1}\,. \tag{4.6}$$

However, it is more conventional to define a renormalized propagator

$$S'(E)=Z_2^{-1}S(E)\,, \tag{4.7}$$

and for consistency a renormalized coupling constant

$$f'=Z_2 f\,. \tag{4.8}$$

As mentioned in Sec. III, Z_2 is the probability that the dressed nucleon looks like a bare nucleon and is therefore less than 1. Thus, as observed by Chew there are two reasons for performing the mass renormalization: (i) It leads to a simpler propagator

$$S'(E)=[E-m-Z_2^{-1}\Sigma^R_{N\pi}(E)]^{-1}\,, \tag{4.9}$$

because as Chew demonstrated numerically this is very well approximated by

$$S'(E)\simeq(E-m)^{-1}\,, \tag{4.10}$$

for low energy pion scattering. (ii) Since $Z_2<1$, renormalizing the coupling constant reduces its magnitude, so that an expansion in powers of the coupling constant is more convergent.

In this theory there is no pion coupling to a nucleon-antinucleon pair, and therefore no renormalization of the pion propagator. Thus the only renormalization remaining is the inclusion of processes as in Fig. 6. As shown by Chew, this leads to a redefinition of the coupling constant

$$f_r=Z_2 Z_1^{-1} f\,. \tag{4.11}$$

Once again $Z_1<1$, but the lowest-order contribution of Fig. 6(a) has only $\frac{1}{9}$ of the effect in increasing Z_1^{-1} that Fig. 5(a) has in lowering Z_2—that is Z_2 is significantly less than Z_1 (indeed Z_1 is very nearly one in Chew's model).

With this renormalization, one has to calculate fewer diagrams in studying πN scattering. Since the renormalized $NN\pi$ coupling constant was relatively small, Chew argued that an expansion in powers of f_r^2 would make sense. The one additional observation which he made was that the pole in diagrams with only one pion in an intermediate state would effectively lower it by a power f_r^2. Thus each term in the infinite series of graphs in Fig. 1 is formally of order f_r^2, whereas those in Fig. 7 are of order f_r^4 or higher, and are dropped.

It is well known that the series of Fig. 1, with $f_r^2\sim 0.08$ and suitable choice of vertex function

$$v_{\text{Chew}}(k)\simeq\theta(m-k)\,, \tag{4.12}$$

leads to the Chew-Low effective range formula, and in particular to a resonance in the P_{33} channel (see, for example, Ref. 36).

B. The cloudy bag model

This involves a very straightforward extension of the theory of Sec. IV A to the more general Hamiltonian (2.35)–(2.44), which was dictated by our considerations of PCAC and the bag model in Sec. II. The key results which we need are the formal expressions (3.14) and (3.25) of Sec. III for the physical nucleon and the πN scattering amplitude.

The nucleon

If for clarity we retain only the two lowest-order nucleon self-energy graphs of Fig. 4 explicitly, and call the rest $\Sigma^{(N)}_{\text{HO}}(E)$ (HO = higher order), the nucleon propagator will be

$$S_N(E)=[E-m_b^{(N)}-\Sigma^{(N)}_{N\pi}(E)-\Sigma^{(N)}_{\Delta\pi}(E)-\Sigma^{(N)}_{\text{HO}}(E)]^{-1}\,. \tag{4.13}$$

The large number of virtual pions in $\Sigma^{(N)}_{\text{HO}}(E)$ means that it will be effectively constant in the energy region of interest. Thus these terms will shift the mass down, but [cf., the discussion near Eq. (4.6)] have a negligible affect on the coupling constant. With this assumption the renormalization can be carried out as before, with

$$S'_N(E)\simeq(E-m_N)^{-1}\,, \tag{4.14}$$

and

$$f'_m=Z_2 f\,, \tag{4.15a}$$

FIG. 6. Contributions to vertex renormalization.

FIG. 7. Typical higher-order irreducible diagrams contributing to πN scattering.

$$Z_2^{(N)} = [1 + \Sigma_{N\pi}'^{(N)}(m) + \Sigma_{\Delta\pi}'^{(N)}(m)]^{-1}. \qquad (4.15b)$$

Once again Z_2 is the probability that the physical nucleon looks like the three-quark bag. (Technically it also includes the possibility that the nucleon looks like a bag with *more than one* virtual pion, but by assumption this is small.)

The Δ

As we have defined it, the physical Δ is a resonance in the πN P_{33} scattering amplitude. Thus we are led to consider the series of diagrams generated by the perturbative expansion of the exact scattering amplitude (3.25). This is done by using the formally exact expression for the wave function $|\tilde{N}'k'\rangle_-$,

$$|\tilde{N}'k'\rangle_- = a_{k'}^\dagger |N'\rangle Z^{1/2} + \frac{1}{m_N + \omega_{k}' - \tilde{H}_0} \tilde{H}_I |\tilde{N}'k'\rangle_- . \qquad (4.16)$$

As Eq. (3.25) represents the solution to the Low equation, the correct solution of the linear equation (4.16) along with a solution of Eq. (3.10) for the physical nucleon must yield a solution of the Low equation. As we discuss below, our t matrix is indeed a solution of the Low equation.

Some terms of order coupling constant to the fourth (or lower) are shown in Fig. 8. Note that the nucleon mass renormalization is assumed done, in the manner described above.

By the criterion suggested by Chew for treating low energy pion-nucleon scattering, all of the graphs in Fig. 8 [except Fig. 8(g)] are formally of order coupling constant squared. That is, all those with four vertices, except Fig. 8(g), have one pion which can be on shell in an intermediate state. Terms like Fig. 8(g) can easily be retained as an essentially energy-independent shift in the bare-Δ mass. If apart from such higher-order self-energy graphs we adopt Chew's one-meson approximation, the πN t matrix is easily seen to be the solution of effectively a two-potential problem

$$t(E) = (v_{CL} + v_\Delta) + (v_{CL} + v_\Delta) G_0(E) t(E). \qquad (4.17)$$

Here v_{CL} (CL ≡ Chew Low) is the Chew driving term of Fig. 9(a), and v_Δ involves formation and decay of a Δ bag [Fig. 9(b)],

$$v_\Delta = g_{\pi N\Delta}(\vec{k}')S_\Delta^{(0)}(E) g_{\Delta N\pi}(\vec{k}), \qquad (4.18)$$

with

$$S_\Delta^{(0)}(E) = [E - m_b^{(\Delta)} - \Sigma_{HO}^\Delta(E)]^{-1}. \qquad (4.19)$$

As the higher-order Δ self-energy terms [$\Sigma_{HO}^\Delta(E)$—see e.g., Figs. 8(g) and 8(h)] contain many virtual pions, they should be essentially independent of energy in the low-energy region. Thus we can define

$$m_0^{(\Delta)} = m_b^{(\Delta)} + \Sigma_{HO}^\Delta, \qquad (4.20)$$

and hence

$$S_\Delta^{(0)}(E) = (E - m_0^{(\Delta)})^{-1}. \qquad (4.21)$$

Although we could solve Eq. (4.17) as it stands, the problem is greatly simplified by using the approximation[35,36] for the nucleon propagator in v_{CL} [Fig. 9(a)]:

$$(E - m_N - \omega_k - \omega_{k'})^{-1} = (\omega - \omega_k - \omega_{k'})^{-1}$$
$$= -\frac{\omega}{\omega_k \omega_{k'}} + \frac{(\omega - \omega_k)(\omega - \omega_{k'})}{\omega_k \omega_{k'}(\omega - \omega_k - \omega_{k'})}, \qquad (4.22)$$
$$\simeq -\frac{\omega}{\omega_k \omega_{k'}}. \qquad (4.22')$$

Note that the correction term in (4.22) vanishes when either the incident or outgoing pion is on shell. For fully one-shell kinematics our crossed Born term is proportional to $1/\omega_{k'}$, and gives the pole term of the Low equation (3.34). In the usual Chew model, Eq. (4.22') leads to the standard Chew-Low effective range formula.[35] With this approximation v_{CL} is also separable, and t is the solution of the Schrödinger equation for a rank-2 separable potential, which can be written analytically.[37]

In fact, with the usual Chew-Low normalization conventions,[31]

$$v_\Delta(\vec{k}', \vec{k}; \omega) = 4\pi P_{33} v_\Delta(k', k; \omega), \qquad (4.23)$$

with P_{33} the usual projection operator,[31] and

$$v_\Delta(k', k; \omega) = \frac{k' u_\Delta(k') u_\Delta(k) k}{(2\omega_{k'} 2\omega_k)^{1/2}} \frac{f_{\Delta N\pi}^2}{3m_\pi^2} S_\Delta^{(0)}(\omega). \qquad (4.24)$$

The potential v_{CL} with approximation (4.22) is

$$v_{CL}(\vec{k}', \vec{k}; \omega) = 4\pi P_{33} v_{CL}(k', k; \omega), \qquad (4.25)$$

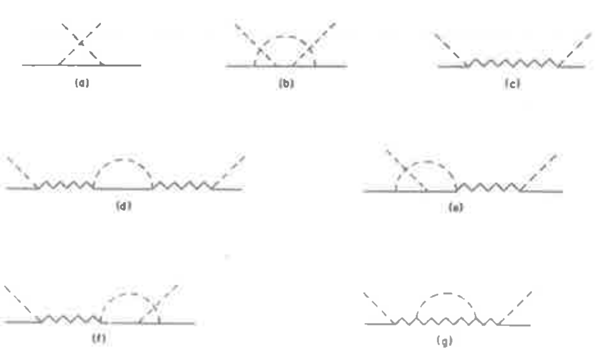

FIG. 8. Terms of Eq. (3.25) after renormalization.

FIG. 9. Born terms of Eq. (4.17).

and

$$v_{\rm CL}(k',k;\omega) = -\frac{4}{3}\frac{f_{NN\pi}^2}{m_\pi^2}\frac{k'u_N(k')u_N(k)k}{(2\omega_{k'}2\omega_k)^{1/2}}\frac{\omega}{\omega_k\omega_{k'}}\,. \tag{4.26}$$

The nucleon and Δ form factors are related to the Fourier transform of the quark wave functions in the bag by Eq. (2.34). From Eqs. (4.24), (4.26), and (4.21) it is easily seen that $(v_{\rm CL}+v_\Delta)$ is a rank-2, energy-dependent, separable potential

$$v_{\rm CL}(k',k;\omega)+v_\Delta(k',k;\omega) = \omega g(k')g(k)+(\omega-\omega_\Delta^{(0)})^{-1}h(k')h(k)\,, \tag{4.27}$$

$$\omega_\Delta^{(0)} = m_\Delta^{(0)} - m_N\,. \tag{4.28}$$

The solution to the Lippmann-Schwinger equation (4.17) for a rank-2 separable potential can easily be obtained [cf., Eq. (9) of Mongan[37]] as

$$t(k',k;\omega) = N(k',k;\omega)/D(\omega)\,, \tag{4.29}$$

with

$$N(k',k;\omega) = g(k')g(k)D_2(\omega)+h(k')h(k)D_1(\omega) + \omega[g(k')h(k)+h(k')g(k)]D_3(\omega)\,, \tag{4.30a}$$

$$D(\omega) = D_1(\omega)D_2(\omega) - \omega D_3^2(\omega)\,. \tag{4.30b}$$

Here D_1 is very closely related to the Chew-Low propagator,

$$D_1(\omega) = 1-\frac{2\omega}{\pi}\int_0^\infty \frac{dq\,q^2g^2(q)}{\omega^+-\omega_q}\,. \tag{4.31}$$

and D_2 is the propagator for the dressed Δ:

$$D_2(\omega) = \omega-\omega_\Delta^{(0)}-\frac{2}{\pi}\int_0^\infty\frac{dq\,q^2h^2(q)}{\omega^+-\omega_q} \tag{4.32}$$

$$= S_\Delta^{-1}(\omega) = \omega-\omega_\Delta^{(0)}-\frac{f_{\Delta N\pi}^2}{3m_\pi^2\pi}\int_0^\infty\frac{dq\,q^4}{\omega_q}\frac{u_\Delta^2(q)}{\omega^+-\omega_q}\,. \tag{4.33}$$

Finally, D_3 involves the interference between Chew-Low and Δ terms

$$D_3(\omega) = \frac{2}{\pi}\int_0^\infty\frac{dq\,q^2g(q)h(q)}{\omega^+-\omega_q}\,. \tag{4.34}$$

As for the Chew model of Sec. IV A, all the quantities in the cloudy bag model are finite and no renormalization is absolutely necessary. However, just as for that case, there are advantages to carrying out the Δ mass renormalization here. In particular, we readily identify the term $S_\Delta(\omega)$ in Eq. (4.33) as the Δ propagator. Formally,

$$S_\Delta(E) = [E-m_b^{(\Delta)}-\Sigma_{\rm HO}^{(\Delta)}-\Sigma_{N\pi}^{(\Delta)}(E)]^{-1}\,. \tag{4.35}$$

where $\sum_{N\pi}^{(\Delta)}$ is given by the self-energy diagram of Fig. 4(c) involving a nucleon and a pion in the intermediate state. The renormalization consists of replacing $S_\Delta(\omega)$ by $S'_\Delta(\omega)$:

$$S'_\Delta(\omega) = Z_2^{(\Delta)-1}S_\Delta(\omega)\,, \tag{4.36}$$

where

$$S'_\Delta(E) = [\omega-\omega_\Delta-\bar\Sigma_{N\pi}^{(\Delta)}(E)]^{-1}\,, \tag{4.37}$$

and

$$Z_2^\Delta = [1+\Sigma_{N\pi}^{(\Delta)\prime}(\omega_\Delta)]^{-1}\,. \tag{4.38}$$

Although m_Δ in Eqs. (4.37) and (4.38) will not necessarily be the exact position of the observed P_{33} resonance, because of the interference with the Chew-Low-type graphs, we expect it to be rather close. The one minor difficulty with Eqs. (4.37) and (4.38) is that m_Δ is above the $N\pi$ threshold, so that in fact we must carry out the subtraction procedure on the principal-value part of the self-energy integral only:

$$Z_2^\Delta(\omega) = \left[1+\frac{{\rm P}\Sigma_{N\pi}^{(\Delta)}(\omega)-{\rm P}\Sigma_{N\pi}^\Delta(\omega_\Delta)}{\omega-\omega_\Delta}\right]^{-1}, \tag{4.39a}$$

$$Z_2^\Delta = \lim_{\omega\to\omega_\Delta} Z_2^\Delta(\omega)\,, \tag{4.39b}$$

where P means that only the Cauchy principal value

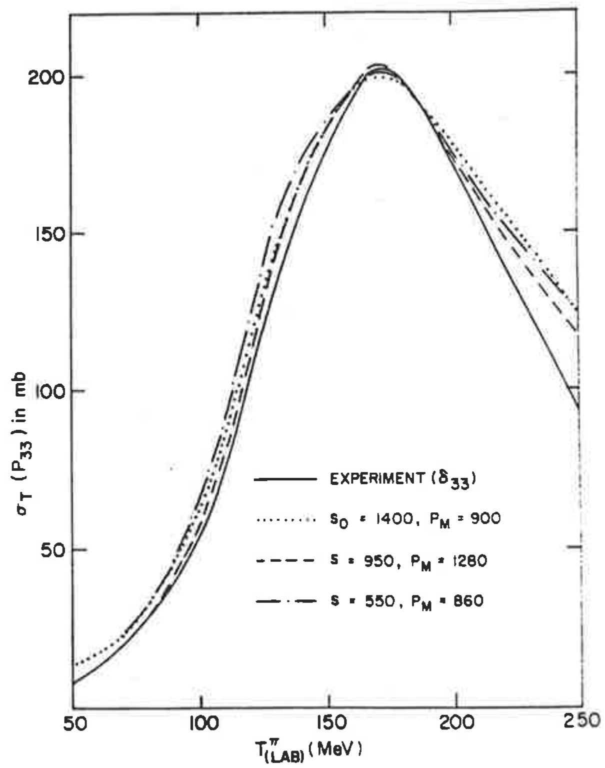

FIG. 10. Multiplicity of solutions of Ref. 27. The vertex function was a simple cutoff at p_M, and s (s_0) was the renormalized (unrenormalized) Δ mass, with respect to the mass of the nucleon.

of the integral is included. We also have

$$\bar{\Sigma}^{(\Delta)}_{N\pi}(\omega)=\frac{Z_2^{\Delta}(\omega)-Z_2^{\Delta}(\omega_\Delta)}{\omega-\omega_\Delta}-i\frac{f_{\Delta N\pi}{}^2}{3m_\pi{}^2}u^2(p)p^3, \quad (4.40)$$

where

$$p=(\omega^2-m_\pi{}^2)^{1/2}.$$

Finally we observe that this renormalization of the Δ mass also leads to a renormalization of the $\Delta N\pi$ coupling constant.

C. Summary

In terms of the renormalized Δ propagator $S'_\Delta(\omega)$ of Eq. (4.37), and the renormalized coupling constants[39] $(f_{NN\pi}, f_{\Delta N\pi})$, the pion-nucleon P_{33} scattering amplitude for the CBM may be written analytically [using the standard approximation (4.22′)] as

$$t_{\mathrm{CBM}}(k',k;\omega)=\frac{g(k')g(k)[S'_\Delta(\omega)]^{-1}+h(k')h(k)D_1(\omega)+[g(k')h(k)+h(k')g(k)]D_3(\omega)}{D_1(\omega)[S'_\Delta(\omega)]^{-1}-\omega D_3{}^2(\omega)}\,. \quad (4.41)$$

All the quantities in Eq. (4.41) were defined in Sec. IV B, but for convenience we recall that $g(k)$ represents the $NN\pi$ vertex, $h(k)$ the $\Delta N\pi$ vertex, D_1 is effectively the Chew-Low propagator, and D_3 represents the interference between the Chew-Low and Δ type of graphs.

The parameters in Eq. (4.41) are ω_Δ, $f_{NN\pi}$, $f_{\Delta N\pi}$, and implicitly the bag radius R. While we cannot fix the renormalized Δ mass at the position of the experimental resonance because of the interference from v_{CL}, we nevertheless expect ω_Δ $(\equiv m_\Delta - m_N)$ to be in the region of 290 MeV. The overall magnitude of $f_{NN\pi}$ and $f_{\Delta N\pi}$ is to be determined, but we do not expect the ratio $(f_{\Delta N\pi}/f_{NN\pi})$ to be altered much from the bag-model values. Finally the bag radius appearing in $u(k)$ must be considered an unknown, although everyone has his own prejudices.

Now that we have our solution we can show that it is a solution of the Low equation (3.34). The amplitude (4.41) satisfies the criteria of Castillejo *et al.*[6] for an amplitude to be a solution of the Low equation. This solution is different from the Chew-Low solution, but it has long been known that there are many such solutions. Indeed the fact that different choices for the discrete spectrum of states of the unperturbed Hamiltonian lead to different solutions of the Low equation was pointed out by Dyson[38] in 1957.

V. NUMERICAL RESULTS

As we explained in Sec. IV, the parameters of our theory are the mass of the dressed Δ bag $(\omega_\Delta = m_\Delta - m_N)$, which we expect to be near 290 MeV, the strength of the renormalized $\Delta N\pi$ coupling constant $f_{\Delta N\pi}$, and the bag radius R. The latter, through the form factor (2.34), serves to cut off the contribution of the high-energy virtual pions.

In our first calculations,[27] we followed the suggestion of Brown and Rho[14] by using simply a sharp cut off, $\theta(1/R-k)$, at the $\Delta N\pi$ and $NN\pi$ vertices. This gave a multiplicity of solutions, each of which fit the P_{33} scattering data equally well. For example, Fig. 10 shows the fits to the experimental P_{33} total cross section for two possible combinations of (ω_Δ, R), namely (950 MeV, 0.15 fm) and (550 MeV, 0.23 fm). In general, as ω_Δ decreased, the bag radius for the best fit increased. In the limit of very large Δ mass, the solution was essentially the Chew-Low result, and the percentage of Δ in the observed P_{33} resonance [as measured by the relative strength of the *gg* and *hh* terms in Eq. (4.41) at the pole] decreased to zero.

From many points of view this multiplicity of solutions was unsatisfactory. We needed some constraint other than πN scattering to choose between the solutions. Fortunately, this problem disappears when the theoretically derived form factor (2.34) is used. Indeed, in that case it is very hard to find a solutions. With $f_{\Delta N\pi}$ anywhere near the usually accepted range [and $(f_{\Delta N\pi}/f_{NN\pi})^2 = \frac{72}{25}$] we were able to find only *one* acceptable solution. This fit is shown in Fig. 11. It is an extremely good fit, corresponding to (ω_Δ, R) equal to (294 MeV, $R=0.72$ fm). The coupling constant $f_{\Delta N\pi}$ is 0.42, and the delta carries about 80% of the strength at the P_{33} resonance. We stress that this minimum in χ^2 space corresponding to this fit was quite sharp, and to the best of our knowledge it is unique.

The bag radius for the CBM fit is intermediate between the Brown-Rho suggestion of ~0.3 fm and the MIT value of about 1 fm. It is more in line with the suggestions of many of the early papers dealing with quark confinement. The mass $\omega_\Delta = 294$ MeV corresponds to a dressed Δ mass very close to the observed P_{33} resonance position, but slightly lighter ($m_\Delta \sim 1232$ MeV, compared with $m_R = 1236$ MeV from experiment).

In our model the bag radius is extremely well determined. (A shift of only one-tenth of a fermi would destroy the fit.) However, there are many features of a complete theory of πN scattering which we have omitted in this initial work. In par-

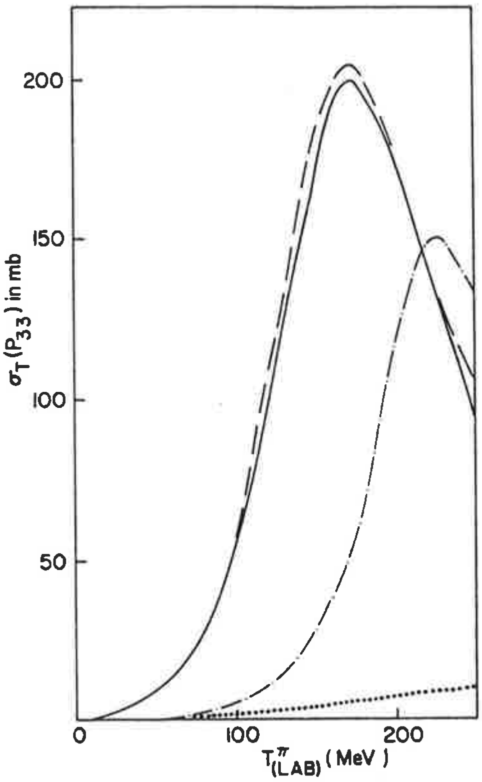

FIG. 11. Best fit in the cloudy bag model (dashed curve) to the experimental P_{33} total cross section (solid). The dash-dotted line shows the effect of arbitrarily setting $f_{NN\pi}(f_{\Delta N\pi})$ to zero, with all other parameters unchanged.

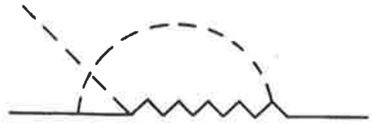

FIG. 12. A possible higher-order vertex correction to the $NN\pi$ coupling constant obtained in this work.

As we have emphasized our model keeps both pionic and Δ terms. One may investigate the relative importance of the two kinds of effects by setting $f_{NN\pi}$ or $f_{\Delta N\pi}$ equal to zero. This is shown in Fig. 11. If $f_{NN\pi}=0$, the position of the calculated resonance peak moves up by about 50 MeV. Thus pionic terms are important. If $f_{\Delta N\pi}=0$ the calculated resonance goes away; hence, Δ terms are much more important than pionic terms.

ticular, the inclusion of crossing and inelasticities would probably increase the size of the source somewhat. From our experience with the Chew-Low model, this could increase R by as much as 20%. Thus if forced to quote some estimate of the possible systematic error in the determination of R in the CBM, we would guess 0.72 ± 0.14 fm.

We also note that the $NN\pi$ coupling constant for the CBM solution is about 10% lower than the experimental value $f_{NN\pi}{}^2=0.06$. [Recall that $f_{NN\pi}$ is given in terms of $f_{\Delta N\pi}$, by demanding that $(f_{\Delta N\pi}/f_{NN\pi})^2$ be $\frac{72}{25}$.] Since we do not claim better than perhaps 20% accuracy in the determination of the bag radius, this level of agreement is acceptable for the moment. Future work may involve the explicit calculation of vertex corrections like Fig. 12.

The essential feature of the CBM is that one must keep both couplings $f_{\Delta N\pi}$ and $f_{NN\pi}$ nonzero. Nevertheless, it is interesting to turn one of these off to obtain either an elementary Δ model or an equivalent Chew-Low model. In both cases only one good solution could be found. For the "elementary Δ model" ω_Δ was 265 MeV. and R was 0.16 fm. In the effective Chew-Low case, R was 0.22 fm. These two cases are shown in Fig. 13.

VI. CONCLUSION

By incorporating chiral invariance in the MIT bag model, we obtain a theory in which the pion field is coupled to the confined quarks only at the bag surface. This leads us naturally to a theory of bare (bag-state) nucleons and Δ's interacting with a quantized pion field. Renormalization of

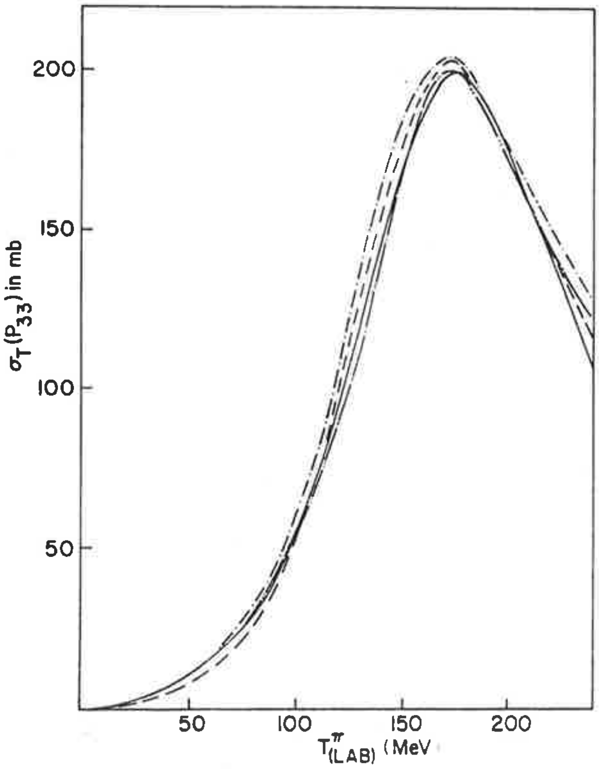

FIG. 13. Best-fit calculations using the CBM form factor but retaining only the delta graphs (dash-dot-dot curve, $R=0.16$ fm), or only Chew-type graphs (dot-dash, $R=0.22$ fm)—the solid line is the experimental result, and the dashed curve is the full CBM curve of Fig. 11.

this theory is necessary and is carried out. Explicit equations were derived for the physical nucleon and the πN scattering amplitude. This scattering amplitude satisfies the Low equation.

In the present model the Δ resonance is given by the coherent contribution of elementary Δ and Chew-Low-type graphs. Although the Δ is not an exact eigenstate of the Hamiltonian, by examining the residue of the (3,3) t matrix at the pole it is found that 80% of the strength is carried by the elementary Δ contributions, and only 20% by Chew-Low. This is a very satisfying result, because it unifies the two apparently contradictory theories of πN scattering, namely the quark model and the Chew-Low model, which have existed side by side in the literature for many years. In a later paper we intend to examine the consequences of this new model of the off-shell behavior of the πN interaction for pion-nucleus scattering—particularly the Lorentz-Lorenz effect.

The bag radius which we obtain (0.72 fm) is interesting for a number of reasons. It lies below the MIT value ($R \gtrsim 1$ fm) but considerably above the value of ~0.3 fm oringinally suggested by Brown and Rho.[14] (However, recent self-consistent calculations by the Stony Brook group[16] have suggested that any value of R from 0.5–1.5 fm could be acceptable.)

One of the most fascinating observations concerns the charge distribution of the neutron. In our model (to lowest order) the physical nucleon is 61% of the time a nucleon bag, 25% an $N\pi$ state, and 14% a $\Delta\pi$ state. In the absence of quark-quark interactions the neutron bag has no charge distribution. In higher order their spin-spin interaction would tend to give a negative charge radius, but with R=0.72 fm this effect is far too small. On the other hand, the N-bag-plus-pion state has a probability $\frac{2}{3}$ of being a proton bag with a π^- cloud at the surface (hence the cloudy bag model). Since this cloud is very much localized at the surface, we see that there is a very natural explanation of the positive charge core of the neutron and its negative tail. Most important for the moment, we see that the *bag radius* will be very naturally associated with the *zero* of the *neutron charge distribution*. Experimentally this occurs at about 0.8 fm, which is surprisingly close to our bag radius. Detailed calculations of the nucleon (and Δ) charge form factors and magnetic moments will be reported in a forthcoming paper, but there is reason to believe that this relatively small pion admixture will help to cure a number of quantitative failures in the pure quark bag models.

Another interesting feature of this bag radius is that it is no longer so difficult to accommodate classical nuclear physics with the bag model of nucleon structure. Our nucleon bags will only occupy about 35% of the typical nuclear volume, and there will certainly be long-range pion exchange forces between them. In addition, the lower limit on the critical density for percolation, as discussed by Baym,[25] is $\rho_c = 0.34/\frac{1}{3}4\pi R^3 = 0.22$ nucleons/fm^3, which is some 30% above normal nuclear densities.

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LETTER TO THE EDITOR

Low-energy pion scattering in the cloudy-bag model

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Abstract. By applying a unitary transformation to the Lagrangian density of the cloudy-bag model, we obtain a generalisation of the Weinberg effective Lagrangian for pion–nucleon scattering. The new Lagrangian incorporates the Weinberg–Tomozawa result for s-wave pion scattering from *any* hadronic bag in a very simple but elegant fashion.

In the late 1960's Fubini and Furlan (1968) and many others achieved a deep understanding of low-energy pion–hadron scattering on the basis of current algebra. Nevertheless, for some purposes it is useful to have a Lagrangian which includes the general results in a transparent way. Thus Weinberg was led to derive (from the sigma model) an effective Lagrangian for the pion–nucleon system (Weinberg 1966, 1967), which incorporated the well known result that the s-wave pion–nucleon scattering length is purely isovector. This is a special case of the general Weinberg–Tomozawa result (Fubini and Furlan 1968, Weinberg 1966, 1967, Adler and Dashen 1968) for soft-pion–hadron scattering that the scattering length is

$$a_{\mathrm{T}}=\left(\frac{g}{2m}\right)^2\left(\frac{g_{\mathrm{V}}}{g_{\mathrm{A}}}\right)^2\frac{m_\pi}{2\pi}(1-m_\pi/m_{\mathrm{t}})^{-1}[T(T+1)-T_{\mathrm{t}}(T_{\mathrm{t}}+1)-2] \tag{1}$$

where T (T_{t}) is the total (target) isospin, g the pion–nucleon coupling constant and (g_{V}, g_{A}) the vector and axial-vector coupling constants of the nucleon.

In the last two or three years there has been a great deal of emphasis on the violation of chiral symmetry (Chodos and Thorn 1975) in the MIT bag model (Chodos *et al* 1974, DeGrand *et al* 1975, Donoghue *et al* 1975). A number of hybrid models have been invented (Chodos and Thorn 1975, Jaffe 1979, Barnhill 1979, Barnhill and Halprin 1980, Brown and Rho 1979, Vento *et al* 1980, DeTar 1980a, b, Miller *et al* 1980, 1981, Théberge *et al* 1980, 1981, Thomas *et al* 1981) in which elementary (σ and) π fields are coupled to the bag in such a way that chiral symmetry is restored. While most of these models have been solved in the classical limit, the cloudy-bag model (Miller *et al* 1980, 1981, Théberge *et al* 1980, 1981, Thomas *et al* 1981) (CBM) has been specifically constructed in order that the pion field can be quantised in the standard way. Thus we can readily make contact with conventional nuclear and medium-energy physics (Miller *et al* 1981). Indeed, the first application of the CBM was to pion–nucleon scattering in the (3, 3) channel (Théberge *et al* 1980, 1981). However, the conceptual difficulty in the work of Théberge *et al* (1980, 1981) was that (although chiral symmetry was guaranteed) it was

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not obvious how the current algebra result, equation (1), could be obtained in the model. In this Letter we give a new derivation of equation (1) in the CBM, and make clear the connection with Weinberg's earlier work.

For the case of two (u and d) massless quarks the full Lagrangian density of the CBM is (Thomas *et al* 1981)

$$L_{\mathrm{CBM}}(x)=(\mathrm{i}\bar{q}(x)\not{\partial}q(x)-B)\theta_{\mathrm{v}}-\tfrac{1}{2}\bar{q}(x)\exp(\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{\varphi}(x)\gamma_5/f)q(x)\Delta_{\mathrm{s}}+\tfrac{1}{2}(\mathrm{D}_\mu\boldsymbol{\varphi})^2 \quad (2)$$

where θ_{v} is one inside the bag volume and zero outside, Δ_{s} is a surface delta function ($\delta(r-R)$ in the static, spherical case), B the usual phenomenological energy density, f the pion decay constant (93 MeV), and $q(x)$, $\boldsymbol{\varphi}(x)$ the quark and pion fields. The covariant derivative

$$\mathrm{D}_\mu\boldsymbol{\varphi}=\partial_\mu\boldsymbol{\varphi}-[1-j_0(\varphi/f)]\hat{\boldsymbol{\varphi}}\times(\partial_\mu\boldsymbol{\varphi}\times\hat{\boldsymbol{\varphi}}) \quad (3a)$$

$$\varphi=|\boldsymbol{\varphi}| \qquad \hat{\boldsymbol{\varphi}}=\boldsymbol{\varphi}/\varphi \quad (3b)$$

(where j_0 is a spherical Bessel function) is chosen such that L_{CBM} is invariant under the non-linear, global, chiral transformation

$$q(x)\rightarrow(1+\tfrac{1}{2}\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{\varepsilon}\gamma_5)q(x) \quad (4)$$

$$\boldsymbol{\varphi}\rightarrow\boldsymbol{\varphi}-\boldsymbol{\varepsilon}f+f[1-(\varphi/f)\cot(\varphi/f)]\hat{\boldsymbol{\varphi}}\times(\boldsymbol{\varepsilon}\times\hat{\boldsymbol{\varphi}}). \quad (5)$$

This invariance guarantees a conserved axial current. When the pion mass term is included in equation (2) L is no longer invariant, but one still has a partially conserved axial current (PCAC).

In our work on the (3, 3) resonance (Théberge *et al* 1980, 1981), we simply assumed that $\boldsymbol{\varphi}$ was small, and expanded equation (2) to lowest order in $\boldsymbol{\varphi}$. This led to a surface coupling of the pion field to the bag

$$H_{\mathrm{int}}=-\frac{\mathrm{i}}{2f}\int \mathrm{d}\boldsymbol{x}\,\bar{q}(x)\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}(x)q(x)\Delta_{\mathrm{s}}. \quad (6)$$

After taking matrix elements of H_{int} between baryonic bag states, this yielded a theory very like the old meson source theories, but now the form factor $3j_1(kR)/kR$ was related to the bag size, and the $\Delta\Delta\pi$, $\Delta N\pi$, $NN\pi$ coupling constants were all related. Unfortunately the model said nothing about s-wave scattering.

To make the connection with current algebra clearer, let us make a unitary transformation to a new quark field:

$$q\rightarrow q_{\mathrm{w}}=Sq \qquad \bar{q}=\bar{q}_{\mathrm{w}}S^\dagger, \quad (7)$$

with

$$S=\exp(\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{\varphi}(x)\gamma_5/2f). \quad (8)$$

With these relations the Lagrangian density becomes

$$L(x)=(\mathrm{i}\bar{q}_{\mathrm{w}}S^\dagger\not{\partial}S^\dagger q_{\mathrm{w}}-B)\theta_{\mathrm{v}}-\tfrac{1}{2}\bar{q}_{\mathrm{w}}q_{\mathrm{w}}\Delta_{\mathrm{s}}+\tfrac{1}{2}(\mathrm{D}_\mu\boldsymbol{\varphi})^2 \quad (9)$$

$$=(\mathrm{i}\bar{q}_{\mathrm{w}}\not{\partial}q_{\mathrm{w}}-B)\theta_{\mathrm{v}}-\tfrac{1}{2}\bar{q}_{\mathrm{w}}q_{\mathrm{w}}\Delta_{\mathrm{s}}+\tfrac{1}{2}(\mathrm{D}_\mu\boldsymbol{\varphi})^2+\bar{q}_{\mathrm{w}}\gamma^\mu\mathrm{i}(S\,\partial_\mu S^\dagger)q_{\mathrm{w}}\theta_{\mathrm{v}}. \quad (10)$$

At this stage we can make use of the Au–Baym (1974) identity

$$S\,\partial_\mu S^\dagger=\int_0^1 \mathrm{d}\lambda\, S^\lambda\,\partial_\mu(\ln S^\dagger)(S^\dagger)^\lambda \quad (11)$$

to prove that

$$\mathrm{i}S\,\partial_\mu S^\dagger = \frac{\gamma_5}{2f}\,\boldsymbol{\tau}\cdot(\mathrm{D}_\mu \boldsymbol{\varphi}) + \left(\frac{\cos(\varphi/f)-1}{2}\right)\boldsymbol{\tau}\cdot(\hat{\boldsymbol{\varphi}}\times\partial_\mu\hat{\boldsymbol{\varphi}}). \tag{12}$$

Thus the CBM Lagrangian density, after this unitary transformation, has the form

$$L(x) = (\mathrm{i}\bar{q}_\mathrm{w}(x)\not{\mathrm{D}}q_\mathrm{w}(x) - B)\theta_\mathrm{v} - \tfrac{1}{2}\bar{q}_\mathrm{w}q_\mathrm{w}\Delta_\mathrm{s} + \tfrac{1}{2}(\mathrm{D}_\mu\boldsymbol{\varphi})^2 + \frac{1}{2f}\bar{q}_\mathrm{w}(x)\gamma^\mu\gamma_5\boldsymbol{\tau}q_\mathrm{w}(x)\cdot(\mathrm{D}_\mu\boldsymbol{\varphi})\theta_\mathrm{v} \tag{13}$$

where the quark field covariant derivative is

$$\not{\mathrm{D}}q_\mathrm{w} = \not{\partial}q_\mathrm{w} - \mathrm{i}\left(\frac{\cos(\varphi/f)-1}{2}\right)\boldsymbol{\tau}\cdot(\hat{\boldsymbol{\varphi}}\times\not{\partial}\hat{\boldsymbol{\varphi}})q_\mathrm{w}. \tag{14}$$

The connection with Weinberg's effective Lagrangian can now be clarified. The $\mathrm{NN}\pi$ vertex is described by pseudovector coupling throughout the bag volume. The strength (at $k=0$) is simply $(g_\mathrm{A}^\mathrm{bag}/2f)$—identical to that of Théberge *et al* (1980, 1981)—and the form factor is easily shown to be very close to that of Théberge *et al* (1980, 1981). It should be emphasised that while we reproduce the result of Weinberg's effective Lagrangian, ours is considerably more general. Equation (13) describes the pion coupling to *any* hadron which can be described by the bag model. (The pion is an obvious exception for this theory!) In particular, the important results of Théberge *et al* (1980, 1981) for the ratio of $\mathrm{NN}\pi$, $\Delta\mathrm{N}\pi$, $\Delta\Delta\pi$ coupling constants will remain.

Of course the Lagrangian in equation (13) is still an effective Lagrangian. It constitutes the basis of a physical theory only when one realises that $\boldsymbol{\varphi}$ describes the behaviour of the centre of mass of a composite pion—that is, a $\mathrm{q}\bar{\mathrm{q}}$ state arising through some unspecified, dynamical symmetry-breaking mechanism. Thus, in writing either equation (2) or (13) we have already made a long-wavelength approximation. It would be inconsistent with this if multi-pion effects were large. Indeed this was the reason why equation (2) was expanded to lowest order in $\boldsymbol{\varphi}$ in earlier work. (One might expect that the form factors associated with each pion at, say, a four-pion vertex would reduce it drastically.)

Expanding equation (13) to lowest order in $\boldsymbol{\varphi}$, we find (with a pion mass term)

$$\begin{aligned} L'_\mathrm{CBM}(x) = {} & (\mathrm{i}\bar{q}_\mathrm{w}\not{\partial}q_\mathrm{w} - B)\theta_\mathrm{v} - \tfrac{1}{2}\bar{q}_\mathrm{w}q_\mathrm{w}\Delta_\mathrm{s} - \frac{\theta_\mathrm{v}}{4f^2}\bar{q}_\mathrm{w}\boldsymbol{\tau}\gamma^\mu q_\mathrm{w}(\boldsymbol{\varphi}\times\partial_\mu\boldsymbol{\varphi}) \\ & + \frac{\theta_\mathrm{v}}{2f}\bar{q}_\mathrm{w}\gamma^\mu\gamma_5\boldsymbol{\tau}q_\mathrm{w}\;\partial_\mu\boldsymbol{\varphi} + \tfrac{1}{2}(\partial_\mu\boldsymbol{\varphi})^2 - \tfrac{1}{2}m_\pi^2\boldsymbol{\varphi}^2. \end{aligned} \tag{15}$$

This Lagrangian density is completely renormalisable, and from our earlier experience (Thomas *et al* 1981) we expect the renormalisation to be small. Therefore, not only does equation (15) describe the pion coupling to any bag, rather than just the nucleon, but it is no longer merely an effective Lagrangian. It now provides a meaningful basis for a systematic calculation of higher-order corrections.

In addition to the pseudovector coupling of the pion to the bag there is also a quadratic term—arising from the covariant quark derivative—which can give rise to pion scattering. Considering for the present only zero-energy pion scattering, we clearly have a purely isovector s-wave scattering term

$$L_\mathrm{s} = -\frac{1}{2f^2}\,[\bar{q}_\mathrm{w}\gamma^0(\tfrac{1}{2}\boldsymbol{\tau})q_\mathrm{w}](\boldsymbol{\varphi}\times\boldsymbol{\pi}). \tag{16}$$

Recognising that the term in square brackets is the isospin operator for *any hadronic bag* (again the pion is excepted—its isospin operator is $\boldsymbol{\varphi} \times \boldsymbol{\pi}$), this leads to an s-wave pion–baryon interaction at threshold of the form

$$H_s = \frac{1}{2f^2} \boldsymbol{t} \cdot \boldsymbol{t}_\pi . \tag{17}$$

For the nucleon this gives an isovector scattering length $b_1 \simeq -0.09\, m_\pi^{-1}$, in excellent agreement with experiment. More significantly, we stress that equation (17) holds for *any* hadron which can be described in the bag model (except the pion). It is completely equivalent to equation (1) (if we use the Goldberger–Treiman relation between f, g_A and g), with the advantage that it is derived from a simple, renormalisable Lagrangian field theory.

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4

Pionic Corrections in the MIT Bag Model

The MIT bag model[1,2] has provided a highly successful phenomenological description of the hadronic spectrum. It has been reviewed very well in a number of places,[3,4] and so for our present purposes it suffices to say that the model postulates a spherical confinement region, a bag, within which the quarks are absolutely confined, yet move freely. In order that four momentum be conserved at the boundary of the confinement region, the MIT group postulated that there be, inside the bag, a phenomenological, positive energy density B that is equal to the Dirac pressure of the constituents at the bag boundary.

In its simplest form, without even lowest-order gluonic corrections and for massless up and down quarks, the bag model is described by the Lagrangian density

$$\mathcal{L}(x) = \left[\frac{i}{2}\bar{q}(x)\overleftrightarrow{\not{\partial}}\, q(x) - B\right]\theta_B - \frac{1}{2}\bar{q}(x)q(x)\,\delta_s, \tag{1}$$

where q and $\bar{q}$ are quark field operators; $\not{\partial} \equiv j^\mu \nabla_\mu$; θ_B is 1 inside and 0 outside the confinement volume; δ_s is a surface delta function — $\delta(r - R)$ for a static, spherical bag, with R the bag radius. By demanding that the associated action be invariant under variations in q and $\bar{q}$, and also under changes of the confinement volume normal to the surface, we get the three equations of the MIT odel. The first is the free Dirac equation for massless quarks inside the bag. The second (linear boundary condition) ensures that no color flux leaves the bag, while the third is the previously mentioned pressure balance at the surface.

One of the first outstanding successes of the model was the prediction that the axial vector coupling constant, g_A, has the value 1.09 for the nucleon. This is in remarkably good agreement with the experimental value of 1.24, and constitutes a significant improvement on the value of 5/3 given by the nonrelativistic quark model. However, there is still a problem, because the axial charge in the bag is not conserved, and the divergence of the axial vector current $(\partial_\mu A^\mu)$ is not zero.

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Formally this can be understood from Eq. (1), where the third term, which is associated with reflection at the bag boundary, is seen not to be invariant under the global chiral transformation (by infinitestimal, constant isovector ϵ)

$$q \to q + \frac{i}{2} \tau \cdot \epsilon \gamma_5 q, \qquad \bar{q} \to \bar{q} + \frac{i}{2} \bar{q} \gamma_5 \tau \cdot \epsilon \tag{2}$$

An apparently unrelated problem which has been discussed in recent years is the possible incompatibility of the bag model with nuclear physics. Without corrections for the zero-point and center-of-mass energies, the MIT bag had a radius of order[1] 1.2–1.3 fm, but most recently[2] a value near 1.0 fm has come to be preferred. Since the average internucleon separation in a nucleus is of order 1.8 fm we will clearly have problems in justifying many of the familiar and successful results of low-energy nuclear physics. For example, it is difficult to see how an independent-particle or shell model could describe a system of overlapping quark bags.

Last year, Brown and Rho[5] suggested that the problems of the nonvanishing divergence of the axial current, and the excessively large bag radius may be related. In particular, the source of axial current at the surface of the bag could act as a source of pion field. This leads to a conserved axial vector current (if $m_\pi = 0$), a long range one-pion-exchange (OPE) force and, in view of the extra pressure exerted by the pions, possibly to a smaller confinement region. Indeed, they took the rather extreme position that the bag should be of order 0.3 fm in radius, thus hoping to provide a naive explanation of the hard core in the N–N interaction.

If the very small bag radius were correct, one might have to radically alter conventional quark models. For example, the root mean-square charge radius of the proton $\langle r^2 \rangle_p^{1/2}$ is about 0.8 fm. Thus, pions outside the bag must play a crucial role in increasing the calculated value of $\langle r^2 \rangle_p^{1/2}$ from the contribution ($\sim$0.2 fm) due to the quarks inside the little bag; and pion dynamics would be a dominant feature in understanding hadronic structure. Thus, it is very important to check that a pionically amended bag model is consistent with hadronic properties.

From the preceding paragraph one might assume that there is an inverse relationship between R and the importance of pionic effects. This can be seen by paraphrasing Jaffe's[4] argument based on a classical treatment. The pion field obeys a Klein–Gordon equation with a pseudoscalar (p-wave) source term. If one neglects m_π (a reasonable approximation here), the wave equation becomes simply Laplace's equation, and for positions just outside the surface the isovector pseudoscalar pion field ϕ is proportional to $\tau \vec{\sigma} \cdot \hat{r}/R^2$. Thus if R is large, ϕ and pionic effects are small.

The idea of introducing pions in bag models is not so new. As early as 1975, Chodos and Thorn[6] at MIT had realized the problem of the source of axial current, and shown how to cure it in a simple extension of the σ model. In this case the (σ, ϕ) fields were coupled to the quarks at the bag surface in the minimal way to yield a conserved A_μ. Similar models were constructed by Vento *et al.*[7] Barnhill *et al.*[7] and Jaffe last year, except that, in sympathy with a two-phase picture[8] of QCD, the π and σ fields were kept outside the bag.

So far, none of these three groups has attempted to quantize the pion field. Thus, Jaffe has relied on perturbative solutions of the classical problem (around a bag of radius 1 fm). The other groups tried to solve the nonlinear classical field equations exactly. Unfortunately, exact solutions have only been obtained for pion fields which have the rather strange "hedgehog" property in which ϕ is parallel to $\hat{r}$. For such solutions the "nucleon" is not an eigenstate of isospin or total angular momentum. It has been speculated that this object is some combination of nucleon and delta states, and that its remaining properties (e.g., mass) are similar to those of the physical nucleon. In the most recent work of Vento *et al.*, the authors find a variety of solutions for bag radii between 0.3 and 1.0 fm. Unfortunately, the value of g_A for the nucleon in this classical theory is of order 5/3.

The TRIUMF-University of Washington group[9] took very similar steps to restore chiral invariance in the Lagrangian by introducing a new, pseudoscalar isovector field ϕ in a minimal way:

$$\mathcal{L}_{\mathrm{CBM}}(x) = \left[\frac{i}{2}\bar{q}(x)\overleftrightarrow{\not\partial}q(x) - B\right]\theta_B - \frac{1}{2}\bar{q}(x)e^{i\tau\cdot\phi(x)\gamma_5/f}q(x)\delta_s + [D_\mu\phi(x)]^2, \tag{3}$$

where f is the pion decay constant. (The subscript CBM is for cloudy bag model, which takes its name because the three-quark bag is surrounded by a cloud of pions.) The chiral transformation is nonlinear, and, for simplicity, we keep only terms linear in ϕ, in which case

$$A^\mu = \bar{q}(x)\gamma^\mu\gamma_5\frac{\tau}{2}q(x)\theta_B + f\partial_\mu\phi. \tag{4}$$

Note that in this simple model, as in the work of Chodos and Thorn, the pion exists both inside and outside the bag. This allows one to quantize the pion field very easily, but a procedure for excluding the pion from the bag is being developed. It is easy to show that, if we now break the chiral symmetry by explicitly adding a pion-mass term to $\mathcal{L}_{\mathrm{CBM}}(x)$, then we obtain the usual PCAC relationship

$$\partial_\mu A^\mu = f m_\pi^2 \phi. \tag{5}$$

At first sight, the theory presented here is indecently simple. The pion is treated as a structureless elementary particle. For example, at this level there is no fundamental connection between chiral-symmetry breaking at the quark level – they get a mass – and PCAC – the introduction of the pion mass term in $\mathcal{L}_{CBM}(x)$. Nevertheless, this model seems to have powerful implications for our understanding of low- and intermediate-energy nuclear physics.

As a first example of a crucial problem in medium-energy physics, we consider the (3, 3) resonance. In one part of the literature (typically *Physical Review C*), this is essentially a potential resonance generated by the graphs in Figure 1a. In the high-energy-physics literature (such as *Physical Review D*), it is a three-quark state which decays "weakly" into πN, so that πN scattering is described by Figure 1b. These two pictures have never been reconciled before. But, if the pion field is in some sense small (so that the exponential can be expanded to first order in ϕ), the Lagrangian density of Eq. (3) leads to a Hamiltonian

$$H_{CBM} = H_{MIT} + H_\pi + H_{int}. \tag{6}$$

Here H_{MIT} and H_π describe a free MIT bag and a free pion field, respectively. The interaction term describes the coupling of the pion field to a $q\bar{q}$ pair at the surface of the bag. If the bag states are restricted to N and Δ only, then H_{int} contains the vertices shown in Figure 1c. However, now the coupling constants are all related – e.g., $f_{\Delta N\pi} = \sqrt{(72/25)}\, f_{NN\pi}$ – and the cut-off (or vertex) function $u(k)$, calculated in terms of the quark wave functions at the bag surface, is the same in each case

$$u(k) = j_0(kR) + j_2(kR) \equiv \frac{3j_1(kR)}{kR}. \tag{7}$$

Therefore, one cannot arbitrarily increase the strength of the graphs in Figure 1a

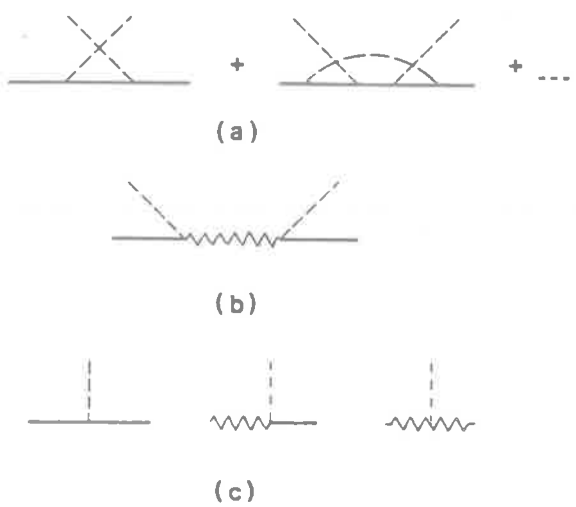

FIGURE 1. Diagrams of the (3, 3) resonance, where the solid lines represent nucleons, dashed ones pions and the curved line the delta: (a) Chew–Low series; (b) delta model; (c) pion–baryon vertex functions.

by increasing the momentum cut-off without also increasing the strength of Figure 1b. (Note that there will also be interference terms in this model.[10])

This model is an explicit, physically well-motivated example of an alternate solution to the Low equation[11] as discussed by Castillejo, Dalitz and Dyson.[12] It is fascinating that, whereas $H_0 \equiv H_{\mathrm{MIT}} + H_\pi$ has two discrete (bag) states, the interacting Hamiltonian has only one – the nucleon. The other becomes an unstable resonance in the πN system. A fit to the (3, 3) scattering data using the CBM leads to a unique bag radius $R \simeq 0.8$ fm, for which the graph of Figure 1b dominates; that is, if $f_{\mathrm{NN}\pi}$ is set to zero (with all other parameters unaltered) the (3, 3) resonance would move up by only 50 MeV, whereas if $f_{\Delta \mathrm{N}\pi}$ was turned off there would be no resonance at all. This discussion of the nature of the Δ is a specific example of a very general problem of handling unstable particles in the quark model. In a rather different language, a very similar discussion of the $\Lambda(1405)$ was recently presented by Dalitz *et al.*[13]

Only preliminary work has yet been carried out to re-do hadronic spectroscopy in this hybrid model, but the first indications are that there will be little trouble. In fact, one interesting result already in hand[14] is that the pion self-energy is a little more attractive for the nucleon (~ -400 MeV) than the delta (~ -340 MeV). Thus, the magnetic q–q interaction needs to give only some 240 MeV splitting between N and Δ in this model (instead of the entire amount of 293 MeV, as in Ref. 2). With the 15–20% smaller radius – 0.8 fm as against 0.95–1.0 fm – and the idea that the splitting is proportional to α_c/R, this means the color coupling constant, α_c, should be 0.36 rather than the value of 0.55 cited in the earlier MIT work.[2] The smaller number is considerably easier to reconcile with recent very-high-energy data – although the uncertainty in defining an appropriate momentum transfer in the bag model makes quantitative comparison impossible at the present time. A small value of α_c is also more consistent with the perturbative treatment of the q–q interaction.

The pionic corrections to the bag's electromagnetic properties can also be calculated to lowest order (such a procedure is valid only for a reasonably large value of R). For the strange baryons, Rho[15] presented some "back-of-the-envelope" calculations at the Berkeley conference. In all cases but Σ^+, the pionic correction resulted in some improvement over the unrefined bag model. For the nucleon, much more detailed calculations have been made in the cloudy bag model.[14] The results for $R = 0.8$ fm are summarized in Table I. Similar results have been obtained recently by deTar.[16] In every case except $\langle r^2 \rangle_p$, where there is little effect, the correction improves the agreement with experiment significantly. Note also that our value of g_A is in good agreement with the experimental one.

The agreement with the neutron electric form factor $G_{En}(q^2)$ is extremely significant. Just as in all the old static-source theories the process $n \to p\pi^-$ gave rise to a negative tail for the intrinsic neutron charge distribution, so does the

TABLE I
Comparison of cloudy-bag-model results with experiment and MIT results

	$\langle r^2 \rangle_p$ (fm^2)[a]	$\langle r^2 \rangle_n$ (fm^2)[b]	μ_p[c]	μ_n[c]	g_A
CBM (R = 0.8)	0.50	−0.16	2.27	-1.96	1.19
Experiment	0.69	−0.12	2.79	-1.91	1.24
MIT (R = 1.0)	0.53	0.0	1.9	-1.2	1.09

[a] The mean-square charge radius of the proton.
[b] The mean-square charge ratio of the neutron.
[c] The nucleon magnetic moments, μ_p and μ_n, are given in units of nuclear magnetons.

CBM. Those prehistoric models had, however, two basic problems. First, the core was not understood, and its properties were incalculable. Secondly the interpretation of $G_{En}(q^2)$ was always clouded by the presence of the Darwin-Foldy term, whereby a Dirac particle with an anomalous magnetic moment appears, because of zitterbewegung, to have an intrinsic charge distribution. Indeed μ_n explains essentially all of the value of $\langle r^2 \rangle_n$ measured experimentally. For a beautiful discussion of this phenomenon, including especially the ambiguities in the comparison with meson source theories, we strongly recommend the review by Foldy.[17]

In the quark model the photon interacts not with a Dirac nucleon, but with three confined quarks (and the pion in the CBM), and there is no Darwin-Foldy term. Thus, the interpretation of $G_{En}(q^2)$ in terms of an intrinsic charge distribution is unambiguous in this model, and the agreement with $\langle r^2 \rangle_n$ is very significant. Further, if we take seriously the phenomenological fits[18] of $\rho^n_{ch}(r)$ to the admittedly very poor data for $G_{En}(q^2)$, we see that they tend to give the zero in $\rho^n_{ch}(r)$ (where it switches from positive to negative) at about 0.8 fm (with at least 20% uncertainty either bigger or smaller). In the cloudy bag model the pion field is a maximum at the surface of the bag and this switch in sign should occur very close to R, the bag radius. The qualitative agreement between the experimental value quoted above (~0.8 fm) and the value of 0.8 fm extracted from πN scattering is at the very least a remarkable coincidence. Better data for $G_{En}(q^2)$ would be extremely valuable.

Because we began this review with a discussion of the difficulties of describing the long-range N–N force in a quark model, we must comment on this problem again. In models where the pion field is included explicitly as we have described, the conventional picture of one- and two-pion exchange applies unless the bags overlap. The only technical question is whether the NNπ and N$\Delta\pi$ vertex functions derived here are significantly different from other work. The preliminary results of an analysis of N–N scattering data by Gersten and collaborators[19] leads to a bag radius in the range 0.7–1.0 fm. (A similar value of R is obtained in a bag-model analysis of $N\bar{N}$ scattering.[20])

Once the nucleon bags touch, we are in a new regime, and the techniques of deTar,[21] Harvey[22] and others[23,24] can be applied – albeit with pion-field modifications. (Note that in none of these calculations is the naive association of bag radius with hard core radius found to be correct.) In short, we live in an exciting time, where there is actually a chance of understanding the N–N interaction at the quark level. Of course, the original physical problem of cramming nucleons into a finite nucleus without percolation[25] (or something worse) is not so critical if the bag radius is of order 0.8 fm or less. However, as the density rises beyond its average value, new physical phenomena[24] may occur as the bags do begin to overlap.

It would be inappropriate to end our review without mentioning some of our present worries. There is a clear contradiction between our assumption of an elementary pion field and the experimental observation of a finite charge radius for the pion. It is a fundamental problem for the hybrid models to reconcile these two aspects of pion physics. Finally, we recall that the cloudy bag model relies on first-order expansions both for the coupling of the pion at the bag surface and in the expression for the physical nucleon wave function. Higher-order corrections should be calculated for both of these approximations.

This new approach, in which baryons are regarded as a three-quark bag surrounded by a pionic cloud, has raised many fundamental questions. It has also provided answers to some old ones. Most importantly, it promises a new way of approaching the deepest questions at the heart of nuclear physics.

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Cloudy bag model of the nucleon

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A previously derived model in which a baryon is treated as a three-quark bag that is surrounded by a cloud of pions is used to compute the static properties of the nucleon. The only free parameter of the model is the bag radius which is fixed by a fit to pion-nucleon scattering in the (3,3)-resonance region to be about 0.8 fm. With the model so determined the computed values of the root-mean-square radii and magnetic moments of the neutron and proton, and g_A, are all in very good agreement with the experimental values. In addition, about one-third of the Δ-nucleon mass splitting is found to come from pionic effects, so that our extracted value of α_s is smaller than that of the MIT bag model.

I. INTRODUCTION

The MIT bag model[1-3] provides a reasonable description of hadronic properties. In its original form, however, the model did not possess chiral symmetry. Chodos and Thorn[4] and Brown and Rho[5] overcame this difficulty by introducing pions into the Lagrangian that defines the bag model. Thus the baryon can be thought of as containing three quarks inside a bag that is surrounded by a cloud of pions. An additional benefit of including pions, pointed out by Brown and Rho, is that two nucleons can interact by the exchange of pions.

If a baryon is to be regarded as partially consisting of pions, an important question can be raised: How large are the effects of pions on the properties of baryons? One might also ask: How large is the bag radius R? A rough equivalence between a small value of R and large pionic effects (or between a large value of R and small pionic effects) may be established by paraphrasing a classical-physics argument of Jaffe.[6] For positions just outside the bag surface the pion field $\vec{\phi}(\vec{r})$ may be written as

$$\phi_j(R\hat{r})=\frac{\vec{P}_j\cdot\hat{r}}{R^2}\,, \qquad (1.1)$$

where $\vec{P}_j$ is a certain[6] vector-isovector quantity. The form (1.1) arises from neglecting the pion mass m_π in the pion wave equation ($m_\pi R$ is fairly small in any of the present models) and then using the P-wave solution of the resulting Laplace's equation. Hence the strength of the pion field, which determines the importance of pionic effects, is inversely proportional to the square of the bag radius.

In the MIT work $R\approx 1.0$ fm and pionic effects are expected to be small.[6] In the Brown and Rho work, the bag is little with $R\approx 0.35$ fm and pionic effects are very large. In the present work our radius $R\approx 0.80$ fm, and pion effects are modest, but not negligible.

In previous publications[7,8] we obtained a quantized theory of nucleons (N) and Δ's interacting with pions by incorporating chiral invariance in the MIT bag model. With a bag radius of about 0.72 fm we obtained a pion-nucleon (πN) scattering that gives a good fit to experimental data in the (3, 3)-resonance region.

In this work we extend our theory by calculating the renormalized πNN and $\pi N\Delta$ coupling constants. Use of these coupling constants gives the same πN scattering amplitude as in Ref. 8, but the bag radius is determined to be 0.82 fm. With this value of R we find that the charge radii and magnetic moments of the proton and neutron are very well described by our cloudy bag model (CBM). The computed value of the axial-vector coupling constant g_A is also found to be in agreement with the experimental value. Approaches similar to ours have been used by Cottingham *et al.*[9] and DeTar.[9]

The outline of the paper is as follows. In Sec. II the theory of Ref. 8 is reviewed. The calculation of the renormalized coupling constants and the resulting πN scattering cross section is presented in Sec. III.

The computation of the nucleonic charge radii and magnetic moments and comparison of the results with experiment provides a severe test of our model. This is done in Sec. IV where it is shown that the calculated electromagnetic properties of the nucleon are in very good agreement with the experimental ones.

The value of g_A provided by our model is discussed in Sec. V. Again we find that the predictions of our model agree with the experimental data.

The difference between the physical nucleon and Δ masses, ω_Δ, has two sources in our model. As in the MIT model a splitting is caused by one-gluon exchange between quarks.[2] However, there is also a contribution to ω_Δ because the pionic self-energies of the nucleon and Δ are different. In the MIT work the entire value of ω_Δ is assumed to come from gluon exchange alone. This can occur only with a very large value (0.55) of the quantum-chromodynamics (QCD) coupling constant α_s. In Sec. VI, we show that, within our model, about 30% of ω_Δ is due to pionic effects. If we ascribe the remainder of ω_Δ to gluon-exchange effects we obtain a value of α_s that is about 60% that of the MIT work.

A few concluding remarks are made in Sec. VIII. Some technical details are given in an appendix.

II. THE CLOUDY BAG MODEL

The cloudy bag model is defined by a Hamiltonian that is obtained, approximately, from a Lagrangian density with a partially conserved axial-vector current. This Lagrangian density $\mathcal{L}_{\rm CBM}$ is given by

$$\mathcal{L}_{\rm CBM}(x)=\left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/}q_a(x)-B\right]\theta_V-\tfrac{1}{2}\sum_a \bar{q}_a(x)e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f}q_a(x)\Delta_s+\tfrac{1}{2}[D_\mu\vec{\phi}(x)]\cdot[D_\mu\vec{\phi}(x)]-\frac{m_\pi^2}{2}\vec{\phi}^2(x)\,. \tag{2.1}$$

where

$$D_\mu\vec{\phi}=\partial_\mu\vec{\phi}-[1-j_0(\phi/f)]\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\,. \tag{2.2}$$

The term $q_a(x)$ is the Dirac wave function (color a) of the quarks in the bag, θ_V a function which is one inside the confinement volume and zero outside, Δ_s is a surface δ function, $\vec{\phi}$ is the isovector pseudoscalar pion field operator, and f is the pion decay constant, 93 MeV. The present $\mathcal{L}_{\rm CBM}$ is not identical to the one of Ref. 8. An explanation for this is given in the Appendix.

The nonlinear pion-quark coupling is too difficult to handle in an exact manner so two approximations are made: (1) the terms nonlinear in $\vec{\phi}$ occurring in the exponential of (2.1) and in $D_\mu\vec{\phi}$ are neglected; and (2) the quark wave functions are taken as those of the MIT bag model. Thus for quarks in a $1s$ state we take

$$q_a(\vec{r})=\frac{N}{\sqrt{4\pi}}\begin{pmatrix} j_0\left(\frac{\omega r}{R}\right) \\ j_1\left(\frac{\omega r}{R}\right)i\vec{\sigma}\cdot\hat{r}\end{pmatrix}v_a\,, \tag{2.3}$$

where v_a is a spin and isospin wave function, we take $\omega=2.04$ (for the mode of lowest frequency), and

$$N^2=\frac{\omega^2}{R^3}\frac{1}{1-j_0^2(\omega)}\,. \tag{2.4}$$

There are several motivations and justifications for the two approximations. The linear pion-quark coupling leads to the familiar linear πNN and $\pi N\Delta$ couplings which suffice to explain a wide variety of phenomena. The quadratic term $\hat{\phi}^2$ leads to s-wave pion-nucleon scattering which is not under consideration here. Finally, the use of (2.3) gives a surface quark flux that vanishes identically. We are currently studying ways of improving on these approximations, but the current simple theory is of considerable interest—especially for nuclear-physics applications.

Employing our approximations and constructing the Hamiltonian in the usual manner we find

$$\hat{H}=\hat{H}_{\rm MIT}+H_\pi+H_I\equiv H_0+H_I\,. \tag{2.5}$$

The first term $\hat{H}_{\rm MIT}$ is the Hamiltonian describing baryons of the original MIT bag model, and is given by

$$\hat{H}_{\rm MIT}=\sum_\alpha m_\alpha\alpha^\dagger\alpha\,. \tag{2.6}$$

The operator α ($\alpha^\dagger$) destroys (creates) the bare baryon. The Hamiltonian for a free quantized pion field is H_π which is given by

$$H_\pi=\sum_j\int d\vec{k}\,\omega_k a^\dagger_{j\vec{k}}a_{j\vec{k}}\,, \tag{2.7a}$$

where $a_{j\vec{k}}$ destroys a free pion of quantum numbers $j,\vec{k}$. The free quantized pion field is then described by

$$\phi_j(\vec{x})=(2\pi)^{-3/2}\int\frac{d\vec{k}}{(2\omega_k)^{1/2}}(a_{j\vec{k}}e^{i\vec{k}\cdot\vec{x}}+a^\dagger_{j\vec{k}}e^{-i\vec{k}\cdot\vec{x}})\,. \tag{2.7b}$$

Using the above definitions of $\hat{H}_{\rm MIT}$ and H_π we may obtain the matrix elements of H_I in the set of basis states of H_0. We find

$$H_I=\sum_{\alpha,\beta,j}\int\frac{d\vec{k}}{(2\pi)^{3/2}}(v_{\vec{k}}^{\alpha\beta}\alpha^\dagger\beta a_{j\vec{k}}+\text{H.c.})\,, \tag{2.8}$$

with

$$v_{\vec{k}}^{NN}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2}i\frac{f_{\pi NN}}{m_\pi}u(k)\tau_j\vec{\sigma}\cdot\vec{k} \tag{2.9a}$$

and

$$v_k^{\Delta N}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\frac{f_{\pi\Delta N}}{m_\pi}u(k)T_k\vec{S}\cdot\vec{k}\,, \quad (2.9b)$$

$$v_k^{\Delta\Delta}=\left(\frac{4\pi}{2\omega_k}\right)^{1/2} i\frac{f_{\pi\Delta\Delta}}{m_\pi}u(k)\mathcal{T}_k\vec{\Sigma}\cdot\vec{k}\,. \quad (2.9c)$$

The form factor $u(k)$ is given by

$$u(k)=\frac{3j_1(kR)}{kR}\,. \quad (2.10)$$

The operator $\vec{S}(\vec{T})$ changes spin (isospin-)-$\frac{1}{2}$ states to spin (isospin-)-$\frac{3}{2}$ states, and is defined by the reduced matrix element

$$\langle\tfrac{3}{2}\|S\|\tfrac{1}{2}\rangle=\langle\tfrac{3}{2}\|T\|\tfrac{1}{2}\rangle=2\,. \quad (2.11)$$

[The convention of Messiah[10] (p. 1076) is used here.] The constants $f_{\pi NN}$ and $f_{\pi N\Delta}$ are unrenormalized coupling constants and have the relation

$$\frac{f_{\Delta N\pi}}{f_{NN\pi}}=\left(\frac{72}{25}\right)^{1/2} \quad (2.12a)$$

in the quark model. Finally $\mathcal{T}_k$ and $\vec{\Sigma}$ are the spin and isospin operators for a spin-$\frac{3}{2}$—isospin-$\frac{3}{2}$ object and

$$\frac{f_{\Delta\Delta\pi}}{f_{NN\pi}}=\frac{4}{5}\,. \quad (2.12b)$$

In principle $f_{NN\pi}$ may be obtained from $q_a(x)$ and H_I, with

$$\sqrt{4\pi}\,\frac{f_{NN\pi}}{m_\pi}=\frac{5}{18f}\,\frac{\omega}{\omega-1}\,. \quad (2.13)$$

However, we treat $f_{\pi NN}$ as a free parameter, to be determined in a fit to πN scattering in the (3,3)-resonance region. In Sec. III we find that the phenomenologically determined value of $f_{NN\pi}$ agrees with that of (2.13) to within 20%.

The cloudy bag model is thus defined by the relations (2.5)—(2.12) and the specification of the parameters R and $f_{\pi NN}$.

An additional number needed for computations is the difference between the physical masses of the Δ and nucleon, ω_Δ. In practice a fit to πN scattering data is used to determine the value of ω_Δ. However, the difference between the Δ and nucleon bag masses, $m_\Delta^{(0)}-m_N^{(0)}$; and the difference between the pion contributions to the Δ and nucleon self-energies (which we calculate) make up ω_Δ. Thus, in our theory there is a relationship between ω_Δ and $m_\Delta^{(0)}-m_N^{(0)}$, and either one may be regarded as a free parameter. However, $m_\Delta^{(0)}-m_N^{(0)}$ can be estimated in one-gluon-exchange models, so that ω_Δ can be obtained from a calculation. We find (Sec. VI) that the theoretical expectation for the value of ω_Δ corresponds closely to the phenomenologically determined value, so that ω_Δ is not to be regarded as a (totally) free parameter.

In the theory presented above, as in Ref. 4, the pion field $\phi_\pi(\vec{r})$ does not vanish for $r<R$. That is, the pion penetrates the confinement region. However, when the pion is inside the bag it does not interact with quarks so that the asymptotic freedom property of QCD is maintained. A version of the CBM in which the pion does *not* enter the bag is being developed and will appear elsewhere.

The theory presented above employs a static bag: the quarks are contained within, and pions interact with the surface of a fixed sphere. Thus the eigenfunctions of energy are not eigenfunctions of the total-momentum operator. Recently Donoghue and Johnson[11] have presented a method for improving such eigenstates by projecting these on to states of good momentum. In Secs. IV, V, and VI we use the static model to compute properties of the nucleon, then discuss how the "recoil corrections" of Ref. 11 change the results. The importance of these corrections has been emphasized by DeTar.[9] We are also investigating alternate ways of making such corrections.

III. RENORMALIZED COUPLING CONSTANTS AND PARAMETER SPECIFICATION

In previous work we discussed the renormalization procedures which simplify the use of our Hamiltonian. However, the renormalized πNN and $\pi N\Delta$ coupling constants, $f_{\pi NN}^{(R)}$, $f_{\pi N\Delta}^{(R)}$ were taken to be independent of energy with

$$f_{\pi NN}^{(R)\,2}=0.06 \quad (3.1a)$$

and

$$f_{\pi N\Delta}^{(R)}=(\tfrac{72}{25})^{1/2}f_{\pi NN}^{(R)}\,. \quad (3.1b)$$

In this work we take $f_{\pi NN}$ as a free parameter, use (2.12) to obtain $f_{\pi N\Delta}$, and employ the theory of Théberge *et al.*[8] to obtain the renormalized coupling constants. These depend on an energy parameter ϵ. Finally the parameter $f_{\pi NN}$ (for a given R and ω_Δ) is limited by the condition that the renormalized πNN coupling constant (for pions of zero four-momentum) has approximately the correct value

$$f_{\pi NN}^{(R)\,2}(\epsilon=0)\approx 0.08\,. \quad (3.2)$$

Then the parameters R and ω_Δ are specified by the πN scattering data. The fitted value of $f_{\pi NN}$ is then compared with the theoretical value of (2.13).

According to Ref. 8 (but in a slightly different notation) the relationship between $f_{\pi NN}^{(R)}(\epsilon)$ and $f_{\pi NN}$ is

$$f_{\pi NN}^{(R)}(\epsilon)=\frac{V_N(\epsilon)f_{\pi NN}}{Z_N}\,. \quad (3.3)$$

where Z_N^{-1} is the probability (less than one) that the physical nucleon is a bare three-quark state. To lowest order ($f_{\pi NN}^2$), the factor Z_N may be obtained by using (2.9) in Eq. (3.17) of Ref. 8. (The quantity Z_N is Z^{-1} of that equation.) We find

$$Z_N = 1 + \frac{3f_{\pi NN}^2}{\pi m_\pi^2}\int_0^\infty dq\,\frac{q^4u^2(q)}{\omega_q^3} + \frac{96}{25\pi}\frac{f_{\pi NN}^2}{m_\pi^2}\int_0^\infty dq\,\frac{q^4u^2(q)}{\omega_q(\omega_q+\omega_\Delta)^2}. \tag{3.4a}$$

The second and third terms of (3.4) arise from $N\pi$ and $\Delta\pi$ components of the wave function of the physical nucleon, respectively. The quantity Z_N can also be given in terms of the nucleon self-energy $\Sigma_N(E)$ (Fig. 1),

$$Z_N = 1 - \frac{\partial\Sigma_N(E)}{\partial E}\bigg|_{E=m_N}. \tag{3.4b}$$

For future reference we also display $\Sigma_N(E)$:

FIG. 1. Nucleon self-energy. In all the figures the pion, nucleon, and Δ are represented by dashed, solid, and wiggly lines, respectively.

$$\Sigma_N(E) = \frac{3f_{\pi NN}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q(E^+-\omega_q-m_N)} + \frac{96}{25}\frac{f_{\pi NN}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q(E^+-\omega_q-m_\Delta)}. \tag{3.4c}$$

The suppression due to the wave-function-renormalization constant Z_N is mitigated by the inclusion of the vertex correction $V_N(\epsilon)$. This quantity is displayed, to order $f_{\pi NN}^2$, in Fig. 2. A straightforward set of manipulations gives the result

$$V^N(\epsilon) = 1 + \frac{f_{\pi NN}^2}{3\pi m_\pi^2}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q^2(\omega_q-\epsilon)} + \frac{32}{15}\frac{f_{\pi NN}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q(\omega_q+\omega_\Delta)(\omega_q+\omega_\Delta-\epsilon)} + \frac{128}{75}\frac{f_{\pi NN}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q(\omega_q-\epsilon)(\omega_q+\omega_\Delta)} + \frac{128}{75}\frac{f_{\pi NN}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q^2(\omega_q+\omega_\Delta-\epsilon)}. \tag{3.5}$$

The expressions (3.3)—(3.5) specify the renormalized πNN coupling constant. Prior to obtaining the renormalized $\pi N\Delta$ coupling constant it is worthwhile to discuss the parameter ϵ used in obtaining the crossed Born graph of Fig. 3(a). By considering the energy denominators of Fig. 3(b), one may show that for the calculation of Fig. 2(a), $V(\epsilon)$ must be evaluated at the point

$$\epsilon = -E\,, \tag{3.6}$$

where E is the energy of the incident and outgoing pion.

The Δ is not an eigenstate of the Hamiltonian. However, in quark models the Δ and N are members of an SU(6) multiplet. In order to maintain the SU(6) symmetry and treat the Δ and N in the same manner, we must apply the renormalization techniques of Ref. 1 to the Δ as well as the nucleon. Thus we write in analogy with (3.3)

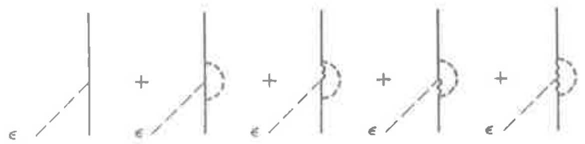

FIG. 2. The $\pi N\rightarrow N$ vertex function. ϵ is the energy of the incident pion.

$$f^R_{\pi N\Delta}(\epsilon) = \frac{V_\Delta(\epsilon)}{\sqrt{Z_\Delta}\sqrt{Z_N}}f_{\pi N\Delta}. \tag{3.7}$$

The Δ wave-function-renormalization constant Z_Δ is given by

$$Z_\Delta = 1 - \frac{\partial\,\mathrm{Re}\,\Sigma_\Delta(E)}{\partial E}\bigg|_{E=m_\Delta}, \tag{3.8}$$

where $\Sigma_\Delta(E)$ is the pion contribution to the Δ self-energy, Fig. 3. The evaluation of $\Sigma_\Delta(E)$ gives

$$\mathrm{Re}\,\Sigma_\Delta(E) = \frac{24}{25}\frac{f_{\pi NN}^2}{m_\pi^2\pi}\,\mathrm{P}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q(E-\omega_q-m_N)} + 3\frac{f_{\pi NN}^2}{m_\pi^2\pi}\,\mathrm{P}\int_0^\infty \frac{q^4dq\,u^2(q)}{\omega_q(E-m_\Delta-\omega_q)}. \tag{3.9}$$

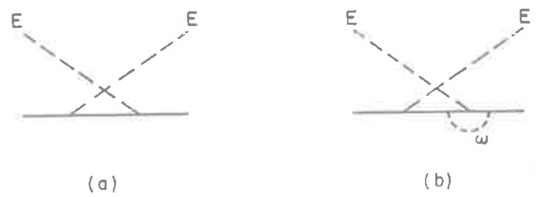

FIG. 3. Pion-nucleon crossed Born term. (a) Lowest order. (b) With a vertex correction. The energy of the virtual pion is ω.

The vertex correction $V_\Delta(\epsilon)$ is shown in Fig. 4. Some care must be used in evaluating the terms of Fig. 4. For example, the term of Fig. 4(e) is already included in the pion-nucleon t matrix of Ref. 1 [see Fig. 8 therein and Fig. 4(f) here]. It should not be recomputed in calculating the pion-nucleon scattering amplitude. Similarly the term of Fig. 4(b) would have been included if the crossed Δ term, Fig. 5, had been included in the driving term of the integral equation of the pion-nucleon t matrix. Indeed the term of Fig. 5 may be regarded as a correction to the Chew term of Fig. 3(a). However, the ratio of the term of Fig. 5 to that of Fig. 3(a) is ~ 0.03. We therefore neglect the term of Fig. 4(b) in treating πN scattering. We do, however, include the terms of Figs. 4(b) and 4(e) in computations of the renormalized $N\Delta\pi$ coupling constant to be used in calculating the electromagnetic properties of the nucleon (Sec. IV) and the Δ-N mass splitting (Sec. VI).

The evaluation of the terms of Figs. 4(a), 4(c), and 4(d) gives

$$V_\Delta(\epsilon)=1+\frac{5}{3}\frac{f_{NN\pi}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q^2(\omega_q+\omega_\Delta-\epsilon)} + \frac{4}{3}\frac{f_{NN\pi}^2}{\pi m_\pi^2}\int_0^\infty \frac{q^4 dq\, u^2(q)}{\omega_q(\omega_q+\omega_\Delta)(\omega_q+\omega_\Delta-\epsilon)}\,. \tag{3.10}$$

A consideration of the term of Fig. 5(b) leads to the result that the value of ϵ to be used in (3.10) appropriate for computing πN scattering is the pion energy E. We further note that for energies of our current interest the denominators in the second and third terms of (3.10) do not vanish.

The equations (3.7)–(3.10) specify the renormalized $\pi N\Delta$ coupling constant. The next step is to determine the parameters R and ω_Δ by computing the scattering phase shifts as a function of energy

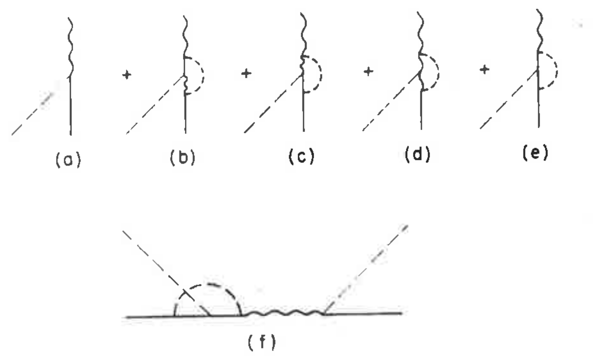

FIG. 4. (a)–(e) The $\pi N\to\Delta$ vertex function. (f) A term included previously in the pion-nucleon T matrix.

FIG. 5. (a) Small correction to pion-nucleon Born term. (b) Vertex correction in π-nucleon scattering.

and obtaining the best possible fit. This is done by using Eq. (4.41) of Ref. 8, but replacing the energy-independent coupling constants of (3.1) by the ones of (3.3) and (3.7). The best fit shown in Fig. 6 is obtained with the parameters

$$R=0.82\text{ fm}\,,\quad \omega_\Delta=280\text{ MeV}\,,\quad f_{NN\pi}^2=0.078\,. \tag{3.11}$$

The above value of $f_{NN\pi}^2$ leads to

$$f_{NN\pi}^{(R)2}(\epsilon=0)=0.064\,. \tag{3.12}$$

For comparison with the usual value of 0.08 one should multiply the results of (3.12) by $u^2(|\vec{q}|=im_\pi)$. This leads to an enhancement of the value of (3.11) by about 7%. The result (3.12) is then in reasonable agreement with the usual value of 0.080. The energy dependence of the renormalized coupling constants is displayed in Table I.

The phenomenological value of $f_{\pi NN}$ is [by (3.11)] 0.28, whereas the theoretical value [from

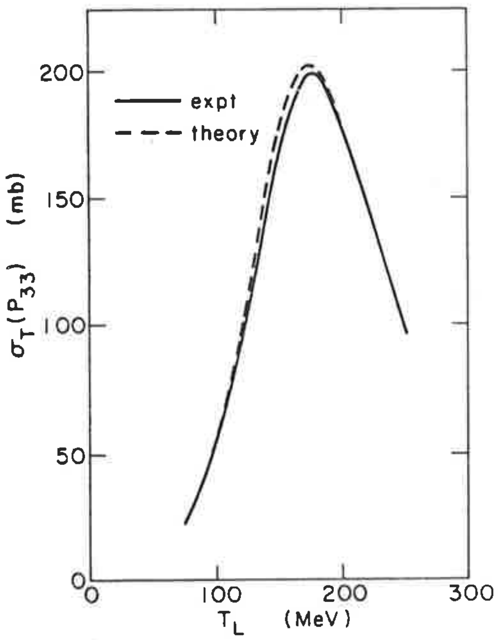

FIG. 6. Best fit (dashed curve) to the experimental P_{33} total cross section (solid).

(2.13)| is 0.23, so that the phenomenological value agrees with the theoretical one to within 20%. Such a deviation is to be expected from the inherent inaccuracies of the Goldberger-Trieman relation, and the static approximations we use. Thus the phenomenological value of $f_{\pi NN}$ is reasonably well predicted by the CBM.

This specification of R, ω_Δ, and $f_{\pi NN}$ completely defines the theory. Thus the computation of additional observables is free of arbitrary parameters.

IV. ELECTROMAGNETIC PROPERTIES OF THE NUCLEON

The comparison of the computed electromagnetic properties of the nucleon with the experimental values should provide a severe test of our model. In particular, the root-mean-square (rms) charge radius of the neutron is expected to be extremely sensitive to the pionic components of the neutron.

The organization of this section is as follows. First the pionic charge $\hat{\rho}_\pi(x)$ and current $\vec{j}_\pi(x)$ density operators are defined. Then the pionic contribution to the charge density is obtained by evaluating $\hat{\rho}_\pi(x)$ in the physical nucleon state. The bag charge density $\rho_B(x)$ is computed, and the total charge density is the sum of $\rho_B(x)$ and the pion charge density. The rms radii of the proton and neutron are then computed. The expectation value of $\vec{j}_\pi(x)$ in the physical nucleon state is evaluated to obtain the pionic contribution to the magnetic moment of the nucleon. The quark contribution to the nucleon magnetic moment is calculated and added to the pionic contribution to get the nucleon magnetic moments. The parameters of (3.11) are used in these computations.

TABLE I. Energy-dependent renormalized coupling constants. The quantity $C(E)$ is defined by $C(E) = \frac{25}{72} f^{(R)\,2}_{\pi N\Delta}/f^{(R)\,2}_{\pi NN}(E)$. The terms of Figs. 4(b) and 4(e) are not included in the computation of $C(E)$.

E (MeV)	$f^{R\,2}_{\pi NN}(-\epsilon)$	$C(E)$
0	0.064	0.85
260	0.054	1.13
285	0.054	1.16
312	0.053	1.19
337	0.053	1.21
368	0.053	1.25
391	0.052	1.27

To establish our notational conventions we present a few definitions. The matrix element of the electromagnetic current operator $\hat{j}_\mu(0)$ is given by

$$\langle p'|\hat{j}_\mu(0)|p\rangle = \frac{1}{(4EE')^{1/2}}\,\bar{u}(p')\{[F_1^S(q^2)+F_1^V(q^2)\tau_3]\gamma_\mu + i\sigma_{\mu\nu}q^\nu[F_2^S(q^2)+F_2^V(q^2)\tau_3]\}u(p)\,, \tag{4.1}$$

where $|p\rangle$ represents a physical nucleon state of four-momentum p and $q=p'-p$. For future reference the $q^2=0$ values of the form factors are listed below:

$$\begin{aligned} F_1^S(0)&=e/2\,,\\ F_1^V(0)&=e/2\,,\\ F_2^S(0)&=-0.06\,e/2m_N\,,\\ F_2^V(0)&=+1.85\,e/2m_N\,. \end{aligned} \tag{4.2}$$

Because we use the static solution of the free bag equations it is appropriate to compute $\langle p'|\hat{j}^\mu|p\rangle$ in the static limit ($m_N\to\infty$). We define $\hat{J}^\mu(q^2=-\vec{q}^{\,2})$ by the relation

$$\langle\chi_{\lambda'}|\hat{J}^\mu(-\vec{q}^{\,2})|\chi_\lambda\rangle = \lim_{m_N\to\infty}\langle p'|\hat{j}_\mu(0)|p\rangle\,, \tag{4.3}$$

where $|\chi_\lambda\rangle$, $|\chi'_\lambda\rangle$ are the spin-isospin wave functions of the nucleon. By taking the $m_N=\infty$ limit of the matrix elements on the right-hand side of (4.1) we find

$$\hat{J}^0(-\vec{q}^{\,2}) = F_1^S(-\vec{q}^{\,2}) + F_1^V(-\vec{q}^{\,2})\tau_3 - [F_2^S(0)+F_2^V(0)\tau_3]\,\frac{\vec{q}^{\,2}}{2m_N} \tag{4.4a}$$

$$\equiv G_{ES}(-\vec{q}^{\,2}) + G_{EV}(-\vec{q}^{\,2})\tau_3 \tag{4.4b}$$

and

$$\vec{J}(-\vec{q}^{\,2}) = i(\vec{\sigma}\times\vec{q})[F_2^S(-\vec{q}^{\,2})+F_2^V(-\vec{q}^{\,2})\tau_3]\,. \tag{4.5}$$

In taking the static limit of (4.1) we have kept a term that arises from the $\sigma_{0\nu}q^\nu$ operator and gives rise to the F_2 term of (4.4a). Because of the large value of the anomalous magnetic moment, this is effectively of order zero in m_N, and makes an important contribution to the effective rms charge radius of the neutron.

It is our task, in this paper, to compute the theoretical values of the nucleonic electromagnetic properties at small $\vec{q}^2$. Thus we employ the CBM to compute the theoretical value of the current $\hat{J}^{\mu}_{\rm CBM}(q^2)$. In making the computations it is useful to define the Fourier transform $\hat{j}^{\mu}(r)$ of $\hat{J}^{\mu}_{\rm CBM}(q^2)$ as

$$\hat{j}^{\mu}(\vec{r}) = \int \frac{d^3q}{(2\pi)^3}\, \hat{J}^{\mu}_{\rm CBM}(q^2) e^{-i\vec{q}\cdot\vec{r}} \tag{4.6}$$

with

$$\hat{j}^{\mu}(\vec{r}) = (\hat{\rho}(\vec{r}), \vec{j}(\vec{r})) \,. \tag{4.7}$$

By using the Fourier inverse of (4.6) and comparing the result with (4.5) we obtain

$$G_{ES}^{\rm CBM}(-\vec{q}^{\,2}) + G_{EV}^{\rm CBM}(-\vec{q}^{\,2})\tau_3 = \left\langle p' \right| \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, \hat{\rho}(\vec{r}) \left| p \right\rangle \tag{4.8}$$

and

$$[F_2^S(-\vec{q}^{\,2}) + F_2^V(-\vec{q}^{\,2})\tau_3]_{\rm CBM} = -i\,\frac{(\vec{\sigma}\times\vec{q})}{2\vec{q}^{\,2}} \cdot \left\langle p' \right| \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, \vec{j}(\vec{r}) \left| p \right\rangle . \tag{4.9}$$

To obtain the theoretical values of the various electromagnetic form factors, $G_{ES}^{\rm CBM}(-\vec{q}^{\,2})$, etc., we compute $\hat{\rho}(\vec{r})$ and $\vec{j}(\vec{r})$.

We must first obtain the pion contribution to $\hat{j}$. The pion current operator $\hat{j}^{\mu}_{\pi}(\vec{r})$ is the conserved current of that part of $\mathcal{L}_{\rm CBM}(x)$ (2.1) corresponding to the free pion field. We have

$$\hat{j}^{\mu}_{\pi}(x) = -ie[\phi(x)\partial^{\mu}\phi^*(x) - \phi^*(x)\partial^{\mu}\phi(x)] \,, \tag{4.10a}$$

where

$$\phi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) - i\phi_2(x)] \,. \tag{4.10b}$$

By using the expression for the free pion field (2.7a) in (4.10) we can obtain expressions for the pion charge density $\hat{\rho}_{\pi}(x)$ and current density $\vec{j}_{\pi}(x)$ in terms of pion creation and destruction operators. After suitable manipulations one derives

$$\hat{\rho}_{\pi}(\vec{x}) = -\frac{ie}{2}\sum_{i,j=1,3}\frac{\epsilon_{ij3}}{(2\pi)^3}\int d^3k\, d^3k' \left(\frac{\omega_{k'}}{\omega_k}\right)^{1/2} e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}(-a_{i,-\vec{k}'} + a^{\dagger}_{i,\vec{k}'})(a_{j,\vec{k}} + a^{\dagger}_{j,-\vec{k}}) \tag{4.11}$$

and

$$\vec{j}_{\pi}(\vec{x}) = -\frac{ie}{2}\sum_{i,j=1,3}\frac{\epsilon_{ij3}}{(2\pi)^3}\int \frac{d^3k\, d^3k'}{(\omega_k\omega_{k'})^{1/2}}\, \vec{k}(a^{\dagger}_{i\vec{k}'} + a_{i,-\vec{k}'})(a_{j,\vec{k}} + a^{\dagger}_{j,-\vec{k}})\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} . \tag{4.12}$$

To compute the expectation value of $\hat{\rho}_{\pi}(x)$ we use the expression

$$|\tilde{N}\rangle = Z_N^{-1/2}\left[1 + \Lambda\frac{1}{m_N - \tilde{H}_0}\,\tilde{H}_I + \Lambda\left(\frac{1}{m_N - \tilde{H}_0}\,\Lambda\tilde{H}_I\right)^2\right]|N\rangle \tag{4.13}$$

for the physical state $|\tilde{N}\rangle$, valid to second order in $f_{\pi NN}$. (The ket $|\tilde{N}\rangle$ describes $|p\rangle$ in the limit that m_N is infinite.) In (4.13) $|N\rangle$ is the bare nucleon state and $\Lambda = 1 - |N\rangle\langle N|$. The operators $\tilde{H}_0$ and $\tilde{H}_I$ are given by

$$\tilde{H}_I = H_I - \Sigma_N \,, \qquad \tilde{H}_0 = H_0 - \Sigma_N \,. \tag{4.14}$$

The pion-charge-density operator can be defined by

$$\rho_{\pi}(x)\tau_3 = \langle\tilde{N}|\hat{\rho}_{\pi}(\vec{x})|\tilde{N}\rangle \tag{4.15}$$

because pions contribute only to the isovector form factors. The application of (4.13) to (4.15) leads to four terms, but only two are different from zero. These are depicted in Fig. 7. The contributions of the terms of Figs. 7(a)–7(b) to $\rho_{\pi}(x)$ are defined according to the relevant intermediate state as $\rho_{\pi,N}(\vec{x})$ and $\rho_{\pi,\Delta}(\vec{x})$ so that

$$\rho_{\pi}(\vec{x}) = \rho_{\pi,N}(\vec{x}) + \rho_{\pi,\Delta}(\vec{x}) \,. \tag{4.16}$$

After a lengthy evaluation we find

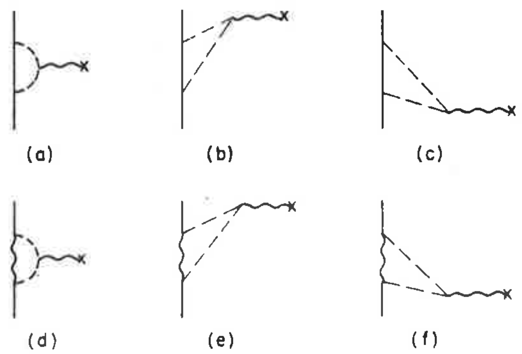

FIG. 7. Photon-pion interactions. The wiggly line with a × at the end is used to describe the photon.

$$\rho_{\pi,N}(\vec{x}) = \frac{4e}{(2\pi)^3 m_\pi^2} f^R_{\pi NN}{}^2 \int\int \frac{d^3k\, d^3k' u(k)u(k')}{\omega_k \omega_{k'}(\omega_k + \omega_{k'})} \vec{k}\cdot\vec{k}' e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} \tag{4.17a}$$

and

$$\rho_{\pi,\Delta}(\vec{x}) = -\frac{e}{(2\pi)^5}\frac{8}{9}\frac{f^R_{\pi\Delta N}{}^2}{m_\pi^2}\frac{d^3k\, d^3k'}{(\omega_\Delta + \omega_k)}\frac{u(k)u(k')}{(\omega_\Delta + \omega_{k'})(\omega_k + \omega_{k'})}\vec{k}\cdot\vec{k}' e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} \tag{4.17b}$$

Only terms of second order in $f^R_{\pi NN}$ have been kept in obtaining (4.17). The renormalized coupling constants of (4.17) are evaluated at $\epsilon = 0$ [see (3.3) and (3.7)]. Using the parameters obtained in Sec. III we compute $\rho_{\pi,N}(\vec{x})$ and $\rho_{\pi,\Delta}(\vec{x})$ and present the results in Fig. 8.

The computation of the nucleon charge density is completed by the specification of the contribution due to the photon-quark interactions. These are shown schematically in Fig. 9.

The electromagnetic current of the quarks, $j^\mu_Q(\vec{x})$, is given by the expression

$$j^\mu_Q(\vec{x}) = \sum_a eQ_a \bar{q}_a(x)\gamma^\mu q_a(x). \tag{4.18}$$

For the moment we are interested in the zeroth component of $j^\mu_Q(\vec{x})$, which is the quark charge density $\rho_Q(\vec{x})$. We have

$$\rho_Q(\vec{x}) = \sum_a eQ_a q_a^\dagger(x) q_a(x). \tag{4.19}$$

The matrix element of ρ_Q between the Δ and nucleon bag states vanishes, so the terms of Figs. 9(d) and 9(e) do not contribute to the quark charge density. The expression for the bag contribution may be further simplified by the observation that all quarks, whether in the nucleon or Δ, have the wave function of (2.3). The use of that expression in (4.19) gives the result

$$\rho_Q(x) = eC\left[j_0^2\left(\frac{\omega x}{R}\right) + j_1^2\left(\frac{\omega x}{R}\right)\right]\theta(R-x), \tag{4.20}$$

where C is obtained from the condition that

$$Q^{p,n} = \int d^3x\, \rho^{p,n}(\vec{x}), \tag{4.21a}$$

with

$$\rho^{p,n}(\vec{x}) = \rho_Q(\vec{x}) \pm \rho_\pi(\vec{x}). \tag{4.21b}$$

The upper (lower) sign of the right-hand side of (4.21b) refers to the proton (neutron). The charge density of the proton, as well as $\rho_Q(r)$, is shown in Fig. 10. The charge density of the neutron is shown in Fig. 11. It must be noted that no corrections for center-of-mass motion have been applied in the results of Figs. 8, 10, and 11.

From the densities of Figs. 10 and 11 we may compute the rms charge radii of the proton and neutron. We find $\langle r_c^2\rangle_p^{1/2} = 0.69$ fm and $|\langle r_c^2\rangle_n^{1/2}| = 0.34$ fm in very good agreement with the experimental values[3,12] of 0.83 and 0.34 fm. The expectation value $\langle r_c^2\rangle_n$ has a negative sign, also in agreement with experiment.

It is worthwhile to estimate the effects of

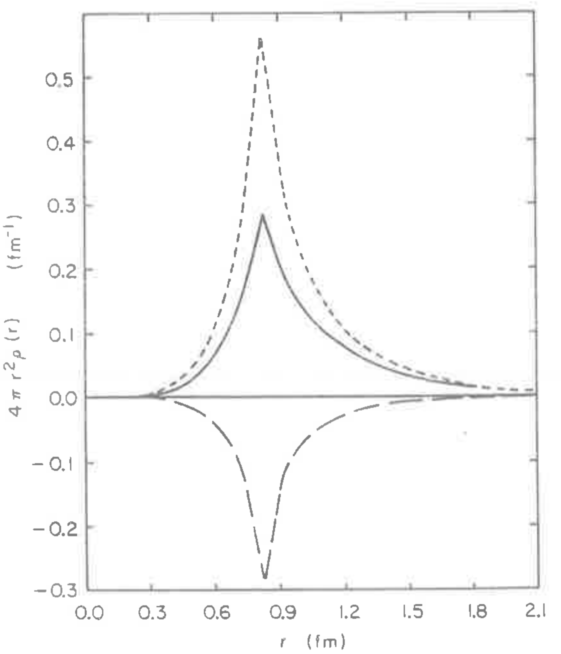

FIG. 8. Pionic contributions to the nucleon charge density. Short dashed curve, $4\pi r^2\rho_{\pi,N}$. Long dashed curve, $4\pi r^2\rho_{\pi,\Delta}$. Solid curve, $4\pi r^2\rho_\pi(r)$.

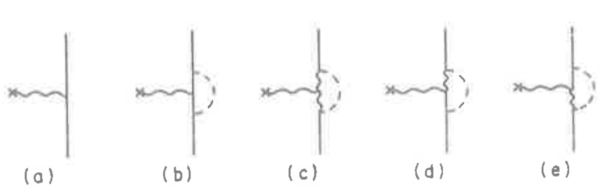

FIG. 9. Photon-quark interactions.

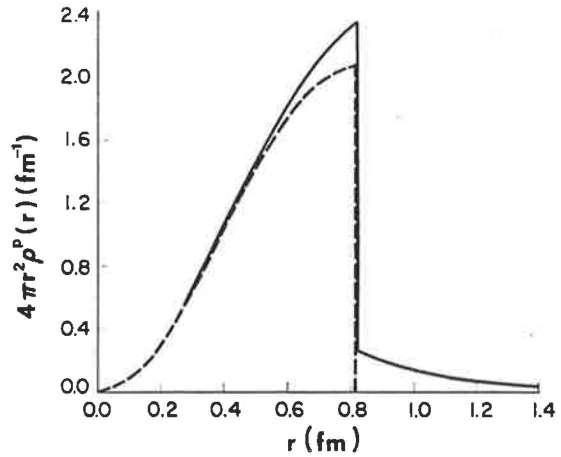

FIG. 10. Proton charge density, solid curve. $\rho_Q(r)$, dashed curve.

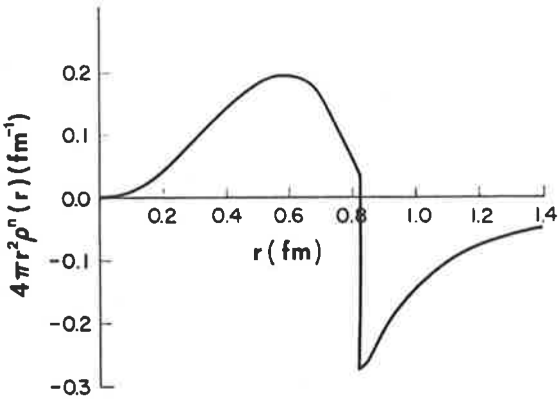

FIG. 11. Neutron charge density.

center-of-mass motion on our results. If we apply the Donoghue-Johnson[11] correction procedure to the calculation of $\langle r_c^2\rangle_{n,p}^{1/2}$ we find small changes: the value of $\langle r_c^2\rangle_p^{1/2}$ is changed from 0.69 to 0.73 fm, and the value of $|\langle r_c^2\rangle_n^{1/2}|$ is changed from 0.34 to 0.36 fm.

The agreement with the low-q^2 behavior of the neutron electric form factor $G_{En}(q^2)$ is significant. Just as in all the old static source theories the process $n\to p\pi^-$ gave rise to a negative tail for the intrinsic neutron charge distribution, so does our model. These early models had, however, one essential problem, namely that the core was not understood, and its properties were incalculable. In our model the core is a simple three-quark bag. Second the interpretation of $G_{En}(q^2)$ was always clouded by the presence of the Darwin-Foldy term, whereby a Dirac particle with an anomalous magnetic moment appears, because of the *Zitterbewegung* to have an intrinsic charge distribution.

In the quark model the photon interacts not with a Dirac nucleon, but with three confined quarks (and the pion in the CBM) and there is no Darwin-Foldy term. Thus the interpretation of $G_{En}(q^2)$ in terms of an intrinsic charge distribution is unambiguous in this model, and the agreement with $\langle r_c^2\rangle_n$ is very significant. Further, if we take seriously the phenomenological fits of $\rho_{\rm ch}^n(r)$ to the admittedly very poor data for $G_{En}(q^2)$, we see that they tend to give the zero in $\rho_{\rm ch}^n(r)$ (where it switches from positive to negative) at radii between 0.7 and 0.9 fm. In the cloudy bag model the pion field is a maximum at the surface of the bag and this switch in sign should occur very close to R, the bag radius, Fig. 11. The qualitative agreement between the experimental value quoted above and the value of 0.82 fm extracted from πN scattering is at the very least a remarkable coincidence. Better data for $G_{En}(q^2)$ are essential.

In the CBM pions penetrate the confinement region, and one might wonder about the fraction λ of $\langle r_c^2\rangle_p$ that results from the contribution pions with $r<R$. To do this define the quantity λ as

$$\lambda=\frac{\int_0^R r^4\rho_\pi(r)dr}{\int_0^\infty r^4\rho_\pi(r)dr}\ . \qquad (4.22)$$

We find that λ is about 15%, so pions inside the bag contribute only a negligible amount to $\langle r_c^2\rangle_p$.

Next we evaluate the magnetic moments of the nucleon. To do this we need the expectation value of $\vec{j}_\pi(\vec{x})$. We define a vector $\vec{J}_\pi(\vec{x})$ so that

$$\tau_3\vec{J}_\pi(\vec{x})=\langle \bar{N}|\vec{j}_\pi(\vec{x})|\bar{N}\rangle\ . \qquad (4.23)$$

The terms contributing to $\vec{J}_\pi(\vec{x})$ are shown in Fig. 7. We define $\vec{j}_{\pi,N}(\vec{x})$ and $\vec{j}_{\pi,\Delta}(\vec{x})$ with the two terms coming from nucleon and Δ intermediate states:

$$\vec{J}_\pi(\vec{x})=\vec{j}_{\pi,N}(\vec{x})+\vec{j}_{\pi,\Delta}(\vec{x})\ . \qquad (4.24)$$

By defining the pion contributions to $F_2^{S,V}(q^2)$ in analogy with (4.9) as

$$F_{2,N}^{V,\pi}(-\vec{q}^{\,2})=-\frac{i(\vec{\sigma}\times\vec{q})}{2\vec{q}^{\,2}}\int d^3x\,\vec{j}_{\pi,N}(\vec{x}) \qquad (4.25a)$$

and

$$F_{2,\Delta}^{V,\pi}(-\vec{q}^{\,2})=-\frac{i(\vec{\sigma}\times\vec{q})}{2\vec{q}^{\,2}}\int d^3x\,\vec{j}_{\pi,\Delta}(\vec{x})\ , \qquad (4.25b)$$

and performing a lengthy manipulation we find

$$F^{V,\pi}_{2,N}(-\vec{q}^2) = \frac{e}{2\pi^2} \frac{f^{R\,2}_{\pi NN}}{m_\pi^2} \int d^3k \, \frac{\sin^2\theta \, k^2 u(k) u(k')}{\omega_k^2 \omega_{k'}^2} \tag{4.26}$$

and

$$F^{V,\pi}_{2,\Delta}(-\vec{q}^2) = +\frac{e}{9} \frac{f^{R\,2}_{\pi N\Delta}}{m_\pi^2} \frac{1}{2\pi^2} \int d^3k \, \frac{k^2 \sin^2\theta \, u(k) u(k')}{\omega_k \omega_k'} \frac{(\omega_\Delta + \omega_k + \omega_{k'})}{(\omega_\Delta + \omega_k)(\omega_\Delta + \omega_{k'})(\omega_k + \omega_{k'})} . \tag{4.27}$$

In (4.26) and (4.27) θ is the angle between $\vec{k}$ and $\vec{q}$, and $\vec{k}' = \vec{k} + \vec{q}$. (There is no contribution to the isoscalar form factors.) Once again the renormalized coupling constant are evaluated at $\epsilon = 0$. The contributions to the magnetic moment are obtained by taking the $|\vec{q}| = 0$ limit of (4.26) and (4.27).

Next we must compute the contribution of the magnetic moment due to the photon-quark interactions. The magnetic-moment operator $\hat{\mu}$ is given by

$$\hat{\mu} = \mu_0 \sum_a U_a^\dagger \sigma_3 Q U_a , \tag{4.28}$$

where U_a is a quark spin-isospin wave function, Q is the quark charge matrix

$$Q = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} . \tag{4.29}$$

and

$$\mu_0 = \frac{R}{12} \frac{1}{\omega^2 - \sin^2\omega} (4\omega - 3 \sin 2\omega + 2\omega \cos 2\omega) . \tag{4.30}$$

Unlike the charge operator, $\hat{\mu}$ can cause transitions between the nucleon and Δ bag states. We obtain the quark contribution to the magnetic moment by a lengthy evaluation of the terms of Fig. 9. The photon-quark contribution to the magnetic moment is defined as $F_2^{p,n}(0, \text{bag})$. The result is

$$\begin{pmatrix} F_2^p(0,\text{bag}) \\ F_2^n(0,\text{bag}) \end{pmatrix} = \mu_0 \left[\begin{pmatrix} 1 \\ -\frac{2}{3} \end{pmatrix} \tilde{Z}_N^{-1} + \frac{1}{27} \begin{pmatrix} 1 \\ -4 \end{pmatrix} P_{N\pi} + \frac{5}{27} \begin{pmatrix} 4 \\ -1 \end{pmatrix} P_{\Delta\pi} + \frac{4}{9} \begin{pmatrix} 1 \\ -1 \end{pmatrix} P_{N\Delta\pi} . \right] \tag{4.31}$$

The quantity $\tilde{Z}_N$ is defined by replacing the unrenormalized coupling constants of (3.4a) by renormalized ones evaluated at $\epsilon = 0$, and has the value 1.55. $P_{N\pi}$ and $P_{\Delta\pi}$ are the probabilities that the physical nucleon have $N\pi$ and $\Delta\pi$ components. These are given by the expressions

$$P_{N\pi} = \frac{3}{\pi} \tilde{Z}_N^{-1} \frac{f^{(R)\,2}_{\pi NN}}{m_\pi^2} \int_0^\infty \frac{k^4 dk \, u^2(k)}{\omega_k^3} \tag{4.32}$$

and

$$P_{\Delta\pi} = \frac{4}{3\pi} \tilde{Z}_N^{-1} \frac{f^{(R)\,2}_{\pi N\Delta}}{m_\pi^2} \int_0^\infty \frac{k^4 dk \, u^2(k)}{\omega_k(\omega_k + \omega_\Delta)^2} . \tag{4.33}$$

For our parameters we find $P_{N\pi} = 0.20$ and $P_{\Delta\pi} = 0.15$. The term $P_{N\Delta\pi}$ comes from the graphs of Figs. 7(d) and 7(e) and has the expression

$$P_{N\Delta\pi} = \frac{8\sqrt{2}}{3\pi} \frac{f^R_{\pi NN} f^R_{\pi N\Delta}}{m_\pi^2} \int_0^\infty \frac{k^4 dk \, u^2(k)}{\omega_k^2(\omega_\Delta + \omega_k)} . \tag{4.34}$$

The magnetic moment of the nucleon is given by the expression

$$\mu^{p,n} = \pm [F^{V,\pi}_{2,N}(0) + F^{V,\pi}_{2,\Delta}(0)] + F_2^{p,n}(0, \text{bag}) , \tag{4.35}$$

where the plus (minus) sign refers to the proton (neutron). The evaluation of (4.31) leads to the values

$$\begin{aligned} \mu_p &= 2.2 e/2m_N , \\ \mu_n &= -1.7 e/2m_N , \end{aligned} \tag{4.36}$$

which are in fairly good agreement with the experimental values of $2.79e/2m_N$ and $-1.91e/2m_N$.

It is worthwhile to examine the effects of correcting for the motion of the center of mass. According to Ref. 11, the static values of μ_p and μ_n are increased by 18%, so that the corrected values of μ_p and μ_n are given by

$$\begin{aligned} \mu_p &= 2.60 \, e/2m_N , \\ \mu_n &= -2.01 e/2m_N . \end{aligned} \tag{4.37}$$

Thus the inclusion of recoil effects substantially improves the agreement with experiment.

A summary of the calculated electromagnetic properties of the nucleon along with a comparison to experiment and the results of the MIT bag model is given in Table II. We stress that these results are obtained in a parameter-free calculation.

Our proton rms charge radius is about the same as that of the MIT bag, even though our bag radius (0.82 fm) is smaller than theirs (1.0 fm). This is due to the presence of positively charged pions outside the bag.

There is no mechanism in the original MIT bag model that gives a nonzero value of the neutron

TABLE II. Static electromagnetic properties of the nucleon. Magnetic moments are given in units of Bohr magnetons.

Quantity	This work	Experiment	MIT bag model[2]
$\langle r_c^2\rangle_p^{1/2}$	0.73 fm	0.83 fm	0.73 fm
$\lvert\langle r_c^2\rangle_n\rvert^{1/2}$	0.36 fm	0.35 fm	0.00 fm
μ_p	2.60	2.79	1.9
μ_n	-2.01	-1.91	-1.2

rms charge radius. In our model the π^- components give a value of $\langle r^2\rangle_n^{1/2}$ in good agreement with experiment. There have been attempts to explain the value of $\langle r^2\rangle_n^{1/2}$ in terms of charge segregation caused by the gluon-exchange interaction.[12] The size of such effects is proportional to α_s and the bag radius R. In our theory both R and α_s (Sec. VI) are smaller than in the MIT model. Hence for our model it is reasonable to expect that such contributions to $\langle r_c^2\rangle_n$ would be fairly small.

The inclusion of the pion cloud and recoil corrections leads to a substantial enhancement of the MIT-bag-model magnetic moment. Indeed the experimental values of μ_n and μ_p are very well reproduced.

It seems that the pion contributions make modest but significant corrections to quantities derived from the MIT bag model. Furthermore, from the relatively small values $P_{N\pi}=0.20$ and $P_{\Delta\pi}=0.15$ we conclude that the nucleon wave function is reliably calculated in lowest order. (A more detailed study of the convergence of our perturbation treatment is in progress, and preliminary results show that the lowest-order treatments we employ are reliable.) Thus the cloudy bag model provides a very reasonable description of the structure of the nucleon and Δ.

V. AXIAL-VECTOR COUPLING CONSTANT g_A

Quark-model calculations of the quantity g_A, which is the ratio of effective strengths of axial-vector and vector currents in nucleon β decay, have been of considerable interest. Evaluations of g_A using the nonrelativistic quark model[13] give $g_A=1.67$ which is significantly larger than the experimental value $g_A^{\exp}=1.24$. Use of the MIT bag model[2] leads to a value of $g_A=1.09$. The reduction can be understood by a consideration of the quark contribution to the axial-vector $\vec{A}_Q^i$:

$$\vec{A}_Q^i(x)=\sum_a \bar{q}_a(x)\vec{\gamma}\gamma_5\tfrac{1}{2}\tau^i q_a(x)\,, \tag{5.1}$$

where

$$\vec{\gamma}\gamma_5=\begin{pmatrix}\vec{\sigma} & 0\\ 0 & -\vec{\sigma}\end{pmatrix}. \tag{5.2}$$

From the quark wave function (2.3) and with $x=2.04$, we note that the lower component of $q_a(x)$ is comparable in magnitude with the upper component. Hence the minus sign in the lower diagonal term of (5.2) leads to a reduction in the calculated value of g_A.

To obtain g_A in our model, observe that our model has a partially conserved axial-vector current $\vec{A}_\mu$ such that

$$\vec{A}_\mu=\sum_a \bar{q}_a(x)\gamma_\mu\gamma_5\tfrac{1}{2}\vec{\tau}q_a(x)-f\partial_\mu\vec{\phi} \tag{5.3}$$

and

$$\partial^\mu\vec{A}_\mu=m_\pi^2 f\vec{\phi}\,. \tag{5.4}$$

[The relation (5.4) is valid even if one uses the approximations discussed in Sec. II.] It is well known that using (5.4) and standard techniques of taking various matrix elements between physical states one may derive the Goldberger-Treiman relation. In our notation we have

$$g_A=\frac{2f}{m_\pi}\sqrt{4\pi}f_{\pi NN}^{(R)}=\frac{2f}{m_\pi}\sqrt{4\pi}\,f_{\pi NN}\frac{V_N(\epsilon=0)}{Z_N}\,, \tag{5.5}$$

where the quantity in parentheses is the renormalized pion-nucleon coupling constant. The result (5.5) can also be obtained from the direct evaluation of the matrix element of $e^{i\vec{q}\cdot\vec{x}}A_\mu(x)$ for small $\vec{q}$. Using our value of $f_{\pi NN}$ ($f_{\pi NN}^2=0.078$) (which is consistent, within the error[14] in the Goldberger-Treiman[15] relationship, with our theory) and Eqs. (3.4) and (3.5) we find

$$g_A=1.19 \tag{5.6}$$

in excellent agreement with the experimental value of 1.24.

If the Donoghue-Johnson procedure for center-of-mass corrections is applied to our result for g_A we find

$$g_A=1.33\,, \tag{5.7}$$

which is also in good agreement with the experimental value of 1.24.

Before concluding this section we compare our results with the calculation of Jaffe.[6] In a classical lowest-order calculation he finds that pionic effects increase the value of g_A (1.09, as obtained in a bag calculation) by a factor of $\frac{3}{2}$. Our value for g_A is smaller then Jaffe's because pions enter the bag and also because of renormalization

TABLE III. Values of g_A.

Source	g_A
Experiment	1.24
Nonrelativistic quark model	1.67
MIT bag model	1.09
Jaffe	1.63
This work	1.33

effects.

The values of g_A discussed in this section are presented in Table III.

VI. GLUON-EXCHANGE CONTRIBUTION TO ω_Δ

The difference between the physical masses of the Δ and nucleon has been determined from our fit to scattering data in the (3,3)-resonance region to be 280 MeV. In the MIT bag work of DeGrand *et al.*,[3] this splitting is entirely ascribed to the difference between the one-gluon-exchange contribution, Fig. 12 to the Δ and nucleon masses. With a bag radius of 1.0 fm DeGrand *et al.* find that the strong coupling constant $\alpha_s = 0.55$, a value that is somewhat too large compared with more recent determinations[16] and the expectation that perturbative treatments of QCD are valid at short distances. The one-gluon-exchange energy is expected to be a perturbative correction to the bag energy and the large value of α may not be consistent with this expectation. In this work there is an additional contribution to ω_Δ, namely the difference between the pion self-energies of the Δ ($\bar{\Sigma}_\Delta$) and nucleon ($\bar{\Sigma}_N$). Hence the extracted value of α_s is reduced.

If one assumes that the entire contribution to ω_Δ comes from pionic and one-gluon-exchange effects one may write

$$\omega_\Delta = \mathrm{Re}\,\bar{\Sigma}_\Delta - \bar{\Sigma}_N + \omega_\Delta^{\mathrm{QCD}}, \qquad (6.1)$$

where $\omega_\Delta^{\mathrm{QCD}}$ is the mass splitting caused by gluon exchange. The quantities $\bar{\Sigma}_\Delta$ and $\bar{\Sigma}_N$ are defined by replacing the unrenormalized coupling constants of (3.4a) and (3.9) by renormalized ones evaluated at the energies ω_Δ for $\bar{\Sigma}_\Delta$ and zero for $\bar{\Sigma}_n$. Using the parameters obtained in Sec. III, and Eqs. (3.4c) and (3.9) we find $\omega_\Delta^{\mathrm{QCD}} = 200$ MeV. The parameter $\omega_\Delta^{\mathrm{QCD}}$ is well determined; slight shifts in R and $f_{\pi NN}$, which change the calculated phase shifts by modest amounts, do not change $\omega_\Delta^{\mathrm{QCD}}$. The corresponding value of $\omega_\Delta^{\mathrm{QCD}}$ used by DeGrand *et al.* is 300 MeV. Using the fact that $\omega_\Delta^{\mathrm{QCD}}$ is proportional to α_s and inversely proportional to R we find

$$\alpha_s = \frac{200}{300} \times \left(\frac{0.82\ \mathrm{fm}}{1.0\ \mathrm{fm}}\right) \times 0.55 \qquad (6.2)$$

or

$$\alpha_s = 0.30\,. \qquad (6.3)$$

The result (6.3) is in better agreement with the idea that QCD effects in the bag can be treated in a perturbative manner.

The smaller value of α_s has important implications for bag-model calculations of the nucleon-nucleon (NN) force. In DeTar's work[17] the color-electrostatic interaction between quarks leads to a minimum in the value of the deformation energy of about -200 MeV which occurs at an internucleon separation of about 1 fm. This is a feature not found in phenomenological NN forces. However, this attraction is very sensitive to the value of α_s, and the use of $\alpha_s = 0.30$ would very significantly reduce the magnitude of the calculated attraction.[17]

VII. CONCLUSIONS

In the cloudy bag model the baryon is described as three quarks in a bag surrounded by a pion cloud. There is only one uncalculated parameter in our treatment of the field equations. This is the bag radius R, which is determined from π-nucleon scattering to be about 0.82 fm. With this radius the calculated electromagnetic properties (at zero momentum transfer) are in very good agreement with the experimental ones. The computed value of the axial-vector coupling constant g_A is also in good agreement with experiment. Thus, the cloudy bag model provides a very good description of the static properties of the nucleon and Δ.[18]

Still to be answered is the question of whether the cloudy bag model provides a good description of the energy spectrum of the baryons. In this

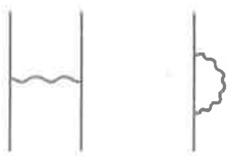

FIG. 12. Gluon contributions to the nucleon self-energy. Here the wiggly line describes a gluon, and the solid lines the quarks.

model the energy splitting due to pion contributions to baryonic mass splitting essentially replaces the splittings (of the MIT bag model) caused by gluon exchanges. Very good agreement with the data has been achieved in the MIT bag model,[2] and it will be interesting to see if similar results can be obtained in the cloudy bag model.

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APPENDIX

The CBM equations presented in Ref. 8 (CBM 1) were sufficient to guarantee PCAC (partial conservation of axial-vector current), but even in the limit $m_\pi \to 0$ were not exactly chirally symmetric. As discussed in many places,[6,19] exact chiral symmetry usually implies considerable nonlinearity in both the transformation on the pion field, and in the Lagrangian density. For completeness we shall give the full, nonlinear Lagrangian density and field equations in this appendix. We observe, however, that from the point of view adopted in CBM 1, and also in this paper, where only terms of order ϕ are retained in actual calculation, the formalism presented here changes nothing.

The lack of exact chiral symmetry in the Lagrangian density (2.8) of CBM 1 under the chiral transformation

$$q(x) \to q(x) + \frac{i}{2}\vec{\tau}\cdot\vec{\epsilon}\,\gamma_5 q(x)\,, \tag{A1}$$

$$\vec{\phi} \to \vec{\phi} - f\vec{\epsilon} \tag{A2}$$

comes from the bag surface term. In fact, under the transformation (A1) and (A2) we find

$$\mathcal{L}_{CBM}(x) \to \mathcal{L}_{CBM}(x) + \frac{1}{2f}\sum_a \bar{q}_a(x) i\gamma_5 j_1(\phi/f) \times \vec{\tau}\cdot\vec{\phi}(x) \times [\vec{\epsilon}\times\hat{\phi}(x)] q_a(x)\,\Delta_s, \tag{A3}$$

where

$$\phi = |\vec{\phi}|\,; \quad \hat{\phi} = \vec{\phi}/\phi\,, \tag{A4}$$

and Δ_s is a surface δ function. This problem can be cured by making a more complicated transformation on $\vec{\phi}(x)$, namely

$$\vec{\phi} \to \vec{\phi} - f\vec{\epsilon} + f[1 - (\phi/f)\cot(\phi/f)]\hat{\phi}\times(\vec{\epsilon}\times\hat{\phi})\,. \tag{A5}$$

Having cured the problem with the surface term, we have now introduced difficulties with the pion kinetic energy term. In order to cure that we must finally introduce the covariant derivative

$$D_\mu\vec{\phi} = \partial_\mu\vec{\phi} - [1 - j_0(\phi/f)]\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\,. \tag{A6}$$

It is a difficult, but instructional algebraic exercise to prove that the new Lagrangian density

$$\mathcal{L}'_{CBM}(x) = \left[\frac{i}{2}\sum_a \bar{q}_a(x)\overleftrightarrow{\not\partial} q_a(x) - B\right]\theta_V - \frac{1}{2}\sum_a \bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)\,\Delta_s + \tfrac{1}{2}(D_\mu\vec{\phi})^2 \tag{A7}$$

is invariant under the chiral transformations (A1) and (A5). This invariance is of course associated with a conserved axial-vector current ($\vec{A}^\mu$) which one can calculate in the canonical way

$$\vec{A}^\mu = \tfrac{1}{2}\sum_a \bar{q}_a\gamma^\mu\gamma_5\vec{\tau} q_a\theta_V - f\hat{\phi}(\partial^\mu\phi) - f\frac{\sin(2\phi/f)}{2\phi/f}\hat{\phi}\times(\partial^\mu\vec{\phi}\times\hat{\phi})\,. \tag{A8}$$

Finally we write down the field equations which follow when one demands that the action associated with the Lagrangian density (A7) be invariant under arbitrary variations in the quark and pion fields and under variations along the normal to the bag surface:

$$i\not\partial q_a(x) = 0, \quad x\in V\,, \tag{A9}$$

$$i\gamma\cdot n q_a(x) = e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x), \quad x\in S\,, \tag{A10}$$

$$B = -\tfrac{1}{2} n\cdot\partial\sum_a [\bar{q}_a(x) e^{i\vec{\tau}\cdot\vec{\phi}(x)\gamma_5/f} q_a(x)], \quad x\in S\,, \tag{A11}$$

$$\partial^2\vec{\phi}(x) - \partial_\mu\left[\left(1 - \frac{\sin(2\phi/f)}{2\phi/f}\right)\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\right] = -\frac{i}{2f}\sum_a \bar{q}_a\gamma_5\left[\cos(\phi/f)\hat{\phi}\times(\vec{\tau}\times\hat{\phi}) + \frac{i\gamma_5}{f}\vec{\phi}\cos(\phi/f) + \frac{\vec{\tau}\cdot\vec{\phi}}{f}\hat{\phi}\frac{\cos(\phi/f)}{\tan(\phi/f)}\right] q_a\Delta_s, \quad \forall x\,. \tag{A12}$$

These equations are very closely related to those of Jaffe, but we do not exclude the pion field from the bag at this stage.

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Cloudy bag model: Convergent perturbation expansion for the nucleon

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A previously published bound on the probability of finding n pions in the dressed nucleon in Chew-Low theory is improved. The proof is then extended to the recently derived cloudy-bag-model Hamiltonian. Together with a bound on the average number of pions (0.9 ± 1.0), our result strongly suggests a rapid convergence of the perturbation expansion in the cloudy bag model.

I. INTRODUCTION

The concept of chiral symmetry has been of great importance in elementary-particle physics for many years.[1-3] In the context of massless quarks and quantum chromodynamics it is, of course, an exact symmetry, and should survive the proof of confinement in some way. It is not surprising, therefore, that immediately after the presentation of the original MIT bag model[4] Chodos and Thorn[5] attempted to repair its obvious violation of chiral symmetry. Their method relied on introducing massless, elementary σ and $\vec{\pi}$ fields which coupled to the quarks, only at the bag surface, in such a way as to restore exact chiral symmetry.

In the past year or so, interest in this problem has been dramatically revived. For a longer review of these developments we refer to the discussion of Ref. 6, but a few brief comments will be useful here. Brown and collaborators have argued that the $\vec{\pi}$ field is actually a crucial aspect of the confinement process for the nucleon.[7] That is, chiral symmetry should be manifest in the Wigner-Weyl mode inside (no pions), and in the Nambu-Goldstone mode outside (the pion is the Goldstone boson).[3] In their purely classical model this external pion exerts a large pressure on the bag, resulting in a confinement volume of a few tenths of a fermi for the nucleon. They argue further that such a picture (the "little bag") would be more consistent with classical nuclear physics.

On the other hand, Jaffe[8] and others[9] have developed classical models of a bag surrounded by a pion field which merely acts as a small perturbation on the usual MIT ground state. Once again the pion field appears as a Goldstone boson, excluded from the interior of the bag. Recent work by Johnson has also shown the importance of collective $q\bar{q}$ excitations in the volume about the MIT bag.[10] However, the phenomenological replacement of such excitations by a pion field has not yet been clarified.

At the same time as these developments using a classical pion field were taking place, the TRIUMF–University of Washington group has constructed a quantized version of the theory—the "cloudy bag model" (CBM).[11-13] In order to avoid technical problems, the CBM (like that of Chodos and Thorn) allows the pion field to penetrate the bag. By working only to lowest order in the pion field, which is assumed to be small, it was possible to derive a Hamiltonian (see Sec. II) describing an interacting system of (bare) nucleons, deltas, and pions. In Ref. 12, hereafter CBM-1, this Hamiltonian was used to settle the longstanding problem of the nature of the (3, 3) resonance. In CBM-2 (Ref. 13) this model has been used, with considerable success, to calculate pionic corrections to the MIT model of the nucleon (g_A, magnetic moment, and charge radii).

In the CBM work the contribution from the pion field inside the bag was rather small, and could be justified as a crude approximation to the effect of virtual $q\bar{q}$ pairs inside the bag. Indeed it is just this point which was made recently by DeTar.[14,15] His work provides some formal link between the theory of Jaffe[8] with the pion excluded from the bag and the CBM Hamiltonian, which he also used in a calculation of nucleon properties.

There is a great deal of interesting physics in these developments, but for our present purpose we note only that all groups, except Stony Brook, rely on a perturbative treatment of pionic effects. This perturbative treatment has two aspects. First, the exponential coupling at the bag surface

$\bar{q}\exp(i\vec{\tau}\cdot\vec{\phi}\gamma_5/f)q$ is replaced by $\bar{q}(1+i\vec{\tau}\cdot\vec{\phi}\gamma_5/f)q$, and the covariant derivative of the pion field reduces to a normal derivative $(D_\mu\vec{\phi}\to\partial_\mu\vec{\phi})$. Second, the resulting linear Hamiltonian describing the coupling of a pion to a baryon is used only in lower-order perturbation theory to obtain the pionic corrections for nucleon observables. In this paper we shall only address the second aspect of this problem.

In his classical treatment of the problem Jaffe extracted a parameter ϵ, related to the strength of the pion field at the bag surface $[\epsilon=g_A/(8\pi f^2R^2)]$, which should be small if perturbation theory is to work. For the usual MIT parameters his ϵ is quite small, but it certainly is not small for the "little bag". In the calculations using a quantized pion field, that is the CBM Hamiltonian, only the one- and two- pion terms have been retained.

Until now there has been no rigorous proof of convergence in any of these calculations. This paper takes the CBM Hamiltonian as given, and provides such a proof. Of course, if the $\Delta N\pi$ coupling were omitted from this model it would be identical in form to the static Chew-Low model[16,17] for the πN system For that model a great deal of formal work has been done to establish convergence properties. For example, Alvarez-Estrada[18-20] has proven rigorously that the perturbation expansion of the physical nucleon state in the Chew-Low model does converge, in the sense that a rigorous least upper bound (LUB) P_n can be placed on the probability of finding n pions in it. P_n does tend to zero as n goes to infinity, but the *convergence is very slow*. For example, his LUB on P_n is not a useful limit, that is, it is not less than one, until $n=5$.

Henley and Thirring were aware of this problem[21]: "For a long time it has been one of the main goals of meson theory to analyze the physical nucleon in terms of the bare nucleon and its surrounding meson cloud. This problem led into a dead-end road.... The reason is that the ... resonant state of the nucleon is not important for the ground state." It is exactly on this point that the CBM has something new to say. As stressed in CBM-1, the quark model has an elementary Δ which carries most of the strength of the P_{33} scattering. Therefore, we do not need such a large bare coupling constant, or such a high cutoff momentum.[13] Consequently, one is led to hope that the theory may be more convergent.

In this paper we first improve the original bound of Alvarez-Estrada (for the Chew-Low model) by a factor of 4, corresponding to the spin-isospin degeneracy of the nucleon. The proof is also generalized to the CBM Hamiltonian by extending the space of bare baryon states. For the parameters of the CBM (Refs. 12 and 13)—or indeed any reasonable parameters near those of the MIT bag model—this leads to a remarkable proof of convergence of the perturbation expansion of the dressed nucleon state. Indeed we find that the probability of finding three pions about the (bag) core of the nucleon is strictly less than 12%. In view of the weakness of the bound, the real probability is almost certainly a factor (2–3) smaller. Even more impressive is the bound and standard deviation on the mean number of pions in the physical nucleon. For the CBM (Ref. 13) these numbers are 0.9 and 1.03, in comparison with the Chew-Low values of 2.16 and 2.22, respectively.

This rapid convergence of perturbation theory for strong interactions is a novel feature of the CBM. It comes about because of the large size of the pion source. As we point out in the final section, this rapid convergence has important consequences not only for the calculation of nucleon properties[13,15] and the N-N force,[22] but also for such exotic questions as the proposed tests of grand unified theories in the search for proton decay.

II. THE CLOUDY-BAG-MODEL HAMILTONIAN

The Hamiltonian of the cloudy bag model (CBM) of Ref. 12 takes the form

$$H=H_0+H_I, \tag{2.1a}$$

$$H_0=\sum_\alpha m_\alpha N_\alpha^\dagger N_\alpha+\sum_k \omega_k a_k^\dagger a_k, \tag{2.1b}$$

and

$$H_I=\sum_{\alpha,\beta,k}(v_k^{\alpha\beta}N_\alpha^\dagger N_\beta a_k+\text{H.c.}). \tag{2.1c}$$

Here N_α $(N_\alpha^\dagger)$ are annihilation (creation) operators for the static baryon bag states $|\alpha\rangle$ of bare mass m_α. In our application the states $|\alpha\rangle$ include the single-particle states $|n,s,t\rangle$ of the nucleon with spin $s_n=\frac{1}{2}$ and isospin $t_n=\frac{1}{2}$, and the single-particle states $|\Delta st\rangle$ of the Δ with spin $s_\Delta=\frac{3}{2}$ and isospin $t_\Delta=\frac{3}{2}$. The labels s and t are the spin and isospin projections, respectively. The sum over the index k represents the integration over the momentum $\vec{k}$ and the sum over isospin projections j of the pion,

$$\sum_k\equiv\sum_{j=1,2,3}\int\frac{d^3k}{(2\pi)^3}\,.$$

Since there is no renormalization of the pion in the theory, the rest mass μ and the bare mass of the pion are identical, and the pion energies in Eq. (2.1b) are given by

$$\omega_k=(\vec{k}^2+\mu^2)^{1/2}\,.$$

The interaction Hamiltonian (2.1c) allows transitions between a nucleon and a Δ with the emission or absorption of a pion. An important feature of the cloudy bag model is that the interaction matrix elements $v^{\alpha\beta}$ are highly constrained by the underlying quark structure of the baryons, and are determined by a single coupling strength f and a single form factor $u(kR)$. Explicitly,

$$v_k = \frac{i\mathfrak{F} u(kR)}{\mu(2\pi)^{3/2}(2\omega_k)^{1/2}} T_j \vec{\mathrm{S}}\cdot\vec{\mathrm{k}}\,, \tag{2.2a}$$

with

$$\mathfrak{F} \equiv \begin{pmatrix} \mathfrak{F}^{nn} & \mathfrak{F}^{n\Delta} \\ \mathfrak{F}^{\Delta n} & \mathfrak{F}^{\Delta\Delta}\end{pmatrix} = \tfrac{6}{5} f \begin{pmatrix} 5 & +4\sqrt{2} \\ 4\sqrt{2} & 10 \end{pmatrix} \tag{2.2b}$$

and

$$u(kR) = \frac{3j_1(kR)}{kR}\ . \tag{2.2c}$$

The parameter R is the bag radius and j_1 is a spherical Bessel function of order one.

The Hermitian transition spin operator $\vec{\mathrm{S}}$ of Eq. (2.2a), which acts in the spin subspace of the baryon states, is defined by

$$\langle s_\alpha s|\vec{\mathrm{S}}\cdot\vec{\mathrm{k}}|s_\beta s'\rangle = \sum_m (-1)^{1/2-s}\begin{pmatrix} s_\alpha & s_\beta & 1 \\ -s & s' & m\end{pmatrix} k_m\,. \tag{2.3}$$

Here, k_m is the spherical component of $\vec{\mathrm{k}}$, and the $3j$ symbol couples the spins s_α and s_β (nucleons or Δ's) to the angular momentum of the pion. The isospin transition operator T_j, coupling the isospins of α, β, and the pion, is defined similarly.[23]

The standard Hamiltonian of the Chew theory may be recovered from Eqs. (2.1) by taking f as the unrenormalized pseudovector coupling constant[16,17] and restricting the sum over α to the nucleon states only or, alternatively, by setting

$$\mathfrak{F}^{n\Delta} = \mathfrak{F}^{\Delta n} = \mathfrak{F}^{\Delta\Delta} = 0\,. \tag{2.4}$$

The transition operators for n-n transitions are proportional to the usual Pauli spin and isospin operators

$$\sqrt{6}\ \vec{\mathrm{S}} = \vec{\sigma}, \quad \sqrt{6}\ \vec{\mathrm{T}} = \vec{\tau}\,, \tag{2.5}$$

so in this case we obtain the standard interaction of the Chew-Low theory,

$$v_k^{nn} = \frac{if}{\mu}\,\frac{u(k)}{(2\pi)^{3/2}(2\omega_k)^{1/2}}\,\tau_j\vec{\sigma}\cdot\vec{\mathrm{k}}\,. \tag{2.6}$$

III. BOUNDS

The physical nucleon of mass $\tilde{m}$ with isospin projection t and spin projection s is described in the model by a state $|\tilde{n}st\rangle \equiv |\tilde{n}\rangle$ which is a solution of [12,16,17]

$$H|\tilde{n}\rangle = \tilde{m}|\tilde{n}\rangle\,. \tag{3.1}$$

Since the interaction (2.1c) conserves baryon number, the expansion of this state in terms of the eigenstates of the bare Hamiltonian (2.1b) may be restricted to states containing a single baryon $|\alpha\rangle$ and arbitrary numbers of the field quanta:

$$|\tilde{n}st\rangle = Z_2^{1/2}|nst\rangle + \sum_{r=1}^{\infty}\sum_\alpha \sum_{k_1\ldots k_r} c_r(\alpha;k_1\ldots k_r;\tilde{n}st) \times \frac{1}{(r!)^{1/2}} a^\dagger_{k_1}a^\dagger_{k_2}\cdots a^\dagger_{k_r}|\alpha\rangle\,, \tag{3.2}$$

with

$$\delta_{s's}\delta_{t't}Z_2^{1/2} = \langle nst|\tilde{n}s't'\rangle \tag{3.3}$$

and

$$c_r = \frac{1}{(r!)^{1/2}}\langle\alpha|a_{k_r}a_{k_{r-1}}\cdots a_1|\tilde{n}st\rangle\,. \tag{3.4}$$

The bare Δ (with no pions) does not appear on the right-hand-side of (3.2), since it has a different total spin and isospin from the nucleon,

$$\langle\Delta st|\tilde{n}st\rangle = 0\,. \tag{3.5}$$

The matrix element c_r is the probability amplitude for finding r pions with momenta $\vec{\mathrm{k}}_1$, $\vec{\mathrm{k}}_2$, $\ldots$, $\vec{\mathrm{k}}_r$ and isospin projections j_1, j_2, $\ldots$, j_r surrounding the bag state α (either a nucleon or a Δ with spin s' and isospin t', depending on the index α) in the physical nucleon with spin s and isospin t.

The probability of finding r pions of any momenta and isospin surrounding the bag state α is then

$$\eta_r^\alpha = \sum_{k_1\ldots k_r}|c_r(\alpha;k_1\ldots k_r;\tilde{n}st)|^2\,. \tag{3.6}$$

The normalization condition from Eqs. (3.2), (3.4), and (3.6) is

$$\langle\tilde{n}|\tilde{n}\rangle = Z_2 + \sum_{r=1}^{\infty}\sum_\alpha \eta_r^\alpha = 1\,. \tag{3.7}$$

The probability of finding r pions in and around the cloudy bag is then

$$P_r = \sum_\alpha \eta_r^\alpha \tag{3.8}$$

In order to construct bounds on P_r, it is useful to define[18-20] a state $|\phi_r\rangle$ by removing r pions of prescribed momenta and isospin from the physical nucleon,

$$|\phi_r\rangle = \frac{1}{(r!)^{1/2}}a_{k_1}\cdots a_{k_r}|\tilde{n}st\rangle\,. \tag{3.9}$$

Then, from Eq. (3.4), $c_r = \langle\alpha|\phi_r\rangle$ and

$$P_r = \sum_\alpha \eta_r^\alpha = \sum_\alpha\sum_{k_1\ldots k_r}\langle\phi_r|\alpha\rangle\langle\alpha|\phi_r\rangle\,. \tag{3.10}$$

Interchanging the sums over pion states and bag states, and using the completeness of the state $|\alpha\rangle$ in the single-baryon subspace, we have

$$p_r = \sum_{k_1 \ldots k_r} \langle \phi_r | 0\rangle \langle 0 | \phi_r\rangle \leq \sum_{k_1 \ldots k_r} \langle \phi_r | \phi_r\rangle = \sum_{k_1 \ldots k_r} \|\phi_r\|^2 \qquad (3.11)$$

where $|0\rangle\langle 0|$ is the projector for the pion vacuum times the unit operator in the baryon subspace.

Our aim is to find simple, explicit expressions for $\|\phi_r\|$ in order to place upper bounds on the probabilities P_r using Eq. (3.11). First, consider ϕ_1. The identity

$$|\phi_1\rangle = a_{k_1}|\bar{n}\rangle = \frac{1}{\bar{m}_n - \omega_{k_1} - H}[a_{k_1}, H_I]|\bar{n}\rangle \qquad (3.12)$$

is easily established using Eq. (3.1) and the relation $[H_0, a_k] = -\omega_k a_k$. For brevity let us introduce the notation $k_l \equiv l$, and denote the commutator in Eq. (3.12) by $C_1 = [a_{k_1}, H_I]$. For the particular interaction (2.1c), we have

$$C_1 = \sum_{\alpha\beta} N_\beta^\dagger v_k^{\alpha\beta\dagger} N_\alpha . \qquad (3.13)$$

By applying a_2 to Eq. (3.12) and using the identity

$$a_k \frac{1}{z-H} = \frac{1}{z-\omega_k - H} a_k + \frac{1}{z-\omega_k-H} C_k \frac{1}{z-H} , \qquad (3.14)$$

we find that

$$\begin{aligned} a_2 a_1 |\bar{n}\rangle &= a_2 \frac{1}{\bar{m}_n - \omega_1 - H} C_1 |\bar{n}\rangle \\ &= \frac{1}{\bar{m}_n - \omega_1 - \omega_2 - H}\left(C_1 a_2 + C_2 \frac{1}{\bar{m}_n - \omega_1 - H} C_1\right)|\bar{n}\rangle \\ &= \frac{1}{\bar{m}_n - \omega_1 - \omega_2 - H}\left(C_1 \frac{1}{\bar{m}_n - \omega_2 - H} C_2 + C_2 \frac{1}{\bar{m}_n - \omega_1 - H} C_1\right)|\bar{n}\rangle = \sqrt{2}\,|\phi_2\rangle . \end{aligned} \qquad (3.15)$$

Repeated application of the identity (3.15) yields the following result: Let $\gamma_1\gamma_2\ldots\gamma_r$ be an arbitrary permutation of 1, 2, ... r, then

$$|\phi_r\rangle = \frac{1}{(r!)^{1/2}} \frac{1}{\bar{m}_n - \sum_{l=1}^r \omega_l - H} \sum_{\gamma_1 \ldots \gamma_r} C_{\gamma_1} \frac{1}{\bar{m}_n - \sum_{l=1, l\neq\gamma_1}^r \omega_l - H} \times C_{\gamma_2} \frac{1}{\bar{m}_n - \sum_{l=1, l\neq\gamma_1,\gamma_2}^r \omega_l - H} C_{\gamma_3} \cdots \frac{1}{m_n - \omega_{\gamma_r} - H} C_{\gamma_r}|\bar{n}\rangle . \qquad (3.16)$$

Taking norms throughout Eq. (3.16), we have our key result:

$$\frac{\|\phi_r\|}{\||\bar{n}\rangle\|} \leq \frac{1}{(r!)^{1/2}} \frac{1}{\sum_{l=1}^r \omega_l} \sum_{\gamma_1 \ldots \gamma_r} \frac{1}{\sum_{l=1, l\neq\gamma_1}^r \omega_l \sum_{l=1, l\neq\gamma_1,\gamma_2}^r \omega_l} \cdots \frac{1}{\omega_{\gamma_r}} \|C_{\gamma_1}\| \, \|C_{\gamma_2}\| \cdots \|C_{\gamma_r}\|$$

$$= \frac{1}{(r!)^{1/2}} \prod_{l=1}^r \frac{\|C_l\|}{\omega_l} \qquad (3.17)$$

In deriving (3.17), we have assumed that the spectrum of the total Hamiltonian H begins at $\bar{m}_n$, the physical mass of the nucleon, so that for any $\omega > 0$ the inequality

$$\left\| \frac{1}{\bar{m}_n - \omega - H} \right\| \leq \frac{1}{\omega} \qquad (3.18)$$

holds.

From the inequalities (3.11) and (3.17), we obtain the central result of this section: The probability of finding r pions in the pion cloud is bounded by

$$P_r \leq \frac{\Lambda^r}{r!} , \qquad (3.19)$$

where

$$\Lambda = \sum_k \frac{\|C_k\|^2}{\omega_k^2} . \qquad (3.20)$$

Consequently an upper bound for the mean number of pions present in the pion cloud is

$$\langle r\rangle = \sum_r r P_r \leq \Lambda e^{\Lambda} . \qquad (3.21)$$

A much tighter bound on the mean number of pions present is given by considering the expectation value of the number operator directly,

$$\langle r\rangle = \frac{1}{\|\,|\bar{n}\rangle\,\|^2}\left\langle \bar{n}\left|\sum_k a_k^\dagger a_k\right|\bar{n}\right\rangle$$
$$= \sum_k \frac{\|\phi_k\|^2}{\|\,|\bar{n}\rangle\|^2} \leq \Lambda . \qquad (3.22)$$

It is shown in Appendix A that the uncertainty in the number of pions in the cloud $(\Delta r)^2 = \langle r^2\rangle - \langle r\rangle^2$ is bounded by

$$\Delta r \leq \left(\Lambda^2 + \tfrac{1}{4}\right)^{1/2} . \qquad (3.23)$$

With the specific interaction of the CBM, Eq. (2.2), we find that

$$\Lambda = \tfrac{57}{25} f^2 I(R) , \qquad (3.24)$$

where

$$I(R) = \frac{3}{\mu^2 (2\pi)^2}\int_0^\infty \frac{k^4 u(kR)^2}{\omega_k^{\,3}}\, dk . \qquad (3.25)$$

Some details of the evaluation of Λ are presented in Appendix B.

IV. NUMERICAL RESULTS AND DISCUSSION

First let us consider the case where the Δ is excluded from the single-particle space, i.e., the system is described by the Chew Hamiltonian (2.6). With the coupling matrix (2.4) Eq. (B4) of Appendix B gives

$$\Lambda = f^2 I(R) . \qquad (4.1)$$

[A bound of the same form was derived originally by Alvarez-Estrada.[18-20] However, Λ of (4.1) is improved by a factor of 4—through the use of completeness in Eq. (3.11)—corresponding to the spin and isospin degeneracy of the nucleon.]

We also note that Λ of Eq. (4.1) occurs in the expression for the probability Z_2 of Eq. (3.3) when it is evaluated to second order in perturbation theory. In the Chew-Low theory, unlike the cloudy bag model, the functional form of the factor $u(kR)$ is not well determined, and often a simple step function with a cutoff $k = k_{max}$ is adopted. As described by Henley and Thirring, analysis of experimental data leads (with some ambiguity) to values of about $f^2/4\pi = 0.22$ (compared with $f^2/4\pi = 0.08$ for the renormalized coupling constant) and $k_{max} \sim 5\mu$. For these values $\Lambda = 2.16$, which, according to Eq. (3.22), is also a bound on the mean number of pions in the nucleon. The corresponding bounds on the probabilities P_r are limited to small values only for $r > 6$, and the uncertainty in the number of pions present in the cloud is from Eq. (3.23) bounded by 2.22. Also listed in Table I are numerical results for the Lorentzian form factor $u(k) \sim \xi^2/(\xi^2 + k^2)$ used by Fubini and Thirring.[24]

For the Chew Hamiltonian our bounds give no reason to expect that perturbation theory is valid for the values of the coupling constant and form factors required by experiment. Defining a dimensionless parameter λ by $\lambda = k_{max}/\mu$ we find $I(R) \approx \lambda^2/2$ for reasonable form factors, and if we take from Eq. (4.1) $\lambda f < 1$ as the criterion for the validity of perturbation theory, the unrenormalized coupling constant is restricted to values

$$f < \mu R , \qquad (4.2)$$

where $R = k_{max}^{-1}$. In the Chew-Low theory the "radius" of the nucleon is small, $R \approx 0.28$ fm, and the

TABLE I. Upper bounds for the probabilities P_r of finding r pions surrounding the nucleon in the Chew-Low and cloudy bag models. The column headed $\langle r\rangle$ gives the mean value of the number of pions present, calculated using Eq. (3.22), while the column headed Δr lists the uncertainties in pion numbers, Eq. (3.23). The values a for the Chew-Low model were calculated with a step-function form factor, and for case b a Lorentzian form factor was used. In the cloudy bag model, the results labeled c correspond to the values of the coupling constant f and bag radius R determined in Ref. 13. Those labeled d are constrained, as in Ref. 13 (Théberge *et al.*) to yield in perturbation theory the renormalized value $f_r^2/4\pi = 0.08$.

Theory		$f^2/4\pi$	R (fm)	$I(R)$	P_1	P_2	P_3	P_4	P_5	P_6	$\langle r\rangle$	Δr
Chew-Low	a	0.22	0.28	9.80	2.16	2.33	1.67	0.90	0.39	0.14	2.16	2.22
	b	0.22	0.30	9.03	1.99	1.97	1.31	0.65	0.26	0.09	1.99	2.05
CBM	c	0.078	0.82	5.04	0.90	0.40	0.12	0.03	0.005		0.90	1.03
	d	0.113	0.6	10.44	2.69	3.62	2.73	1.55	0.71	0.27	2.69	2.74
	d	0.109	0.7	7.16	1.78	1.58	0.94	0.56	0.20	0.06	1.78	1.85
	d	0.103	0.8	5.26	1.24	0.76	0.31	0.10	0.024	0.005	1.24	1.34
	d	0.100	0.9	3.98	0.91	0.41	0.12	0.03	0.005	0.001	0.91	1.04
	d	0.096	1.0	3.08	0.68	0.23	0.05	0.009	0.001		0.68	0.85
	d	0.093	1.1	2.43	0.52	0.14	0.023	0.003			0.52	0.72

unrenormalized coupling constant f far too large to satisfy the criterion (4.2).

Turning now to the cloudy bag model, we note that there is an additional factor $\frac{57}{25}$ in Eq. (3.23) due to the presence of the Δ, which tends to increase the bound. However, in the CBM, smaller values of the unrenormalized coupling constant f are needed to fit the observed quantities. In the calculations of Ref. 13, it was found that, with the form factor (2.2c) fixed by the bag model, there was very little freedom in fitting the P_{33} phase shift through the Δ resonance. A bag radius of 0.82 fm and a coupling constant $f^2/4\pi = 0.078$ were determined. Evaluating Λ for these values and the form factor (2.2c) we find that the mean number of pions present in the nucleon is bounded by $\Lambda = 0.9$, and P_r tends to zero quite rapidly, the root-mean-square fluctuation in the pion number being bounded by 1.03. The renormalized coupling constant calculated with these values of f and R, using perturbation theory, is $f_r^2/4\pi = 0.071$, somewhat less than the accepted value of $f_r^2/4\pi = 0.080$.

In Table I under the entries labeled d, we have also listed bounds for the values of f and R taken in the perturbative calculations of Théberge *et al.*[13] of the static properties of the nucleon in the CBM. Here, f and R are constrained to produce the value $f_r^2/4\pi = 0.080$ for the renormalized coupling constant.

It is seen from Table I that for reasonable values of the bag radius $R \approx 0.9$ fm, the use of perturbation theory, or other approximate methods which truncate the number of pions in the pion cloud, is much more acceptable in the case of the CBM than in the Chew-Low theory. The criterion (4.2) is much closer to being satisfied, and the bound on the mean number of pions in the cloud is about unity.

Our bounds are simple, but quite crude, and probably overestimate P_r significantly. The probability of finding one pion in the physical nucleon takes the value 35% in the perturbation calculations of Ref. 13. This value may be compared with our bound of 0.9.

V. CONCLUSION

Within the framework of a static source theory, we have established an improved, rigorous bound on the probability of finding the physical nucleon to contain n pions. For the recently developed Hamiltonian of the cloudy bag model, this bound goes rapidly to zero as n goes to three or more pions. In this model the mean number of pions about the nucleon is less than about 0.9, and the standard deviation is less than 1.0. This represents a remarkable improvement in convergence over earlier models such as the Chew-Low model—essentially because of the inclusion of the bare Δ isobar in the CBM.

It is certainly true that the calculation of pionic corrections to nucleon properties such as magnetic moments and charge radii is more complicated than simple probabilities. This is because of the interference between amplitudes with different numbers of pions. Thus, even though the probability of finding three pions is very small, it is conceivable that the three-pion terms could alter the calculations of Refs. 13 and 15 at a noticeable level. Nevertheless, the convergence properties of the CBM seem to be so good that we do not expect any major change in their conclusions.

Not only do our results give great support to the perturbative approach to single-baryon properties, but one may hope for new insight in several other areas. For example, one might now expect to make progress in the understanding of the long- and intermediate-range N-N force using similar techniques.[22] We might also mention the proposed tests of the various grand unified theories.[25] In particular, there are many experiments under way which look for proton decay modes, such as $p \to e^+\pi^0$. With few exceptions (e.g., Ref. 26), the assumption is usually made that the nucleon consists of just three quarks, two of which annihilate to an antiquark and a lepton. If the dressed nucleon actually had a cloud of pions like that in the Chew-Low model, the theoretical predictions based on the three-quark picture would be quite unreliable, because of the dominance of multipion decay modes. However, within the CBM our bounds strongly suggest that decays to a lepton and one or two pions will dominate. Detailed calculations on this problem would be very useful.

Our purpose in this paper has been to put bounds on the pion content of the dressed nucleon, within the framework of the linearized equations (2.1). This is a worthwhile exercise in itself, in view of the interest in such Hamiltonians in low- and medium-energy nuclear physics. However, we did remark in the Introduction that Eqs. (2.1) are an approximation to a highly nonlinear, exactly chiral-symmetric theory.[8,13] Unlike the truncated version discussed here that theory is not renormalizable, and discussions of it (e.g., the nonlinear σ model) usually rely on the tree approximation. It is worth observing though, that the reason for this problem is the treatment of the pion as an elementary, pointlike object. Our underlying motivation for introducing the pion is that one expects in the limit of exact SU(2) × SU(2) symmetry, that the pion should appear as a massless Goldstone boson associated with the dynamical breaking of the symmetry of the vacuum. Once the pion has some internal structure the pion sector of the theory will have a natural cutoff too, and one might expect a fairly rapid truncation of the higher-order terms (in f^{-1}), required formally for exact chiral symmetry. Thus, although our results may at first appear to be of somewhat limited interest because they rely on a linearized version of the equations, they may be rather close to reality.

In conclusion we must remark that the convergence of this expansion in number of pions is essential to the internal consistency of the CBM. At present, the internal structure of the pion is ignored in our model, and therefore we should only expect to describe the long-range piece of the pion field about the nucleon—that is, the one- and two-pion pieces. By the time we get to three or more pions we are probing phenomena within one- or two-tenths of a fermi of the bag surface—where the bag model itself, and particularly the static cavity approximation, is probably unrealistic.

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APPENDIX A

The expectation value of the square of the number operator for the pion field is related to the vector $|\phi_2\rangle$ of Eq. (3.9) in the following way:

$$\left\langle \bar{n} \left| \left(\sum_k a_k^\dagger a_k \right)^2 \right| \bar{n} \right\rangle = \left\langle \bar{n} \left| \sum_{kk'} a_k^\dagger a_{k'}^\dagger a_k a_{k'} \right| \bar{n} \right\rangle + \left\langle \bar{n} \left| \sum_k a_k^\dagger a_k \right| \bar{n} \right\rangle = 2 \sum_{kk'} \langle \phi_2 | \phi_2 \rangle + \langle \bar{n} | \bar{n} \rangle \langle r \rangle , \tag{A1}$$

where we have used the commutator $[a_k^\dagger, a_{k'}] = \delta_{kk'}$ and $\langle r \rangle$ is the mean number of pions present.

Consequently for $(\Delta r)^2 = \langle r^2 \rangle - \langle r \rangle^2$, we have the expression

$$(\Delta r)^2 = 2 \sum_{kk'} \frac{\|\phi_2\|^2}{\| |\bar{n}\rangle \|^2} + \langle r \rangle - \langle r \rangle^2 . \tag{A2}$$

Since the maximum value of $\langle r \rangle - \langle r \rangle^2$ is $\frac{1}{4}$, and from Eq. (3.17) and definition (3.20),

$$2 \sum_{kk'} \frac{\|\phi_2\|^2}{\| |\bar{n}\rangle \|^2} \leq \Lambda^2 , \tag{A3}$$

the uncertainty in the number of pions in the cloud is bounded by

$$\Delta r \leq (\Lambda^2 + \tfrac{1}{4})^{1/2} . \tag{A4}$$

APPENDIX B: EVALUATION OF Λ

To evaluate Λ, we seek the maximum value of the magnitude of the vector $C_k|\psi\rangle$, C_k given by Eq. (3.13), as the normalized vector $|\psi\rangle$ ranges over the complete single-particle space, i.e., if $|\psi\rangle = \sum_\alpha d_\alpha |\alpha\rangle$ and $\sum_\alpha |d_\alpha|^2 = 1$, the expansion coefficients d_α must be chosen to maximize the quantity

$$\sum_k \frac{\rho_k^2}{\omega_k^2} = \sum_k \frac{1}{\omega_k^2} \sum_\gamma \left| \sum_\alpha d_\alpha \langle \gamma | v_k^{\gamma\alpha\dagger} | \alpha \rangle \right|^2 . \tag{B1}$$

After introducing spin and isospin labels explicitly by setting $|\alpha\rangle = |\alpha s t\rangle$, $|\beta\rangle = |\beta s'' t''\rangle$, and $|\gamma\rangle = |\gamma s' t'\rangle$, and substituting the interaction of Eq. (2.2a), expression (B1) becomes

$$\sum_k \frac{\rho_k^2}{\omega_k^2} = \sum_j \int d^3k \frac{u(kR)^2}{\mu^2 (2\pi)^3 2\omega_k^3} \sum_{\substack{\alpha s t \\ \beta s'' t''}} d_{\alpha s t}^* d_{\beta s'' t''} \sum_\gamma \mathfrak{F}^{\alpha\gamma} \mathfrak{F}^{\dagger\gamma\alpha} \sum_{s'} \langle s_\alpha s | \vec{S} \cdot \vec{k} | s_\gamma s' \rangle \langle s_\gamma s' | \vec{S} \cdot \vec{k} | s_\beta s'' \rangle \times \sum_{t'} \langle t_\alpha t | T_j | t_\gamma t' \rangle \langle t_\gamma t' | T_j | t_\beta t'' \rangle . \tag{B2}$$

The evaluation is simplified by integrating over the angles of $\vec{k}$ first, using

$$\int k_i k_j d\Omega_k = 4\pi/3 \delta_{ij} k^2 .$$

The spin and isospin sums may then be performed with the help of the identities

$$\sum_i \sum_{s'} \langle s_\alpha s | S_i | s_\gamma s' \rangle \langle s_\gamma s' | S_i | s_\beta s'' \rangle = \delta_{s' s''} \delta_{\alpha\gamma} (2s_\alpha + 1)^{-1}$$

and

$$\sum_j \sum_{t'} \langle t_\alpha t | T_j | t_\gamma t' \rangle \langle t_\gamma t' | T_j | t_\beta t'' \rangle = \delta_{t' t''} \delta_{\alpha\gamma} (2t_\beta + 1)^{-1} , \tag{B3}$$

which follow from the definition (2.3) of the spin and isospin transition operators. The result is

$$\sum_k \frac{\rho_k^2}{\omega_k^2} \sum_{\alpha s t} \frac{|d_{\alpha st}|^2}{2(2\pi)3\mu^2} \frac{1}{(2s_\alpha+1)(2t_\alpha+1)} \times \sum_\gamma \mathfrak{F}^{\alpha\gamma}\mathfrak{F}^{\dagger\gamma\alpha} \frac{4\pi}{3} \int \frac{k^4}{\omega_k^3} u(kR)^2 dk \,. \quad (B4)$$

With the specific values of the coupling matrix of Eq. (2.2b), we find the maximum value of (4.4) is attained when $d_{\Delta st}=0$, giving the results, Eqs. (3.24) and (3.25), of the text.

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New source of charge-symmetry violation in the nucleon-nucleon system

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Within the framework of the cloudy bag model we show that the small difference of up- and down-quark masses leads to a small increase in the $\pi^0 nn$ coupling constant relative to $\pi^0 pp$. There may be many experimental consequences of this observation but, as an example, $|a_{nn}| - |a_{pp}| \cong 0.3$ fm because of this effect.

In the past two of three years there has been great interest in the so-called chiral bag models.[1-8] Essentially the idea is that whereas chiral $SU(2) \times SU(2)$ is one of the best symmetries of the strong interaction,[9] most phenomenological models of quark confinement violate it rather badly. The MIT bag model is a very useful case study, because the whole source of chiral-symmetry violation is localized on the bag surface. By introducing a pion field (at the first, crude stage elementary, and no longer describable in a bag model) into the theory, it is possible to compensate for the surface term so that the $SU(2) \times SU(2)$ symmetry is restored—at least in the absence of quark and pion mass terms.

The cloudy bag model (CBM) is one theory of this kind which has proven rather successful in a number of applications.[3-5] It is a basic assumption of the CBM that pionic corrections can be calculated as a relatively small perturbation on the original MIT model. Thus it should make sense to expand the full Lagrangian density in powers of $\vec{\phi}$ (the pion field). This leads to expressions for the pion coupling to bare nucleons, Δ's, and so on. Once the truncation in powers of $\vec{\phi}$ has been made the theory is completely renormalizable, and the renormalizations are, moreover, finite and small. We refer to Refs. 10, 11, and 5 for discussion of the rigorous bounds on the probability of finding n pions in the physical nucleon ($\langle n \rangle < 0.9$), and for calculations of nucleon electromagnetic properties. In the latter case there is a significant improvement over the original MIT model,[12,13] for both the magnetic moments and the neutron charge radius.

At the same time as these developments involving the substructure of the nucleon have been going on, there has been a great deal of activity, both experimental and theoretical, on the question of the violation of charge symmetry in the nucleon-nucleon system. The classical system in which this has been studied a great deal is the ${}^1S_0$ scattering length for nn compared with pp (Coulomb corrected). Currently the best experimental values, -17.1 ± 0.2 fm (Ref. 14) and -18.6 ± 0.6 fm (Ref. 15), respectively, indicate a small violation of charge symmetry. However, there is considerable discussion of the interpretation of the errors quoted.

A recent experiment at LAMPF failed to see a charge-symmetry-violating forward-backward asymmetry in the reaction $np \to d\pi^0$ at the level of 0.5%.[16] Complex experiments under way at Indiana and TRIUMF hope to see a small difference in the position of the zero in P and A in np elastic scattering, which should provide the most sensitive test to date.

On the theoretical side, apart from electromagnetic effects which are relatively straightforward, most calculations of charge-symmetry violation are based on boson-exchange models of the nucleon-nucleon force—e.g., ρ-ω and π-η mixing. An excellent summary of the present experimental and theoretical situation is to be found in the proceedings of the workshop held at TRIUMF earlier this year[17]; see also Ref. 18.

In the absence of a theory which connects the nucleon-nucleon force with the substructure of the nucleon, there has never been an estimate of the direct effects on NN scattering of the violation of $SU(2) \times SU(2)$ at the quark level. In particular, in order to explain the mass splittings within multiplets such as the nucleon, Σ, Ξ, Δ, and so on, one expects a splitting of the u- and d-quark masses.[19] Within the framework of the bag model we recently obtained a value of $(m_d - m_u)$ of 4–5 MeV.[20] Of course it is common to calculate the effects of n-p mass differences (an indirect inclusion of effects of $m_u \neq m_d$).

What we shall now show is that the π^0 coupling constant to the neutron is about 0.4% bigger than that to the proton as a consequence of these small quark mass differences.

In the original formulation of the CBM, in the case where $SU(2)\times SU(2)$ is still an exact symmetry, the Lagrangian density has the form[4]

$$\mathcal{L}_{CBM}(x)=(i\bar{q}_0\partial\!\!\!/ q_0-B)\theta_v-\tfrac{1}{2}\bar{q}_0 e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/f}q_0\Delta_s+\tfrac{1}{2}(D_\mu\phi)^2\ . \quad (1)$$

Here q_0 is the two-isospin-component (u and d) quark field, B is the phenomenological energy density associated with making the bag, and $\vec{\phi}$ is the pion field. The bag volume is defined by θ_v, and Δ_s is a surface δ function [$\theta_v=\theta(R-r)$ and $\Delta_s=\delta(r-R)$ in the static, spherical case]. The parameter f is readily identified as the pion decay constant (93 MeV). Finally, $D_\mu\phi$ is the appropriate covariant derivative, so that the whole Lagrangian density is invariant under the chiral tranformation

$$q_0\rightarrow q_0+\frac{i}{2}\vec{\tau}\cdot\vec{\epsilon}\gamma_5 q_0,\quad \bar{q}_0\rightarrow\bar{q}_0+\frac{i}{2}\bar{q}_0\vec{\tau}\cdot\vec{\epsilon}\gamma_5\ , \quad (2)$$

$$\vec{\phi}\rightarrow\vec{\phi}-\vec{\epsilon}f+f\left[1-\left(\frac{\phi}{f}\right)\cot\left(\frac{\phi}{f}\right)\right]\hat{\phi}\times(\vec{\epsilon}\times\hat{\phi})\ ; \quad (3)$$

that is

$$D_\mu\vec{\phi}=\partial_\mu\vec{\phi}-\left[1-j_0\left(\frac{\phi}{f}\right)\right]\hat{\phi}\times(\partial_\mu\vec{\phi}\times\hat{\phi})\ . \quad (4)$$

Very recently it has been discovered[21] that by making a transformation on the quark fields one obtains a Lagrangian density in which the current-algebra constraints are somewhat easier to see. In fact, defining

$$q=Sq_0,\quad \bar{q}=\bar{q}_0 S\ , \quad (5)$$

with

$$S=e^{i\vec{\tau}\cdot\vec{\phi}\gamma_5/2f}\ , \quad (6)$$

the Lagrangian density becomes

$$\mathcal{L}'(x)=(iq\not{D}q-B)\theta_v-\tfrac{1}{2}\bar{q}q\Delta_s+\tfrac{1}{2}(D_\mu\phi)^2+\frac{1}{2f}\bar{q}\gamma^\mu\gamma_5\vec{\tau}q\cdot(D_\mu\vec{\phi})\theta_v\ , \quad (7)$$

where the covariant derivative on the (new) quark fields is

$$\not{D}q=\partial\!\!\!/ q-i\left[\frac{\cos(\phi/f)-1}{2}\right]\vec{\tau}\cdot(\hat{\phi}\times\partial\!\!\!/\hat{\phi})q\ . \quad (8)$$

A full discussion of Eq. (7) can be found elsewhere,[..] but some remarks are necessary here. First, the covariant derivative on the quark fields [Eq. (8)] leads to the Weinberg-Tomozawa relation for low-energy pion scattering from any hadron describable in the bag model. Second, the pion coupling to the bag is a derivative coupling—once again in agreement with current algebra. (For the present time we avoid a discussion of exactly how derivative coupling to the quarks translates into coupling to the hadrons.)

Of course it is true that the original quark fields q_0 are not equivalent to the new q. What the transformation has in fact done is to change from the simple representation of $SU(2)\times SU(2)$ shown in Eq. (2), to a new, nonlinear representation in which chiral transformations mix quark states with different numbers of pions. (In this representation $\bar{q}q$ is, in fact, chiral invariant.) The new representation is preferable in many ways; for example, the Lagrangian density (7) decomposes much more readily as $\mathcal{L}_{MIT}+\mathcal{L}_\pi+\mathcal{L}_{interaction}$ (as used in earlier work). Moreover, the key elements which make Eq. (7) chiral invariant, namely, $\partial\!\!\!/ q\rightarrow\not{D}q$ and derivative coupling, can be used for *any* model of quark confinement—they are not specific to the bag model.

In order to obtain an $NN\pi$ vertex from Eq. (7), we follow the earlier procedure[4] of defining a P space (of three-quark baryons) and a Q space (the rest), and expanding to lowest order in ϕ. The appropriate $NN\pi$ interaction vertex is then simply the matrix element of the derivative-coupling term in Eq. (7) between MIT-bag nucleons. Clearly the $NN\pi$ coupling constant (at $k=0$) is $(1/2f)$ times the expectation value of $\gamma^\mu\gamma_5\vec{\tau}$ in the nucleon bag. That is simply $(g_A^{bag}/2f)$, where g_A^{bag} is the MIT-bag-model value of the axial-vector coupling constant. Since it was shown in Refs. 5 and 11 that in the truncated theory (higher powers of ϕ dropped) renormalization of coupling constants is small ($\sim$10% for $NN\pi$), we shall not consider this further. (As we have pointed out explicitly in Ref. 5 the large surface correction to g_A found by Jaffe[2] and others is not present in the CBM, because the pion field is not excluded from the bag volume.)

Of course the enormous improvement in the prediction of g_A (1.09 for massless quarks) in comparison with nonrelativistic quark models ($\frac{5}{3}$) was one of the great successes of the MIT bag model. (With the recoil correction of Donoghue and Johnson[23] the bag value becomes 1.22 for massless quarks.) The reason for the suppression of g_A is the presence of small Dirac components in the quark wave functions. Obvioulsy, as the quark mass gets larger this suppression is less effective and g_A goes up.

The result promised earlier, namely, $f_{\pi^0 nn}>f_{\pi^0 pp}$, is now trivially obtained. The d quark is 5 MeV more massive than the u, and hence, for neutral-current coupling,

$$\frac{g_A^n}{g_A^p}=1+\tfrac{3}{5}\delta \ , \tag{9}$$

where the ratio of the appropriate spatial matrix elements for single up and down quarks is $(1-\delta)$. Using the equations of Donoghue *et al.*,[13] $(m_d-m_u)=5$ MeV (Ref. 20) implies $\delta=6.4\times10^{-3}$, and hence g_A^n/g_A^p is 0.4% greater than one.

At the present level of accuracy of neutral-current experiments it is not possible to imagine seeing such a small effect. For the future one may hope. We prefer to examine the effects on pion production through the relationship $\sqrt{4\pi}\,f_{NN\pi}/m_\pi=g_A^{\text{bag}}/2f$, noted above. This leads to a prediction that the $nn\pi^0$ coupling constant is 0.4% greater than that for $pp\pi^0$. In order to estimate the significance of this charge-symmetry violation, we have calculated the implied difference in nn and pp scattering lengths. For the Bryan-Gersten potential[24] (model D) this leads to $|a_{nn}|-|a_{pp}^{\text{no Coul}}|=+0.3$ fm, which is in the same direction as the experiment, although a little small. (Recall, however, that the errors may be somewhat low.) Of course, we must add the caution that this is only one possible source of charge-symmetry violation in the low-energy N-N system.[17,25]

In summary, we stress that the source of charge-symmetry violation shown here is new. It is a long-range effect, which should survive in even the most recent models of N-N scattering which take into account the large size of the nucleon bag.[26] (A pessimist might doubt whether ρ-ω and π-η mixing will survive, since they are of short range and the bags overlap at 1.5–2.0 fm.) The effect of this violation should be seen at an appropriate level in many systems. In particular we think of different widths for Δ decay to π^0, of forward-backward asymmetry in $np\rightarrow d\pi^0$ (which may be enhanced in polarization measurements), and so on.

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The cloudy bag model. IV. Pionic corrections to the nucleon properties

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A detailed formulation of the Hamiltonian formalism, together with a consistent renormalization procedure, is described for the cloudy bag model. The electromagnetic properties of the nucleon are calculated with center-of-mass corrections included. Good agreement with the experimental results is obtained for bag radii ranging from 0.8 to 1.0 fm.

Le formalisme Hamiltonien et la procédure de renormalisation sont décrits pour le modèle du "bag pionique". Les propriétés électromagnétiques du nucléon sont calculées, et ce en incluant les corrections du centre de masse. D'excellents resultats sont obtenus pour des rayons du bag variant de 0,8 à 1,0 fm.

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I. Introduction

The cloudy bag model (CBM) (1–3) is a phenomenological model which incorporates most of the known features of the nucleon. In this model, the nucleon consists of a spherical static cavity of radius R filled with three massless free quarks. A vacuum pressure B, first introduced by the MIT group (4–6) allows the pressure of the quark field on the surface to be balanced, and then enables the system to reach an equilibrium position. Around and inside the nucleon circulates a cloud of pions, moving freely everywhere except at the bag surface where they can be emitted or absorbed.

This emission and absorption process of pions is related to the chiral symmetry property of the CBM Lagrangian density. In the absence of pions, the CBM is identical to the MIT bag model, which violates chiral symmetry. This follows from the necessary reflection of the quarks on the bag surface, which flips their helicity. However, by introducing a pion field coupled to the quarks in a specific way on the bag surface, one can restore chiral symmetry and generate a conserved axial current $A^{\mu}(x)$ (7–10). Experimentally, $A^{\mu}(x)$ is not exactly conserved, and hence chiral symmetry should be violated. We do this by adding a pion mass term to our Lagrangian density which then generates the well known PCAC relation (11).

This paper is intended to give the reader a complete description of the Hamiltonian formalism in the CBM, and this with a detailed formulation of the renormalization procedure. The bag radius dependence of this renormalization procedure and of the various nucleon electromagnetic properties, is also studied in order to derive a consistent picture of the nucleon.

In Sect. II of this paper, we write down the Lagrangian density of the cloudy bag model. We derive from it the equations of motion, which reflect the confinement of massless free quarks, the conservation of energy–momentum, and the Klein–Gordon equation for the pion field with a source term on the bag surface. Finally, the conserved vector current $J^{\mu}(x)$ and the partially conserved axial current $A^{\mu}(x)$ are derived from Noether's theorem.

The Hamiltonian formalism of the theory is derived in Sect. III. We use the MIT solution for the quark field inside the bag cavity, and quantize the pion field $\boldsymbol{\phi}(x)$ in momentum space to obtain a Hamiltonian quark operator. Being concerned primarily with $SU(2)$ flavour (up and down quarks), we project this Hamiltonian quark operator onto the space of bare colourless baryon bags: the nucleons and the deltas. This results in a free Hamiltonian H_0, and a 3-point interaction Hamiltonian H_I which couples these baryons to the pion field. The internal dynamics of the quarks appear in H_I as a form factor $u(kR)$. The Hamiltonian theory then obtained is in fact a modern version of the Chew–Low theory of the nucleon. "The cut-off factor in the static approximation, which is equivalent to spreading out the region of the pion–nucleon interaction in space, may have a real physical significance", says Chew in his 1954 paper (12). This statement is now understandable in the context of the cloudy bag model.

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In Sect. IV we are concerned with renormalization. The physical nucleon has a probability Z_2^N to be a bare (no pions around) three quark nucleon bag, a probability $P_{N\pi}$ to be a nucleon bag core surrounded with one pion, etc. The presence of the pion in the model obliges us to take into account self-energies (mass renormalization) and higher order corrections to the coupling constants (vertex renormalization). These are done by requiring that the renormalized nucleon mass be the physical nucleon mass, and the renormalized πNN coupling constant, f_r^2, be the experimental πNN coupling constant. This renormalization procedure to order f_r^2 is convergent for $R \gtrsim 0.7$ fm.

Section V is concerned with the electromagnetic properties of the nucleon. Here, the photon current can couple either to the quarks inside the bag or to the pion field. We develop formal expressions to order f_r^2 for the magnetic moments, charge radii, and electric form factors. A simple model for the center-of-mass corrections is also described, and we finally state all these results as a function of the bag radius R, which is in fact the only parameter of the theory. Very good agreement with experimental results is obtained for bag radii of 0.8 to 1.0 fm. These values are consistent with our previous bag radius of 0.82 fm obtained from pion–nucleon scattering calculations.

II. The Lagrangian density

In the cloudy bag model (CBM), the Lagrangian density is defined in the following way:

$$\mathcal{L}(x) = \left[\sum_{a=1}^{3} \frac{i}{2}\bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/}q_a(x) - B\right]\theta_v - \frac{1}{2}\sum_{a=1}^{3}\bar{q}_a(x)\exp(i\boldsymbol{\tau}\cdot\boldsymbol{\phi}(x)\gamma_5/f_{0\pi})\,q_a(x)\Delta_s + \frac{1}{2}(D_\mu\boldsymbol{\phi}(x))\cdot(D^\mu\boldsymbol{\phi}(x)) - \frac{1}{2}m_{0\pi}^2\boldsymbol{\phi}(x)\cdot\boldsymbol{\phi}(x) \tag{2.1}$$

where D_μ is the covariant derivative defined by

$$D_\mu\boldsymbol{\phi} = \partial_\mu\boldsymbol{\phi} - [1 - j_0(\phi/f_{0\pi})]\hat{\boldsymbol{\phi}} \times (\partial_\mu\boldsymbol{\phi} \times \hat{\boldsymbol{\phi}}) \tag{2.2}$$

In this equation, $q_a(x)$ is the quark field of colour index a; $\boldsymbol{\phi}(x)$ is the isovector pseudoscalar pion field ($\phi = |\boldsymbol{\phi}|$, $\hat{\boldsymbol{\phi}} = \boldsymbol{\phi}/\phi$); $m_{0\pi}$ is the bare mass of the pion; $f_{0\pi}$ is closely related to the pion decay constant f_π; B is the energy density of the vacuum; θ_v is a step function equal to one inside the bag volume and zero outside; Δ_s is a surface delta function and finally, $j_0(x)$ is the spherical Bessel function of order zero.

By varying the quark fields, the pion field and the bag volume, and requiring that the corresponding variation of the action vanish, we obtain the Euler–Lagrange equations of motion. The first is

$$i\partial\!\!\!/ q_a(x) = 0, \qquad x \in V \tag{2.3}$$

This is the Dirac equation for the free quarks inside the confining volume V. It is in accord with the Bjorken scaling phenomenon (13) which tells us that at short distances, the nucleon behaves as if it were made of massless free quarks. In QCD this is obtained from the asymptotic freedom behaviour of the theory (14). The second equation is

$$i\gamma\cdot n q_a(x) = \exp(i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5/f_{0\pi})\,q_a(x), \qquad x \in S \tag{2.4}$$

This boundary condition guarantees that the quarks remain confined inside the bag. By using this equation and its conjugate, we see that there is no flux of quark current through the surface

$$n\cdot J_a(x) = \bar{q}_a(x)\gamma\cdot n q_a(x) = 0, \qquad x \in S \tag{2.5}$$

There is a second (nonlinear) boundary condition,

$$B = -\frac{1}{2}n\cdot\partial\left[\sum_{a=1}^{3}\bar{q}_a(x)\exp(i\boldsymbol{\tau}\cdot\boldsymbol{\phi}\gamma_5/f_{0\pi})\,q_a(x)\right], \qquad x \in S \tag{2.6}$$

which incorporates energy–momentum conservation in the theory. Finally we have the pion field equation,

$$(\partial^2 + m_{0\pi}^2)\boldsymbol{\phi}(x) = \partial_\mu\left[(1 - j_0(2\phi/f_{0\pi}))\hat{\boldsymbol{\phi}} \times (\partial_\mu\boldsymbol{\phi} \times \hat{\boldsymbol{\phi}})\right] - (i/2f_{0\pi})\sum_{a=1}^{3}\bar{q}_a\gamma_5\Big[\cos(\phi/f_{0\pi})\hat{\boldsymbol{\phi}} \times (\boldsymbol{\tau}\times\hat{\boldsymbol{\phi}}) + i\gamma_5(\boldsymbol{\phi}/f_{0\pi})\cos(\phi/f_{0\pi}) + \hat{\boldsymbol{\phi}}(\boldsymbol{\tau}\cdot\boldsymbol{\phi}/f_{0\pi})\cos(\phi/f_{0\pi})\cot(\phi/f_{0\pi})\Big]q_a\Delta_s \tag{2.7}$$

which, when expanded to order $\phi(x)$, has the form

$$[2.8] \quad (\partial^2 + m_{0\pi}^2)\boldsymbol{\phi}(x) = -(i/2f_{0\pi})\sum_{a=1}^{3}\bar{q}_a(x)\gamma_5\boldsymbol{\tau}q_a(x)\Delta_s + (\boldsymbol{\phi}/2f_{0\pi}^2)\sum_{a=1}^{3}\bar{q}_a(x)q_a(x)\Delta_s$$

This linearized version is the Klein–Gordon equation for the pion field with a source term on the surface of the bag.

From the Lagrangian density [2.1], one can derive two important conserved currents. The first one is the usual vector current $J_a^\mu(x)$ associated with each quark, and obtained from our Lagrangian density by the infinitesimal global transformation:

$$[2.9] \quad q_a(x) \to q_a(x) + i\epsilon_a q_a(x); \qquad \boldsymbol{\phi}(x) \to \boldsymbol{\phi}(x)$$

The conserved vector current is

$$[2.10] \quad J_a^\mu(x) = \bar{q}_a(x)\gamma^\mu q_a(x)\theta_v$$

For a massless pion field, the theory also contains a conserved axial vector current $A^\mu(x)$. It is obtained from the following infinitesimal transformation

$$[2.11a] \quad q_a(x) \to q_a(x) + \frac{i}{2}\boldsymbol{\tau}\cdot\boldsymbol{\epsilon}\gamma_5 q_a(x)$$

$$[2.11b] \quad \boldsymbol{\phi}(x) \to \boldsymbol{\phi}(x) - f_{0\pi}\boldsymbol{\epsilon} + f_{0\pi}[1 - (\phi/f_{0\pi})\cot(\phi/f_{0\pi})]\hat{\boldsymbol{\phi}} \times (\boldsymbol{\epsilon}\times\hat{\boldsymbol{\phi}})]$$

The corresponding conserved axial current is

$$[2.12] \quad A^\mu(x) = \frac{1}{2}\sum_{a=1}^{3}\bar{q}_a\gamma^\mu\gamma^5\boldsymbol{\tau}q_a\theta_v - f_{0\pi}(\hat{\boldsymbol{\phi}}\cdot\partial^\mu\boldsymbol{\phi})\hat{\boldsymbol{\phi}} - f_{0\pi}j_0(2\phi/f_{0\pi})\hat{\boldsymbol{\phi}}\times(\partial^\mu\boldsymbol{\phi}\times\hat{\boldsymbol{\phi}})$$

which can be expanded to first order in $\boldsymbol{\phi}(x)$,

$$[2.13] \quad A^\mu(x) \cong \frac{1}{2}\sum_{a=1}^{3}\bar{q}_a\gamma^\mu\gamma^5\boldsymbol{\tau}q_a\theta_v - f_{0\pi}\partial^\mu\boldsymbol{\phi}$$

Now, if we leave the pion mass term in the Lagrangian density, the axial vector current $A_\mu(x)$ is no longer conserved, and this results in the relation

$$[2.14] \quad \partial_\mu A^\mu(x) = f_{0\pi}m_{0\pi}^2\boldsymbol{\phi}(x)$$

which we identify with the PCAC relation

$$[2.15] \quad \partial_\mu A^\mu(x) = f_\pi m_\pi^2\boldsymbol{\phi}(x)$$

where f_π is the pion decay constant and m_π is the physical mass of the pion. For later use, we notice that

$$[2.16] \quad f_{0\pi} = f_\pi\left(\frac{m_\pi^2}{m_{0\pi}^2}\right)$$

It is in fact through the PCAC relation [2.15] that one can identify the $\boldsymbol{\phi}$ field introduced earlier as being the pion field.

III. The Hamiltonian

In the absence of the pion field, the CBM Lagrangian density and the equations of motion are reduced to the MIT ones:

$$[3.1] \quad \mathscr{L}_{\mathrm{MIT}}(x) = \left[\frac{i}{2}\sum_{a=1}^{3}\bar{q}_a(x)\overleftrightarrow{\partial\!\!\!/}q_a(x) - B\right]\theta_v - \frac{1}{2}\sum_{a=1}^{3}\bar{q}_a(x)q_a(x)\Delta_s$$

$$[3.2] \quad i\partial\!\!\!/ q_a(x) = 0, \qquad x\in V$$

$$[3.3] \quad i\gamma\cdot n q_a(x) = q_a(x), \qquad x\in S$$

$$[3.4] \quad B = -\frac{1}{2}n\cdot\partial\left[\sum_{a=1}^{3}\bar{q}_a(x)q_a(x)\right], \qquad x\in S$$

These equations can be solved exactly for a static spherical bare bag (no pion cloud around the bag) of radius R:

$$[3.5]\quad q_a(r) = \left[\frac{\Omega^2}{4\pi R^3(1-j_0^2(\Omega))}\right]^{1/2}\begin{pmatrix} j_0(\Omega r/R) \\ i\underline{\sigma}\cdot\hat{r}j_1(\Omega r/R)\end{pmatrix} e^{-i\Omega t/R}\upsilon_a$$

where υ_a is the spin–isospin–colour wave function and Ω is the quark frequency ($\Omega = 2.04$ for lowest lying state).

The Hamiltonian operator $\hat{H}_{\mathrm{MIT}}$ for the bare bag is obtained from the energy–momentum tensor $T^{\mu\nu}$ to be

$$[3.6]\quad \hat{H}_{\mathrm{MIT}} = \int d^3x\, T^{00}(x)$$

This quark operator is now projected onto a truncated space of bare colourless baryons (nucleons and deltas). A bare bag free Hamiltonian H_{MIT} is then derived as

$$[3.7]\quad H_{\mathrm{MIT}} = m_{0N}N^{\dagger}N + m_{0\Delta}\Delta^{\dagger}\Delta$$

Here m_{0N} and $m_{0\Delta}$ are the bare masses of the nucleon and the delta, which are defined by the mass formula (15) (including gluon corrections):

$$[3.8]\quad m_0 = \frac{3\Omega}{R} + \frac{4\pi}{3}BR^3 + \frac{0.24\alpha_c}{R}[-9+S(S+1) + 3I(I+1)]$$

where α_c is the colour coupling constant, S the total spin, and I the total isospin of the specific baryon. The last term generates a mass splitting between bare nucleon and delta bags.

If we assume that the pion field is relatively weak, we can then expand the exponential present in the CBM Lagrangian density so that

$$[3.9]\quad \mathcal{L}(x) = \mathcal{L}_{\mathrm{MIT}}(x) + \mathcal{L}_{\pi}(x) - \frac{i}{2f_{0\pi}}\sum_{a=1}^{3}\bar{q}_a\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}q_a\Delta_s$$

where $\mathcal{L}_{\pi}(x)$ is the free pion term. We can construct the Hamiltonian quark operator from the energy-momentum tensor corresponding to [3.9]

$$[3.10]\quad \hat{H} = \int d^3x\, T^{00}(x)$$

Now, if we use in $\hat{H}$ the quark wave function of the unperturbed bare baryon bags [3.5], and expand the pion field $\phi(x)$ in momentum space according to

$$[3.11]\quad \boldsymbol{\phi}(x) = (2\pi)^{-3/2}\int d^3k\,(2\omega_k)^{-1/2}(\boldsymbol{a}_k e^{-ik\cdot x} + \boldsymbol{a}_k^{\dagger}e^{ik\cdot x})$$

we find

$$[3.12]\quad \hat{H} = \hat{H}_{\mathrm{MIT}} + \hat{H}_{\mathrm{I}} + \int d^3k\,\omega_{0k}\boldsymbol{a}_k^{\dagger}\boldsymbol{a}_k$$

The interaction Hamiltonian $\hat{H}_{\mathrm{I}}$ is given by

$$[3.13]\quad \hat{H}_{\mathrm{I}} = \int d^3k\,(\hat{\boldsymbol{V}}_{0k}\cdot\boldsymbol{a}_k + \hat{\boldsymbol{V}}_{0k}^{\dagger}\cdot\boldsymbol{a}_k^{\dagger})$$

$$[3.14]\quad \hat{\boldsymbol{V}}_{0k} = \frac{i}{6f_{0\pi}}\frac{\Omega}{\Omega-1}\frac{u(kR)}{\sqrt{2\omega_{0k}}\sqrt{(2\pi)^3}} \times \sum_{a=1}^{3}\upsilon_a^{\dagger}\underline{\sigma}\cdot\underline{k}\boldsymbol{\tau}\upsilon_a$$

and $u(kR)$ is the form factor:

$$[3.15]\quad u(kR) = j_0(kR) + j_2(kR)$$

We can now write down the full Hamiltonian of the cloudy bag model by again projecting the quark Hamiltonian $\hat{H}$ onto the baryonic space of nucleons and deltas

$$[3.16]\quad H = H_0 + H_{\mathrm{I}}$$

The terms H_0 and H_{I} are defined by the relations

$$[3.17]\quad H_0 = m_{0N}N^{\dagger}N + m_{0\Delta}\Delta^{\dagger}\Delta + \int d^3k\,\omega_{0k}\boldsymbol{a}_k^{\dagger}\boldsymbol{a}_k$$

$$[3.18]\quad H_{\mathrm{I}} = \int d^3k\,(\boldsymbol{V}_{0k}\cdot\boldsymbol{a}_k + \boldsymbol{V}_{0k}^{\dagger}\cdot\boldsymbol{a}_k^{\dagger})$$

$$[3.19]\quad \boldsymbol{V}_{0k} = \sum_{\alpha,\beta}\alpha^{\dagger}\boldsymbol{\upsilon}_{0k}^{\alpha\beta}\beta, \qquad \alpha,\beta\in\{N,\Delta\}$$

$$[3.20]\quad \boldsymbol{\upsilon}_k^{\alpha,\beta} = \frac{if_0^{\alpha\beta}}{m_\pi}\frac{\sqrt{4\pi}u(kR)}{\sqrt{2\omega_{0k}}\sqrt{(2\pi)^3}}(\underline{S}^{\alpha\beta}\cdot\underline{k})\boldsymbol{T}^{\alpha\beta}$$

$$[3.21]\quad \underline{S}^{\alpha\beta} = \sum_{m} C(S_\beta 1 S_\alpha/s_\beta m s_\alpha)\hat{\underline{s}}_m^{*}\cdot\sqrt{3}$$

$$[3.22]\quad \boldsymbol{T}^{\alpha\beta} = \sum_{n} C(T_\beta 1 T_\alpha/t_\beta n t_\alpha)\hat{\boldsymbol{t}}_n^{*}\cdot\sqrt{3}$$

$$[3.23]\quad f_0^{NN} = f_0^{\Delta\Delta} = \frac{5}{4\sqrt{2}}f_0^{N\Delta} = \frac{5}{2\sqrt{2}}f_0^{\Delta N} = \frac{5}{18\sqrt{4\pi}}\frac{m_\pi}{f_{0\pi}}\frac{\Omega}{\Omega-1}$$

The $C(j_1 j_2 j/m_1 m_2 m)$ are the usual Clebsch–Gordan coefficients and $\hat{s}_m$ and $\hat{t}_n$ are spherical basis vectors in spin and isospin space respectively. The ratio of the coupling constants are those obtained from the $SU(6)$ quark model.

If we consider only the nucleon sector of the theory, we obtain the well known Chew–Low model of the nucleon.

$$[3.24] \quad V_{0k} = i\sqrt{4\pi}\,\frac{f_0^{NN}}{m_\pi}\,\frac{u(kR)}{\sqrt{2\omega_{0k}}\,\sqrt{(2\pi)^3}}\,\underline{\sigma}\cdot\underline{k}\tau$$

The only true free parameter in the CBM is then the radius R of the bag.

IV. Renormalization of the CBM

(a) Physical nucleon and physical delta

Our first task now is to relate the eigenstates of the full Hamiltonian H to the one of the bare Hamiltonian H_0. Let: $|N_0\rangle$, $|N_0,\underline{k}\rangle, \ldots, |\Delta_0\rangle, |\Delta_0,\underline{k}\rangle, \ldots,$ be eigenstates of H_0, and $|N\rangle$, $|N,\underline{k}\rangle$ be eigenstates of H, where $|N_0\rangle$ ($|\Delta_0\rangle$) is a bare nucleon (delta) bag and $|N\rangle$ is the physical nucleon. There are also "approximate" eigenstates of H involving dressed deltas: $|\Delta\rangle$, $|\Delta,\underline{k}\rangle$, etc. For a lengthy discussion about these delta states, we refer the reader to one of our previous papers (1–3). Here we simply mention that our delta is the P_{33} resonance state from which the decay channel has been removed. This procedure allows us to treat the nucleon and the delta on the same footing. In the following we will use the notation $|\alpha\rangle$ or $|\beta\rangle$ to refer to both nucleon and delta dressed bags.

The physical nucleon $|N\rangle$ consists, in our model, of a bare bag surrounded by a cloud of pions. Part of the time, there will be no pion "in the air". If $Z_2^N(E_N)$ is such a probability, then

$$[4.1] \quad |N\rangle = \sqrt{Z_2^N(E_N)}\,|N_0\rangle + \Lambda(N_0)|X_N\rangle$$

where $|X_N\rangle$ involves those components of the dressed nucleon containing at least one pion, and $\Lambda(N_0)$ is a projection operator defined as

$$[4.2] \quad \Lambda(N_0) = 1 - |N_0\rangle\langle N_0|$$

The physical nucleon bag $|N\rangle$ and the bare nucleon bag $|N_0\rangle$ obey the relations

$$[4.3] \quad H|N\rangle = E_N|N\rangle$$

$$[4.4] \quad H_0|N_0\rangle = E_{0N}|N_0\rangle$$

where $E_N = m_N$ for an on-shell dressed nucleon. Using the previous relations and similar ones for the delta, we derive the following integral equation for $|N\rangle$ and $|\Delta\rangle$:

$$[4.5] \quad |N\rangle = \sqrt{Z_2^N(E_N)}\,|N_0\rangle + (E_N - H_0)^{-1}\Lambda(N_0)H_1|N\rangle$$

$$[4.6] \quad |\Delta\rangle = \sqrt{Z_2^\Delta(E_\Delta)}\,|\Delta_0\rangle + P(E_\Delta - H_0)^{-1}\Lambda(\Delta_0)H_1|\Delta\rangle$$

where P indicates the principal value of the integrals. Thus our removal of the delta decay channel corresponds mathematically to keeping only the principal part of $(E_\Delta - H_0)^{-1}$ (16).

(b) Mass renormalization

One effect of the presence of pion cloud around the bag is to lower the mass of the hadron. If we use α to represent either the nucleon or the delta, the mass correction $\Sigma^\alpha(E_\alpha)$ to any order can be obtained from the relation

$$[4.7] \quad \Sigma^\alpha(E_\alpha) = (Z_2^\alpha(E_\alpha))^{-1/2}\,\langle\alpha_0|H_1|\alpha\rangle$$

which is easily verified by using $H_1 = H - H_0$ and the relations [4.3], [4.4] in [4.7]. Notice that we have considered the general case where the hadron $|\alpha\rangle$ may be off the energy shell. We approximate the physical state $|\alpha\rangle$ by replacing it by $|\alpha_0\rangle$ in the terms on the right-hand side of [4.5] and [4.6]. This is then equivalent to keeping only the one-pion component of the wave function

$$[4.8] \quad \Sigma_0^\alpha(E_\alpha) = \sum_\beta \sum_k \int d^3k\, \frac{v_k^{\alpha\beta}(v_k^\dagger)^{\beta\alpha}}{E_\alpha - m_\beta - \omega_k}$$

This is represented graphically in Fig. 1. More explicitly we have

$$[4.9] \quad \Sigma_0^N(E_N) = \frac{3}{\pi}\frac{f_0^2}{m_\pi^2}\int_0^\infty \frac{dk\,k^4u^2(kR)}{\omega_k(E_N - m_N - \omega_k)} + \frac{4}{3\pi}\frac{g_0^2}{m_\pi^2}\int_0^\infty \frac{dk\,k^4u^2(kR)}{\omega_k(E_N - m_\Delta - \omega_k)}$$

$$[4.10] \quad \Sigma_0^\Delta(E_\Delta) = \frac{1}{3\pi}\frac{g_0^2}{m_\pi^2}P\int_0^\infty \frac{dk\,k^4u^2(kR)}{\omega_k(E_\Delta - m_N - \omega_k)} + \frac{3}{\pi}\frac{h_0^2}{m_\pi^2}\int_0^\infty \frac{dk\,k^4u^2(kR)}{\omega_k(E_\Delta - m_\Delta - \omega_k)}$$

where

$$[4.11] \quad f_0 \equiv f_0^{NN}$$

$$[4.12] \quad g_0 \equiv 3f_0^{\Delta N}$$

$$[4.13] \quad h_0 \equiv f_0^{\Delta\Delta}$$

(c) The bare bag probability, $Z_2^\alpha(E_\alpha)$

The bare bag probability $Z_2^\alpha(E_\alpha)$ can be obtained by either normalizing the state $|\alpha\rangle$ to unity in [4.5], or using the equivalent expression of Chew (11)

$$[4.14]\quad Z_2^\alpha(E_\alpha) = \left[1 - \frac{\partial}{\partial E}\sum{}^\alpha(E)\right]^{-1}_{E=E_\alpha}$$

Again, we write down explicitly this expression in our one-pion approximation

$$[4.15]\quad Z_2^N(E_N) = \left[1 + \frac{3}{\pi}\frac{f_0^2}{m_\pi^2}\int_0^\infty \frac{\mathrm{d}k\, k^4u^2(kR)}{\omega_k(E_N - m_N - \omega_k)^2} + \frac{4}{3\pi}\frac{g_0^2}{m_\pi^2}\int_0^\infty \frac{\mathrm{d}k\, k^4u^2(kR)}{\omega_k(E_N - m_\Delta - \omega_k)^2}\right]^{-1}$$

and

$$[4.16]\quad Z_2^\Delta(E_\Delta) = \left[1 + \frac{3}{\pi}\frac{h_0^2}{m_\pi^2}\int_0^\infty \frac{\mathrm{d}k\, k^4u^2(kR)}{\omega_k(E_\Delta - m_\Delta - \omega_k)^2} - \frac{\partial}{\partial E}\left\{\frac{1}{3\pi}\frac{g_0^2}{m_\pi^2} P\int_{m_\pi}^\infty \frac{\mathrm{d}\omega_k\, k^3u^2(kR)}{E_\Delta - m_\Delta - \omega_k}\right\}_{E=E_\Delta}\right]^{-1}.$$

Notice that in [4.16] we are not allowed to commute the derivative and the integral operator since the integrand is not a continuous function of the energy.

(d) The vertex renormalization, $f_r^{\beta\alpha}(E_\beta,E_\alpha)$

The renormalized coupling constants $f_r^{\beta\alpha}$ are related

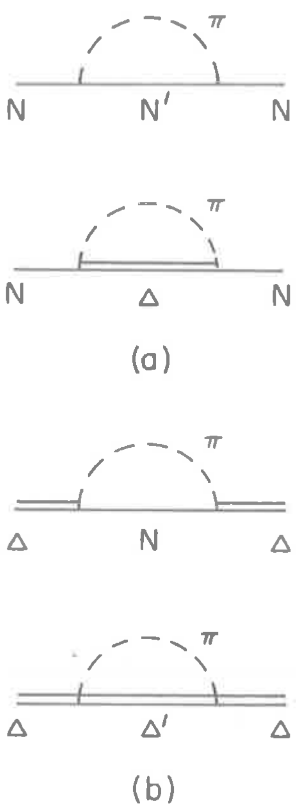

FIG. 1. (*a*) Nucleon self-energy. (*b*) Delta self-energy. In all figures the pion, nucleon, and delta are represented by dashed, solid, and double lines respectively.

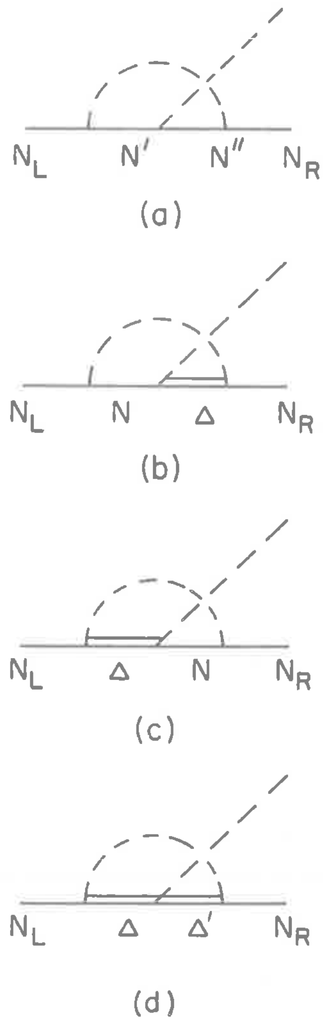

FIG. 2. The order f_0^2 corrections to the πNN vertex. (*a*) $\lambda(N_L,N|N,N_R)$; (*b*) $\lambda(N_L,N|\Delta,N_R)$; (*c*) $\lambda(N_L,\Delta|N,N_R)$; (*d*) $\lambda(N_L,\Delta|\Delta,N_R)$.

to the bare coupling constants $f_0^{\beta\alpha}$ via the relation defined by Chew (12):

$$[4.17] \quad \langle \beta_0 | V_{rq} | \alpha_0 \rangle = \langle \beta | V_{0q} | \alpha \rangle$$

where V_{rq} is the same as V_{0q} except that it contains the renormalized coupling constants. Writing $V_{rq} = f_r \cdot \bar{V}_{0q}$ then

$$[4.18] \quad f_r^{\beta\alpha}(E_\beta,E_\alpha)\bar{V}_{0q}^{\beta\alpha} = \langle \beta | V_{0q} | \alpha \rangle.$$

Using our one-pion approximation for $|\alpha\rangle$ and $|\beta\rangle$, one obtains

$$[4.19] \quad \langle \beta | V_{0q} | \alpha \rangle = \sqrt{Z_2^\beta(E_\beta)Z_2^\alpha(E_\alpha)} \times \left\{ \bar{V}_{0q}^{\beta\alpha} + \bar{V}_{1q}^{\beta\alpha} \right\} f_0^{\beta\alpha}$$

with $\bar{V}_{1q}^{\beta\alpha}$ defined as:

$$[4.20] \quad \bar{V}_{1q}^{\beta\alpha} = \sum_{\alpha'\beta'} \bar{V}_{0q}^{\beta\alpha} \cdot \lambda(\beta,\beta'|\alpha'\alpha)$$

where $\lambda(N,\beta'|\alpha',N)$ for example, is represented graphically to order f_0^2 in Fig. 2. Combining [4.18]–[4.20] allows us to obtain a relation between the renormalized coupling constant and the bare coupling constant

$$[4.21] \quad f_r^{\beta\alpha}(E_\beta,E_\alpha) = \sqrt{Z_2^\beta(E_\beta)Z_2^\alpha(E_\alpha)} \cdot Z_1^{-1}(E_\beta,E_\alpha) f_0^{\beta\alpha}$$

with

$$[4.22] \quad Z_1^{-1}(E_\beta,E_\alpha) = 1 + \sum_{\alpha'\beta'} \lambda(\beta,\beta'|\alpha',\alpha)$$

To lowest order in f_0^2, $f_r^{\beta\alpha}(E_\beta,E_\alpha)$ can be calculated explicitly. To be complete, we give here the expression for the λ's. Let us define:

$$[4.23] \quad \mathrm{D}\omega_k = \frac{1}{27\pi} \frac{k^3u^2(kR)}{m_\pi^2} \mathrm{d}\omega_k$$

$$[4.24] \quad S_k(E) = (E - \omega_k)^{-1}$$

then:

$$[4.25] \quad \begin{aligned}
\lambda(N_\mathrm{L}, N|N, N_\mathrm{R}) &= 9f_0^2 \int \mathrm{D}\omega_k\, S_k(E_{N\mathrm{L}}-m_N)S_k(E_{N\mathrm{R}}-m_N) \\
\lambda(N_\mathrm{L}, N|\Delta, N_\mathrm{R}) &= 16g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{N\mathrm{L}}-m_N)S_k(E_{N\mathrm{R}}-m_\Delta) \\
\lambda(N_\mathrm{L}, \Delta|N, N_\mathrm{R}) &= 16g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{N\mathrm{L}}-m_\Delta)S_k(E_{N\mathrm{R}}-m_N) \\
\lambda(N_\mathrm{L}, \Delta|\Delta, N_\mathrm{R}) &= 20g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{N\mathrm{L}}-m_\Delta)S_k(E_{N\mathrm{R}}-m_\Delta)
\end{aligned}$$

$$[4.26] \quad \begin{aligned}
\lambda(\Delta_\mathrm{L}, N|N, N_\mathrm{R}) &= 36f_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_N)S_k(E_{N\mathrm{R}}-m_N) \\
\lambda(\Delta_\mathrm{L}, N|\Delta, N_\mathrm{R}) &= g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_N)S_k(E_{N\mathrm{R}}-m_\Delta) \\
\lambda(\Delta_\mathrm{L}, \Delta|N, N_\mathrm{R}) &= 45f_0h_0 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_\Delta)S_k(E_{N\mathrm{R}}-m_N) \\
\lambda(\Delta_\mathrm{L}, \Delta|\Delta, N_\mathrm{R}) &= 36h_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_\Delta)S_k(E_{N\mathrm{R}}-m_\Delta)
\end{aligned}$$

$$[4.27] \quad \begin{aligned}
\lambda(\Delta_\mathrm{L}, N|N, \Delta_\mathrm{R}) &= 5g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_\mathrm{N})S_k(E_{\Delta\mathrm{R}}-m_N) \\
\lambda(\Delta_\mathrm{L}, N|\Delta, \Delta_\mathrm{R}) &= g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_N)S_k(E_{\Delta\mathrm{R}}-m_\Delta) \\
\lambda(\Delta_\mathrm{L}, \Delta|N, \Delta_\mathrm{R}) &= g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_\Delta)S_k(E_{\Delta\mathrm{R}}-m_N) \\
\lambda(\Delta_\mathrm{L}, \Delta|\Delta, \Delta_\mathrm{R}) &= \frac{121}{8}g_0^2 \int \mathrm{D}\omega_k\, S_k(E_{\Delta\mathrm{L}}-m_\Delta)S_k(E_{\Delta\mathrm{R}}-m_\Delta)
\end{aligned}$$

(e) Results of the calculations

The renormalization equations obtained in the previous sections involve three important parameters: the physical mass of the delta, the bare πNN coupling constant f_0^{NN}, and the bag radius R.

The physical mass of the delta m_Δ is chosen here to be the position of the P_{33} resonance even though part of the contribution to the resonance comes from the pion–nucleon cross term. However, it is shown in ref. 2 that such a term gives rise only to a small contribution in the cross section. We then choose m_Δ = 1232 MeV. The πNN bare coupling constant $f_0 = f_0^{NN}$ is defined in [3.23] as a function of the physical pion mass m_π, the quark frequency Ω, and the bare decay constant $f_{0\pi}$. The latter is related by [2.16] to the pion decay constant f_π and the pion bare mass $m_{0\pi}$. In this paper, we do not present a renormalization procedure for the pion propagator and we shall use $\omega_{0k} = \omega_k = (k^2+m_\pi^2)^{1/2}$, and also leave f_0 as a free parameter. An interesting point to notice is that (from [2.16]) $f_0^2(m_{0\pi}=m_\pi) = 0.05$, and $f_0^2(m_{0\pi}=1.2m_\pi) = 0.10$, showing thus that the value of f_0 is very sensitive to the renormalization of the pion

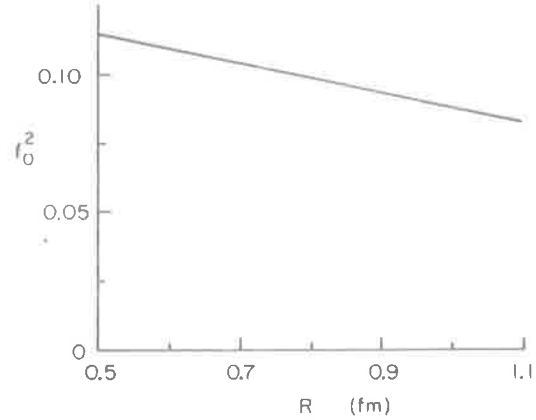

FIG. 3. Radius dependence of the bare πNN coupling constant f_0^2.

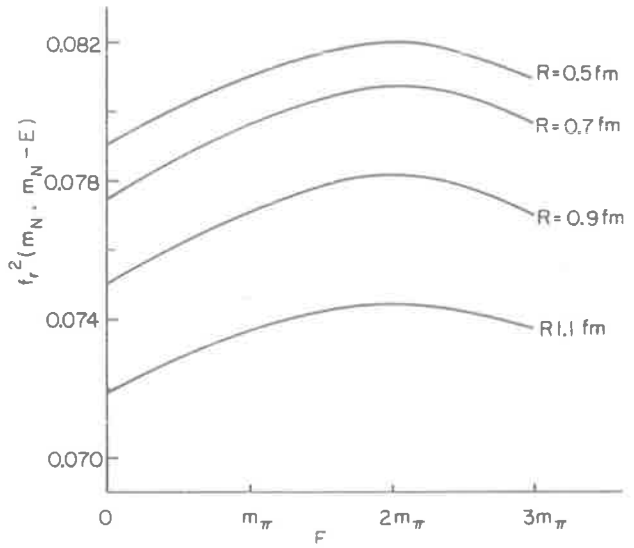

FIG. 4. Energy dependence of the renormalized coupling constant f_r^2 for different bag radii.

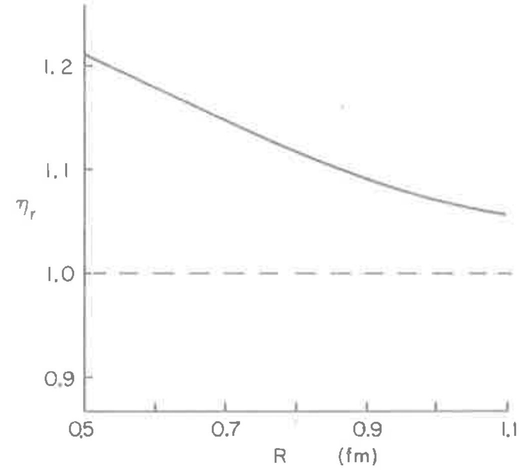

FIG. 5. Dependence of η_r on the bag radius as defined in [4.30].

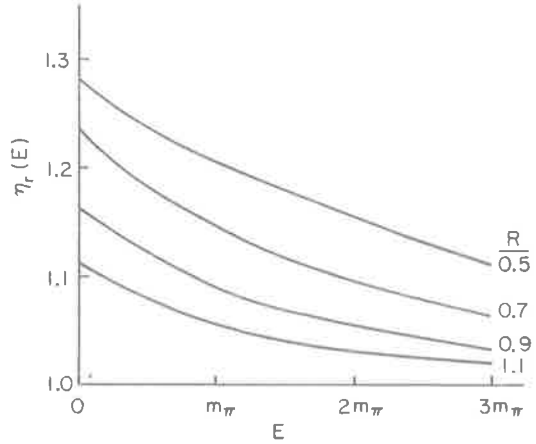

FIG. 6. Energy dependence of η_r as defined in [4.31] for different bag radii.

propagator. This gives us the freedom to choose f_0 such that the renormalized coupling constant f_r^{2*} at the pion pole be the experimental one of 0.081 for each value of R.

$$[4.28] \qquad f_r^{2*} \equiv u^2(im_\pi R)\, f_r^2(m_N, m_N) = 0.081$$

So, by this constraint, f_0 is not a free parameter but a function of R. Finally, the bag radius R is left in this paper as a totally free parameter so that the dependence of the theory on it can be studied.

Figure 3 shows the radius dependence of the bare coupling constant f_0^2. The most interesting result appearing on this graph is how very close the bare coupling constant is to the renormalized one. One can then conclude that, in contrast to the Chew–Low static theory, the renormalization due to the pion field is small. This is the direct consequence of both the shape of our form factor and the presence of the delta in our calculations.

In Fig. 4 we present the energy dependence of the πNN renormalized coupling constant $f_r^2(m_N, m_N - E)$. Observe that this energy dependence is relatively weak. This feature enables us to simplify our calculations of the electromagnetic properties of the nucleon by evaluating the coupling constants at $E = m_\pi$.

We mentioned earlier in this paper (see [4.12]) that in the absence of the pion field, the ratio g_0^2/f_0^2 is given by the $SU(6)$ quark model. Using the definitions [3.23] and [4.12], this ratio is 72/25. An interesting quantity to calculate is the ratio of the corresponding renormalized coupling constants

$$[4.30] \qquad \eta_r = \frac{25}{72}\frac{g_r^2(E_N = m_N, E_\Delta = m_N - m_\pi)}{f_r^2(E_{NL} = m_N, E_{NR} = m_N - m_\pi)}$$

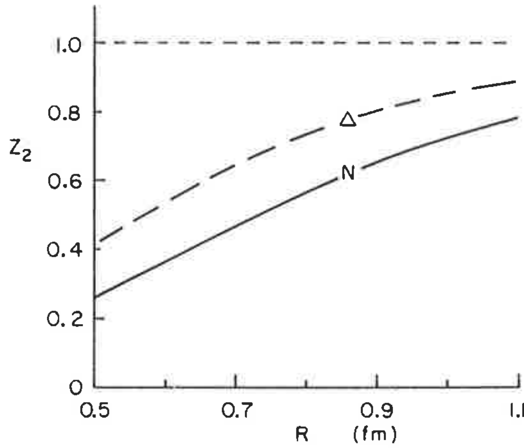

FIG. 7. Bare nucleon bag probability (solid curve) and bare delta bag probability (dashed curve).

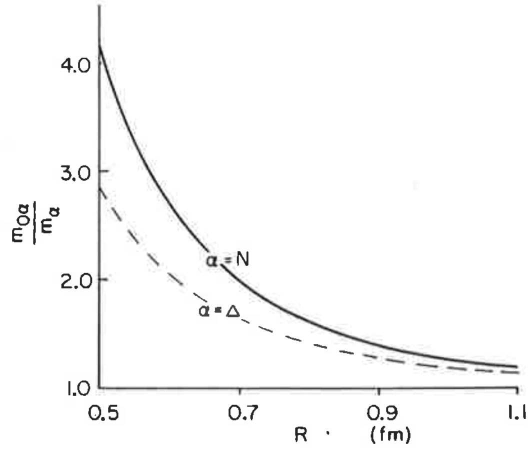

FIG. 8. Ratio of the bare to the physical mass of the nucleon (solid line) and delta (dashed line).

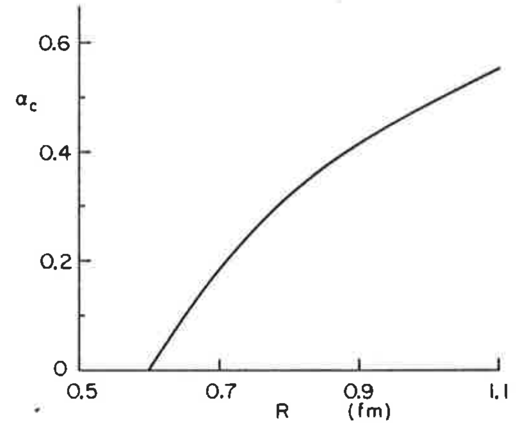

FIG. 9. The colour coupling constant α_c versus the bag radius.

which is a function of the radius of the bag. This dependence is displayed in Fig. 5, which shows a relatively important dependence of η_r on the radius. As the radius gets larger η_r approaches 1, which reflects the fact that the pion field is negligible for large bags.

We can also examine the energy dependence of η_r defined as

$$[4.31] \quad \eta_r(E) = \frac{25}{72}\frac{g_r^2(E_N=m_N, E_\Delta=m_N-E)}{f_r^2(E_{NL}=m_N, E_{NR}=m_N-E)}$$

This is shown in Fig. 6. We again observe that $\eta_r(E)$ tends towards 1 for large energies. This can be understood physically by noticing that for large energies, the pion probes the core of the bag, and therefore $\eta_r(E)$ for E large approaches its bare value, 1.

Throughout this paper, we have assumed that the pion field is "small". We mean by this that there are not too many pions "floating around". The bare nucleon probability $Z_2^N(m_N)$ allows us to judge if the one pion expansion of the Lagrangian density makes sense. In Fig. 7, we give this probability as well as $Z_2^\Delta(m_\Delta)$, as a function of the radius R. From this graph, we see that $Z_2^\alpha(m_\alpha)$ is rather small for bag radii less than 0.7 fm. Thus, for such radii, we expect that terms involving two or more pions will be important, and therefore that the theory developed here loses its accuracy.

In Fig. 8, we show the radius dependence of the ratio of the bare to physical mass for both the nucleon and the delta using [4.9] and [4.10]. Again, we see very large pionic effects for small bag radii which makes the one-pion expansion meaningless. There is then no way, in perturbation theory, to treat little bags.

Having now the bare mass of the nucleon and the delta, we can estimate the colour coupling constant α_c as given in [3.8]. The results are presented in Fig. 9. Particular care should be taken when analyzing this graph since it is obtained by subtracting two large numbers (the bare masses) which were calculated only to lowest order in the theory. Experimentally, α_c is about 0.2 for momentum transfer q^2 around 30 GeV^2.

(f) Conclusion

In summary, the renormalization of the cloudy bag model can be treated reasonably to lowest order and all results make sense as long as we deal with bag radii larger than 0.7 fm. This is in agreement with the calculations of Dodd *et al.* (17) who recently showed how to place a rigorous bound on the probability of finding n pions in the physical nucleon.

V. Electromagnetic properties of the nucleon

The electromagnetic properties of the nucleon are extracted from the elastic scattering amplitude of electrons on free nucleons. To lowest order in the interaction, a single virtual photon is exchanged. The transition amplitude (Fig. 10) is then given by

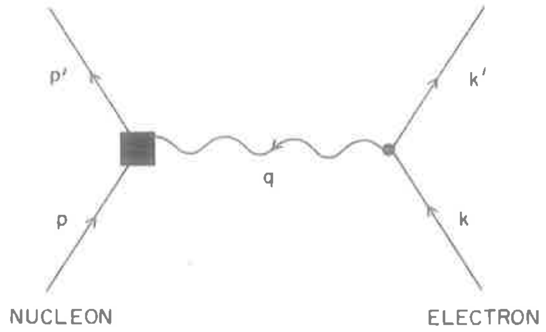

FIG. 10. Electron–nucleon interaction via one-photon exchange.

$$[5.1] \quad M = \int \frac{d^4q}{(2\pi)^4} \langle p'| \int d^4x\, j_N^{\mu}(x) e^{-iq\cdot x}|p\rangle \times \frac{g_{\mu\nu}}{q^2} \langle k'| \int d^4y\, j_e^{\nu}(y) e^{iq\cdot y}|k\rangle$$

where the electron current is given by QED to be

$$[5.2] \quad \langle k'| \int d^4y\, j_e^{\nu}(y)|k\rangle = (2\pi)^4 e(4E_kE_{k'})^{-1/2} \times \bar{u}_e(k')\gamma^{\nu} u_e(k)\delta^4(k' - k + q)$$

The matrix element for the nucleon current is restricted by the requirement of gauge invariance and covariance under the improper Lorentz group to the form

$$[5.3] \quad \langle p'| \int d^4x\, j_N^{\mu}(x)|p\rangle = (2\pi)^4 e(4E_pE_{p'})^{-1/2} \times \bar{u}_N(p') j_N^{\mu}(o) u_N(p)\delta^4(p' - p + q)$$

$$[5.4] \quad j_N^{\mu}(0) = F_1^N(q^2)\gamma^{\mu} + K^N(ie/2m_N)F_2^N(q^2)\sigma^{\mu\nu}q_{\nu}$$

It was shown by Yennie *et al.* (18) that this expression holds for any spin one-half particle, composite or not. $F_{1,2}^N(q^2)$ are form factors related to the electric form factor $G_E^N(q^2)$ and the magnetic form factor $G_m^N(q^2)$ by

$$[5.5] \quad G_E^N(q^2) = F_1^N(q^2) - K^N(|q^2|/4m_N^2)F_2^N(q^2)$$

$$[5.6] \quad G_m^N(q^2) = F_1^N(q^2) + K^N F_2^N(q^2)$$

where K^N is the nucleon anomalous magnetic moment.

The magnetic moment of the nucleon is defined by the expectation value of the magnetic moment operator in the rest frame of the nucleon:

$$[5.7] \quad \langle M\rangle = \langle N|\frac{1}{2} \int d^3r\, r \times j_N|N\rangle$$

$$[5.8] \quad M = (e/2m_N)G_m^N(0)\boldsymbol{\sigma}$$

The Fourier transformation of the charge distribution in the rest frame of the nucleon can be obtained using $j_N^0(\underline{x})$ in [5.3]. The use of [5.4] in [5.5] leads to the result

$$[5.9] \quad G_E^N(q^2) = \langle N(q)| \int d^3r\, j_N^0(\underline{r}) e^{i\underline{q}\cdot\underline{r}}|N\rangle$$

The second moment of the charge distribution is obtained by taking the expectation of the mean square charge radius operator again in the rest frame of the nucleon.

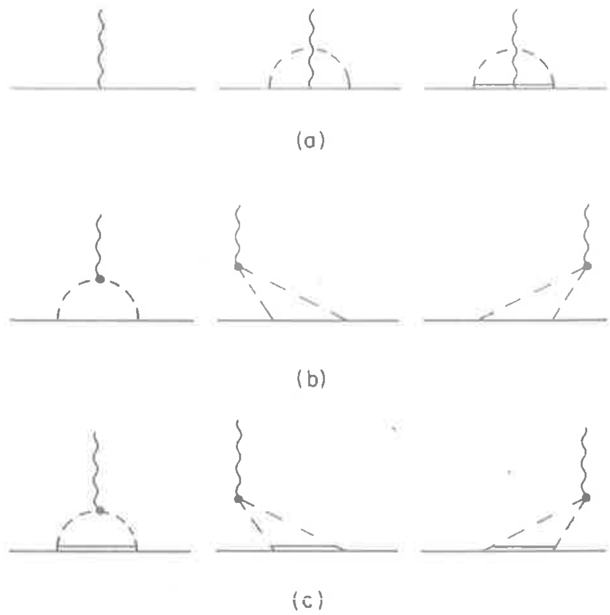

FIG. 11. Contribution to the electric form factor of the nucleon from (*a*) the quark current, (*b*) the pion current with intermediate nucleon present, (*c*) the pion current with intermediate delta present.

$$[5.10] \quad \langle R^2\rangle = \langle N| \int d^3r\, r^2 j_N^0(\underline{r})|N\rangle$$

which is related to [5.9] by

$$[5.11] \quad \langle R^2\rangle = -6\frac{\partial}{\partial|q^2|} G_E^N(q^2)|_{q^2=0}$$

Notice that $G_E^N(q^2)$ measures the charge distribution, not $F_1(q^2)$ as often asserted (19). In the Cloudy Bag Model, we can evaluate explicitly the expectation of the operators M and R^2 for a nucleon at rest, since the photon couples to the pion and to the quarks in a well-defined way. Such a procedure allows us to compare our results directly with G_E^N without adding the Foldy term.

Let us first write the nucleon current as

$$[5.12] \quad j_N^{\mu} = j_Q^{\mu} + j_{\pi}^{\mu}$$

where j_Q^{μ} is the quark current, and j_{π}^{μ} is the pion one. Formally they are given by the following equations:

$$[5.13] \quad j_Q^{\mu} = \sum_{a=1}^{3} e_a\bar{q}_a\gamma^{\mu}q_a$$

$$[5.14] \quad j_{\pi}^{\mu} = -ie(\Phi\partial^{\mu}\Phi^* - \Phi^*\partial^{\mu}\Phi)$$

$$[5.15] \quad \Phi(x) = \frac{1}{\sqrt{2}}(\phi_1(x) - i\phi_2(x))$$

The final ingredient needed is the expansion of the physical nucleon $|N\rangle$ in terms of the bare states, on which the action of the operator j_N^μ is calculable (given by [4.5]).

(a) The charge distribution of the nucleon

From the previous discussion of the charge radius of the nucleon, we understand that the electric form factor $G_E^N(q^2)$ is simply the Fourier transform of the charge distribution for a nucleon at rest (see [5.9]). Expanding the physical nucleon on the eigenstates of H_0, (eq. [4.5]) allows us to obtain the quark contribution $G_E^N(q^2,Q)$ (Fig. 11*a*); the pion contribution with intermediate nucleon $G_E^N(q^2,N\pi)$ (Fig. 11*b*); and the pion contribution with intermediate delta $G_E^N(q^2,\Delta\pi)$ (Fig. 11*c*). Their expressions to second order are the following:

$$[5.16] \quad G_E^N(q^2) = G_E^N(q^2,Q) + G_E^N(q^2,N\pi) + G_E^N(q^2,\Delta\pi)$$

$$[5.17] \quad G_E^N(q^2,Q) = \frac{C_N}{4\pi}\int_{\mathrm{BAG}} \mathrm{d}^3r\,(j_0^2(\Omega r/R) + j_1^2(\Omega r/R))\,\mathrm{e}^{i\underline{q}\cdot\underline{r}}$$

$$[5.18] \quad G_E^N(q^2,N\pi) = \frac{1}{\pi^2}\frac{f_r^2}{m_\pi^2}\int \mathrm{d}^3k\,\frac{u(kR)u(k'R)\underline{k}\cdot\underline{k}'\langle N|\tau_3|N\rangle}{\omega_k\omega_{k'}(\omega_k+\omega_{k'})}$$

$$[5.19] \quad G_E^N(q^2,\Delta\pi) = \frac{2}{9\pi^2}\frac{g_r^2}{m_\pi^2}\int \mathrm{d}^3k\,\frac{u(kR)u(k'R)\underline{k}\cdot\underline{k}'\langle N|\tau_3|N\rangle}{(\omega_\Delta+\omega_k)(\omega_\Delta+\omega_{k'})(\omega_k+\omega_{k'})}$$

where $\underline{k}' = \underline{k} + \underline{q}$ and C_N is chosen such that $G_E^p(0) = 1$ and $G_E^n(0) = 0$.

(b) The magnetic moment μ_N of the nucleon

If we consider the case of a spin-up nucleon, then from [5.8] we have

$$[5.20] \quad \mu_N = G_m^N(0) = \langle M_Z\rangle$$

The first contribution to the magnetic moment will come from the photon coupling to the quarks inside the bag. From the definition of $\boldsymbol{M}$, and using the quark wave functions in [3.5], we get

$$[5.21] \quad M_Q = \mu_Q\sum_{a=1}^{3} e_a v_a^\dagger \sigma v_a$$

where e_a is the charge of the ath quark, and μ_Q is given by

$$[5.22] \quad \mu_Q = \frac{R(4\Omega-3)}{12\Omega(\Omega-1)}$$

When M_Q acts on a bag core, there may be no pions or one pion "in the air". In the latter case, we have to add the contribution of all the intermediate states allowed as shown in Fig. 12*a*. This is fully calculable in our model and the result is

$$[5.23] \quad G_m^N(0,Q) = \mu_Q\left[\begin{pmatrix}1\\-\frac{2}{3}\end{pmatrix}Z_N + \frac{1}{27}\begin{pmatrix}1\\-4\end{pmatrix}P_{N\pi} + \frac{5}{27}\begin{pmatrix}4\\-1\end{pmatrix}P_{\Delta\pi} + \frac{4}{9}\begin{pmatrix}1\\-1\end{pmatrix}P_{N\Delta\pi}\right]$$

with

$$[5.24] \quad P_{N\pi} = Z_N\cdot\frac{3}{\pi}\frac{f_r^2}{m_\pi^2}\int_0^\infty \mathrm{d}k\,\frac{k^4u^2(kR)}{\omega_k^3}$$

$$[5.25] \quad P_{\Delta\pi} = Z_N\cdot\frac{4}{3\pi}\frac{g_r^2}{m_\pi^2}\int_0^\infty \mathrm{d}k\,\frac{k^4u^2(kR)}{\omega_k(\omega_\Delta+\omega_k)^2}$$

$$[5.26] \quad P_{N\Delta\pi} = Z_N\cdot\frac{8\sqrt{2}}{3\pi}\frac{f_rg_r}{m_\pi^2}\int_0^\infty \mathrm{d}k\,\frac{k^4u^2(kR)}{\omega_k^2(\omega_\Delta+\omega_k)}$$

and

$$[5.27] \quad Z_N + P_{N\pi} + P_{\Delta\pi} = 1$$

The contribution of the pion to the magnetic moment is expressed by the intermediate nucleon contribution $G_m^N(0,N\pi)$ in Fig. 12*b* and the intermediate delta contribution $G_m^N(0,\Delta\pi)$ in Fig. 12*c*, and is given explicitly by:

$$[5.28] \quad G_m^N(0,N\pi) = 2m_N \cdot \frac{4}{3\pi}\frac{f_r^2}{m_\pi^2}\int_0^\infty dk\,\frac{k^4u^2(kR)}{\omega_k^4}\langle N|\tau_3|N\rangle$$

$$[5.29] \quad G_m^N(0,\Delta\pi) = 2m_N \cdot \frac{2}{27\pi}\frac{g_r^2}{m_\pi^2}\int_0^\infty dk\,\frac{k^4u^2(kR)(\omega_\Delta + 2\omega_k)}{\omega_k^3(\omega_\Delta + \omega_k)^2}\langle N|\tau_3|N\rangle$$

and

$$[5.30] \quad \mu_N = G_m^N(0) = G_m^N(0,Q) + G_m^N(0,N\pi) + G_m^N(0,\Delta\pi)$$

(c) Center-of-mass corrections

In this section we estimate the effects of the center-of-mass corrections in the Cloudy Bag Model. We will follow the prescription of Donoghue and Johnson (20) in treating the bag core, and neglect those on the pion field, since the latter was already assumed to be small. First we write

$$[5.31] \quad \langle E_{CM}\rangle = \langle P_{CM}^2\rangle/2m_0$$

where $\langle P_{CM}^2\rangle$ is the square of the CM momentum, and m_0 is the bare mass of the nucleon. The most trivial form to use to estimate $\langle P_{CM}^2\rangle$ is to add the contribution from each quark according to

$$[5.32] \quad \langle P_{CM}^2\rangle \simeq \sum_{a=1}^{3}\langle p_a^2\rangle$$

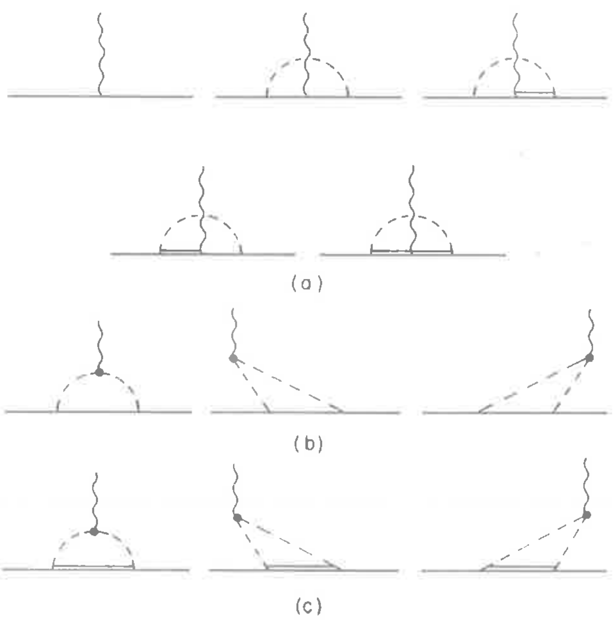

FIG. 12. Contribution to the magnetic moment of the nucleon from (*a*) the quark current, (*b*) the pion current with intermediate nucleon present, (*c*) the pion current with intermediate delta present.

but for each quark, $\langle p_a^2\rangle = \Omega^2/R^2$

$$[5.33] \quad \langle P_{CM}^2\rangle \simeq 3\Omega^2/R^2$$

For the bare bag mass, we use Fig. 8 which can be parametrized in the form

$$[5.34] \quad m_0 = 5.9/R$$

Using $\Omega = 2.04$ and combining all the previous equations:

$$[5.35] \quad \langle E_{CM}\rangle \simeq 1.06/R$$

which has the same form as the Z/R of DeGrand *et al.* (6).

The CM corrections to the electromagnetic properties of the nucleon have been evaluated by Donoghue and Johnson:

$$[5.36] \quad \delta\mu^{CM} = \mu^{BAG}\cdot\frac{1}{2}\frac{\langle P_{CM}^2\rangle}{m_0^2}$$

$$[5.37] \quad \delta\langle r^2\rangle^{CM} = \langle r^2\rangle^{BAG}\cdot\frac{1}{3}\frac{\langle P_{CM}^2\rangle}{m_0^2}$$

where μ^{BAG} and $\langle r^2\rangle^{BAG}$ are the bag core contributions to the magnetic moment and charge radius square. Using [5.33] and [5.34] gives finally:

$$[5.38] \quad \delta\mu^{CM} = 0.18\mu^{BAG}$$

$$[5.39] \quad \delta\langle r^2\rangle^{CM} = 0.12\,\langle r^2\rangle^{BAG}$$

This crude but simple prescription for the CM corrections allows us to refine the results as described in the next section.

(d) Results

In the equations for the electromagnetic properties of the nucleon, the parameters involved are: the physical mass of the delta, the πNN renormalized coupling constant f_r^2, and the bag radius R. Again, we choose m_Δ to be 1232 MeV, $f_r^2 = f_r^2(m_N, m_N - m_\pi)$ as described in Fig. 4, and leave the bag radius as a free parameter.

The results of the calculation are summarized in Tables 1 and 2 (21, 22) where the center-of-mass corrections have been added. Our equations also give expressions for $G_E^p(q^2)$ and $G_E^n(q^2)$ which, without recoil corrections, are presented in Figs. 13 and 14. These two

TABLE 1. Radius dependence of proton and neutron magnetic moments

(*a*) Experiment: $\mu_p = 2.793$ (ref. 21)

R	μ_p^B	μ_p^π	$\mu_p^{(0)}$	$\delta\mu_p^{CM}$	μ_p
0.7	1.164	1.122	2.286	0.210	2.496
0.8	1.356	0.833	2.189	0.244	2.433
0.9	1.557	0.699	2.256	0.280	2.536
1.0	1.759	0.568	2.329	0.317	2.646
1.1	1.964	0.462	2.427	0.354	2.781

(*b*) Experiment: $\mu_n = -1.913$ (ref. 21)

R	μ_n^B	μ_n^π	$\mu_n^{(0)}$	$\delta\mu_n^{CM}$	μ_n
0.7	−0.835	−1.222	−2.057	−0.150	−2.207
0.8	−0.966	−0.833	−1.799	−0.174	−1.973
0.9	−1.096	−0.699	−1.795	−0.197	−1.992
1.0	−1.228	−0.568	−1.796	−0.221	−2.017
1.1	−1.361	−0.462	−1.823	−0.245	−2.068

TABLE 2. Radius dependence of proton and neutron charge radii

(*a*) Experiment: $\langle r_p^2\rangle^{1/2} = 0.836$ fm (ref. 22)

R	$\langle r^2\rangle_p^B$	$\langle r^2\rangle_p^\pi$	$\langle r^2\rangle_p^0$	$\delta\langle r^2\rangle_p^{CM}$	$\langle r^2\rangle_p^{1/2}$
0.7	0.193	0.249	0.442	0.023	0.682
0.8	0.273	0.228	0.501	0.033	0.731
0.9	0.365	0.207	0.572	0.044	0.785
1.0	0.468	0.190	0.658	0.056	0.845
1.1	0.582	0.173	0.755	0.070	0.908

(*b*) Experiment: $\langle r_n^2\rangle^{1/2} = -0.342$ fm (ref. 22)

R	$\langle r^2\rangle_n^B$	$\langle r^2\rangle_n^\pi$	$\langle r^2\rangle_n^0$	$\delta\langle r^2\rangle_n^{CM}$	$\langle r^2\rangle_n^{1/2}$
0.7	0.067	−0.249	−0.182	0.008	−0.417
0.8	0.067	−0.228	−0.161	0.008	−0.391
0.9	0.064	−0.207	−0.143	0.008	−0.367
1.0	0.062	−0.190	−0.128	0.007	−0.348
1.1	0.059	−0.173	−0.114	0.007	−0.327

graphs show clearly that the Cloudy Bag Model fits the electric form factors very well for bag radii within the range of 0.8 to 1.0 fm. Notice that the neutron form factor, $G_E^n(q^2)$, provides a significative test of our model of the nucleon. However, the precision of the experimental data (23) is insufficient to allow a more precise selection of the bag radius.

VI. Conclusion

In summary, the Cloudy Bag Model provides us with a consistent microscopic picture of the nucleon. The Hamiltonian formalism has been presented, together with a lowest order renormalization procedure, which was shown to be adequate for bag radii larger than

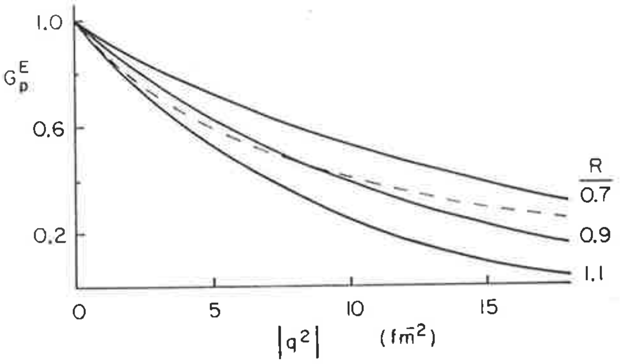

FIG. 13. Proton electric form factor for different bag radii (solid lines) and experimental electric form factor (dashed line).

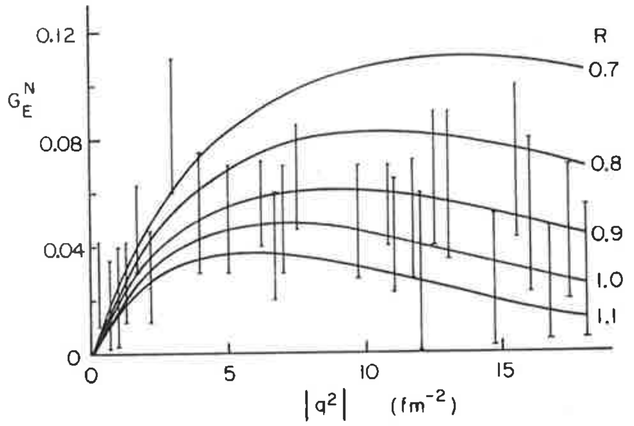

FIG. 14. Neutron electric form factor for different bag radii (solid lines) and experimental electric form factor (vertical lines).

0.7 fm. The electromagnetic properties of the nucleon were also shown to agree very well with experiments for bag radii of 0.8 to 1.0 fm.

In the forthcoming papers, we will add a new ingredient to the CBM, the strange quark. Properties of the strange baryons will then be calculable in this extended form of the cloudy bag model, and thus will provide more tests of its predictive power.

VII. Acknowledgements

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Magnetic moments of the nucleon octet calculated in the cloudy bag model

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We calculate the lowest-order pionic corrections to the magnetic moments of the strange members of the nucleon octet. The overall agreement is remarkably good, but one would like to see an improvement in the data for Σ^- and Ξ^-.

In our search to understand the structure of the hadrons, their magnetic moments provide some very significant information. The large moments of neutral baryons like the n and Ξ^0, as well as the anomalous magnetic moments of charged particles like the proton, are clear evidence for important internal structure. Indeed, a major success of the naive quark model was its prediction of the ratio μ_p/μ_n as $-\frac{3}{2}$.[1] More sophisticated dynamical theories, such as the potential model of Isgur and Karl, have hardly altered this result.[2] Although one might question the validity of an essentially nonrelativistic approach, some justification for the procedures of the constituent quark model has been provided recently by the bag model,[3] or more general relativistic considerations.[4]

With the availability of excellent hyperon beams in the last couple of years there has been a dramatic improvement in the precision with which we know the hyperon magnetic moments.[5] The only exception is the Σ^-, for which the decay asymmetry is very small. Therefore in this case we must rely on the less precise determinations using exotic-atom techniques.[6] Unfortunately, from the theoretical point of view the Σ^- magnetic moment is quite significant, as is that of the Ξ^-, for which only a preliminary number is presently available. In a recent Letter,[7] Franklin discussed some inconsistencies in the theoretical understanding of the magnetic moments in the nucleon octet, and concluded that the "present Σ^- moment determination [was] incompatible with [his] analysis."

Even more interest in the value of the Σ^- magnetic moment was engendered by the study of Brown and co-workers.[8] Within the framework of one particular chiral bag model,[9] they obtained values of $\mu(\Sigma^-)$ in the range $-0.54\mu_N$ to $-0.64\mu_N$ (μ_N=nuclear magneton), in comparison with the experimental determination of $(-1.41 \pm 0.25)\mu_N$ (Ref. 6). In agreement with Lipkin[4] the authors suggested that a remeasurement of $\mu(\Sigma^-)$ would provide "a crucial test of [their] model."

The hybrid bag models[10–13] have been constructed in the past two years as a response to the observation that the MIT bag model (or indeed any model which confines quarks through a scalar potential) badly violates chiral symmetry.[14] By introducing the pion as an approximate Goldstone boson, associated with an as-yet-unknown dynamical symmetry-breaking mechanism, one can restore the $SU(2)\times SU(2)$ symmetry (when $m_\pi=0$). The cloudy bag model (CBM) developed at TRIUMF and the University of Washington, has already been applied to the problem of the nucleon magnetic moments,[15,16] which were a long-standing puzzle for the MIT bag model.

One of the beauties of the bag model[17] is that there are no extra parameters (such as constituent quark masses) which can be used to fit (say) the p, n, and Λ magnetic moments. The massless quarks in the bag model have a magnetic moment as a result of confinement, which is proportional to the radius of the confining volume. It was one of the major puzzles of the original MIT work[17] that with a radius of order 1 fm, as determined by spectroscopy, the proton magnetic moment was only $1.9\mu_N$. (If one scaled all predictions by μ_p, however, the predictions for all other members of the octet were invariably an improvement on naive quark-model predictions.) The recently calculated recoil correction of Donoghue and Johnson[18] improved the situation [μ_p (corrected) $=2.24\mu_N$], but there was still a significant discrepancy. It was therefore satisfying that the inclusion of lowest-order pion-loop corrections improved the situation even more, giving $\mu_p=2.60\mu_N$ (Ref. 15). In fact, the remaining corrections from configuration mixing[2] and sea quarks[19] are sufficiently large that one could not really expect better agreement from the model.

In view of this success it seems natural to extend the CBM calculation to the rest of the octet. As in Refs. 12, 15, and 16, we use the linearized Lagrangian density (for a static spherical bag of radius R)

$$\mathcal{L}(x)=(i\bar{q}\not{\partial}q-B)\theta(R-r)-\tfrac{1}{2}\bar{q}q\,\delta(r-R)-\frac{i}{2f}\bar{q}\gamma_5\vec{\tau}q\cdot\vec{\phi}\,\delta(r-R)+\tfrac{1}{2}(\partial_\mu\vec{\phi})^2-\tfrac{1}{2}m_\pi^2\phi^2\ . \qquad (1)$$

Here q and ϕ are the quark and pion fields, and f the pion decay constant (93 MeV). Equation (1) clearly leads to a Hamiltonian of the form

$$H=H_{\rm MIT}+H_{\rm int}+H_\pi\ , \qquad (2)$$

where $H_{\rm int}$ describes the surface coupling of the (for the present) elementary pion field to the quarks. By defining a P space of three-quark baryons, and ignoring the corrections due to Q space (essentially the effects of sea quarks), we obtain a Hamiltonian describing the emission and absorption of pions by extended (bag model) hadrons. The resultant theory is completely renormalizable, and the convergence properties have been rigorously established for the nucleon.[20]

In order to generalize Eqs. (1) and (2) to the other members of the nucleon octet we simple redefine q as a three-component field (u,d,s) where the s has a mass of 279 MeV. While one could generalize the CBM to $SU(3)_L\times SU(3)_R$ and calculate corrections from a virtual-kaon "cloud," we have chosen not to do so. The mass of the kaon is so much larger than that of the pion that there is no longer such a clean separation between the phenomenology of the bag surface and the mesonic corrections. For the present we calculate only the longest-range (that is, pionic) corrections.

The coupling of the quark and pion fields to the photon occurs through the usual minimal coupling, and the pionic current was presented in detail in Ref. 15. For simplicity we take SU(6) wave functions for all octet members in order to calculate both the ratio of the coupling constants (e.g., $\Sigma\Lambda\pi$, $\Xi\Xi\pi$, etc.) to the $NN\pi$ coupling constant, and the magnetic coupling to the bag ($\gamma\Lambda\Lambda$, $\gamma\Lambda\Sigma^0$, etc.). The $NN\pi$ coupling constant itself was fixed at the usual value of $f^2/4\pi=0.081$, and the radius of the bag for all hyperons was taken to be 1 fm, in agreement with the MIT analysis. [Of course the radii may change when pionic corrections are included. However, as the pion self-energy is a factor of 2 smaller for the Σ (6 for the Ξ), we expect the changes for the hyperons to be much smaller than for the nucleon. In view of the insensitivity to bag radius noted below, the neglect of such corrections seems quite reasonable.]

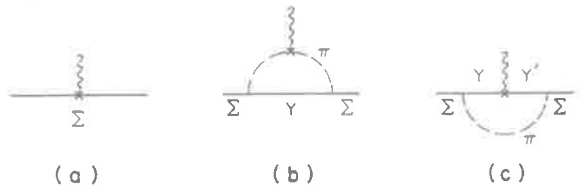

FIG. 1. Contributions to the magnetic moment of the Σ hyperon, including pionic corrections to $O(f^2)$ ["Y" denotes either Σ, Λ, or Σ^*, and the combinations (Y,Y') include (Λ,Λ), (Σ,Σ), (Σ^*,Σ^*), (Λ,Σ), (Λ,Σ^*), and (Σ,Σ^*)].

At this stage the model has no free parameters! For the Σ, for example, we calculate all the graphs shown in Fig. 1 in exactly the way described in Ref. 15. We observe that although there are a large number of graphs in which the photon couples to the bag with the pion "in the air," these are usually small, and in any case there is considerable cancellation. (In calculating the Donoghue-Johnson[18] recoil correction, the appropriate value of $\langle p^2\rangle$ and mass is used for each baryon.) The results of the calculation are summarized in Table I, together with the most recent experimental values.[5,6]

TABLE I. Comparison of the magnetic moments of the members of the nucleon octet calculated in the CBM, in comparison with the most recent data (Refs. 5 and 6) (all numbers in nuclear magnetons).

	CBM	Experiment
p	2.60[a]	2.793
n	−2.01[a]	−1.913
Λ	−0.58	−0.614 ± 0.005
Σ^-	−1.08	−1.41 ± 0.27
Σ^+	2.34	2.33 ± 0.13
Ξ^-	−0.51	−0.75 ± 0.07[b]
Ξ^0	−1.27	−1.25 ± 0.014

[a]From Ref. 15, using $R=0.82$ fm as determined from pion-nucleon scattering.
[b]Preliminary result.

Clearly the overall agreement with experiment is excellent. One remarkable feature of the calculation not shown in Table I is that once pionic corrections are included there is little sensitivity to a small change in the bag radius. For example, arbitrarily reducing R from 1.0 to 0.9 fm changes $\mu(\Sigma^+)$ and $\mu(\Sigma^-)$ to $2.21\mu_N$ and $-1.07\mu_N$ (i.e., by 5% and 1%), respectively. To some extent, therefore, the extra pion contribution for a small bag compensates for the decrease in the contribution from the core.

Unlike Brown *et al.*,[8] we do not find any significant disagreement with the Σ^- magnetic moment. This would appear to rule out the phenomenological isoscalar contribution assumed in Ref. 8. With regard to the analysis of Franklin we note that the combination

$$\mu_s'(\Sigma) = -\Sigma^+ - 2\Sigma^- \ , \tag{3}$$

[Eq. (2′) of Ref. 7] includes a pionic-loop correction or order $0.6\mu_N$. Such corrections would be expected to violate SU(6) constraints, and thus reduce the usefulness of such sum rules in extracting quark moments.

It must be emphasized that there has been a great deal of work on other corrections to baryon magnetic moments, arising from effects such as configuration mixing[2] and sea quarks.[19] In view of the theoretical uncertainties associated with both these effects and our pionic corrections,[21] it appears unlikely that theory will match experiment in precision for some time. Nevertheless, it does seem reasonable to conclude from Table I that the inclusion of the lowest-order pionic corrections associated with chiral $SU(2)\times SU(2)$ results in good overall agreement with the presently available data. More accurate measurements for the Σ^- and the Ξ^- would certainly be welcome.

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PION-NUCLEUS SCATTERING THEORY

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Abstract: We briefly review progress in the theory of pion nucleus scattering and reactions over the past two or three years. This begins with the realization that low and medium energy nuclear physics can no longer proceed without recognition of the underlying quark structure of the nucleons themselves. In particular, we review attempts to understand the structure of the nucleon, and the pion-nucleon interaction, within the framework of one model of quark confinement. Then we pass to the exciting progress in pion-few-nucleon systems, where great advances have been made in understanding so-called "effects of absorption". Finally we turn to pion elastic scattering and reactions with real nuclei.

1. Introduction

Alas, the reviewer's lot is not a happy one! On the one hand this session has a large number of contributions (thirty nine) which all deserve some comment. However, merely devoting equal time to each contribution would be inadequate, not only because of the ridiculous amount of time per contribution, but also because a vast amount of other relevant work, some of which appears in other parallel sessions would be ignored. On the other hand, the collection of work submitted to any conference is the result of many random processes, and it would be a disservice to the community to ignore important, recent work merely because the author could not get travel money. Finally, the limitations of space and time make it impossible to review all significant work.

The point of view adopted here is that there are certain critical issues in nuclear and medium energy physics at the present time. Our aim is to pull together as much of the material available on these topics, taking first but not exclusively that which has been submitted to session G of this conference. (Such contributions will not be included in the list of references but will be referred to as "Gx" in the text.)

Amongst the large issues to be addressed in a session on pion-nucleus theory, one might ask whether we yet know enough to reliably extract information on nuclear sizes or transition densities. At the more speculative level there has been much discussion of the possibility of making new phases of nuclear matter such as a pion condensate, or at least seeing precursor effects of such a state at normal nuclear densities. In this case the essential issue is our understanding of the nuclear many-body problem, including the effects of the strong tensor force arising from one pion exchange.

In the past few years there has been a slow realization that there is great difficulty justifying the usual approach to the nuclear many-body problem. That is, essentially any model of hadron structure results in a nucleon with a radius of order 1 fm. At nuclear matter densities such objects are clearly overlapping, and one is (to say the least) hard put to justify the standard approximation of point-like objects interacting through potentials. Consequently, no discussion of the nuclear many-body problem can begin without an understanding of the nucleons themselves. This has been realized by a number of people for a long time, for example in 1970 Wilets and collaborators wrote a paper[1]) entitled "Nucleon Dynamics and the Nuclear Many-Body Problem. I. *The Free Nucleon Problem*". Nevertheless, it is only in the past few years that models relevant to nuclear structure have been constructed, which include the quark structure of the nucleons.

Once one admits quark degrees of freedom into the nuclear many-body problem another hypothetical state of matter, quark matter, must be considered. Indeed one might ask whether the centre of a large nucleus is not quark matter for some percentage of the time[2]). This question may be most amenable to experiment in the baryon number two system, and the question of exotic (six quark) states and their relationship to the so-called dibaryon resonances is now highly controversial.

With this brief outline of the major issues let us begin with the fundamentals.

2. The pion nucleon interaction

The exact nature of the Hamiltonian describing pion-baryon interactions is basic both to pion nucleus scattering and to nuclear structure physics. One set of calculations of pion-nucleus scattering, for example those at Erlangen[3]), Regensburg[4]), Helsinki- and Argonne-Vancouver[5,6]) (to name a few) begin with an effective Lagrangian describing a $\Delta N\pi$ vertex. The Δ is viewed variously as a stable particle, or the (unstable) (3,3) resonance. Only in the work of Betz and Coester[6]) is a serious attempt made to describe the pion-nucleon dynamics, but in that work the $NN\pi$ vertex is omitted. An alternative approach is to construct a potential model of pion-nucleon scattering, with various degrees of sophistication in the treatment of relativistic corrections. The latter method has been used in all three-body treatments of the πNN system [except ref.[6])][7]), in many optical model calculations of pion nucleus scattering[7]), and in the Δ-hole calculations of our distinguished chairman and his collaborators[8]). At this conference Nakano (G15) has presented a new set of separable interactions. Bhakar (G1) has constructed an impact parameter representation for the πN scattering amplitude for use in eikonal calculations of pion-nucleus scattering. Of most interest to this discussion is the fit to πN scattering in the P_{11} channel obtained by Mizutani (G35). Following procedures set out as long ago as 1974 by Mizutani and Koltun[9]), he divides the P_{11} amplitude into a piece corresponding to the nucleon pole, and a background term which interferes with it. In this way, as we shall discuss briefly in sect. 3, one can separate the 'effect of absorption' on elastic πd scattering from the purely potential scattering contribution.

Unlike the work of Rinat on the same subject (but placed in another section), Mizutani makes no attempt to derive the P_{11} interaction from a microscopic theory. This is not intended as a criticism, because the relativistic three-body problem is very difficult, and it makes sense to treat the two-body subsystems in as simple a phenomenological way as possible. Rather, we wish to stress that there has been a paucity of attention paid to the underlying theory of pion-nucleon dynamics. Indeed, the major efforts that have been made, have been within the framework of Chew-Low theory[10,11]). In ref.[7]) Landau and I gave arguments relating the range of phenomenological, separable πN interactions to the analytic structure of the Chew-Low amplitudes.

For most of its existence, medium energy physics existed happily oblivious to the great progress made in high energy physics, and in particular the quark models of baryon structure. At the Houston meeting I asked Gerry Miller a rather confused question: "While the Chew-Low model is a useful model of the P_{33} resonance, it is very dated. Since then we have discovered ... quarks etc. In that model there is unambiguously an elementary $\Delta \equiv (qqq)$ state. ... Is it not possible that the truth about the πN interaction is that the elementary Δ contributes a short-range piece, while the πN rescattering ... results in a relatively long range piece of the interaction? On a more philosophical level, why must physics be split into two non-overlapping camps ... ?" Since that meeting, with the help of a Ph.D. student, S. Théberge, and prompted initially by the letter of Brown and Rho[12]), we have gone a long way toward joining these two areas of physics[13-15]).

The essential ingredient in the development of the cloudy bag model, is the observation that chiral symmetry, $SU(2) \times SU(2)$, is one of the best symmetries of the strong interaction[16]). On the other hand, the MIT bag model[17,18]) violates this symmetry very badly[19]). (In fact, although we take the MIT bag as the starting point because of its pedagogical advantages as well as its success, the same symmetry considerations apply to all models of quark confinement.) As discussed by a number of people[12-15,19-22]), the most economical way to restore this symmetry is to introduce the pion as a compensating field. In the present, crude stage of development the pion is elementary, and its structure enters only through the decay constant "f" (93 MeV). Needless to say, such an approximation is like a "long wavelength" approximation and will fail if multi-pion (or short range) effects are significant.

For the cloudy bag model (CBM), it has been rigorously established that the theory is self-consistent. That is, Dodd, Alvarez-Estrada and myself[23]) have managed to put a rigorous bound on the probability (P_n) of finding n pions in the physical nucleon, namely $P_n < \Lambda^n/n!$ Furthermore the average number of pions, $\langle n \rangle$, is less than Λ. For the parameters of the CBM[14]), $\Lambda = 0.9$, in comparison with Chew-Low for which $\Lambda \sim 2.2$. It appears that even these bounds are generous as the calculated values of P_1 and P_2 are of order 35% and 5% respectively. Thus to speak of a pion cloud about the nucleon is a little exaggerated—50% of the time it is just a bag. The implications of this model for nucleon electromagnetic properties have been calculated. For the magnetic moments, and for the neutron charge radius, the CBM[14,24]) provides a significant improvement over the MIT bag model[25]). The model also gives, by construction, the correct induced pseudoscalar piece of the axial current (which is absent in the bag model). The very interesting consequences of the model for proton decay are presently being calculated.

From the present point of view we are most interested in the consequences of the CBM for πN scattering. These are relatively straightforward to estimate, because in the limit where non-linear pion effects are eliminated (consistent with a long wavelength approximation) the full Lagrangian density[14,15])

$$\mathcal{L}_{CBM}(x) = (i\bar{q}\not{\partial} q - B)\theta_V - \frac{1}{2}\bar{q}\, e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f} q\Delta_s + \frac{1}{2}(D_\mu\underset{\sim}{\phi})^2 - \frac{1}{2}m_\pi^2\,\underset{\sim}{\phi}^2 , \qquad (2.1)$$

reduces to

$$\mathcal{L} \simeq \mathcal{L}_{MIT} + \mathcal{L}_\pi + \mathcal{L}_{int} . \qquad (2.2)$$

The first two pieces describe free MIT bags and pions respectively, while $\mathcal{L}_{int}$ describes how they interact

$$\mathcal{L}_{int} = -\frac{i}{2f}\bar{q}\,\gamma_5\underset{\sim}{\tau}\, q \cdot \underset{\sim}{\phi}\,\Delta_s , \qquad (2.3)$$

where Δ_s is $\delta(r-R)$ in the case of a static bag of radius R. If we finally define a P space of non-exotic baryons (3q) and mesons ($q\bar{q}$—except the pion)—all stable—and neglect effects of coupling to Q(Q = 1-P), the Hamiltonian corresponding to (2.2) and (2.3) looks like most people's starting place

$$H = \sum_\alpha m_\alpha^{bag}\,\alpha^+\alpha + \sum_k a_k^+ a_k w_k + \sum_{\alpha\beta k}(V_k^{\alpha\beta}\,\alpha^+\,\beta\,a_k + \text{h.c.}) . \qquad (2.4)$$

Here α destroys (e.g.) a stable three-quark N or Δ bag, and a_k destroys a pion of isospin and momentum k. The key point is that the coupling constants and vertex functions can be calculated from (2.3) and the appropriate wave functions.

Using SU(6) wave functions for the N and Δ one of course obtains coupling constants ($NN\pi$, $\Delta N\pi$, $\Delta\Delta\pi$) with the SU(6) ratios. Moreover, since the N and Δ bags have essentially the same radius, the vertex function (or high momentum cut-off), is the same at all these vertices

$$u(k) = j_0(kR) + j_2(kR) \xrightarrow[k\to\infty]{} \frac{1}{k^2} . \qquad (2.5)$$

Clearly in calculating πN scattering in the region of the (3,3) resonance one is now not free to calculate *either* the series of Chew-Low graphs, fig. 1(a), (b) etc., or the Δ-formation and decay, fig. 1(c), (d), (f) etc. But one must calculate both sets and in addition interference terms—fig. 1(e) and (f). (The latter could be incorporated into a renormalisation of the $\Delta N\pi$ vertex, however, they contribute to unitarity in a very significant way and are therefore treated explicitly.)

The results of such a calculation of πN scattering in the (3,3) channel are shown in fig. 2. Clearly those graphs involving Δ-excitation dominate, but the

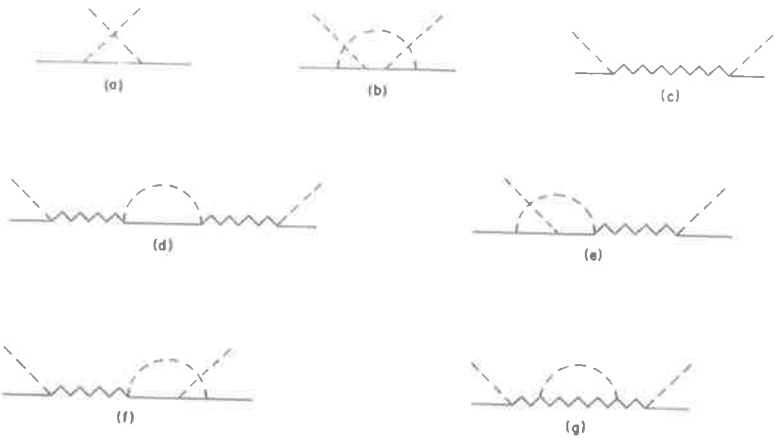

Fig. 1. Various low order contributions to πN scattering in the (3,3) channel in the CBM[13].

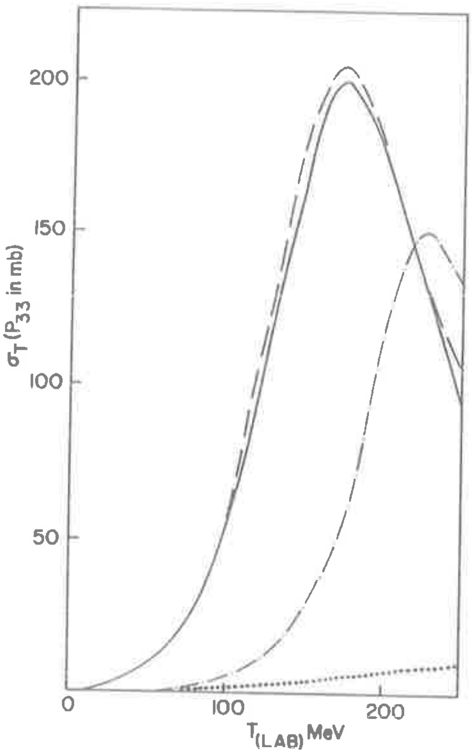

Fig. 2. Best fit (dashed curve) to πN scattering data (solid) in the (3,3) channel using the CBM[13]. The dash-dotted (dotted) line shows the effect of arbitrarily setting $f_{NN\pi}$ ($f_{\Delta N\pi}$) to zero, with all other parameters unchanged.

interference terms are not negligible. In our original work the bag radius found from this fit was R = 0.72 fm[13]. More recently a more sophisticated fitting procedure, in which all vertex renormalisations were calculated to lowest order before πN scattering was calculated, gave R = 0.82 fm[14]. The latter was used in the calculations of nucleon properties noted earlier.

A remarkable feature of the CBM, consistent with the convergence noted above, is that the renormalisation of coupling constants is not only finite, but small! Unlike Chew-Low theory, where the ratio of renormalised to unrenormalised $NN\pi$ coupling constants is approximately $\sqrt{1/3}$ (ref.[26]), in the CBM (with R > 0.7 fm) the ratio is larger than 0.9. This is essentially because of the contributions to Z_1 from the intermediate Δ (see fig. 3) which are not there in Chew-Low. Finally we remark that although the underlying Hamiltonians differ for Chew-Low and the CBM, the πN t-matrix resulting from both satisfies the Low equation[13,27]. It is probably for that reason that the πNN vertex functions derived by us (eq. (2.5), R = 0.82 fm) and by Ernst and Johnson[10] are essentially identical to $k = 4\ fm^{-1}$, which is all that matters in practical calculations.

Let us briefly review what has been achieved so far. We have constructed a very simple theory which includes the quark degrees of freedom at short distances, in addition to the long range pion effects known and loved by nuclear physicists. All of this has been motivated on the basis of known symmetry properties of nature. Of course, the price we have paid is that for the moment the bag is static, so we have none of the sophistication of (say) the Blankenbecler-Sugar equation. The extension of the theory to include recoil[28] and relativistic corrections can and should be carried out. However, the remarkable feature is that eqs. (2.1)-(2.5) provide a framework

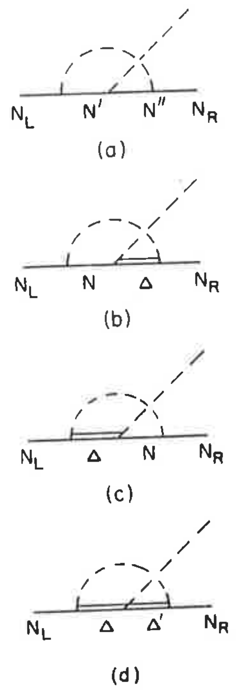

Fig. 3. Contributions to the vertex renormalisation of the NNπ coupling constant in the CBM[24]. The presence of the Δ greatly reduces the overall difference between $f^{(r)}_{NN\pi}$ and $f^{(o)}_{NN\pi}$.

(with outstanding convergence features in the one nucleon sector) within which we can reasonably expect to formulate, and perhaps resolve, many of the major issues raised in sect. 1!

It is quite obvious that the techniques mentioned above for the P_{33} resonance can be extended to other unstable hadrons in a straightforward way. Rinat has recently applied these ideas to the problem of πN scattering in the P_{11} channel[29]. As we mentioned in connection with Mizutani's contribution (G35) this is of interest because it contains the s-channel nucleon pole. For that reason the P_{11} phase shifts start off small and repulsive (negative) before passing through zero at about 150 MeV and finally resonating (the Roper resonance) at 520 MeV. In the CBM description one must include a stable Roper bag state, plus coupled Nπ and Δπ channels. Rinat finds that he can fit the P_{11} data with parameters quite consistent with those expected in the CBM, provided the Roper is predominantly a $(1s^2,2s)$ configuration.

In another recent application, the CBM has been used to estimate the σ-commutator in pion nucleon scattering[30]. They conclude that when all corrections are included, the data is consistent with a value of 25 MeV suggested earlier by Banerjee[27]. Of particular interest with respect to s-wave scattering is the discovery[31] that it is possible to define a transformation on the quark fields ($q \to q_w$) so that $\mathcal{L}(x)$ in eq. (2.1) becomes

$$\mathcal{L}'(x) = (i\bar{q}_w \not{D}\, q_w - B)\theta_V - \frac{1}{2}\bar{q}_w q_w + \frac{1}{2}(D_\mu \underline{\phi})^2 - \frac{1}{2} m_\pi^2 \underline{\phi}^2 + \frac{1}{2f}\bar{q}_w \gamma^\mu \gamma_5\, \underline{\tau}\, q_w \cdot D_\mu \underline{\phi} \ . \qquad (2.6)$$

Some consideration of the two possible forms (2.1) and (2.6) has led us to consider the fascinating possibility that perhaps q_w should be identified as the MIT-bag-quarks. In this case the new quark fields belong to a different, non-linear representation of SU(2) × SU(2) and $\bar{q}_w q_w$ is in fact invariant. Equation (2.6) incorporates the results of current algebra much more clearly, as the pion coupling is pseudovector, and the covariant derivative on the quark fields gives rise, in a very natural way, to the Weinberg-Tomozawa relation for s-wave pion scattering from any bag.

In concluding this section we mention two other attempts to derive the NNπ coupling from quark dynamics[32,33]. Weise calculates the amplitude for emission of a composite ($q\bar{q}$) pion from the nucleon, and finds that this happens most readily at the surface[32]. Weber attempts to derive a one boson exchange model of N-N scattering from QCD[33]. In neither case is it possible to tackle anything like the range of problems already treated by the CBM, and neither used SU(2) × SU(2) as a constraint. Nevertheless the inclusion of the structure of the pion itself is perhaps the major obstacle which the CBM must overcome in the near future.

3. The πNN system

One of the problems of a broad conference like ICOHEPANS is that physics tends to become compartmentalised. The problem of pion-deuteron scattering provides an excellent example. In many ways the πd system is a prototype for the general pion-nucleus problem. We can test many of the approximations common to the general problem (e.g. the static approximation, semi-relativistic treatments, binding corrections etc.), in a system where the answer can be exactly calculated. However, the problem of πd scattering can not be decoupled from the reaction $\pi d \rightarrow NN$, and indeed N-N scattering itself. It is a real shame that these connections are lost by putting relevant papers in at least three sessions, dealing with the N-N interaction, pion production and this one! This is particularly relevant in the discussion of dibaryon resonances in N-N scattering, because the gross features of this phenomenon are undoubtedly associated with the opening of $N\Delta$ inelasticity, and sensible conclusions are impossible unless the theoretical treatment guarantees two- and three-body unitarity.

Since the three-body theory of the πNN system was reviewed very thoroughly in ref.[7]), including a concise definition of the effect of absorption, we shall review only the most recent developments. In preparing these comments I have benefitted greatly from the simultaneous presence at TRIUMF of M. Betz, B. Blankleider, J. Niskanen and A.S. Rinat, and in particular from several lively group discussions.

In ref.[7]) we placed greatest emphasis on the attempts to derive equations for πd scattering which guaranteed exact two- and three-body unitarity. These are usually based on equations of the Freedman-Lovelace-Namyslowski type[34]), but as we mentioned, such equations do have factorisation problems. Several attempts have been made to overcome this[6,35]), and one recent attempt has been reported at this conference (G4). The real achievement for the πNN system has been the rigorous derivation (from an underlying field theory, but in the one-pion approximation) by many groups[36]) of equations of the type Afnan and I postulated almost ten years ago[37]). [Kobayashi *et al.* have reported another derivation at this conference (G11).] These equations couple *in a unitary way* the amplitudes for $\pi d \rightarrow \pi d$, $\pi d \rightarrow NN$ and $\pi d \rightarrow NN\pi$ (e.g. through $N\Delta$).

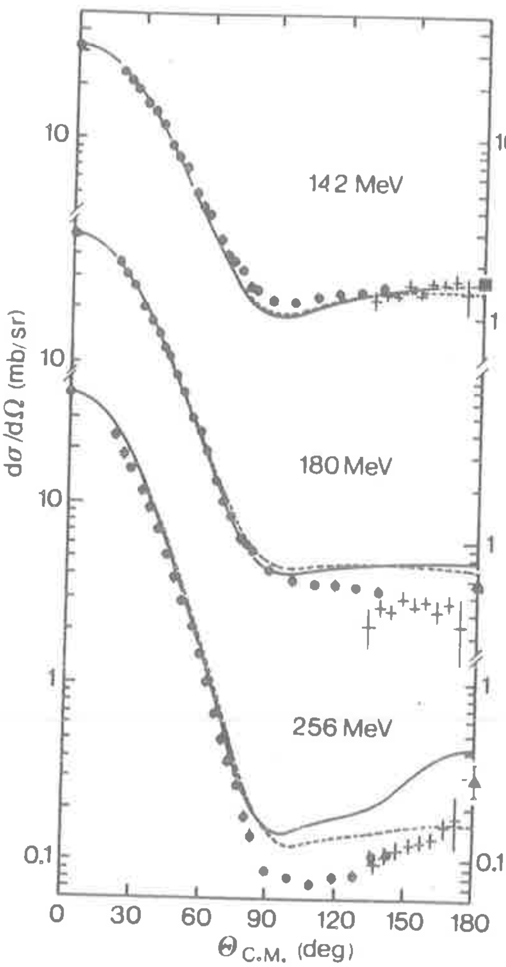

Fig. 4. Calculations of πd elastic scattering [by Fayard *et al.*[38])] without the P_{11} interaction (dotted curve), and with the P_{11} interaction (solid)—the latter includes the effect of absorption.

Figures 4 and 5 show the most recent predictions of the Lyon[38]) and Flinders-TRIUMF[39]) groups for πd elastic scattering, including the "effect of absorption". Similar calculations were submitted to the conference by Rinat (G2), improving slightly upon earlier work where the treatment of NN intermediate states was not completely self-consistent[40]). The remarkable feature of the Afnan-Blankleider calculation[39]) is that the infamous dip at 100° at 256 MeV[41]) is now fitted! Nevertheless, the backward angle differential cross section is still not well reproduced.

Realizing how sneaky theorists can be when given only differential cross sections, our experimental colleagues have made heroic efforts to measure polarisation observables for the πd system. Boschitz and collaborators managed to measure the vector polarisation, it_{11}, at 142 MeV and 256 MeV last summer[42]). The dramatic disagreement with three-body predictions at 256 MeV provided a great deal of excitement, because the oscillations coincided exactly with a published prediction of the effect of a 3^- dibaryon resonance, coupling to πd in $\ell_\pi = 4$

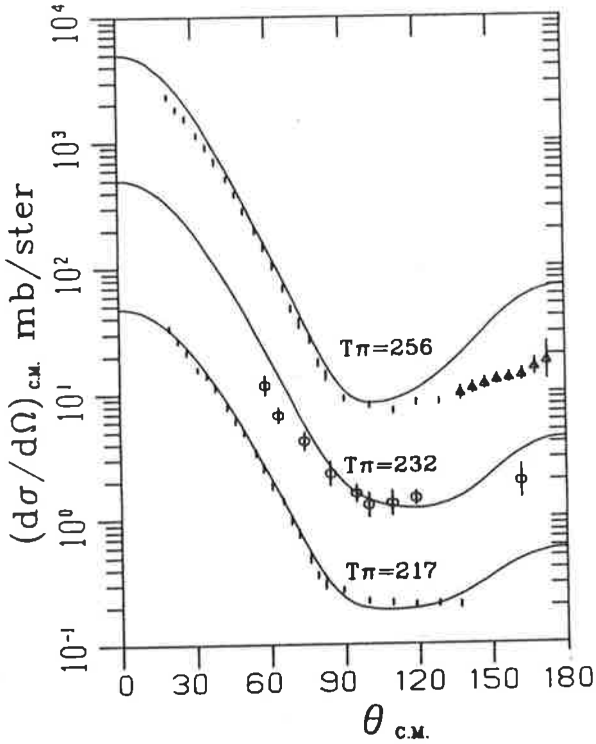

Fig. 5. Calculations of πd elastic scattering including the effects of absorption—from Blankleider and Afnan[39]).

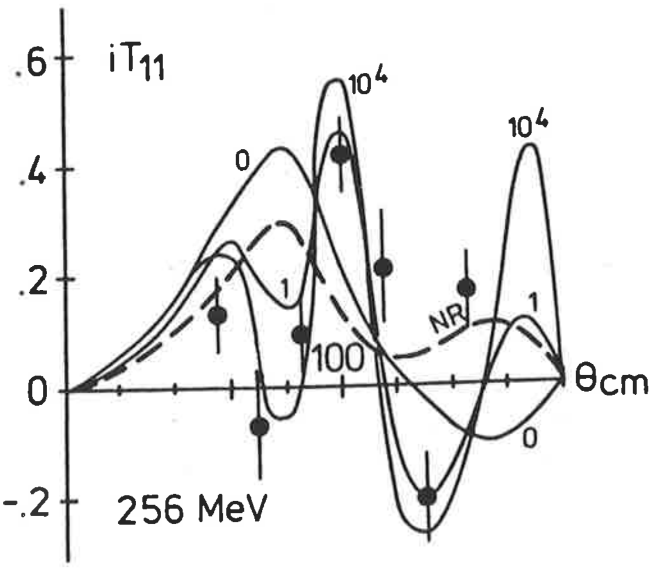

Fig. 6. Predictions[43]) of the effect of the supposed 3^- dibaryon resonance on the vector polarisation in πd scattering. The strength is put arbitrarily into $\ell_\pi = 2$ (label 0), $\ell_\pi = 4$ (10^4) and equally in each (1). The data is from Boschitz *et al.*[42]).

(ref.[43])—see fig. 6.

At this conference Arvieux—contribution G3, see also ref.[44])—has attempted to bring the community back to earth by pointing out that his phase-shift analysis is more consistent with a change in the real part of the dominant 2^+ amplitude. Now, it must be remarked that because of the strong spin dependence and inelasticity in πd scattering, a phase-shift analysis is a non-trivial exercise. Indeed one must rely on a theoretical calculation for most of the 15 or so (n,δ) pairs, leaving only a selected few on which one searches. Thus one can never deny the possibility of bias in the starting values, so that one does not actually find the true χ^2 minimum. Nevertheless, Arvieux's observation is both timely and sobering.

Even more recent is the extension by Holt *et al.*[45]) of their measurement of t_{20} away from 180°[46]). As we see in fig. 7, the pure three-body theory (with or without absorption) fails again. Nevertheless, with the previous analysis of Arvieux fresh in our minds we should not be too hasty in attributing the difference to dibaryon resonances—as much as we may believe in exotic, six quark states! All polarisation observables are sensitive to interference between amplitudes, and a relatively small change in the real part of an apparently small amplitude can sometimes produce dramatic effects.

At this stage it is worthwhile to recall an alternate approach to the problem of NN coupling to NΔ due to Niskanen and Green[47-49]). While their approach does not guarantee exact two- and three-body unitarity, much greater care is taken in the construction of the transition potentials (e.g. NN → NΔ). For example, all time orderings are included, whereas in the three-body case only forward going pions are included. We have chosen to mention this here because, whereas unitarity constrains the imaginary parts of the transition potentials, the real parts are sensitive to everything included. Thus it is quite conceivable that the discrepancies found in it_{11} and t_{20}, which we have just described, could be remedied by a

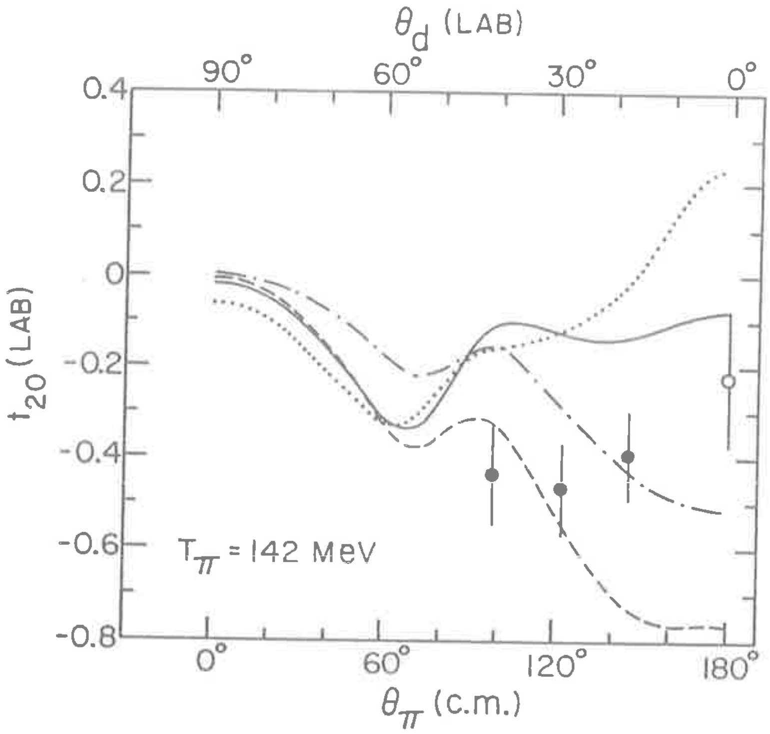

Fig. 7. Recent experimental results from Holt *et al.*[45]) in comparison with the calculations of refs.[40]), [40]) without absorption, [38]) and [6])—dotted, dashed, solid and dash-dot respectively.

slightly less strict adherence to the one-pion approximation.

In closing this section we must comment on the calculation of the absorption channel $\pi^+d \rightarrow pp$. It is important for two reasons. Firstly, if one is to assess a calculation of the effect of absorption on elastic scattering, it is important to know how well the absorption is described. Secondly, in the next section we shall comment on several contributions dealing with pion absorption on nuclei. Obviously we should like to carry into that problem any relevant information from the few-body system.

By far the most extensive calculations of $\pi^+d \rightarrow pp$ have been carried out by Niskanen. For a detailed discussion of the status of the coupled channels calculations vis à vis the many polarisation data that have so far accumulated, we refer to his talk at last year's polarisation conference[47]). From the theoretical point of view there are two main lessons. Firstly, it is not sufficient to calculate simply one rescattering through the Δ. The multiple scattering series is rather slowly convergent. Secondly, the type of closure approximation employed in ref.[50]) can be very inaccurate, particularly at the energy of the (3,3) resonance.

Several groups have applied relativistic three-body equations to the $\pi^+d \rightarrow pp$ reaction, but the agreement with data has not been impressive[51]). However, Afnan and Blankleider have reported some calculations at this conference [H5 and ref.[39])], which show extremely impressive agreement with the $\pi^+d \rightarrow pp$ differential cross section over a very wide energy region. A sample of these results is shown in fig. 8. It is not yet completely clear why this fit is so much better than that of Rinat and collaborators [G2 and ref.[51])] but this reaction does seem to be very sensitive to the off-shell behaviour of the P_{11} interaction, and particularly the $NN\pi$ vertex function. It would be worthwhile to consider imposing some theoretical constraints on the vertex function—say using the CBM.

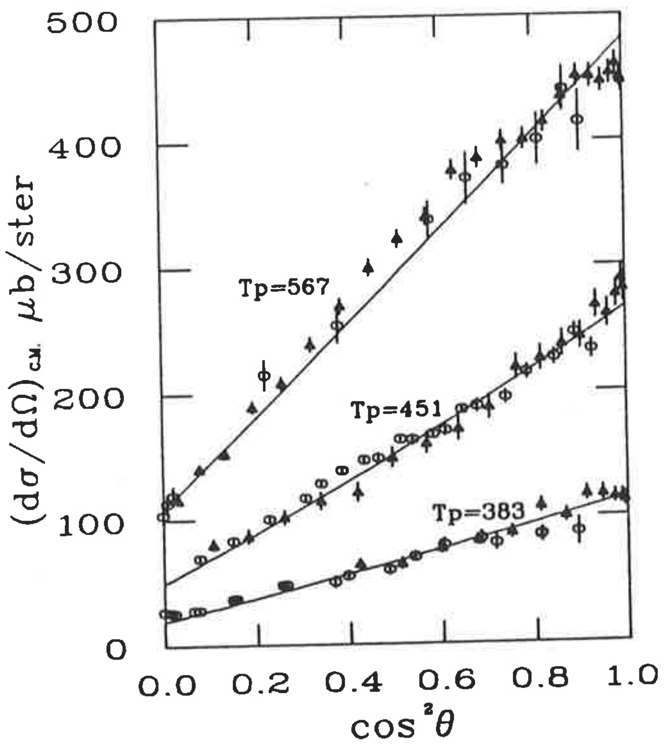

Fig. 8. Calculations of the differential cross section for $pp \to \pi^+d$ by Afnan and Blankleider[39]).

4. Pion nucleus scattering and reactions

4.1 ELASTIC SCATTERING

It is very natural that the first problem one should tackle in this field is elastic scattering. For that the standard technique is a systematic development of the microscopic optical potential[7,52]). A popular alternative in pion physics is the calculation of the eigenmodes of a Δ in the nucleus[3,4,8,53]). In principle these two approaches are identical—in the first it is the pion self-energy, and in the second the Δ self-energy, which is calculated in the medium. The only real question is which method provides the most convenient framework to incorporate the essential physics of the problem.

There is little disagreement over which corrections to the impulse approximation need to be included. At this conference we have reports on new calculations of Pauli corrections (G12, G16, G17), binding corrections (G8), relativistic effects—based (G18) on the work in few body systems mentioned in sect. 3—and, of course, absorption (G14, G16, G19). There is little doubt that the exact inclusion of binding corrections[54]) is extremely difficult within the optical model formulation, but up to order (m_π/m_N) it appears very naturally in the Δ-h formulation.

On the other hand, the large number of states appearing in the Δ-h formulation for a nucleus with $A > 16$ appears prohibitive. This technical difficulty has led the University of Washington group [contribution to this conference and ref.[55])] to develop an alternative calculational technique (not limited by A) in which one directly solves a differential equation for the effective Δ wave function, and from that obtains an optical potential. (In many ways this is closely related to the even more phenomenological approach of Kisslinger and collaborators[56,57]). For the present it has been necessary to approximate the Δ-nucleus self-energy as a local potential—despite the fact that Pauli blocking, true absorption and so on are certainly highly non-local. It remains to be seen whether this assumption can be improved in future.

While a great deal of progress has been made in including most of the corrections listed above, the effect of absorption has intimidated almost everyone. With the exception of the crude attempts of refs.[58] and [59]—see also Phatak's contribution, G19, to this meeting—the effect of absorption on elastic scattering is not calculated, but merely parametrized either as a ρ^2-potential[7,60], or as a local Δ-nucleus "spreading potential" (ref.[8]) and also contribution G14). For this reason, it is not possible to claim to understand the pion-nucleus interaction in any detail.

Of course, at relatively low pion energies the elementary πN interaction is quite weak, so that the pion has a mean free path of (5-6) fm—comparable with a K^+. For this reason one expects the higher order corrections to the optical potential to converge quickly. [Explicit results supporting this were reported to the conference by Mach (G7).] Thus, if we are ever to claim to understand pion-nucleus dynamics well enough to use pions as a probe of nuclear structure, 50 MeV is the most likely energy region. There is still the poorly understood effect of absorption, but most (crude) estimates support the approximation of an approximately local ρ^2-term below 50 MeV[50,59]. In addition, one can use reaction content arguments[61-63] to constrain the parameters of this phenomenological term—see contribution G16 for some recent analysis along these lines.

The classical testing ground for the low energy pion-nucleus optical potential is the pionic atom[60,61]. Only recently has the semi-phenomenological approach taken there run into difficulty—particularly for 3d (but also 4f) orbits of heavy nuclei. Ericson has made a suggestion that there should be an extra contribution to the pion-nucleus interaction as a result of gauge invariance (G10). Apparently this correction has its greatest influence in the 3d state, for larger Z, and so we do not yet have grounds to doubt the pionic atom phenomenology.

At the Berkeley conference, the use of low energy pion scattering was compared with all other probes for sensitivity and apparent model independence in the extraction of nuclear density information[65]. The major advantage of low energies, apart from the long mean free path mentioned earlier, is the selectivity of the πN p-wave interaction for neutrons (in $\ell = 1$, $f_{\pi-n} : f_{\pi-p} \cong 10:1$). Figure 9 shows the results of a TRIUMF experiment[66] for π^- scattering on ^{18}O and ^{16}O, from which we deduced a difference in rms neutron radii of 0.21 ± 0.03 fm—where the error includes all model dependence imagined by the authors.

In contrast with the apparent model independence of density parameters extracted at low energy (and a lot of work remains to be done!), the quantitative analysis of data in the region of the (3,3) resonance is extremely uncertain. Although Dedonder *et al.*[67] in their analysis of the $^{16,18}O$ data of Jansen *et al.*[68] find a difference in neutron radii of perhaps 5% [a little smaller than, but not inconsistent with, ref.[66]], they do remark on the strong model dependence. The general reasons for this model dependence were discussed in ref.[65], but the essential problem is the lack of convergence of the higher order corrections to

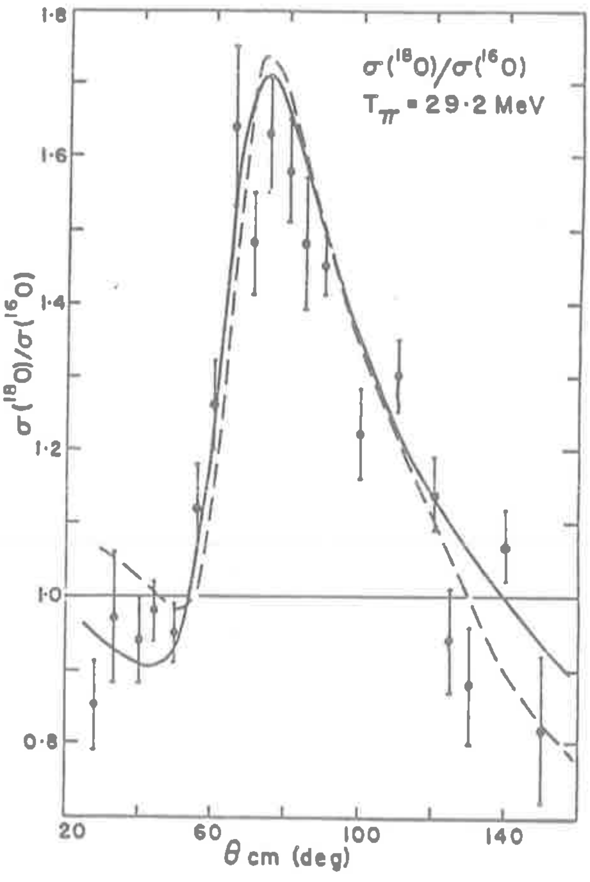

Fig. 9. The ratio of the π^- differential cross sections on ^{18}O and ^{16}O at 29 MeV measured at TRIUMF, in comparison with two different theoretical models[66].

the optical potential. One infamous example of this is the reflection, or local field correction involving multiple pion rescattering between correlated nucleons[69]. Although I have seen no contribution to this conference on that problem, the most recent calculations of the Maryland group suggest it is a sizeable correction[70].

4.2 REACTIONS

From our point of view there are two major reasons for studying reactions on nuclei induced by pions. The most compelling reason is that, as discussed in sect. 4.1, the "effect of absorption" is the worst understood feature of elastic scattering. It is eminently sensible, therefore, to study the absorption process itself with an aim to understanding that directly. From a more phenomenological point of view, the effects of absorption are variously parametrized as Δ-spreading potentials (G14, refs.[8], [53], [55] etc.), or pion potentials with a non-linear density dependence. Given good search programs, elastic scattering alone can not distinguish between different models. Thus one searches for reactions which should show some sensitivity to the off-shell behaviour of the pion-nucleus interaction[71].

Amongst the reactions most likely to yield information on the off-shell behaviour of the pion nucleus interaction one might think of inelastic scattering (G21-G23)[72], photoproduction [see session C, especially the (γ,π^0) reaction C23 and ref.[73]], quasi-elastic scattering [G36 and ref.[74]], and single and double charge exchange (G37-G39). With respect to inelastic scattering we recall Keister's study of the sensitivity to phase-shift-equivalent transformations on the pion distorted waves[72]. He found little off-shell sensitivity for experimentally accessible states, where the nuclear structure is well known. The contributions to this conference deal mainly with the effect of channel coupling on the elastic channel (G21 and G23). For ^{6}Li Mach found significant effects from channel coupling, which could be important in the analysis of new TRIUMF data for ^{6,7}Li at low energy.

Single (SCX) and double charge exchange (DCX) provide a remarkable contrast in sensitivity to details of the theory[75-78]. It is now well known that, for example, small changes in the phenomenological Δ-spreading potential in different isospin channels can lead to order of magnitude changes in SCX, and particularly DCX cross sections[76,77]. The essential problem is that the charge exchange amplitude (for analogue transitions) is the difference of two amplitudes (or three for DCX) which are almost identical. Thus one is sensitive to very small effects. At this conference this has been emphasised by Liu (G37) and Oset and Strottmann (G39). In the former case it is shown that channel coupling caused by the Coulomb interaction can remedy the famous anomaly in the position of the first minimum of the $^{18}O(\pi^+,\pi^-)^{18}Ne$ reaction[79,80]. In the latter it is shown that meson exchange currents, of the type considered earlier by Germond and Wilkin[81], can even dominate the DCX amplitude at some angles! Unfortunately, it would seem that at the present time charge exchange exhibits too much sensitivity to provide a useful constraint on the theory used for the pion-nucleus interaction.

In concluding this necessarily abbreviated discussion of pion reactions let us return to the description of absorption per se. Obviously the next simplest system after πd is π^3He, and a detailed calculation of this system has been submitted to the conference by Schucan and Nägeli (G27). This system was also discussed by Yoo and Landau in a recent preprint[81]—although that work was rather more phenomenological. [Incidentally Yoo and Landau did find a significant spin-flip component in the ρ^2-potential used to approximate the effect of absorption. This is of some importance if one hopes to learn about the neutron distribution in ^{3}He using pion scattering[82].]

The calculation by Schucan and Nägeli involved several versions of the effective two-nucleon operator for pion absorption[83], and different off-shell behaviour in the πN system. While the total absorption rate for a stopped π^- is essentially the same for all these models, the differential spectra are rather different. A detailed comparison with the recent SIN data[84] should be most informative. One further advantage of π^3He is that it is not unthinkable that one could actually calculate explicitly the dispersive part of the absorptive contribution to elastic scattering. This would be a significant step towards understanding the effect of absorption in the general case.

Let us mention several other contributions (G20, G28 and G30) in which calculations are made for the (π,N) reaction. Of most interest is the calculation of Liu Xian-hui and Chen Xue-jun (G20) in which the Δ-h model is used to describe the distortion of the outgoing pion. Of course, a number of important effects have been omitted, such as distortion of the outgoing nucleon wave function. Nevertheless this work constitutes an attempt to come to grips with the most difficult reaction in intermediate physics using a microscopic description of the pion-nucleus interaction.

Finally we note that a large number of cascade calculations of pion absorption and inelastic scattering were submitted to the meeting (G24, G25, G29, G31 and G33). It is hoped that such calculations can give some overall guidance as to the mechanism whereby the pion deposits its energy in the nucleus. In my view such calculations do serve a useful purpose, but in the end there is no substitute for the full quantum mechanical treatment. This attitude should encourage lively discussion at the meeting!

5. Conclusion

In the many details of the preceding pages it is easy to lose sight of our purpose and achievements in pion-nucleus theory. We are slowly discovering new insights into the nuclear many body problem. These insights involve new techniques for including quark degrees of freedom and isobar excitation. In the near future we may hope to see not only corrections to old ideas, but perhaps new and unexpected phenomena. The crudeness of the models in many areas should be an encouragement to do better. It is an exciting and challenging time to work in this field of physics.

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Up-down quark mass differences in the MIT bag model

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The contribution of quark masses to the splittings of hadron isospin multiplets is calculated in a phenomenological way within the framework of the MIT bag model. A value for the up-down quark mass difference is extracted.

It is generally accepted that the mass splitting of hadrons within an isospin multiplet is due to electromagnetic (EM) interactions between quark pairs combined with effects due to the difference between the u- and d-quark masses, i.e.,

$$\Delta M=(\Delta M)_{\rm EM}+(\Delta M)_q \; . \tag{1}$$

Deshpande *et al.*[1] have calculated $(\Delta M)_{\rm EM}$ in the MIT bag model and have obtained contributions comparable to those of the naive quark model.[2] However, as is well known, $(\Delta M)_{\rm EM}$ is of the wrong sign to explain the mass difference between the proton and neutron (as well as several other important mass differences) and the term $(\Delta M)_q$ must be invoked, with m_u less than m_d.

Deshpande *et al.* attribute $(\Delta M)_q$ to electromagnetic self-energies of the quarks but, rather than attempt to calculate the differences directly, they prefer to parametrize them by

$$(\Delta M)_q=A/R+Bn_s+Cn_c \; , \tag{2}$$

where R is the bag radius, n_s and n_c are the number of s and c quarks, respectively, and A, B, and C are constants. They did not have any data with which to determine C but otherwise obtained the following fit for the baryons using the p-n, Σ^+-Σ^0, Σ^0-Σ^-, and Ξ^0-Ξ^- mass differences:

$$A_B=-8.95\times10^{-3} \; , \tag{3a}$$

$$B_B=-1.64\pm0.12 \text{ MeV} \; . \tag{3b}$$

However, for the mesons they took $(\Delta M)_q=0$ for the π and ρ (in line with usual quark-model expectations) and made a different fit to the K^+-K^0 and K^{*+}-K^{*0} mass differences obtaining:

$$A_M=-3.38\times10^{-3} \; , \tag{4a}$$

$$B_M=-4.10 \text{ MeV} \; . \tag{4b}$$

Thus they made no nontrivial predictions for $(\Delta M)_q$ in the meson sector. This work has been extended to b-flavored mesons by Singh[3] who, following earlier work by Deshpande *et al.*,[4] makes the even simpler assumption that $(\Delta M)_q$ is a constant.

The reasoning behind Eq. (2) was that the existence of a quark mass difference due to electromagnetic self-energies depends on the quarks being bound and therefore should vary as $1/R$. It was then argued that this $1/R$ variation may be modified by the presence of heavy quarks since these "provide an alternative mass scale." There are several points here which might prompt one to investigate further. First, the parametrization in Eq. (2) is, as Deshpande *et al.*[1] state, "a purely phenomenological one unrelated to the bag model." Second, the role of self-energies (even electromagnetic ones) in the bag model is not altogether clear and different mass scales are not normally considered in the standard MIT model.[5] Third, and we consider this most significant, the above parametrization does not yield a value for the difference in the u- and d-quark masses, $\Delta m=m_u-m_d$. We would like to use Δm as input to bag-model calculations of some dynamical effects due to m_u not being equal to m_d, which will be described elsewhere.[6]

One of the advantages of the MIT model is that it can be formulated in terms of a Lagrangian density of essentially QCD type inside the bag.[7] That is,

$$\mathscr{L}(x)=\left[\frac{i}{2}\bar{q}(x)\overleftrightarrow{\partial\!\!\!/}q(x)-\bar{q}(x)mq(x)-\tfrac{1}{4}F^{a\mu\nu}(x)F^a_{\mu\nu}(x)+g\bar{q}(x)A^a_\mu\gamma^\mu\lambda^a q(x)-B\right]\theta_v-\tfrac{1}{2}\bar{q}(x)q(x)\delta_s \; , \tag{5}$$

where $q(x)$ describes the quarks, m is a (conventionally) diagonal mass matrix, A_μ^a ($F_{\mu\nu}^a$) describes the gluon fields, B is the phenomenological energy density associated with the volume V, δ_s is a surface delta function, and θ_v is one inside V, zero outside. If n of the quarks have small masses one naturally has a fairly good $SU(n)\times SU(n)$ symmetry inside—of course the boundary condition $(-\frac{1}{2}\bar{q}q\delta_s)$ badly violates this (but that is another story).[7–10] Within this framework it is quite natural to assign the u and d quarks small, unequal masses. Furthermore one would expect to treat them as constant, just as the s-quark mass is kept constant,[5] even in the presence of c quarks.[11] The insertion of unequal u and d masses leads to mass splittings within multiplets, just as m_s leads to splittings within SU(3)-flavor multiplets.

At this stage we stress that we are only interested in mass splittings and not absolute masses. To calculate these splittings we proceed as follows. The MIT spherical-cavity Hamiltonian is given by a sum of four terms[5]:

$$E_v=\tfrac{4}{3}\pi R^3 B\ , \tag{6a}$$

$$E_0=-Z_0/R\ , \tag{6b}$$

$$E_k=\sum_{\text{flavors } a} n_a\omega(m_a R)/R\ , \tag{6c}$$

$$E_m=(\alpha_c/R)\sum_{\substack{\text{flavor}\\ \text{pairs } a\geq b}}\Delta^{ab}M(m_aR,m_bR) \tag{6d}$$

and the energy of a hadron is found by minimizing the energy eigenvalue with respect to the bag radius R. Now, within any isospin multiplet the radius from hadron to hadron changes very little and we can, because we are close to a minimum, replace R in Eq. (6) by an average value for the multiplet, R_{av}. The errors introduced by this approximation are very small—less than about 0.5%. (A similar approximation has been made, by Aerts *et al.*[12] over entire $SU(3)^{\text{fl}}$ multiplets, with reasonable success.) Thus the energy difference between two members X and Y of a multiplet is given by

$$(\Delta M)_q=\sum_{\text{flavors}}(n_a^X-n_a^Y)\omega(m_aR_{\text{av}})/R_{\text{av}}+\sum_{\text{flavor pairs}}(\Delta_X^{ab}-\Delta_Y^{ab})M(m_aR_{\text{av}},m_bR_{\text{av}})\alpha_c/R_{av}\ , \tag{7}$$

where n_a is the number of quarks of flavor a, ω and M are mode frequencies and interaction strengths respectively (for which analytic expressions may be obtained from Ref. 5), and Δ^{ab} is a coefficient representing the color-hyperfine interaction between the type-a quarks and the type-b quarks. The evaluation of Δ is a purely group-theoretic problem[5,13,14] and the required values are listed in Table I. We merely note here that to satisfy the flavor-symmetry limit the sum of the Δ^{ab} must equal -8 for spin-$\frac{1}{2}$ baryons, $+8$ for spin $-\frac{3}{2}$ baryons, -16 for pseudoscalar mesons, and $+\frac{16}{3}$ for vector mesons.

Thus we see from Eq. (7) that $(\Delta M)_q$ arises from two terms. The first is a kinetic-energy term and the second comes from the color-hyperfine interaction. The splittings of isospin multiplets that the latter introduces are of the same type as the Λ-Σ splitting introduced by $SU(3)^{\text{fl}}$ violations.[5] Such a term is also important in studies of exotic hadrons.[14] To apply Eq. (7) we use (in line with Deshpande *et al.*)[1] the values of R_{av} listed in Table III of DeGrand *et al.*,[5] $\alpha_c=0.55$, $m_s=279$ MeV (Ref. 5), and m_u and m_d as determined below.

The important ingredient is the quark mass difference Δm. This may be determined from the experimental proton-neutron difference by noting that for this case, provided that Δm is not too large, $(\Delta M)_q$ is conveniently expressed by

$$(E_p-E_n)_q=\left[\frac{d\omega(mR)}{d(mR)}+\frac{8}{3}\alpha_c\frac{dM(mR,mR)}{d(mR)}\right]\Bigg|_{\bar{m}R_{\text{av}}}\Delta m\ . \tag{8}$$

One has only to evaluate these derivatives for various values of the average quark mass, $\bar{m}=\frac{1}{2}(m_u+m_d)$, using $R_{\text{av}}=5.00\ \text{GeV}^{-1}$ and noting that $(\Delta M)_q$ must be -1.79 MeV, in order to determine Δm. The results are given in Table II. We remark that the color-hyperfine contribution is about 14% of that from kinetic energies and it has the important effect of increasing Δm from the value it would have in the presence of the kinetic term alone.

We can now proceed to calculate the multiplet splittings for all hadrons. The results are given in Table III using both $\bar{m}=30$ MeV and $\bar{m}=10$ MeV [alongside the results for $(\Delta M)_q$ and $(\Delta M)_{\text{EM}}$ obtained in Ref. 1]. As can be seen, the mass differences depend very little on the value of $\bar{m}$ used, the

TABLE I. Quark content and values of the color-magnetic coefficients Δ^{ab} for the hadrons.

Particle	Quark content	Δ^{uu}	Δ^{ud}	Δ^{dd}	Δ^{us}	Δ^{ds}	Δ^{ss}
p	uud	$+\frac{8}{3}$	$-\frac{32}{3}$				
n	udd		$-\frac{32}{3}$	$+\frac{8}{3}$			
Δ^{++}	uuu	$+8$					
Δ^{+}	uud	$+\frac{8}{3}$	$+\frac{16}{3}$				
Δ^{0}	udd		$+\frac{16}{3}$	$+\frac{8}{3}$			
Δ^{-}	ddd			$+8$			
Σ^{+}	uus	$+\frac{8}{3}$			$-\frac{32}{3}$		
Σ^{0}	uds		$+\frac{8}{3}$		$-\frac{16}{3}$	$-\frac{16}{3}$	
Σ^{-}	dds			$+\frac{8}{3}$		$-\frac{32}{3}$	
Σ^{*+}	uus	$+\frac{8}{3}$			$+\frac{16}{3}$		
Σ^{*0}	uds		$+\frac{8}{3}$		$+\frac{8}{3}$	$+\frac{8}{3}$	
Σ^{*-}	dds			$+\frac{8}{3}$		$+\frac{16}{3}$	
Ξ^{0}	uss				$-\frac{32}{3}$		$+\frac{8}{3}$
Ξ^{-}	dss					$-\frac{32}{3}$	$+\frac{8}{3}$
Ξ^{*0}	uss				$+\frac{16}{3}$		$+\frac{8}{3}$
Ξ^{*-}	dss					$+\frac{16}{3}$	$+\frac{8}{3}$
π^{+},π^{-}	$u\bar{d},\bar{u}d$		-16				
π^{0}	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	-8		-8			
ρ^{+},ρ^{-}	$u\bar{d},\bar{u}d$		$+\frac{16}{3}$				
ρ^{0}	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	$+\frac{8}{3}$		$+\frac{8}{3}$			
K^{+}	$u\bar{s}$				-16		
K^{0}	$d\bar{s}$					-16	
K^{*+}	$u\bar{s}$				$+\frac{16}{3}$		
K^{*0}	$d\bar{s}$					$+\frac{16}{3}$	

TABLE II. Values of the derivatives of the quark eigenfrequencies ω and the interaction strengths M between equal-mass quarks and the ensuing values for the up-down mass difference Δm for various values of the average mass, $\bar{m}=\frac{1}{2}(m_u+m_d)$. The multiplet radius is that of the nucleon, i.e., $R_{\rm av}=5.00$ GeV^{-1}.

$\bar{m}$ (MeV)	$\left.\frac{d\omega}{d(mR)}\right\vert_{\bar{m}R_{\rm av}}$	$\left.\frac{dM}{d(mR)}\right\vert_{\bar{m}R_{\rm av}}$	$\Delta m = m_u - m_d$ (MeV)
10	0.487	-0.047	-4.28
20	0.495	-0.047	-4.20
30	0.503	-0.047	-4.12
40	0.511	-0.047	-4.05
50	0.519	-0.047	-3.98

largest difference being 0.06 MeV in the K. We note that any value of $\overline{m}$ much larger than 30 MeV would be unreasonable. In fact, by the time $\overline{m}$ reaches 109.5 MeV the pion no longer exists in the MIT model.[5] (Donoghue and Johnson[15] obtain a best fit of 33 MeV in their study of chiral-symmetry breaking and center-of-mass corrections. Similarly, Frank *et al.*[16] estimate that $\overline{m} \leq 30$ MeV.) Thus we make all our predictions with essentially only one new parameter, Δm. We could easily extend these results to hadrons containing c and b quarks, without introducing any more new parameters, but we feel that the results would not be reliable enough to warrant doing so at this stage.

Inspection of Table III reveals that the mass differences obtained in our approach are somewhat different from the phenomenological fit in Ref. 1. In particular, there is no term in our approach corresponding to Bn_s. Also, while Eq. (7) does yield a $1/R$ behavior, the coefficient A which we obtain is not constant. If our $(\Delta M)_q$ were fitted by an A/R term, then A would vary from -8.5×10^{-3} to -12.6×10^{-3}. An important feature of the present calculation is the presence of the color-hyperfine term, which gives a contribution of as much as 30% of the kinetic-energy term. In its absence $(\Delta M)_q$ = constant would be a reasonable approximation—except for π and ρ.

In the baryon sector, our result for the splitting of the Ξ^* is better than that of Deshpande *et al.* However, the Σ and Ξ results are systematically worse because of the absence of a term like Bn_s. In the meson sector both models are bad for π and ρ, and while our results for K and K^* are not good, one cannot compare with Ref. 1 because there the two data points were fitted with two parameters. (This observation is not intended as a

TABLE III. Predicted mass differences. The first row for each entry corresponds to $\overline{m} = 30$ MeV and the second row to $\overline{m} = 10$ MeV. Quantities in parentheses have been used as input.

Particle difference	R_{av} (GeV^{-1})	$(\Delta M)_{\mathrm{kinetic}}$ (MeV)	$(\Delta M)_{\mathrm{color}}$ (MeV)	$(\Delta M)_q$ (MeV)	$(\Delta M)_q$[a] (MeV)	$(\Delta M)_{\mathrm{EM}}$[a] (MeV)	$(\Delta M)_{\mathrm{total}}$ (MeV)	$(\Delta M)_{\mathrm{experimental}}$ (MeV)
$p-n$	(5.00)	−2.07	+0.28	(−1.79)	(−1.79)	+0.50	(−1.29)	−1.293 43±0.000 04
		−2.09	+0.30					
$\Delta^{++}-\Delta^{+}$	(5.48)	−2.08	+0.28	−1.80	−1.63	+2.03	+0.23	
		−2.09	+0.30	−1.79			+0.24	
$\Delta^{+}-\Delta^{0}$			+0.28	−1.80	−1.63	+0.42	−1.38	
			+0.30	−1.79			−1.37	
$\Delta^{0}-\Delta^{-}$			+0.28	−1.80	−1.63	−1.06	−2.86	
			+0.29	−1.79			−2.85	
$\Sigma^{+}-\Sigma^{0}$	(4.95)	−2.07	−0.08	−2.15	(−3.45)	+0.32	−1.83	−3.10±0.14
		−2.09	−0.08	−2.17			−1.85	
$\Sigma^{0}-\Sigma^{-}$			−0.08	−2.15	(−3.45)	−1.30	−3.45	−4.860±0.077
			−0.08	−2.17			−3.47	
$\Sigma^{*+}-\Sigma^{*0}$	(5.43)	−2.08	+0.25	−1.83	−3.29	+0.39	−1.44	
		−2.09	+0.26	−1.83			−1.44	
$\Sigma^{*0}-\Sigma^{*-}$			+0.25	−1.83	−3.29	−1.11	−2.94	
			+0.26	−1.83			−2.94	
$\Xi^{0}-\Xi^{-}$	(4.91)	−2.07	−0.45	−2.52	(−5.10)	−1.50	−4.02	−6.34±0.08
		−2.08	−0.46	−2.55			−4.05	
$\Xi^{*0}-\Xi^{*-}$	(5.39)	−2.08	+0.22	−1.86	−4.94	−1.15	−3.01	−2.90±0.99
		−2.09	+0.22	−1.86			−3.01	
$\pi^{\pm}-\pi^{0}$	(3.34)	0	0.00	0.00	0	+1.16	+1.61	+4.6043±0.0040
			0.00				+1.61	
$\rho^{\pm}-\rho^{0}$	(4.71)	0	0.00	0.00	0	+0.94	+0.94	−0.3±2.1
			0.00				+0.94	
$K^{+}-K^{0}$	(3.26)	−2.04	−0.73	−2.77	(−5.14)	+1.15	−1.62	−3.91±0.15
		−2.07	−0.75	−2.83			−1.68	
$K^{*+}-K^{*0}$	(4.65)	−2.06	+0.23	−1.84	(−4.83)	+0.73	−1.11	−6.7±1.3
		−2.08	+0.23	−1.85			−1.12	

[a]These columns show the results of Deshpande *et al.*[1]

criticism of Deshpande *et al.*, who were interested in extending the predictions to charmed mesons.)

It is beyond the scope of this article to provide a solution to the discrepancies in Table III. Perhaps, as suggested in Ref. 1 there should be some R dependence of the quark masses arising from the electromagnetic self-energies of the confined quarks. However, such an effect was not found to be necessary in the work of Isgur.[17] Using a constant (constituent) mass difference for u and d, in the nonrelativistic quark model, he was able to obtain excellent agreement with data. His calculations are analogous to ours except for the inclusion of a "second-order" color hyperfine term, which provides a substantial contribution in the right direction. The origin of this term is the one-gluon-exchange—induced mixing with higher radial modes.[18] Such effects are usually very much smaller in the bag model.[19]

Of course, the mass differences are sensitive to the bag radius (R_{av}) used, and it has been observed in the context of some chiral bag models[8–10] that some of the MIT radii may be a little large. It could be worthwhile considering the consequences of such corrections for the present problem. Center-of-mass corrections will also affect the calculation of R_{av}, and this could be very significant for the pion.[15,16] Finally we note that if an effect of the kind proposed by Isgur did survive in the bag model, it would act to increase Δm and improve our value of -1.14 MeV for the Δ^{++}-Δ^0 splitting, the experimental value of which is -2.7 ± 0.3 MeV.[20] One would then have to explain why the combination $C_W = (\Delta^- - \Delta^{++}) + \frac{1}{3}(\Delta^0 - \Delta^+)$ agrees so well—that is, 4.47 MeV from Table III, compared with 4.6 ± 0.2 MeV.[20]

In summary, we have shown that the contribution of quark mass differences to the splitting of hadron isospin multiplets can be easily calculated using the MIT bag model by treating the u and d quark masses as constant parameters. The presence of a color-hyperfine splitting is an important feature of our calculation. An up-down quark mass difference of between -4 and -5 MeV is found. The agreement with data is not as good as that found in an analogous calculation using the harmonic-oscillator model.[17] Further work seems to be necessary to clarify this question within the bag model.

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(12)

THE CLOUDY BAG MODEL*)

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We review recent developments in the bag model, in which the constraints of chiral symmetry are explicitly included. The model has profound implications for nuclear, medium energy and high energy physics.

1. INTRODUCTION

The study of pion-few-nucleon systems is an honorable endeavour. As I have stressed many times, when viewed as a purely three-body system, πd scattering offers a prototype of the general pion-nucleus scattering problem [1, 2]. One can test many aspects of the general problem, such as relativistic corrections, convergence of the multiple scattering series and so on, without the need for the usual approximations of many-body theory. The detailed experimental tests of predictions for the πd system have just begun, with very accurate total [3] differential cross sections [4] (including $\pi^{\pm}$ differences [5]). Recently polarisation studies have also begun [6, 7], and we shall return to the SIN measurement of it_{11} at the end of this paper.

In these studies the πN interaction itself is generally taken for granted. It is typically approximated by a simple, separable form with no reference to any underlying dynamical theory. This is understandable given the difficulty of the three-body problem [8], and certainly in line with the philosophy of the first non-relativistic three-body calculations on the nd system [9]. Nevertheless it is the duty of the few-body theorist to remain alert to the implications of his work for nuclear and medium energy physics in general. In the past few years our group at TRIUMF and the University of Washington has been led to think much more deeply about the πN interaction itself — in the context of recent discoveries in particle physics.

In section 2 I shall review the bag model of hadronic structure, and the implications of imposing chiral symmetry on it. This leads in section 3 to a new understanding of the Δ-resonance which unifies the old Chew-Low and quark models. In section 4 the connection of this new theory with current algebra will be clarified by rederiving the Weinberg-Tomozawa relationship. The implications of the model for the structure of the nucleon itself are discussed in section 5, whilst in the final section we speculate on the changes our point of view may impose on nuclear physics.

2. FUNDAMENTALS

The MIT bag model [10] was invented some seven years ago as a simple and not necessarily realistic, phenomenological model of confined quarks-consistent with the experimental facts of deep inelastic scattering. In this paper we shall be concerned

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with just the u- and d-quarks which will be taken to be massless. These fields obey the free Dirac equation inside the bag; the confinement is imposed by a linear boundary condition at the surface (which also leads to quantised energy levels). Energy-momentum conservation at the surface is imposed by assuming that there is some energy density (B) associated with the confinement volume (which leads to a non-linear boundary condition at the surface).

The phenomenological successes of the model have been numerous, and these are reviewed in many places [11, 12]. These successes include hadronic spectroscopy, magnetic moments, charge radii, and so on. Of particular significance to us is the success of the bag model in predicting a value of $g_A \simeq 1{\cdot}09$ ($1{\cdot}24$ experimentally) – which is to be compared with the standard non-relativistic prediction of 5/3.

It is a convenient feature of the model that the appropriate Lagrangian density is very simple, viz.:

$$\mathscr{L}(x) = (\mathrm{i}\bar{q}\, \partial_\mu \gamma_\mu q - B)\, \theta_v - \tfrac{1}{2}\bar{q}q\Delta_s\,, \tag{2.1}$$

where q is the two-component quark field (u and d), Δ_s a surface delta function [$\delta(r - R)$ in the static case] and θ_v is unity inside the bag volume and zero outside. This Lagrangian density is invariant under usual $SU(2)$ transformations and therefore gives rise to a conserved vector current. However, there will obviously be problems with the axial current associated with chiral $SU(2)$. At the Lagrangian level this is easily seen, because the Lagrange multiplier term in eq. (2.1) (i.e. $-\frac{1}{2}\,\bar{q}q\Delta_s$) is chirally odd. That is, it is not invariant under the chiral transformation

$$q \to q + \tfrac{1}{2}\mathrm{i}\boldsymbol{\tau} \,.\, \boldsymbol{\varepsilon}\gamma_5 q\,; \quad \bar{q} \to \bar{q} + \tfrac{1}{2}\mathrm{i}\bar{q}\gamma_5\boldsymbol{\tau} \,.\, \boldsymbol{\varepsilon}\,, \tag{2.2}$$

where $\boldsymbol{\varepsilon}$ is an infinitesimal constant.

At the intuitive level this problem is also obvious. From the point of view of conventional weak interaction phenomenology we know that in the non-relativistic limit the nucleon axial current has two pieces g_A and g_P. In the limit $m_\pi \to 0$ this current is conserved, and the strength of the axial and induced pseudoscalar couplings are related. That is

$$\langle p|\, \mathbf{A}(q^2)\, |n\rangle \underset{(m_\pi = 0)}{\propto} -2Mg_A(q^2)\, \boldsymbol{\sigma} + (q^2 g_P(q^2))\, \boldsymbol{\sigma} \,.\, \hat{q}\hat{q}\,, \tag{2.3}$$

and hence

$$\langle p|\, \nabla \,.\, \mathbf{A}|n\rangle \propto (-2Mg_A(q^2) + g_P(q^2)\, q^2)\, \boldsymbol{\sigma} \,.\, \mathbf{q}\,, \tag{2.4}$$

which vanishes as required at $q = 0$ if

$$g_P(0) = \frac{2Mg_A(0)}{q^2}\,. \tag{2.5}$$

While g_A is beautifully described by the bag model, the necessary pion pole (replace q^{-2} in eq. (2.5) by $(q^2 + m_\pi^2)^{-1}$ if $m_\pi \neq 0$) is absent.

The conventional method of restoring a symmetry is to introduce compensating fields, such as (σ, π) in the standard σ-model [13, 14]. However, the σ-meson is not seen experimentally, and one is therefore forced to work with a non-linear representation of chiral $SU(2)$. That is, the chiral transformation will connect states with different numbers of pions. The appropriate Lagrangian density is [15, 16] (without a pion mass term)

$$\mathscr{L}_{\mathrm{CBM}}(x) = (\mathrm{i}\bar{q}(x)\,\partial_\mu\gamma_\mu\, q(x) - \mathrm{B})\,\theta_5 - \tfrac{1}{2}\bar{q}(x)\exp(\mathrm{i}\boldsymbol{\tau}\,.\,\boldsymbol{\phi}(x)\,\gamma_5/f) \times \times\, q(x)\,\Delta_s + \tfrac{1}{2}(\mathrm{D}_\mu\boldsymbol{\phi})^2\,, \tag{2.6}$$

where $\boldsymbol{\phi}$ is the three-component compensating (pion) field, and $\mathrm{D}_\mu\boldsymbol{\phi}$ is a covariant derivative

$$\mathrm{D}_\mu\boldsymbol{\phi} = \partial_\mu\boldsymbol{\phi} - [1 - j_0(\phi/f)]\,\hat{\boldsymbol{\phi}} \times (\partial_\mu\boldsymbol{\phi} \times \hat{\boldsymbol{\phi}})\,. \tag{2.7}$$

This Lagrangian density is invariant under the chiral transformation (2.2), provided that

$$\boldsymbol{\phi} \to \boldsymbol{\phi} - \boldsymbol{\varepsilon} f + f[1 - (\phi/f)\cot(\phi/f)]\,\hat{\boldsymbol{\phi}} \times (\boldsymbol{\varepsilon} \times \hat{\boldsymbol{\phi}})\,. \tag{2.8}$$

Because of the invariance one can define a conserved axial current ($\partial_\mu\boldsymbol{A}^\mu = 0$), which to lowest order in the pion field is

$$\boldsymbol{A}^\mu(x) \simeq \tfrac{1}{2}\bar{q}(x)\,\gamma^\mu\gamma_5\boldsymbol{\tau}\,q(x)\,\theta_v - f\,\partial^\mu\boldsymbol{\phi}\,. \tag{2.9}$$

If we add a pion mass term to eq. (2.6), thereby breaking the symmetry, we find that $\boldsymbol{A}^\mu$ obeys the PCAC relationship

$$\partial_\mu\boldsymbol{A}^\mu = f m_\pi^2\boldsymbol{\phi} + o(\phi^2)\,, \tag{2.10}$$

with f the pion decay constant (93 MeV).

Lest the reader feel that this procedure is too arbitrary, we briefly recall the Gell-Mann-Renner-Oakes view of QCD [17, 18]. That is, the ideal QCD Lagrangian (for the moment we consider only three flavours, u, d and s) embodies an exact $SU(3) \times SU(3)$ symmetry. If this were the whole story, each physical particle would have a partner of opposite parity. Because this is not observed, by the Goldstone theorem we know that the vacuum symmetry must be broken [18]. Thus we are led, at this level, to the existence of 8 massless, pseudoscalar Goldstone bosons $(\pi, \eta, \bar{K}, K)$. As a result of processes involving heavy vector bosons there is a successive breaking of the $SU(3) \times SU(3)$ symmetry. First the s-quark (and hence the K- and η-mesons) acquires a mass, leading to $SU(2) \times SU(2)$. Next the u and d get equal masses, leading to $SU(2)$, and massive pions. In fact, there is excellent support for the idea that $SU(2) \times SU(2)$ is good to (5–7)%. The exceptionally light mass of the pion provides excellent support for describing it as an approximate Goldstone boson. The Lagrangian (2.6) is a phenomenologically reasonable way to incorporate these ideas into a practical theory with exact $SU(2) \times SU(2)$ symmetry. The final symmetry-breaking step is easily achieved by adding explicit pion and quark mass terms.

3. PION-NUCLEON SCATTERING: THE (3,3) RESONANCE

If the pion field is small, we might expect to be able to calculate pionic corrections to the MIT bag model perturbatively. Keeping only terms of lowest order in ϕ eq. (2.6) becomes

$$\mathscr{L}(x) \simeq (i\bar{q}\,\partial_\mu \gamma_\mu q - B)\,\theta_v + \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\bar{q}q\Delta_s - \frac{i}{2f}\,\bar{q}\tau\,.\,\phi\gamma_5 q\Delta_s\,. \tag{3.1}$$

From this we can easily write down a Hamiltonian for a pion interacting with any hadron describable in the bag model. If we restrict the space of bag states to non-exotic hadrons (e.g. three quark baryons[1]), denoted $(\alpha, \beta \ldots)$, this Hamiltonian is [15]:

$$H = H_{\mathrm{MIT}} + H_\pi + H_{\mathrm{int}}\,, \tag{3.2}$$

$$H_{\mathrm{MIT}} = \sum_\alpha m_\alpha^{\mathrm{bag}} \alpha^+ \alpha\,, \tag{3.3}$$

$$H_\pi = \sum_k a_k^+ a_k w_k, \tag{3.4}$$

and

$$H_{\mathrm{int}} = \frac{-i}{2f} \sum_{\alpha,\beta,k} \int \mathrm{d}\boldsymbol{x} \langle\beta|\; \bar{q}(x)\,\tau\,.\,\phi(x)\,\gamma_5 q(x)\,\Delta_s\,|\alpha\rangle\; \beta^+ \alpha a_k + \text{h.c.}\,. \tag{3.5}$$

In the limited space of N and Δ, which is of most interest in low and intermediate energy nuclear physics, H_{int} contains $\Delta\Delta\pi$, $\Delta N\pi$ and $NN\pi$ vertices. The ratio of the bare coupling constants can be calculated in terms of the bag model wave functions. There is also a form factor, or high momentum cut-off, because the pion is no longer coupling to a point, but to the surface of a large object. If we neglect the small difference between the N and Δ bag radii, this form factor is identical for all three vertices, that is

$$u(k) = j_0(kR) + j_2(kR) = 3j_1(kR)/kR\,. \tag{3.6}$$

Now we are able to tackle one of the most neglected problems of quark physics — the description of unstable resonances. In the Gell-Mann-Okubo mass formula, m_Δ is usually taken as the energy parameter in a Breit-Wigner fit to the pion nucleon data (i.e. $m_\Delta \sim 1231$ MeV). Jaffe and Low have mentioned this problem in their discussion of the p-matrix [19], but in our view their dynamical framework is over-simplified. In the context of our Lagrangian, the correct procedure for describing the delta is to compute the series of graphs contributing to πN scattering [15, 20]. It is easily seen that this includes the usual "Physical Review C" picture of the Δ as a series of crossed Born graphs, *in addition to* direct delta formation and decay and self-energy terms.

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[1]) A similar analysis could and should be carried out for non-exotic mesons ($\bar{q}q$ states) which couple strongly to pions — e.g. the rho-meson.

It is fascinating that while our non-interacting Hamiltonian contains two discrete states N and Δ, after the interactions are turned on the Δ-pole moves into the complex plane. Thus the Δ is an unstable resonance in the πN (3,3) channel, it is *not* an eigenstate of any Hamiltonian. For a detailed discussion of the calculation of πN scattering we refer to ref. [15], where the renormalisation procedures are shown in great detail. One of the beauties of the CBM is that all renormalisation are finite and calculable. A self-consistent fitting procedure leads to a unique best fit to the (3,3) data with a bag radius of $R = 0{\cdot}82$ fm [16]. Of course, systematic errors in such an analysis are very difficult to estimate, and we expect at least a ten per cent uncertainty in R.

The fitting procedure also leads to a unique value of the difference of the renormalised nucleon and delta bag masses, namely 280 MeV. In the usual bag model this splitting is assigned to the spin dependence of the one gluon exchange interaction, leading to a rather large value of the colour coupling constant $\alpha_S^{\mathrm{MIT}} = 0{\cdot}55$ [10]. In the CBM, this mass difference includes pion self-energy terms which by explicit calculation we find to be 70 MeV more attractive for the nucleon than the delta. Thus in the CBM the QCD splitting is only 200 MeV, and with the smaller radius this leads to a new, smaller value of $\alpha_S^{\mathrm{CBM}} = 0{\cdot}30$. This is much more consistent with the idea that gluonic effects should be treated as a first-order perturbation.

In conclusion we note that a similar result concerning the delta was recently obtained in a less complete calculation by Lichtenburg and Wills [21]. These authors also considered the rho-meson, which couples strongly to two pions. Once again the effect of the coupled channel was to lower the required QCD splitting (of the π and ϱ in this case).

4. MORE PION-NUCLEON SCATTERING

Satisfying though it may be to have such a unified description of the (3,3) resonance, one would also expect to learn something about s-wave scattering from a chiral symmetric theory. The classic paper on this subject is of course Weinberg's derivation of an effective Lagrangian for πN scattering based on the σ-model [14].

We could include nucleon recoil in eqs. (3.2–3.4), and hence compute s-wave scattering; however, there is a much more elegant way [22]. Suppose we define a new quark field by the unitary transformation S:

(4.1) $$q_w = Sq\,; \quad S = \exp\left(\mathrm{i}\boldsymbol{\tau}\,.\,\boldsymbol{\phi}(x)\,\gamma_5/2f\right).$$

Then after straightforward algebraic manipulation one finds

(4.2) $$\mathscr{L}(x) = \left(\mathrm{i}\bar{q}_w(x)\,\partial_\mu\gamma_\mu q_w(x) - B\right)\theta_v - \tfrac{1}{2}\bar{q}_w(x)\,q_w(x)\,\Delta_s + \\ + \tfrac{1}{2}(\mathrm{D}_\mu\phi(x))^2 + \bar{q}_w(x)\,\gamma^\mu\,\mathrm{i}[S\,\partial_\mu S^+]\,q_w\theta_v\,.$$

At this stage the Au-Baym [23] identity

(4.3) $$S\,\partial_\mu S^+ = \int_0^1 \mathrm{d}\lambda\, S^\lambda\,\partial_\mu(\ln S^+)\,(S^+)^\lambda$$

can be used to prove that

(4.4) $$iS\,\partial_\mu S^+ = \frac{\gamma_5}{2f}\,\tau\,.\,(D_\mu\phi) + \left(\frac{\cos(\phi/f) - 1}{2}\right)\tau\,.\,(\hat{\phi} \times \partial_\mu\hat{\phi})\,.$$

Clearly the surface γ_5-coupling of the pion field has become a volume, derivative coupling, with strength $(g_A^{\mathrm{Bag}}/2f)$ at $|\mathbf{k}_\pi| = 0$. That is the CBM Lagrangian incorporates the Goldberger-Treiman relation *with the correct value of* g_A. Furthermore, the second term in eq. (4.4), when expanded to $o(\phi^2)$, leads to an s-wave pion-bag interaction (as $\mathbf{k}_\pi \to 0$)

(4.5) $$\mathscr{L}_s(x) \simeq -\frac{1}{2f^2}\left[\bar{q}_w(x)\,\gamma^0\boldsymbol{\tau}/2q_w(x)\right].\,(\boldsymbol{\phi}(x) \times \boldsymbol{\pi}(x))\,.$$

However, the pieces in brackets are respectively the bag and pion isospin operators, so that

(4.6) $$\mathscr{L}_s = -\mathbf{t}\,.\,\mathbf{t}_\pi/2f^2\,.$$

Thus we obtain from the CBM Lagrangian not only the Weinberg result for πN scattering, but also the Weinberg-Tomozawa prediction [24, 25] for the pion scattering length on any hadronic target describable in the bag model:

(4.7) $$a_T = -\frac{1}{8\pi f^2}\left[T(T+1) - t(t+1) - 2\right].$$

To summarise, the CBM Lagrangian incorporates many of the results of current algebra, allows internal excitation of the nucleon, and should provide an excellent starting point for the study of modern nuclear physics — including such exotic phenomena as pion condensation.

5. THE NUCLEON

The nucleon bag will of course be dressed by its interaction with the pion field, but the nucleon does remain as the one discrete eigenvector of the Hamiltonian (3.2), satisfying

(5.1) $$H|N\rangle = m_N|N\rangle\,.$$

In the old meson source theories such as Chew-Low one could also write an equation like (5·1), but H would not include the Δ. The convergence properties of these old source theories were very poor. For example the ratio of bare to renormalised coupling constant squared was about three [26]; the average number of pions in the cloud about the nucleon was large. Finally the properties of the core itself were completely unknown.

For the CBM, on the other hand, the convergence properties have been shown to be excellent. The renormalisation of the $NN\pi$ coupling constant has been shown by explicit calculation to be of order 10% [16]. More formally, starting with eq. (5·1), it has been proven by Dodd, Alvarez-Estrada and the author [27] that one can place rigorous bounds on the number of pions present in the physical nucleon. Indeed, for $R = 0{\cdot}82$ as described above for πN scattering, the average number of pions is rigorously less than or equal to 0·9 and this seems to be a generous upper bound. This should be compared with Chew-Low where the same bound is 2·2 pions.

The implications of this model for proton decay are under investigation, but it is clear that *in lowest order* it justifies the usual assumption that the nucleon consists of just three quarks.

Although historically the order was reversed(!) it is natural after establishing the formal convergence of the perturbation expansion of the nucleon wave function to compute the pionic corrections to its electromagnetic properties. This has been carried out by our group [16], and the results are summarised in table 1. Similar results were obtained by DeTar, also using the CBM Hamiltonian [28]. Given the approximations of the calculation — such as the static bag — the theoretical numbers should probably carry a (5—10)% error, and the agreement with the measured nucleon properties is excellent!

Table 1

Electromagnetic properties of the nucleon (magnetic moments are in units of Bohr magnetons).

Quantity	CBM ($R = 0{\cdot}82$ fm)	Experiment	MIT Bag ($R = 1{\cdot}0$ fm)[a]
$\langle r_{\text{ch}}^2 \rangle_p^{1/2}$	0·73 fm	0·83 fm	0·73 fm
$\lvert\langle r_{\text{ch}}^2 \rangle_n\rvert^{1/2}$	0·36 fm	0·35 fm	0·0 fm
μ_p	2·60 fm	2·79 fm	2·24 fm
μ_n	−2·01 fm	−1·91 fm	−1·42 fm

[a] We have included the c.m. correction of J. Donoghue and K. Johnson [Phys. Rev. D*21* (1980) 1975].

The prediction for the neutron charge radius is of particular interest. In the harmonic-oscillator model, the *one gluon exchange* spin-spin interaction leads to a repulsion of the two *d*-quarks in the neutron, and hence a negative charge radius [29, 30]. However, in the bag model this effect is not large enough to explain the observed rms charge radius [3]. It is of course true in any meson theory of nucleon structure that the π^- yields a negative tail to the charge distribution, but as we have already mentioned the core is completely unknown. In the CBM the core is simply a three--quark bag and is completely understood. Indeed, the CBM provides the solution to a very old problem.

For the experimentalists it is important to emphasise the significance of a good measurement of the neutron charge distribution [15, 16, 32]. In fact, in our model it is inescapable that the zero in the neutron charge distribution would measure the bag size — that is the size of the confinement region! Finally we note that the model is presently being applied to the problem of hyperon magnetic moments where there is currently a problem of interpretation.

6. CONCLUSION

The major applications of the CBM have so far been to pion-hadron scattering and the weak and electromagnetic properties of the nucleon. It is in low and intermediate energy nuclear physics, however, that this approach may have its most profound influence. We refer to Baym's discussion of percolation [33], for example — that is the possibility that beyond some critical density there could be considerable linking of bags, so that quark currents would no longer be localised in individual nucleons. As we have observed in refs. [15] and [32], with the smaller radius of the nucleon in the CBM, Baym's critical density is near nuclear matter density. Thus we are led to a very natural explanation of why nucleons can move independently in valence orbits [as seen in $(p,2p)$, (d, p), etc.], but may not be so independent in the nucleon interior. The CBM Lagrangian density is an ideal starting point for a new discussion of pion condensation, which according to standard calculational techniques should occur well above the critical density for percolation — thereby throwing considerable doubt on the use of the standard techniques!

Some work has already been carried out on the long range $N - N$ force in the CBM. At least as long as the bags do not overlap, the standard one- and two-pion-exchange potentials (including delta excitation) should be a good approximation. At shorter distances (that is inside about 1·6 fm) the quarks will play a critical role, and unhappily there is, as yet, no reliable calculational technique. In a very interesting piece of analysis, Gersten [34] has been able to set limits on the value of R, appearing in the $NN\pi$ form factor (3·6), on the basis of $N - N$ phase shift analysis alone. His limits of R between 0·60 and 0·90 fm are quite consistent with the CBM result of 0·82 fm ($\pm 10\%$).

In conclusion, we return to the rather topical question of dibaryon resonances — originally discovered in the Argonne polarised total cross section measurements [35]. In my view the clearest (but by no means definitive) experimental support for such a resonance is the rapid oscillation found in the recent SIN measurement of it_{11} in pion scattering from a polarised deuterium target [7, 36]. While it is clear that many bag model predictions of such exotic states have been made, it is impossible to calculate their properties reliably without including the possiblity of decay through pion-producing channels. Once again the CBM is ideal for such a problem. Hopefully the interplay getween theory and experiment in this area, in particular, will greatly expand our phenomenological understanding of QCD.

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Partial wave expansions of modified one-particle-exchange diagrams

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Expressions are derived for the partial wave expansion coefficients of the scattering matrix elements corresponding to one-particle-exchange Feynman diagrams which are modified by form factors or by Reggeization. A few examples are presented, including the application to nucleon-nucleon scattering in the cloudy bag model. A value of the bag radius $R \simeq 0.8$ fm seems to be consistent with the phase-shift analysis in selected partial waves.

NUCLEAR REACTIONS Modified one particle exchange diagrams applied to N-N scattering.

I. INTRODUCTION

In practical applications of Feynman diagrams one often needs the partial wave expansions of S-matrix elements corresponding to one-particle-exchange diagrams. In the spinless case the result is rather simple and well known. If we denote by θ the c.m. scattering angle the amplitude is proportional to $1/(z_0-\cos\theta)$, where z_0 depends on the momenta and masses of the particles involved. In this case the partial wave expansion is proportional to

$$1/(z_0-\cos\theta)=\sum_{l=0}^{\infty}(2l+1)Q_l(z_0)P_l(\cos\theta)\ , \tag{1.1}$$

and

$$Q_l(z_0)=\tfrac{1}{2}\int_{-1}^{1}[P_l(\cos\theta)/(z_0-\cos\theta)]d\cos\theta\ , \tag{1.2}$$

where P_l are the Legendre polynomials and Q_l are the Legendre functions of the second kind. In the case of scattering of particles with spin the amplitudes may involve spherical harmonics (associated Legendre polynomials) or Wigner $d^J_{\lambda\mu}(\theta)$ functions (in the helicity representation) and the expansion coefficients are combinations of the $Q_l(z_0)$ functions of different orders.

The subject of this paper is the partial wave expansion corresponding to one-particle-exchange diagrams modified by form factors at the vertices or by Reggeization. In general, this modification will lead to amplitudes proportional to

$$f(t)/(z_0-\cos\theta)\ , \tag{1.3}$$

where $(-t)$ is the four-momentum transfer squared and f is a related function. In general, it will be sufficient to replace the $Q_l(z_0)$ functions appearing in the expansion coefficients (even in the case of the scattering of particles having spin) by the new functions

$$A_l(z_0)=\tfrac{1}{2}\int_{-1}^{1}[f(t)P_l(\cos\theta)/(z_0-\cos\theta)]d\cos\theta\ . \tag{1.4}$$

In Sec. II we derive formulas suitable for an evaluation of these integrals and in Sec. IV a few examples are discussed. In Sec. III a simple recurrence relation for the $A_l(z_0)$ is derived for the case where the partial wave expansion of $f(t)$ is known. As an important application of our scheme we discuss in Sec. V the possibility of determining the radius R of the cloudy bag nucleon from the nucleon-nucleon phase-shift analysis. An estimate of $R\simeq 0.8$ fm is obtained.

II. GENERAL FORMULAS

In this section we derive general formulas for evaluating the integrals of Eq. (1.4). Let us assume that we can expand

$$f(t)=F(\cos\theta)=\sum_{n=0}^{\infty} a_n \left[\frac{\gamma-\cos\theta}{2}\right]^n . \tag{2.1}$$

Let us denote $x=\cos\theta$ and

$$S_N(x)=\sum_{n=0}^{N} a_n \left[\frac{\gamma-x}{2}\right]^n . \tag{2.2}$$

In the Appendix we prove that

$$\begin{aligned} A_n(z) &= \tfrac{1}{2}\int_{-1}^{1} \frac{F(x)P_n(x)dx}{z-x} \\ &= F(z)Q_n(z)+\tfrac{1}{2}\sum_{m=n}^{\infty}\left\{[F(z)-S_m(z)]/\left[\frac{\gamma-z}{2}\right]^{m+1}\right\}C_{mn}(\gamma) , \end{aligned} \tag{2.3}$$

where

$$C_{mn}(\gamma)=\tfrac{1}{2}\int_{-1}^{1}\left[\frac{\gamma-x}{2}\right]^m P_n(x)dx . \tag{2.4}$$

The advantage of this formula appears when $F(x)$ is regular; then all singularities are retained in the Legendre function $Q_n(z)$ of which the properties are well known.

The integrals which appear in Eq. (2.4) can be evaluated for $\gamma=0,\pm 1$:

$$C_{mn}(1)=(-1)^n(m!)^2/[(m+n+1)!(m-n)!] , \tag{2.5}$$

$$C_{mn}(-1)=(-1)^{m+1}(m!)^2/[(m+n+1)!(m-n)!] , \tag{2.6}$$

$$C_{mn}(0)=0 \text{ if } m-n \text{ is odd} , \tag{2.7}$$

otherwise

$$C_{mn}(0)=(-1)^n m!/4^m \left[\frac{m-n}{2}\right]!(\tfrac{3}{2})_{(n+m)/2} , \tag{2.8}$$

where

$$(\tfrac{3}{2})_i=\tfrac{3}{2}(\tfrac{3}{2}+1)\cdots(\tfrac{3}{2}+i-1) . \tag{2.9}$$

For $\gamma\neq 0,\pm 1$, the binomial formula can be used in order to evaluate the integral (2.4). Thus for $\gamma=0$ we have from Eqs. (2.7), (2.8), and (2.3):

$$\begin{aligned} A_n(z)=F(z)Q_n(z)+\frac{(-1)^n}{2}&\left\{\frac{n!}{4^n 0!(3/2)_n}\left[a_{n+1}+a_{n+2}\left[\frac{-z}{2}\right]+a_{n+3}\left[\frac{-z}{2}\right]^2+\cdots\right]\right.\\ &\left.+\frac{(n+2)!}{4^{n+2}1!(3/2)_{n+1}}\left[a_{n+3}+a_{n+4}\left[\frac{-z}{2}\right]+\cdots\right]\right\} . \end{aligned} \tag{2.10}$$

Similarly for $\gamma=1$

$$\begin{aligned} A_n(z)=F(z)Q_n(z)+\frac{(-1)^n}{2}&\left\{\frac{(n!)^2}{0!(2n+1)!}\left[a_{n+1}+a_{n+2}\left[\frac{1-z}{2}\right]+\cdots\right]\right.\\ &\left.+\frac{[(n+1)!]^2}{1!(2n+2)!}\left[a_{n+2}+a_{n+3}\left[\frac{1-z}{2}\right]+\cdots\right]+\cdots\right\} . \end{aligned} \tag{2.11}$$

Equation (2.11) can be rearranged in the form

$$A_n(z)=F(z)Q_n(z)+\frac{(-1)^n}{2}\frac{(n!)^2}{(2n+1)!}\sum_{k=1}^{\infty} a_{n+k}B_{k,n}(z) . \tag{2.12}$$

Where $B_{k,n}(z)$ are polynomials of degree $k-1$ in z, they satisfy the following recursion relations:

$$\begin{aligned} B_{k+1,n}(z) &= \left[\frac{1-z}{2}\right] B_{k,n}(z) + b_{k+1,n} - \delta_{k1} , \\ b_{k+1,n} &= b_{k,n} \frac{(n+k)^2}{(2n+k+1)k} , \\ B_{1,n} &= 1, \quad b_{1,n} = 1 , \end{aligned} \tag{2.13}$$

where δ_{k1} is the Kroneker delta.

The case $\gamma = z$ is obviously simple. Substituting Eq. (2.1) in Eq. (1.4) we obtain

$$A_n(z) = F(z) Q_n(z) + \tfrac{1}{4} \sum_{k=n+1}^{\infty} a_k \int_{-1}^{1} \left[\frac{z-x}{2}\right]^{k-1} P_n(x) dx . \tag{2.14}$$

The integrals of Eq. (2.14) can be majorized

$$\left| \int_{-1}^{1} \left[\frac{z-x}{2}\right]^m P_n(x) dx \right| < 2 \left[\frac{z+1}{2}\right]^m , \tag{2.15}$$

and the expansion (2.14) will be convergent if

$$\lim_{m\to\infty} \left| \frac{a_{m+1}}{a_m} \right| < \frac{z+1}{2} . \tag{2.16}$$

III. RECURSION RELATIONS

If the expansion of the modifying function $F(x)$ in terms of Legendre polynomials is known, one can derive recursion relations for the $A_l(z)$ coefficients in the following way. We start with the Christoffel-Darboux formula[1]

$$\sum_{m=0}^{n} (2m+1) P_m(x) P_m(z) = (n+1)[P_n(x) P_{n+1}(z) - P_{n+1}(x) P_n(z)]/(z-x) \tag{3.1}$$

and assuming that

$$F(x) = \sum_{m=0}^{\infty} (2m+1) \phi_m P_m(x) \tag{3.2}$$

is known, we multiply Eq. (3.1) by $F(x)$ and integrate both sides from -1 to 1. If we rewrite Eq. (1.4) as

$$A_l(z) = \tfrac{1}{2} \int_{-1}^{1} dx\, F(x) P_l(x)/(z-x) , \tag{3.3}$$

we obtain

$$\sum_{m=0}^{n} (2m+1) \phi_m P_m(z) = (n+1)[A_n(z) P_{n+1}(z) - A_{n+1}(z) P_n(z)] \tag{3.4}$$

or

$$A_{n+1}(z) = \frac{1}{P_n(z)} \left\{ P_{n+1}(z) A_n(z) - \frac{1}{n+1} \sum_{m=0}^{n} (2m+1) \phi_m P_m(z) \right\} , \tag{3.5}$$

and if

$$A_{n+1}(z) = Q_{n+1}(z) + R_{n+1}(z) , \tag{3.6}$$

then

$$R_{n+1}(z) = \frac{1}{P_n(z)} \left\{ P_{n+1}(z) P_n(z) + \frac{1}{n+1} - \frac{1}{n+1} \sum_{n=0}^{n} (2m+1) \phi_m P_m(z) \right\} . \tag{3.7}$$

We can evaluate the first term $A_0(z)$ using the expansion

$$1/(z-x)=\sum_{m=0}^{\infty}(2m+1)Q_m(z)P_m(x) \ . \qquad (3.8)$$

Using Eqs. (3.8) and (3.2) we obtain

$$A_0(z)=\tfrac{1}{2}\int_{-1}^{1}dx\,F(x)/(z-x)$$
$$=\sum_{m=0}^{\infty}(2m+1)\phi_m Q_m(z) \ . \qquad (3.9)$$

Equation (3.9) can be converted to a form similar to Eq. (2.2) by noting that

$$Q_n(z)=P_n(z)Q_0(z)+W_{n-1}(z) \ , \qquad (3.10)$$

where $W_{n-1}(z)$ are well known polynomials[1] of degree $n-1$.

Substituting Eqs. (3.10) and (3.2) in (3.9) we obtain

$$A_0(z)=F(z)Q_0(z)$$
$$+\sum_{m=0}^{\infty}(2m+1)\phi_m W_{m-1}(z) \ . \qquad (3.11)$$

IV. EXAMPLES

A. Example 1

Let us assume

$$F(x)=(y-z)/(y-x) \ , \qquad (4.1)$$

then

$$F(x)/(z-x)=1/(z-x)-1/(y-x) \qquad (4.2)$$

and

$$\tfrac{1}{2}\int_{-1}^{1}dx\,P_n(x)F(x)/(z-x)$$
$$=Q_n(z)-Q_n(y) \ . \qquad (4.3)$$

Therefore this case will lead us to an expansion for $Q_n(y)$. For this purpose we will use Eq. (2.3) with $\gamma=1$. Expanding Eq. (4.1) we have

$$F(x)=\sum_{n=0}^{\infty}a_n\left[\frac{1-x}{2}\right]^n ;$$

$$a_n=\frac{y-z}{y-1}\left[\frac{-2}{y-1}\right]^n, \quad F(z)=1 \ , \qquad (4.4)$$

$$S_n(z)=\sum_{m=0}^{n}a_m\left[\frac{1-z}{2}\right]^m$$
$$=1-\left[\frac{z-1}{y-1}\right]^{n+1} , \qquad (4.5)$$

and

$$[F(z)-S_n(z)]/\left[\frac{1-z}{2}\right]^{n+1}=\left[\frac{-2}{y-1}\right]^{n+1} \qquad (4.6)$$

Substituting Eq. (4.4) in Eq. (2.3) we obtain from Eqs. (4.3) and (2.11)

$$Q_n(y)=-\frac{(-1)^n}{2}\sum_{m=n}^{\infty}\frac{(m!)^2}{(m+n+1)!(m-n)!}$$
$$\times\left[\frac{-2}{y-1}\right]^{m+1} , \qquad (4.7)$$

which is a well known result. The convergence in this case is limited to $y>3$. This has to be linked with the singularity of $F(x)$ at $x=y$ in Eq. (4.1). One may notice that the expansion (4.4) is uniformly convergent for $-1\le x\le 1$ and for $y>3$. In this case one can conjecture that the convergence of the expansion (2.4) is related to the uniform convergence of the expansion (2.1) in $-1\le\cos\theta\le 1$ and for $\gamma=1$.

B. Example 2

Reggeization leads to functions $f(t)$ of the form

$$f(t)\simeq e^{\alpha t+\beta} \ , \qquad (4.8)$$

where α and β may be complex numbers depending on energies and $-t$ is the four-momentum transfer squared. Equation (4.8) can be rewritten as

$$F(\cos\theta)=e^{A-B\cos\theta}$$
$$=e^A\sum_{n=0}^{\infty}\frac{(-B)^n}{n!}(\cos\theta)^n \ , \qquad (4.9)$$

where A and B depend on the energies of the particles involved.

We can transform Eq. (4.9) to the form

$$F(\cos\theta)=e^{A-Bz}e^{2B(z-\cos\theta/2)}$$
$$=e^{A-Bz}\sum_{n=0}^{\infty}\frac{(2B)^n}{n!}\left[\frac{z-\cos\theta}{2}\right]^n \qquad (4.10)$$

and use Eq. (2.14). One can majorize the integrals of Eq. (2.14) and in Eqs. (2.15) and (2.16), and hence one can see that for Eq. (4.10) the expansion (2.14) will always be convergent. In practice, the use of Eq. (2.10) or (2.11) would be more convenient. Example 2 can be treated efficiently by using the techniques of Sec. III. We can use the expansion

$$e^{-B\cos\theta}=\sum_{0}^{\infty}(-1)^n(2n+1)f_n(B)P_n(\cos\theta)\,, \tag{4.11}$$

where

$$f_n(B)=\sqrt{\pi/2B}\,I_{n+1/2}(B)$$
$$=(i)^n j_n(iB),\ \text{for } -\pi\le \arg B\le\frac{\pi}{2} \tag{4.12}$$

and I_m and j_n are the Bessel function of the second kind and the spherical Bessel function, respectively. The functions $f_n(B)$ can be easily calculated from the recursion relations

$$f_{n-1}(B)-f_{n+1}(B)=(2n+1)f_n(B)/B \tag{4.13}$$

with

$$f_0(B)=\sinh B/B \tag{4.14}$$

and

$$f_1(B)=\cosh B/B-\sinh B/B^2\,. \tag{4.15}$$

C. Example 3

In the cloudy bag model of the nucleon the following form factor emerges for the $NN\pi$ vertex[2]

$$F(qR)=3j_1(qR)/(qR)$$
$$=3[\sin qR/(qR)-\cos qR]/(qR)^2\,, \tag{4.16}$$

where R is the bag radius and q the momentum transfer. For the one-pion-exchange diagram this form factor will appear in two vertices. Therefore, the pion propagator will be modified by

$$[F(qR)]^2=9[(1-\cos 2qR)/(2q^2R^2)-\sin 2qR/(qR)+(1+\cos 2qR)/2]/(qR)^4$$
$$=72\sum_{n=0}^{\infty}\frac{(2n+2)(2n+5)(-4)^n}{(2n+6)!}(qR)^{2n}$$
$$=72\sum_{n=0}^{\infty}\frac{(2n+2)(2n+5)(-8p^2R^2)^n}{(2n+6)!}(1-\cos\theta)^n\,. \tag{4.17}$$

In the last stage we have used the substitution

$$q^2=2p^2(1-\cos\theta)\,, \tag{4.18}$$

where p is the c.m. momentum and θ the c.m. scattering angle. Now Eq. (4.17) can be used in the framework of Eq. (2.11) to calculate the partial wave expansions of the one-pion-exchange diagram modified by the form factor squared. These partial wave expansions are presented in Sec. V.

V. THE MODIFIED ONE-PION-EXCHANGE DIAGRAM

In this example we present the partial waves of the modified nucleon-nucleon one-pion-exchange diagram.[3] We do it in order to demonstrate a case of scattering of particles with spin where the coefficients of the partial waves are given by combinations of integrals as in Eq. (1.4). Let us introduce the notations

$$\gamma_J(z_0)=A_J(z_0)-\frac{J+1}{2J+1}A_{J+1}(z_0)$$
$$-\frac{J}{2J+1}A_{J-1}(z_0) \tag{5.1}$$

and

$$\eta_J(z_0)=-A_J(z_0)+\frac{J}{2J+1}A_{J+1}(z_0)$$
$$+\frac{J+1}{2J+1}A_{J-1}(z_0)\,, \tag{5.2}$$

where $A_J(z_0)$ are integrals as given by Eq. (1.4).

$$z_0=1+\mu^2/(2p^2)\,, \tag{5.3}$$

where μ is the pion mass and p the c.m. momentum of the nucleons. Let us introduce the abbreviated notation

$$\alpha=\frac{g^2}{4\pi}\langle\vec{\tau}_1\cdot\vec{\tau}_2\rangle/(4E_p)\,, \tag{5.4}$$

where g is the pion nucleon pseudoscalar coupling constant ($g^2/4\pi\simeq 14.5$), $\langle\vec{\tau}_1\cdot\vec{\tau}_2\rangle=-3$ or 1 for states with isotopic spin 0 or 1, respectively, and E_p is the c.m. total energy of one nucleon.

Using the bar phase-shift notation $\bar{\delta}_{LJ}$, the T-matrix elements of the modified one-pion-exchange diagram become

$$T_J=\frac{1}{2i}(e^{2i\bar{\delta}_J}-1)=-\alpha\gamma_J(z_0)\ ,$$
$$T_{J,J}=\frac{1}{2i}(e^{2i\bar{\delta}_{JJ}}-1)=-\alpha\eta_J(z)\ ,$$
$$T_{J-1,J}=\frac{1}{2i}(\cos 2\bar{\epsilon}_J e^{2i\bar{\delta}_{J-1,J}}-1)=\alpha\left[\frac{J}{2J+1}\gamma_J(z_0)+\frac{J+1}{2J+1}\eta_J(z_0)\right],\qquad(5.5)$$
$$T_{J+1,J}=\frac{1}{2i}(\cos 2\bar{\epsilon}_J e^{2i\bar{\delta}_{J+1,J}}-1)=\alpha\left[\frac{J+1}{2J+1}\gamma_J(z_0)+\frac{J}{2J+1}\eta_J(z_0)\right],$$
$$T^J=\tfrac{1}{2}\sin 2\bar{\epsilon}_J\exp[i(\bar{\delta}_{J-1,J}+\bar{\delta}_{J+1,J})]=\alpha\sqrt{J(J+1)}[\gamma_J(z_0)-\eta_J(z_0)]/(2J+1)\ .$$

To the lowest order in the coupling constant

$$\bar{\delta}_J\simeq T_J,\ \ \bar{\delta}_{LJ}\simeq T_{LJ},\ \text{and}\ \bar{\epsilon}_J\simeq T^J\ .\qquad(5.6)$$

The approximation (5.6) (which satisfies the unitarity condition) can be applied only for sufficiently small phase shifts when the result is more or less independent of the unitarization scheme.

In order to get an insight into the form factor of the πNN vertex one should consider only partial waves for which the two and more pion exchanges give very small contributions to the phase shifts. One can not go to values of L which are too high, because the effect of the form factor becomes very small, and also the experimental determination of the phase shifts is inaccurate.[4] The two-pion-exchange contributions for the peripheral partial waves were estimated by several groups.[5] On the basis of their results it was impossible to select partial waves satisfying the criteria mentioned above. This is understandable as we expect that the effect of the two-pion exchanges should not be smaller than the effect of the form factor. Therefore, we had to find a more refined method.

We have looked for partial waves for which the two-pion-exchange contributions are well approximated by the box diagram. For these partial waves we have used the quasipotential equation approach of Ref. 3 by which the ladder diagrams are generated in an approximate way. We have selected only the partial waves for which the first iterated Born term was a good approximation to the box diagram. This led us to consider the $^3D_2$, ϵ_3, $^3G_3$, and $^3G_4$

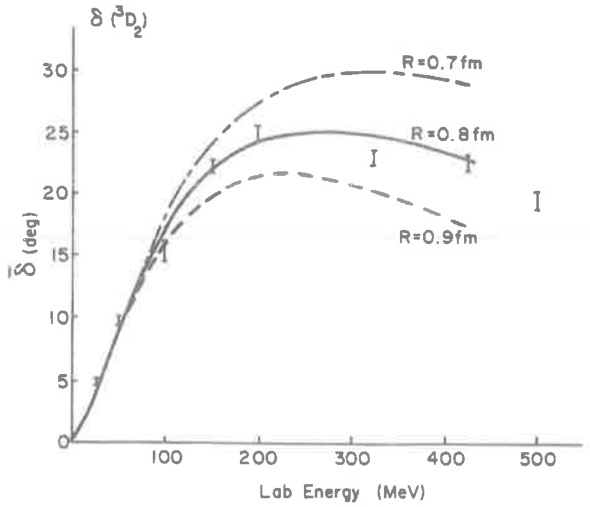

FIG. 1. The $^3D_2$ bar phase shift evaluated for different bag radii R. The experimental phase shifts are from Ref. 4.

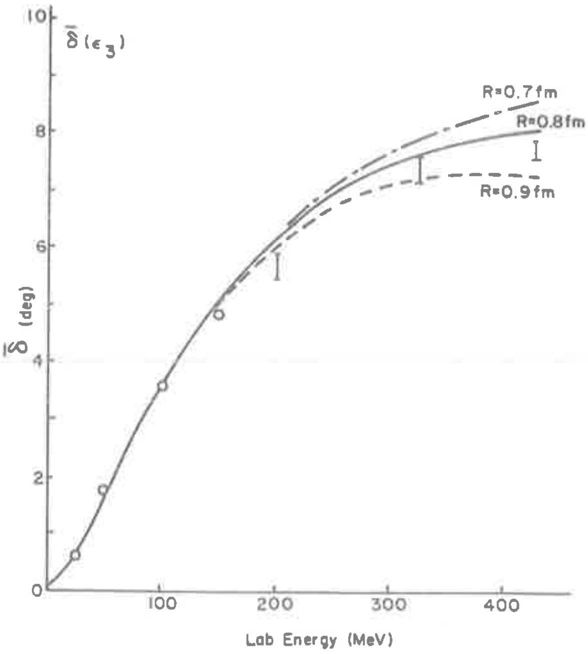

FIG. 2. The ϵ_3 bar phase shift evaluated for different bag radii R. The experimental phase shifts are from Ref. 4.

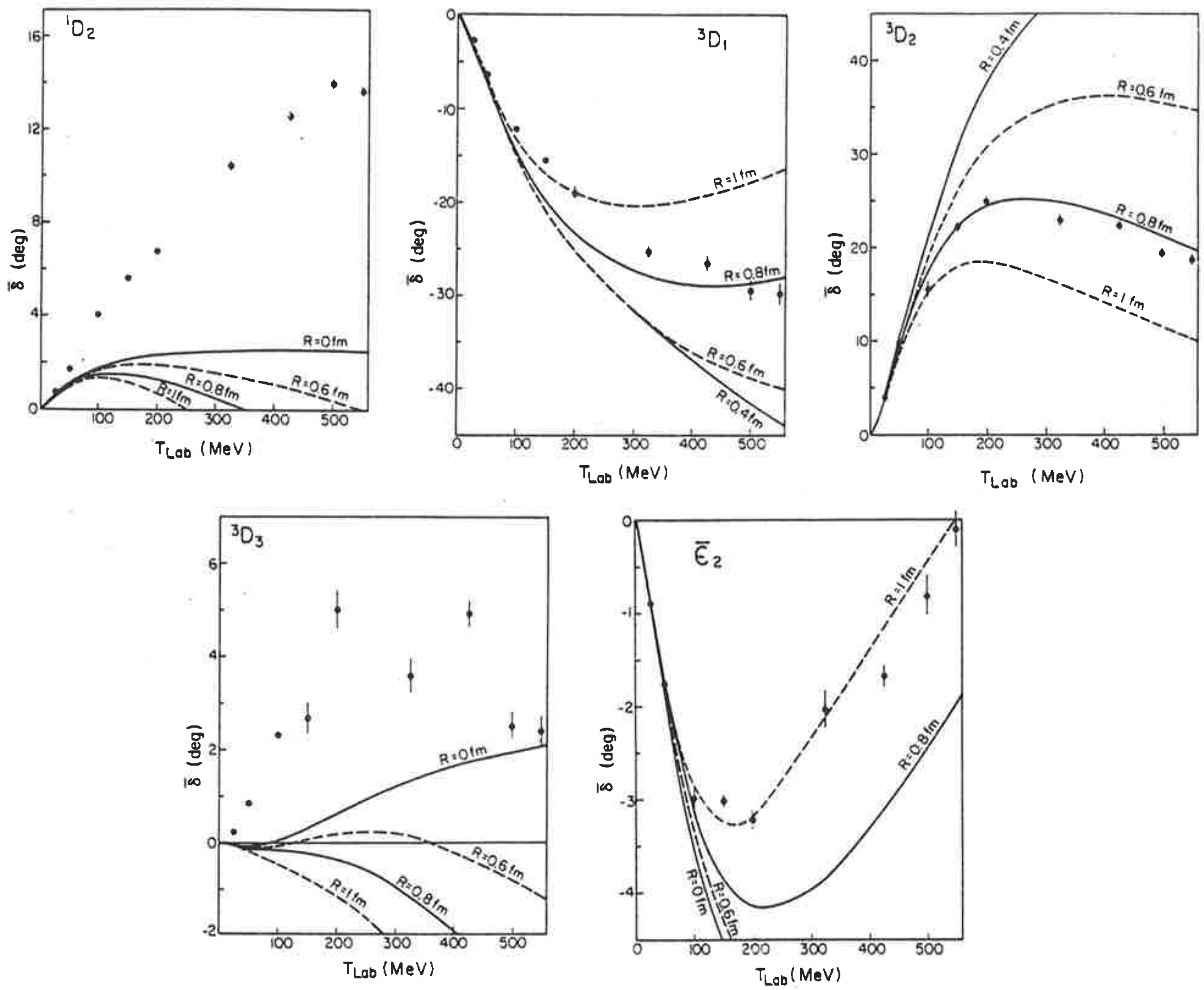

FIG. 3. The $l=2$ and $l=3$ bar phase parameters for different bag radii R. The experimental values are from Ref. 4.

partial waves as the best candidates. Next we used the one-pion-exchange potential modified by the form factor of Eq. (4.16). The resulting $^3D_2$ phase shift is displayed in Fig. 1 showing preference for a bag radius of $R \simeq 0.8$ fm. The determination of the phases $^3G_3$ and $^3G_4$ in the phase-shift analysis is still not accurate. In Fig. 2 we display the ϵ_3 phase shift. The result should be interpreted with caution as the determination of the ϵ_3 phase shift is quite unstable in spite of the relatively small statistical error bars. Nevertheless, the result $R \simeq 0.8$ fm seems to be adequate for both $^3D_2$ and ϵ_3 phases.

In choosing the $^3D_2$, ϵ_3, $^3G_3$, and $^3G_4$ partial waves we considered only uncorrelated and correlated two-pion exchanges. At the present state of the art it is impossible to calculate the contributions of exchanges of a higher number of mesons. Therefore, we cannot estimate how sensitive the result $R \simeq 0.8$ fm is to the inclusion of three or more uncorrelated (irreducible) and correlated pion exchanges.

It might be of interest to the expert in the field to see the effect of the cutoffs on all the $l=2$ and $l=3$ phase parameters. They are displayed in Fig. 3.

VI. SUMMARY AND CONCLUSIONS

General formulas have been derived in Sec. II [Eqs. (2.2), (2.10), and (2.11)] which allow an efficient expansion of modified one-particle-exchange diagrams. The coefficients of the partial wave expansions consist of two parts. One part is proportional to the Q_l function which would have been obtained from the one-particle-exchange diagram without modifications. The second part includes an expansion depending only on the modifications.

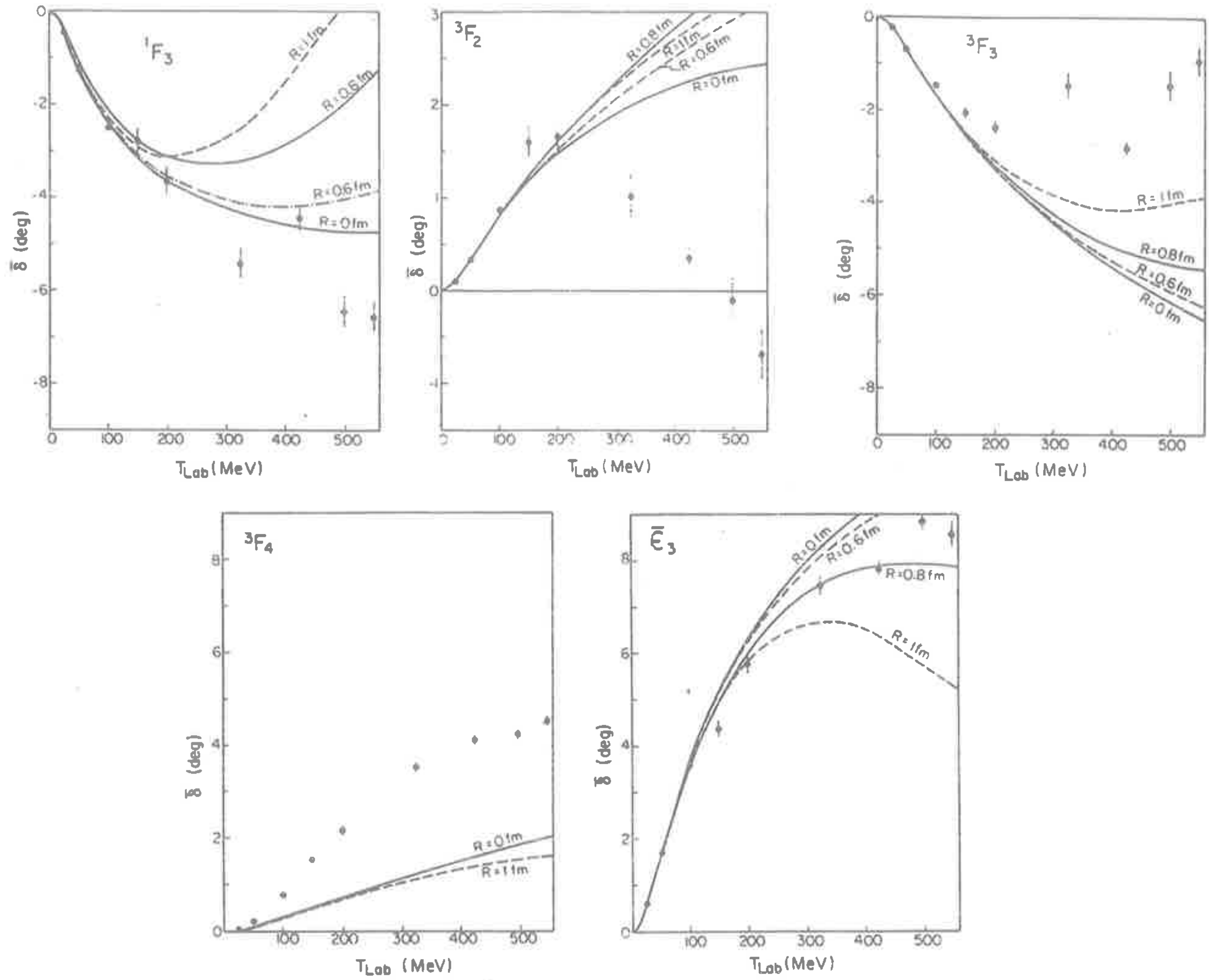

FIG. 3. (*Continued.*)

This expansion usually converges fast if the modification is done through analytic functions. In Sec. III an efficient scheme is developed for the case when the partial wave expansion of the modification is known. For this case we derive a simple recurrence relation, Eq. (3.7), for the partial wave coefficients of the modified diagrams. The Reggeized amplitude belongs to that category as is demonstrated in example 2 of Sec. IV. As a further application of our scheme (perhaps the most important in our paper) we give an estimate of the radius of the cloudy bag nucleon. This particular case is considered first in example 3 of Sec. IV, and further in Sec. V by getting results via a pseudopotential equation. Our results are consistent with a bag radius of about 0.8 fm.

APPENDIX

Here we prove Eq. (2.2). For this purpose we rewrite $A_n(z)$ as

$$A_n(z)=\frac{1}{2}\int_{-1}^{1}\frac{\{F(z)+[F(x)-F(z)]\}}{z-x}dx \ .$$

We use Eq. (1.2) and substitute Eq. (2.1) in Eq. (A1) to obtain

$$A_n(z)=F(z)Q_n(z)+\tfrac{1}{2}\int_{-1}^{1}\sum_{m=0}^{\infty}a_m\left[\left[\frac{\gamma-x}{2}\right]^m-\left[\frac{\gamma-z}{2}\right]^m\right]\frac{p_n(x)dx}{z-x} \tag{A1}$$

$$=F(z)Q_n(z)+\tfrac{1}{4}\int_{-1}^{1}\sum_{m=1}^{\infty}a_m\left[\left[\frac{\gamma-x}{2}\right]^{m-1}+\left[\frac{\gamma-x}{2}\right]^{m-2}\left[\frac{\gamma-z}{2}\right]+\cdots+\left[\frac{\gamma-z}{2}\right]^{m-1}\right]P_n(x)dx\ . \tag{A2}$$

After some rearrangements of the sums in the integrand, we obtain

$$A_n(z)=F(z)Q_n(z)+\tfrac{1}{4}\int_{-1}^{1}\sum_{m=1}^{\infty}\left\{[F(z)-S_m(z)]\Big/\left[\frac{\gamma-z}{2}\right]^{m+1}\right\}\left[\frac{\gamma-x}{2}\right]^n P_n(x)dx\ . \tag{A3}$$

We should note[1] that if $R_m(x)$ is a polynomial of degree m, then

$$\int_{-1}^{1}R_m(x)P_n(x)dx=0 \text{ if } m<n\ ; \tag{A4}$$

therefore, the summation in (A3) should start with $m=n$.

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A NEW APPROACH TO LOW-ENERGY STRONG INTERACTION PHYSICS

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ABSTRACT

We briefly review the philosophy behind the creation of chiral bag models, and then present some of the highlights of the application of one particular model, the Cloudy Bag Model. We touch on the charge and magnetic properties of the baryons, pion-nucleon scattering, *N-N* scattering, proton decay, and exotic states. Some new results on the question of convergence of the pion self-energy are also presented.

Introduction

At this moment we stand at the threshold of a new way of looking at low-energy physics. For the purposes of this paper, low energy means below charm, but the absence of charm does not mean a lack of excitement. The divergence of nuclear and particle physics since the early 1960s has been a rather unsatisfactory feature of modern physics, and it is time to put this right. Of course, it would be naive in the extreme to assume that at the mention of quarks all the outstanding problems of nuclear physics would vanish. Nevertheless, many of us are optimistic enough to hope that after a lot of hard work (which has barely begun!) we may be able both to explain some old mysteries and to find some more aesthetically pleasing explanations of phenomena that already have a conventional explanation.

For a much more detailed and pedagogically useful summary of the subjects touched on here, the interested reader is referred to Ref. 1. Briefly, the structure of the talk is as follows. The major part is the Chiral Symmetry and the Bag Model section, where we first outline the philosophy behind chiral bag models and the Cloudy Bag Model (CBM) in particular. Then we summarize the results of that model for hadronic properties and pion scattering. In a following section, Convergence Properties, we discuss the convergence properties of the CBM and show that it has some consequences for nucleon decay. We also have some new comments on the question of the supposed divergence of the self-energy in the chiral bag models. Two following sections deal with *N-N* scattering (and particularly with charge symmetry violation) and possible exotic bag states.

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At no stage will we question the underlying bag model itself. For a discussion of both its defects and its relationship to quantum chromodynamics (QCD) and to soliton models, we again refer to Ref. 1 and the references therein. The MIT bag model has proved phenomenologically very successful.[2,3] There are reasons for believing it may really resemble the true QCD description of hadron structure. Finally, in the static, spherical limit, it is simple enough to permit application to a wide variety of low-energy phenomena.

Chiral Symmetry and the Bag Model

Chiral symmetry is an important property of QCD with massless quarks. Because the physical particles we see do not come with degenerate, negative-parity partners, we know that it must be realized in the Goldstone mode.[4] In particular the pion is very close to being the massless, pseudoscalar Goldstone boson required in that idealized world. Much of the work on chiral symmetry in nuclear physics has been based on models related to the σ model of Gell-Mann and Lévy,[5] models that have nothing to do with quarks and QCD.

All the chiral bag models follow from the early observation by Inoue and Maskawa, and by Chodos and Thorn, that the MIT bag model violated chiral symmetry.[6] In all cases the basic idea is to include a pion (and in some cases a σ) field to restore chiral symmetry — by analogy with the original work of Gell-Mann and Lévy. For a simple pedagogical treatment of these ideas, we refer the interested reader to Ref. 1. Our intention here is merely to review some of the highlights of the calculations made in the chiral bag model nearest to our heart, the CBM.[7] However, because the underlying philosophy of the model has often been misunderstood, it seems worthwhile to make some general remarks before presenting those results.

Our aim in developing the CBM was to develop a new theoretical framework for low- and medium-energy nuclear physics by building on the success of the MIT bag model. As we mentioned, the major theoretical shortcoming of that model was its lack of chiral symmetry. To restore global, chiral invariance, we introduced an "elementary" pion field coupled to the confined quarks in the minimal way.[1,8,9] To lowest order in the pion field, that coupling is unique, and because of partially conserved axial current (PCAC) involves no new parameters.

Although the pion field is formally treated as a structureless object, the appearance of the pion decay constant (f) is the signal of its internal structure — being essentially related to the $q\bar{q}$ wave function at the origin.[10] It is certainly not expected that deep inelastic scattering, for example, should reveal pointlike, pseudoscalar particles inside the nucleon! As we have continually stressed, the CBM was constructed to apply to typical nuclear problems, involving relatively low momentum transfer. There is no question that it should break down as one probes shorter and shorter distances. This appears to have been forgotten in recent studies of its convergence properties — as we shall discuss in the following section, Convergence Properties.

When looking to the high-energy literature for guidance in constructing such a phenomenological model, there is wisdom in Shakespeare's admonition that "the devil can cite scripture for his purpose." Not long ago the popular creed was a two-phase picture of hadron structure in which the interior region could not possibly contain pions. On the other hand, recent lattice calculations[11] suggest that the transition to the Goldstone mode occurs at significantly higher temperatures (smaller distances) than true confinement. Today we have a three-phase picture, tomorrow, ...?

This concept of the pion field having its source in the bag surface, but nevertheless leaking inside, is a fundamental difference between the CBM and little bag models. It is consistent (in a qualitative sense, and all of the arguments on this question are no more than that) also with attempts to construct the pion in a scheme where chiral symmetry is broken by the exceptionally strong one-gluon-exchange attraction in the $0^- \; I = 1, \bar{q}q$ channel.[12] Such a model naturally leads to a compact $q\bar{q}$ component of the pion, and provides the motivation for its special treatment in the CBM. That is why the pion is allowed to propagate freely inside as well as outside the bag. (A more recent version by Chin and Miller[13] based on the work of Ref. 11 would exclude the pion from a third interior region, but would hardly alter the successful CBM phenomenology.) Future, more sophisticated theories may improve on this by building in some effective potential for the coherent $\bar{q}q$ pair inside a bag. However, the phenomenological success of the CBM, as well as the similarity of the results when m_π is arbitrarily set to zero, suggests that this should not greatly alter our results.

A Hamiltonian for Low-Energy Physics

There is little point to repeating in the space available the formal developments necessary to restore chiral symmetry in the bag model. The details can be found in the original papers[6-9,14] and in Ref. 1. Either the resulting Lagrangian involves both σ and pion fields, or the pion field enters in a highly nonlinear way. In any case, our philosophy has been to build on the success of the MIT model by assuming that the pionic effects will act as a small perturbation. A similar approach has been taken by Jaffe,[8] De Tar,[15] and others.[16] Then one can expand in powers of the pion field, retaining only lowest order terms. The resulting linearized CBM Lagrangian density breaks very nicely into three separate pieces,

$$\mathcal{L}_{\mathrm{CBM}}(x) = \mathcal{L}_{\mathrm{MIT}}(x) + \mathcal{L}_{\pi}(x) + \mathcal{L}_{\mathrm{int}}(x) \quad , \tag{2.1}$$

where $\mathcal{L}_{\mathrm{MIT}}$ is the usual Lagrangian density for the MIT bag model, $\mathcal{L}_{\pi}$ describes a free pion field, and

$$\mathcal{L}'_{\mathrm{int}} = -\frac{i}{2f}\,\bar{q}\,\gamma_5 \tau\, q \cdot \phi\, \delta_s \quad . \tag{2.2}$$

Without $\mathcal{L}_{\mathrm{int}}$, which was dictated by chiral symmetry, the theory would describe stable MIT bag states and free pions.

Once gluon degrees of freedom are included in $\mathcal{L}'_{\mathrm{MIT}}$, only colorless states have finite energy. We shall be concerned here with only baryon structure, although similar ideas could be applied to the heavy mesons. Thus we are naturally led to consider at first colorless bag states with baryon number one — that is, $3q$, $4q\text{-}\bar{q}$, $5q\text{-}2\bar{q}$, and so on. In view of the success of the bag model in describing the low-lying baryons without exotic components, it is reasonable to divide the space of baryon number-one hadrons into two pieces $(P + Q)$,

$$P = \sum_{\substack{\alpha\,=\,\text{nonexotic}\\ \text{baryons}}} |\alpha\rangle\langle\alpha| \quad ; \quad Q = 1 - P \quad . \tag{2.3}$$

That is, P is a projection operator onto nonexotic bag states such as N, Δ, R (the Roper resonance), etc. The wave functions for these states are simply the usual bag-model $SU(6)$ wave functions. The unit operator 1 refers to the space of $B = 1$ bag states, and Q is a projection operator onto exotic states.

Formally, the inclusion of corrections arising from coupling to the Q space is equivalent to evaluating the lowest order sea quark corrections. Such corrections have been shown numerically to be rather small, so for the present purposes we shall neglect off-diagonal terms connecting P and Q. In that case the Hamiltonian obtained from $\mathcal{L}_{\mathrm{MIT}}$ in the canonical way is simply

$$H_{\mathrm{MIT}} \simeq P\, H_{\mathrm{MIT}}\, P = \sum_{\alpha} |\alpha\rangle\, m_{\alpha}^{(b)} \langle\alpha| \quad . \tag{2.4}$$

In terms of more conventional second quantization, this becomes

$$H_{\mathrm{MIT}} = \sum_{\alpha} \alpha^{+}\alpha\; m_{\alpha}^{(b)} \ , \tag{2.5}$$

where α^{+} creates a three-quark bag state with the quantum numbers of N, Δ, R, etc. (There is one rather innocent assumption implicit in the last step, namely that two different bag states α and β, with different masses, are orthogonal. Unfortunately, this is not completely correct in the naive bag model because the radii of those two bag states will not be exactly equal as a result of the nonlinear boundary condition. Nevertheless, one expects on physical grounds that the orthogonality must hold in a more sophisticated formulation, such as the soliton bag model, and we simply impose it here.)

In the canonical way, we obtain the usual Hamiltonian for a free pion field, that is,

$$H_{\pi} = \sum_{i} \int d\underline{k}\; w_{\underline{k}}\;\; a^{+}_{\underline{k}i}\;\; a_{\underline{k}i} \quad . \tag{2.6}$$

Finally, and of course this was the whole point of the exercise, there is an interaction term

$$P\, H_{\mathrm{int}}\, P = \frac{i}{2f} \sum_{\alpha,\beta} \int d^3x \times \langle\beta|\,\bar{q}(x)\tau \cdot \phi(\underline{x})\gamma_5\; q(x)|\,\alpha\rangle\, \delta_s\, \beta^{+}\alpha \quad . \tag{2.7}$$

Using the usual expansion for the pion field, and assuming static, spherical bags of equal radii $[\delta_s \equiv \delta(x - R)]$, Eq. (2.7) becomes

$$P\, H_{\mathrm{int}}\, P = (2\pi)^{-3/2} \sum_{\alpha,\beta,i} \int d\underline{k}\; \left(v^{\beta\alpha}_{\underline{k}\,i}\;\; \beta^{+}\alpha\, a_{\underline{k}i} + \mathrm{h.c.} \right) \quad . \tag{2.8}$$

where h.c. denotes Hermitian conjugate, and

$$v^{\beta\alpha}_{\underline{k}\,i} = \frac{i}{2f}\frac{1}{(2w_k)^{1/2}}\int d^3x\, e^{i\underline{k}\cdot\underline{x}}\ \delta(x-R)$$
$$\times <\beta|\bar{q}(\underline{x})\ \tau_i\gamma_5\, q(\underline{x})|\alpha> \quad . \tag{2.9}$$

Thus, as promised, all $B'B\pi$ couplings can be calculated in terms of the pion decay constant, $f = 93$ MeV.

To see what is involved in Eq. (2.9) let us consider the $NN\pi$ vertex. Then the spatial orbits of all quarks in the initial and final hadrons are the same, namely, $1s_{1/2}$. The spatial matrix element gives rise to a form factor $u(k)$ of the form[9]

$$u(k) = 3j_1(kR)/kR \quad . \tag{2.10}$$

The overall strength is simply related to the axial charge for the bag model g_A, so that in putting all of this together we find a very natural expression for the operator at the $NN\pi$ vertex,

$$v^{NN}_{\underline{k}\,i} = i\ (2w_{\underline{k}})^{-1/2}(g_A^{BAG}/2f)\ u(k)\ \tau_i\ \underline{\sigma}\cdot\underline{k} \quad . \tag{2.11}$$

If for the present we ignore questions of renormalization, it is clear that the CBM makes a remarkably accurate prediction for $f_{NN\pi}$. Using $g_A = 1.09$ gives a value of 0.23 in comparison with the observed value of 0.28. However, including c.m. corrections (about 20% increase in g_A),[17] we find that theory and experiment agree within a few percent! (In fact, the effect of renormalization is to reduce the bare coupling constant by 10% or less.[18] Thus a small additional contribution to g_A as proposed by Chin and Miller[13] is not unwelcome.)

In addition to predicting the $NN\pi$ coupling constant, we see that the CBM provides a very beautiful explanation for what was previously an *ad hoc* high-momentum cutoff. The form factor $u(k)$ simply reflects the fact that the violation of chiral symmetry, and therefore pion coupling to the bag, is associated with its surface. Because the bag is far from being pointlike, there is a natural cutoff in the theory with a range related to the radius of the source R. We expect such a cutoff (far from being specific to the CBM) to be a general feature of

Fig. 1.
Pion-baryon couplings that appear naturally in the CBM Hamiltonian.

any model that treats the quark structure of the hadrons explicitly.

Let us return to the general pion-absorption vertex [Eq. (2.9)]. If the hadrons α and β have the same radii, it is well defined. But, as we have remarked already, this will not usually be the case because of the nonlinear boundary condition. Nevertheless, the radii of the members of the lowest baryon octet and decuplet do not vary by more than about 10% from the mean value. Thus in computing ratios of coupling constants, we have assumed that these radii are all equal. (A more satisfying procedure would be to use the pseudovector volume coupling described in Ref. 19.)

A very basic example of an interaction that is extremely important in medium-energy physics is the $\Delta N\pi$ vertex. In the CBM the pion induces this transition by flipping the spin and isospin of a quark at the bag surface ($I = 1/2, J = 1/2 \rightarrow I = 3/2, J = 3/2$). Figure 1 illustrates some of these fundamental vertices. The form factor at all such vertices will be the same function $u(k)$ derived above. In the general case, the vertex function associated with the $B'B\pi$ process is

$$v^{B'B}_{\underline{k}i} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}$$
$$\times\ (f^{(0)}_{B'B\pi}/m_\pi)u(k)\ \underline{S}^{B'B}\cdot\underline{k}\ T_i^{B'B} \quad , \tag{2.12}$$

where $\underline{S}$ and T are standard transition spin and isospin operators.

The coupling constants appropriate to transitions between all members of the nucleon octet have been summarized in the paper of Théberge and Thomas[20] (see also Ref. 21). In the specific case that is of most interest to us after the nucleon, namely the Δ, the bare coupling constants are in the $SU(6)$ ratios

$$f^{(0)}_{NN\pi} : f^{(0)}_{\Delta N\pi} : f^{(0)}_{\Delta\Delta\pi} = 1 : \sqrt{\frac{72}{25}} : \frac{4}{5} \quad . \tag{2.13}$$

Nucleon Properties

We have seen that the practical effect of imposing chiral symmetry on the bag model is to dictate the pion coupling term in the Hamiltonian. Thus the physical hadrons will be dressed by a pion cloud. As we discuss later, the Δ becomes unstable once the interaction with the pion field is turned on and it is no longer strictly an eigenstate of the Hamiltonian. The nucleon must, of course, remain a discrete eigenstate, with eigenvalue m_N. Given the excellent convergence properties of the CBM, which will be discussed below, the calculation of the electromagnetic properties of dressed nucleons (and other members of the nucleon octet) is straightforward. One is justified in making a perturbative expansion of the state $|\tilde{N}\rangle$ as

$$|\tilde{N}\rangle \cong Z^{1/2}|N\rangle + c|N\pi\rangle + c'|\Delta\pi\rangle \quad . \qquad (2.14)$$

Perhaps the most significant observation concerning nucleon electromagnetic structure in this model is the charge form factor of the neutron, G_{En}.

In the MIT bag model the neutron bag has three quarks, whose charges sum to zero, in identical spatial orbits, and therefore no charge distribution. There are a number of higher order effects that tend to mix other configurations into the ground state, but none of these give even the right order of magnitude for $\langle r^2\rangle^n_{ch}$ in the bag model.

On the other hand, if we truncate the perturbation expansion of the physical neutron wave function in the CBM at one pion, we find

$$|\tilde{n}\rangle = Z^{1/2}|n\rangle + c_{N\pi} \times \left(\sqrt{\frac{2}{3}}\,|p\pi^-\rangle - \sqrt{\frac{1}{3}}\,|n\pi^0\rangle\right) \quad , \qquad (2.15)$$

where $|c_{N\pi}|^2$ is the probability for finding the nucleon to consist of a nucleon bag and a pion (approximately 20%, depending on the size of the bag). There is also a $|\Delta\pi\rangle$ component included in all calculations. However, it is much less important for the charge distribution because the $\Delta^-\pi^+$ piece tends to cancel against $\Delta^+\pi^-$, and the 300-MeV excitation energy of the Δ also makes the range of the pion field much smaller. Equation (2.15) shows quite explicitly that the charge distribution of the neutron in the CBM is a first-order effect of the pion coupling, arising directly from the $|p\pi^-\rangle$ component. This was first observed by Thèberge et al.[9] Earlier calculations in classical models missed this because time derivatives of the pion field vanish in the classical limit. Thus one really needs an explicit treatment of the quantum fluctuations of the pion field — as in the CBM — in order to see the effect.

Because the charge of the proton bag is confined inside the bag volume (that is, radii less than R), and the pion field has its source at the bag surface and extends outside, the model obviously predicts a positive core and a negative tail. The details are illustrated in Fig. 2, from Ref. 18. It is clearly an inescapable conclusion of the CBM that the zero in the neutron charge distribution necessarily occurs at the bag radius. An accurate experimental determination of G_{En} thus would provide a direct measure of the size of the confinement volume! (Note that there is certainly no physical significance to the discontinuity of $\rho^n_{ch}(r)$ at $r = R$; it is a consequence of the oversimplification of the description of the bag surface as a rigid sphere. It is unlikely that any more realistic treatment would do more than smooth out

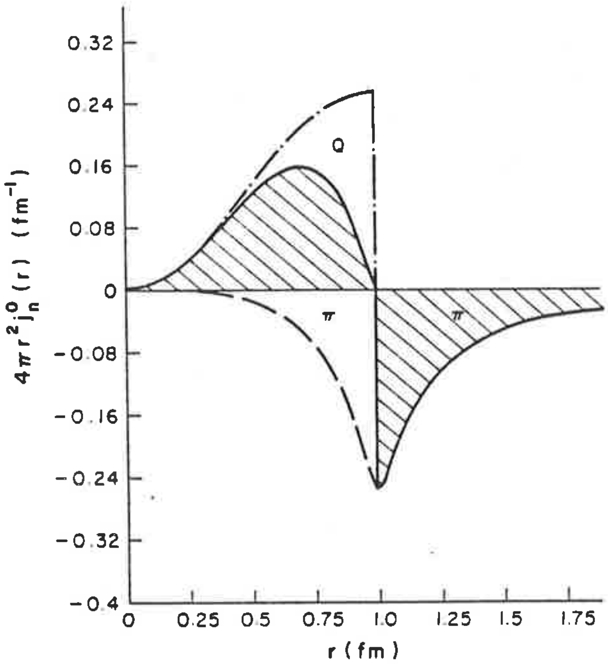

Fig. 2.
The neutron charge distribution $4\pi r^2 j^0_n(r)$ vs the radial distance r (shaded area) (Ref. 20). Also shown are the quark (Q) and the pion (π) charge distribution inside the neutron. The neutron bag radius is set at 1 fm.

the charge density in the surface region without altering our conclusion.) The rms radius of the neutron is not strongly dependent on R, varying from -0.391 fm at 0.8 fm to -0.327 at 1.1 fm — in excellent agreement with the experimental value of -0.342 fm (obtained by dropping thermal neutrons on an electron target).[22]

It is of course of great interest to calculate the other nucleon electromagnetic properties, such as the proton charge radius ($\langle r^2\rangle^{p}_{ch}$), and proton and neutron magnetic moments (μ_p and μ_n), even though the pionic contribution is not the leading term there. Théberge et al.[21] found a proton rms charge radius between 0.73 and 0.91 fm for R between 0.8 and 1.1 fm. However, the c.m. correction to the bag contribution is somewhat controversial.[1,17] Without any c.m. correction, the results of Théberge et al. lay between 0.71 and 0.87 fm. This is still in rather good agreement with the experimental value of 0.836 fm. Finally, we note that very similar results have been obtained by DeTar[15] and Myhrer.[16]

The pionic contribution to the magnetic moments involves the spatial component of the pion current evaluated between nucleon wave functions of the form given in Eq. (2.14) — that is, including both nucleon and Δ intermediate states. The bag contribution itself, while the pion is "in the air," is also interesting. It is possible for the quark magnetic moment operator (unlike the charge operator) to induce an N-Δ transition. Thus one must compute all of the processes shown in Fig. 3. Unlike the direct interaction with the pion cloud, the core interactions will have both an isoscalar and an isovector piece. It is therefore not true, as one can find in the literature, that the pionic contribution is purely isovector.

Once again the comparison of calculational results with experiment is somewhat clouded by the uncertainty over c.m. corrections. Nevertheless, this uncertainty is smaller than for the charge radii. Including the Donoghue-Johnson correction,[1,17] μ_p and μ_n range between (2.43,2.78) and (−1.97,−2.07) nuclear magnetons, respectively,[18] for $R \in (0.8,1.1)$ fm. With no c.m. corrections, the corresponding values are (2.20,2.43) and (−1.80, −1.82) μ_N. Recalling that the MIT results with and without c.m. corrections were (2.24, −1.49) μ_N and (1.9, −1.26) μ_N, respectively, we see that the inclusion of pionic corrections has made a tremendous quantitative improvement in the agreement with data. In particular, the residual discrepancy of (5-10)% is well within the uncertainties of the calculation — for example, from sea quarks, configuration mixing, and so on.

In concluding this section it is worthwhile to point out that the CBM produces an acceptable description of the axial current. The excellent MIT prediction of g_A is not much altered by renormalization[18] (see also Ref. 13). Even the axial form factor $g_A(q^2)/g_A(0)$ is rather well reproduced for a bag radius between 1.0 and 1.1 fm.* The presence of the explicit pion field means that there is an induced pseudoscalar term in $A^{\mu}(x)$, with a somewhat softer form factor.* Furthermore, the imposition of chiral symmetry implies that the relative strengths of the axial and induced pseudoscalar terms are consistent with the Goldberger-Treiman relation.

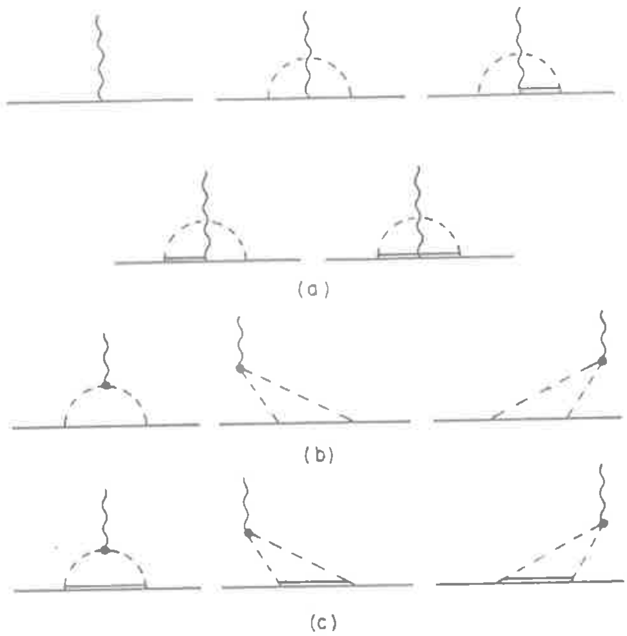

Fig. 3.
Contribution to the magnetic moment of the nucleon from (a) the quark current, (b) and (c) the pion current with an intermediate nucleon or Δ.

Magnetic Moments of the Strange Baryons

Looked at objectively, there is not a great deal of data at our disposal for testing models of hadron structure. One important data set that has seen a dramatic improvement in quality recently, as a result of improved hyperon beams, is the set of magnetic moments of the stable hyperons. In view of the success of the CBM with the nucleon magnetic moments, it is reasonable to ask what the CBM predictions might be for the strange partners of the nucleon. This is even more critical in view

*P. Guichon, G. A. Miller, and A. W. Thomas, to be published in Phys. Lett.

of the findings of Brown and co-workers[23] that the Σ^- moment was in the range (−0.54, −0.64) μ_N, in comparison with the experimental values of −1.41 ± 0.25 μ_N (Ref. 24), and −0.89 ± 0.14 μ_N.*

It is a rather beautiful feature of the CBM Hamiltonian that there is very little freedom in the calculation of these magnetic moments. Equation (2.12) can be used to relate all of the $B'B\pi$ coupling constants to that for $NN\pi$ (Ref. 20). Furthermore, once the strange quark mass is chosen, the photon coupling to the bag is determined. The calculation involves exactly the same diagrams as that for the nucleon except that the intermediate bag states [while the pion is in the air (see Fig. 3)] must have the correct strangeness. For example, for the Σ^- we can have intermediate Λ, Σ, Σ^*, (Λ,Σ), (Λ,Σ^*), and (Σ,Σ^*) baryons. (Such terms were first discussed by Pilkuhn and Eeg from a different point of view, with quite different numerical results.[25])

The results of a calculation[20] using the same bag radii and strange quark mass as the original MIT work[26] are shown in Table I. Clearly the overall agreement of the CBM with data is excellent. A more detailed study of the dependence on bag radius and strange quark mass has confirmed that this is no accident.[20]

In view of the theoretical uncertainties associated with configuration mixing, sea quarks, virtual kaons, and c.m. corrections, it appears unlikely that a more accurate description of the data is likely in the near future. Nevertheless, it does seem that the inclusion of the lowest order pionic corrections results in a good overall description. Clearly a definitive experimental result for both the Σ^- and Ξ^- would be most welcome.

Table I. Comparison of the Predictions of the CBM for the Magnetic Moments (in Nuclear Magnetons) of the Nucleon Octet — from Ref. 20.

	CBM	Experiment
p	2.60	2.79
n	−2.01	−1.91
Λ	−0.58	−0.61
Σ^-	−1.08	−1.41 ± 0.25 −0.89 ± 0.14[a]
Σ^+	2.34	2.33 ± 0.13
Ξ^-	−0.51	−0.69 ± 0.04
Ξ^0	−1.27	−1.25

[a]Preliminary results from T. Devlins, private communication (December 1981).

Meson-Nucleon Scattering

The P_{33} Resonance. Once the constraint of chiral symmetry is imposed on the bag model, there is a qualitative change in the interpretation of the Δ. Whereas N, Δ, R, and so on are eigenstates of H_{MIT}, once the pionic coupling is turned on, only N (actually $\tilde{N}$ in our earlier notation) remains as an eigenstate of the full H. (Of course the other members of the nucleon octet should also remain stable under strong interactions.) The Δ is sufficiently high in mass that it can decay in $N\pi$ and can therefore at best be regarded as an approximate eigenstate of the full Hamiltonian with complex eigenvalue. In this case it seems most appropriate to discuss directly the predictions of the CBM for πN scattering in the P_{33} channel.

When the first crude calculation of pion-nucleon scattering was made in the original Brown-Rho bag model,[27] there was considerable concern in the medium-energy community about double counting. That is, the old Chew-Wick meson theory, which involves just an $NN\pi$ vertex function, can generate a resonance in the P_{33} channel. The reason is that the crossed Born graph (u-channel nucleon pole) shown in Fig. 4(a) produces a strongly attractive, effective potential in the (3,3) channel. When iterated [as in Fig. 4(b)], this potential produces a good description of the P_{33} scattering

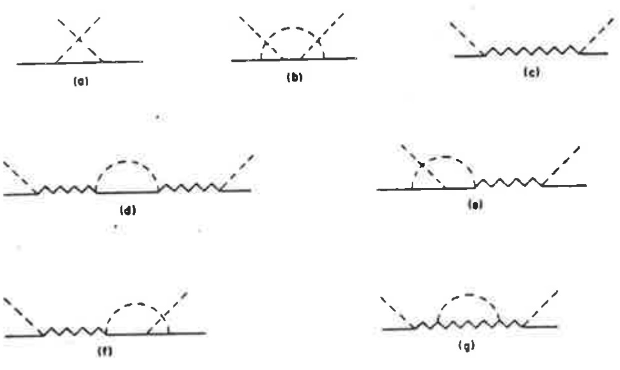

Fig. 4.
Some low-order contributions to πN scattering in the CBM — from Ref. 9.

*T. A. Devlins, private communication.

phase shifts up to 300 MeV, with a suitable choice of cutoff [for example, $\nu(k) \approx \theta(m_N - k)$]. Such a model of the P_{33} resonance is still widely used in the medium-energy physics literature.

The apparent problem with the CBM is that it naturally incorporates both this crossed graph and a direct coupling to the Δ bag [Fig. 4(c)] because both $NN\pi$ and $\Delta N\pi$ couplings occur on the same footing. One might ask whether there is not some double counting, or perhaps even two Δ resonances! The answer is simply that there is no double counting and the pion-nucleon t matrix defined by the CBM satisfies the Low equation[9] as it should. Both the Chew-Wick and direct-Δ mechanisms contribute to πN scattering in the (3,3) channel (and interfere with each other) with a relative strength dictated directly by the CBM Hamiltonian, as illustrated in Fig. 4. One is no longer free to arbitrarily adjust the $NN\pi$ vertex function so that the Chew-Wick mechanism produces a resonance by itself, because the same vertex function occurs at the $\Delta N\pi$ vertex.

To summarize, far from raising problems of double counting, the CBM provides an explicit and physically well motivated example of an alternate solution to the (nonlinear) Low equation, as discussed by Castillejo, Dalitz, and Dyson.[28] Moreover, it provides a precise answer to the rather confused question asked of Gerry Miller at the Houston meeting some 3 years ago[29]: "While the Chew-Low model is a useful model of the P_{33} resonance, it is very dated. Since then we have discovered ... quarks, etc. In that model there is unambiguously an elementary $\Delta \equiv (qqq)$ state .... Is it not possible that the truth about the πN interaction is that the elementary Δ contributes a short-range piece, while the πN rescattering ... results in a relatively long-range piece of the interaction?"

For details of the actual calculation we refer to Ref. 9. It is possible to sum the graphs of Fig. 4 to all orders, and obtain a closed form for the scattering amplitude. The motivation for retaining this particular set of graphs is precisely the same as in Chew's original work; namely, that they are necessary for unitarity. A major advantage of the closed expression for the P_{33} t matrix is that one can explicitly show how the renormalization scheme goes through. Whereas the best fit to the data was obtained with a bag radius of 0.82 fm (Ref. 9), any bag radius between 0.7 and 1.1 fm gave a fair description.[21] Of course, the calculation was made only for static nucleons and therefore should be quantitatively improved. However, the essential physical idea (that the participation of a relatively large three-quark Δ, treated on the same footing as the N, dominates the P_{33} scattering process) will not be altered.

It also turns out that the pionic contribution to the N and Δ masses has an important consequence for the bag model parameters.[9,20] In the conventional bag the entire N-Δ mass splitting is attributed to the spin-spin interaction resulting from one-gluon exchange. Indeed that splitting determines the color coupling constant, α_c, to be 0.55. However, the pionic self-energy for the Δ is considerably less than for N, thus providing an additional mechanism to break their degeneracy. After including this effect, the preferred value of α_c is in the range (0.3, 0.4), which is more consistent with the bag idea of using low-order perturbation theory for the gluons. We also shall see in the Exotic States section that this change has important consequences for exotic states.

In conclusion, we note that there is a considerable amount of loose discussion about the Δ. For example, it is often claimed that the quark model $\Delta N\pi$ coupling constant $[f_{\Delta N\pi} = (72/25)^{1/2} f_{NN\pi}]$ is not sufficient to explain the width of the Δ. However, it should be clear from our discussion of the CBM that this is not the only contribution to the width. For example, the intermediate pion in Fig. 4(b) or 4(e) also can be on-shell. Niskanen has given a nice summary of this recently.[30] It is quite possible that the solution to the problem of the difference between predicted and extracted values of the $\Delta\Delta\pi$ coupling constant raised recently by Duck and Umland,[31] also may be related to the subtlety of the structure of the P_{33} resonance. In any case, this problem deserves more work.

It also may be a source of confusion to some readers that processes such as in Fig. 4(e) and (f) are not simply incorporated into a renormalized $\Delta N\pi$ coupling constant. The answer is that above the $N\pi$ threshold such terms contribute an imaginary part to the πN scattering amplitude. Any theory that seriously expects to explain the width of the Δ must include them explicitly. A similar observation also must be made about the magnetic moment of the Δ. The photon can couple to any of the intermediate-pion legs in Fig. 4, just as for the nucleon. For the reasons we have just outlined, the effective magnetic moment of an on-shell Δ necessarily will be complex. It is absolutely pointless to expect to test so-called quark models of the Δ magnetic moment without incorporating pionic effects.

We might make also some brief remarks concerning the behavior of the Δ in dense nuclear matter. For example, it is commonly believed that the Δ^- should be an important component of nuclear matter at the core of a neutron star. It is very easy to see that imbedding a Δ in nuclear matter would severely inhibit the self-energy contribution involving an intermediate $N\pi$ state. As that contribution is approximately 160 MeV for $R = 0.8$ fm, this can obviously be a large effect. Of course, the tendency to raise the mass of the Δ may be counteracted by the interaction with other nucleons in the medium. It is not even clear that one can simply Pauli block the intermediate nucleon once its quark structure is being considered and the density is high. At the very least we shall have to develop a many-body theory of confined quarks and pions.

Small p Waves. One of the attractive features of the Chew-Low model was that it not only explained the resonant behavior of the P_{33} interaction, but that it also explained (qualitatively at least) the behavior of the other p-wave πN phase shifts at low energy. It is therefore not unreasonable to ask that any theory purporting to replace Chew-Low should do as well. For the small repulsive P_{13} and P_{31} phase shifts, this has been established by Israilov and Musakhanov.[32]

The P_{11} is rather more interesting for a number of reasons. This channel contains the nucleon pole, as a result of which the low-energy phase shifts are negative. However, at about 150 MeV the phase shift changes sign and rises rapidly through 90° at the highly inelastic Roper resonance (520 MeV). Within the MIT bag model we expect that the Roper (R) should be predominantly a $(1s^2,2s)$ configuration. Just like the Δ, the R is stable in the absence of pion coupling. Once the full Hamiltonian is used, R will of course move into the complex plane, obtaining its width predominantly from the coupling to $N\pi$ and $\Delta\pi$. Both Rinat and Nogami have independently shown that the CBM can provide quite a good description of the P_{11} data.[33]

s Waves. So far we have described the success of the CBM for πN p waves. However, there is no obvious prediction for s waves, in contrast with the soft-pion ideas of the 60's, which led to the Weinberg-Tomozawa relationship. Much of the popularity of the nonlinear σ model followed from Weinberg's proof that it provided a convenient effective Lagrangian incorporating the Weinberg-Tomozawa relationship.[34] In fact, as one might have hoped, it is possible to make a unitary transformation[19] on the original nonlinear CBM Lagrangian density in such a way that the Weinberg-Tomozawa relationship for pion scattering from any bag appears explicitly. In particular, the scattering length for a pion incident on a bag of isospin I_t, when the total isospin is I, is simply

$$a_1 = -m_\pi^2 \frac{I(I+1) - I_t(I_t+1) - 2}{8\pi(m_\pi + m_N)f^2} \quad . \tag{2.16}$$

In conclusion, we have only mentioned the results that follow from retaining those terms linear and quadratic in the pion field. If one wishes to deal with reactions like $(\pi,2\pi)$, it will be necessary to keep terms approximately ϕ^3. At that level, as Eisenberg and Kalbermann have stressed,[35] one is sensitive to the particular nonlinear realization of chiral symmetry chosen. Thus without changing the successes of the CBM for elastic scattering, one is led to different predictions for $(\pi,2\pi)$. One might hope that sophisticated coincidence measurements of this reaction, such as those presently being made by the Omicron collaboration at CERN, might help distinguish between models. Unfortunately the analysis will not be easy because the threshold for $(\pi,2\pi)$ sits right on the Δ resonance. It is nevertheless an important problem.

Convergence Properties

As we pointed out in the introduction to the second section, Chiral Symmetry and the Bag Model, one of the critical assumptions of all chiral bag models is that it makes sense in some limited range of momentum transfer to neglect the internal structure of the pion. If, after linearizing the CBM equations and solving for the structure of the nucleon, we discovered that multipion states were important, the approach would not be consistent. Fortunately it has proved possible to make a rather precise statement on this problem as a result of the work by Dodd et al.[36]

In particular, if one restricts the allowed three-quark baryons in Eqs. (2.4)-(2.9) to N and Δ only (no radial excitation of the quarks), one can show rigorously, independent of renormalization details, that the probability of finding n virtual pions in the physical nucleon P_n is bounded. That is,

$$P_n \leq \lambda^n/n! \quad , \qquad \langle n \rangle = \sum_n nP_n \leq \lambda \quad , \tag{3.1}$$

with λ given by the bare pion bag coupling constant and an integral over the $NN\pi$ form factor.[36] For the CBM with bag radii approximately 0.8-1.1 fm, λ is typically 0.9. Clearly the chance of finding three or more pions is negligible. An actual calculation with renormalized coupling constant calculated in lowest order[18] gives $(P_1,P_2,P_3) = (0.35, 0.05, <0.01)$ compared with the bounds of (0.9,0.40,0.12). Thus it may be possible to improve on the bounds even further. Notice also that the average number of pions about the nucleon is about 0.5, compared with the bound of 0.1. The nucleon's cloud cover is rather thin!

In conclusion, we must comment on the renormalization of the $NN\pi$ coupling constant. Because the Δ enters this model on the same footing as the N itself (unlike the old Chew-Wick model), it happens that the increase in the renormalized strength caused by vertex renormalization compensates the decrease caused by Z_2. The net result is that for any bag radius greater than 0.8 fm, the renormalized coupling constant is within 10% of the bare value.[18]

To summarize, if QCD does lead to relatively large baglike baryons, with chiral symmetry realized in the way we have described, the familiar nuclear, strong-interaction regime can be solved by perturbation theory.

Proton Decay

A relatively large number of our experimental colleagues are at this very moment hidden in old gold mines looking for evidence that the eventual fate of the universe is to end with a true whimper — decaying away with a lifetime around 10^{30} years.[37,38] In terms of our conventional nuclear physics picture of the proton, the usual calculations of proton decay [Fig. 5(a)] make little sense. In that case the nucleon is viewed as a relatively small source surrounded by a dense cloud of pions.

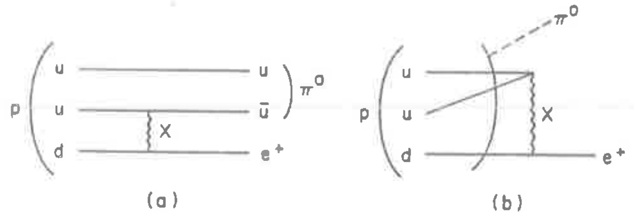

Fig. 5.
(a) The conventional mechanism for proton decay to $e^+\pi^0$. (b) The pion pole term that dominates in the CBM.

Within such a picture, the chance of finding just a (three-quark) core, or the core plus one pion, is negligible.

On the other hand, given the convergence properties of the CBM (as we have just outlined), it is clear that in zeroth order the conventional calculation makes sense. To next order it suggests that there also will be a pole diagram [Fig. 5(b)], where a pion is emitted in the surface of the bag (with a strength and form factor dictated by chiral symmetry) followed by conversion of the off-shell proton into a positron.

Starting from this line of reasoning, McKellar and Thomas have estimated the proton lifetime and its two-body decay modes in a chiral model.[39] Rather than the bag, they used the harmonic oscillator model to estimate the conventional mechanism, Fig. 5(a), and the $p \rightarrow e^+$ piece of Fig. 5(b). However, the strength and form factor for pion emission were given by the CBM. For details and references to related work, we refer to Ref. 39. The inclusion of the pole graph has dramatic consequences. First, the direct and pole graphs for $p \rightarrow e^+\pi^0$ are in the ratio 1:2.2 for the preferred parameters. Thus the lifetime is decreased by a factor of $(3.2)^2$, or about one order of magnitude. For $SU(5)$ this gives $\tau_p = 2.7 \times 10^{29}$ years even when the unification mass is as high as 4×10^{14} GeV. In consequence, the simplest $SU(5)$ GUT is very close to being eliminated already.

The second important implication of the chiral models is for the branching ratios of the possible two-body decay modes. There has been considerable discussion of whether the form factor associated with the $e^+\pi^0$ mode would lead to some suppression in that channel compared with, say, $e^+\omega$ because of the large momentum in the former case. As shown in Table II, which is taken from Ref. 39, in a chiral model the pion modes definitely dominate. The essential difference between the two columns is that in the second, an $S = -1$ pole has been included for the kaon modes ($p \rightarrow \mu^+K^0$) too.[40] Perhaps future work on kaon scattering and the properties of strange baryons will clarify whether or not such a pole term should be included. See Ref. 20 for a discussion of our reservations on this matter. In any case, the $e^+\pi^0$ mode is clearly dominant.

Divergence of the Self-Energy?

This is not the occasion for a detailed technical discussion. However, at least three preprints have appeared recently pointing to a divergence in the lowest order self-energy loop for the nucleon.[41] If we sum over all possible

Table II. Two-Body Branching Ratios for Proton Decay in a Quark Model with Chiral Symmetry (Ref. 39).

	Branching Ratio	
Mode	$SU(2) \times SU(2)$	$SU(3) \times SU(3)$
$e^+\pi^0$	60	49
$e^+\eta$	2.4	0.9
$e^+\rho^0$	0.8	1.2
$e^+\omega$	5.8	9.4
$\bar{\nu}_e\pi^+$	24	19
$\bar{\nu}_e\rho^+$	0.4	0.3
μ^+K^0	6.4	20
$\bar{\nu}_\mu K^+$	0	0

states (n,ℓ) for the quark in the intermediate state, rather than keeping just the $1s$ state as in the CBM calculations described previously, there is a linear divergence of the self-energy. Such a result should not surprise us. As we mentioned in the introduction, the CBM was constructed as an approximate theory to be used for processes involving low-momentum transfer. If at any time we encounter distances small compared with the size of the $q\bar{q}$ component of the pion, it is not consistent to use the CBM Hamiltonian.[42]

Simple arguments based on the uncertainty principle imply that as n and/or ℓ increase in the intermediate state, the associated pion field is compressed closer to the bag surface,

$$\phi_{n\ell}(r) \sim e^{-(\epsilon_{n,\ell}-\epsilon_{l,s})|r-R|} \quad . \tag{3.2}$$

Because $\epsilon_{n\ell}$ increases linearly with n and ℓ in steps of about 400 MeV, it is clear that even for a virtual $3s$ excitation, the CBM is probably becoming unreliable. The formal divergence as n and ℓ go to infinity is physical nonsense. Nevertheless, if one wants to perform precise calculations of the consequences of the chiral bag models, the theory should be generalized to include an appropriate cutoff. In the harmonic oscillator model (for example, Ref. 43) the model space is simply truncated at $n = 2$ or 3 by hand. (The MIT bag model is really not much better, with its instruction to calculate the one-gluon exchange once and only once.)

The clue to the physical origin of such a cutoff is the observation that if the pion has a finite size it will not be possible to squeeze it closer and closer to the bag surface. For example, let us start from the generalization (by de Kam and Pirner) of the alternate CBM Lagrangian[19] to include finite pion size. In that case the interaction Lagrangian density has the form[44]

$$\mathcal{L}_{\rm int}(\vec{r}) \propto \int d\vec{\eta}\, q(\vec{r}+\vec{\eta})\gamma^\mu\gamma_5 \tau\, q(\vec{r}-\vec{\eta})\, P(\vec{\eta}) \times \partial_\mu \Phi(\vec{r}) \quad , \tag{3.3}$$

where $P(\vec{\eta})$ is the probability to find a $q\bar{q}$ pair with relative separation $\vec{\eta}$ in the pion; for example,[45]

$$P(\vec{\eta}) \propto \exp(-\vec{\eta}^2/\rho^2) \quad . \tag{3.4}$$

The motivation for de Kam and Pirner[44] was to stabilize the chiral bag against collapse (see also Ref. 13). However, in the present context Eq. (3.3) is significant because it is straightforward to show that the nucleon self-energy is finite for such a coupling. As we have suggested before,[9,36] the finite pion size provides an additional cutoff. Thus the formal problem of the divergence has been cured, but the practical problem of what to use for $P(\vec{\eta})$, or alternatively $(n_{\rm max},\ell_{\rm max})$, is not. For that, one needs a theory of pion structure related to a dynamic symmetry-breaking scheme for QCD.

Another aspect of this problem is the use of a fixed bag radius, independent of the intermediate state occupied by the quark. The motivation is that at least we are guaranteed a complete set of wave functions in that cavity. On the other hand, those intermediate states do not respect the nonlinear boundary condition of the MIT model which implies that the bag radius should increase roughly as $E^{1/3}$. In fact, this goes right to one of the weakest points of the bag model. The Hamiltonian is not a true Hamiltonian, but is also subject to constraints. As discussed by Rebbi,[46] a correct lowest order treatment of these constraints for excited bag states implies that the surface of the bag should move. Unfortunately this perturbative treatment is suspect even for a single $1p$ excitation.[47] For higher excitations there is no consistent formulation of bag transition amplitudes.

Within the soliton bag model[48] there is a true Hamiltonian, without constraints, and it is relatively easy to see that a correct treatment of the intermediate states would involve a larger hole in the vacuum for the quarks, that is, a larger bag. In that case, there is an additional suppression of the contribution from excited states. To summarize, the much cited divergence of the self-energy in the chiral bag models results from pushing them beyond their region of validity. The introduction of finite pion size and a better treatment of the intermediate states cure the formal divergence. However, there is a need for more theoretical work on the treatment of transition amplitudes to excited bag states, and on the structure of the pion. In the meantime, numerical studies of the effect of phenomenological truncations higher than simply the $(1s)^3$ configuration also would be useful.

The Nucleon-Nucleon Interaction

Attempts to understand the *N-N* force have occupied a lot of bright minds for the past half-century. Through the application of sophisticated techniques relying on crossing and analyticity, the Paris group has arrived at a remarkable description of the two-pion exchange *N-N* force in free space.[49] It is often claimed that the *N-N* potential is known to distances approximately 0.8 fm on the basis of such calculations. On the other hand, it seems self-evident that, if nucleons have a radius approximately 1 fm, quark degrees of freedom should be significant inside 2 fm.

For a brief review of attempts to describe the short-range *N-N* force in terms of quarks, we refer to Ref. 1. Unlike the nonrelativistic quark model where unobserved van der Waals forces occur naturally at large distance, in the MIT bag model there is no interaction for non-overlapping bags. With the restoration of chiral symmetry and the necessary inclusion of the pion field, we do have a natural mechanism for the long-range interaction.

As shown originally by Gross,[50] and rederived many times since, a form factor of the CBM type (being an entire function of q^2) does not alter the usual one-pion-exchange (OPE) force for $r > 2R$. However, if it is still meaningful to use the CBM Hamiltonian when two bags overlap, this form factor will cut down the OPE potential. Unfortunately, it is very difficult to isolate such effects in *N-N* scattering. For example, Gersten[51] has only been able to put limits of $R\epsilon(0.65, 1.0)$ fm by analysis of the *N-N* Fermi invariant amplitudes (see also Ref. 52). As this form factor is omitted in all *N-N* phase-shift analyses, we cannot help wondering whether or not there may be some systematic error in them.

Because the CBM includes the excitation of the Δ in a very natural way, we would expect it to yield essentially the Paris two-pion-exchange (TPE) potential for $r > 2R$. Even as the bags begin to overlap, it may be some time, provided nothing dramatic happens at the quark level, before the conventional TPE breaks down. That is, even if $R \sim 1.0$ fm, it is conceivable that the usual OPE plus TPE potential is not badly wrong to (1.0-1.3) fm. A major challenge for the future is to make this statement more quantitative.

Charge Symmetry Violation

Over the years we have come to appreciate the value of symmetry principles in nuclear physics, and violations of any fundamental symmetry are studied in great detail. It is not unreasonable to expect that the CBM, with its more fundamental description of nucleon structure, should have something new to say about symmetry violation. It is even possible that predictions of symmetry violation made in this model might survive the improvements necessary to obtain quantitative fits to nuclear data.

Whether or not a symmetry is fundamental depends, of course, on one's point of view. In a quark model it is quite apparent that conventional isospin is an accidental symmetry. Indeed the *u* and *d* quark masses are typically about 5 and 10 MeV, respectively, so $SU(2)$ is badly broken at the Lagrangian level. However, these masses are much smaller than the eigenvalue of the Dirac equation for a light, confined quark. Thus the microscopic breaking of the symmetry gets hidden and isospin looks good at the hadronic level. Because charge symmetry is a special case of isospin invariance, corresponding to rotations by 180° about the *y* axis in isospin space, it is clearly no longer "fundamental." Nevertheless there is a great deal of experimental activity presently aimed at finding charge symmetry violation (CSV) in the *N-N* system[53] — so far without success. The classic case studied at length is the $^1S_0$ scattering length, where the best experimental values for *nn* and *pp* are -18.6 ± 0.6 fm and -17.1 ± 0.2 fm, respectively. While this apparently indicates a small CSV, there is considerable discussion of the meaning of the errors quoted.

In a recent LAMPF experiment, Hollas and coworkers[54] failed to see a charge-symmetry-violating forward-backward asymmetry in the process $np \rightarrow d\pi^0$ at a level of 0.5%. The most sensitive tests so far should come from experiments presently under way at both Indiana University Cyclotron Facility (IUCF) and TRIUMF, where they are looking for a small difference in the position of the zero in *P* and *A* in *np* elastic scattering.[53]

Conventional theoretical models for CSV typically involve ρ-ω and π-η mixing in a one-boson-exchange picture. The presence of such mixing is a result of the *u-d* mass difference necessary to reproduce the *p-n* mass splitting. However, in a quark model of the short- and medium-range *N-N* force, it is not obvious that such mixing for real mesons has anything to do with *N-N* scattering. It would seem more appropriate to calculate *N-N* scattering using $m_u \neq m_d$ directly. This has not yet been done.

What has been looked at[55] is the possibility of a direct source of CSV in the OPE interaction caused by $m_u \neq m_d$. Because of the explicit appearance of quarks and pions in the Lagrangian density, and its excellent convergence properties, the CBM is ideally suited to this problem. We recall from Eq. (2.11) that the pion-nucleon coupling had strength $g_A/2f$, where g_A is the axial charge of the nucleon calculated in the bag model. In the MIT bag model, the presence of the lower piece of the Dirac spinor for the quark gives a maximum suppression of about 34% of the nonrelativistic value of g_A (5/3) in the case $m_{\text{quark}} = 0$. Of course, in the nonrelativistic limit of infinite quark mass, the lower component vanishes and the value of 5/3 is restored. If one has two masses in between the ultrarelativistic and nonrelativistic limits, the suppression factor will be smaller, and hence g_A larger, for the heavier of the two.

In particular, if m_d is (4-5) MeV heavier than m_u — as we require to fit the *n-p* mass difference — g_A will be larger for the *d* than for the *u* quark. If we consider π^0 coupling to the *n* and *p*, it should now be clear that the coupling to the neutron will be larger than that to the proton because the former contains more *d* quarks. Using the spin-flavor wave functions for distinguishable *u* and *d* quarks, we can easily show that

$$\frac{g_A^n}{g_A^p} = 1 + \frac{3}{5}\delta \quad , \tag{4.1}$$

where (1-δ) is the ratio of g_A for a single *u* quark to that for a single *d* quark. This leads to a value δ = 0.64% for $(m_d - m_u) = 5$ MeV, and hence g_A^n/g_A^p is >1 by 0.4%.

This is outside the level of accuracy for present neutral current experiments. However, one may hope to see this effect through the difference in $f_{\pi^0 nn}$ and $f_{\pi^0 pp}$. Clearly we expect that the $nn\pi^0$ coupling constant should be about 0.4% bigger than that for $pp\pi^0$ — in direct violation of charge symmetry. For the *N-N* scattering length this implies[55] $|a_{nn}| - |a_{pp}^{No\ Coul}| = +0.3$ fm, which is in the same direction as experiment but a little small (although we stress again that the experimental numbers are not conclusive). Other systems, in which we might hope to see this CSV include the decay widths of the Δ and the forward-backward asymmetry in $np \rightarrow d\pi^0$, which may be enhanced for an appropriate polarization observable.

The ^{3}He-^{3}H Mass Difference

Within the framework of nonrelativistic potential theory, the three-nucleon system has been amenable to exact solution for about a decade. However, we still do not have a satisfactory explanation of the ^{3}H-^{3}He mass difference. After removing the *n-p* mass difference, we find a residual 760-keV splitting between these mirror nuclei. Potential model calculations using charge-independent forces give typically 640 keV and never more than 680 keV (see the proceedings of the TRIUMF workshop[53]). The remaining 80 keV has been a mystery for at least 15 years.

To see what a quark-level description would imply for this problem, we first need to recall where the *n-p* mass difference comes from. As in all quark models, the Coulomb contribution is about 0.5 MeV in the wrong direction, tending to make the proton heavier than the neutron. The only freedom in the bag model description is to take the *u* and *d* quarks to have different masses. With a *u*-quark mass about (4-5) MeV less than that of the *d* quark, the necessary 1.79-MeV mass difference (1.29 MeV experimental plus 0.5 MeV from electromagnetic effects) can be explained.

Next we recall that ^{3}He is one of the most dense nuclear systems available. Its rms charge radius is only about 1.8 fm, compared with 0.83 fm for the nucleon. It is therefore highly likely that in a random snapshot of the nucleus we would find two nucleons overlapping. Thus one obvious difference between ^{3}H and ^{3}He is that with some probability *P*, we shall find the contents of two neutrons in one bag in the former, whereas in the latter we would find two protons. The essential point is that the mass splitting between a 2*p* bag and a 2*n* bag is not $2(m_p - m_n)$, but must be calculated.

First the nonlinear boundary condition implies that the radius of a six-quark bag is bigger than that of a three-quark bag. (In general one can show that $R \sim M^{1/3}$, with *M* the mass of the multiquark system.) Therefore we find at once a 30% reduction in the *n-p* mass splitting caused by $m_u \neq m_d$. In addition, a simple calculation shows that even allowing for the increase in average interquark separation, the Coulomb splitting increases in the wrong direction. The net result is that the $2n$ and $2p$ bags are split by only 0.9 MeV instead of $2(m_n\text{-}m_p) = 2.6$ MeV. Alternatively, the effective n-p mass difference for the fraction of time P that the bags overlap is only 0.45 MeV.

A probability P of 10% would therefore explain the 80-keV discrepancy. This is a perfectly reasonable probability. Indeed, if we assume that when the center of one bag is within one bag radius of the center of another that they have coalesced, we obtain a probability $(1.0/1.8)^3 = 17\%$ for ^{3}He. It is clearly difficult to make this argument more quantitative at the present time, but the $A = 3$ system does provide a beautiful example of just how different the quark-model perspective may be, even for a familiar problem. Further work along these lines is presently being carried out* to see to what extent such ideas can contribute to an explanation of the Nolen-Schiffer anomaly.

Exotic States

It is an unavoidable consequence of the bag model that not only will three-quark ($3q$) baryons exist, but in fact any color singlet combination will — $6q$, $4q$-$\bar{q}$, etc. Were such states to be discovered as relatively long-lived identifiable particles, it would be a real triumph for QCD. Much theoretical effort has been devoted to calculating the spins, parities, and masses of such states.[56] Obviously it was very tempting to attribute the rapid energy dependence observed in $\Delta\sigma_L$ and $\Delta\sigma_T$ at the Argonne Zero Gradient Synchrotron (ZGS) to such a dibaryon resonance; certainly the energy regions coincided. However, the dibaryon example reveals the essential problem of almost all exotics. The structure in the $^3F_3$ N-N channel coincides with the opening of the N-Δ p wave, and the inclusion of this coupled channel alone can qualitatively reproduce the observed structure.[57] To reach this conclusion, we must perform rather complicated three-body calculations (involving two nucleons and a pion), which decently respect unitarity. The moral of the story is simply that when an exotic is connected with several open channels it cannot be discussed in isolation.

One rather simple attempt to deal with this is the p-matrix formalism of Jaffe and Low. Using this, it has been suggested that indeed a number of $B = 0$ and $B = 2$ exotics would not be expected to produce dramatic effects in π-π and N-N phase shifts.[58] However, ideally we would like to see a consistent, unitary, coupled-channels calculation. At least for those cases where pion production is significant (like the dibaryons), the CBM should provide the basis for such a treatment.

One very important exception is the doubly strange Λ-Λ bag, which is actually predicted to be bound by about 80 MeV and therefore to have no strong decay channels.[56] The experimental observation of this state would be very exciting, but it has not yet been seen. One possible reason for its nonappearance is provided by the chiral bag models. For example, in the CBM[19] the pionic self-energy contribution is approximately −130 MeV for the Λ, but the di-lambda would be some 30% larger (because of the nonlinear boundary condition). Because the pionic self energy decreases as $R^{-3.5}$, one would naively expect the pion self-energy for the di-lambda to be cut in half. In fact, this argument has been confirmed in a recent calculation of Mulders and Thomas.[59] They found a mass of about 2.22 to 2.23 GeV for the ($\Lambda\Lambda$), compared with the MIT bag prediction of 2.15 GeV. Because of the uncertainties inherent in such a calculation, it is not possible to say definitively whether the ($\Lambda\Lambda$) is weakly bound or just unbound. Certainly in the latter case it would be much harder to identify experimentally.

Conclusion

This has been a very rapid excursion into quark-model ideas of hadron structure and the N-N interaction. We could therefore only provide a flavor of the challenge and excitement felt by those of us involved in this work. For the next few years our effort must go in (at least) two diverse directions. First, we should expect to see a deepening of the theoretical basis of the model, making its connection to QCD clearer and undoubtedly leading to some modifications of the phenomenology. At the same time we should expect more realistic studies of the nuclear many-body problem — at nuclear matter density and beyond — using the existing phenomenological theory. There is much work to be done.

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MAGNETIC MOMENTS OF THE NUCLEON OCTET CALCULATED IN THE CLOUDY BAG MODEL

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Abstract: The formalism of the cloudy bag model is used to calculate the pion coupling to the strange members of the nucleon octet (and delta decuplet). We then calculate the magnetic moments of all members of the octet, including lowest-order pionic corrections. Results are presented as a function of the radius of the bag, R, which is the only true free parameter of the model. Excellent agreement is obtained with experiment for bag radii ranging from 0.8 to 1.1 fm.

1. Introduction

One of the major constraints when constructing a model for the internal structure of baryons is its ability to predict the right values for the known baryon magnetic moments. This constraint has become even more restrictive recently with the new measurements of the hyperon magnetic moments, which have greatly improved in precision [1,2]).

It has long been understood that baryons do possess internal structure. The earliest evidence of this was the measurement of the anomalous magnetic moment of the nucleon. When Gell-Mann and Zweig presented the quark model of hadrons in 1964 [ref. [3])] not only could this model explain the success of the "eightfold way" [4]) classification of baryons and mesons, but it could also predict the ratio μ^{p}/μ^{n} to be $-\frac{3}{2}$ [ref. [5])], which compares rather well with the experimental value of −1.46. However, the quark model had Fermi statistics problems with the Δ^{++} for example, which required the introduction of a new quantum number, colour [6]). The antisymmetry of the baryon wave function could then be supplied by a totally antisymmetric colour wave function. In this scheme, all hadrons are colour singlets as observed experimentally.

In 1973 various authors suggested that quark dynamics could be governed by a local gauge symmetry similar to QED [7]). This scheme would confine colour inside hadrons. Since three colours are sufficient to describe the hadron spectrum, the gauge group was chosen to be SU(3). The eight massless vector gluons would then carry the interaction between coloured quarks. This model, quantum chromodynamics (QCD), was soon realized to have "asymptotic freedom" which guarantees that the quarks are free at short distances, in agreement with the observed Bjorken phenomenon in high-energy electron-nucleon scattering [8]). However, in the low-energy regime, QCD has not yet been proved to confine quarks within hadrons.

Since QCD cannot supply us with a quantitative framework with which to derive the properties of hadrons, some simplified picture is necessary. Guided by QCD, quark confinement, asymptotic freedom, chiral symmetry (which arises naturally in massless QCD) and our knowledge of nuclear physics, it is possible to derive a consistent phenomenological model for the internal structure of baryons which allows quantitative predictions to be made.

The phenomenology of quark dynamics has followed two different paths. The first one is the non-relativistic quark model which has culminated recently with the potential model of Isgur and Karl [9]). In short, non-relativistic quarks with masses of 300 to 400 MeV are confined in a harmonic oscillator potential whose specific form is dictated by considerations of QCD. The model dramatically improves the simple SU(6) quark model, but there are many reasons to believe that this approach is an over-simplification of the physics within hadrons [10]). For example, their massive quarks are in contradiction with the results of current algebra, chiral symmetry, and Bjorken scaling, which all point towards very light up and down quark masses.

A more plausible model for hadrons was suggested in 1974 by the MIT group [11]). In agreement with current algebra, the u- and d-masses are chosen to be very light – a few tens of MeV at most. These light quarks are confined permanently inside a static spherical cavity called the "bag", whose radius R (or equivalently the vacuum pressure B) fixes the scale for the whole model. The energy of the quarks is then ω/R with $\omega \sim 2.04$ for massless quarks. For a bag radius of 1 fm, this quark energy is of the order of 400 MeV which corresponds to the constituent mass of the non-relativistic quark model discussed above. The ratio of the magnetic moments of all baryons can be calculated explicitly in this framework, and the results are a major improvement on the SU(6) quark model. However, the prediction of the proton magnetic moment of 1.9 nuclear magnetons (μ_N) [2.24 when centre-of-mass (c.m.) corrections are included [12])], is still far from the experimental value of 2.79 μ_N.

One major success of the MIT model is the prediction of 1.09 for the axial vector constant (1.27 with c.m.c. included). This compares very well with the experimental value of 1.25 and constitutes a major improvement on the non-relativistic quark model prediction of 1.67. An important problem with the MIT bag model is the neutron charge radius which comes out to be exactly zero. Finally, we mention that the MIT bag model does not provide any mechanism for interaction between baryons, which makes it rather inappropriate for treating nuclear physics problems [13,14]. These problems can be solved if we add a vital ingredient to the MIT bag model, namely chiral symmetry.

Chiral symmetry has long been known to be an important symmetry of the strong interaction [15,16]. Introduced first in the context of current algebra, it gives rise to important results such as the Goldberger–Treiman relation, the Adler–Weisberger sum rule for pion–nucleon cross sections, and the pion–nucleon scattering lengths, all of which have been verified experimentally within 7%. Chiral symmetry is an intrinsic property of massless fermions, and is clearly realized in the context of massless QCD. However, it was less apparent in hadronic physics until Gell-Mann and Levy presented the σ-model [17], where the nucleon acquires a mass via the spontaneous breaking of chiral symmetry. There is also a massless Goldstone boson, the pion which can acquire a mass if chiral symmetry [here $SU(2)\times SU(2)$] is slightly violated.

The hybrid bag models [refs. [13,18–29]] offer versions of the MIT bag model which incorporate chiral symmetry. If we assume that the up and down quarks are massless, a quark in an helicity eigenstate will stay there, and the axial current, $\boldsymbol{A}^{\mu}(x)$, is conserved. However, because the bag surface is scalar, the quark helicity is flipped when it is reflected, thus violating chiral symmetry. By coupling the pion field to the quarks on the bag surface, the hybrid bag models restore chiral symmetry, thereby allowing the bags to interact via the one-pion-exchange mechanism.

In the cloudy bag model (CBM) [13,22], the pion field is coupled to the quarks at the bag surface via a non-linear realization of $SU(2)_L\times SU(2)_R$. This removes the need to introduce a fictitious sigma field as in the model of Chodos and Thorn [18]. Next, the pion field is assumed to be small. That is, the bag radius is large enough that the non-linearities of the model can be neglected. This was not possible in the model of Brown and Rho for example [19], where a strong pion field was supposed to shrink the bag to about 0.3 fm – making the use of perturbation theory unreliable.

From the linearized lagrangian density in the CBM, we extract a hamiltonian in the canonical way using the energy-momentum tensor $T^{00}(x)$. We then quantize the pion field and project the resulting hamiltonian onto the space of colourless, non-exotic baryons (the octet and decuplet of baryons). The resulting theory is similar to the Chew–Low model of the nucleon [30], with the form factor $u(kR)= j_0(kR)+j_2(kR)$ in the πAB vertex function ($A, B\in N, \Delta, \ldots$) providing a natural cut-off for the integrals involved in the calculations. The πAB coupling constants f_0^{AB} are all related to the πNN one, f_0^{NN}, via SU(6) coefficients, while f_0^{NN} itself is related to the quark frequency Ω, the pion mass m_π and the pion decay constant f_π (as we shall see later). This model of quantized pions coupled to baryons appears to us to be far better suited to the study of hadronic properties than using the classical static pion field solution as advocated by various authors.

Crucial to our quantization of the pion is having the pion field allowed to leak inside the bag. There are many scenarios available to explain this [13]. First, if QCD behaves more like quarks coupled with strings, the probability of creating $q\bar{q}$ pion-like objects is non-zero for a finite distance between the quarks, and therefore a realistic pion field will have to leak inside the bag. If QCD has a confining phase (the true vacuum), and a non-confining phase (the chirally symmetric vacuum inside the bag), a correct treatment of the surface will have to be non-static. Due to the dynamical motion of the surface, which will occur in a covariant treatment of the bag, the time-averaged pion field will leak inside. There is also a possibility, suggested by the recent work of Goldman and Haymaker [31]), that the pion may be bound by the strongly attractive one-gluon-exchange force in that channel. In that case $q\bar{q}$ pairs with pion quantum numbers might be better treated as a coherent pair – even inside a baryonic bag [13]). Of course we expect such pairs to be preferentially produced near the surface of the bag, where the coupling constant (α_c) is increasing rapidly. Indeed, in our earlier calculations of the pion charge distribution in the CBM [24]), the pion field did strongly peak at the bag surface. To summarize, we consider it physically very reasonable that the pion field be non-zero inside the baryonic bag volume.

Of course the CBM goes one step further and treats the pion as an elementary particle. This can only be reconciled with its undeniable quark structure if the pion field is weak, and we deal with low momentum transfer. One measure of the reasonableness of this approach is the ratio of the pion charge radius [0.56 ± 0.04 fm, ref. [32])] to its Compton wavelength (1.4 fm), namely $0.56/1.4 \sim 0.4$. As long as we deal with low energy phenomena, and the number of virtual pions is low [28]), we expect the CBM to be a good approach.

In sect. 2, we present a summary of the cloudy bag model formalism, and derive the CBM hamiltonian which involves the pion–baryon–baryon couplings (πAB vertices). We shall concern ourselves strictly with the chiral group $SU(2)_L \times SU(2)_R$, and therefore neglect entirely the contribution of the kaon whose large mass violates badly the larger chiral group $SU(3)_L \times SU(3)_R$.

In sect. 3, we study carefully the stable eigenstates of the full hamiltonian H, $|A\rangle$. Here $|A\rangle$ stands for all members of the stable nucleon octet $\{N, \Lambda, \Sigma, \Xi\}$. We shall expand $|A\rangle$ on the basis of the bare eigenstates of $H_0|A_0, n\rangle$ (we use the subscript "0" for bare quantities), where A_0 stands for the members of both the octet *and* the decuplet of baryons. We shall look at Z_2^A, the probability that the physical baryon $|A\rangle$ is a bare bag (no pion) $|A_0\rangle$; the mass renormalization Σ^A caused by the pion field; and the vertex renormalization function Z_1^{AB}.

Sect. 4 deals with the magnetic moment of all members of the baryon octet. We extract explicitly the contribution of the quark and pion field in the "no more than one pion in the air" approximation. We also argue about the two prescriptions available for the c.m. corrections. The predicted magnetic moments are compared with the experimental ones, with and without c.m. corrections, as a function of the bag radius R.

Finally, concerning the conventions used in this work, we shall use the Bjorken and Drell conventions for all relativistic quantities such as four-vectors, Dirac matrices, etc. [33]). We use the conventions of Rose for the Clebsch–Gordan coefficients and their identities [34]). All other conventions and definitions will be described in the text as we need them.

2. The cloudy bag model formalism

The cloudy bag model is entirely defined by its lagrangian density [23,24]):

$$\mathscr{L}(x)=\bar{q}(x)\{\tfrac{1}{2}i\overleftrightarrow{\partial}-m_q\}q(x)\theta_V-B\theta_V$$
$$-\tfrac{1}{2}\bar{q}(x)\,e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_5/f_\pi}q(x)\Delta_S+\tfrac{1}{2}(\mathscr{D}_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2\,, \tag{2.1}$$

where $q(x)$ is the quark field of mass m_q, $\boldsymbol{\pi}(x)$ is the pion field of mass m_π, f_π is the pion decay constant with $|f_\pi|\approx 93$ MeV; B is the constant "vacuum pressure"; $\mathscr{D}_\mu$ is a covariant derivative, θ_V is one inside the bag volume V and 0 outside and Δ_S is a surface delta function. The meaning of $\mathscr{L}(x)$ is more transparent in its equations of motion:

$$(i\not{\partial}-m_q)q(x)=0\,,\qquad x\in V\,, \tag{2.2}$$

$$i\gamma\cdot nq(x)=e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_5/f_\pi}q(x)\,,\qquad x\in S\,, \tag{2.3}$$

$$B=-\tfrac{1}{2}n\cdot\partial[\bar{q}(x)\,e^{i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_5/f_\pi}q(x)]\,,\qquad x\in S\,, \tag{2.4}$$

$$(\partial^2+m_\pi^2)\boldsymbol{\pi}(x)=-\frac{i}{2f_\pi}\bar{q}(x)\gamma_5\boldsymbol{\tau}q(x)\Delta_S+\text{higher order}\,. \tag{2.5}$$

Eq. (2.2) is simply the free Dirac equation for the quarks inside the bag; eq. (2.3) is the linear, surface boundary condition which guarantees that the quarks remain permanently confined inside the bag volume; eq. (2.4) is the non-linear boundary condition which gives pressure stability to the bag, and therefore guarantees energy momentum conservation in the theory. The last equation, (2.5), is a highly non-linear Klein–Gordon equation for the pion field, which we give here only to lowest order.

The lagrangian density (2.1) also gives rise to conserved currents when applying Noether's theorem. First,

$$J^\mu_{(x)}=\bar{q}(x)\gamma^\mu q(x)\theta_V\,, \tag{2.6}$$

is the conserved quark matter current. There is also a conserved vector current

$$\boldsymbol{V}^\mu_{(x)}=\tfrac{1}{2}\bar{q}(x)\gamma^\mu\boldsymbol{\tau}q(x)\theta_V+\boldsymbol{\pi}(x)\times\partial^\mu\boldsymbol{\pi}(x)+\text{higher order}\,, \tag{2.7}$$

and, in the limit of $m_q=m_\pi=0$, the axial current

$$\boldsymbol{A}^\mu_{(x)}=\tfrac{1}{2}\bar{q}(x)\gamma^\mu\gamma_5\boldsymbol{\tau}q(x)\theta_V+f_\pi\partial^\mu\boldsymbol{\pi}+\text{higher order}\,, \tag{2.8}$$

is also conserved, but for $m_\pi\neq 0$, we recover the PCAC result

$$\partial_\mu\boldsymbol{A}^\mu(x)=-f_\pi m_\pi^2\boldsymbol{\pi}(x)\,. \tag{2.9}$$

The exact form of the higher-order terms in eqs. (2.5)–(2.9) is given in refs. [13,22,24]).

Our next approximation in the CBM consists of keeping only the terms of order $\boldsymbol{\pi}/f_\pi$ in the lagrangian density (2.1):

$$\mathscr{L}(x)=\mathscr{L}_{\mathrm{MIT}}(x)+\mathscr{L}_\pi(x)+\mathscr{L}_{\mathrm{int}}(x)\,, \tag{2.10}$$

with

$$\mathscr{L}_{\mathrm{MIT}}(x)=\bar{q}(x)\{\tfrac{1}{2}i\overleftrightarrow{\partial}-m_{\mathrm{q}}\}q(x)\theta_{\mathrm{V}}-B\theta_{\mathrm{V}}-\tfrac{1}{2}\bar{q}q\Delta_{\mathrm{S}}\,, \tag{2.11}$$

$$\mathscr{L}_\pi=\tfrac{1}{2}(\partial_\mu\boldsymbol{\pi})^2-\tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2\,, \tag{2.12}$$

$$\mathscr{L}_{\mathrm{int}}=-\frac{i}{2f_\pi}\bar{q}(x)\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)q(x)\Delta_{\mathrm{S}}\,. \tag{2.13}$$

Our weak-pion-field approximation described in the introduction will then consist in doing a perturbation expansion around the MIT solution.

The next step towards getting a hamiltonian formalism involving baryons coupled to pions is to extract the quark space hamiltonian from eq. (2.10) via the canonical quantization procedure

$$\hat{H}=\int \mathrm{d}^3x T^{00}(x)\,, \tag{2.14}$$

where $T^{00}(x)$ is the usual stress-momentum tensor defined as

$$T^{\mu\nu}_{(x)}=\sum_\psi \frac{\partial\mathscr{L}}{\partial(\partial_\mu\psi)}(\partial^\nu\psi)-\mathscr{L}g^{\mu\nu}\,. \tag{2.15}$$

For example, the interaction term is simply

$$H_{\mathrm{int}}=\int \mathrm{d}^3x\frac{i}{2f_\pi}\bar{q}(x)\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_5 q(x)\Delta_{\mathrm{S}}\,. \tag{2.16}$$

If we now recall the MIT solution for massless quarks in the lowest mode, for a static, spherical bag of radius R [where $\Delta_{\mathrm{S}}\equiv\delta(R-r)$] then

$$q(\boldsymbol{r},t)=\left[\frac{\Omega/(\Omega-1)}{8\pi R^3 j_0^2(\Omega)}\right]^{1/2}\begin{pmatrix} j_0(\Omega r/R)\\ i\boldsymbol{\sigma}\cdot\hat{\boldsymbol{r}}j_1(\Omega r/R)\end{pmatrix}\mathrm{e}^{-i\Omega t/R}b\theta(R-r)\,, \tag{2.17}$$

with $\Omega\simeq 2.0428$ from the linear boundary condition, and b is the spin–isospin wave function of the quark. Using eq. (2.17), and the Fourier transform of the pion field

$$\boldsymbol{\pi}(\boldsymbol{r},t)=\int\frac{\mathrm{d}^3k}{(2\omega_k[2\pi]^3)^{1/2}}\{\boldsymbol{a}(\boldsymbol{k})\,\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}}+\boldsymbol{a}^\dagger(\boldsymbol{k})\,\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\}\,, \tag{2.18}$$

we can project the resulting hamiltonian on the space of colourless non-exotic baryonic bags

$$H=\sum_{\mathrm{A}_0,\mathrm{B}_0}A_0^\dagger\langle\mathrm{A_Q}|\hat{H}|\mathrm{B_Q}\rangle B_0\,,\qquad \mathrm{A,B}\in\{\mathrm{N},\Delta,\Lambda\ldots\}\,. \tag{2.19}$$

TABLE 1

The πAB bare coupling constants f_0^{AB}/f_Q defined in eq. (2.24)

f_0^{AB}/f_Q	N	Δ	Λ	Σ	Σ^*	Ξ	Ξ^*
N	5	$4\sqrt{2}$	0	0	0	0	0
Δ	$2\sqrt{2}$	5	0	0	0	0	0
Λ	0	0	0	$2\sqrt{3}$	$2\sqrt{6}$	0	0
Σ	0	0	-2	$\frac{4}{3}\sqrt{6}$	$-\frac{4}{3}\sqrt{3}$	0	0
Σ^*	0	0	2	$\frac{2}{3}\sqrt{6}$	$\frac{2}{3}\sqrt{30}$	0	0
Ξ	0	0	0	0	0	-1	$-2\sqrt{2}$
Ξ^*	0	0	0	0	0	2	$\frac{1}{3}\sqrt{5}$

Here B_0 destroys a three-quark MIT bag of type B_0, $A_0^\dagger$ creates a three-quark MIT bag of type A_0, $|A_Q\rangle$ and $|B_Q\rangle$ are the SU(6) baryonic quark wave functions. Then the resulting hamiltonian can be written

$$H = H_0 + H_{\text{int}}\,, \tag{2.20}$$

$$H_0 = \sum_{A_0} m_{0A} A_0^\dagger A_0 + \sum_j \int d^3k\, \omega_k a_j^\dagger(\boldsymbol{k}) a_j(\boldsymbol{k})\,, \tag{2.21}$$

with m_{0A} being the MIT bare bag mass (no pion around)

$$H_{\text{int}} = \sum_j \int d^3k \{V_{0j}(\boldsymbol{k}) a_j(\boldsymbol{k}) + V_{0j}^\dagger(\boldsymbol{k}) a_j^\dagger(\boldsymbol{k})\}\,, \tag{2.22}$$

with the vertex function given by

$$V_{0j}(\boldsymbol{k}) = \sum_{A_0, B_0} A_0^\dagger v_{0j}^{AB}(\boldsymbol{k}) B_0\,, \tag{2.23}$$

$$v_{0j}^{AB}(\boldsymbol{k}) = \frac{i f_0^{AB}}{m_\pi} \frac{u(kR)}{[2\omega_k (2\pi)^3]^{1/2}} C_{S_B 1 S_A}^{s_B m s_A} (\hat{\boldsymbol{s}}_m^* \cdot \boldsymbol{k}) C_{T_B 1 T_A}^{t_B n t_A} (\hat{\boldsymbol{t}}_n^* \cdot \boldsymbol{e}_j)\,. \tag{2.24}$$

The coupling constants f_0^{AB} are given in table 1 as a function of f_Q with

$$f_Q = \frac{m_\pi}{6 f_\pi} \frac{\Omega}{\Omega - 1} \simeq 0.49\,, \tag{2.25}$$

and the form factor $u(kR)$ in eq. (2.24) is given formally by

$$u(kR) = j_0(kR) + j_2(kR)\,, \tag{2.26}$$

and is shown explicitly in fig. 1.

In the nucleon sector of the theory, $v_{0j}^{NN}(\boldsymbol{k})$ has the form

$$v_{0j}^{NN}(\boldsymbol{k}) = i\sqrt{4\pi}(f_0/m_\pi) \frac{u(kR)}{[2\omega_k (2\pi)^3]^{1/2}} \boldsymbol{\sigma} \cdot \boldsymbol{k} \tau_j\,, \tag{2.27}$$

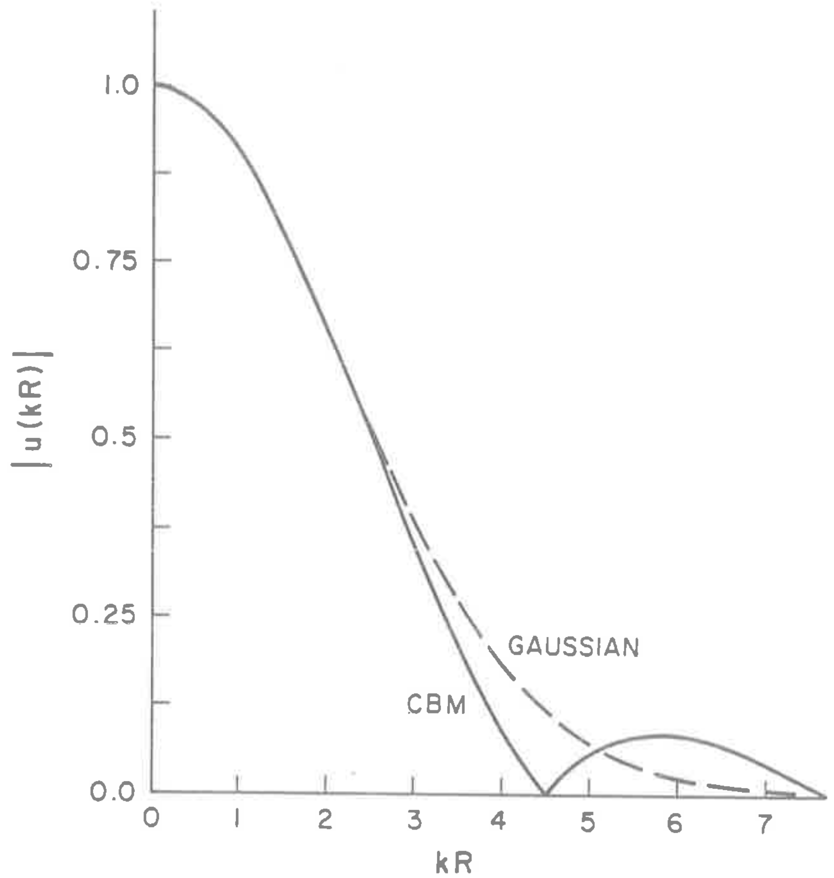

Fig. 1. The CBM form factor $|u(kR)|$, from eq. (2.26), and a gaussian fit, $v(kR) = \exp(-0.106k^2R^2)$.

with

$$f_0 = \frac{1}{3\sqrt{4\pi}} f_0^{NN} \simeq 0.23 . \quad (2.28)$$

This is the usual form for the pseudoscalar pion coupling to the nucleon except for the presence of the form factor $u(kR)$ – as in the Chew–Low model of the nucleon mentioned in the introduction.

In summary, we have a theory of pions coupled to bare three-quark bags where the underlying presence of the quarks reveals itself in the strength and ratios of the coupling constants, and in the form factor $u(kR)$ which arises because the baryons are extended objects. Notice that the only free parameter in this hamiltonian is the bag radius R!

3. The physical baryon $|A\rangle$

3.1. THE PHYSICAL BARYON EXPANSION

In the CBM, the physical baryons are "dressed" bags. That is, because of the coupling of the pion field to the MIT bag, the physical nucleon $|N\rangle$ will be part of the time, Z_2^N, a bare three-quark MIT nucleon bag $|N_0\rangle$, a bare nucleon bag with one pion "in the air", a bare delta with one pion "in the air", etc. Therefore, we can expand any physical baryon state $|A\rangle$ ($A \in \{N, \Lambda, \Sigma, \Xi\}$) in terms of the bare eigenstates of H_0, for example

$$|A\rangle = \sqrt{Z_2^A}|A_0\rangle + \Lambda|A\rangle\,, \tag{3.1}$$

where the on-shell baryon $|A\rangle$ obeys the equation

$$H|A\rangle = m_A|A\rangle\,. \tag{3.2}$$

Here m_A is the physical mass of the baryon $|A\rangle$, and Λ is an operator which projects out all the components of $|A\rangle$ with at least one pion:

$$\Lambda = 1 - \sum_{B_0 \in 56} |B_0\rangle\langle B_0|\,. \tag{3.3}$$

Combining (3.1) to (3.3), and using the identity $\Lambda^2 = \Lambda$, gives the following integral equation for the physical baryon $|A\rangle$:

$$|A\rangle = \sqrt{Z_2^A}|A_0\rangle + (m_A - H_0)^{-1}\Lambda H_{\text{int}}|A\rangle\,. \tag{3.4}$$

Expanding eq. (3.4) with $[\Lambda, H_0] = 0$ gives after recombination

$$|A\rangle = \sqrt{Z_2^A}\{1 + (m_A - H_0 - \Lambda H_{\text{int}}\Lambda)^{-1}H_{\text{int}}\}|A_0\rangle\,. \tag{3.5}$$

This last relation is essential in the calculation of the baryon magnetic moments in sect. 4 of this paper. However, we shall first consider other quantities of interest such as the baryon self-energy Σ^A, the bare bag probability Z_2^A, and the vertex renormalization function Z_1^{AB}.

3.2. THE BARYON SELF-ENERGY

If we denote the bare mass (i.e., the mass of the three-quark MIT bag) of the baryon A, as m_{0A}, which is the eigenvalue of H_0:

$$H_0|A_0\rangle = m_{0A}|A_0\rangle\,, \qquad A \in \{N, \Delta, \Lambda \ldots\}\,, \tag{3.6}$$

then using eqs. (3.1), (3.2) and (3.6) we have

$$\langle A|H_{\text{int}}|A_0\rangle = \langle A|(H - H_0)|A_0\rangle = \sqrt{Z_2^A}(m_A - m_{0A}) \equiv \sqrt{Z_2^A}\Sigma^A\,. \tag{3.7}$$

Replacing $\langle A|$ in (3.7) by its expression in (3.5) gives

$$\Sigma^A \equiv \Sigma^A(m_A) = \langle A_0|H_{\text{int}}(m_A - H_0 - \Lambda H_{\text{int}}\Lambda)^{-1}H_{\text{int}}|A_0\rangle\,, \tag{3.8}$$

and Σ^A then gives rise to the one-particle irreducible self-energy graphs (because of the presence of Λ in the propagator). Some of the low-order terms contributing to Σ^A are given explicitly in fig. 2 for the nucleon.

Since only the terms in the expansion of Σ^A with an even number of H_I can contribute to the sum (each pion created by an H_I has to be destroyed by another

Fig. 2. Lowest-order self-energy diagrams for the nucleon.

H_I), then eq. (3.8) can be simplified by first defining an operator Σ_0 as

$$\Sigma_0 \equiv \Sigma_0(m_A) = H_{int}\Lambda(m_A - H_0)^{-1}\Lambda H_{int}\,, \tag{3.9}$$

giving then for Σ^A:

$$\Sigma^A = \langle A_0 | H_{int}(m_A - H_0 - \Sigma_0)^{-1} H_{int} | A_0 \rangle\,. \tag{3.10}$$

Since Σ_0 will generate self-energy loops within Σ^A, we simplify the procedure by replacing $H_0 + \Sigma_0$ by the operator $\tilde{H}_0$ which is defined to be the bare hamiltonian with the bare bag masses replaced by the physical ones:

$$\tilde{H}_0 = \sum_A m_A A_0^\dagger A_0 + \sum_j \int d^3k\, \omega_k a_j^\dagger(\boldsymbol{k}) a_j(\boldsymbol{k})\,, \tag{3.11}$$

then

$$\Sigma^A \simeq \langle A_0 | H_{int}(m_A - \tilde{H}_0)^{-1} H_{int} | A_0 \rangle\,. \tag{3.12}$$

Eq. (3.12) can be evaluated explicitly using the expressions (2.22)–(2.24) for H_I. After some algebraic manipulations we get

$$\Sigma^A \equiv \Sigma^A(m_A) = \sum_B \left(\frac{f_0^{AB}}{m_\pi}\right)^2 \frac{1}{12\pi^2} \int_0^\infty dk\, \frac{k^4 u^2(kR)}{\omega_k(m_A - m_B - \omega_k)}\,, \tag{3.13}$$

which can be calculated explicitly. However, we postpone the discussion of the results until the end of this section.

3.3. THE BARE BAG PROBABILITY Z_2^A

For the bare bag probability, or wave function renormalization function Z_2^A, we only have to use the normalization condition on the physical state:

$$\langle A | A \rangle = 1\,, \tag{3.14}$$

together with the expansion (3.5) of $|A\rangle$ to get

$$Z_2^A \equiv Z_2^A(m_A) = [1 + \langle A_0 | H_{int}(m_A - H_0 - \Lambda H_{int}\Lambda)^{-2} H_{int} | A_0 \rangle]^{-1}\,, \tag{3.15}$$

which is related to the expression (3.8) of Σ^A via

$$Z_2^A(m_A) = \left[1 - \frac{\partial}{\partial E}\Sigma^A(E)\right]^{-1}_{E=m_A}\,. \tag{3.16}$$

Using the same approximation as in eq. (3.12), we then have

$$Z_2^{\mathrm{A}}=\left[1-\frac{\partial}{\partial E}\left\{\sum_{\mathrm{B}}\left(\frac{f_0^{\mathrm{AB}}}{m_\pi}\right)^2\frac{1}{12\pi^2}\int_0^\infty\frac{\mathrm{d}k\,k^4u^2(kR)}{\omega_k[E-m_{\mathrm{B}}-\omega_k]}\right\}\right]^{-1}_{E=m_{\mathrm{A}}}, \tag{3.17}$$

or more simply

$$Z_2^{\mathrm{A}}=\left[1+\sum_{\mathrm{B}}\left(\frac{f_0^{\mathrm{AB}}}{m_\pi}\right)^2\frac{1}{12\pi^2}\int_0^\infty\frac{\mathrm{d}k\,k^4u^2(kR)}{\omega_k(m_{\mathrm{A}}-m_{\mathrm{B}}-\omega_k)^2}\right]^{-1}. \tag{3.18}$$

3.4. THE "DRESSED", UNSTABLE BARYONS

We mentioned previously that the bare hamiltonian H_0 has two sets of eigenstates, namely the baryon octet $\{|\mathrm{N}_0\rangle, |\Lambda_0\rangle, |\Sigma_0\rangle, |\Xi_0\rangle\}$ and the baryon decuplet $\{|\Delta_0\rangle, |\Sigma_0^*\rangle, |\Xi_0^*\rangle, |\Omega_0\rangle\}$. However, the physical hamiltonian H has only one set of eigenstates, the members of the physical baryon octet $\{|\mathrm{N}\rangle, |\Lambda\rangle, |\Sigma\rangle, |\Xi\rangle\}$. If the physical delta mass satisfied $m_\Delta < m_{\mathrm{N}}+m_\pi$ then the "dressed" delta would also be an eigenstate of H, since it would not decay spontaneously into a pion and a physical nucleon. However, we really have $m_\Delta > m_{\mathrm{N}}+m_\pi$, and the delta propagator in the expansion (3.9) can vanish. In this case, we can still define a "dressed" delta state which is not an eigenstate of H but which satisfies

$$|\Delta\rangle=\sqrt{Z_2^\Delta}\{1+\mathrm{P}(m_\Delta-H_0-\Lambda H_{\mathrm{int}}\Lambda)^{-1}H_{\mathrm{int}}\}|\Delta_0\rangle\,, \tag{3.19}$$

where the P indicates the principal value. This prescription originates from the Lee model of an unstable V-particle [35]), and the motivation comes from the requirement that the dressed delta should be a single-particle state, and therefore must be time-reversal invariant.

What is the mass m_Δ of the dressed delta? At first, one would think that m_Δ should be the resonance energy of 1232 MeV in the P_{33} channel. However, the formation and decay of the dressed delta bag does not account for the full cross section, in fact, crossed nucleon graphs also contribute to the P_{33} resonance [24]). We found previously that this last contribution is rather small for large bags ($R \geqslant 0.8$ fm), and we shall then use the resonance mass of 1232 MeV for the dressed delta mass, and similarly for the Σ^* and Ξ^*.

With eq. (3.19) describing the dressed unstable baryons, the self-energy relation (3.13) is correct, provided that the integral is replaced by a principal value integral. For Z_2^{A}, eq. (3.16) remains valid for all states belonging to the octet and decuplet.

3.5. THE VERTEX RENORMALIZATION FUNCTION Z_1^{AB}

The renormalized vertex function $v_j^{\mathrm{AB}}(\boldsymbol{k})$ is defined as follows:

$$v_j^{\mathrm{AB}}(\boldsymbol{k})=\langle\mathrm{A}|V_{0j}(\boldsymbol{k})|\mathrm{B}\rangle\,. \tag{3.20}$$

That is, it is the matrix element of $V_{0j}(\boldsymbol{k})$ taken between physical states rather than the bare ones

$$v_{0j}^{AB}(\boldsymbol{k})=\langle A_0|V_{0j}(\boldsymbol{k})|B_0\rangle . \tag{3.21}$$

Since $|A_0\rangle$ and $|A\rangle$ ($|B_0\rangle$ and $|B\rangle$) have the same quantum numbers, $v_j^{AB}(\boldsymbol{k})$ is proportional to $v_{0j}^{AB}(\boldsymbol{k})$, and the proportionality constant is independent of the magnetic quantum numbers. It is customary to write

$$v_j^{AB}=\frac{\sqrt{Z_2^A}\sqrt{Z_2^B}}{Z_1^{AB}}v_{0j}^{AB}(\boldsymbol{k}), \tag{3.22}$$

for on-shell baryons (for the off-shell behaviour, replace m_A and m_B everywhere by E_A and E_B). Then, by expanding the physical states $\langle A|$ and $|B\rangle$ in eq. (3.19) using eq. (3.9) gives

$$Z_1^{AB}=[1+\lambda^{AB}]^{-1}, \tag{3.23}$$

with

$$v_{0j}^{AB}\lambda^{AB}=\langle A_0|H_{int}(m_A-H_0-\Lambda H_{int}\Lambda)^{-1}V_{0j}(\boldsymbol{k})(m_B-H_0-\Lambda H_{int}\Lambda)^{-1}H_{int}|B_0\rangle . \tag{3.24}$$

Some explicit terms for $v_{0j}^{AB}\lambda^{AB}$ are given for the NN vertex in fig. 3. Again we use the approximation (3.11), and after manipulations we get

$$\lambda^{AB}=\sum_{C,D}\left(\frac{f_0^{AC}f_0^{CD}f_0^{BD}}{f_0^{AB}m_\pi^2}\right)U_{CD}^{AB}(12\pi^2)^{-1}\int_0^\infty\frac{dp\,p^4u^2(pR)}{\omega_p(m_A-m_C-\omega_p)(m_B-m_D-\omega_p)}, \tag{3.25}$$

with

$$U_{CD}^{AB}=(-)^{S_B+S_C+T_B+T_C}[(2S_B+1)(2S_C+1)(2T_B+1)(2T_C+1)]^{1/2}$$
$$\times\begin{Bmatrix}S_A & 1 & S_C\\ S_D & 1 & S_B\end{Bmatrix}\begin{Bmatrix}T_A & 1 & T_C\\ T_D & 1 & T_B\end{Bmatrix}, \tag{3.26}$$

where the brackets are the usual 6-j symbols [34]).

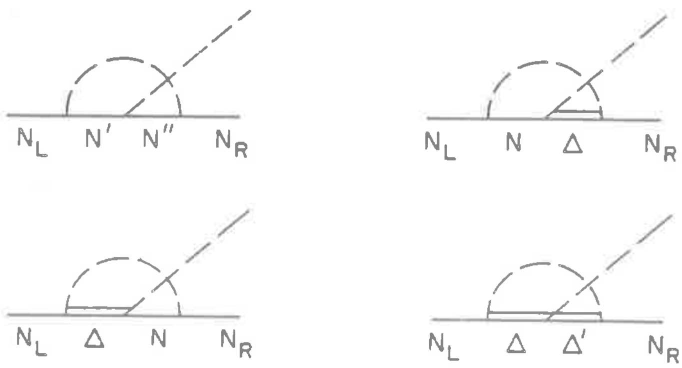

Fig. 3. Lowest-order diagrams contributing to $v_{0j}^{N_LN_R}\lambda^{N_LN_R}$ given in eq. (3.24).

3.6. RESULTS OF THE CALCULATIONS

The second order expressions for Σ^{A}, Z_2^{A} and Z_1^{AB} obtained in the previous sections involve the following parameters: the dressed bag mass m_A, the bare πAB coupling constant f_0^{AB}, and the bag radius R, which arises explicitly in the form factor $u(kR)$.

The dressed bag mass m_A is chosen to be the physical mass if A is a stable baryon, and the resonance mass if A is an unstable baryon. The bare f_0^{AB} coupling constants are all related via SU(6) coefficients given in table 1 to the bare NNπ coupling constant, f_0^{NN}. Since it was shown previously that f_0^{NN} is very close to f^{NN} [ref. [24])], we then approximate

$$f_0^{AB} \simeq f^{AB} \simeq \left(\frac{f_0^{AB}}{f_0^{NN}}\right) f^{NN}\,, \tag{3.27}$$

with f^{NN} being given experimentally to be $\sim$3 in our notation. Finally, the radius R of the bag is left as a free parameter so that the dependence of the predictions of the CBM on R can be studied.

In our lagrangian density (2.10), we have assumed π/f_π' to be small – that is, not too many pions are "floating around". The bare baryon probability Z_2^A is a good indicator of the validity of this approximation. In fig. 4, we give Z_2^A for the stable baryons. From this graph we conclude that the CBM is expected to work better

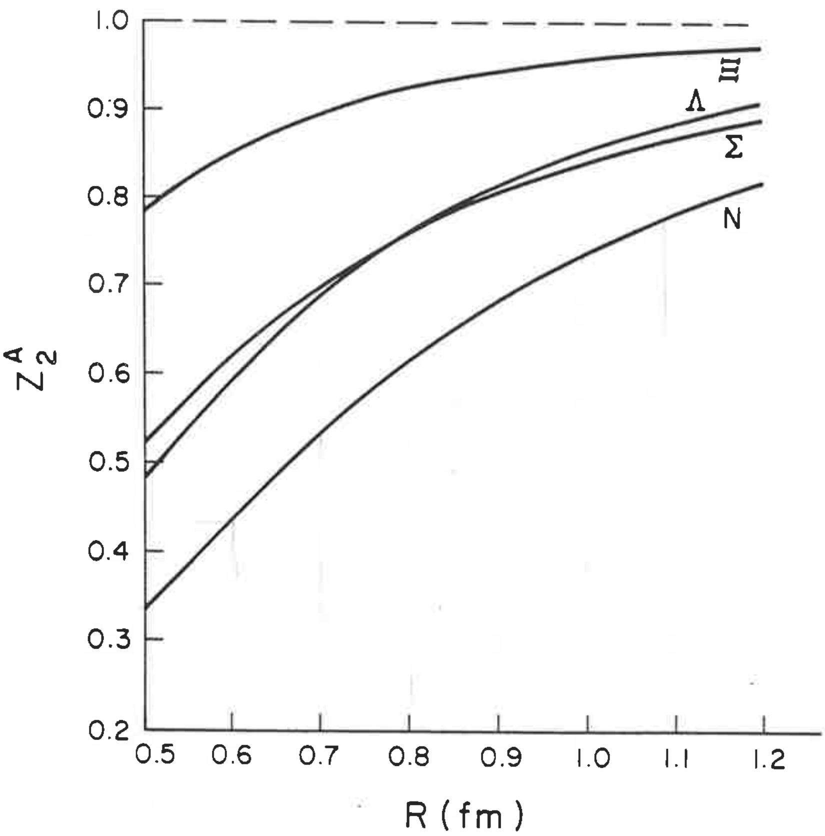

Fig. 4. Bare baryon bag probability Z_2^A versus the bag radius R for all members of the baryon octet obtained via eq. (3.17) but with the renormalized coupling constants defined in eq. (3.27).

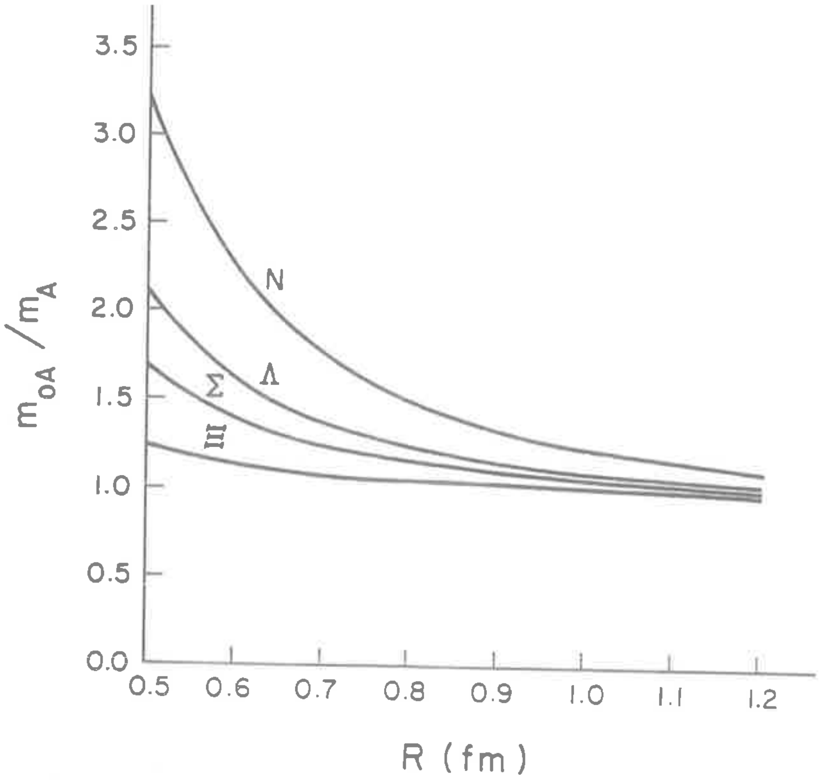

Fig. 5. Ratio of the bare to renormalized baryon mass m_{0A}/m_A for all members of the baryon octet.

for large bags of 0.8 fm or more than for smaller ones. In fig. 5, we give the radius dependence of the bare to physical mass ratio for the stable baryons. Again, we observe large pionic effects for small bags. However, if we limit ourselves to large bags of 0.8 fm or more, we can use the difference between the Ξ and Σ bare masses to extract the strange quark mass m_s. Using the mass formula of Myhrer *et al.* [20]), we have

$$m_{0\Xi} - m_{0\Sigma} = \frac{\Omega_s - \Omega_0}{R} - 0.043 m_s\,, \tag{3.28}$$

with $(\Omega_s - \Omega_0)$ being the strange to non-strange quark energy difference, and

$$\Omega_i \simeq 2.04 + 0.36 m_i R\,, \tag{3.29}$$

which can be solved for m_s. A graph of the solution for m_s as a function of R is given in fig. 6. The value of $m_s = 144$ MeV for $R = 1$ fm will be preferred in the calculation of the hyperon magnetic moments in the following section – unlike our earlier work [26]), where we simply took the old MIT value $m_s = 279$ MeV.

4. Calculation of the baryon octet magnetic moments

4.1. INTRODUCTION

The photon–hadron interaction, when analyzed in the context of the CBM, leads to predictions of the electromagnetic properties of hadrons. We shall in fact devote

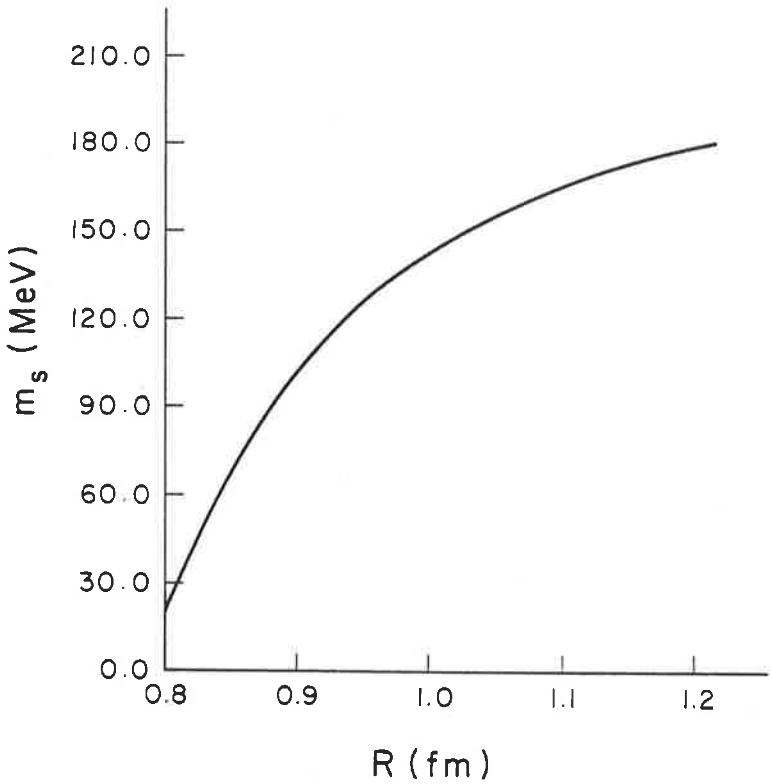

Fig. 6. The strange quark mass m_s which obeys the mass relation eq. (3.28).

our attention to obtaining formal expressions for the magnetic moments of all members of the baryon octet. Quantitative results can be obtained in the CBM because the photon couples to both the quarks and the pion field in a well-defined way.

4.2. THE ELECTROMAGNETIC FORM FACTORS: $G_E^A(q^2)$ AND $G_M^A(q^2)$

In order to study the electromagnetic form factors $G_{EM}^A(q^2)$, we consider first the elastic scattering of electrons on free spin-$\frac{1}{2}$ baryons. To lowest order in the interaction, a single photon is exchanged. The transition amplitude can be written as

$$T_{fi} = \int \frac{d^4q}{(2\pi)^4} \langle p', m'| \int d^4x \hat{j}_A^\mu(x)\, e^{-iq\cdot x} |p, m\rangle$$

$$\times \frac{g_{\mu\nu}}{q^2} \langle k', s'| \int d^4y\, \hat{j}_e^\nu(y)\, e^{iq\cdot y} |k, s\rangle\,, \tag{4.1}$$

where $p, m(p', m')$ are the four-momentum and spin projection of the incoming (outgoing) baryon A, and $k, s(k', s')$ are the four-momentum and spin projection of the incoming (outgoing) electron.

The electron current is known from QED to have the form

$$\langle k', s'| \int d^4y \hat{j}_e^\nu(y)\, e^{iq\cdot y}|k, s\rangle = \frac{(2\pi)^4 e}{(4E_k E_{k'})^{1/2}} \delta^{(4)}(k'-k+q) \bar{u}_e^{s'}(k') \gamma^\nu u_e^s(k)\,, \tag{4.2}$$

where $u_e^s(k)$ is the positive energy Dirac spinor for the electron field. From translational invariance, the baryon current operator $\hat{j}_A^\mu(x)$ can be rewritten as

$$\hat{j}_A^\mu(x) = e^{i\hat{P}\cdot x}\hat{j}_A^\mu(0)\, e^{-i\hat{P}\cdot x}, \tag{4.3}$$

where $\hat{P}^\mu$ is the four-momentum operator. The nucleon current in eq. (4.1) becomes

$$\langle p', m'| \int d^4x\, \hat{j}_A^\mu(x)\, e^{-iq\cdot x}|p, m\rangle = \frac{(2\pi)^4 e}{(4E_pE_{p'})^{1/2}}\delta^{(4)}(p'-p-q)\bar{u}_A^{m'}(p')\Gamma_A^\mu u_A^m(p), \tag{4.4}$$

where Γ_A^μ is a 4×4 Dirac matrix.

From the requirement of relativistic covariance, the condition of gauge invariance which requires that

$$q_\mu \Gamma_A^\mu = 0, \tag{4.5}$$

and the knowledge that the states $|p, m\rangle$ form a basis for a representation of the inhomogeneous Lorentz group corresponding to a definite spin, the most general form of the operator Γ_A^μ is [36,37])

$$e\Gamma_A^\mu = eF_1^A(q^2)\gamma^\mu + \frac{ie}{2m_A}F_2^A(q^2)K_A\sigma^{\mu\nu}q_\nu. \tag{4.6}$$

Here $F_{1,2}^A(q^2)$ are real functions, and K_A is the anomalous magnetic moment of the baryon A.

Following Ernst *et al.* [38]), we next consider the expectation value of the magnetic moment operator $\boldsymbol{M}$ in the rest frame of the baryon A,

$$\boldsymbol{M}_A = \langle A(\boldsymbol{p}=0)\left|\frac{1}{2}\int d^3r\, \boldsymbol{r}\times\hat{\boldsymbol{j}}_A(\boldsymbol{r})\right|A(\boldsymbol{p}=0)\rangle. \tag{4.7}$$

Substituting in eq. (4.7) $\hat{\boldsymbol{j}}_A(\boldsymbol{r})$ from eqs. (4.3), (4.4), (4.6) gives after a few manipulations

$$\boldsymbol{M}_A = \frac{e}{2m_A}[F_1^A(0)+K_AF_2^A(0)]\bar{u}_A^m(0)\boldsymbol{\sigma}u_A^m(0), \tag{4.8}$$

or

$$\boldsymbol{M}_A = eG_M^A(0)\boldsymbol{\sigma}. \tag{4.9}$$

This leads to the definition of the magnetic form factor $G_M^A(q^2)$ as

$$G_M^A(q^2) = \frac{1}{2m_A}[F_1^A(q^2)+K_AF_2^A(q^2)]. \tag{4.10}$$

We can also consider the charge radius of the baryon A [e.g., the nucleon – refs. [23,24])], which is given by the expectation value of the charge radius operator in the rest frame of the baryon considered. We have

$$R_A^2 = \langle A(\boldsymbol{p}=0)| \int d^3r\, r^2 \hat{j}_A^0(\boldsymbol{r})|A(\boldsymbol{p}=0)\rangle. \tag{4.11}$$

Again, substituting in eq. (4.11) the expression for $\hat{j}^0_{\mathrm{A}}(\boldsymbol{r})$ gives

$$R^2_{\mathrm{A}} = -6\frac{\partial}{\partial|q^2|}\left[F^{\mathrm{A}}_1(q^2) - \frac{K_{\mathrm{A}}}{4m^2_{\mathrm{A}}}|q^2|F^{\mathrm{A}}_2(q^2)\right]_{q^2=0}. \tag{4.12}$$

From this geometrical result, Ernst *et al.* define the electric form factor $G^{\mathrm{A}}_{\mathrm{E}}(q^2)$ as

$$G^{\mathrm{A}}_{\mathrm{E}}(q^2) = F^{\mathrm{A}}_1(q^2) - \frac{K_{\mathrm{A}}}{4m^2_{\mathrm{A}}}|q^2|F^{\mathrm{A}}_2(q^2). \tag{4.13}$$

Sachs has shown that the form factors $G^{\mathrm{A}}_{\mathrm{E,M}}(q^2)$ are a measure of the interaction of the baryon A with weak, static electric and magnetic fields [39]). Finally, Sachs also showed that in the Breit frame (where $q^0 = 0$), $G^{\mathrm{A}}_{\mathrm{E,M}}(q^2)$ are related to the Fourier transform of the spatial current via

$$j^0_{\mathrm{A}}(\boldsymbol{r}) = \langle \mathrm{A}|\hat{j}^0_{\mathrm{A}}(\boldsymbol{r})|\mathrm{A}\rangle = \frac{e}{(2\pi)^3}\int \mathrm{d}^3q G^{\mathrm{A}}_{\mathrm{E}}(q^2)\,\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}, \tag{4.14}$$

$$\boldsymbol{j}_{\mathrm{A}}(\boldsymbol{r}) = \langle \mathrm{A}|\hat{\boldsymbol{j}}(\boldsymbol{r})|\mathrm{A}\rangle = \frac{ie}{(2\pi)^3}\int \mathrm{d}^3q\, G^{\mathrm{A}}_{\mathrm{M}}(q^2)\langle S_{\mathrm{A}}|\boldsymbol{\sigma}\times\boldsymbol{q}|S_{\mathrm{A}}\rangle\,\mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{r}}. \tag{4.15}$$

We shall need the inverse of these relations, i.e.,

$$G^{\mathrm{A}}_{\mathrm{E}}(q^2) = \frac{1}{e}J^0_{\mathrm{A}}(q^2), \tag{4.16}$$

$$G^{\mathrm{A}}_{\mathrm{M}}(q^2) = \frac{1}{ie}\frac{1}{2\boldsymbol{q}^2}\langle S_{\mathrm{A}}|\boldsymbol{\sigma}\times\boldsymbol{q}|S_{\mathrm{A}}\rangle \boldsymbol{J}_{\mathrm{A}}(q^2), \tag{4.17}$$

where $J^{\mu}_{\mathrm{A}}(q^2)$ is the Fourier transform of $j^{\mu}_{\mathrm{A}}(\boldsymbol{r})$ in the Breit frame

$$J^{\mu}_{\mathrm{A}}(q^2) = \int \mathrm{d}^3r\, j^{\mu}_{\mathrm{A}}(\boldsymbol{r})\,\mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}}. \tag{4.18}$$

4.3. FORMAL EXPRESSION FOR $j^{\mu}_{\mathrm{A}}(\boldsymbol{r})$ IN THE CBM

If we introduce the photon field in the CBM lagrangian density (2.10), using the requirement of local U(1) gauge invariance, we get a conserved, local current which we identify as the baryonic electromagnetic current $\hat{j}^{\mu}(x)$. This current can be separated explicitly into its quark and pionic sectors:

$$\hat{j}^{\mu}(x) = \hat{j}^{\mu}_{\mathrm{Q}}(x) + \hat{j}_{\pi}(x), \tag{4.19}$$

with $\hat{j}^{\mu}_{\mathrm{Q}}(x)$ being the quark contribution,

$$\hat{j}^{\mu}_{\mathrm{Q}}(x) = e_q\bar{q}(x)\gamma^{\mu}q(x)\theta_{\mathrm{V}}, \tag{4.20}$$

and $\hat{j}^{\mu}_{\pi}(x)$ the pion contribution,

$$\hat{j}^{\mu}_{\pi}(x) = ie(\phi^{+}(x)\partial^{\mu}\phi(x) - \phi(x)\partial^{\mu}\phi^{+}(x)). \tag{4.21}$$

where

$$\phi(x) = \sqrt{\tfrac{1}{2}}(\pi_1(x) + i\pi_2(x)) . \tag{4.22}$$

If we quantize the pion field in the usual way [as given in eq. (2.18)]

$$\pi_j(\boldsymbol{r}, t=0) = \int \frac{d^3k}{[2\omega_k(2\pi)^3]^{1/2}} \{a_j(\boldsymbol{k})\, e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + a_j^\dagger(\boldsymbol{k})\, e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\} , \tag{4.23}$$

then the pion current in eq. (4.21) becomes

$$j_\pi^0(\boldsymbol{r}) = \tfrac{1}{2}ie \sum_{j,j'} \varepsilon_{jj'3} \int \frac{d^3k\, d^3k'\, \omega_k}{(2\pi)^3(\omega_k\omega_{k'})^{1/2}} (a_{j'}(-\boldsymbol{k}') + a_{j'}^\dagger(\boldsymbol{k}'))(a_j(\boldsymbol{k}) - a_j^\dagger(-\boldsymbol{k}))\, e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} , \tag{4.24}$$

$$\boldsymbol{j}_\pi(\boldsymbol{r}) = \tfrac{1}{2}ie \sum_{j,j'} \varepsilon_{jj'3} \int \frac{d^3k\, d^3k'\, \boldsymbol{k}}{(2\pi)^3(\omega_k\omega_{k'})^{1/2}} (a_{j'}(-\boldsymbol{k}') + a_{j'}^\dagger(\boldsymbol{k}'))(a_j(\boldsymbol{k}) + a_j^\dagger(-\boldsymbol{k}))\, e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} . \tag{4.25}$$

This form of $\boldsymbol{j}_\pi(\boldsymbol{r})$ is the key equation for determining the pion contribution to the electromagnetic form factors.

The quark current contribution to $\hat{j}^\mu(\boldsymbol{r})$ is extracted from eq. (4.20) for $\hat{j}_{\mathrm{Q}}^\mu(x)$ by using the MIT quark wave functions from eq. (2.17) to get

$$\hat{j}_{\mathrm{Q}}^0(\boldsymbol{r}) = \sum_{a=1}^{3} e_a N_a^2 \left[(\alpha_a^+)^2 j_0^2\left(\frac{\Omega_a r}{R}\right) + (\alpha_a^-)^2 j_1^2\left(\frac{\Omega_a r}{R}\right)\right] b_a^\dagger b_a \theta(R-r) , \tag{4.26}$$

$$\hat{\boldsymbol{j}}_{\mathrm{Q}}(\boldsymbol{r}) = \sum_{a=1}^{3} e_a N_a^2 \frac{2\Omega_a}{\alpha_a} j_0\left(\frac{\Omega_a r}{R}\right) j_1\left(\frac{\Omega_a r}{R}\right) b_a^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{r} b_a . \tag{4.27}$$

Finally, since the quantity of physical interest is the expectation value of the baryon current operator, we define $j_{\mathrm{A}}^\mu(\boldsymbol{r})$ as

$$j_{\mathrm{A}}^\mu(\boldsymbol{r}) \equiv \langle \mathrm{A}|\hat{j}^\mu(\boldsymbol{r})|\mathrm{A}\rangle , \tag{4.28}$$

where $|\mathrm{A}\rangle$ is the physical on-shell baryon A state. We also define the quantities $j_{\mathrm{QA}}^\mu(\boldsymbol{r})$ and $j_{\pi\mathrm{A}}^\mu(\boldsymbol{r})$ as

$$j_{\mathrm{QA}}^\mu(\boldsymbol{r}) = \langle \mathrm{A}|\hat{j}_{\mathrm{Q}}^\mu(\boldsymbol{r})|\mathrm{A}\rangle , \tag{4.29}$$

$$j_{\pi\mathrm{A}}^\mu(\boldsymbol{r}) = \langle \mathrm{A}|\hat{j}_\pi^\mu(\boldsymbol{r})|\mathrm{A}\rangle , \tag{4.30}$$

with

$$j_{\mathrm{A}}^\mu(\boldsymbol{r}) = j_{\mathrm{QA}}^\mu(\boldsymbol{r}) + j_{\pi\mathrm{A}}^\mu(\boldsymbol{r}) . \tag{4.31}$$

4.4. THE PION CONTRIBUTION TO $j_{\mathrm{A}}^\mu(\boldsymbol{r})$

The formal expression for the pion current operator eqs. (4.24), (4.25) can be rewritten as

$$\hat{j}_\pi^\mu(\boldsymbol{r}) = \tfrac{1}{2}ie \sum_{j,j'} \varepsilon_{jj'3} \int \frac{d^3k\, d^3k'}{(2\pi)^3(\omega_k\omega_{k'})^{1/2}} k^\mu \hat{S}_\pi(\boldsymbol{k}_{j'}', \boldsymbol{k}_j; \mu)\, e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}} , \tag{4.32}$$

with

$$\hat{S}_\pi(\boldsymbol{k}'_{j'}, \boldsymbol{k}_{j;\mu}) = (a_{j'}(-\boldsymbol{k}') + a^\dagger_{j'}(\boldsymbol{k}'))(a_j(\boldsymbol{k}) - g^{\mu\mu} a^\dagger_j(-\boldsymbol{k}))\,, \tag{4.33}$$

and we recall that $g^{00} = 1$ and $g^{ii} = -1$ in our convention. The next step is to rewrite $\hat{S}_\pi$ in terms of the operators V_{0j}. For this purpose, we use the identity

$$[a_j(\boldsymbol{k}), H] = \omega_k a_j(\boldsymbol{k}) + V^\dagger_{0j}(\boldsymbol{k})\,, \tag{4.34}$$

which leads to the results

$$a_j(\boldsymbol{k})|\mathrm{A}\rangle = (m_\mathrm{A} - \omega_k - H)^{-1} V^\dagger_{0j}(\boldsymbol{k})|\mathrm{A}\rangle\,, \tag{4.35}$$

and

$$\begin{aligned} a_{j'}(\boldsymbol{k}')a_j(\boldsymbol{k})|\mathrm{A}\rangle = \{&(m_\mathrm{A} - \omega_k - \omega_{k'} - H)^{-1} V^\dagger_{0j}(\boldsymbol{k})(m_\mathrm{A} - \omega_{k'} - H)^{-1} V^\dagger_{0j'}(\boldsymbol{k}') \\ &+ (m_\mathrm{A} - \omega_k - \omega_{k'} - H)^{-1} V^\dagger_{0j'}(\boldsymbol{k}')(m_\mathrm{A} - \omega_k - H)^{-1} V^\dagger_{0j}(\boldsymbol{k})\}|\mathrm{A}\rangle. \end{aligned} \tag{4.36}$$

The expectation value of the operators $\hat{S}_\pi(\boldsymbol{k}'_j, \boldsymbol{k}_j; \mu)$ between physical baryon states A has then the form

$$S_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = \langle\mathrm{A}|\hat{S}_\pi(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mathrm{A})|\mathrm{A}\rangle\,, \tag{4.37}$$

$$S_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = \sum_{\nu=1}^{3} S^{(\nu)}_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) \tag{4.38}$$

with

$$\begin{aligned} S^{(1)}_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = &\langle\mathrm{A}|(m_\mathrm{A} - \omega_k - \omega_{k'} - H)^{-1} V^\dagger_{0j}(\boldsymbol{k})(m_\mathrm{A} - \omega_{k'} - H)^{-1} V^\dagger_{0j'}(-\boldsymbol{k}')|\mathrm{A}\rangle \\ &+ \langle\mathrm{A}|(m_\mathrm{A} - \omega_k - \omega_{k'} - H)^{-1} V^\dagger_{0j'}(-\boldsymbol{k}')(m_\mathrm{A} - \omega_k - H)^{-1} V^\dagger_{0j}(\boldsymbol{k})|\mathrm{A}\rangle\,, \end{aligned} \tag{4.39}$$

$$\begin{aligned} S^{(2)}_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = &\langle\mathrm{A}|V_{0j}(\boldsymbol{k}')(m_\mathrm{A} - \omega_{k'} - H)^{-1}(m_\mathrm{A} - \omega_k - H)^{-1} V^\dagger_{0j}(\boldsymbol{k})|\mathrm{A}\rangle \\ &- g^{\mu\mu}\langle\mathrm{A}|V_{0j}(-\boldsymbol{k})(m_\mathrm{A} - \omega_k - H)^{-1}(m_\mathrm{A} - \omega_{k'} - H)^{-1} V^\dagger_{0j'}(-\boldsymbol{k}')|\mathrm{A}\rangle, \end{aligned} \tag{4.40}$$

$$\begin{aligned} S^{(3)}_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = -g^{\mu\mu}\{&\langle\mathrm{A}|V_{0j}(-\boldsymbol{k})(m_\mathrm{A} - \omega_k - H)^{-1} V_{0j'}(\boldsymbol{k}')(m_\mathrm{A} - \omega_k - \omega_{k'} - H)^{-1}|\mathrm{A}\rangle \\ &+ \langle\mathrm{A}|V_{0j'}(\boldsymbol{k}')(m_\mathrm{A} - \omega_{k'} - H)^{-1} V_{0j}(-\boldsymbol{k})(m_\mathrm{A} - \omega_k - \omega_{k'} - H)^{-1}|\mathrm{A}\rangle\,. \end{aligned} \tag{4.41}$$

If we expand the physical baryons A on the basis of the eigenstates of the bare hamiltonian H_0, insert the completeness relation for the bare eigenstates between each operator in eqs. (4.39)–(4.41), and use the renormalization procedure described in the previous section, we get

$$S^{(1)}_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = -\sum_\mathrm{B}\left\{\frac{\omega^{\mathrm{AB}}_j(\boldsymbol{k})\omega^{\mathrm{BA}}_{j'}(-\boldsymbol{k}')}{(\omega_k + \omega_{k'})(\omega_\mathrm{AB} - \omega_{k'})} + \frac{\omega^{\mathrm{AB}}_{j'}(-\boldsymbol{k}')\omega^{\mathrm{BA}}_j(\boldsymbol{k})}{(\omega_k + \omega_{k'})(\omega_\mathrm{AB} - \omega_{k'})}\right\}, \tag{4.42}$$

$$S^{(2)}_{\pi\mathrm{A}}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = \sum_\mathrm{B}\left\{\frac{v^{\mathrm{AB}}_{j'}(\boldsymbol{k}')\omega^{\mathrm{BA}}_j(\boldsymbol{k})}{(\omega_\mathrm{AB} - \omega_{k'})(\omega_\mathrm{AB} - \omega_k)} - g^{\mu\mu}\frac{v^{\mathrm{AB}}_j(-\boldsymbol{k})\omega^{\mathrm{BA}}_{j'}(-\boldsymbol{k}')}{(\omega_\mathrm{AB} - \omega_k)(\omega_\mathrm{AB} - \omega_{k'})}\right\}, \tag{4.43}$$

$$S^{(3)}_{\pi A}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; \mu) = -g^{\mu\mu} \sum_B \left\{ \frac{v_j^{AB}(-\boldsymbol{k}) v_{j'}^{BA}(\boldsymbol{k}')}{(\omega_{AB}-\omega_k)(\omega_k+\omega_{k'})} + \frac{v_{j'}^{AB}(\boldsymbol{k}') v_j^{BA}(-\boldsymbol{k})}{(\omega_{AB}-\omega_{k'})(\omega_k+\omega_{k'})} \right\}, \tag{4.44}$$

which when added together give $S_{\pi A}$:

$$S_{\pi A}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; 0) = \sum_B \left\{ \frac{2\omega_{k'}[v_{j'}^{AB}(\boldsymbol{k}')\omega_j^{BA}(\boldsymbol{k}) - v_j^{AB}(\boldsymbol{k})\omega_{j'}^{BA}(\boldsymbol{k}')]}{(\omega_k+\omega_{k'})(\omega_{BA}+\omega_{k'})(\omega_{BA}+\omega_k)} \right\}, \tag{4.45}$$

$$S_{\pi A}(\boldsymbol{k}'_{j'}, \boldsymbol{k}_j; m) = \sum_B \left\{ \frac{2(\omega_{BA}+\omega_k+\omega_{k'})[v_{j'}^{AB}(\boldsymbol{k}')\omega_j^{BA}(\boldsymbol{k}) + v_j^{AB}(\boldsymbol{k})\omega_{j'}^{BA}(\boldsymbol{k}')]}{(\omega_k+\omega_{k'})(\omega_{BA}+\omega_k)(\omega_{BA}+\omega_{k'})} \right\}, \tag{4.46}$$

where we made use of

$$\omega_j^{AB}(\boldsymbol{k}) = (v_j^{\dagger}(\boldsymbol{k}))^{AB} = v_j^{*BA}(\boldsymbol{k}) = -v_j^{AB}(\boldsymbol{k}) = v_j^{AB}(-\boldsymbol{k}). \tag{4.47}$$

Since $v_j^{AB}(\boldsymbol{k})$ is known [from eqs. (2.24) and (3.20)], we can evaluate $S_{\pi A}$ explicitly. After manipulations of the Clebsch–Gordan coefficients, we get for the pion current $j^{\mu}_{\pi A}(\boldsymbol{r})$:

$$j^0_{\pi A}(\boldsymbol{r}) = \frac{e}{3(2\pi)^6} \sum_B \left(\frac{f^{AB}}{m_\pi}\right)^2 t_A(B) \int d^3k\, d^3k' \frac{u(kR)u(k'R)\boldsymbol{k}\cdot\boldsymbol{k}'\, e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot\boldsymbol{r}}}{(\omega_k+\omega_{k'})(\omega_{BA}+\omega_k)(\omega_{BA}+\omega_{k'})}, \tag{4.48}$$

$$\boldsymbol{j}_{\pi A}(\boldsymbol{r}) = \frac{ie}{3(2\pi)^6} \sum_B \left(\frac{f^{AB}}{m_\pi}\right)^2 s_A(B) t_A(B)$$
$$\times \int \frac{d^3k\, d^3k'\, \boldsymbol{k}(\omega_{BA}+\omega_k+\omega_{k'})u(kR)u(k'R)\boldsymbol{k}\cdot(\boldsymbol{\sigma}\times\boldsymbol{q})}{\omega_k\omega_{k'}(\omega_k+\omega_{k'})(\omega_{BA}+\omega_k)(\omega_{BA}+\omega_{k'})} e^{i(\boldsymbol{k}-\boldsymbol{k})\cdot\boldsymbol{r}}, \tag{4.49}$$

with $s_A(B)$ and $t_A(B)$ given respectively by

$$s_A(B) = \begin{cases} 1 & \text{if } S_B = \frac{1}{2} \\ -\frac{1}{2} & \text{if } S_B = \frac{3}{2}, \end{cases} \tag{4.50}$$

$$t_A(B) = \begin{cases} t_A/T_A & \text{if } T_B = T_A - 1 \\ t_A/T_A(T_A+1) & \text{if } T_B = T_A \\ -t_A/(T_A+1) & \text{if } T_B = T_A + 1, \end{cases} \tag{4.51}$$

and the momenta are all related via

$$\boldsymbol{k}' = \boldsymbol{k} + \boldsymbol{q}. \tag{4.52}$$

4.5. THE PION CONTRIBUTION TO μ^A

For the pion contribution to the magnetic moment, we recall from eq. (4.17) that

$$G^A_{M\pi}(q^2) = \frac{1}{ie2\boldsymbol{q}^2} \langle s_A|\boldsymbol{\sigma}\times\boldsymbol{q}|s_A\rangle \int d^3r\, \boldsymbol{j}_{\pi A}(\boldsymbol{r})\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}, \tag{4.53}$$

Substituting for $\boldsymbol{j}_{\pi A}(\boldsymbol{r})$ from eq. (4.49) gives

$$G^{A}_{M\pi}(q^2)=\frac{1}{24\pi^2}\sum_{B}\left(\frac{f^{AB}}{m_\pi}\right)^2 t_A(B)s_A(B)\int\frac{d^3k\,(\omega_{BA}+\omega_k+\omega_{k'})u(kR)u(k'R)(\boldsymbol{q}\times\boldsymbol{k})^2}{\omega_k\omega_{k'}(\omega_k+\omega_{k'})(\omega_{BA}+\omega_k)(\omega_{BA}+\omega_{k'})}\,. \tag{4.54}$$

If we define μ^{A}_{π} to be the pion contribution to the baryon A magnetic moment, then

$$\mu^{A}_{\pi}=G^{A}_{M\pi}(0)=\frac{1}{18\pi^2}\sum_{B}\left(\frac{f^{AB}}{m_\pi}\right)^2 t_A(B)s_A(B)\int_0^\infty\frac{dk\,k^4u^2(kR)(\omega_{BA}+2\omega_k)}{2\omega_k^3(\omega_{BA}+\omega_k)^2}\,. \tag{4.55}$$

As a specific example we consider the nucleon, for which

$$\mu^{N}_{\pi}=\mu^{N}_{\pi}(N)+\mu^{N}_{\pi}(\Delta)\,, \tag{4.56}$$

$$\mu^{N}_{\pi}(N)=(27\pi^2)^{-1}\left(\frac{f^{NN}}{m_\pi}\right)^2\int_0^\infty dk\,\frac{k^4u^2(kR)}{\omega_k^4}\langle N|\tau_3|N\rangle\,, \tag{4.57}$$

$$\mu^{N}_{\pi}(\Delta)=(216\pi^2)^{-1}\left(\frac{f^{N\Delta}}{m_\pi}\right)^2\int_0^\infty\frac{dk\,k^4u^2(kR)(\omega_{\Delta N}+2\omega_k)}{\omega_k^3(\omega_{\Delta N}+\omega_k)^2}\langle N|\tau_3|N\rangle\,, \tag{4.58}$$

and we therefore recover, as expected, the results obtained previously [24]).

4.6. THE QUARK CONTRIBUTION TO μ^A

As mentioned in subsect. 4.3, the quark contribution to the baryon spatial current is the expectation value of the quark operator $\hat{\boldsymbol{j}}_Q(\boldsymbol{r})$ in the dressed baryon A,

$$\boldsymbol{j}_Q(\boldsymbol{r})=\sum_{a=1}^{3}e_aN_a^2\frac{2\Omega_a}{\alpha_a}j_0\left(\frac{\Omega_a r}{R}\right)j_1\left(\frac{\Omega_a r}{R}\right)\theta(R-r)\{b_a^{+}\boldsymbol{\sigma}\times\hat{\boldsymbol{r}}b_a\}\,. \tag{4.59}$$

The quark magnetic moment operator $\hat{\boldsymbol{\mu}}_Q$ is related to $\hat{\boldsymbol{j}}_Q(\boldsymbol{r})$ via

$$\hat{\boldsymbol{\mu}}_Q=\frac{1}{2}\int d^3r\,\boldsymbol{r}\times\hat{\boldsymbol{j}}_Q(\boldsymbol{r})\,. \tag{4.60}$$

The integration can be performed exactly and gives [11])

$$\boldsymbol{\mu}_Q=R\sum_{a=1}^{3}\mu_a b_a^{+}\boldsymbol{\sigma}b_a\,, \tag{4.61}$$

with

$$\mu_a=\frac{4\alpha_a+2\lambda_a-3}{12\alpha_a(\alpha_a-1)+6\lambda_a}\,. \tag{4.62}$$

We shall use $\mu_a=\mu_0$ for the massless up and down quarks, and $\mu_a=\mu_s$ for the massive strange quark.

The contribution of the quarks to the magnetic moment of baryon A can be calculated as being the expectation value of the z-component of the quark magnetic moment operator $\hat{\mu}_Q$ in the baryon state A of spin projection $+\frac{1}{2}$

$$\mu_Q^A = \langle A, s_A = +\tfrac{1}{2} | \hat{\mu}_{Qz} | A, s_A = +\tfrac{1}{2} \rangle \,, \tag{4.63}$$

with

$$\hat{\mu}_{Qz} = R \sum_{a=1}^{3} \mu_a b_a^\dagger \sigma_z b_a \,. \tag{4.64}$$

However, the matrix elements of $\hat{\mu}_{Qz}$ are known only for the bare bags. We need therefore to expand the physical baryon A wave function in terms of the bare eigenstates of H_0 according to the formal expansion, eq. (3.5),

$$|A\rangle = Z_2^A(m_A)^{1/2}\{1 + (m_A - H_0 - \Lambda H_{\rm int} \Lambda)^{-1} H_{\rm int}\} |A_0\rangle \,. \tag{4.65}$$

Then we find

$$\begin{aligned}\mu_Q^A = Z_2^A(m_A)\{\langle A_0|\hat{\mu}_{Qz}|A_0\rangle + \langle A_0|H_{\rm int}(m_A - H_0 - \Lambda H_{\rm int}\Lambda)^{-1} \\ \times \hat{\mu}_{Qz}(m_A - H_0 - \Lambda H_{\rm int}\Lambda)^{-1} H_{\rm int}|A_0\rangle\} \,,\end{aligned} \tag{4.66}$$

and the two other terms not present in this expansion vanish because $\hat{\mu}_{Qz}$ does not create or destroy any pions.

Next, we make a "no more than one pion in the air" approximation – i.e., consider only the terms in eq. (4.66) which have no more than one pion in the air when the photon couples to the bag. Let us define as in eq. (3.9)

$$\Sigma_0(E) = H_{\rm int}\Lambda(E - H_0)^{-1}\Lambda H_{\rm int} \,, \tag{4.67}$$

$$\mu_{Qz}(B, C) = \langle B_0|\hat{\mu}_{Qz}|C_0\rangle \,, \tag{4.68}$$

then eq. (4.66) becomes

$$\begin{aligned}\mu_Q^A = Z_2^A(m_A)\,\{\mu_{Qz}(A, A) + \sum_{B,C}\sum_j \int d^3k\, \langle A_0|H_{\rm int}(m_A - H_0 - \Sigma_0(m_A))^{-1} \\ \times |B_0, \boldsymbol{k}_j\rangle \mu_{Qz}(B, C) \langle C_0, \boldsymbol{k}_j|(m_A - H_0 - \Sigma_0(m_A))^{-1} H_{\rm int}|A_0\rangle \,.\end{aligned} \tag{4.69}$$

The next approximation consists of replacing $H_0 + \Sigma_0$ by $\tilde{H}_0$ as we did in eq. (3.12). After a few manipulations, eq. (4.69) reads

$$\mu_Q^A = \mu_{Qz}^A(A) + \sum_{B,C} \mu_{Qz}^A(B, C) \,, \tag{4.70}$$

with

$$\mu_{Qz}^A(A) \equiv Z_2^A(m_A)\mu_{Qz}(A, A) \,, \tag{4.71}$$

$$\mu_{QZ}^A(B, C) = Z_2^A(m_A) \sum_j \int d^3k \frac{v_j^{AB}(\boldsymbol{k})\mu_{Qz}(B, C)\, \omega_j^{CA}(\boldsymbol{k})}{(\omega_{BA} + \omega_k)(\omega_{CA} + \omega_k)} \,, \tag{4.72}$$

where we also made use of eq. (3.27). Replacing v_j^{AB} and ω_j^{CA} by their formal expressions gives for eq. (4.73)

$$\mu^{\mathrm{A}}_{\mathrm{Q}z}(\mathrm{B},\mathrm{C})=\frac{RZ^{\mathrm{A}}_{2}(m_{\mathrm{A}})}{36\pi^2}\frac{f^{\mathrm{AB}}f^{\mathrm{AC}}}{m_\pi^2}\int_0^\infty\frac{\mathrm{d}k\,k^4u^2(kR)}{\omega_k(\omega_{\mathrm{BA}}+\omega_k)(\omega_{\mathrm{CA}}+\omega_k)}$$
$$\times\eta(S_{\mathrm{B}},S_{\mathrm{C}})\sum_{t_{\mathrm{B}}}C^{t_{\mathrm{B}}(t_{\mathrm{A}}-t_{\mathrm{B}})t_{\mathrm{A}}}_{T_{\mathrm{B}}1T_{\mathrm{A}}}C^{t_{\mathrm{B}}(t_{\mathrm{A}}-t_{\mathrm{B}})t_{\mathrm{A}}}_{T_{\mathrm{C}}1T_{\mathrm{A}}}\tilde{\mu}_{\mathrm{Q}z}(\mathrm{B},\mathrm{C})\,, \tag{4.73}$$

with

$$\eta(S_{\mathrm{B}},S_{\mathrm{C}})=\begin{cases}-1 & \text{if } S_{\mathrm{B}}=S_{\mathrm{C}}=\frac{1}{2}\\ -2 & \text{if } S_{\mathrm{B}}\neq S_{\mathrm{C}}\\ 5 & \text{if } S_{\mathrm{B}}=S_{\mathrm{C}}=\frac{3}{2}\,,\end{cases} \tag{4.74}$$

$$R\tilde{\mu}_{\mathrm{Q}z}(\mathrm{B},\mathrm{C})\equiv\mu_{\mathrm{Q}z}(\mathrm{B}(s_{\mathrm{B}}=\tfrac{1}{2}),\,\mathrm{C}(s_{\mathrm{C}}=\tfrac{1}{2}))\,. \tag{4.75}$$

The $\tilde{\mu}_{\mathrm{Q}z}(\mathrm{B},\mathrm{C})$ are given explicitly in table 2 with the symmetry relation

$$\tilde{\mu}_{\mathrm{Q}z}(\mathrm{B},\mathrm{C})=\tilde{\mu}_{\mathrm{Q}z}(\mathrm{C},\mathrm{B})\,. \tag{4.76}$$

As an example, we consider the nucleon case for which the quark contribution to the magnetic moment is found from eq. (4.73) to be

$$\mu^{\mathrm{N}}_{\mathrm{Q}}=\mu^{\mathrm{N}}_{\mathrm{Q}z}(\mathrm{N})+\mu^{\mathrm{N}}_{\mathrm{Q}z}(\mathrm{N}',\mathrm{N}')+\mu^{\mathrm{N}}_{\mathrm{Q}z}(\Delta,\Delta)+2\mu^{\mathrm{N}}_{\mathrm{Q}z}(\mathrm{N}',\Delta)\,, \tag{4.77}$$

or

$$\mu^{\mathrm{N}}_{\mathrm{Q}}=\frac{R\mu_0}{27}\left[\begin{pmatrix}27\\-18\end{pmatrix}Z^{\mathrm{N}}_{2}+\begin{pmatrix}1\\-4\end{pmatrix}P^{\mathrm{N}}_{\mathrm{NN}\pi}+\begin{pmatrix}20\\-5\end{pmatrix}P^{\mathrm{N}}_{\Delta\Delta\pi}+\begin{pmatrix}16\sqrt{2}\\-16\sqrt{2}\end{pmatrix}P^{\mathrm{N}}_{\Delta\mathrm{N}\pi}\right], \tag{4.78}$$

TABLE 2

The quark magnetic moment matrix elements $\tilde{\mu}_{\mathrm{Q}z}(\mathrm{A},\mathrm{B})$ as defined in eq. (4.76)

(A, B)	$\tilde{\mu}_{\mathrm{Q}z}(\mathrm{A}^{++},\mathrm{B}^{++})$	$\tilde{\mu}_{\mathrm{Q}z}(\mathrm{A}^{+},\mathrm{B}^{+})$	$\tilde{\mu}_{\mathrm{Q}z}(\mathrm{A}^{0},\mathrm{B}^{0})$	$\tilde{\mu}_{\mathrm{Q}z}(\mathrm{A}^{-},\mathrm{B}^{-})$
(N, N)		μ_0	$-\frac{2}{3}\mu_0$	
(N, Δ)		$\frac{2}{3}\sqrt{2}\mu_0$	$\frac{2}{3}\sqrt{2}\mu_0$	
(Δ, Δ)	$\frac{2}{3}\mu_0$	$\frac{1}{3}\mu_0$	0	$-\frac{1}{3}\mu_0$
(Λ, Λ)			$-\frac{1}{3}\mu_s$	
(Λ, Σ)			$-\sqrt{\frac{1}{3}}\mu_0$	
(Λ, Σ^*)			$\sqrt{\frac{2}{3}}\mu_0$	
(Σ, Σ)		$\frac{1}{9}(8\mu_0+\mu_s)$	$\frac{1}{9}(2\mu_0+\mu_s)$	$\frac{1}{9}(-4\mu_0+\mu_s)$
(Σ, Σ^*)		$\frac{1}{9}\sqrt{2}(4\mu_0+2\mu_s)$	$\frac{1}{9}\sqrt{2}(\mu_0+2\mu_s)$	$\frac{2}{9}\sqrt{2}(\mu_0-\mu_s)$
(Σ^*, Σ^*)		$\frac{1}{9}(4\mu_0-\mu_s)$	$\frac{1}{9}(\mu_0-\mu_s)$	$\frac{1}{9}(-2\mu_0-\mu_s)$
(Ξ, Ξ)			$\frac{1}{9}(-2\mu_0-4\mu_s)$	$\frac{1}{9}(\mu_0-4\mu_s)$
(Ξ, Ξ^*)			$\frac{1}{9}\sqrt{2}(4\mu_0+2\mu_s)$	$\frac{2}{9}\sqrt{2}(\mu_0-\mu_s)$
(Ξ^*, Ξ^*)			$\frac{1}{9}2(\mu_0-\mu_s)$	$\frac{1}{9}(-\mu_0-2\mu_s)$

with

$$P^{\rm N}_{\rm BC\pi}=\frac{Z^{\rm N}_2}{12\pi^2}\left(\frac{f^{\rm NB}f^{\rm NC}}{m_\pi^2}\right)\int_0^\infty \frac{{\rm d}k\, k^4 u^2(kR)}{\omega_k(\omega_{\rm BN}+\omega_k)(\omega_{\rm CN}+\omega_k)}\,, \tag{4.79}$$

and the probability conservation condition

$$Z^{\rm N}_2+P^{\rm N}_{\rm NN\pi}+P^{\rm N}_{\Delta\Delta\pi}=1\,. \tag{4.80}$$

4.7. CENTRE-OF-MASS CORRECTIONS

In the most naive version of the bag model, where the bag itself carries no momentum, there is an obvious problem of spurious c.m. motion. A number of attempts have been made to correct for this [12,40,41]). Donoghue and Johnson found about a 16% increase in the magnetic moment calculated in the usual bag model [12]). Improvements in the Donoghue–Johnson work by Wong [41]) and later by Carlson and Chachkhunashvili [CC [40])] have led to a correction between −15% and +8%. For a detailed discussion of the ambiguities in defining the c.m. correction we refer to ref. [25c]). Here we must be content with a brief outline of the method.

The technique for making the c.m. correction is known as the Peierls–Yoccoz projection in nuclear physics. One assumes that the independent particle model wave function can be written as a superposition of momentum eigenstates. If $|{\rm B}(\boldsymbol{R})\rangle$ denotes a bag located at position $\boldsymbol{R}$, one writes

$$|{\rm B}(\boldsymbol{R})\rangle=\int\frac{{\rm d}^3p}{W(p)}\,{\rm e}^{i\boldsymbol{p}\cdot\boldsymbol{R}}\phi(\boldsymbol{p})|{\rm b},\boldsymbol{p}\rangle\,, \tag{4.81}$$

where $|{\rm b},\boldsymbol{p}\rangle$ is a momentum eigenstate of particle b. This is normalized as

$$\langle{\rm b},\boldsymbol{p}'|{\rm b},\boldsymbol{p}\rangle=(2\pi)^3W(p)\delta(\boldsymbol{p}-\boldsymbol{p}')\,, \tag{4.82}$$

where

$$W(p)=\begin{cases}2\omega_p, & \text{mesons}\\ (m_{\rm B}^2+p^2)^{1/2}/m_{\rm B}\,, & \text{baryons}\,.\end{cases} \tag{4.83}$$

From eqs. (4.81) and (4.82) one can solve immediately for the wave packet $\phi(p)$,

$$\phi^2(\boldsymbol{p})=\frac{W(p)}{(2\pi)^6}\int{\rm d}^3r\,{\rm e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}}\langle B(-\tfrac12\boldsymbol{r})|B(\tfrac12\boldsymbol{r})\rangle\,. \tag{4.84}$$

Finally, one is able to evaluate the matrix element of $j_{\rm em}$ in the Breit frame to obtain the magnetic form factor (subsect. 4.2). In this way CC found a 15% reduction from the static bag value of the nucleon magnetic moment.

One of the problems with the Peierls–Yoccoz technique is that the internal state corresponding to $|{\rm b},\boldsymbol{p}\rangle$ is not guaranteed to be independent of $\boldsymbol{p}$! One indication of such a problem in the bag model, with its sharp boundary, is that $\langle p^2\rangle$ is infinite in the CC calculation. When they used a smoothly behaved gaussian approximation to the bag model wave function

$$\psi(\boldsymbol{r}) = N\, \mathrm{e}^{-r^2/2R^2} \begin{pmatrix} 1 \\ i(\beta/R)\boldsymbol{\sigma}\cdot\boldsymbol{r} \end{pmatrix} \chi\,, \tag{4.85}$$

they found an 8% increase over the static magnetic moment. Thus the correction to the magnetic moment is much more model-dependent than the correction for either g_A or the charge radius [for which the approximation (4.85) made little difference].

One further problem which should be mentioned does not arise in the nuclear case. The bag radius itself is determined by the non-linear boundary condition, and thus depends on whether or not a c.m. correction is included in the expression for the total energy. Indeed, the inclusion of the familiar $-Z_0/R$ term with $Z_0 = 1.84$ as in the original MIT work decreases the bag radius by at least 10%. Since in the work of both Wong and CC it seems appropriate to use a non-corrected bag radius, and since the magnetic moment is proportional to R in the bag model, this would lead to another 10% increase. Combined with the 8% increase for a gaussian wave function noted above, this leads to a total increase of about 18%, which is very close to the 16% of Donoghue and Johnson.

In view of the problems in unambiguously defining the c.m. correction we shall present our results with and without the Donoghue–Johnson c.m. correction to the quark contribution.

5. Discussion of the results

The parameters entering our expressions for the magnetic moments of the nucleon octet are renormalized masses and coupling constants, the strange quark mass (m_s) and the bag radius (R). For the reasons we presented earlier, the renormalized masses are taken to be the observed masses of the stable baryons, and the resonance masses for the unstable ones. The renormalized coupling constants, f^{AB}, are all determined relative to f^{NN} by the appropriate SU(6) factor – see eq. (3.27) and table 1. In our notation the renormalized NNπ coupling constant is given by

$$\left(\frac{f^{NN}}{6\sqrt{\pi}}\right)^2 = 0.081\,. \tag{5.1}$$

For the mass of the strange quark we shall use $m_s = 144$ MeV, which we obtained in subsect. 3.5 (for $R = 1$ fm). However, in table 3 we also present results (with $R = 1$ fm) for $m_s = 210$ MeV [as found by Myhrer *et al.* [20]], and $m_s = 279$ MeV [as in the original MIT work [11,26]]. Finally we have decided to vary the bag radius freely in the range 0.8 to 1.2 fm, in order to show explicitly the dependence on this parameter.

TABLE 3

The baryon octet magnetic moments in the SU(6), MIT and CBM models

μ^A/μ^p_{exp}	p	n	Σ^+	Σ^-	Λ	Ξ^0	Ξ^-
$\mu^A_{SU(6)}$	$1.00^\dagger$	−0.67	1.00	−0.33	−0.33	−0.67	−0.33
μ^A_{MIT}	$1.00^\dagger$	−0.67	0.97	−0.36	−0.26	−0.56	−0.23
$\mu^A_{CBM}(279)$	0.95	−0.73	0.84	−0.39	−0.22	−0.46	−0.19
$\mu^A_{CBM}(210)$	0.95	−0.73	0.84	−0.38	−0.23	−0.47	−0.20
$\mu^A_{CBM}(144)$	0.95	−0.73	0.84	−0.38	−0.24	−0.49	−0.22
μ^A_{exp}	1.00	−0.68	0.83	-0.32 ± 0.05	−0.22	−0.45	-0.25 ± 0.01

All SU(6) and MIT values scaled so μ^p is correct.

In figs. 7 to 10 we show the ratio of the theoretical to experimental magnetic moments (μ^A/μ^A_{exp}), as a function of the bag radius R, with and without the c.m. correction discussed above. We stress again that for all these curves m_s is fixed at 144 MeV. Table 3 shows the ratio of the various baryon magnetic moments to the experimental value for the proton in each of the SU(6), MIT and CBM. In table 4, we give explicitly the quark, c.m. and pion contribution to μ^A for the specific bag radius of 1 fm with $m_s = 144$ MeV.

Fig. 7 shows clearly that the theoretical prediction for the nucleon magnetic moment agrees very well with the experimental values (within 10%) for the wide

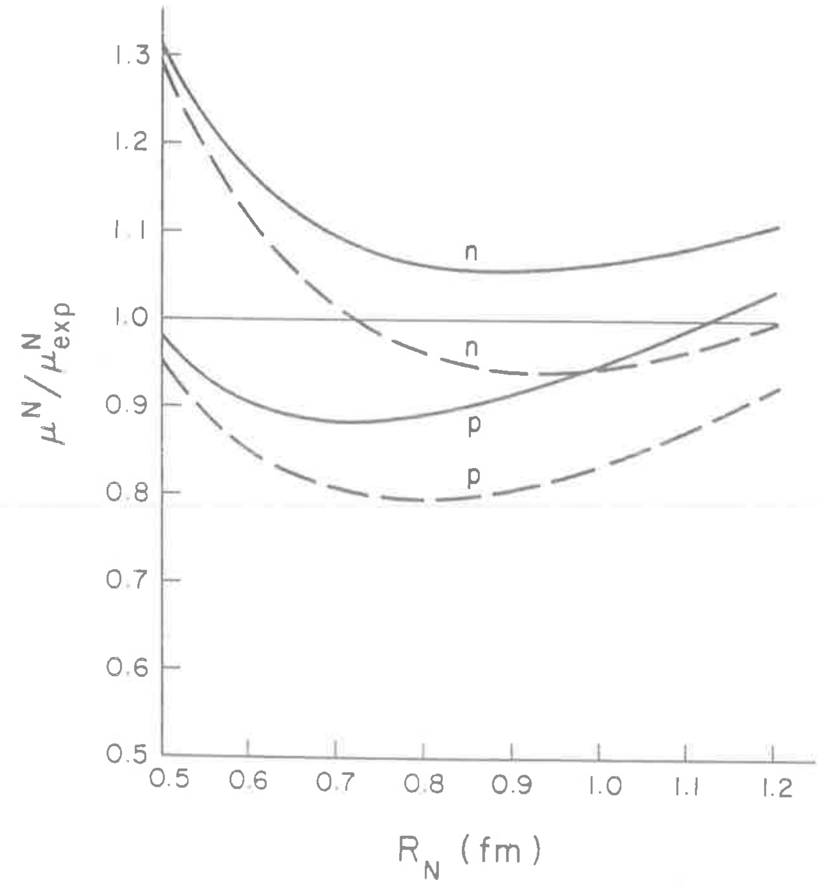

Fig. 7. The nucleon theoretical to experimental magnetic moment ratio μ^N/μ^N_{exp} as a function of the nucleon bag radius R. (The dashed curves do not include c.m. corrections.)

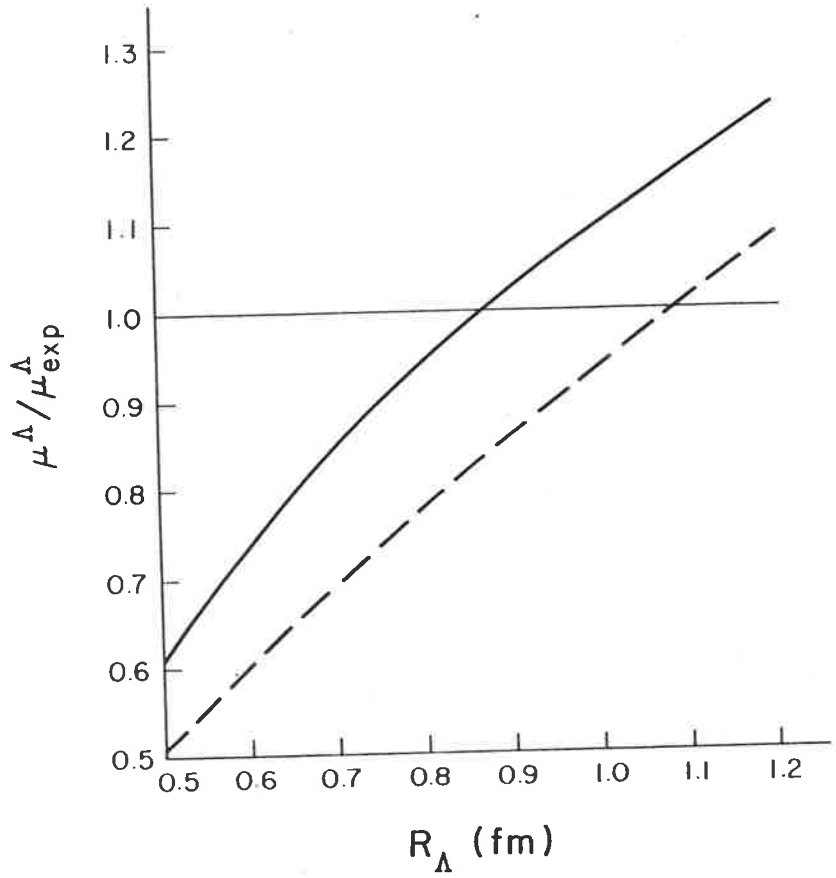

Fig. 8. The lambda magnetic moment ratio $\mu^{\Lambda}/\mu^{\Lambda}_{exp}$ as a function of the lambda bag radius R_{Λ}. (The dashed curve does include c.m. corrections.)

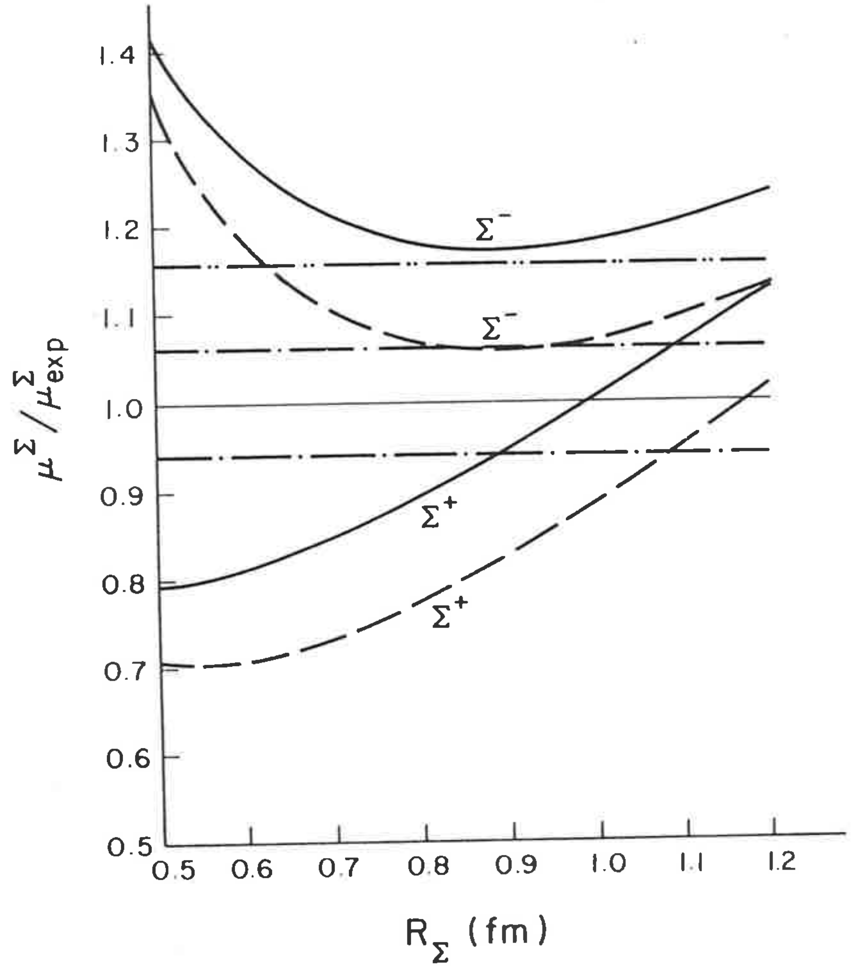

Fig. 9. The dependence of the sigma magnetic moment ratios $\mu^{\Sigma}/\mu^{\Sigma}_{exp}$ on the bag radius R_{Σ} using the recent value of $\mu^{\Sigma^-}_{exp} = -0.89 \pm 0.14\mu_N$. The dash-dot and dash-dot-dot lines are the experimental limits for $\mu^{\Sigma+}_{exp}$ and $\mu^{\Sigma-}_{exp}$ respectively. (The dashed curves do not include c.m. corrections.)

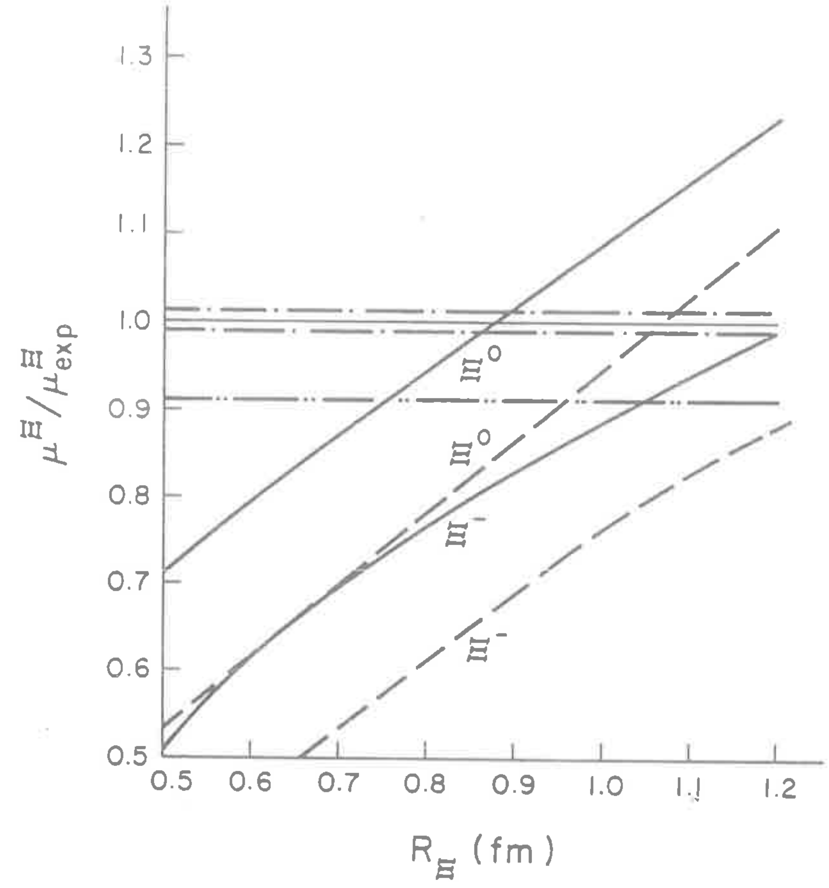

Fig. 10. The dependence of the cascade magnetic moment ratios $\mu^{\Xi}/\mu^{\Xi}_{exp}$ on the bag radius R_{Ξ}. The dash-dot and dash-dot-dot lines are the experimental limits for $\mu^{\Xi^0}_{exp}$ and $\mu^{\Xi^-}_{exp}$ respectively. (The dashed curves do not include c.m. corrections.)

range of bag radii of 0.85 to 1.15 fm – although the c.m. correction is essential. Notice also from table 4 that the pion contribution of $0.6\mu_N$ (for $R = 1.0$ fm) is very important – contributing roughly one-quarter of the theoretical value of $2.65\mu_N$. This contrasts with the MIT result of $\mu^p = 2.24\mu_N$ which does not contain any pion field contribution[11,12]).

For the lambda magnetic moment our results (shown in fig. 8) are consistent with a lambda bag radius of 1 fm. Again, the results are quite insensitive to the

TABLE 4

Contribution of the pion, quark and c.m. correction to the baryon magnetic moments in the CBM with $R_A = 1$ fm and $m_s = 144$ MeV

$\mu^A(\mu_N)$	p	n	Σ^+	Σ^-	Λ	Ξ^0	Ξ^-
μ^A_π	0.60	−0.60	0.34	−0.34	0.00	−0.02	0.02
μ^A_Q	1.74	−1.22	1.73	−0.62	−0.57	−1.16	−0.54
μ^A_{CM}	0.31	−0.22	0.27	−0.09	−0.10	−0.18	−0.09
$\mu^A_{CBM}(144)$	2.65	−2.04	2.34	−1.05	−0.67	−1.36	−0.61
μ^A_{exp}	2.79	−1.91	2.33±0.13	−0.89±0.14	−0.61	−1.25	−0.69±0.04

bag radius since for the whole radius range of 0.8 to 1.0 fm, the theoretical prediction for μ^{Λ} agrees within 10% with the experimental value of $\mu^{\Lambda}_{\rm exp} = -0.614 \pm 0.005 \mu_{\rm N}$. Notice from table 3 that a 25% decrease of $m_{\rm s}$ from 279 MeV to 210 MeV changes μ^{Λ} by only 10%, showing therefore that the strange quark mass is not such a critical parameter.

Next comes the sigma magnetic moment. For Σ^+ the theoretical predictions agree very well with the experimental value of $\mu^{\Sigma+}_{\rm exp} = 2.33 \pm 0.13 \mu_{\rm N}$ for a bag radius $R = 1.0 \pm 0.1$ fm, independent of $m_{\rm s}$. This result is consistent with the lambda case – one expects R_{Λ} and R_{Σ} to be about equal because they have the same quark content. For the Σ^-, whose magnetic moment is determined via exotic atom techniques, the experimental value has changed recently from $\mu^{\Sigma-}_{\rm exp} = -1.41 \pm 0.25 \mu_{\rm N}$ [ref. [1]] to $\mu^{\Sigma-}_{\rm exp} = -0.89 \pm 0.14 \mu_{\rm N}$ [ref. [2]]. Our theoretical result of $-1.05 \mu_{\rm N}$ shown in fig. 9 lies between these two results, and is slightly more than one standard deviation from the new experimental value for any bag radius between 0.7 and 1.2 fm. More accurate measurements of $\mu^{\Sigma-}$ would certainly be welcome.

Finally, for the Ξ^0 and Ξ^-, table 4 shows clearly that the pion contribution is negligible. We welcome the recent determination of the Ξ^0 and Ξ^- magnetic moments based on the decay asymmetry [1]. Our results shown in fig. 10 for μ^{Ξ^0} are in excellent agreement with the experimental value of $-1.250 \pm 0.014 \mu_{\rm N}$ provided that the Ξ^0 bag radius is $R = 0.95 \pm 0.10$ fm. For the Ξ^- magnetic moment, our theoretical value of $-0.61 \pm 0.02 \mu_{\rm N}$, with $R = 1.0 \pm 0.1$ fm, and $m_{\rm s} = 144$ MeV, agrees reasonably well with the new experimental value of $-0.69 \pm 0.04 \mu_{\rm N}$ [ref. [1]].

We have already commented on some other, recent calculations of baryon magnetic moments [42–44] in ref. [26]. In particular, our explicit calculation reveals no evidence for the phenomenological, isoscalar contribution assumed by Brown and Rho [42]. As a consequence our value for the Σ^- magnetic moment is about one standard deviation from each of the two rather different experimental values. We also note that the extraction of effective quark magnetic moments [43] is complicated by the pion contributions, which break SU(6) symmetry. For example, the effective strange-quark moment extracted as

$$\mu'_{\rm s}(\Sigma) = -\mu(\Sigma^+) - 2\mu(\Sigma^-) , \tag{5.2}$$

contains (from table 4) a pionic contribution of $0.34 \mu_{\rm N}$. Results very similar to ours have recently been obtained in a non-relativistic quark model which included pionic corrections [45]. However, it must be said that there is much less theoretical motivation for introducing pions in such an approach. Moreover the form of the coupling is not uniquely prescribed by symmetry considerations as it is here.

It is straightforward to extend the CBM to $\rm SU(3)_L \times SU(3)_R$, and this model has been applied to the coupled $\bar{\rm K}{\rm N} \leftrightarrow \Sigma\pi$ system in the region of the $\Lambda(1405)$ [ref. [46]]. However, we are less convinced that it makes sense to treat virtual kaons in the same way as virtual pions, because of their larger mass and hence shorter range. It is not clear whether one is justified in treating such fluctuations as coherent pairs or whether one should rather include them as sea quarks (virtual pairs in the bag). (This is the reason why vector meson contributions have been omitted.)

Over-all the level of agreement with data which we have obtained is at the 10% level in the region $R \sim 1$ fm, where low-order perturbation theory should work. In view of the complications which we have not included this is excellent. We think for example of the ambiguity in the c.m. correction [40]), the effects of configuration mixing [44]), and the sea quarks [47–49]).

Of the results presented here those most deserving comment are the p, Σ^- and Ξ^-. Clearly in the case of the proton the c.m. correction is essential. For the Σ^- the experimental situation is very unclear, and we badly need a decisive experiment. In any case the pion contribution is very important for the Σ^- because one can have both $\Sigma^0\pi^-$ and $\Lambda\pi^-$ intermediate states. The latter was first calculated by Pilkuhn and Eeg [50]), although their numerical results were much larger than ours – because of the soft form factor in the CBM. Finally the Ξ^- looks bad because of the very small experimental error. However, the discrepancy is only of order $0.1\mu_N$, which could easily arise from one of the higher order effects mentioned above. It is difficult to imagine how the theoretical precision can match that of the experiments in the next few years.

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PROTON POLE CONTRIBUTION IN PROTON DECAY

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We show that nucleon pole terms contribute significantly to proton decay, increasing the branching ratio for $e^+\pi^0$ and $\bar{\nu}_R\pi^+$ modes. The total lifetime is decreased to the point where keeping the SU(5) value of the lifetime below the experimental upper bound forces us to the choice of QCD scale parameter and proton radius parameter which are near the edge of the allowed region.

1. Introduction

Since the introduction of grand unified theories [1], and the subsequent realization that the proton maybe unstable [2], much effort has been devoted to the calculation of the proton lifetime [3]. Such a calculation contains three essential ingredients:

(i) a calculation of the mass m_X of the vector bosons mediating the decay;

(ii) a calculation of the four-fermion operators mediating the decay, with coefficients scaled by a renormalization group calculation from m_X to typical hadronic masses;

(iii) a calculation of the matrix element of the decay process from the basic four-fermion operators.

In this paper we take a new approach to item (iii), the calculation of the decay matrix element once m_X and the effective, four-fermion (baryon-number violating) operator are given. Until now, the assumption has usually been made that the spectator process of fig. 1 should dominate this matrix element [4-6]. A few authors have been motivated by soft-pion considerations to include, in addition, the nucleon pole graph of fig. 2, either using current algebra [7, 8], or an SU(3) invariant effective lagrangian [9]. In our calculations of the matrix element of the baryon-number violating effective lagrangian we use the harmonic oscillator model for the

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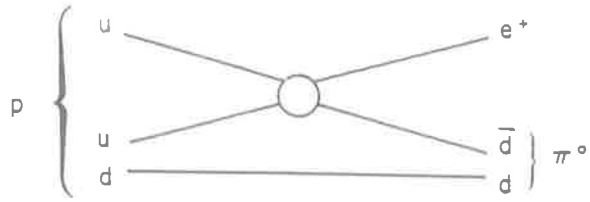

Fig. 1. The spectator diagram, generally assumed to dominate the proton-decay process.

structure of the baryon. However the motivation for the work, and the $NN\pi$ interaction (including the form factor), come from the "cloudy bag model" [10].

It is worthwhile to notice that, contrary to general expectations, the nucleon pole contribution should have been anticipated to be of the same order of magnitude as the spectator term. As a rough estimate of the covariant matrix element M_{pole} of fig. 2 we set

$$M_{\text{pole}} \sim \langle e^+|\mathcal{L}|p\rangle \omega^{-1}(g/2m)\boldsymbol{\sigma}\cdot\boldsymbol{k}\,, \tag{1a}$$

$$\sim G_{\text{GUT}}(g/2m)\psi(0,0)\,, \tag{1b}$$

$$\sim G_{\text{GUT}} f_\pi^{-1}\psi(0,0)\,, \tag{1c}$$

where the ω^{-1} factor in eq. (1a) is the propagator, and the other two factors come from the vertices. In eq. (1b) G_{GUT} is the grand unified theory four-fermion coupling constant ($G_{\text{GUT}}/\sqrt{2} = g^2/8m_X^2$) and $\psi(\boldsymbol{\xi},\boldsymbol{\eta})$ is the intrinsic quark wave function of the proton, defined in more detail below. The Goldberger–Treiman relation has been used to go from eqs. (1b) to (1c).

The spectator term of fig. 1 has an invariant matrix element of order

$$M_{\text{spec}} \sim G_{\text{GUT}}(2\omega)^{1/2}\int \mathrm{d}\boldsymbol{\eta}\,\psi(0,\boldsymbol{\eta})\,, \tag{2}$$

where the factor of $(2\omega)^{1/2}$ is simply the normalization factor between matrix elements and invariant matrix elements. A glance at eqs. (1b) and (2) shows that M_{pole} is proportional to the amplitude for all three quarks in the proton to be at the same point, whereas M_{spec} is proportional to the amplitude for just the two u-quarks to be at the same point. For this reason, M_{pole} has generally been neglected. However, we note that [11]

$$f_\pi \sim \psi_{\text{m}}(0)/\mu_\pi^{1/2}\,, \tag{3}$$

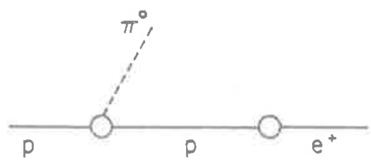

Fig. 2. The nucleon pole contribution to $p \to \pi^0 e^+$.

where $\psi_m(\boldsymbol{r})$ is the quark wave function in the meson. Making the reasonable estimates

$$|\psi(0,0)|^{1/2} \sim \int d\boldsymbol{\eta}\, \psi(0,\boldsymbol{\eta}) \sim \psi_m(0)\,, \tag{4}$$

we see that M_{pole} and M_{spec} are of the same order:

$$M_{\text{pole}} \sim (\mu_\pi/\omega)^{1/2} M_{\text{spec}} \sim M_{\text{spec}}\,. \tag{5}$$

We shall see that this conclusion is borne out by the detailed calculations below.

From the point of view of conventional nuclear physics, the calculation of proton decay through either the spectator or pole graphs makes little sense. There the nucleon is viewed as a relatively small source surrounded by a dense cloud of pions, so that the chance of finding just three quarks, or three quarks plus one pion is negligible. However in the cloudy bag model (CBM), which naturally incorporates the coupling of the pion field to a relatively large (MIT-like) bag, it has proven possible to place rigorous bounds on the number of virtual pions in the cloud about the physical nucleon [12]. For example, Dodd et al. have shown that the average number of pions in the physical nucleon ($\langle n \rangle$) is rigorously bounded by their parameter Λ, which is of order one for the CBM [12]. Explicit calculation yields $\langle n \rangle \simeq 0.5$. As a first approximation this justifies the conventional calculations for the proton decay process being considered here. To next order it suggests that fig. 2 should also be considered. These considerations [12] led naturally to the present calculation.

In sect. 2 we describe the evaluation of the spectator and pole matrix elements in detail, paying particular attention to their relative phase. Then, in sect. 3, we present our results for branching ratios and partial decay rates. In the final section we draw our conclusions and indicate possible further applications of our method.

We shall conclude this introduction by specifying the effective lagrangian which we shall use. It is

$$\begin{aligned}\mathscr{L}_{\text{GUT}} = 2\sqrt{2}\, G_{\text{GUT}}\{&\varepsilon_{ijk}\overline{u^{c}_{k\text{L}}}\gamma^\mu u_{j\text{L}}[A_{\text{L}}(r\overline{e^{+}_{\text{L}}}\gamma_\mu d_{i\text{L}} + \overline{\mu^{+}_{\text{L}}}\gamma_\mu s_{i\text{L}})\\ &+A_{\text{R}}(\overline{e^{+}_{\text{R}}}\gamma_\mu d_{i\text{R}} + \overline{\mu^{+}_{\text{R}}}\gamma_\mu s_{i\text{R}})]\\ &+A_{\text{R}}\varepsilon_{ijk}\overline{u^{c}_{k\text{L}}}\gamma^\mu d_{j\text{L}}(\overline{\nu^{c}_{e\text{R}}}\gamma_\mu d_{i\text{R}} + \overline{\nu^{c}_{\mu\text{R}}}\gamma_\mu s_{i\text{R}})\} + \text{h.c.}\,,\end{aligned} \tag{6}$$

where we have ignored Cabibbo-like mixing, and including only the first two generations of quarks [3],

$$\sqrt{\tfrac{1}{2}}\, G_{\text{GUT}} = g^2/8m_{\text{X}}^2 = g^2/8m_{\text{Y}}^2\,. \tag{7}$$

The ratio of the left to right-handed pieces, r, depends on the particular model chosen [13]. In SU(5) $r=2$ at the tree level, while in SO(10) broken to $\text{SU(2)}_\text{L} \times \text{U(1)} \times \text{SU(3)}$ $r \approx 0$, and in theories broken to $\text{SU(2)}_\text{L} \times \text{SU(2)}_\text{R} \times \text{U(1)} \times \text{SU(3)}$ $r \approx 1$. The factors A_{L} and A_{R} are renormalization group coefficients, which are given in

SU(5) by [13–16]*

$$A_{L(R)} = \Gamma \left[\frac{\alpha_1(100\ \mathrm{GeV})}{\alpha_5(m_X)} \right]^{\gamma_{L(R)}}, \tag{8}$$

where

$$\Gamma = \left[\frac{\alpha_3(1\ \mathrm{GeV})}{\alpha_5(m_X)} \right]^{6/(33-4N_G)} \left[\frac{\alpha_2(100\ \mathrm{GeV})}{\alpha_5(m_X)} \right]^{27/(86-16N_G)}, \tag{9a}$$

$$\gamma_R = -33/(6+80N_G), \tag{9b}$$

$$\gamma_L = -69/(6+80N_G), \tag{9c}$$

where N_G is the number of generations, and $\alpha_n(q^2)$ is the "fine structure constant" of the SU(n) theory. Numerically we adopt the values [16, 3]**

$$(g^2/4\pi) = 0.0244, \qquad A_L \approx A_R \approx 3.4 \approx A, \tag{10a}$$

$$m_X = (2.5 \pm 1.5) \times 10^{14}\ \mathrm{GeV}, \tag{10b}$$

$$G_{GUT} \approx (0.34\ \text{to}\ 5.4) \times 10^{-30}\ \mathrm{GeV}^2, \tag{10c}$$

and turn to the calculation of M_{pole}.

2. The nucleon pole contribution to the $p \to \pi^0 e^+$ matrix element

There are two basic ingredients for the calculation of M_{pole}, the matrix element for proton decay arising from the pole diagram of fig. 2. The first is $\langle e^+|\mathscr{L}|p\rangle$, represented in the quark model by the three-quark annihilation diagram of fig. 3. This we calculate directly from the effective lagrangian of eq. (6), in a harmonic oscillator quark model (so that our results are comparable with those of Gavela et al. [5]). The second ingredient is the πNN coupling constant and form factor, which

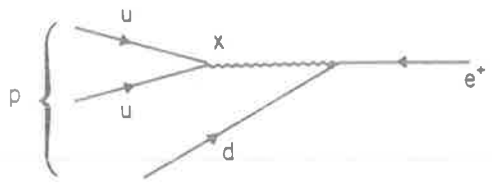

Fig. 3. The three-quark annihilation diagram representing $\langle e^+|\mathscr{L}|p\rangle$.

* The relative phase between right- and left-handed pieces in eq. (6) is chosen following Gavela et al. [5]; see also ref. [16]. This was given incorrectly in refs. [14, 15].

** In fact (ref. [16]) the "best" values of A_L and A_R are $A_L \approx 3.5$, $A_R \approx 3.3$. In view of the other uncertainties (especially that in m_X), there seems little point in maintaining the distinction between A_L and A_R in our calculation. The smaller value of m_X compared to that of ref. [16] reflects the smaller value of the QCD scale parameter ($\Lambda_{\overline{MS}} \approx 100$ to 200 MeV for four flavours; see: Buras, ref. [17]) currently in favour, through the approximate relation $m_X/\Lambda_{\overline{MS}} \approx (1.5 \pm 0.5) \times 10^{15}$. See the review of Ellis (ref. [3]) for more details. The smaller value of Λ will also reduce A by 15% or so, but we have not included this refinement in the present calculation.

are taken from the cloudy bag model [10]. It is useful to emphasize at this stage that the CBM incorporates chiral invariance at the quark level, and that we should expect the results obtained from it to be compatible with calculations which invoke chiral symmetry at the hadronic level [7–9]. We will later use the principle to determine the relative phase of $M_{\rm pole}$ and $M_{\rm spec}$.

The calculation of $\langle e^+|\mathscr{L}|p\rangle$ proceeds straightforwardly from the effective lagrangian of eq. (6). We choose to follow Gavela et al. [5] and calculate in the limit of non-relativistic quarks (and ultra-relativistic positrons). In this way we obtain a two-component effective lagrangian which is

$$\begin{aligned}\mathscr{L}_{2,p\to e^+\pi^0} = -2G_{\rm GUT}A\varepsilon_{\alpha\beta\gamma}b_{1u\gamma}b_{2u\beta}b_{3d\gamma}d^{+}_{4e}\\ \times[r\chi_1^+\chi^+_{4e_L}(-i\sigma_y^{(1)})\{\tfrac{1}{2}(1+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\}\chi_2\chi_3\\ +\chi_1^+\chi^+_{4e_R}(-i\sigma_y^{(1)})\{\tfrac{1}{2}(1+\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\}\chi_2\chi_3],\end{aligned} \tag{11}$$

where $\chi_1^+\chi_4^+O^{(1)}O^{(2)}\chi_2\chi_3=\chi_1^+O^{(1)}\chi_2\chi_4^+O^{(2)}\chi_3$ is used in writing eq. (11) in a more compact notation. The operators b and d are annihilation operators for particles and anti-particles, respectively.

We then use an SU(6) wave function for the proton [4, 6]*

$$|p\uparrow\rangle=\tfrac{1}{3}\sqrt{\tfrac{1}{2}}\,\varepsilon^{\alpha\beta\gamma}_{\alpha'\beta'\gamma'}(b^+_{u\alpha'\uparrow}b^+_{d\beta'\downarrow}-b^+_{u\alpha'\downarrow}b^+_{d\beta'\uparrow})b^+_{u\gamma'\uparrow}|0\rangle\,, \tag{12}$$

and combine the results for e_L^+ and e_R^+ to write the final matrix element in the form required by T-invariance, viz.

$$\langle e^+|\mathscr{L}|p\rangle=-12G_{\rm GUT}AF\,\overline{u_e}(\alpha+\beta\gamma_5)u_p\,. \tag{13}$$

In eq. (13) we have used the definitions

$$\alpha=\tfrac{1}{2}(1-r)\,,\qquad (=-\tfrac{1}{2}\text{ in SU(5)})\,, \tag{14a}$$

$$\beta=\tfrac{1}{2}(1+r)\,,\qquad (=+\tfrac{3}{2}\text{ in SU(5)})\,, \tag{14b}$$

$$F=3^{-3/4}\psi(0,0)\,. \tag{14c}$$

Here, the spatial wave function of the proton is defined to be

$$\psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3)=3^{-3/4}\,e^{i\boldsymbol{k}_p\cdot\boldsymbol{R}/\sqrt{3}}\psi(\boldsymbol{\xi},\boldsymbol{\eta})\,, \tag{15}$$

with

$$\boldsymbol{R}=\sqrt{\tfrac{1}{3}}(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3)\,, \tag{16a}$$

$$\boldsymbol{\xi}=\sqrt{\tfrac{1}{2}}(\boldsymbol{r}_1-\boldsymbol{r}_2)\,, \tag{16b}$$

$$\boldsymbol{\eta}=\sqrt{\tfrac{1}{6}}(\boldsymbol{r}_1+\boldsymbol{r}_2-2\boldsymbol{r}_3)\,, \tag{16c}$$

* $\varepsilon^{\alpha\beta\gamma}_{\alpha'\beta'\gamma'}$ is the parity of the permutation $(\alpha'\beta'\gamma')$ of colour indices with respect to the permutation $(\alpha\beta\gamma)$. We write the wave function in this form to explicitly indicate the colour degrees of freedom of the wave function.

the Jacobi co-ordinates for the three quark system. We use Isgur–Karl harmonic oscillator wave functions for the proton

$$\psi(\boldsymbol{\xi}, \boldsymbol{\eta}) = \pi^{-3/2} R_p^{-3} \exp\{-\tfrac{1}{2} R_p^{-2} (\xi^2 + \eta^2)\}, \tag{17}$$

so that

$$F = 1/(3^{1/4} \pi^{1/2} R_p)^3, \tag{18}$$

in terms of the proton radius parameter, R_p.

Unfortunately the value of $R_p = \alpha_p^{-1}$ which should be used in these calculations is subject to some controversy [19]. Difficulties arise in attempting to fit a variety of different data relevant to the proton with a one-parameter wave function. To illustrate this effect consider fitting the calculated harmonic oscillator electromagnetic form factor, $F_{HO} = e^{-k^2/6\alpha_p^2}$, to the "observed" form factor which we may parameterise as $F_{exp} = [1 + k^2/0.71\ \mathrm{GeV}^2]^{-2}$. The usual approach is to demand equality of the k^2 terms in the expansion of F_{HO} and F_{exp}; in other words to adjust α_p to fit the r.m.s. charge radius. This gives

$$\alpha_p = 0.23 \pm 0.02\ \mathrm{GeV}, \qquad R_p = 0.87 \pm 0.08\ \mathrm{fm}, \tag{19a}$$

and clearly amounts to reproducing the observed long-distance behaviour of the wave function. On the other hand, as Hara [20] has emphasised we could alternatively choose α_p to fit the value of $\int_0^\infty k^2\, dk\, F_{exp}(k^2)$. This gives

$$\alpha_p = 0.41\ \mathrm{GeV}, \qquad R_p = 0.48\ \mathrm{fm}, \tag{19b}$$

and amounts to choosing α_p to reproduce the short-distance behaviour of the wave function since [20]

$$\int_0^\infty dk\, k^2 F(k^2) = 3\pi^2 \sqrt{\tfrac{3}{2}} \int d\boldsymbol{\xi} |\psi(\boldsymbol{\xi}, \mathbf{0})|^2 .$$

From a study of the electromagnetic mass differences of hadrons Hara [20] has obtained further constraints on the short-distance behaviour of the proton wave functions, viz.

$$2^{-3/2} \int d\boldsymbol{\eta} |\psi(\mathbf{0}, \boldsymbol{\eta})|^2 = (1.29 \pm 0.10) \times 10^{-2}\ \mathrm{GeV}^3,$$

$$\sqrt{\tfrac{1}{2}}\, \alpha_{e.m.} \int \frac{1}{|\boldsymbol{\xi}|} |\psi(\boldsymbol{\xi}, \boldsymbol{\eta})|^2\, d\boldsymbol{\xi}\, d\boldsymbol{\eta} = 3.6\ \mathrm{MeV};$$

which imply for our wave function

$$\alpha_p = 0.59\ \mathrm{GeV}, \qquad R_p = 0.34\ \mathrm{fm}, \tag{19c}$$

$$\alpha_p = 0.61\ \mathrm{GeV}, \qquad R_p = 0.32\ \mathrm{fm}, \tag{19d}$$

respectively. De Rujula et al. [21] constrained the short-distance behaviour of the hadron wave function from the N-Δ mass difference, obtaining

$$2^{-3/2}\int d\boldsymbol{\eta}|\psi(\mathbf{0},\boldsymbol{\eta})|^2 = 7.6\times 10^{-3}\ \mathrm{GeV}\,,$$

which gives

$$\alpha_p = 0.49\ \mathrm{GeV}\,, \qquad R_p = 0.40\ \mathrm{fm}\,. \tag{19e}$$

Other methods of determining R_p have been used. Electroproduction and the baryon spectrum give [22]

$$\alpha_p = 0.36\pm 0.05\ \mathrm{GeV}\,, \qquad R_p = 0.56\pm 0.08\ \mathrm{fm}\,, \tag{19f}$$

but it should be noted that this emphasises the long-distance properties of the wave function. Fits to hyperon decays [23] quote values for R_p which vary from 0.6 for a fit to S-wave amplitudes to 0.45 fm for a fit to P-wave decays. Other fits give values in this range. The discrepancy is not the fault of the wave function, but is a reflection of the fact that there is still no generally accepted way of reconciling S- and P-wave amplitudes, although most attempts to do so retain P-wave values of $\langle B'|H_w|B\rangle$ and modify the S-wave amplitudes.

The mutual agreement of (19b) through (19e) (the values of R_p determined from "measurements" of $\psi(\boldsymbol{\xi},\boldsymbol{\eta})$ when one of $|\boldsymbol{\xi}|$ and $|\boldsymbol{\eta}|$ is constrained to be small) gives us confidence in using a value

$$R_p = 0.35\ \mathrm{fm}\,,$$

to determine $\psi(0,0)$ and $\int d\boldsymbol{\eta}|\psi(0,\boldsymbol{\eta})|^2$. We attribute the discrepancy between (19b) through (19e) on the one hand, and (19a) and (19f) on the other, to a deficiency in the harmonic oscillator (or any other one parameter) wave function. Such a wave function cannot at the same time reproduce the long- and short-distance behaviour of the wave function. Quite plausibly there are short-distance correlations between the quarks, induced by gluon exchange. In the absence of a wave function including these correlations we give our results for a range of values of R_p. We also attempt to include correlations in a phenomenological way by introducing a "hybrid" model in which $R_p = 0.87$ fm is used when we calculate long-distance form factor effects, and $R_p = 0.35$ fm is used when we calculate short-distance effects such as the amplitude for two or three quarks to be at the same point.

Having obtained the amplitude $\langle e^+|\mathscr{L}|p\rangle$, we can now proceed to compute the nucleon pole term of fig. 2. We do so in the framework of the cloudy bag model [10], in which the interaction hamiltonian coupling the pion to the nucleon

$$\underset{\sim}{V}_k = i\frac{g_A}{2f_\pi}\frac{u(kR_B)}{\sqrt{2\omega_k}}\boldsymbol{\sigma}\cdot \boldsymbol{k}\underset{\sim}{\tau}\,, \tag{20}$$

where R_B is the bag radius,

$$u(kR_B) = j_0(kR_B) + j_2(kR_B)\,, \tag{21}$$

is the πNN form factor, g_A is the axial vector strength of the nucleon ($g_A \approx 1.25$), and f_π is the pion decay constant (with the normalization $\sqrt{2} f_\pi \approx m_\pi$). Using eq. (20) we find for the invariant matrix element M_{pole}

$$M_{\text{pole}} = -i\frac{g_A}{2f_\pi} u(kR_B) \frac{k}{\omega_k + k^2/2m_p} \frac{12G_{\text{GUT}}A}{3^{3/4}\pi^{3/2}R_p^3} \bar{u}_e(\beta + \alpha\gamma_5)u_p \,, \tag{22}$$

in the rest frame of the decaying proton.

To obtain the complete matrix element for the decay process $p \to e^+\pi^0$ we must add the spectator term of fig. 1 to the pole term of fig. 2. The former is readily calculated following the methods of Gavela et al. [5] and Kane and Karl [6], but particular attention must be paid to the relative sign of the amplitudes $p \to e_L^+\pi^0$ and $p \to e_R^+\pi^0$*. The resulting invariant amplitude is

$$M_{\text{spec}} = -i\eta \frac{\phi(k)2\sqrt{3}\,AG_{\text{GUT}}}{(2\pi)^{3/4}R_p^{3/2}}(2\omega_k)^{1/2}\bar{u}_e(\beta + \alpha\gamma_5)u_p \,, \tag{23}$$

where η is a phase factor ($|\eta|^2 = 1$) associated with the pion wave function. The latter is taken to be

$$|\pi^0\rangle = \frac{-i\eta^*}{2\sqrt{3}}\varepsilon^{\uparrow\downarrow}_{\lambda\mu}(d^\dagger_{ui\lambda}b^\dagger_{ui\mu} - d^\dagger_{di\lambda}b^\dagger_{di\mu})|0\rangle \,, \tag{24}$$

where $\varepsilon^{\uparrow\downarrow}_{\lambda\mu}$ is the permutation tensor ($\varepsilon^{\uparrow\downarrow}_{\uparrow\downarrow} = +1$, $\varepsilon^{\uparrow\downarrow}_{\downarrow\uparrow} = -1$, other $\varepsilon^{\uparrow\downarrow}_{\lambda\mu} = 0$), $d^\dagger_{ui\lambda}$ is the creation operator of the $\bar{u}$ quark with colour index i and spin λ. In eq. (23) $\phi(k)$ is a form factor associated with the spectator decay mechanism, and is given by [5]

$$\phi(k) = \frac{[\frac{3}{4}R_p^2R_m^2]^{3/4}}{[\frac{1}{2}(R_m^2 + \frac{3}{4}R_p^2)]^{3/2}} \exp\left(\frac{-3k^2R_p^2R_m^2}{12R_p^2 + 16R_m^2}\right), \tag{25}$$

with R_m the harmonic oscillator parameter for the meson (analogous to R_p for the proton).

The phase factor, η, is undetermined at this stage of the calculation. We note that the total amplitude ($M_{\text{pole}} + M_{\text{spec}}$) is T-invariant whatever value is assigned to η. For the total amplitude, we write

$$\begin{aligned} M &= M_{\text{pole}} + M_{\text{spec}} \\ &= -iG_{\text{GUT}}AB\bar{u}_e(\beta + \alpha\gamma_5)u_p \,, \end{aligned} \tag{26}$$

with

$$B = B_{\text{pole}} + \eta B_{\text{spec}} \,, \tag{27a}$$

* Most calculations reported in the literature have not given the relative sign of these amplitudes, which turns out to be *negative*. In the context of these earlier calculations, this sign enters only the electron polarization in the decay of polarized protons (the γ parameter of hyperon decays) and was not experimentally relevant. However, we need it so that the amplitudes M_{pole} and M_{spec} can be correctly combined.

TABLE 1

Values of the form factor $\phi(k)$, which arises from the proton–meson wave function overlap (see eq. (25) in the spectator process)

R_p (fm)	$\phi(k)$	B_{spec} (GeV^2)	B_{pole} (GeV^2)	ρ
0.87	0.68	0.063	0.037	0.59
0.56	0.85	0.153	0.137	0.90
0.35	0.94	0.341	0.560	1.64
0.30	0.96	0.436	0.890	2.04
hybrid	0.68	0.247	0.560	2.27

The pole (fig. 2) and spectator (fig. 1) matrix elements (B_{pole} and B_{spec}, respectively), and their ratio ρ ($\equiv B_{pole}/B_{spec}$), are shown as a function of the harmonic oscillator radius parameter R_p (see eq. (17)). The "hybrid" model, which we prefer, is explained in the text.

$$B_{pole} = \frac{g_A}{2f_\pi} u(kR_B) \frac{k}{\omega_k + k^2/2m_p} \frac{6\sqrt{2}}{3^{3/4}\pi^{3/2}R_p^3}, \tag{27b}$$

$$B_{spec} = \phi(k) \frac{1}{(2\pi)^{3/4}R_p^{3/2}} 2\sqrt{3}(2\omega_k)^{1/2}. \tag{27c}$$

The values of $\phi(k)$ obviously depend on the ratio of meson radius R_m to proton radius R_p. Values of R_m^2/R_p^2 used in the literature range from 0.75 to 1.0. As ϕ varies by only a few per cent over this range, we simply follow Gavela et al. [5] in setting $R_m^2 = \frac{3}{4}R_p^2$. Table 1 then lists the values of $\phi(k)$, B_{spec}, B_{pole} and B_{pole}/B_{spec} for the range of values of R_p discussed above. It should be noted that a bag radius of 0.8 fm has been used [10], for which the CBM form factor at the $NN\pi$ vertex is $u(kR_B) = 0.69$.

The extreme sensitivity of the results to the value of the radius parameter R_p is evident. This is unfortunate because of the uncertainties in the value of the parameter [19]. Taken at its face value, this would lead to a factor of one hundred uncertainty in the partial rate $\Gamma(p \to e^+\pi^0)$ (for $\eta = 1$).

However, we believe the uncertainty can be reduced by noting that R_p enters eqs. (27) in two ways: the explicit dependence, R_p^{-3} and $R_p^{-3/2}$, where R_p is measuring the magnitude of the wave function at short distances, and the implicit dependence in the form factor $\phi(k)$ where R_p is measuring wave function overlaps, or large-distance effects. This suggests that we use the charge radius value of R_p (0.87 fm) in $\phi(k)$, but the short-distance value of R_p, say 0.35 fm in the explicit dependence. This gives the results marked "hybrid" in table 1, and represents an attempt to overcome the limited nature of the one-parameter harmonic oscillator wave functions.

Before proceeding to calculate the decay rate, we need to know the phase η. We can obtain this phase by one of two methods; either:

(a) using U(6, 6) wave functions for the meson [24]* or

(b) by comparison with the phenomenological, chirally symmetric theory of Claudson et al. [9].

In method (a) the pion wave function is written as

$$\langle 0|\psi_\alpha \bar{\psi}^\beta|\pi\rangle = -\left[\left(\frac{\not{p}+\mu}{2\mu}\right)i\gamma_5\right]_\alpha^\beta, \tag{28}$$

and evaluating the left-hand side using eq. (24), and taking the non-relativistic limit of the right-hand side, gives $\eta = +1$. In method (b) the phenomenological, $\Delta B = -1$ lagrangian of ref. [9] is (for $p \to e(n\pi)$, $n = 0, 1$)

$$\mathscr{L}^{\Delta B=-1} = K\left[C^{(1)}\left(\bar{e}_L p + \frac{i}{\sqrt{2}f_\pi}\bar{e}_L p\pi^0\right) + C^{(2)}\left(\bar{e}_R p - \frac{i}{\sqrt{2}f_\pi}\bar{e}_R p\pi^0\right)\right], \tag{29}$$

where K is an undetermined parameter of order $G_{GUT}m_p^2$. With the strong interaction lagrangian

$$\mathscr{L}_{\text{strong}} = -\frac{g_A}{2f_\pi}\bar{p}\gamma^\mu\gamma_5 p(\partial_\mu\pi^0), \tag{30}$$

the matrix element is

$$M_{\text{phen}} = \frac{-iK}{2\sqrt{2}f_\pi}(1+g_A)\bar{e}(C_- + C_+\gamma_5)p, \tag{31}$$

with

$$C_\pm = C^{(1)} \pm C^{(2)}. \tag{32}$$

In eq. (31) the unit term comes from the direct π^0 emission and g_A comes from the nucleon pole term. Note that the spinor structure coincides with that of eq. (26) for $C^{(2)}/C^{(1)} = -r$ which also suggests that $\eta = 1$. We note in passing that our favoured value of $\rho = B_{\text{pole}}/B_{\text{spec}}$ is 2.27 which is somewhat larger than the value $\rho = g_A = 1.25$ implied by eq. (31).

To summarize, we set the phase $\eta = +1$ and proceeded to compute partial decay rates and branching ratios.

3. Partial decay rates and branching ratios

From the values of B given in table 1, and the other parameters defined in sect. 2, we can compute the partial decay rates for $p \to e^+\pi^0$ which are given in table

* Note that γ_5 in this reference is to be identified with $i\gamma_5$ in this paper.

TABLE 2

(a) Partial decay rates for $p \to e^+\pi^0$ in units of $10^{-30}\,y^{-1}$, both pole and spectator terms included

G_{GUT} $10^{-30}\,GeV^{-2}$ \ R_p/fm	0.85	0.56	0.35	0.30	hybrid
0.33	0.025	0.22	2.2	4.7	1.7
1.33	0.41	3.6	35	76	28
5.4	6.8	59	580	1290	450

Note that $G_{GUT} = 5.4 \times 10^{-30}\,GeV^{-2}$ corresponds to $m_X = 1 \times 10^{14}$ GeV.

(b) Partial decay rates for $p \to e^+\pi^0$ in units of $10^{-30}\,y^{-1}$, spectator terms only

G_{GUT} $10^{-30}\,GeV^{-2}$ \ R_p/fm	0.85	0.56	0.35	0.30	hybrid
0.33	0.01	0.02	0.32	0.50	0.16
1.33	0.17	0.95	5.1	8.2	2.6
5.4	2.7	16	84	140	43

2a. The relevant formula is (neglecting the electron mass)

$$\Gamma(p \to e^+\pi^0) = \frac{k}{8\pi} \frac{(m_p^2 - \mu^2)}{m_p^2} G_{GUT}^2 A^2 B^2 \left\{ \alpha^2 + \frac{m_p^2 - \mu^2}{m_p^2 + \mu^2} \beta^2 \right\} . \tag{33}$$

In this section we give results only for SU(5), $r = 2$. For comparison, we give in table 2b the two-body decay rates given by the traditional, spectator diagram of fig. 1. To compare our results to those of ref. [5], we note that their eqs. (4.3), (5.2)–(5.5) imply that they used $R_p = 0.49$ fm, $A = 5.0$, and set $\phi = 1$. For $m_X = 1 \times 10^{14}$ GeV they give a partial rate for $p \to e^+\pi^0$ of $62 \times 10^{-30}\,y^{-1}$. Our nearest value is that for $G_{GUT} = 5.4 \times 10^{-30}\,GeV^{-2}$, and $R_p = 0.56$, which is $16 \times 10^{-30}\,y^{-1}$. To allow for the parameter variation, we multiply this by $(0.56/0.49)^3(5/3.4)^2(1/0.85)^2$, giving $69 \times 10^{-30}\,y^{-1}$ in reasonable agreement with the value of ref. [5]. In fact, our "hybrid" rate agrees quite well with the rate of ref. [5], and we will use it for further comparisons.

We now have to discuss the extent of pole term modifications of other two-body decay rates of the proton. Clearly, we can compute $p \to \bar{\nu}_R \pi^+$ using the same technique as in sect. 2. The rates of $p \to \mu^+ K^0$ or $p \to e^+\eta$ are not quite so obviously calculated in our model, in that the CBM has not yet been fully extended to an SU(3)×SU(3) chiral theory (although there has been one application to low-energy $\bar{K}p$ scattering [25]). Instead of making this extension here, we simply note that the phenomenological SU(3)×SU(3) chiral model of Claudson et al. [9], see eq. (29),

TABLE 3
Branching ratios for two-body decays of the proton

Decay mode	Gavela et al. [5] modified by addition of pion pole graph		Phenomenological chiral model
	$R_p = 0.56$ fm	hybrid	hybrid
$e^+\pi^0$	54	60	49
$e^-\eta$	3.9	2.4	0.9
$e^+\rho^0$	1.3	0.8	1.2
$e^-\omega$	9.5	5.8	9.4
$\bar{\nu}_e\pi^-$	22	24	19
$\bar{\nu}_e\rho^-$	0.6	0.4	0.3
μ^-K^0	10	6.4	20
$\bar{\nu}_\mu K^-$	0	0	0

is very close to our model for $p \to \pi^0 e^+$, when K is chosen correctly. In fact, sect. 2 could be regarded as providing a microscopic justification of that model, including a calculation of the value of K. Thus one approach is to use the model of ref. [9] to determine the ratio of matrix elements for those decay modes involving (π, η, K); those involving ρ and ω are taken from ref. [5]. Finally, after including the differences in phase space, as well as form factor effects for pion modes, we obtain the last column of table 3 (phenomenological chiral model, hybrid). To obtain absolute rates in table 4 we have used our hybrid calculation of the $p \to e^+\pi^0$ mode to set the scale.

A second extreme model would be to use only the spectator term for those modes which do not involve pions. These results could be obtained from the results in table 2 of Gavela et al. [5] as follows. For modes involving a pion, we multiply the rate of ref. [5] by the ratio of our total $\Gamma(p \to e^+\pi^0)$ (from table 2a) to theirs. For those modes which do not involve a pion, we simply multiply their rate by the ratio of our spectator rate $\Gamma(p \to e^+\pi^0)$ (from table 2b) to their $\Gamma(p \to e^+\pi^0)$, and

TABLE 4
The partial lifetime (in years) for two-body decay, as a function of the unification mass; the branching ratios were given in table 3

$\dfrac{G_{GUT}}{10^{-30}\,GeV^{-2}}$	Gavela et al. [5]	Gavela et al. [5] (modified by pole term)		Phenomenological chiral model
		$R_p = 0.56$ fm	hybrid	hybrid
0.33	1.6×10^{30}	2.4×10^{30}	3.3×10^{29}	2.7×10^{29}
1.33	9.9×10^{28}	1.5×10^{29}	2.1×10^{28}	1.7×10^{28}
5.4	6.0×10^{27}	9.1×10^{27}	1.3×10^{27}	1.1×10^{27}

then divide by $[\phi(k)]^2$ from table 1. This last step accounts for the fact that we have included the form factor effect [19], which influences only the pion modes. Incidentally, we must point out the amazing similarity of the CBM form factor, which occurs in the pole term, and $\phi(k)$. The former is in fact well approximated for $kR_B \leq 3$ by [26]

$$u(kR_B) \simeq \exp(-0.106k^2R_B^2)\,, \tag{34}$$

whereas with $R_m^2 = \frac{3}{4}R_p^2$ we find

$$\phi(k) = \exp(-0.094k^2R_p^2)\,. \tag{35}$$

Since these functions arise from quite different calculations, it is remarkable how close they are. From the practical point of view, it is convenient that these form factors do not significantly alter the ratio of pole and spectator matrix elements.

Using this second model we generate the branching ratios in columns 2 and 3 of table 3. The addition of the pion pole term overcomes the tendency of the form factor to suppress pionic modes. The over-all result is that the experimentally important pion modes should dominate the two-body decay modes. The results of tables 2 and 3 can be combined to give our estimates of the proton lifetime in table 4. (Of course, the two-body decay modes only give an upper bound.) The effect of including the nucleon pole terms has been to reduce the estimate of the proton lifetime by a factor of four or so.

The experimental limit on the proton lifetime is [27] $\tau_p \geq (1-2)\times 10^{30}$ y. To satisfy this limit in the SU(5) case requires us to select a large value of m_X (i.e. of Λ), and a large value of R_p. In view of the extreme sensitivity of our results to $m_X(\tau_p \propto m_X^4)$ and $R_p(\tau_p^{-1} \propto aR_p^{-3} + bR_p^{-6} + cR_p^{-9/2})$ it is not yet possible to claim with authority that the present data rule out the SU(5) model. But it does force us into a small corner of parameter space, and a modest improvement in either proton decay experiments or the independent limits on m_X and R_p would bring the SU(5) model to judgement. (At the time of proof correction the lower limit for $p \to e^+\pi^0$ was 10^{32} years (IMB collaboration). For a discussion of this model, in the light of the new limit, see our contribution to the EPS conference on h.e.p. in Brighton (July, 1983).)

4. Remarks

Motivated by considerations of chiral symmetry, we have calculated the effect of the nucleon pole graph on proton decay. Independent of the parameters used, we find that the ratio of the pole to spectator terms is of O(1). This leads to an enhancement of the pionic decay modes, and a substantial reduction of the proton lifetime. Our results raise a number of open questions with which we conclude our discussion:

(a) what is the influence of other spin-$\frac{1}{2}$ resonances?
(b) how much does the Δ pole of fig. 4 contribute to two pion decay modes?
(c) should one also include $S=-1$ pole terms?

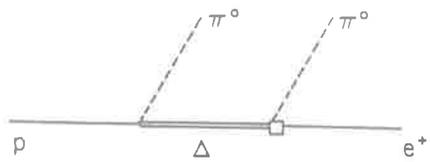

Fig. 4. A possible Δ pole contribution to $\langle e^+\pi^0\pi^0|\mathscr{L}|p\rangle$.

We speculate that higher spin-$\frac{1}{2}$ resonances will not contribute significantly to the $e^+\pi^0$ decay rate, because the πNR couplings are usually smaller than πNN. There is also likely to be an angular momentum suppression of the $R\to e^+$ amplitude. On the other hand, we would expect the Δ pole to contribute at least as much to the $\pi\pi e$ decay mode as the N pole considered in ref. [9].

Finally, there is the fundamental question of whether the K is to be treated as a Goldstone boson in the same way as the pion. The essential difference between columns 2 and 3 and column 4 of table 3 is the inclusion of an $S=-1$ pole term in the latter. This significantly enhances the μ^+K^0 decay mode, and is therefore of relevance to the experimental situation. It is even more critical if, instead of the conventional GUT, one believed in supersymmetry. In that case, the dominant decay modes would be [28] μ^+K^0 and $\bar{\nu}_\mu K^+$, and by analogy with the present work we would expect $S=-1$ pole terms to give a substantial enhancement in this decay rate.

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Note added

On completing this paper we received a preprint from J.F. Donoghue and E. Golowich who also investigated the pole contribution to proton decay. They used a bag model to compute $\langle e|\mathscr{L}_{GUT}|p\rangle$, and in our notation give as the bag equivalent of eq. (18)

$$F=0.247R_B^{-3}=0.482\ \mathrm{fm}^{-3}, \qquad \text{with } R_B=0.8\ \mathrm{fm}.$$

This is close to the numerical value of F in our calculation for $R_p=0.56$ fm, when $F=0.45\ \mathrm{fm}^{-3}$. They quote a lifetime in the range 2.2 to 6.2×10^{29} y in qualitative agreement with our results. It should be noted that Donoghue and Golowich did not add the pole and spectator terms coherently in their calculation (nor did they include πNN form factor effects). As shown in table 1, the two terms are comparable for $p\to e^+\pi^0$, and the interference is therefore important.

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THE AXIAL FORM FACTOR OF THE NUCLEON AND THE PION–NUCLEON VERTEX FUNCTION

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The axial form factor of the nucleon in the Cloudy Bag Model is harder than the NNπ vertex function, and consistent with data for a bag radius about 1 fm. We examine how these results are altered by excluding the pion field from a smaller, interior volume.

There has recently been considerable discussion about the calculation of the axial charge of the nucleon in various chiral bag models [1–5]. However little attention has been given to the axial-vector form factor, $g_A(q^2)$, which is known from measurements of muon production cross sections ($\nu_\mu + n \to \mu^- + p$) and pion electroproduction [6]. Phenomenologically the best fit is a dipole with mass $m_A = 0.95 \pm 0.14$ GeV. In fact Amaldi et al. [6] quote an error of ± 0.07 GeV from the neutrino reaction, and ± 0.14 GeV from electroproduction. We take the more conservative error in order to improve the overlap with the pion production data from the ZGS, which yielded values of m_A between 1.0 and 1.4 GeV [7]. Of course these data are most sensitive to the low $-q^2$ behaviour, which can be parametrized as

$$g_A(q^2)/g_A(0) = 1 - q^2 r_A^2/6, \tag{1}$$

with $r_A = 0.72 \pm 0.12$ fm.

In the Cloudy Bag Model [2,8] the pion field is allowed to penetrate the interior of the MIT bag. The appropriate expression for the axial-vector current density is

$$A_i(\underline{x}) = \tfrac{1}{2}\bar{q}(\underline{x})\gamma\gamma_5\tau_i\, q(\underline{x})\theta(R-r) - f_\pi \nabla\phi_i(\underline{x}), \tag{2}$$

where q and ϕ are respectively the quark and pion fields, R the bag radius and f_π is the pion decay constant (93 MeV). For this model $g_A(q^2)$ is entirely determined [8] as the expectation value of the Fourier transform of the first term in eq. (2), in the physical nucleon. Ignoring the corrections arising from renormalization, which are expected to be small [‡1], we find

$$g_A^{\mathrm{CBM}}(q^2)/g_A^{\mathrm{CBM}}(0) = 1.5\, j_0^{-2}(\omega) \times \int_0^R \mathrm{d}r\, r^2 \{[j_0^2(\omega r/R) - j_1^2(\omega r/R)]\, j_0(qr) + 2j_1^2(\omega r/R) j_1(qR)/(qR)\}. \tag{3}$$

As usual, $\omega = 2.04$ is the eigenfrequency for massless quarks.

Expanding eq. (3) to order q^2 for comparison with eq. (1) we find $r_A^{\mathrm{CBM}} = 0.62\,R$, and hence $R = 1.16 \pm 0.20$ fm. Of course we must add a caution that we have also omitted corrections arising from possible spurious centre of mass motion and recoil. Surprisingly several groups [9] have found quite a large correction to $g_A(0)$ from CM corrections (of order 20%) – using the Peierls–Yoccoz technique. Because the deviation of $g_A(0)$ from $\frac{5}{3}$ is a purely relativistic effect in the bag model we have limited confidence in the Peierls–Yoccoz procedure. In any case we expect the

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[‡1] In the static approximation they are identically zero for the ratio.

corrections in the ratio $g_A(q^2)/g_A(0)$ to be less than in $g_A(0)$ alone.

Recent lattice calculations of Kogut et al. [10] have suggested that the temperature at which confinement occurs (T), may be somewhat smaller than that at which chiral symmetry breaking occurs (T_{ch}). This has been translated by Chin and Miller [4] and Vento [5] into a simple modification of the CBM in which pions are coupled to the quarks at $r = R$, but excluded from the region $r < R_{ch}$ (where $R_{ch}/R \sim T/T_{ch} < 1$). As a caution we would note that the calculation of Kogut et al. omits fermion loops, so that it is hard to see how dynamical symmetry breaking could appear in the model. There is no doubt that their calculation gives an indication of the surface thickness of the confinement volume, but the naive association of the interior with the Wigner phase and the exterior as the Goldstone phase is not so straightforward [11].

Nevertheless it is interesting to pursue the interpretation of Chin and Miller [4] and Vento [5]. As shown by those authors the axial charge of the nucleon was modified to

$$\tilde{g}_A(0) = g_A^{CBM}(0)\,(1 + \xi^3/2)\,, \quad (4)$$

with $\xi = R_{ch}/R$. For the case $\xi = \frac{2}{3}$ suggested by Kogut et al. [10] this means a 15% increase over the CBM value of 1.09 – which is identical to that of the original MIT bag model [12]. The effect on charge radii and magnetic moments is expected to be much smaller because these quantities get their biggest contribution from pions outside the bag [2,8].

Clearly it is interesting to check whether this ne version of the CBM significantly alters the predictic for $g_A(q^2)/g_A(0)$. Following Chin and Miller we wr the expectation value of the pion field as

$$f_\pi \langle \underset{\sim}{\phi}(r)\rangle = g_A(0)\Delta_1(r,R)(3/4\pi R)\langle N|\tfrac{1}{2}\boldsymbol{\sigma}\cdot\hat{r})\underset{\sim}{\tau}|N\rangle\ , \quad ($$

where $\Delta_1(r,R)$ is the static Green function for p-w: pions excluded from the region $r < R_{ch}$. This Greer function has the form [4]

$$\Delta_1(r,r') = \tfrac{1}{3}m_\pi\,[j_1(im_\pi r_<) + \gamma h_1^{(1)}(im_\pi r_<)]$$

$$\times\, h_1^{(1)}(im_\pi r_>)\,, \quad (6$$

where

$$\gamma = -\,\{[\partial j_1(im_\pi r)/\partial r]\,/\,[\partial h_1^{(1)}(im_\pi r)/\partial r]\}_{r=R_{ch}}\,. \quad (6$$

It is now straightforward to show that in this model the q^2-dependence is given by

$$\tilde{g}_A(q^2) = g_A^{CBM}(q^2) + g_A^{CBM}(0)\,\tfrac{1}{2}\xi^3 u(qR_{ch})(1 - y^2 \quad ($$

where $u(x) = 3j_1(x)/x$ is the familiar CBM form fact and $y \equiv m_\pi R$. The $q^2 = 0$ limit of eq. (7) differs fro eq. (4) because the pion mass (m_π) has not been tak to zero. In eq. (7) small terms of order y^3 or higher are ignored.

If we expand eq. (7) to order q^2 the new mean square radius of the axial-vector charge distribution ($\tilde{r}_A^2$) is

$$\tilde{r}_A^2 = (1 + \tfrac{1}{2}\xi^3)^{-1}\,[0.38 + 0.30\,\xi^5(1 - \tfrac{1}{2}y^2)]\,R^2\,. \quad ($$

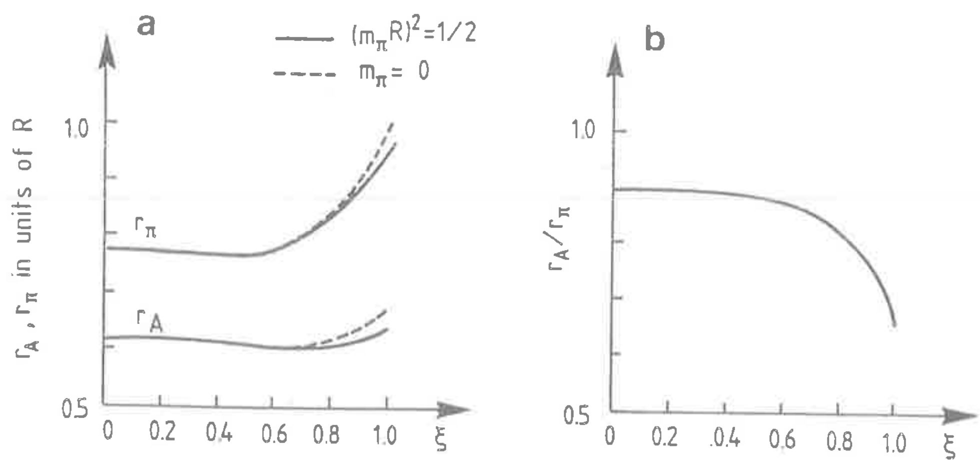

Fig. 1. (a) $\tilde{r}_\pi$ and $\tilde{r}_A$ in units of R. (b) $\tilde{r}_A/\tilde{r}_\pi$. This quantity is essentially independent of y.

As shown in fig. 1a $\tilde{r}_A$ is almost independent of ξ over its entire range. [The two curves refer to two values of y ($y = 0$ and $y^2 = \frac{1}{2}$) used in eq. (8).] For small $m_\pi R$ and for $\xi = 1$, $\tilde{r}_A/(r_A)_{CBM} = 1.05$. For all other parameters $\tilde{r}_A/(r_A)_{CBM}$ is closer to unity. Thus $g_A(q^2)$ offers no discrimination between models with different values of ξ.

Another property of the nucleon which has been intensely discussed in the context of chiral bag models is the form factor for coupling to a pion, $g_{\pi NN}(q^2)$. As shown in eq. (3), in such models $g_A(q^2)$ arises from an integral over the bag volume. On the other hand, $g_{\pi NN}(q^2)$ is associated with the surface of the bag, and one might intuitively expect it to fall more rapidly with q^2. In the CBM this is certainly the case, because the form factor there is

$$g_{\pi NN}^{CBM}(q^2) = 3j_1(qR)/(qR)$$
$$= 1 - q^2(0.6\,R^2)/6\,, \tag{9}$$

and hence r_π $[g_{\pi NN}(q^2) \equiv 1 - q^2 r_\pi^2/6 + O(q^4)]$ is $0.77\,R$. This should be compared with the value of r_A ($= 0.62\,R$) calculated from eq. (3). That is in the CBM (r_A/r_π) equals 0.80.

One can also compute the NNπ form factor in the model of Chin and Miller. This is more complicated than in the CBM because the pion field cannot be expanded in plane waves. The NNπ form factor can be identified from the coefficient of the pion pole term which appears in the expectation value of the pion field operator for the physical nucleon. By using the field equations, or by considering directly

$$(-\nabla^2 + m_\pi^2)\partial_\mu \underset{\sim}{A}^\mu = (-\nabla^2 + m_\pi^2)\theta(r - R_{ch}) m_\pi^2 f_\pi \underset{\sim}{\phi}\,, \tag{10}$$

one finds

$$(q^2 + m_\pi^2)\int d^3r\, \exp(i q\cdot r)\langle \underset{\sim}{\phi}(r)\rangle$$
$$= \int d^3r \exp(i q\cdot r)\langle (i/2f_\pi)\, \bar{q}(r)\gamma_5 \underset{\sim}{\tau} q(r)\delta(r - R)$$
$$- i(q\cdot \hat{r})\delta(r - R_{ch})\underset{\sim}{\phi}(r)\rangle\,. \tag{11}$$

Direct evaluation of the integrals in eq. (11) gives the result

$$g_{\pi NN}(q^2)/g_{\pi NN}(0) = \{u(qR) + \tfrac{1}{2}\xi^3[u(qR_{ch})$$
$$- 3j_2(qR_{ch})](1 - \tfrac{1}{2}y^2)\}/[1 + \tfrac{1}{2}\xi^3(1 - \tfrac{1}{2}y^2)]\,. \tag{12}$$

Using the small q^2 expansions of the spherical Bessel functions it is then easy to show that

$$\tilde{r}_\pi^2 = \{[1 + \tfrac{3}{2}\xi^5(1 - \tfrac{1}{2}y^2)]/[1 + \tfrac{1}{2}\xi^3(1 - \tfrac{1}{2}y^2)]\}\, r_\pi^2\,. \tag{13}$$

For $\xi \lesssim 0.75$ (fig. 1a) the variation with ξ is rather mild just as for $\tilde{r}_A$, and indeed $\tilde{r}_\pi$ is essentially equal to r_π. For values of ξ closer to one, on the other hand $\tilde{r}_\pi/r_\pi$ tends towards 1.28. Thus for a given bag radius, in models with pions totally excluded from the bag interior ($R_{ch}/R = \xi = 1$) the πNN vertex function is significantly softer than for the cloudy bag model.

At this stage we recall the infamous discrepancy in the Goldberger–Treiman relation [13]

$$5.97 \pm 0.03 = g_A(0) m_N \neq g_{\pi NN} f_\pi = 6.36 \pm 0.10\,, \tag{14}$$

where we have used the measured value of the NNπ coupling constant [14] ($g^2/4\pi = 14.3 \pm 0.4$) *at the nucleon pole* ($q^2 = -m_\pi^2$). On the other hand one should use $g_{\pi NN}$ at $q^2 = 0$. It is amusing to ask what radius (r_π^{GT}) would give enough variation between $q^2 = -m_\pi^2$ and $q^2 = 0$ to resolve all of this discrepancy. Clearly that would be $(r_\pi^{GT})^2 m_\pi^2/6$ equal to $6.1 \pm 1.7\%$, or

$$r_\pi^{GT} = 0.86 \pm 0.12 \text{ fm}. \tag{15}$$

Using the experimental value for r_A given in eq. (1), and the CBM ratio $r_A/r_\pi = 0.80$, we find $r_\pi = 0.90 \pm 0.15$ fm. (We hope that the use of the ratio of these quantities will reduce the unknown error associated with centre of mass motion.)

Thus, within the rather large experimental errors, the variation of the NNπ form factor with q^2 in the CBM is enough to remove all of the Goldberger–Treiman discrepancy. From the discussion of the modified version of the CBM, with pions excluded from a region $r < R_{ch}$, and in particular using eqs. (8) and (13) (see also fig. 1b), it is easily seen that this agreement is maintained for all values of ξ. For example, with $\xi = \frac{2}{3}$ as suggested by the first calculations of Kogut et al. [10] $\tilde{r}_a/\tilde{r}_\pi = 0.75$ and hence $\tilde{r}_\pi = 0.96 \pm 0.16$ fm. Even in the extreme case where $\xi = 1$ [15], $\tilde{r}_A/\tilde{r}_\pi = 0.65$ and $\tilde{r}_\pi = 1.11 \pm 0.18$ fm, there is no discrepancy within the errors.

Let us summarize briefly. We have calculated the q^2-dependence of the axial-vector form factor of the nucleon, and the NNπ vertex function, using both the original CBM and a recent modification due to Chin and Miller [4] and Vento [5]. These calculations used a static bag, without CM or recoil corrections. All of these approximations should be improved in future work. Nevertheless our central result is very simple, and probably correct independent of these approximations. That is, in both versions of the theory the NNπ form factor is considerably softer than the axial form factor. (The corresponding RMS radius is at least 25% larger.) If one then uses the experimental axial form factor, the q^2 dependence of the resulting NNπ form factor is enough to entirely remove the Goldberger–Treiman discrepancy – within the rather large experimental uncertainty. At this stage we might remark that the NNπ form factor obtained in this fashion is rather soft, and certainly raises interesting questions in the context of the nucleon–nucleon interaction [16]. Clearly it would be very valuable to have more precise data on $g_A(q^2)$, which is of fundamental importance in chiral bag models. Finally we must point out that there may well be additional contributions to the Goldberger–Treiman discrepancy – see, for example, refs. [17–19]. Our point is merely that given the rather large experimental errors at present, these additional corrections are not essential.

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Towards a Description of Nuclear Physics at the Quark Level

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CONTENTS

Lecture 1: From QCD to Chiral Bags

INTRODUCTION

There has been a vast change in our understanding of particle physics over the past 20 years. The classification of almost one hundred "elementary particles" according to the eightfold way was in many ways the first step. It was then natural to propose that the fundamental representation of the symmetry group, SU(3)-flavour, could actually exist (Gell-Mann, 1964; Zweig, 1964). This set into motion a long series of quark searches, in spite of a theoretical problem which such an interpretation posed -- namely the existence of the Δ^{++}. The simple picture of the Δ^{++} as three identical up-quarks, with spin up, in the same spatial orbital, violated the Pauli principle. Thus an extra, apparently arbitrary quantum number called colour was assigned.

From that rather crude state to the present day, where Quantum Chromodynamics (QCD) is regarded almost universally as *the* theory of the strong interactions, there have been a number of astounding experimental discoveries. The deep inelastic scattering experiments at SLAC in the late 60's revealed point-like constituents inside the nucleon through the phenomenon of scaling. (In lecture 3 we shall review the quark-parton description of DIS within the context of the chiral bag models and the nuclear many-body problem.) In addition, the discovery of neutral currents at CERN in 1973 was interpreted as a very strong sign that gauge theories contained some fundamental truth. Further confirmation of both the quark model and the gauge principle has come from the discovery of the J/ψ, the investigation of e^+e^- annihilation into hadrons, and the continued success of the Weinberg-Salam model.

Within the framework of QCD, the apparently arbitrary addition of colour as a property of quarks becomes an essential element. By demanding that the theory should be invariant under arbitrary transformations amongst the colours at each point in space, one arrives at a very powerful dynamical theory. Just as in electromagnetism the gauge principle based on U(1) demands the existence of massless vector photons, in QCD [based on SU(3)] we have eight massless gluons which mediate the interaction [see, for example, Marciano and Pagels (1978) and Abers and Lee (1973)].

Our purpose in these lectures is not to discuss the many attempts made to obtain solutions to the QCD equations. In the long term the most likely method for producing accurate numerical predictions for the structure of the proton based on QCD is to use a discrete space-time lattice. However, the present state of the art involves a lattice of 20 points in time and 10 points in each spatial direction --

see Lipps and co-workers (1983). Since the lattice spacing is of order 0.1-0.15 fm a single baryon is not wholly contained, and the answers appear to depend on the presence of the boundary. The calculation of the magnetic moment of the proton on a smaller lattice required some 9 hours of CDC 7600 time (Martinelli and others, 1982). Moreover, in all of these calculations the effects of virtual q-$\bar{q}$ fluctuations (e.g. pionic corrections) had to be omitted.

Clearly, for the next few years at least, these techniques cannot be applied to the nuclear many-body problem. Nevertheless, as we shall argue later in this lecture, there is good reason to believe that an acceptable microscopic description of nuclear physics should involve the quarks explicitly. The only avenue open therefore is to use a phenomenological model of hadron structure. Of course, whatever the model it should incorporate as much as possible of what we know about QCD.

The first crucial property of QCD is usually referred to as asymptotic freedom. Unlike QED where the renormalized coupling constant $\alpha(Q^2)$ grows as a function of Q^2; in QCD it actually decreases logarithmically. Indeed at the one-loop-level we find

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - {}^2\!/\!{}_3 n_F) \ln (Q^2/\Lambda^2)} , \tag{1.1}$$

with n_F the number of quark flavours and Λ the scale parameter. The proof to all orders relies on renormalization group arguments. This rather weak decrease of the coupling constant at high energy is the ultimate justification of the success of the quark-parton description of DIS.

A second property of QCD which is not yet proven rigorously, but is almost universally accepted is that quarks are confined to colour singlet configurations only. That is if we start with a bound q-$\bar{q}$ pair and try to move them apart the energy of the system rises approximately linearly, and in fact they can only be free of each other at finite energy by making another q-$\bar{q}$ pair in between, and hence two colour singlet q-$\bar{q}$ pairs.

The last property of QCD which will concern us is that at least for the first three flavours, the quark masses seem to be very light (Pagels, 1975; Gasser and Leutwyler, 1982). In fact, with $(m_u + m_d)/2$ less than 10 MeV, which is tiny compared with typical hadronic masses, QCD has an almost exact chiral symmetry. That is to say, that in the limit $m_u = m_d = 0$ there is no interaction which can mix left-handed with right-handed fermions -- hence the notation $SU(2)_L \times SU(2)_R$. In such a theory the Goldstone theorem then says we have two choices. Either for every eigenstate of the theory there is a degenerate partner of opposite parity, or the symmetry of the vacuum is broken and massless pseudoscalar bosons appear. For SU(2) × SU(2) this Goldstone boson is the pion.

In order to finish this introduction let us summarize briefly. We would like to ask what are the consequences for nuclear physics of taking QCD as our starting point. Because of its complexity we cannot actually use QCD, but rather phenomenological models which incorporate its main features, namely

a. confinement,

b. asymptotic freedom, (1.2)

c. chiral symmetry.

In the rest of this lecture we discuss the ambiguities associated with implementing (1.2) in a phenomenological model. We then introduce the essential features of the so-called chiral bag models. In lecture 2 we discuss the properties of one particular model of this type -- the Cloudy Bag Model (CBM). Finally in lecture 3 we return to DIS and show that it can place severe restrictions on the parameters of any chiral bag model (Thomas, 1983a). Furthermore, the modification of the structure function of Fe expected on the basis of the ideas presented here is consistent with the recent observations of the European Muon Collaboration (Aubert and others, 1983).

General Phenomenological Considerations

The earliest and probably the most widely successful QCD motivated phenomenology is the non-relativistic quark model (NRQM) (De Rújula, Georgi, Glashow, 1975; Isgur, 1980). There one deals with a system once removed from QCD in the sense that the u and d masses are of the order of 300 MeV. That is the quarks must be considered as having been dressed by higher order processes. Confinement is incorporated through a phenomenological potential, and the hyperfine q-q interaction arising from one-gluon-exchange is then obtained as an expansion in powers of v/c. By construction the model is not intended to incorporate asymptotic freedom, nor does the notion of chiral symmetry appear naturally.

The MIT bag model was one of several somewhat more ambitious schemes developed in the mid-70's (Chodos and others, 1974a, 1974b). Both confinement and asymptotic freedom were incorporated by letting the quarks (now almost massless and therefore highly relativistic) move freely inside, but only inside, a cavity. That such a model intrinsically destroyed the third of our favourite properties of QCD, namely chiral symmetry, was realized immediately, and some possible extensions were proposed (Chodos and Thorn, 1975; Inoue and Maskawa, 1975). However, the phenomenological consequences of these chiral bag models were not explored for another five years.

In the rest of this lecture we shall summarize the properties of the MIT bag model and show how chiral symmetry is implemented. Since there are strongly felt differences between models, the crucial question we would like to address now is whether QCD offers any more specific guidelines than the global constraints of Eq. (1.2). For example, if the "true" solution resembles a bag, how thick is the surface, where do the Goldstone bosons appear, and so on?

The lattice calculations of Kogut and collaborators (1982) strongly indicate that there is a first order phase transition at the surface of the hadron. For SU(3)-colour this transition occurs in essentially the same place (actually the same temperature) for both the Wilson line and the chiral symmetry breaking parameter $\langle 0|\bar{\psi}\psi|0\rangle$. The rapid variation with temperature near the critical point strongly suggests that the surface region in a phenomenological representation should be rather thin. Of course one must add some cautions in interpreting this calculation. The number of time steps was only 2 and 4, compared with 20 in the work of Lipps and co-workers (1983). In addition fermion loops were neglected, and one cannot draw any conclusions related to the question of where Goldstone bosons may or may not exist.

Shuryak (1983) has presented a rather different picture, in many ways closer to the NRQM. Because of the very large values obtained for the gluon condensate from QCD sum rules, he suggests that there are probably three scales inside hadrons. At very short distances the theory is asymptotically free. As we move further from a quark (few $^1/_{10}$ fm) there are very strong fluctuations and chiral symmetry is broken. Finally at larger distances ($\sim$ 1 fm) this dressed quark (with a constituent mass) is confined. Within this model the mechanism for dynamical symmetry breaking is a residual instanton induced attraction for any q-$\bar{q}$ pair with quantum numbers of a pseudoscalar meson (t'Hooft, 1976). The resulting (almost) Goldstone bosons then dress each of the "constituent" quarks, and certainly exist at distances much less than those at which quark confinement takes over.

Clearly even this very brief summary indicates that QCD still gives us very little guidance in constructing phenomenological models. There is as yet no appeal to any higher authority, save one, to justify a model. The one exception is of course experiment, and as long as any model satisfies the global constraints of Eq. (1.2), and reproduces all the data for which it is suitable, it has as much right to be believed as any other. Since there will inevitably be more models than correct descriptions of Nature, our job must be to find as many experimental tests as possible to eliminate pretenders.

With these preliminary remarks at last complete, let us briefly review the MIT bag model.

THE MIT BAG MODEL

There have been a number of quite thorough reviews of the MIT bag model over the past seven years. The two most recent by De Tar and Donoghue (1983) and Thomas (1983b) contain full references to earlier work on the MIT bag. Needless to say our conventions are identical with Thomas (1983b), which we shall refer to as I. There is little point in reproducing algebra which already appears in I, so here we simply present the key equations.

In its most common form, the static cavity approximation, massless u and d quarks move freely inside a fixed spherical cavity. They are then described by the free Dirac equation in that region

$$\vec{\alpha}\cdot\vec{p}\psi = i\,\frac{\partial\psi}{\partial t}\,, \tag{1.3}$$

which has solutions

$$\psi^{\mu}_{\kappa} = N_{\kappa}\begin{bmatrix} j_{\ell}(Er) \\ -iS_{\kappa}j_{\bar{\ell}}(Er)\vec{\sigma}\cdot\hat{r}\end{bmatrix}\chi^{\mu}_{\kappa}\,e^{-iEt}\,. \tag{1.4}$$

Here $\kappa = \ell$ if $j = \ell - \frac{1}{2}$ and $\kappa = -\ell - 1$ if $j = \ell + \frac{1}{2}$,

$$|\chi^{\mu}_{\kappa}\rangle = \sum_{m\nu} C^{m\ \nu\ \mu}_{\ell\ \frac{1}{2}\ j}\,|\ell m\rangle|\tfrac{1}{2}\nu\rangle\,, \tag{1.5}$$

$\bar{\ell} = \kappa - 1$ if $\kappa > 0$ and $\bar{\ell} = -\kappa$ if $\kappa < 0$, and finally S_{κ} is the sign of κ. For example, for a quark in an s-state

$$\psi^{\mu}_{n,-1}(\vec{r}) = \frac{N_{n,-1}}{\sqrt{4\pi}}\begin{bmatrix} j_0(Er) \\ i\vec{\sigma}\cdot\hat{r}j_1(Er)\end{bmatrix}\chi^{\mu}_{\frac{1}{2}}\,, \tag{1.6}$$

with $\chi^{\mu}_{\frac{1}{2}}$ a Pauli spinor.

Of course at this stage any value of the energy is permitted. The imposition of confinement amounts to saying that for $r > R$ the particle moves in a scalar potential well, and so acts as if it had mass $M \to \infty$. If we write the corresponding solution of the Dirac equation and demand that both its upper and lower components are continuous with (1.4) at $r = R$ we find the following *boundary condition* (see I for details)

$$j_\ell(\omega_{n,\kappa}) = -S_\kappa j_{\bar{\ell}}(\omega_{n,\kappa}) , \tag{1.7}$$

where

$$E_{n,\kappa} = \omega_{n,\kappa}/R . \tag{1.8}$$

The $1s_{1/2}$ and $2s_{1/2}$ levels correspond to $\omega_{1,-1} = 2.04$ and $\omega_{2,-1} = 5.40$, respectively, while the $1p_{1/2}$ level has $\omega_{1,+1} = 3.81$, and so on. The general expression for the $ns_{1/2}$-level normalization constant is

$$N^2_{n,-1} = \frac{\omega^3_{n,-1}}{2R^3(\omega_{n,-1} - 1)\sin^2\omega_{n,-1}} . \tag{1.9}$$

It is interesting that unlike the corresponding situation for the Schrödinger equation it is not the density which vanishes at $r = R$ -- in fact for an s-state:

$$\begin{aligned}\rho(R) &= \bar{q}(R)\gamma^0 q(R)\\ &\propto j_0^2(\omega) + j_1^2(\omega) = 2j_0^2(\omega) .\end{aligned} \tag{1.10}$$

On the other hand, it is easily shown that

$$\hat{r}\cdot\vec{j}(R) = 0 , \tag{1.11}$$

where

$$\vec{j}(\vec{r}) = \bar{q}(\vec{r})\vec{\gamma}q(\vec{r}) , \tag{1.12}$$

because of the boundary condition (1.7). Thus it is not the density but the flow of colour current normal to the surface of the bag which vanishes.

In the general case, where we are not restricted to a static, spherical bag, the condition (1.11) is replaced by

$$n^\mu\bar{q}\gamma_\mu q = 0 \text{ on } S , \tag{1.13}$$

where n^μ is an outward normal to the surface. In fact, in order to guarantee Eq. (1.13) it is sufficient that q satisfy a *linear boundary condition* (see I for details)

$$i\gamma\cdot nq = q \text{ on } S . \tag{1.14}$$

Up to this stage the MIT bag model is equivalent to the confining potential model of Bogolioubov (1967). However, the MIT group realized that this solution was not stable. The reason is simply that as with any gas the relativistic quarks exert a pressure on the surface of the bag and nothing balances this pressure. As a solution the MIT group proposed that because of the different structure of the vacuum inside and outside the bag it should cost energy to make a bag. In its simplest form this energy was taken to be simply $(4\pi/3)R^3B$, with B a phenomenological constant. Formally the pressure balance is then imposed by demanding that the divergence of the energy-momentum tensor of the bag vanish. This means that

$$\partial_\mu T^{\mu\nu} = (B - P_D)n^\nu \delta_s = 0 \;, \tag{1.15}$$

with δ_s a surface δ-function. Here P_D is the Dirac pressure of the quark gas, and more explicitly Eq. (1.15) can be seen to be a *non-linear boundary condition*

$$B = P_D = -\frac{1}{2}\, n\cdot\partial(\bar{q}q) \text{ on } S \;. \tag{1.16}$$

Putting all these ingredients together we find that the mass of a nucleon or delta (degenerate in the absence of spin-dependent interactions) is

$$M(R) = 3\,\frac{\omega_{1,-1}}{R} + \frac{4\pi}{3}\,BR^3 \;. \tag{1.17}$$

It is a simple exercise using Eq. (1.6) to show that Eq. (1.16) is equivalent to the stability condition

$$\frac{\partial M}{\partial R} = 0 \;. \tag{1.18}$$

Thus B and R are not independent parameters. Indeed, the stable solution satisfies

$$R^4 = \frac{3\omega_{1,-1}}{(4\pi B)} \;. \tag{1.19}$$

If we then adjust R to fit the average of N and Δ masses we easily find

$$\begin{aligned} R_N &= 1.4 \text{ fm} \;, \\ B^{1/4} &= 120 \text{ MeV} \;. \end{aligned} \tag{1.20}$$

Having once determined B in this way, it is supposed to be a universal constant, and through Eq. (1.18) it then determines the radii of all other baryons.

Further Corrections to the Model

What we have described so far might be called the bare bones of the MIT bag model. If it truly represents an approximation to QCD one would expect additional terms in the expression for the mass. For example, after cavity independent mass renormalizations have been performed there will remain a finite, R-dependent piece of the self-energy. Similarly there should be an R-dependent energy associated with zero point motion. Finally, there is almost certainly some correction to the mass because of spurious c.m. motion of the three quarks. All of these (and possibly other) contributions are usually included in a phenomenological contribution to the mass, $-Z_0/R$, with Z_0 constant -- typically $Z_0 \varepsilon$ (1.3-1.8). We refer to I, and to Breit (1982) and Baacke and co-workers (1983) for further discussion of the origin of Z_0.

For reasons essentially the same as those presented by De Rújula, Georgi and Glashow (1975), it is assumed in the MIT bag model that the long range confining force resulting from QCD is spin-independent. In order to preserve asymptotic freedom in a simple way, the only interaction allowed between quarks inside the bag is one gluon exchange. The colour electric piece of this interaction was dropped in the original MIT work -- see Breit (1982) for a more recent discussion. However the magnetic piece, which gives rise to an interaction of the form

$$\Delta H_{F-S} = -\frac{\alpha_s}{R}\sum_{i<j}\vec{\sigma}_i\lambda_i^a\cdot\vec{\sigma}_j\lambda_j^a\,\mathcal{M}(m_i,m_j,R)\ , \qquad (1.21)$$

is essential if the model is to produce reasonable spectroscopic predictions.

In Eq. (1.21) the function $\mathcal{M}$ is the result of a radial integration of the bag model wave functions of quarks i and j, whose masses we now allow to be different from zero. (For example, $m_s \neq 0$ is necessary in order to fit the masses of the strange mesons and baryons.) It is weakly dependent on the bag radius R. Using the fact that the observed mesons and baryons are colour singlets, we find

$$\begin{aligned}\lambda_i^a\lambda_j^a &= -8/3,\ \text{baryons}\\ &= -16/3,\ \text{mesons}\end{aligned} \qquad (1.22)$$

and hence

$$\Delta H_{F-S} \propto \lambda\frac{\alpha_s}{R}\sum_{i<j}\vec{\sigma}_i\cdot\vec{\sigma}_j\,\tilde{\mathcal{M}}(m_i,m_j,R)\ , \qquad (1.23)$$

with

$$\begin{aligned}\lambda &= +1\ \text{baryons}\\ &= +2\ \text{mesons}\ .\end{aligned} \qquad (1.24)$$

Clearly this implies much larger spin-dependent forces for mesons than for baryons. In addition, spin triplet states are energetically unfavoured. For the N-Δ system the interaction (1.23) moves the N up and the Δ down by equal amounts of about 150 MeV. Indeed this essentially determines $\alpha_s = 2.2$. For future reference we also mention the ρ-π pair which would be degenerate at 650 MeV without Eq. (1.23). In its presence the ρ is moved up by about 130 MeV -- a 20% correction, which is probably not too large for first order perturbation theory. On the other hand for the spin-zero pion $\langle\vec{\sigma}_i\cdot\vec{\sigma}_j\rangle$ is negative and three times as big, and hence it gets shifted down by about 400 MeV! Clearly one-gluon-exchange is not going to be a very good approximation in this case and one should consider going to higher order -- see, for example, Goldman and Haymaker (1981), Miransky and Fomin (1981) and Brockmann, Weise and Werner (1983). This observation will be important when we come to consider mechanisms for dynamically breaking chiral symmetry.

Summary

For massless quarks the eigenfunctions of the MIT bag model were given in Eq. (1.4), with the quantized energies determined by Eq. (1.7). (It is a simple extension to write down the corresponding solutions for non-zero mass, for example, for the strange quark with $m_s \sim 200$-300 MeV -- see I.) Including the additional terms mentioned above, the total mass of a bag state is

$$M(R) = \frac{\sum_i \omega_i}{R} + \frac{4\pi}{3} R^3 B - Z_0/R + \Delta H_{F-S} \; . \tag{1.25}$$

The radius of the state is determined by the stability condition (1.18).

Now it is a very attractive feature of the MIT bag model that it can be summarized in a QCD-like Lagrangian density

$$\mathcal{L}_{MIT} = \Big[i\bar{q}\not{\partial}q - \bar{q}mq - \frac{1}{4} F^{a,\mu\nu}F^{a}_{\mu\nu} + g\bar{q}A^{a}_{\mu}\gamma^{\mu}\lambda^{a}q - B \Big] \theta_V - \frac{1}{2}\bar{q}q\delta_s \; . \tag{1.26}$$

Here θ_V is one inside the cavity and zero outside, and δ_s is a surface δ-function. The equations of motion are obtained by demanding that the corresponding action should be invariant under arbitrary variations in q, $\bar{q}$, A^a_μ and the size of the cavity.

It is not our intention to discuss Eq. (1.24) at any length in these lectures except as it illustrates that the model violates chiral symmetry. Even that we postpone for a short time while we make a general point.

RELEVANCE TO NUCLEAR PHYSICS

Even with the refinements to the mass formula the radius of the nucleon in the MIT bag model remains at about 1 fm or a little larger. This immediately raises some questions with regard to our usual perception of nuclear physics in terms of point-like nucleons interacting through two-body potentials. Since the average inter-nucleon separation is about 2 fm at nuclear matter density (ρ_0) it is clear that the nucleons must overlap with each other a good fraction of the time. How then is it possible that the independent particle shell model works so well, or indeed that within 1% $m_A = Am_N$?

Baym (1979) has presented this problem another way. He considered first some cubic blocks, some Cu and some wooden, arranged on an infinite cubic lattice. When 31% of the blocks are Cu there must be at least one infinite conducting chain. For spherical balls arranged on regular lattices the critical density is 15 ± 1.5%. While finally for conducting blocks alone, arranged at random, when they occupy 34% of space one is guaranteed an infinite conducting network.

By analogy one would expect that when bags touch a colour current could flow. Thus at a density of

$$\rho_c = 0.34/\left(\frac{4\pi R^3}{3}\right) \text{ nucleons/fm}^3 \; , \tag{1.27}$$

nuclei might begin to show non-trivial collective effects, associated with the linking together of a chain of bags. With R = 1 fm ρ_c is one half of nuclear matter density! Of course if R drops to 0.8 fm, $\rho_c = \rho_0$ and so on. Clearly the bag size must be a critical parameter in such a discussion, and there has been considerable controversy over this issue.

Brown and Rho (1979) originally proposed that the coupling of Goldstone pions to the MIT bag -- next section -- might compress it to perhaps $^1/_{10}$th the volume it would otherwise have ($\sim$ 0.3 fm). In that case $\rho_c >> \rho_0$ and quark physics is essentially irrelevant to nuclear physics -- except through the coupling constants and masses of the heavy bosons which then mediate the N-N force.

In our view the controversy over the nucleon bag size is something that can only be resolved experimentally. In lecture 2 we shall present considerable indirect evidence to suggest that $R_N \in (0.8-1.1)$ fm, while in lecture 3 we show that deep-inelastic scattering puts a lower *limit* on R_N of 0.87 ± 0.10 fm (Thomas, 1983a) -- at least in the Cloudy Bag Model. Thus to anticipate a little, the conclusion of these lectures is that it is *wrong* [not just in principle, as stated by Brodsky (1982), but for very simple physical reasons] to think of nuclei as composed of isolated nucleons exchanging mesons. In fact, the nucleon radius is about half of the average internucleon separation in nuclear matter, and if one may speculate a little, this is probably not an accident. Almost certainly the saturation properties of nuclear matter are intimitely linked to the overlap of finite size quark bags.

Let us now turn to the discussion of chiral symmetry breaking in the bag model where this controversy had its beginnings.

CHIRAL SYMMETRY IN THE BAG MODEL

Let us return to the Lagrangian density (1.26) describing the MIT bag model, and keep those pieces involving only the quark fields.

$$\mathcal{L} = (i\bar{q}\not{\partial}q - B)\theta_V - \frac{1}{2}\bar{q}q\delta_s \ . \tag{1.28}$$

We recall that in general for every transformation which leaves $\mathcal{L}$ invariant there is a conserved current. It is a simple exercise for example to show that (1.28) is invariant under rotations of u into d quarks, for example,

$$\begin{aligned} q &\to q + i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}q \ , \\ \bar{q} &\to \bar{q} - i\bar{q}\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha} \ . \end{aligned} \tag{1.29}$$

Therefore there is a conserved vector current

$$I_a^\mu = \bar{q}\gamma^\mu(\tau_a/2)q\theta_V \ , \tag{1.30}$$

associated with the MIT bag ($\partial_\mu I_a^\mu = 0$).

On the other hand under chiral transformations

$$\begin{aligned} q &\to q + i\,\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\gamma_5 q \ , \\ \bar{q} &\to \bar{q} + i\bar{q}\gamma_5\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2} \ , \end{aligned} \tag{1.31}$$

the piece of Eq. (1.28) involving δ_s is not invariant. Thus the corresponding axial current

$$A_a^\mu = \bar{q}\gamma^\mu\gamma_5(\tau_a/2)q\theta_V \tag{1.32}$$

is not conserved. In fact, a simple calculation shows that

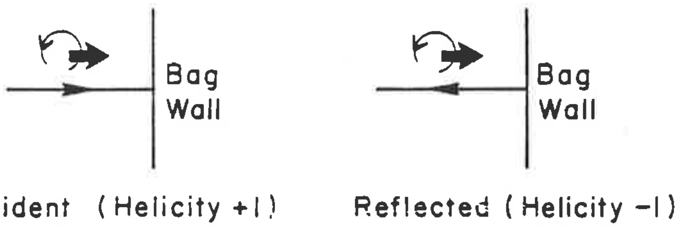


Fig. 1. Illustration of the intrinsic violation of chiral symmetry in the MIT bag model.

$$\partial_\mu A_a^\mu = -\frac{i}{2}\bar{q}\gamma_5\tau_a q\delta_s . \tag{1.33}$$

Thus the MIT bag model, while incorporating two of the three basic properties of QCD [Eq. (1.2)] badly violates the third. Furthermore, as illustrated in Fig. 1, this violation is an intrinsic property of the model. Because quarks are confined they must be reflected at the boundary, and when they are reflected their helicity is flipped. Technically the problem is associated with the $\bar{q}q\delta_s$ piece of Eq. (1.28), which is essential in order to get the linear boundary condition (1.14).

As we mentioned already, this problem was recognized almost immediately by Chodos and Thorn (1975) and Inoue and Maskawa (1975). They proposed a solution along the lines of the classical σ-model of Gell-Mann and Lévy (1960). That is, they introduced four new fields (σ, π_i, $i \in 1, 2, 3$) which had appropriate transformation properties under $SU(2)_L \times SU(2)_R$ so that the new Lagrangian was invariant. The new Lagrangian density replacing Eq. (1.28) is then

$$\begin{aligned}\mathcal{L}' &= (i\bar{q}\partial\!\!\!/ q - B)\theta_V \\ &\quad - \frac{1}{2}\bar{q}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q\delta_s + \frac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 \\ &\qquad + \frac{1}{2}(\partial_\mu\sigma)^2 .\end{aligned} \tag{1.34}$$

The appropriate transformations of σ and $\underset{\sim}{\pi}$ under Eqs. (1.29) and (1.31) may be found in I, together with the expressions for the corresponding conserved vector and axial-vector currents.

Notice that in order to preserve the chiral symmetry the new scalar field σ, and pseudoscalar-isovector field $\underset{\sim}{\pi}$, must be massless. The pion is the obvious candidate as the latter field -- its mass being much less than other hadrons. On the other hand there is no realistic candidate for the σ-field. For this reason it is quite common to eliminate it by a very simple trick. That is, the combination $(\sigma^2 + \underset{\sim}{\pi}^2)$ happens to be invariant under the transformations of $SU(2) \times SU(2)$. By setting $(\sigma^2 + \underset{\sim}{\pi}^2)$ equal to some constant, f^2, we can then eliminate all mention of σ *in a chiral invariant way*. The resulting theory is however necessarily non-linear. Indeed there is not one resulting theory but an infinite number, depending on how σ is eliminated. For example, one could eliminate it by substituting

$$\sigma = \sqrt{f^2 - \underset{\sim}{\pi}^2} , \tag{1.35}$$

in Eq. (1.34). Alternatively, one could introduce a new pion field, $\underset{\sim}{\phi}$, through the definitions

$$\sigma = f \cos (\phi/f)$$
$$\underset{\sim}{\pi} = f\hat{\phi} \sin (\phi/f) \;, \tag{1.36}$$

and so on.

To first order in the pion field all of these interactions lead to a surface coupling of the pion field to the confined quarks of the following form

$$H_{int} = - \frac{i}{2f} \bar{q}\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5 q\delta_s \;. \tag{1.37}$$

However, in higher order, they can be quite different and it is essentially an empirical problem to find which best represents Nature.

Lecture 2: The Cloudy Bag Model

INTRODUCTION

In the last lecture we briefly showed that one could formally restore chiral symmetry to the MIT bag model by coupling an apparently elementary pion field to the surface of the bag. At first sight this seems even further removed from our ideal starting point, QCD. In reality this is not the case. On very general grounds we have already argued that QCD must produce very light pseudoscalar mesons in order to break the symmetry of the vacuum. The effects associated with such bosons can never be obtained in perturbation theory in terms of the original quark fields. Of course, it is truly an idealization to treat the pion as pointlike. This should not be taken to mean that one should expect to see pointlike spin-0 objects in deep-inelastic scattering (DIS) on nucleons. Instead it limits the range of momentum transfer (Thomas, 1983c; Gasser and Lleutwyler, 1982) within which the model can be used without some more sophistication -- see lecture 3 for an example of how to extend the model for DIS.

Unfortunately, once we have agreed to construct a chiral symmetric bag model with quarks and pions, the controversy begins. In order to illustrate the difficulty consider the particular realization following from Eqs. (1.34) and (1.36). [For full details see Thomas (1983b) -- referred to as I.] The full non-linear Lagrangian density is

$$\begin{aligned}\mathcal{L}_{CBM}(x) &= (i\bar{q}\not{\partial}q - B)\theta_V \\ &\quad - \frac{1}{2}\,\bar{q}\,\exp\,(i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f)q\delta_S \\ &\quad + \frac{1}{2}\,(D_\mu\phi)^2\,, \end{aligned} \tag{2.1}$$

where $D_\mu\phi$ is the covariant derivative

$$\begin{aligned} D_\mu\underset{\sim}{\phi} &= \partial_\mu\underset{\sim}{\phi} - [1 - j_0(\phi/f)]\hat{\phi} \times (\partial_\mu\underset{\sim}{\phi} \times \hat{\phi})\,, \\ &\equiv (\partial_\mu\phi)\hat{\phi} + f\,\sin\,(\phi/f)\partial_\mu\hat{\phi}\,, \end{aligned} \tag{2.2}$$

and as usual

$$\phi = |\underset{\sim}{\phi}| \;\; ; \;\; \hat{\phi} = \underset{\sim}{\phi}/\phi \; . \tag{2.3}$$

The conserved axial current associated with this is

$$\underset{\sim}{A}^{\mu} = \bar{q}\gamma^{\mu}\gamma_5\underset{\sim}{\tau}/2q\theta_V - f\hat{\phi}(\partial^{\mu}\phi) - f\,\frac{\sin\,(2\phi/f)}{(2\phi/f)}\,\hat{\phi}\times(\partial^{\mu}\underset{\sim}{\phi}\times\hat{\phi}), \tag{2.4}$$

from which by expanding to lowest order in $\underset{\sim}{\phi}$ we recognize f as the pion decay constant, f = 93 MeV.

Now the two major models, the Little Bag Model (LBM) and the Cloudy Bag Model (CBM) differ in two essential respects. In the former it is *assumed* (mainly with a view to avoiding the difficulties in nuclear physics to which we referred last lecture) that:

> LBM(a) the pion is strictly forbidden from the interior of the bag, so that there is a clear separation of phases; and LBM(b) the non-linear coupling of the pion is essential, and compresses the bag to perhaps $^1/_{10}$th of the volume that it has in the MIT bag model.

On the other hand, in the CBM it is *assumed* that:

> CBM(a) to a first approximation the pion should be allowed to propagate freely inside the bag; and CBM(b) the size of the bag is determined by non-linear effects in QCD unrelated to the pion, whose effects can then be treated as a weak perturbation on the MIT bag model.

Note that whatever may be said later to motivate either choice, both models are truly phenomenological and involve working hypotheses which can only be accepted or discarded on the basis of comparison with experimental data. As we explained in lecture 1, the present state of theoretical QCD cannot eliminate one model or the other.

From a very practical point of view the difference is that in the LBM one keeps all the non-linearity of Eq. (2.1) and multiplies the pion kinetic energy term by $\theta_{\bar{V}}$ (which is zero inside and one outside the bag). In the CBM there is no $\theta_{\bar{V}}$ and we expand systematically in powers of $\underset{\sim}{\phi}/f$. Clearly the latter is far easier to use in practical calculations. Indeed very few LBM calculations have actually retained condition LBM(a), excluding the pion -- see Chin (1982) for a rare example. Furthermore, the full non-linearity has only been kept in unphysical examples like the hedgehog which conserves neither isospin, charge nor angular momentum.

We shall leave the description of the merits of the LBM to its major proponents (Brown and Rho, 1979; Brown, Rho and Vento, 1979, 1980; Brown, 1982; Vento and others, 1980), and concentrate on the CBM from this point.

Motivation

The primary motivation for the working assumption CBM(b) was the success of the MIT bag model itself. Given the fact that this model incorporates two of the three properties (1.2), and reproduces a lot of data, it would be almost indecent if it turned out to be completely wrong. Thus, preferring to believe that Nature is not devious, we have made the minimal extension of the bag model necessary to guarantee chiral symmetry. The pionic effects are treated as quantum fluctuations about the stable MIT bag.

The first assumption, CBM(a) has proven most controversial even though it is a technical matter. If we view the MIT bag as an approximate classical solution corresponding to a saddle point in a path integral formulation of QCD, it is clear that there will be quantum corrections, and that the expansion functions for these fluctuations should be calculated in some effective potential. The LBM assumes that this effective potential gives the pion an infinite mass inside the bag, while the CBM assumes that the effective potential is negligible. In practice the former is very difficult to handle because the expansion functions are complicated. Indeed, in many applications of the LBM the exclusion of the pion from the bag interior has been abandoned in the numerical work.

The first advantage of the CBM is therefore its simplicity of application. One simply expands in plane waves. A somewhat deeper reason for this choice is that it automatically preserves the correct prediction of g_A in the MIT bag model. To see why, consider Eq. (2.4) for the axial current to lowest order in $\underset{\sim}{\phi}$

$$\vec{A}_i^{CBM} = \bar{q}\vec{\gamma}\gamma_5 \frac{\tau_i}{2} q\theta_V - f\vec{\nabla}\phi_i \ . \tag{2.5}$$

If we take the matrix element in a dressed nucleon and integrate over all space, we know that

$$\int d^3x \ \vec{A}_i(\underline{x}) = g_A \frac{\vec{\sigma}\tau_i}{2} \ , \tag{2.6}$$

with g_A the axial charge (1.26 experimentally).

We can, of course, replace the integral of $\vec{\nabla}\phi$ by a surface integral which vanishes. Thus in the CBM g_A is given only by the confined quarks in the MIT bag, and hence $g_A = 1.09$ (or 1.31 with centre-of-mass corrections) -- see Chodos and others (1974b) and Donoghue and Johnson (1980). If on the other hand we exclude the pion from the bag interior,

$$\vec{A}_i^{LBM} = \bar{q}\vec{\gamma}\gamma_5 \frac{\tau_i}{2} q\theta_V - f\vec{\nabla}\phi_i\theta_{\bar{V}} \ , \tag{2.7}$$

then the integral over the pion field reduces to an integral of the pion source over the bag surface, which in turn is related to the axial current of the quarks. Overall the effect is to augment g_A in the LBM by about 50% (Jaffe, 1979). Thus $g_A = 1.63$ in the LBM (1.85 with c.m. corrections), and a major improvement of the bag over the NRQM (which gives 1.66) is lost. The essential point is that there should be no discontinuity in $\underset{\sim}{\phi}$.

More recently a compromise has been proposed by Vento (1983) and by Chin and Miller (1983), wherein the pion is excluded from a smaller cavity [of radius R_{ch}, $\xi = R_{ch}/R \in (0,1)$] inside the bag. In this case one finds

$$g_A = g_A^{MIT} (1 + \xi^3/2) \ , \tag{2.8}$$

and the agreement with experiment is not badly affected provided $\xi \lesssim 0.8$. Once again for this model the numerical calculations are difficult. Indeed, the πNN form-factor was not given correctly until this year [Guichon, Miller and Thomas (1983)]. Moreover, since most physical quantities involve an integral in which the pion field is weighted by a power of r, the numerical differences between the CBM and this hybrid model should be no greater than for g_A -- and hence unimportant for $\xi \lesssim 0.8$.

To summarize, we stress that the motivation for the CBM is intimately connected with the phenomenological success of the MIT bag model. We have made the minimal changes necessary to guarantee essential symmetry properties, and the agreement with a fundamental piece of data -- namely the axial charge of the nucleon.

PIONS IN THE BAG: AN ALTERNATE FORMULATION

Up to this point we have avoided epistomological discussion, preferring to stay close to data in formulating the CBM. However, it must be said that there are theoretical indications that pions should exist inside the bag. First, the fixed bag surface is a mathematical idealization and it is certain that the actual surface would be deformed when hit by a pion, which would therefore penetrate. Secondly, it has been suggested that iterated one-gluon exchange (which is extremely attractive for the pion) could lead to dynamical symmetry breaking -- Goldman and Haymaker (1981) and Miransky and Fomin (1981). Such a mechanism could lead to coherent $q\bar{q}$-pairs with pion quantum numbers inside a nucleon bag. The effects of such pairs could never be obtained in perturbation theory!

Finally we recall Shuryak's (1983) argument that chiral symmetry breaking could occur on a much smaller scale than confinement. The pion would then be produced by dynamical symmetry breaking as a result of instanton effects deep inside the bag. At first sight this picture may seem almost orthogonal to the CBM, but appearances can be deceptive. In fact the CBM with its surface coupling can be transformed to look very much like the picture proposed by Shuryak.

In order to see this we follow the discussion of Thomas (1981) -- see I for more algebraic detail, and Szymacha and Tatur (1981) for a similar model. The idea is to define a new, dressed, quark field q_W as

$$q_W = Sq = \exp\,(i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/2f)q\;. \tag{2.9}$$

Then it is straightforward to show that the surface coupling in Eq. (2.1) becomes simply $-\frac{1}{2}\bar{q}_W q_W\,\delta_S$. On the other hand, the kinetic energy term written in terms of the new fields ($q = S^+ q_W$, $\bar{q} = \bar{q}_W S^+$) is more complicated. After quite a bit of algebra (see I) we obtain

$$i\bar{q}\partial\!\!\!/ q\theta_V = i\bar{q}_W D\!\!\!\!/\, q_W\theta_V + \frac{1}{2f}\,\bar{q}_W\gamma^\mu\gamma_5\underset{\sim}{\tau}q_W\theta_V\cdot D_\mu\underset{\sim}{\phi}\;, \tag{2.10}$$

where $D_\mu\underset{\sim}{\phi}$ is the covariant derivative given in Eq. (2.2), and the corresponding derivative on the new quark fields $D\!\!\!\!/$ is

$$D\!\!\!\!/\, q_W = \partial\!\!\!/ q_W - i\,\frac{[\cos\,(\phi/f) - 1]}{2}\,\underset{\sim}{\tau}\cdot(\hat{\phi}\times\partial\!\!\!/\hat{\phi})q_W\;. \tag{2.11}$$

The new Lagrangian density is therefore

$$\begin{aligned}\mathcal{L}'_{CBM}(x) &= (i\bar{q}_W D\!\!\!\!/\, q_W - B)\theta_V - \frac{1}{2}\,\bar{q}_W q_W\delta_S \\ &\quad + \frac{1}{2}\,(D_\mu\underset{\sim}{\phi})^2 + \frac{1}{2f}\,\bar{q}_W\gamma^\mu\gamma_5\underset{\sim}{\tau}q_W\theta_V\cdot(D_\mu\underset{\sim}{\phi})\;.\end{aligned} \tag{2.12}$$

To order (ϕ/f) this is simply the MIT bag model for the new fields q_w, with pseudo-vector coupling to the pion field throughout the volume of the bag

$$H_{PV} = \frac{1}{2f} \int d^3x \, \underset{\sim}{A}^{\mu}_{quarks}(\underline{x}) \; \partial_\mu \underset{\sim}{\phi}(\underline{x}) \; . \tag{2.13}$$

For a transition in which the quark does not change its radial orbit (e.g. N → Nπ, Δ → Nπ, etc.) *this gives rise to exactly the same form-factor* as the surface coupling Eq. (1.37), namely

$$u(kR) = 3j_1(kR)/kR \; . \tag{2.14}$$

The strength of the coupling as k → 0 is obviously given by the axial charge [the volume integral of $\vec{A}(x)$] divided by 2f. That is,

$$\frac{g}{2m_N} = \frac{g_A^{MIT}}{2f}$$

$$m_N g_A^{MIT} = fg \; , \tag{2.15}$$

which is the Goldberger-Treiman relation.

Notice that in any chiral bag model the πNN form-factor arises because the pion couples to an extended hadron, of size R, which provides a natural high momentum cut-off. Of course the actual form (2.14), with its oscillatory behaviour at large k, arises because of the fixed bag surface. As shown in Fig. 2 it can be very well approximated by a Gaussian (for kR ≲ 3)

$$u(kR) \simeq \exp\,(-0.106\; k^2R^2) \; , \tag{2.16}$$

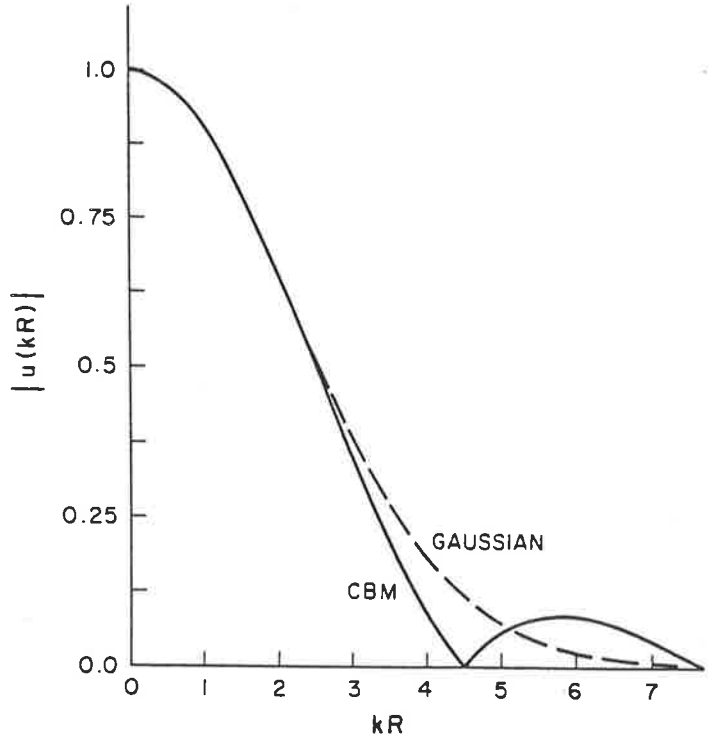


Fig. 2. The NNπ vertex function in the CBM, compared with the "best fit" exponential form-factor given in Eq. (2.) -- from Théberge and Thomas (1983).

which gives the same answers to within 1% for all static properties, and may even be more realistic at large values of the pion momentum. Finally, we must point ou that the form-factors arising from (2.13) and (1.37) differ in the case where the quark orbital changes as a result of pion absorption or emission. There is some indirect evidence [Eisenberg and Kalbermann (1983)] that Eq. (2.13) may be better phenomenologically.

If we go to next order in ϕ/f, the covariant derivative on q_W gives rise to an interaction

$$H_s = \frac{1}{4f^2}\int d^3x\ \bar{q}_W\gamma^\mu \underset{\sim}{I} q_W \theta_V \cdot (\underset{\sim}{\phi} \times \partial_\mu \underset{\sim}{\phi}) \ , \tag{2.17}$$

which can scatter pions from the bag. In particular, for pion-bag scattering at threshold ($\vec{k} = 0$), only $\partial_0\underset{\sim}{\phi} \equiv \underset{\sim}{\pi}$ is non-zero, and the interaction takes the form

$$H_s = \underset{\sim}{I}_t \cdot \underset{\sim}{i}_\pi / 2f^2 \ . \tag{2.18}$$

Here I_t is the isospin of the target. Equation (2.18) is in fact the Weinberg-Tomozawa result for pion-nucleon scattering. It is quite a general result following from current algebra, and the fact that we obtain it is an indirect indication that the CBM satisfies correct commutation relations. What is remarkable, however, is how easily the result is obtained from the new Lagrangian density. It is very difficult to see how to extract it from the original Lagrangian density (2.1).

There has so far been no application of Eq. (2.17) to πN s-wave scattering at higher energies, although this should definitely be tried. The $SU(3)_L \times SU(3)_R$ generalization of Eq. (2.12) has been applied to the coupled $\bar{K}p - \Sigma\pi$ system at threshold -- that is, in the region of the $\Lambda'(1405)$ -- although so far the results are only preliminary [Barrett and co-workers (1983)].

It is important to stress that both q and q_W *cannot* satisfy canonical commutation relations. That is one can choose either Eq. (2.1) or (2.12) as the starting point but not both. Nevertheless to order ϕ/f they are identical in many applications, and presumably one could think of the q_W as in some sense a "quasi-quark". In this regard it is remarkable how similar the new model is to that proposed by Shuryak (1983). In both cases the quarks can be thought of as dressed by higher order effects, and the pion field couples to these quarks throughout the confining volume. While we find Eq. (2.12) more attractive in many ways, it was discovered later than Eq. (2.1), and most of the applications we discuss in the rest of this lecture use the latter. Fortunately the (limited) equivalence at $O(\phi/f)$ means that most of our results would not be altered by changing to Eq. (2.12).

THE NUCLEON

Our approach to calculating the pionic corrections to the MIT bag model of the nucleon was described at length in I. Therefore we shall just summarize the key results and report only recent results in more detail.

Briefly the idea is to expand Eq. (2.1) to $O(\phi/f)$, in which case it breaks into three terms corresponding to the usual MIT bag, a free pion field, and an interaction term. From $\mathcal{L}(x)$ we can construct $T^{00}(x)$ and hence the Hamiltonian, which has the form

$$H = H_{MIT} + H_\pi + H_{int} \ . \tag{2.18}$$

If for the present we work only in the P-space of (non-exotic) three-quark baryon bags -- one can do the same thing for mesons other than the pion -- we find (in the notation of I)

$$H_{int} \to PH_{int}P = (2\pi)^{-3/2} \sum_{\alpha,\beta,i} \int d^3k \, (v_{\underline{k}i}^{\beta\alpha} \beta^+ \alpha a_{\underline{k}i} + \text{h.c.}) \; . \qquad (2.19)$$

Here $\alpha(\alpha^+)$ is a destruction (creation) operator for the non-exotic bag state $|\alpha\rangle$, $a_{\underline{k}i}$ destroys a pion of momentum $\underline{k}$ and isospin i, and the coefficient is given by the matrix element of Eq. (1.37) between bag states $|\alpha\rangle$ and $|\beta\rangle$. For example, using bag model SU(6) wave functions we find

$$v_{\underline{k}i}^{NN} = i(4\pi/2w_k)^{1/2}(f_{NN\pi}^{(0)}/m_\pi)u(k)\vec{\sigma}\cdot\vec{k}\tau_i \; ,$$

$$v_{\underline{k}i}^{\Delta N} = i(4\pi/2w_k)^{1/2}(f_{\Delta N\pi}^{(0)}/m_\pi)u(k)\vec{S}\cdot\vec{k}T_i \; , \qquad (2.20)$$

$$v_{\underline{k}i}^{\Delta\Delta} = i(4\pi/2w_k)^{1/2}(f_{\Delta\Delta\pi}^{(0)}/m_\pi)u(k)\vec{\Sigma}\cdot\vec{k}\mathcal{J}_i \; .$$

Here $\vec{S}$ and $\underline{T}$ are transition spin and isospin operators describing a change from spin $1/2$ to $3/2$. The vertex function u(k) was given in Eq. (2.14) and the bare baryon-pion couplings are in the ratios

$$f_{NN\pi}^{(0)} : f_{\Delta N\pi}^{(0)} : f_{\Delta\Delta\pi}^{(0)} = 1 : \sqrt{72/25} : 4/5 \; . \qquad (2.21)$$

The physical nucleon $|\tilde{N}\rangle$ is then the solution of the equation

$$H|\tilde{N}\rangle = m_N|\tilde{N}\rangle \; , \qquad (2.22)$$

which formally can be expanded in terms of bare bag states (e.g. $|N\rangle$, $|\Delta\rangle$) in the form

$$|\tilde{N}\rangle = Z^{1/2}|N\rangle + c|N\pi\rangle + c'|\Delta\pi\rangle + \ldots \; . \qquad (2.23)$$

Of course such an expansion is only useful if if converges relatively quickly and that is indeed the case for the CBM. If we restrict the bare bag states to N and Δ, it has been shown by Dodd, Alvarez-Estrada and Thomas (1981) that the probability of finding n pions in $|\tilde{N}\rangle$ is bounded by $\Lambda^n/n!$, with $\Lambda = 0.68$ for the MIT bag radius. (At the CBM radius determined from pion nucleon scattering, R = 0.82 fm [Thomas, Théberge and Miller (1981)] we find $\Lambda = 0.9$.) Thus $P_1 \leq 68\%$, $P_2 \leq 23\%$, and P_3 is bounded by 5%. Furthermore, the average number of pions is bounded by Λ. Explicit calculation in the case R = 0.82 yields $P_1 = 35\%$, $P_2 \sim 5\%$, and $P_3 < 1\%$, so it is clear the bound could be improved even more. Nevertheless, the convergence properties of the CBM are remarkable. The pion cloud cover around the nucleon is more like a halo!

The Nucleon Self-Energy

With confidence in the convergence of Eq. (2.23) we can evaluate pionic self-energy corrections to the mass of the nucleon -- see Fig. 3. The first term of Fig. 3 was considered explicitly by Gasser and Leutwyler in their "improved chiral perturbation theory". Certainly we agree very strongly with their idea that highly excited intermediate states should be cut out in order not to double count higher order QCD effects. (The recent storm in a tea-cup over the so-called divergence of the nucleon self-energy seems to us to be misguided. However, it is true that the CBM gave no recipe for cutting off those contributions involving intermediate states with one quark highly excited. It is precisely because of this ambiguity that we placed little emphasis on spectroscopy as a way to determine bag size.) One the other hand, the Δ intermediate state does not involve a change of orbital or angular momentum quantum number and should certainly be calculated at the same level as the N intermediate state.

The total contribution of Fig. 3 is about -200 MeV at R = 1 fm and about -400 MeV at 0.82 fm. (In general it behaves as $\sim R^{-3.5}$ when form-factor and pion mass

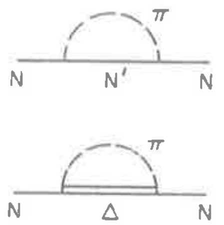


Fig. 3. Pionic self-energy contribution to the mass of the nucleon involving intermediate nucleon and delta bag states [from Théberge and Thomas (1983)].

effects are included.) Most important from the point of view of the comparison with QCD, the corresponding pionic self-energy for the Δ is somewhat smaller. Thus part of the N-Δ splitting which was attributed to gluon-exchange in the MIT bag model actually comes from the pionic or chiral corrections, and one can use a value of $\alpha_c \sim 1.5$ rather than 2.2 -- see I for details. While still very large, at least the change is in the right direction.

The Renormalized NNπ Coupling Constant

Because the Goldberger-Treiman relation (2.15) implies that the bare πNN coupling constant is rather close to the observed renormalized coupling constant, it is important to consider this renormalization in the CBM. This involves computing the wave function renormalization [Z in Eq. (2.23)], and the vertex renormalization shown in Fig. 4. In the Chew-Low model the latter does not compensate for the reduction caused by Z and $(f^{(r)}/f^{(0)})^2$ is about ⅓ [Henley and Thirring (1962)].

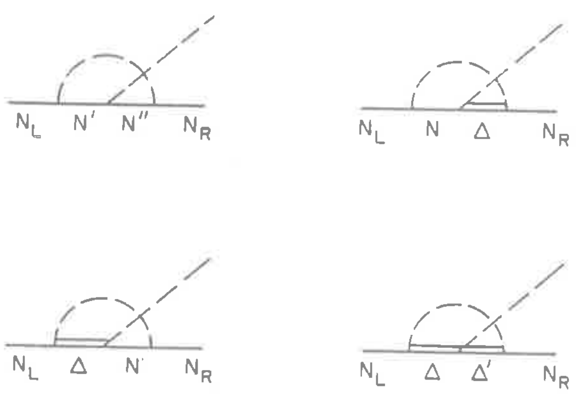


Fig. 4. Vertex renormalization of the NNπ coupling constant in the CBM [from Théberge and Thomas (1983)].

In the CBM, however, the presence of the explicit Δ (in Fig. 4 in particular), combined with the softer form-factor, means that the renormalization is very small for bag radii near 1 fm. To illustrate this we show in Fig. 5 the unrenormalized NNπ coupling constant f_0^2, required to reproduce the observed, renormalized coupling constant for a given bag radius. For R $\gtrsim$ 0.8 fm the bare and renormalized coupling constants are within 10% of each other! Thus conventional strong interactions are amendable to low order perturbation theory!

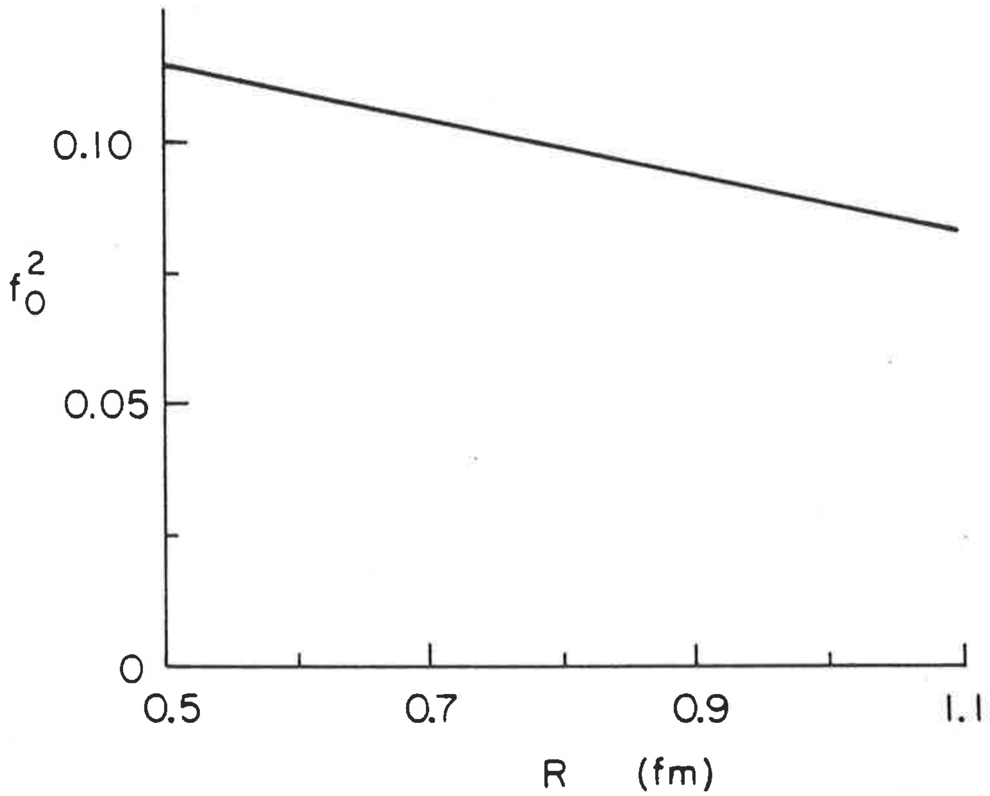


Fig. 5. The unrenormalized NNπ coupling constant needed to reproduce the observed renormalized coupling as a function of the bag radius in the CBM [from Théberge, Miller and Thomas (1982)].

Electromagnetic Properties

Given the rapid convergence of the perturbation expansion (2.23), it is natural to use the model to calculate the electromagnetic properties of the nucleon [DeTar (1981a, 1981b); Thomas, Théberge and Miller (1981); Théberge, Miller and Thomas (1982)]. Here we shall concentrate on the magnetic moment of the nucleon in the CBM. Figure 6 shows the diagrams which must be evaluated.

It is important to notice that the only parameter in these calculations if the bag radius. (That could also be fixed once we have a consistent scheme for cutting-off highly excited intermediate states in the evaluation of the pionic self-energy -- as discussed earlier.) We typically show results for R between 0.7 fm, where first order perturbation theory is not so reliable, up to 1.1 fm.

One final caution concerns the centre-of-mass problem. As shown by Carlson and Chachkhunashvili (1981), this correction is very sensitive to the quark wave functions in the surface of the bag. Even the sign is not agreed upon. Nevertheless, with some slight smoothing of the bag wave functions in the surface, their results were close to those of Donoghue and Johnson (1980). In order to avoid the

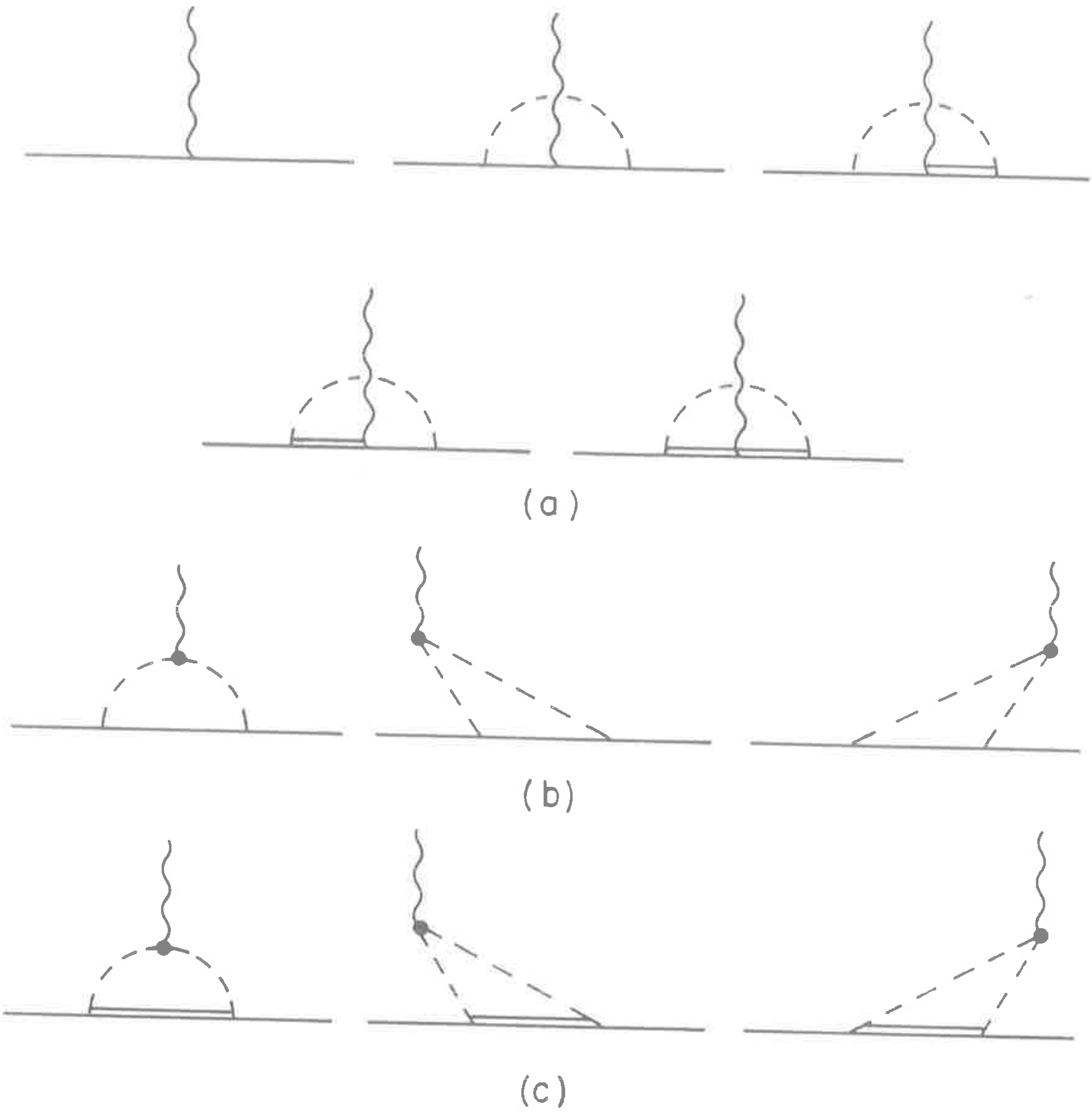


Fig. 6. Contribution to the magnetic moment of the nucleon from (a) the quark current, (b) the pion current with intermediate nucleon present, (c) the pion current with intermediate delta present [from Théberge, Miller and Thomas (1982)].

controversy we show the CBM predictions for μ_p and μ_n both with (solid) and without the Donoghue-Johnson correction. [For a thorough discussion of the more sophisticated question of the cancellation between retardation and spin-precession we refer to the work of Guichon (1983).]

In order to put the results shown in Fig. 7 in perspective, we note that the original MIT bag gave $\mu_p \sim 1.9\ \mu_N$ and $\mu_n \sim -1.26\ \mu_N$ without c.m. corrections (2.24 μ_N and -1.49 μ_N, respectively with the c.m. correction). Expressed as a ratio to the experimental values (as in Fig. 7) this would be (0.68, 0.66) [and (0.80, 0.78) with c.m. corrections]. Furthermore, there are corrections due to the creation of virtual q-$\bar{q}$ pairs in the bag whose size is generally found to be of order $(0.2\text{-}0.3)\mu_N$, but which are not yet generally agreed upon -- e.g. see Maxwell and Vento (1983) and Donoghue and Golowich (1977). Thus at the present stage of the theory, agreement with data at the 10% level should be considered quite good.

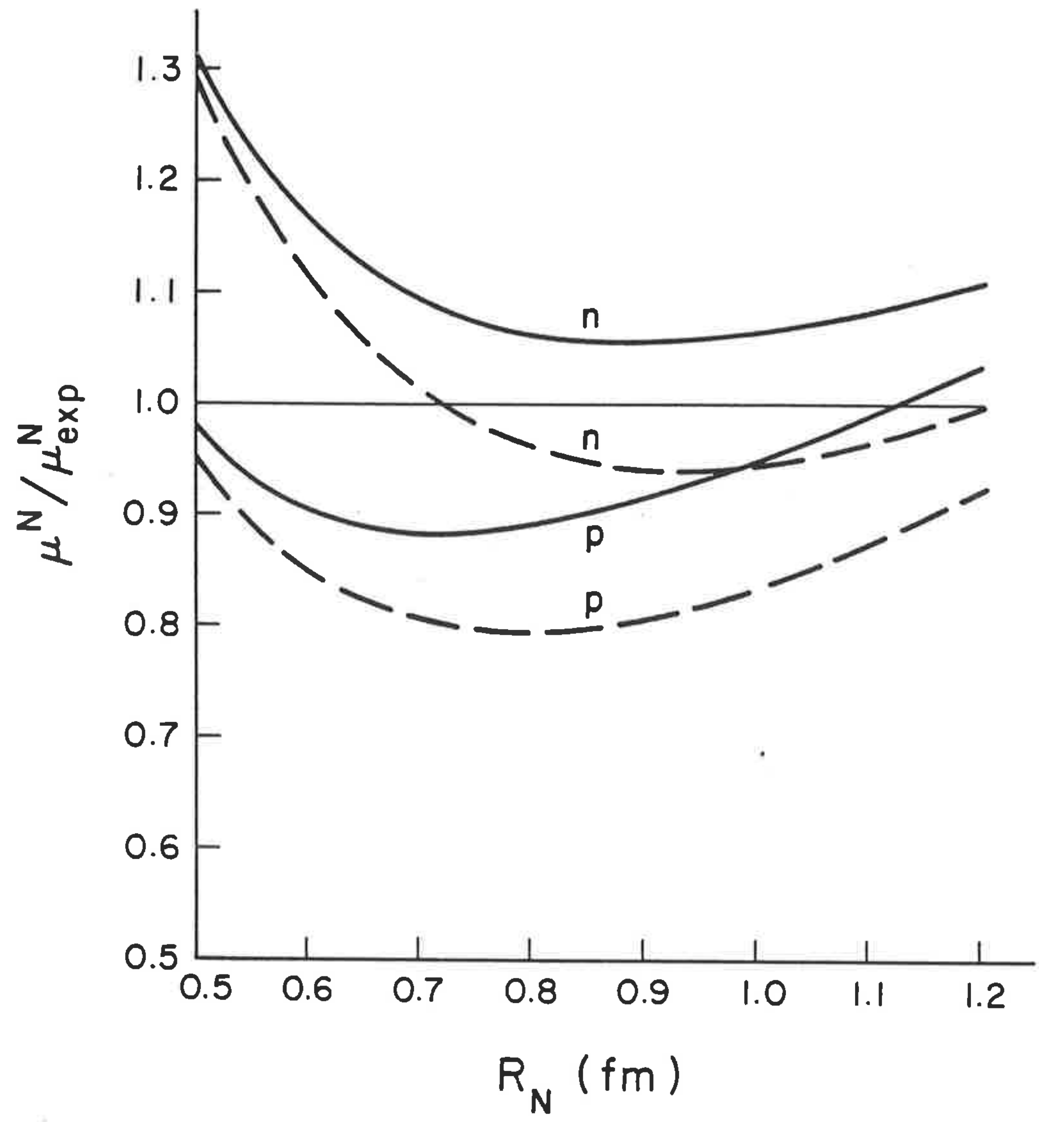


Fig. 7. Predictions of the CBM for the magnetic moments of the proton and neutron, as a function of the bag radius, with (solid) and without (dash) centre-of-mass corrections [Théberge and Thomas (1983)].

Clearly the results shown in Fig. 7 are a *significant quantitative improvement* over the original bag model. The best agreement with data is undoubtedly found when the c.m. correction is included -- although this is no argument in favour of including it. It may be instead that there is other physics at the 10% level -- for example, a small residual rho-meson coupling.

It is remarkable that the results for both μ_p and μ_n in the CBM are quite insensitive to variations in R, the bag radius. As R decreases the pionic corrections increase to compensate the reduction in the bare bag contribution (both Z and μ^{MIT} decrease as R decreases). This illustrates a point which we have made often, but which has not been widely appreciated. That is, the pionic corrections are *not* purely isovector because of the coupling to the extended core while the pion is in the air! One must calculate all of the terms shown in Fig. 6 in order to obtain a reliable result.

The Axial Form-Factor

As we explained in detail at the beginning of this lecture, in the CBM only the quarks contribute to the axial (as opposed to the induced pseudoscalar) current of the bag. Thus, unlike the electromagnetic properties for which there are pionic contributions, the axial form-factor is a direct measure of the quark distribution in the nucleon. At present the data on $g_A(q^2)$ comes from two sources, the reaction $\nu_\mu + n \to \mu^- + p$ and pion electroproduction -- see Amaldi, Fubini and Furlan (1979). It is usually represented as a dipole

$$g_A(q^2) = (1 + q^2/m_A^2)^{-2} , \tag{2.24}$$

with $m_A = 0.95 \pm 0.14$ GeV.

If we calculate $g_A(q^2)$ for the CBM we find this corresponds to a bag radius $R = 1.16 \pm 0.20$ fm [Guichon, Miller and Thomas (1983)]. Clearly there should be corrections to this value arising from c.m. and recoil effects, but as a first estimate this strongly suggests a bag size similar to that expected in the original MIT bag model.

One can also investigate $g_A(q^2)$ in the hybrid model discussed earlier, where the pion is excluded from a region $r < R_{ch}$ inside the bag [i.e. $\xi = R_{ch}/R \in (0,1)$]. For $\xi \neq 0$ the pion also contributes to $g_A(q^2)$. However, as shown by Guichon, Miller and Thomas (1983) the slope of $g_A(q^2)$ changes by less than 10% over the whole range of values of ξ. Thus the result $R = 1.16 \pm 0.20$ fm is a general result for all chiral bag models.

Finally we note that we can also calculate the πNN form-factor in the hybrid model. If we parametrize $g_A(q^2)$ as $[1 - q^2 r_A^2/6]$, and $g_{\pi NN}(q^2)$ as $[1 - q^2 r_\pi^2/6]$, then for all values of ξ $r_\pi > r_A$. That is, the πNN form-factor in all chiral bag models is softer than $g_A(q^2)$. In the CBM, where $\xi = 0$, $g_{\pi NN}(q^2)$ would correspond to a dipole of mass 0.90 ± 0.14 GeV ($r_A/r_\pi \sim 0.9$), which is very soft. For ξ in the range 0.0 to 0.8 this hardly changes, but in the range 0.8 to 1.0 $g_{\pi NN}(q^2)$ becomes rapidly softer, with r_A/r_π dropping to 0.65 and the corresponding dipole mass to about 0.76 GeV!

Clearly it would be very valuable to have more precise data for $g_A(q^2)$. Nevertheless, even at the present accuracy, we regard the arguments which we have just reviewed as the most direct indication (apart from the discussion of DIS in lecture 3) that the nucleon bag is of the order of 1 fm in radius.

MAGNETIC MOMENTS OF THE NUCLEON OCTET

As a further test of the CBM one can also calculate the pion couplings to the hyperons (Λ, Σ, Σ^*, etc.). Then we can calculate the properties of the hyperons using exactly the same perturbative expansion as for the nucleon, e.g.

$$|\tilde{\Lambda}\rangle = \sqrt{Z_\Lambda}|\Lambda\rangle + c|\Sigma\pi\rangle + c'|\Sigma^*\pi\rangle . \tag{2.25}$$

Just as in the case of the nucleon, we include the coupling to those baryons where no orbital or angular momentum excitation is involved. For example, for the Σ we include $\pi\Lambda$, $\pi\Sigma$ and $\pi\Sigma^*$ as possible coupled channels.

The corresponding probabilities of finding a bare three-quark bag in the physical baryon are shown in Fig. 8 as a function of the bag radius. Even for bag radii

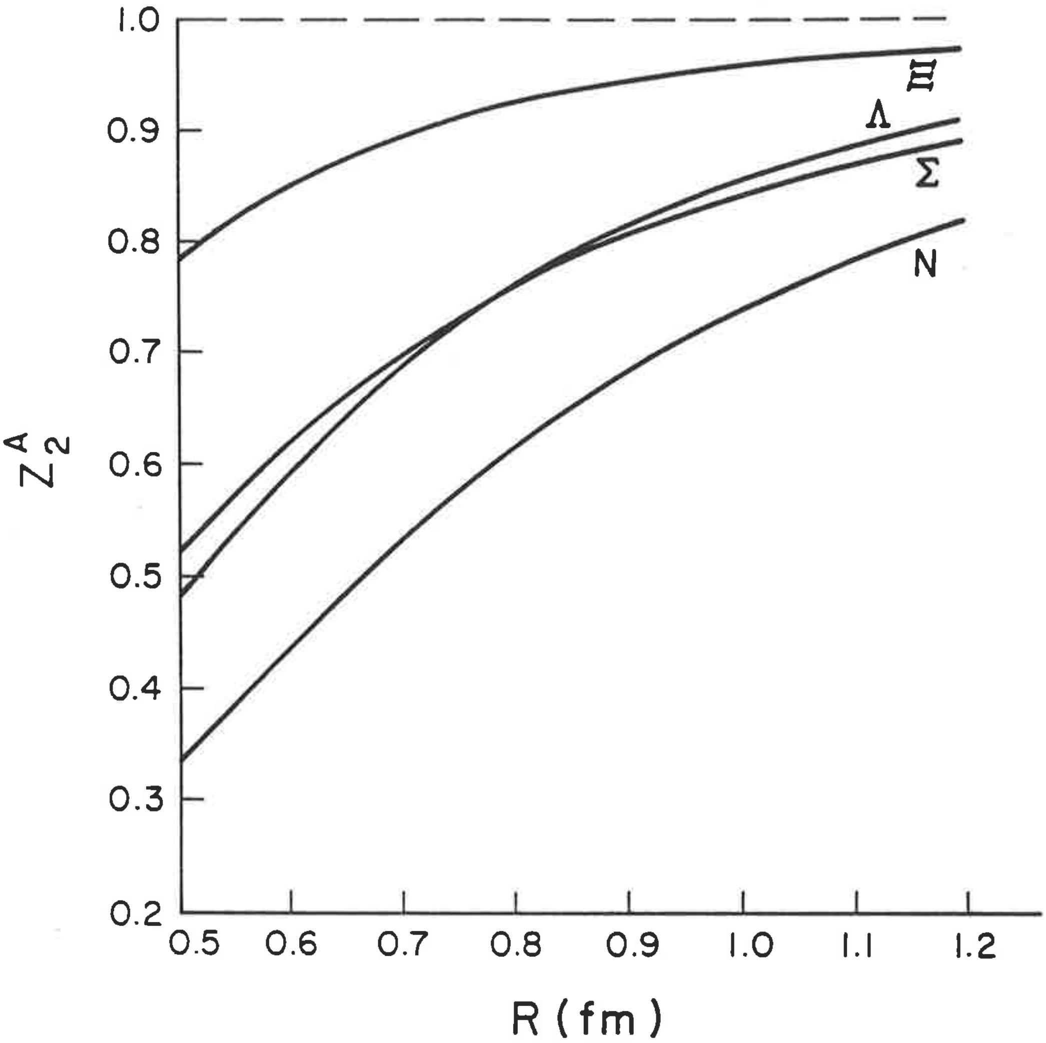


Fig. 8. Probability of finding a bare bag in the dressed baryon, as a function of bag radius [Théberge and Thomas (1983)].

near 1 fm there is a sizeable pionic component in the physical Λ and Σ. It will be very interesting to see how this modifies the description of hyperon decays in the bag model.

As a further indication of the size of pionic corrections, in Fig. 9 we show the ratio of the "bare bag mass m_{0A}", to the observed baryon mass m_A, as a function of the radius of the bag. Our operational definition of m_{0A} is the physical mass minus the pionic self-energy (Fig. 2). Even at R = 1 fm the pionic self-energy of the Λ is of the order of -100 MeV. We shall mention this a little later in connection with the existence of possible exotic (six-quark) bound states.

Finally in Fig. 10 we show the results obtained in the CBM [Théberge and Thomas (1982, 1983)] for the magnetic moments of the Σ^+ and Σ^- hyperons. This calculation involved no free parameters because once the strange quark mass is specified (we used 144 MeV), all the $\pi YY'$ and $\gamma YY'$ couplings are uniquely determined. (There are however a lot of graphs -- see Fig. 6, and insert all possible Y or Y^* intermediate states.) The experimental values were taken to be $\mu(\Sigma^+) = 2.33 \pm 0.13\ \mu_N$ and $\mu(\Sigma^-) = -0.89 \pm 0.14$. More recently the Σ^- measurement using exotic atoms has become much more accurate [Roberts and Welsh (1983)], and the best value is now $\mu(\Sigma^-) = -1.10 \pm 0.03\ \mu_N$. With this improved value, both the Σ^+ and Σ^- magnetic

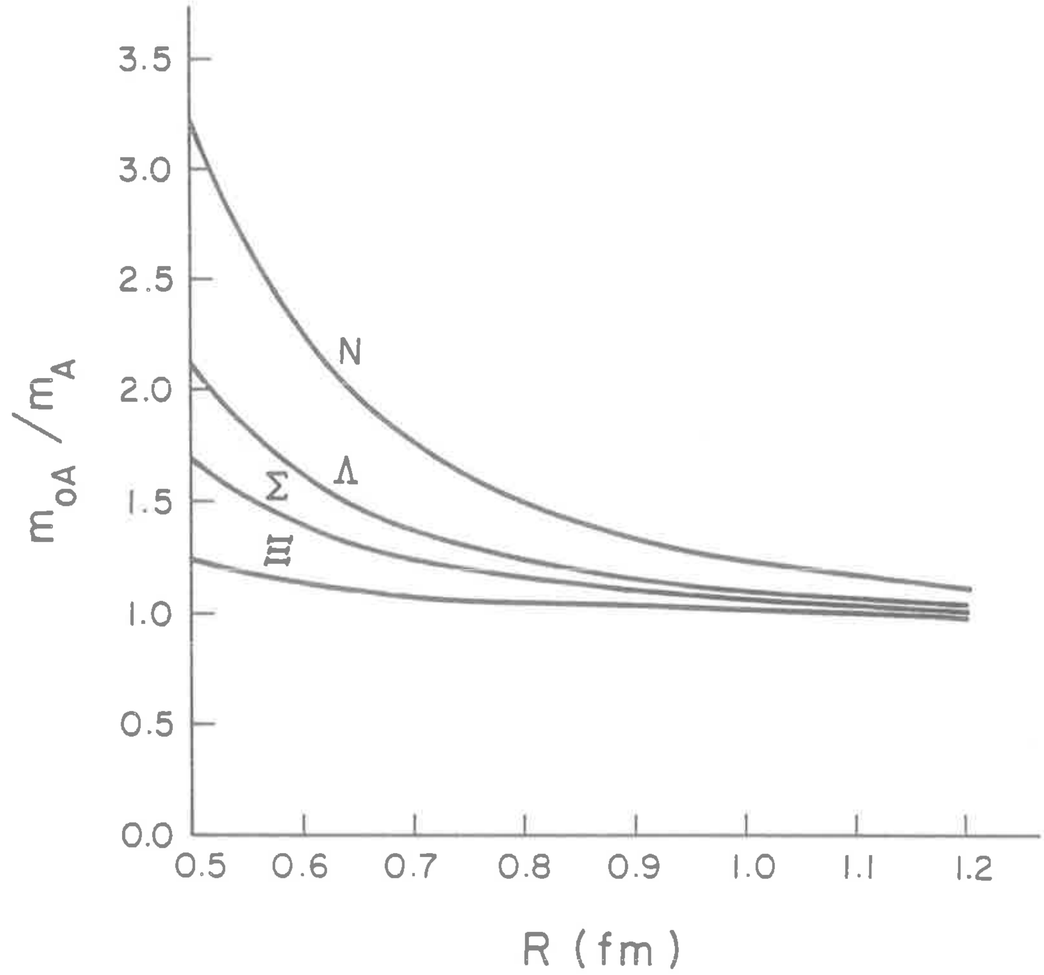


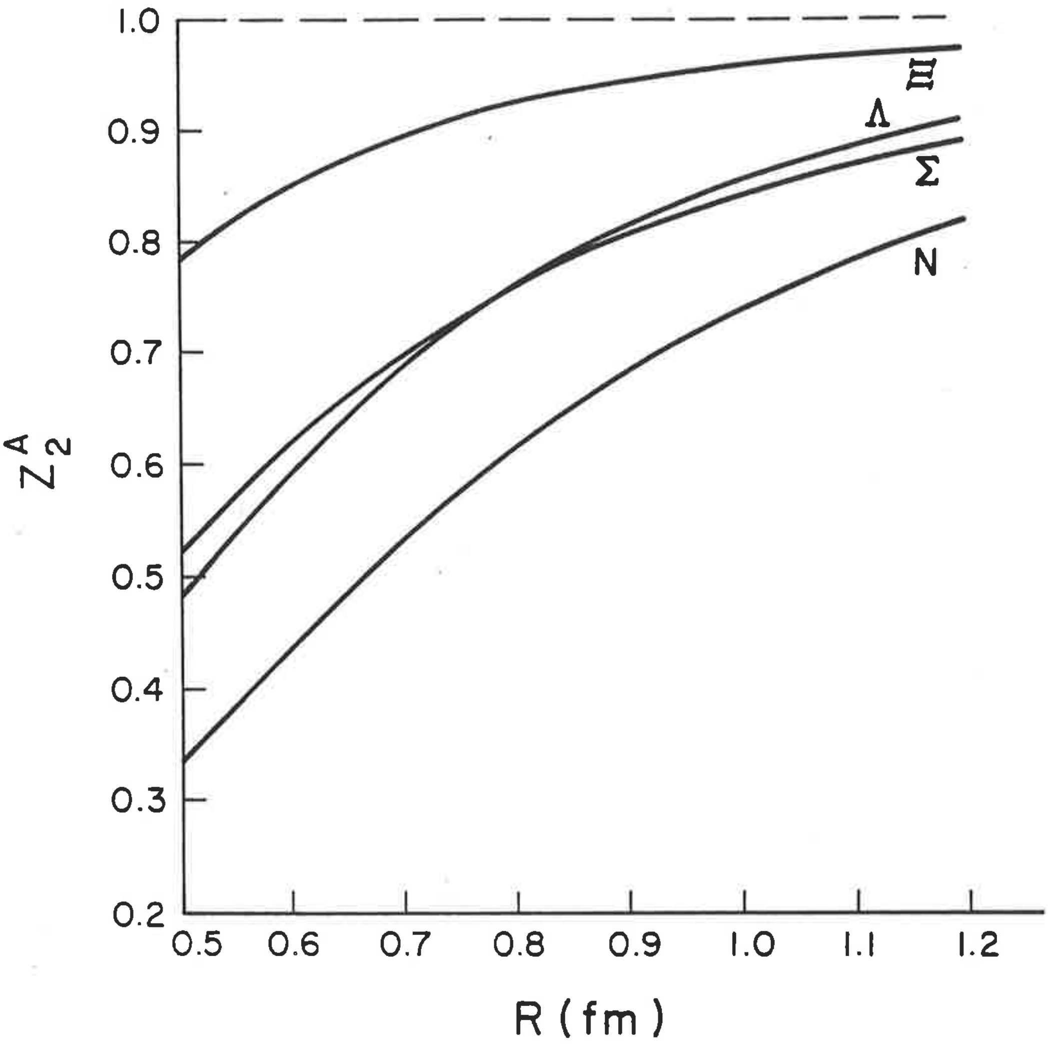
Fig. 9 Ratio of the bare mass to the observed mass of the nucleon octet. The bare mass is defined to be m_A minus the second order pionic self-energy [from Théberge and Thomas (1983)].

moments calculated in the CBM are in excellent agreement with the data. (Again the solid curve includes the Donoghue-Johnson c.m. correction, which is not as important here as for the proton.)

It is worth while to stress that the charged-Σ magnetic moments provide an important test of pionic corrections. Because of the near degeneracy of the Σ^0 and Λ, the pion contribution is twice as big as it might otherwise be (i.e. $\Sigma^- \to \Lambda\pi^-$ and $\Sigma^- \to \Sigma^0\pi^-$). This was first pointed out by Eeg and Pilkuhn (1978), but they used very hard form-factors and hence obtained a value of $\mu(\Sigma^-) = -1.66\ \mu_N$ which is far too large. In the CBM with R = 1 fm, m_s = 279 MeV (as in the original MIT work) we find $\mu(\Sigma^-) = -1.08\ \mu_N$ [Théberge and Thomas (1982)] whereas the MIT bag model gave $\mu(\Sigma^-) = -0.80\ \mu_N$ (including the c.m. correction).

PION-NUCLEON SCATTERING

The first major achievement of the CBM was to provide a unified framework within which to understand the P_{33} resonance. Before the analysis of Théberge, Thomas and Miller (1980), we had either the Chew-Wick description entirely in terms of the πNN coupling, or the quark model within which the three-quark Δ decayed to $N\pi$. In the CBM both $NN\pi$ and $\Delta N\pi$ couplings appear from the same Hamiltonian, with the same form-factor and related coupling constants. Without the pion-bag coupling all



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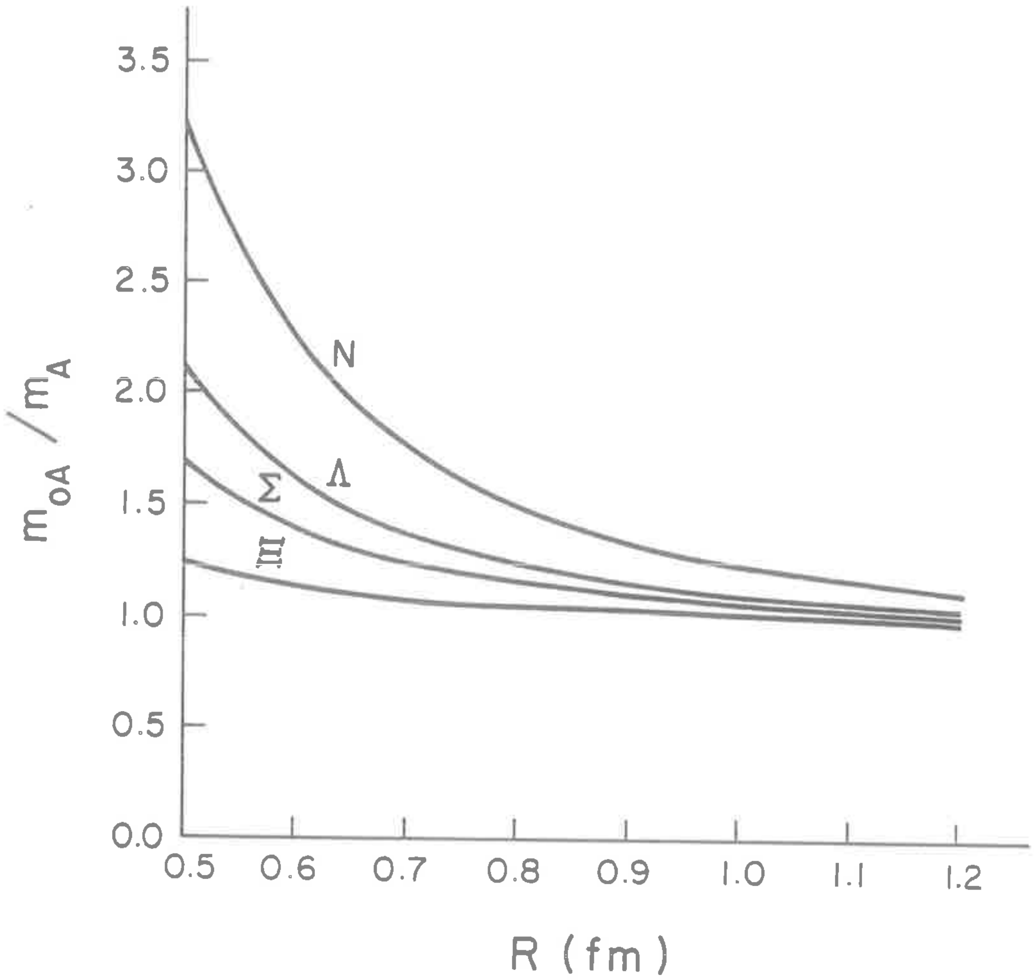

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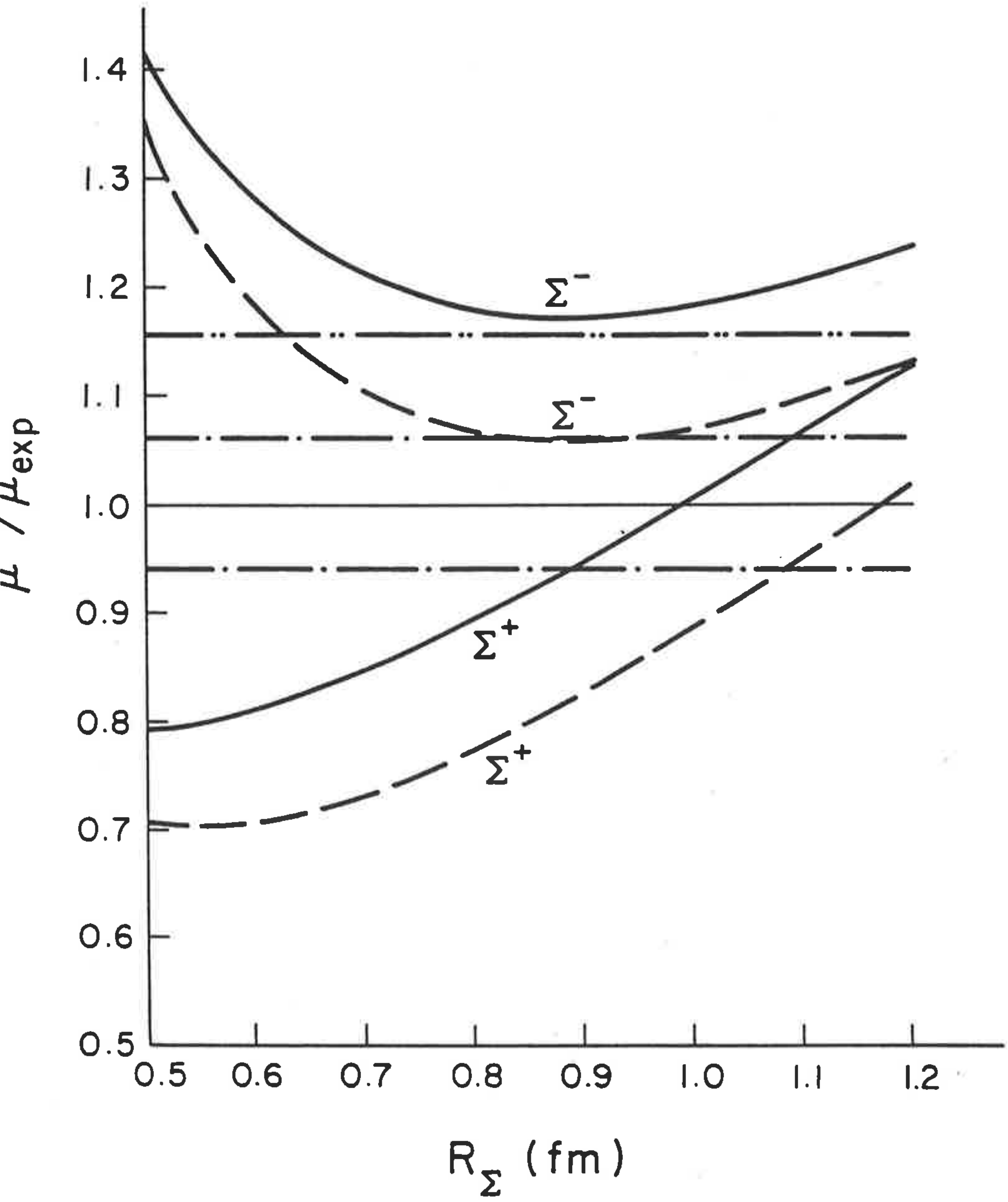

Fig. 10. Ratio of calculated to experimental magnetic moment as a function of bag radius. The Σ^- magnetic moment data has since been improved -- see the text [Théberge and Thomas (1983)].

of the baryons N, Δ, R (Roper) and so on would be stable. However, once we impose chiral symmetry, and hence the coupling of pions, only the nucleon and the other members of the nucleon octet remain stable. All of the others become unstable and cannot be treated with any accuracy without explicit consideration of the channels to which they can decay. Thus the CBM treatment of the Δ is a prototype for understanding all of the unstable resonances within a bag model framework. Similar analyses have been made for the Roper resonance (Rinat, 1982; Nogami and Ohtsuka, 1982; Brown, Durso and Johnson, 1983).

Since the CBM of the P_{33} resonance is now quite ancient and was reviewed extensively in I we shall not repeat ourselves.

Perhaps the only point which should be emphasized is that contrary to claims in the literature the ratio $f_{\Delta N\pi}:f_{NN\pi} = (72/25)^{1/2}$ does explain the observed width of the delta provided one includes *all* the relevant diagrams [Théberge, Thomas and Miller (1980); Thomas, Théberge and Miller (1981); Niskanen (1981)]. This is of some relevance to other lecturers at this school when fixing their parameters for the calculation of (e.g.) the suppression of Gamow-Teller strength.

EXOTICS

One of the more exciting possibilities raised by the MIT bag model was that there might be stable, exotic states. For example, it was suggested that the so-called H-dibaryon (a Λ-Λ state) might be bound by (50-80) MeV -- Jaffe (1977). In view of the relatively large self-energy corrections associated with pions for single hadrons (see Fig. 9), it is reasonable to ask how those corrections affect the masses of exotic states.

In order to check this in a scheme consistent with the philosophy of the CBM, Mulders and Thomas (1982) refitted the usual hadron spectrum with the phenomenological form

$$E(R) = E_Q + E_V + E_M + E_p \ . \tag{2.26}$$

Here E_Q, E_V, and E_M are, respectively, the standard kinematic energy, volume and colour magnetic contributions to the bag energy. The last term, E_p, is a phenomenological representation of the pion self-energy which has the form

$$E_p = \frac{-1}{pR^3} \sum_{i,j} (\vec{\sigma}\underset{\sim}{\tau})_i \cdot (\vec{\sigma}\underset{\sim}{\tau})_j \ . \tag{2.27}$$

The spin-isospin structure corresponds to keeping only the lowest orbital in the intermediate state, and treating all such states as degenerate. Finally, p is a phenomenological constant.

There were several notable features associated with the best fit parameters. As expected from the earlier discussion of the N-Δ splitting the colour coupling constant was reduced by about 35%. The strange quark mass also came down to 218 MeV (from 279 MeV) -- a little closer to the usual current algebra value of 150 MeV. Lastly, we observe that, although treated as an adjustable parameter, the value of p agreed very well with that calculated for a nucleon in the chiral bag models.

For the non-strange, B = 2, exotic bag states, the pionic corrections had little effect. In ${}^3S_0$ and ${}^1S_0$ the bag masses were 2.18 and 2.24 GeV, respectively (c.f. 2.16 and 2.23 in the original MIT bag model). Since these lie well above the appropriate thresholds they will be quite broad, and should not have dramatic experimental consequences.

On the other hand, for the doubly strange H-dibaryon the change is dramatic. The combination of decreased colour attraction (smaller α_c), and the R^{-3} dependence of the pionic self-energy result in a larger mass for the H -- 2.22 instead of 2.15 GeV. Thus the H is almost certainly unbound, and it is no longer a mystery why recent searches have failed to reveal it.

CONCLUSION

In this lecture we first reviewed the similarities and the essential differences of various chiral bag models. We then showed how the CBM could be transformed to look very much like the picture of hadron structure proposed by Shuryak. Within that framework we saw that the model incorporated both the Weinberg-Tomozawa result for low-energy pion-nucleon scattering, and the Goldberger-Treiman relation for the bare πNN coupling constant. Next we examined the structure of the nucleon in the CBM, showing that there are in fact very few pions in the cloud, that the renormalization of the πNN coupling constant is small, and that both its magnetic moments and axial form-factor are well described for a bag radius in the range 0.8 to 1.1 fm. An extension to the properties of the stable hyperons was also very successful.

We briefly reviewed the application of the CBM to pion-nucleon scattering in the region of the $\Delta(1231)$. Finally we discussed recent results for multiquark bag states. Because it would go beyond the scope of these lectures, we did not discuss one of the more exciting applications of the CBM, namely an estimate of the effect on the lifetime of the proton in the simplest SU(5) Grand Unified Theory of imposing chiral symmetry (Thomas and McKellar, 1983). In addition, we avoided any discussion of the intermediate range N-N force in the quark model, since that was discussed by Faessler at this school -- see also I. We will however raise this matter in the next lecture in connection with the EMC effect.

Lecture 3: Deep Inelastic Scattering

INTRODUCTION

In a certain sense in this lecture we come full circle. Early in lecture 1 we referred to the fact that DIS provided the first hard evidence that there were point-like constituents, with the properties expected of quarks, inside hadrons. We then showed how phenomenological quark models suggest that we might have to take a new view of the microscopic structure of the nucleus. Indeed, if the confinement radius for the valence quarks in a nucleon is of order 1 fm, rather than the classical model of point-like nucleons interacting through potentials generated by heavy meson exchange, one would expect to find two (or more) nucleons overlapping, and therefore sharing their valence quarks a large fraction of the time. [Many authors have suggested ideas like this in the past few years, as examples and sources of further references we cite Bergström and Fredriksson (1980), Pirner and Vary (1981), Miller (1983) and Thomas (1983b).]

If such ideas make any sense one might expect to be able to test them experimentally using DIS. Therefore in this lecture we shall briefly review the ideas of scaling in DIS. This is already well treated in text books (Feynman, 1972; Close, 1979), and we shall merely summarize the main ideas. We then apply these ideas to the nucleon itself, to see whether DIS can test or constrain the chiral bag models. In fact this could be regarded as something of a challenge in view of statements like the following (Vento, 1983). "The other version of the model includes in addition ... a pion like degree of freedom in the interior of the bag ... from an experimental point of view such a model is in clear contradiction with the fact that short distance probes do not see any pseudoscalar constituent objects. Due to these inconsistencies the latter model would not be of any relevance if it were not because it seems to reproduce a fair amount of data." We shall show that while DIS gives not one jot of evidence about whether pions exist inside or outside of bags, it does put severe constraints on the form-factor at the $NN\pi$ vertex. Within the CBM it places a *lower limit* on the radius of the nucleon bag of 0.87 ± 0.10 fm (Thomas, 1983a).

Thus we almost certainly live in a universe where nucleons overlap frequently in nuclei. A smart theorist might have even expected the distribution of valence quarks in a nucleus to be modified from that in a free nucleon. Unfortunately in this case the experimenters were smarter (or luckier). The European Muon Collaboration has recently published fairly convincing evidence for just such a modification. As the finale to this set of lectures we discuss the EMC effect and speculate on how we might interpret it.

SCALING

Consider the scattering of an electron or muon of initial energy E in the laboratory to a final energy E′ and scattering angle θ. According to convention we define the energy transfer (E - E′) to be ν, and $Q^2 = -q^2$, where q is the four-momentum transfer to the target ($Q^2 > 0$ for space-like momentum transfer). If the lepton scatters elastically from a target of initial momentum p [$p = (m_t,\vec{0})$ in the laboratory], then

$$\begin{aligned} m_t^2 &= (p + q)^2 \\ &= m_t^2 + 2m_t\nu - Q^2 \end{aligned}$$

and hence

$$Q^2 = 2m_t\nu \; . \tag{3.1}$$

This is the crucial kinematic relation behind all scaling arguments, because if a target is elementary, elastic scattering is the only channel available in such a collision. It then follows that Q^2 and ν must be related by Eq. (3.1) in scattering from an elementary target.

In an electromagnetic interaction of an electron or muon, the most general form for the cross-section is

$$\sigma \sim \frac{\alpha^2}{Q^4} L_{\mu\nu}W^{\mu\nu} \; , \tag{3.2}$$

where $L_{\mu\nu}$ describes the lepton photon vertex and $W^{\mu\nu}$ the photon target vertex. (The factor α^2 then just tells us the charge and Q^{-4} comes from the photon propagator.) The lepton tensor is simply

$$\begin{aligned} L_{\mu\nu} &= \frac{1}{2}\,\mathrm{Tr}[(\not{k}' + m)\gamma_\mu(\not{k} + m)\gamma_\nu] \\ &= 2[k'_\mu k_\nu + k_\mu k'_\nu - g_{\mu\nu}(k\cdot k' - m^2)] \; , \end{aligned} \tag{3.3}$$

where k and k′ are the initial and final lepton four-momentum (q = k - k′). It is a purely kinematic factor.

On the other hand, for a general hadronic target $W^{\mu\nu}$ is defined as

$$\begin{aligned} W^{\mu\nu} = \frac{1}{2}\sum_n \; &\langle p|J^{\mu+}|n\rangle\delta^{(4)}(p + q - p_n) \\ &\times \langle n|J^\nu|p\rangle \; , \end{aligned} \tag{3.4}$$

where the sum over n includes all possible hadronic final states. Using just the principles of gauge invariance and parity conservation one can show that the most general form allowed for $W^{\mu\nu}$ is

$$W^{\mu\nu} = W_1(\nu,Q^2)\left[q^\mu q^\nu/q^2 - g^{\mu\nu}\right] + \frac{W_2(\nu,Q^2)}{m_t^2}\left[(p^\mu - p\cdot q q^\mu/q^2)(p^\nu - p\cdot q q^\nu/q^2)\right] . \quad (3.5)$$

Using Eqs. (3.2), (3.3) and (3.4) one can show that in general

$$\frac{d^2\sigma}{dE'd\theta} = \frac{4E'^2\alpha^2}{Q^4}\left[\sin^2\theta/2 \; 2W_1 + \cos^2\theta/2 \; W_2\right] . \quad (3.6)$$

The arbitrary functions W_1 and W_2, which have dimensions of $(\text{energy})^{-1}$ contain all the information we can obtain about the structure of the target from such a reaction. They are called the structure functions.

It is actually more convenient for many purposes to consider the dimensionless structure functions

$$F_1 = m_t W_1 ,$$
$$F_2 = \nu W_2 . \quad (3.7)$$

As a simple example we take the case of electron scattering from a structureless target of mass m_t. Then from the arguments presented at the beginning of this section we know that F_1 and F_2 cannot depend on Q^2 and ν independently. In fact, reference to your favourite text-book on Quantum Electrodynamics reveals that

$$2F_1 = \delta(1 - Q^2/2m_t\nu) ,$$
$$F_2 = \delta(1 - Q^2/2m_t\nu) . \quad (3.8)$$

That is, Q^2 and ν may vary independently, but F_1 and F_2 are unaltered if the dimensionless combination x,

$$x = Q^2/2m_t\nu \quad (3.9)$$

is the same. This phenomenon is known as scaling.

Although it is not often stressed, scaling is a fairly common phenomenon. For a very beautiful discussion we refer to the text of Close (1979). Here we content ourselves with a single example, namely electron proton elastic scattering. Once again by referring to standard texts one can find the result

$$2F_1(\nu,Q^2) = G_M(Q^2)\delta(1 - Q^2/2m_p\nu) , \quad (3.10)$$

where m_p is the proton mass and $G_M(Q^2)$ its magnetic form-factor. This has the general structure $F_1 = F(Q^2)f(x)$. Clearly if one is in a range of Q^2 where $F(Q^2)$ is essentially constant, we will see scaling. In the proton example this would correspond to $1/Q >>$ proton radius (i.e. $Q^2 \lesssim 0.01$ GeV^2).

In order to see how DIS can reveal the substructure of a target, let us consider electron scattering from ^{56}Fe, which for the present we think of as 56 nucleons. Elastic scattering on ^{56}Fe will occur at $x_{56} = Q^2/2m(^{56}\text{Fe})\nu = 1$. On the other hand, elastic scattering on a nucleon in ^{56}Fe (ignoring small binding and Fermi motion corrections) would occur at $x_N = Q^2/2m_N\nu = 1$, which means $x_{56} \sim 1/56$.

Following the argument we gave above we would then expect that for $1/Q \gg$ size of ^{56}Fe the structure function of Fe would look like $\delta(1 - x_{56})$. That is it would exhibit scaling in terms of a structureless particle of mass $56m_N$. On the other hand, if we raise Q^2 to the region where the elastic scattering from Fe is small, but the proton form-factor is still near unity, the structure function would exhibit scaling and have the form $\delta(1/56 - x_{56})$. [This is simply quasi-elastic scattering, (e,e'p).] Finally, if a nucleon consisted of three point-like quarks, we would expect that for $1/Q \ll$ size of the proton (and $\nu \gg$ excitation energy of the proton) the structure function would peak at $x_{56} = 1/168$, or $x_N = 1/3$.

Of course in any field theory no particle is ever exactly point-like, because its effective charge varies with Q^2. However, for QCD the logarithmic decrease in the coupling constant -- see Eq. (1.1) -- means that at the quark level scaling violations happen slowly. Over quite a large range of Q^2 one can in fact write the structure function of the nucleon as a function of x alone.

THE QUARK-PARTON MODEL AND THE STRUCTURE FUNCTION OF THE NUCLEON

For the rest of this lecture, in the interest of simplicity, we shall ignore the evolution of the structure function of the nucleon with Q^2 which must happen in QCD (Altarelli and Parisi, 1977). What remains in this approximation is usually called the naive quark parton model. Its basic premises are (i) that DIS from a hadron can be described entirely in terms of the interaction with point-like, spin-½ constituents which are on-mass-shell, and (ii) this interaction can be treated in impulse approximation. The justification of point (i) in terms of the quark model is clear, and we will assign these constituents the charges and weak couplings expected of quarks. The second assumption is meant to incorporate asymptotic freedom, so that during the collision the quarks are free. It is only after the collision, when the outgoing lepton is far away, that the effects of confining forces determine the hadronic final state.

It is conceptually easier to think of the DIS process in the limit where both Q^2 and ν go to infinity (with $x = Q^2/2m_N\nu$ fixed). In that case one can view the reaction in an infinite momentum frame, where the nucleon has a momentum far larger than any of its constituents when it is at rest.

In this frame the nucleon four-momentum is simply $(P,0_\perp;\ P)$, and each constituent has a momentum $(x_iP,0_\perp;\ x_iP)$, with $x_i \geq 0$ and $\sum_i x_i = 1$. At any finite momentum transfer there will be corrections of order P^{-1} from the transverse momentum of the mass of the partons. Again we ignore these complications in the interest of simplicity.

In this simple picture all that happens in DIS muon scattering, for example, is that the exchanged photon, with definite x, can only be absorbed on a parton with $x_i = x$ [because of Eq. (3.1) and the assumption that partons are *structureless*]. Thus DIS of muons measures directly the product $q_i^2 f_i(x)$, where q_i is the charge of the quark, and $f_i(x)\,dx$ is the number of quarks of flavour i carrying a momentum between xP and $(x + dx)P$ in an infinite momentum frame. It follows that the entire information about the structure of a nucleon which is provided by DIS can be summarized by the twelve functions $\{u^p(x), d^p(x) \ldots \bar{s}^p(x), u^n(x), d^n(x) \ldots \bar{s}^n(x)\}$ -- the superscript indicates either neutron or proton.

On the basis of charge symmetry one usually reduces this to six independent quark distributions

$$u^p(x) = d^n(x) = u(x) ,$$
$$d^p(x) = u^n(x) = d(x) , \tag{3.11}$$

and so on, through $s(x)$, $\bar{u}(x)$, $\bar{d}(x)$ and $\bar{s}(x)$. Clearly by integrating these distributions weighted by x we can find the total fraction of the momentum of a nucleon (in an infinite momentum frame!) carried by each parton. These are labelled by capital letters U, D ... $\bar{S}$ in an obvious notation

$$U = \int_0^1 u(x) \times dx ,$$
$$D = \int_0^1 d(x) \times dx , \ldots \tag{3.12}$$
$$\bar{S} = \int_0^1 \bar{s}(x) \times dx .$$

If this description is to make sense one would expect these momentum fractions to satisfy a sum rule, namely

$$U + D + S + \bar{U} + \bar{D} + \bar{S} = 1 - G , \tag{3.13}$$

where

$$G = \int_0^1 g(x) \times dx \tag{3.14}$$

is the fraction of the momentum of the nucleon carried by gluons.

Of course G cannot be measured directly in DIS of leptons for the obvious reason that gluons have no charge for electroweak interactions. On the other hand, the measurement of the momentum dependence of three independent structure functions, such as $F_2(x,Q^2)$, $F_3(x,Q^2)$ and $\bar{q}(x,Q^2)$, using beams of ν and $\bar{\nu}$ (see below) does allow the determination of $g(x,Q^2)$ through the Altarelli-Parisi equations -- see, for example, Bergsma and collaborators (1983). Alternatively, one could try to extract g(x) from multi-jet events at the ISR, or better the SP$\bar{\text{P}}$S.

However, at the moment this game is played the other way. That is one takes a g(x), albeit indirectly determined, and uses perturbative QCD to try to understand the observations in the colliding beam experiments.

A second, more direct, sum rule can be derived simply by conserving baryon number. Because each quark has $B = 1/3$ the total excess of quarks over anti-quarks should be three. In fact we would expect two u-quarks and one d-quark. These are usually called valence quarks. Of course in a DIS experiment one cannot tell if a given u quark (say) is a valence quark or part of the sea of virtual q-$\bar{q}$ pairs present in any hadron. Operationally one *defines* the valence distributions $u_V(x)$ and $d_V(x)$ as

$$u_V(x) = u(x) - \bar{u}(x) \;, \qquad d_V(x) = d(x) - \bar{d}(x) \;. \tag{3.15}$$

Then the sum rule giving the baryon number of the nucleon is

$$\int_0^1 u_V(x)\,dx + \int_0^1 d_V(x)\,dx = \int_0^1 \left[u(x) - \bar{u}(x)\right] + \left[d(x) - \bar{d}(x)\right]\,dx = 3 \;. \tag{3.16}$$

In fact, if one allows for QCD corrections to order (α_s/π), the theoretical expectation is about 2.8, and the experimental data gives 2.56 ± 0.41 (statistics) ± ± 0.10 (systematics) (Bergsma and collaborators, 1983).

In terms of the quark distribution functions, the structure functions F_1 and F_2 defined earlier can be written down directly

$$2xF_1^{e,\mu}(x) = F_2^{e,\mu}(x) = \sum_i q_i^2 f_i(x)\; x \;. \tag{3.17}$$

For example, for muon scattering on the proton we have

$$F_{2p}^{\mu}(x) = \frac{4x}{q}\left[u(x) + \bar{u}(x)\right] + \frac{x}{q}\left[d(x) + \bar{d}(x)\right] + \frac{x}{q}\left[s(x) + \bar{s}(x)\right] \;, \tag{3.18}$$

while for the neutron [see Eq. (3.11)]

$$F_{2n}(x) = \frac{x}{q}\left[u(x) + \bar{u}(x)\right] + \frac{4x}{q}\left[d(x) + \bar{d}(x)\right] + \frac{x}{q}\left[s(x) + \bar{s}(x)\right] \;. \tag{3.19}$$

Clearly one cannot use an electromagnetic probe to separate the quark and antiquark distributions. For that the weak interaction is perfect, because it couples only left-handed particles and right-handed antiparticles.

Consider the reaction

$$\nu_\mu + d \rightarrow \mu^- + u \;, \tag{3.20a}$$

which involves only left-handed particles. In the c.m. system of the ν and d there is therefore no angular momentum and the angular distribution is isotropic. On the other hand for a neutrino with an antiquark the angular distribution is proportional to $|d^1_{11}(\theta^*)|^2$, or $(1 + \cos\theta^*)^2$ -- where θ^* is the scattering angle in the ν-$\bar{q}$ c.m. system. Thus there is a clear experimental signature for the antiquark distribution. What is actually measured is conventionally labelled $\bar{q}^{\nu}(x)$, defined as

$$\bar{q}^{\bar{\nu}}(x) = x\left[\bar{u}(x) + \bar{d}(x) + 2\bar{s}(x)\right] \;. \tag{3.20}$$

This is directly obtained from the isotropic piece of the $\bar{\nu}_\mu \rightarrow \mu^+$ cross-section on an isoscalar target.

In Fig. 11 we show the results for $\bar{q}^{\bar{\nu}}$ obtained by the CERN-Dortmund-Heidelberg-Saclay (CDHS) group using the ν_μ and $\bar{\nu}_\mu$ beams from the SPS at CERN. The shape is qualitatively consistent with the form $(1 - x)^7$ expected on the basis of the Drell-Yan-West relation [see, for example, Close (1979) or Brodsky (1982)]. For a more accurate parametrization we refer to the original papers (Abramowicz and co-workers, 1982,1983). Notice that the sea distribution essentially vanishes for $x > 0.3$. In addition, the fact that $\bar{q}^{\bar{\nu}}(x)$ does not vanish at $x = 0$, together with Eq. (3.20), implies that the actual number of sea quarks is logarithmically divergent. The momentum carried by the sea is however finite. In fact, if we define $\bar{Q}^{\bar{\nu}}$ [by analogy with Eq. (3.12)] as the integral of $\bar{q}^{\bar{\nu}}$ we obtain 5.95 ± 0.4% [at $Q^2 = 5\ \text{GeV}^2$ (Eisele, 1982)].

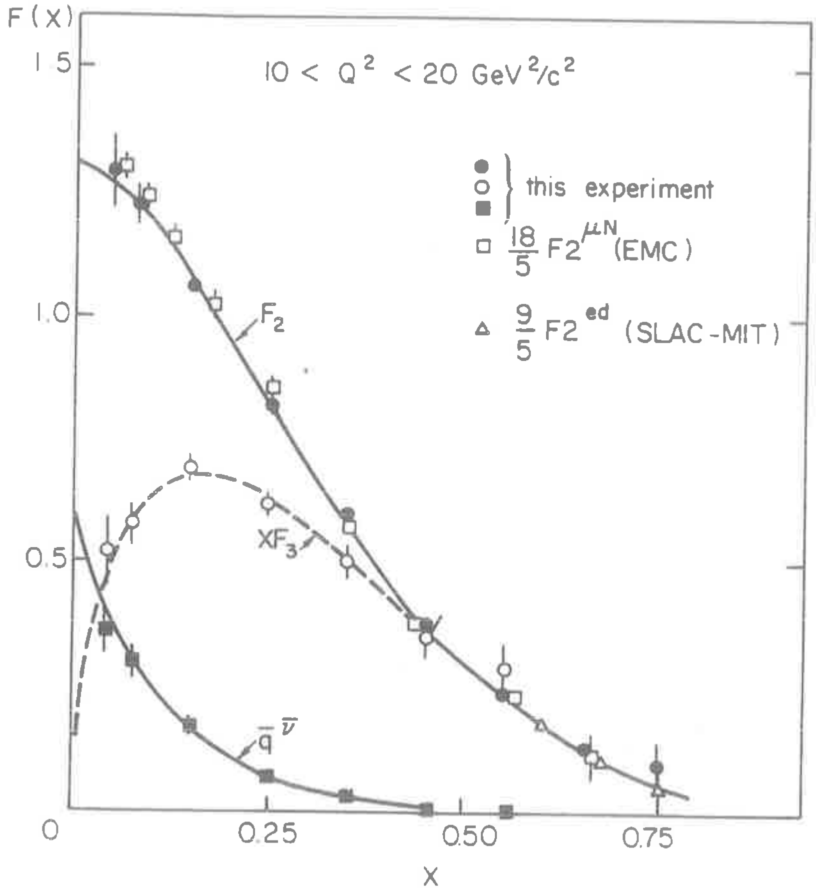

Fig. 11. Results of the CDHS group (Abramowicz and co-workers, 1983) for the structure functions F_2 and F_3 (using neutrino beams), and the antiquark distribution $\bar{q}^{\bar{\nu}}$ defined in Eq. (3.20).

We also show in Fig. 11 the comparison between the structure function F_2 extracted from neutrino reactions (on an iron target)

$$2xF_1(x) = F_2(x) = \{u + d + s + \bar{u} + \bar{d} + \bar{s}\} , \tag{3.21}$$

with the electromagnetic structure functions $(F^\mu_{2n} + F^\mu_{2p})$. From Eqs. (3.18) and (3.19) we find

$$(F^\mu_{2n} + F^\mu_{2p}) \equiv F_2^{\mu N} = \frac{5}{18} \{u + d + \bar{u} + \bar{d} + \frac{2}{5} (s + \bar{s})\} . \tag{3.22}$$

Thus to within a small correction from the strange part of the sea the quark model predicts

$$F_2^{\mu N}(x) = \frac{5}{18} F_2(x) \ . \qquad (3.23)$$

As we see from the figure, Eq. (3.23) is remarkably well confirmed.

Finally we observe that unlike the electromagnetic case, where parity conservation restricted us to two structure functions, for weak interactions there is a third, called $F_3(x)$. For an isoscalar nucleus the sum F_3^{ν} and $F_3^{\bar{\nu}}$, denoted $F_3(x)$, is exactly the distribution of valence quarks $[u_V(x) + d_V(x)]$. This is also shown in Fig. 11. It is clear that the valence quarks dominate over the sea for x beyond about 0.1. We shall recall this again soon in connection with the EMC effect.

The final piece of information which we need to pin down the properties of the nucleon is the ratio of strange to non-strange quarks in the sea. Fortunately there is a direct measurement of this available through the reaction

$$\bar{\nu}_\mu + \bar{s} \rightarrow \mu^+ + \bar{c} \qquad (3.24)$$
$$\hookrightarrow \mu^- \ ,$$

with its unique di-muon signal. Again we show the results of the CDHS group (Fig. 12). The shape of $\bar{s}(x)$ is identical with that of $\bar{q}^{\bar{\nu}}(x)$ within the experimental errors. The actual magnitude of the strange sea is best determined from the *shape* of the ν_μ and $\bar{\nu}_\mu$ x distributions, as described by Abromowicz and co-workers (1982). In fact what they found is

$$\frac{2S}{(\bar{U} + \bar{D})} = 0.52 \pm 0.09 \ . \qquad (3.25)$$

It seems to be an almost universal assumption*) that $s(x) = \bar{s}(x)$ and hence $S = \bar{S}$, so we shall also use S and $\bar{S}$ interchangeably.

Summary

We have seen how to determine the momentum distributions of all the types of parton in the nucleon. Briefly we can summarize the data (at $Q^2 \simeq 5\ \mathrm{GeV}^2$) as**)

$$G \simeq 54\% \ , \qquad (3.26a)$$

$$Q + \bar{Q} \simeq 46\% \ , \qquad (3.26b)$$

$$\bar{Q}^{\bar{\nu}} = 5.95 \pm 0.4\% \ , \qquad (3.26c)$$

$$\bar{S} = 1.0 \pm 0.3\% \ , \qquad (3.26d)$$

$$(\bar{U} + \bar{D})/2 = 1.95 \pm 0.3\% \ , \qquad (3.26e)$$

$$Q - \bar{Q} \simeq 36\% \ . \qquad (3.26f)$$

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*) Certainly wrong in principle, but the error is probably not large.

**) I benefited a great deal from discussions with F. Eisele concerning the CDHS data.

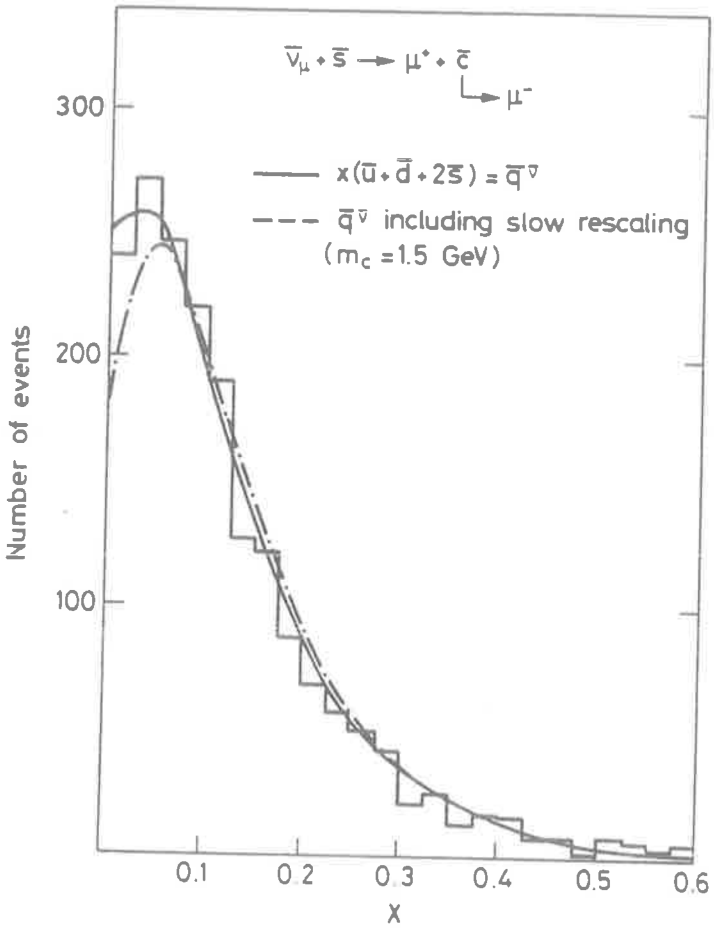

Fig. 12. Comparison of the shape of the $\bar{s}$ distribution in Fe with that of $\bar{q}^\nu$ described earlier -- from Abramowicz and co-workers (1983).

That is, the glue carries a little more than half the momentum of the nucleon [Eq. (3.26a)], the valence quarks a little more than one third [Eq. (3.26f)], and the sea about 10% [Eqs. (3.26d) and (3.26e)]. Finally we note the obvious breaking of SU(3)-flavour [$SU(3)_F$] by the sea -- there are on average four times as many non-strange as strange quarks [c.f. twice as many in the $SU(3)_F$ limit]. To some extent this was expected because of the higher mass of the strange quark. In the next section we shall show that this symmetry breaking can be used to constrain the strength of the pion field about the nucleon in any chiral bag model.

A CONSTRAINT ON THE PION FIELD OF THE NUCLEON

Let us begin with the observation by Sullivan (1972) that there is a contribution to the structure function of the nucleon from the process illustrated in Fig. 13. The virtual photon (γ^*) finds a quark in the pion with fraction x of the momentum of the original nucleon. Naively, at Q^2 of order 10 GeV^2 one would expect to probe distances of order $1/20$th of a Fermi or less. Thus the dressing of the nucleon by pions, which is a relatively long range effect, would not be expected to play a role. The resolution of this apparent contradiction is that one is probing the short distance structure of the pion which is described by the pion's own structure function ($F_{2\pi}$) -- the latter evaluated at x/y, with y the fraction of the momentum of the nucleon carried by the pion in the infinite momentum frame.

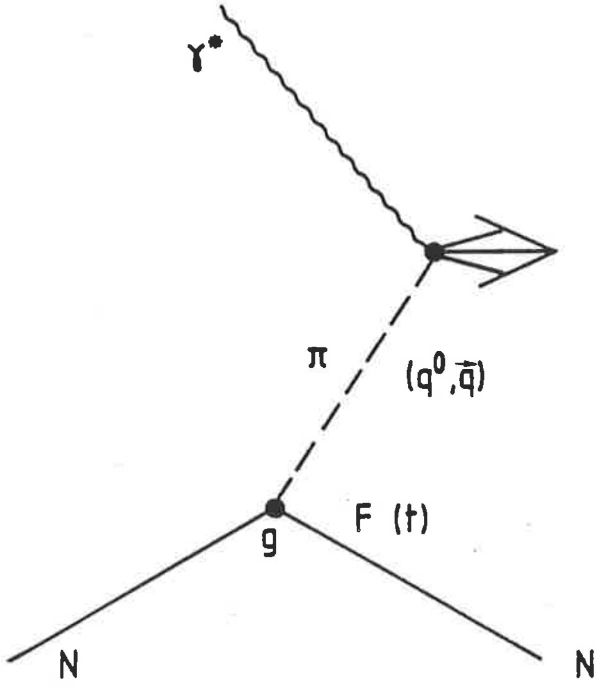

Fig. 13. The contribution of the pion to the structure function of the nucleon.

Putting all this together we find the contribution of Fig. 13, $\delta F_{2N}(x)$, is given by the expression

$$\delta F_{2N}(x) = \int_x^1 f(y) F_{2\pi}(x/y)\, dy \;. \tag{3.27}$$

The function f(y) is the number density of pions carrying fraction y of the momentum of the nucleon. It is calculated very simply from the lower portion of Fig. 13 in terms of the πNN coupling constant (g = 13.5), and the high-momentum cut-off at the πNN vertex, F(t).

$$f(y) = \frac{3g^2}{16\pi^2} \int_{\frac{m_N^2 y^2}{1-y}}^{\infty} \frac{dt\; t|F(t)|^2}{(t + m_\pi^2)^2} \;. \tag{3.28}$$

Here $t = (\vec{q}^{\,2} - q^{0\,2})$ is minus the four-momentum of the exchanged pion, and the lower limit on t in Eq. (3.28) follows from demanding that the final nucleon be on mass shell -- see Sullivan (1972). [If instead of a nucleon one had some resonance in the final state, the lower limit on the integral over t would become even higher. This would reduce the corresponding f(y) and force it to peak at smaller y. From the form of Eq. (3.27) this moves the corresponding contribution to smaller x. For this reason the region $x < 0.05$ is quite complicated to analyse. However, for $x > 0.05$ numerical calculations show that the $N \rightarrow N\pi$ process is dominant, and we shall treat only that -- again see Sullivan (1972).]

In order to estimate f(y) we took a simple Gaussian for F(t) (actually it is a Gaussian in $\vec{q}^{\,2}$, where $\vec{q}^{\,2} \gg q^{0\,2}$ for Fig. 13)

$$F(t) = \exp\left[-\lambda(t + m_\pi^2)/m_\pi^2\right] . \tag{3.29}$$

From our discussion of the CBM, see Fig. 2, we recall that Eq. (3.29) is an excellent approximation the πNN form-factor provided $\lambda = 0.106\ m_\pi^2 R^2$. Of course the CBM form-factor is based on the static, spherical cavity approximation to the MIT bag model, and therefore applies only for spacelike momenta. Fortunately, as we have already pointed out, $q^0 \sim -\vec{q}^{\,2}/2m_N$ if the final nucleon is on-shell, and hence $|q^0| \ll |\vec{q}|$. Thus it is meaningful to talk about the CBM prediction for the πNN form-factor in the present context.

In Fig. 14 we show the function f(y) which results from evaluating Eq. (3.28) using Eq. (3.29), for several values of λ (or R). There are two essential features to which we would like to draw attention. First, f(y) has its maximum value at $y \simeq 0.25$ for all reasonable values of λ. Second, the value at the peak increases rapidly as λ decreases -- that is, as the form-factor becomes harder.

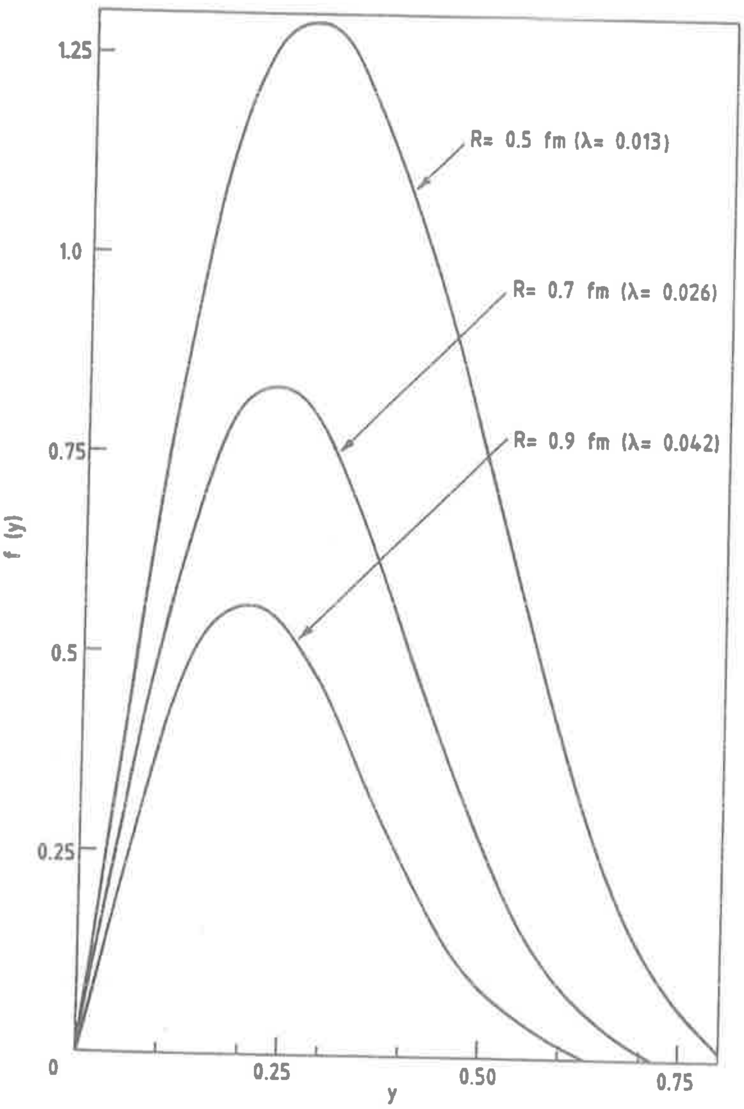

Fig. 14. The probability f(y) of finding a pion carrying a fraction y of the momentum of the nucleon, for several values of the cut-off parameter λ (or bag radius R) -- from Thomas (1983).

Returning to Eq. (3.27), we see that the pion structure function is evaluated at x/y. As usual we expect that the valence component of the pion should dominate for $x/y > 0.1$. Since y is typically 0.25, this implies that the pionic contribution to the nucleon structure function for $x > 0.03$ involves only non-strange quarks. Thus, if the pion is an important component of nucleon structure, it should contribute to breaking the SU(3) flavour symmetry of the sea. Of course, as we mentioned earlier, it is generally expected that $SU(3)_F$ will be broken because of the larger strange quark mass, and it would be unreasonable to attribute the entire excess of non-strange sea quarks to the pion. Nevertheless, it seems quite reasonable to use any evidence for $SU(3)_F$ breaking to impose a limit on the pionic contribution to the nucleon structure function.

Integrating Eq. (3.27) over x, we find that

$$\int_0^1 \delta F^{\mu}_{2N}(x)\, dx = \left[\int_0^1 F^{\mu}_{2\pi}(\xi)\, d\xi\right]\left[\int_0^1 dy\, y\, f(y)\right]. \tag{3.30}$$

Thus, if we knew the structure function of the pion, we could use the measured excess of u- and d-quarks in the sea to put an upper limit on the average momentum fraction of the nucleon carried by the pion ($\langle y\rangle_\pi$) -- the latter being given by the second bracket on the right of Eq. (3.30). Let us call the excess of non-strange over strange quarks in the sea $q^{(2)}(x)$. [We conservatively assume that $(\bar{u} - \bar{s}) = (\bar{d} - \bar{s}) = (u - u_V - s) = (d - d_V - s) = q^{(2)}$, even though there is some evidence from the Gottfried sum rule that $\bar{d} > \bar{u}$. If this were used we would get an even lower upper bound on $\langle y\rangle_\pi$.] Then the contribution to the integral over $F_{2N}(x)$ from the excess of non-strange sea quarks, which we shall call $F_{2N,\text{excess}}$, is found, from Eq. (3.17), to be

$$\begin{aligned} F_{2N,\text{excess}} &= \int_0^1 \left(\frac{4}{9} + \frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) q^{(2)}(x)\, x\, dx \\ &= \frac{10}{9}\left\{\frac{\bar{U} + \bar{D}}{2} - \bar{S}\right\}. \end{aligned} \tag{3.31}$$

Thus our bound on $\langle y\rangle_\pi$ is, from Eq. (3.30) and the argument following it

$$\langle y\rangle_\pi \equiv \int_0^1 f(y)\, y\, dy \le \frac{\frac{10}{9}\left\{\frac{\bar{U} + \bar{D}}{2} - \bar{S}\right\}}{\int_0^1 F_{2\pi}(\xi)\, d\xi}. \tag{3.32}$$

Determination of the structure function of the pion

Because the pion is unstable one cannot measure its structure function directly using lepton DIS. However, there is another technique available, the Drell-Yan process. In a collision of a high energy pion with a nucleon, an antiquark of the former can annihilate on a quark of the latter to give a massive photon which then decays to $\mu^+\mu^-$. The invariant mass distribution of $\mu^+\mu^-$ pairs produced this way gives a measure of the product of the quark distributions in the pion and the nucleon (see, for example, Kenyon, 1982). Since we know the structure function of the nucleon we can extract that of the pion.

The results obtained by the NA3 group at CERN are shown in Fig. 15. Just to show that the method works we also show (Fig. 16) the proton structure function extracted by the same group from the process $q\bar{q} \to \mu^+\mu^-$ in $p\bar{p}$ collisions. The result agrees very well with the proton structure function (at the same $|Q^2|$) known from DIS up to a renormalization factor of 2.3. This can be explained in terms of the next to leading log corrections in perturbative QCD. The main correction seems to come from the larger phase space in the Drell-Yan process ($q^2 = -Q^2 \sim$ (4-8.5) GeV^2 in UA3) compared with DIS ($Q^2 = -q^2 \sim 5\ \text{GeV}^2$).

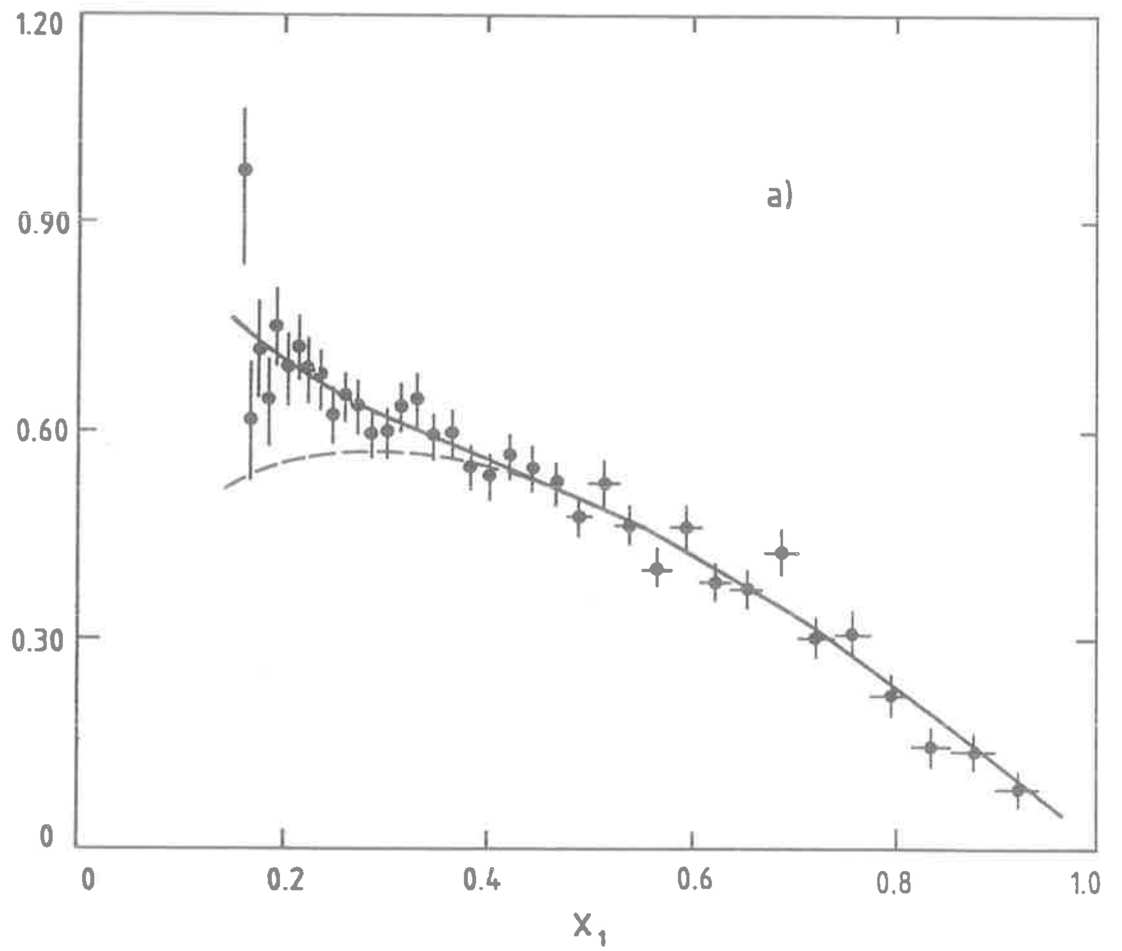

Fig. 15. The structure function of the pion extracted by the UA3 group at CERN from the Drell-Yan process (Badier and others, 1983).

From the results quoted in the very readable review by Kenyon (1982), and also from Badier and co-workers (1983), we find (at $|Q^2| \sim 5\ \text{GeV}^2$)

$$\int_0^1 F_{2\pi}(\xi)\ d\xi = 0.015 \pm 0.004\ . \tag{3.33}$$

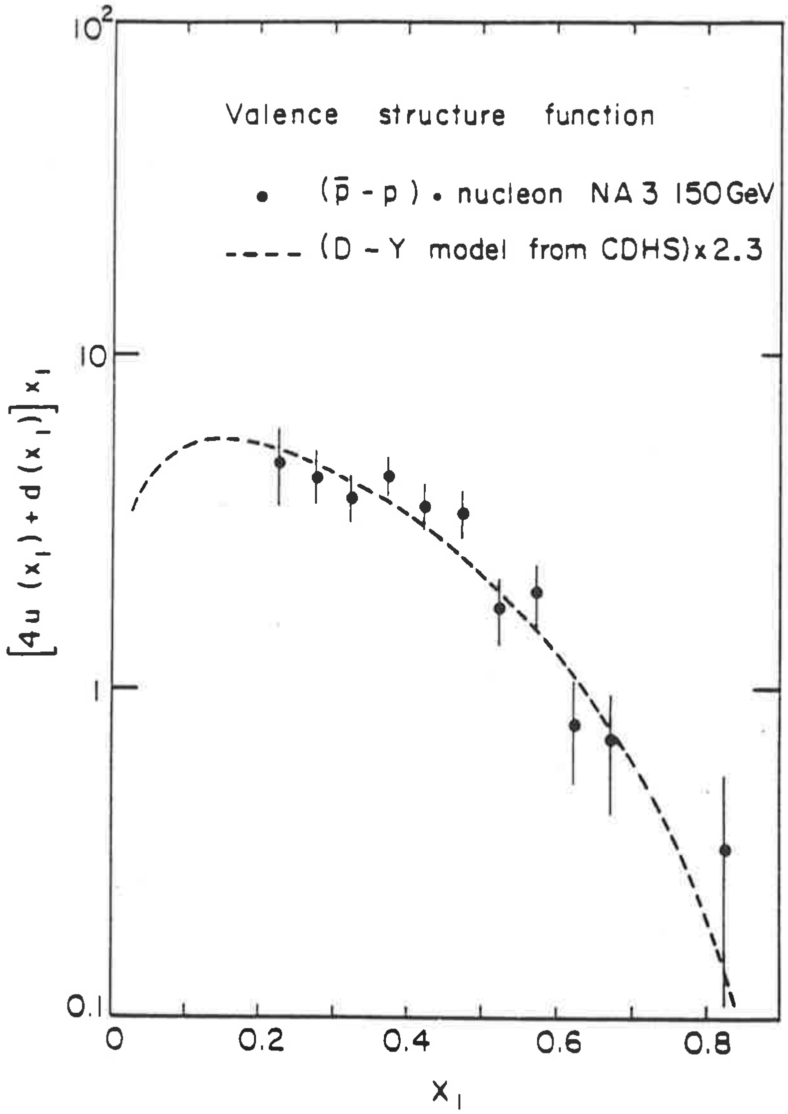

Fig. 16. Comparison of the structure function of the proton extracted from the Drell-Yan process in $p\bar{p}$ collisions (points), with that from DIS (dashed line) -- from Kenyon (1982).

<u>Summary</u>

But for one small problem we could put together Eqs. (3.33), (3.32) and (3.26) to find an upper limit on $\langle y \rangle_\pi$ of 7 ± 4%. The little problem is known as the EMC effect, and will be the subject of the last part of this lecture. For our present purposes the essential point is that the sea per nucleon in Fe is enhanced by between 20 and 40% over that for a free nucleon (see Fig. 18). Unfortunately, all of the CDHS neutrino data, on which Eq. (3.26) was based, was taken on Fe.

The most naive correction for the EMC effect would simply lower the whole sea by 15%, leading to a non-strange excess in a free nucleon $[(\bar{U} + \bar{D})/2 - \bar{S}]_N =$ $= 0.67 \pm 0.4\%$. On the other hand, some of the proposed explanations, including the one we describe in the next section (Llewellyn Smith, 1982, 1983; Ericson and Thomas, 1983), would lead to an enhancement of only the non-strange part of the sea in a nucleus. In that case the 30% correction would be applied only to $(\bar{U} + \bar{D})/2$, leading to an excess of non-strange sea quarks of about 0.4 ± 0.4% in a free nucleon. Then the upper limit allowed for the fraction of the momentum of the nucleon carried by pions ($\langle y \rangle_\pi$) would be 3 ± 3%.

From this brief analysis it should be clear that there is not yet a very reliable, direct, experimental determination of the non-strange excess. Nevertheless, the qualitative consistency of the numbers with those of Field and Feynman (FF, 1977) is remarkable. Using the DIS data from SLAC together with general theoretical constraints (including the Gottfried sum rule) they concluded that $(\bar{U} + \bar{D})/2 = 1.8\%$ and $\bar{S} = 1.1\%$ in a free nucleon. These numbers are very close to those obtained by the CDHS group, and may even be more accurate, despite the fact that they were obtained indirectly. Because of the uncertainty over the EMC correction, we actually used the FF value for $[(\bar{U} + \bar{D})/2 - \bar{S}]_N = 0.7\%$. This puts an upper limit on $\langle y \rangle_\pi$ of (Thomas, 1983a)

$$\langle y \rangle_\pi \lesssim 5 \pm 1.5\% \ , \tag{3.34}$$

[from Eqs. (3.32) and (3.33)]. We regard this as somewhat more conservative than the value 3 ± 3% obtained from the CDHS data.

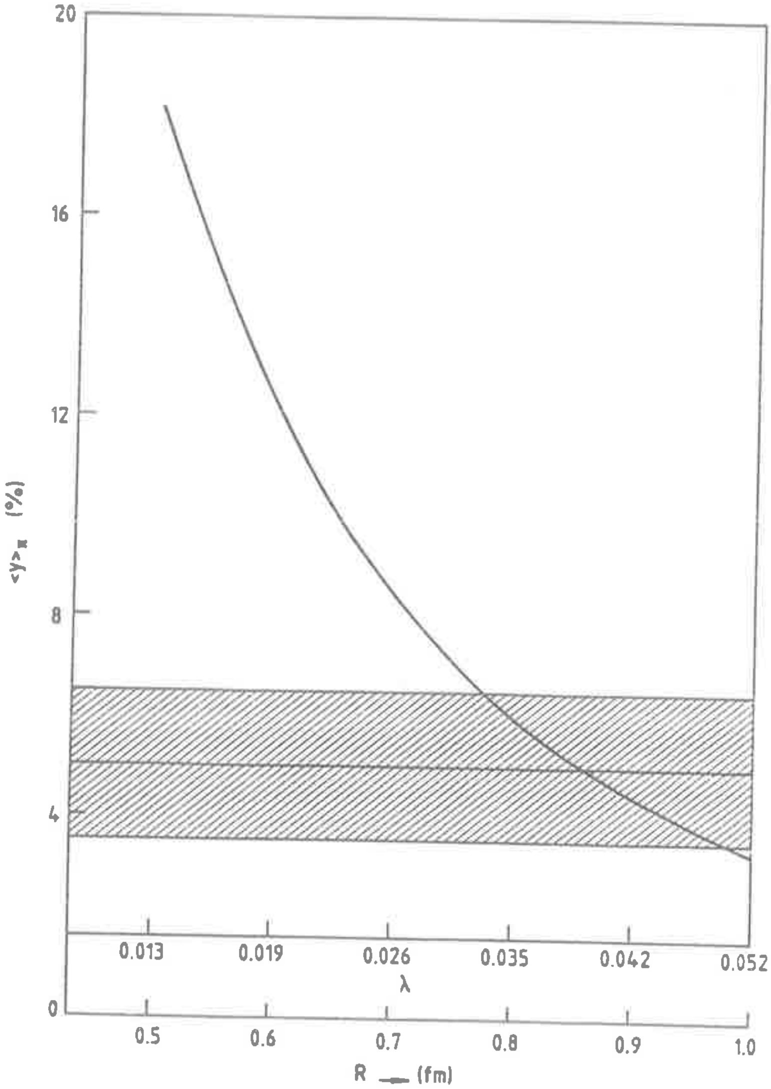

Fig. 17. The average fraction of the nucleon's momentum carried by the pion as a function of λ (or bag radius R). The shaded area represents the upper bound obtained from $SU(3)_F$ breaking in the nucleon's sea (Thomas, 1983a).

In Fig. 17 we compare this upper limit on $\langle y \rangle_\pi$ (shaded area) with the theoretical value (obtained numerically) for a range of values of λ. This analysis clearly puts *quite a severe lower limit* on the range of values allowed, namely $\lambda \geq 0.039\,^{+0.012}_{-0.006}$. Also shown in the figure is the corresponding value of the bag radius in the CBM. *The lower bound implied by Eq. (3.34) is $R \geq 0.87 \pm 0.10$ fm.* Of course, we must add the caution that the static bag model has many defects, and insistence on a precise value of R would be meaningless. (For example, the bag should have finite surface thickness, and this together with c.m. and recoil corrections could change the exact relationship between R and λ.) Nevertheless, this lower bound is a very strong indication that the essential assumption of the CBM [CBM(b) in the notation of lecture 2] is correct. For the general reasons outlined in lecture 1 we must therefore allow the possibility of a rather new picture of the nucleus -- one in which we find "nucleons" sharing their quarks a considerable fraction of the time. One of the most natural explanations for the EMC effect involves just this! It is therefore quite appropriate to devote the last part of this series of lectures to that dramatic discovery.

THE EMC EFFECT

Figure 18 shows the ratio of the structure function per nucleon in Fe to that in D, as a function of $x = Q^2/2m_N\nu$. Because of its low density we shall henceforth drop the distinction between data on D and free nucleons. (An analysis at the level of

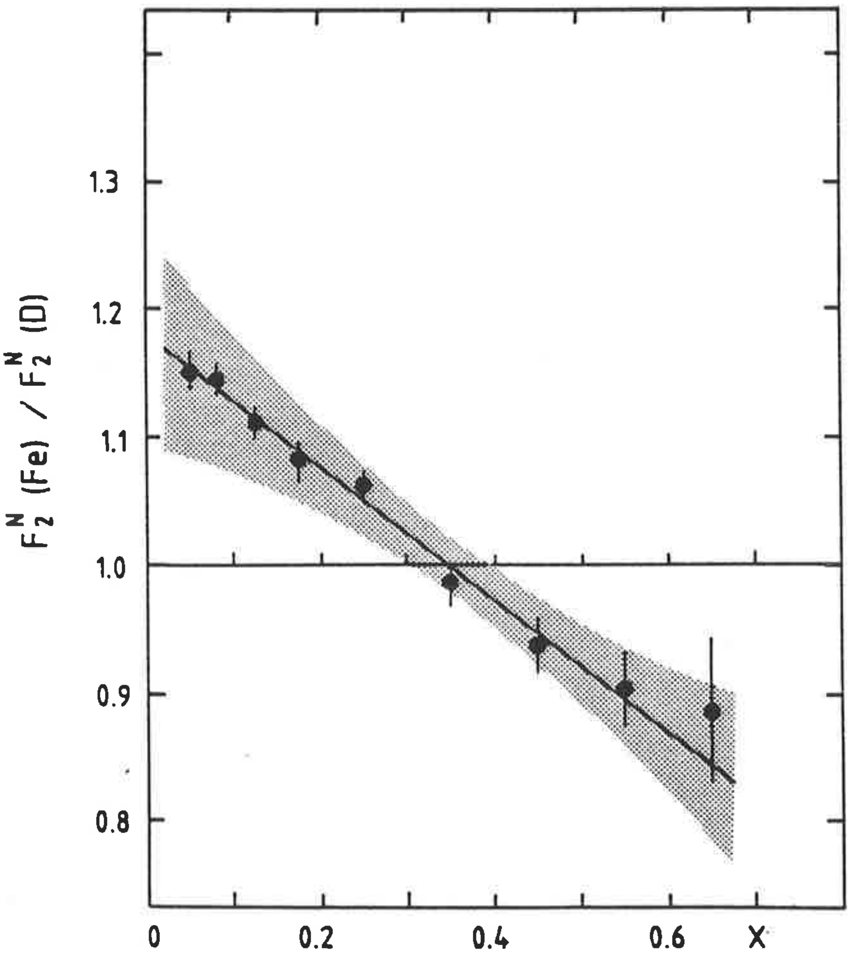

Fig. 18. The structure function of a "nucleon" in Fe, relative to that in D, as measured by the European Muon Collaboration (Aubert and others, 1983).

one or two percent would have to take the difference into account.) There is a clear softening of the structure function in Fe, with the sea region ($x < 0.3$) enhanced by about 15%, and the valence region depressed by about the same fraction. The data in the region $0.3 < x < 0.65$ was confirmed by a SLAC group (Bodek and others, 1983) using background subtraction data almost ten years old. That data also extended the range of x to about 0.9 where the ratio does climb back above unity.

In view of the enormous range of Q^2 covered by the two experiments ($3 \le Q^2 \le$ ≤ 170 GeV^2), and the observation that the effect shows no Q^2-dependence within the experimental errors, this is almost certainly not a higher twist effect. In order to put this result in the right perspective, we show in Fig. 19 the theoretical predictions for the ratio, made before the data were taken. The variation in the predictions is certainly an overestimate of the theoretical uncertainties involved in the Fermi-averaging procedure. We believe that the dot-dash curve is probably the most reasonable. However, even using that curve, the discrepancy between theory and experiment in the valence region ($x \sim 0.6$) is as much as 30%. That is an enormous change.

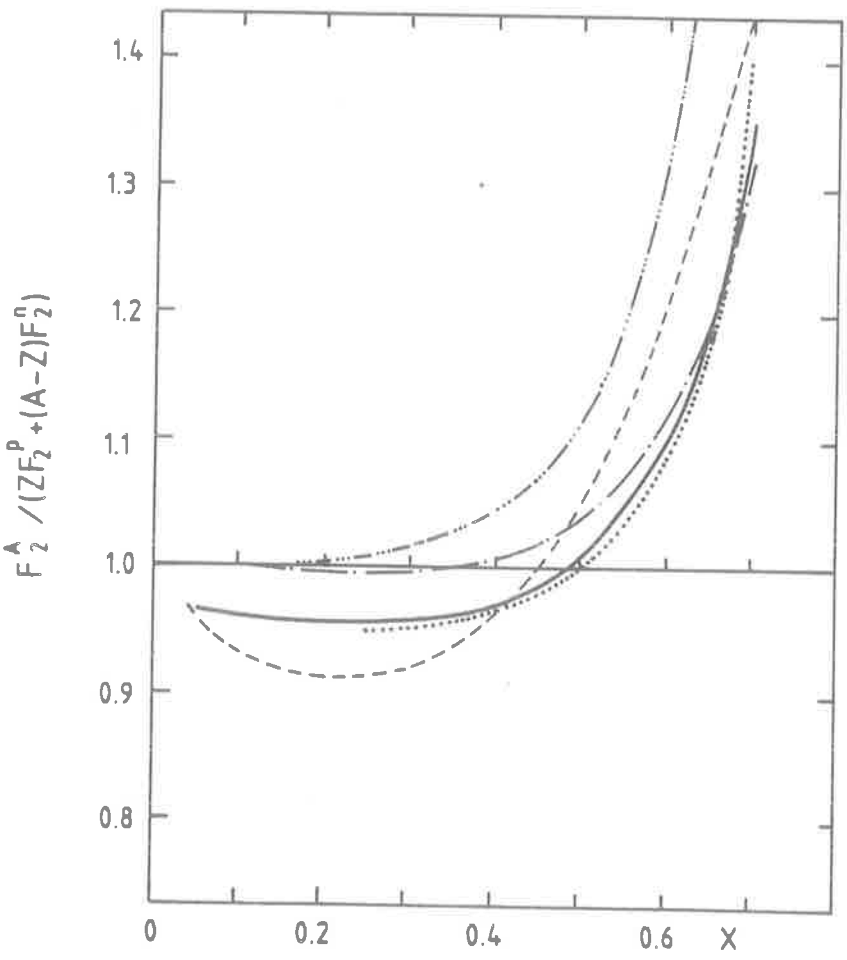

Fig. 19. Predictions for the ratio of the structure function per nucleon in Fe to that in D, predicted by various Fermi-averaging prescriptions (again from the compilation of Aubert and co-workers, 1983).

The first theoretical paper to discuss the EMC data was by Jaffe (1983), and it went right to the heart of the matter. Using the MIT bag model, he showed that the softening of the valence distribution could be qualitatively understood if a given quark spent a relatively large fraction of its time in a six quark state -- say (10-20)%. [A similar number was obtained by Pirner and Vary (1983).] Although Jaffe used the bag model, his result is quite general. According to the Drell-Yan-West relation the large x behaviour for an (N + 1) quark bag is

$$F_{N+1}(\tilde{x}) \sim (1 - \tilde{x})^{2N-1} \tag{3.35}$$

where $\tilde{x} = Q^2/2m_{N+1}\nu \in (0,1)$. In terms of the experimentalists' x we then find the ratio of the structure functions for a six-quark and a three-quark bag to be

$$\frac{F_{6q}(x)}{F_{3q}(x)} \sim \frac{\left(1 - \frac{x}{2}\right)^9}{\left(1 - x\right)^3}\ , \tag{3.36}$$

(where we assumed that $m_6 \sim 2m_3$). It is a trivial exercise to show that this ratio has a minimum value at x = 0.5 -- just like the data.

One of the simplest ways to approach the many-body problem for composite nucleons is a boundary condition model. That is, one would assume that for a centre-to-centre separation r > b the bags can be treated as distinct hadrons. However, for r ≤ b, when the overlap is large (we expect b ∿ R), we might treat the system as a six-quark state. For $b \sim R_{CBM} \sim 1$ fm the probability of finding a given quark in a six-quark state is indeed of order 20%, even in light nuclei (Greben and Thomas, 1983; Pirner and Vary, 1983).

It is unfortunately more difficult to make predictions about the sea in a six-quark bag. Of course one could always choose it to fit the EMC data, but it is probably fair to say that the multi-quark-bag idea does not provide a natural explanation of the EMC effect in the region x < 0.3.

In order to understand the suggestion of Llewellyn-Smith (1982, 1983) for the region x < 0.3, we return to Eq. (3.27) giving the contribution of the pion cloud to the structure function of the nucleon. Forgetting about shadowing corrections and so forth, which could alter the predictions for x very near zero, we see that Eq. (3.27) implies

$$\delta F_{2N}(0)/F_{2N}(0) \simeq \delta F_{2N}(0)/F_{2\pi}(0) = \int_0^1 f(y)\, dy\ . \tag{3.37}$$

Thus for a free nucleon the fractional contribution to the structure function at x = 0 from the process shown in Fig. 13 is simply the number of pions (in the infinite momentum frame). It is then clear, that if in a nucleus the number of pions/nucleon was enhanced by about 0.15, one would have a qualitative explanation for the EMC data for x < 0.3.

In order to check this quantitatively, Ericson and Thomas (1983) calculated the modified momentum distribution $\tilde{f}(y)$ of pions in Fe. The mechanism for changing the distribution is illustrated in Fig. 20. We see that after being emitted by one nucleon a pion can interact with other nucleons in the medium, making nucleon particle-hole, or delta particle-hole excitations [Figs. 20(b) and 20(c), respectively]. When iterated in the RPA such processes could by themselves produce pion

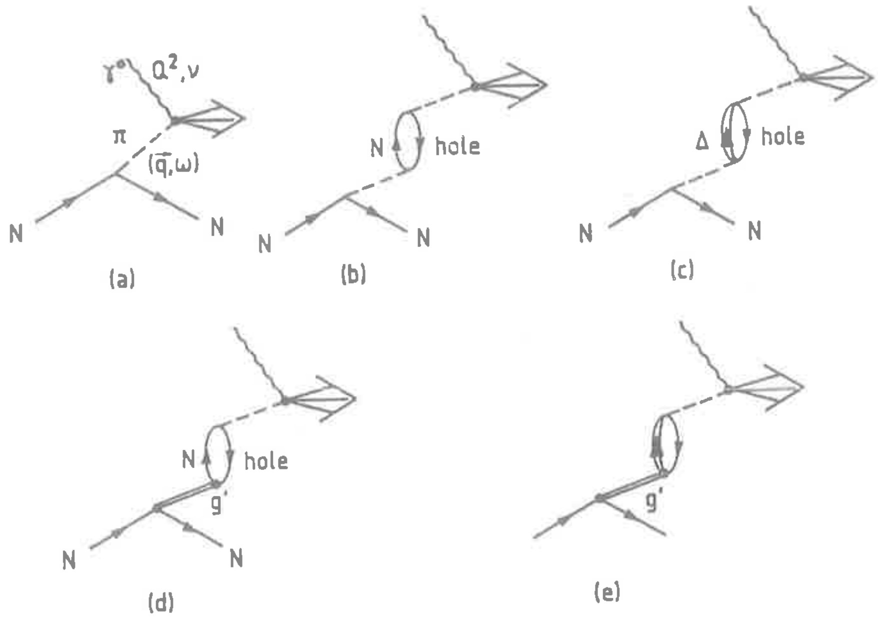

Fig. 20. Illustration of (a) the basic pion contribution to the nucleon structure function (the $\gamma^*\pi$ vertex involves the structure function of the pion itself); (b) and (c) other coherent processes involving pion rescattering in the nucleus which lead to enhancement for $|\vec{q}| \sim 300$-400 MeV/c; (d) and (e) the phenomenological short-range repulsion which damps the enhancement arising from (b) and (c) -- from Ericson and Thomas (1983).

condensation at $[q^0 = 0,\ |\vec{q}| \sim 400$ MeV/c$]$, at nuclear matter density (Migdal, 1972, 1978; Meyer-ter-Vehn, 1981). Since pion condensation is not observed there must be some suppression of the attraction, and this is often described phenomenologically by a short-range repulsive interaction g' (the Landau-Migdal parameter).

There has been a great deal of phenomenological work to determine the values of the three parameters $g'_{NN}, g'_{N\Delta}$ [Fig. 20(d) and 20(e), respectively] and $g'_{\Delta\Delta}$. Much of this school has dealt with the controversy over the role of $g'_{N\Delta}$ in the suppression of Gamow-Teller strength, and we shall not add to the confusion. Suffice it to say that as a first estimate $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} \equiv g' \sim 0.7$ is fairly conservative -- although some theorists argue that $g'_{N\Delta}$ could be as small as 0.4. For a discussion of this formalism as it relates to the calculation of $\tilde{f}(y)$ we refer to the lectures of M. Ericson.

Briefly, Ericson and Thomas use the approximation of treating Fe as nuclear matter with $k_F = 1.30$ fm (Smith and Moniz, 1972), which should not be unreasonable for inclusive processes at momentum transfer of 400 MeV/c. They use $g' = 0.7$ for the reasons just mentioned, and solve for the nuclear response in RPA. Before discussing the results, the only other choice of parameter which should be mentioned is the $NN\pi$ (and $\Delta N\pi$) form-factor. We argued in the last section that for a *free nucleon* the $NN\pi$ form-factor corresponded to $R \geq 0.87 \pm 0.10$ fm. However, there is a great deal of phenomenological evidence for a much harder form-factor in a conventional description of the N-N interaction (see the lectures of T.E.O. Ericson). The resolution of this may be related to the simple fact that the CBM form-factor only modifies the OPE interaction between two nucleons when they start to overlap. However, at that stage the OPE interaction in the CBM would get additional nonlocal contributions from exchange terms. Thus there is no reason why the $NN\pi$ form-factor for a free nucleon, calculated in the CBM, should be directly applicable to the OPE N-N interaction. In order to be consistent with the usual Landau-Migdal phenomenology we therefore choose a dipole form-factor of mass 1.67 GeV (monopole of 1.2 GeV) in all exchanges involving two baryons, but at the last vertex, after which the pion is smashed by the photon, we use the CBM with $R \sim 0.8$ fm. (In fact, the standard curve was defined before our lower limit was proven, and involved $R = 0.7$ fm. We now consider this to be a little small.)

Finally we show in Fig. 21 the predictions of Ericson and Thomas for the enhancement of the pionic part of the sea in Fe. Although there is considerable sensitivity to parameters it is clear that the EMC data is consistent with the much searched for enhancement of the nuclear, pionic response function in the region $|\vec{q}| \sim 400$ MeV/c (Meyer-ter-Vehn, 1981; Oset, Toki and Weise, 1982). What may be most interesting to a medium-energy physics audience is the fact that *lowering* $g'_{N\Delta}$ below 0.6 would give far too much enhancement!

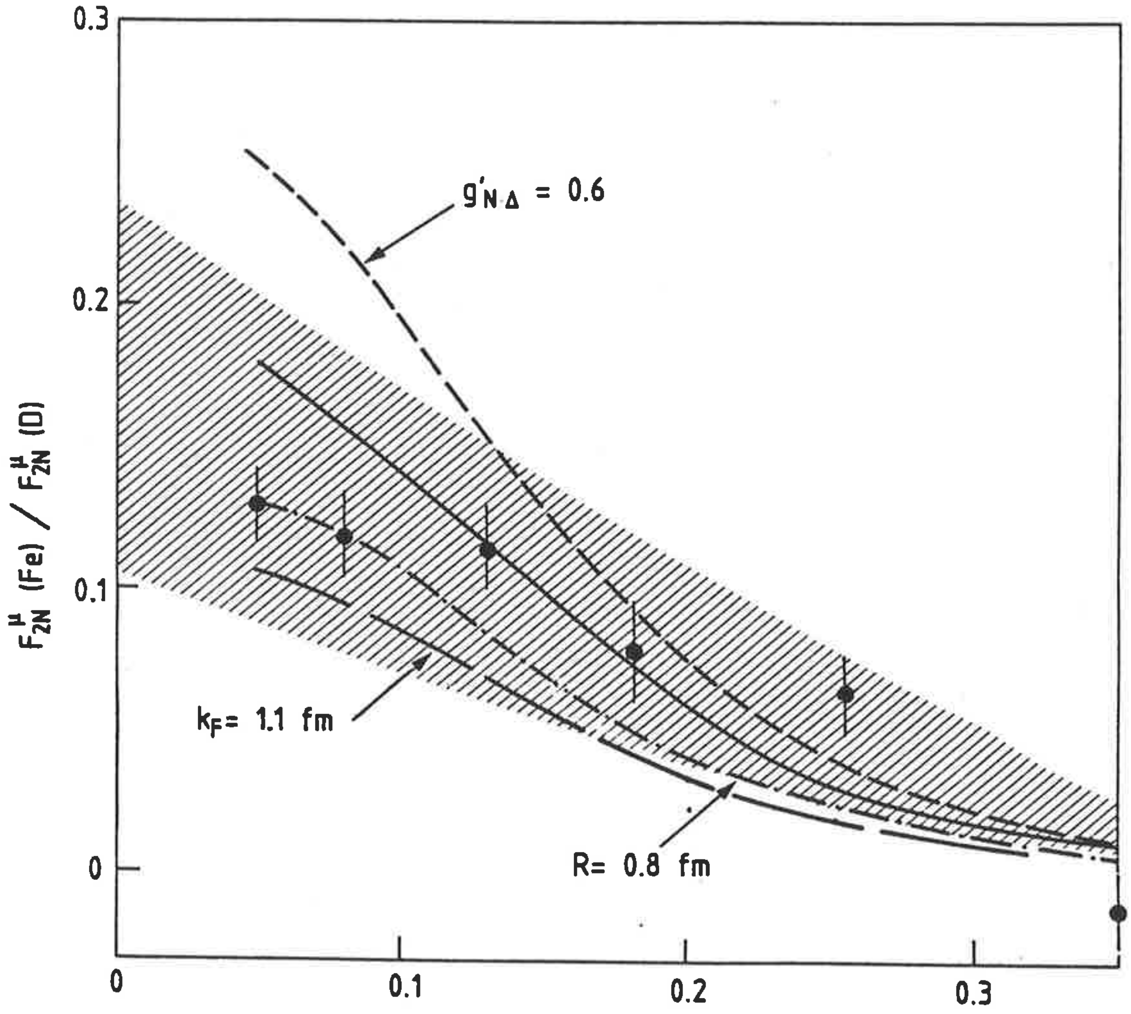

Fig. 21. The fractional increase in the ratio of the structure function in Fe compared with D, as a function of x ($= Q^2/2m_N\nu$), caused by the multinucleon pion emission graphs of Figs. 3(b)-(e). The data are from the European Muon Collaboration, and the shaded area indicates possible systematic errors. The standard input (solid curve) for Fe is $k_F = 1.30$ fm^{-1}, $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} = 0.7$, a bag radius of 0.7 fm in $F(q^2)$, and $\Gamma(q^2)$ is a dipole of mass 1.67 GeV. We show in the other curves the effect of altering any single one of these parameters (Ericson and Thomas, 1983).

Obviously, at the present stage of the analysis, we can draw no firm conclusions. On the experimental side we need further studies of the systematic behaviour of the effect -- with A and Q^2. (Our model predicts that the enhancement of the sea should vary as the density, k_F^3 -- so that in ^{12}C, $k_F = 1.1\ fm^{-1}$ in Fig. 21, the effect is cut almost in half.) One would like to know whether it is only the non-strange part of the sea which is enhanced (as our model predicts), and so on. A series of follow-up experiments is planned at both CERN and SLAC.

From the theoretical point of view we have the uncertainties in correcting for Fermi motion, illustrated in Fig. 19. In addition, as observed by Llewellyn-Smith (1982, 1983), an enhancement of the sea almost certainly implies a softening of the valence distribution -- again difficult to calculate reliably. Finally, since April when the only theoretical papers of which we were aware were those described above, there have been at least eight more -- Nachtmann and Pirner (1983), Close, Roberts and Ross (1983), Faissner and Kim (1983), Date (1983), Carlson and Havens (1983), Szwed (1983), Furmanski and Krzywicki (1983) and finally Friman, Pandharipande and Wiringa (1983). It will be a long time before the constraints of theoretical consistency and new data allow us to choose definitively which explanation is correct.

Let us close by simply observing that the EMC effect is qualitatively in agreement with our naive expectations for the structure of the nucleus based on the CBM. For the present the explanations of the change of the sea as an enhancement in the number of pions, and the change in the valence distribution in terms of overlapping bags, are quite separate. Nevertheless it may prove possible to explain the parameter g' in terms of the dynamics of overlapping bags. Success in that effort would go a long way towards unifying the two ideas. We have a lot of hard work ahead. However, the reward, a new and deeper understanding of the structure of the nucleus than we have ever possessed, is surely worth the effort.

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A LIMIT ON THE PIONIC COMPONENT OF THE NUCLEON THROUGH SU(3) FLAVOUR BREAKING IN THE SEA

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It is shown that deep inelastic scattering data provide a very strong bound ($5.0 \pm 1.5\%$) on the fraction of the momentum of the nucleon which is carried by pions. In the CBM this translates into a bag radius greater than 0.87 ± 0.10 fm.

There is currently a great deal of interest in phenomenological models of hadron structure of relevance to nuclear physics [1–4]. In particular, much attention has been paid to those extensions of the MIT bag model which incorporate chiral symmetry by coupling the pion field to the bag surface [1–3,5]. Considerable indirect evidence has been accumulated, through calculations of static baryon properties [6,7], pion–nucleon scattering [8], and photoproduction [9], which indicates that this is a sensible approach. However, it is rather disturbing that no one has yet provided direct experimental evidence of a pionic component in the nucleon.

This is even more worrying in view of the enormous differences between the Little Bag Model (LBM) and the Cloudy Bag Model (CBM). In the former [3] it is proposed that the pion coupling to the bag surface should be highly non-perturbative, compressing the bag to a radius of order 0.4 fm. In the latter [1,2] the hadron size is supposed to be given by non-perturbative QCD effects (perhaps related to, but not dominated by pionic corrections) to be of the order (0.8, 1.0) fm. Given that the average internucleon separation in nuclear matter is of order 1.8 fm, it is a priori obvious that the consequences of the two models in a microscopic theory of nuclear structure could be quite different [2,10]. The purpose of this note is to show that existing deep-inelastic scattering data provide quite a stringent limit on the pionic content of the nucleon. To be specific, pions do not carry more than about 5% of the momentum of a nucleon in an infinite momentum frame. Within the CBM this indeed corresponds to a bag radius greater than 0.87 ± 0.1 fm.

In order to obtain this result, let us begin with the observation by Sullivan [11] that there is a contribution to the sea component of the nucleon structure function associated with the process shown in fig. 1. (This has been discussed recently in connection with the EMC effect [12–14], because a relatively small enhancement in the number of pions per nucleon in Fe can explain the observed increase in the structure function for $x < 0.3$.) This contribution has the form [11]

$$\delta F^{\mu}_{2\mathrm{N}}(x) = \int_x^1 \mathrm{d}y\, f(y) F^{\mu}_{2\pi}(x/y)\,, \tag{1}$$

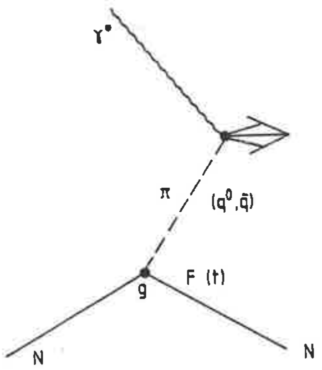

Fig. 1. The contribution of the pion to the structure function of the nucleon.

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where $F^{\mu}_{2\pi}$ is the structure function of the pion, and $f(y)$ is the momentum distribution of the pion in an infinite momentum frame. {In writing eq. (1) we have omitted any explicit mention of the Q^2 dependence of δF^{μ}_{2N} and $F^{\mu}_{2\pi}$ which is required by QCD [15,16]. Its inclusion would not change our discussion in any significant way.} Very simply, eq. (1) says that we should integrate over the probability, $f(y)$, of finding a pion with a fraction y of the momentum of the nulceon *and* finding a quark in that pion with momentum fraction x (that is x/y of y).

A simple calculation gives $f(y)$ in terms of the coupling constant, $g = 13.5$, and the form factor $F(t)$, at the πNN vertex. That is [11]

$$f(y) = \frac{3g^2}{16\pi^2} y \int_{M^2y^2/(1-y)}^{\infty} \frac{\mathrm{d}t\, t|F(t)|^2}{(t+m_\pi^2)^2} , \qquad (2)$$

with $t = (q^2 - q^{02})$ equal to minus the four-momentum of the exchanged pion. In order to illustrate the behaviour of $f(y)$ we have assumed a simple form factor

$$F(t) = \exp\,[-\lambda(t + m_\pi^2)/m_\pi^2] . \qquad (3)$$

{It has been shown [7] that eq. (3) is an excellent approximation to the form factor in the CBM, with $\lambda = -0.106 m_\pi^2 R^2$.} Fig. 2 shows the pion momentum distribution for a range of values of λ. It has two essential features. Firstly, $f(y)$ peaks very near $y = 0.25$ over the entire range of λ considered. Secondly, the maximum value of $f(y)$ increases rather rapidly as λ decreases.

Returning to eq. (1) we see that the pion structure function is evaluated at x/y. As usual we expect that the valence component of the pion should dominate for $x/y > 0.1$. Since y is typically 0.25, this implies that the pionic contribution to the nucleon structure function for $x > 0.03$ involves only non-strange quarks. Thus, if the pion is an important component of nucleon structure it should contribute to breaking the SU(3) flavour symmetry [$\mathrm{SU(3)_F}$] of the sea. Of course, it is generally expected that $\mathrm{SU(3)_F}$ will be broken because of the larger strange quark mass, and it would be unreasonable to attribute the entire excess of non-strange sea quarks to the pion. Nevertheless it seems quite reasonable to use any evidence for $\mathrm{SU(3)_F}$ breaking to impose a limit on the pionic contribution to the nucleon structure function.

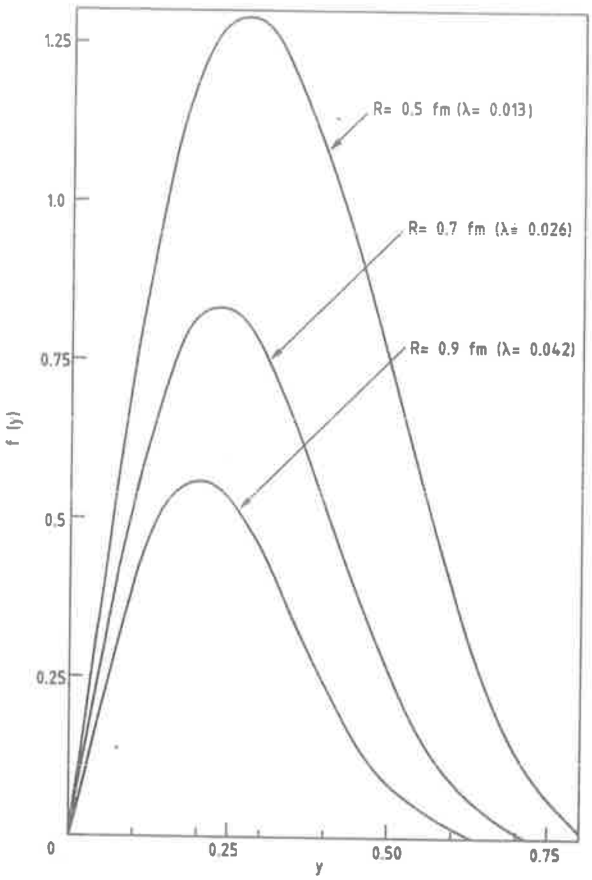

Fig. 2. The probability $f(y)$ of finding a pion carrying a fraction y of the momentum of the nucleon, for several values of the cut-off parameter λ (or bag radius R) – see eq. (3).

Integrating eq. (1) over x, we find that

$$\int_0^1 \delta F^{\mu}_{2N}(x)\,\mathrm{d}x = \left(\int_0^1 F^{\mu}_{2\pi}(\xi)\,\mathrm{d}\xi\right)\left(\int_0^1 \mathrm{d}y\, y f(y)\right) . \qquad (4)$$

There is now quite good data on the pion structure function from Drell–Yan [17–19], which gives $\int_0^1 F^{\mu}_{2\pi}(\xi)\,\mathrm{d}\xi = 0.15 \pm 0.04$. Further, from the definition of $f(y)$ we recognize the second integral on the right of eq. (4) as the average fraction of the nucleon momentum carried by the pion $-\langle y\rangle_\pi$. Finally, if as we argued above the SU(2) excess in the sea makes a contribution, $F_{2N,\mathrm{excess}}$, greater than that from the pion alone, we obtain the bound

$$\langle y\rangle_\pi \leqslant F_{2N,\mathrm{excess}}/(0.15 \pm 0.04) . \qquad (5)$$

As a simple estimate of the SU(3) breaking in the sea we take the estimate of Feynman and Field (FF) that for a free proton [20]

$$\bar{\mathrm{U}} = \int_0^1 \mathrm{d}x\, x\, \bar{\mathrm{u}}(x) = 0.015\,, \tag{6a}$$

$$\bar{\mathrm{D}} = \int_0^1 \mathrm{d}x\, x\, \bar{\mathrm{d}}(x) = 0.021\,, \tag{6b}$$

$$\bar{\mathrm{S}} = \int_0^1 \mathrm{d}x\, x\, \bar{\mathrm{s}}(x) = 0.011\,. \tag{6c}$$

In fact, their estimate of the excess of $\bar{\mathrm{D}}$ over $\bar{\mathrm{U}}$ was based on the Gottfried sum rule [15,20]

$$\int_0^1 \frac{\mathrm{d}x}{x}\,[F_2^{\mathrm{ep}}(x) - F_2^{\mathrm{en}}(x)] = \frac{1}{3} + \frac{2}{3}\int_0^1 [\bar{\mathrm{u}}(x) - \bar{\mathrm{d}}(x)]\,\mathrm{d}x\,, \tag{7}$$

where the left-hand side was taken as 0.27. The CFS group at Fermilab has found supporting evidence for $\bar{\mathrm{D}} > \bar{\mathrm{U}}$ by about 30% on the basis of the Drell–Yan process in proton-nucleus collisions [21].

In contrast, there is essentially no direct evidence to support the claim that $\bar{\mathrm{S}} < \bar{\mathrm{U}}$. Of course, the CDHS group has reported that [22]

$$2\bar{\mathrm{S}}/(\bar{\mathrm{U}} + \bar{\mathrm{D}})|_{\mathrm{Fe}} = 0.52 \pm 0.09\,, \tag{8}$$

but this is in Fe, where the EMC results have shown that the sea is modified [12]. It is nevertheless striking that taken at face value FF would predict $2\bar{\mathrm{S}}/(\bar{\mathrm{U}}+\bar{\mathrm{D}}) = 0.61$. If one attributes all of the $\mathrm{SU(3)_F}$ breaking in the nucleon to pionic effects, which are doubled in Fe, this ratio becomes 0.44. Clearly the FF estimate of $\mathrm{SU(3)_F}$ breaking is near to being correct and we shall use it to estimate $F_{2\mathrm{N,excess}}$. However, high statistics di-muon data on $\mathrm{H_2}$ and $\mathrm{D_2}$ to really pin down S and $\bar{\mathrm{S}}$ would be extremely valuable, and should be given a high priority.

Clearly the pion exchange process of fig. 1 does predict that the excess of $\bar{\mathrm{D}}$ to $\bar{\mathrm{U}}$ should be in the ratio 5 to 1 in the proton. To first order this is consistent with eqs. (6a) and (6b). If one were less circumspect this might suggest that some other mechanism must provide an equal excess of $\bar{\mathrm{U}}$ and $\bar{\mathrm{D}}$ over $\bar{\mathrm{S}}$ of about 20%. While this would sharpen our bound considerably we shall take the more conservative approach, and use all of the excess of non-strange over strange quarks $[(\bar{\mathrm{U}}+\bar{\mathrm{D}})/2 - \bar{\mathrm{S}}] = 0.007$ to estimate $F_{2\mathrm{N,excess}}$. A trivial calculation then yields $F_{2\mathrm{N,excess}} = (10/9) \times 0.007 = 0.008$. Together with eq. (5) this gives the required bound on the average fraction of the momentum of the nucleon carried by the pion

$$\langle y \rangle_\pi \leqslant 5 \pm 1.5\%\,. \tag{9}$$

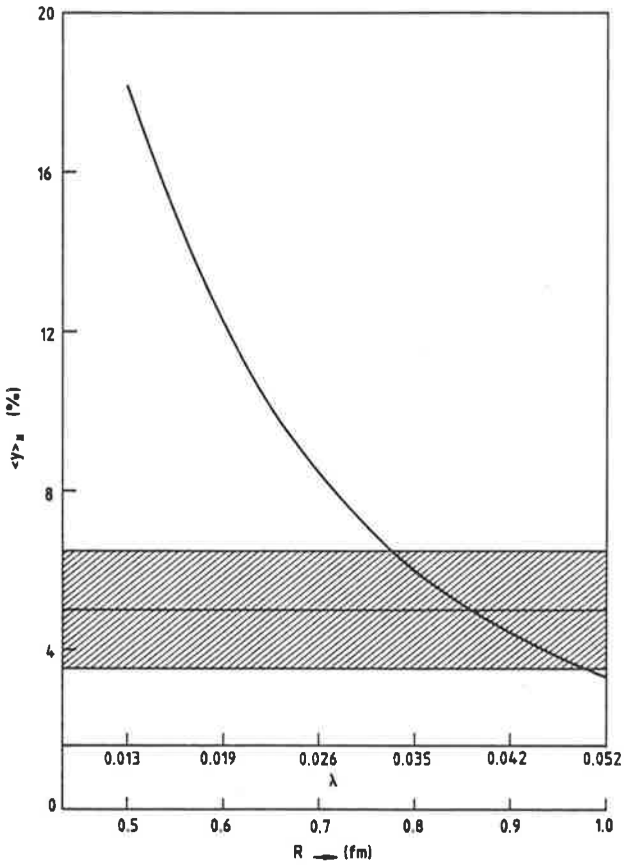

Fig. 3. The average fraction of the nucleon's momentum carried by the pion as a function of λ (or bag radius R). The shaded area represents the bound obtained in this work.

In fig. 3 we show the average fraction of the momentum of the nucleon carried by pions, $\langle y \rangle_\pi$, as a function of the cut-off parameter, λ, at the NNπ vertex [see eq. (3)]. Clearly eq. (9) is a very strong constraint on that parameter. It is not possible to accept a value of λ smaller than $0.039^{+0.012}_{-0.006}$.

We also show in fig. 3 the CBM radius corresponding to each value of λ. The lower bound on the bag radius in the CBM implied by eq. (9) is $R = 0.87 \pm 0.10$ fm. Of course, there are many defects in the static bag model, and one cannot insist too strongly on an absolute value of R. One expects the bag to have some surface thickness [23–25], and this together with centre-of-mass and recoil corrections could change the simple relationship between R and λ. Nevertheless, we expect this upper bound to be a good indication of the size of the region within which quarks are confined in the nucleon. The concept of a little bag with a size of order 0.3–0.5 fm is definitely excluded.

Let us briefly consider some objections which might be raised to the present calculation. For example, we have considered only that graph involving a direct interaction between the γ^* and the pion. There are a number of other graphs which should be included in principle. While those graphs could have important effects on the valence distribution, the small-x or sea contribution should be adequately described by fig. 1. Secondly, we have neglected the off-shell behaviour of the pion structure function ($t \neq -m_\pi^2$). However, the typical momentum of the virtual pion is $\sqrt{t} \sim$ 300 MeV/c, which is not far off-shell.

One might also ask whether we have a rigorous upper bound. It is logically possible that the strange sea could be bigger than the non-strange sea apart from pionic effects. However, with m_s some 200 MeV heavier than $m_{u,d}$, no one has ever suggested a physical mechanism for it. Finally, all our arguments relied on integrated quantities, such as $\bar{D}$, $\bar{U}$ and $\bar{S}$. This was unavoidable in view of the lack of experimental information about the shapes of $s(x)$, $\bar{s}(x)$ and $[\bar{d}(x) - \bar{u}(x)]$. It is therefore appropriate to close with a plea for better measurements of these three quantities in the free proton. Such data would not only allow us to sharpen the bound on $\langle y \rangle_\pi$ considerably, but would also deepen our understanding of how chiral symmetry is realized in nature.

I would like to thank Chris Llewellyn Smith for informative discussions related to this work.

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PIONIC CORRECTIONS AND THE EMC ENHANCEMENT OF THE SEA IN IRON

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It is shown that the EMC enhancement in the region $x < 0.3$ can be explained by an increase in the pion field in iron if there is an attractive force which mixes nucleon-hole and delta-hole states at momenta of order 300–400 MeV/c. Some experimental consequences of this explanation are discussed.

The recent report [1] by the European Muon Collaboration (EMC) of a significant difference between the structure function ($F_2^{\mu N}$) of a "nucleon" in Fe and D, has been confirmed at SLAC [2]. In view of the enormous range of momentum covered by the experiments ($3.3 \leqslant Q^2 \leqslant 170$ GeV^2), this is almost certainly not a higher twist effect. It has been suggested by Llewellyn Smith [3] that the observed enhancement of the ratio $F_2(\mathrm{Fe})/F_2(\mathrm{D})$ at small values of x ($= Q^2/2m_N\nu < 0.3$) could be the result of an increase in the number of pions per nucleon. This suggestion is of considerable interest in medium energy physics where the study of spin–isospin excitations, in particular mediated by the pion, is a central occupation.

Our aim is to investigate this question quantitatively. Let us begin with the observation by Sullivan [4] that the pion exchange process shown in fig. 1a contributes to $F_2(x)$ for a free nucleon. (The parton which is struck is part of the internal structure of the pion, while the recoiling nucleon remains intact.) For a pion at rest this contribution is restricted to $x < m_\pi/m_N$, but a full calculation gives the following contribution to the nucleon structure function

$$\delta F_2^{\mu N}(x,Q^2) = \int_x^1 f(y) F_2^{\mu\pi}(x/y,Q^2)\,\mathrm{d}y, \qquad (1)$$

where $F_2^{\mu\pi}$ is the (electromagnetic) structure function of the pion, and $f(y)$ is the probability that the pion carries a fraction y of the momentum of the nucleon in an infinite momentum frame. It is given by [3,4]

$$f(y) = \frac{3g^2}{16\pi^2} y \int_{m_N^2 y^2/(1-y)}^{\infty} \frac{t|F(t)|^2\,\mathrm{d}t}{(t+m_\pi^2)^2}, \qquad (2)$$

where $-t = w^2 - \boldsymbol{q}^2$ is the four-momentum squared of the exchanged pion, $g = 13.5$ is the coupling con-

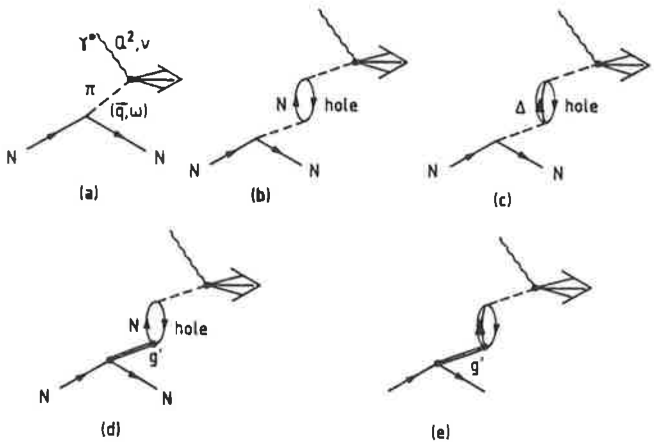

Fig. 1. Illustration of (a) the basic pion contribution to the nucleon structure function (the $\gamma^*\pi$ vertex involves the structure function of the pion itself); (b) and (c) other coherent processes involving pion rescattering in the nucleus which lead to enhancement for $|\boldsymbol{q}| \sim 300$–$400$ MeV/c; (d) and (e) the phenomenological short-range repulsion which damps the enhancement arising from (b) and (c).

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stant, and $F(t)$ the form factor, at the NNπ vertex. We take the latter to be $\exp[-\lambda(t+m_\pi^2)/m_\pi^2]$. (In the Cloudy Bag Model [5,6] λ is related to the bag R by $\lambda = 0.106\, m_\pi^2 R^2$.) It is worth noting that $\delta F_2^{\mu N}$ shows the same scaling properties as $F_2^{\mu\pi}$.

Following the argument of ref. [3] we observe that eq. (1) implies that (at $x = 0$)

$$\delta F_2^{\mu N}(0,Q^2)/F_2^{\mu\pi}(0,Q^2) = \int_0^1 f(y)\,\mathrm{d}y = N_\pi , \qquad (3)$$

where N_π is the number of pions per (free) nucleon. It is then easily seen that in the nuclear case the EMC enhancement of about 15% at small x [‡1] would be explained if there were an extra (6–10) pions in Fe. (It is natural to attribute the effect to Fe and not D, which is a dilute system.) This possibility seems quite natural to a nuclear physicist.

While the ultimate description of the nucleus will probably be based on QCD [5,6], the idea that nucleons are close packed but non-overlapping most of the time, suggests that it is their long range structure – the pion cloud – which will be most distorted. Certainly it is quite a simplification to attribute the whole change in the sea to $q\bar{q}$ pairs in the form of pions. However, there is a qualitative difference in the range of pionic corrections ($m_\pi^{-1} \sim 1.4$ fm) as compared with any other possible fluctuations [mass scale $|O(m_\rho^{-1}) \equiv O(2\omega_{1,-1}/R) \sim 0.2$ fm].

Because we are dealing with a quasi-free process the exchanged pion in fig. 1a is spacelike ($\omega \sim q^2/2m_N \ll |\boldsymbol{q}|$). In addition, the low momentum region contributes only for small y, where $f(y)$ is suppressed by an explicit factor of y. Large momenta are suppressed by the pion propagator and the form factor. Thus the most important momenta for δF_2 are typically 300–400 MeV/c. This is precisely the region where a significant enhancement of the nuclear pion field has been predicted for many years [10–12] – so far without experimental confirmation.

In a nucleus the emission of a pion with momentum 300–400 MeV/c can involve more than one nucleon. For example it could be emitted by a correlated pair of nucleons, leaving the nucleus in a state with two particles knocked out of the Fermi sea (a two-particle–two-hole, 2p–2h, state). Another possibility is that the final nuclear excitation is of the one-particle–one-hole, 1p–1h, type, but the emitted pion scatters from a second nucleon before being hit by the photon. In this case it can either lift the nucleon above the Fermi sea (fig. 1b) – a NN^{-1} excitation) or turn it into a Δ (ΔN^{-1} excitation – fig. 1c). This can happen any number of times, and in the absence of further physics would lead (even at nuclear density, $\rho_0 = 0.17\ \mathrm{fm}^{-3}$) to a pion condensed state [13] – that is a long range spin–isospin ordering in the region $\omega = 0$, $|\boldsymbol{q}| \approx 400$ MeV/c.

A more realistic calculation recognizes the existence of repulsive, short range, NN, NΔ and $\Delta\Delta$ interactions (see figs. 1d and 1e). They are conventionally parameterized as a contact interaction of strength g', the Landau–Migdal parameter – see ref. [12]. This repulsion kills most of the enhancement that would be generated by the rescattering processes of figs. 1b, 1c and their iterations. As a consequence the density for pion condensation moves well above nuclear matter density ($\rho_c > 3\rho_0$). Nevertheless some enhancement of the pion field in the region $|\boldsymbol{q}| \sim 400$ MeV/c should survive.

All of these processes leading to pion emission, as well as Pauli blocking of final states below the Fermi surface, can be incorporated through the longitudinal (i.e. $\boldsymbol{\sigma}\cdot\boldsymbol{q}$ coupling) spin–isospin response function [12,14,15]. We take a nuclear matter description which is expected to be good for inclusive reactions on finite nuclei at large momenta. (Momenta less than 200 MeV/c, where such a description is dubious, are quite unimportant.)

The pion distribution function (per nucleon) in a nucleus then has the form

$$\tilde{f}(y) = \frac{3g^2}{16\pi^2}\, y \int_{m_N^2 y^2}^{\infty} \int_0^{q - m_N y} \mathrm{d}\omega\, \frac{q^2 |F(q^2)|^2 R(q,\omega)}{(t+m_\pi^2)} . \qquad (4)$$

Here $R(q,\omega)$ is the response to a longitudinal spin–isospin excitation of momentum q and frequency ω – normalized in such a way that the expression for a free nucleon is recovered in a dilute system. We

‡1 We add the caution that the $x \to 0$ limit is artificial in this model because we ignore a number of effects there. Processes with a recoiling baryon (e.g., Δ) in fig. 1a, which can be large [4] for $x < 0.05$ are omitted. Also, shadowing may be important at small x, and this may depend on A [7,8].

have made the replacement $t \to q^2$ $(\omega \ll q)$ in order to match pion–nucleus phenomenology. Finally, because the response function guarantees that only on-shell final nuclear states contribute, the limits of integration are simply given by the condition that $q^2 > q_3^2$, where q_3 is the component of pion momentum along the direction of the photon. With $y = (q_3 - \omega)/m_N$ this implies $q > \omega + m_N y$.

In the random phase approximation (RPA), including only 1p–1h excitations the response function (per nucleon) is [12,14,15]

$$R(q,\omega) = -(3\pi/k_F^3)\,\mathrm{Im}\,\{(2m_N^2/g^2\Gamma^2(q^2)q^2)\Pi^0(q,\omega)/\{1 - [g' - q^2/(t+m_\pi^2)]\,\Pi^0(q,\omega)\}. \tag{5}$$

Here k_F is the Fermi momentum, and $\Pi^0(q,\omega)$ is the self-energy of the pion arising from NN^{-1} and ΔN^{-1} excitations (respectively Π_N^0, fig. 1b, and Π_Δ^0, fig. 1c). {The effects of anti-symmetrization in the ring approximation [eq. (5)] must be incorporated in the empirical value of g'.} To be explicit, Π_N^0 is given by

$$\Pi_N^0(q,\omega) = 2(g^2/4m_N^2)\Gamma^2(q^2)q^2\Pi_{FW}^0(q,\omega), \tag{6}$$

where Π_{FW}^0 is given by Fetter and Walecka [16]. We have also allowed an effective vertex function, $\Gamma(q^2)$, in the baryon–baryon interaction, which is not necessarily the same as $F(q^2)$ – the latter being a property of one nucleon. {As usual we take $\Gamma(q^2)$ to be quite hard [11,12], a dipole of mass 1.67 GeV.}

For Π_Δ^0 we have used an approximation from pionic atoms [17,18]

$$\Pi_\Delta^0(q,\omega) = -4\pi c_0 \rho \Gamma^2(q^2)q^2, \tag{7}$$

with $c_0 = 0.22\ m_\pi^{-3}$. Because this is purely real, the region of response for 1p–1h excitations in the residual nucleus is restricted to NN^{-1} alone. The ΔN^{-1} excitations which lie about 300 MeV higher are ignored. They also have a many-body character, and could in principle modify F_2. However the cut-off condition $\omega < q - My$ restricts this contribution mainly to small x ($x < 0.05$ for a free nucleon) [4].

In eq. (5) we have used a single value of g', however the forces g'_{NN}, $g'_{N\Delta}$ and $g'_{\Delta\Delta}$ play different roles, and need not be equal. After the ω integration the enhancement of $\tilde{f}(y)$ over $f(y)$ comes mainly because of mixing of the ΔN^{-1} states with NN^{-1}. This is controlled by $g'_{N\Delta}$. The mixing between NN^{-1} states (controlled by g'_{NN}) distorts the response from that of a free Fermi gas – enhancing it at small ω and quenching it for large ω. However, because we are far from a pion condensed state the average effect is small. We are therefore mainly sensitive to $g'_{N\Delta}$ and to a lesser extent to $g'_{\Delta\Delta}$. In the final analysis we have adopted a conservative attitude and taken $g'_{N\Delta}$ and $g'_{\Delta\Delta}$ as large as g'_{NN} – that is $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} \equiv g' = 0.7$ ‡2. The numerical sensitivity to the input parameters will be discussed later.

The pionic contribution to the effective structure function of a nucleon in a nucleus is given by an equation analogous to eq. (1), with $f(y)$ replaced by $\tilde{f}(y)$. In principle y could run from x to A in a nucleus, because a single pion could carry all the momentum of the nucleus. However, in practice $\tilde{f}(y)$ is very small for $y \gtrsim 0.5$. The pion structure function [19,20] is

‡2 In the process considered here we are sensitive to $q \sim 400$ MeV/c, where the values of g' can differ from those at $q = 0$.

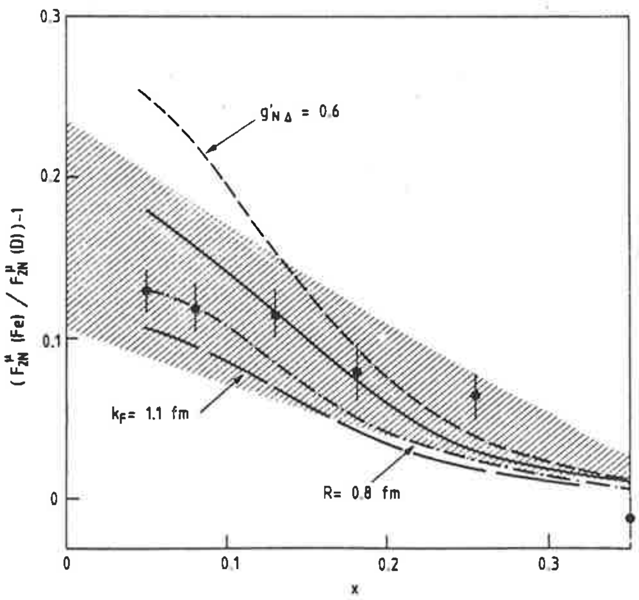

Fig. 2. The fractional increase in the ratio of the structure function in Fe compared with D, as a function of x $(= Q^2/2m_N\nu)$, caused by the multi-nucleon pion emission graphs of figs. 1b–1e. The data are from the EMC collaboration [1], and the shaded area indicates possible systematic errors. The standard input (solid curve) for Fe is $k_F = 1.30\ \mathrm{fm}^{-1}$, $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} = 0.7$, a bag radius of 0.7 fm in $F(q^2)$, and $\Gamma(q^2)$ is a dipole of mass 1.67 GeV. We show in the other curves the effect of altering any single one of these parameters.

taken to be constant for $x < 0.3$ and then to go as $(1-x)^2$.

The results of the calculation are displayed in fig. 2, which shows the fractional increase in the effective nucleon structure function, together with the experimental data. Both the magnitude and the x dependence of the effect are well reproduced. The value of the Fermi momentum of Fe was taken as $k_F = 1.30\ \mathrm{fm}^{-1}$. We have also checked that the deuteron does not contribute by more than 1% to the modification of the ratio. (In the deuteron the major modification arises from the interference term between the probability amplitudes for pion emission by the two nucleons, an interference which vanishes in a spin saturated nucleus.)

The sensitivity of the calculation to the input parameters is fairly strong. Consider first the Landau–Migdal parameters. Lowering $g'_{N\Delta}$ to 0.6 from 0.7 raises $\delta F_2(x)$ uniformly by about 30%. (For $g'_{N\Delta} = 0.4$ the enhancement is doubled.) The sensitivity to g'_{NN} and $g'_{\Delta\Delta}$ is weaker for the reasons given above, lowering either of these to 0.6 gives an increase of only 10% in $\delta F_2(x)$. Finally if g' is raised beyond 0.8 the enhancement is killed.

The πNN form factor $F(q^2)$ also plays a crucial role. In fig. 2 we show the effect of increasing λ from 0.026 to 0.035 (i.e. from $R = 0.7$ to 0.8 fm). This leads to a 25% reduction of the effect. Notice that if the last NNπ vertex $[F(q^2)]$ were made as hard as the effective vertex function $\Gamma(q^2)$ the effect would be more than doubled. However, such hard vertex functions for a single nucleon are ruled out by deep-inelastic scattering data [21]. We consider the results for $R \sim 0.7$–0.8 fm the most reliable.

We have also estimated the role of the 2p–2h excitations discussed earlier. Since these have no counterpart for a free nucleon they certainly participate in the enhancement of F_2. In order to estimate this effect we have used the relationship between 2p–2h excitations and the absorptive part of the p wave pionic atom potential discussed in ref. [22]. Using a reasonable extrapolation of this parameter with energy and momentum, we find that the 2p–2h excitations can easily provide an extra enhancement of 50% over that obtained with the standard parameter set. More details of this estimate will be given in a longer report [23].

In this study of the EMC effect we have concentrated entirely on the small x enhancement. The large x region is both exciting and intrinsically more difficult to deal with. For example, the Fermi motion corrections are extremely model dependent [1,3]. In addition, as observed by Llewellyn Smith [3], because of momentum conservation the presence of additional pions in the nucleus must soften the valence quark distribution. Finally, Jaffe has suggested that the presence of six-quark bags in nuclei would modify the large x behaviour in a way which agrees qualitatively with the data [24].

Let us now summarize. We have shown that the EMC enhancement in the region $x < 0.3$ can be explained if there is an attractive force which mixes NN^{-1} and $\Delta\mathrm{N}^{-1}$ states at momenta of order 300–400 MeV/c. Although it has long been anticipated there has hitherto been no experimental hint that this attractive force exists. Experiments in preparation at LAMPF on the polarization transfer in $(\vec{p},\vec{p}\,')$ inclusive scattering should help resolve this issue [25].

In the meantime one important test of this mechanism would be provided by its A dependence. Since Π^0_Δ is linear in the density, we expect that the enhancement should also be roughly linear. For example, in fig. 2 we show the effect of lowering k_F to $1.1\ \mathrm{fm}^{-1}$, which is appropriate [26] for ^{12}C. This cuts the enhancement in half. Even for ^{20}Ne ($k_F \sim 1.2\ \mathrm{fm}^{-1}$) [26] the enhancement is reduced by some 30%.

We also note that the enhancement of the sea proposed here is in the form of pions and therefore is not $SU(3)_F$ invariant. {This has been used in ref. [21] to put limits on the cut-off parameter, λ (or R), for a free nucleon.} The CDHS data [27], which suggest a greater excess of $\bar{u}$ and $\bar{d}$ over $\bar{s}$ in Fe than in hydrogen, are consistent with this [21], but the errors are too large to permit a firm conclusion. Finally, we note that in nuclei with a large neutron excess (e.g. ^{208}Pb), the isovector character of the πN interaction means that $\bar{u}d(\pi^-)$ pairs are favoured over $u\bar{d}(\pi^+)$. This may have a signature in inclusive hadron production.

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Pionic corrections and multi-quark bags

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Abstract. We investigate the influence of pionic corrections on multi-quark hadrons. After determining the bag parameters from ordinary baryons and mesons, we discuss the implications for six-quark configurations.

1. Introduction

In this paper we wish to investigate the masses of multi-quark bag states, including the contribution from the pion field. For the ordinary baryons and mesons these contributions have been shown to amount to 100 MeV or more (Jaffe 1979, Théberge *et al* 1980, Thomas *et al* 1981, Cottingham *et al* 1981, Myhrer *et al* 1981, Thomas 1983). Furthermore, the pion energy provides another mechanism for splitting the N and Δ, Σ and Λ, and so on—a job usually reserved for the one-gluon-exchange interaction (De Rujula *et al* 1975, DeGrand *et al* 1975). Both because of its size and its spin dependence one expects substantial changes in the parameters of the MIT bag model, and hence in the predictions for the more exotic multi-quark bags.

In order to determine the new bag-model parameters we shall use the spherical, static cavity approximation (DeGrand *et al* 1975). The pionic corrections will be treated as a perturbation to the masses found by applying the non-linear boundary condition ($\partial M/\partial R = 0$) to the rest of the bag energy. In this way we avoid the collapse of the bag (Vento *et al* 1980), caused by the $R^{-3.5}$ behaviour (Théberge 1982, Théberge and Thomas 1982a, b) of the attractive pionic self-energy term. Such a term would be dominant at small R, driving the overall bag mass to zero. Since the chiral bag models neglect the finite size of the pion itself, the calculations for small R cannot be trusted (De Kam and Pirner 1982). Moreover, it seems unlikely on the basis of QCD that the pion should play a major role in determining hadronic sizes. Clearly in our work the bag size is still determined by B, the energy density required to make a bubble in the QCD vacuum (Thomas 1983, DeGrand *et al* 1975).

There is one other uncertainty in determining the bag parameters—that is deciding which masses to use for the unstable hadrons. Previous bag-model calculations have usually used the resonance energy (in, say, a Breit–Wigner fit) for unstable hadrons. For example, the Δ is usually taken to have a mass of about 1.23 GeV. Of course, if one has a complete dynamical model for the background in a resonant system, the underlying resonance position can be determined unambiguously. This idea was illustrated by the cloudy-bag-model (CBM) description of the Δ resonance (Théberge *et al* 1980, Thomas *et al* 1981, Thomas 1983), although even there only the most important (Chew–Low) background terms were included. In general one would not expect to have such a clear idea of the most important background. Moreover, it would be impractical in a global fit of the kind which we are undertaking to first make a coupled-channel calculation for each unstable resonance.

A much simpler approach was proposed by Jaffe and Low (1979). They suggested identifying bag-model masses as 'primitives', or poles in the P matrix rather than the S matrix. In the case of the Δ there is a large shift from the resonance position to the P-matrix pole (Moniz 1982; see also Heller *et al* 1983). For instance, with a matching radius of the order 1.3 fm, the P-matrix pole of the Δ occurs at 1.31 GeV. The value of 1.31 GeV, however, takes into account only the open Nπ channel. If one were to include closed channels, like $\Delta\pi$, the shift would be even greater. Thus, the analysis using the P-matrix formalism is also model dependent. Furthermore, the primitive masses for stable particles like the nucleon are also shifted because of closed channels.

In view of these ambiguities we have decided to follow the usual practice of using observed resonance positions in the determination of bag-model parameters. To some extent this pragmatic approach is supported by the CBM analysis of the Δ. There one could unambiguously define the mass of the Δ bag, including pionic self-energy corrections, and it turned out to be very close to the observed resonance energy. This certainly does not establish the result in the general case, but it is indicative.

For the exotic, six-quark bags we do not know in general how to calculate their experimental consequences. Instead we shall compare our results with the predictions of the original MIT model (Jaffe 1977, Aerts *et al* 1978). Of special interest is the lowest double strange ($Y=0$) dibaryon (H dibaryon), which according to Jaffe's (1977) initial work should be bound by 80 MeV. Once pionic corrections are included this state moves much closer to threshold, and it is either unbound or very weakly bound. It may therefore be much harder to identify, which may explain why our experimental colleagues have not been successful in finding it (Pauli 1982; see also Aerts and Dover 1982).

2. Bag energy including pions—ordinary baryons and mesons

In the limit of a static, spherical cavity (DeGrand *et al* 1975, Thomas 1983) the usual expression for the energy of the MIT bag is

$$E(R)=E_{\mathrm{v}}+E_{\mathrm{Q}}+E_{\mathrm{M}}. \tag{2.1}$$

Here E_{v} is the energy required to make a hole in the vacuum (BV) minus a phenomenological term ($-Z_0/R$), originally attributed to zero-point energy (DeGrand *et al* 1975). More recently the latter has been associated with centre-of-mass (Donghue and Johnson 1980, Wong 1981, Carlson and Chachkhunashvili 1981, Thomas 1983) and colour electric (Chin *et al* 1982, Breit 1982a, b, Hansson and Jaffe 1982) contributions. We shall comment on the latter in the final section. However, all calculations have been performed in the same way as the original MIT bag model, using

$$E_{\mathrm{v}}=\tfrac{4}{3}\pi BR^3-(Z_0/R) \tag{2.2}$$

with Z_0 constant.

The quark kinetic energy is

$$E_{\mathrm{Q}} = \sum_i \frac{\varepsilon(m_i R)}{R} \tag{2.3}$$

where ε is the usual eigenfrequency of the lowest mode in the cavity resulting from the linear boundary condition, which is a function of the product of quark mass and bag radius, $\mu = m_i R$. We have

$$\varepsilon^2 = \mu^2 + x^2 \tag{2.4}$$

where x is a function of μ satisfying

$$\tan(x) = x/(1 - \mu - \varepsilon). \tag{2.5}$$

For massless quarks $\varepsilon = x = 2.043$.

The last term in equation (2.1) is the colour magnetic interaction associated with the exchange of a single gluon between two quarks inside the bag. It is given by the expression

$$E_{\mathrm{M}} = -\sum_{i>j} \alpha_{\mathrm{s}} \frac{M(m_i R, m_j R)}{R} (F^{\mathrm{c}}\boldsymbol{\sigma})_i \cdot (F^{\mathrm{c}}\boldsymbol{\sigma})_j \tag{2.6}$$

where α_{s} is the effective quark–gluon coupling constant, and F^{c} and $\boldsymbol{\sigma}$ are respectively the colour and spin of the quark. The function $M(\mu_i, \mu_j)$ is a wavefunction overlap. Its precise form was given by DeGrand *et al* (1975); for $\mu \leqslant 1.5$ it can be well approximated as

$$\begin{aligned} M(0, \mu) &\simeq 0.177 - 0.025\,\mu \\ M(\mu, \mu) &\simeq 0.177 - 0.043\,\mu. \end{aligned} \tag{2.7}$$

As explained in the introduction we obtain the masses of baryons and mesons from

$$M = \min_R \{E(R)\} + E_{\mathrm{p}} \tag{2.8}$$

where E_{p} is the pion self-energy. We have chosen to use the simple phenomenological form (Jaffe 1979)

$$E_{\mathrm{p}} = -\frac{1}{pR^3_{\mathrm{min}}} \sum_{i,j} (\boldsymbol{\sigma\tau})_i \cdot (\boldsymbol{\sigma\tau})_j \tag{2.9}$$

where p is an adjustable constant. This corresponds to keeping only intermediate states with quarks in the lowest radial state, and treating all such states as degenerate. The eigenvalues of the operator

$$\Sigma_{\mathrm{op}} = -\sum_{(i,j)\in(\mathrm{u},\mathrm{d})} (\boldsymbol{\sigma\tau})_i \cdot (\boldsymbol{\sigma\tau})_j \tag{2.10}$$

were given by Cottingham *et al* (1981) and Myhrer *et al* (1981) for the ordinary baryons and mesons—see also the appendix to this paper.

Using equation (2.8) we choose to fit the masses of the $\omega(782)$, N(939), $\Delta(1232)$ and $\Omega(1672)$, as well as the mass splitting of the Δ and Σ (77 MeV) in order to fix the five parameters of the model (B, Z_0, α_{s}, m_{s}, p)—as usual, $m_{\mathrm{u}} = m_{\mathrm{d}} = 0$. These parameters are given in table 1 together with the predictions for all the low-lying mesons and baryons. For the mesons the results are not very satisfactory, suggesting (not surprisingly) that we need

Table 1. Results for bag parameters and masses of baryons and mesons including pionic corrections (equations (2.9) and (2.10)). R_{min} is given in GeV^{-1}, other quantities in GeV. $B^{1/4}=0.151$ GeV, $Z_0=1.31$, $\alpha_s=1.41$, $m_s=0.218$ GeV, $p^{1/2}=1.49$ GeV.

Particle	R_{min}	E_v	E_Q	E_M	E_p	M	M_{expt}
N	5.058	0.025	1.212	−0.099	−0.199	0.939	0.939
Λ	5.034	0.020	1.339	−0.099	−0.127	1.132	1.116
Σ	5.034	0.020	1.339	−0.079	−0.071	1.209	1.193
Ξ	5.009	0.014	1.467	−0.088	−0.032	1.361	1.318
Δ	5.328	0.086	1.150	0.094	−0.098	1.232	1.232
Σ*	5.306	0.081	1.278	0.085	−0.060	1.383	1.385
Ξ*	5.283	0.076	1.405	0.076	−0.028	1.529	1.533
Ω	5.261	0.071	1.533	0.069	0.0	1.672	1.672
π	4.049	−0.178	1.009	−0.247	−0.163	0.420	0.138
η_n†	4.049	−0.178	1.009	−0.247	0.0	0.583	0.549(η)
η_s†	3.987	−0.190	1.262	−0.198	0.0	0.873	0.958(η')
K	4.018	−0.184	1.135	−0.218	−0.063	0.670	0.496
ρ	4.659	−0.060	0.877	0.072	−0.071	0.818	0.776
ω	4.659	−0.060	0.877	0.072	−0.107	0.782	0.782
φ	4.606	−0.070	1.128	0.055	0.0	1.113	1.020
K*	4.632	−0.065	1.003	0.062	−0.041	0.959	0.892

† The n and s denote the pure non-strange and pure strange η meson, respectively.

a more sophisticated treatment of the pionic corrections to the mesons. (The pion itself should really be excluded, and the η, η' problem is not unique to the bag model. However we show these for completeness.)

On the other hand, the resulting fit to the baryon spectrum is quite good. The size of the pionic correction, E_p, and the bag radii are in qualitative agreement with the results of Cottingham *et al* (1981) and Myhrer *et al* (1981). A number of interesting features of the parameters emerge. First, as discussed by a number of people (Théberge *et al* 1980, 1982, Thomas *et al* 1981, Cottingham *et al* 1981, Myhrer *et al* 1981), the colour coupling constant is significantly reduced; we find a reduction of about 35%. In addition the strange-quark mass is reduced to 218 MeV (from 280 MeV)—see also Théberge (1982) and Théberge and Thomas (1982a, b)—which is closer to the 150 MeV preferred by current algebra. Finally we note that the agreement between the phenomenological value of $p^{1/2}$, namely 1.49 GeV, and that computed on the basis of chiral symmetry (Jaffe 1979, Thomas 1983),

$$p^{1/2}=(\tfrac{400}{3}\pi)^{1/2} f_\pi/g_A = 1.52 \text{ GeV}, \tag{2.11}$$

is excellent.

We cannot resist the temptation to mention a fit to the P-matrix positions of baryons and mesons, although there are many questions involved about the procedure, as discussed in the introduction. Taking the simplest and least model-dependent approach we determine the P-matrix poles from scattering in open channels. Of the five particles we used to fit the baryon and meson spectra, only the (unstable) Δ has a P-matrix pole with a position different from the S-matrix pole. We thus fit to the ω(782), N(939), Δ(1310), Ω(1672), and the Λ–Σ mass difference (77 MeV). It is surprising—and maybe accidental—that an excellent fit is obtained with the parameters $B^{1/4}=0.169$ GeV, $Z_0=1.80$, $\alpha_s=1.69$, $m_s=0.181$ GeV and $p^{1/2}=1.85$. The kaon in this fit, for example, has a mass of 498 MeV and the Λ has a mass of 1109 MeV.

3. Bag-model predictions for the dibaryons

In order to calculate the masses of the bag states with baryon number 2, we follow the work of Mulders *et al* (1979). The colour magnetic interaction is approximated by

$$E_M = m\Delta_{op} \tag{3.1}$$

where $m(R)$ is the strength averaged over non-strange and strange quarks and Δ_{op} is the operator

$$\Delta_{op} = -\sum_{i>j} (F^c\sigma)_i \cdot (F^c\sigma)_j, \tag{3.2}$$

which for N quarks has the expectation value

$$\Delta = \tfrac{1}{4}N(10-N) + \tfrac{1}{3}S(S+1) + f_F^2 + \tfrac{1}{2}f_C^2. \tag{3.3}$$

Here S is the total spin and f_F^2 and f_C^2 are the eigenvalues of the SU(3) quadratic Casimir operators for flavour and colour.

Once again the pionic corrections are calculated after minimising the rest of the bag energy. They are give by

$$\Sigma_p = \Sigma_{op}/pR^3_{min} \tag{3.4}$$

where Σ_{op} was defined in equation (2.10). For N non-strange quarks the expectation value of Σ_{op} is given by (see equation (A.6))

$$\Sigma = +\tfrac{7}{3}N^2 - 28N + 8f_C^2 + 4S(S+1) + 4I(I+1) \tag{3.5}$$

where S and I are the total spin and isospin of the non-strange quarks and f_C^2 is the eigenvalue of the SU(3) quadratic Casimir operator for colour—again for the non-strange quarks only. As discussed in the appendix, equation (3.5) agrees with the result of Jaffe (1979) for $N=3$, but is different for larger N.

The results for the non-strange ($Y=2$) dibaryons are given in table 2. In this case the inclusion of pionic corrections (column B) does not greatly alter the original predictions based on the MIT model (Jaffe 1977, Aerts *et al* 1978) (column A). There is some tendency for the smaller colour coupling constant in case B to yield lower masses for dibaryons with large, positive Δ.

For the strange dibaryons the inclusion of pionic corrections is more complicated because we need to know the spin and colour of the non-strange quarks alone. One finds

Table 2. Masses of the non-strange ($Y=2$) dibaryons in the original MIT bag-model calculation (Jaffe 1977, Aerts *et al* 1978) (A) and in the present calculation including pionic corrections (B).

				A		B	
I	S	Δ	Σ	$R(\mathrm{GeV}^{-1})$	$M(\mathrm{GeV})$	$R(\mathrm{GeV}^{-1})$	$M(\mathrm{GeV})$
0	1	$\frac{2}{3}$	−76	6.60	2.16	6.41	2.18
1	0	2	−76	6.68	2.23	6.45	2.24
1	2	4	−52	6.79	2.35	6.52	2.36
0	3	4	−36	6.79	2.35	6.52	2.38
2	1	$\frac{20}{3}$	−52	6.93	2.50	6.61	2.46
3	0	12	−36	7.19	2.79	6.78	2.69

that the colour magnetic contribution E_M (equation (2.6)) and the pionic correction (equation (2.9)) do not commute. Instead of doing the full calculation of the mixing between all dibaryons with a given spin and isospin—total spin and isopin still do commute with E_M and E_p—we only indicate the minimum and maximum values of the correction. For the low-lying states of greatest interest these upper and lower estimates are very close. We show the calculated masses of the lowest $Y=1$ and $Y=0$ dibaryons in tables 3 and 4 respectively. By far the most dramatic change is the increase in mass of the lowest $Y=0$ dibaryon (H). While this is still the most interesting state to look for, it appears quite likely that it may not be bound.

There are two main reasons that we find to make the H dibaryon less bound once pion corrections are included—although actually all contributions to the energy do change. First, as mentioned earlier, the colour coupling constant α_s is significantly reduced; this reduces the colour magnetic attraction for the H dibaryon. Second, a free Λ receives some -130 MeV self-energy because of its pion cloud. The H dibaryon has a radius about 20% larger than the Λ. Because of the strong dependence of the pion self-energy on the bag radius, $\sim R^{-3}$, we expect the correction to be about half the correction for two Λ's (we actually find -110 MeV). The result is that the H has a higher mass than in the original MIT calculation where this pion correction was not included.

4. Discussion

The main results of this work are summarised in tables 2–4. By far the most significant result is the increase in mass of the lowest $Y=0$ dibaryon, discussed in detail in § 3. In this final section we will not repeat what has already been said. Instead, we shall make some brief comments on the possible relevance of our results to experiments, including the implications of recent work on the quark self-energy (Chin *et al* 1982, Breit 1982a, b, Hansson and Jaffe 1982).

There is little chance that there will be any dramatic experimental consequences of the non-strange states listed in table 2. They all lie far above the appropriate threshold—be it NN, NΔ (and NNπ) or $\Delta\Delta$ (and NN$\pi\pi$)—and will be very broad. This has been demonstrated explicitly within the framework of the P-matrix formalism (see, e.g., Mulders 1982). On the other hand, if the masses were lower—that is, near or below the thresholds—dibaryons might produce striking consequences in NN or πd scattering. In this case the small width of, for example, the $I=2$, $S=1$ state might compensate enough for its small (isospin violating) coupling to those channels to produce a clear signal over a narrow energy region. One is tempted to suggest such a possibility to explain the discrepancy between the recent SIN and LAMPF measurements of t_{20} in πd scattering (Grüebler *et al* 1982, Holt *et al* 1981)—although it is probably an experimental problem.

We would hesitate to even mention this unlikely possibility were it not for the recent

Table 3. Masses of lowest two $Y=1$ dibaryons in cases A and B (see caption to table 2).

				A		B	
I	S	Δ	Σ	$R(\mathrm{GeV}^{-1})$	M(GeV)	$R(\mathrm{GeV}^{-1})$	M(GeV)
$\frac{1}{2}$	1	$-\frac{7}{3}$	$-67/-57$	6.38	2.16	6.28	2.20/2.22
$\frac{1}{2}$	2	-1	$-57/-39$	6.47	2.23	6.33	2.27/2.31

Table 4. Mass of lowest $Y=0$ dibaryon (H) in cases A and B (see caption to table 2).

				A		B	
I	S	Δ	Σ	$R(\mathrm{GeV}^{-1})$	$M(\mathrm{GeV})$	$R(\mathrm{GeV}^{-1})$	$M(\mathrm{GeV})$
0	0	−6	−56/−48	6.09	2.15	6.11	2.22/2.23

appearance of a mechanism which might conceivably produce a downward shift in mass for exotic states—with respect to our calculation.

It has been argued by Chin *et al* (1982), and by Breit (1982a, b), on the basis of the soliton bag model (Friedberg and Lee 1978, Goldflam and Wilets 1982), that the $-Z_0/R$ term (equation (2.2)) in the usual MIT model results from the quark self-energy. The constant Z_0 is then given by

$$Z_0 = N\lambda\alpha_s \tag{4.1}$$

where N is the number of confined quarks and λ is some—still controversial (Chin *et al* 1982, Breit 1982a, b, Hansson and Jaffe 1982)—number. In Breit's work $\lambda = 0.25$, which for baryons and with the MIT value of $\alpha_s = 2.2$ is in excellent agreement with $Z_0 = 1.8$. Clearly if all of the phenomenological $-Z_0/R$ term were interpreted this way, we would get twice as big a contribution for dibaryon states (with $N=6$ instead of 3), a reduction in energy of typically 100–200 MeV.

On a more realistic level, it seems to us that a fairly convincing case can be made that centre-of-mass corrections contribute of the order 0.6–0.8 to Z_0 for the usual baryons (Myhrer *et al* 1981, Donoghue and Johnson 1980, Wong 1981, Carlson and Chachkhunashvili 1981). With our coupling constant, $\alpha_s = 1.4$, and $Z_0 = 1.31$ a value $\lambda \approx 0.15$ is required. If this combination of effects is indeed the origin of the infamous Z_0 term, there will be no major change in the energies of the exotic bag states from those given in tables 2–4. A more precise statement than this will have to wait until we understand all of the difficulties—centre-of-mass corrections, quark and gluon self-energies and the appropriate mass parameter for comparison with experiment—much better than we do presently.

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Appendix

In order to find the expectation value of the spin–isospin operator Σ_{op} appearing in equation (2.10) we use the antisymmetry of the full N-quark wavefunction. That is, we use the fact that (for $i \neq j$)

$$P^C_{ij} P^I_{ij} P^S_{ij} = -1 \tag{A.1}$$

and hence

$$P^C_{ij} = -P^I_{ij} P^S_{ij} \tag{A.2}$$

where the P_{ij} are permutation operators for colour, isospin, and spin (Mulders *et al* 1979):

$$\begin{aligned} P^C_{ij} &= \tfrac{1}{3} + 2F^c_i \cdot F^c_j \\ P^I_{ij} &= \tfrac{1}{2}(1 + \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \\ P^S_{ij} &= \tfrac{1}{2}(1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j). \end{aligned} \tag{A.3}$$

Substituting (A.3) into (A.2) we find (for $i \neq j$)

$$(\boldsymbol{\sigma\tau})_i \cdot (\boldsymbol{\sigma\tau})_j = -\tfrac{7}{3} - 8F^c_i F^c_j - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j - \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j. \tag{A.4}$$

Finally, using

$$(\boldsymbol{\sigma\tau})_i \cdot (\boldsymbol{\sigma\tau})_i = 9, \tag{A.5}$$

we obtain the desired result

$$\left\langle -\sum_{i,j} (\boldsymbol{\sigma\tau})_i \cdot (\boldsymbol{\sigma\tau})_j \right\rangle = \tfrac{7}{3}N^2 - 28N + 8f^2_C + 4S(S+1) + 4I(I+1) \tag{A.6}$$

where f^2_C is the eigenvalue of the colour, quadratic Casimir operator.

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Further studies of convergence in the cloudy-bag model

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Abstract. We generalise an earlier bound on the virtual-pion content of the physical nucleon to include momentum conservation through the non-relativistic recoil of the baryon core. In addition, we discuss the possibility, and physical meaning, of obtaining such a bound for virtual ρ mesons.

1. Introduction

At the present time there is a great deal of interest in the development of phenomenological models which link conventional nuclear physics with the quark substructure of the nucleon (Baym 1979, Miller *et al* 1981, Thomas 1982a, b). One of the more interesting of these models which has proven successful in a number of applications (Théberge *et al* 1980, 1982, Thomas 1981, Thomas *et al* 1981, Théberge and Thomas 1982a) is the cloudy-bag model (CBM). In an earlier study (Dodd *et al* 1981, hereafter referred to as I) we examined the convergence properties of the static CBM and obtained rigorous bounds on the probability of finding n pions in the cloud about the nucleon (MIT bag) core, as well as a bound on the average number of pions itself.

In this paper we first present a generalisation of the results of I to the case where the baryon bag is allowed to recoil non-relativistically in such a way that overall momentum conservation is guaranteed. (A preliminary account of this work was given at the Versailles conference—see Alvarez-Estrada and Thomas (1981).) Note that we do not tackle the outstanding problem of spurious CM motion in the bag (Wong 1981, Carlson and Chachkhunashvili 1981, Betz 1982) and the associated corrections. We assume that such corrections would not change the basic shape of the static NNπ vertex function, although they may alter the effective bag radius.

An additional restriction of the present analysis is that the possible dependence of the NNπ vertex function on nucleon momentum has not been included. As a consequence, the Hamiltonian used here is not Galilean invariant. Let us only make the observation that it is by no means obvious that the non-relativistic limit of an operator in a Lorentz-invariant theory should be Galilean invariant (Friar 1974)—consider, for example, the difference in the reductions of the pseudoscalar and pseudovector forms. Intuitively one would expect such corrections to be of order $(m_\pi/m_{\rm N})$, and therefore that they would not alter our conclusions. However, we have not been able to prove this. To summarise, our aim in § 2 is merely to modify the CBM Hamiltonian in baryon space to ensure momentum conservation. The resulting bounds are identical with those proven in I, and constitute an improvement with respect to those of Alvarez-Estrada and Thomas (1981).

In the second part of this paper we address the question of vector-meson coupling to the bag. Under the assumption that it makes physical sense to talk about a virtual ρ-meson cloud surrounding the extended core (which has also been made more-or-less explicitly by other authors), we are able to obtain bounds analogous to those found for pions. Such bounds will turn out to be consistent, essentially, with previous phenomenological estimates of the ρ-meson contribution in various situations by other authors. At a more fundamental level, some arguments are also presented which question the real physical meaning of such ρ-meson couplings.

2. CBM with momentum conservation

One of the attractive features of the CBM for the purposes of intermediate-energy applications is that we arrive very naturally at a Hamiltonian involving only pion and baryon degrees of freedom. The considerations at the quark level serve to constrain the parameters of the theory—bare masses, coupling constants and form factors. Since the MIT-bag model itself deals with a static, fixed bag, the original CBM necessarily involved a static baryon source. The simplest extension of such a model to guarantee total three-momentum conservation is

$$H_{\mathrm{R,CBM}} = H_{\mathrm{R},0} + H_{\mathrm{R,I}} \tag{2.1}$$

$$H_{\mathrm{R},0} = \sum_{\substack{\alpha=\mathrm{N},\Delta \\ p}} [m_\alpha + (\boldsymbol{p}_\alpha^2/2m_\alpha)]\alpha_p^+ \alpha_p + \sum_k \omega_k a_k^+ a_k \tag{2.2}$$

$$H_{\mathrm{R,I}} = \sum_{k,p} \sum_{\alpha,\beta=\mathrm{N},\Delta} \alpha_{p+k}^+ \beta_p a_k v_k^{\alpha\beta} + \mathrm{HC} \tag{2.3}$$

$$\omega_k = (k^2 + \mu^2)^{1/2}. \tag{2.4}$$

Here m_α, $\boldsymbol{x}_\alpha$ and $\boldsymbol{p}_\alpha$ are respectively the bare mass, position and momentum operators of the baryon bag state of type α ($=\mathrm{N}, \Delta$), a_k (a_k^+) destroys (creates) one pion of momentum $\boldsymbol{k}$, energy ω_k and isospin projection j ($k \equiv (\boldsymbol{k}, j)$) and $v_k^{\alpha\beta}$ is the bare interaction matrix element for the pion α–β vertex. (Its detailed expression is given by Théberge *et al* (1980, 1982), Thomas *et al* (1981), Miller *et al* (1981) and Thomas (1982a, b).) As we discussed in the first section, we ignore the possible dependence of this vertex on the momentum of the baryon, and the model is therefore not Galilean invariant (Friar 1974). Finally, α_p (α_p^+) destroys (creates) a bag of type α with momentum p.

The total momentum operator is given by

$$\boldsymbol{P}_{\mathrm{R}} = \sum_p \sum_{\alpha=\mathrm{N},\Delta} \alpha_p^+ \alpha_p \boldsymbol{p} + \sum_k \boldsymbol{k} a_k^+ a_k. \tag{2.5}$$

Equations (2.2) and (2.3) differ from equations (2.1*b*–*c*) in I (the static limit) by the inclusion of the non-relativistic, baryon kinetic energy $\boldsymbol{p}_\alpha^2/2m_\alpha$. It is easy to show by direct computation that $[\boldsymbol{P}_{\mathrm{R}}, H_{\mathrm{R,CBM}}] = 0$. We shall characterise the pionic content of the physical nucleon by suitable generalising techniques and results presented in I. For that purpose it will be useful to introduce successively:

(*a*) the subspace $\mathscr{H}_{\boldsymbol{\pi}}$ of all eigenstates ψ of $\boldsymbol{P}_{\mathrm{R}}$ with eigenvalue $\boldsymbol{\pi}$, $|\boldsymbol{\pi}|$ being suitably small (say $\boldsymbol{\pi}^2/2m_\alpha < \mu$);

(b) the following basis in $\mathscr{H}_\pi$:

$$\begin{aligned} &\psi(k_1, \ldots, k_r; \alpha, \boldsymbol{q}_\pi) = (r!)^{-1/2} a_{k_1}^+ \ldots a_{k_r}^+ |\alpha, \boldsymbol{q}_\pi\rangle \\ &\boldsymbol{q}_\pi + \sum_{i=1}^{r} \boldsymbol{k}_i = \boldsymbol{\pi} \end{aligned} \tag{2.6}$$

where $|\alpha, \boldsymbol{q}_\pi\rangle$ denotes a bare bag state of type α ($=\mathrm{N}, \Delta$) with given spin and isospin projections (not written explicitly) and momentum $\boldsymbol{q}_\pi$;

(c) the restricted scalar product in $\mathscr{H}_\pi$:

$$\langle \psi(k_1, \ldots, k_r; \alpha, \boldsymbol{q}_\pi) | \psi(k_1', \ldots, k_r'; \alpha', \boldsymbol{q}_\pi') \rangle_\pi$$

$$= \delta_{\alpha\alpha'} \delta_{rr'} (r!)^{-1} \left(\sum_{\text{permutations}} [\delta(\boldsymbol{k}'_{\sigma(1)} - \boldsymbol{k}_1') \delta_{j'_{\sigma(1)} j_1}] \ldots [\delta(\boldsymbol{k}'_{\sigma(r)} - \boldsymbol{k}_r) \delta_{j_{\sigma(r)} j_r}] \right) \tag{2.7}$$

where $(\sigma(1) \ldots \sigma(r))$ denotes a generic permutation of $(1 \ldots r)$. Equation (2.7) defines immediately the restricted norm, $\|\psi\|_\pi$, of a generic ket belonging to $\mathscr{H}_\pi$. The usual scalar products and norms in the full Hilbert space are equal to the restricted ones introduced above times $\delta^{(3)}(\mathbf{0})$;

(d) the following restricted norm for an operator A which commutes with $\boldsymbol{P}_\mathrm{R}$: $\|A\|_\pi$ = least upper bound of $\|A\psi\|_\pi / \|\psi\|_\pi$ as ψ varies in $\mathscr{H}_\pi$. Let

$$|\tilde{n}st; \boldsymbol{\pi}\rangle = Z_2^{1/2} |\alpha = nst; \boldsymbol{\pi}\rangle + \sum_{r=1}^{\infty} \sum_{\beta} \sum_{k_1 \ldots k_r} \langle \psi(k_1, \ldots, k_r; \beta, \boldsymbol{q}_\pi) | \tilde{n}st; \boldsymbol{\pi}\rangle_\pi \psi(k_1, \ldots, k_r; \beta, \boldsymbol{q}_\pi) \tag{2.8}$$

be the physical one-nucleon state with spin and isospin projections s and t, total momentum $\boldsymbol{\pi}$ and energy $E(\boldsymbol{\pi})$:

$$\begin{aligned} &(H_{\mathrm{R,CBM}} - E(\boldsymbol{\pi})) |\tilde{n}st; \boldsymbol{\pi}\rangle = 0 \\ &(\boldsymbol{P}_\mathrm{R} - \boldsymbol{\pi}) |\tilde{n}st; \boldsymbol{\pi}\rangle = 0. \end{aligned} \tag{2.9}$$

The probability for finding r pions in the physical nucleon state is

$$P_{r,\mathrm{R}} = \sum_{\beta} \sum_{k_1 \ldots k_r} |\langle \psi(k_1, \ldots, k_r; \beta; \boldsymbol{q}_\pi) | \tilde{n}st; \boldsymbol{\pi}\rangle_\pi|^2 \tag{2.10}$$

if the physical nucleon state is normalised:

$$\langle \tilde{n}st; \boldsymbol{\pi} | \tilde{n}st; \boldsymbol{\pi}\rangle = 1. \tag{2.11}$$

Notice that equations (2.8) and (2.10) are the natural generalisation of equations (3.2) and (3.8) in I.

Reference to the arguments of § 3 and the appendices of I makes it clear that the same arguments can be applied to the present model. The only caution necessary is that: (i) one uses the restricted norm for kets and operators and (ii) one replaces $\tilde{m}_\mathrm{n}$ by $E(\boldsymbol{\pi})$. Clearly the smallest eigenvalue of the new Hamiltonian $H_{\mathrm{R,CBM}}$ in the restricted subspace $\mathscr{H}_\pi$ corresponds to the energy of a physical nucleon with momentum $\boldsymbol{\pi}$. Therefore one easily finds the result

$$\|(E(\boldsymbol{\pi}) - \omega - H_{\mathrm{R,CBM}})^{-1}\|_\pi \leqslant (\omega + E(\boldsymbol{\pi}) - E(\boldsymbol{\pi}))^{-1} = \omega^{-1}. \tag{2.12}$$

Thus one finally obtains the following bounds for $P_{r,\mathrm{R}}$, the mean number of pions

($\langle r\rangle_{\mathrm{R}}$) and the uncertainty in the number of pions (Δr_{R}):

$$P_{r,\mathrm{R}} \leqslant \Lambda^r/r! \qquad \langle r\rangle_{\mathrm{R}} \leqslant \Lambda \qquad \Delta r_{\mathrm{R}} \leqslant (\Lambda^2+\tfrac{1}{4})^{1/2} \tag{2.13}$$

where

$$\Lambda = \frac{57}{25}\,\frac{27f^2}{\mu^2(2\pi)^2}\int_0^\infty \omega_k^{-3}\,\mathrm{d}k\,k^4\left(\frac{\sin(kR)}{(kR)^3}-\frac{\cos(kR)}{(kR)^2}\right)^2. \tag{2.14}$$

The analytical expression for Λ turns out to coincide with that for the static CBM (see equations (3.24)–(3.25) in I) and constitutes a slight improvement with respect to the bound $\Lambda(\boldsymbol{q})$ obtained by Alvarez-Estrada and Thomas (1981). As in I, R is the bag radius and f is the unrenormalised pion–nucleon coupling constant—which in the CBM is within 10% of the renormalised value (Thomas *et al* 1981, Théberge *et al* 1982).

If one uses values for R and f close to $R \simeq 0.82F$, $f^2/4\pi \simeq 0.078$ (which were obtained by Thomas *et al* (1981) and Théberge *et al* (1982) from a best fit of the static CBM predictions to the pion–nucleon scattering data), one finds $\Lambda < 1$. This suggests a rather rapid convergence of the perturbation expansion for $|\tilde{n}st; \boldsymbol{\pi}\rangle$ as in the static CBM case.

3. Rho-meson coupling to the bag

In this section we extend the earlier bounds on the virtual (non-resonating) π-meson cloud to include virtual ρ mesons (resonating pairs of pions) as well. We tackle this problem in two parts. First we assume that it makes sense to talk about the coupling of a point-like ρ-meson field to the bag, with conventional strength (Brown and Weise 1975). Even in this case we shall see that the average number of ρ mesons about the dressed nucleon is bounded by a rather small number, consistent with other phenomenological studies. Having dealt with the problem in the conventional way, we then ask whether such an approach is consistent with the quark model itself. We shall in fact argue that within the context of a theory where the composite nature of the nucleon is taken seriously (Miller *et al* 1981, Thomas 1982a, b), one should not perhaps introduce the ρ meson explicitly. Such contributions are probably better treated as uncorrelated $q\bar{q}$ excitations in the bag—or sea quarks (Donoghue and Golowich 1977, Maxwell and Vento 1981).

3.1. A new bound

Let us consider the following Hamiltonian which generalises equation (2.1) to include a purely phenomenological interaction between a ρ-meson field and the confined quarks. If we define the formal sum over isospin and three-momentum of the ρ as

$$\sum_q \equiv \sum_{n=1}^{3}\int \mathrm{d}^3 q \tag{3.1}$$

then the new Hamiltonian becomes

$$H = H_0 + H_{\mathrm{I}} \tag{3.2}$$

with

$$H_0 = H_{\mathrm{R},0} + \sum_{q,l} b_q^+(l)b_q(l)w_{\rho,q} \tag{3.3}$$

where $H_{R,0}$ was given in equation (2.2), $w_{p,q}$ is the kinetic energy of a ρ meson of momentum q and spin projection l and b_q is the appropriate destruction operator. The interaction Hamiltonian has the form

$$H_I = H_{R,I} + \sum_{l=1}^{3} \sum_{p,q} \sum_{\alpha,\beta=N,\Delta} \alpha^+_{p+q} \; \beta_p b_q(l) v^{\alpha\beta}_{p,q}(l) + \text{HC} \tag{3.4}$$

where again $H_{R,I}$ was given in § 2 (equation (2.3)) and the interaction of the ρ with the bag is given by

$$v_{p,q}(l) = \frac{\mathrm{i}\mathscr{F} v_\rho(q) f'}{(2\pi)^{3/2} m_N (2w_{p,q})^{1/2}} T_h \boldsymbol{S} \cdot \boldsymbol{a}(l) \tag{3.5}$$

where the matrix of coupling constants (in SU(6)) is

$$\mathscr{F} = \frac{6}{5}\begin{pmatrix} 5 & 4\sqrt{2} \\ 4\sqrt{2} & 10 \end{pmatrix} = \begin{pmatrix} \mathscr{F}^{nn} & \mathscr{F}^{n\Delta} \\ \mathscr{F}^{\Delta N} & \mathscr{F}^{\Delta\Delta} \end{pmatrix}. \tag{3.6}$$

As in the work of Brown and Weise (1975), we consider only the transverse coupling of the ρ meson, which conventionally has the largest coupling constant,

$$f' = \tfrac{1}{2} g_\rho (1 + K_V) \qquad 0.4 < g_\rho^2 < 0.6 \qquad K_V = 3.7. \tag{3.7}$$

Of course the transverse coupling involves the vector $\boldsymbol{a}$ given by

$$\boldsymbol{a}(l) = \boldsymbol{q} \times \boldsymbol{\varepsilon}(\boldsymbol{q}, l) \tag{3.8}$$

where $\boldsymbol{\varepsilon}(\boldsymbol{q}, l)$ is the usual polarisation vector for a non-relativistic, spin-1 particle satisfying

$$\sum_{l=1}^{3} (\boldsymbol{\varepsilon}(\boldsymbol{q}, l))_i \cdot (\boldsymbol{\varepsilon}^*(\boldsymbol{q}, l))_j = \delta_{ij} \qquad (i, j = 1, 2, 3) \tag{3.9}$$

$$\boldsymbol{q} \cdot \boldsymbol{\varepsilon}(\boldsymbol{q}, l) = 0 \qquad l = 1, 2. \tag{3.10}$$

The transition spins $\boldsymbol{T}$ and $\boldsymbol{S}$ are standard—see, for example, equations (2.3) in I. We also remark that the structure of the above model for the coupling of virtual ρ mesons to N's and Δ's is, essentially, the one considered by Niskanen (1981).

Finally $v_\rho(q)$ is a new form factor describing the coupling of the ρ meson to the extended, composite nucleon. Unlike the pion coupling, which is completely determined by chiral symmetry, the coupling of the ρ meson is not dictated by symmetry considerations. We can envisage two possible ways of describing this coupling, both of which lead to a relatively soft form factor—similar to that at the NNπ vertex. These arguments will be presented in § 3.2. For the present we simply note that our expectations in this matter do agree with the findings of Niskanen (1981) in a phenomenological study of the ρ meson on the width of the delta.

The total momentum operator in this theory is

$$\boldsymbol{P} = \boldsymbol{P}_R + \sum_{l=1}^{3} \sum_{q} \boldsymbol{q} b_q^+(l) b_q(l) \tag{3.11}$$

and again one finds that $\boldsymbol{P}$ is a constant of the motion:

$$[\boldsymbol{P}, H] = 0. \tag{3.12}$$

In order to determine the ρ-meson content of the physical nucleon, we generalise directly steps (a)–(d) of § 2 by replacing $\boldsymbol{P}_R$ by $\boldsymbol{P}$, and using the following basis vectors with any numbers of π's and ρ's:

$$\psi(k_1, \ldots, k_r; q_1 l_1, \ldots, q_s l_s; \alpha; \boldsymbol{q}_\pi)$$
$$=(s!)^{-1/2} b^+_{q_1}(l_1) \ldots b^+_{q_s}(l_s) \psi(k_1, \ldots, k_r; \alpha, \boldsymbol{q}_\pi). \tag{3.13}$$

The wavefunction $\psi(k_1, \ldots, k_r; \alpha, \boldsymbol{q}_\pi)$ is given in equation (2.6), but now $\boldsymbol{q}_\pi = \boldsymbol{\pi} - \Sigma^r_{i=1} \boldsymbol{k}_i - \Sigma^s_{i=1} \boldsymbol{q}_i$. The new restricted scalar product for these basis vectors is similar to the right-hand side of equation (2.7), but a new factor for ρ mesons

$$\delta_{ss'}/s! \sum_{\text{permut}} \delta^{(3)}(\boldsymbol{q}'_{\sigma(1)} - \boldsymbol{q}_1) \delta_{h'_{\sigma(1)}; h_1} \delta_{l'_{\sigma(1)}, l_1} \ldots$$

has to be included. The normalised physical one-nucleon state $|\tilde{n}st; \boldsymbol{\pi}\rangle$ satisfies equations (2.9), with $H_{\mathrm{R, CBM}}$, $\boldsymbol{P}_{\mathrm{R}}$ replaced by H, $\boldsymbol{P}$, and is given by an expansion similar to equation (2.8), with $\psi(k_1, \ldots, k_r; \alpha \boldsymbol{q}_\pi)$ replaced by $\psi(k_1, \ldots, k_r; q_1 l_1 \ldots q_s l_s; \alpha \boldsymbol{q}_\pi)$. Now, we are interested in the probability for finding r pions and s ρ's in $|\tilde{n}st; \boldsymbol{\pi}\rangle$, which reads (cf equation (2.10))

$$P_{r,s} = \sum_{\beta} \sum_{k_1 \ldots k_r} \sum_{q_1 \ldots q_s} \sum_{l_1 \ldots l_s} |\langle \psi(k_1 \ldots k_r; q_1 l_1 \ldots q_s l_s; \alpha, \boldsymbol{q}_\pi | \tilde{n}st; \boldsymbol{\pi}\rangle_\pi|^2. \tag{3.14}$$

By generalising the techniques in I, one finds

$$P_{r,s} \leqslant \frac{\Lambda^r}{r!} \frac{\Lambda^s_\rho}{s!} \tag{3.15}$$

where Λ is given in equation (2.14), and

$$\Lambda_\rho = \sum_q \sum_l w^{-2}_{\rho, q} \left\| \sum_{\alpha, \beta = \mathrm{N}, \Delta} v^{\alpha\beta+}_{\rho, q} \beta^+_{\boldsymbol{\pi} - q} \alpha_{\boldsymbol{\pi}} \right\|^2_{\pi} \tag{3.16}$$

which should be compared with equation (3.20) in I. Λ_ρ can be evaluated by extending the arguments given in appendix B of I. By choosing linearly polarised ε's and integrating over the angles of $\boldsymbol{q}$, one finds (cf equation (B2) in I)

$$\sum_{l, l'=1}^{3} \int \mathrm{d}\Omega_q \, \langle s_\alpha s | \boldsymbol{S} \cdot \boldsymbol{a}(l) | s_\gamma s' \rangle \langle s_\gamma s' | \boldsymbol{S} \cdot \boldsymbol{a}(l') | s_\beta s'' \rangle$$
$$= \tfrac{8}{3}\pi |\boldsymbol{q}|^2 \sum_{l=1}^{3} \langle s_\alpha s | S_l | s_\gamma s' \rangle \langle s_\gamma s' | S_l | s_\beta s'' \rangle. \tag{3.17}$$

After this, the calculation proceeds as in appendix B in I, and one finds

$$\Lambda_\rho = \frac{57}{25} \frac{6 f'^2}{(2\pi)^2 m^2_{\mathrm{N}}} \int_0^\infty \frac{\mathrm{d}q \, q^4 v^2_\rho(q)}{w^3_{\rho, q}}. \tag{3.18}$$

Similarly, the mean number of ρ's and the uncertainty in the number of ρ's are bounded as $\langle s \rangle \leqslant \Lambda_\rho$, $\Delta s \leqslant (\Lambda^2_\rho + \frac{1}{4})^{1/2}$.

To obtain a numerical estimate of Λ_ρ we use one of the form factors of Niskanen (1981), namely

$$v_\rho(\boldsymbol{q}) = \frac{\Lambda^2_0 - m^2_\rho}{\Lambda^2_0 + \boldsymbol{q}^2} \tag{3.19}$$

with $\Lambda_0 = 1$ GeV. This leads to the result $\Lambda_\rho = 0.037$, which is extremely small. Using the somewhat larger value of $K_{\mathrm{V}} = 6.7$, this would be $\Lambda_\rho = 0.10$—again a very small number.

Of course this would increase rapidly if the cut-off Λ_0 were increased, but both Niskanen's (1981) phenomenological results and the arguments given in the next section strongly suggest that our choice is most reasonable.

3.2. Discussion of ρ-meson coupling to an extended nucleon

In the previous section we suggested that one might *a priori* expect a relatively soft form factor at the ρNN, ρNΔ vertices—in agreement with Niskanen's (1981) phenomenological findings. Suppose, for example, we couple the ρ meson directly to the quarks. As the quarks are structureless there is no $\sigma^{\mu\nu}$ term, but we can write a pure vector coupling, $\gamma^\mu \rho_\mu$. After taking bag-model matrix elements this will reduce to the general form given in equations (3.5)–(3.8). The key point with regard to the form factor is that one does not expect the q̄q constituents of the ρ meson to propagate as a coherent pair inside the baryon. Indeed the one-gluon-exchange interaction is quite strongly repulsive for this case (De Rújula *et al* 1975, De Grand *et al* 1975)—unlike the case of the pion (Goldman and Haymaker 1981a, b, Thomas 1982b). Thus it seems much more appropriate to put the q̄q pair into bag eigenstates once the ρ meson and baryon overlap.

In that case the phenomenological ρ-meson field should vanish inside the baryon bag radius and one naturally finds a cloudy-bag-type of form factor for the ρ, namely

$$v_\rho(\boldsymbol{q}) = 3j_1(qR)/qR \tag{3.19}$$

(where R is the radius of the baryon bag) which for practical purposes can be approximated as (Théberge 1982, Théberge and Thomas 1982b)

$$v_\rho(\boldsymbol{q}) \simeq \exp(-0.106\boldsymbol{q}^2 R^2). \tag{3.20}$$

This can be compared with the best-fit form factor of Niskanen (1981):

$$v_\rho^{\text{Niskanen}}(\boldsymbol{q}) = \exp[-0.077(m_\rho^2 + \boldsymbol{q}^2)] \tag{3.21}$$

with m_ρ and q in fm^{-1} corresponding to $R \sim 0.82$ fm.

An alternate possibility would be to allow the ρ meson to couple to the bag only through a two-pion intermediate state. Using the Gaussian approximation (3.20), it is relatively easy to show that if we do not exclude the ρ meson from the hadronic bag $R \to R_{\text{eff}} = R/\sqrt{2}$—that is, the ρ form factor becomes a little harder. However, the most probable effect of allowing the pions to form a ρ only outside the nucleon bag would be to soften it again.

So far our discussion has only served to explain the choice of ρNN and ρNΔ form factors in the numerical work of § 3.1. However, there is an even more fundamental issue, namely whether it makes sense to talk about a cloud of virtual ρ mesons at all. Since the typical range of such a cloud is $(m_\rho)^{-1} \sim 0.2$ fm, the answer to this question is probably no. The static surface of the MIT-bag model is a phenomenological simplification of the true dynamical situation. One naturally expects a hadron to have some surface thickness, and a few tenths of a fermi is not an unreasonable scale for such a transition region—see, for example, the soliton-bag models (Lee 1979, Goldflam and Wilets 1982). On this sort of scale (unlike the 1.4 fm ($\equiv \lambda_\pi$) associated with a virtual pion) there is no clean separation between bag-model phenomenology and the meson cloud. Consequently it may be far more meaningful to simply deal with such effects as isovector q̄q fluctuations in baryon-bag eigenstates. Clearly it will not be possible to resolve this question completely until the problem can be formulated quantitatively. Nevertheless, it is reassuring that in either extreme such effects are relatively small, as the comparison of the estimates of Niskanen (1981) with our suggests.

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Chapter 1

CHIRAL SYMMETRY AND THE BAG MODEL: A NEW STARTING POINT FOR NUCLEAR PHYSICS

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1. INTRODUCTION

They must often change who would be constant in happiness or wisdom—
CONFUCIUS

Classical nuclear theory deals with a many-body system of neutrons and protons interacting nonrelativistically through two-body potentials. It has, of course, long been realized that there must be corrections to this simple picture—for example, the meson exchange effects which preclude a simple interpretation of the magnetic moment of the deuteron in terms of *d*-state probability. Nevertheless, the availability of beams of pions, and the consequent ability to study the excitation of real isobars in nuclei, has been critical in the realization that for many problems one must develop a theoretical model which explicitly includes pion and isobar degrees of freedom (see, for example, the proceedings of recent topical conferences Cat+ 82, MT 80).

While these developments have been taking place in intermediate energy physics, and particularly since the discovery of the J/ψ, our colleagues in high-energy physics have become thoroughly convinced of the quark model of hadron structure. This approach to the structure of hadrons began in the early 1960s. On the basis of symmetry considerations Gell-Mann, Ne'eman, and Zweig suggested that all hadrons might be made from more elementary components—the quarks (or aces) (GN 64). These constitute the fundamental representation of the group SU(3). All of the low-mass hadrons were found to fall into low-dimensional representations of SU(3). In the case of the mesons they could be thought of as being made of quark–antiquark, while for the baryons three quarks were required. Nevertheless, at that stage it was not clear whether the quarks were real particles or simply a mathematical trick.

One of the initial problems of the quark model was that, for example, the $J_Z = +\frac{3}{2}$ state of the Δ^{++} would necessarily be made from three identical up quarks in the same spin and spatial state. Since the quarks should be fermions, this would violate Fermi statistics. In order to overcome this difficulty the quarks were assigned a new, unobserved property called "color"—each quark having three possible colors. (A somewhat older, but equivalent explanation involved parastatistics.) This apparently ad hoc explanation became a great strength of the model when it was realized that one could build a theory of strong interactions (quantum chromodynamics or QCD) based on a gauge theory of color—the symmetry group again being SU(3) (AL 73, MP 78).

It was soon established that because of the non-Abelian nature of the theory it had two novel features. First, at short distances, or high-momentum transfer, the interactions become weaker—"asymptotic freedom." This realization was crucial in the identification of partons—the elementary, apparently free constituents of the proton observed in deep inelastic e- and ν-scattering—with quarks (Clo 79). Second, it seems that at large distances the interaction grows stronger. This property is generally believed, though not yet proven, to lead to confinement of the quarks into color-singlet objects—hence three-quark baryons and no free quarks.

At the present time a great deal of theoretical effort is being devoted to attempts to solve the QCD equations—by brute force mainly, using Monte Carlo techniques. In the absence of exact solutions, we must either abandon all hope of tackling nuclear problems or rely on phenomenological models. Fortunately, we have at our disposal a variety of successful, phenomenological models which incorporate the features expected from QCD. Of all these models the MIT bag model is perhaps the most attractive. As we shall see, it incorporates the facts that quarks are confined, pointlike, and essentially massless. The model is therefore relativistic and can be summarized in a relatively concise Lagrangian formalism. This feature has proven essential in the recent developments involving chiral symmetry, which we shall review in Sections 5 and 6.

We shall see that in the bag model, as in any other quark model of nucleon structure, the nucleon is far from pointlike. Its radius is about 1.0 fm, so that at the average internucleon separation of 1.8 fm at nuclear matter density ($\varrho_0 \sim 0.17\ \text{fm}^{-3}$) the nucleon bags overlap! This is a rather different state of affairs from that envisaged in most modern N–N potential models. As Baym has discussed (Bay 79) (see also Section 7), with a bag radius of 1.0 fm one would expect to find considerable linking of different bags in a nucleus (and hence free flow of color current between bags), even at half nuclear matter density! In that case even the independent particle shell model behavior of valence nucleons is mysterious.

By lowering the bag size just a little—e.g., to $R \sim (0.8$–$0.9)$ fm, as in the cloudy bag model—the critical density can be made about the same as nuclear matter density. In this way the problem with the independent particle shell model becomes less severe. Nevertheless, it seems inevitable that there should be considerable linking of bags in finite nuclei. Thus, we are forced to suggest that a precise description of many phenomena in nuclear physics may require the explicit inclusion of the quarks themselves. This seems to us the natural extension of the developments involving isobars to which we referred earlier. Such a suggestion deserves urgent theoretical (and, when the questions are clearly formulated, experimental) attention in the next few years. [Incidentally, there has been some discussion of quark degrees of freedom in nuclei by Robson (Rob 78), who derived effective many-body forces on the basis of a nonrelativistic quark model. Our approach will be rather different.]

One of the defects of the MIT bag model from the nuclear physics point of view is the absence of any mechanism for long-range N–N interactions. In fact, this is just one indication of a fundamental problem in the model, namely that it badly violates chiral symmetry. Since chiral symmetry is a property of QCD itself, this is quite worrying. The chiral bag models have been developed in response to this difficulty. At the present stage of the phenomenology the pion appears as fundamental as the quarks, although eventually this must be improved. Recent work which suggests that the pion exists as a consequence of dynamical symmetry breaking in short-distance QCD will be discussed and related to the chiral bag ideas.

In summary, we shall see that whereas a great deal of progress has been made towards understanding single-hadron properties, we are just beginning to make progress on the problem of two or more interacting hadrons. We have little doubt that for the next five to ten years this will be one of the major areas of research in nuclear physics (if not the major one). With this in mind the time is right for a graduate level introduction to the concepts and models that will be used. We hope that this review may help to provide such a bridge between the high-energy and nuclear communities.

In general the tone of the first major sections (Sections 2–5) is quite pedagogical. Full details of the algebra are often given in order that the reader can concentrate on the ideas and concepts being presented. After studying these sections carefully, the keen student should have a fairly good working knowledge of the MIT bag model, as well as a degree of familiarity with chiral symmetry. By the end of Section 6, which is more in the nature of a review, he will be essentially *au courant* with all published chiral bag models, and particularly the cloudy bag model. Section 7 is of quite general interest and in it we attempt to set the stage for future work in the physics of many-body systems of composite nucleons.

This review will have succeeded if a good number of its readers decide to take part in this fascinating new approach to a very old subject. Needless to say we welcome all constructive comments concerning anything said here.

2. THE BASIC BAG MODEL

In order to have a sound basis for the later developments of direct relevance to nuclear physics, we must first describe the original MIT bag model. The discussion in Section 2.1 is meant to lay this basis in considerable detail. It follows closely the pedagogical approach of Hey (77), to which we refer for more discussion of excited-state spectroscopy. Section 2.2 deals with the application of the model to the mass spectrum of the low-lying hadrons. In Section 2.3 we briefly review some recent attempts to justify the bag model starting from QCD. Finally, in Section 2.4 we discuss the relationship to the popular, nonrelativistic quark models.

2.1. The MIT Bag Model

2.1.1. Bogolioubov

The MIT bag model actually had its beginnings in the late 1960s in the attempts to describe phenomenologically a system of confined, relativistic quarks. In particular, Bogolioubov (Bog 67) considered the simplest possible case of a massless Dirac particle moving freely inside a spherical volume of radius R, outside of which there was a scalar potential of strength m. Clearly, by taking the limit $m \to \infty$ we can confine the quarks to the spherical volume.

Let us therefore begin with the Dirac equation for a particle of mass m:

$$H\psi = i\frac{\partial \psi}{\partial t} \tag{2.1}$$

with the Hamiltonian

$$H = \vec{\alpha} \cdot \vec{\mathbf{p}} + \beta m \tag{2.2}$$

(Our convention for Dirac matrices is summarized in Appendix I.) There are two operators which commute with H and can therefore be used to classify its eigenstates. These are

$$\vec{\mathbf{j}} = \vec{\mathbf{l}} + \vec{\sigma}/2 \tag{2.3}$$

where, when necessary we have

$$\vec{\sigma} = \begin{bmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{bmatrix} \tag{2.4}$$

and the relativistic analog of the operator k described in Appendix I. The analog K is

$$K = \beta(\vec{\sigma} \cdot \vec{\mathbf{l}} + 1) \tag{2.5}$$

With these definitions it is straightforward to prove that

$$[\vec{\mathbf{j}}, K] = 0 = [H, \vec{\mathbf{j}}] = [H, K] \tag{2.6}$$

and

$$K^2 = \beta^2[1 + (\vec{\sigma} \cdot \vec{\mathbf{l}})^2 + 2\vec{\sigma} \cdot \vec{\mathbf{l}}] = \vec{\mathbf{j}}^2 + \tfrac{1}{4} \tag{2.7}$$

Clearly, K has eigenvalues $\varkappa$, where

$$\varkappa = \pm(j + \tfrac{1}{2}) \tag{2.8}$$

In the case of a central, scalar field $W(r)$, the Dirac equation becomes

$$H\psi(\vec{\mathbf{r}}) = \{\vec{\alpha} \cdot \vec{\mathbf{p}} + \beta[m + W(r)]\}\psi(\vec{\mathbf{r}}) = E\psi(\vec{\mathbf{r}}) \tag{2.9}$$

where

$$\psi(\vec{\mathbf{r}}, t) = \psi(\vec{\mathbf{r}})e^{-iEt} \tag{2.10}$$

$$\vec{\mathbf{j}}^2\psi_\varkappa{}^\mu = j(j+1)\psi_\varkappa{}^\mu; \qquad j_z\psi_\varkappa{}^\mu = \mu\psi_\varkappa{}^\mu \tag{2.11}$$

and

$$K\psi_\varkappa{}^\mu = -\varkappa\psi_\varkappa{}^\mu \tag{2.12}$$

Let the solution of Eq. (2.9) have the form

$$\psi = \begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} \tag{2.13}$$

Then the structure of K

$$K = \begin{bmatrix}k & 0\\ 0 & -k\end{bmatrix} \tag{2.14}$$

implies that ψ can be written, without loss of generality, as

$$\psi_\varkappa{}^\mu = \begin{bmatrix}g(r) & \chi_\varkappa{}^\mu\\ if(r) & \chi^\mu_{-\varkappa}\end{bmatrix} \tag{2.15}$$

Then, using

$$\vec{\boldsymbol{\nabla}} = \hat{r}\frac{\partial}{\partial r} - i\frac{\hat{r}}{r}\times\vec{\mathbf{l}} \tag{2.16}$$

we can write the kinetic energy piece of H as

$$\begin{aligned}\vec{\boldsymbol{\alpha}}\cdot\vec{\mathbf{p}} &= -i\vec{\boldsymbol{\alpha}}\cdot\hat{r}\frac{\partial}{\partial r} + \frac{i}{r}\vec{\boldsymbol{\alpha}}\cdot\hat{r}\vec{\boldsymbol{\sigma}}\cdot\vec{\mathbf{l}}\\ &= -i\vec{\boldsymbol{\alpha}}\cdot\hat{r}\frac{\partial}{\partial r} + \frac{i}{r}\vec{\boldsymbol{\alpha}}\cdot\hat{r}(\beta K - 1)\end{aligned} \tag{2.17}$$

Substituting Eqs. (2.15) and (2.17) into the Dirac equation (2.9) it becomes

$$\begin{aligned}(E - W - m)g &= -\left(\frac{df}{dr} + \frac{f}{r}\right) + \frac{\varkappa f}{r}\\ (E + W + m)f &= \left(\frac{dg}{dr} + \frac{g}{r}\right) + \frac{\varkappa g}{r}\end{aligned} \tag{2.18}$$

Bogolioubov's simple model of confinement (Bog 67) corresponded to the scalar potential

$$\begin{aligned}W(r) &= -m, \qquad r \leq R\\ W(r) &= \quad 0, \qquad r > R\end{aligned} \tag{2.19}$$

If we now define

$$U = m + W \tag{2.20}$$

Eq. (2.18) becomes

$$\begin{aligned}\frac{df}{dr} &= \frac{\varkappa - 1}{r}f - (E - U)g\\ \frac{dg}{dr} &= (E + U)f - \frac{\varkappa + 1}{r}g\end{aligned} \tag{2.21}$$

Consider the case $\varkappa = -1$, which is the $s_{1/2}$ level. Equation (2.21) implies

$$f = (E + U)^{-1} \frac{dg}{dr} \tag{2.22}$$

so that defining $g = u/r$, the equation for the upper component of $\psi_\varkappa^\mu$ is

$$\frac{d^2u}{dr^2} + (E^2 - U^2)u = 0 \tag{2.23}$$

Inside the scalar potential well this means

$$\ddot{u} + E^2 u = 0 \tag{2.24}$$

and hence

$$u(r) = A \sin Er \tag{2.25}$$

Outside the scalar well $u(r)$ satisfies

$$\ddot{u} - (m^2 - E^2)u = 0 \tag{2.26}$$

and hence

$$u(r) = A(\sin ER)e^{-(m^2-E^2)^{1/2}(r-R)} \tag{2.27}$$

This is an eigenvalue problem because u (and of course g) must be continuous at $r = R$. If we also demand that $f(r)$ [defined by Eq. (2.22)] be continuous, we obtain the matching condition

$$\cos(ER) + \frac{[1 - (E/m)^2]^{1/2}}{1 + (E/m)} \sin ER = \frac{\sin ER}{ER}\left(1 - \frac{E}{E + m}\right) \tag{2.28}$$

Clearly, in the limit $m \to \infty$ (corresponding to confinement) this becomes

$$\frac{\sin ER}{ER} = \left(\frac{\sin ER}{ER} - \cos ER\right)\Big/ER \tag{2.29}$$

and hence

$$j_0(ER) = j_1(ER) \tag{2.30}$$

This is the appropriate boundary condition for massless, confined quarks.

If we parametrize the energy levels as

$$E_{n\varkappa} = \omega_{n\varkappa}/R \tag{2.31}$$

where n is the principal quantum number, we find $\omega_{1-1} = 2.04$, $\omega_{2-1} = 5.40$, and so on. The solution has the form

$$\psi^{\mu}_{n s_{1/2}} \equiv \psi^{\mu}_{n\kappa=-1} = \frac{N_{n,-1}}{(4\pi)^{1/2}} \begin{bmatrix} j_0\left(\dfrac{\omega r}{R}\right) & \chi^{\mu}_{-1} \\ -ij_1\left(\dfrac{\omega r}{R}\right) & \chi_1^{\mu} \end{bmatrix} \tag{2.32}$$

and using Eq. (I.15) from Appendix I this may be written as

$$\psi_{n,-1}(\underline{r}) = \frac{N_{n,-1}}{(4\pi)^{1/2}} \begin{bmatrix} j_0\left(\dfrac{\omega r}{R}\right) \\ i\vec{\sigma}\cdot\hat{r}j_1\left(\dfrac{\omega r}{R}\right) \end{bmatrix} \chi^{\mu}_{1/2} \tag{2.33}$$

with $\chi^{\mu}_{1/2}$ a Pauli spinor, and the normalization constant given by

$$N^2_{n,-1} = \frac{\omega^3_{n,-1}}{2R^3(\omega_{n,-1} - 1)\sin^2(\omega_{n,-1})} \tag{2.34}$$

The density of quarks is readily calculated as

$$j^0 \equiv \varrho = \bar{\psi}\gamma^0\psi \sim \left[j_0^2\left(\frac{\omega r}{R}\right) + j_1^2\left(\frac{\omega r}{R}\right)\right]\theta(R-r) \tag{2.35a}$$

where

$$\begin{aligned} \theta(x) &= 1, \qquad x \geq 0 \\ &= 0, \qquad x < 0 \end{aligned} \tag{2.35b}$$

Thus, the density [Eq. (2.35a)] certainly does not vanish at $r = R$. Clearly, although the lower component is suppressed for small r, it does make a sizeable contribution near the surface of the bag. Of course it is natural to ask whether this is not unusual in comparison with nonrelativistic experience, where $\psi(R)$ would be zero. However, such a solution would not be consistent with the linear Dirac equation. What counts is that there should be no current flow through the surface of the confining region. For example, in the MIT bag model it is required that [we use $q(x)$ for the quark wave function in the MIT model, but it is identical to $\psi(x)$ in the static, spherical case]

$$n_\mu \bar{q}\gamma^\mu q = 0 \tag{2.36}$$

at the surface—where n^μ is a unit four vector normal to the surface of the confining region.

In the MIT bag model (Cho+ 74, DeG+ 75, Joh 75, JJ 77, HK 78) the condition (2.36) is imposed through a linear boundary condition

$$i\gamma \cdot nq = q \tag{2.37}$$

at the surface. This implies

$$q^{+} = -iq^{+}\gamma^{+} \cdot n \tag{2.38}$$

and hence

$$\bar{q} = -i\bar{q}\gamma \cdot n \tag{2.39}$$

because

$$\gamma^{\mu} = \gamma^{0}\gamma^{\mu+}\gamma^{0} \tag{2.40}$$

Consider now the normal flow of current through the bag surface:

$$in_{\mu}j^{\mu} = in_{\mu}\bar{q}\gamma^{\mu}q$$

which from Eqs. (2.39) and (2.37), respectively, is

$$\begin{aligned} in_{\mu}j^{\mu} &= (\bar{q}i\gamma \cdot n)q = \bar{q}(i\gamma \cdot nq) \\ &= -\bar{q}q = \bar{q}q \\ &= 0 \end{aligned} \tag{2.41}$$

Thus, it is not the density, but $\bar{q}q$ which should vanish at the boundary in a relativistic theory. If we now return to the model of Bogolioubov, Eq. (2.33) implies that

$$\begin{aligned} \bar{\psi}\psi\,|_{r=R} &= [j_0(\omega), i\vec{\sigma} \cdot \hat{r}j_1(\omega)] \cdot \begin{pmatrix} j_0(\omega) \\ i\vec{\sigma} \cdot \hat{r}j_1(\omega) \end{pmatrix} \\ &= j_0^2(\omega) - j_1^2(\omega) = 0 \end{aligned} \tag{2.42}$$

by Eq. (2.30). That is, the matching condition of Bogolioubov is exactly equivalent to the *linear boundary condition* (l.b.c.) for the static spherical MIT bag

$$i\gamma \cdot n\psi = -i\gamma \cdot \hat{r}\psi = \psi \tag{2.43}$$

where

$$n^{\mu} = (0, \hat{r}) \tag{2.44}$$

Suppose we now demand that the lowest energy state of Bogolioubov's model reproduce the nucleon mass. Just as in the independent particle shell model, we add the energies of each of the three quarks in the $1s$ level giving

$$M_N = \frac{3\omega_{1-1}}{R} \tag{2.45}$$

Using $\omega_{1-1} = 2.04$ we find that the radius of the nucleon bag is

$$R_N = 1.3 \text{ fm} \tag{2.46}$$

Then the first excited state of the nucleon, namely the Roper resonance, in which one quark is simply raised from the $1s_{1/2}$ to the $2s_{1/2}$ state, should have a mass M_R where

$$\frac{M_R}{M_N} = \frac{2\omega_{1-1} + \omega_{2-1}}{3\omega_{1-1}} = \frac{4.08 + 5.40}{6.12} = 1.55 \tag{2.47}$$

This is in remarkable agreement with the experimental ratio 1.56—to quote Bogolioubov, "*une telle coincidence est un peu surprenante.*"¶

2.1.2. *Energy-Momentum Conservation*

Up to this point there is little practical difference between the bag model of the MIT group and that of Bogolioubov. Although the MIT model is decently covariant, and this will be put to use in later sections, in practical calculations one is forced to work with the static spherical case. Nevertheless, the more rigorous formal approach did help the MIT group to recognize a fundamental problem in the Bogolioubov model. In order to see this we consider the energy-momentum tensor for that model, $T^{\mu\nu}_{\text{Bog}}$:

$$T^{\mu\nu}_{\text{Bog}} = T_D{}^{\mu\nu}\theta_V \tag{2.48}$$

where θ_V defines the bag volume

$$\begin{aligned} \theta_V &= 1 \text{ inside} \\ &= 0 \text{ outside} \end{aligned} \tag{2.49}$$

and $T_D{}^{\mu\nu}$ is the familiar energy-momentum tensor for a *free* Dirac field

$$T_D{}^{\mu\nu} = \frac{i}{2}\,\bar{q}(x)\gamma^\mu \overleftrightarrow{\partial}^\nu q(x) \tag{2.50}$$

¶ Such a "coincidence" is a little surprising.

(As usual we have defined

$$\overleftrightarrow{\partial}^{\nu} = \overrightarrow{\partial}^{\nu} - \overleftarrow{\partial}^{\nu} \tag{2.51}$$

where the first and second terms on the right-hand side act to the right and left, respectively.) The condition for overall energy and momentum conservation is that the divergence of the energy-momentum tensor should vanish, and this is certainly true for $T_D{}^{\mu\nu}$, as is easily proven from the free Dirac equation [$i\partial\!\!\!/ q(x) = 0$, for a massless quark]

$$\partial_\mu T_D{}^{\mu\nu} = 0 \tag{2.52}$$

However, the fact that these quarks move freely only inside the restricted region of space V leads to problems. Indeed

$$\partial_\mu \theta_V = n_\mu \Delta_s \tag{2.53}$$

where Δ_s is a surface delta function

$$\Delta_s = -n \cdot \partial(\theta_V) \tag{2.54}$$

In the static spherical case [see Eq. (2.44)] we find that Δ_s is simply $\delta(r - R)$. Putting all this together we obtain

$$\partial_\mu T^{\mu\nu}_{\text{Bog}} = \frac{i}{2} \bar{q}\gamma \cdot n \overleftrightarrow{\partial}^{\nu} q \Delta_s \tag{2.55}$$

and using the l.b.c. [Eq. (2.37)]

$$\partial_\mu T^{\mu\nu}_{\text{Bog}} = -\tfrac{1}{2}\partial^\nu[\bar{q}q]\Delta_s = -P_D n^\nu \Delta_s \tag{2.56}$$

where P_D is the pressure exerted on the bag wall by the contained Dirac gas

$$P_D = -\tfrac{1}{2}\, n \cdot \partial(\bar{q}q)\,|_{\text{surface}} \tag{2.57}$$

Clearly, the model of Bogolioubov violates energy-momentum conservation! Furthermore, this violation is an essential result of the confinement process. We shall see in Section 4 that a similar problem arises in connection with the axial current.

The resolution of this problem is possible only if we are willing to add a new ingredient. In particular, we simply add a phenomenological energy density term $-B\theta_V$ to the Lagrangian density (see Section 4). Then (since $T^{\mu\nu}$ involves $-\mathscr{L}g^{\mu\nu}$) the new energy-momentum tensor $T^{\mu\nu}_{\text{MIT}}$ has the form

$$T^{\mu\nu}_{\text{MIT}} = (T_D{}^{\mu\nu} + Bg^{\mu\nu})\theta_V \tag{2.58}$$

Therefore, the divergence of the energy-momentum tensor is

$$\partial_{\mu} T^{\mu\nu}_{\text{MIT}} = (-P_D + B) n^{\nu} \Delta_s \tag{2.59}$$

which will vanish if

$$B = P_D = -\tfrac{1}{2}\, n \cdot \partial[\bar{q}(x) q(x)]_{\text{surface}} \tag{2.60}$$

Equation (2.60) involves the square of the quark fields, and hence is referred to as the *nonlinear boundary condition* (n.l.b.c.) of the MIT bag model. Because of this condition the introduction of a constant B involves no new parameters.

By taking the explicit solutions of the free Dirac equation (massless case)

$$\psi_{\varkappa}{}^{\mu} = N_{\varkappa} \begin{pmatrix} j_l(\omega r/R) \chi_{\varkappa}{}^{\mu} \\ \pm i j_{l\mp 1}(\omega r/R) \chi_{-\varkappa}^{\mu} \end{pmatrix} \tag{2.61}$$

[where the upper (lower) sign refers to $\varkappa$ positive (or negative)], it is easily verified that only $\varkappa = 1$ leads to an angle-independent result on the right-hand side of Eq. (2.60). Thus, only states with $j = \frac{1}{2}$ can satisfy the n.l.b.c. as given. In fact, for applications to excited-state spectroscopy the strict n.l.b.c. [Eq. (2.60)] must be relaxed in favor of an angle-averaged version (Reb 76, DJ 76, DeG 76). We shall not pursue this topic further because it is of little direct relevance to the low-lying baryons of interest in nuclear physics.

The meaning of this addition to $T^{\mu\nu}$ can be clarified by considering the total energy of the bag state

$$P^0 = \int d^3x\, T^{00}(x) \tag{2.62}$$

which we shall label $E(R)$ rather than $M(R)$ as a precursor to our discussion of center of mass (c.m.) corrections later

$$E(R) = \frac{3\omega_{1-1}}{R} + \frac{4\pi}{3} R^3 B \tag{2.63}$$

The first term is the kinetic energy, which also appeared in Bogolioubov's model, while the second is a volume term. Essentially it implies that it costs an energy BV to make this bubble in the vacuum within which the quarks move freely. It should be intuitively clear that energy-momentum conservation is related to pressure balance at the bag surface, so that a small change in radius should not significantly increase $E(R)$. Nevertheless, it is a recommended exercise for the reader to show explicitly that the n.l.b.c.

implies that

$$\partial E/\partial R = 0 \tag{2.64}$$

In concluding this section we wish to stress that it is an *assumption* of the model that B should be constant for all hadrons (see Section 2.3.1). Furthermore, as all hadronic bags have radii in the region 0.8–1.1 fm, this assumption has really not been severely tested. For example, Hasenfratz and Kuti (HK 78) show that a surface tension (or some linear combination of the two) can produce similar results. Clearly, any simple phenomenological device like B is a crude representation of the complicated QCD mechanism which leads to confinement, and one must be cautious about taking it too seriously outside the limited range where it has been used so far.

2.2. The Spectroscopy of Low-Lying States

2.2.1. General Features—Massless Quarks

We have seen that the only change in the calculation of the energy in the MIT bag model with respect to Bogolioubov is the addition of a volume term, BV. It is *assumed* that B is a universal constant, chosen to fit one piece of data. Once B is chosen, because of the n.l.b.c. (which as we have seen implies $\partial E/\partial R = 0$) the radius of the bag is uniquely determined for each hadron. Generalizing Eq. (2.63) to include excited states, the n.l.b.c. implies

$$\frac{\partial E(R)}{\partial R} = \frac{-\sum_i \omega_i}{R^2} + 4\pi R^2 B = 0 \tag{2.65}$$

and hence

$$R^4 = \sum_i \omega_i/(4\pi B) \tag{2.66}$$

Using Eq. (2.66) we can then simplify the expression for $E(R)$:

$$E(R) = \frac{4}{3}\left(\sum_i \omega_i\right)\Big/R = \frac{4}{3}\left(\sum_i \omega_i\right)^{3/4}(4\pi B)^{1/4} \tag{2.67}$$

Clearly, the remarkable result obtained by Bogolioubov for the mass of the Roper resonance was indeed a coincidence! In the absence of spin-dependent corrections to be discussed below, M_R/M_N is $(1.55)^{3/4} = 1.39$, which is still not bad. If once again we choose the average of nucleon and delta masses to set the mass scale, we now find

$$R_N \simeq 1.46 \text{ fm} \tag{2.68}$$

$$B^{1/4} \simeq 113 \text{ MeV}, \qquad B = 21 \text{ MeV/fm}^3 \tag{2.69}$$

2.2.2. Hyperfine Structure

Since the N and Δ are split by about 300 MeV, one is straining the atomic label of "hyperfine" a little. Nevertheless, it is generally accepted that the degeneracy between these two baryons is broken by the spin–spin interaction. Within the context of QCD, DeRujula *et al.* (DeR+ 75) were the first to observe that one-gluon exchange could provide just this kind of interaction. The ideas of DeRujula, Georgi, and Glashow have been developed over the last few years into a tremendously successful phenomenological description of hadronic properties using a harmonic oscillator basis and nonrelativistic quarks—most notably by Isgur, Karl, and collaborators (IK 77, IK 78, IK 79, Gut+ 79, For 81, and the whole proceedings of the Baryon 1980 Conference, Isg 80). In essence these calculations involve a diagonalization of the one-gluon exchange interaction in a very limited harmonic oscillator basis.

The philosophy of the bag model is rather different. It is hoped that the bag itself provides a suitable, phenomenological description of the nonperturbative gluon interactions—including gluon self-coupling. All that remains is the (hopefully weak) one-gluon exchange interaction, which is first order in the color-coupling constant α_c. Thus, there is no diagonalization procedure in the bag model. One simply uses first-order perturbation theory to estimate the energy shifts. As far as this procedure has been tested (which is really not far because of the technical complexity!), the use of perturbation theory does seem justified. For example, Close and Horgan (CH 80, CH 81), and more recently Maxwell and Vento (MV 81), have shown that the admixture of higher configurations in the usual $(1s_{1/2})^3$ nucleon ground state is very small. This is quite different from the large effects found in the nonrelativistic models, and shows up most dramatically in the attempts to understand the negative charge radius of the neutron—as we shall discuss in detail in Section 6.2.

If we keep only terms of order α_c, the problem reduces to the evaluation of the graphs shown in Fig. 2.1, where both transverse and Coulombic gluons are included. Let us identify $(\vec{\mathbf{E}}_i^a, \vec{\mathbf{B}}_i^a\colon a \in 1, \ldots, 8)$ as the color-electric and magnetic fields generated by the ith quark. Since the nonperturbative vacuum outside the bag is supposed to be a perfect color-dia-

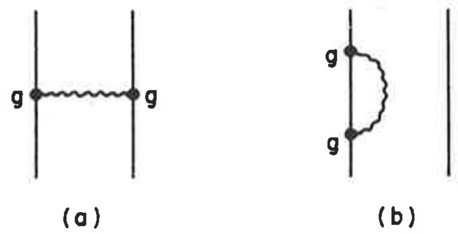

Fig. 2.1. One-gluon exchange contributions to the energy of the MIT bag.

electric medium (e.g., Cho+ 74a, Lee 79), the appropriate boundary conditions are (for a static spherical bag)

$$\hat{r} \cdot \left(\sum_i \vec{\mathbf{E}}_i^a\right) = 0; \qquad \hat{r} \times \left(\sum_i \vec{\mathbf{B}}_i^a\right) = 0 \tag{2.70}$$

at the surface. These fields obey the Maxwell equations

$$\vec{\nabla} \times \vec{\mathbf{B}}_i^a = j_i^a; \qquad \vec{\nabla} \cdot \vec{\mathbf{B}}_i^a = 0 \tag{2.71}$$

and

$$\vec{\nabla} \cdot \vec{\mathbf{E}}_i^a = j_i^{0a}; \qquad \vec{\nabla} \times \vec{\mathbf{E}}_i^a = 0 \tag{2.72}$$

inside the bag volume. Here the quark color current is simply

$$j_i^{\mu a}(x) = g\bar{q}_i(x)\gamma^\mu \lambda^a q_i(x) \tag{2.73}$$

with λ^a the usual Gell-Mann SU(3) matrices and g the strong QCD coupling constant. Having solved this rather straightforward problem in classic electromagnetism, the one-gluon exchange contribution to the energy, ΔE_g, is

$$\Delta E_g = \alpha_c \sum_{a=1}^{8} \frac{1}{2} \int_{\text{bag}} d\vec{\mathbf{x}}\, (\vec{\mathbf{E}}^a \cdot \vec{\mathbf{E}}^a - \vec{\mathbf{B}}^a \cdot \vec{\mathbf{B}}^a) \tag{2.74}$$

$$= \Delta E_g^E + \Delta E_g^M \tag{2.75}$$

There is one rather shady feature of the present discussion. That is, the quark self-energy shown in Fig. 2.1(b) should be included as part of the renormalization of the quark mass and therefore treated separately. For the magnetic term this is in fact what is done. That is, instead of Eqs. (2.74) and (2.75) one actually uses

$$\Delta E_g^M = -\alpha_c \sum_{a=1}^{8} \sum_{i<j} \frac{1}{2} \int d\vec{\mathbf{x}}\, \vec{\mathbf{B}}_i^a \cdot \vec{\mathbf{B}}_j^a \tag{2.76}$$

This is possible because *each* $\vec{\mathbf{B}}_i^a$ *separately* satisfies the boundary condition (2.70). On the other hand, for a uniformly charged sphere the electric field is necessarily in the radial direction, in fact

$$\vec{\mathbf{E}}_i^a(\vec{\mathbf{r}}) \sim \hat{r} \int_0^r \bar{q}_i(\vec{\mathbf{x}})\gamma^0\lambda^a q_i(\vec{\mathbf{x}})\, d^3x \tag{2.77}$$

Thus, it is possible to satisfy the boundary condition (2.70) only for a color singlet, for which

$$\sum_i \lambda_i^a = 0 \tag{2.78}$$

Therefore, in order to preserve the boundary conditions we are forced to keep those self-energy terms [Fig. 2.1(b)] which involve Coulomb-like gluons.

For hadrons in which all quarks have the same mass and are in the same orbit, the radial distributions will be identical. From Eq. (2.78) the total color-electric contribution to the energy will then be zero. Even in the case of strange hadrons, in which case $m_s \neq m_{u,d}$ (see Section 2.2.3), the color-electric contribution is of the order 5 MeV or less and is usually ignored (DeG+ 75). Finally then the one-gluon exchange quark–quark interaction gives a spin–spin contribution to the energy. Solving Eq. (2.71) for $\vec{\mathbf{B}}_i^a$ and substituting in Eq. (2.76) one finds

$$\Delta E_g^M = \frac{-3\alpha_c}{R} \sum_{a=1}^{8} \sum_{i<j} (\vec{\sigma}_i\lambda_i^a)(\vec{\sigma}_j\lambda_j^a) M(m_i, m_j, R) \tag{2.79}$$

where M is a function of the masses of quarks i and j, and the bag radius, which can be obtained in closed form (DeG+ 75).

Using the fact that physical hadrons are color singlets, so that

$$\sum_i \lambda_i^a = 0 \tag{2.80}$$

and the property of the SU(3) matrices

$$\sum_a (\lambda_i^a)^2 = 16/3 \tag{2.81}$$

one easily finds ($i \neq j$)

$$\begin{aligned} \sum_a \lambda_i^a \lambda_j^a &= -8/3 \quad \text{baryons} \\ &= -16/3 \quad \text{mesons} \end{aligned} \tag{2.82}$$

Thus, the one-gluon exchange interaction is

$$\Delta E_g^M = \frac{\lambda\alpha_c}{R} \sum_{i<j} \bar{M}(m_i, m_j, R)\vec{\sigma}_i \cdot \vec{\sigma}_j \tag{2.83}$$

where $\lambda = 1$ for baryons and 2 for mesons. The fact that the sign of the force is the same for both baryons and mesons is a direct consequence of using a non-Abelian theory. Clearly, the effect of this interaction is to move m_N down and m_Δ up, because the Δ contains only triplet states ($\vec{\sigma}_i \cdot \vec{\sigma}_j = +1$). In fact, in this case the amounts up and down are equal and, of course, proportional to α_c. This splitting essentially determines α_c, and in the old MIT fits it was 2.2.

As another example consider the Λ and Σ. Because $m_s \neq m_{u,d}$ these will also be split. Basically, in the Λ the u and d are in a spin singlet state, so $\vec{\sigma}_u \cdot \vec{\sigma}_d = -3$ and $\vec{\sigma}_s \cdot (\vec{\sigma}_u + \vec{\sigma}_d) = 0$. In the Σ the u and d have $S = 1$, so $\vec{\sigma}_u \cdot \vec{\sigma}_d = +1$ and $\vec{\sigma}_s \cdot (\vec{\sigma}_u + \vec{\sigma}_d) = -4$. Consequently

$$\Delta E_g{}^M(\Lambda) = -3\alpha_c \bar{M}(0, 0) \tag{2.84}$$

[where $\bar{M}(0,0)$ refers to the masses of the u and d quarks] while

$$\Delta E_g{}^M(\Sigma) = +1\alpha_c \bar{M}(0, 0) - 4\alpha_c \bar{M}(0, m_s) \tag{2.85}$$

Clearly, $\Delta E_g{}^M(\Lambda) = \Delta E_g{}^M(\Sigma)$ if $m_s = 0$, but with $m_s > 0$, $\bar{M}(0, m_s)$ is somewhat suppressed and hence m_Λ is less than m_Σ.

2.2.3. Nonzero Quark Masses

If the strange quark was massless, the other members of the nucleon octet, namely the Σ, Λ, and Ξ, would all be degenerate with the nucleon. From prebag phenomenology one might expect that giving the strange quark a mass would solve the problem and that is indeed the case. We might add that there is presently no understanding of the quark mass problem; these can only be regarded as free parameters of the theory (see, however, CT 74, Fri 77, Gun+ 77, Wei 77).

In the case where the mass of a quark is not precisely zero inside the bag, all of the formalism of Section 2.1.1 can again be applied—we need only change the form of $W(r)$ in the Dirac equation (2.9). Inside the bag we then have (Bar 75, DeG+ 75)

$$(-i\vec{\gamma} \cdot \vec{\nabla} + \gamma^0 E + m)q(\vec{r}) = 0 \tag{2.86}$$

with the boundary condition

$$-i\vec{\gamma} \cdot \hat{r} q = q \quad \text{at} \quad r = R \tag{2.87}$$

This has the solution for $\varkappa = -1$ (i.e., an $s_{1/2}$ level)

$$q(\vec{\mathbf{r}}) = \frac{N(x)}{(4\pi)^{1/2}} \begin{bmatrix} \left(\dfrac{E+m}{E}\right)^{1/2} & j_0\left(\dfrac{xr}{R}\right) \\ \left(\dfrac{E-m}{E}\right)^{1/2} & i\vec{\sigma}\cdot\hat{r}j_1\left(\dfrac{xr}{R}\right) \end{bmatrix} \chi \tag{2.88}$$

where

$$E(m, R) = \frac{1}{R}\,[x^2 + (mR)^2]^{1/2} \tag{2.89}$$

and the normalization constant is

$$N^{-2}(x) = R^3 j_0^2(x)\,\frac{2E(E - 1/R) + m/R}{E(E-m)} \tag{2.90}$$

The eigenfrequency, x, resulting from the imposition of the l.b.c. satisfies

$$\tan x = \frac{x}{1 - mR - [x^2 + (mR)^2]^{1/2}} \tag{2.91}$$

Obviously we have $x = 2.04$ when $m = 0$, as before. It rises to π as $mR \to \infty$. At $mR = 1.5$, corresponding approximately to the best fit for the strange quark mass, $m_s \sim 300$ MeV (DeG+ 75), we obtain $x = 2.5$ and $ER = 2.92$. Thus, forgetting about the n.l.b.c. for the moment, we see that replacing one up or down quark by a strange quark raises the mass of the hadron by $(2.92-2.04)/R$, or about 170 MeV, which by construction agrees with the observed Λ–N mass splitting.

2.2.4. Other Corrections to Hadronic Masses

There are two other possible contributions to the mass of the hadron which have been discussed in the literature. As always when one quantizes a radiation field, there is some infinite zero-point term. However, when the quantization is carried out in a finite cavity there will in general be additional, finite pieces which depend on the size of the cavity. It has not yet proven possible to calculate the finite remainder for a spherical cavity. [See, however, Section 2.3.1 for a recent discussion by Johnson (Joh 79) based on an analogy with QED.] For phenomenological simplicity it has been parametrized (Cho+ 74b, DeG+ 75) as a constant Z_0 divided by the bag radius R:

$$\Delta E_Z^{(0)} = -Z_0/R \tag{2.92}$$

The constant was determined to be $Z_0 = 1.8$ in the fit of DeGrand and co-workers (DeG+ 75).

The second correction would be the most obvious to nuclear physicists. That is, we have adopted the equivalent of the "independent particle shell model" for a three-quark hadron. For ^{3}He every nuclear theorist would recognize that there would be a sizeable spurious contribution to the energy from motion of the center of mass. To estimate the size of this effect let us assume that the (relativistic) energy of the bag $E(R)$ is related to the mass $M(R)$ by

$$E^2(R) \simeq \langle \vec{\mathbf{p}}^2_{\text{c.m.}} \rangle + M^2(R) \tag{2.93}$$

so that

$$M(R) \simeq E(R) - \langle \vec{\mathbf{p}}^2_{\text{c.m.}} \rangle / 2E(R) \tag{2.94}$$

But the total c.m. momentum is

$$\begin{aligned} \langle \vec{\mathbf{p}}^2_{\text{c.m.}} \rangle &= \left\langle \left(\sum_i \vec{\mathbf{p}}_i \right)^2 \right\rangle, \\ &\simeq \sum_i \langle p_i^2 \rangle \end{aligned} \tag{2.95}$$

Using the fact that for a massless quark $\langle p_i^2 \rangle = \omega_i^2/R^2$, we find for the nucleon [using Eq. (2.67), (2.94), and (2.95)]

$$\Delta E_{\text{c.m.}} \simeq \frac{-3\omega^2/R^2}{(8\omega/R)} = -\frac{3}{8}\frac{\omega}{R} \simeq -0.75/R \tag{2.96}$$

For a radius of 1.4 fm this is of the order 110 MeV, which is a sizeable correction. It becomes even more important as R decreases.

From the phenomenological point of view we notice that Eq. (2.96) has the same dependence on R as Eq. (2.92) and is about one-half as big. Thus, a good part of the original "zero-point energy" can be understood as a correction for spurious c.m. motion. Of course, our derivation should make it obvious that Z_0 should not be strictly constant, and this has been approximately taken into account in recent fits (DJ 80, Myh+ 81) by multiplying Eq. (2.96) by m_N/m_B, with m_B the physical mass of the appropriate baryon.

For the ground-state baryons this c.m. correction is simply an inconvenience and requires some correction to the energy. However, when we deal with excited states it becomes critical. In particular, the first applications of the bag model to negative-parity baryons found many more states than are seen experimentally. Some of these, explicitly the members of the $(56,1^-)$ representation of $SU(4)\times O(3)$, correspond to translation of the center of mass of the bag and are spurious (DJ 76, DeG 76, Reb 76, Hey 77, DR 78). The detailed discussion of how to eliminate spurious c.m. motion for excited bag states is technically very complicated (unlike the nonrelativistic harmonic oscillator calculations!). Moreover, the numerical results (DeG 76) are really rather poor—possibly because the MIT bag overestimates the spin–orbit force splitting of the $p_{1/2}$ and $p_{3/2}$ levels (DeG 80). We refer the interested reader to the literature already cited and particularly to the proceedings of Baryon 1980 Conference (Isg 80) for further discussion.

2.2.5. Summary

The complete mass formula for the original MIT bag model can then be summarized as

$$M(R) = \sum_i \frac{\omega_i}{R} + \frac{4\pi}{3} BR^3 + \Delta E_g{}^M - Z/R \tag{2.97}$$

with the spin-dependent one-gluon exchange interaction given by Eq. (2.83). The last term in Eq. (2.97) is now understood to include both c.m. and zero-point energies. There are four adjustable parameters in this mass formula, namely m_s, B, α_c, and Z and the radius R is determined for each hadron by the n.l.b.c. (the requirement of stability)

$$\frac{\partial M}{\partial R} = 0 \tag{2.98}$$

The original fit by DeGrand and co-workers (DeG+ 75) is shown in Fig. 2.2. It really gives an excellent description of the lowest baryon octet and decuplet, as well as the two lowest meson octets. The only exception is the pion, which is too heavy. However, it should be obvious from the discussion of Section 2.2.4 that the approximate correction for spurious c.m. motion will be meaningful only for fairly heavy states. For the pion it is not inconceivable that the entire bag model mass is spurious (DJ 80)! In particular, as we shall discuss further in Section 5, considerations of chiral symmetry strongly suggest that in the limit $m_u = m_d = 0$, the pion mass should also vanish. In this case it would be a true Goldstone boson associated with dynamically broken chiral symmetry (Gel+ 68, Pag 75, CJ 80, HG 81, GH 81, Joh 79).

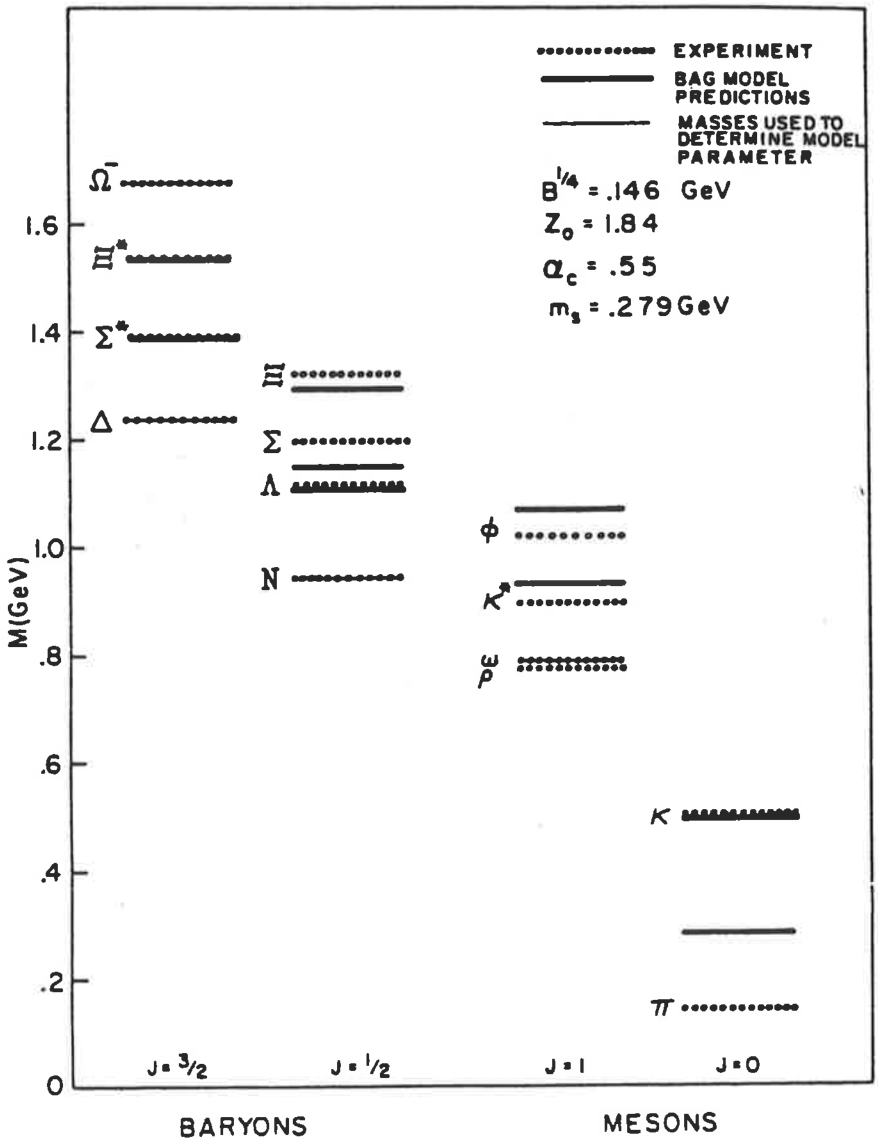

Fig. 2.2. The mass spectrum of the low-lying hadrons calculated in the MIT bag model (DeG+ 75).

2.3. Attempts to Derive a Bag Model

The proof of quark confinement on the basis of QCD has not yet been achieved. Thus, there is no derivation of a bag or its properties or anything like it from a fundamental theory. Nevertheless, there have been a number of very suggestive arguments which lead one to believe that the MIT bag model may not be far from the truth. A strictly personal collection of those arguments the author finds most compelling will be briefly reported here.

2.3.1. The Bubbly Vacuum

Johnson recently presented some rather simple considerations (Joh 79) which suggest that the most stable vacuum configuration in QCD should be a collection of bubbles of size R of order Λ^{-1} (with Λ the QCD scale parameter).

The starting point for this work is the recent solution of a long-standing problem in QED. Suppose one has a cavity of radius R with conducting walls, that is, with the boundary condition

$$\hat{r} \times \vec{\mathbf{E}} = 0 = \hat{r} \cdot \vec{\mathbf{B}} \tag{2.99}$$

at the surface. Then the piece of the total energy which depends on R is (Mil+ 78, BD 78, Boy 68)

$$E_{\mathrm{QED}} = a_{\mathrm{QED}}/R, \qquad a_{\mathrm{QED}} = 0.04618 \tag{2.100}$$

That this answer is finite is the result of a natural high-frequency cutoff arising from the cancellation of small wavelength effects just inside and outside the conducting boundary. It must be stressed that the nature of the boundary is critical.

For QCD the analogous boundary conditions are given in Eq. (2.70), but since Maxwell's equations are invariant under $\vec{\mathbf{E}} \to \vec{\mathbf{B}}$ and $\vec{\mathbf{B}} \to -\vec{\mathbf{E}}$, we can take this result over. Now, of course, there are eight gluon fields and we assume that R is small enough to permit the use of perturbative QCD. To lowest order we then find

$$E_{\mathrm{QCD}}^{(0)} = a_{\mathrm{QCD}}/R = 8a_{\mathrm{QED}}/R = 0.369/R \tag{2.101}$$

The difference in QCD is, of course, that the gluons have self-interactions. Interactions of the sort shown in Fig. 2.3 are known to be attractive for the color-singlet state. Thus, there is a pairing-type force which tends to favor color singlets. Furthermore, this attraction should grow rapidly with R.

Johnson parametrizes the higher-order non-Abelian effects in terms of a running coupling constant

$$\alpha_c(\Lambda R) = \frac{1}{(9/2\pi)\ln[(\Lambda R)^{-1} + 1]} \tag{2.102}$$

The total energy of the bubble of radius R would then be

$$E_{\mathrm{QCD}}^{(1)}(R) = a_{\mathrm{QCD}}/R - (b/R)\alpha_c(\Lambda R) \tag{2.103}$$

Fig. 2.3. Some low-order gluon self-interactions.

with $b(>0)$ an unknown constant. Clearly, as R grows, eventually $\alpha_c(\Lambda R)$ will be greater than a_{QCD}/b and the bubble has an energy density below the noninteracting case. Finally, $E(R)$ eventually vanishes as R goes to infinity. We therefore expect the most stable bubble at some finite radius R_0 which can be found by minimizing the energy density

$$\partial(E_{\mathrm{QCD}}^{(1)}/V)/\partial R \mid R = R_0 = 0 \tag{2.104}$$

The QCD vacuum tends to break spontaneously into a set of bubbles of size R_0!

By extending this argument to include quark degrees of freedom, Johnson was able to derive a formula for hadronic masses very close to the static MIT bag model. In particular, the bag constant (B) is simply the energy per unit volume of the empty bags surrounding the hadron. From a simple phenomenological analysis he found $R_0 \simeq 0.5$ fm with $\Lambda = 500$ MeV. While this picture is very much simplified—for example, it is not Lorentz invariant—it has many suggestive features. Most importantly there is a volume energy, the hadron is stable [see, e.g., Eq. (2.98)], perturbative QCD is permitted inside the bag, and there is a very rapid phase transition at the surface. Of course, the physical nature of the surface which would provide the color-dia-electric boundary conditions is beyond the scope of this treatment.

2.3.2. Soliton Bag Models

Many groups have proposed that bag formation should be associated with a phase transition. In the presence of the strong color fields inside the bag the vacuum is very simple and the quarks are essentially free. However, at some critical field strength there is a phase transition to a highly complicated vacuum state with color-dielectric constant $\varkappa \to 0$. thus confining color fields. In the Princeton picture the pion appears as an essential part of this process (Cal+ 78, Cal+ 79). As we shall discuss further in Sections 4 and 5, in their picture, it is a Goldstone boson associated with the breaking of chiral symmetry in the complicated vacuum outside the bag. It contributes to the bag pressure.

Goldman and Haymaker (GH 81, HG 81) have recently demonstated how pion and sigma (scalar–isoscalar) fields can appear as a result of dynamical symmetry breaking in a model of the Jona-Lasinio–Nambu type (NJ 61). Although it was not strictly QCD, the model was sufficiently realistic to be highly suggestive. We shall return to the need for the pion again in Section 5. The appearance of an effective σ-field interacting with the quarks is, however, directly relevant here. In particular, Friedberg and Lee have shown that it is possible to obtain baglike states as soliton solutions of a relativistic, local field theory containing just q and σ (Bar+ 75, IM 75, Cre 74, CS 75, FL 77, FL 78, Lee 79).

A complete discussion of soliton models of elementary particles is far beyond the scope of the present review. The interested reader should refer first to the recent text by Lee (81) and then to the references therein. For our purposes it is sufficient to summarize the recent discussion of Goldflam and Wilets (GW 82), which has by far the most detailed numerical results for the soliton bag.

Consider the following Lagrangian density for interacting σ and quark fields;

$$\mathscr{L}(x) = i\bar{q}\not{\partial}q - g\bar{q}\sigma q + \tfrac{1}{2}(\partial_\mu\sigma)^2 - U(\sigma) \tag{2.105}$$

The first and third terms are standard kinetic energy operators and the second is the simplest possible coupling. The existence of solitonlike solutions is a consequence of the nonlinear form of the potential $U(\sigma)$:

$$U(\sigma) = \frac{c}{24}\sigma^4 + \frac{b}{6}\sigma^3 + \frac{a}{2}\sigma^2 + p \tag{2.106}$$

whose general form is illustrated in Fig. 2.4. [Equation (2.106) is the most general self-coupling permitted in a renormalizable field theory.] The energy of the σ-field alone will be a minimum at the minimum of $U(\sigma)$ (recall $T^{00} \sim -\mathscr{L} \sim +U$), namely

$$\sigma_v = \frac{3}{2c}\left[-b + \left(b^2 - \frac{8}{3}ac\right)^{1/2}\right] \tag{2.107}$$

[It is usual to choose p so that $U(\sigma_v) = 0$.] In the absence of a coupling to quark fields the lowest energy state would be simply a constant classical field $\sigma = \sigma_v$ throughout space.

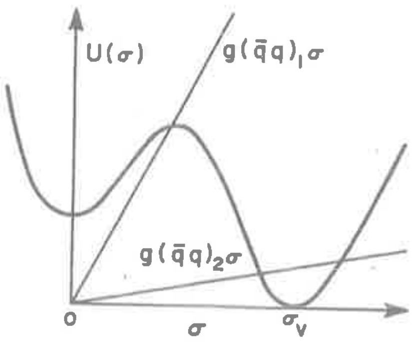

Fig. 2.4. A typical form for the σ-potential energy, $U(\sigma)$, in a soliton bag model.

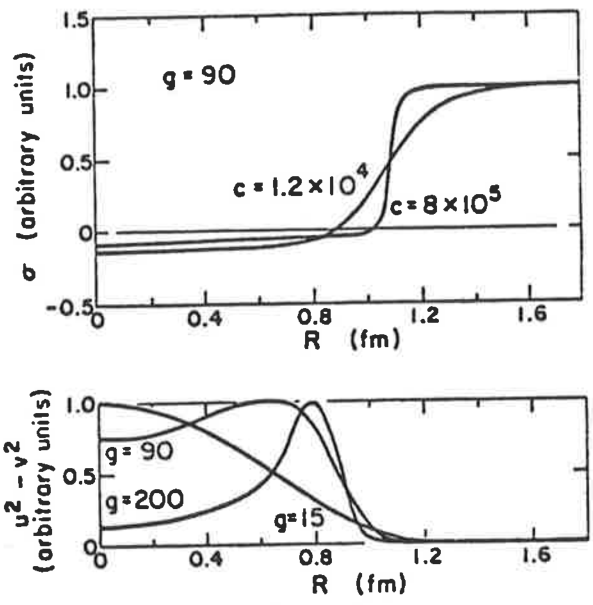

Fig. 2.5. Numerical results from the soliton bag model calculations of Goldflam and Wilets (GW 82) showing (a) the σ-field for MIT-like solutions and (b) the quark density for MIT-like ($g = 15$), SLAC-like ($g = 200$), and intermediate bags.

However, suppose that there is a nonzero quark density at some point in space, which we can choose to be $\vec{r} = 0$. (Strictly we want $\bar{q}q \neq 0$.) The second term on the right-hand side of Eq. (2.105) is then linear in σ as shown in Fig. 2.4. Clearly, if either g or $\bar{q}q$ is large enough, it is possible that the minimum energy will occur at $\sigma = 0$ rather than $\sigma = \sigma_v$. In this region the quark and sigma fields obey coupled linear equations

$$\begin{aligned}(\vec{\alpha} \cdot \vec{p} + g\gamma^0\sigma_0)\psi_k &= \varepsilon_k\psi_k \\ -\nabla^2\sigma_0 + U'(\sigma_0) &= -g\sum_k \bar{\psi}_k\psi_k\end{aligned} \tag{2.108}$$

where σ_0 is the time-independent, mean σ-field. Some typical solutions of these equations are plotted in Fig. 2.5.

In all cases $\bar{\psi}\psi$ eventually vanishes as $r \to \infty$ so that asymptotically σ returns to its usual vacuum expectation value. Inside, however, σ is very small and the quarks are essentially free ($\sigma(0) \approx 0$). That is, the quarks "dig a hole" in the complicated vacuum represented by large σ_v within which things are simple. Case 1 in Fig. 2.5 represents an MIT-bag type of solution where the quarks are distributed through the bag volume, while case 2 is a SLAC-bag (Bar+ 75) with its strongly surface-peaked quark distribution.

Many other intermediate solutions are possible depending on the choice of parameters (a, b, c). However, the baglike properties, namely that the quarks are essentially free inside and that the transition region from inside to outside is quite sharp, is true in all confining solutions. That is, the transition is sharp in all solutions where $g\sigma_v$ [the quark mass outside the bag from Eq. (2.105)] is chosen to be extremely large. Finally, we note that as discussed by Lee the color-dielectric constant $\varkappa$ is

$$\varkappa = \left(1 - \frac{\sigma}{\sigma_v}\right) \tag{2.109}$$

and therefore vanishes outside the bag (Lee 79). It will be close to one inside, and the gluon fields are therefore essentially free, if $\sigma \ll \sigma_v$ in that region. In that case a perturbation expansion of hadronic properties in powers of the color-coupling constant α_c should make sense. That is precisely the philosophy of the phenomenological bag model we have discussed!

2.4. Relationship to the Nonrelativistic Quark Models

Although it is not our purpose to review the nonrelativistic quark model here, it is so widely used and generally regarded as being so successful that some comments must be made about the relationship to the bag model. Some of the comments found here have also been made by Thomas DeGrand (DeG 80).

While the identification is not so straightforward, it may be helpful to consider the bag model quarks with essentially zero mass (for u and d) to be what is usually referred to as "current quarks." It is these objects that are confined in an infinite scalar potential as we have seen. The result of this confinement is an energy level of the scale of typical hadronic masses. This eigenfunction can be thought of as a "constituent quark." Now if there is some truth to such a translation, there are important consequences for the usual diagonalization procedures of the nonrelativistic quark models, and this augments the surprise at their success. We defer further discussion of this until Sections 6 (neutron charge radius) and 7 (N–N force).

One major objection to the nonrelativistic (or harmonic oscillator) quark model calculations is the tendency to ignore relativistic corrections. In computing μ_p and μ_n, for example, the up and down quark masses are chosen so that the corresponding Dirac moments ($e/2m_q$) when added nonrelativistically yield a good fit. Relativistic corrections are simply omitted despite the fact that typically $\langle p^2\rangle/2m_q$ is bigger than m_q!

As Litchfield remarked (Isg 80, p. 216), "despite the theoretical bricks thrown at Isgur and Karl's model the amazing thing is how well their formulae actually fit an extremely large and varied data set. This would seem to imply that there must be a basis of truth in the arithmetic and maybe more effort should go into seeing why their formula is so nearly correct." In the case of magnetic moments the bag model does just this. As we shall see in Section 3.2, there is no free parameter in computing the magnetic moment of a massless quark in a bag. The answer is, however, proportional to $R/\omega_{n,\varkappa}$, which we recognize as one over the energy of the appropriate level. This includes *all* relativistic effects. However, as we remarked earlier the bag model quark energy is essentially the constituent quark mass! Thus, both models arrive at an answer proportional to m_q^{-1}, but the bag helps us to understand why there are no "relativistic corrections."

The nonrelativistic models really have tremendous practical advantages in dealing with excited states. As DeGrand observes (DeG 80), "(although I hate to admit it) bag models are calculationally much more unwieldy than the non-relativistic quark models." The major problem is to deal with spurious c.m. motion. It does not appear likely that this problem with bag models will improve in the near future.

Finally, as we shall re-emphasize in Section 4, the bag model can be formulated (at least in principle) as a relativistic local field theory. In particular, it can readily be described by a Lagrangian and all the standard technology can be applied to it. This has been extremely important in discussions of symmetry properties (conserved currents) and was certainly an important factor in spurring the further development of chiral bags. It is dubious whether such considerations would ever have arisen out of the potential model calculations.

3. HADRONIC PROPERTIES IN THE MIT BAG MODEL

In the last section we gave some arguments to support the idea that QCD could lead to baglike hadrons. We discussed the quark wave functions in the bag in great detail and showed that the fit to at least the low-lying baryon and meson masses was rather good. From the purist's point of view it is an attractive feature of the bag model that once the fit to the mass of a hadron has been made there are no further parameters to adjust. Either the calculated properties, r.m.s. charge radius, magnetic moment, axial-coupling constant, and so on agree with the data or they don't. In this section we shall show how these three basic properties are calculated. For the axial-coupling constant (Section 3.3) the agreement with experiment is excellent—as realized originally by Bogolioubov. For the charge radii and magnetic moments (Sections 3.1 and 3.2, respectively), the model is not quite so successful, particularly when the question of c.m. corrections is raised again (Section 3.4).

3.1. Charge Radii

As we remarked in Section 2, the matter density for a particular quark i is given (as usual for a Dirac particle) by

$$j_i^0(\underline{r}) = \bar{q}_i(\underline{r})\gamma^0 q_i(\underline{r})\theta_V = q_i^+(\underline{r})q_i(\underline{r})\theta_V \tag{3.1}$$

which is plotted in Fig. 3.1 for the $1s_{1/2}$ level. Clearly then the charge density for a given hadron is

$$\varrho(\underline{r}) = \sum_i q_i^+(\underline{r})Q_i q_i(\underline{r}) \tag{3.2}$$

where Q_i is the charge of the ith quark. The r.m.s. charge radius is therefore

$$\langle r^2\rangle_{\text{ch}} = \sum_i Q_i \int_{\text{bag}} d\underline{r}\; q_i^+(\underline{r})r^2 q_i(\underline{r}) \tag{3.3}$$

which is proportional to R^2. In fact, for the proton it is easily shown, using Eq. (2.33), (2.34), and (3.3), that

$$\langle r^2\rangle_{\text{ch}}^p = \left\{\frac{\omega_{1-1}^3}{2(\omega_{1-1}-1)\sin^2\omega_{1-1}} \int_0^1 dx\, x^4[j_0^2(\omega_{1-1}x) + j_1^2(\omega_{1-1}x)]\right\}R^2 \tag{3.4}$$

With $\omega_{1-1} = 2.04$ this gives

$$\langle r^2\rangle_{\text{ch}}^{p\,1/2} = 0.73 \text{ fm} \tag{3.5}$$

for $R = 1$ fm, as found in the fit of DeGrand *et al.* (DeG+ 75). This is to be compared with the experimental value of 0.82 fm.

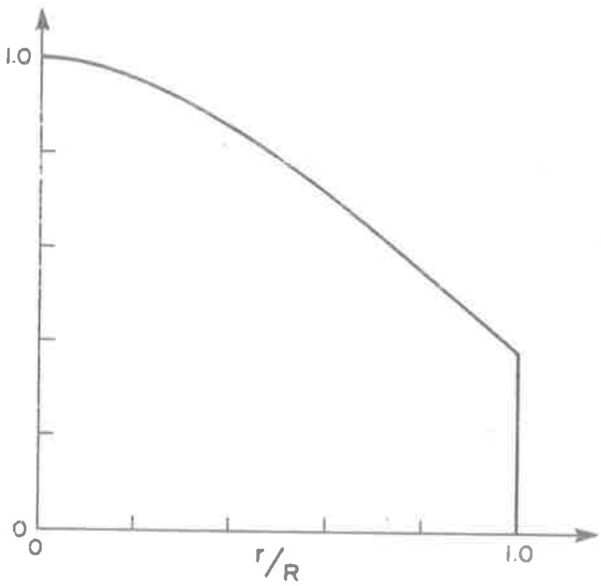

Fig. 3.1. Matter density in the $1s_{1/2}$ orbit for the MIT bag model.

Of more profound importance, as we shall discuss at length in Section 6, is the neutron charge distribution. Because each quark occupies the same spatial state, and the sum of the quark charges is zero, the mean square charge radius of the bag model neutron is zero in lowest order. Experimentally $\langle r^2\rangle_{\text{ch}}^{n}$ is known to be $-0.116\ \text{fm}^2$ from very accurate experiments with thermal neutrons. Attempts have been made to obtain this negative tail of the charge distribution through the perturbation of the ground-state wave function by one-gluon exchange. In the neutron the *dd* pair necessarily has isospin one and hence spin one (because their color wave function is antisymmetric). From Eq. (2.83) we see that the *dd* interaction is therefore repulsive and will tend to push the *dd* pair out from the center of the bag—hence a negative tail for the charge distribution. Quantitatively this idea fails for the bag. For example, Close and Horgan (CH 81) and Maxwell and Vento (MV 81) both find that this effect explains only about 6% of the observed ratio of $\langle r^2\rangle_{\text{ch}}^{n}/\langle r^2\rangle_{\text{ch}}^{p}$.

3.2. Magnetic Moments

In the bag model the quarks are structureless Dirac particles which therefore have no intrinsic moments. Indeed they yield a magnetic moment only because of the confinement. As for any current loop the magnetic moment is given by

$$\underline{\mu} = \tfrac{1}{2} \int (\underline{r} \times \underline{j}_{\text{em}})\, d\underline{r} \tag{3.6}$$

Using the usual form for the Dirac electromagnetic current

$$= \frac{1}{2} \int_{\text{bag}} d\underline{r}\ \underline{r} x \Big[\sum_i q_i^{+}(\underline{r}) \underline{\alpha}_i Q_i q_i(\underline{r}) \Big] \tag{3.7}$$

and the $1s_{1/2}$ wave functions of Section 2 we find (for massless quarks)

$$\underline{\mu} = \frac{N^2}{2} \sum_i Q_i \int_0^R dr\, r^2 [j_0(\omega r/R), -i\underline{\sigma}_i \cdot \underline{\hat{r}} j_1(\omega r/R)]$$
$$\times \begin{pmatrix} 0 & \underline{r} \times \underline{\sigma}_i \\ \underline{r} \times \underline{\sigma}_i & 0 \end{pmatrix} \begin{pmatrix} j_0(\omega r/R) \\ i\underline{\sigma}_i \cdot \hat{r} j_1(\omega r/R) \end{pmatrix} \tag{3.8}$$

Finally, after a little spin algebra, Eq. (3.8) reduces to the form

$$\underline{\mu} = \mu_0 \sum_i \underline{\sigma}_i Q_i \tag{3.9}$$

where μ_0 is directly proportional to the radius of the confinement region, that is

$$\mu_0 = \frac{4\omega - 3}{\omega(\omega - 1)} \frac{R}{12}$$

$$= \frac{2.43}{12}(2m_N R)\mu_N \tag{3.10}$$

Here μ_N is the usual nuclear magneton $(e/2m_N)$.

Therefore, just as in the nonrelativistic constituent quark models we are left to evaluate a simple spin–isospin matrix element

$$\mu_p = \mu_0 \langle p\uparrow | \sum_{i=1}^{3} \sigma_{iz} Q_i | p\uparrow \rangle \tag{3.11}$$

However, instead of μ_0 being adjusted to fit experiment it is calculable in terms of the quark radial wave functions yielding Eq. (3.10). Using the standard mixed-symmetry spin–flavor wave functions (Kok 69)

$$\begin{aligned} \psi' &= 2^{-1/2}(duu - udu) \\ \psi'' &= -\left(\frac{2}{3}\right)^{1/2}\left[uud - \frac{1}{2}(udu + duu)\right] \end{aligned} \tag{3.12}$$

and similarly the mixed-symmetry spin states χ' and χ'' [substitute $u \to \uparrow$ and $d \to \downarrow$ in Eq. (3.12)], we have

$$| p\uparrow \rangle = 2^{-1/2}(\psi'\chi' + \psi''\chi'') \tag{3.13}$$

It is a straightforward exercise to prove that

$$\langle p\uparrow | \sum_{i=1}^{3} Q_i \sigma_{iz} | p\uparrow \rangle = +1 \tag{3.14}$$

and hence

$$\mu_p = \mu_0 \tag{3.15}$$

With the original radius $R = 1.3$ fm this gives $\mu_p = 2.6\,\mu_N$, but with the "best-fit" parameters of DeGrand *et al.* $\mu_p = 1.9\,\mu_N$ (DeG+ 75). As recognized by those authors, the failure to reproduce the magnitude of the proton magnetic moment was the most serious discrepancy of all the predictions of the model. Nevertheless, if one normalizes all other moments to that of the proton, it is a remarkable fact that the bag model is invariably an improvement over the naive SU(3) predictions (see Table 3.1).

TABLE 3.1

Magnetic Moments of the Nucleon Octet in Units of the Proton Magnetic Moment[a]

	MIT bag	Experiment	Naive SU(3)
μ_n	−2/3	−0.685	−2/3
μ_Λ	−0.26	−0.219	−1/3
μ_{Σ^-}	−0.36	−0.51	−1/3
μ_{Σ^+}	+0.97	+0.84	+1.0
μ_{Ξ^0}	−0.56	−0.45	-2/3
μ_{Ξ^-}	−0.23	−0.27	−1/3

[a] From DeG+ 75.

3.3. The Axial Current

The accurate prediction of the axial coupling constant, g_A, is certainly one of the major successes of the MIT bag model. It is a direct consequence of the correct, relativistic treatment of the quarks. In view of the importance of the axial current in our later development of a chiral-symmetric model of hadronic structure, we shall discuss the calculation of g_A in detail. First, we briefly review the standard phenomenological treatment of weak interactions.

The usual weak-interaction Hamiltonian has a current–current form (Mar+ 69)

$$H_W = (G/2)J^\mu J_\mu{}^+ \tag{3.16}$$

where the coupling constant is

$$G \simeq 10^{-5} m_p{}^{-2} \tag{3.17}$$

The current is a sum of hadronic and leptonic pieces:

$$J^\mu = J_h{}^\mu + J_l{}^\mu \tag{3.18}$$

where (assuming $V - A$)

$$J_l{}^\mu = \bar{\nu}_e \gamma^\mu (1 - \gamma_5) e + (e \to \mu;\, \nu_e \to \nu_\mu) \tag{3.19}$$

and

$$J_h{}^\mu = V^\mu - A^\mu \tag{3.20}$$

The hadronic vector and axial vector components have both strangeness conserving ($\Delta S = 0$, proportional to $\cos\theta_c$) and nonconserving

($\Delta S = 1$, proportional to $\sin\theta_c$) pieces. For our purposes only the $\Delta S = 0$ piece is relevant and we shall effectively set $\theta_c = 0$ for pedagogical purposes. Consider the semileptonic matrix element

$$i \to f + \text{leptons} \tag{3.21}$$

which would appear (for example) in β-decay. This matrix element is proportional to

$$\langle f, l \mid J_h^\mu J_\mu^{l\dagger} \mid i\rangle = \langle f \mid J_h^\mu \mid i\rangle\langle l \mid J_\mu^{l\dagger} \mid 0\rangle \tag{3.22}$$

where the leptonic piece is known exactly. In the simple case of neutron β-decay

$$\begin{aligned}\langle f \mid J_h^\mu \mid i\rangle &= \langle p \mid J_h^\mu \mid n\rangle \\ &\sim e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}\bar{u}_p[\gamma^\mu g_v(k^2) - \gamma^\mu\gamma_5 g_A(k^2) + \cdots]u_n\end{aligned} \tag{3.23}$$

where the corrections are of order k^2 and therefore suppressed. In fact, in the limit of very small $\underset{\sim}{k}$ Eq. (3.23) is simply

$$\langle p \mid J_h^\mu \mid n\rangle \simeq g_v(0)\delta_{\mu 0} - g_A(0)\sigma_i\delta_{\mu i} \tag{3.24}$$

where

$$g_v \simeq 1 \quad \text{and} \quad g_A/g_v = 1.24 \tag{3.25}$$

The fact that $g_v = 1$ even in the presence of strong interactions is, of course, of great significance and is "explained" by the CVC hypothesis of Feynman and Gell-Mann (BD 64). That is, the vector current is assumed to be directly proportional to the isospin current

$$V_\mu{}^a = 2g_v I_\mu{}^a \tag{3.26}$$

where $I_\mu{}^a$ is the isospin current which is *conserved in strong interactions.*

The fact that g_A is so nearly one is also highly suggestive, as we shall discuss in the next section. For the moment we ask only how to calculate this in the bag model. The isospin current in the MIT bag model is

$$\underset{\sim}{I}^\mu(\underset{\sim}{x}) = \sum_i \bar{q}_i(\underset{\sim}{x})\gamma^\mu \frac{\underset{\sim}{\tau}}{2} q_i(x)\theta_v \tag{3.27}$$

and the axial current

$$\underset{\sim}{A}^\mu(\underset{\sim}{x}) = \sum_i \bar{q}_i(\underset{\sim}{x})\gamma^\mu\gamma_5 \frac{\underset{\sim}{\tau}}{2} q_i(\underset{\sim}{x})\theta_v \tag{3.28}$$

[By analogy with Eq. (3.26) we let $\underset{\sim}{A}^\mu = 2\underset{\sim}{A}^\mu$.]

Clearly then at $\underset{\sim}{k} = 0$, the matrix element

$$\underset{\sim}{t}_{pn} = \int d^3r \, \langle p \mid \underset{\sim}{I}^0(r) \mid n\rangle\Big|_{\underset{\sim}{k}=0} = \int d^3r \sum_i \langle p \mid q_i^+(\underset{\sim}{r}) \frac{\underset{\sim}{\tau}}{2} q_i(\underset{\sim}{r}) \mid n\rangle$$

and because the quark radial wave functions are normalized we find

$$\underset{\sim}{t}_{pn} = {}_{s-f}\langle p \mid \sum_i \frac{\underset{\sim}{\tau}_i}{2} \mid n\rangle_{s-f} \tag{3.29}$$

where the subscript indicates a spin–flavor matrix element only. Using the wave function given earlier (and the analogous one for the neutron) this becomes

$$\underset{\sim}{t}_{pn} = \langle p \mid \underset{\sim}{\tau}/2 \mid n\rangle \tag{3.30}$$

On the other hand, again with $\underset{\sim}{k} = 0$, we find

$$\int d^3x \, \langle p \mid \vec{\underset{\sim}{A}}(\underset{\sim}{x}) \mid n\rangle\Big|_{\underset{\sim}{k}=0} = \int d^3x \sum_i \langle p \mid q_i^+(\underset{\sim}{x})\gamma^0\vec{\underset{\sim}{\gamma}}\gamma_5 \frac{\underset{\sim}{\tau}}{2} q_i(\underset{\sim}{x}) \mid n\rangle \tag{3.31}$$

However, with our conventions (Appendix I)

$$\gamma^0\vec{\underset{\sim}{\gamma}}\gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & \underset{\sim}{\sigma} \\ -\underset{\sim}{\sigma} & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \underset{\sim}{\sigma} & 0 \\ 0 & \underset{\sim}{\sigma} \end{pmatrix} \tag{3.32}$$

and hence

$$\begin{aligned}\int_{\text{bag}} d\underset{\sim}{x} \, q^+(\underset{\sim}{x})\gamma^0\vec{\underset{\sim}{\gamma}}\gamma_5 q(\underset{\sim}{x}) &= \int_{\text{bag}} d\underset{\sim}{x} \frac{N^2}{4\pi} (j_0, -i\underset{\sim}{\sigma} \cdot \hat{x} j_1)\begin{pmatrix} \underset{\sim}{\sigma} & 0 \\ 0 & \underset{\sim}{\sigma} \end{pmatrix}\begin{pmatrix} j_0 \\ i\underset{\sim}{\sigma} \cdot \hat{x} j_1 \end{pmatrix} \\ &= \frac{N^2}{4\pi} \int_0^R dx \, x^2 \int d\hat{x} \, (j_0^2 \underset{\sim}{\sigma} + j_1^2 \underset{\sim}{\sigma} \cdot \hat{x}\underset{\sim}{\sigma}\underset{\sim}{\sigma} \cdot \hat{x})\end{aligned} \tag{3.33}$$

However, after a little spin algebra we find

$$\int d\hat{x} \, \underset{\sim}{\sigma} \cdot \hat{x}\underset{\sim}{\sigma}\underset{\sim}{\sigma} \cdot \hat{x} = (-4\pi/3)\underset{\sim}{\sigma} \tag{3.34}$$

and hence

$$\begin{aligned}\int d^3x \, \langle p \mid \vec{\underset{\sim}{A}}(\underset{\sim}{x}) \mid n\rangle\Big|_{\underset{\sim}{k}=0} &= N^2 \int_0^R dx \, x^2 \left[j_0^2\left(\frac{\omega x}{R}\right) - \frac{1}{3} j_1^2\left(\frac{\omega x}{R}\right)\right] \\ &\quad \times {}_{s-f}\langle p \mid \sum_{i=1}^{3} \vec{\boldsymbol{\sigma}}_i \frac{\underset{\sim}{\tau}_i}{2} \mid n\rangle_{s-f}\end{aligned} \tag{3.35}$$

The first term can be evaluated analytically, and for massless u and d quarks one finds (DeG+ 75)

$$N^2 \int_0^R dx\, x^2\left(j_0^2 - \frac{1}{3} j_1^2\right) = 1 - \frac{1}{3}\left(\frac{2\omega - 3}{\omega - 1}\right) = 0.65 \qquad (3.36)$$

This is the crucial difference from nonrelativistic (constituent) quark models.

In the usual nonrelativistic quark model

$$\vec{\underset{\sim}{A}}_{NR} = \sum_{i=1}^{3} \vec{\sigma}_i \frac{\underset{\sim}{\tau}_i}{2} \qquad (3.37)$$

and the spin–flavor matrix element is easily evaluated using the wave functions given earlier with the result

$$_{\text{s-f}}\langle p \mid \sum_{i=1}^{3} \underset{\sim}{\sigma}_i \frac{\underset{\sim}{\tau}_i}{2} \mid n\rangle_{\text{s-f}} = \frac{5}{3} \langle p \mid \underset{\sim}{\sigma} \frac{\underset{\sim}{\tau}}{2} \mid n\rangle \qquad (3.38)$$

and hence $g_A^{NR} = \frac{5}{3}$. The bag model reduces this to 1.09, which is in much better agreement with the experimental result $g_A/g_V = 1.24$.

3.4. Center of Mass Corrections

As it is usually presented, the bag model is effectively an *i*ndependent *p*article *s*hell *m*odel (IPSM) of hadron structure. In the nuclear context it is widely known that the IPSM is a terrible way to treat ^{3}He. For example, if one uses harmonic oscillator wave functions the removal of the spurious motion of the center of mass reduces the value of the squared charge radius by a factor of $\frac{2}{3}$. Thus, one might expect that c.m. corrections should be extremely important for the bag model.

The procedures for removing spurious kinetic energy associated with the motion of the c.m. were discussed in Section 2.2.4. Here we are concerned with the effect on observables associated with the bag. In Section 3.4.1 we shall briefly describe what seems to be the most reasonable correction procedure, while in Section 3.4.2 we discuss the ambiguities which do not arise in the nuclear case.

3.4.1. Center of Mass Corrections in the Independent-Particle Approximation

The recent discussions of c.m. corrections to observables began with the work of Donoghue and Johnson (DJ 80). These authors attempted to calculate the pion decay constant (f) in the bag model. Further work, with rather different conclusions, was carried out by Wong (Won 81).

Finally, Carlson and Chachkhunashvili (CC 81) followed the same approach as Wong in order to derive corrections for hadronic properties—charge radii, magnetic moments, and so on.

The technique used by Wong to remove the spurious c.m. motion is known as the Peierls–Yoccoz projection in nuclear physics (PY 57). The assumption is that the independent particle model wave function can be written as a supersposition of momentum eigenstates, whose internal structure describes the true hadron. Suppose we have a bag fixed at the position $\underline{R}$, which we denote by $| B(\underline{R})\rangle$, then we have

$$| B(\underline{R})\rangle = \int \frac{d\underline{p}}{W(\underline{p})} e^{i\underline{p}\cdot\underline{R}}\phi(\underline{p}) \mid b, \underline{p}\rangle \tag{3.39}$$

where $| b, \underline{p}\rangle$ is the momentum eigenstate representing particle b. This will be normalized as usual:

$$\langle b, \underline{p}' \mid b, \underline{p}\rangle = (2\pi)^3\delta(\underline{p} - \underline{p}')W(\underline{p}) \tag{3.40}$$

$$\begin{aligned} W(\underline{p}) &= 2\omega_p \quad \text{meson} \\ &= (m_N{}^2 + \underline{p}^2)^{1/2}/m_N \quad \text{baryon} \end{aligned} \tag{3.41}$$

Finally, $\phi(\underline{p})$ is the wave packet describing the momentum distribution in the bag. It can be obtained simply by inverting Eq. (3.39) to obtain

$$| b, \underline{p}\rangle = (2\pi)^{-3} \frac{W(\underline{p})}{\phi(\underline{p})} \int d\underline{R}\, e^{-i\underline{p}\cdot\underline{R}} \mid B(\underline{R})\rangle \tag{3.42}$$

and substituting into Eq. (3.40) with the result

$$\phi^2(\underline{p}) = \frac{W(\underline{p})}{(2\pi)^6} \int d\underline{r}\, e^{-i\underline{p}\cdot\underline{R}} \langle B(-\underline{r}/2) \mid B(\underline{r}/2)\rangle \tag{3.43}$$

Equation (3.43) will, of course, only receive contributions when r is less than twice the bag radius.

Having constructed an eigenstate of momentum we can now calculate any matrix element required. For example, the electric and magnetic form-factors of the nucleon are given by the matrix elements of j_0 and $\mathbf{j}$, respectively, in the Breit frame (CC 81, Bet 82)

$$G_E(Q^2) = \langle b, \underline{Q}/2 \mid j^0(0) \mid b, -\underline{Q}/2\rangle \tag{3.44}$$

and

$$G_M(Q^2)\chi_{\lambda'}^{+} \frac{i\sigma x Q}{m} \chi_\lambda = \langle b_{\lambda'}, \underline{Q}/2 \mid \mathbf{j}(0) \mid b_\lambda, -\underline{Q}/2\rangle \tag{3.45}$$

(In the second case we have explicitly shown spin labels, λ, for the hadronic state.) Carlson and Chachkhunashvili (CC 81) explicitly calculated the correction to the naive bag model predictions for the charge radius, magnetic moment, and axial charge using this approach. For the r.m.s. charge radius they found about 20% reduction—in rough agreement with the factor $\frac{2}{3}$ (for r^2) of the nonrelativistic harmonic oscillator. In the case of the magnetic moment there was a 15% reduction. The axial charge, g_A, increased by about 20%.

An important difficulty with the Peierls–Yoccoz procedure is that there is no guarantee that the internal state of the momentum eigenstate, $|b, \underline{p}\rangle$, will be independent of $\underline{p}$. Indeed, the two complications of the bag model, namely its sharp boundary and the fact that its wave functions are highly relativistic, make it less likely that this technique will be reliable for the bag. One practical indication of this, suggested by Carlson and Chachkhunashvili, is to compute the correction for slightly altered wave functions. For example, one might use the approximate Gaussian wave function of Duck (Duc 78):

$$\psi(\underline{r}) = \left[R_0{}^3\pi^{3/2}\left(1 + \frac{3}{2}\beta^2\right)\right]^{-1/2} e^{-r^2/2R_0{}^2}\begin{pmatrix} 1 \\ i\beta\dfrac{\underline{\sigma}\cdot\underline{r}}{R_0}\end{pmatrix}\chi \tag{3.46}$$

Whereas the results for the r.m.s. charge radius and g_A were not altered significantly by using Eq. (3.46) instead of Eq. (2.33), the magnetic moment increased by 8% for the former in comparison with a 15% decrease noted above. Clearly, the correction for the magnetic moment at least is untrustworthy. The ultimate c.m. correction which allows one to correct any spurious momentum dependence was developed by Peierls and Thouless (PT 62). This has never been applied to the bag model, mainly because of the complications introduced by relativity (Won 81).

3.4.2. Ambiguities Associated with the Center of Mass Correction

In the previous section the discussion was based on the nuclear physics analogy to the bag. However, as we discussed in Section 2.3 the bag itself might be expected to have some reality. Indeed, as discussed by Bardeen *et al.* (Bar+ 75), there is some momentum associated with the soliton bag. Thus, even though the MIT cavity does not carry momentum, in a better dynamical model one could conceive of the bag playing an important dynamical role.

With this in mind, Duck constructed a pion wave function in which the momentum of the bag balanced that of the two quarks (Duc 76). In that way the quarks were allowed to move independently of each other inside the hadron. While it was still necessary to construct momentum eigenstates, there was no c.m. correction in that approach.

An excellent illustration of the dilemma to be faced if the bag does not carry momentum has been raised by Betz (Bet 82). Consider the physically unreasonable case of a single quark confined in the bag. In the IPSM approach all of its motion would be spurious c.m. motion, which should be removed by the Peierls–Yoccoz procedure. On the other hand, if the soliton ideas are a reasonable representation of the physics, the single quark could dig a hole in the vacuum, and there should be a form-factor associated with the internal structure of the system. We shall mention this again in connection with the cloudy bag model form-factor for pion–hadron coupling in Sections 5 and 6.

One other important aspect of this problem concerns the n.l.b.c. As have discussed, the term $-Z_0/R$ is now thought to arise mainly as a c.m. correction. Including this in the stability calculation [$\partial M/\partial R = 0$, see Eq. (2.98)] produces a bag radius that is smaller than that which would be obtained by first setting $\partial E/\partial R$ to zero and then correcting for c.m. motion. For the nucleon it is readily seen that this gives about a 5% reduction (for $Z_0 = 0.75$) in R.

In the absence of a truly covariant bag model it is not at all clear which of these choices of bag radii is most appropriate for computing hadronic properties. However, since both the r.m.s. radius and the magnetic moment are proportional to R, the answers depend crucially on the choice made. The parallel with nuclear physics is of no help because there is no analog to the n.l.b.c. As a practical matter our choice has been to use the smaller radius but then omit further c.m. corrections. But to be honest any of the four possible options is equally acceptable and one has to accept an uncertainty of at least $\pm 10\%$ on bag model predictions of r.m.s. radii and magnetic moments. To end on a note of balance we might point out that this is still considerably better than the uncertainties associated with relativistic corrections in the nonrelativistic quark models (see Section 2.4).

4. CHIRAL SYMMETRY

In this section we first present the Lagrangian formulation of the MIT bag model. One of the most attractive features of this model as a basis for a pedagogical discussion is that it can be summarized in an extremely simple Lagrangian density. Using this we are able to formally derive conserved electromagnetic and isospin currents. However, in Section 4.3 we show that the axial current associated with the bag is not conserved. This is something of a disaster in view of the experimental successes of *p*artially *c*onserved *a*xial *c*urrent (PCAC). Indeed, chiral SU(2)×SU(2) is known to be one of the best symmetries of the strong interaction (Pag 75). The classical representation of SU(2)×SU(2) is the so-called "σ-model" we describe in Section 4.4. This discussion should also provide some background from which the later development of the cloudy bag model can be better appreciated.

4.1. Lagrangian Formulation of the MIT Bag Model

It is extremely convenient to have a concise mathematical summary of the MIT bag model as a Lagrangian density. In the limit of massless quarks, which we have seen to be a good starting point for dealing with nonstrange hadrons, the following very simple expression gives the essential content for the fermions (CT 75, DeT 80a, Jaf 79):

$$\mathscr{L}(x) = \left[\frac{i}{2}\bar{q}(x)\overleftrightarrow{\not\partial}q(x) - B\right]\theta(R-r) - \frac{1}{2}\bar{q}(x)q(x)\delta(r-R) \tag{4.1}$$

For pedagogical reasons we have specialized to the case of a static spherical bag of radius R. Of course, the whole problem is usually formulated in a covariant fashion by replacing $\theta(R-r)$ by θ_v, which is one inside the bag and zero outside, and $\delta(r-R)$ by a general surface δ-function, Δ_s. As usual B denotes the phenomenological energy density of the bag. We also have

$$\overleftrightarrow{\not\partial} = \gamma^\mu(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu) \tag{4.2}$$

where the arrow indicates the direction in which the derivative acts. Lastly $q(x)$ is the Dirac spinor describing the quarks. It actually has four Dirac components for each of two flavors (u and d) and three colors. (The extension to include strangeness, charm, and so on requires no essential change, but one must then introduce a mass matrix.)

The last term in Eq. (4.1) may seem a little strange until we recall that the l.b.c., which ensured no current flow through the surface of the bag, amounted to the condition that $\bar{q}q$ should be zero on the surface [Eq. (2.41)]. This term is a Lagrange multiplier guaranteeing that $\bar{q}q$ is zero at the bag surface.

As usual the field equations are obtained by demanding that

$$S = \int d^4x \mathscr{L}(x) \tag{4.3}$$

should be stationary under arbitrary changes in the fields

$$\begin{aligned} q_a &\to q_a + \delta q_a \\ \bar{q}_a &\to \bar{q}_a + \delta \bar{q}_a \end{aligned} \tag{4.4}$$

and in this case under changes in bag size (without change of shape). In the static spherical case this means

$$R \to R + \varepsilon \tag{4.5}$$

In the general case such a variation leads to the following changes in θ_v and Δ_s:

$$\delta\theta_v = \varepsilon \Delta_s \tag{4.6}$$

$$\delta\Delta_s = -\varepsilon n \cdot \partial \Delta_s \tag{4.7}$$

where n is the unit normal outward from the bag surface $[n^\mu = (0, \hat{r})$ for a static spherical bag]. The coefficients of δq, $\delta q\ \Delta_s$, and ε in the expression for δS give the three bag model equations discussed at such length in Section 2, respectively

$$i\partial\!\!\!/ q(x) = 0, \qquad r \leq R \tag{4.8}$$

$$i\gamma \cdot n q(x) = q(x), \qquad r = R \tag{4.9}$$

$$B = -\tfrac{1}{2} n \cdot \partial[\bar{q}(x)q(x)], \qquad r = R \tag{4.10}$$

4.2. Conserved Currents in Lagrangian Field Theory

In the previous section we referred to the mathematical convenience of a Lagrangian formulation. A prime example of this convenience is Noether's theorem, which states that an invariance of the Lagrangian density is associated with a conserved quantity. Consider, for example, the Lagrangian density

$$\mathscr{L} = \mathscr{L}(\phi_i, \partial^\mu \phi_i) \tag{4.11}$$

for which the equations of motion are determined by Hamilton's principle

$$\delta S = \delta \int d^4x \mathscr{L} = 0 \tag{4.12}$$

for arbitrary variations of the fields $\{\phi_i\}$, which vanish on the boundary (usually at infinity).

Suppose that we make a variation in these fields by an amount

$$\delta\phi_i(x) = f_i(\phi_j(x))\varepsilon \tag{4.13}$$

where f_i is an arbitrary function of the fields $\{\phi_j\}$ at x, and ε is an infinitesimal constant. If $\mathscr{L}$ is invariant under the transformation (4.13), we have

$$\delta\mathscr{L} = \left[\frac{\partial\mathscr{L}}{\partial\phi_i} f_i + \frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)} \partial_\mu f_i\right]\varepsilon = 0 \tag{4.14}$$

where there is an implicit summation over repeated indices. If ε is no longer constant $[\varepsilon = \varepsilon(x)]$, $\delta\mathscr{L}$ has an extra, nonvanishing term:

$$\delta\mathscr{L} = \frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)} f_i\partial_\mu\varepsilon(x) \tag{4.15}$$

However, from Eq. (4.12) the integral of $\delta\mathscr{L}$ still vanishes, and integrating by parts we find

$$\int d^4x\, \partial_\mu\left[\frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)} f_i\right]\varepsilon(x) = 0 \tag{4.16}$$

As this is true for arbitrary $\varepsilon(x)$, clearly we have constructed a *conserved current*. That is, if we define the current $j^\mu(x)$ as

$$j^\mu(x) = \frac{\partial\mathscr{L}}{\partial(\partial_\mu\phi_i)} f_i \tag{4.17}$$

then it is locally conserved

$$\partial_\mu j^\mu(x) = 0 \tag{4.18}$$

Finally, we note that if $\mathscr{L}$ has two pieces, as is often the case in examples of physical interest, so that only $\mathscr{L}_0$ is invariant under the transformation while $\mathscr{L}_b$ is not, that is

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_b \tag{4.19}$$

If $\mathscr{L}$ is a function of ϕ_j, only its divergence is readily shown to be

$$\partial_\mu j^\mu = \frac{\partial\mathscr{L}_b}{\partial\phi_j} f_j \tag{4.20}$$

4.2.1. The Usual Charge Current

As the simplest possible example of a conserved current in the bag model, consider the following simple gauge transformation:

$$\begin{aligned} q(x) &\to q(x) + i\varepsilon q(x) \\ [q^+(x) &\to q^+ - i\varepsilon q^+]\gamma^0 \end{aligned} \tag{4.21}$$

$$\bar{q}(x) = \bar{q}(x) - i\varepsilon\bar{q}(x) \tag{4.22}$$

Clearly, $\mathscr{L}(x)$ in Eq. (4.1) is invariant under this transformation because it contains only the combinations $\bar{q}q$ ($\to \bar{q}q - i\varepsilon\bar{q}q + \bar{q}i\varepsilon q = \bar{q}q$). Therefore, there is a conserved current easily found from Eq. (4.17):

$$j^\mu = \frac{i}{2}\,\bar{q}(x)\gamma^\mu[iq(x)]\theta_v - \frac{i}{2}\,[-i\bar{q}(x)]\gamma^\mu q(x)\theta_v \tag{4.23}$$

or up to a minus sign

$$j^\mu = \bar{q}(x)\gamma^\mu q(x)\theta_v \tag{4.24}$$

This has already been used in calculating the charge distribution (j^0) and magnetic moment ($\mathbf{j}$) of a bag model hadron in Section 3.

4.2.2. Isospin Conservation—Invariance under SU(2)

Now let us make an arbitrary, infinitesimal rotation in isospin:

$$\begin{aligned} q &\to q + i(\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}/2)q \\ \bar{q} &\to \bar{q} - i\bar{q}(\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}/2) \end{aligned} \tag{4.25}$$

with $\underset{\sim}{\varepsilon}$ constant. Once again $\mathscr{L}(x)$ is invariant, and hence $\underset{\sim}{I}^\mu$, given by

$$\underset{\sim}{I}^\mu(x) = \bar{q}(x)\gamma^\mu(\underset{\sim}{\tau}/2)q(x) \tag{4.26}$$

is a conserved current. Of course, the total isospin of the bag ($\underset{\sim}{t}$) is the integral of the isospin density

$$\underset{\sim}{t} = \int d^3x\, \underset{\sim}{I}^0(x) \tag{4.27}$$

and because $\partial_\mu \underset{\sim}{I}^\mu$ is zero, it is a constant of the motion.

4.3. The Axial Current

At last we have sufficient background material to begin consideration of the most recent developments in the bag model which are of direct relevance in nuclear physics. The natural starting point for this discussion is the axial current in the MIT bag model. As we shall see, unlike the charge and isospin currents that were decently conserved, the axial current is far from being conserved. Moreover, this problem seems to be inescapably linked with the concept of confinement. For this reason we believe that the ideas presented here have a far more general validity than the MIT model on which the discussion is based.

4.3.1. Nonconservation of the Axial Current

Suppose that instead of just rotating in isospin space, as in Eq. (4.25), we also operate with γ_5, thereby introducing a dependence on the quark's helicity

$$\begin{aligned} q &\to q - i(\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}/2)\gamma_5 q \\ q^+ &\to q^+ + iq^+\gamma_5(\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}/2) \end{aligned} \tag{4.28a}$$

and therefore

$$\bar{q} \to \bar{q} - i\bar{q}\gamma_5(\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}/2) \tag{4.28b}$$

Under this transformation we find

$$\mathscr{L} \to \mathscr{L} + \frac{1}{2}\bar{q}(\gamma_5\gamma^\mu + \gamma^\mu\gamma_5)\overleftrightarrow{\partial}_\mu \frac{\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}}{2} q\theta_v + \frac{i}{2}\bar{q}(x)\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}\gamma_5 q(x)\,\Delta_s \tag{4.29}$$

but whereas the second term vanishes because $\{\gamma^\mu, \gamma_5\} = 0$, the last is definitely nonzero. The jargon for this is that the surface term $-\frac{1}{2}\bar{q}q\,\Delta_s$ is "*chirally odd.*" Figure 4.1 illustrates in a very simple way what this lack of invariance means physically. Confinement implies that any quark impinging on the bag surface must be reflected. However, there is no spin-flip associated with the reflection, and hence the chirality, or handedness, of the quark is changed. Formally this is known as a violation of chiral symmetry.

Fig. 4.1. Violation of chiral symmetry at the bag surface.

Because of the lack of invariance of the Lagrangian density under the transformation (4.28) we do not have a conserved current. In fact, the axial current associated with Eq. (4.28) is

$$\underset{\sim}{A}^{\mu}(x) = \bar{q}(x)\gamma^{\mu}\gamma_5(\underset{\sim}{\tau}/2)q(x)\theta(R - r) \tag{4.30}$$

and using Eq. (4.20) we find easily that its divergence is

$$\partial_{\mu}A^{\mu}(x) = -\frac{i}{2}\bar{q}(x)\gamma_5\underset{\sim}{\tau}q(x)\delta(r - R) \tag{4.31}$$

This emphasizes once more that the essential problem is the confining wall at $r = R$. It also serves to remind us of Bogolioubov's relativistic potential model without the phenomenological energy density B. In that case [see Eqs. (2.53) and (2.54)] the divergence of the energy-momentum tensor was proportional to a surface δ-function times the Dirac pressure exerted by the quarks. Indeed, this was the observation that necessitated the introduction of B. In the same way we expect that something new will be required here. For guidance we recall the conventional description of the hadronic weak current.

4.3.2. Partially Conserved Axial Current PCAC

In section 3.3 we reviewed the conventional theoretical description of the weak interaction. As an example we considered neutron β-decay, which involves rather low-momentum transfer. Accordingly Eq. (3.23) did not contain all the pieces of the vector and axial-vector currents. The most general expression for the hadronic axial current is

$$\langle p \mid A^{\mu} \mid n\rangle \sim \bar{u}_p[\gamma^{\mu}\gamma_5 g_A(k^2) + k^{\mu}\gamma_5 g_P(k^2)]u_n \tag{4.32}$$

where u_n and u_p are Dirac spinors for the nucleons and the second term in brackets is the induced pseudoscalar term. If the momentum transfer k^2 is spacelike and small, we find the nonrelativistic limits

$$\gamma^{\mu}\gamma_5 \to \underset{\sim}{\sigma}; \qquad k^{\mu}\gamma_5 \to -\frac{\underset{\sim}{\sigma}\cdot\underset{\sim}{k}}{2m_N}\underset{\sim}{k} \tag{4.33}$$

and hence

$$\langle p \mid \underset{\sim}{A} \mid n\rangle = \chi_p{}^{+}\left[g_A(k^2)\underset{\sim}{\sigma} - \frac{g_P(k^2)}{2M}\underset{\sim}{\sigma}\cdot\underset{\sim}{k}\underset{\sim}{k}\right]\chi_n \tag{4.34}$$

where χ_n and χ_p are Pauli spinors.

Now the problem of concern to us in Section 4.3.1 was the nonconservation of the axial current in the bag model—or more specifically the fact that $\partial_\mu A^\mu$ was nonzero. In the present case $\partial_\mu A^\mu$ becomes simply $\underline{k} \cdot \underline{A}$, and we see from Eq. (4.34) that $\underline{k} \cdot \underline{A}$ is zero only if

$$\left[g_A(k^2) - \frac{g_P(k^2)k^2}{2M}\right]\underline{\sigma} \cdot \underline{k} = 0 \qquad (4.35)$$

which implies that g_P is related to g_A by

$$g_P(k^2) = \frac{2Mg_A(k^2)}{\underline{k}^2} \qquad (4.36)$$

Since g_A is simply a constant as $\underline{k}^2 \to 0$ we see that *if the axial current is to be conserved the induced pseudoscalar term must have a pole corresponding to the propagation of a massless exchanged particle.* Furthermore, the quantum numbers of the exchanged particle are those of the pion.

If we then accept that the axial current may not be exactly conserved, it seems very natural to replace Eq. (4.36) by

$$g_P(k^2) = \frac{2Mg_A(k^2)}{\underline{k}^2 + m_\pi^2} \qquad (4.37)$$

Since m_π is unusually small on the scale of hadronic masses, the axial current is said to be almost, or partially, conserved. In fact the correct statement of the PCAC (*p*artially *c*onserved *a*xial *c*urrent) hypothesis (Col 68) is that the extrapolation from zero pion mass to m_π should be smooth.

In the limited space available here we cannot do justice to the depth of physics investigated using the PCAC hypothesis. At best we can refer to some excellent textbook presentations (GL 60, AD 68, Lee 68, Col 68, Zum 68, ER 72, Bro 79). In addition, we can get some physical insight into the structure of $\underline{A}^\mu$ by referring to Fig. 4.2. As we see there are two essential contributions to it. The first is a direct term, which reduces to $g_A\underline{\sigma}$ and is included in the bag model. Secondly, there is the possibility that the nucleon emits a pion which then decays via the axial current with am-

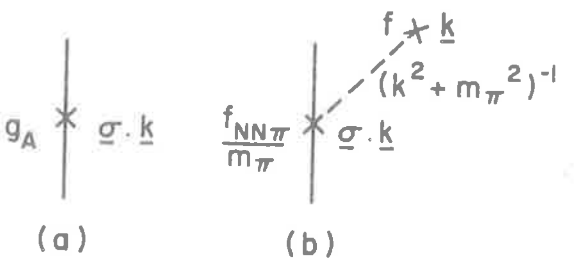

Fig. 4.2. The direct and pion pole contributions to the nucleon axial current.

plitude $\sqrt{2}f\underset{\sim}{k}$, where $f = 93$ MeV is the pion decay constant. If as suggested by Eq. (4.37) we equate these two terms, when m_π^2 is zero we obtain

$$(g_A \underset{\sim}{\sigma}) \cdot \underset{\sim}{k} = \sqrt{2}\left(\frac{f_{NN\pi}}{m_\pi}\right) \underset{\sim}{\sigma} \cdot \underset{\sim}{k} \frac{1}{\underset{\sim}{k}^2} (\sqrt{2}\, f \underset{\sim}{k}) \cdot \underset{\sim}{k} \tag{4.38}$$

and hence

$$\frac{g_A}{2f} = \frac{f_{NN\pi}}{m_\pi} \tag{4.39}$$

Equation (4.39) provides a remarkable connection between weak and strong coupling constants and is known as the Goldberger–Trieman relationship. In conclusion let us re-emphasize that the massless pion pole term is essential if one wants $\partial_\mu A^\mu = 0$. In the real world where m_π is nonzero we have instead the relationship

$$\partial_\mu A^\mu = f m_\pi^2 \underset{\sim}{\phi} \tag{4.40}$$

where $\underset{\sim}{\phi}$ is the pion field (Col 68).

4.4. The σ-Model and Spontaneous Symmetry Breaking

4.4.1. *General Discussion of $SU(2) \times SU(2)$*

In the preceding sections we have discussed separately the vector and axial-vector currents in the bag model. However, the quantities of more general interest in particle physics are the combinations $(V \pm A)$. In the case of massless fermion fields these are the left- and right-handed currents. The original significance of these combinations lay in the current algebra hypothesis of Feynman and Gell-Mann (Gel 64, Fey+ 64, AD 68). This significance has only grown with the development of QCD over the past decade.

In particular, the underlying Lagrangian density for QCD contains a kinetic energy term for free massless quarks. As we have seen in Sections 4.2.2 and 4.3.1 such a Lagrangian density leads to conserved vector and axial-vector currents:

$$V_\mu{}^i = \bar{q}(x)\tau^i\gamma_\mu q(x) \tag{4.41}$$

and

$$A_\mu{}^i = \bar{q}(x)\tau^i\gamma_\mu\gamma_5 q(x) \tag{4.42}$$

The combinations $V \pm A$ then describe the isospin structure of left- and right-handed quarks, respectively

$$L_\mu{}^i = \bar{q}(x)\tau^i\gamma_\mu(1 - \gamma_5)q(x)$$
$$R_\mu{}^i = \bar{q}(x)\tau^i\gamma_\mu(1 + \gamma_5)q(x) \tag{4.43}$$

Because of the commutation relations amongst the V and A currents, L and R form independent algebras under equal-time commutation (AD 68, Sak 69). That is, we have two independent representations of SU(2), one for left-handed particles and the other for right-handed particles. The invariance of the theory under separate transformations for left- and right-handed particles is referred to as chiral SU(2)×SU(2) symmetry, or $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, in an obvious notation.

To restate this simply, the theory is chirally symmetric if no piece of the Lagrangian density mixes left- and right-handed particles. Figure 4.1 illustrated exactly why SU(2)×SU(2) is violated by the MIT bag model, or indeed any model where quarks are reflected by a boundary. Such a reflection changes helicity and thus mixes the left- and right-handed parts of the theory.

Thus, the first argument that something is missing from the usual bag model is that it does not have a symmetry which is present in what is generally believed to be the correct theory of strong interactions, namely QCD. The second indication is rather more pragmatic. That is, there is an extremely successful phenomenology which has been built on the idea that chiral SU(2)×SU(2) is a good symmetry of strong interactions. An excellent discussion of the evidence can be found in the review by Pagels (Pag 75). Based on the comparison between theory and experiment for the Goldberger–Treiman relationship [Eq. (4.39)], the πN Σ-commutator, and so on, Pagels concludes that, "SU(2)×SU(2) is a good hadron symmetry to within 7%. *This makes chiral SU(2)×SU(2) the most accurate hadron symmetry after isotopic invariance*" (Pagel's italics) (Pag 75, p. 242).

We are therefore faced with a problem very similar to that encountered by Gell-Mann and Lévy in 1960. They had to reconcile the fact that the axial current for the nucleon was partially conserved with the fact that the nucleon has a large mass. That is, the Lagrangian density for a free nucleon is

$$\mathscr{L}(x) = i\bar{\psi}\partial\!\!\!/\psi - m_N\bar{\psi}\psi \tag{4.44}$$

where the mass term [as we saw in Eq. (4.29)] is "chirally odd." Their solution to the problem was the so-called "σ-model", to which we turn in the following section. Although it is a very simple model, it is of more than academic interest. It has been used as a method of incorporating the constraints of chiral symmetry in many applications in conventional nuclear theory, such as the following:

(a) exchange current corrections, e.g., for the axial charge density in nuclei (Gui+ 78, Ose 80);
(b) the two-pion exchange N–N force (Bro 78, Bro 79);
(c) many-body forces (MR 79); and
(d) exotic states of matter, such as Lee–Wick matter and pion condensation (LW 74, Cam 78, Bay 78, Mey 81).

For the present we simply observe that the cloudy bag model, which will be described in Section 5 and 6, is the natural generalization of the σ-model to the case where the nucleon has structure. It invites application in each of the areas (a)–(d).

4.4.2. The σ-Model

As we have remarked many times the essential problem in constructing a chiral-symmetric theory containing fermions is the mass term proportional to $\bar{\psi}\psi$. The simplest way to avoid this problem is to introduce new fields $(\sigma, \underset{\sim}{\pi})$—an isoscalar–scalar field and an isovector–pseudoscalar field—in addition to the nucleon, ψ. The generalization of the infinitesimal transformation (4.28) [replace $q(x)$ by $\psi(x)$] is

$$\psi \rightarrow e^{-i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2}\psi \tag{4.45a}$$

$$\bar{\psi} \rightarrow \bar{\psi}e^{-i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2} \tag{4.45b}$$

Then the idea is to replace $m_N\bar{\psi}\psi$ in Eq. (4.44) by $g\bar{\psi}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)\psi$, where σ and $\underset{\sim}{\pi}$ are defined to transform in exactly the right way to cancel the transformation (4.45). In particular, we demand that

$$(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5) \rightarrow e^{+i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)e^{+i\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}\gamma_5/2} \tag{4.46}$$

If we now consider the case where $\underset{\sim}{\alpha}$ is infinitesimal, Eq. (4.46) implies that

$$\sigma \rightarrow \sigma - \underset{\sim}{\alpha}\cdot\underset{\sim}{\pi} \tag{4.47a}$$

$$\underset{\sim}{\pi} \rightarrow \underset{\sim}{\pi} + \sigma\underset{\sim}{\alpha} \tag{4.47b}$$

and of course

$$\psi \rightarrow \psi - i\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}}{2}\gamma_5\psi; \qquad \bar{\psi} \rightarrow \bar{\psi} - i\bar{\psi}\gamma_5\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\alpha}}{2} \tag{4.48}$$

It is a simple exercise to show that Eq. (4.47) implies that $\sigma^2 + \underset{\sim}{\pi}^2$ is invariant under this chiral transformation. That is, we are merely making a rotation in a four-dimensional (4D) space.

We mentioned above that under the familiar SU(2) of isospin, σ is a scalar and $\underset{\sim}{\pi}$ a vector. Under an infinitesimal rotation in isospin space

$$\sigma \to \sigma; \qquad \underset{\sim}{\pi} \to \underset{\sim}{\pi} - \underset{\sim}{\beta} \times \underset{\sim}{\pi} \tag{4.49}$$

and [recall Eq. (4.25)]

$$\psi \to \psi + i\,\frac{\underset{\sim}{\tau} \cdot \underset{\sim}{\beta}}{2}\,\psi; \qquad \bar{\psi} \to \bar{\psi} - i\bar{\psi}\,\frac{\underset{\sim}{\tau} \cdot \underset{\sim}{\beta}}{2} \tag{4.50}$$

Equation (4.49) also leaves $\sigma^2 + \underset{\sim}{\pi}^2$ constant. Thus, the most general transformation under SU(2)×SU(2) involves two parameters ($\underset{\sim}{\alpha}$, $\underset{\sim}{\beta}$) and amounts to nothing more than a rotation in 4D space. Indeed, as discussed in detail by Lee (68), SU(2)×SU(2) is isomorphic to the rotation group in four dimensions, R(4). The basis of the regular (adjoint) representation of R(4) (Car 66) is in fact (σ, $\underset{\sim}{\pi}$).¶

The most general renormalizable Lagrangian density involving nucleon, σ- and $\underset{\sim}{\pi}$-fields which is consistent with chiral symmetry is therefore

$$\begin{aligned}\mathscr{L}(x) = {} & i\bar{\psi}\partial\!\!\!/\psi + g\bar{\psi}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)\psi + \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\partial_\mu\pi)^2 \\ & -\tfrac{1}{4}\lambda^2[(\sigma^2 + \underset{\sim}{\pi}^2) - \nu^2]^2\end{aligned} \tag{4.51}$$

In case it is not clear, we stress that the σ and $\underset{\sim}{\pi}$ kinetic energy terms are invariant under SU(2)×SU(2) because we are only discussing *global transformations*, that is, $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ constants, not functions of x.

Let us consider the potential energy term in Eq. (4.51) in more detail:

$$V(\sigma, \underset{\sim}{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \underset{\sim}{\pi}^2)^2 - \frac{\lambda^2\nu^2}{2}(\sigma^2 + \underset{\sim}{\pi}^2) + \frac{\lambda^2}{4}\nu^4 \tag{4.52}$$

(There is a change of sign because the Hamiltonian goes as $-g^{00}\mathscr{L}$.) If the system is ever to be stable, we obviously need $\lambda^2 > 0$. Then there are two possibilities. First, it is possible that $\nu^2 \leq 0$, in which case the coefficient of the σ^2 and $\underset{\sim}{\pi}^2$ terms is positive and therefore an acceptable mass term. The σ- and $\underset{\sim}{\pi}$-fields have the same mass, $(-\lambda^2\nu^2)^{1/2}$, and the potential

¶ The regular representation is 4D because only four of the six operators $\underset{\sim}{\tau}_i$ and $\gamma_5\underset{\sim}{\tau}_i$ are independent. As we discussed in Section 4.4.1 the combinations $(1 \pm \gamma_5)\underset{\sim}{\tau}_i$ are the operators for left- and right-handed SU(2).

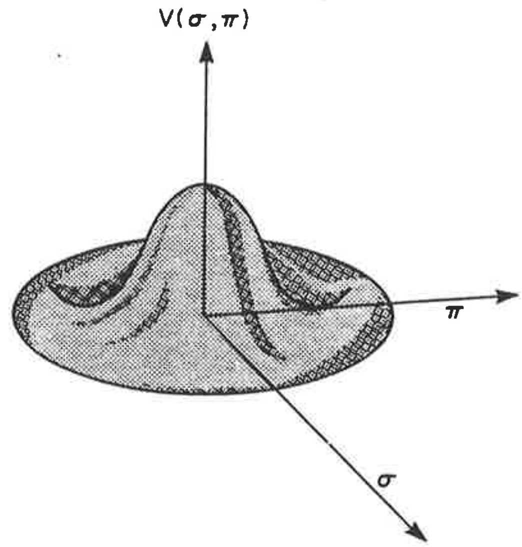

Fig. 4.3. The potential energy density $V(\sigma, \pi)$ with $\mu^2 < 0$, $\nu^2 > 0$, and $c_\pi = 0$.

$V(\sigma, \underset{\sim}{\pi} = 0)$ has only one minimum, at $\sigma = 0$. One could then deal with fluctuations of the σ- and $\underset{\sim}{\pi}$-fields in the normal vacuum.

A second possibility, which is far more interesting, is the case $\nu^2 > 0$. In this case the potential has the "Mexican hat" shape shown in Fig. 4.3. Remarkably, the point $\sigma = 0$ is no longer stable and it would be meaningless to talk about quantum fluctuations about that point! Instead there is a minimum on the surface $(\sigma^2 + \underset{\sim}{\pi}^2) = \nu^2$. Since a nonzero classical expectation value for $\underset{\sim}{\pi}$ would violate parity, it is natural to think of expanding about either of the equivalent minima of $V(\sigma, \underset{\sim}{\pi} = 0)$ at $\sigma = \pm\nu$, for example

$$\sigma \rightarrow \sigma + \nu, \qquad \underset{\sim}{\pi} \rightarrow \underset{\sim}{\pi} \tag{4.53}$$

Once this transformation is made the symmetry of the original Lagrangian density (4.51) is hidden.

However, Goldstone's theorem (Gol 61, Gol+ 62, Ber 74, Pag 75, Lee 81) tells us that when a continuous symmetry is hidden, a Goldstone boson or massless excitation of the system appears. Mathematically we find upon substituting Eq. (4.53) into Eq. (4.51)

$$\begin{aligned}\mathscr{L}(x) = \bar{\psi}(i\partial\!\!\!/ + g\nu)\psi + g\bar{\psi}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)\psi + \tfrac{1}{2}(\partial_\mu\sigma)^2 - \tfrac{1}{2}(2\lambda^2\nu^2)\sigma^2 \\ + \tfrac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 - \nu\lambda^2\sigma(\sigma^2 + \underset{\sim}{\pi}^2) - \tfrac{1}{4}\lambda^2(\sigma^2 + \underset{\sim}{\pi}^2)^2\end{aligned} \tag{4.54}$$

and the explicit SU(2)×SU(2) symmetry has certainly been lost. The nucleon, for example, now has a mass term with

$$m_N = -g\nu \tag{4.55}$$

which arises because the vacuum state is now complicated and the nucleon always meets resistance. Similarly, the σ now has a mass corresponding to the second derivative of $V(\sigma, \underset{\sim}{\pi} = 0)$, at $\sigma = \pm\nu$, in the σ-direction

$$m_\sigma{}^2 = 2\lambda^2\nu^2 \tag{4.56}$$

Furthermore, as advertised, there is no mass term for the pion which is now a massless Goldstone boson corresponding to massless excitations around the rim of the "hat." We also observe that there are now σ–π–π and σ–σ–σ interaction vertices with strength proportional to the expectation value of the original σ-field.

Let us recall that the whole purpose of this exercise was to produce a chiral-symmetric theory with a massive nucleon. Although it may not be obvious, we have succeeded, and the whole key is the spontaneous breaking of chiral symmetry associated with Eq. (4.53). Actually a much more appropriate term would be hidden chiral symmetry because Eq. (4.54) is invariant under the chiral transformation

$$\begin{aligned} \sigma &\to \sigma - \underset{\sim}{\alpha} \cdot \underset{\sim}{\pi} \\ \underset{\sim}{\pi} &\to \underset{\sim}{\pi} + (\sigma + \nu)\underset{\sim}{\alpha} \end{aligned} \tag{4.57}$$

and hence there is still a conserved axial current

$$\underset{\sim}{A}^\mu(x) = \bar{\psi}\gamma^\mu\gamma_5\underset{\sim}{\tau}/2\psi - \underset{\sim}{\pi}\partial^\mu\sigma + \sigma\partial^\mu\underset{\sim}{\pi} \tag{4.58}$$

Finally, the conserved vector current associated with Eq. (4.54) is

$$\underset{\sim}{V}^\mu(x) = (\bar{\psi}\gamma^\mu\underset{\sim}{\tau}/2\psi) + \underset{\sim}{\pi} \times \partial^\mu\underset{\sim}{\pi} \tag{4.59}$$

4.4.3. PCAC in the σ-Model

Having obtained a chiral-symmetric theory with a nucleon mass, all we need to do to make contact with the real world is to introduce a mass for the pion. This is done by explicitly breaking the chiral symmetry of the original Lagrangian density (4.51) by "tipping" the Mexican hat

$$\mathscr{L} \to \mathscr{L} + c\sigma \tag{4.60}$$

In this case there is a preferred direction in $(\sigma, \underset{\sim}{\pi})$ space and the minimum about which we expand is σ_0, where

$$\sigma_0(\sigma_0{}^2 - \nu^2) = c/\lambda^2 \tag{4.61}$$

If we now let

$$\sigma \to \sigma + \sigma_0 \tag{4.62}$$

in Eq. (4.60) it is a straightforward exercise to show that the nucleon, σ, and $\underset{\sim}{\pi}$ all get masses, with

$$m_N = -g\sigma_0 \tag{4.63}$$

$$m_\sigma^2 = \lambda^2(3\sigma_0^2 - \nu^2) \tag{4.64}$$

and

$$m_\pi^2 = \lambda^2(\sigma_0^2 - \nu^2) \tag{4.65}$$

Because we broke the chiral SU(2) symmetry with the $-c\sigma$ term, the axial current is no longer conserved. Instead, from Eq. (4.20) we find

$$\partial_\mu \underset{\sim}{A}^\mu = -c\underset{\sim}{\pi}^\mu \tag{4.66}$$

This is exactly the form given in Section 4.3.2 provided we identify

$$c = -fm_\pi^2 \tag{4.67}$$

Using Eq. (4.67), (4.61), and (4.65) we find that the minimum about which we have expanded σ_0 is equal to the pion decay constant:

$$\sigma_0 = -f \tag{4.68}$$

Hence Eq. (4.63) becomes

$$m_N = gf \tag{4.69}$$

which we recognize as the Goldberger–Treiman relation [see Eq. (4.39)] *with* $g_A = 1$. This is a defect of the σ-model usually overcome in practice by introducing $g_A = 1.24$ as a fudge-factor whenever needed!

In summary, we emphasize that the σ-model was presented not as the best one can do in imposing chiral symmetry but in order to motivate what follows. In order to appreciate what is really new and advantageous about the CBM we need to understand what has been done in the past. Nevertheless, the σ-model is a beautiful case study, presenting as it does simple examples of chiral symmetry, spontaneous symmetry breaking, PCAC, and the Goldberger–Treiman relation. The serious student should follow up our brief presentation by reading the appropriate sections of Lee 68, Bro 79, IZ 80, and Lee 81.

5. BAG MODELS WITH CHIRAL SYMMETRY

In Sections 2 and 3 we took great care to explain the MIT bag model, its application to particle properties, and the attempts to derive it from a more fundamental theory. The concept of chiral symmetry was explained in Section 4. In particular, we showed that the essential effect of confinement was to lead to the nonconservation of the axial current. We then examined the classical σ-model as an example of how chiral symmetry can be restored through the appearance of a Goldstone boson. The purpose of this section is to show how a number of groups have attempted to put these concepts together to create a hybrid model in which the experimental fact of PCAC is preserved. However, such a review would be incomplete without some discussion of the relationship of this phenomenology to QCD. It is the purpose of Section 5.1 to provide that background.

5.1. Motivation

Finding the solution of QCD, which is widely accepted as the correct theory of strong interactions, poses a very difficult problem (AL 73, MP 78). It is quite likely that some genuine physical insight will be required if we are ever to solve the QCD equations. Symmetry arguments may be of great importance in developing that insight. In the innocent days of 1968, when only three quark flavors were known, Gell-Mann, Oakes, and Renner (GOR) proposed the following scheme (Gel+ 68). Beginning with three massless quarks, QCD (for the reasons reviewed in Section 4.4) would have an exact SU(3)$\times$SU(3) symmetry. Because physical particles have definite parity the vacuum symmetry in this theory must be hidden—leading to an octet of massless Goldstone bosons (π, η, k, and $\bar{k}$).

If the strange quark is then given a mass, the symmetry group is broken to SU(2)$\times$SU(2)—with only the pion still massless. Next, one sets the masses of the u and d quarks to be nonzero ($m_{\mathrm{u}} = m_{\mathrm{d}} \neq 0$) leaving only SU(2) (isospin) and $m_\pi \neq 0$. Finally, in order to explain mass splittings in isospin multiplets one must set $m_{\mathrm{u}} \neq m_{\mathrm{d}}$ leaving U(1), or charge conservation as the only exact symmetry.

For the present we shall ignore chiral SU(3)$\times$SU(3) because of the large mass of the kaon. [Nevertheless there may be a great deal to be learnt by extending the hybrid bag models to include strangeness (RT 82).] On the other hand, as we have stressed many times, SU(2)$\times$SU(2) is found experimentally to be an excellent symmetry. It should therefore make a firm foundation for model building. As GOR observed, on very general grounds the physical realization of chiral SU(2)$\times$SU(2) must be the Goldstone mode. To see this, suppose

$$\partial_\mu \underset{\sim}{A}^\mu = 0 \tag{5.1}$$

in all space. Therefore, if we integrate over all space

$$\int d^3x\, \partial_\mu \underset{\sim}{A}^\mu = 0 \tag{5.2}$$

and use Gauss's theorem on the $\vec{\nabla} \cdot \vec{\underset{\sim}{A}}$ piece we find

$$\partial_0 \underset{\sim}{Q}_5 = \partial_0 \int d^3x\, \underset{\sim}{A}^0(\underset{\sim}{x}, t) = 0 \tag{5.3}$$

Thus, the axial charge is a constant of the motion, and therefore commutes with the Hamiltonian

$$[H, \underset{\sim}{Q}_5] = 0 \tag{5.4}$$

If an eigenstate of H, namely $| N^+\rangle$, exists with mass m

$$H \,| N^+\rangle = m \,| N^+\rangle \tag{5.5}$$

then $| N^-\rangle$ defined as

$$| N^-\rangle = \underset{\sim}{Q}_5 \,| N^+\rangle \tag{5.6}$$

also has mass m, viz:

$$\begin{aligned} \underset{\sim}{Q}_5 H \,| N^+\rangle = H \underset{\sim}{Q}_5 \,| N^+\rangle &= H \,| N^-\rangle \\ &= m \,| N^-\rangle \end{aligned} \tag{5.7}$$

Since $| N^-\rangle$ necessarily has opposite parity from $| N^+\rangle$ there is an unobserved, opposite-parity partner for each hadron!

The only way around this theorem is the Goldstone representation of chiral symmetry in which $\underset{\sim}{Q}_5$ does not annihilate the vacuum (Pag 75, Gol+ 62), i.e.

$$\underset{\sim}{Q}_5 \,| O\rangle \neq 0 \tag{5.8}$$

In that case, rather than being a parity partner of $| N^-\rangle$, the state $| N^-\rangle$ contains an arbitrary number of massless, pseudoscalar Goldstone bosons. [Recall Section 4.4.2 where we showed explicitly how such bosons can appear as a result of *s*pontaneous *s*ymmetry *b*reaking (SSB).] Thus on very general grounds *the pion must be present as a Goldstone boson in this ideal chiral-symmetric world* (*with* $m_\pi = 0$).

While the σ-model was pedagogically very useful for introducing the ideas of SSB and chiral symmetry, it is physically very unsatisfactory. The nucleon is pointlike and there is no way to relate it to QCD. Thus, it certainly does not help to resolve the problem of $\partial_\mu A^\mu \neq 0$, which we found in the bag model. We recall that the essential difficulty there was the confining surface of the bag, and this has led to speculation of a phase change at the bag surface. Briefly the idea is that chiral symmetry would be realized in the Wigner–Weyl mode inside the bag (massless quarks, no pions) and in the Goldstone mode outside (Cal+ 78, Cal+ 79, BR 79). In such a picture the pion field outside the bag could (but need not) play an essential role in the confinement process—even contributing significantly to the bag pressure.

Very recently Goldman and Haymaker have taken some steps which may provide the link between QCD and the appearance of the Goldstone mode (GH 81, HG 81). Their considerations were based upon an effective Lagrangian of the Nambu–Jona-Lasinio type

$$\mathscr{L}_{\text{eff}} = i\bar{q}\partial\!\!\!/ q - g[(\bar{q}q)^2 + (i\bar{q}\gamma_5\boldsymbol{\tau}q)^2] \tag{5.9}$$

Actually they used a rather more general form than this with the δ-function 4-quark interaction replaced by exchange of a massive vector particle, but the idea is the same. Moreover there have been indications that such an effective Lagrangian density could come out of QCD after transforming away the gluons (Cal+ 79). The properties of Eq. (5.9) have been well studied (NJ 61, GN 74, GH 81), indeed it provides the classic example of a dynamically broken symmetry. Beyond a certain critical value of the coupling constant, g_c, one finds that the quarks become massive and the pion appears as a massless, composite Goldstone boson. The breaking of chiral symmetry as a result of the dynamics of the system is (not surprisingly) referred to as dynamical symmetry breaking (DSB).

Put very briefly the essential idea of Goldman and Haymaker is the following. The one-gluon exchange is very strongly attractive in the state with pion quantum numbers [see Eq. (2.83)]. It is quite conceivable that the one-gluon-exchange ladder graphs alone could bind a $q\bar{q}$ pair in that channel. Then the large-distance, nonperturbative aspects of QCD responsible for confinement need not alter the properties of the pion very much. Chiral symmetry could be dynamically broken, with the appearance of a Goldstone pion, independently of the usual mechanism of confinement. Naturally this leads to a rather small pion, with a hydrogenlike relative $\bar{q}q$ wave function. At present the only experimental problem this presents would be the measured r.m.s. charge radius of 0.56 ± 0.04 fm (Dal+ 81). However, theoretical corrections to the charge distribution from processes like $\pi \to 3\pi$ have not been estimated.

Whatever the nature of the pion, there is strong theoretical justification for treating it as a Goldstone boson arising from some DSB mechanism. In addition, it is unique amongst hadrons in having a size (less than or equal to its r.m.s. charge radius) considerably less than its Compton wavelength. Thus, in first approximation, it should be reasonable to construct a theory in which chiral symmetry is retained in the Goldstone mode but the internal structure of the pion is neglected. This would be essentially a long-wavelength approximation.

5.2. Chodos and Thorn

The lack of chiral symmetry in the MIT bag model was recognized immediately by the MIT group. One attempt was made to deal with this problem as early as 1975 by Chodos and Thorn (CT 75)—see also Inoue and Maskawa (IW 75). Their proposal was a simple generalization of the σ-model we described in Section 4. That is, the surface term in the MIT Lagrangian density [Eq. (4.1)], $\bar{q}q\delta_s$, is replaced by the chiral invariant form $\bar{q}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)q\delta_s$. The new Lagrangian density involving the extra elementary fields σ and $\underset{\sim}{\pi}$ is therefore

$$\mathscr{L}_{\mathrm{CT}}(x) = (i\bar{q}\overleftrightarrow{\partial\!\!\!/}q - B)\theta_v - \frac{\lambda}{2}\bar{q}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)q\delta_s + \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 \tag{5.10}$$

where λ is a Lagrange multiplier which turns out to be simply $(\sigma^2 + \underset{\sim}{\pi}^2)^{-1/2}$. By construction, Eq. (5.10) is invariant under the chiral transformations [Eqs. (4.47) and (4.48)] ($\psi \to q$), and the conserved axial current analogous to Eq. (4.58) is

$$\underset{\sim}{A}^\mu = \tfrac{1}{2}\bar{q}(\gamma^\mu\gamma_5\underset{\sim}{\tau}/2)q\theta_v - \underset{\sim}{\pi}\partial^\mu\sigma + \sigma\partial^\mu\underset{\sim}{\pi} \tag{5.11}$$

Having written down the classical field equations corresponding to Eq. (5.10) for the case of a static spherical bag, namely

$$i\partial\!\!\!/ q = 0, \qquad r < R \tag{5.12}$$

$$i\underset{\sim}{\gamma} \cdot \hat{r}q = -\frac{1}{(\sigma^2 + \underset{\sim}{\pi}^2)^{1/2}}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)q, \qquad r = R \tag{5.13}$$

$$\nabla^2\sigma = \frac{1}{2}\frac{1}{(\sigma^2 + \underset{\sim}{\pi}^2)^{1/2}}\bar{q}q\delta(r - R) \tag{5.14}$$

$$\nabla^2\underset{\sim}{\pi} = \frac{1}{2}\frac{1}{(\sigma^2 + \underset{\sim}{\pi}^2)^{1/2}}i\bar{q}\gamma_5\underset{\sim}{\tau}q\delta(r - R) \tag{5.15}$$

Chodos and Thorn attempted to find an exact classical solution. The only case for which this was feasible was a highly idealized baryon called the "hedgehog." If we define a spin–flavor wave function ν as (u and d are up and down, and the arrows describe spin direction)

$$|\nu\rangle = (|u\downarrow\rangle - |d\uparrow\rangle)/\sqrt{2} \tag{5.16}$$

that is a mixed spin–flavor singlet, then the hedgehog has the spin–flavor wave function

$$|h\rangle_{s-f} = |\nu\rangle_1 |\nu\rangle_2 |\nu\rangle_3 \tag{5.17}$$

Such an animal clearly has no place in the real world, as it is an eigenstate of neither isospin nor angular momentum. In fact, with three quarks in $1s_{1/2}$ orbitals it is a linear superposition of N and Δ states of all charges. However, the choice of h leads to a very simple form for the source term in Eq. (5.15). In fact one can easily show that $\bar{q}\gamma_5\underset{\sim}{\tau}q$, a vector in isospin space, always points in the radial direction $\hat{r}$! That is, for a quark in a $1s_{1/2}$ hedgehog orbit (q_h)

$$\bar{q}_h\underset{\sim}{\tau}\gamma_5 q_h = -2ij_0j_1\nu^+\nu\hat{r} \tag{5.18}$$

It can then be seen that the set of Eq. (5.12)–(5.15) allow a solution of the form

$$q(\underset{\sim}{r}) = \begin{pmatrix} j_0(\omega r/R) \\ i\underset{\sim}{\sigma}\cdot\hat{r} \quad j_1(\omega r/R)\end{pmatrix}\nu e^{-i(\omega/R)t} \tag{5.19}$$

$$\underset{\sim}{\pi}(\underset{\sim}{r}) = \hat{r}g(r) \tag{5.20}$$

$$\sigma(\underset{\sim}{r}) = f(r) \tag{5.21}$$

Although no explanation for the hedgehog was given by Chodos and Thorn, the form [Eq. (5.20)] is identical to the monopole solutions under investigation at about the same time. Solving explicitly for the pion field they found

$$g(r) = -\beta\left[\theta(R-r)r + \theta(r-R)\frac{R^3}{r^2}\right] \tag{5.22}$$

where β measures the strength of coupling at the bag surface. The quark frequency ω is obtained by solving a transcendental equation.

If it was not obvious from Eqs. (5.14) and (5.15), it is obvious from the explicit solution Eq. (5.22) that the $\underset{\sim}{\pi}$-field has a discontinuous derivative at the bag surface. [Although we do not show it, there is a similar discontinuity in the derivative of $\sigma(r)$.] Such a discontinuity is actually essential if the axial current is to be conserved and simply serves to balance the

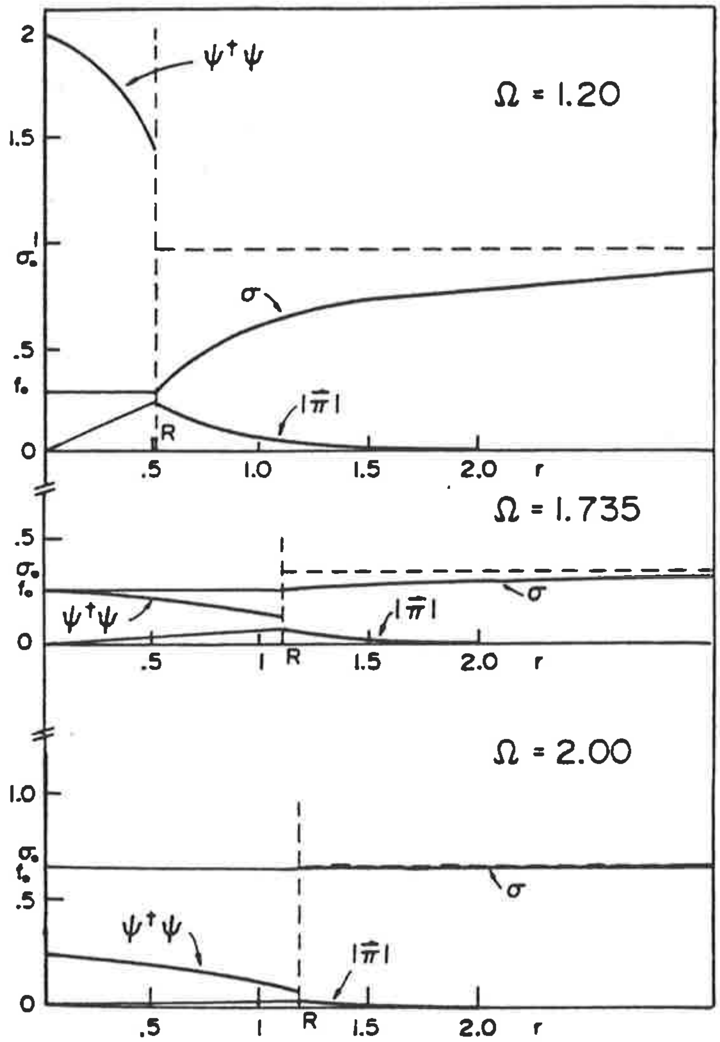

Fig. 5.1. Behavior of the quark density and the σ- and π-fields for various choices of parameters in the hedgehog model of Chodos and Thorn (CT 75).

source of axial current arising from quark reflection at the surface. We mention it here because in the nonlinear boundary condition

$$\frac{\partial}{\partial r}\left[\bar{q}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)q\right] = -2(\sigma^2 + \underset{\sim}{\pi}^2)^{1/2}B, \qquad r = R \qquad (5.23)$$

consistency with energy-momentum conservation requires that one use the average of the $\underset{\sim}{\pi}$- and σ-field derivatives inside and outside the bag surface. The solutions for several values of the bag radius are displayed in Fig. 5.1.

As we have hinted, although the existence of hedgehoglike solutions is fascinating, they are not of much physical significance because of the lack of rotational invariance in space and isospin. An alternative approach suggested by Chodos and Thorn, which was not inconsistent with the model results for the hedgehog, was to make a perturbative expansion about the MIT solution, with a constant classical σ-field and zero classical pion field. Since the same approach was used by Jaffe, whose work is discussed in Section 5.3.3 below, we shall defer discussion of the perturbative approach.

5.3. Further Developments

5.3.1. General Considerations

Of course the form of the classical σ-field obtained by Chodos and Thorn is rather different from what we obtained in a soliton bag model in Section 2.3. That discussion suggested that the bag should correspond to a region where $\langle\sigma\rangle$ was zero. From the phenomenological point of view it is possible to impose this simply by multiplying the kinetic energy for the σ-field, $(\partial_\mu\sigma)^2$, in Eq. (5.10) by $\theta_{\bar{v}}$ (which is zero for $r < R$ and unity elsewhere). Indeed, if one identifies σ_v, the expectation value of the σ-field in free space with f [the pion decay constant, see Eq. (4.68)], it is easily seen that one gets volume energy contribution $-B\theta_v$ with B about 20 MeV/fm^3, or about one-half of the phenomenological MIT value (Ros 81).

It seems to us extremely worthwhile to extend the soliton bag model of Section 2.3 by making it at least approximately chirally symmetric. For example, one might start with the symmetric form,

$$\begin{aligned}\mathscr{L}_s(x) = i\bar{q}\not{\partial}q + g\bar{q}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q + \tfrac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 \\ +\tfrac{1}{2}(\partial_\mu\sigma)^2 - \tfrac{1}{4}\lambda^2(\sigma^2 + \underset{\sim}{\pi}^2 - v^2)^2 - p\end{aligned} \tag{5.24}$$

and then explicitly break the symmetry with, say

$$\mathscr{L}_b(x) = C\sigma \quad \text{or} \quad C\sigma^3 \tag{5.25}$$

In either case the sum $\mathscr{L}_s + \mathscr{L}_b$ is equivalent to the form used by Lee, or Goldflam and Wilets, when $\underset{\sim}{\pi}$ is set to zero [see Eq. (2.105)]. Unfortunately, none of the parameters actually used by Goldflam and Wilets gives the right pion decay constant, but their study covered a very limited range of parameters.

Another persistent problem in any version of the σ-model is that the mass of its quantum fluctuations must be large. In fact, the lightest isoscalar two-pion resonances are the narrow S^* (980) which is possibly exotic and the $\varepsilon(1300)$ which does at least have a large width. It is not at all clear that either of these should be identified with the fluctuations in the σ-field. Such problems led even the earliest investigators (GL 60) to consider eliminating the σ-field altogether. In that case one is forced to deal with *nonlinear representations of* SU(2)$\times$SU(2).

One example of such a nonlinear representation is obtained by the Cayley transformation, in which σ and $\underset{\sim}{\pi}$ are replaced by a new pion field

$\underset{\sim}{\xi}$ (Zum 68). That is

$$\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5 \to \frac{1 - i\xi\gamma_5}{1 + i\xi\gamma_5} = \Xi \tag{5.26}$$

where

$$\xi \equiv \underset{\sim}{\tau} \cdot \underset{\sim}{\xi} \tag{5.27}$$

Just as we discussed in Section 4.4.2 the transformation properties of $\underset{\sim}{\xi}$ must be such as to keep $\bar{q}\Xi q$ invariant under a chiral transformation [Eq. (4.28)]. It is an easy algebraic exercise to show that this implies

$$\xi \to \xi - \delta\xi; \qquad \delta\xi = \varepsilon + \xi\varepsilon\xi \tag{5.28}$$

which is clearly nonlinear, involving ξ^2 on the right [by analogy with (5.27) $\underset{\sim}{\tau} \cdot \underset{\sim}{\varepsilon}$ is denoted ε].

An alternate approach introduced by Gell-Mann and Lévy was to use the fact that chiral transformations leave $\sigma^2 + \underset{\sim}{\pi}^2$ constant to eliminate σ^2. In fact, we saw that in the σ-model $(\sigma^2 + \underset{\sim}{\pi}^2)$ was equal to f^2 (Section 4.4). Substituting the relation

$$\sigma^2 = f^2 - \underset{\sim}{\pi}^2 \tag{5.29}$$

in the σ-model Lagrangian density (4.51) with $v \equiv f$, we obtain the nonlinear sigma model. Clearly there are many other possibilities which use Eq. (5.29). For a general discussion of nonlinear representations of $SU(2) \times SU(2)$ we refer to the work of Weinberg (Wei 67, Wei 68) and the lectures of Zumino (Zum 68).

5.3.2. The Little Brown Bag

For a period of about four years the work of Chodos and Thorn was more or less forgotten. [A notable exception was the calculation of $B \to B\pi$ matrix elements in the MIT bag model by LeRoy (LeR 78).] Then in early 1979 several groups returned to this problem of imposing chiral symmetry (Bar+ 79, BR 79). Undoubtedly the largest shock wave was associated with the Stony Brook group. Brown and Rho proposed that one should take seriously the idea of a two-phase picture of physical hadrons. The interior of the static MIT bag was to contain asymptotically free massless quarks while the exterior would contain pions—the Goldstone bosons of $SU(2) \times SU(2)$.

Most notably from the point of view of this review Brown and Rho addressed the problem we raised in Section 1—namely, the compatibility of the bag model of the nucleon (with its large radius) with classical nuclear physics. Their proposal was that the pion coupling should have a dramatic effect on the bag, compressing it to a radius of say $\frac{3}{10}$ fm! In that way the nucleon structure would be irrelevant at normal nuclear matter density.

In order to avoid the σ-meson the Stony Brook group worked with a nonlinear version of the σ-model (Ven+ 80, Ven 80). In fact, their Lagrangian density can be obtained from that of Chodos and Thorn [Eq. (5.10)] by multiplying $\theta_{\bar{v}}$ (1 outside, 0 inside the bag volume V) into the $\underset{\sim}{\pi}$ and σ kinetic energy terms

$$\begin{aligned}\mathscr{L}_{\mathrm{SB}} = (i\bar{q}\partial q - B)\theta_v - \frac{1}{2f}\,\bar{q}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)q\delta_s \\ + \frac{1}{2}\,(\partial_\mu\sigma)^2\theta_{\bar{v}} + \frac{1}{2}\,(\partial_\mu\underset{\sim}{\pi})^2\theta_{\bar{v}}\end{aligned} \tag{5.30}$$

and eliminating $(\sigma, \underset{\sim}{\pi})$ in favor of a new pion field, $\underset{\sim}{\xi}$, defined by

$$\begin{aligned}\underset{\sim}{\pi} &= \underset{\sim}{\xi}(1 + \xi^2/f^2)^{-1/2} \\ \sigma &= f(1 + \xi^2/f^2)^{-1/2}\end{aligned} \tag{5.31}$$

Equation (5.31) is just one of the many nonlinear transformations consistent with Eq. (4.59).

If the pion field is to drastically alter the equilibrium radius it is clear that a nonperturbative treatment must be used. There's the rub! The only case for which a nonperturbative treatment is feasible is once again the hedgehog. Even then the solution is no longer algebraic, instead Vento *et al.* obtained an ordinary second-order differential equation for the classical pion field [i.e., for $G(r)$, where $\underset{\sim}{\xi}(r) \equiv \hat{r}G(r)$].

We display in Fig. 5.2 some typical results from the Stony Brook group. For all these curves the πNN coupling constant, as measured by the asymptotic strength of the pion field, has been fixed at $f_{\pi NN}^2 = 0.081$ by varying f. There are clearly two rather different regions. For large values of R the graph of mass versus R is very flat and the result would be much like the usual MIT solution. Alternatively, the hedgehog tends to collapse as R goes below about 0.6 fm. Indeed, within the crude model proposed there is nothing to stabilize it, and the mass goes to zero at $R \lesssim 0.3$ fm! It has been suggested that this problem can be overcome by coupling the ω-meson to the bag, in which case there is a stable minimum at about 0.5 fm (Ven 81).

As a model system the hedgehog is great fun to play with. For systems of six quarks it has provided some insight into the physics of the short-

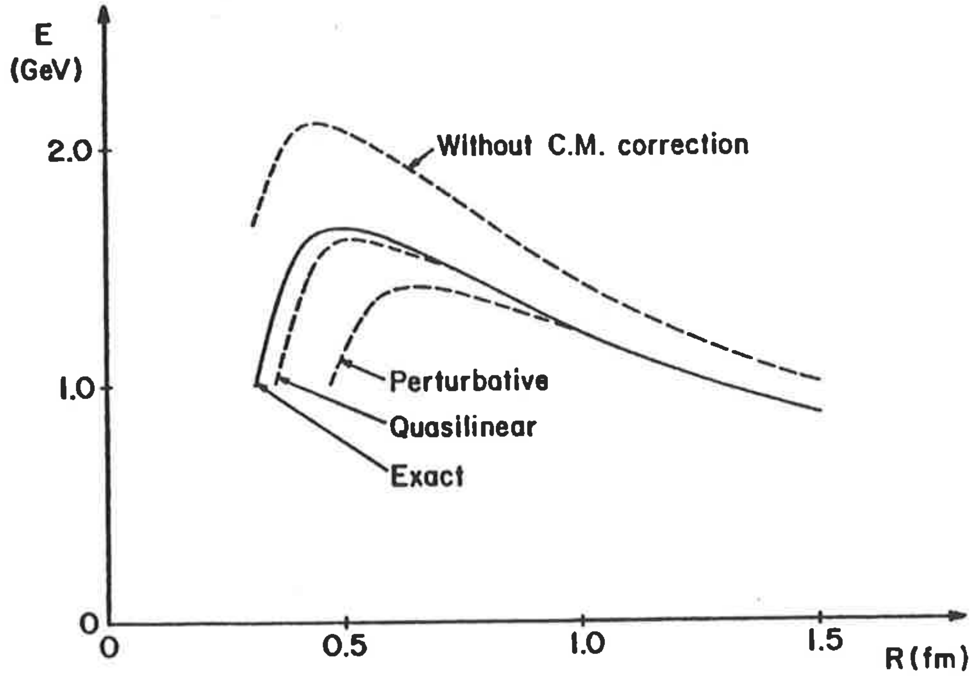

Fig. 5.2. Rest energy of the hedgehog bag versus radius—with $f^2 = 0.081$ (Ven÷ 80).

distance in N–N scattering (Section 7). However, there are too many inconsistencies in this approach for it to be considered realistic. In particular, all of the successes of the MIT bag model, which motivated the whole discussion of chiral symmetry breaking are lost in the small-R, nonperturbative limit. Moreover, as we shall argue in more detail in presenting the cloudy bag model (Section 5.4), when multipion effects are important the long-wavelength approximation breaks down and one can no longer justify neglecting the internal structure of the pion itself.

5.3.3. Classical Perturbation Theory

The revival of interest in "hybrid" bag models (pions and quarks) continued through 1979. In his lectures at the Erice school, Jaffe continued the work of Brown and collaborators in a different direction (Jaf 79). He too worked with a classical pion field, but (not surprisingly!) took the view that the MIT bag should not be drastically altered by its pion couplings. [A similar approach was taken by Musakhanov (Mus 80).] Thus, he developed a systematic expansion of the pionic corrections in terms of a small parameter ε

$$\varepsilon = \frac{g_A}{8\pi f^2 R^2} \tag{5.32}$$

which essentially measures the strength of the classical pion field at the bag surface.

Formally, Jaffe's work is almost identical to that of Vento *et al.* (Ven+80). Instead of the nonlinear transformation (5.31) Jaffe chose to define a new pion field, $\underset{\sim}{\phi}$, using the relations

$$\begin{aligned} \underset{\sim}{\pi} &= f\hat{\phi}\sin(\phi/f) \\ \sigma &= f\cos(\phi/f) \end{aligned} \tag{5.33}$$

which obviously respects the condition $\sigma^2 + \underset{\sim}{\pi}^2 = f^2$. In Eq. (5.33) ϕ is the magnitude of the three-component vector $\underset{\sim}{\phi}$, and $\hat{\phi}$ is the unit vector giving its direction in isospin space

$$\phi = (\underset{\sim}{\phi} \cdot \underset{\sim}{\phi})^{1/2}; \qquad \hat{\phi} = \underset{\sim}{\phi}/\phi \tag{5.34}$$

It will be useful to have the following simple identities:

$$\partial_\mu \phi = \hat{\phi} \cdot \partial_\mu \underset{\sim}{\phi} \tag{5.35}$$

$$\partial_\mu \hat{\phi} = [\hat{\phi} \times (\partial_\mu \underset{\sim}{\phi} \times \hat{\phi})]/\phi \tag{5.36}$$

if the reader intends to follow the original papers in detail.

With the transformation (5.33) the surface-coupling term becomes

$$-\frac{1}{2f}\bar{q}(\sigma + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}\gamma_5)q\delta_s \rightarrow -\frac{1}{2}\bar{q}e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}q\delta_s \tag{5.37}$$

To prove this, simply make a power-series expansion of the right-hand side of Eq. (5.37) and use the identities

$$\gamma_5{}^2 = (\underset{\sim}{\tau} \cdot \hat{\phi})^2 = +1 \tag{5.38}$$

The kinetic energy pieces of the usual σ-model can be written in the form

$$\tfrac{1}{2}(\partial_\mu \sigma)^2 + \tfrac{1}{2}(\partial_\mu \underset{\sim}{\pi})^2 = \tfrac{1}{2}[(\partial_\mu \phi)^2 + f^2 \sin^2(\phi/f)(\partial_\mu \hat{\phi})^2] \tag{5.39}$$

However, if we define a "covariant derivative" as

$$D_\mu \underset{\sim}{\phi} \equiv (\partial_\mu \phi)\hat{\phi} + f\sin(\phi/f)\partial_\mu \hat{\phi} \tag{5.40}$$

it is easy to see from the orthogonality of $\hat{\phi}$ and $\partial_\mu \hat{\phi}$ [see Eq. (5.36)] that

$$\tfrac{1}{2}(\partial_\mu \sigma)^2 + \tfrac{1}{2}(\partial_\mu \underset{\sim}{\pi})^2 \rightarrow \tfrac{1}{2}(D_\mu \underset{\sim}{\phi})^2 \tag{5.41}$$

Finally, if we exclude the pion field from the interior of the bag—in line with the simple-minded, two-phase picture—the Lagrangian density is [use Eqs. (5.41) and (5.37) in Eq. (5.30)]

$$\mathscr{L}(x) = (i\bar{q}\partial\!\!\!/ q - B)\theta_v - \tfrac{1}{2}\bar{q}e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}q\delta_s + \tfrac{1}{2}(D_\mu \underset{\sim}{\phi})^2\theta_{\bar{v}} \tag{5.42}$$

By construction we know that Eq. (5.42) must be invariant under a nonlinear chiral transformation. We leave it as an exercise for the reader to show that the appropriate transformation is

$$\begin{aligned} q &\to q - i\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\varepsilon}}{2}\gamma_5 q \\ \underset{\sim}{\phi} &\to \underset{\sim}{\phi} + \underset{\sim}{\varepsilon} f + f(\underset{\sim}{\varepsilon}\times\hat{\phi})\times\hat{\phi}[1-(\phi/f)\cot(\phi/f)] \end{aligned} \tag{5.43}$$

and the corresponding, conserved axial current has the form

$$\underset{\sim}{A}^\mu = \bar{q}\gamma^\mu\gamma_5(\underset{\sim}{\tau}/2)q\theta_v + \left[f\hat{\phi}\partial^\mu\phi + \frac{f^2}{2}\partial^\mu\hat{\phi}\sin(2\phi/f)\right]\theta_{\bar{v}} \tag{5.44}$$

[The latter is easily obtained by direct substitution for $\underset{\sim}{\pi}$ and σ in terms of $\underset{\sim}{\phi}$ in Eq. (5.11).] For completeness we also give the expression for the conserved vector current which, by analogy with the discussion of the σ-model in Section 4.4.2 [Eqs. (4.49) and (4.50)], arises from the invariance of Eq. (5.42) under the transformation

$$\begin{aligned} q &\to q + i\frac{\underset{\sim}{\tau}\cdot\underset{\sim}{\beta}}{2}q \\ \underset{\sim}{\phi} &\to \underset{\sim}{\phi} - \underset{\sim}{\beta}\times\underset{\sim}{\phi} \end{aligned} \tag{5.45}$$

It is

$$\underset{\sim}{V}^\mu = \bar{q}\gamma^\mu\underset{\sim}{\tau}/2q\theta_v + j_0^2(\phi/f)(\underset{\sim}{\phi}\times\partial^\mu\underset{\sim}{\phi})\theta_{\bar{v}} \tag{5.46}$$

with $j_0(\phi/f)$ the s-wave spherical Bessel function ($j_0(x) = \sin x/x$).

The corresponding nonlinear field equations were written down by Jaffe (Jaf 79) "in their full non-linear ugliness," to emphasise that "if no sensible approximation scheme exists the situation is hopeless." We do not repeat those equations here, but simply summarize the results of the perturbative solution of the classical problem. The small parameter in the expansion is taken to be $(\underset{\sim}{\phi}/f)$. Then in the lowest order only the quark fields are nonzero and we have the MIT solution (q_0). To next order we have to solve the free Klein–Gordon equation for a massless pion field outside the bag

$$\nabla^2\underset{\sim}{\phi}_1(\underset{\sim}{r}) = 0, \qquad r \geq R \tag{5.47}$$

subject to the boundary condition

$$\frac{\partial}{\partial r}\phi_1(\underset{\sim}{r}) = \frac{i}{2f}\bar{q}_0(\underset{\sim}{r})\gamma_5\underset{\sim}{\tau}q_0(\underset{\sim}{r}), \qquad r = R \tag{5.48}$$

This is exactly the phenomenon we observed earlier, that the discontinuity in the derivative of the pion field compensates for the source of axial charge due to the quarks at the surface of the bag.

Equations (5.47) and (5.48) are easily solved for the nucleon

$$\frac{\phi_1(\underset{\sim}{r})}{f} = \frac{3}{2}\,\varepsilon\left(\frac{R}{r}\right)^2 \underset{\sim}{\sigma} \cdot \hat{r}\underset{\sim}{\tau}, \qquad r \geq R \tag{5.49}$$

Thus, as we advertised below Eq. (5.32), ε measures the strength of the pion field at the bag surface. Using $f = 93$ MeV and $g_A = 1.24$ we see that for the typical MIT bag radius, $R \approx 1$ fm, ε is about 0.2, and one would expect this perturbation expansion to work very well. However, for a "little bag" ($R \simeq 0.3$ fm) ε would be about 2 and perturbation theory useless.

In order to be consistent in the classical scheme one must calculate the first-order correction to the quark fields as well:

$$q_0 \to q_0 + \varepsilon q_1 + O(\varepsilon^2) \tag{5.50}$$

The correction q_1 must be calculated from the equations

$$\begin{aligned} \not{\partial} q_1 &= 0, \qquad & r \leq R \\ (i\gamma \cdot \hat{r} - 1)q_1 &= 3i\underset{\sim}{\sigma} \cdot \hat{r}\gamma_5 q_0, \qquad & r = R \end{aligned} \tag{5.51}$$

in order to consistently obtain the lowest-order corrections to the energy, axial-coupling constant, and so on.

$$\begin{aligned} E &= E_0 + \varepsilon E_1 + O(\varepsilon^2) \\ g_A &= g_A{}^{(0)} + \varepsilon g_A{}^{(1)} + O(\varepsilon^2) \end{aligned} \tag{5.52}$$

For example, Jaffe obtained the result

$$\varepsilon E_1 = \frac{-3g_A\varepsilon}{50R} \sum_{i,j} \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \underset{\sim}{\tau}_i \cdot \underset{\sim}{\tau}_j \tag{5.53}$$

where for an N-quark bag of total spin S and isospin I (with all quarks in the same spatial state)

$$\left\langle \sum_{i,j} \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \underset{\sim}{\tau}_i \cdot \underset{\sim}{\tau}_j \right\rangle = 3N^2 + 12N - 4S(S+1) - 4I(I+1) \tag{5.54a}$$

(Actually Eq. (5.54a) is only correct for $N = 3$, when the color wave function is totally antisymmetric. For example, when $N = 6$ one finds instead (Mul+ 82)

$$\left\langle \sum_{i,j} \underset{\sim}{\sigma}_i \cdot \underset{\sim}{\sigma}_j \underset{\sim}{\tau}_i \cdot \underset{\sim}{\tau}_j \right\rangle = 20N - N^2 - 4S(S+1) - 4I(I+1) \tag{5.54b}$$

which implies somewhat smaller pionic corrections.)

There are several satisfying features of this classical treatment. None of the major features of the MIT bag model are altered much. For example, Eqs. (5.53) and (5.54) give changes in the N and Δ masses by -100 MeV and -65 MeV, respectively (for $R \approx 1$ fm) (Jaf 79). In addition the pion current also contributes to the magnetic moment of the hadron (BH 80). Indeed, Myhrer and collaborators have recently shown (using the classical approach still) that not only are the proton and neutron moments improved by the addition of pionic corrections, but that the Λ magnetic moment also comes out rather well (Myh+ 81). We shall not discuss the calculation of magnetic moments further here as the most extensive investigations have been carried out in the cloudy bag model, which will be discussed in Section 6.

On the other hand, the model proposed by Jaffe does raise some problems. We recall from Section 3.3 that the correct prediction of the axial charge of the nucleon, g_A, was a major triumph of the MIT bag. Once the pion field is included, however, there is a contribution to $\vec{\underset{\sim}{A}}(\underset{\sim}{x})$ from the gradient of the pion field—see Eq. (5.44), which to first order in $\underset{\sim}{\phi}/f$ is simply

$$\vec{\underset{\sim}{A}}(\underset{\sim}{x}) = \bar{q}(x)\vec{\gamma}\gamma_5(\underset{\sim}{\tau}/2)q(x)\theta_v + f\vec{\nabla}\underset{\sim}{\phi}(\underset{\sim}{x})\theta_{\bar{v}} \tag{5.55}$$

Now if the pion field were not excluded from the bag (by $\theta_{\bar{v}}$) the integral over $\vec{\nabla}\underset{\sim}{\phi}$ could be converted to a vanishing surface integral.[¶] In the presence of $\theta_{\bar{v}}$ there remains a nonvanishing contribution from the integral of the pion field over the bag surface. As verified by a number of groups, this surface contribution from the pion field increases the overall value of g_A by a factor of approximately $\frac{4}{3}$ (Jaf 79, BH 80, Ven+ 80). Thus, the hybrid bag model gives a value of g_A very close to the $\frac{5}{3}$ of the "good old (nonrelativistic) quark model"—a retrograde step to be sure. Further investigation of higher-order corrections only makes the situation worse, with g_A rising above 2 (Hul+ 81).

¶ The astute reader may have observed that this is not actually true in the case of massless pions because $\phi(\underset{\sim}{r}) \propto r^{-2}$ and hence there is a constant contribution from the surface at infinity. However, for any finite pion mass (no matter how small) this will vanish.

Quite apart from the disaster for g_A one might expect to find some contribution to hadronic charge densities from the pion field. Unfortunately the charge density involves the time derivative of the pion field which vanishes in the classical limit.

Finally, classical models of the type considered by Jaffe offer little connection with nuclear physics. Indeed, Jaffe seemed to feel that the hybrid bag models, although an entertaining sidelight to serious physics, were rather sterile. To quote directly, "it should be clear to the reader that hybrid chiral models are of limited theoretical interest. They are entirely ad hoc . . . and restricted to the low energy regime."

We have taken a rather different and far more optimistic point of view. It seems to us that understanding the transition to the Goldstone realization of chiral symmetry will be an essential step in the solution of the QCD equations. Moreover, as we shall demonstrate, one particular hybrid model, the cloudy bag model (CBM), overcomes all of the objections raised above, while retaining the positive features. Most significantly for the present review it goes further, offering a basis for optimism in low- and medium-energy nuclear physics which has simply not been conceivable before. The CBM will be introduced in Section 5.4 and its applications for hadronic properties described in Section 6. First, however, we summarize some of the other attempts to deal with pion-bag interactions.

5.3.4. Other Bag Model Calculations

As we have already remarked, the first in what we may regard as modern investigations of hybrid bags after Chodos and Thorn was the work of LeRoy (LeR 78). He used the Chodos–Thorn surface coupling to estimate the strength of various $B'B\pi$ couplings. This was then compared with the more conventional Melosh analysis (Mel 74). In spite of the simplicity of this first analysis of a wide range of decays in the bag model, rather good qualitative agreement with experiment was obtained. For the specific examples of the nucleon and delta we shall see in Section 6 some of the corrections which would need to be incorporated in a more detailed investigation.

The first studies of the effect of pion coupling, dictated by chiral symmetry, on hadronic properties were those of Brown and Rho (BR 79) and Barnhill *et al.* (Bar+ 79). As Jaffe demonstrated at length (Jaf 79) neither of these works gave a fully consistent set of field equations for the coupled quark-pion system. In particular Barnhill *et al.* omitted the first-order correction to the quark field [q_1 in Eq. (5.50)] caused by the nonzero pion field. This actually leads to the wrong sign for the first-order correction to the energy [E_1 in Eq. (5.52)]. We have already shown above that the later Stony Brook work on the hedgehog was based on a suitable hybrid extension of the nonlinear σ-model.

Several other groups have used essentially the linearized version of Jaffe's equations in redoing the MIT spectroscopy for low-lying states (Cot+ 80, McM 81, Myh+ 81, Thé 82). Although the details of these fits vary a little, the overall conclusion is that there is no difficulty refitting the mass spectrum with pionic corrections. If anything, there is some improvement.

In concluding this section we note that there have been a number of other attempts to deal with pion coupling to the MIT bag which have *not* been motivated by considerations of chiral symmetry. In the sense that it is a nonperturbative treatment the work of Weber (Web 80, Web 81) is probably the most closely related to our present discussion. Both Duck (Duc 76) and Weise (WW 81) attempted to calculate the pion emission perturbatively. It is interesting that Weise also finds the pions to be predominantly created in the surface region of the bag. The surprising aspect of the calculation, since it involved only low-order perturbation theory, was that the coupling constant had the right order of magnitude. In fact, the work of Genz (Gen 70, and private communication) and Weber (Web 82, and private communication), suggests that a complete calculation to this order should give $g_{\pi NN} \sim 2.4$ instead.

In comparison with these rather ambitious calculations, the hybrid bag models in general and the CBM in particular are more phenomenological. On the other hand, chiral symmetry is imposed as a crucial guide in constructing the theory, and one is in that sense not compelled to rely on perturbation theory in the bag surface which is not an asymptotically free region.

5.4. The Cloudy Bag Model

The starting point for the development of a model of hadron structure of relevance to nuclear physics is the model of Jaffe. As we saw in Section 5.3.3, even in its linearized form that model had some unfortunate features. The *c*loudy *b*ag *m*odel (CBM) (Thé+ 80, Tho+ 81a, Tho 81, Mil+ 81) also relies on a perturbative approach. However, it overcomes all of the problems encountered in Jaffe's model by (a) dealing with a quantized pion field and (b) not explicitly excluding the pion from the static bag volume. Some compelling but nevertheless qualitative arguments will be given to suggest that not only does this approach yield good results, but that it may also be the best approximation to the underlying physics.

5.4.1. The Nonlinear Equations

We have already explained in great detail how to obtain a chiral invariant Lagrangian density involving only quark and pion fields by making the substitution (5.33) in the Chodos–Thorn Lagrangian density. In the case where the pion is not excluded from the interior of the static bag volume this yields [cf., Eq. (5.42)]

$$\mathscr{L}(x) = (i\bar{q}\partial q - B)\theta_v - \tfrac{1}{2}\bar{q}e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}q\delta_s + \tfrac{1}{2}(D_\mu\underset{\sim}{\phi})^2 \tag{5.56}$$

All of the formal results of Section 5.3.3 hold and need not be repeated here. [The covariant derivative, $D_\mu\underset{\sim}{\phi}$, was given in Eq. (5.40).] The only change is that wherever $\theta_{\bar{v}}$ appeared in Section 5.3.3 it should be replaced by 1. If an explicit symmetry-breaking term, $-\frac{1}{2}m_\pi{}^2\underset{\sim}{\phi}^2$, were introduced in Eq. (5.56) the axial current of the model [cf., Eq. (5.44)]

$$\underset{\sim}{A}^\mu = \bar{q}\gamma^\mu\gamma_5(\underset{\sim}{\tau}/2)q\theta_v + \left[f\hat{\underset{\sim}{\phi}}\partial^\mu\phi + \frac{f^2}{2}\,\partial^\mu\hat{\underset{\sim}{\phi}}\,\sin(2\phi/f)\right] \tag{5.57}$$

would satisfy the PCAC condition

$$\partial_\mu\underset{\sim}{A}^\mu = -fm_\pi{}^2\underset{\sim}{\phi} + O(\underset{\sim}{\phi}^2) \tag{5.58}$$

5.4.2. Pions inside the Bag?

None of the hybrid bag models which have been developed so far have really constituted a dynamical description of the process of pion emission. It is difficult enough to believe that the static MIT bag model itself, with its rigid, spherical boundary is more than a mathematically convenient idealization of a real hadron. However, it is impossible to believe that the boundary remains static and unperturbed by the creation of a pion with several hundred MeV/c momentum. Thus, the very concept of interior and exterior, which was taken to be sacrosanct in the models discussed in Section 5.3, is by no means clear-cut.

A useful model to consider at this stage is the soliton bag model discussed in Section 2.3.2. There we saw that with a suitable interaction between an effective σ-field and a fermion field it is possible for the fermions to dig themselves a "hole" (or bag). Within the hole the vacuum would be simple, with the expectation value of the σ-field very near zero. Outside the bag, where $\bar{q}q$ is zero, the σ-field has a nonvanishing expectation value. The transition region between these two extremes is the bag surface. It has been shown that results very similar to those of the MIT bag model can be obtained for a variety of parameters and surface thicknesses (GW 82).

Suppose that a $\bar{q}q$ pair is produced by some perturbative interaction in the surface of such a bag. This pair could also start to dig a hole and eventually move into the vacuum as a new particle, as illustrated schematically in Fig. 5.3. It is clear that creation of such a pair could occur anywhere inside the bag, although our ideas of asymptotic freedom suggest it would be most likely in the surface where the effective value of α_s is growing rapidly.

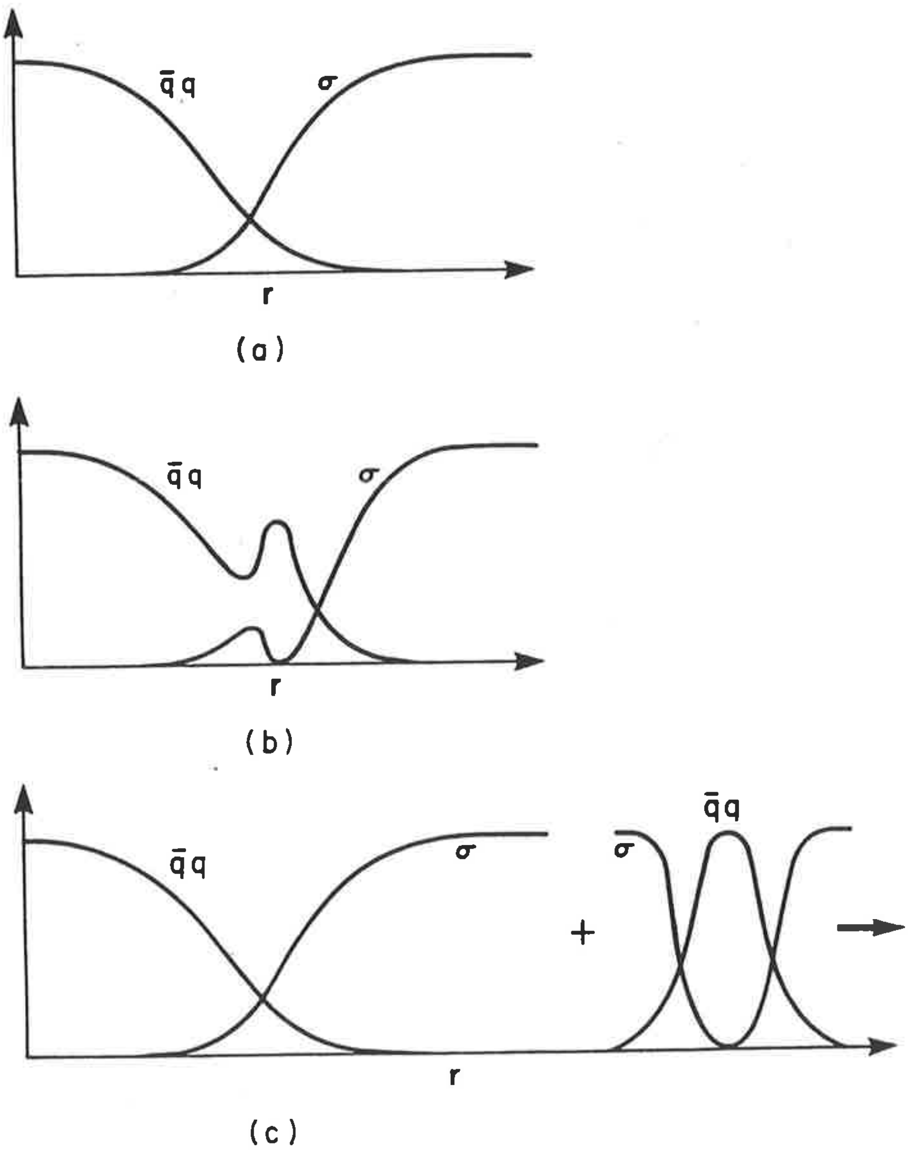

Fig. 5.3. Schematic illustration of the soliton bag: (a) before, (b) during, and (c) after emission of a $q\bar{q}$ pair.

Of course this sort of pair creation process in a cavity has been studied in other ways (MV 81, CH 81, DG 77) and such pairs are referred to as "sea quarks." Usually such pairs are treated like exchange current corrections in nuclear physics with the quarks being put in cavity eigenstates, rather than exhibiting any coherence. DeTar (DeT 81) suggested, without much conviction, that one might be able to treat pairs with pion quantum numbers as though they were coherent—in that way deriving a model identical to the CBM (Thé+ 80). However, the essential justification for such a procedure can come only from dynamical symmetry breaking (DSB)—in particular, a model such as that proposed by Goldman and Haymaker (GH 81, HG 81; see also MF 81). If their idea (see Section 5.1) that short-distance one-gluon exchange suffices to bind a $\bar{q}q$ pair with pion quantum numbers (thereby producing DSB and a Goldstone boson) is correct, then it would be essential to treat such pairs coherently—even inside another bag!

Thus, it should be clear for a number of reasons that the insistence on excluding pions from the interior of a static, spherical MIT bag is not only an unreasonable simplification, it may be wrong. On the other hand, it is clearly an approximation to treat the pion as a free particle through all space, as we assumed in writing Eq. (5.56). A more sophisticated treatment would perhaps involve the expansion of the pion field in eigenfunctions of some effective potential. Nevertheless, incorporating exact chiral symmetry and the concept of DSB, the CBM seems, *a priori*, to be a good place to begin.

5.4.3. *Linearization of the CBM Equations*

If the discussion towards the end of Section 5.1 did not make it clear, let us stress again that it will only make sense to write down a hybrid model if the problem to be examined is one where the internal structure of the pion can reasonably be ignored. In this sense we are making a long-wavelength approximation from the beginning. Therefore, we must agree with Jaffe that either perturbation theory about the usual MIT solution is adequate or we should attack the problem in a different way.

As it stands, the Lagrangian density in Eq. (5.86) is probably not renormalizable. However, if it could be generalized to include the internal structure of the pion there would be a natural mechanism for cutting off higher-order terms. This is a challenging problem for the future. For now, bearing all of these arguments in mind we have chosen (like Jaffe) to deal with small fluctuations in the pion field about the point $\underset{\sim}{\phi} = 0$. In that case we find the simplifications

$$\frac{1}{2}(D_\mu \underset{\sim}{\phi})^2 \rightarrow \frac{1}{2}(\partial_\mu \underset{\sim}{\phi})^2 \tag{5.59}$$

$$-\frac{1}{2}\bar{q}e^{i\underset{\sim}{\tau}\cdot\underset{\sim}{\phi}\gamma_5/f}q\delta_s \rightarrow -\frac{1}{2}\bar{q}q\delta_s - \frac{i}{2f}\bar{q}\gamma_5\underset{\sim}{\tau}q\cdot\underset{\sim}{\phi}\delta_s \tag{5.60}$$

in Eq. (5.86). The resulting Lagrangian density (Thé+ 80, DeT 81)

$$\begin{aligned}\mathscr{L}_{\text{CBM}}(x) = (i\bar{q}\not{\partial}q - B)\theta_v - \frac{1}{2}\bar{q}q\delta_s + \frac{1}{2}(\partial_\mu\underset{\sim}{\phi})^2 - \frac{1}{2}m_\pi^2\underset{\sim}{\phi}^2 \\ - \frac{i}{2f}\bar{q}\gamma_5\underset{\sim}{\tau}q\cdot\underset{\sim}{\phi}\delta_s\end{aligned} \tag{5.61}$$

will be treated in great detail in Section 6.

It will be an essential part of the discussion in Section 6 to show that the hadronic states resulting from Eq. (5.61) do not contain large multipion components. As long as perhaps one or two pions dominate, the large Compton wavelength of the pion ensures that the internal structure of the pion can be neglected. If, on the other hand, we find that there is an appreciable probability of finding, say, four or five pions, the distance scale of $5m_\pi^{-1} \sim 0.1$–0.2 fm would simply make nonsense of our long-wavelength approximation. This is also the reason why we oppose the inclusion of vector mesons as an explicit component of the hadronic wave function—such heavy $\bar{q}q$ pairs are best treated as sea quarks. (This will be discussed further in Sections 6 through 8 because it impacts severely on the conventional description of nuclear physics!)

Fortunately, we shall find that over a wide range of bag sizes a perturbative expansion in the number of pions converges extremely rapidly and the linearization and long-wavelength approximation do produce a consistent solution! Indeed we shall show that Eq. (5.91) constitutes a renormalizable theory of bare bags coupled to a pion field within which the renormalizations are not only finite but small. For example, the bare $NN\pi$ coupling constant is within 10% of the renormalized value for any bag radius greater than 0.8 fm.

In motivating the present model, rather than those considered in Section 5.3, we noted that the CBM would overcome all of the problems connected with the classical model of Jaffe. Hopefully, it is obvious that as there is no exclusion of the pion from the bag interior there is no surface contribution to g_A from the pion field. Thus, in lowest order the good bag model result that g_A is 1.09 is retained. Of course, we have a Goldberger–

Treiman relation and g_A will be renormalized in exactly the same way as the $NN\pi$ coupling constant. However, as we remarked above, such renormalizations are small in the CBM. Incidentally, it is interesting to contrast this beautifully simple picture of PCAC and the fact that g_A is near one with the classical version described in Section 4. In the CBM g_A is near one because that's what three confined, relativistic, massless quarks give. The renormalization is small because the cavity containing the quarks is large and low-order perturbation theory in the pion field makes sense!

5.4.4. An Alternative Formulation

The implications of Eq. (5.61) for pion–nucleon scattering, particularly in the P_{33} channel, will be discussed in detail in the next section. However, it is worth noting at this stage that the one disappointing feature of the CBM Lagrangian density is that there is no obvious prediction for low-energy pion–baryon scattering. One of the triumphs of the soft-pion ideas of the late sixties was the Weinberg–Tomozawa relationship (Wei 66, AD 68). That is, the prediction that in a chirally symmetric world the scattering length for a pion on *any* target of isospin T_t, with total isospin T, is exactly

$$a_T = \left(\frac{g}{2m}\right)^2 \left(\frac{g_V}{g_A}\right)^2 \frac{m_\pi}{2\pi}\left(1 - \frac{m_\pi}{m_t}\right)^{-1} [T(T+1) - T_t(T_t+1) - 2] \tag{5.62}$$

where $(g/2m)$ is the pseudoscalar $NN\pi$ coupling constant and m_t the target mass. Thus, the scattering length is purely isovector in the soft-pion limit. Much of the popularity of the nonlinear sigma model in fact followed from Weinberg's proof (Wei 67) that it provided a convenient dynamical framework which incorporated Eq. (5.62) explicitly in an effective Lagrangian.

It is possible to make a unitary transformation on the original, nonlinear Lagrangian density (5.56) in such a way that the Weinberg–Tomozawa result appears explicitly (Tho 81b). However, the price is a redefinition of the quark fields which essentially get dressed by the pions. Only one of these two quark fields can be canonical and one must make a choice.

To be specific, consider the new quark field q_w, defined by the transformation

$$q \to q_w = Sq \tag{5.63}$$

$$\bar{q} \to \bar{q}_w = \bar{q}S \tag{5.64}$$

with

$$S = \exp[i\underset{\sim}{\tau} \cdot \underset{\sim}{\phi}(\gamma_5/2f)] \tag{5.65}$$

Then $\mathscr{L}(x)$ becomes

$$\mathscr{L}(x) = (i\bar{q}_w S^+ \partial\!\!\!/ S^+ q_w - B)\theta_v - \tfrac{1}{2}\bar{q}_w q_w \delta_s + \tfrac{1}{2}(D_\mu \underset{\sim}{\phi})^2 \tag{5.66}$$

(As usual the explicit, symmetry-breaking pion mass is omitted, but it can of course be put in with no change in our argument.) The $\partial\!\!\!/$ in Eq. (5.66) acts both on S^+ and q_w, so it is convenient to separate the two pieces with the result

$$\mathscr{L}(x) = (i\bar{q}_w \partial\!\!\!/ q_w - B)\theta_v - \tfrac{1}{2}\bar{q}_w q_w \delta_s + \tfrac{1}{2}(D_\mu \underset{\sim}{\phi})^2 + \bar{q}_w \gamma^\mu i(S\partial_\mu S^+) q_w \theta_v \tag{5.67}$$

(We have used $\{\gamma^\mu, \gamma_5\} = 0$ to change $S^+\gamma^\mu$ to $\gamma^\mu S$.)

At this stage there is an extremely useful identity which appears in a paper by Au and Baym (AB 74):

$$S\partial_\mu S^+ = \int_0^1 d\lambda\, S^\lambda \partial_\mu (\ln S^+)(S^+)^\lambda \tag{5.68}$$

The essential feature of Eq. (5.68) is that the logarithm reduces $\exp[i\underset{\sim}{\tau} \cdot \underset{\sim}{\phi} \times (\gamma_5/2f)]$ to a form linear in $\underset{\sim}{\phi}$. We leave it as a fairly straightforward algebraic exercise using Eq. (5.68) and

$$S = \cos(\phi/2f) + i\underset{\sim}{\tau} \cdot \hat{\underset{\sim}{\phi}} \gamma_5 \sin(\phi/2f) \tag{5.69}$$

to prove that

$$iS\partial_\mu S^+ = \frac{\gamma_5}{2f}\, \underset{\sim}{\tau} \cdot D_\mu \underset{\sim}{\phi} + \left[\frac{\cos(\phi/f) - 1}{2}\right] \underset{\sim}{\tau} \cdot (\hat{\underset{\sim}{\phi}} \times \partial_\mu \hat{\underset{\sim}{\phi}}) \tag{5.70}$$

Thus, if we define the covariant derivative on the quark fields as

$$D_\mu q_w = \partial_\mu q_w - i\left[\frac{\cos(\phi/f) - 1}{2}\right] \underset{\sim}{\tau} \cdot (\hat{\underset{\sim}{\phi}} \times \partial_\mu \hat{\underset{\sim}{\phi}}) q_w \tag{5.71}$$

the transformed Lagrangian density takes the form

$$\begin{aligned} \mathscr{L}'_{\mathrm{CBM}}(x) = {} & (i\bar{q}_w D\!\!\!\!/\, q_w - B)\theta_v - \frac{1}{2}\, \bar{q}_w q_w \delta_s + \frac{1}{2}\, (D_\mu \underset{\sim}{\phi})^2 \\ & + \frac{1}{2f}\, \bar{q}_w \gamma^\mu \gamma_5 \underset{\sim}{\tau} q_w \cdot (D_\mu \underset{\sim}{\phi}) \theta_v \end{aligned} \tag{5.72}$$

Clearly the surface coupling of the pion has been transformed into volume pseudovector coupling. This is exactly what one expects from current algebra considerations (AD 68). At $k = 0$ the strength of the coupling is simply related to the axial charge of the bag state

$$(4\pi)^{1/2} f_{NN\pi}/m_\pi = g_A/2f \tag{5.73}$$

The Goldberger–Treiman relation is thereby made explicit. It has been proven by Betz (Bet 82) that the form-factor associated with this $NN\pi$ vertex is identical to that in the first version of the CBM, namely $3j_1(kR)/kR$ (see Section 6.1 for details of the $NN\pi$ form-factor). Thus, both versions are identical in all predictions associated with single pion emission and absorption.

To illustrate the consequences for s-wave pion scattering from a bag let us consider the zero energy limit and as suggested in Section 5.4.3 work to lowest nontrival order in $\underset{\sim}{\phi}$. Then the covariant derivative on the quark fields [Eq. (5.71)] leads to an interaction term quadratic in the pion field

$$\mathscr{L}_s(x) = -\frac{1}{2f^2}\,[\bar{q}_w(\gamma^0\underset{\sim}{\tau}/2)q_w](\underset{\sim}{\phi}\times\partial_0\underset{\sim}{\phi})\theta_v \tag{5.74}$$

But the term in square brackets is just the isospin density of the bag target [see Eq. (4.26)] and $(\underset{\sim}{\phi}\times\partial_0\underset{\sim}{\phi})$ the usual pion isospin density. For pions of zero three-momentum, $\underset{\sim}{\phi}\times\partial_0\underset{\sim}{\phi}$ is independent of $\underset{\sim}{x}$, and integrating Eq. (5.104) to give the matrix element of the Hamiltonian between pion states of zero momentum

$$\begin{aligned} H_s &= -\int d^3x\,\mathscr{L}_s(x) \\ &= \underset{\sim}{T}_t\cdot\underset{\sim}{t}_\pi/2f^2 \end{aligned} \tag{5.75}$$

in an obvious notation. Thus, to lowest order we obtain a general relationship for pion scattering from any hadronic bag (except another pion!) which is identical to the Weinberg–Tomozawa result, Eq. (5.62). [To see this, use the Goldberger–Treiman relationship (5.73) and the familiar equivalence of pseudoscalar and pseudovector coupling constants $g/2m = (4\pi)^{1/2}f_{NN\pi}/m_\pi$.] This result has been obtained independently by Szymacha and Tatur (ST 81).

Thus, the alternate form of CBM Lagrangian density (Tho 81b)

$$\begin{aligned} \mathscr{L}'_{\mathrm{CBM}}(x) = {} & (i\bar{q}_w\not{\partial}q_w - B)\theta_v - \frac{1}{2}\bar{q}_w q_w\delta_s - \frac{\theta_v}{4f^2}\bar{q}_w\gamma^\mu\underset{\sim}{\tau}q_w\cdot(\underset{\sim}{\phi}\times\partial_\mu\underset{\sim}{\phi}) \\ & + \frac{\theta_v}{2f}\bar{q}_w\gamma^\mu\gamma_5\underset{\sim}{\tau}q_w\cdot\partial_\mu\underset{\sim}{\phi} + \frac{1}{2}(\partial_\mu\underset{\sim}{\phi})^2 - \frac{1}{2}m_\pi{}^2\underset{\sim}{\phi}^2 \end{aligned} \tag{5.76}$$

incorporates both major results of the current algebra for low-energy pion scattering and generalizes the Weinberg Lagrangian (which applied to the

$NN\pi$ system only) to any hadron describable by the MIT bag model. Furthermore, with the cautions given in the next section, rather than being used simply as an effective Lagrangian it defines a renormalizable theory of strong interactions, thereby permitting the systematic calculation of higher-order corrections.

6. APPLICATIONS OF THE CLOUDY BAG MODEL

At last we have established the chiral-bag formalism and can begin to apply it to cases of physical interest. Our starting point will be the linearized version of the cloudy bag model given in Eq. (5.61). Essentially all of the applications so far have relied on the linear coupling of the form $B'B\pi$, and as we have remarked the alternate form of the Lagrangian density [Eq. (5.76)] would give identical results. On the other hand, if one is concerned with s-wave π–π scattering, or reactions like $(\pi, 2\pi)$ it would be necessary to retain terms of higher order in $\underset{\sim}{\phi}$. In that case, as we have already seen in obtaining the Weinberg–Tomozawa relationship, it is most fruitful to simply go to higher order in the expansion of the alternate Lagrangian density [Eq. (5.72)]. For example, to $O(\phi^3)$ there will be an explicit term describing $\pi + B \rightarrow \pi + \pi + B'$, which arises from the pseudovector coupling to the axial current (KE 81, Tho 81b).

The natural first step in making practical calculations is to obtain a Hamiltonian from the underlying Lagrangian density. This Hamiltonian can be written entirely in terms of bags with the quantum numbers of observed particles, rather than in terms of quarks. At that stage the theory will look very much like the starting point for many calculations in medium-energy physics. To first order, what we have gained is a microscopic understanding of the high-momentum cutoff in the theory and relationships between the relevant coupling constants. Looked at in more detail, we shall see that the model is conceptually quite different, and the difference should have important consequences for our understanding of nuclear physics, particularly at high density.

6.1. A Hamiltonian for Low- and Medium-Energy Physics

The linearized CBM Lagrangian density [Eq. (5.61)] breaks very nicely into three separate pieces

$$\mathscr{L}_{\mathrm{CBM}}(x) = \mathscr{L}_{\mathrm{MIT}}(x) + \mathscr{L}_{\pi}(x) + \mathscr{L}_{\mathrm{int}}(x) \tag{6.1}$$

where $\mathscr{L}_{\text{MIT}}$ was given in Eq. (4.1), $\mathscr{L}_\pi$ describes a free pion field

$$\mathscr{L}_\pi = \tfrac{1}{2}(\partial_\mu \underset{\sim}{\phi})^2 - \tfrac{1}{2}m_\pi{}^2\underset{\sim}{\phi}^2 \tag{6.2}$$

and

$$\mathscr{L}_{\text{int}} = -\frac{i}{2f}\,\bar{q}\gamma_5\underset{\sim}{\tau}q \cdot \underset{\sim}{\phi}\delta_s \tag{6.3}$$

Without $\mathscr{L}_{\text{int}}$, which was dictated by chiral symmetry, the theory would describe stable MIT bag states and free pions.

Once gluon degrees of freedom are included in $\mathscr{L}_{\text{MIT}}$ only colorless states have finite energy.¶ Our emphasis in this review will be on baryon structure and interactions, although similar ideas could be applied to the heavy mesons. Thus, we are naturally led to consider first colorless bag states with baryon number one. These will contain $3q$ (three quarks), $4q - \bar{q}$, $5q - 2\bar{q}$, and so on. In view of the success of the bag model in describing the low-lying baryons without exotic components, it is reasonable to divide the space of baryon number one hadrons into two pieces $(P + Q)$

$$P = \sum_{\substack{\alpha=\text{nonexotic}\\ \text{baryons}}} |\,\alpha\rangle\langle\alpha\,| \tag{6.4}$$

$$Q = \underset{\sim}{1} - P \tag{6.5}$$

That is, P is a projection operator onto nonexotic bag states such as N, Δ, R (the Roper resonance), etc. The wave functions for these states are simply the usual bag model SU(6) wave functions [see, for example, Eq. (3.13) for the nucleon]. The unit operator $\underset{\sim}{1}$ refers to the space of $B = 1$ bag states, and Q is a projection operator onto exotic states.

Formally, the inclusion of corrections arising from coupling to the Q-space is equivalent to evaluating the lowest-order sea quark corrections. Such corrections have been shown numerically to be rather small (MV 81, CH 81, DG 77), so for the present purposes we shall neglect off-diagonal terms connecting P and Q. In that case the Hamiltonian obtained from $\mathscr{L}_{\text{MIT}}$ in the canonical way is simply (Thé+ 80)

$$H_{\text{MIT}} = \int d^3x\, T^{00}_{\text{MIT}}(x) \tag{6.6}$$

¶ For pedagogic simplicity our discussion has ignored the role of gluons in the MIT bag model, except where they are absolutely essential—as in Section 3 for hyperfine splitting of hadronic levels. Nevertheless, any realistic calculation must include the gluons. Since they play no role with respect to chiral symmetry they will only appear in $\mathscr{L}_{\text{MIT}}$.

where the energy-momentum tensor ($T^{\mu\nu}$) is

$$T^{\mu\nu}_{\mathrm{MIT}}(x) = \frac{\partial \mathscr{L}_{\mathrm{MIT}}(x)}{\partial(\partial_\mu q)}\,(\partial^\nu q) - g^{\mu\nu}\mathscr{L}_{\mathrm{MIT}}(x) \tag{6.7}$$

Explicitly this gives a bag model Hamiltonian

$$H_{\mathrm{MIT}} = \int d^3x\left[\bar{q}(-i\underset{\sim}{\gamma}\cdot\underset{\sim}{\nabla})q + B + \frac{1}{2}\sum_{a=1}^{8}(E_a{}^2 - B_a{}^2)\right]\sigma_v \tag{6.8}$$

However, the states α are eigenstates of Eq. (6.8) with masses $m_\alpha{}^{(\mathrm{b})}$, where the superscript means "bag." Thus, we obtain

$$\begin{aligned} H_{\mathrm{MIT}} &\simeq PH_{\mathrm{MIT}}P \\ &= \sum_\alpha |\,\alpha\rangle m_\alpha{}^{(\mathrm{b})}\langle\alpha\,| \end{aligned} \tag{6.9}$$

In terms of more conventional second quantization this can be written

$$H_{\mathrm{MIT}} = \sum_\alpha \alpha^+\alpha m_\alpha{}^{(\mathrm{b})} \tag{6.10}$$

where α creates a three-quark bag state with the quantum numbers of N, Δ, R, etc. (There is one rather innocent assumption implicit in the last step, namely that two different bag states α and β, with different masses, are orthogonal. Unfortunately, this is not completely correct in the naive bag model because the radii of those two bag states will not be exactly equal—as a result of the nonlinear boundary condition. Nevertheless, one expects on physical grounds that the orthogonality must hold in a more sophisticated formulation, such as the soliton bag model, and we simply impose it here.)

In the canonical way we also obtain the Hamiltonian for a free pion field corresponding to Eq. (6.2), namely

$$H_\pi = \tfrac{1}{2}\int d^3x\,[(\partial_0\underset{\sim}{\phi})^2 + (\underset{\sim}{\nabla}\underset{\sim}{\phi})^2 + m_\pi{}^2\underset{\sim}{\phi}^2] \tag{6.11}$$

with $\underset{\sim}{\phi}$ the quantized pion field

$$\phi_i(\underset{\sim}{x}) = (2\pi)^{-3/2}\int \frac{d\underset{\sim}{k}}{(2w_k)^{1/2}}\,(a_{\underset{\sim}{k}i}e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} + a^+_{\underset{\sim}{k}i}e^{-i\underset{\sim}{k}\cdot\underset{\sim}{x}}) \tag{6.12}$$

The creation and destruction operators obey the usual commutation relations

$$\begin{aligned} [a_{\underset{\sim}{k}i}, a_{\underset{\sim}{k}'j}] &= [a^+_{\underset{\sim}{k}i}, a^+_{\underset{\sim}{k}'j}] = 0 \\ [a_{\underset{\sim}{k}i}, a^+_{\underset{\sim}{k}'j}] &= \delta_{ij}\delta(\underset{\sim}{k} - \underset{\sim}{k}') \end{aligned} \tag{6.13}$$

After normal ordering, Eq. (6.11) takes the rather simple and familiar form

$$H_\pi = \sum_i \int d\underset{\sim}{k}\; w_{\underset{\sim}{k}} a^+_{\underset{\sim}{k}i} a_{\underset{\sim}{k}i} \tag{6.14}$$

Finally, and of course this was the whole point of the exercise, there is an interaction term

$$PH_{\text{int}}P = \frac{i}{2f} \sum_{\alpha,\beta} \int d^3x \, \langle \beta \mid \bar{q}(x)\underset{\sim}{\tau} \cdot \underset{\sim}{\phi}(\underset{\sim}{x})\gamma_5 q(x) \mid \alpha \rangle \delta_s \beta^+ \alpha \tag{6.15}$$

Using the expansion [Eq. (6.12)] for the pion field, and assuming static spherical bags of equal radii [$\delta_s \equiv \delta(x - R)$], Eq. (6.15) becomes

$$PH_{\text{int}}P = (2\pi)^{-3/2} \sum_{\alpha,\beta,i} \int d\underset{\sim}{k}\, (v^{\beta\alpha}_{\underset{\sim}{k}i} \beta^+ \alpha a_{\underset{\sim}{k}i} + \text{H.c.}) \tag{6.16}$$

where H.c. denotes Hermitian conjugate and

$$v^{\beta\alpha}_{\underset{\sim}{k}i} = \frac{i}{2f} \frac{1}{(2w_{\underset{\sim}{k}})^{1/2}} \int d^3x \, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}} \delta(x - R) \langle \beta \mid \bar{q}(\underset{\sim}{x}) \tau_i \gamma_5 q(\underset{\sim}{x}) \mid \alpha \rangle \tag{6.17}$$

Thus, as promised, all $B'B\pi$ couplings can be calculated in terms of the pion decay constant, $f = 93$ MeV.

6.1.1. The $NN\pi$ Vertex

To see what is involved in Eq. (6.17) let us consider the $NN\pi$ vertex in this theory. In that case the spatial orbits of all quarks in the initial and final hadrons are the same, namely $1s_{1/2}$. The spatial portion of Eq. (6.17) is therefore [from Eqs. (2.33) and (2.34)]

$$\begin{aligned} \bar{q}_{1,-1}(\underset{\sim}{x})\gamma_5 q_{1,-1}(\underset{\sim}{x}) \Big|_{x=R} &= \frac{N^2_{1,-1}}{4\pi} 2ij_0(\omega) j_1(\omega) \underset{\sim}{\sigma} \cdot \hat{r} \\ &= \frac{\omega}{(\omega - 1)} \frac{i}{4\pi R^3} \underset{\sim}{\sigma} \cdot \hat{r} \end{aligned} \tag{6.18}$$

and we have used the fact that the surface δ-function restricts the integral to $x = R$. Using Eq. (6.18) to perform the integral over coordinates in Eq. (6.17) we find that

$$v^{NN'}_{\underset{\sim}{k}i} = (2w_{\underset{\sim}{k}})^{-1/2} \frac{i}{2f} \frac{\omega}{(\omega - 1)} \frac{j_1(kR)}{kR} \, {}_{\text{s-f}}\langle N \mid \sum_{a=1}^{3} \tau_{ai} \underset{\sim}{\sigma}_a \cdot \underset{\sim}{k} \mid N' \rangle_{\text{s-f}} \tag{6.19}$$

where the sum over a runs over the three quarks, and the subscript s–f denotes the spin–flavor part of the nucleon wave function. Now we recognize that the combination $\sum_{a=1}^{3} \underset{\sim}{\tau}_a \underset{\sim}{\sigma}_a$ appeared in our discussion of the axial current. Indeed, from Eq. (3.38) we know that

$$_{\text{s-f}}\langle N \mid \sum_{a=1}^{3} \tau_{ai}\underset{\sim}{\sigma}_a \cdot \underset{\sim}{k} \mid N' \rangle_{\text{s-f}} = \frac{5}{3} \langle N \mid \tau_i \underset{\sim}{\sigma} \cdot \underset{\sim}{k} \mid N' \rangle \tag{6.20}$$

Let us now define a form-factor $u(k)$ which goes to one as $k \to 0$, namely

$$u(k) = 3j_1(kR)/kR \tag{6.21}$$

and recognize [from Eqs. (3.36) and (3.38)] that

$$g_A{}^{\text{bag}} = \frac{5}{9} \frac{\omega}{(\omega - 1)} \tag{6.22}$$

Putting all of this together we find a very natural expression for the operator at the $NN\pi$ vertex

$$v_{\underset{\sim}{k}i}^{NN} = i(2w_{\underset{\sim}{k}})^{-1/2}(g_A{}^{\text{bag}}/2f)u(k)\tau_i \underset{\sim}{\sigma} \cdot \underset{\sim}{k} \tag{6.23}$$

This should be compared with the usual static interaction (Wic 55, HT 62, Che 54, CL 55)

$$v_{i\underset{\sim}{k}} = i(4\pi)^{1/2}(2w_k)^{-1/2}(f_{NN\pi}^{(0)}/m_\pi)v(k)\tau_i \underset{\sim}{\sigma} \cdot \underset{\sim}{k} \tag{6.24}$$

where $f_{NN\pi}^{(0)}$ is the bare pseudovector $NN\pi$ coupling constant whose renormalized value is $f_{NN\pi}^2 = 0.081$—if the phenomenological cutoff function, $v(k)$, is defined to be one at $k = im_\pi$.

If for the present we ignore questions of renormalization and so forth, it is clear that the CBM makes a remarkably accurate prediction for $f_{NN\pi}$. Using $g_A = 1.09$ gives a value of 0.23 in comparison with the observed value of 0.28. However, including the c.m. correction discussed in Section 3.4.1 (about 20% increase in g_A), we find that theory and experiment agree within a few percent! We shall see in Section 6.2.1 that renormalization will not significantly alter this success.

In addition to predicting the $NN\pi$ coupling constant, we see that the CBM provides a very beautiful explanation for what was previously an ad hoc high-momentum cutoff. The form-factor $u(k)$, which is plotted in Fig. 6.1, simply reflects the fact that the violation of chiral symmetry, and therefore pion coupling to the bag, is associated with its surface. Since the

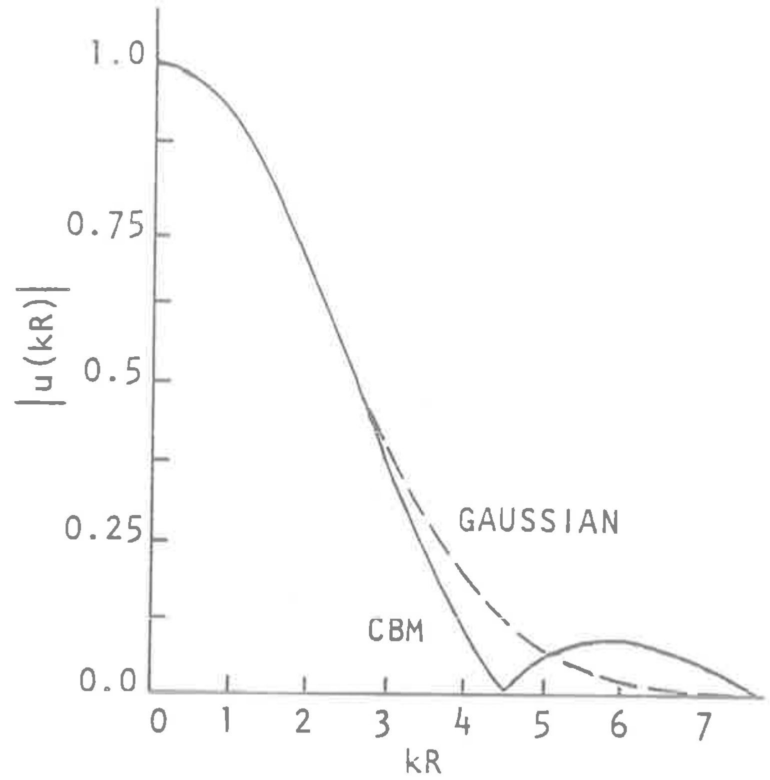

Fig. 6.1. The form-factor in the CBM compared with a best-fit Gaussian (from TT 82b).

bag is far from being pointlike there is a natural cutoff in the theory with a range related to the radius of the source, R. Far from being specific to the CBM we expect such a cutoff to be a general feature of any model which treats the quark structure of the hadrons explicitly.

6.1.2. The General $B'B\pi$ Vertex

Let us return to the general pion absorption vertex [Eq. (6.17)]. If the hadrons α and β have the same radii it is well defined. But, as we have already remarked, this will not usually be the case because of the nonlinear boundary condition. Nevertheless, the radii of the members of the lowest baryon octet and decuplet do not vary by more than about 10% from the mean value. Thus, in computing ratios of coupling constants we have assumed that these radii are all equal. [A more satisfying procedure would be to use the pseudovector volume coupling described in Section 5.4.3 (Tho 81b).]

A very basic example of an interaction, which is extremely important in medium-energy physics, is the $\Delta N\pi$ vertex. In the CBM the pion induces this transition by flipping the spin and isospin of a quark at the bag surface

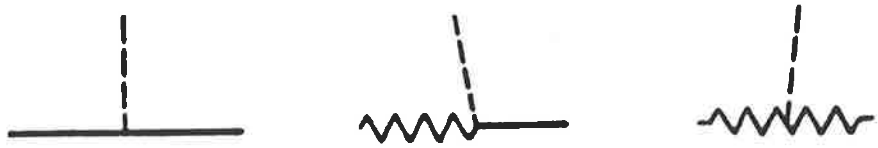

Fig. 6.2. Pion-baryon couplings which appear naturally in the CBM Hamiltonian.

$(I = \frac{1}{2},\ J = \frac{1}{2} \to I = \frac{3}{2},\ J = \frac{3}{2})$. Figure 6.2 illustrates some of these fundamental vertices. The form-factor at all such vertices will be the same function $u(k)$ derived above. In the general case the vertex function associated with the $B'B\pi$ process is [from Eq. (6.17)]

$$v_{\underset{\sim}{k}i}^{B'B} = \frac{i}{2f}(2w_k)^{-1/2}\int d^3x\, e^{i\underset{\sim}{k}\cdot\underset{\sim}{x}}\delta(x - R)\langle B' \mid \bar{q}(\underset{\sim}{x})\tau_i\gamma_5 q(\underset{\sim}{x}) \mid B\rangle \tag{6.25}$$

which can always be summarized as

$$v_{\underset{\sim}{k}i}^{B'B} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}(f_{B'B\pi}^{(0)}/m_\pi)u(k)\underset{\sim}{S}^{B'B}\cdot\underset{\sim}{k}T_i^{B'B} \tag{6.26}$$

In general $\underset{\sim}{S}$ and $\underset{\sim}{T}$ are transition spin and isospin operators defined by

$$\begin{aligned}\underset{\sim}{S} &= \sum_{m=-1}^{+1} S_m\hat{s}_m^{*}\\ \underset{\sim}{T} &= \sum_{m=-1}^{+1} T_m\hat{t}_m^{*}\end{aligned} \tag{6.27}$$

with $\hat{s}_m$ and $\hat{t}_m$ unit vectors in a spherical basis (Edm 60)

$$\varepsilon_{\pm 1} = \mp(\hat{x} \pm i\hat{y})/\sqrt{2}, \qquad \varepsilon_0 = \varepsilon_z \tag{6.28}$$

The transition spins are given in terms of their reduced matrix elements, for example

$$\langle S_{B'}s_{B'} \mid S_m \mid S_B s_B\rangle = C_{S_B 1 S_{B'}}^{s_B m s_{B'}} \tag{6.29}$$

and similarly for T_m. (For a more symmetric definition, which is not so widely used, see Dod+ 81.)

The coupling constants appropriate to transitions between all members of the nucleon octet have been summarized in the paper of Théberge and Thomas (TT 82b)—see also Thé 82. In the specific case that is of most interest to us after the nucleon, namely the Δ, the appropriate vertex functions are

$$v_{\underline{k}i}^{\Delta N} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}\left(\frac{f_{\Delta N\pi}^{(0)}}{m_\pi}\right)u(k)\underline{S}\cdot\underline{k}T_i \tag{6.30}$$

$$v_{\underline{k}i}^{\Delta\Delta} = i\left(\frac{4\pi}{2w_k}\right)^{1/2}\left(\frac{f_{\Delta\Delta\pi}^{(0)}}{m_\pi}\right)u(k)\underline{\Sigma}\cdot\underline{k}\mathscr{T}_i \tag{6.31}$$

where $\underline{S}$ and $\underline{T}$ are the transition spins and isospins of Brown and Weise ($S_B = \frac{1}{2}$, $S_{B'} = \frac{3}{2}$) in Eq. (6.29), $\underline{\Sigma}$ and $\mathscr{T}$ are the usual $-\frac{3}{2}$ spin and isospin operators, and the bare coupling constants are in the SU(6) ratios

$$f_{NN\pi}^{(0)} : f_{\Delta N\pi}^{(0)} : f_{\Delta\Delta\pi}^{(0)} = 1 : \left(\frac{72}{25}\right)^{1/2} : \frac{4}{5} \tag{6.32}$$

6.2. The Nucleon

We have seen that the practical effect of imposing chiral symmetry on the bag model is to dictate the pion coupling term in the Hamiltonian. Thus, the physical hadrons will be dressed by a pion cloud. As we discuss in the next section, the Δ becomes unstable once the interaction with the pion field is turned on and is no longer strictly an eigenstate of the Hamiltonian. The nucleon must, of course, remain as a discrete eigenstate. Denoting the dressed nucleon as $|\tilde{N}\rangle$ (and the bare three-quark nucleon as $|N\rangle$) we have

$$H|\tilde{N}\rangle = m_N|\tilde{N}\rangle \tag{6.33}$$

where from the discussion in Section 6.1

$$H = H_{\text{MIT}} + H_\pi + H_{\text{int}} \tag{6.34}$$

[see Eqs. (6.10), (6.14), and (6.16)].

In recognition of the central importance of the nucleon in nuclear physics we shall discuss its properties in great detail. We shall see that unlike older static meson theories (HT 62), the convergence properties of the CBM are excellent. Whereas in the Chew–Low model the ratio of bare to renormalized coupling constants squared was about three, in the CBM this ratio is within 20% of unity (Thé+ 82)! Moreover, the average number of pions in the "cloud" has been rigorously proven to be small. The average number of pions of any charge or momentum ($\langle n\rangle$) is less than or equal to a parameter Λ which is of order 0.9 for a bag radius bigger than 0.8 fm (Section 6.2.1). A low-order perturbative calculation actually yields $\langle n\rangle \simeq 0.5$. Thus, the pion "cloud" about the nucleon is rather sparse!

Given these excellent convergence properties the calculation of electromagnetic properties of dressed nucleons (and other members of the nucleon octet) is straightforward. One is justified in making a perturbative expansion of the state $|\tilde{N}\rangle$ as¶

$$\tilde{N}\rangle \cong Z^{1/2} \,|\, N\rangle + c \,|\, N\pi\rangle + c' \,|\, \Delta\pi\rangle \tag{6.35}$$

Perhaps the most significant observation concerning nucleon electromagnetic structure in this model is the charge form-factor of the neutron, $G_{\mathrm{E}n}$. It is discussed at length in Section 6.2.2, where we stress the significance of a good experimental determination. In the CBM it is inescapable that the measurement of the zero in the neutron charge distribution measures the bag size—modulo surface thickness corrections.

Finally, in Section 6.2.5 we note that the CBM has obvious implications for calculations of nucleon decay—as suggested by grand unification. In particular, considerations of chiral symmetry suggest a rather strong enhancement of the $p \rightarrow e^+\pi^0$ decay mode.

6.2.1. Convergence Properties of the CBM

In this section we briefly indicate how the bounds on the pion content of the nucleon were obtained. Then we look at the pionic self-energy contribution for the nucleon. Finally we discuss the renormalization of the bare $NN\pi$ coupling constant and show that it is small for two reasons: first, because of the rather strong cutoff provided by the vertex function $u(k)$, and second because of the explicit appearance of the Δ.

Following the discussion of Dodd, Thomas, and Alvarez-Estrada (Dod+ 81) we write the most general solution of Eq. (6.33) as

$$\begin{aligned} |\,\tilde{N}st\rangle = Z^{1/2} \,|\, Nst\rangle + \sum_{r=1}^{\infty} \sum_{\alpha} \sum_{k_1 \cdots k_n} c_n(\alpha; k_1 \cdots k_n; \tilde{N}st) \\ \times (n!)^{-1/2} a_{k_1}^{+} \cdots a_{k_n}^{+} \,|\, \alpha\rangle \end{aligned} \tag{6.36}$$

where $|\,\alpha\rangle$ represents a colorless, three-quark bag state, s and t are spin and isospin labels for the nucleon, and the $\{c_n\}$ are expansion coefficients. For notational convenience we have also followed the common practice (Wic 55) of replacing the sum over pion isospin and integral over momenta

¶ See also the recent discussion of Bolsterli (Bol 81, Bol 82) who describes the use of "coherent meson pair states" to simplify the calculations when first-order perturbation theory is not adequate.

by a formal sum:

$$\sum_{k} \equiv \sum_{\substack{\text{isospin}\\ \text{labels}}} \int \frac{d\underline{k}}{(2\pi)^3} \tag{6.37}$$

Clearly, the coefficients c_n are given by

$$c_n = (n!)^{-1/2} \langle \alpha \mid a_{k_n} a_{k_{n-1}} \cdots a_1 \mid \tilde{N}st \rangle \tag{6.38}$$

and we can see that it may be useful to define a state $\mid \phi_n \rangle$ by removing n pions with specific isospin and momentum from a physical nucleon

$$\mid \phi_n \rangle = (n!)^{-1/2} a_{k_1} \cdots a_{k_n} \; \tilde{N}st \rangle \tag{6.39}$$

so that

$$c_n = \langle \alpha \mid \phi_n \rangle \tag{6.40}$$

Since the physical nucleon state must be normalized we find

$$\langle \tilde{N}st \mid \tilde{N}st \rangle = Z + \sum_{n=1}^{\infty} P_n = 1 \tag{6.41}$$

where P_n, the probability of finding n pions in the physical nucleon, is

$$P_n = \sum_{\alpha} \sum_{k_1 \cdots k_n} \mid \langle \alpha \mid \phi_n \rangle \mid^2 \tag{6.42}$$

Now, from the completeness of the states $\mid \alpha \rangle$ in the single baryon subspace, Eq. (6.42) implies

$$P_n \leq \sum_{k_1 \cdots k_n} \langle \phi_n \mid \phi_n \rangle = \sum_{k_1 \cdots k_n} \| \phi_n \|^2 \tag{6.43}$$

One can now use the defining equation for $\mid \tilde{N} \rangle$ [Eq. (6.33)], and the commutation relations of the pion creation and destruction operators to manipulate the expression for ϕ_n. For example, in the case $n = 1$, using Eq. (6.33) and the relation

$$[H_\pi, a_k] = -w_k a_k \tag{6.44}$$

we find

$$\phi_1 = a_{k_1} \mid \tilde{N} \rangle = (\tilde{m}_N - w_{k_1} - H)^{-1} [a_{k_1}, H_{\text{int}}] \mid \tilde{N} \rangle \tag{6.45}$$

However, the commutator in Eq. (6.45) is readily found from Eq. (6.16)

$$C_1 \equiv [a_{k_1}, H_{\text{int}}] = \sum_{\alpha\beta} (v_{k_1}^{\beta\alpha})^+ \beta^+ \alpha \tag{6.46}$$

In addition, the spectrum of the full Hamiltonian H begins at $\tilde{m}_N$, so that $(H - \tilde{m}_N + w_{k_1})$ is greater than or equal to w_{k_1}. Thus, we have a bound on P_1:

$$P_1 \leq \sum_{k_1} \| \phi_1 \|^2 \leq \sum_{k_1} \frac{\| C_1 \|^2}{w_{k_1}^2} \tag{6.47}$$

and in general one finds (Dod+ 81)

$$P_n < (n!)^{-1}\Lambda^n \tag{6.48}$$

$$\Lambda = \sum_{k_1} \frac{\| C_1 \|^2}{w_{k_1}^2} \tag{6.49}$$

It is also rather easy to obtain a bound on the average number of pions. Consider the normalized expectation value of the number operator

$$\langle n \rangle = \| \, | \tilde{N} \rangle \, \|^{-2} \langle \tilde{N} \, | \sum_k a_k{}^+ a_k \, | \tilde{N} \rangle \tag{6.50}$$

$$= \sum_{k_1} \| \phi_1 \|^2 \leq \Lambda \tag{6.51}$$

All that remains is to evaluate the norm of the commutator, which is simply the maximum value of the vector $C_{k_1} \, | \, \psi \rangle$, with $| \, \psi \rangle$ any normalized linear combination of baryon number-one bags. As shown by Dodd *et al.* (Dod+ 81), if we include only N and Δ states [which we expect to dominate because of their closeness in mass and radius (see Section 6.1.2)] Λ has the form

$$\Lambda = \frac{57}{25} \, 4\pi \left(\frac{f^{(0)}_{NN\pi}}{m_\pi} \right)^2 \frac{3}{(2\pi)^2} \int_0^\infty \frac{k^4 u^2(k)}{w_k{}^3} \, dk \tag{6.52}$$

where $f^{(0)}_{NN\pi}$ is the bare coupling constant, and the CBM form-factor $u(k)$ was given in Eq. (6.21). In Table 6.1 we give the value of Λ and the corresponding bounds on P_n for a bag radius of 0.82, and $f^{(0)2}_{NN\pi} = 0.078$, as found by Thomas *et al.* (Tho+ 81a) from pion–nucleon scattering. For comparison we show the results for $R = 1$ fm (the MIT bag radius), and also a bound for the old static Chew–Wick meson theory. Clearly, the convergence properties of the model are remarkable. Indeed, this bound is probably still a little loose, for the calculated values of P_1, P_2, and $\langle n \rangle$ in the second column of Table 6.1 are 0.35, ≤ 0.05, and ~ 0.5, respectively.

The coupling of the pion field to the bag will of course shift its energy, as we have already discussed in Section 5.3.3. In the present quantized description of the problem, the lowest-order self-energy corrections are

TABLE 6.1

Bounds on the Pion Content of the Dressed Nucleon[a]

	CBM[b]	CBM[c]	Chew–Wick[d]
$f_{NN\pi}^{(0)2}$	0.078	0.096	0.22
R	0.82	1.0	0.28
Λ	0.9	0.68	2.16
$P_1 \leq$	0.9	0.68	2.16
$P_2 \leq$	0.40	0.23	2.33
$P_3 \leq$	0.12	0.05	1.67
$P_4 \leq$	0.03	0.009	0.90
$\langle n \rangle \leq$	0.90	0.68	2.16

[a] Data from Dod+ 81.
[b] Using parameters of Tho+ 81a.
[c] Using MIT radius and value of $f_{NN\pi}^{(0)}$ necessary to reproduce the observed renormalized coupling constant $f_{NN\pi}^2 = 0.081$ (from Thé+ 82).
[d] Bare coupling constant and sharp cutoff (HT 62).

the single-loop contributions shown in Fig. 6.3, for N and Δ. For the nucleon this mass shift is

$$\delta m_N^{(2)} = -\sum_k \left(\frac{v_k^{NN} v_k^{NN*}}{w_k} + \frac{v_k^{N\Delta} v_k^{\Delta N*}}{m_\Delta - m_N + w_k} \right) \tag{6.53}$$

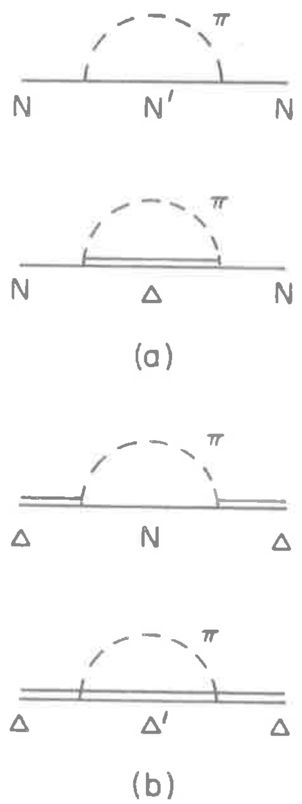

Fig. 6.3. Lowest-order nucleon and delta self-energy corrections.

This behaves roughly as $R^{-3.5}$ (Thé 82), and therefore grows rapidly as the bag radius decreases. With $R = 1$ fm the pionic contribution to the self-energy of the nucleon is about 200 MeV, which is comparable to the one-gluon exchange, volume, and c.m. corrections. As we mentioned, a number of groups have shown that quite respectable fits can be obtained for the masses of the low-lying baryons when this correction is included (e.g., Myh+ 81). We shall discuss the mass splitting of the nucleon and delta further in Section 6.3.

To conclude this section we consider the renormalization of the $NN\pi$ coupling constant. In a theory without antinucleons (and therefore with no renormalization of the pion propagator) the renormalized coupling constant is given by

$$f_{NN\pi}^{(r)} = Zf^{(0)}/Z_1 \tag{6.54}$$

The factor Z (usually Z_2) measures [see Eq. (6.41)] the probability that the physical nucleon contains a bare nucleon; it therefore reduces $f^{(r)}$ from the bare value. The dressing of the vertex, which tends to increase the coupling strength, is described by Z_1. Figure 6.4 shows the first-order dressing of the

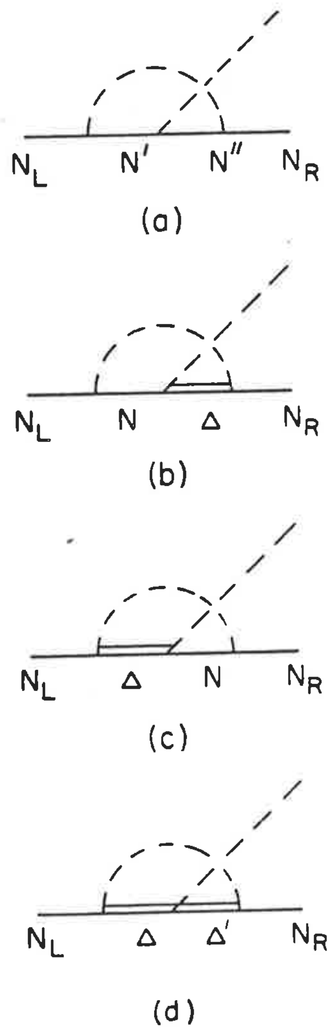

Fig. 6.4. Lowest-order contributions to the vertex renormalization of the $NN\pi$ coupling constant in the CBM.

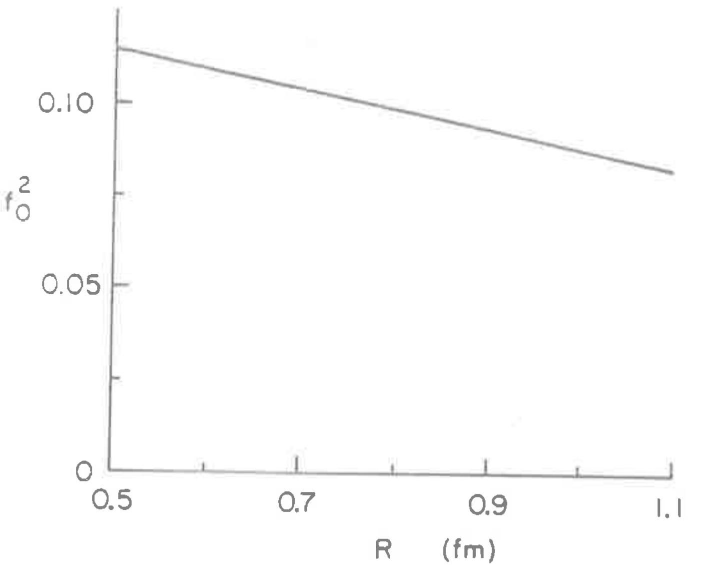

Fig. 6.5. Radius dependence of the bare $NN\pi$ coupling constant necessary to reproduce the observed, renormalized coupling constant $f^2 = 0.081$.

bare $NN\pi$ vertex. We see that the Δ bag enters very naturally in this model, unlike the earlier static theories where only Fig. 6.4(a) would appear (HT 62).

Complete expressions for Z_1 can be found in the article of Théberge *et al.* (Thé+ 82). In Fig. 6.5 we show the corresponding bare coupling constant squared necessary to reproduce the observed renormalized $NN\pi$ coupling constant squared (0.081) as a function of bag radius. It is remarkable that for any radius greater than about 0.8 fm, $f^{(0)}$ is within 10% of $f^{(r)}$! This should be compared with the old static meson theories (HT 62) where, as shown in the fourth column of Table 6.1, $f^{(0)2} : f^{(r)2}$ was about 3 : 1. There are two reasons for this dramatic improvement. First, the nucleon is now a rather large object, and the form-factor $u(k)$ cuts off the integrals describing Z. [Iterative solution of Eq. (6.33) implies

$$|\,N\rangle \simeq Z^{1/2}\,|\,N\rangle - Z^{1/2}\sum_k \left(\frac{v_k^{NN*}}{w_k}\,\Big|N, k\rangle + \frac{v_k^{\Delta N*}}{m_\Delta - m_N + w_k}\Big|\,\Delta, k\rangle\right) \tag{6.55}$$

so that

$$Z^{-1} \simeq 1 + \sum_k \left[\frac{v_k^{NN} v_k^{NN*}}{w_k^2} + \frac{v_k^{N\Delta} v_k^{\Delta N*}}{(m_\Delta + w_k - m_N)^2}\right] \tag{6.56}$$

and we see that the summation term is just the derivative with respect to energy of the self-energy term $\delta m^{(2)}$—Eq. (6.53)—as it must be in general.] Thus, Z is typically greater than about $\frac{2}{3}$ for $R \geq 0.8$ fm, compared with $\frac{1}{3}$ for the Chew–Wick case.

The second reason why $f^{(r)}$ is so close to $f^{(0)}$ is the occurrence of the Δ in the vertex renormalization (Fig. 6.4). To see this consider first Fig.

6.4(a) which goes like

$$\lambda_{NN} \simeq \sum_k \underset{\sim}{\sigma} \cdot \hat{k}\tau_k \underset{\sim}{\sigma} \cdot \underset{\sim}{q}\tau_q \underset{\sim}{\sigma} \cdot \hat{k}\tau_k = \frac{1}{9} \underset{\sim}{\sigma} \cdot \underset{\sim}{q}\tau_q \Big[\sum_k (\underset{\sim}{\sigma} \cdot \hat{k})^2 \tau_k{}^2\Big] \tag{6.57}$$

where

$$Z_1{}^{-1} = 1 + \lambda_{NN} + \lambda_{N\Delta} + \lambda_{\Delta N} + \lambda_{\Delta\Delta} \tag{6.58}$$

and we have used the commutation relations of $\underset{\sim}{\sigma}$ and $\underset{\sim}{\tau}$. The factor of $\frac{1}{9}$ essentially kills any compensation for the small value of Z in the Chew–Wick theory. This does not happen for those terms involving an explicit Δ. Indeed, if the N and Δ were degenerate, the ratio of the four terms in Fig. 6.4 would be, respectively (Thé+ 82)

$$\begin{aligned} a : b : c : d &\equiv \lambda_{NN} : \lambda_{N\Delta} : \lambda_{\Delta N} : \lambda_{\Delta\Delta} \\ &= 1 : 16\left(\frac{f^{\Delta N}}{f^{NN}}\right)^2 : 16\left(\frac{f^{\Delta N}}{f^{NN}}\right)^2 : 20\left(\frac{f^{\Delta N}}{f^{NN}}\right)^2 \end{aligned} \tag{6.59}$$

To summarize, the CBM with bag radii greater than about 0.8 fm is remarkably convergent. This convergence arises because of the rapid cutoff of high-momentum components and the explicit treatment of the Δ. In fact, we shall see in Section 6.3 that these factors are related; it is only because of the presence of the explicit Δ that one can understand pion–nucleon scattering with a strong high-momentum cutoff. In this light the pessimism of Henley and Thirring (HT 62, p. 179) should rather be regarded as a clue for future development: "For a long time it has been one of the main goals of meson theory to analyse the physical nucleon in terms of the bare nucleon and its meson cloud. This led to a dead end road. . . . The reason is that the . . . resonant state of the nucleon is not important for the ground state."

In case the point has not been made clear let us repeat it briefly. We have been led to the remarkable conclusion that if QCD results in large composite baryons with a structure like the MIT idealization, the usual world of so-called "strong" interactions is amenable to solution by low-order perturbation theory!

6.2.2. The Neutron Charge Distribution

Let us briefly recall the discussion of hadronic charge distributions in the MIT bag model given in Section 3.1. We noted that since the neutron bag has three quarks in lowest order, whose charges sum to zero, in identical spatial orbits it has no charge distribution. There are a number of higher-order effects which tend to mix other configurations into the ground state (CH 81, MV 81) but none of these give even the right order of magnitude for $\langle r^2\rangle_{\text{ch}}^n$ in the bag model.

On the other hand, if we truncate the perturbation expansion of the physical neutron wave function in the CBM at one pion we find

$$|\tilde{n}\rangle = Z^{1/2}\,|n\rangle + c_{N\pi}\left[\left(\frac{2}{3}\right)^{1/2}|p\pi^-\rangle - \left(\frac{1}{3}\right)^{1/2}|n\pi^0\rangle\right] \tag{6.60}$$

where $|c_{N\pi}|^2$ is the probability for finding the nucleon to consist of a nucleon bag and a pion [of order 20% depending on R (Tho+ 81a, Thé+ 81)]. As indicated in Eq. (6.35) there is also a $|\Delta\pi\rangle$ component which is included in all calculations. However, it is much less important for the charge distribution because the $\Delta^-\pi^+$ piece tends to cancel against $\Delta^+\pi^-$, and the 300-MeV excitation energy of the Δ also makes the range of the pion field much smaller. Equation (6.60) shows quite explicitly that the charge distribution of the neutron in the CBM is a *first-order effect* of the pion coupling, arising directly from the $|p\pi^-\rangle$ component.

This was first observed by Théberge *et al.* (Thé+ 80, Mil+ 81). Earlier calculations in classical models missed this because time derivatives vanish in the classical limit, and the pion charge current is

$$\begin{aligned} j_\pi{}^0(x) &= -ie[\phi(x)\partial_0\phi^*(x) - \phi^*(x)\partial_0\phi(x)] \\ \phi(x) &= [\phi_1(x) - i\phi_2(x)]/\sqrt{2} \end{aligned} \tag{6.61}$$

Thus, one really needs an explicit treatment of the quantum fluctuations of the pion field (as in the CBM) in order to see the effect. In terms of the creation and annihilation operators for pions of specific momentum and isospin, Eq. (6.61) becomes (Tho+ 81a)

$$\begin{aligned} j_\pi{}^0(\underline{x}) = \frac{-ie}{2}\sum_{i,j=1}^{2}\frac{\varepsilon_{ij3}}{(2\pi)^3}\int d\underline{k}\,d\underline{k}'\left(\frac{w_{k'}}{w_k}\right)^{1/2} e^{i(\underline{k}-\underline{k}')\cdot\underline{x}} \\ \times(-a_{i,-\underline{k}'} + a_{i,\underline{k}'}^{+})(a_{j,\underline{k}} + a_{j,-\underline{k}}^{+}) \end{aligned} \tag{6.62}$$

The calculation of the pion contribution to the charge distribution then amounts to evaluating the expectation value of the operator in Eq. (6.62) in the state of Eq. (6.55) [i.e., essentially Eq. (6.60)]. The quark contribution was already explained in Section 3.1.

Since the charge of the proton bag in Eq. (6.60) is confined inside the bag volume (i.e., radii less than R) and the pion field has its source at the

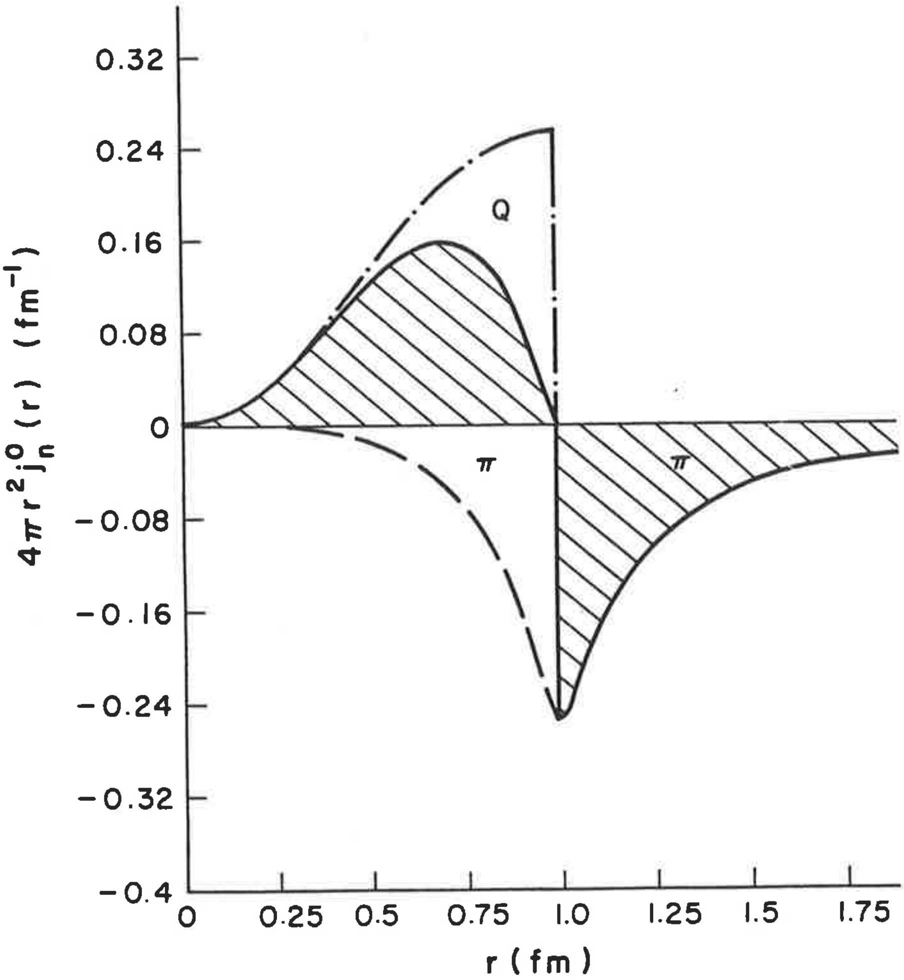

Fig. 6.6. The neutron charge distribution $4\pi r^2 j_n{}^0(r)$ versus the radial distance r (shaded area). Also shown are the quark (Q) and the pion (π) charge distribution inside the neutron. The neutron bag radius is set at 1.0 fm (Thé 82).

bag surface and extends outside, the model obviously predicts a positive core and a negative tail. The details are illustrated in Fig. 6.6. It is clearly an inescapable conclusion of the CBM that the zero in the neutron charge distribution necessarily occurs at the bag radius. An accurate experimental determination of G_{En} would thus provide us with a direct measure of the size of the confinement volume! [Note that there is certainly no physical significance to the discontinuity of $\varrho_{\text{ch}}^n(r)$ at $r = R$; it is a consequence of the oversimplification of the description of the bag surface as a rigid sphere. It is unlikely that any more realistic treatment would do more than smooth out the charge density in the surface region without altering our conclusion.] The r.m.s. radius of the neutron is not strongly dependent on R, varying from -0.391 fm at 0.8 fm to -0.327 at 1.1 fm (Thé+ 82), in excellent agreement with the experimental value of -0.342 fm obtained by dropping thermal neutrons on an electron target (Eri 78). Similar results have since been obtained by DeTar (DeT 81) and Myhrer (Myh 82).

Of course the idea of associating the negative tail of the neutron charge distribution with the process $n \to p\pi^-$ is very old—dating back to the late fifties and static meson theory (HT 62). However, that approach had two very important problems. First, the properties of the core of the nucleon were unknown. Second, the interpretation of G_{En} was complicated by the presence of the Darwin–Foldy term, whereby a Dirac particle with an anomalous magnetic moment appears to have a charge distribution—because of *Zitterbewegung*. Indeed, the observed neutron magnetic moment is sufficient to explain all of $\langle r^2\rangle_{\mathrm{ch}}^n$ (Fol 58, Eri 78).

In the quark model there is no Darwin–Foldy term. The photon interacts with three confined quarks and the pion. Thus, there is no ambiguity in the interpretation of G_{En} in the CBM and the agreement with the data both for $\langle r^2\rangle_{\mathrm{ch}}^n$ and G_{En} is significant. In conclusion, let us stress once more the importance of a better measurement of G_{En} in determining the size of the confinement region.

6.2.3. Further Nucleon Electromagnetic Properties

It is, of course, of great interest to calculate the other nucleon electromagnetic properties such as the proton charge radius ($\langle r^2\rangle_{\mathrm{ch}}^p$) and proton and neutron magnetic moments (μ_p and μ_n), even though the pionic contribution is not the leading term there. The calculation of the proton charge radius proceeds exactly as we described above for the neutron, except that the bare bag makes a major contribution. Théberge *et al.* found a proton r.m.s. charge radius between 0.73 and 0.91 fm for R between 0.8 and 1.1 fm (Thé+ 82). However, the c.m. correction to the bag contribution is somewhat controversial as we described in Section 3.4.1. Without any c.m. correction the results of Théberge *et al.* lay between 0.71 and 0.87 fm. This is still in rather good agreement with the experimental value of 0.836 fm (Nag+ 79). Finally, we note that very similar results have been obtained by DeTar (DeT 81) and Myhrer (Myh 82).

The pionic contribution to the magnetic moments involves the spatial component of the pion current

$$\mathbf{j}_\pi(\underset{\sim}{x}) = ie[\phi(\underset{\sim}{x})\vec{\nabla}\phi^*(\underset{\sim}{x}) - \phi^*(\underset{\sim}{x})\vec{\nabla}\phi(\underset{\sim}{x})] \tag{6.63}$$

which eventually can be written as (Sal 57, HT 62)

$$\mathbf{j}_\pi(\underset{\sim}{x}) = \frac{-ie}{2}\sum_{i,j=1}^{2}\frac{\varepsilon_{ij3}}{(2\pi)^3}\int\frac{d^3k\,d^3k'}{(w_k w_{k'})^{1/2}}\,\vec{\mathbf{k}}(a^{+}_{i\vec{\mathbf{k}}'} + a_{i-\underset{\sim}{k}'})$$
$$\times(a_{j,\vec{\mathbf{k}}} + a^{\dagger}_{-\underset{\sim}{k}})e^{i(\underset{\sim}{k}-\underset{\sim}{k}')\cdot\underset{\sim}{x}} \tag{6.64}$$

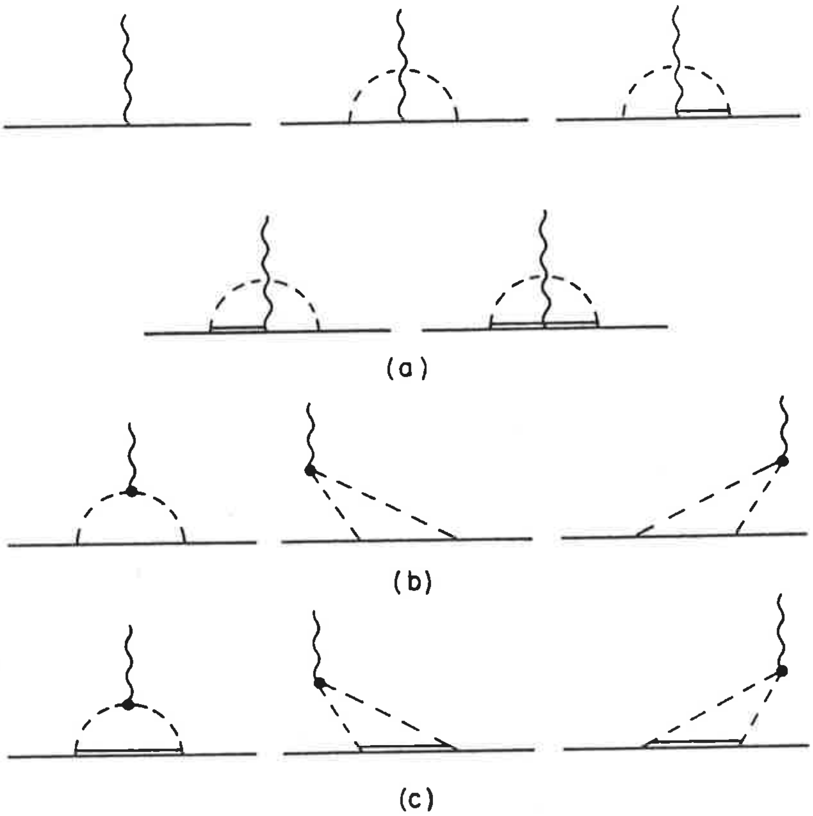

Fig. 6.7. Contribution to the magnetic moment of the nucleon from (a) the quark current, and [(b) and (c)] the pion current with an intermediate nucleon or delta.

Once again we need to evaluate this operator between nucleon wave functions of the form given in Eq. (6.35)—that is, including both nucleon and delta intermediate states.

The bag contribution itself (while the pion is "in the air") is also rather interesting. It is possible for the quark magnetic moment operator (unlike the charge operator) to induce an N–Δ transition. Thus, one must compute all of the processes shown in Fig. 6.7. Unlike the direct interaction with the pion cloud, the core interactions will have both an isoscalar and an isovector piece. Thus, it is not true, as one can find in the literature, that the pionic contribution is purely isovector.

Once again the comparison of calculational results with experiment is somewhat clouded by the uncertainty over c.m. corrections. Nevertheless, this uncertainty is smaller than for the charge radii. Including the Donoghue–Johnson correction (DJ 80) μ_p and μ_n range between 2.43 and 2.78 and -1.97 and -2.07 nuclear magnetons, respectively, for R at 0.8 and 1.1 fm (Thé+ 82). With no c.m. corrections the corresponding values are 2.20–2.43 and $-1.80\,\mu_N$ and $-1.82\,\mu_N$. Recalling that the MIT results with and without c.m. corrections were 2.24 μ_N and $-1.49\,\mu_N$ and 1.9 μ_N and $-1.26\,\mu_N$, respectively (DeG+ 75), we see that *the inclusion of pionic*

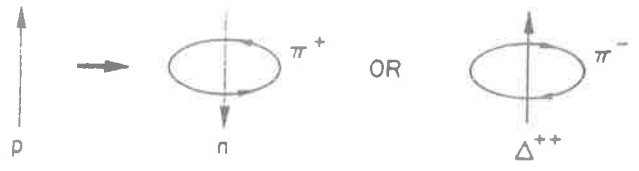

Fig. 6.8. Illustration of the pionic contribution to the proton magnetic moment with an intermediate nucleon or delta.

corrections has made a tremendous quantitative improvement in the agreement with data. In particular, the residual discrepancy of 5–10% is well within the uncertainties of the calculation—e.g., from sea quarks, configuration mixing, and so on.

In conclusion, we make a couple of qualitative remarks about the role of the intermediate Δ in these calculations. For the charge distribution the Δ_π contribution tends to reduce that associated with $N\pi$. For example, the proton goes predominantly to $\pi^+ n$ and $\pi^-\Delta^{++}$. However, for magnetic moments the spin of the Δ is very important. The π^+ cloud around the n-core obviously gives a positive contribution to the magnetic moment. But when the proton with spin-up goes to Δ^{++} the Δ tends to have spin $+\frac{3}{2}$ so that the π^- orbits in the opposite direction to the π^+. Therefore, we get a positive contribution from the pion cloud (see Fig. 6.8). Again we see that the explicit presence of the Δ bag is rather important for the quantitative success of the model.

6.2.4. Weak Interactions

In view of the long development of the ideas of chiral symmetry and PCAC in Section 4.3 it should be clear that the chiral bag models necessarily produce an acceptable description of the axial current. The presence of an explicit pion field means that there is an induced pseudoscalar term in $A^\mu(\underline{x})$. Furthermore, the imposition of chiral symmetry implies that the relative strengths of the axial and induced pseudoscalar terms is consistent with the Goldberger–Treiman relation. In both these respects the chiral bag models are, by construction, superior to the original MIT bag model.

A somewhat deeper feature of the cloudy bag model (CBM) is the interpretation of PCAC implicit in it. From Section 4.3 we recall that the correct statement of PCAC is that the dependence of physical quantities on the pion mass should be smooth. Both this and the nearness of g_A to one are directly related to the remarkable convergence properties of the model. We demonstrated in Section 5.3.3 that there is no direct pionic contribution to g_A in the CBM—as opposed to those models where the pion is excluded from the interior of the static bag. In addition, the renormalization of g_A is identical to that of the $NN\pi$ coupling constant. But we showed in Section 6.2.1 that the large size of the nucleon bag, plus the presence of the Δ, means that this renormalization is 10% or less for a bag radius of 0.8 fm or larger—consistent with nucleon electromagnetic properties. In summary, the successful prediction of g_A in the MIT bag model (see Section 3.3) is preserved by the CBM.

6.2.5. Proton Decay

There has been a great deal of excitement in the last couple of years since it was realized that the beautiful ideas of grand unification (PS 73, GG 74, Bur+ 78) may actually lead to the decay of the proton at an observable rate (Wei 79, WZ 79, Lan 81). From the practical point of view of our experimental colleagues the interesting question is what are the dominant decay modes—i.e., to what should a detector be sensitive. For someone with a classical nuclear physics background the idea of $p \to e^+\pi^0$, $e^+\varrho^0$, and $e^+\omega$ being the dominant processes (KK 80, Gav+ 81, Lan 81) seems absurd! For example, I had always believed that the pointlike nucleon of nuclear textbooks must carry a large number of virtual pions with it—just as predicted by the Chew–Wick static meson theory mentioned earlier (see Table 6.1). If that were the case, even in the "unlikely" event that the small core contains three quarks which convert to $e^+\pi^0$ [Fig. 6.9(a)] all those pions in the cloud would be observed too. Thus, because of phase space the only decay mode would be e^+ with many pions.

Because of its remarkable convergence properties the CBM provides a rather beautiful resolution of this difficulty. Most of the time the proton consists of a three-quark bag for which the usual calculations apply. However, there is also a chance of about one in three that the physical nucleon consists of a pion in the air with a three-quark core. The latter, being off-shell, can decay directly to e^+ as shown in Fig. 6.9(b). The probability of finding more than one pion in the cloud is negligibly small as we showed

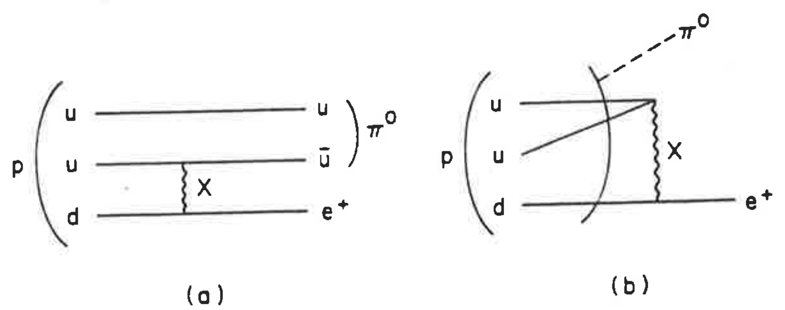

Fig. 6.9. (a) The conventional mechanism for proton decay to $e^+\pi^0$; (b) the nucleon pole term which dominates in the CBM.

above. Calculations of the process in Fig. 6.9(b) have been made on the basis of current algebra (Tom 81) and chiral SU(3)×SU(3) (Wis+ 81, Cla+ 81). However, the question of proton structure was not addressed in either of these approaches.

McKellar and Thomas recently carried out a calculation motivated by the CBM (MT 82a). The pole graph [Fig. 6.9(b)] enhances the matrix element by a model-dependent factor of 3 to 6, and hence decreases the proton lifetime by at least an order of magnitude. (Both Tomozawa and Claudson *et al.* found a model-independent enhancement like $1 + g_A$.) Thus, within the SU(5) model of grand unification, considerations of chiral symmetry seem to imply both that $e^+\pi^0$ should be the dominant mode of decay, and that for a unification mass of order 4×10^{14} GeV the proton lifetime is about 3×10^{29} years (MT 82a). The deep-mine physicists can live in hope of seeing daylight soon!

6.3. Pion–Nucleon Scattering

6.3.1. The P_{33} Resonance

We recall from Section 3 that the Δ played a role as important as the nucleon in fixing the parameters of the MIT bag model. Indeed, the color-coupling constant α_c was essentially determined from the hyperfine splitting of N and Δ. Once the constraint of chiral symmetry is imposed on the bag model, leading to the Hamiltonian given in Eq. (6.34), there is a qualitative change in the interpretation of the Δ. Whereas N, Δ, R, and so on are eigenstates of H_{MIT}, once the pionic coupling is turned on only N (actually $\tilde{N}$ in our earlier notation) remains as an eigenstate of the full H. (Of course the other members of the nucleon octet should also remain stable under strong interactions.) The Δ is sufficiently high in mass that it can decay into $N\pi$, and can therefore at best be regarded as an approximate eigenstate of the full Hamiltonian with complex eigenvalue (FP 58, GK 57). In this case it seems most appropriate to discuss directly the predictions of the CBM for πN scattering in the P_{33} channel.

When the first crude calculation of pion–nucleon scattering was made in the original Brown–Rho bag model (Mil+ 80) there was considerable concern in the medium-energy community about double counting. That is, the old Chew–Wick meson theory, which involves just an $NN\pi$ vertex function can generate a resonance in the P_{33} channel (Che 54, Wic 55). The reason is that the crossed Born graph (u-channel nucleon pole) shown in Fig. 6.10(a), produces a strongly attractive, effective potential in the (3,3) channel

$$v_C(\underline{k}', \underline{k}; w) = 4\pi P_{33}\left[-\frac{4}{3}\frac{f_{NN\pi}^2}{m_\pi^2}\frac{k'kv(k')v(k)}{(2w_{k'}2w_k)^{1/2}}\frac{1}{w_{k'}+w_k-w}\right] \quad (6.65)$$

where P_{33} is the usual projection operator onto the isospin $-\frac{3}{2}$, spin $-\frac{3}{2}$ πN channel (Sch 64), and $v(k)$ provides the high-momentum cutoff. When iterated [as in Fig. 6.10(b) and so on], this potential produces a good description of the P_{33} scattering phase shifts up to 300 MeV—with a suitable choice of cutoff—e.g., $v(k) \approx \theta(m_N - k)$. Such a model of the P_{33} resonance is still widely used in the medium-energy physics literature [typified by *Phys. Rev. C* (e.g., Ban 79, Mil 79, EJ 80)] and is often (somewhat incorrectly) referred to as the Chew–Low model.

The apparent problem with the CBM is that it naturally incorporates *both* this crossed graph and a direct coupling to the delta bag [Fig. 6.10(c)] because both $NN\pi$ and $\Delta N\pi$ couplings occur on the same footing. One might ask whether there is not some double counting, or perhaps even two Δ resonances! The answer is simply that there is no double counting and the pion–nucleon t-matrix defined by the CBM satisfies the Low equation (Low 55) as it should (Thé+ 80). Both the Chew–Wick and direct-Δ mechanisms contribute to πN scattering in the (3,3) channel (and interfere with each other) with a relative strength dictated directly by the CBM Hamiltonian, as illustrated in Fig. 6.10. One is no longer free to arbitrarily adjust the $NN\pi$ vertex function so that the Chew–Wick mechanism produces

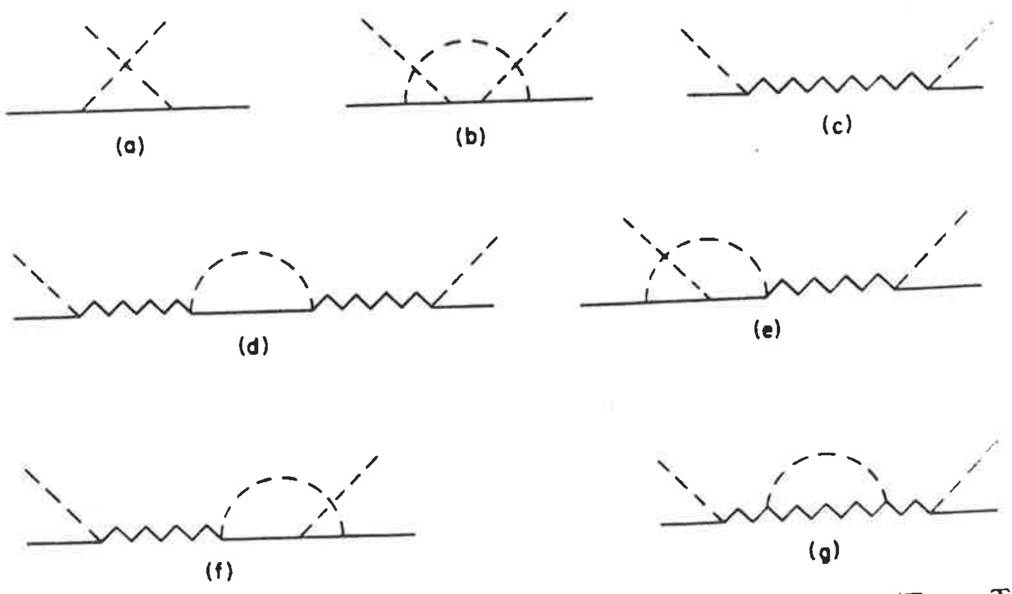

Fig. 6.10. Some low-order contributions to πN scattering in the CBM. (From Thé+ 80.)

a resonance by itself, because the same vertex function occurs at the $\Delta N\pi$ vertex (see Section 6.1.2).

To summarize, far from raising problems of double counting, the CBM provides an explicit and physically well-motivated example of an alternate solution to the (nonlinear) Low equation, as discussed by Castillejo, Dalitz, and Dyson (Cas+ 56). Moreover, it provides a precise answer to the rather confused question I asked Gerry Miller at the Houston meeting some three years ago (Mil 79, p. 575):

> While the Chew–Low model is a useful model of the P_{33} resonance, it is very dated. Since then we have discovered . . . quarks, etc. In that model there is unambiguously an elementary $\Delta \equiv (qqq)$ state. . . . Is it not possible that the truth about the πN interaction is that the elementary Δ contributes a short-range piece, while the πN rescattering . . . results in a relatively long-range piece of the interaction? On a more philosophical level, why must physics be split into two nonoverlapping camps . . . ?

The treatment of πN scattering in the CBM therefore involves solving the scattering equation

$$t = (v_C + v_\Delta) + (v_C + v_\Delta)G_0 t \tag{6.66}$$

Here v_Δ is given by Fig. 6.10(c):

$$v_\Delta(\underset{\sim}{k}', \underset{\sim}{k}; w) = 4\pi P_{33}\left[\frac{f_{\Delta N\pi}^{(0)2}}{3m_\pi^{\ 2}} \frac{k'ku(k')u(k)}{(2w_{k'}2w_k)^{1/2}} S_\Delta^{(0)}(w)\right] \tag{6.67}$$

where

$$S_\Delta^{(0)}(w) = [w - (m_\Delta^{\text{bag}} - m_N) - \Sigma_\Delta^{\text{h.o.}}(w)]^{-1} \tag{6.68}$$

and $\Sigma_\Delta^{\text{h.o.}}$ is the sum of all the irreducible pionic self-energy contributions for the Δ, which do not involve an intermediate $N\pi$ state. The Chew–Wick driving term v_C is identical to that given in Eq. (6.64), except that the CBM form-factor $u(k)$ replaces $v(k)$. Considerable numerical simplification is obtained by approximating the propagator of the crossed Born graph as

$$(w - w_k - w_{k'})^{-1} \simeq -\frac{w}{w_k w_{k'}} \tag{6.69}$$

which has been shown by Miller and Henley to be good to $\sim$15% in the usual Chew–Wick theory (MH 80). In that case we find

$$v_C(\underset{\sim}{k}', \underset{\sim}{k}; w) \simeq 4\pi P_{33}\left[-\frac{4}{3}\frac{f_{NN\pi}^{(0)2}}{m_\pi^{\ 2}} \frac{k'ku(k')u(k)}{(2w_{k'}2w_k)^{1/2}} \frac{w}{w_{k'}w_k}\right] \tag{6.70}$$

and both v_C and v_Δ are separable. Then the solution to Eq. (6.66) can be written down in closed form (Thé+ 80).

Another advantage of the analytic form for the pion–nucleon t-matrix is that one can very easily see what is involved in the renormalization process. In fact, one can explicitly show that the bare coupling constants in Eqs. (6.67) and (6.70) are replaced by their renormalized values and the bare nucleon and delta masses get dressed by pionic interactions—as illustrated (in lowest order at least) in Fig. 6.3. The one free parameter of the model is the bag radius which can be adjusted to fit the P_{33} scattering data. While the best fit is obtained with $R = 0.82$ fm (Tho+ 81a), any bag radius between 0.7 and 1.1 fm provides a fairly good description (Thé 82).

Of course, the model we have described is fairly crude. The $B'B\pi$ vertices have all been calculated for a static bag. Nucleon kinetic energies have been neglected in all propagators and so on. It would certainly be worthwhile to repeat this work using (say) the Blankenbecler–Sugar equation, with improved vertex functions. In that case one might be able to pin down the bag radius somewhat more reliably. However, the essential physics, which is the participation of a relatively large composite Δ on the same footing as the nucleon will not be altered.

From the point of view of the bag model it is very interesting to ask whether the pionic self-energy corrections affect the Δ–N mass splitting. To lowest order in the pion coupling (which should be a rather good approximation for large bag radii¶) the self-energy loops shown in Fig. 6.3 give rise to the following corrections:

$$\Sigma_N(E) = \frac{3f_{NN\pi}^2}{\pi m_\pi^2}\int_0^\infty \frac{k^4u^2(k)\,dk}{w_k(E - w_k - m_N)} + \frac{4}{3}\,\frac{f_{\Delta N\pi}^2}{\pi m_\pi^2}\int_0^\infty \frac{k^4u^2(k)\,dk}{w_k(E - w_k - m_\Delta)} \tag{6.71}$$

(which was called $\delta m_N^{(2)}$ earlier) and

$$\Sigma_\Delta(E) = \frac{f_{\Delta N\pi}^2}{3\pi m_\pi^2}\int_0^\infty \frac{k^4u^2(k)\,dk}{w_k(E - w_k - m_N)} + \frac{75}{16}\,\frac{f_{\Delta\Delta\pi}^2}{\pi m_\pi^2}\int_0^\infty \frac{k^4u^2(k)\,dk}{w_k(E - w_k - m_\Delta)} \tag{6.72}$$

¶ See, however, the recent discussion of Hoodbhoy (Hoo 82).

where the "physical" N and Δ masses are defined by

$$\begin{aligned} m_N &= m_N^{\text{bag}} + \Sigma_N(m_N) \\ m_\Delta &= m_\Delta^{\text{bag}} + \operatorname{Re} \Sigma_\Delta(m_\Delta) \end{aligned} \tag{6.73}$$

Whenever the energy denominators in Eqs. (6.71) and (6.72) can vanish the self-energy becomes complex (corresponding to the width of the Δ, for example), and the real part is given by the principal value prescription. The difference $(m_\Delta - m_N)$ was used as a fitting parameter in the CBM work (because the interference with Chew–Wick terms could shift the resonance position), but in fact the best fit value of 280 MeV is very close to the value one would naively extract from the particle data book (1231 − 940 = 291 MeV).

Recalling the CBM relationships between coupling constants from Eq. (6.32) we see that the first term in Eq. (6.71) at $E = m_N$ equals the second term in Eq. (6.72) at $E = m_\Delta$. On the other hand, the $N\pi$ contribution to the Δ self-energy and the $\Delta\pi$ effect on N can only be compared numerically because of the principal value in the former. For the parameters of Thomas *et al.* [i.e., $R = 0.82$ fm, $m_\Delta - m_N = 280$ MeV (Tho+ 81a)] $\Sigma_\Delta(m_\Delta)$ is actually 80 MeV less attractive than $\Sigma_N(m_N)$. Consequently, the QCD splitting of the N and Δ bag masses is only 200 MeV. Since the hyperfine splitting due to one-gluon exchange goes as $1/R$ [Eq. (2.83)] this means one does not need anywhere near as large a value of α_c as in the original MIT work. Indeed α_c of order 0.3 to 0.4 (rather than 0.55—as in DeG+ 75) is sufficient (Thé+ 82). This is much more consistent with the idea of treating gluon exchange in the bag in low-order perturbation theory.

Very similar conclusions regarding the N–Δ mass splitting have been reached by Lichtenberg and Wills on the basis of a nonrelativistic quark model (LW 81). They also treated the strong coupling of the ϱ-meson to two pions in a coupled channels formalism. Once again the effect of the channel coupling was to reduce the splitting between π and ϱ required from one-gluon exchange. If, as we strongly suspect, the same result were to hold in a bag model description this would also be consistent with a smaller value of α_c.

In concluding this discussion we note that there is a considerable amount of loose discussion about the delta. For example, it is often claimed that the quark model $\Delta N\pi$ coupling constant [i.e., $f_{\Delta N\pi} = (72/25)^{1/2} f_{NN\pi}$] is not sufficient to explain the width of the Δ. That is, the δ-function piece of Eq. (6.72) contributes only about 80 rather than 110 MeV to the width of the P_{33} resonance. However, it should be clear from our discussion of the CBM that this is not the only contribution to the width. For example, the intermediate pion in Fig. 6.10(b) or 6.10(e) can also be on-shell. Niskanen has given a rather nice summary of this recently (Nis 81). It is quite possible that the solution to the problem of the difference between predicted (Tho+ 81a) and extracted (Arn+ 79) values of the $\Delta\Delta\pi$ coupling constant raised recently by Duck and Umland (DU 82) may also be related to the subtlety of the structure of the P_{33} resonance. But in any case this problem deserves more work.

It may also be a source of confusion to some readers that processes such as Fig. 6.10(e), (f), etc., which appear naturally when Eq. (6.66) is iterated, are not simply incorporated into a renormalized $\Delta N\pi$ coupling constant. The answer is unitarity! That is, above the $N\pi$ threshold such terms contribute an imaginary part to the πN scattering amplitude. Any theory which seriously expects to explain the width of the Δ must include them explicitly! A similar observation must also be made about the magnetic moment of the Δ. The photon can couple to any of the intermediate pion legs in Fig. 6.10, just as we explained for the nucleon in Section 6.2.3. For the reasons we have just outlined the effective magnetic moment of an on-shell Δ will necessarily be complex. It is absolutely pointless to expect to test so-called "quark models" of the Δ magnetic moment without incorporating pionic effects [e.g., see the rather simple model of Moniz (Mon 82), which could easily be extended along CBM lines].

We might also make some brief remarks concerning the behavior of the Δ in dense nuclear matter. For example, it is commonly believed (BP 75, CL 78, BP 79) that the Δ^- should be an important component of nuclear matter at the core of a neutron star. It is very easy to see that imbedding a Δ in nuclear matter would severely inhibit the self-energy contribution involving an intermediate $N\pi$ state (Saw 72, Tho+ 80). Since this term is of order 160 MeV for $R = 0.8$ fm this can obviously be a large effect! Of course, the tendency to raise the mass of the Δ may be counteracted by the interaction with other nucleons in the medium. It is not even clear that one can simply Pauli-block the intermediate nucleon once its quark structure is being considered and the density is high. In short we shall have to develop a many-body theory of confined quarks and pions—at least! This will be discussed a little more in Section 7. For the present we merely note that the internal structure of the isobar (and the nucleon) may significantly modify our predictions for dense nuclear matter (Tho+ 80, Dre+ 82).

6.3.2. Other Partial Waves

One of the attractive features of the Chew–Low model was that it not only explained the resonant behavior of the P_{33} interaction but that it also explained (qualitatively at least) the behavior of the other P-wave πN phase shifts at low energy. It is therefore not unreasonable to ask that any theory which purports to replace Chew–Low should do as well. For the small repulsive P_{13} and P_{31} phase shifts this has been established by Israilov and Musakhanov (IM 81).

The P_{11} is rather more interesting for a number of reasons. This channel contains the nucleon pole, as a result of which the low-energy phase shifts are negative. However, at about 150 MeV the phase shift changes sign and rises rapidly through 90° at the highly inelastic Roper resonance (at 520 MeV). Within the MIT bag model we expect that the Roper (R) should be predominantly a $1s^2$, $2s$ configuration (although as mentioned in Section 2.4 the MIT bag model is not overwhelmingly successful for excited hadrons). Just like the Δ, the R is stable in the absence of pion coupling. Once the full Hamiltonian is used R will of course move into the complex plane, obtaining its width predominantly from the coupling to $N\pi$ and $\Delta\pi$. Although the Roper necessarily involves higher energies, which means that the neglect of recoil corrections (and the difference in R and N bag radii) will be more drastic than for the Δ, Rinat has shown that the CBM can provide quite a good qualitative description of the P_{11} data (Rin 81). As we have stressed several times, the development of the CBM description of this channel will be crucial in the rather ambitious attempts to develop a microscopic understanding of the prototype π–nucleus system, namely the pion–deuteron system including absorption (Bet+ 82, Tho 82).

These results combined with the excellent fits to the P_{33} phase shifts and the derivation of the Weinberg–Tomozawa relationship in s-wave mean that the overall description of low-energy πN scattering is in rather good shape.

6.4. Magnetic Moments of the Nucleon Octet

Looked at objectively there is not a great deal of data at our disposal for testing models of hadron structure. One important data set which has seen a dramatic improvement in quality recently, as a result of improved hyperon beams, is the magnetic moments of the stable hyperons (Ove 81, Lip 81). In view of the success of the CBM with the nucleon magnetic moments described above it is reasonable to ask what its predictions might be for the strange partners of the nucleon. This is even more critical in view of the findings of Brown and co-workers that the Σ^- moment was in the range $-0.54\mu_N$ to $-0.64\mu_N$ (Bro+ 80), in comparison with the experimental value of $-1.41 \pm 0{,}25\mu_N$ (Han+ 78) [see also the discussion of Franklin (Fra 80) and Lipkin (Lip 81)].

It is a rather beautiful feature of the CBM Hamiltonian that there is very little freedom in the calculation of these magnetic moments. Equation (6.17) can be used to relate all of the $B'B\pi$ coupling constants to that for $NN\pi$. The results are summarized in the paper of Théberge and Thomas (TT 82). Furthermore, once the strange quark mass is chosen (see Section 2.2.3) the photon coupling to the bag is determined (Section 3.2). The calculation involves exactly the same diagrams as that for the nucleon except that the intermediate bag states [while the pion is in the air (see Fig. 6.7)] must have the correct strangeness—e.g., for the Σ^- we can have intermediate Λ, Σ, Σ^*, (Λ,Σ), (Λ,Σ^*), and (Σ,Σ^*) baryons. [Such terms were first discussed by Pilkuhn and Eeg from a different point of view, with quite different numerical results (EP 78).]

The results of a calculation using the same bag radii and strange quark mass as the original MIT work are shown in Table 6.2 (TT 82a). Clearly, the overall agreement of the CBM with data is excellent. A more detailed study of the dependence on bag radius and strange quark mass has confirmed that this is no accident (TT 82b).

TABLE 6.2

Comparison of the Predictions of the CBM for the Magnetic Moments (in Nuclear Magnetons) of the Nucleon Octet[a]

	CBM	Experiment
p	2.60	2.79
n	−2.01	−1.91
Λ	−0.58	−0.61
Σ^-	−1.08	−1.41 ± 0.25 −0.89 ± 0.14[b]
Σ^+	2.34	2.33 ± 0.13
Ξ^-	−0.51	−0.69 ± 0.04
Ξ^0	−1.27	−1.25

[a] Data from TT 82a.
[b] Preliminary result from T. Devlins, private communication (December 1981).

In view of the theoretical uncertainties associated with configuration mixing (Isg 80), sea quarks (DG 77, MV 81), and c.m. corrections it appears unlikely that a more accurate description of the data is likely in the near future.¶ Nevertheless, it does seem that the inclusion of the lowest-order pionic corrections does result in a good overall description. Clearly, a definitive experimental result for both the Σ^- and Ξ^- would be most welcome.

6.5. Summary

Our considerations of chiral symmetry and the MIT bag model have led us to a remarkably optimistic new theory of strong interactions. There is hope that, once the nonperturbative region of QCD is understood and quarks are confined in baglike objects, the conventional strong interactions may converge in low-order perturbation theory.

To illustrate this we discussed the renormalization properties of the CBM Hamiltonian in detail. It is a remarkable fact that in every case where the CBM has been applied, it has either led to better agreement with data than the original MIT model, or in other cases provided new insight to old problems. The results that we have described strongly support our belief that the CBM is an excellent model on which to begin to build a new, unified description of nuclear medium- and high-energy physics.

7. TOWARD A NEW VIEW OF NUCLEAR PHYSICS

In the preceding sections we have attempted to put together a thorough and, as far as possible, objective review of bag models, chiral symmetry, and the applications to single-hadron properties. This task was made relatively easy by the fact that the successes described in Section 6 are the culmination of many years of theoretical effort. On the other hand, there have been only a few tentative steps made towards our ultimate goal of defining a consistent, unified picture of nuclear and particle physics. It is our aim in this section to present a blatantly optimistic view of how this search may go. If we achieve nothing more than generating an interest in

¶ One might also consider generalizing the CBM to $SU(3)_L \times SU(3)_R$ and including a kaon cloud. We chose not to do so because the large mass of the kaon means there is no longer such a clean separation between the phenomenology of the bag and the mesonic corrections (see the introduction to Section 7).

the nuclear community in tackling some old problems in a new framework, this review will have succeeded.

In view of the successes of chiral bag models it seems a natural next step to attempt to derive the properties of many-nucleon systems from the same starting point. That is, each nucleon should be treated as a relatively large quark bag with a rather thin pion cloud. In contrast with the conventional models of the N–N interaction we see little room or necessity for vector mesons. To explain this consider the early sixties picture of the nucleon anomalous moment. Basically, this was interpreted in a vector dominance model as the photon coupling to a ϱ-meson which is then absorbed by the nucleon through the interaction

$$\mathscr{L}_{\varrho NN} = g_{\varrho NN}\left(\frac{1+K_v}{2M}\right)\psi^\dagger(\underline{\sigma} \times \underline{q}) \cdot \underline{\varrho}\tau_3\psi \tag{7.1}$$

where $K_v = 3.7$ is the isovector–nucleon anomalous moment $(\mu_p - \mu_n - 1)$. In Eq. (7.1) we have shown only the nonrelativistic limit of the anomalous coupling $(\varrho_\mu \sigma^{\mu\nu} q_\nu)$. If the direct vector coupling $(\varrho_\mu \gamma^\mu)$ to both the ϱ and ω is also included one has at least a qualitative explanation of the neutron and proton charge distributions too (Hof 63).

By the mid-sixties one had an alternate explanation of μ_p/μ_n based on the static quark model (Beg+ 64). The seventies saw the refinement of the harmonic oscillator quark model calculations—still nonrelativistic, but "QCD motivated." Also in the seventies came the bag models, which predicted μ_p/μ_n correctly *without vector mesons* and furthermore (Section 2.5) explained why the nonrelativistic quark models worked. Most recently we have seen the development of chiral bag models, and particularly the CBM, which improved the overall agreement with data for the nucleon octet without altering any of the earlier successes. Once again there was no need for vector meson contributions.

In the triplet state the $\bar{q}q$ interaction associated with one-gluon exchange is strongly repulsive. Thus, unlike the pion, it is quite likely that the vector mesons are large [$R \sim 1.0$ fm in the MIT bag model (DeG+ 75)]. Their large mass implies that virtual vector mesons should have ranges of a few tenths of a fermi about the bag. Since the sharp bag surface in the MIT model is in any case a phenomenological simplification, it seems to make little physical sense to talk of virtual vector mesons [with a propagator like $(q^2 - m_\varrho^2)^{-1}$] about the bag. It would be more physically reasonable to treat such terms as virtual $\bar{q}q$ excitations in the nucleon bag—i.e., "sea quarks." Finally, we might observe that even if one agrees to include vector mesons as a working hypothesis, the ϱ (for example) could only couple through two pions, and therefore with a soft form-factor. In that case its effects would be very small (Web 80, Nis 81, AT 82).

The moral of all this is that quite different theoretical pictures can often reproduce a limited data set. One's preference for a particular model must be determined not just by convenience but also by the range of phenomena with which it is consistent. We saw in Section 2.3 that the MIT bag model embodies by construction the concept of asymptotic freedom, suggested by deep inelastic scattering, as well as confining the quarks and gluons. It is consistent with how we believe the solution of QCD should look. Supplemented with a pion field it also incorporates the $SU(2) \times SU(2)$ symmetry of QCD. A mechanism has been suggested by which the pion could develop from QCD as a Goldstone boson associated with dynamical symmetry breaking. In short, the chiral bag models are consistent with a great deal of data ranging from high-energy electron and neutrino scattering down to static properties like magnetic moments. They also match our theoretical prejudices. In the form described in Section 6, namely the CBM, it is quite straightforward to make calculations.

For all these reasons it seems to us absolutely compelling that we begin the long job of replacing the old meson exchange picture by one in which the internal structure of the nucleon is taken seriously. Naturally, for several years it will not be possible to duplicate the quality of fits achieved over more than 20 years work, by hundreds of theorists, culminating in the Paris potential (Vin 82). Nevertheless, the rewards in the long term will be great. For example, one might hope for a new and deeper understanding of nuclear matter and phenomena like pion condensation associated with high density.

7.1. The Nucleon–Nucleon Force

Attempts to understand the nucleon–nucleon force have probably occupied more man-years of effort than almost any other single scientific problem—except perhaps the creation of better weapons. Through the application of sophisticated techniques relying on analytic properties of scattering amplitudes, the Paris group has arrived at a remarkably accurate description of the *N–N* force in free space (Vin 82). The claim is often made that the *N–N* potential is known to distances of order 0.8 fm on the basis of such calculations. There are some fascinating questions connected with the analytic behavior of wave functions and scattering amplitudes in a theory with confinement (Wol 82). Eventually one would hope to put together the concepts of QCD and dispersion relations. However, for the present we simply note that there are conceptual problems to be overcome. In particular, in a collision of two bags of radius 1.0 fm it would appear self-evident that quark degrees of freedom could be significant inside 2.0 fm.

A number of attempts have already been made to derive a N–N interaction from quark models. From the introductory discussion to Section 7 it should be clear that the standard heavy boson exchange picture of the N–N interaction is no longer satisfactory. This does not mean that one will not have *effective* isovector–vector, isoscalar–vector interactions when two bags overlap (Web 80, Web 81). It simply means that (as for the anomalous magnetic moments of the nucleons) these are better treated directly in terms of quarks. Our discussion will center on work like that of DeTar (DeT 78–80) and also of Harvey (Har 81).

There is already enough excellent work on the short-range N–N force in quark models that a full review of that alone would not be out of place. Our purpose in this section is merely to outline briefly that work which we find most promising. Unfortunately this discussion cannot be considered complete.

7.1.1. The Short-Range Force in a Bag Model

The pioneering work in the application of the MIT bag model to the N–N force is that of DeTar (DeT 78–80). Although much of his work was very sophisticated, involving calculations in a deformed bag (in fact a major finding was that the deformation made little difference), one can understand his essential results on the basis of a spherical bag approximation.

Briefly then it is supposed that once two nucleon bags overlap sufficiently they coalesce to form a six-quark bag.¶ Although it is no longer correct to think of the quark clusters in such a bag as nucleons, DeTar was nevertheless able to calculate the total energy of the system as a function of the separation between the clusters. The difference between the total energy of the six-quark system and two nucleon masses was compared with conventional N–N potentials. While there is no rigorous justification for comparing this energy with conventional N–N potentials, in fact there are many similarities. In particular there is a repulsive core of about 300 MeV, which arises from the color-magnetic one-gluon exchange interaction.

As mentioned above it would divert us too much to review DeTar's

¶ The model of DeTar says nothing about the N–N force outside this coalescence radius.

work in detail. Instead let us sketch how the calculation would proceed in the spherical approximation. That is, when the six quarks are in the same bag it is assumed to be spherical. (In practice this gives fairly reliable answers.) Then the left and right clusters have wave functions

$$\begin{aligned} q_L &= q_s - \sqrt{\mu}\, q_P \\ q_R &= q_s + \sqrt{\mu}\, q_P \end{aligned} \tag{7.2}$$

where q_s is the $1s_{1/2}$ and q_P (an odd function of z) the $1p_{3/2}$ state in the same large spherical bag. (From our discussion in Section 2 we recall that a state with $j \neq 1/2$ can satisfy the n.l.b.c. only in an angle-averaged sense.) The parameter $\mu \varepsilon (0, 1)$ determines the average separation of the left and right clusters. It serves as a variational parameter in the sense we now describe.

For given μ one can calculate the parameter δ:

$$\delta = \frac{2\mu^{1/2}(1+\mu)}{1+\mu^2} \int q_s{}^+(\underset{\sim}{x})\, q_P(\underset{\sim}{x}) z\, d^3x \tag{7.3}$$

which corresponds to the internucleon distance at large separations, and in any case serves as a measure of the cluster separation. Given some value of the Lagrange multiplier C, and a separation δ_0 of interest, one can evaluate

$$H(C, \delta_0; \mu) = \langle H_{\text{MIT}} + C(\delta - \delta_0) \rangle \tag{7.4}$$

For fixed C and δ_0 one can minimize Eq. (7.4) as a function of μ. By varying C, $\delta(\mu)$ at the minimum can be made equal to δ_0. The expectation value of the MIT Hamiltonian at this constrained minimum is called $E(\delta_0)$. By repeating the whole process for a new δ_0 one can actually map out the function $E(\delta_0)$. Note that this calculation is complicated by the fact that for each μ, $\langle H_{\text{MIT}} \rangle$ can only be evaluated subject to the n.l.b.c.—so that R can also vary with δ_0 in principle. Fortunately, the radius of the six-quark bag is essentially independent of δ_0—i.e., about 1.3 fm.

In Fig. 7.1 we show the value of $E(\delta)$ calculated by DeTar in a number of spin–isospin channels. The repulsive core which we mentioned earlier is clear. However, so is the very strong attraction at slightly larger separations. The latter seems to be the result of a cancellation that doesn't quite happen. A slight reduction of α_c from 0.55 to 0.36 [consistent with the CBM description of the Δ–N mass splitting (see Section 6.3.1)] essentially kills this attraction without significantly affecting the repulsive core (DeT 80c).

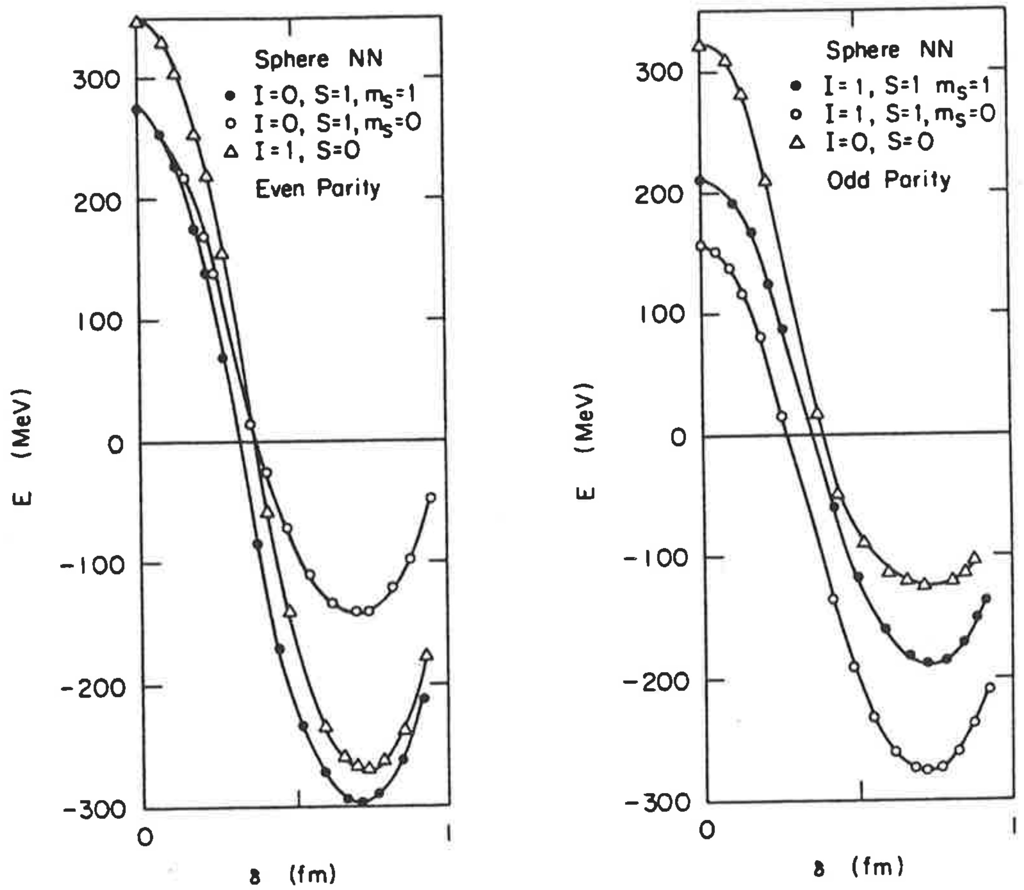

Fig. 7.1. Interaction energy of a spherical six-quark bag as a function of the separation parameter δ. (From DeT 78b.)

Given the obvious qualitative similarities between Fig. 7.1 and conventional *N–N* potentials it is rather disappointing that not much more has been done. The next stage requires some dynamical scheme for bringing bags together, letting them coalesce and fission again. Unfortunately no realistic method of calculating this has yet been formulated. This is certainly a very important problem to resolve.

7.1.2. Nucleon–Nucleon Force in the Nonrelativistic Quark Model

In view of the success of the nonrelativistic quark model (NRQM) in hadronic spectroscopy (Isg 80), it is quite natural to consider extending it to treat the scattering of two composite hadrons. Moreover, because the model is essentially nonrelativistic, the standard nuclear technique for scattering of two clusters (resonating group method) can readily be applied. The group theory is a little more complicated by the extra color degree of freedom, but these details have all been worked out by Harvey (Har 80, Har 81a). For our present purposes it is sufficient to realize that once two composite nucleons overlap, it is not enough to consider just N–N configurations. There will also be a Δ–Δ component as well as a "hidden-color" C–$\bar{C}$ configuration.

Harvey's first work (Har 81a, Har 81b), like that of DeTar in the bag model, involved simply calculating the total energy of the system as a function of the intercluster distance. As we mentioned in Section 7.1.1 there is no compelling reason for comparing this with phenomenological N–N potentials since the effective interaction in a quark model would be highly nonlocal. Nevertheless, in DeTar's work this procedure did produce strong, short-range repulsion, and it was therefore quite disturbing when Harvey found no such effect. Indeed the energy of his 6-quark system at zero separation ($r = 0$) was very close to $2m_N$. The reason for this difference seems to be DeTar's insistence on having all six quarks in the $1s_{1/2}$ orbit (s^6) at $r = 0$, whereas Harvey had quite a large s^4p^2 (hidden-color) component. However, it must also be pointed out that the definition of "separation" in these two calculations is quite different. Whereas DeTar's definition actually means the separation between peaks in the matter distribution, Harvey's is the distance between the origins for two sets of basis functions. Thus, "zero separation" may not be the same in the two calculations (DeT 82).

Recent work by Arima and collaborators has suggested an explanation for this apparent discrepancy (Oka+ 81, OY 80). The essential problem was already discussed in Section 2. That is, the NRQM consists of a one-gluon exchange potential and a recipe for restricting the space in which the diagonalization is to be performed. Furthermore, only the baryon spectrum with respect to the nucleon is fitted—the nucleon mass itself is put in by hand. Arima *et al.* (Oka+ 81) used the quark cluster model of Oka and Yazaki (OY 80) with Harvey's Hamiltonian to confirm his results in the six-quark system. However, they found that if $2\hbar\omega$ excitations were included in a variational calculation of the nucleon mass itself, then the effective nucleon mass would be lowered by 540 MeV. In that case the six-quark system would again be appreciably heavier than $2m_N$ at $r = 0$, a net repulsion of 760 and 850 MeV in the $(S, T) = (1,0)$ and $(0,1)$ channels, respectively. Clearly, one needs to formulate an unambiguous truncation procedure that is equivalent in a system of three and six quarks.

A much more sophisticated program, which was begun recently by Harvey and LeTourneux (Har 81c),¶ involves a direct solution of the

¶ See also the very similar recent work by Faessler and collaborators (Fae+ 82).

Schrödinger equation (Lib 77, WS 80):

$$H\psi(\underset{\sim}{x}) = E\psi(\underset{\sim}{x}) \tag{7.5}$$

where

$$H = \sum_i T_i + \sum_{i<j} \sum_{a=1}^{8} \lambda_i{}^a \lambda_j{}^a F(r_{ij}) + \Delta E_g^M \tag{7.6}$$

is the NRQM Hamiltonian (Isg 80). The radial form of the potential is a harmonic oscillator

$$F(r) = Br^2 \tag{7.7}$$

where $B < 0$ guarantees confinement for a colorless hadron [see Eq. (2.82)]. As usual, the solution $\psi(\underset{\sim}{x})$ is constructed in terms of a set of antisymmetrized cluster wave functions $\phi_\alpha(\underset{\sim}{x}, \underset{\sim}{r})$ describing two three-quark clusters separated by a distance $\underset{\sim}{r}$

$$\psi(\underset{\sim}{x}) = \sum_\alpha \int d\underset{\sim}{r}\, \phi_\alpha(\underset{\sim}{x}, \underset{\sim}{r}) f_\alpha(\underset{\sim}{r}) \tag{7.8}$$

The solution of the Griffin–Hill–Wheeler equations (OL 80) for $f_\alpha(\underset{\sim}{r})$ (with appropriate boundary conditions)

$$\int d\underset{\sim}{r}\, [H_{\alpha'\alpha}(\underset{\sim}{r}', \underset{\sim}{r}) - EN_{\alpha'\alpha}(\underset{\sim}{r}', \underset{\sim}{r})] f_\alpha(\underset{\sim}{r}) = 0 \tag{7.9}$$

yields the N–N phase shifts. (The function $N_{\alpha'\alpha}$ is simply the overlap of two clusters located at $\underset{\sim}{r}$ and $\underset{\sim}{r}'$.)

With the addition of a long-range pionlike interaction (for which there is no compelling theoretical argument in the NRQM), Harvey was able to obtain quite a good qualitative fit to the ${}^3S_1$ N–N phase shifts. Significantly, this fit reproduced the change of sign at about 250 MeV. Thus, the model clearly does incorporate a repulsive short-range interaction.

The major advantage of this approach is that one can directly follow the collision of two clusters without assumptions about the radial configuration at $r = 0$ (e.g., s^6 only). There are, however, a number of fundamental objections to overcome. As observed by Greenberg and Lipkin the NRQM gives rise to unobserved, strong van der Waals forces between hadrons—in contradiction with experiment (GL 81). On a more technical level we have already recorded the ambiguity in restricting the harmonic oscillator space in which the diagonalization should be carried out—in the three- and six-quark systems. Finally, the treatment of the quarks as nonrelativistic is fundamental to this method. They necessarily have a mass of about 360 MeV, one-third of the average N and Δ masses. Thus, each cluster has a dynamical mass of 1080 MeV at all intercluster separations—unaffected by the dynamics. This is clearly a crude approximation, and as Harvey has observed could be removed only in a truly relativistic treatment. That is a major challenge for the future.

7.1.3. The Long-Range Force

Unlike the NRQM, where unobserved van der Waals forces occur naturally at large distances, in the naive bag model there is no interaction at all for nonoverlapping bags. Of course, the static spherical bag is an idealization and in reality one would expect to deal with a finite surface thickness and surface fluctuations. However, it is probably reasonable to ignore this fuzziness in first order. Then the only mechanism for interaction in the region $r > 2R$ is pion exchange. For this the chiral bag models are ideally suited.

The first discussion of the long-range N–N force generated by pion coupling at the bag surface was that of Gross (Gro 79). Following the first paper of Brown and Rho he considered the interaction between bags resulting from a linear combination of pseudoscalar (λ) and pseudovector ($1 - \lambda$) pion coupling at the surface. He showed that this gave rise to an $NN\pi$ vertex function

$$\Gamma(q^2) \sim j_0(qR) + (3\lambda - 2)j_2(qR) \tag{7.10}$$

which reduces to that of the CBM [see Eq. (6.21)] in the case $\lambda = 1$. Moreover, he observed—as many others have done since—that *this form-factor did not alter the radial dependence of the OPE force for* $r > 2R$, because $\Gamma(q^2)$ is an entire function of q^2.

If for the moment we suppose that the OPE interaction can be calculated using the interaction Hamiltonian (6.24), even when two bags overlap, then the CBM form-factor will cut down the OPE potential for $r < 2R$. It is interesting to see what evidence there is to support the existence of such a form-factor. Clearly, the matter will be complicated by the tendency of ϱ-mesonlike exchanges at short distance to also damp the OPE. Nevertheless, by using experimental data to construct the Fermi invariant amplitudes for N–N scattering (Gol+ 60, BJ 76), and taking the appropriate linear combination of amplitudes to isolate the isovector–pseudoscalar pole term, Gersten was able to pick out the one-pion-exchange contribution (Ger 81). The data are consistent with a form-factor of the CBM type with a radius between 0.65 and 1.0 fm—although it is only the initial slope that is determined.

In another attempt to see such effects, Gersten and Thomas (GT 81) looked for specific partial waves in which the first iterated OPE Born term was a good approximation to the two-pion-exchange box diagram—namely $^3D_2$, ε_2, $^3G_3$, and $^3G_4$. (One cannot consider L too high or else the form-factor has no effect at all.) Unfortunately, the experimental determination of the $^3G_3$ and $^3G_4$ phase shifts is not good. But for both $^3D_2$ and ε_3 a bag radius $R \sim 0.8$ fm produces a good fit to the data.

However, the fundamental question in all this is what happens to the one- and two-pion exchange force when the two bags do overlap. More specifically, how much must the bags overlap before the "Cheshire bag approximation"¶ breaks down? It may well be that the answer to this question is quite a lot! From DeTar's work (Section 7.1.1) we know that (with $\alpha_c \sim 0.36$) nothing very dramatic happens when two bags begin to overlap. Moreover, the $NN\pi$ coupling strength goes as $g_A/2f$, and g_A depends on the spin–isospin structure, *not* on the radial size of a hadron (or quark cluster). Finally, as we have argued, the pion is not excluded from the bag interior (although it may have a somewhat different mass there). Thus, even with an individual nucleon of radius 0.8–1.0 fm, it is conceivable that the usual OPE-plus-TPE potential is not too far wrong down to 1.0–1.3 fm. The challenge in the next years will be to turn qualitative statements like "not too far wrong" into a quantitative theory.

For the present, one attractive, phenomenological option is to extend the old Feshbach–Lomon boundary condition model (LF 68), to include $N\Delta$ and $\Delta\Delta$ (and perhaps even C–C) components outside the boundary radius R_0 (Lom 81). Inside the boundary radius one would describe the system purely as six quarks (Hog+ 80, Kis 81, Mil 82). Naively, one might identify the boundary radius R_0 with the size of a six-quark bag (i.e., about 20% bigger than the nucleon bag). For the backward electro-disintegration of the deuteron, Kisslinger has shown that the quarks can make an important contribution—particularly at high-momentum transfer ($q^2 > 10$ fm^{-2}). The elastic deuteron form-factor seems to scale as expected for a six-quark bag at high-momentum transfer and there has been a similar success for the deep inelastic structure function, with about a 6% admixture of the six-quark component (BF 80). Even at very low energy, such as the circular

¶ The "Cheshire bag approximation" is a term coined by Fritz Coester to describe the use of the CBM Hamiltonian even when two bags overlap. Like Lewis Carroll's Cheshire cat, there is nothing to the bag except a "smile."

polarization in thermal neutron capture (DO 81), it has been suggested that the quark contribution could be crucial.

7.1.4. Nucleon–Antinucleon Scattering

With the expectation of large quantities of high-quality data from LEAR in the near future, there is a renewed interest in the $N\bar{N}$ system. Conventionally one obtains the $N\bar{N}$ potential from that for NN by G-parity. One simply changes the sign of the N–N meson coupling constant for those mesons of odd G-parity (π, ω, etc.). Thus, the strong short-range repulsion generated by ω-exchange in the N–N system becomes a very strongly attractive potential for $N\bar{N}$—which can support many bound states. Clearly, in the case of large composite N and $\bar{N}$ even this feature of the $N\bar{N}$ interaction may be in doubt. However, our present interest is not with that problem, but rather with the major ambiguity of any potential model, namely the effect of annihilation. The annihilation in the $N\bar{N}$ system is in fact so strong that the deeply bound states mentioned above would be unobservably broad (MT 76). This unfortunate conclusion can only be avoided if for some reason (a) the annihilation potential is extremely short range, (b) strongly state dependent, or (c) the optical model treatment is invalid.

It was noticed by Wilets and collaborators (Wil+ 81) that the bag model should yield a fairly definite idea of the shape of the annihilation potential. Before the bags overlap there is no annihilation at all. When the bags do overlap, the process

$$q\bar{q} \to \text{gluon} \tag{7.11}$$

becomes possible, and the remaining four quarks and gluon will arrange themselves into mesons. The probability for the process (7.11) obviously depends on the amount of overlap of the N and $\bar{N}$ bags. Thus, although a perturbative calculation based on (7.11) would not be expected to yield the correct magnitude of the annihilation process, one might expect the geometry to be well represented. Just as DeTar found nothing dramatic when two nucleon bags start to overlap, so Wilets *et al.* found little annihilation at $r = 2R$. Most of the strength of the annihilation seems to occur in the region $r\ \varepsilon(0.5R, R)$. From their extensive analysis of the presently available $p\bar{p}$ scattering data Wilets *et al.* found a range of bag radius parameters between 0.7 and 1.0 fm, with the overall best fit at 0.86 ± 0.06 fm. This is in excellent agreement with the radius expected in the CBM, as we discussed in Section 6. We can expect to hear much more about this problem in the next few years.

7.1.5. Exotic States

It is an unavoidable consequence of the bag model that not only will three-quark ($3q$) baryons exist, but in fact any color-singlet combination—$6q$, $4q\bar{q}$, etc. Were such states to be discovered as relatively long-lived identifiable particles, it would be a real triumph for QCD. Much theoretical effort has been devoted to calculating the spins, parities, and masses of such states (Joh 75, Jaf 77, WL 78, Mul+ 79, Mul 80). Obviously, it was very tempting to attribute the rapid energy dependence observed in $\Delta\sigma_L$ and $\Delta\sigma_T$ at the Argonne ZGS (Apr+ 80) to such a dibaryon resonance—certainly the energy regions coincided.

However, the dibaryon example reveals the essential problem of almost all exotics. The structure in the $^3F_3$ N–N channel coincides with the opening of the N–Δ p-wave, and the inclusion of this coupled channel alone can qualitatively reproduce the observed structure (Bet+ 82). In order to reach this conclusion one must perform rather complicated three-body calculations (involving two nucleons and a pion), which decently respect unitarity. The moral of the story is simply that when an exotic is connected with several open channels it cannot be discussed in isolation. One rather simple attempt to deal with this is the P-matrix formalism of Jaffe and Low (JL 79). Using this, it has been suggested that indeed a number of $B = 0$ and $B = 2$ exotics would not be expected to produce dramatic effects in π–π and N–N phase shifts (Low 79). However, one would ideally like to see a consistent, unitary coupled-channels calculation. At least for those cases where pion production is significant (like the dibaryons) the CBM should provide the basis for such a treatment.

One very important exception is the doubly strange Λ–Λ bag, which is actually predicted to be bound by about 80 MeV (Jaf 77) and therefore to have no strong decay channels. The experimental observation of this state would be very exciting but it has not yet been seen (Car 78, Pau 82). One possible reason for its nonappearance is provided by the chiral bag models. For example, in the CBM the pionic self-energy contribution is of order -130 MeV for the Λ (Thé 82, TT 82b). But the dilambda would be some 30% larger (because of the n.l.b.c.). Because the pionic self-energy decreases like $R^{3.5}$ as R increases, one would naively expect the pion self-energy for the dilambda to be cut in half. That alone would be enough to unbind the dilambda and make it rather difficult to see. A more refined calculation of the pionic corrections to the exotics is presently under way (MT 82b).

In closing this very brief discussion of N–N forces we recall that in

Sections 7.1.1 and 7.1.2 we reviewed two attempts to describe the short-distance N–N force in terms of quarks, either in the NRQM or the bag model. However, at no time did we discuss corrections associated with chiral symmetry (because neither DeTar nor Harvey considered this). Nevertheless, for exactly the reasons we have just outlined for the dilambda, the inclusion of pion self-energies will tend to provide some short-range repulsion! This will be true for the CBM and bags of the MIT size, although a similar point was made by Vento *et al.* in the context of the little hedgehog (Ven+ 81, see Section 5.3.2).

7.2. Symmetry Breaking as a Clue

Ultimately one might hope to start from a microscopic model of the nucleon (including chiral symmetry) and derive a precision fit to N–N scattering data. But, as we hope is clear from the discussion in Section 7.1 such a precision fit is a long way off. Moreover, it would be stretching one's hopes too far to expect to convince unbelievers that a quark level description is necessary on the basis of even an excellent fit to N–N data alone. Nevertheless, the situation is not as bad as it may first appear—there are more subtle avenues of attack.

We have come to hold symmetry principles rather dear in nuclear and particle physics, and violations of any fundamental symmetry are studied in great detail. It is not unreasonable to expect that the new view of nuclear physics proposed here should have something new to say about symmetry violation. It is conceivable that predictions of symmetry violation made in our present crude models might survive the improvements necessary to obtain quantitative fits to nuclear data. We might even hope to find cases where the quark model suggests a new and beautifully simple explanation for a problem that has hitherto been a puzzle for conventional nuclear theory. In this section we briefly report on one example of each kind. Although these are the only ones of which we are aware at present, the reader is graciously invited to find more!

7.2.1. Charge Symmetry Violation in OPE

Whether or not a symmetry is fundamental depends, of course, on one's point of view. In a quark model it is quite apparent that conventional isospin is an accidental symmetry. Indeed, the u and d quark masses are typically of order 5 and 10 MeV, respectively (Wei 77, BT 82), so SU(2) is badly broken at the Lagrangian level (see Section 7.2.2). However, these masses are much smaller than the eigenvalue of the Dirac equation for a light, confined quark [if $w_u/R \sim 400$ MeV, $w_d/R \sim 402$ MeV—see Eqs. (2.89)-(2.91)] the constituent quark mass (Section 2.4). Thus the microscopic breaking of the symmetry gets hidden and isospin looks good at the hadronic level.

Since charge symmetry is a special case of isospin invariance, corresponding to rotations by 180° about the y-axis in isospin space (HM 79), it is clearly no longer "fundamental." Nevertheless there is a great deal of experimental activity presently aimed at finding charge symmetry violation (CSV) in the N–N system (Dav+ 81). So far there is no clear indication of CSV there. The classical case which has been studied at length is the ${}^1S_0$ scattering length. At present the best experimental values for nn and pp are -18.6 ± 0.6 fm (Gab+ 79) and -17.1 ± 0.2 fm (Gur+ 80; after Coulomb corrections), respectively. While this apparently indicates a small CSV, there is considerable discussion of the meaning of the errors quoted.

In a recent LAMPF experiment Hollas and co-workers failed to see a charge-symmetry-violating forward-backward asymmetry in the process $np \to d\pi^0$ at a level of 0.5% (Hol+ 81). The most sensitive tests so far should come from experiments presently underway at both IUCF and TRIUMF, where one is looking for a small difference in the position of the zero in P and A in np elastic scattering (Dav+ 81).

Conventional theoretical models for CSV typically involve ϱ–ω and π–η mixing in a one-boson-exchange picture (HM 79). The presence of such mixing is a result of the u–d mass difference mentioned earlier (LS 79). However, in view of our discussion of the short- and medium-range N–N force in Section 7.1, it is not obvious that such mixing for real mesons has anything to do with N–N scattering. It would seem more appropriate to directly calculate N–N scattering in one of the ways discussed in Section 7.1 using $m_{\mathrm{u}} \neq m_{\mathrm{d}}$ directly. This has not yet been done.

What has been looked at is the possibility of a direct source of CSV in the OPE interaction caused by $m_{\mathrm{u}} \neq m_{\mathrm{d}}$ (Tho+ 81b). Because of the explicit appearance of quarks and pions in the Lagrangian density, and its excellent convergence properties, the CBM is ideally suited to this problem. We recall from Eq. (5.103) that the pion–nucleon coupling had strength $g_A/2f$, where g_A is the axial charge of the nucleon calculated in the bag model. In Section 3.3.1 we calculated g_A explicitly for the MIT bag model and showed why it gave such an improvement over the naive quark model. The presence of the lower piece of the Dirac spinor for the quark gave a maximum suppression of about 34% of the nonrelativistic value ($\frac{5}{3}$) in the case $m_{\mathrm{quark}} = 0$ [Eq. (3.36)]. Of course, in the nonrelativistic limit of infinite quark mass the lower component vanishes and the value of $\frac{5}{3}$ is restored. If one has two masses in between the ultra-relativistic and nonrelativistic limits, the suppression factor will be smaller, and hence g_A larger, for the heavier of the two.

In particular, if m_d is 4–5 MeV heavier than m_u—as we require in order to fit the n–p mass difference (BT 82, LW 78)—then g_A will be larger for the d than the u quark. If we consider π^0 coupling to the n and p, it should now be clear that the coupling to the neutron will be larger than that to the proton, because the former contains more d quarks. In fact, using the spin–flavor wave functions

$$\begin{aligned} |\,p\uparrow\rangle_{s-f} &= u_1u_2d_3(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)/\sqrt{6} \\ |\,n\uparrow\rangle_{s-f} &= d_1d_2u_3(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow)/\sqrt{6} \end{aligned} \tag{7.12}$$

for distinguishable u and d quarks one can easily show that

$$g_A{}^n/g_A{}^p = 1 + \tfrac{3}{5}\delta \tag{7.13}$$

where $1 - \delta$ is the ratio of g_A for a single u-quark to that for a single d quark. Using the results of Golowich and collaborators (Gol 75, Don+ 75) we find $\delta = 0.64\%$ for $m_d - m_u = 5$ MeV, and hence $g_A{}^n/g_A{}^p$ is greater than one by 0.4%.

This is outside the level of accuracy for present neutral current experiments. However one may hope to see this effect through the difference in f_{π^0nn} and f_{π^0pp}, implied by Eq. (5.103), viz:

$$(4\pi)^{1/2}f/m_\pi = g_A/2f$$

Clearly, we expect that the $nn\pi^0$ coupling constant should be about 0.4% bigger than that for $pp\pi^0$—in direct violation of charge symmetry. For the N–N scattering length this implies $|\,a_{nn}\,| - |\,a_{pp}^{\text{no coul}}\,| = +0.3$ fm (Tho+ 81b), which is in the same direction as experiment but a little small. (Although we stress again that the experimental numbers are not conclusive.) Other systems in which we might hope to see this CSV include the decay widths of the Δ, and the forward-backward asymmetry in $np \rightarrow d\pi^0$—which may be enhanced for an appropriate polarization observable.

7.2.2. The ^{3}He–^{3}H Mass Difference—A New Perspective

Within the framework of nonrelativistic potential theory the three-nucleon system has been amenable to exact solution for about a decade.

As we observed in Section 1 the discrepancy between the experimental binding energy of the triton and that obtained with realistic potentials has usually been attributed to relativistic or off-shell effects. However, a much more disturbing problem is the failure to fit the ^{3}H–^{3}He mass difference. After removing the n–p mass difference there is a residual 760 keV splitting between these mirror nuclei. Potential model calculations using charge-independent forces give typically 640 keV and never more than 680 keV—see the Proceedings of the TRIUMF workshop (Dav+ 81). The remaining 80 keV has been a mystery for at least 15 years. If one takes all possible sources of CSV in a conventional OBE potential model, and they all add coherently with maximum permissible strength one can just about get the 80 keV. However, it is not a very compelling explanation.

In order to see what a quark level description would imply for the same problem, we first need to review the n–p mass difference itself. The calculation of the electromagnetic shift in the bag model is rather complicated (Des+ 77) but the answer can be understood quite simply. Within about 10%

$$\Delta M_{e\text{-}m} = \sum_{i<j} \frac{Q_i Q_j}{R} \tag{7.14}$$

where the bag radius R is a measure of the average interquark distance. For $(\Delta E^p_{e\text{-}m} - \Delta E^n_{e\text{-}m})$ this gives about 0.5 MeV (with $R = 1$ fm), in agreement with Deshpande *et al.* Note that this effect acts in the *wrong* way, tending to make the proton heavier than the neutron.

The only freedom in the bag model description is to take the u and d quarks to have different masses. With a u quark mass about 4–5 MeV less than that of the d quark the necessary 1.79-MeV mass difference (1.29 MeV experimental plus 0.5 MeV from electromagnetic effects) can be explained (LW 78, BT 82). About 80% of the shift is simply associated with the change in quark eigenfrequency [see Eqs. (2.89)–(2.91)], and the rest with the change in the color-magnetic term (Section 2.2.2).

Next we recall that ^{3}He is one of the most dense nuclear systems available. Its point nucleon distribution has an r.m.s. radius of only 1.6 fm. With the nucleon itself having a radius of about 1.0 fm, it is highly likely that in a random snapshot of the nucleus we shall find two nucleons overlapping. Thus one obvious difference between ^{3}H and ^{3}He is that with some probability P we shall find the contents of two neutrons in one bag in the former, whereas in the latter we would find two protons. *The essential point is that the mass splitting between a 2p-bag and a 2n-bag is not* $2(m_p - m_n)$.

First the n.l.b.c. implies that the radius of a six-quark bag is bigger than that of a three-quark bag. We recall from Section 7.1.1 that DeTar found $R_6 \sim 1.3$ fm, compared with $R_3 \sim 1.0$ fm. (In general, one can show that $R \sim M^{1/3}$, with M the mass of the multiquark system.) Therefore we find at once a 30% reduction in the n-p mass splitting caused by $m_u \neq m_d$. In addition, a simple calculation with Eq. (7.14) shows that even allowing for the increase in average interquark separation, the Coulomb splitting increases in the wrong direction. The net result is that the $2n$ and $2p$ bags are split by only 0.9 MeV, instead of $2(m_n - m_p) = 2.6$ MeV. Alternatively, the effective n–p mass difference for the fraction of time P that the bags overlap is only 0.45 MeV.

A probability P of 10% would therefore suffice to explain the 80-keV discrepancy [(2.6-0.9/2)$\times$10% $\approx$ 80 keV]. This is a perfectly reasonable probability and indeed if we assume that when the center of one bag is within R_3 of the center of another they have coalesced, one obtains a probability $(1.0/1.6)^3 = 24\%$ for ^{3}He.¶ It is clearly difficult to make this argument more quantitative at the present time, but the $A = 3$ system does provide a beautiful example of just how different the quark model perspective may be—even for a familiar problem. Further work along these lines is presently being carried out (TG 82) to see to what extent such ideas can contribute to an explanation of the famous Nolen–Schiffer anomaly (NS 69).

7.3. The Nuclear Many-Body Problem

As there is no published calculation of the properties of a many-nucleon system near nuclear matter density (ϱ_0) in the sort of model which we have presented, this will be a brief section. (We exclude from the present discussion the very high-density limit of quark matter, where there are no individual bags at all.) Nevertheless it does seem appropriate to collect together some of the ideas which may eventually be applied to the problem.

In a very stimulating attempt to understand how a system of finite-size bags might behave, Baym introduced the idea of percolation (Bay 79). To introduce the concept, consider an infinite array of cubic children's blocks, some of which are copper and some wooden. If they are arranged at random there is a critical percentage of the blocks ($P_c = 31\%$) which must be copper in order to *guarantee* that there is an infinite conducting chain through the array. If instead of being cubic we have spheres arranged on a regular lattice, P_c is $15 \pm 1.5\%$. Finally, for conducting spheres only,

¶ Similar arguments have been made by Pirner and Vary (PV 81).

arranged at random through space, the critical percentage of space which must be occupied by spheres is 34%.

The analogy is, of course, that if two bags touch we expect that the quarks (i.e., a color current) will be able to flow between them. [This was exactly the assumption made by DeTar (see Section 7.1.2).] Consequently, in infinite nuclear matter above a certain critical density (ϱ_c), we expect that there should be at least one infinite conducting chain along which the quarks flow freely. This free flow of quarks is known as "percolation." Since the volume of a spherical bag is just $(4\pi R^3/3)$, we expect that

$$\varrho_c = 0.34\bigg/\left(\frac{4\pi}{3}R^3\right) \tag{7.15}$$

and hence (with $\varrho_0 = 0.17\ \mathrm{fm}^{-3}$), ϱ_c is $1/2\varrho_0$, ϱ_0, and $1.4\varrho_0$ for $R = 1.0$, 0.8, and 0.7 fm, respectively.

We see that in the center of a large nucleus like ^{208}Pb, any acceptable nucleon bag radius (following the considerations of Section 6, $R \geq 0.8$ fm) will imply the presence of conducting chains. More to the point, for a radius near the MIT value ($R \sim 1.0$ fm) ϱ_c is of order $\varrho_0/2$, and even the nuclear surface should contain such chains. Such is our ignorance at present that it is not even clear whether this would have observable consequences! Qualitatively at least, it does seem easier to reconcile the success of the conventional shell model for valence nucleons with a somewhat smaller bag radius—say $R \sim 0.8$ fm. In that case $\varrho_c \sim \varrho_0$ and one would expect little effect in the nuclear surface where $\varrho \sim \varrho_0/2$. On the other hand, one might expect that single-particle ideas could fail in the nuclear interior.

7.3.1. Dense Nuclear Matter

There has been considerable theoretical and experimental interest in the past few years in the possibility of exotic phenomena at densities higher than ϱ_0—phenomena like pion condensation and Lee–Wick matter. Chiral symmetry plays a crucial role in the conventional description of such processes. Indeed the σ-model, which we described at length in Section 4.4 is the starting point for most of the work in this area (LW 74, Bay 78, Cam 78, Mey 81). Clearly, if we are to be concerned about effects of the finite size of the nucleon in the center or even the surface of finite nuclei, it is unthinkable to ignore such effects at densities twice that of nuclear matter or greater! Indeed it would seem that pion condensation or Lee–Wick matter in the usual scenario of pointlike nucleons with spin–isospin ordering is quite unlikely. Nevertheless the phenomenon which replaces it, namely overlapping bags with a free flow of color through linked bags may be more interesting!

Incidentally, if it makes sense to talk of finite-size nucleons exchanging pions even when they overlap a little (as discussed in Section 7.1), the CBM should provide an admirable successor to the σ-model. As we observed in Sections 5 and 6, it naturally incorporates the Δ-degree of freedom on the same footing as the nucleon. One does not have to put in $g_A \neq 1$ by hand (as in the σ-model). Finally in its linearized form the CBM is a rapidly convergent renormalizable theory and one does not have the ambiguities of using a tree-level Lagrangian in a many-body system. Self-energy corrections are meaningful in the CBM. In the final part of this section we wish to outline a new approach to the nuclear many-body problem designed to exploit these advantages of the CBM.

7.3.2. The e^s Formalism—A Generalization

In attempting to solve for the properties of a many-body system for a given Hamiltonian it is essential that one use a technique which allows for systematic improvement. The coupled-cluster expansion, or e^s formalism, has played this role in conventional nuclear theory (Coe 69, Kum+ 78, ZE 79). While making no attempt at a serious review of the formalism (the quoted articles fulfil that purpose) it is worthwhile to outline its essential features here. Given a many-body Hamiltonian

$$H = H_0 + V \tag{7.16}$$

where V includes all two-body interactions, the linked-cluster expansion amounts to writing the exact eigenfunction of H, namely Ψ, as

$$\Psi = e^s \Phi \tag{7.17}$$

where Φ is a Slater determinant describing the noninteracting Fermi gas.

If we define creation operators for particles and holes [$a^+(x)$, $b^+(x)$, respectively] in the usual way, the operator s is

$$s = \sum_{n>1} s_n \tag{7.18}$$

$$s_n = \frac{1}{(n!)^2} \int dx_1 \cdots dx_n \int dy_n \cdots dy_1\, a^+(x_1) \cdots a^+(x_n) \\ \times b^+(y_n) \cdots b^+(y_1) s_n(x_n, \ldots, x_1; y_1, \ldots, y_n) \tag{7.19}$$

Clearly s_n is related to the amplitude for creating n particle-hole pairs. What is less obvious is that it is the amplitude for creating *correlated* particle-hole pairs. This is crucial in a low-density system because one can prove rigorously that the importance of the nth order piece goes as $(h^3\varrho)^{n-1}$ where ϱ is the density and h a "healing distance"—related to the range of the two-body interaction. With $h \sim 1$ fm and $\varrho_0 \sim 0.17\ \text{fm}^{-3}$, one has a systematically convergent expansion at nuclear matter density. For completeness we note that in the case of pure two-body interactions in infinite nuclear matter, the total energy can be calculated entirely in terms of s_2 ($s_1 = 0$ by translational invariance). That is, the total energy per particle is given by (Coe 69)

$$(E/A) = \varrho_0^{-1}(\Phi \mid H \mid \Phi) + \tfrac{1}{4} \int dk \int dp\, dP\, (p \mid V \mid k)s(k, p; P) \qquad (7.20)$$

where

$$s_2(k_1k_2; p_2p_1) = \delta(k_1 + k_2 - p_2 - p_1)s(k, p; P) \qquad (7.21)$$

Of course, in order to obtain s_2 one must solve a set of coupled-cluster equations involving all amplitudes $\{s_n\}$. These equations are easily obtained by noting that

$$H\Psi = E\Psi \qquad (7.22)$$

and by Eq. (7.17)

$$e^{-s}He^{s}\Phi = E\Phi \qquad (7.23)$$

But any particle or hole destruction operator, d, acting on Φ gives zero, so that

$$\langle\Phi \mid de^{-s}He^{s} \mid \Phi\rangle = 0 \qquad (7.24)$$

More generally

$$\langle\Phi \mid b(y_1) \cdots b(y_n)a(x_n) \cdots a(x_1)e^{-s}He^{s} \mid \Phi\rangle = 0, \qquad \forall n \qquad (7.25)$$

which are the coupled-cluster equations. After truncation at some order N (because of the proof of convergence noted above) one obtains a closed set of nonlinear integral equations. The convergence of the iterative solution of those equations can be formally established for certain conditions on V.

In recent years we have come to realize the importance of the Δ in nuclear physics. A suitable generalization of the e^s formalism to include the Δ explicitly was recently developed by Coester (Coe 81) for the Betz–Lee model (BL 81). In their model the only pion emission and absorption allowed are the processes $\Delta \leftrightarrow N\pi$. In such a simple field theory there is no renormalization of the nucleon, but the properties of the Δ, and hence the intermediate range N–N force, will be density dependent.

The excellent convergence properties of the CBM, and the fact that the Δ (and other $B = 1$ resonances) appears so naturally there, have prompted us to develop a coupled-cluster expansion including pion degrees of freedom explicitly (Coe+ 82). Formally all that is required is to replace Eqs. (7.18) and (7.19) by

$$s = \sum_{n,m\geq 1} s_{n,m} \tag{7.26}$$

where

$$\begin{aligned} s_{n,m} = \frac{1}{m!}\,\frac{1}{(n!)^2} \int dk_1 \cdots dk_m \int dx_1 \cdots dx_n \int dy_n \cdots dy_1 \\ \times \alpha^+(k_1) \cdots \alpha^+(k_m) a^+(x_1) \cdots a^+(x_n) b^+(y_n) \cdots b^+(y_1) \\ \times s_{n,m}(k_1 \cdots k_m; x_n \cdots x_1; y_1 \cdots y_n) \end{aligned} \tag{7.27}$$

In Eq. (7.27) $\alpha^+(k_1)$ creates a pion of momentum and isospin k_1 and $s_{n,m}$ is, of course, the amplitude for creating m pions and n particle-hole pairs all correlated. The generalization of Eq. (7.25) to obtain the new coupled-cluster equations is obvious.

Of course, in order to obtain equations which one can solve numerically one must again be able to justify a truncation at some maximum value of n and m. The cutoff in n will again be justified in terms of powers of $(h^3\varrho)$. However, the cutoff in number of pions is a unique feature of the CBM and its justification was presented in Section 6. We expect that retaining all five amplitudes with m and $n \leq 2$ should be sufficient at nuclear matter density (Coe+ 82).

Unfortunately, there are no numerical results available yet from this formalism, so one cannot judge yet whether it will throw any new light on the nuclear many-body problem. Nevertheless, there are solid physical reasons for believing that it might. Because the nucleon bag is relatively large, we have seen that the $NN\pi$ form-factor $[3j_1(kR)/kR]$ is quite soft. An equivalent dipole, $(k^2 + \Lambda^2)^{-1}$, would have a range parameter $\Lambda \sim 640/R$ MeV (with R in fm). Thus the cutoff in all renormalization integrals is of the order of the fermi momentum ($k_F \sim 275$ MeV/c). In such an intermediate situation one might expect that the properties of the many-body system as a function of density would be inextricably linked with the renormalization process. This problem does not appear to have been seriously addressed before.

We cannot conclude this section without a note of caution. There are many more subtleties in describing a system of composite nucleons than we have been able to address. The e^s formalism deals with the creation of N, Δ, ... obeying standard fermion anticommutation relations and dressed with a pion cloud. As we have argued in the earlier sections, it is possible that for a bag radius in the lower range of that permitted in a chiral bag model ($R \sim 0.8$–0.9 fm), this may be a reasonable approximation even up to nuclear matter density. However, it must break down as the density increases and the quarks begin to percolate. It becomes increasingly difficult to assign a meaning to exchange terms, for example, as the density goes up. If we are lucky, we will begin to learn how to formulate this problem in a respectable way in the next few years. It is a noble endeavor!

8. CONCLUSION

This is a moment of dramatic change in our conception of nuclear physics. In the next decade the impact of the discoveries made by our colleagues in high-energy physics will have to be reconciled with the conventional view of the nucleus. At the present stage we can only begin to guess at how much richer and more fascinating our subject may be. Amongst the admittedly crude models available to us in this detective work, we argued that the MIT bag model is a promising place to start. In particular, we outlined the ideas which have led a number of investigators to believe that it may have many of the properties of the eventual solution of QCD (incorporating both confinement and asymptotic freedom very concisely). For this reason we gave a detailed summary of the model, its underlying assumptions, its solutions, its predictions for the properties of single hadrons, and finally its unresolved problems.

Next we explained the concept of chiral symmetry and why it must be broken in nature—even though it is exact in pure QCD. The linear σ-model was used as the classic example of a spontaneously broken symmetry—with the appearance of the pion as a Goldstone boson. On a more fundamental level we mentioned the possibility that the pion may be the result of dynamical symmetry breaking caused by the strongly attractive one-gluon-exchange force in that channel. In that case its appearance would be independent of the usual mechanism for confinement. Then we reviewed the various attempts which have been made over the last three years to make a bag model incorporating chiral symmetry.

We saw that the cloudy bag model (CBM) in particular has produced a number of striking results for the properties of single hadrons—e.g., the neutron electric form-factor, the magnetic moments of the neutron, proton, and other members of the nucleon octet, and finally the proton lifetime. The CBM has led to a new and deeper understanding of the Δ-resonance which, like all the other baryons, enters in a natural, unified manner consistent with chiral symmetry. It was possible to transform the Lagrangian of the CBM so that it is a generalization of the Weinberg Lagrangian and naturally incorporates the Weinberg–Tomozawa relationship for low-energy pion scattering. Most significant for nuclear physics applications are the excellent convergence properties of the CBM. For example, the bare $NN\pi$ coupling constant is renormalized by less than about 10% for any bag radius bigger than 0.7–0.8 fm.

Armed with a chiral bag model which had proven so successful in one-body systems, we made some observations in the last section about the N–N interaction and the nuclear many-body problem. Clearly that discussion was by far the most speculative. However, we did suggest that with a little subtlety one might, even now, be able to see some hints of the quark substructure in processes involving symmetry violation.

In order to be useful to the community a review must not only point out the achievements of a particular model, but also its faults and problems—the cutting edge of research often lies there. We have tried to pinpoint such problems throughout the review, but let us stress a few of the major questions again. One would be to firmly establish a relationship between the MIT bag model, soliton bag models, and QCD. Of course, the nature and origin of the pion itself (particularly in relation to QCD) is an absolutely crucial question to answer. The formal problems associated with doing many-body calculations in a dense system of composite nucleons are formidable but must be addressed. Finally there is a whole set of questions of a more technical nature, such as how to include recoil corrections, whether the CBM ideas can be generalized to $SU(3)\times SU(3)$, and so on. There is no shortage of work or challenge, and this whole review should be considered an invitation to take part.

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APPENDIX I

Throughout these notes we follow the conventions of Bjorken and Drell (BD 64).

$$\beta = \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \qquad \underset{\sim}{\gamma} = \begin{pmatrix} 0 & \underset{\sim}{\sigma} \\ -\underset{\sim}{\sigma} & 0 \end{pmatrix} \tag{I.1}$$

$$x^\mu \text{ is a contravariant vertor—}(x^0, x^1, x^2, x^3) = (t, \underset{\sim}{x}) \tag{I.2a}$$

$$g^{\mu\nu} = \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \tag{I.2b}$$

$$\underline{\gamma} = \gamma^0 \underline{\alpha} \tag{I.3}$$

$$\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{I.4}$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \tag{I.5a}$$

so that

$$\sigma^{ij} = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \tag{I.5b}$$

$$\sigma^{0i} = i\alpha^i = i\begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix} \tag{I.5c}$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \tag{I.6}$$

$$\not{p} = \gamma_\mu p^\mu = \gamma^\mu p_\mu = i\not{\partial} \tag{I.7}$$

The Dirac equation is

$$\begin{aligned} (\not{p} - m)u(p, s) &= 0 \\ \bar{u}(p, s)(\not{p} - m) &= 0 \end{aligned} \tag{I.8}$$

where

$$\bar{u} = u^+\gamma^0 \tag{I.9}$$

To conclude this section on notation we briefly review a useful classification scheme for nonrelativistic angular momentum eigenfunctions

$$\left| l \frac{1}{2} j\mu \right\rangle \equiv | \chi_\varkappa{}^\mu \rangle = \sum_m C^{(\mu-m)m\ \mu}_{l(1/2)j} \left| \frac{1}{2} m \right\rangle | l(\mu - m) \rangle \tag{I.10}$$

If we define

$$k = \underline{\sigma} \cdot \underline{l} + 1 \tag{I.11}$$

then, because $\underline{\sigma} \cdot \underline{l}$ has eigenvalues $\{j(j+1) - l(l+1) - \frac{3}{4}\}$, k has eigenvalues $\varkappa$

$$k\chi_\varkappa{}^\mu = -\varkappa\chi_\varkappa{}^\mu \tag{I.12}$$

with

$$\begin{aligned} \varkappa &= l, & j &= l - \tfrac{1}{2} \\ \varkappa &= -l - 1, & j &= l + \tfrac{1}{2} \end{aligned} \tag{I.13}$$

Thus $\varkappa$ alone specifies l and j, for example

$$\begin{aligned} s_{1/2} &\text{ is } \varkappa = -1 \\ P_{1/2} &\text{ is } \varkappa = +1 \\ P_{3/2} &\text{ is } \varkappa = -2 \end{aligned} \tag{I.14}$$

and so on.

In conclusion we note that $(\sigma \cdot \hat{r})^2 = +1$, and $\sigma \cdot \hat{r}$ is pseudoscalar, thus

$$\sigma \cdot \hat{r}\chi_\varkappa^{\mu} = -\chi_{-\varkappa}^{\mu} \tag{I.15}$$

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CHIRAL SYMMETRY AND THE BAG MODEL

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We give a brief review of the connection between QCD and the more phenomenological, chiral bag models, which have generated so much excitement recently. Some recent results from the cloudy bag model are then presented, together with a discussion of the evidence from deep-inelastic scattering which supports this choice. We close with a few comments on the relevance of these ideas in interpreting the EMC effect.

1. INTRODUCTION

There is now an almost universal acceptance that quantum chromodynamics (QCD) is the theory of strong interactions. It is therefore the only truly fundamental starting point from which to develop a consistent theoretical description of nuclear phenomena. Unfortunately it is too difficult to solve the QCD equations except in some limits. At high Q^2 it has been established (using the renormalization group) that QCD is "asymptotically free". That is, if we determine the strength of the quark-gluon coupling, $\alpha_s(Q_0^2)$, at some momentum scale Q_0^2, then $\alpha_s(Q^2)$ decreases logarithmically as Q^2 increases beyond Q_0^2. Thus at high Q^2 (or small distances) quarks should behave essentially like free particles. This is the main reason for the success of the naïve quark-parton model for deep inelastic scattering (DIS)[1,2].

Another limit where it is believed that we know something about QCD is in the infra-red - large separation. There the non-Abelian nature of QCD is supposed to lead to confinement of coloured objects. On a time scale of many years it is possible that brute-force numerical work on a space-time lattice may unambiguously yield the structure of the nucleon implied by QCD. However, even the most ardent lattice advocates do not foresee the day when one could calculate (e.g.) the properties of finite nuclei in this way. For that we need phenomenological models. A great variety of such models exist, ranging from the non-relativistic quark models[3], through variants of the kind proposed by Shuryak[4], to the relativistic bag models[5-7].

Our discussions will concern only the recent generalizations of the MIT bag model, but it should be realized that this is largely a matter of taste. The

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major advantages of the bag model are that it incorporates two key features of QCD, namely confinement and asymptotic freedom in a simple, phenomenological Lagrangian. As with essentially all other phenomenological models proposed by high energy theorists, the radius of the region within which the quarks are confined is of order 1 fm. This brings us to the question of what is meant by short-range nuclear physics. A very natural definition would be that inter-nucleon separation at which it is no longer sufficient to describe nucleon-nucleon scattering in terms of nucleons and pions alone. Within conventional nuclear theory the exchange of the massive ω meson leads to a repulsive core at distances of order 0.3 to 0.5 fm. Because of its large mass, the ρ meson does not contribute much beyond 1 fm. Nevertheless, there is tremendous model dependence in (for example) the calculation of short-range exchange currents because of the interplay between correlations and heavy meson exchange (ρ, $\rho\pi$, $\omega\pi$, etc.). (These ambiguities are even worse in calculations of electromagnetic processes because of the difficulty of imposing gauge invariance in the presence of *ad hoc* form factors at the meson-nucleon vertices.) Within this framework the no-man's-land of uncontrolled short-distance corrections is typically 0.3 to 1.0 fm.

On the other hand, if one thinks of nucleons as composite bags of quarks with a radius of order 1 fm, it is clear that short-distance physics begins at 2 fm ! Certainly at an inter-nucleon separation of 1 fm nuclear phenomena should deeply involve quark degrees of freedom. Rather than being more complicated than the conventional meson exchange picture, because of the property of asymptotic freedom, there is reason to hope that calculations at the quark level might prove simpler and less ambiguous.

With these long-term aims in mind we now turn to the most recent extensions of the bag model, which have centered on incorporating a third fundamental property of QCD - namely chiral symmetry.

2. CHIRAL BAG MODELS

It is firmly established empirically[8] that the masses of the u and d quarks are very small (less than about 10 MeV) compared with the typical hadronic energy scale. Thus to a good approximation the strong interactions should preserve chiral symmetry. Simply put, this implies that the equations of motion should be invariant under separate SU(2) transformations for left- and right-handed particles [i.e., under $SU(2)_L \times SU(2)_R$]. Unfortunately, the MIT bag model necessarily violates this third fundamental property of QCD[9]. The reason, illustrated in Fig. 1, is simply that the very act of confining the quarks mixes the left-handed and right-handed sectors.

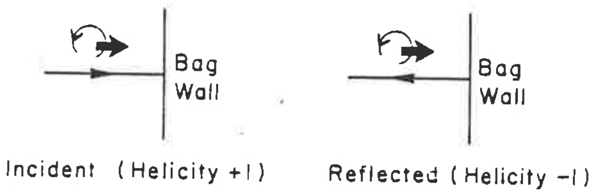

FIGURE 1
Illustration of the intrinsic violation of chiral symmetry in the MIT bag model

A possible solution to this problem is suggested by the following general consideration[10]. The Goldstone theorem tells us that if $SU(2)_L \times SU(2)_R$ is an exact symmetry, either all the particles occurring in nature come with degenerate, negative parity partners or the symmetry must be realized in the Goldstone mode. There are very good reasons for believing that the pion, with its remarkably low mass, is very close to being a Goldstone boson. Unfortunately, one of the mysteries of QCD is that we do not yet understand the dynamical mechanism whereby this collective $q\bar{q}$ state appears. Certainly the one-gluon exchange is extremely strong in the pion channel - without it the ρ and ω would be degenerate at $\sim$ 650 MeV in the MIT model, and in first order, one-gluon exchange lowers the pion mass to some 280 MeV. Several groups have been led by this to suggest that iterated gluon exchange could be the mechanism for dynamical symmetry breaking[11]. Others have shown that instanton effects can produce a strong attraction in the pion channel[12,13]. Whatever the mechanism for producing the pion, all of the recent extensions of the MIT bag model which restore chiral symmetry, do so (by analogy with the work of Gell-Mann and Levy[14]) by coupling an elementary pion field to the confined quarks[14,15]. Of course, this does not mean that we expect to see pointlike, pseudoscalar objects in deep inelastic lepton-nucleon scattering[16,17]. Instead, we are constructing a phenomenological model meant to be applied at momentum transfers low compared with the internal structure of the pion. There are many examples in physics where the introduction of such collective pairing effects are essential in order to describe observed phenomena.

Whereas these very general arguments tell us that pions are intimately involved in the restoration of chiral symmetry, it is unfortunate that QCD gives little practical guidance in constructing phenomenological models. (For a much more detailed discussion, see Ref. 17.) There is therefore room for quite different phenomenology and hence considerable controversy. In the absence of any higher authority the ultimate test of which model is best must be a comparison with as much experimental data as possible.

Essentially all of the chiral bag model calculations performed so far correspond to one of two main working hypotheses, the little bag model (LBM)[14] or the cloudy bag model (CBM)[6,15]. In the latter it is *assumed* that hadron sizes are determined by non-perturbative QCD effects which are not significantly altered by pionic corrections. Then it makes sense to calculate pionic corrections as a small perturbation about the MIT bag model solutions. In the former, on the other hand, the pionic effects are supposed to be intimately linked with the process of confinement, compressing the bag to perhaps one tenth the volume of the MIT model. In this way, one would of course revive the conventional nuclear physics picture of essentially pointlike nucleons exchanging heavy mesons.

A second difference between the models, which has recently faded to insignificance[18] was the original insistence in the LBM on excluding the pion from the interior of the bag - a strict two-phase model. In the CBM, this was not the case. The pion was allowed throughout all space for two reasons. Firstly, the theoretical case for a strict two-phase picture is by no means universally accepted, and secondly the exclusion of the pion field destroys one of the major successes of the MIT bag model, namely the quite accurate prediction for the axial charge of the nucleon[19]. In the CBM this correct prediction is preserved in a very simple and natural way[6,15].

Since the mathematical details of the pion coupling to confined quarks in both models have been described in great detail elsewhere[2,20] we shall not repeat that material here. Instead, in the next section we review a few of the more recent results obtained in the CBM. Only then shall we discuss the recent test of these models using DIS, which strongly supports the CBM.

3. RECENT RESULTS IN THE CLOUDY BAG MODEL

A fairly recent summary of results from the CBM can be found in Refs. 6 and 17. It is not unreasonable to say that in every case where pionic corrections have been computed the agreement with experiment is as good as, and usually better than, the original MIT bag model. Of course, the major underlying defect of the bag, namely the spurious centre-of-mass motion, is not solved by adding pionic corrections. Thus for magnetic moments, and particularly for the charge radii, there are corrections at the level of 10% or so, upon whose sign there is no general agreement. It remains to be seen whether a thorough theoretical analysis can lead to a generally acceptable correction procedure, or whether what we really need is a better relativistic model of confinement. For the present, agreement of any bag model calculation at a level better than (5-10)% must be regarded as random. At that level, however, its success is still striking.

Because of the fact that the CBM results have been reviewed elsewhere, we shall only discuss those cases where there has been a significant new development.

3.1. The Σ^- magnetic moment

This is of particular interest for the chiral bag models because of the so-called Pilkuhn-Eeg effect[21]. That is, the pionic correction for the Σ^- is twice as big as one might naïvely expect, because as well as the process $\Sigma^- \to \Sigma^0\pi^-$, one has also $\Sigma^- \to \Lambda\pi^-$. In the CBM, using the same bag parameters as the MIT bag model, we find $\mu(\Sigma^-) = -1.08\ \mu_N$[22]. This answer is quite insensitive to the actual strange quark mass or bag radius[23]. (For comparison the corresponding value without pionic corrections is about $-0.81\ \mu_N$.) On the other hand the LBM prediction is of the order $-0.58\ \mu_N$[24].

Until recently, the experimental situation was unclear, with older atomic physics measurements giving $-1.41\pm0.27\ \mu_N$, and a Σ^- beam measurement giving $-0.89\pm0.14\ \mu_N$. The new generation of Σ^- atom measurements made by the William and Mary group have made an order of magnitude improvement in this. Indeed, the accuracy of the most recent value of $\mu(\Sigma^-)$ [25], namely $-1.09\pm0.03\ \mu_N$, is too good for the present theory! Nevertheless, the confirmation of the CBM prediction is very welcome.

3.2. The axial form factor of the nucleon

For reasons explained in detail in Ref. 6, in the CBM only the quarks contribute to the axial (as opposed to the induced pseudoscalar) current of the bag. Thus, unlike the electromagnetic properties for which there are pionic contributions, the axial form factor is a direct measure of the quark distribution in the nucleon. At present the data on $g_A(q^2)$ come from two sources, the reaction $\nu_\mu + n \to \mu^- + p$ and pion electroproduction - see, e.g., Ref. 26. It is usually represented as a dipole

$$g_A(q^2) = (1 + q^2/m_A^2)^{-2}, \tag{3.1}$$

with $m_A = 0.95\pm0.14$ GeV.

If we calculate $g_A(q^2)$ for the CBM we find this corresponds to a bag radius $R = 1.16\pm0.20$ fm[27]. Clearly there should be corrections to this value arising from centre-of-mass and recoil effects, but as a first estimate this strongly suggests a bag size similar to that expected in the original MIT bag model.

Guichon et al.[27] also investigated $g_A(q^2)$ in the hybrid model of Chin and Miller and Vento, where the pion is excluded from a region $r < R_{ch}$ inside the bag [i.e., $\xi = R_{ch}/R \in (0,1)$]. For $\xi \neq 0$ the pion also contributes to $g_A(q^2)$. However, as shown in Ref. 27, the slope of $g_A(q^2)$ changes by less than 10% over the whole range of values of ξ. Thus the result $R = 1.16\pm0.20$ fm is a general result for all chiral bag models.

Finally we note that we can also calculate the πNN form factor in the hybrid model. If we parametrize $g_A(q^2)$ as $[1-q^2r_A^2/6]$ and $g_{\pi NN}(q^2)$ as $[1-q^2r_\pi^2/6]$, then for all values of ξ, $r_\pi > r_A$. That is, the πNN form factor in all chira bag models is softer than $g_A(q^2)$. In the CBM, where $\xi = 0$, $g_{\pi NN}(q^2)$ would correspond to a dipole of mass 0.90 ± 0.14 GeV ($r_A/r_\pi \sim 0.9$), which is very soft. For ξ in the range 0.0 to 0.8 this hardly changes, but in the range 0.8 to 1.0 $g_{\pi NN}(q^2)$ becomes rapidly softer, with r_A/r_π dropping to 0.65 and the corresponding dipole mass to about 0.76 GeV!

Clearly it would be very valuable to have more precise data for $g_A(q^2)$. Nevertheless, even at the present accuracy, we regard the arguments which we have just reviewed as the most direct indication (apart from the discussion of DIS in Section 4) that the nucleon bag is of the order of 1 fm in radius.

3.3. Exotic states

One of the more exciting possibilities raised by the MIT bag model was that there might be stable, exotic states. For example, it was suggested that the so-called H dibaryon (a Λ-Λ state) might be bound by (50-80) MeV[28]. In view of the relatively large self-energy corrections associated with pions for single hadrons, it is reasonable to ask how those corrections affect the masses of exotic states.

In order to check this in a scheme consistent with the philosophy of the CBM, Mulders and Thomas refitted the usual hadron spectrum with the phenomenological form[29]

$$E(R) = E_Q + E_V + E_M + E_p \tag{3.2}$$

Here E_Q, E_V and E_M are respectively the standard kinetic energy, volume and colour magnetic contributions to the bag energy. The last term E_p, is a phenomenological representation of the pion self-energy which has the form

$$E_p = \frac{-1}{pR^3} \sum_{i,j} (\vec{\sigma}\,\underset{\sim}{\tau})_i \cdot (\vec{\sigma}\,\underset{\sim}{\tau})_j \; . \tag{3.3}$$

The spin-isospin structure corresponds to keeping only the lowest orbital in the intermediate state, and treating all such states as degenerate. Finally, p is a phenomenological constant.

There were several notable features associated with the best fit parameter. The rather large value of the colour coupling constant α_s in the bag model was reduced by some 35%, which is a step in the right direction. The strange quark mass also came down to 218 MeV (from 279 MeV) - a little closer to the usual current algebra value of 150 MeV. Lastly, we observe that, although treated as an adjustable parameter, the value of p agreed very well with that calculated for a nucleon in the chiral bag models.

For the non-strange, B = 2, exotic bag states, the pionic corrections had little effect. In $^3S_0$ and $^1S_0$ the bag masses were 2.18 and 2.24 GeV respectively (cf. 2.16 and 2.23 in the original MIT bag model[28,30]). Since these lie well above the appropriate thresholds they will be quite broad, and should not have dramatic experimental consequences.

On the other hand, for the doubly strange H dibaryon the change is dramatic. The combination of decreased colour attraction (smaller α_s), and the R^{-3} dependence of the pionic self-energy result in a larger mass for the H - 2.22 instead of 2.15 GeV. From this, Mulders and Thomas conclude that the H is almost certainly unbound, and thus it is no mystery that experimental searches have failed to find it. In conclusion, we must remark that this matter is not yet completely closed, as Kerbikov[31] has recently claimed that the coupling of the six quark bag to hadronic channels could lower the mass again. This deserves further study.

3.4. Pion photoproduction

The initial motivation for, and the first success of, the CBM was to reconcile[32] the two orthogonal views of the Δ(1232) which existed side by side - namely the Chew-Wick and the quark models. Next it was established that the CBM also reproduced s wave πN scattering at low energy[33,34]. Given these successes with the elastic channel it is natural to ask whether the model is also able to reproduce existing pion photoproduction data. This is of particular interest because of the claims in the LBM of a very large d wave component in the small nucleon bag which could lead to a sizeable E2 amplitude[35].

As shown by Kälbermann and Eisenberg[36] the CBM does indeed provide a "consistent and reasonable" picture of the M1 photoproduction amplitude in the Δ(1232) energy region. The same calculation yields a ratio of E2/M1 amplitudes of -0.9% of which only a fifth comes from the d state admixture in the Δ. Most importantly this rather small result is quite consistent with existing experimental data, which could of course be profitably improved.

3.5. Other developments

While the coupling of the pion to the bag is uniquely determined up to order ϕ^2 in Refs. 33 and 34, the terms of next order can be altered by redefinitions of the physical pion field[6,37]. The $\pi N \to \pi\pi N$ reaction near threshold provides an interesting testing ground for alternative versions of the CBM which differ at that order. For an initial discussion of this problem, which indicates that Weinberg's choice[37] for the pion field may be preferable at order ϕ^3, we refer to the recent discussion of Kälbermann and Eisenberg[38].

Another very exciting development, based on the Thomas formulation of the CBM[33], is the work of Miller and Singer[39]. Whereas the CBM was initially applied to baryon properties, they have been able to successfully derive (e.g.) the $\omega\rho\pi$, $\rho\pi\pi$, $K^*K\pi$ and $K^*K^*\pi$ coupling constants. A more recent extension to radiative decays of the vector mesons also seems to agree very well with existing data[40].

There are a number of other interesting developments related to the CBM which we simply do not have space to describe here. Instead we refer to the recent review by Miller for details[41].

4. A TEST USING DIS

The phenomenon of Bjorken scaling in deep inelastic scattering (DIS) of leptons from nucleons was discovered at SLAC in the late 60's. We now understand fairly well why scaling violations must occur if QCD is the theory of the strong interactions, and these violations have been studied systematically[1,2]. Nevertheless the property of asymptotic freedom also explains why the naïve quark-parton model works so well over a large range of Q^2. For our purposes it will be sufficient to use this language. For a relatively simple and up-to-date review of the present knowledge of the nucleon structure function, and its interpretation in the naïve quark-parton model we refer to Ref. 17.

In order to relate what is known about DIS to chiral bag models we begin with the observation by Sullivan that there is a contribution to the nucleon structure function arising from the process shown in Fig. 2[42]. This contribution can be written as

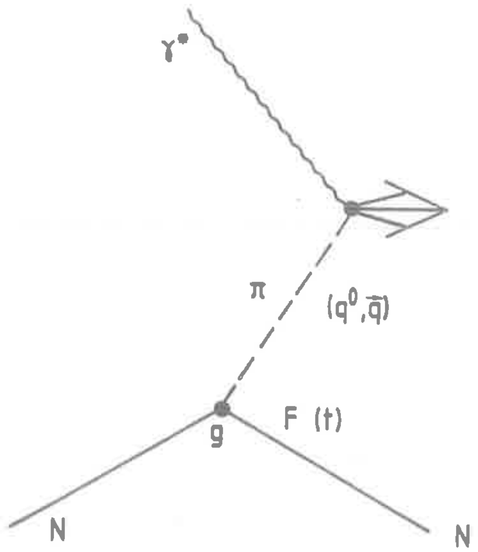

FIGURE 2
The contribution of the pion
to the structure function of the nucleon

$$\delta F_{2N}(x) = \int_x^1 dy\, f(y)\, F_{2\pi}(x/y), \tag{4.1}$$

where $F_{2\pi}$ is the pion structure function and $f(y)$ is the momentum distribution of the pion in an infinite momentum frame. (For the present purposes we can omit the Q^2 dependence of δF_{2N} and $F_{2\pi}$.)

The physical interpretation of Eq. (4.1) is that we sum over all y the product of the probability $[f(y)]$ of finding a pion carrying a fraction y of the momentum of the nucleon, with the probability $[F_{2\pi}(x/y)]$ of finding a quark in the pion with a fraction x of the nucleon's momentum. Since $F_{2\pi}(\xi)$ has been measured by the NA3 collaboration at CERN in the Drell-Yan process[43], all we need is $f(y)$. This is very easily calculated in terms of the πNN coupling constant g, and the πNN vertex function $F(t)$ - with $t = \vec{q}^{\,2} - q^{02}$ = minus the four-momentum transfer. For simplicity, we take a simple exponential for $F(t)$

$$F(t) = \exp[-\lambda(t + m_\pi^2)/m_\pi^2] \tag{4.2}$$

and seek to put some bounds on λ. However, we should point out that in the CBM the form factor is very well approximated by Eq. (4.2) if $\lambda = 0.106\, m_\pi^2 R^2$ - with R the bag radius[6,23]. The final expression for $f(y)$ is

$$f(y) = \frac{3g^2}{16\pi^2} \int_{\frac{m_N^2 y^2}{1-y}}^{\infty} \frac{dt\, t\, |F(t)|^2}{(t + m_\pi^2)^2} \tag{4.3}$$

A straightforward numerical calculation of Eq. (4.3) reveals two essential features. First, $f(y)$ peaks at about 0.25 for any reasonable value of λ. Second the maximum value of $f(y)$ increases rapidly as λ decreases. Returning to Eq. (4.1) we see that the pion structure function is evaluated at x/y. As usual we expect that the valence component of the pion should dominate for $x/y > 0.1$. Since y is typically 0.25, this implies that the pionic contribution to the nucleon structure function for $x > 0.03$ involves only non-strange quarks. Thus, if the pion is an important component of nucleon structure, it should contribute to breaking the SU(3) flavour symmetry $[SU(3)_F]$ of the sea. Of course, it is generally expected that $SU(3)_F$ will be broken because of the larger strange quark mass, and it would be unreasonable to attribute the entire excess of non-strange sea quarks to the pion. Nevertheless, it seems quite reasonable to use any evidence for $SU(3)_F$ breaking to impose a limit on the pionic contribution to the nucleon structure function.

Integrating Eq. (4.1) over x, we find that

$$\int_0^1 \delta F_{2N}^{\mu}(x)dx = \left[\int_0^1 F_{2\pi}^{\mu}(\xi)d\xi\right]\left[\int_0^1 dy\, y\, f(y)\right] . \tag{4.4}$$

Using the Drell-Yan data for the pion structure function we find the first integral on the right of Eq. (4.4) is 0.015±0.004. From the physical interpretation of $f(y)$, the second integral - which we denote $\langle y\rangle_\pi$ - is the average fraction of the momentum of the nucleon carried by pions. Clearly if we use the observed excess of non-strange over strange quarks[17,44] to give an upper bound on the value of the left-hand side of Eq. (4.4), we obtain an upper bound on $\langle y\rangle_\pi$[16]. Modulo some discussion of nuclear corrections to the experimental value of $\bar{S}/(\bar{U}+\bar{D})$[44], which was obtained in Fe, we find $\langle y\rangle_\pi \leq 5\pm1.5\%$.

In Fig. 3, we show the average fraction of the momentum of the nucleon carried by pions, $\langle y\rangle_\pi$, as a function of the cut-off parameter λ, at the $NN\pi$ vertex

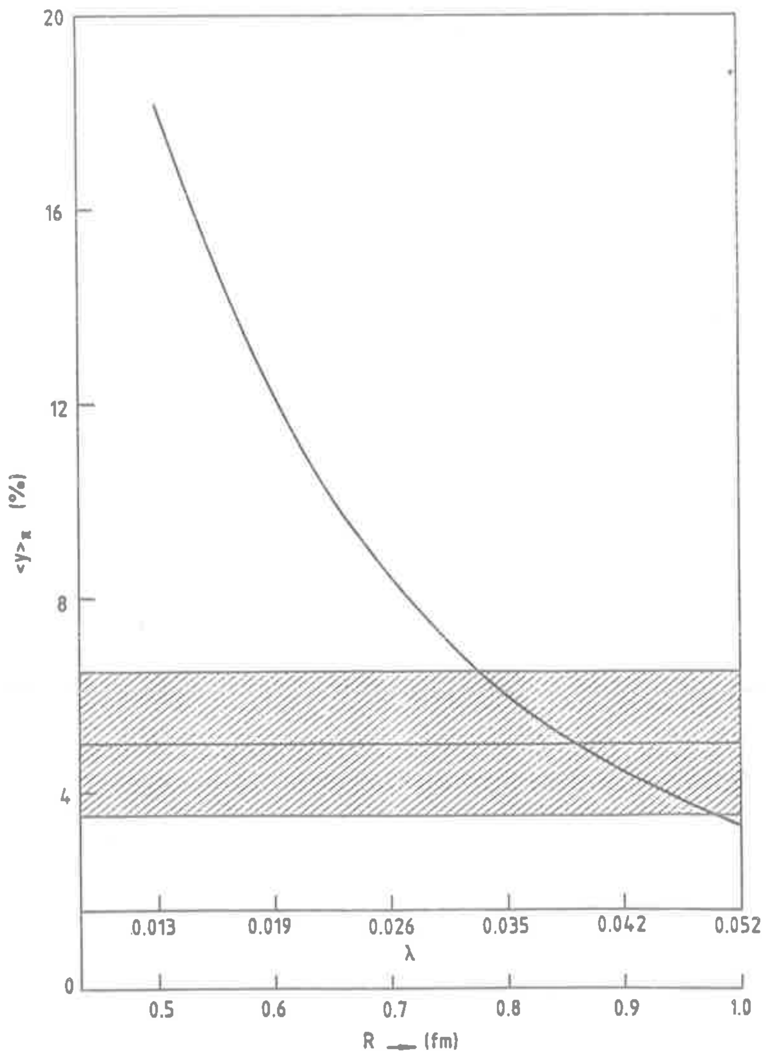

FIGURE 3
The average fraction of the nucleon's momentum carried by the pions as a function of λ (or bag radius R). The shaded area represents the bound obtained in Ref. 16

Clearly our bound is a very strong constraint on that parameter. It is not possible to accept a value of λ smaller than $0.039^{+0.012}_{-0.006}$. We also show in Fig. 3 the CBM radius corresponding to each value of λ. The lower bound on the bag radius in the CBM is $R = 0.87 \pm 0.10$ fm. Of course, there are many defects in the static bag model, and one cannot insist too strongly on an absolute value of R. One expects the bag to have some surface thickness, and this together with centre-of-mass and recoil corrections could change the simple relationship between R and λ. Nevertheless, we expect this upper bound to be a good indication of the size of the region within which quarks are confined in the nucleon. The concept of a little bag with a size of order (0.3-0.5) fm is definitely excluded.

For a nuclear audience, it is worthwhile to put this result in perspective. From the measurements on Fe we know that the valence quarks carry some 36% of the momentum of the nucleon, while the whole sea carries about 10%. Our bound says very simply that the pionic contribution should not be more than about 20% of the sea. (The quarks carry about 40% of the pion's momentum, and $0.40 \times 0.05/ /0.10 = 0.20$.) Even this may seem quite large to a number of high energy physicists!

In conclusion we note one corollary to this discussion, which may turn out to be more important. The study of the evolution of structure functions is quite an industry at present. Within experimental errors this evolution is consistent with the Altarelli-Parisi equations[1,45]. However, the evolution of the sea is inextricably linked to the unmeasurable gluon momentum distribution, and one must solve for these self-consistently[45]. From the earlier discussion of Fig. 2 we know that it is the *valence* distribution in the pion which is mainly responsible for the pionic contribution to the *nucleon sea*. The former should decrease as Q^2 goes up, whereas the latter is known to increase. We intend to investigate the consequences of this intriguing observation in the next few months.

5. THE EMC EFFECT

In the early part of this lecture we gave as one of the major motivations for the development of the CBM that it might lead us to a somewhat deeper understanding of the nuclear many-body problem. Indeed a rather natural picture which one might consider involves a collection of relatively large nucleons ($R \sim 0.8$-1.0 fm) moving independently some of the time, but also merging and fissioning. Thus at any given instant there will be a non-negligible probability of finding a given quark in a six-quark rather than a three-quark bag. It is therefore quite gratifying that recent data from the European Muon Collaboration (EMC)[46] has revealed a dramatic difference in the effective structure function of a nucleon in Fe compared with that in D. (Throughout the rest of this discussion, we shall not distinguish between the structure function of a free-nucleon and that of a nucleon in deuterium - because of the latter's low density.)

Essentially the EMC data, which has been partially confirmed at SLAC[47], shows a softening of the structure function in Fe. For $x \lesssim 0.1$, $F^{\mu}_{2N}(x)$ is enhanced by about 15%, while at $x \sim 0.6$, it is depressed by the same amount. Eventually at large x fermi motion takes over and the ratio rises above one.

Late last year, it was suggested by Llewellyn Smith[48], on the basis of Eq. (4.1), that an increase in the number of pions per nucleon in Fe could explain the enhancement at small x. To see this we evaluate Eq. (4.1) at $x = 0$, with the result

$$\frac{\delta F_{2N}(0)}{F_{2N}(0)} = \int_0^1 \delta f(y)\, dy. \tag{5.1}$$

Here $\delta f(y)$ is the change in the distribution of pions (per nucleon) in Fe compared with a free (isoscalar) nucleon. Thus, the right-hand side of Eq. (5.1) is the extra number of pions per nucleon in Fe, and in order to explain the extrapolated experimental value at $x = 0$ of 0.18 ± 0.07, one would need between 6 and 14 *extra* pions in Fe.

Having said this we should immediately add a caution about interpreting this extra number of pions too literally[49]. Equation (4.1) is meaningless if one goes too near $x = 0$, firstly because of shadowing, but also because additional processes where a nucleon turns into a pion and a baryon resonance should be considered inside $x \sim 0.05$ [42]. Thus, while the argument of Llewellyn Smith was very important in motivating further work, the only reasonable way to use the EMC data is to calculate $\delta F_{2N}(x)$ using a model of the nuclear response to a pionic excitation[50] and compare directly with the data for $x \gtrsim 0.05$. Of course if a fit is found one could a posteriori calculate the number of extra pions. Even then this "number" is defined in an infinite momentum frame[42,48,50] and does not correspond to the simple expectation value of the number operator in the rest frame[49,51].

Explicit calculation with Eq. (4.3) reveals that the most important contribution comes from pions with a three-momentum, $|\vec{q}|$, of order 300 to 400 MeV/c and low energy $(\omega \sim -|\vec{q}|^2/2m_N)$. This region has been of tremendous interest in medium energy physics for the past decade in connection with possible pion condensation[52-54]. The mechanism for this enhancement of the pion field is shown in Figs. 4b and 4c. If iterated in RPA these processes would lead to pion condensation at nuclear matter density if it were not for a short-range repulsive interaction which is conventionally parametrized as the Landau-Migdal parameter g' - shown in Figs. 4d and 4e.

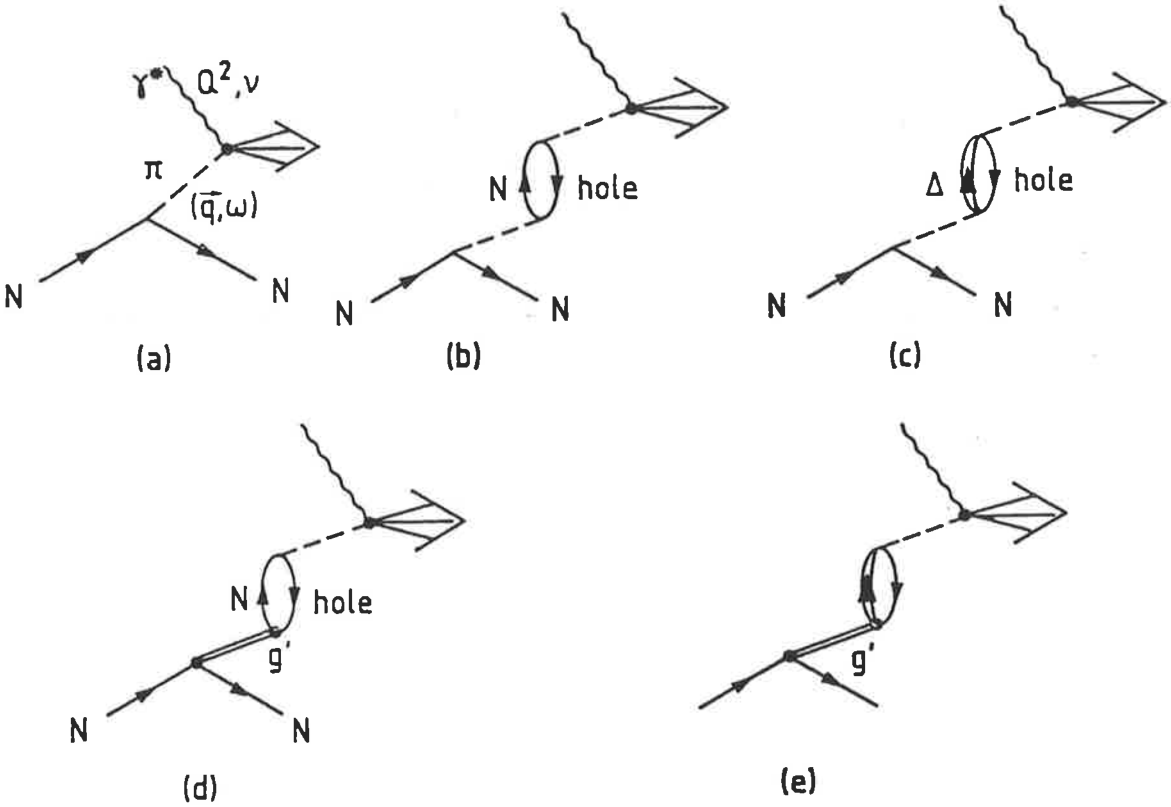

FIGURE 4
Illustration of (a) the basic pion contribution to the nucleon structure function (the $\gamma^*\pi$ vertex involves the structure function of the pion itself); (b) and (c) other coherent processes involving pion rescattering in the nucleus which lead to enhancement for $|\vec{q}| \sim$ 300-400 MeV/c; (d) and (e) the phenomenological short-range repulsion which damps the enhancement arising from (b) and (c) - from Ericson and Thomas[50].

Our intention is not to pursue the justification of the Landau-Migdal force, or to discuss its consequences in the famous suppression of Gamow-Teller strength[52,53]. We merely note that as shown by Ericson and Thomas[50] it is possible to generalize Eq. (4.1) to the nuclear case by introducing the nuclear spin-isospin response function. Then, within the conventional RPA with $g' \sim 0.7$, we obtain the solid curve of Fig. 5. Clearly, the shape and magnitude of the enhancement of the sea is reproduced. In view of the controversy over the microscopic calculation of $g'_{N\Delta}$, we point out that any value of this parameter significantly less than 0.7 would give an enhancement that was far too big.

Obviously the model which we have described says nothing directly about the decrease in the structure function of Fe in the valence region $(x \sim 0.6)$. On the simple grounds of momentum conservation, if the momentum carried by pions

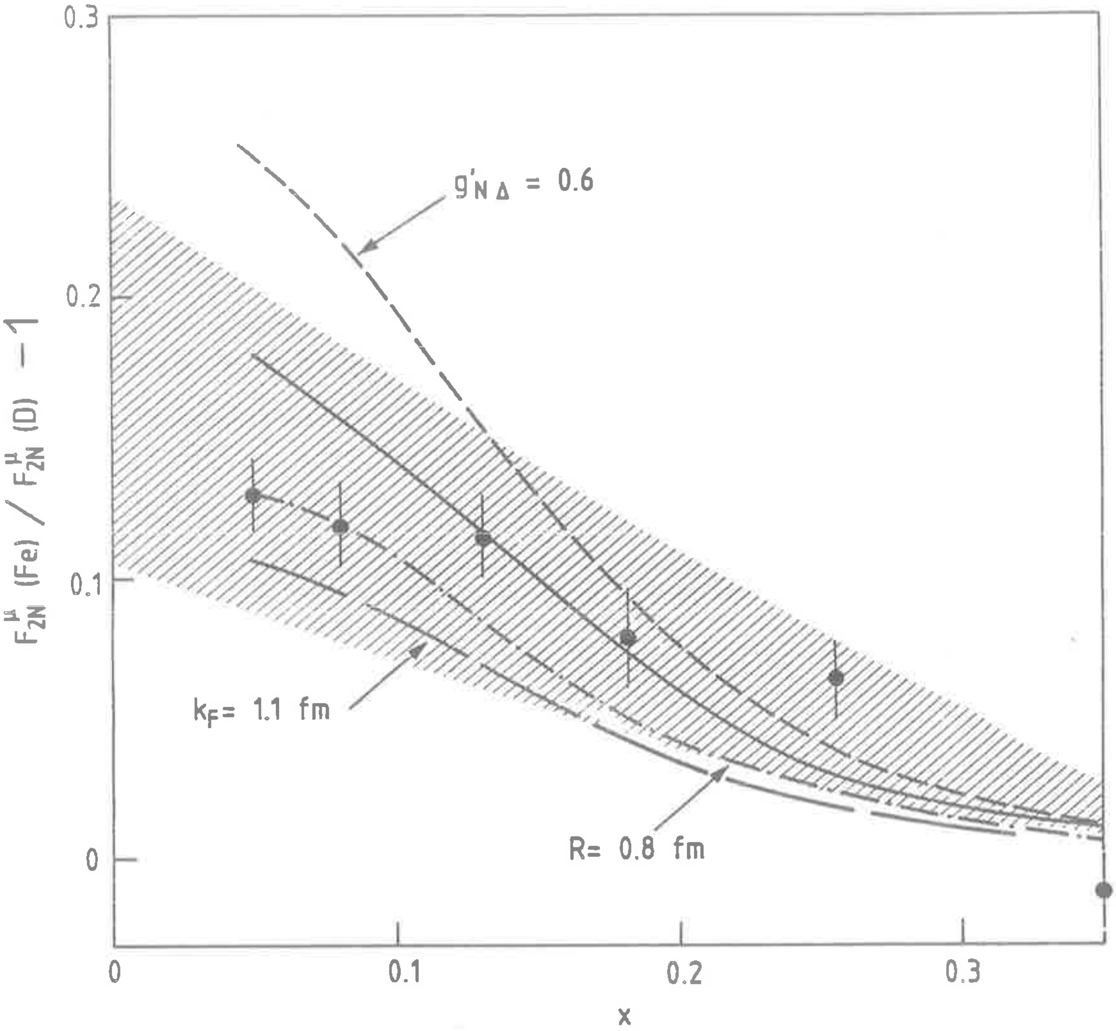

FIGURE 5
The fractional increase in the ratio of the structure function in Fe compared with D, as a function of x $(= Q^2/2m_N\nu)$, caused by the multi-nucleon pion emission graphs of Figs. 4b-e. The data are from the EMC collaboration, and the shaded area indicates possible systematic errors. The standard input (solid curve) for Fe is $k_F = 1.30\ \mathrm{fm}^{-1}$, $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta} = 0.7$, a bag radius of 0.7 fm in $F(q^2)$, and $\Gamma(q^2)$ is a dipole of mass 1.67 GeV. We show in the other curves the effect of altering any single one of these parameters - taken from Ericson and Thomas[50]

goes up, something else must lose momentum. Simple estimates of this effect have been made by several groups - either by lowering the average momentum per nucleon[48], or by a naive calculation of the photon coupling to the nucleon instead of the pion in Fig. 3[49,55]. Both methods give similar results, depressing the structure function at $x \sim 0.6$ in agreement with the data, but also lowering it at $x \sim 0.2$, thereby worsening the fit there.

In Ref. 50, we decided to calculate only the processes shown in Fig. 4, for the following reasons. These terms are gauge invariant by themselves. They represent the modification of the longest range part of the structure of the nucleon. Calculating all of the couplings of the photon to the baryons in Fig. 4

would hopelessly complicated. Finally a mechanism for balancing a large part of the momentum taken by pions had already been suggested by Jaffe[56]. Since that mechanism matches perfectly with the picture of the nucleus which we have advocated as a consequence of the CBM, we prefer to pursue that first. Only after it has been calculated, if there is still some small momentum imbalance, would we resort to purely phenomenological corrections.

Although Jaffe's suggestion was based on the MIT bag model the result is more general[57,58]. All one needs is that there is a significant probability of finding a given quark in a six-quark rather than a three-quark bag. The Drell-Yan-West relation[1,2] then tells us that the structure function of a six-quark state must behave as $(1-x/2)^9$, while that of a three-quark bag goes as $(1-x)^3$. It is then trivial to show that

$$\frac{F_{2,6q}(x)}{F_{2,3q}(x)} \sim \frac{(1 - x/2)^9}{(1 - x)^3} , \tag{5.2}$$

has a minimum at $x = 0.5$ - exactly as in the data.

Clearly the essential qualitative features of the data can be understood. The real difficulty is to make the analysis quantitative. For example, even the fermi motion corrections seem to be fairly model-dependent. A program of experiments to map out the dependence of this effect on atomic number has just been completed at SLAC, and we eagerly await the results. This information, together with a measurement of whether the enhancement of the sea is SU(3) symmetric or not, should distinguish between most of the numerous theoretical models which have appeared in the last few months[48-51,56,58,59].

While it will be some time before the EMC effect is fully understood, we should take some pleasure in what has been achieved. It is quite conceivable that we are seeing confirmation of a new and deeper understanding of the structure of the nucleus than we have ever had before. Through phenomenological models of hadron structure, like the CBM, we may at last be near to a unified theoretical description of nuclear and particle physics.

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NUCLEON POLE TERMS IN PROTON DECAY

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We show that nucleon pole terms can increase the proton decay rate by factors of 2 to 10 from that obtained from two quark fusion. The range of values corresponds to the uncertainty in the proton wave function at the origin — fixing parameters of the wave function to fit other short distance phenomena favours the large enhancement factors. In any case, this effect decreases the predicted lifetime in the minimal SU(5) model to the point where it is impossible to reconcile with recent limits on the lifetime.

Immediately following the development of grand unified theories it was realised that such theories predicted the decay of the nucleon with a lifetime of order 10^{30} years.[1] Following this observation, many groups made calculations of the lifetime to be expected[2], particularly in the minimal SU(5) model in which the calculation of the GUT mass scale m_X depends on no further assumption, except that there are no surprises in store for us between 10 GeV and 10^{14} GeV!

All of these calculations utilised the "spectator" diagram of figure 1 as the decay mechanism. More recently, the pole diagram of figure 2 was also included as a contribution to the decay process. This was done at a phenomenological level through current algebra[3] and chiral symmetry[4], and at a microscopic level[5,6] through a quark model calculation of the amplitude $< e^+|\mathcal{H}_{GUT}|p >$. Invariably inclusion of this additional term enhances the decay rate, reducing the lifetime.

In this contribution it is our aim to discuss the sensitivity of the results of these calculations to the proton wavefunction, and to show that, independently of detailed assumptions about the wavefunction, it is not possible to reconcile the predictions of minimal SU(5) with the recent result of the IMBH group[7], that the partial lifetime for $p \to e^+\pi^0$ exceeds 6.5×10^{31} years.

It is clear from the diagrams of figures 1 and 2 that the spectator and pole terms depend on $\int \psi(\underset{\sim}{\xi} = 0, \underset{\sim}{\eta}) d\underset{\sim}{\eta}$ and $\psi(\underset{\sim}{\xi} = 0, \underset{\sim}{\eta} = 0)$, where $\underset{\sim}{\xi} = \dfrac{\underset{\sim}{r}_1 + \underset{\sim}{r}_2}{\sqrt{2}}$, $\underset{\sim}{\eta} = \dfrac{\underset{\sim}{r}_1 + \underset{\sim}{r}_2 - 2\underset{\sim}{r}_3}{\sqrt{6}}$ are the Jacobi coordinates in the three quark system. To illustrate this we adopt the simple Gaussian wavefunction exploited extensively by Igsur and Karl[8],

$$\psi(\underset{\sim}{\xi}, \underset{\sim}{\eta}) = \pi^{-3/2} R_p^{-3} \exp\left\{- \frac{1}{2} R_p^{-2} (\underset{\sim}{\xi}^2 + \underset{\sim}{\eta}^2)\right\} \tag{1}$$

in which case the pole term becomes[9]

$$M_{pole} = - i \frac{g_A}{2f_\pi} u(k R_B) \frac{k}{\omega_k + k^2/2m_p} \times$$

$$\times \frac{12 G_{GUT} A}{3^{3/2} \pi^{3/2} R_p^3} \bar{u}_e (\beta + \alpha \gamma_5) u_p, \tag{2}$$

in the rest frame of the decaying proton.

In equation (2)

$$u(x) = j_0(x) + j_2(x) = \frac{3j_1(x)}{x} \tag{3}$$

is the cloudy bag model form factor associated with the NNπ vertex, g_A anf f_π have their usual significance ($g_A \approx 1.25$, $f_\pi \approx m_\pi/\sqrt{2}$), $G_{GUT} = g^2\sqrt{2}/(8m_X^2)$ is the GUT "Fermi Constant", and α and β depend on the chiral structure of the GUT currents. In SU(5) they take the values -1/2 and +3/2 respectively. The factor A takes into account the renormalisation group scaling from $Q^2 = m_X^2$ to $Q^2 = m_p^2$ of the 4 fermion effective Hamiltonian describing the decay, and is approximately 3.

In Donoghue and Golowich[6] it is noted that M_{pole} varies by a factor of 2 depending on whether pseudoscalar or pseudovector coupling is used for the NNπ vertex. This is well known in the context of pole models of hyperon decays[10], where it is shown that to reconcile the results with current algebra, it is necessary to use pseudovector coupling or equivalently take account of the variation in the pseudoscalar term between the physical point $q_\pi^2 - m_\pi^2$ and the soft pion point $q^2 = 0$. In fact eqn.(2) uses the non relativistic form of the NNπ vertex of the Cloudy Bag Model, but this differs by a few percent from the pseudovector result.

The spectator term in the same model is

$$M_{spec} = -i\eta \frac{\phi(k)\; 2\sqrt{3}\; A\; G_{GUT}}{(2\pi)^{3/2}\; R_p^{3/2}} \; (2\omega_k)^{\frac{1}{2}}\; \bar{u}_e \; (\beta + \alpha\,\gamma_5)\; u_p \;, \tag{4}$$

where $\phi(k)$ is a form factor associated with the spectator decay mechanism

$$\phi(k) = \frac{[3/4\; R_p^2\; R_m^2]^{3/4}}{[1/2\;(R_m^2 + 3/4\; R_p^2)]^{3/2}} \; \exp\left(\frac{-3\; k^2\; R_p^2\; R_m^2}{12\; R_p^2 + 16\; R_m^2}\right) \;, \tag{5}$$

with R_m the harmonic oscillator parameter for the meson (analogous to

R_p for the proton), and η is a phase factor ($|\eta|^2 = 1$) which determines the relative phase of the pole and spectator terms. Agreement with the chiral symmetry calculations requires $\eta = +1$, which we adopt.

It is immediately apparent that the pole and spectator matrix elements have the same spinor structure, and that $\rho = M_{pole}/M_{spec}$ depends sensitively on the value chosen for R_p. This sensitivity is illustrated in figure 3. It is however important to emphasise that for all reasonable values of R_p (say from 0.3 fm to 0.6 fm) the value of ρ is not very different from the value $g_A \approx 1.25$ which it takes in the phenomenological chiral models[4].

The sensitivity of ρ, and hence τ_p which varies as $|1 + \rho|^{-2}$, to the value of R_p, demands that we take a critical look at methods suggested for fixing this parameter.

One widely used approach is to select R_p to fit the proton charge radius[11]. This gives

$$R_p = (0.87 \pm 0.08) \text{ fm}$$

We reject this determination because we believe a significant contri- to the proton charge radius is generated by the pion cloud (as in the Cloudy Bag Model, for example[12]), and that it is unrealistic to take R_p so large as to generate the entire charge distribution from the bare, 3 quark proton wavefunction.

Determination of the spectrum of the excited states of the nucleon in the harmonic oscillator model[13] gives a value of

$$R_p = 0.56 \pm 0.08 \text{ fm.}$$

This leads us to suggest $R_p = 0.6$ fm as an extreme upper limit to the values of R_p which it is reasonable to use in calculating the proton lifetime.

Determining R_p from the nucleon spectrum has the disadvantage for us that the spectrum depends on the large scale structure of the wave-function, whereas in calculating the proton decay rate we are explicitly interested in the short distance behaviour of the wavefunction. There are other observables which also depend on the nucleon wavefunction when the quark-quark separation is small. These offer the possibility of a more "relevant" determination of R_p.

First we mention the analysis of the N-Δ mass difference by de Rujula et al.[14], which for $\alpha_s(m_p^2) \approx 0.5$ (see[15]) gives

$$2^{-3/2} \int d\underset{\sim}{\eta}\, |\psi(\underset{\sim}{\xi} = 0, \underset{\sim}{\eta})|^2 = 7.6 \times 10^{-3}\ \mathrm{GeV}^3 .$$

This fixes

$$R_p = 0.40\ \mathrm{fm}.$$

Hara[16] has analysed the electromagnetic form factor of the proton and the electromagnetic mass differences of the octet baryons to obtain three independent constraints on the short distance wave-function, viz

$$\frac{1}{2} \int d\underset{\sim}{\xi}\, |\psi(\underset{\sim}{\xi}, \underset{\sim}{\eta} = 0)|^2 = 0.026\ \mathrm{GeV}^3$$

$$2^{-3/2} \int d\underset{\sim}{\eta}\, |\psi(\underset{\sim}{\xi} = 0, \underset{\sim}{\eta})|^2 = (0.013 \pm 0.001)\ \mathrm{GeV}^3$$

$$2^{-1/2} \int \frac{d\underset{\sim}{\xi}\, d\underset{\sim}{\eta}}{|\underset{\sim}{\xi}|}\, |\psi(\underset{\sim}{\xi}, \underset{\sim}{\eta})|^2 = 490\ \mathrm{MeV},$$

which give

$$R_p = 0.48\ \mathrm{fm}$$

$$R_p = 0.34\ \mathrm{fm}$$

$$\text{and} \quad R_p = 0.32\ \mathrm{fm},$$

respectively.

Finally we mention various attempts to estimate R_p from fits to non leptonic hyperon decay rates[17]. In the literature, s and p wave decays are fitted independently to give $R_p = 0.60$ fm (for s wave decays) and $R_p = 0.45$ fm (for p wave decays). However attempts to reconcile the calculated s and p wave amplitudes suggest that $< B'|\mathcal{H}_w|B >$ should be determined from the p wave decays[18], favouring the smaller values of R_p. The situation is further complicated in that the QCD enhancement factors which enter these calculations are dependent on $\Lambda_{\overline{ms}}$, which has been reduced from 0.4 GeV (for 4 flavours) to 0.15 GeV since the calculations were done. While it has been suggested to us[19] that "it is by no means clear how the change in $\Lambda_{\overline{ms}}$ would affect the hyperon decay matrix elements," we feel it is useful to emphasise that the most naive approach, namely a recalculation of the enhancement factors for the new $\Lambda_{\overline{ms}}$,[20] with a readjustment of R_p to fit the data, alters the above values of R_p to 0.45 fm and 0.35 fm.

That it is impossible to fit both short distance and global properties of the wavefunction at the same time should come as no surprise. The Gaussian wavefunction we are using is the wavefunction appropriate to harmonic oscillator potentials between the quarks. This may be regarded as a useful representation of the long range confining potential, but the one gluon exchange potential with its Coulombic form at short distances remains as a residual interaction. It is plausible that this residual interaction induces short range correlations between the nucleons which enhance the wavefunction at short distances.

To summarise, we believe that the value of R_p to be used in proton decay calculations should be restricted to the range 0.3 fm $\leq R_p \leq$ 0.6 fm, and we believe the evidence we have on the short range wavefunction favours the more restricted range 0.3 fm $\leq R_p \leq$ 0.5 fm.

The other parameter upon which the proton decay rate depends sensitively is the GUT mass scale m_X. In minimal SU(5) with F families and n_H light Higgs doublets[2,21] the prediction of $\sin^2\theta_W$ is

$$\sin^2\theta_W(m_W) = 0.2138 \pm 0.0025$$

$$+ 0.006\ \ln\left[\frac{0.16\ \text{GeV}}{\Lambda_{\overline{ms}}\ (4)}\right] + 0.004(n_H-1) - 0.0007\ (F-3)$$

while m_X is given in GeV by

$$m_X = 2.4 \times 10^{14} \times (1.5)^{\pm 1} \left[\frac{\Lambda_{\overline{ms}(4)}}{0.16 \text{ GeV}}\right] \times \left(\frac{1}{1.5}\right)^{n_H - 1} \times (1.2)^{F-3}$$

Presently favoured values for $\Lambda_{\overline{ms}(4)}$ are[22]

$$\Lambda_{\overline{ms}(4)} = 0.16^{+0.10}_{-0.08} \text{ GeV},$$

which leads to good agreement with the observed value of the weak angle:

$$\sin^2\theta_W(m_W) = 0.215 \pm 0.012.$$

The reasonable range of values for m_X is thus

$$0.8 \times 10^{14} \text{ GeV} \leq m_X \leq 5.9 \times 10^{14} \text{ GeV}.$$

We plot the constant lifetime curves (isochrons(?)) for the partial lifetime of the branch $p \to e^+\pi^o$ as a function of m_X and R_p in figure 4, on which the IMBH limit is also indicated. Values below and to the left of the IMBH curve are excluded. Even when the large range 0.3 fm $\leq R_p \leq 0.6$ fm is allowed the minimal SU(5) theory is excluded. If, as we argue, R_p should be restricted to 0.3 fm $\leq R_p \leq 0.5$ fm the exclusion is even stronger, as is illustrated by the fact that, at $R_p = 0.4$ fm $m_X \geq 12 \times 10^{14}$ GeV is required to satisfy the IMBH bound on the $p \to e^+\pi^o$ partial lifetimes. It is difficult to see how such a large value of m_X can be accommodated in SU(5) theories without upsetting the agreement between the predicted and observed values of $\sin^2\theta_W$.

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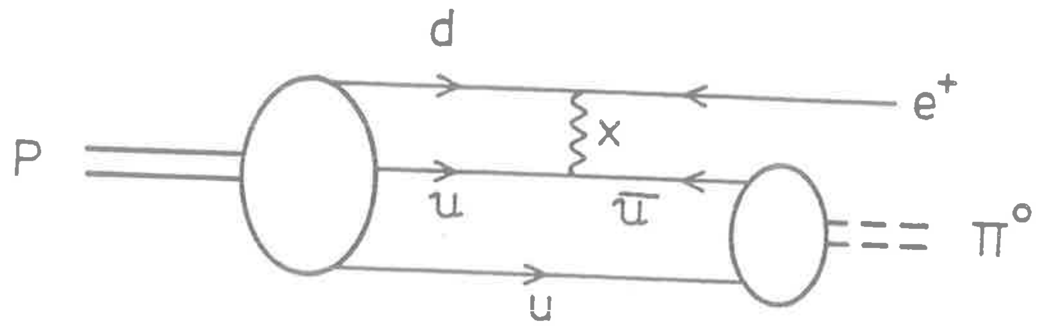


Figure 1

Spectator contributions to nucleon decay.

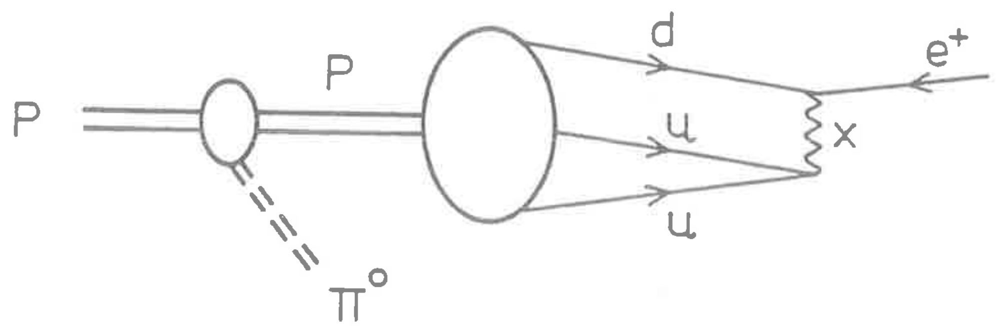
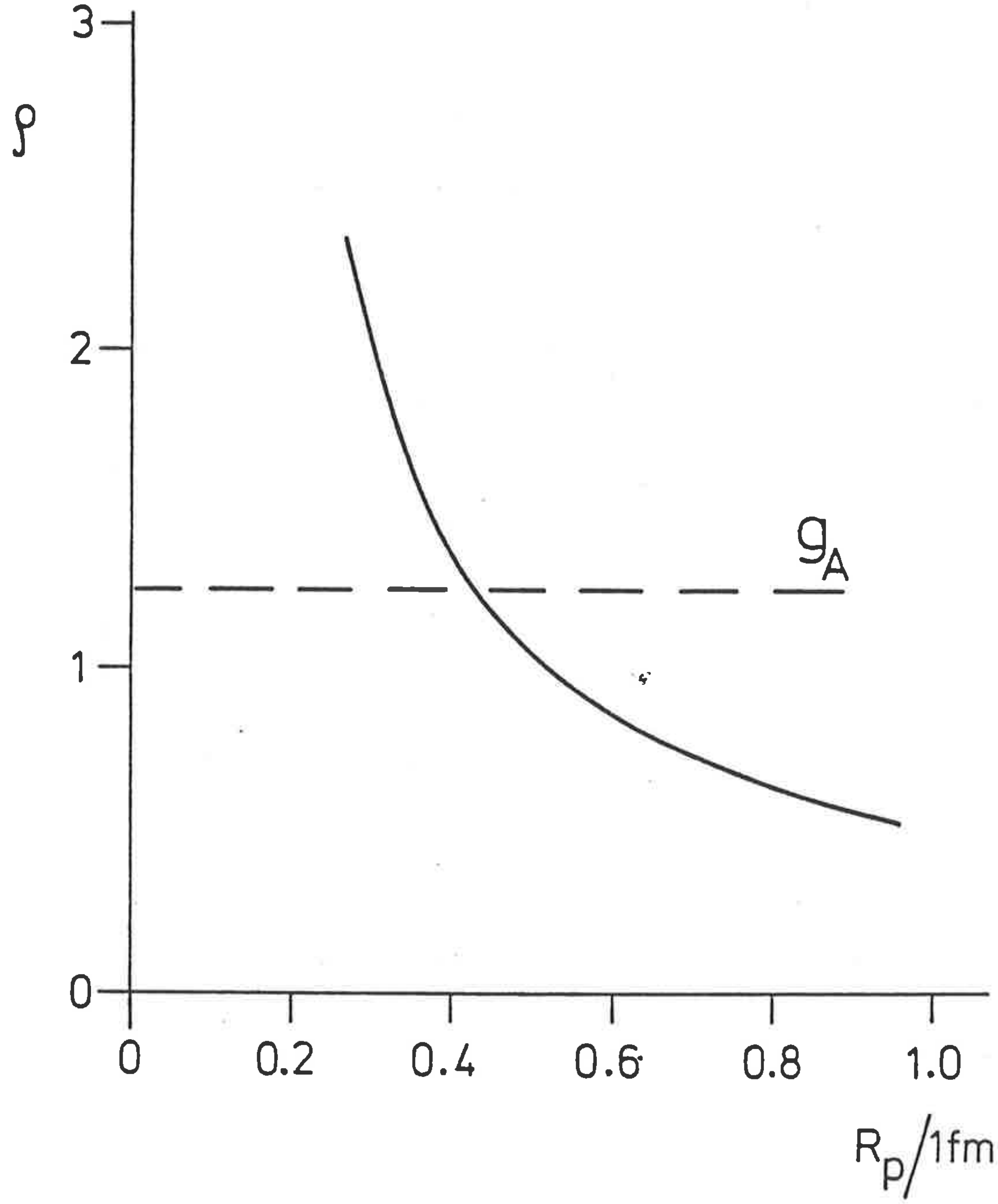


Figure 2

Pole term contributions to nucleon decay.



<u>Figure 3</u>

The ratio $M_{pole}/M_{spectator} = \rho$ as a function of R_p.

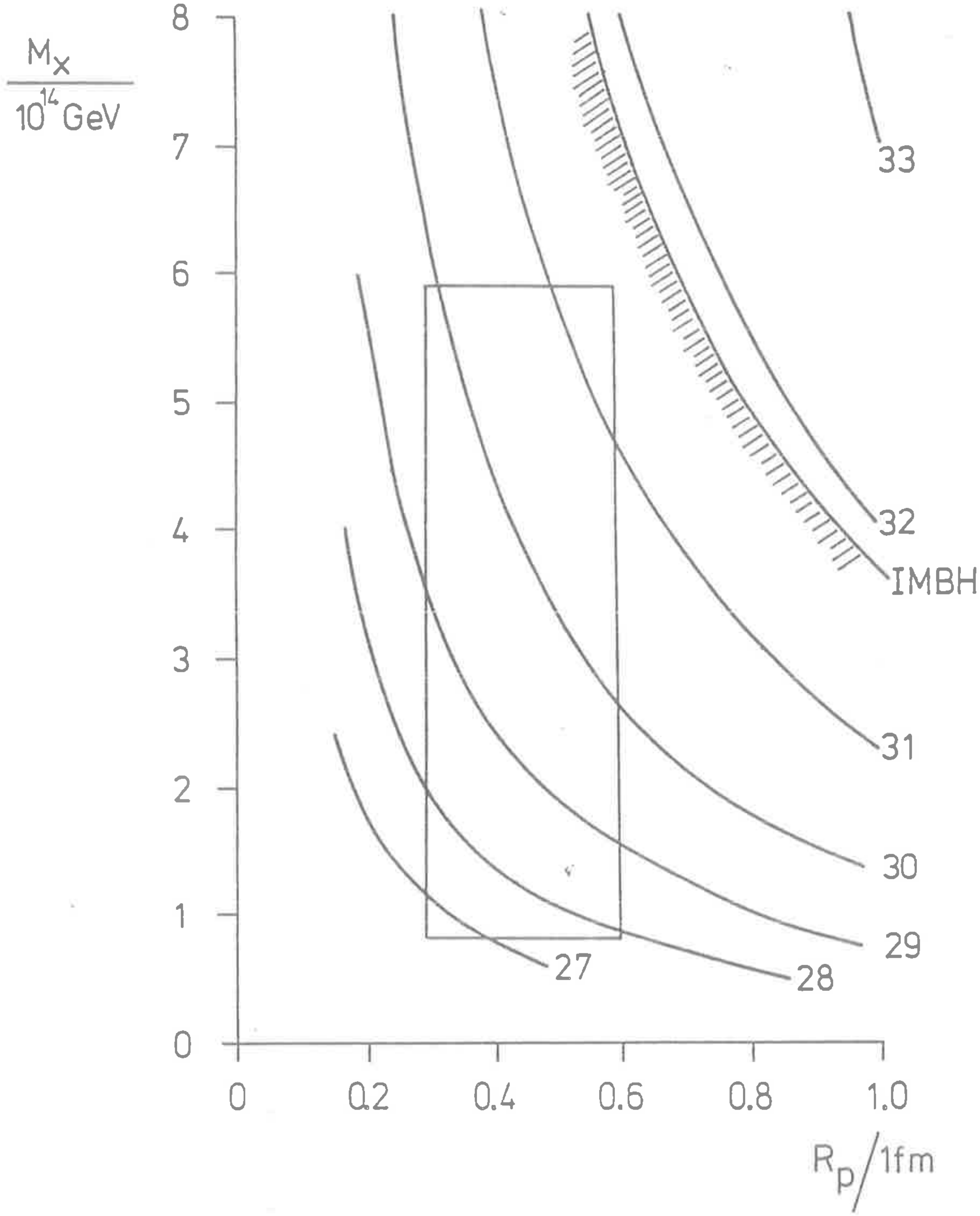


Figure 4

Isochrons (curves of equal lifetime) for the partial lifetime $p \to \pi^o e^+$ in the R_p, m_X plane. The isochrons are labelled by $\log_{10} \tau$. The region to the left and below the curve labelled IMBH ($\log_{10} \tau = 31.8$) is excluded by the result of ref. 7. In the minimal SU(5) model the parameters are confined to the rectangle.

"Six-Quark" Component in the Deuteron from a Comparison of Electron and Neutrino/Antineutrino Structure Functions

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We discuss a way to measure the "six-quark" component in the deuteron from a comparison of the structure functions in ep and ed deep-inelastic scattering and the structure in νp and $\bar{\nu} p$ scattering. Such a determination is obtained by looking at the deviation from 1 in the ratio $T = d(x)\tilde{u}(x)/u(x)\tilde{d}(x)$, where u and d are the quark distributions determined from νp and $\bar{\nu} p$, and $\tilde{u}$ and $\tilde{d}$ are the effective quark distributions determined from ep and ed by neglect of coherent six-quark effects.

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Our present understanding of hadrons as extended objects containing colored quarks and gluons suggests that a nucleus might not always behave as a collection of nucleons. Even in the loosely bound deuteron there is a few percent probability that the nucleons are separated by a distance less than their radius. In such a situation it seems reasonable that instead of talking of two clusters of three quarks one should speak of a single six-quark system.[1,2] Of course, if we were to decompose the six-quark system into clusters they could be either color singlet or octet.[3,4] A specific estimate of about 5% is obtained from models for the deuteron form factor.[5,6] Boundary-condition models yield about 5% for the difference between 1 and the integrated deuteron wave function squared from 1 fm to infinity.[7,8]

Although one might consider fitting low-energy reactions and static deuteron properties in order to determine this probability, it seems to us that deep-inelastic scattering (DIS) is the tool likely to provide the least ambiguous answer.[9] The quark distribution functions in a six-quark system are different from those of a bound proton-neutron system, whose intrinsic quark distributions suffer no polarization correction. One obvious difference is the structure function for $x > 1$ ($x = Q^2/2M_N\nu$ in the usual notation). For the deuteron the kinematically allowed range for x is $0 \leq x \leq 2$. Although taking the momentum of the nucleons in the deuteron into account (the so-called smearing correction) yields structure functions which extend beyond $x = 1$, there will be no typical behavior near $x \simeq 2$ as one would expect from quark counting rules. The high-Q^2 behavior of the deuteron form factor, however, seems to indicate that quark counting rules work quite well.[9-11] The structure functions near $x \simeq 2$ would definitely show the coherent six-quark effects that we are after,[12] but it is doubtful that reliable results can be achieved experimentally.[13]

For x sufficiently large, say $x > 0.3$, we believe that it is not necessary to worry about the contributions of sea quarks. We then have (assuming isospin symmetry)

$$F_2^{ep}(x)/x = [4u(x) + d(x)]/9, \tag{1}$$

$$F_2^{en}(x)/x = [u(x) + 4d(x)]/9, \tag{2}$$

where $u(x)$ and $d(x)$ are the up- and down-valence-quark distributions in the proton. Following the arguments given above we assume that in addition to the smearing correction, one should add a contribution to $F_2^{ed}(x)$ because of the probability of scattering coherently off six quarks (which are not restricted to be in color singlets),

$$F_2^{ed}(x)/x = (1-\delta_6)[F_{2s}^{ep}(x)/x + F_{2s}^{en}(x)/x] + \delta_6[4u^D(x) + d^D(x)]/9. \tag{3}$$

Here $u^D(x) = d^D(x) = n(x)$ are the up- and down-quark distributions in an isosinglet six-quark state (equal because of isospin symmetry); the index s indicates that a smearing correction has been applied.[14] The quantity δ_6 measures the probability that the deuteron behaves like a system of six quarks.

In order to be able to learn something about δ_6

we need to know the quark distributions in Eqs. (1)–(3). The functions $u(x)$ and $d(x)$ may be obtained from νp and $\bar{\nu} p$ scattering. The accuracy with which these functions are extracted, however, is not very high. Perhaps the most accurately known quantity is the ratio $d(x)/u(x)$, which for $x>0.3$ is obtained as the ratio $F_2^{\nu p}(x)/F_2^{\bar{\nu} p}(x)$. Statistically much more accurate determinations of the quark distributions are usually obtained from *ep* and *ed* scattering—but they are not obtained by use of Eqs. (1)–(3). Rather, one customarily uses

$$F_2^{ep}(x)/x=[4\tilde{u}(x)+\tilde{d}(x)]/9, \tag{4}$$

$$\tilde{F}_2^{en}(x)/x=[\tilde{u}(x)+4\tilde{d}(x)]/9, \tag{5}$$

$$F_e^{ed}(x)/x=F_{2s}^{ep}(x)/x+\tilde{F}_{2s}^{en}(x)/x, \tag{6}$$

where we put $\tilde{u}(x)$, $\tilde{d}(x)$, and $\tilde{F}_2^{en}$ to indicate that these are effective distributions deduced from proton and deuteron data. Equating Eqs. (1) and (4), and Eqs. (3) and (6), and assuming a simple smearing correction[14]

$$S(x)=F_2^{ep}(x)/F_{2s}^{ep}(x)$$
$$=\tilde{F}_2^{en}(x)/\tilde{F}_{2s}^{en}(x) \tag{7}$$

one finds the following expressions for the distribution functions $\tilde{u}$ and $\tilde{d}$, extracted from electron scattering (*ep* and *ed*) in terms of the correct distribution functions u and d, extracted from (anti)neutrino scattering (νp and $\bar{\nu} p$):

$$\tilde{u}(x)=u(x)+\delta_6[u(x)+d(x)$$
$$-S(x)n(x)]/3, \tag{8}$$

$$\tilde{d}(x)=d(x)-4\delta_6[u(x)+d(x)$$
$$-S(x)n(x)]/3. \tag{9}$$

For the parametrization of the distribution functions we use the normalized $[\int_0^1 dx\, q(x)=1]$ function

$$q(x;\alpha,\beta)=\frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha)\Gamma(\beta+1)}x^{\alpha-1}(1-x)^{\beta}. \tag{10}$$

We then have $u(x)=2q(x;\alpha_u,\beta_u)$, $d(x)=q(x;\alpha_d,\beta_d)$, and $n(x)=1.5q(x/2;\alpha_6,\beta_6)$.

For the up- and down-quark distributions we have used the functions found from neutrino/antineutrino-hydrogen scattering in Parker *et al.*[15] They are parametrized as $u(x)=2q(x;0.53,2.85)$, and $d(x)=q(x;0.63,3.9)$. Quark counting rules, consistent with the Drell-Yan-West relation,[11,16] indicate that for six quarks the coefficient β_6 in Eq. (10) is equal to $2N_{\text{quarks}}-3=9$. Arguments from Regge theory indicate that the coefficient α_6 is of order 0.5, just as for the distribution functions in the proton. In Fig. 1 we have plotted the distribution functions $xu(x)$, $xd(x)$, and $xn(x)$. For the last function a number of values of the parameters α_6 and β_6 have been considered in order to check the sensitivity to them. For a 5% six-quark probability ($\delta_6=0.05$) the differences between $xu(x)$ and $x\tilde{u}(x)$ and between $xd(x)$ and $x\tilde{d}(x)$ are very small as one may check from Eqs. (8) and (9). To see the effect one would need to determine these functions to very great precision.

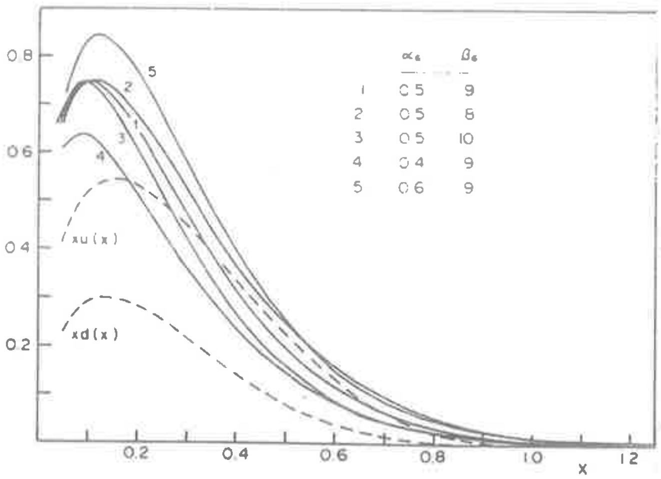

FIG. 1. The up- and down-valence-quark momentum distribution functions $xu(x)$ and $xd(x)$ in the proton, and the nonstrange-quark momentum distribution $xn(x)$ in a six-quark system for various values of the parameters α_6 and β_6 in Eq. (10).

A much more useful quantity is the ratio

$$T=\frac{d(x)/u(x)}{\tilde{d}(x)/\tilde{u}(x)}, \tag{11}$$

which has the following features:

(1) For $\delta_6=0$ it is 1, irrespective of any corrections which are applied to relate the *ed* structure function to the *ep* and *en* structure functions, like the smearing correction, relativistic effects, shadowing, etc.[14]

(2) For $\delta_6\neq 0$ small changes in the way the above corrections are applied are an order of magnitude smaller than the effects of putting in the "six-quark" contribution itself. This is demonstrated in Fig. 2, where the effect for $n(x)=1.5q(x/2;0.5,9.0)$ including the smearing correction[14] (solid line 1) is compared with the same choice for $n(x)$ without any smearing (dashed line).

(3) The ratio $d(x)/u(x)$ is expected to be much less dependent on Q^2 than the quark distributions themselves.[17]

(4) The ratio $d(x)/u(x)$ can be obtained more

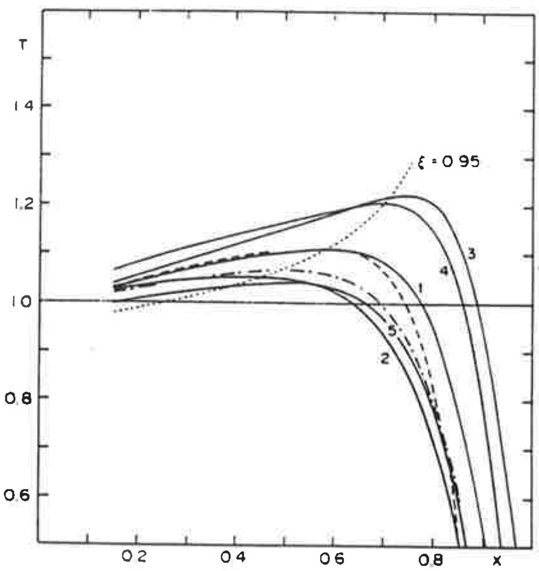

FIG. 2. The calculated value for the ratio $T(x)$ [see Eq. (11)] for various choices for $xn(x)$ (solid lines 1–5; see Fig. 1 for parameters). The smearing correction is taken into account. Neglecting this correction for curve 1 gives the dashed line. The dot-dashed line shows how curve 1 is modified if we take $u(x) = 2q(x;0.5, 3.0)$ and $d(x) = q(x;0.6, 4.0)$. The dotted line shows the result for a scale change in the deuteron [see Eq. (12)].

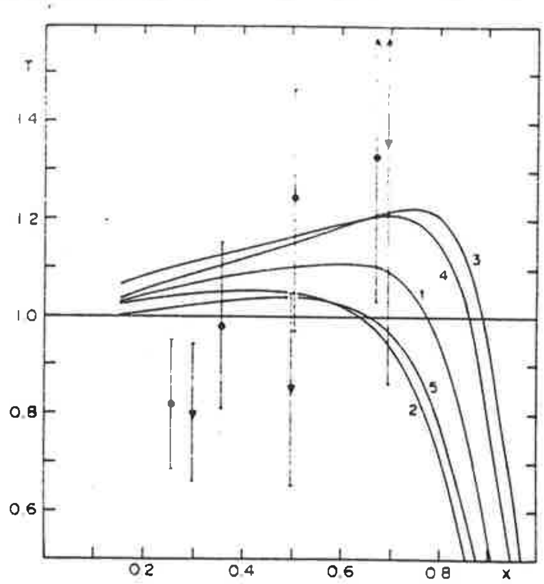

FIG. 3. The comparison of some calculated values for $T(x)$ (solid lines 1–5 from Fig. 2) with the experimental values from Refs. 18 (triangles) and 15 (dots).

accurately from the neutrino data than the quark distributions itself.

(5) Unfortunately, there is a strong dependence on the form of $n(x)$, the nonstrange-quark distribution in a six-quark system. Although the value $\beta_6 = 9$ may be trusted near $x \simeq 2$, the effective form for $n(x)$ in the relevant region $0.3 < x < 0.8$ may be better described with slightly different parameters. The effect of various choices for $n(x)$, and also for different forms for $u(x)$ and $d(x)$, are shown in Fig. 2.

Qualitatively we always find an enhancement of T in the region $0.3 < x < 0.7$. For δ_6 equal to 5% this enhancement is (5–20)%. A quantitative determination of δ_6 is not possible because of the sensitivity to the quark distribution functions. The most optimistic point of view is, of course, that a more accurate experimental determination of T may teach us about both the magnitude of the six-quark contribution and about the distribution function $n(x)$. At this stage one is still far from this, as is shown in Fig. 3, where some of the results for T (see Fig. 2) are compared with the experimentally determined ratio.[15,18]

Recently, it has been conjectured that the difference in structure functions in nuclei as compared to those in the nucleon indicates a change of scale taking place.[19] For the deuteron this means that in the range $0.2 < x < 0.6$ one would have

$$F_2^{ed}(x, \xi Q^2)/x = F_2^{ep}(x, Q^2)/x + F_2^{en}(x, Q^2)/x, \tag{12}$$

where $\xi = \xi(Q^2)$ is proportional to the change of scale squared with a Q^2 dependence caused by the strong coupling constant. Using $F(x, \xi Q^2) \sim \xi^{0.25-x} F_2(x, Q^2)$ (Ref. 9) we can again find $\bar{u}$ and $\bar{d}$ by comparing Eqs. (1), (2), and (12) with Eqs. (4)–(6). The result for T for a rather arbitrarily chosen $\xi = 0.95$ is also shown in Fig. 2. In the region $0.3 < x < 0.7$ such a change of scale has the same qualitative effect on T as a six-quark distribution as discussed by us. At any Q^2 the effect of a change in scale as in Eq. (12) can, of course, be considered as a six-quark contribution as in Eq. (3). Because of the Q^2 dependence of ξ, however, T in this case has a much stronger Q^2 dependence.

Finally we would like to discuss what the effect in the deuteron implies for the "EMC effect," where the structure function F_2^{eA} for some nucleus is compared with F_2^{ed}.[20] We have compared F_2^{ed} with the idealized structure function "F_2^{ed}," which does not contain any six-quark effects, i.e., is given by Eqs. (4)–(6), but with the correct quark distributions u and d instead of the effective ones u and d. The ratio F_2^{ed}/"F_2^{ed}," which might be called the "deuterium EMC effect," is given in Fig. 4 for a set of reasonable parameters ($\delta_6 = 0.05$, $\alpha_6 = 0.5$, $\beta_6 = 9.0$) and is indeed small. From this we can conclude that the error made in analyzing the EMC effect in heavier nuclei[21] (in a six-quark model) be-

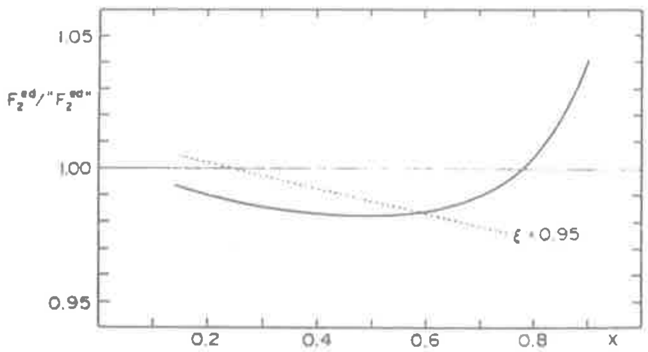

FIG. 4. The "EMC effect for deuterium" for a six-quark contribution (solid line, parameters for curve 1 in Figs. 1–3) and for a scale change in the deuteron [dotted line, see Eq. (12)].

cause of neglect of the same effect in the deuteron is not larger than a few percent, in agreement with results found by Bodek.[22] We have also plotted the effect when $F_2^{\xi d}$ is given by Eq. (12) and come to the same conclusion. We note that in both cases the deviation from 1 in the ratio $F_2^{ed}/``F_2^{ed}"$ is about a factor of 6 smaller than the deviation from 1 in the ratio T. This makes T much more suitable to extract the six-quark effects in the deuteron. For this reason we would very much like to have new high-precision neutrino and antineutrino measurements on hydrogen.

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KAON–NUCLEON SCATTERING IN AN EXTENDED CLOUDY BAG MODEL

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We describe low-energy kaon–nucleon scattering by generalizing the cloudy bag model (CBM) to SU(3) chiral symmetry. We restrict our attention to $I = 0$, s-wave scattering. We obtain a reasonable fit to both the $\Sigma\pi$ cross section in the region of the $\Lambda^*(1405)$ and low-energy kaon–nucleon scattering.

The cloudy bag model (CBM) has had considerable success in describing low-energy pion–nucleon scattering [1]. In this letter we use a chiral SU(3) × SU(3) extension of the cloudy bag model to describe kaon–nucleon scattering near threshold. The original version of the CBM has the mesons coupling to the baryons at the bag surface. Here we use an alternative version [2] obtained by a chiral rotation of the quark wave functions, in which the pions couple throughout the bag volume. This gives much faster convergence for the sum over intermediate quark states [3], and also yields the current algebra results in a transparent manner [2], although it does introduce the complication of a contact term.

In the present letter we ignore backward-going lines and crossed meson lines (the Chew–Low series). From our experience in pion–nucleon scattering [1] this is expected to be a reasonable first approximation. We restrict our attention to $I = 0$ s-wave $\bar{K}N$ scattering, including the $\bar{K}N$ and $\pi\Sigma$ channels. We allow for one excited baryon state which we take to be an SU(3) singlet (with a quark excited to a 1p level). Thus in our model the $\Lambda^*(1405)$ could in principle be either a pure quark state or a $\bar{K}N$ bound state depending on the parameters. In practice we find that both aspects play a role with the $\bar{K}N$ bound state being the more important.

The kaon–nucleon and kaon–nucleus field has been recently reviewed by Dover and Walker [4]. An outstanding problem in kaon–nucleon physics is the difference in the sign [5–8] of the real part of the scattering length obtained from kaon–nucleon scattering and from the kaonic hydrogen energy shift. Analysis of the kaon–nucleon scattering yields a negative sign [6] while the kaonic hydrogen data indicate a positive sign [7]. Attempts to explain this discrepancy using potential models [8] have not been successful.

Kumar and Nogami [9] suggest that it is possible to get a positive sign for the scattering length from the interference between a pole term and the background producing a zero in scattering amplitude near threshold. Our model has a similar zero, but at a much

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higher energy and it is probably associated with the $\Lambda^*(1670)$ resonance.

It is interesting that the $\Lambda^*(1405)$ stands out both in the non-relativistic quark models [10] and in the bag model [11] as one state that is hard to fit. It is usually predicted to be too high. In our model the $\Lambda^*(1405)$ occurs much below the bare quark state mass. However, with one bare quark state (not two) we get not just the $\Lambda^*(1405)$ but also a second resonance which may well represent the $\Lambda^*(1670)$.

For a pedagogic review of the SU(2) × SU(2) version of the cloudy bag model we refer to ref. [12]. The straightforward extension to chiral SU(3) × SU(3), with volume coupling, yields the lagrangian:

$$\mathcal{L}_{\theta_\mathrm{v}} = [\mathrm{i}\bar{\mathrm{q}}\not{D}\mathrm{q} - B]\,\theta_\mathrm{v} - \tfrac{1}{2}\bar{\mathrm{q}}\mathrm{q}\delta_\mathrm{s} + \tfrac{1}{2}[\mathrm{D}_\mu\boldsymbol{\phi}]^2 + (1/2f)\mathrm{q}\gamma^\mu\gamma_5\,\boldsymbol{\lambda}\cdot\mathrm{q}\,[\mathrm{D}_\mu\boldsymbol{\phi}]\,\theta_\mathrm{v}\,. \tag{1}$$

Here q is the quark field (with colour indices suppressed) and $\boldsymbol{\phi}$ is the meson-octet field. The energy density of the vacuum is B, θ_v is a step function which vanishes outside the bag, and δ_s is a surface delta function which reduces to $\delta(r-R)$ for a static spherical bag. The $\boldsymbol{\lambda}$ are the SU(3) Gell-Mann matrices and f the meson-octet decay constant. Consistent with the CBM philosophy we expand the lagrangian in powers of $1/f$ and we obtain the interaction hamiltonian which to order f^{-2} has two terms. The first-order term is:

$$H_1 = -(\theta_\mathrm{v}/2f)\bar{\mathrm{q}}\gamma^\mu\gamma_5\boldsymbol{\lambda}\mathrm{q}\cdot\partial_\mu\boldsymbol{\phi} \tag{2}$$

which couples the $\pi\Sigma$ or $\bar{\mathrm{K}}\mathrm{N}$ to the bare Λ^*. The second-order term is:

$$H_2 = (\theta_\mathrm{v}/4f^2)\bar{\mathrm{q}}\gamma^\mu\boldsymbol{\lambda}\cdot\mathrm{q}\,\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi}, \tag{3}$$

which is a contact term or four-point interaction.

As usual [12] we project this hamiltonian onto the space of non-exotic baryon bags. Then neglecting all diagrams with backward-going lines and crossed meson lines, we obtain a "potential":

$$V_{\alpha\beta} = \langle\alpha|H_1|\Lambda^*\rangle(E-m_0)^{-1}\langle\Lambda^*|H_1|\beta\rangle + \langle\alpha|H_2|\beta\rangle \tag{4}$$

and solve the corresponding Lippmann–Schwinger equation. The states $\langle\alpha|$ and $\langle\beta|$ stand for the various $\bar{\mathrm{K}}\mathrm{N}$ and $\pi\Sigma$ states. The $\langle\Lambda^*|$ indicates the bare Λ^*, a pure quark state, and m_0 its bare mass. The Λ^* is treated on the same level as the N and Σ. We assume it to be in a $(70,1^-)$ representation and a SU(3) singlet [11,13]. For the propagator in the Lippmann–Schwinger equation we use $G(k) = (E - E' - W + \mathrm{i}\epsilon)^{-1}$, where $E'^2 = k^2 + M^2$ is the energy of the propagating baryon and $W^2 = k^2 + m^2$ the energy of the corresponding meson.

Because of the form-factor at the vertices, arising from the finite size of the baryons, the renormalizations are all finite and there is no need to explicitly eliminate m_0 and the bare coupling constants in terms of renormalized ones. In principle we should have a complete set of quark intermediate states rather than just the Λ^* in eq. (4). For the volume coupling we believe these other states give us a relatively small contribution and this has been confirmed in the case of pion–nucleon scattering [3].

It now just remains to calculate the matrix elements needed in eq. (4). The quark wave functions are well known [13]. We begin with the matrix elements for H_1. Rather than evaluating them directly it is useful to do an integration by parts. Using the Dirac equation and the quark boundary condition this yields for massless quarks:

$$\begin{aligned}\langle BM(k)|H_1|\Lambda^*\rangle &= -(\lambda_{B\Lambda^*}/2f)C^{i_{B_3}i_{M_3}0}_{i_B i_M 0}\{N_\mathrm{s}N_\mathrm{p}/[(2\pi)^3 2\omega_M(k)]^{1/2}\} \\ &\times\Big(2R^2 j_0(\omega_\mathrm{s}R)j_0(\omega_\mathrm{p}R)j_0(kR) \\ &- [\omega_\mathrm{s} - \omega_\mathrm{p} + \omega_M(k)]\int_0^R \mathrm{d}x\, x^2[j_0(\omega_\mathrm{s}x)j_0(\omega_\mathrm{p}x) \\ &+ j_1(\omega_\mathrm{s}x)j_1(\omega_\mathrm{p}x)]j_0(kx)\Big),\end{aligned} \tag{5}$$

where $B(M)$ stands for $\mathrm{N}(\bar{\mathrm{K}})$ or $\Sigma(\pi)$, $\lambda_{\mathrm{N}\Lambda^*} = \sqrt{2}$ and $\lambda_{\Sigma\Lambda^*} = \sqrt{3}$.

The energies $\omega_\mathrm{s} = 2.04/R$ and $\omega_\mathrm{p} = 3.81/R$ refer to the ground state and first excited p-wave quark state, respectively. The quark state normalizations N_s and N_p are well known [12]. For surface coupling we would have just the first term in the curly brackets in eq. (5).

In contrast to scattering through the Λ^*, scattering through the contact term is not pure $I = 0$. Thus we must project out the $I = 0$ part. For the case of s-wave scattering the spatial part of the covariant derivative in H_2 does not contribute, so we just quote the result coming from the time derivative for $I = 0$:

$$\langle BM(k)|H_2|B'M'(k')\rangle$$
$$= -(\lambda_{BB'}/2f^2) C^{i_{B_3} i_{M_3} 0}_{i_B i_M 0} C^{i_{B'_3} i_{M'_3} 0}_{i_{B'} i_{M'} 0} N_s^2$$
$$\times \{[\omega_M(k) + \omega_{M'}(k')]/(2\pi)^3 [2\omega_M(k) 2\omega_{M'}(k')]^{1/2}\}$$
$$\times \int_0^R \mathrm{d}x\, x^2 [j_0^2(\omega_s x) + j_1^2(\omega_s x)] j_0(kx) j_0(k'x)\,, \quad (6)$$

where $\lambda_{NN} = 3/2$, $\lambda_{\Sigma\Sigma} = 2$ and $\lambda_{N\Sigma} = \lambda_{\Sigma N} = \sqrt{6}/4$.

Let us now consider the results for $\bar{K}N$ and $\pi\Sigma$ s-wave $I = 0$ scattering. The threshold are taken to be 1332.1 and 1432.6 MeV for the $\pi\Sigma$ and $\bar{K}N$ channels, respectively.

In our calculation the contact term plays a very important role, giving (by itself) a bump in the $\pi\Sigma$ elastic cross section around 1390 MeV and producing a large scattering amplitude near the $\pi\Sigma$ threshold. As a consequence the results are very sensitive to 10% variations in the value of the meson decay constant (f). In fact, there is not one decay constant but two, with f_π (93 MeV) being some 20% lower than f_K (112 MeV) [14]. However, at the present stage we prefer to take the interaction given in eq. (3) in the limit of exact SU(3) symmetry as a guide, without specifying how the symmetry is broken in the real world. In that case it seems reasonable to tolerate some phenomenological variation in f in the region of 100 MeV. Happily the best fit values, namely 110 MeV with $R = 1.1$ fm and 120 MeV with $R = 1.0$ fm do satisfy this criterion. Finally we note that the bare mass of the Λ^* is also treated as an adjustable parameter, but it turns out that the results are insensitive to its exact value.

In fig. 1 we compare our results with the $\pi\Sigma$ mass distribution given by Chao et al. [15] for $\pi^- p \to (\pi\Sigma)^0 K^0$. The theoretical curves are obtained by multiplying the $\pi\Sigma$ cross section by a phase space factor and an arbitrary normalization factor chosen to get roughly the right height for the distribution. The solid curve corresponds to $R = 1.0$ fm, $f = 120$ MeV, and the dashed one to $R = 1.1$ fm and $f = 110$ MeV. In both cases the bare mass is 1600 MeV. In the present model the Λ^* is largely a $\bar{K}N$ bound state, since most of the contribution to this resonance comes from the contact term. It is due to the strong attraction of the contact term that the $\Lambda^*(1405)$ resonance occurs almost 200 MeV below the bare quark state mass.

It must be pointed out that the present model has

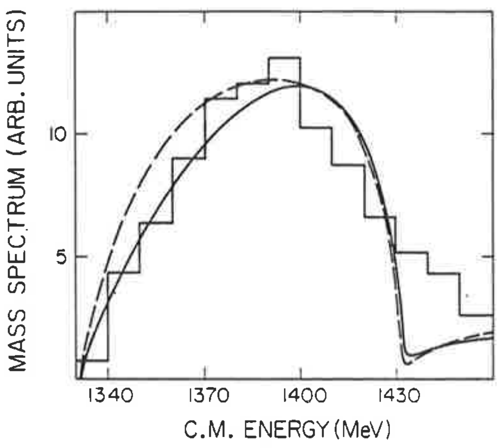

Fig. 1. A plot of the final-state $\pi\Sigma$ mass distribution. The histogram is the data from Chao et al. [15]. The theoretical curves are calculated as discussed in the text, with an arbitrary normalization. The parameters are $R = 1.0$ fm, $f = 120$ MeV (solid curve) and $R = 1.1$ fm, $f = 110$ MeV (dashed curve). The bare mass (m_0) is taken as 1600 MeV.

not only the $\Lambda^*(1405)$ resonance but also a resonance at a higher energy presumably corresponding to the $\Lambda^*(1670)$, although the present model is not sufficiently accurate at such high energies to make more than qualitative statements. [It leaves out the $\eta\Lambda$ channel and the wide $\Lambda^*(1800)$.]

It is interesting to compare the cross sections with the experimental results. Unfortunately the experimental data do not separate the s-waves from higher partial waves and, in the $K^- p$ elastic scattering, the $I = 0$ from the $I = 1$ contribution. For this reason the

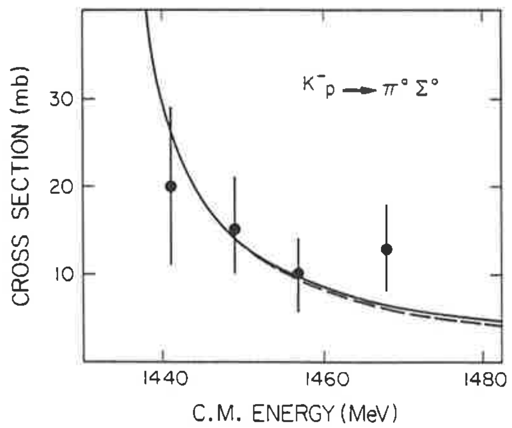

Fig. 2. The $I = 0$ s-wave $K^- p \to \pi^0 \Sigma^0$ cross sections for the sets of parameters. The same convention is used as in fig. 1. The data are from ref. [16].

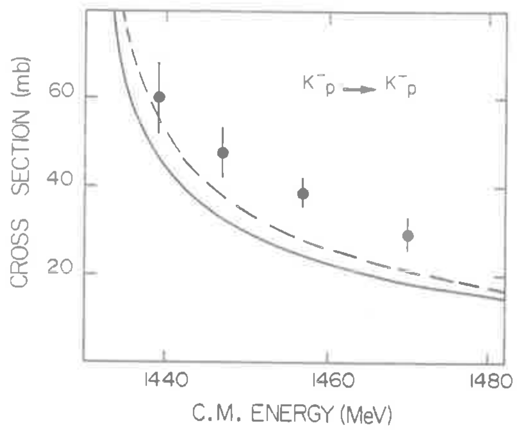

Fig. 3. The curves are the $I = 0$ s-wave K^-p elastic cross sections. The convention is the same as in fig. 1. The data, taken from ref. [16], refer to the $I = 0 + I = 1$.

comparison can be made only qualitatively. Potential calculations [16] indicate that below about 250 MeV/c the $\bar{K}N$ elastic and absorptive cross sections are predominantly s-wave. In fig. 2 we plot the $I = 0$ $K^-p \rightarrow \pi^0\Sigma^0$ cross section for the same set of parameters discussed above. The curves, which are very insensitive to the parameters, are in fairly good agreement with the data.

In fig. 3 we plot the $I = 0$ K^-p elastic cross section. The experimental data plotted are not $I = 0$ but rather $\frac{1}{2}[\sigma(K^-p \rightarrow K^-p) + \sigma(K^-p \rightarrow \bar{K}^0 n)]$. The fits are reasonable, particularly if one takes account that the data have a small $I = 1$ contribution (see ref. [16]).

The $I = 0$ $\bar{K}N$ scattering lengths for the two sets of parameters are:

$$a = -1.16 + 1.44\mathrm{i}\ \mathrm{fm}$$

with $R = 1.0$ fm, $f = 120$ MeV,

and

$$a = -1.36 + 1.55\mathrm{i}\ \mathrm{fm}$$

with $R = 1.1$ fm, $f = 110$ MeV.

For comparison the value obtained by Martin [6] from scattering using dispersion relation constraints is $-1.70 + 0.68\mathrm{i}$ fm. Although the real part of the scattering length is smaller in our calculation, it is still negative – in apparent contradiction with the K^-p atomic data. On the other hand, the K^-p amplitude calculated here undergoes a very rapid variation below threshold which may resolve the problem. This certainly deserves to be investigated further.

In conclusion, we note that in spite of the large mass of the kaon the SU(3) CBM seems to be a good starting point for describing low energy $\bar{K}N$ scattering.

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Mass differences between mirror nuclei in a hybrid quark-nucleon model

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A hybrid quark-nucleon model of nuclei is developed in which nucleons merge into multiquark bags at short distances. This model is applied to calculate mass differences between ^{3}He and ^{3}H and a number of other mirror nuclei. For light nuclei we obtain a reduction of the discrepancy between experiment and conventional theory. Probabilities for the formation of six-quark bags and nine-quark bags in these nuclei are evaluated, and the consequences of our results are discussed. In particular we comment on the compatibility of conventional and the hybrid quark-nucleon results.

I. INTRODUCTION

In the last few years considerable excitement has been generated by the idea that we may one day achieve a truly unified description of nucleon and nuclear structure.[1-4] For the present most attempts at such a unification are necessarily based on quantum chromodynamic (QCD) motivated phenomenology, rather than QCD itself. Given the present diversity of models of hadron structure, it is hardly surprising that there is no agreement on how to deal with nuclei. Nevertheless, a number of attempts have already been made to describe the short-distance N-N force at the quark level.

Undoubtedly the most sophisticated calculations of the N-N force using the quark model have been based on the nonrelativistic (constituent) quark model.[5-8] There, one has the tremendous technical advantage that one can draw on long experience in the application of resonating group methods to light ion reactions. With the addition of a long range interaction associated with pion exchange, this approach has even achieved semiquantitative agreement with *S*-wave N-N data.

The first application of the bag model (which has the advantage of avoiding color van der Waals forces) was similar in spirit to the nonrelativistic quark model.[9] But recently the *P*-matrix method[10] has been more commonly used—again with considerable success.[11-16] The essential idea is that inside some boundary radius the N-N system should be described as a six-quark ($6q$) bag. By analogy with the old *R*-matrix theory, one demands that the exterior wave function vanishes at the boundary when the total energy of the system matches the mass of the internal $6q$ state.

There is no doubt that in order to be credible in nuclear physics, any quark model must eventually provide a fit to N-N elastic scattering data at least as good as that provided by the Paris potential.[17] However, that day may be some years away. In the meantime it is tempting to assume that eventually such a fit will be achieved, and to ask whether this new description of the short distance physics may have other consequences.[4,18] Examples of such applications include the electrodisintegration of the deuteron[19] and ^{3}He,[20] parity violation in the N-N system,[21,22] the EMC effect,[23-25] and so on.

In this paper we address the theoretical question of how to make consistent calculations in finite nuclei using such a hybrid quark-nucleon model. As an example we investigate the consequences for the systematic discrepancy in the energy differences of mirror nuclei—the Nolen-Schiffer anomaly.[26-28] In particular, it has already been observed that the mass difference between $6q$ bags formed from two protons and two neutrons is not equal to twice the proton-neutron mass difference.[4] Similar arguments apply to $9q$ bags if the overlap of more than two nucleons is important. These mass differences amount to a somewhat different model of the charge-symmetry violating N-N force, and therefore could contribute to reducing the size of the Nolen-Schiffer anomaly.

The plan of the paper is as follows. In Sec. II we discuss the mass differences of $6q$ bags. In Sec. III we develop the formalism for a hybrid description of nuclear systems. For the present problem, where we deal only with energy differences, the probability that two nucleons are within the critical radius b is the essential quantity. This probability is defined and calculated for several values of b in Sec. IV. Because it is amenable to exact treatment the 3N system is dealt with separately in Secs. V and VI. The results for larger nuclei are presented in Sec. VII, and a brief discussion follows in Sec. VIII.

II. MASS DIFFERENCES IN THE BAG MODEL

At the quark level the n-p mass difference must have at least two sources.[29] In fact, the Coulomb interaction would make the proton heavier than the neutron—typically by ~ 0.5 MeV.[29,30] Within about 10% this result can be represented by

$$\Delta M_{\mathrm{em}} = 1.44 \sum_{i<j} \frac{Q_i Q_j}{R} , \tag{2.1}$$

which yields $m_{\mathrm{p}} - m_{\mathrm{n}} = 0.48$ MeV with $R = 1$ fm. In order to explain the observed mass difference one needs to assign different masses to the quarks themselves. Within the bag model the energy of a quark is $E=\omega/R$, with ω the eigenfrequency implied by the nonlinear boundary condition.[4,31] By numerical solution of the Dirac equation for a quark of mass m, in a cavity of radius R, one finds[32] for $R\approx 1$ fm:

$$\frac{d\omega}{d(mR)}\approx 0.49 \ . \tag{2.2}$$

A quark mass difference of $(m_d-m_u)\sim 4$ MeV (Refs. 32 and 33) then gives the correct n-p mass difference, provided the difference in the color hyperfine interaction for a d and u quark[32] is taken into account as well. The hyperfine interaction term can approximately be accounted for by using the value 0.42 instead of 0.49 in the left-hand side of Eq. (2.2).

Let us now consider the region where two nucleons overlap sufficiently to be considered a $6q$ bag—in the boundary condition model this is when $r<b$. Clearly the Coulomb interaction (2.1) will now involve a sum over $i\in(1,6)$, which will, e.g., not be simply twice the value obtained in the proton for a bag containing $4u$ and $2d$. A further correction arises because the radius of a $6q$ bag is about 20% bigger than a $3q$ bag. For the Coulomb force this is trivial to include, but for the quark mass effect it is much more unclear.

If the quark mass m really was a scalar number independent of the environment, $(\omega_u-\omega_d)/R$ would not change between a $3q$ and a $6q$ bag, because of Eq. (2.2). However, we know that the light quark masses are still a mystery. They are presumably the residual effect of renormalization due to interactions with a much larger energy scale, and in a cavity they may depend on the size. For dimensional reasons it would be natural to set $m\propto R^{-1}$ (the weaker assumption $m_d-m_u\sim R^{-1}$ is in fact sufficient), in which case we find

$$(E_d-E_u)_6=(E_d-E_u)_3\left[\frac{R_3}{R_6}\right] . \tag{2.3}$$

[Here $(E_d-E_u)_i$ is the difference in the total energy of a d and u quark in a bag of i quarks, and (R_3,R_6) are the corresponding bag radii.] It is interesting that an R-dependent mass was phenomenologically necessary for Deshpande *et al.* to reproduce the mass differences for the strange members of the nucleon octet.[30] We shall adopt Eq. (2.3) here as a working hypothesis. However, we stress that it is no more than that in the absence of a deeper theoretical understanding of light quark masses.

Even worse, from the point of view of serious quantitative predictions we note that there are other mass dependent corrections to the mass of the MIT bag, for which there is as yet no theoretical consensus on the sign.[34,35] Clearly we are at an early stage of understanding quark dynamics, and one cannot expect high precision in the predictions. Nevertheless, it is our belief that Eqs. (2.1) and (2.3) should provide at least an indication of the magnitude of the charge-symmetry violation to be expected in a quark bag model.

III. HYBRID MODEL OF QUARKS IN NUCLEI

Symbolically we represent the N-N system as follows:

$$\begin{aligned}&\Psi=\mathscr{A}\Psi_1\Psi_2\phi_{12}(\vec{r}),\ \ r_{12}>b\ ,\\&\Psi=C\Phi_6(\xi_1,\ldots,\xi_6),\ \ r_{12}<b\ .\end{aligned}\tag{3.1}$$

In these equations Ψ_1 and Ψ_2 represent normalized nucleon wave functions, i.e., nonrelativistic Pauli spinors, while $\phi_{12}(\vec{r})$ is the relative two-nucleon wave function. The six-quark wave function is written as a product of a normalized wave function ϕ_6 and a probability amplitude C. Larger nuclei are then described using conventional models modified to account for the short-range behavior implied by Eqs. (3.1).

Clearly this does not provide for a complete description of the strong dynamics in nuclei; however, before developing the model in further detail we want to consider what information is needed to calculate the mass differences according to our prescriptions in Sec. II. Essential for these calculations will be the six quark probability $|C|^2$, the transition radius b, the bag radii R_3 and R_6, and to a smaller extent the radius R_9. Obviously these five quantities are not independent, although their exact relationship depends on the details of the model or theory. If we assume that the quark density in the $(3n)$-quark bags is constant, then $R_{3n}=n^{1/3}R_3$. On the other hand, if we consider a multiquark MIT bag with just a volume and a mass term $(\sim 1/R)$, and assume that all quarks are in an S state, then the nonlinear boundary condition leads to $R_{3n}=n^{1/4}R_3$. We have used a conservative value for the exponent between these two extremes, namely 0.27. In a more detailed description one would also expect to find a relation between b and the bag radii; however, we treat b as a free parameter (within reasonable limits). The six-quark probability $|C|^2$ depends strongly on b and will in general not be treated as a free parameter. In the following we discuss various different approaches, all of which give a unique determination of $|C|^2$ for a specific b.

If $\phi_{12}(\vec{r})$ in Eq. (3.1) is taken to be identical to the conventional nuclear wave function, then $|C|^2$ automatically equals the probability defect of this wave function for $r<b$. This is the simplest prescription for the six-quark probability. We also consider the following modifications. First, because of the different strong dynamics for $r<b$, the probability to find six quarks with $r<b$ does not have to be the same as that of finding two nucleons with $r<b$ in the conventional picture. To accommodate this change we could allow for a different normalization of the external wave function, even though its shape remains the same. Second, the effective potential for $r>b$ may have to be modified to accommodate the different dynamics for $r<b$. This would even lead to a different shape of the external wave function.

In order to decide which approximations are most appropriate for calculating $|C|^2$, we look for guidance in the nonrelativistic quark potential calculations.[5-8] All of these calculations indicate that there is no sudden decrease in the six-quark probability for small r. The sign change of the S-wave phase shifts, which is usually explained by short-range repulsion or equivalently by the vanishing of the short-range N-N wave function, can then be interpreted as the absence of N-N components in the short-range six-quark wave function or as a node in the conventional wave function for small r. Therefore, if we want to determine the six-quark probability from the conventional wave function defect, we should not use strongly repulsive N-N potentials for the short-distance behavior. In nuclei we can, therefore, use uncorrelated shell-model wave functions rather than correlated ones, unless the correlation function only represents a modest short range repulsion. We thus see that the use of a hybrid quark-nucleon model can even lead to a simpler description of nuclei. In most of our calculations we have employed the uncorrelated wave functions; however, to check the stability of our results against this particular assumption we have also performed some calculations with correlation functions, and some calculations in which $|C|^2$ is basically treated as a free parameter.

For the three-body system, where exact conventional wave functions are available, we have also opted for a simpler uncorrelated wave function. Because of the quark-potential argument above it does not seem appropriate to base the wave function on conventional potentials; in addition the three-body calculations have been rather unsuccessful in reproducing the major physical properties of the three-body system (the three-body binding energy and the charge form factor), so that the arguments for using the "exact" wave functions are not particularly strong. We note in this connection that exact three-body wave functions have been used for the study of quark effects in deep inelastic scattering by Vary.[20]

In a recent study,[36] where a similar boundary condition model was used for the description of continuum wave functions in the two-body system, it was shown that current conservation guarantees the identity of the six-quark probability and the conventional wave function defect for $r<b$ as long as we do not change the interaction for $r>b$. Thus it may seem that the six-quark probability is independent of the internal dynamics. This is of course not the case. It is simply that in Ref. 36 only those descriptions for $r<b$ are allowed which together with the conventional potential for $r>b$ lead back to the original phase shifts. Whether there exists a model of the internal dynamics which can satisfy such a constraint is still an open question. While this identity was derived for the continuum case, it has subsequently also been stated to hold in the bound state case.[36,37] This conclusion is clearly subject to the same caution expressed for the continuum case. If true, it guarantees automatically the correctness of our first prescription for calculating $|C|^2$.

For larger nuclei, where we have little information on the normalization of the conventional wave function, we do not have any strong constraints on the six-quark probability and we have to rely on our physical intuition to decide which of the possible options for determining $|C|^2$ are most reasonable.

Let us now describe in some more detail our hybrid model of nuclei. Using the uncorrelated shell model wave function we can easily write the conventional part of the wave function of the $(A+1)$-nucleon system as the following:

$$\Psi_N(1,2,\ldots,A+1)=\mathscr{A}\left\{\prod_{i<j}^{A+1}[1+f_{ij}(r_{ij})]\times\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right\}, \tag{3.2}$$

where

$$f_{ij}(r_{ij})=-\theta(b-r_{ij}), \tag{3.3}$$

and the $\phi_{\alpha_i}(i)$ are normalized single-particle states with quantum numbers α_i. For the correlated shell-model wave function, f_{ij} should be nonzero for $r_{ij}>b$ and the wave function should be renormalized. The third possible description is to change the normalization of Ψ_N arbitrarily, and to maintain the correct overall normalization by adjusting the six-quark probability.

Since the valence particle, in which we are interested mostly, is characterized by its single-particle quantum numbers α_v, we prefer to represent ψ_N as follows:

$$\Psi_N(1,2,\ldots,A)=\mathscr{A}\left\{\prod_{\alpha_i<\alpha_j}^{\alpha_v}[1-\theta(b-r_{\alpha_i\alpha_j})]\times\prod_{i'=1}^{A+1}\phi_{\alpha_{i'}}(i')\right\}. \tag{3.4}$$

The radius $r_{\alpha_i\alpha_j}$ should now be considered as an operator defined by the following:

$$r_{\alpha_i\alpha_j}\phi_{\alpha_i}(m)\phi_{\alpha_j}(n)=r_{mn}\phi_{\alpha_i}(m)\phi_{\alpha_j}(n). \tag{3.5}$$

This notation has the advantage that $\mathscr{A}$ can operate directly on the single-particle wave functions as it commutes with $r_{\alpha_i\alpha_j}$. Since we are mainly concerned with the state of the valence particle and do not care whether the core particles form six-quark bags between themselves, we define the new "conventional" wave function

$$\psi_N^v(1,2,\ldots,A)=\mathscr{A}\left\{\prod_{\alpha_i<\alpha_v}[1-\theta(b-r_{\alpha_i\alpha_v})]\times\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right\}, \tag{3.6}$$

which is constructed to guarantee that the valence particle is not in a six-quark bag. In Eq. (3.6) we have represented the core wave function by a single determinant, so that it also includes the six-quark configurations for $r_{\alpha_i\alpha_j}<b$ with $\alpha_i<\alpha_j<\alpha_v$. This is why we put our "conventional" in quotation marks. If we now define the full Slater determinant by

$$\Psi^0(1,2,\ldots,A)=\mathscr{A}\left[\prod_{i=1}^{A+1}\phi_{\alpha_i}(i)\right], \tag{3.7}$$

then we can interpret

$$P_Q^v = \langle \Psi^0 | \Psi^0 \rangle - \langle \Psi_N^v | \Psi_N^v \rangle = 1 - \langle \psi_N^v | \Psi_N^v \rangle \tag{3.8}$$

as the probability of the valence particle being part of one or more six-quark bags. Since we have only discussed the situation that two nucleons merge into a single six-quark bag, it is convenient to define

$$| \psi_{Q_1}^v \rangle = \mathcal{A} \left\{ \sum_{\alpha_i} \theta(b - r_{\alpha_i \alpha_v}) \prod_{\alpha_j \neq \alpha_i}^{\alpha_A} [1 - \theta(b - r_{\alpha_j \alpha_v})] \times \prod_{i=1}^{A+1} \phi_{\alpha_i}(i) \right\} , \tag{3.9}$$

which represents that part of the full wave function for which the valence particle forms a six-quark bag with a single core nucleon. The corresponding probability

$$P_{Q_1}^v = \langle \psi_{Q_1}^v | \psi_{Q_1}^v \rangle \tag{3.10}$$

can therefore be used in connection with our model of mass differences between six-quark bags. Calculating $P_{Q_1}^v$ implies calculating the expectation value of an $(A+1)$-body operator for $|\psi_0\rangle$. This is only feasible for the three-body case ($A=2$), as we will demonstrate in Sec. V. For now we deal with the large A case, and we rely on an expansion in the correlation function, observing that matrix elements of $\theta(b-r_{ij})$ will be small if b is small. The lowest order result in $\theta(b-r_{ij})$, which will be denoted by P_b^v, is a sum of single particle terms:

$$P_b^v = \sum_{\alpha_m = \alpha_1}^{\alpha_A} P_{\alpha_m}(b) = \sum_{nlji_z} (2j+1) P_{nlji_z}(b) , \tag{3.11}$$

where

$$P_{\alpha_m}(b) = \langle \phi_{\alpha_v}(1) \phi_{\alpha_m}(2) | \theta(b - r_{12}) | \phi_{\alpha_v}(1) \phi_{\alpha_m}(2) - \phi_{\alpha_v}(2) \phi_{\alpha_m}(1) \rangle . \tag{3.12}$$

Notice that despite the fact that $|\psi_{Q_1}\rangle$ is first order in $\theta(b-r_{ij})$, the quadratic expression (3.10) is still first order in $\theta(b-r_{ij})$ because of the identity

$$\theta(b - r_{ij}) \theta(b - r_{ij}) = \theta(b - r_{ij}) . \tag{3.13}$$

The lowest order result (3.12) is identical to what we would have obtained from Eq. (3.8) in lowest order. Although it looks remarkably similar to the matrix element of a residual short-range interaction, it would be wrong to identify the operator $\sum_{i<j} \theta(b-r_{ij})$ this way, since for the higher-order terms such an interpretation breaks down.

The calculation of higher order terms becomes more and more difficult. We can avoid these complications by giving a classical interpretation to (3.11), namely by interpreting $P_{nlji_z}(b)$ as the probability for the valence particle to be within a distance b of a *specified* core particle with quantum numbers $nlji_z$. Then, by assuming that the chance for the valence particle to overlap with a core particle does not depend on whether it already overlaps with other core particles, we can calculate all required probabilities in a straightforward fashion. For example, the chance for the valence nucleon to form exactly one and only one pair with a core nucleon is given by the following:

$$P_{Q_1}^v = \sum_{nlji_z} (2j+1) P_{nlji_z}(b) / [1 - P_{nlji_z}(b)] \times \prod_{n'l'j'i_z'} [1 - P_{n'l'j'i_z'}(b)]^{2j'+1} . \tag{3.14}$$

In practice the dependence on single-particle quantum numbers is completely insignificant in calculating these average properties, and one might just as well use the average probability P_b^v/A. With this simplification we can write

$$P_{Q_1}^v = P_b^v (1 - P_b^v / A)^{A-1}$$

and

$$P_{Q_2}^v = \begin{bmatrix} A \\ 2 \end{bmatrix} \left[\frac{P_b^v}{A} \right]^2 \left[1 - \frac{P_b^v}{A} \right]^{A-2} . \tag{3.15}$$

Finally, the chance for the valence particle to form at least one six-quark bag is

$$P_Q^v = 1 - \left[1 - \frac{P_b^v}{A} \right]^A . \tag{3.16}$$

In the three-body case we can evaluate all of these quantities explicitly using the quantum mechanical expressions and therefore we can test our semiclassical model explicitly. Not unexpectedly, it will appear that the model does not work very well in the three-body case. However, the suggested modifications for the three-body case, when applied to the many-body case, do not change the results significantly, so that it appears that the semiclassical model can be used with some confidence in the many-body case.

IV. THE PROBABILITY FOR OVERLAP OF THE VALENCE NUCLEON WITH A NUCLEON IN THE CORE

In Eqs. (3.11) and (3.12) we defined the overlap probability $P_{nlji_z}(b)$, which can also be written as follows:

$$P_{nlji_z}(b) = \frac{1}{2j_v+1} \frac{1}{2j+1} \sum_{\alpha_v \alpha_m} \langle \phi_{\alpha_m}(1) \phi_{\alpha_v}(2) | \theta(b - r_{12}) | \phi_{\alpha_m}(1) \phi_{\alpha_v}(2) - \phi_{\alpha_m}(2) \phi_{\alpha_v}(1) \rangle , \tag{4.1}$$

where the sum is over the magnetic substates. We assume that the differences between neutron and proton orbits can be

TABLE I. Probabilities for the valence nucleon to form six-quark bags with one ($P^v_{Q_1}$) or two core nucleons ($P^v_{Q_2}$). The transition radius is $b=0.95$ fm. The values in parentheses are defined using higher order correlations as described in Sec. V.

Core	^{12}C	^{16}O	^{28}Si	^{32}S	^{40}Ca
$P^v_{Q_1}$	0.151	0.117	0.168	0.167	0.137
$P^v_{Q_2}$	0.013	0.0074	0.017	0.017	0.011
	(0.014)	(0.0081)	(0.018)	(0.018)	(0.011)

ignored, and will from now on suppress the isospin index where possible. For the direct term we obtain after some standard angular momentum algebra:

$$P^d_{nlj}(b)=\int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 \,|\phi_{nlj}(r_1)\phi_{n_v l_v j_v}(r_2)|^2 \tfrac{1}{2}\int_{-1}^{1} d\cos\theta\; \theta(b-r_{12}) \,. \tag{4.2}$$

The exchange term is found to be the following:

$$P^e_{nljl_z}(b)=\delta_{i_z i_v}\int_0^\infty dr_1 r_1^2 \int_0^\infty dr_2 r_2^2 e_{ljl_vj_v}(r_1,r_2)\phi^*_{nlj}(r_1)\phi^*_{nlj}(r_2)\phi_{n_vl_vj_v}(r_1)\phi_{n_vl_vj_v}(r_2)\,, \tag{4.3}$$

where

$$e_{ljl_vj_v}(r_1,r_2)=(2l+1)(2l_v+1)\sum_\lambda (2\lambda+1)\left[\begin{pmatrix} l & l_v & \lambda \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} \lambda & l & l_v \\ \frac{1}{2} & j_v & j\end{Bmatrix}\right]^2 \tfrac{1}{2}\int_{-1}^{1} d\cos\theta\; \theta(b-r_{12})P_\lambda(\cos\theta)\,. \tag{4.4}$$

If $b\rightarrow\infty$ the angular integral reduces to a constant ($\delta_{\lambda 0}$), and the exchange integral vanishes because of the orthogonality of the single particle states. In summary we have the following:

$$P^v_b=\sum_{nlj}(2j+1)[2P^d_{nlj}(b)-P^e_{nlj}(b)]\,, \tag{4.5}$$

where the factor 2 stems from the identical proton and neutron direct contributions. Using this average probability, or the individual probabilities $P_{nlj}(b)$, we can evaluate the probability for the valence nucleon to overlap with any one ($P^v_{Q_1}$), or any two ($P^v_{Q_2}$) core nucleons according to Eqs. (3.14) and (3.15).

In Table I we show results for the nuclei ^{12}C, ^{16}O, ^{28}Si, ^{32}S, and ^{40}Ca, which in the present investigation are considered as ideal magic nuclei. The results were obtained for one particular transition radius ($b=0.95$ fm). However, as P^v_b behaves very nearly like b^3, we can easily obtain the results for other values of b. We have also done calculations of $P^v_{Q_1}$ and $P^v_{Q_2}$ using the simpler Fermi gas model. These latter results are somewhat larger (between 2% and 10%) than the shell-model results, but are otherwise similar. In particular, comparison of both calculations shows that the A dependence of the six-quark probabilities is due to the single particle nature of the valence particle, rather than to the single-particle nature of the core nucleons, as the Fermi gas model, which does not take account of the single particle nature of different core nucleons, leads to the same A dependence as the microscopic calculations. Although not shown in the table, it is worth noting that the Pauli exchange term (4.3) and (4.4) is about 21% of the direct term, and therefore leads to a sizable reduction of the six-quark probability.

V. A HYBRID DESCRIPTION OF THE THREE-NUCLEON SYSTEM

Given a simple three-nucleon wave function we can calculate all six-quark probabilities exactly in the three-body system. We can also exactly include the effects of the core (a proton plus neutron in this case) and thereby assess the consequences of our neglect of core effects in the preceding section. For our calculations we use a simple wave function without short-range correlations, namely the wave function given by Wildermuth and Tang:[38]

$$\phi=C\sum_{i=1}^{3}A_i\exp\left[-\tfrac{1}{2}\alpha_i\sum_{j=1}^{3}(\vec{r}_j-\vec{R})^2\right]\,, \tag{5.1}$$

where $\vec{R}$ is the center of mass vector. Since this is a symmetric S-state wave function, the exchange terms in the quark probabilities vanish. We define the basic matrix element

$$p=\langle\phi\,|\,\theta(b-r_{12})\,|\,\phi\rangle\,, \tag{5.2}$$

so that $P^v_b=2p$. Using P^v_b or p we can evaluate all other probabilities with the approximations suggested in Sec. III. We can also calculate these exactly, using the definitions

$$P^v_{Q_1}=2\langle\phi\,|\,\theta(b-r_{12})\theta(r_{13}-b)\,|\,\phi\rangle\,, \tag{5.3}$$

$$P^v_{Q_2}=\langle\phi\,|\,\theta(b-r_{12})\theta(b-r_{13})\,|\,\phi\rangle\,, \tag{5.4}$$

and

$$P^v_N=\langle\phi\,|\,\theta(r_{12}-b)\theta(r_{13}-b)\,|\,\phi\rangle\,, \tag{5.5}$$

with $P^v_Q=1-P^v_N$. For all these probabilities the state of the core nucleons 2 and 3 is not specified. One easily checks that the total probability $P_N^v + P_{Q_1}^v + P_{Q_2}^v = 1$.

In addition we now define exclusive probabilities, for which the state of the core is specified as well:

$$P_N = \langle \phi \mid \theta(r_{12}-b)\theta(r_{13}-b)\theta(r_{23}-b) \mid \phi \rangle , \tag{5.6}$$

$$P_{Q_1}^E = \langle \phi \mid \theta(b-r_{12})\theta(r_{13}-b)\theta(r_{23}-b) \mid \phi \rangle , \tag{5.7}$$

$$P_{Q_2}^E = \langle \phi \mid \theta(b-r_{12})\theta(b-r_{13})\theta(r_{23}-b) \mid \phi \rangle , \tag{5.8}$$

and

$$P_{Q_3} = \langle \phi \mid \theta(b-r_{12})\theta(b-r_{13})\theta(b-r_{23}) \mid \phi \rangle . \tag{5.9}$$

The physical meaning of these probabilities is the following: $P_{Q_1}^E$ is the chance for finding a specific pair close but no other pairs close; $P_{Q_2}^E$ is the chance that two specific pairs are close but the third pair is not; and P_{Q_3} is the chance that all three nucleons are close. The completeness of the wave function is now embodied by the identity

$$P_N + 3P_{Q_1}^E + 3P_{Q_2}^E + P_{Q_3} = 1 .$$

The connection between these inclusive and exclusive probabilities is the following:

$$P_N^v = P_N + P_{Q_1}^E ,$$

$$P_{Q_1}^v = 2(P_{Q_1}^E + P_{Q_2}^E) , \tag{5.10}$$

and

$$P_{Q_2}^v = P_{Q_2}^E + P_{Q_3} .$$

Another useful identity is $P_b^v = P_{Q_1}^v + 2P_{Q_2}^v$, giving a complete breakup of the valence six-quark probability in one and two pair components.

In Table II we list these quantities for $b = 0.95$ fm. Again results for other b values can easily be obtained. In this case $P_{Q_1}^E \sim b^2$ while $P_{Q_2}^E$ and $P_{Q_3} \sim b^5$.

Let us analyze the results in Table II in some detail. First compare the exact results with the classical approximations discussed in Sec. III. We see that the approximation for $P_{Q_2}^v$ is very poor. Clearly there is a very strong center-of-mass correlation in the $A = 3$ system. If two particles are close together, then the chance for the third particle to be close to one of them is enhanced by as much as a factor of 2 (since the particles like to be close we could have expected an enhancement, but the factor 2 is somewhat of a surprise). To compensate for this effect in comparing with our classical calculation for heavy nuclei, the chance for two particles to be close should be reduced if we know that the third particle is far away from one of them. We may also expect that these correlations are A dependent, since the effect of one particle on another will be less if there are many other nucleons around. The following formulae give the correct description of the probabilities in the three-body system ($A = 2$), and for larger A give roughly the expected A dependence:

$$p_> = \frac{A}{A-1} p , \tag{5.11}$$

$$p_< = \frac{1 - \frac{A}{A-1}p}{1-p} p . \tag{5.12}$$

Here $p_<$ has been constructed to satisfy the requirement

$$pp_> + (1-p)p_< = p , \tag{5.13}$$

representing the fact that the average probability for finding a close pair should still be p. We could also consider higher order correlations in the three-body system, e.g., we could consider the chance $p_{\ll}$ to find two particles close together if they are both far away from the third one. For $A = 2$ this $p_{\ll}$ is then determined by the following:

$$P_N = (1-p)(1-p_<)(1-p_{\ll}) . \tag{5.14}$$

However, we find that $p_{\ll}$ equals $p_<$ to within 3%, so that we neglect these higher order correlations and set $p_{\ll} = p_<$.

We can now give the general expressions for the correlated probabilities $P_{Q_1}^v$, $P_{Q_2}^v$, and P_Q^v:

$$P_{Q_1}^v = Ap(1-p_>)(1-p)(1-p_<)^{A-3} , \tag{5.15}$$

$$P_{Q_2}^v = \frac{A(A-1)}{2} pp_>(1-p_>)^2(1-p)(1-p_<)^{A-5} , \tag{5.16}$$

and

$$P_N^v = (1-p)(1-p_<)^{A-1} . \tag{5.17}$$

It should be obvious that the products on the right-hand side (RHS) of (5.15) and (5.16) are truncated in the few-body case, e.g., for $A = 2$ we find $P_{Q_2}^v = [A(A-1)/2]pp_>$ and $P_{Q_1}^v = Ap(1-p_>)$. Analogous expressions for other probabilities in the three-body system are easily written out as well. From Table II we see that we have succeeded in giving an excellent representation of the exact three-body results using these approximate correlated probabilities.

Results for the many-body systems with these correlated expressions were already shown in Table I (entries in

TABLE II. Probabilities for quark configurations in the three-nucleon system for $b = 0.95$ fm. The uncorrelated results are based on the approximation in Sec. III. Correlated results are based on approximations suggested in this section.

	P_b^v	P_N^v	$P_{Q_1}^v$	$P_{Q_2}^v$	P_N	$P_{Q_1}^E$	$P_{Q_2}^E$	P_{Q_3}
Exact	0.179	0.836	0.148	0.0158	0.796	0.0672	0.0068	0.0091
Uncorrelated		0.829	0.163	0.0081	0.754	0.0743	0.0073	0.0007
Correlated		0.837	0.147	0.0161	0.769	0.0670	0.0068	0.0093

parentheses). The effect of the correlations on $P_{Q_1}^{v}$ is negligible and the enhancement of $P_{Q_2}^{v}$ is quite minor, amounting to only 8.7%, 6.5%, 3.7%, 3.2%, and 2.5% for the successive nuclei. Obviously, correlations play a much smaller role in the many-body system than in the three-body system, in accordance with the independent particle model of nuclei. Of course, we should keep in mind that this result was obtained under the natural, but *ad hoc*, many-body generalization of the three-body results. Notice also that the expressions for the correlated probabilities can only be valid in a limited range, since they will exceed unity if p becomes large (e.g., if $b \to \infty$).

VI. MASS DIFFERENCE IN THE THREE-BODY SYSTEM

According to the preceding discussion the three-baryon wave function can be decomposed in a conventional component P_N and the multiquark bag components $P_{Q_1}^{E}$, $P_{Q_2}^{E}$, and P_{Q_3}. In Sec. II we discussed the evaluation of mass differences for six-quark bags, i.e., for those parts of the wave function represented by $P_{Q_1}^{E}$. Since the components $P_{Q_2}^{E}$ and P_{Q_3} are not negligible, we now have to discuss how masses for these components are to be evaluated. If all three particles are close, as they are in P_{Q_3}, it is natural to assume that the three six-quark bags have merged into a nine-quark bag. But even in the case that only two of the three pairs are close (i.e., have merged into a six-quark bag), it seems natural to assume that the system is best described by a nine-quark bag. If quarks can move freely between bag one and two, and between two and three, then they can also move freely between bag one and bag three. This system of nine freely moving quarks is best represented by a nine-quark bag. Simple geometrical considerations also favor a nine-quark bag description of the $P_{Q_2}^{E}$ components; however, we do not want to elaborate on these arguments.

Our next problem is then how to describe mass differences between nine-quark bags. For six-quark bags the mass differences could not simply be expressed in terms of the masses of the "originating" nucleons. Likewise, we do not expect that nine-quark bag mass differences can be expressed in terms of the underlying six-quark bag mass differences. It is more natural to extend our model for the three- and six-quark bag to the general $(3n)$-quark bag by means of the following general equation (we omit terms which do not contribute to the mass differences):

$$E = 1.44 \sum_{i<j}^{3n} \frac{Q_i Q_j}{R_{3n}} + 0.42 \sum_{i=1}^{3n} \frac{c_i}{R_{3n}} , \tag{6.1}$$

where we wrote $m_i = c_i / R$ ($c_d - c_u = 4$ MeV fm). This leads to the following mass differences for $R_3 = 1$ fm, $R_6 = 1.2$ fm, and $R_9 = 1.35$ fm:

$$m_{\mathrm{pp}} - m_{\mathrm{nn}} = -0.94 \text{ MeV}; \quad m_{\mathrm{pp}} - m_{\mathrm{pn}} = 0.13 \text{ MeV} ;$$
$$m_{\mathrm{ppp}} - m_{\mathrm{ppn}} = 1.18 \text{ MeV}; \quad m_{\mathrm{ppp}} - m_{\mathrm{pnn}} = 1.30 \text{ MeV} ; \tag{6.2}$$
$$m_{\mathrm{ppp}} - m_{\mathrm{nnn}} = 0.35 \text{ MeV} .$$

All other mass differences follow trivially. One easily verifies that the nine-quark mass differences cannot easily be represented in terms of nucleon or six-quark mass differences.

We can now write the masses of the three-baryon nuclei. For ^{3}He we obtain the following:

$$M_{^3\mathrm{He}} = P_N(2m_{\mathrm{p}} + m_{\mathrm{n}}) + P_{Q_1}^{E}(2m_{\mathrm{p}} + 2m_{\mathrm{np}} + m_{\mathrm{n}} + m_{\mathrm{pp}}) + (3P_{Q_2}^{E} + P_{Q_3}) m_{\mathrm{ppn}} + V_C' , \tag{6.3}$$

where V_C' is the conventional Coulomb energy reduced by the exclusion of the short-distance contribution. In evaluating V_C' we assume that the Coulomb potential between two protons will not change if one of the protons forms a six-quark bag with the neutron. The Coulomb energy of two protons, when they are closer than b, should naturally be excluded, because it is already included in the six-quark bag mass. The components contributing to the conventional Coulomb energy are therefore P_N, $2 \times P_{Q_1}^{E}$, and perhaps $1 \times P_{Q_2}^{E}$. The sum of these components exactly represents that part of the three-body wave function for which the two protons are further apart than b. In calculating quark probabilities we assumed that we could represent the wave function for $r < b$ by the conventional one. If we now make the same assumption for the proton-neutron wave function when we do the Coulomb potential integration, then this integration can be trivially performed. This assumption is not unreasonable, since the neutron only plays a spectator role as far as the Coulomb integration is concerned.

Although convenient, this Coulomb energy determination suffers from one problem. The $P_{Q_2}^{E}$ component, with two protons not close, contributes both to the conventional Coulomb energy and to the nine-quark bag Coulomb energy, since we decided to treat the $P_{Q_2}^{E}$ components as nine-quark bag states. To correct for this one can subtract the Coulomb energy of four u and two d quarks in the nine-quark bag from the overall mass, or one can suppress the contribution to the conventional Coulomb energy which corresponds to two "distant" protons, both of which are close to the neutron. Since the product sum $\sum_{i<j} Q_i Q_j$ is unity, independent of whether the charges Q_i are proton or constituent quark charges, the numerical value of this correction is not particularly sensitive to the procedure chosen. We have calculated the correction using the former procedure, and included it in the Coulomb energy V_C'.

For the triton we obtain the following expression:

$$M_{^3\mathrm{H}} = P_N(2m_{\mathrm{n}} + m_{\mathrm{p}}) + P_{Q_1}^{E}(2m_{\mathrm{n}} + 2m_{\mathrm{np}} + m_{\mathrm{p}} + m_{\mathrm{nn}}) + (3P_{Q_1}^{E} + P_{Q_3}) m_{\mathrm{pnn}} . \tag{6.4}$$

By subtracting (6.3) and (6.4) we obtain the following:

$$m_{^3\mathrm{He}} - m_{^3\mathrm{H}} - (m_{\mathrm{p}} - m_{\mathrm{n}}) - V_C = \Delta_Q + \Delta_C , \tag{6.5}$$

where

$$\Delta_Q = P_{Q_1}^{E}(m_{\mathrm{pp}} - m_{\mathrm{nn}} + 2m_{\mathrm{n}} - 2m_{\mathrm{p}}) + (3P_{Q_2}^{E} + P_{Q_3})(m_{\mathrm{ppn}} - m_{\mathrm{pnn}} - m_{\mathrm{p}} + m_{\mathrm{n}}) \tag{6.6}$$

and

$$\Delta_C = V'_C - V_C \ . \qquad (6.7)$$

The mass shift Δ_Q can now be calculated from the mass differences in Eq. (6.2), and the probabilities $P^E_{Q_1}$, $P^E_{Q_2}$, and P_{Q_3}, some of which were already listed in Table II. The mass shift Δ_C reflects the reduction of the Coulomb energy, due to the suppression of the interior, plus the small correction term to prevent double counting. If we use the three-body wave function (5.1), and account for the finite size of the proton using the Auerbach *et al.* form factor,[39] then we obtain a Coulomb energy of $V_C = 749$ keV. This value is substantially larger than the value obtained with conventional three-body wave functions, and would be larger yet, if we had considered the protons as pointlike particles. This confronts us with a certain problem in the presentation of our results: We would like to single out the effect of the multiquark bags in our description, however, our "conventional" results (i.e., those for $b=0$) are already substantially different from the usual ones, as they are based on a three-body wave function tailored towards our model of the short-range behavior. Therefore, we rather compare our calculation of V'_C to the best conventional value of V_C (including in this latter value other small conventional isospin breaking corrections). We have taken the value $V_C = 683 \pm 29$ keV from Ref. 40. Since V_C appears both on the left-hand and the right-hand sides of Eq. (6.6), its value is immaterial in determining the quality of our prediction, and only plays a role in the interpretation of our results for Δ_C. This choice for V_C implies that the left-hand side of Eq. (6.5) represents the current discrepancy between experiment and (conventional) theory; i.e., $764 - 683$ keV$=81$ keV. Our model, therefore, could claim a success if the sum of $\Delta_Q + \Delta_C$ lies in the neighborhood of 80 keV.

In Table III we show the results for the quark probabilities, the corresponding values for Δ_Q, and the Coulomb shifts Δ_C. Our three-body wave function does not have any short-range suppression, leading to the fairly large quark content of the wave function ($P_Q = 18\%$ for $b=0.85$ fm and 23% for $b=0.95$ fm). In order to have some results with smaller percentage quark states, and also to acquire some insight in the model dependence of our calculations, we have also performed calculations with a correlated three-body wave function. We follow the procedure of Hadjimichael *et al.*,[41] rescaling the short-range N-N wave function to the Reid soft core wave function with a correlation length of 1 fm. Obviously, we have to renormalize the three-body wave function if we introduce correlations; the results are given under the heading type 2 in Table III. We can also put the wave function defect, arising from the introduction of correlations, directly into the total six-quark probability. In this case our theory no longer dictates the division of the total quark probability over $P^E_{Q_1}$, $P^E_{Q_2}$, and P_{Q_3}; but lacking an alternative we still follow the formulae presented in Sec. V (type 3).

We see that in all cases considered $\Delta_Q + \Delta_C$ is positive and removes part of the original discrepancy of 81 keV. In our main calculation (type 1, $b=0.95$ fm) we obtain virtual agreement with experiment (the difference of 10 keV is not significant considering the uncertainties in the calculation). Notice that our results are not particularly sensitive to b, due to the strong cancellations between Δ_Q and Δ_C.

VII. MASS DIFFERENCES IN THE MANY-BODY CASE

In the many-body case we express the mass differences in terms of the valence probabilities P^v_N, $P^v_{Q_1}$, and $P^v_{Q_2}$, which were defined previously in Sec. III. The completeness of the wave function is now given by

$$P^v_N + P^v_{Q_1} + P^v_{Q_2} + \cdots = 1 \ , \qquad (7.1)$$

but since $P^v_{Q_2}$ is already quite small, we have neglected higher order terms in (7.1) and use the completeness in approximate form. The components $P^v_{Q_1}$ and $P^v_{Q_2}$ can be further broken up into components for which the isospin nature of the core nucleon(s), participating in the multiquark bags, is specified. Assuming that the core proton and core neutron probabilities are identical—a reasonable assumption considering that we deal with $N=Z$ cores—we can write the mass of the core plus one additional nucleon as follows (we omit core contributions which are irrelevant for the mass difference):

TABLE III. Multiquark bag probabilities and their contribution (in keV) to the ^{3}He and ^{3}H mass difference (Δ_Q) and the reduction of the conventional Coulomb energy (Δ_C). Uncorrelated (type 1), renormalized correlated (type 2), and correlated calculations with high quark content (type 3) are shown. Agreement with experiment is obtained if the sum $\Delta_Q + \Delta_C$ equals 81 keV, cancelling the conventional discrepancy (Ref. 40).

Type	b (fm)	$P^E_{Q_1}$	$P^E_{Q_2}$	P_{Q_3}	Δ_Q	Δ_C	$\Delta_Q+\Delta_C$
1	0.85	0.0537	0.0041	0.0051	112	−44	68
2	0.85	0.0272	0.0008	0.0010	49	−12	36
3	0.85	0.0570	0.0045	0.0057	120	−94	26
1	0.95	0.0672	0.0068	0.0091	150	−78	73
2	0.95	0.0433	0.0022	0.0030	84	−42	42
3	0.95	0.0687	0.0073	0.0100	156	−122	34

$$m_{N,Z+1}=P_N^v m_p+\tfrac{1}{2}P_{Q_1}^v(m_{pp}-m_p)+\tfrac{1}{2}P_{Q_1}^v(m_{pn}-m_n)$$
$$+\tfrac{1}{2}P_{Q_2}^v(m_{ppn}-m_p-m_n)+\tfrac{1}{4}P_{Q_2}^v(m_{ppp}-2m_p)$$
$$+\tfrac{1}{4}P_{Q_2}^v(m_{pnn}-2m_n)+V_C' \quad (7.2)$$

and

$$m_{N+1,Z}=P_N^v m_n+\tfrac{1}{2}P_{Q_1}^v(m_{nn}-m_n)+\tfrac{1}{2}P_{Q_1}^v(m_{pn}-m_p)$$
$$+\tfrac{1}{2}P_{Q_2}^v(m_{nnp}-m_p-m_n)+\tfrac{1}{4}P_{Q_2}^v(m_{nnn}-2m_n)$$
$$+\tfrac{1}{4}P_{Q_2}^v(m_{npp}-2m_p)\ . \quad (7.3)$$

Note that we recover the conventional result if we replace the multiquark bag masses by their conventional values (e.g., $m_{pp}=2m_p$, etc.).

After subtracting these masses we obtain the following:

$$m_{N,Z+1}-m_{N+1,Z}-(m_p-m_n)-V_C=\Delta_Q+\Delta_C\ , \quad (7.4)$$

where

$$\Delta_Q=\tfrac{1}{2}P_{Q_1}^v(m_{pp}-2m_p-m_{nn}+2m_n)$$
$$+\tfrac{1}{4}P_{Q_2}^v(m_{ppn}-m_{nnp}+m_{ppp}$$
$$-m_{nnn}-4m_p+4m_n) \quad (7.5)$$

and

$$\Delta_C=V_C'-V_C\ . \quad (7.6)$$

To help the interpretation of our results, we will use values for V_C listed by Nolen and Schiffer[26] [these include various corrections and are called Δ(calc) in Ref. 26]. Some of the more recent conventional results for the Coulomb displacement energies are given in the discussion. With this choice for V_C, the left-hand side of (7.4) represents the original Nolen-Schiffer anomaly.

In Table IV we show our results for a transition radius of 0.85 and 0.95 fm. We include the Coulomb energies V_C for reference. The finite size of the protons in the calculation of V_C' is again implemented using the Auerbach *et al.* form factor.[39] In order to get agreement with experiment, $\Delta_Q+\Delta_C$ should equal the right-hand side of Eq. (7.4), which in Table IV is denoted by Δ. A positive $\Delta_Q+\Delta_C$ represents a reduction of the Nolen-Schiffer anomaly, whereas a negative value represents an increase thereof. For light nuclei there is an improvement, in particular in ^{12}C, where the discrepancy is cut in half. For the two larger nuclei the anomaly has increased. In every case the agreement with experiment improves if b increases; however, the sensitivity to b is not sufficient to get full agreement at some values of b, as the cancellation between Δ_Q and Δ_C is too large. These results will further be discussed in the next section.

VIII. SUMMARY AND DISCUSSION

In this paper we have developed a hybrid description of nuclei in terms of quarks and nucleons. The most detailed description was given of the three-body system, and some of the insights obtained for this system were used to improve the treatment of many-body systems. This new description of nuclei was applied to the calculation of mass differences between ^{3}He and ^{3}H, and between mirror nuclei, motivated by persistent problems in reproducing the experimental Coulomb displacement energies. Unfortunately, the assumptions made in Sec. II concerning the dynamical description of mass differences of bags are not (yet) on solid ground, and clearly require further QCD studies of few-nucleon systems. However, they should provide a reasonable indication of the charge-symmetry violations to be expected in the quark bag model.

Originally we had expected that the mass differences between protons and neutrons would reduce in the multiquark bag environment, thereby reducing the Nolen-Schiffer anomalies. Although this effect is certainly present ($\Delta_Q>0$ in all cases) it is cancelled to a large extent by the reduction of the (conventional) Coulomb energy, represented by the quantity Δ_C. Therefore, the sensitivity to the value of the transition radius in b is not as large as expected. Generally, our results improve if b is increased, however, this improvement is so slow that we cannot use the experimental mass difference to fix the value of b. A remarkable consequence of this result is that, as far as the mass differences are concerned, the conventional and quark-nucleon description are largely compatible. If this result survives further improvements of the model, and if it also has validity for other nuclear properties, then it would explain why conventional nuclear physics could have been so successful despite the presence of large quark components in the wave function.

TABLE IV. The shifts Δ_Q and Δ_C in the mass differences due to the presence of multiquark bags. The conventional discrepancy between experiment and theory (the Nolen-Schiffer anomaly) is represented by Δ; the discrepancy in the present theory is $\Delta-(\Delta_Q+\Delta_C)$. All energies are in MeV.

	b (fm)	^{12}C	^{16}O	^{28}Si	^{32}S	^{40}Ca
V_C		2.79	3.23	5.53	6.11	6.66
Δ_Q	0.85	0.106	0.076	0.118	0.114	0.095
Δ_C	0.85	−0.029	−0.060	−0.091	−0.282	−0.212
$\Delta_Q+\Delta_C$	0.85	0.077	0.016	0.027	−0.168	−0.117
Δ_Q	0.95	0.145	0.108	0.163	0.162	0.128
Δ_C	0.95	−0.055	−0.080	−0.128	−0.313	−0.235
$\Delta_Q+\Delta_C$	0.95	0.090	0.028	0.035	−0.151	−0.107
Δ		0.210	0.310	0.200	0.240	0.620

Our results are most encouraging for the light nuclei. For the three-nucleon system the anomaly is reduced from 80 to 10 keV, for ^{12}C from 210 to 125 keV. For ^{16}O and ^{28}Si there is still a small improvement; however, for the larger nuclei (^{32}S and ^{40}Ca) our simple description of the charge-symmetry breaking effects breaks down. In these latter cases we clearly need a more detailed description of the conventional wave function, and a better study of various other contributions to the mass difference. In Ref. 42 such a study was attempted for ^{40}Ca, and in one variant of their calculations the discrepancy was less than 150 eV.

A disturbing consequence of our present results is that the strong cancellation between Δ_Q and Δ_C, and the modest model dependence indicated in Table III, precludes the acquisition of accurate predictions. It also means that even the hybrid results will remain sensitive to the detailed treatment of the exterior wave function and to small corrections, a situation which became most obvious in the ^{40}Ca case. The uncertainty of these corrections (again there seem to be large cancellations between different effects[42]) adds to the uncertainty in the predictions. It therefore appears necessary to test such hybrid descriptions also in the context of other processes, a task which is presently actively pursued[19–21,36,37] using similar models. Ultimately we may be able to put sufficient constraints on the models to exclude some of them, although the present study also warns us that very different descriptions can lead to similar results, thereby precluding discrimination through experiment alone.

Finally we mention one spin-off of the present study which seems particularly relevant at this time. The announcement last year by the European muon collaboration[43] that the structure function of Fe was not simply 56 times that of an isolated, isoscalar nucleon, has led to a great deal of interest in the topic of quarks in nuclei.[25,44] Although there seems to be general agreement that this measurement indicates a change of scale for quarks in a many-body system, it is not yet clear whether one needs to invoke explicit multiquark configurations or not.[45] However, with respect to the recent calculations of Jaffe *et al.*,[46] it is interesting to note that they estimated the scale change by calculating the overlap of N-N pairs in finite nuclei, but neglected the possibility of multinucleon overlap. While not a rigorous proof of their approximation, our discussion in Secs. IV and V [especially near Eqs. (5.11)—(5.17)] indicates that it is probably a good approximation for nuclei heavier than He. Further studies along the lines introduced here seem worthwhile.

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EVIDENCE FOR AN ENHANCED NUCLEAR SEA FROM THE PROTON–NUCLEUS DRELL–YAN PROCESS

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We show that the experimental data for the slope of the rapidity distribution of the Drell–Yan process p on Pt favour the existence of an increase in the sea for a bound nucleon.

The European muon collaboration recently reported [1] a striking difference between the structure functions of a nucleon in deuterium and iron – the EMC effect. Amongst the many explanations which have been proposed, the pionic model [2–4] provides a very natural explanation of the enhancement in Fe at small x, as an increase in the non-strange sea. However, this interpretation has been questioned by neutrino experiments [5,6] which claim to see no significant enhancement of the nuclear sea – although their (statistical and systematic) uncertainties are large.

Clearly, it is very important to settle the question of whether or not the nuclear sea is enhanced, and the Drell–Yan (DY) process (in which a q–$\bar{\mathrm{q}}$ pair annihilate to form a high mass lepton pair) is an obvious tool. In particular, it is possible to select asymmetric kinematic conditions such that only the sea of the target is probed, and one would expect to be sensitive to any enhancement of it. Unfortunately, this simple idea is complicated in practice by the uncertainty in extrapolating from time-like to space-like values of Q^2 when relating DY with deep-inelastic scattering. This extrapolation involves a factor, usually denoted as K, which cannot be calculated with high precision. Thus only a measurement of the relative cross sections for DY on (say) Fe and D, in the same asymmetric kinematical conditions, would provide the information we want. At the present time, there is no such data.

An alternative idea, which we consider here, is to analyse the logarithmic derivative of the rapidity distribution

$$S = (\mathrm{d}/\mathrm{d}y)\big(\ln(\mathrm{d}^2\sigma/\mathrm{d}M\,\mathrm{d}y)\big)_{y=0} \tag{1}$$

where M is the invariant mass of the dilepton pair. This quantity, which has the desired property of being independent of the K factor, has been measured by Ito et al. [7] for protons on Pt ($Z/A = 0.4$, $N/A = 0.6$). The dominant contribution to the DY cross section is proportional to

$$X = \tfrac{4}{9}\big[\mathrm{u}_\mathrm{p}(x_1)(0.4\,\bar{\mathrm{u}}_A(x_2) + 0.6\,\bar{\mathrm{d}}_A(x_2)) + \bar{\mathrm{u}}_\mathrm{p}(x_1)(0.4\,\mathrm{u}_A(x_1) + 0.6\,\mathrm{d}_A(x_2))\big], \tag{2}$$

where the subscript refers to the projectile (p), or target (A). [In eq. (2), we omit, for simplicity, the smaller terms involving d quarks. The actual numerical calculations include all terms – to be specific, we use for the free proton $u_v(x) = 2.2x^{1/2}(1-x)^3$; $d_v(x) = 0.57(1-x)u_v(x)$; $\bar{u}(x) = \bar{d}(x) = 0.2(1-x)^7$.]

With the conventional definition of rapidity (y),

$$x_1 = \sqrt{\tau}\, e^{y}, \quad x_2 = \sqrt{\tau}\, e^{-y}, \tag{3}$$

eq. (2) implies that

$$S = \tfrac{4}{9}(\sqrt{\tau}/X)\big[u'_p(0.4\bar{u}_A + 0.6\bar{d}_A) + \bar{u}'_p(0.4u_A + 0.6d_A) - \bar{u}_p(0.4u'_A + 0.6d'_A) - u_p(0.4\bar{u}'_A + 0.6\bar{d}'_A)\big], \tag{4}$$

where $u'_p \equiv [du_p(x_1)]/dx_1$, etc.

If the quark momentum distributions were unaltered in the nucleus, the interaction between the incident proton and a nuclear proton would give no contribution to S. Then the dominant term is

$$S \simeq \tfrac{4}{9}(\sqrt{\tau}/X)(d_p\bar{u}'_p - u_p\bar{d}'_p), \tag{5}$$

and the asymmetry between u and d (u > d) yields a positive slope. As shown in fig. 1 (solid curve), while the sign is correct, this effect alone is too small to explain the data. In order to account for the remaining discrepancy, Ito et al. [7] proposed that there might be a basic asymmetry between the $\bar{u}$ and $\bar{d}$ distributions in the proton. Using $\bar{u}(x) = (1-x)^{3.5}\bar{d}(x)$, they were able to obtain a reasonable fit.

This asymmetry between $\bar{u}$ and $\bar{d}$ is rather large (e.g., $\bar{u}/\bar{d} = 0.45$ at $x = 0.2$), with $\bar{d}$ dominating the sea beyond $x = 0.2$. Such a large asymmetry does not seem to be present in the $\bar{u}$ and $\bar{d}$ distributions measured directly in ν and $\bar{\nu}$ interactions with H and D. Indeed, the general conclusion seems to be that statistical and systematic uncertainties would permit no more than a 30% difference between $\bar{u}$ and $\bar{d}$ [8].

In the light of the EMC results, we would like to discuss the effect on S of an asymmetry of a different type, namely that between the quark distributions in a free nucleon and one bound in a nucleus. In particular, it is clear from eq. (4) that an increase in the nuclear sea would tend to increase the slope. [Note that $\bar{u}'$ and $\bar{d}'$ are negative, so the last term in eq. (4) is positive.] In fig. 1, we show the effect of a 40% increase in the non-strange nuclear sea (case 1, dotted curve). As well as giving the small x enhancement seen by EMC (see fig. 2), such a change clearly improves the agreement with the slope data of Ito et al.

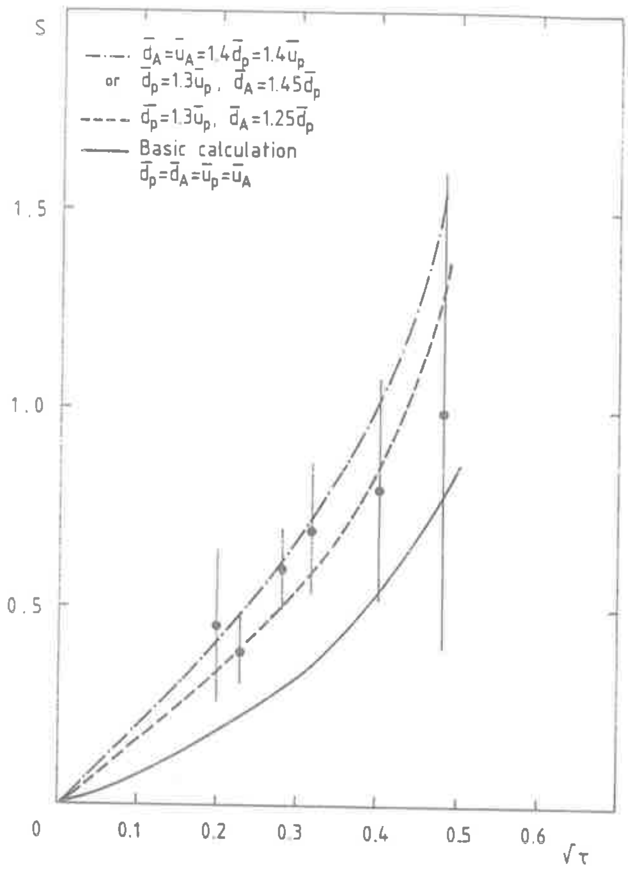

Fig. 1. The slope of the rapidity distribution for various assumptions concerning the antiquark distributions.

It should now be clear that many combinations of these two effects, namely a basic asymmetry between $\bar{u}$ and $\bar{d}$ on the nucleon and an enhancement of the nuclear sea, can reproduce the DY data. To illustrate this, we also show in fig. 1 (case 2, dashed curve) the combined effect of a (maximal) 30% asymmetry between $\bar{d}$ and $\bar{u}$ on the free proton together with an additional 25% enhancement of $\bar{d}$ for a proton ($\bar{d}_A = 1.25\,\bar{d}_p$) in the nucleus. Such a preferential enhancement of $\bar{d}$ compared to $\bar{u}$ occurs naturally [9] in the pionic model because of the virtual process $p \to n\pi^+$.

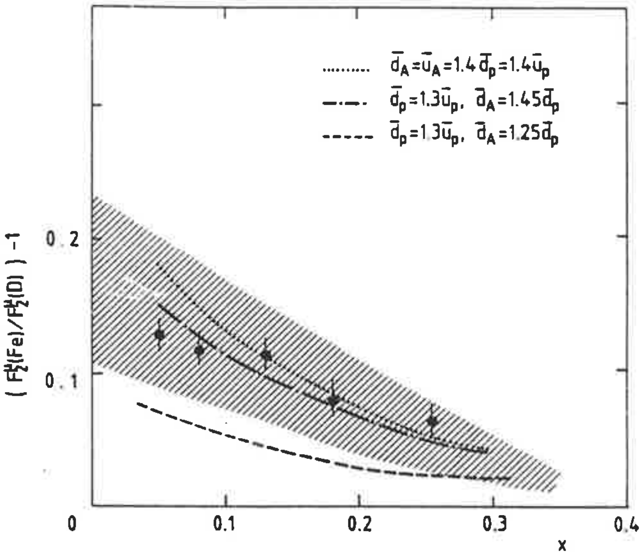

Fig. 2. The EMC effect for various assumptions concerning the antiquark distributions. (For the u and d quarks of the sea, we have taken $u^s + d^s = \bar{u} + \bar{d}$.)

Clearly, the agreement with the data of Ito et al. is rather good, but the corresponding EMC effect is a little small. Finally, we show as the dot–dashed curve (case 3) in fig. 1 the same calculation with a 45% (rather than 25%) enhancement of $\bar{d}$ for a proton in the nucleus. Once again, the slope of the rapidity distribution is well fit and we also find a sizeable EMC effect (see fig. 2).

To summarize, given the experimental constraints on the relative size of $\bar{u}$ and $\bar{d}$, the most reasonable explanation of the slope data of Ito et al. is that there is a substantial increase in the nuclear sea. While one cannot give a tight, quantitative limit on the size of this increase, it is certainly consistent with the small x enhancement seen by EMC.

The essential question remaining is whether this increase in the sea is in disagreement with the neutrino data. In particular, the CDHS group has determined the ratio [6]

$$R = (\bar{u} + \bar{d} + 2\bar{s})_{Fe}/2(\bar{d} + \bar{s})_H \tag{6}$$

to be $R = 1.10 \pm 0.11 \pm 0.07$. For the three cases cited above, we find this ratio to be 1.26, 1.03 and 1.13 respectively. All of these values are compatible with the data. Clearly the sensitivity of eq. (6) to a small asymmetry between $\bar{u}$ and $\bar{d}$ makes it less valuable as a test for an increase in the sea.

In conclusion, the value of the slope of the rapidity distribution for p–Pt DY measured by Ito et al. strongly suggests that the nuclear sea is enhanced. This interpretation has the advantage of not requiring a large asymmetry between $\bar{u}$ and $\bar{d}$ as those authors had suggested. In addition, the enhancement required to explain their data is consistent with the increase at small x seen by EMC, and is also compatible with the measurements of CDHS.

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PION–NUCLEON SCATTERING LENGTHS IN THE CLOUDY BAG MODEL

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We re-examine pion–nucleon s-wave scattering in the volume coupling version of the CBM. If the effects of multiple scattering are taken into account, one needs some phenomenological repulsive interaction in order to reproduce the experimental data. We parametrize this repulsion as a twenty percent increase in the pion mass inside the bag. It is argued that this relatively small change in mass provides strong, a posteriori support for the original assumptions of the CBM.

While the MIT bag model [1–3] combines the key features expected in QCD, namely confinement and asymptotic freedom, in a successful, phenomenological package, it also breaks chiral symmetry [3–5]. A number of phenomenological extensions have been developed recently, which restore chiral symmetry by explicitly introducing the pion as a Goldstone boson [3, 6–8]. At the present time there is little agreement over which of these diverse hybrid models (if any, see refs. [9–11]) is closer to the truth, and on that point we do not intend to take a stand here.

We begin with the observation that one of the major successes of current algebra in the late 60's was to give a fairly deep understanding of low energy, s-wave πN scattering through the Weinberg–Tomozawa relationship [12]. It is therefore natural to ask how this relationship is realized in any model which claims to respect PCAC. One of the frustrations in the early development of the cloudy bag model (CBM) was that it did not provide a transparent explanation of the πN scattering lengths. However, in 1981 several groups [13,14] independently showed that a new version of the CBM (which could be obtained by a unitary transformation of the quark fields), in which the pions coupled to the quarks through derivative coupling over the whole bag volume, did give the Weinberg–Tomozawa result. The transformed lagrangian density has the form

$$\begin{aligned}\mathcal{L}'(x) &= (\mathrm{i}\bar{q}\partial\!\!\!/ q - B)\theta_{\mathrm{v}} - \tfrac{1}{2}\bar{q}q\delta_{\mathrm{s}} \\ &\quad - (\theta_{\mathrm{v}}/4f^2)\bar{q}\gamma^{\mu}\boldsymbol{\lambda}q\cdot(\boldsymbol{\phi}\times\partial_{\mu}\boldsymbol{\phi}) \\ &\quad + (\theta_{\mathrm{v}}/2f)\bar{q}\gamma^{\mu}\gamma_5\boldsymbol{\lambda}q\cdot\partial_{\mu}\boldsymbol{\phi} + \tfrac{1}{2}(\partial_{\mu}\boldsymbol{\phi})^2 - \tfrac{1}{2}m_{\pi}^2\boldsymbol{\phi}^2, \qquad (1)\end{aligned}$$

where as usual q and $\boldsymbol{\phi}$ describe quark and pion fields, θ_{v} is one inside and zero outside the bag, and δ_{s} a surface δ-function.

We shall not dwell on the possible physical interpretations of this new version of the CBM [15]. However, there are three points which should be made explicitly. First, the transformation from surface to volume coupling relied on the CBM hypothesis that the pion looks very much like a free pion inside the bag. Second, the four-point, or contact term in eq. (1) was the piece predominantly responsible for s-wave πN scattering. (In fact it has recently been proven by Jennings and Maxwell [15] that both versions of the CBM yield the same πN scattering amplitude at the tree level. However, the convergence properties of the

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volume coupling version are far superior.) Third, the model only yielded the Weinberg–Tomozawa result in Born approximation.

In the past year or so we have extended the volume coupling version of the CBM to SU(3) × SU(3), and used it to examine the $\bar{K}N-\Sigma\pi$ system near $\bar{K}N$ threshold [16]. Our rather surprising result was that the $\Lambda'(1405)$ is predominantly a $\bar{K}N$ bound-state generated by the contact term. While this is an attractive result for bag model spectroscopy [17,18], the importance of multiple scattering through the contact term in the $S = -1$ system led us to reexamine s-wave πN scattering. Our results were obtained by solving the relativistic Lippmann–Schwinger equation

$$t(\boldsymbol{k},\boldsymbol{k}';E) = v(\boldsymbol{k},\boldsymbol{k}') + \int \mathrm{d}\boldsymbol{k}'' \frac{v(\boldsymbol{k},\boldsymbol{k}'')t(\boldsymbol{k}'',\boldsymbol{k}';E)}{E + \mathrm{i}\epsilon - \omega(k'') - E_N(k'')}, \tag{2}$$

using the contact interaction

$$v^I(\boldsymbol{k},\boldsymbol{k}') = \frac{\lambda^I N_s^2 [\omega(k) + \omega(k')]}{(2\pi)^3 [2\omega(k) 2\omega(k')]^{1/2}} \times \int_0^R \mathrm{d}r\, r^2 [j_0^2(\omega_s r/R) + j_1^2(\omega_s r/R)] j_0(kr) j_0(k'r), \tag{3}$$

which follows from eq. (1) [16]. (Here $\lambda^I = +1/2$ for $I = 3/2$ and -1 for $I = 1/2$, N_s is the normalization constant for the $1s_{1/2}$ bag wave function of eigenfrequency $\omega = 2.04$ ... .)

From the results given in the first three lines of table 1 we see that there is a large, attractive, isoscalar scattering length in contradiction with the experimental result [20]. Since the isoscalar scattering length is zero in Born approximation, it is clear that the multiple scattering has generated a large unwanted attraction. As a matter of interest we observe that while these results were obtained from the contact interaction, they are very close, both in isospin structure and the actual numbers, to what one would find from ρ-meson exchange in the more conventional meson exchange picture.

Phenomenologically it is apparent that a relatively small repulsive interaction would resolve the problem. For example, it would suffice to exclude the pion from a region of radius (R_1) about 0.1 fm at the bag centre

Table 1
Pion–nucleon scattering lengths obtained with a bag radius of 1 fm, for various values of the parameters f and δm_π described in the text.

f	δm	$a_1 + 2a_3$ (fm)	$a_1 - a_2$ (fm)
93	–	0.25	0.51
100	–	0.17	0.41
110	–	0.11	0.31
93	27	0.02	0.42
94	35	−0.05	0.38
97	28	−0.01	0.37
Exp. A [a)]		0.01 ± 0.01	0.43 ± 0.01
Exp. B [b)]		−0.05	0.38

a) Ref. [19]. b) Ref. [20].

[21, 22]. Such a model has been applied, for example, to the calculation of the axial form factor of the nucleon, but no significant constraint on (R_1/R) was obtained [23]. Alternatively, following Shuryak [24] one could imagine excluding the pion from a cavity (again about 0.1 fm in radius) surrounding each quark.

The less drastic proposal which we investigate here, which is more in the spirit of the original CBM, is to imagine that instead of being completely free the pion has an effective mass $(m_\pi + \delta m_\pi)$ inside the bag. By varying δm_π and f we obtain the results shown in table 1. Uncertainties in the experimental values preclude a unique determination of δm_π but we see that it is of the order of 30 MeV. Although f was allowed to vary freely, the fact that it is so near the theoretically expected value of 96 MeV (calculated as 93 MeV modified by renormalization effects [16]) is a reassuring check on the consistency of the model.

Before commenting on the significance of δm_π we should ask what other contributions to s-wave scattering could arise in this model. Scattering through the observed negative parity baryon resonances is very small (<0.01 fm) because of their large masses. We feel that it would be inconsistent to include the exchange of a fictitious scalar meson [25]. One would certainly find a repulsive contribution from scattering through antibaryon intermediate states [25], which would eliminate most of the need for δm_π. However, in a model with composite baryons one might expect such terms to be strongly suppressed [26]. Finally, one might consider scattering via antiparticle states at the quark level i.e. 4q–q̄ states. Jennings and Maxwell [15] found a contribution of –0.03 fm in Born approximation (for $a_1 + 2a_3$) from these, which would account for about 50% of δm_π. However, as with the particle states in their model, this is probably an overestimate. In any case, the sign of these contributions is such as to strengthen our conclusions concerning δm_π.

To summarize, we have shown that the volume coupling version of the CBM can describe low energy πN scattering, provided the pion feels a relatively small repulsive force inside the bag ($\delta m_\pi \sim 30$ MeV). What is most surprising about this result is not the fact that such repulsion exists, but that it is so small! If, as proposed by De Tar [8] for example, allowing pions in the bag was a crude representation of the propagation of uncorrelated qq̄ pairs with pion quantum numbers, one would naively expect δm_π to be of order 800 MeV ($\sim 2\omega/R$). Our finding that δm_π is instead so near to zero provides strong phenomenological evidence that the correlation between such pairs is indeed strong, and as a first approximation it makes sense to talk instead of a free pion inside the bag.

Our conclusions are based on a model where the radius is sufficiently large that the pions can be treated perturbatively. It would be interesting if proponents of various alternative hybrid bag models could test their models against low energy s-wave πN scattering.

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NUCLEON MODELS IN THE CONTEXT OF NUCLEAR PHYSICS

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We have reviewed current ideas of hadron structure in the light of the debate on whether it is necessary or desirable to develop a microsopic understanding of the atomic nucleus at the quark level.

1. INTRODUCTION

This review is prepared as part of a panel discussion on the general topic "quarks in nuclei" chaired by Sir Denys Wilkinson. Because they are necessarily prepared in advance, much of the flavour (not to mention colour) of the actual discussion can not be given here. For that the reader must refer to the edited summary elsewhere in this volume. All we aim to provide here is a pre-conference position paper presenting one point of view quite a bit more forcefully than others.

Much of the debate on nucleon models would of course be superfluous if only we had more guidance from QCD. Unfortunately the practical constraints on lattice gauge calculations mean that we have essentially no guidance on whether (for example) the bag model is a good approximation to the structure of a nucleon. We know that in the limit where virtual $q\bar{q}$ pairs are suppressed, a box 1.5 fm on a side is not enough to free the nucleon mass from exhibiting a strong dependence on the boundary conditions[1]. Thus in that limit it would seem that a bag should have a radius somewhat larger than 0.8 fm. On the other hand, the little baggers[2] would complain that the quenched approximation is unrealistic.

As discussed at greater length in ref. (3), we believe that in the present situation the only approach which makes sense is good phenomenology. That is, we should prefer, of all those models which incorporate the key features expected of QCD, the one which is capable of describing the widest range of physical phenomena.

2. THE FREE NUCLEON

A wealth of phenomenological models of hadron structure have been constructed over the last decade, all of which are supposed to model key features of QCD[4,5]. Of these the constituent, or non-relativistic quark model (NRQM) has been the most widely used. It has also enjoyed the most phenomeno-

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logical success[6] (at the expense of a sizeable number of adjustable parameters). The MIT bag model[7] has also been widely used,[8,9] but suffers because of its greater technical complexity - e.g. centre of mass corrections make a sound treatment of excited states very difficult[10]. In spite of this, the fact that the bag model can reproduce not only masses, magnetic moments and other low energy properties of hadrons, but also allows a description of deep-inelastic scattering (DIS) **without the introduction of further parameters** has led many to prefer it.

The major theoretical blemish on the MIT bag was its failure to respect chiral symmetry[11]. This observation has led to the development of the so-called hybrid chiral bag models[2,8,12,13], where the symmetry is restored by coupling the pion field to the quarks at the surface of the bag. (We refer elsewhere for the discussion of how this can be transformed to pion coupling through the bag volume[14] - a version that appears to be better suited to perturbative treatment[15], and also exhibits a striking resemblance to some constituent quark ideas[3].) It is a remarkable fact that not only did this addition cure a purely theoretical problem of the bag model, but in the cloudy bag model (CBM) in particular, it yielded much improved phenomenology for low energy hadronic properties[3,12].

At the present time a great deal of attention is being given to a very different phenomenology originating with Skyrme[16], but taking its present impetus from the work of Witten and others[17]. The idea is that the nucleon may be a soliton solution of the non-linear σ-model - including an ad-hoc term of higher order to satisfy Derrick's Theorem. Even though the Wess-Zumino term vanishes for SU(2)xSU(2) one can define a topological charge for such a soliton which can be identified with baryon number. This model has been able to reproduce the static properties of the nucleon semi-quantitatively - i.e. as well as the original MIT bag.

A priori it seems that this model may have problems dealing with the properties of strange hadrons because of the large mass of the kaon. (It would be difficult to believe any model which implied a significantly different description of the nucleon from all other baryons.) This will only be resolved by more theoretical work.

Finally we note that the possibility that the meson field of the nucleon can carry baryon number has important implications for the little bag model[2,18]. There the idea was that the non-linear coupling of the pion to a bag might significantly compress it, thereby making explicit quark degrees of freedom irrelevant in nuclear physics[3,8,19]. If the bag radius did become quite small, a large fraction of the baryon number would reside in the meson field.

2.1 WHAT DOES QCD TELL US?

Much of this discussion about which model of hadron structure is the best, misses the point. As a quantum field theory, QCD never contains physical particles which are structureless. The quarks are surrounded by virtual quark and gluon fields whose importance depends on how hard we look - the "layered onion" analogy is helpful here[21]. More precisely, we should ask what is the typical range of momentum transfer, Q^2, over which we wish our model to approximate reality.

At the present time, with new, high energy electron machines under construction and KAON factories being planned, we would like to be able to deal with nuclear systems at momentum transfers ranging from 0-10 $(GeV/c)^2$ at the very least. This includes the region of 5 $(GeV/c)^2$ which is the typical reference point in most analyses of quark and gluon momentum distributions in the nucleon[3,22]. Thus DIS should be a **fundamental constraint** on any model which is to be useful in interpreting the flow of new information on microscopic nuclear structure that we can expect in the next decade[23].

The work of Jaffe and Ross[24] is of fundamental importance in this matter. Motivated by the successes of the bag model in describing the structure of the nucleon at low momentum transfer, they guessed that perhaps there could be some relatively low momentum transfer, $\mu_0{}^2$, at which the bag would give a good approximation to QCD. They calculated the moments of the twist-two contribution to the structure functions F_3 and F_2 in the bag model[25]. Perturbative QCD was then used to extrapolate the moments which had been determined experimentally at 5 $(GeV/c)^2$ - and which were assumed to contain negligible higher twist contributions - down in Q^2. Since $\alpha_s(Q^2)$ grows as Q^2 decreases the reliability of perturbative QCD also decreases. Nevertheless, it was a striking observation that for $\mu_0{}^2 = 0.75 \pm 0.12$ GeV^2 the bag model predictions for the third to seventh moments of the valence distribution (F_3) agreed remarkably well with the extrapolated experimental values. It seems that the existing DIS data, when correctly massaged, supports the bag model picture of a nucleon containing just three valence quarks - at a scale of momentum transfer of roughly one GeV.

Another view of the same problem was given last year in connection with the pion cloud about the nucleon[3,26]. Within the framework of the CBM it was shown that the measured excess of non-strange over strange quarks in the nucleon sea[3,22,27] could be used to put a limit on the strength of the pion field about the nucleon - and hence on the size of the bag. The resulting lower limit, R>0.87±0.10 fm (see Fig. 1), is completely consistent with the results of calculations of other low-energy hadronic properties in the CBM - such as magnetic moments[28], charge radii[12] and the nucleon axial form-factor[29]. Of course it must be said that this analysis was made before the recognition of the growing importance of the non-topological aspects of the pion field as the bag radius decreases. This may (or may not) soften the lower limit on R, but in any case it deserves further work.

It should also be noted that Jaffe[30] has argued that it is inconsistent to calculate the pionic contribution to the nucleon sea in the way proposed by Sullivan[31] and applied in ref. (26). We argue[32] that the condition derived by Jaffe is in fact a sufficient condition for the pionic contribution to be valid, not a necessary condition. In fact, it is the very small mass of the pion and the correspondingly low t-channel momentum transfers associated with pion exchange that we believe justify the calculation as a good first approximation.

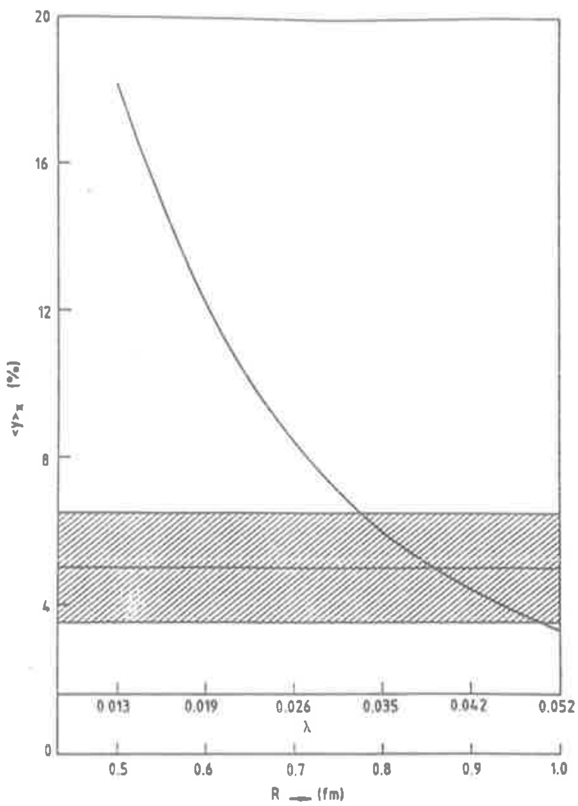

FIGURE 1
The average fraction of the momentum of a nucleon carried by pions as a function of the hardness of the πNN form-factor - controlled by R, the bag radius, in the CBM. The shaded area is the experimental upper bound which clearly implies R > 0.87 ± 0.1 fm (26).

The relation of this work[26] with that of Jaffe and Ross[24] is that the CBM with its relatively small pion cloud has a chance of representing the DIS data at a slightly higher value of μ_0^2. This hypothesis is currently being tested, but if it holds up it may put the Jaffe-Ross analysis on an even firmer theoretical basis, because one would not have to push perturbative QCD quite so far!

An interesting aspect of the discussion over topological solitons is that the original motivation for Witten was the $1/N_c$ expansion for QCD[33,34]. It can be shown that in the limit where the number of colours (N_c) becomes large, QCD (in perturbation theory) is dominated by planar diagrams. Formally the planar diagrams can be related to meson exchanges and indeed it has been claimed that in the large-N limit QCD is a theory of weakly interacting meson fields. Now the existence of soliton solutions of a set of field equations is not related to the strength of the interactions, but to their non-linearity. Thus it is perfectly possible to conceive of hadrons as solitons in the large-N_c limit. Of course no-one has yet evaluated the coefficients of the expansion in $1/N_c$ in order to see whether 1/3 is "large" or "small".

From our point of view a key observation made by Witten in his early work[33] is that the large - N_c limit explains why the quark sea is suppressed - as we noted above. Essentially the meson couplings decrease as N_c goes up so the number of virtual $q\bar{q}$ pairs is suppressed. On the other hand in the later development of the topological soliton this feature of the $1/N_c$ expansion appears to be lost. In the extreme case of the Skyrmion there are no valence quarks, only q-$\bar{q}$ pairs and it is difficult to see how this model could be related to data in the deep-inelastic region - either on nucleons or nuclei.

Unless this problem can be satisfactorily resolved, we believe firmly that the most useful models for future phenomenological applications in nuclear physics will be the MIT bag model and its modern extensions such as the CBM, the relativistic potential models[35] and the Friedberg-Lee non-topological solitons[36] - the latter two with pion coupling.

3. THE NUCLEON IN A NUCLEUS

Clearly if the arguments given in the last section are correct, so that at ρ_0 nearest neighbour nucleons are **on average** only 2R apart, it would be surprising if their quark structure were not altered! For a general discussion of the issue of quarks in nuclei we refer to recent reviews[37,38]. Here we shall mention only the EMC effect[39], which is the only indisputable demonstration of a change in the essential structure of the nucleon when placed in a nucleus.

At face value the EMC data for Fe shows a depletion by about 30% in the valence quark distribution near x=0.65, and an increase of (20-60)% in the sea. Unfortunately while the latter has been confirmed by more recent e- and ν- data[40,41,42], the matter of an increase in the sea is in question - see also ref. (43). As long as the data is uncertain, it is not possible to be definite about its interpretation. Nevertheless some general observations can be made.

As noted by Close and co-workers (at least in the region $0.2<x<0.8$) the data seems to indicate a "change in scale" when we move from a free nucleon (deuterium) to Fe.[44] Technically they found that $F_2(x,Q^2)$ for Fe was approximately equal to $F_2(x,\xi Q^2)$ for D with $\xi \simeq 2$. The essential question is then whether the change in scale is associated with non-coloured degrees of freedom (e.g. a modification of the virtual pion field[45-47] - the longest range structure of the nucleon), or whether it explicitly involves colour (e.g. through the formation of clusters of six[48,49] or more[50] quarks in the nucleus). In the few pages allotted here we can not even refer to all the relevant theoretical papers which have been written on the EMC effect. Instead, following the conclusion of section 2, we shall only report on the relevance of models like the CBM. Those interested in a broader review should see refs. (51) and (52).

Within any chiral bag model it is quite natural that the pion field of the nucleon should be the first aspect of nucleon structure changed in a many-body system. Indeed, at this series of conferences many invited talks over the past decade have dealt with the expectation of an enhancement of the pion field at $q \sim 2\text{fm}^{-1}$. Such an enhancement[53,54] is controlled by the short-range spin-isospin force between two nucleons, which is often parameterised in terms of the Landau-Migdal parameters g'_{NN}, $g'_{\Delta\Delta}$ and $g'_{\Delta N}$. Although we do not know these parameters well in the appropriate region of momentum transfer, $g'_{NN} = g'_{\Delta\Delta} = g'_{\Delta N} = 0.7$ seems a good first guess. In that case we are led to predict a doubling of the non-strange sea in Fe compared with D, in qualitative agreement with the original EMC data[45,46].

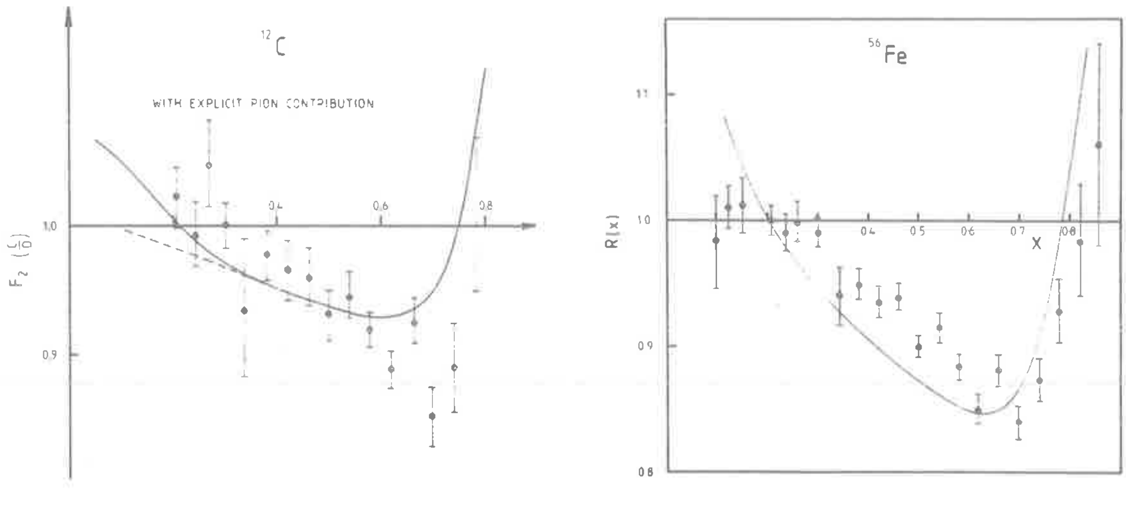

FIGURE 2
Predictions of the EMC effect for ^{12}C and ^{56}Fe in the pionic model[32], in comparison with the data of Arnold et al.[40].

Without a quark level understanding of the short-range N-N force (i.e. a microscopic derivation of g') one can not make a definitive comparison with the data. However, since we do not yet have such a complete calculation, as a first guess it was suggested to simply reduce the momentum fraction carried by nucleons by the extra momentum fraction in virtual pions[45]. The results of such a calculation for ^{56}Fe and ^{12}C are shown in Fig. 2[32]. Because $g'_{\Delta N}$ controls the calculation, this model predicts a linear dependence on the effective nuclear density, in agreement with the recent data of Arnold et al.[40]. Unfortunately, almost all other models make the same prediction!

However, unlike other models there is a possibility to test the pionic model in a way which does not simply involve a repetition of the EMC measurements with higher accuracy and more targets. Following the suggestion of Alberico, Ericson and Molinari[55], Carey et al.[56] have recently reported on the ratio of the longitudinal (i.e. pionic) to transverse response functions for $(\vec{p},\vec{p}')$ on Pb and D at $q \sim 1.7\ fm^{-1}$. As shown in Fig. 3, in contradiction with the expectations of AEM (dash curve - corrected from solid after allowance for proton absorption), the data shows no significant deviation from unity for $\omega \in (30,100)$ MeV. According to Carey et al. this rules out the pionic model for EMC.

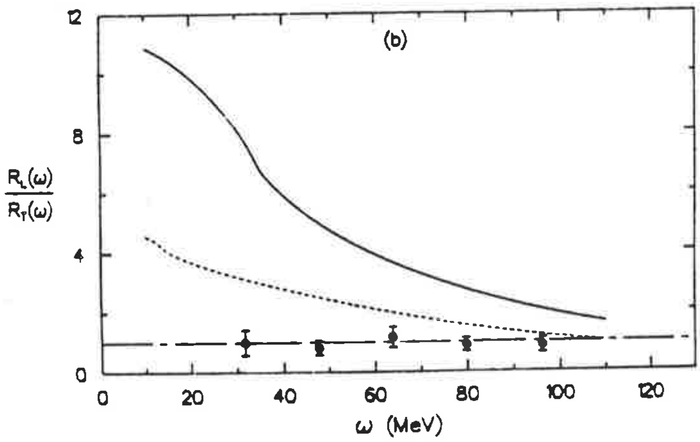

FIGURE 3
Ratio of the longitudinal to transverse response for $(\vec{p},\vec{p}')$ on ^{208}Pb (from Carey et al.[56]) in comparison with the predictions of Alberico et al.[55].

It is our belief that this data in fact supports the pionic model! The calculation of AEM neglects nucleon binding effects which would kill any enhancement of R_L for ω below ~30 MeV. However this does not affect the EMC calculations which get their major contribution at $\omega \sim q^2/2m_N \sim 80$ MeV. Furthermore, at $\omega \sim 80$ MeV the isoscalar, transverse response in $(\vec{p},\vec{p}')$ could be as big as the isovector, whereas the isoscalar longitudinal response is negligible. Thus in the region that counts, Carey et al. are consistent with the expectations of AEM, whose curves do not include isoscalar terms.

Further detailed experimental and theoretical work will be necessary to substantiate this analysis. For the present we merely note that it seems quite likely that the nucleon models which are most attractive according to the criteria of section 2, will prove capable of providing a microscopic description of the EMC effect.

4. CONCLUSION

For the first time in over twenty years nuclear and particle physics are moving together again. Much hard work lies ahead, but we believe that a basis already exists for a truly unified, microscopic description of nucleon and nuclear structure.

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S-wave meson-nucleon scattering in an SU(3) cloudy bag model

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The cloudy bag model (CBM) is extended to incorporate chiral SU(3)×SU(3) symmetry, in order to describe S-wave KN and $\bar{K}N$ scattering. In spite of the large mass of the kaon, the model yields reasonable results once the physical masses of the mesons are used. We use that version of the CBM in which the mesons couple to the quarks with an axial-vector coupling throughout the bag volume. This version also has a meson-quark contact interaction with the same spin-flavor structure as the exchange of the octet of vector mesons. The present model strongly supports the contention that the $\Lambda^*(1405)$ is a $\bar{K}N$ bound state.

I. INTRODUCTION

Over the past five years the various extensions of the MIT bag model which incorporate PCAC (partial conservation of axial-vector current) have attracted a great deal of interest.[1-7] This area remains controversial, and issues such as the nucleon bag size, the penetration of the meson field into the bag interior, and the role of the meson field in generating baryon number[8-10] are still being hotly debated. Nevertheless, these developments have certainly led to some remarkable improvements in our understanding of low-energy hadronic properties (for example, baryon magnetic moments and charge radii,[11-15] and the axial-vector form factor of the nucleon[15-18]) as well as low-energy pion-nucleon scattering[7,14,19,20] and photoproduction.[21]

Given this interest and success in the pionic sector it seems very natural to consider the extension to chiral SU(3)×SU(3). Several groups have already investigated the corrections to hyperon magnetic moments arising from kaon loops.[22,23] However, they turned out to be relatively small. We have been motivated both by the discrepancy between the $\bar{K}p$ atomic shift and the $\bar{K}p$ scattering length,[24] and by the controversial nature of the $\Lambda^*(1405)$ to investigate the consequences of a chiral SU(3)×SU(3) extension of the cloudy bag model (CBM) to the low-energy $\bar{K}N$ and $\Sigma\pi$ systems.

Our first major finding, namely, that (as suggested by Dalitz and co-workers for many years[25]) the $\Lambda^*(1405)$ is not a simple three-quark state, has already been published as a Letter.[26] In this paper we shall present a detailed explanation of this result, including the parameter dependence, the tests to which the model has been subjected, and the calculational technique. There is, of course, also some discussion of the physical assumptions on which the calculation is based.

Briefly the structure of the paper is as follows. In Sec. II we define the model, and derive the appropriate Hamiltonian for low-energy $\bar{K}N$ and $\Sigma\pi$ interactions. After some discussion of the approximations made, and the effects of renormalization, we report on some test of the same model for low-energy KN and πN scattering. The results for the coupled $\bar{K}N$-$\Sigma\pi$ system in the region of the $\Lambda^*(1405)$ are presented in detail in Sec. III. It will be seen that the model provides an excellent description of the new high-quality data of Hemingway *et al.*[27] We reserve Sec. IV for the discussion of several theoretical aspects of the calculation, including the behavior of the K matrices subthreshold, and the fraction of the strength ($\sim 14\%$) at the $\Lambda^*(1405)$ pole associated with a three-quark state. Finally, in Sec. V we summarize our finding and suggest ways of eliminating some of the approximations used here. We also point the way to some interesting new applications of the model.

II. FORMAL DEVELOPMENT OF THE MODEL

The natural generalization of the SU(2)×SU(2) CBM with volume coupling[4,19,20] is

$$L=(i\bar{q}\not{D}q-B)\theta_v-\tfrac{1}{2}\bar{q}q\delta_s+\tfrac{1}{2}(D_\mu\phi)^2+\frac{1}{2f}\bar{q}\gamma^\mu\gamma_5\lambda\cdot q\,(D_\mu\phi)\Theta_v\ . \tag{2.1}$$

Here $q(x)$ and $\phi(x)$ are the quark and meson-octet fields, B is the phenomenological energy density, f is the meson-octet decay constant, and λ are the SU(3) matrices of Gell-Mann. The function Θ_v is 1 inside the bag volume and 0 outside, while δ_s is a surface δ function. For a static, spherical bag, as we assume, these functions reduce to $\theta(R-r)$ and $\delta(R-r)$. The D's denote the appropriate covariant derivatives.

To make the calculations tractable, it is convenient to do a perturbation expansion of the Lagrangian keeping only the terms up to order ϕ^2. The assumption implicit in this approximation is that the meson fields are rather small or, equivalently, that the bag radius is not small ($\gtrsim 0.8$ fm). To this order in ϕ the covariant derivatives reduce to

$$D_\mu \phi \simeq \partial_\mu \phi \ , \tag{2.2}$$

$$\not{D} q \simeq \not{\partial} q + \frac{i}{4f^2} \boldsymbol{\lambda} \cdot (\boldsymbol{\phi} \times \partial_\mu \boldsymbol{\phi}) \gamma^\mu q \ , \tag{2.3}$$

where the SU(3) cross product is

$$(\boldsymbol{\phi} \times \partial_\mu \boldsymbol{\phi})_a = \sum_{bc} \phi_b \partial_\mu \phi_c f_{abc} \ , \tag{2.4}$$

with f_{abc} being the SU(3) structure constants.[28] Hence up to order ϕ^2 the Lagrangian density is

$$L = (i\bar{q}\not{\partial} q - B)\theta_v - \tfrac{1}{2}\bar{q}q\delta_s + \tfrac{1}{2}(\partial_\mu \boldsymbol{\phi})^2 + \frac{\theta_v}{2f} \bar{q}\gamma^\mu \gamma_5 \boldsymbol{\lambda} \cdot q \partial_\mu \boldsymbol{\phi} - \frac{\theta_v}{4f^2} \bar{q}\boldsymbol{\lambda} \cdot \gamma^\mu q (\boldsymbol{\phi} \times \partial_\mu \boldsymbol{\phi}) \ . \tag{2.5}$$

The Hamiltonian is obtained in the canonical way from

$$\hat{H} = \int d^3x \, T^{00}(x) \ , \tag{2.6}$$

where T^{00} is the energy-momentum tensor. From Eq. (2.5) we therefore find

$$\hat{H} = \hat{H}_0 + \hat{H}_s + \hat{H}_c \ , \tag{2.7}$$

where $\hat{H}_0$ describes free bags and mesons, and the interactions are

$$\hat{H}_s = \int d^3x \left[-\frac{\theta_v}{2f} \right] \bar{q}\gamma^\mu \gamma_5 \boldsymbol{\lambda} q \cdot \partial_\mu \boldsymbol{\phi} \tag{2.8}$$

and

$$\hat{H}_c = \int d^3x \frac{\theta_v}{4f^2} \bar{q}\boldsymbol{\lambda}\gamma^\mu q \cdot (\boldsymbol{\phi} \times \partial_\mu \boldsymbol{\phi}) \ . \tag{2.9}$$

The first-order term $\hat{H}_s$ couples a "bare" baryon and a meson to a "bare" baryonic state, while the second-order term $\hat{H}_c$ is a contact or four-point interaction.

It is useful to eliminate the spatial derivatives in $\hat{H}_s$ by rewriting it as a sum of two terms, one of which contains the surface contribution and the other contains just a time derivative. For this purpose, we do the integration in Eq. (2.8) by parts. Using the Dirac equation, the linear boundary condition on the surface

$$i\not{n} q = q \,|_{r=R} \ , \tag{2.10}$$

where n^μ is the unit normal to the surface of the confining region, and the relation

$$\partial_\mu \theta_v(x) = n_\mu \delta_s(x) \ , \tag{2.11}$$

we get for massless quarks

$$\hat{H}_s = \int \left[\frac{i}{2f} \bar{q}\gamma_5 \boldsymbol{\lambda} \cdot q \boldsymbol{\phi} \delta_s - \frac{\theta_v}{2f} \partial_0 (\bar{q}\gamma^0 \gamma_5 \boldsymbol{\lambda} \cdot q \boldsymbol{\phi}) \right] d^3x \ . \tag{2.12}$$

The case where the strange quark is massive is considered in Appendix A.

The interaction Hamiltonian can be projected onto the space of colorless nonexotic baryon states.[4] Following Théberge and Thomas,[11] we write

$$H = H_0 + H_{\rm int} = \sum_{B_0, B_0'} B_0^\dagger \langle B_0 | \hat{H} | B_0' \rangle B_0' \ , \tag{2.13}$$

where $B_0^\dagger$ (B_0') is the creation (annihilation) operator for three quark bags of type B_0 (B_0') and $| B_0 \rangle$ and $| B_0' \rangle$ are baryonic bare wave functions. Using the MIT bag wave functions and the Fourier transform of the meson fields,

$$\phi(\mathbf{x}) = \int \frac{d^3k}{[(2\pi)^3 2\omega_k]^{1/2}} [a(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + a^\dagger(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}] \ , \tag{2.14}$$

where a and $a^\dagger$ represent annihilation and creation operators which obey the usual commutation relations, the unperturbed Hamiltonian at the baryon level is

$$H_0 = \sum_{B_0} (m_{B_0}{}^2 + k^2)^{1/2} B_0^\dagger B_0 + \sum_i \int d^3k' \, \omega_{k'} a_i^\dagger(\mathbf{k}') a_i(\mathbf{k}') \ , \tag{2.15}$$

where m_{B_0} is the MIT bare bag mass. Naturally, the specification of the interaction Hamiltonian depends on the baryons involved in the transition. For the contact piece it is clear that to analyze KN, $\bar{K}N$, and πN scattering we need to consider baryon-octet members transitions ($BM \to B'M'$). For the first-order piece H_s, since we are primarily interested in low-energy $\bar{K}N$ scattering, we shall consider the transition baryon-meson octet to a baryon (Λ^*) composed of u, d, and s quarks in an SU(3) singlet with one quark excited to a $1p$ level. In this case we need only the $1s_{1/2}$ and $1p_{1/2}$ MIT bag wave functions. For a static spherical bag of radius R the $1s$ wave function can be written as

$$q_{1s}(\mathbf{r},t) = \frac{N_s}{\sqrt{4\pi}} \begin{bmatrix} j_0(\omega_s r) \\ i\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} j_1(\omega_s r) \end{bmatrix} e^{-i\omega_s t} b \theta(R-r) \ , \tag{2.16}$$

where b denotes the spin-isospin wave function of the quark (which can be seen in detail in Ref. 29) and $\omega_s = 2.04 \cdots /R$ is the energy of the quark ground state which satisfies the linear boundary condition Eq. (2.10). The $1p_{1/2}$ wave function is

$$q_{1p_{1/2}}(\mathbf{r},t) = \frac{N_p}{\sqrt{4\pi}} \begin{bmatrix} -\boldsymbol{\sigma}\cdot\hat{\mathbf{r}} j_1(\omega_p r) \\ i j_0(\omega_p r) \end{bmatrix} e^{-i\omega_p t} b \theta(R-r) \ , \tag{2.17}$$

with $\omega_p = 3.81 \cdots /R$ being the energy of the first excited quark state. The normalization factors in Eqs. (2.16) and (2.17) are

$$N_{s,p}{}^2 = \frac{1}{2 j_0{}^2(\omega_{s,p} R) R^3} \frac{\omega_{s,p} R}{\omega_{s,p} \mp 1} \ . \tag{2.18}$$

The corresponding wave functions for massive quarks are given in Appendix A.

Using the quantized meson fields and the quark wave functions, the interaction Hamiltonian for the transition $BM \to \Lambda^*$ reduces to

$$H_s=\sum_j \int d^3k\,[V_{0j}(\mathbf{k})a_j(\mathbf{k})+V_{0j}^\dagger(\mathbf{k})a_j^\dagger(\mathbf{k})]\ , \tag{2.19}$$

where j labels the type of meson (including its charge state).

The vertex function is given by

$$V_{0j}(\mathbf{k})=B_0^\dagger v_{0j}^{B\Lambda^*}(\mathbf{k})\Lambda_0^*\ , \tag{2.20}$$

$$v_{0j}^{B\Lambda^*}=\lambda_{B\Lambda^*}\frac{u_{\alpha\Lambda}^*(kR)}{[(2\pi)^3 2\omega_M(k)]^{1/2}}C_{i_B i_M 0}^{i_{B_3} i_{M_3} 0}\ , \tag{2.21}$$

where α labels the meson-baryon pair (e.g., $\bar{K}N$ or $\Sigma\pi$), and the dependence of v_{0j} on j is hidden in the Clebsch-Gordan coefficient. The form factor for s-wave scattering is [using Eq. (2.21)]

$$u_{\alpha\Lambda^*}(kR)=N_sN_p\left[2R^2 j_0(\omega_s R)j_0(\omega_p R)j_0(kR)-[\omega_s-\omega_p+\omega_M(k)]\int_0^R dr\, r^2[j_0(\omega_s r)j_0(\omega_p r)+j_1(\omega_s r)j_1(\omega_p r)]j_0(kr)\right]\ , \tag{2.22}$$

and the coupling constants are given in Table I.

For S-wave scattering the spatial part of the covariant derivative of $\hat{H}_c$ [Eq. (2.9)] does not contribute, and we just present the result corresponding to the time derivative for transitions between baryon-meson-octet members:

$$H_c=\sum_{i,j}\int d^3k\int d^3k'\,V_{0ij}(\mathbf{k},\mathbf{k}')a_i^\dagger(\mathbf{k})a_j(\mathbf{k}')\ , \tag{2.23}$$

with the vertex function

$$V_{0ij}(\mathbf{k},\mathbf{k}')=\sum_{B_0,B_0'} B_0^\dagger v_{0ij}^{BB'}(\mathbf{k},\mathbf{k}')B_0'\ , \tag{2.24}$$

where, once again, (i,j) and (B,B') label the type of meson (and baryon), including its charge state. If we restrict ourselves to purely S-wave scattering we find the following explicit expression for $v_{0ij}^{BB'}$:

$$v_{0ij}^{BB'}(\mathbf{k},\mathbf{k}')=\sum_{I,I_3}\lambda_{\alpha\beta}^I\frac{u'_{\alpha\beta}(k,k',R)}{[(2\pi)^3 2\omega_M(k)]^{1/2}[(2\pi)^3 2\omega_{M'}(k')]^{1/2}}C_{i_B i_M I}^{i_{B_3} i_{M_3} I_3}C_{i_{B'} i_{M'} I}^{i_{B_3'} i_{M_3'} I_3}\ . \tag{2.25}$$

[In Eq. (2.25) α and β are a shorthand notation for the initial or final meson-baryon pair—e.g., $\bar{K}N$ or $\Sigma\pi$.] The form factor is

$$u'_{\alpha\beta}(k,k',R)=N_s^2[\omega_M(k)+\omega_{M'}(k')]\int_0^R dr\, r^2[j_0^2(\omega_s r)+j_1^2(\omega_s r)]j_0(kr)j_0(k'r)\ , \tag{2.26}$$

and the coupling constants are given in Tables I and II.

A. Vertex renormalization

So far the Hamiltonian which we have written down connects bare baryons [see Eq. (2.13)]. However, we want to describe physical baryons which in the CBM consist of "dressed" bags. That is, the physical baryon B is part of the time (Z_2^B) a bare three-quark bag, a bare bag surrounded by a cloud of one meson, two mesons, and so on. Thus the physical baryon satisfies the equation[11]

$$H\,|B\rangle=m_B\,|B\rangle\ , \tag{2.27}$$

and can be expanded in terms of the bare bag states $\{B,B',\dots\}$:

$$|B\rangle=\sqrt{Z_2^B}\,|B_0\rangle+\Lambda\,|B\rangle\ , \tag{2.28}$$

where

$$\Lambda=1-\sum_{B_0'}|B_0'\rangle\langle B_0'| \tag{2.29}$$

TABLE I. The coupling constants for $\bar{K}N$ scattering with isospin 0 and 1. In the first column are the values for $2f\lambda_{\alpha\Lambda^*}$ Eq. (2.21), while the other columns contain $2f^2\lambda_{\alpha\beta}^I$ Eq. (2.25).

I		0			1	
	Λ^*	$\bar{K}N$	$\pi\Sigma$	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$
$\bar{K}N$	$\sqrt{2}$	$-\frac{3}{2}$	$-\sqrt{6}/4$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\sqrt{6}/4$
$\pi\Sigma$	$\sqrt{3}$	$-\sqrt{6}/4$	-2	$-\frac{1}{2}$	-1	0
$\pi\Lambda$	0	0	0	$\sqrt{6}/4$	0	0

TABLE II. The coupling constants for S-wave KN and πN scattering with isospin I. The values quoted are $2f^2\lambda_{\alpha\beta}^I$ in the notation of Eq. (2.25).

I	1	0	$\frac{3}{2}$	$\frac{1}{2}$
	KN	KN	πN	πN
KN	1	0	0	0
πN	0	0	$\frac{1}{2}$	-1

projects out only those states which contain at least one meson. The mass of the dressed state, m_B, is then

$$m_B = m_{B_0} + \Sigma^B . \tag{2.30}$$

In terms of this state Σ^B, the normalization condition on $|B\rangle$ implies

$$Z_2^B(m_B) = \left[1 - \frac{\partial}{\partial E} \Sigma^B(E) \right]^{-1}_{E=m_B}$$

$$= [1 + \Sigma_{B'} Z_2^B(B')]^{-1} . \tag{2.31}$$

The lowest-order self-energy diagram which contributes to $Z_2^B(B')$ is shown in Fig. 1.

Usually the vertex renormalization is discussed in the context of Yukawa couplings, where the renormalized vertex function is

$$v_j^{BB'}(\mathbf{k}) = \langle B \mid V_{0j}(\mathbf{k}) \mid B' \rangle . \tag{2.32}$$

This implies that

$$v_j^{BB'}(\mathbf{k}) = \frac{\sqrt{Z_2^B}\sqrt{Z_2^{B'}}}{Z_1^{BB'}} v_{0j}^{BB'}(\mathbf{k}) , \tag{2.33}$$

with the vertex renormalization constant given by

$$(Z_1^{BB'})^{-1} = 1 + \sum_{C,D} Z_1^{BB'}(C,D) . \tag{2.34}$$

The lowest-order diagram which contributes to $v_{0j}^{BB'}$, namely, $Z_1^{BB'}(C,D)$ is shown in Fig. 2(a).

Hence to calculate the vertex renormalization for a Yukawa coupling it is necessary to calculate the probabilities $Z_2^{B,B',\ldots}$ [defined in Eqs. (2.28) and (2.31)] and the functions $Z_1^{B,B'}(C,D)$. All of these quantities are given in great detail by Théberge and Thomas.[11] We shall briefly indicate how to use a similar procedure to calculate the vertex renormalization of the contact interaction. Once again there is a bare interaction:

$$V_{0j}^{BB'}(\mathbf{k},\mathbf{k}') = \langle B_{0i}(\mathbf{k}) \mid V_0 \mid B'_{0j}(\mathbf{k}') \rangle , \tag{2.35}$$

which is simply the matrix element of H_2 between the initial and final one-meson one-bag states. The renormalized contact interaction is then the matrix element of H_2 between physical baryon states. For a given total isospin and angular momentum (labels suppressed for clarity) we can write

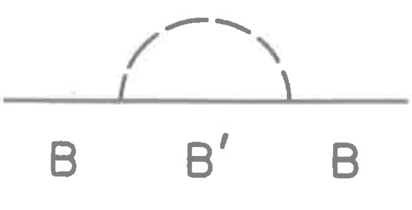

FIG. 1. Lowest-order self-energy diagram for a baryon B. A sum must be made over intermediate baryons B'.

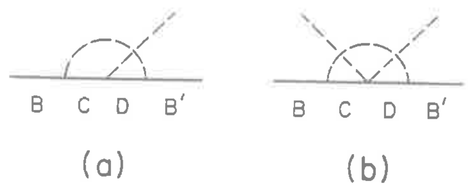

FIG. 2. Lowest-order diagrams contributing to (a) $v_{0j}^{BB'} Z_1^{BB'}(C,D)$ [Eq. (2.34)] and to (b) $V_{0ij}^{BB'}(\mathbf{k},\mathbf{k}') Z_{1MM'}^{BB'}(C,D)$ [Eq. (2.37)].

$$V_{ij}^{BB'}(\mathbf{k},\mathbf{k}') = \langle B \mid V_{0ij}^{BB'}(\mathbf{k},\mathbf{k}') \mid B' \rangle$$

$$= \frac{\sqrt{Z_2^B}\sqrt{Z_2^{B'}}}{Z_{1\alpha\beta}} V_{0,ij}^{BB'}(\mathbf{k},\mathbf{k}') , \tag{2.36}$$

where

$$(Z_{1\alpha\beta})^{-1} = 1 + \sum_{\gamma,\delta} Z_{1\alpha\beta}(\gamma,\delta) . \tag{2.37}$$

The lowest-order diagram which contributes to $V_{0ij}^{BB'}(\mathbf{k},\mathbf{k}')$, namely, $Z_{1\alpha\beta}(\gamma,\delta)$, is shown in Fig. 2(b). In practice, we have included only the dressing associated with virtual pions. However, we have checked that those graphs involving virtual kaons are small.

Basically the difference between the renormalization of a Yukawa interaction and a contact interaction resides in the difference between Figs. 2(a) and 2(b). As the contact interaction is independent of spin, under certain assumptions it is possible to relate the two renormalizations Z_1 and Z_2. For example, if we retain only pion loops, and neglect the $\pi\Lambda \to \pi\Sigma$ contact interaction inside Fig. 2(b), we find

$$(Z_{1\alpha\beta})^{-1} = 1 + \sum_{B'} Z_{1\alpha\beta}(B') , \tag{2.38}$$

where for elastic scattering

$$Z_{1\alpha\beta}(B') = Z_2^B(B') \frac{\sum_{i_B} \left(C_{i_{B'} i_\pi i_B}^{i_{B'_3} i_{\pi_3} i_{B_3}}\right)^2 H_{\text{int}}(i_{B'}, i_M)}{H_{\text{int}}(i_B, i_M)} , \tag{2.39}$$

and

$$H_{\text{int}}(i_{B'}, i_M) \propto i_{B_3} i_{M_3} + \tfrac{3}{4} Y_B Y_M . \tag{2.40}$$

In Eq. (2.40) i_B (i_M) is the isospin of the baryon (meson) and Y_B (Y_M) the corresponding hypercharge.

Let us consider K^+p scattering as an example. Writing the renormalization factors explicitly, we have

$$V_{K^+K^+}^{pp}(\mathbf{k},\mathbf{k}') = \left[\frac{1 + \frac{2}{3} Z_2^N(N) + \frac{7}{6} Z_2^N(\Delta)}{1 + Z_2^N(N) + Z_2^N(\Delta)} \right] V_{0K^+K^+}^{pp}(\mathbf{k},\mathbf{k}')$$

$$= \mathscr{R}^{K^+p} V_{0K^+K^+}^{pp}(\mathbf{k},\mathbf{k}') \tag{2.41}$$

TABLE III. The renormalized value of f ($f_{\rm eff}/f$) for the different channels. The renormalization is independent of isospin for both the πN and $\pi\Sigma$ channels.

	πN	$\pi\Sigma$	KN $I=1$	$\bar{K}N$ $I=0$	$\bar{K}N$ $I=1$
$R=0.8$ fm	1.04	1.11	1.01	1.02	0.98
$R=1.0$ fm	1.03	1.07	1.01	1.01	0.99

and

$$V^{nn}_{K^+K^+}(\mathbf{k},\mathbf{k}')=\left[\frac{1+\frac{5}{3}Z_2^N(N)+\frac{2}{3}Z_2^N(\Delta)}{1+Z_2^N(N)+Z_2^N(\Delta)}\right]V^{nn}_{0K^+K^+}(\mathbf{k},\mathbf{k}')$$

$$=\mathscr{R}^{K^+n}V^{nn}_{0K^+K^+}(\mathbf{k},k')\ . \qquad (2.42)$$

Using the results quoted by Théberge and Thomas[11] for $Z_2^B(B')$, the renormalization factors are $\mathscr{R}^{K^+p}=0.99$ and $\mathscr{R}^{K^+n}=1.03$ for the bag radius $R=1$ fm. It is interesting to note that the renormalization for K^+p and K^+n is different, this causes the isospin-zero K^+n interaction to become nonzero with strength $a_0\simeq a_1/20$ for $R=1$ fm.

Table III contains the renormalization factors needed in the present work. We quote the effective value for f, which is related to $\mathscr{R}$ through

$$\left[\frac{f_{\rm eff}}{f}\right]^2=\frac{1}{\mathscr{R}}\ . \qquad (2.43)$$

B. The scattering problem

In order to solve the scattering problem we define a potential and solve the Lippmann-Schwinger equation. The effect of crossed meson lines is relatively small for pion-nucleon scattering.[14] For kaon-nucleon scattering it is even smaller and for this reason we do not include it here. (In Sec. III we give an estimate of its influence on the $\bar{K}N$ scattering length.) Thus we define the following potential:

$$v_{\alpha\beta}=\sum_{B_0'}\langle\alpha\,|\,H_s\,|\,B_0'\rangle\frac{1}{E-M_0(B_0')}\langle B_0'\,|\,H_s\,|\,\beta\rangle$$
$$+\langle\alpha\,|\,H_c\,|\,\beta\rangle\ , \qquad (2.44)$$

where $|\,\alpha\rangle$ and $|\,\beta\rangle$ stand for baryon-meson states and $|\,B_0'\rangle$ is a baryon bag state with mass $M_0(B_0')$. This potential is iterated in the Lippmann-Schwinger equation:

$$t_{\alpha\beta}(\mathbf{k},\mathbf{k}',E)=v_{\alpha\beta}(\mathbf{k},\mathbf{k}',E)$$
$$+\int v_{\alpha\gamma}(\mathbf{k},\mathbf{k}'';E)\frac{1}{E-E_\gamma(k'')+i\epsilon}$$
$$\times t_{\gamma\beta}(\mathbf{k}'',\mathbf{k}';E)d\mathbf{k}''\ , \qquad (2.45)$$

where

$$E_\gamma(k'')=(M_\gamma{}^2+k''^2)^{1/2}+(m_\gamma{}^2+k''^2)^{1/2}$$

is the energy of the virtual meson-baryon system in the intermediate state. The expansion in partial waves is made in the usual way:

$$t_{\alpha\beta}(\mathbf{k},\mathbf{k}';E)=\sum_{l,m}Y_{lm}(\hat{k})Y^*_{lm}(\hat{k}')t^l_{\alpha\beta}(k,k';E)\ , \qquad (2.46)$$

so that the on-shell, diagonal amplitude is related to the phase shift (δ_l^α) by[30]

$$t^l_{\alpha\alpha}(E)=t^l_{\alpha\alpha}(k_\alpha,k_\alpha;E)=-\frac{e^{i\delta_l^\alpha}\sin\delta_l^\alpha}{\pi\mu_\alpha k_\alpha}\ . \qquad (2.47)$$

In order to avoid the singularity in the denominator of Eq. (2.45) for $E=E_\gamma(k'')$, we do a principal-value subtraction.[31] That is, we use the fact that

$$\mathrm{P}\int_0^\infty\frac{dk'}{k^2-k'^2}=0\ ,$$

to subtract the quantity

$$2k^2\mu(k)v(k,k;E)t(k,k;E)/(k^2-k'^2)\ .$$

This produces a smooth integrand for which it is no longer necessary to calculate a principal-value integral. Then we solve the equation in matrix form by doing the integrations by Gaussian quadrature.

1. KN S-wave scattering

For KN scattering the potential Eq. (2.44) reduces to the contact term, because for S wave there is no candidate with strangeness $+1$ for the state $|\,B_0'\rangle$. Hence the potential for isospin-I, S-wave scattering reduces to Eqs. (2.25) and (2.26) with the coupling constants given in Table I. This table contains the unrenormalized coupling constants extracted under our working hypothesis of exact $SU(3)_F$ symmetry. If we take into account the fact that the renormalizations for K^+p and K^+n are different (using the renormalization procedure explained in Sec. II A) there is a small $I=0$ coupling. For example, for a bag radius of 1 fm the renormalization factors are $\mathscr{R}^{K^+p}=0.99$ and $\mathscr{R}^{K^+n}=1.03$ (see Table III), which leads to an $I=0$ coupling around 5% of that in $I=1$.

By iterating this potential in a Lippmann-Schwinger equation we get the scattering amplitude corresponding to the series shown in Fig. 3. For KN scattering there are only two free parameters, namely, the meson decay constant f and the bag radius. The radius dependence of the $I=1$ scattering length is shown in Fig. 4 for two values of f. Our results are consistent with the data indicated by the dashed region in this figure. It is worth remembering that the renormalization of the coupling constant for K^+p increases the effective value of the meson decay constant by 1%. Hence, assuming a starting value of 93 MeV, the effective value is 94 MeV. Assuming an average value between f_π and f_K, namely, 103 MeV, the renormalized value is 104 MeV. Either value would give a good description of the data. The $I=0$ scattering length is $a_0\simeq a_1/20\simeq-0.01$ fm, while the experimental values

FIG. 3. Series generated by the contact term. Baryons are represented by solid lines and mesons by dashed ones.

vary between -0.11 fm and 0.04 fm,[32] $a_0=+0.02$ fm being the most recent one.[33] Although our result is consistent with existing data, we do not claim to give a good description of the $I=0$ scattering length because, as it is very small, a slight admixture of $I=1$ coming through crossed graphs can be relevant. This comparison is interesting mainly to give an idea of the small effect of higher-order terms not taken into account in the present model.

The mass of the strange quark does not affect the results presented here because this quark is not directly involved in the $KN \to KN$ transition.

2. *S-wave pion-nucleon scattering*

As an additional check on the model we have also calculated the S-wave πN scattering length. Just the contact term has been included because the lowest two possible states which contribute to the separable term, namely, the $N(1520)$ and the $\Delta(1620)$, are already more than 400 MeV above threshold. The isospin $\frac{1}{2}$ and $\frac{3}{2}$ scattering lengths produced by the Born contact term are $a_1=0.22$ fm and $a_3=-0.11$ fm, respectively, for $R=1$ fm and $f=93$ MeV. This is in reasonable agreement with the experimental results,[34] $a_1=0.240$ fm and $a_3=0.145$ fm. Unfortunately this agreement seems somewhat fortuitous. From the renormalization argument we expect f to be effectively increased by 3%, which changes these values to $a_1=0.20$ fm and $a_3=-0.10$ fm. Another point is that the rescattering increases the value of a_1 and decreases a_3 (it becomes less negative), so that the isospin averaged value a_1+2a_3 becomes nonzero. For example, using $f=100$ MeV and $R=1$ fm, $a_1=-0.33$ fm and $a_3=-0.08$ fm.

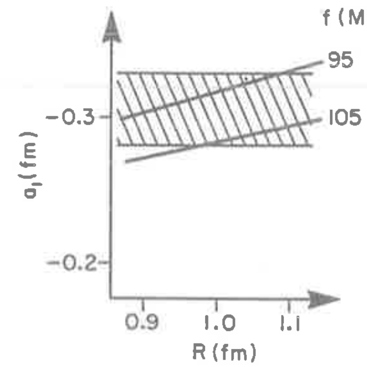

FIG. 4. The $I=1$ KN scattering length plotted against the bag radius for different values of f. The shaded region indicates the range of experimental results, $a_1=-0.33$ fm being the more recent one (Ref. 33).

The reason for the disagreement is not obvious. One possibility is the Z or antiparticle graphs. However, in agreement with Brodsky[35] we would argue that such graphs are suppressed by the finite size of the nucleon. Another possibility is that the multiple scattering is suppressed by the finite size of the pion, as suggested by Crawford and Miller.[36] However, this effect is much too small. A more likely possibility is that the pion interacts with the bag itself. Certainly excluding the pion from the bag would generate much too large a repulsion. Introducing a weak repulsion for the pion inside the bag would, however, cure the problem. Since this leads too far from our main concern of $\bar{K}N$ scattering it will not be considered further here.

III. THE COUPLED $\bar{K}N$-$\Sigma\pi$ SYSTEM IN THE VICINITY OF THE $\Lambda^*(1405)$

To describe $\bar{K}N$ scattering we include the $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$ channels as well as a Λ^* bare three-quark state. For the case of scattering there are several three-quark states which can in principle contribute to the separable part of the potential. However, for the volume coupling version of the CBM the higher excited states give a relatively small contribution for low-energy scattering.[37] Therefore, we restrict the calculation to just one excited baryon state. As a rough guide to the structure of the lowest $\frac{1}{2}^-$ Λ^* state, we note that Isgur and Karl,[38] in their nonrelativistic quark model, find that the lowest $\frac{1}{2}^-$ Λ^* state is 80% an SU(3) singlet with the rest being octet. In our calculation we take the bare state to be a pure SU(3) singlet. Since we find that the $\Lambda^*(1405)$ is predominantly a $\bar{K}N$ bound state, we do not expect the details of the bare state to be important.

The potential for the $\bar{K}N$ S-wave scattering [Eq. (2.44)] reduces to just one separable potential given by Eqs. (2.2) and (2.22), and a contact part given by Eqs. (2.25) and (2.26). The coupling constants are given in Table III. This potential is iterated in the relativistic Lippmann-Schwinger Eq. (2.45) producing the scattering amplitude corresponding to the graphs shown in Figs. (3) and (5).

A. Scattering in S wave

In the case of S-wave $\bar{K}N$ scattering our model has three parameters, namely, f, R, and M_0 (the mass of the bare Λ^* state). These parameters are adjusted in order to

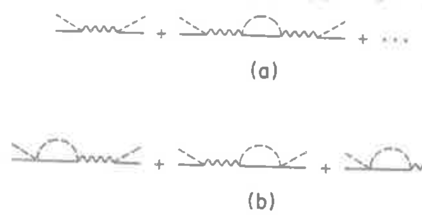

FIG. 5. (a) Series generated by the separable potential and (b) its interference with the contact term. Baryon-octet members are represented by solid lines, mesons by dashed ones, and the Λ^* by wiggly lines.

get a reasonable $\pi\Sigma$ mass spectrum compared to the data. For most of the discussion we will take the strange quark to be massless since we have checked that the mass has much less effect than other uncertainties in the calculation. The thresholds are taken to be 1432.6, 1331.6, and 1254.6 MeV for $\bar{K}N$, $\pi\Sigma$, and $\pi\Lambda$, respectively. When the nondegeneracy of the K^-p and $\bar{K}^0n$ is included, the $\bar{K}^0n$ threshold is 1437.3 MeV.

In Fig. 6 we compare our results with the $\pi\Sigma$ mass spectrum measured by Hemingway *et al.*[27] for $K^-p\to\pi^+\pi^-(\Sigma^-\pi^+)$. The theoretical curves are $k^{\pi}_{\text{c.m.}}\,|\,T_{\pi\Sigma}\,|^2$. The normalization is arbitrary, since we are interested just in the shape of the spectrum. The solid and dashed curves correspond to (R,f) equal to (1.0 fm, 120 MeV) and (1.1 fm, 110 MeV), respectively. The values for f are larger than those obtained with the renormalization procedure. However the mass spectrum is very sensitive to this parameter and it is not possible to get a reasonable mass spectrum without increasing the value of f. The bare mass of the Λ^* turns out to be $M_0=1630$ MeV for the set (1.0 fm, 120 MeV) and $M_0=1650$ MeV for the other set. Variations in these masses around ± 5 MeV are acceptable (we note that in Ref. 26) the bare masses were different because our results were compared with older data). In this comparison we took just the $I=0$ piece. [The $I=1$ $\Sigma\pi$ interaction is less attractive in the S wave (Table I), and the effect of the $\Sigma^*(1385)$ is suppressed by an angular momentum barrier.] In Fig. 7 we show the $I=0$ scattering amplitude for the two sets of parameters. The real piece stays high very near the $\pi\Sigma$ threshold, showing that the cutoff in the mass spectrum at low energy comes from the phase-space factor $k^{\pi}_{\text{c.m.}}$ by which $|\,T_{\pi\Sigma}\,|^2$ is multiplied.

The $\bar{K}N$ elastic-scattering amplitude can be seen in Fig. 8 for $I=0$. We note a rapid variation with the energy near the $\bar{K}N$ threshold, which may be useful in reconciling the kaonic-hydrogen energy shift with the scattering data.

The effective value for f in the $I=1$ piece is kept inside limits compatible with the renormalization procedure, using $f^{\bar{K}N}_{I=1}$ smaller than $f^{\pi\Sigma}_{I=1}$. This gives for $(R,f_{\pi\Sigma,\pi\Lambda},f_{\bar{K}N})$ the values (1.0, 110, 100) and (1.1, 105, 95). The $\bar{K}N$ scattering amplitudes for these two sets of parameters are shown in Fig. 8.

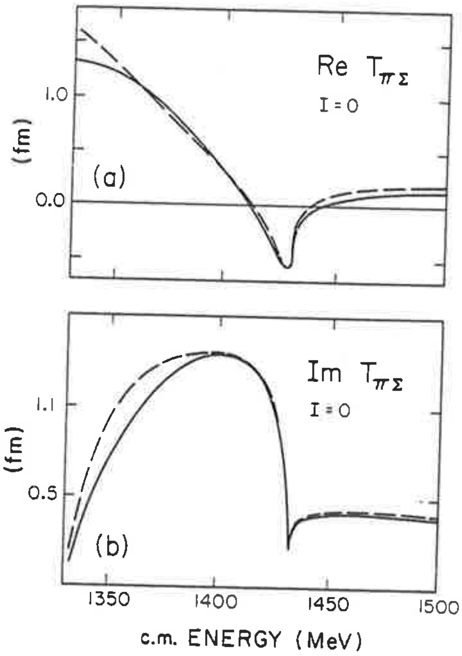

FIG. 7. The $\pi\Sigma$ elastic-scattering amplitude corresponding to parameter set A (solid curve) and B (dashed curve).

To summarize, our two sets of parameters are as follows.

Set A:

$$R=1.0\text{ fm},\quad M_0=1630\text{ MeV}\,,$$

$$f^{I=0}=120\text{ MeV},\quad f^{I=1}_{\pi\Sigma,\pi\Lambda}=110\text{ MeV}\,,$$

$$f^{I=1}_{\bar{K}N}=100\text{ MeV}\,.$$

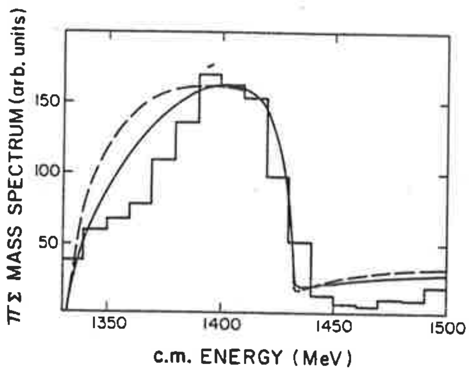

FIG. 6. The $\pi\Sigma$ mass distribution. The histogram is data from Hemingway *et al.* (Ref. 27). The theoretical curves are $k^{\pi}_{\text{c.m.}}\,|\,T_{\pi\Sigma}\,|^2$. The solid curve corresponds to parameter set A and the dashed curve to parameter set B.

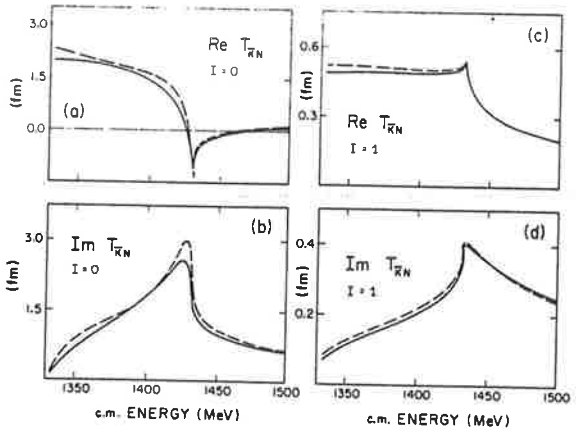

FIG. 8. The $\bar{K}N$ elastic-scattering amplitude for $I=0$ (a) and (b) and $I=1$, (c) and (d), with the parameter set A (solid curve) and B (dashed curve).

Set B:

$$R=1.1\ \text{fm},\quad M_0=1650\ \text{MeV}\ ,$$

$$f^{I=0}=110\ \text{MeV},\quad f^{I=1}_{\pi\Sigma,\pi\Lambda}=105\ \text{MeV}\ ,$$

$$f^{I=1}_{\bar{K}N}=95\ \text{MeV}\ .$$

The comparison between the theoretical and experimental cross sections[39,40] is made in Fig. 9. In these cross sections we include the Coulomb correction and the effect of the nondegenerate masses of the K^-p and $\bar{K}^0n$ as derived by Dalitz and Tuan.[41] We note that Evans *et al.*[42] claim a contribution from p waves for the charge-exchange process at an incident momentum around 230 MeV/c, and for σ production at a momentum as low as 150 MeV/c.

B. The scattering lengths

Table IV contains the $\bar{K}N$ scattering lengths for the two sets of parameters, taking massless u and d quarks, and including the mass of the strange quark (300 MeV). (The mass of the strange quark is introduced in the manner shown in Appendix A.) The u and d quarks are kept massless because even the large mass of the s quark has a small effect. (Its effect on the cross sections presented is smaller than 3%.) In the $\pi\Sigma$ mass spectrum, the s-quark mass tends to decrease the bare mass of the Λ^* by 5 or 6 MeV.

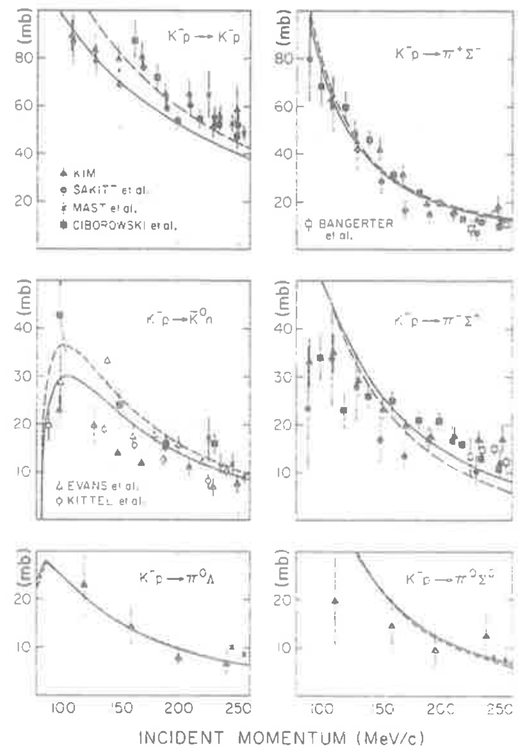

FIG. 9. Comparison of the K^-p cross sections at low energy with data from Refs. 39 and 40. The solid line corresponds to parameter set A and the dashed one to parameter set B.

TABLE IV. The $\bar{K}N$ scattering length for the two sets of parameters taking all the quarks massless and the s quark with mass (M_s).

Parameter set	M_s (MeV)	a_0 (fm)	a_1 (fm)
A	0	$-1.14+1.76i$	$0.53+0.39i$
B	0	$-1.36+2.07i$	$0.54+0.40i$
A	300	$-1.02+1.97i$	$0.55+0.42i$
B	300	$-1.22+2.30i$	$0.56+0.44i$

To compare these scattering lengths with those extracted from the kaonic-hydrogen energy shift $[a_{\bar{K}p}=(0.10\pm0.15)+(0\pm0.28)i\ \text{fm}]$[42] it is necessary to include the splitting of the $\bar{K}p$ and $\bar{K}^0n$ masses and a Coulomb correction. The mass splitting changes the real piece of the average scattering length $\bar{a}=\frac{1}{2}(a_0+a_1)$, e.g., for set A, from -0.31 to -0.26 fm. Including the Coulomb correction just outside the range of the nuclear interaction,[41] this value is shifted to -0.38 fm. It is possible to improve the Coulomb correction,[43] however it is not likely that it can have so large an effect as to reconcile our results with the kaonic-hydrogen data.

For $\bar{K}N$ elastic scattering the Born term involving crossed meson lines vanishes by strangeness conservation. Hence its effect comes only in higher order, and as a consequence has a small effect on the scattering length. For example, including this kind of graph the scattering length for parameter set A is $a_0=-1.02+1.74i$ fm and $a_1=-0.54+0.38i$ fm.

IV. DISCUSSION OF THE RESULTS

The S-wave KN scattering is the cleanest process studied in this paper. As soon as there is no baryon with $S=+1$ to be included there is only the "freedom" to vary f and R. Happily it seems that the values of f and R which describe the data are quite consistent with the renormalization and radius expected in the CBM.

The agreement between experiment and theory for πN scattering is not as good. The contact term gives a contribution quantitatively similar to the ρ exchange included in more conventional models.[44,45] (As pointed out many years ago by Weinberg[46] this is not entirely accidental.) To get better agreement with the data we clearly need some more physics. One might consider some of the standard phenomenological suggestions[45] such as a hard core, or Z graphs. These contributions are outside the scope of the present paper, and we only note that at this stage, the CBM does not seem to cast a new light on the problem.

The S-wave $\bar{K}N$ system has the additional complication of an intermediate state, the $\Lambda^*(1405)$. This state raises problems in both the nonrelativistic quark model[38] and in the MIT bag model.[47] In the nonrelativistic quark model it comes too high in energy and is degenerate with the $\frac{3}{2}^-$ Λ^* state which experimentally is at 1520 MeV. In the bag model the $\frac{1}{2}^-$ state occurs higher than the $\frac{3}{2}^-$ due to the spin-orbit force.

Our model of the $\Lambda^*(1405)$ is very similar to that of Dalitz, Wong, and Rajasekaran.[25] In that calculation the potential was taken from vector-meson exchange with the relative coupling constants taken from SU(3) symmetry. The range of potential was determined by the masses of the vector mesons. In the present calculation the contact interaction has the same spin-flavor structure as the vector mesons, namely, the quark part of the coupling is

$$\bar{q}\gamma^{\mu}\lambda^{a}q\theta_{v} ,$$

the meson part is

$$f_{abc}\phi_{b}\partial_{\mu}\phi_{c} ,$$

and the coupling constants given in Table I for the contact interaction are in fact the same as those given by Dalitz and co-workers[25] (apart from the phase convention). In the present case the range of the potential is given by the bag radius R rather than the vector-meson masses. The net result will be somewhat similar. The main difference is that in our model elementary states (i.e., three-quark states) appear in a rather natural way and it is the model that decides to what extent the $\Lambda^*(1405)$ is a $\bar{K}N$ bound state or a normal quark state. We find that the $\Lambda^*(1405)$ is predominantly a $\bar{K}N$ bound state with only 14% of the strength in the $\pi\Sigma$ cross section around 1405 MeV coming from the bare quark state—see Appendix B. We also have a higher state which presumably corresponds to the $\Lambda^*(1670)$. In agreement with previous analysis[33] we find that the $\Lambda^*(1405)$ is not well described by a Breit-Wigner resonance shape.

The D-wave scattering will be less strongly affected by the potential because of the centrifugal barrier and we do not expect a $\bar{K}N$ bound state in that channel, although the $\frac{3}{2}^-$ state may be shifted down somewhat from its unperturbed value. Thus we would claim that the lowest excited three-quark state seen is the $\frac{3}{2}^-$ $\Lambda^*(1520)$, in agreement with the order suggested by the MIT bag model.

Kumar and Nogami[48] have proposed that a Castillejo-Dalitz-Dyson (CDD) zero in the scattering amplitude would reconcile the scattering and kaonic-hydrogen data. Their model has a pole term and a separable contact potential. The main weakness of this model so far, is that it has not been adjusted to reproduce any experimental data. In our model we also have a CDD zero, however it comes out at much too high an energy to explain the discrepancies at threshold. In Fig. 10 we show the inverse K matrix elements and the determinant of the inverse K matrix. The pole in the determinant at about 1700 MeV is the CDD pole. The inverse K matrix elements, shown over a more restricted energy interval, indicate that these elements do indeed have a linear dependence over a considerable energy range.

The $\bar{K}N$ scattering amplitude given by the CBM shows a strong energy dependence near threshold. It is worth noting that as a consequence of the coupled channel nature of the problem the resonance structure in the $\pi\Sigma$ elastic-scattering amplitude appears in a different position than the change in sign in the $\bar{K}N$ scattering amplitude. In particular, the former occurs at ground 1410 MeV (Fig. 7), while the latter occurs near 1428 MeV (Fig. 8). To explain the disagreement between scattering and kaonic-hydrogen data it would be necessary to move the zero in the $\bar{K}N$ case something like 5 MeV above threshold

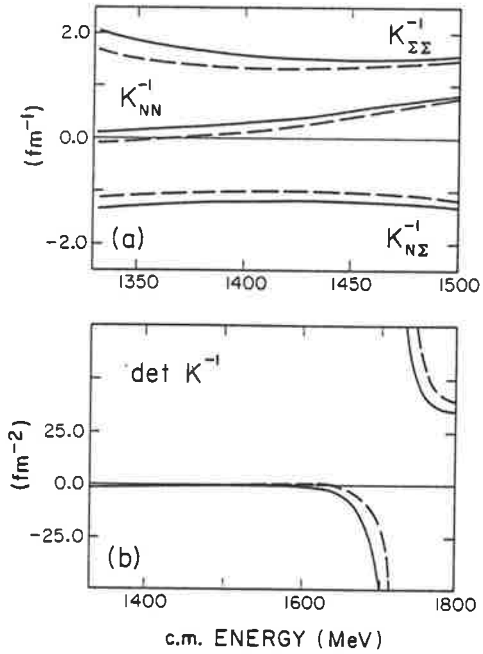

FIG. 10. (a) Matrix elements of the inverse K matrix and (b) the determinant of the inverse K matrix ($I=0$).

without spoiling the $\pi\Sigma$ mass spectrum. However, we were not able to do this with reasonable sets of parameters.

Our results for the $\bar{K}N$ scattering length differ in magnitude from values quoted in the literature (e.g., $a_0=1.70+0.68i$ fm and $a_1=0.37+0.60i$ fm),[25] but with respect to the sign favor the scattering data.

V. CONCLUSIONS

The CBM has been extended to chiral SU(3)$\times$SU(3) and applied to the problem of kaon-nucleon scattering. First some comments must be made on the limitations of the model. We use a static, spherical bag with a sharp surface, so that the deformation of the $\Lambda^*(1405)$ is neglected. We also ignore any center-of-mass correction and use the same radius for all baryons. Some of these limitations can in principle be improved[49] but only at the cost of greatly complicating the calculation. Beyond that, it is not clear how to include the center-of-mass correction. Although interesting, these effects can hardly change the conclusions of the present work.

Crawford and Miller[36] have shown that by using the finite size of the pion, the contribution of excited quark states to the self-energy is finite and small. This explains why the self-energies do not diverge and leads to the conclusion that it is safer when using pointlike mesons to not include a complete set of excited quark states which may yield unphysical results. Additionally, the extended meson decreases the multiple scattering.

In the present model we use pointlike mesons, but do not allow quark excitations beyond the $1p_{1/2}$ level. Thus we do not include those contributions which would anyway be cutoff by smearing the field in space and time. As a first step we have considered only s-wave scattering, which restricts our comparison with the data to very low energy. Clearly the next step is to extend the calculation to other partial waves. The comparison of our results with data for low-energy KN and $\bar{K}N$ scattering is quite good. Thus, in spite of the large mass of the strange quark, and the badly broken SU(3) symmetry in nature, the SU(3) CBM seems to make sense as soon as the physical masses are taken into account. The $\bar{K}N$ scattering length which we find disagrees with that extracted from $\bar{K}p$ atom data, as do all the scattering lengths extracted from scattering data. The remaining point to be checked is whether by using the strong-interaction potential produced by the CBM directly in the kaonic-hydrogen calculations it is possible to get the energy shift measured in the kaonic-hydrogen experiments. Work is in progress on this problem.[50]

The results obtained for low-energy πN scattering are not as good. However, the contact term gives a contribution similar to the ρ meson in meson-exchange descriptions of πN scattering. It is worth commenting at this point that the contact term has many features in common with the exchange of an octet of vector mesons. As pointed out many years ago by Weinberg,[46] the vector mesons can be introduced explicitly in the covariant derivatives that appear in the volume coupling. While that is probably inconsistent with the present model, it does indicate that many effects which are traditionally ascribed to vector mesons may be equally well described by the contact term.

In the controversy over the nature of the $\Lambda^*(1405)$, the present model comes down firmly on the side of the $\Lambda^*(1405)$ being primarily a $\bar{K}N$ bound state.[25] This indicates that the $\Lambda^*(1405)$ should not be included as one of the states fit in simple quark-model descriptions of baryon resonances.

In summary, the CBM provides a useful description of kaon-nucleon scattering at low energy in spite of the fact that SU(3) symmetry is so badly broken in nature.

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APPENDIX A

In this appendix we present the modifications to the formalism required by the massive strange quark. The u and d quarks are kept massless. It would not be hard to include their masses but since even the large mass of the strange quark has little effect there is no reason to. The $1s_{1/2}$ and $1p_{1/2}$ massive-quark wave functions are given, respectively, by [instead of Eq. (2.16) and (2.17)]

$$q_{1s}(\mathbf{r},t)=\frac{N_s'}{\sqrt{4\pi}}\begin{bmatrix}\alpha_s^+ j_0(w_s' r)\\ i\alpha_s^- \sigma\cdot\hat{\mathbf{r}} j_1(w_s' r)\end{bmatrix} e^{-iw_s' t} b\theta(R-r) \tag{A1}$$

and

$$q_{1p_{1/2}}(\mathbf{r},t)=\frac{N_p'}{\sqrt{4\pi}}\begin{bmatrix}\alpha_p^+ \sigma\cdot\hat{\mathbf{r}} j_1(w_p' r)\\ i\alpha_p^- j_0(w_p' r)\end{bmatrix} e^{-iw_p' t} b\theta(R-r) , \tag{A2}$$

where

$$\alpha_{s,p}^{\pm}=\left[\frac{\alpha_{s,p}\pm mR}{\alpha_{s,p}}\right]^{1/2} . \tag{A3}$$

The corresponding normalization constants are given by

$$N_{s,p}'^{\,2}=\frac{1}{j_0^2(w_{s,p}'R)R^3}\,\frac{\alpha_{s,p}(\alpha_{s,p}\mp mR)}{2\alpha_{s,p}(\alpha_{s,p}-1)+mR} , \tag{A4}$$

where

$$\alpha_{s,p}=[(w_{s,p}'R)^2+(mR)^2]^{1/2} . \tag{A5}$$

The energies are $w_s'=2.51\cdots/R$ and $w_p'=3.96\cdots/R$ for $mR=300$ MeV.

The strange-quark mass affects the matrix elements of H_s and H_c in different ways. We consider the case corresponding to H_s first. The massive quark invalidates Eq. (2.12) because that equation was derived using the Dirac equation for massless quarks. When the mass of the strange quark is taken into account the first-order interaction Hamiltonian changes to

$$\hat{H}_s^M=\hat{H}_s+\frac{i}{f}\int d^3x\, \theta_v \bar{q}\gamma_5\lambda q\cdot\phi . \tag{A6}$$

Furthermore, the mass affects the normalization and energy of the quark state as well as the relative weight between upper and lower components of the quark wave function. These effects only modify transitions which involve a strange quark directly, namely, the $\bar{K}N\to\Lambda^*$ transition. (There is no effect on the $\pi\Sigma\to\Lambda^*$ transition.) The form factor for $\bar{K}N\to\Lambda^*$ is given by [instead of Eq. (2.22)]:

$$\begin{aligned}u_{N\bar{K}\Lambda^*}(k,R)=N_sN_s'\{&2R^2\alpha_p^- j_0(w_sR)j_0(w_p'R)j_0(kR)\\ &-[w_s-w_p'+w_{\bar{K}}(k)]\int_0^R dx\,F_+(kx)\\ &+m\int_0^R dx\,F_-(kx)\} ,\end{aligned} \tag{A7}$$

where

$$\begin{aligned}F_\pm(k,x)=x^2[&\alpha_p^- j_0(w_sx)j_0(w_p'x)\\ &\pm\alpha_p^+ j_1(w_sx)j_1(w_p'x)]j_0(kx) .\end{aligned} \tag{A8}$$

The quantities with primes refer to the massive strange quark and those without primes to the massless quarks.

With respect to H_c the mass of the strange quark affects the $\bar{K}N\to\pi\Sigma$ and the $\bar{K}N\to\pi\Lambda$ transitions, without any effect on the elastic transitions. Thus the form factor changes to

$$u'_{\alpha\beta}(k,k',R)=N_sN_s'[w_\alpha(k)+w_\beta(k')] \times \int_0^R dx\, x^2[\alpha_s^+ j_0(w_s x)j_0(w_s' x) +\alpha_s^- j_1(w_s x)j_1(w_s' x)] \times j_0(kx)j_0(k'x)\ , \tag{A9}$$

where α stands for $\bar{K}N$ and β for $\pi\Sigma$ or $\pi\Lambda$.

APPENDIX B

We would like to estimate the relative importance of the bare Λ^* and the contact interaction at the pole of the $\pi\Sigma$ t matrix. Rather than actually doing the calculation at the complex energy where the pole occurs we actually stay on the real axis at 1410 MeV. Using the fact that the pole is associated with the last term in the Lippman-Schwinger equation,

$$t=v+vG_0t\ , \tag{B1}$$

we estimate the relative contribution by comparing $v_{\rm pole}G_0t$ with $v_{\rm con}G_0t$, where $v_{\rm pole}$ and $v_{\rm con}$ are, respectively, the first and second terms on the right-hand side of Eq. (2.44). As noted in the text the pole term gives about 14% of the total width.

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ON THE INTERPRETATION OF THE EMC EFFECT

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ABSTRACT

Since the discovery of a significant change in the structure function of a "nucleon" inside a nucleus there has been a great deal of interest in deep inelastic scattering within the nuclear community. We shall outline a number of proposals which have been made to account for this data. Special emphasis is given to the proposition that the virtual pion field of the nucleus may be enhanced, and to the idea of colour conductivity.

1. INTRODUCTION

Our modern belief in the quark model of hadron structure dates from the discovery of scaling at SLAC in the late 60's. Even though we have been unable so far to liberate quarks from infrared slavery, we have been able to observe them inside hadrons by a judicious choice of leptonic probes.[1)] We know by direct observation that a nucleon consists of three valence quarks (carrying about 36% of its momentum[*]), a sea of virtual quark-antiquark pairs (10% of its momentum), plus glue (carrying the remaining 54% of its momentum).[2,3)] In view of this success it is surprising how little effort has been made to use deep-inelastic scattering (DIS) to constrain models of hadronic - let alone nuclear - structure. The current popularity of Skyrmion-like models of nuclear structure is testimony enough to this.[4,5)]

However, since the revelations of the European Muon Collaboration just two years ago,[6)] this has begun to change. What

[*] All these numbers refer to measurements at $Q^2 = 5\ \mathrm{GeV}^2$.

they found, and this has since been confirmed by three independent experiments,[7] is that the structure function of a nucleon in Fe deviates by as much as 15% from that of a free nucleon. (This becomes 30% after the relatively reliable correction for fermi motion[8].) At present the number of theoretical papers almost outstrips the number of data points. However our task is made easier by the fact that the number of genuinely different ideas is quite small. In Sect. 2 we briefly outline these key ideas. Sections 3 and 4 are devoted to a more comprehensive discussion of two of the ideas on which our group has had something original to say. In Sect. 5 we make some concluding remarks.

2. OVERVIEW

Perhaps the best capsule summary of the EMC data is that given by Close and collaborators[9] (see also Nachtman and Pirner[10]). From the observation that the structure function in Fe at (x, Q^2) is approximately equal to that in d at $(x, 2Q^2)$ $(0.2 < x < 0.8)$ they concluded that there is a change in scale in the nuclear system. In itself this observation was not very satisfying - there was no suggestion by Close et al of how this change of scale might vary with atomic number, nor whether it necessarily involved colour.

Amongst the early models for this change of scale was the possibility that any given quark might find itself in an exotic (6q) state in a nucleus.[11] In either the MIT or cloudy bag models[12], where the nucleon bag has a radius of about (0.8 - 1.0)fm, it is perfectly natural to think of the short range N-N force arising from bag overlap. As the radius of a 6q-bag is about 30% greater than that of a 3q-bag it is clear that this provides one mechanism for a change in the confinement scale for the quarks.

A more extreme version of this idea was actually proposed by Krzywicki before the discovery of the EMC effect.[13] He suggested that for the purposes of describing DIS the nucleus could be treated as one large bag containing 3A valence quarks. Within this framework he actually predicted the enhancement of the nuclear sea seen in the small-x region by EMC. (We shall discuss at greater length in Sect. 3 the evidence for and against such an enhancement in view of the apparent contradiction between the EMC and SLAC data). While we feel that this picture of the nucleus as one large bag is hard to believe (given our present beliefs about nucleon structure), some progress has been made by the Bonn group in deriving conventional shell model behaviour from it.[14)] A somewhat more attractive picture,[15)] in which the quarks only begin to spread throughout the nucleus at higher Q^2, will be critically discussed in Sect. 4.

An alternative to the multi-quark bag picture, which nevertheless leads to an increase in scale in the nucleus, is the idea that the nucleon itself may swell inside the nucleus.[16,17)] Phenomenologically it seems that an increase of some 15% in Fe would be needed to explain the EMC data. As bizarre as this idea seems to nuclear physicists it is amazing how difficult it is to find hard evidence to refute it. Sick has shown that the success of y-scaling in low-energy, inclusive electron scattering from ^{3}He is incompatible with more than a 6% increase in the proton radius there.[18)] However, since the SLAC data shows an A-dependence proportional to the <u>average</u> nuclear density (which is quite low for ^{4}He) Sick's limit does not contradict the possibility of a 15% increase in Fe.

One possible scenario wherein such swelling could take place involves two nucleons, described as Friedberg-Lee solitons,[19)] which approach each other in a nucleus. Clearly at some point the self-consistent scalar field will no longer reach its asymptotic (free space) value in between the nucleons and the quarks will be less effectively confined. Several groups have estimated this effect using a mean field approximation,[17,20)] and claim that a scale change of the required magnitude is quite likely.

From our point of view the main dissatisfaction with such a model is that the quarks were not really confined in the first place. It may be reasonable to argue that giving quarks outside the nucleon a mass of 800 MeV is not unreasonable if one aims to describe the properties of an isolated nucleon.[21] However, in considering the leakage of quarks in a many-body system it is not clear that using such a model does not beg the question.

Finally, in anticipation of the following section, let us state the obvious. Even in a decent quark model of nucleon structure the longest range structure is its pion cloud.[12] In a many-body environment it is that cloud which one would expect to be altered first. Such an alteration would necessarily change the structure function inside a nucleus. We shall see in Sect. 3 that with a little phenomenology, and parameters within the range acceptable to low energy physics, this idea is consistent with the SLAC data. At first sight, however, it does not seem to be consistent with the EMC data which originally suggested the calculation. We defer further discussion of these data sets to the end of Sect. 3.

3. MORE VIRTUAL PIONS IN THE NUCLEUS

The enhancement of the nuclear structure function at small x, where the sea dominates, naturally suggests that there has been an increase in the number of virtual q-$\bar{q}$ pairs. Llewellyn-Smith first suggested that if the number of virtual pions per nucleon in a nucleus were about 0.15 higher than in free space, the EMC data would be explained.[22] That such an increase is not unreasonable was soon established by Ericson and Thomas.[23] Indeed this data may constitute the first evidence for the enhancement of the pion propagator in the region of (300-400) MeV/c momentum transfer once associated with pion condensation.[24,25]

While we know now that the short range repulsion in the spin-isospin channel (often described by the Landau-Migdal parameters g'_{NN}, $g'_{N\Delta}$, $g'_{\Delta\Delta}$) is such that actual pion condensation does not occur, there is still room for a substantial bump in the pion propagator.[24,25] The size of this enhancement is controlled by the process of Δ-h formation which is proportional to the effective nuclear density. Figure 1 shows the variation of fermi momentum,

k_F, with atomic number as determined from low energy quasi-elastic electron scattering.[26] The first scale on the right shows k_F^3, while the scale on the extreme right shows the expected variation of the EMC effect in such a model - normalised to 100% in Fe.[27]

Whereas the A-dependence should be as illustrated in Fig. 2, the actual magnitude of the effect is controlled by $g'_{N\Delta}$. Unfortunately there is little information on this parameter at (300-400) MeV/c, however a value of order 0.6 is not unreasonable. Recent studies of the (p, Δ) reaction by Jain[28] would actually suggest a number nearer 0.5, and hence a much bigger effect than we calculate.

In order to compare with data one must not only compute the sea enhancement, but also balance momentum overall. Following the simple procedure suggested by Llewellyn-Smith[22] we obtain the predictions for the A-dependence of the EMC effect shown in Fig 2.[27] (The $g'_{n\Delta}$ parameter was fine tuned to give a fit to Fe, after which there is no further freedom in the model).

As an example, we show the predictions for Fe in Fig. 3. It is somewhat bizarre that even though the calculation was prompted by the EMC data, the fit is better for the SLAC data! Investigations are presently underway to see whether this is pure chance, or whether there is some physics in this. For example, it is possible[4] that the picture of a pion cloud about a quark core makes sense at $Q^2 \sim 1$ GeV^2, but not at (30-40) GeV^2.

In the remainder of this section we shall briefly mention some other tests of this idea. However to finish the theoretical discussion we recall the argument in favour of the pionic model which was given by Pandharipande et al.[29] They argued that the pion exchange force between nucleons and deltas is relatively well known, because it is the longest range piece. Thus the pion exchange contribution to nuclear binding energies is relatively model independent. However, the pion number operator is just equal to the potential energy associated with one pion exchange, divided by ω_π. In

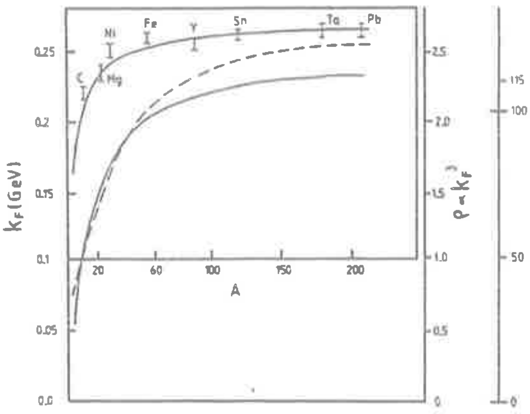

Figure 1: Variation of effective fermi momentum with atomic number [26] (on left), as well as k_F^3 and EMC effect as a percentage of the effect in Fe (right axes)[27].

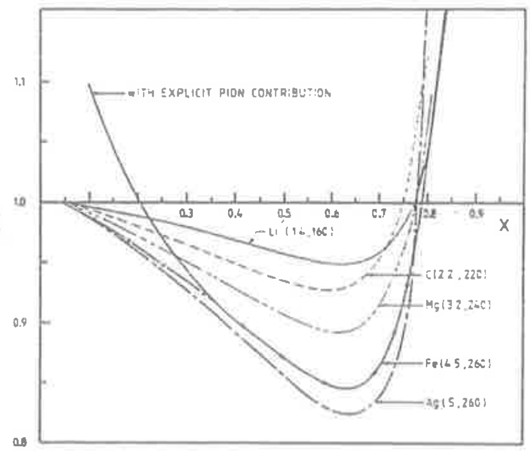

Figure 2: Predicted A-dependence of the contribution to the EMC effect because of loss of momentum to the virtual pion field.[27] The numbers in brackets indicate the extra percentage of momentum carried by pions and the effective fermi momentum used.

this way they estimate an excess of 0.12 pions per nucleon in Fe (for the Argonne V_{28} interaction).

One obvious test of this idea that the sea is enhanced in Fe is to exploit the sensitivity of $\bar{\nu}$ beams to anti-quarks. The CDHS group recently measured the ratio of $\bar{d}$ and $\bar{s}$ quarks in Fe and H to be of the order 1.1 ± 0.2 at low x, and this has been claimed to contradict the pionic model of EMC.[30] In fact, because Fe is an isoscalar target, even a relatively small asymmetry in the $\bar{u}$ and $\bar{d}$ distributions in H[31] enables one to reproduce the CDHS data.[32]

A second experiment at BEBC measured the y-distributions for $\bar{\nu}$ scattering from Ne and d.[33] Despite claims that the relatively flat y-distributions contradict the pionic model for EMC, Bickerstaff, Birse and Miller[34] have shown that it gives quite a good fit (as do several other models).

A rather different test of the pion model alone has been carried out at LAMPF.[35] By a judicious measurement of polarisation observables in proton-nucleus inelastic scattering it is possible to isolate the longitudinal response in the region of (300-400) MeV/c momentum transfer. This would isolate the response of the nucleus to a virtual pion if the exchange were pure isovector. Such a reaction would be possible through the $(\vec{p}, \vec{n})$ reaction, which should have a high priority on the attention of our experimental colleagues in future. Unfortunately, the only experiment performed so far involves the $(\vec{p}, \vec{p}')$ reaction, which mixes the isoscalar and isovector contributions.

Figure 4 shows the ratio of the longitudinal to transverse response for $(\vec{p}, \vec{p}')$ on ^{208}Pb at 340 MeV/c, in comparison with the calculations of Ericson et al for isovector alone at central density (solid line),[36] and weighted according to the effective density probed by the protons (which are strongly surface absorbed) - dashed line.[35] Allowing for some uncertainty because of the isoscalar contribution, the agreement above w = 60 MeV is not bad, whereas the

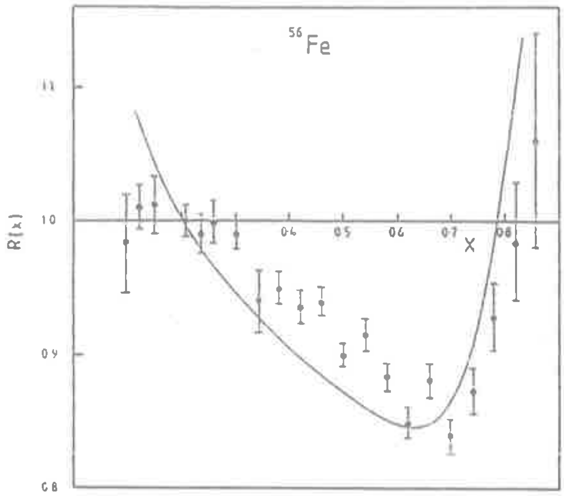

Figure 3: Comparison of the pionic model for the EMC effect in Fe 27), with the data of Arnold et al.[7]

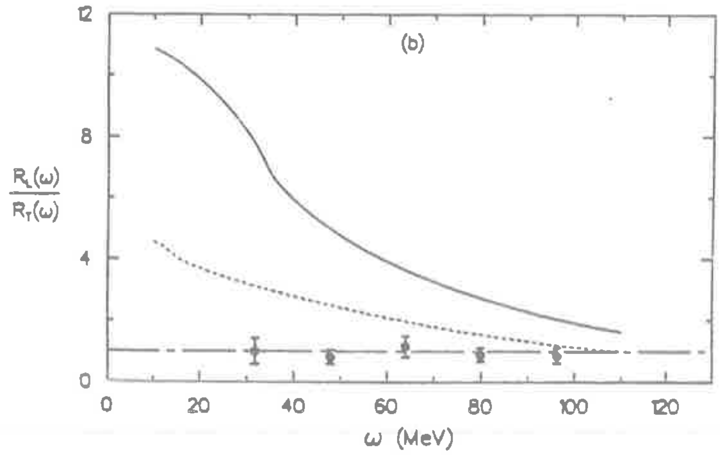

Figure 4: Ratio of the longitudinal to transverse response functions in Pb - see text for details.[35]

low energy region looks terrible. However, the calculations referred to ignore nucleon binding energies which kill the response at low-w. Since the EMC calculation strongly weights the region of $w \sim 80$-100 MeV, this experiment certainly does not rule out the pionic explanation for EMC.

In concluding this section we mention that the 2p-2h contribution to the pionic response has not yet been included in the EMC calculations[25] - nor has the possible shadowing of the Δ contribution at very small x (< 0.05). It should also be said that the apparent dichotomy between the pionic and other explanations of the EMC effect may be illusory. In a quark model the Landau-Migdal parameters, g', would be associated with the regions where two bags overlap. Thus when we learn enough to treat the whole problem consistently, it may be that momentum balance will arise naturally from a softening of the valence quark distribution in a 6q bag, which in turn controls the enhancement of the virtual pion field.

4. DO NUCLEONS SWELL ?

We have already remarked on the unsatisfactory nature of the Friedberg-Lee type of solitons[19] for treating quark deconfinement in nuclei. One model which does not suffer this problem is the model of Nielsen and Patkos where the confining scalar field, χ , is no longer the chiral partner of the pion.[37] In their model true quark confinement is achieved through a term $-(m_q/\chi)\,\bar{q}\,q$ in the Langrangian density. It is assumed that χ feels an effective potential $U(\chi)$ which has an absolute minimum at $\chi=0$, so that m_q/χ becomes infinite outside the "bag".) One's first thought would be to identify m_q with the running quark mass).

Now as pointed out by Pirner and collaborators,[15] if one ignores soliton overlap by approximating a nucleus as a bunch of colourless 3q-clusters ("nucleons") each inside a Wigner-Seitz cell, the lowest energy configuration will not have $\chi=0$ between the "nucleons". Each quark can reduce its kinetic energy by a percentage $\chi_N/(2m_q R)$ by having a non- zero χ-field, χ_N, between the nucleons.

The penalty for not being at the minimum of $U(\chi)$ is $\frac{4\pi}{3}(r_0{}^3 - R^3)\,½$ $m^2_{GB}\,\sigma_v{}^2\,\chi_N{}^2$ (where r_0 = Wigner-Seitz radius, R = bag radius, m_{GB} = glueball mass and σ_v a scale parameter). Minimising the total energy with respect to χ_N solves the problem.

Pirner et al. estimate $\chi_N = 0.024$ which, with a quark mass of 20 MeV, would give the quarks an effective mass of only 800 MeV between the nucleon-like clusters. Thus the quarks leak out of their bags and the nucleon effectively swells (by about 10% in Fe). Even more beautiful, if m_q is the running mass which goes to zero logarithmically as $Q^2 \to \infty$ the quarks become less confined as Q^2 goes up - hence the name "colour conductivity".

Unfortunately, the very feature which makes the model attractive, namely the freedom to identify m_q as a running mass, is its downfall.[38] In particular the appearance of $1/m_q$ in the change of the quark kinetic energy gives an enormous splitting between neutron and proton energy levels (after correcting for the coulomb force). For example, with $m_d - m_u = 5$ MeV, and $\chi = 0.024$ we find an 8.9 MeV shift at nuclear matter density!

The approach of Williams and Thomas[38] has been to use the observed equality of n and p energy levels (within say 1 MeV[39]) to put an upper limit on the allowed value of χ_N. Using more realistic values[40] for $\Delta m_q/m_q = 33\%$ and $(m_u + m_d)/2 = 15$ MeV (at say 1 GeV^2), they found $\chi_N(1\ \mathrm{GeV}^2) < 0.0019$. Thus the effective quark mass between nucleons in a heavy nucleus should be at least 8 GeV! This leads to less than a one percent change in the nucleon size, which is too small to explain the EMC effect by itself.

This discussion does not eliminate the possibility that nucleon swelling as proposed by Pirner et al contributes to the EMC effect. (At very high Q^2 it may eventually dominate.) However, other mechanisms must be invoked to explain most of the presently observed effect. On the positive side, this shift in neutron and proton levels may contribute substantially to the explanation of the Nolen-Schiffer anomaly in low energy nuclear physics.

5. CONCLUSION

Whatever message the EMC effect has for us, there is no doubt that we are being led to profound new insights in strong interaction physics. We can look forward to an exciting interplay between theoretical ideas and new experiments for at least three or four years before we will be sure of exactly what that message may be.

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Quarks in Low Energy (Pion) Physics?

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ABSTRACT

We review the reasons for believing that a well-founded, microscopic theory of the atomic nucleus must explicitly include quarks. We also discuss the possibility that there may be an experiment in low or medium energy physics which can only be explained in a quark model. While we are forced to a negative conclusion on this issue, there are reasons to believe that pion nucleus physics, and double charge exchange in particular, may provide valuable tests of any such model.

INTRODUCTION

One of the advantages of presenting the opening talk at a topical meeting like this is that one is expected to take a grand view. The question to be addressed is not so much where are we are (that will be part of Gerry Miller's unenviable task), but where might we be going. The ambitious goal which I shall describe is one that by now has excited almost everyone working in medium energy physics. That is, it seems possible that we have in our hands the ingredients of a truly consistent and fundamental description of nuclear physics - namely, quarks and QCD.

Dazzling though this idea may be for theorists, our experimental friends (in between gasps of undisguised admiration) unanimously ask one simple question. They would like to know which single experiment will lead to the surrender of conventional nuclear theory, and prove that only the quark model is valid. In our view there is probably no such experiment. Indeed, at the present time one should view all quark model calculations in the low energy regime with some caution. It would take a very brave (not to say incautious) person to pretend that any such calculation can be trusted to better than ten percent.

On the other hand, conventional nuclear theory (involving nucleons, N-N potentials, isobars and meson exchange currents) is precise. That is, given a set of parameters the calculations can be performed with impressive accuracy, including effects like special relativity. It is especially successful when dealing with relatively low momentum transfer processes e.g. low energy n-d elastic scattering and break-up.

The crunch comes only when one pushes to phenomena involving small internucleon separations (1 fm or less). This is the region where the proliferation of difficulties involving very heavy meson exchange, meson-nucleon form-factors, crossed meson exchange (e.g. (π,ρ), (π,ω), (ρ,ω) (ϕ,ρ)...) and so on force one to admit defeat. Conventional nuclear theory is essentially unsound at short distances. Of course this does not mean that it is not possible to make calculations within the framework of reasonable working hypotheses about the short distance behaviour of the N-N system. Indeed it is extremely valuable for everyone in our field that such calculations are pushed to their limits. Nevertheless, it should surprise no-one if it is found that the parameters (coupling constants, form-factors etc.) needed to fit data, vary from one experiment to the next.

For those of us developing the quark level description of nuclear phenomena there are two paths to follow. First, we need to check the consistency of this approach with all low energy phenomena - the N-N force, the shell model and so on. Naturally special emphasis will be given to those areas where short range physics is expected to be probed. A priori, double charge exchange and pion absorption would appear to be excellent candidates. Second, one should pursue those problems where quarks are essential to any discussion - e.g. the EMC effect, Drell-Yan on nuclei etc. It would be most appropriate to follow these parallel paths at the same time, with frequent exchange of information.

With these introductory remarks made, we outline the contents of this paper. We discuss the need for explicit quark models with reference to the non relativistic quark model (NRQM), the MIT bag model, the cloudy bag model (CBM) and finally the Skyrmion. Next we outline some of the evidence that quark models are needed to understand existing low energy phenomena. Finally, as the curtain raiser to the detailed discussions to be held in the next three days, we comment on the suggestion that low energy DCX may be a sensitive test for an explicit quark presence in nuclei.

MOTIVATION

There is little point in repeating material which has already been presented in far greater detail elsewhere. The situation up to mid-1982 is fully discussed in ref. (1) (entitled "Chiral Symmetry and the Bag Model: A New Starting Point for Nuclear Physics") - see also refs. (2) and (3). Very simply put, there are a number of successful models of hadron structure - including the NRQM [4], the MIT bag [5] and its extension the CBM.[1,2] All of these models have the feature that the quarks which make up the nucleon itself occupy a volume of order 1 fm in radius. Thus, even at a separation of 2 fm (which is close to the average separation of nearest neighbours in nuclear matter) the quark wave functions of two nucleons will begin to overlap - and possibly be modified. Such a modification is naturally interpreted as a N-N force.[6]

Within the NRQM, conventional resonating group techniques have already been extensively applied to N-N scattering.[7] The first achievement of this approach has been to provide a simple explanation of the apparent hard-core of the conventional N-N force. Instead of a hard core the N-N piece of the relative wave function has a node - which has the same effect asymptotically. A second claim, which is more controversial, is that the NRQM also reproduces the intermediate range attraction.[8] The central issue here is of course the existence of unobserved van der Waals forces in quark potential models. It is crucial to understand just how much of the attraction found by Isgur and Maltman is spurious in this sense. Nevertheless, this is an exciting claim.

Fewer attempts have been made to treat N-N scattering in terms of overlapping bags, because of the extra technical complications. In this regard it may be preferable to begin with the soliton bag model - as advocated by Wilets and collaborators.[9] Of course, in practical applications of the quark model to other processes most of the calculations are much cruder at the present time - involving perhaps a simple boundary condition connecting a spherical 6q-bag with an exterior N-N wave function.

With the present excitement surrounding models of the type proposed by Skyrme[10] and Witten[11], wherein baryon number is a topological property of Bose fields, we must make some remarks at this stage. In particular, there is an alternate school of thought to that which we have so far advocated. With apologies to its proponents the idea is that every low energy, hadronic phenomenon - baryon masses, magnetic moments, N-N scattering etc. - can be dealt with in terms of a theory involving only meson fields ($\pi,\rho,\omega\ldots$). That is, the quarks themselves need never be explicitly considered.

The motivation for this approach is the 1/N expansion of 't Hooft.[12] If one treats the number of colours available to the quarks as a variable N, which is allowed to be very large, the elementary 3-gluon vertex has a coupling constant g proportional to $1/\sqrt{N}$, as does the quark-quark-gluon vertex. Within this model, Witten has proven that whereas the meson masses are independent of N, the baryons (which must contain N valence quarks) have masses proportional to N.

This result immediately suggested to Witten an analogy with non-linear theories which possess soliton solutions. In general such solutions persist even in the limit of small coupling constant. Indeed one characteristically finds that as the coupling (g) decreases, the mass (M) goes up ($M \sim g^{-1}$). Now in the large-N limit of QCD the meson-meson coupling does behave as $1/N$. Witten's proposal was therefore that the nucleon (for example) should be a soliton solution of a non-linear theory of interacting meson (mainly pion) fields. Numerical solutions have been obtained for a non-linear Lagrangian consistent with current algebra, and the nucleon static properties are reproduced within about 30% (with one parameter).

For our purposes the crucial question is which of these models is right? In order to answer this we must realize that in any field theory (e.g. QCD as opposed to the parton model) no model can be realistic over more than a limited range of Q^2. Thus, our choice of model must be linked to the appropriate region of momentum transfer. For a longer discussion of this issue I refer to the proceedings of the Heidelberg conference last year.[13] Here it is enough to observe, following Jaffe and Ross,[14] that the existing DIS data strongly suggest that at $Q^2 \sim 0.7$ Gev^2 (and presumably also below this) the nucleon looks like simply three valence quarks in a bag. Thus, not only does the bag model reproduce hadronic masses and static properties better than the Skyrmion (particularly if perturbative pion corrections are included), but it is also consistent with DIS in the appropriate region of momentum transfer.

There has been little exploration of DIS from Skyrmions. It would seem to be more consistent with Witten's ideas to actually not do so at all. A priori it seems contradictory to apply a model which contains equal numbers of quarks and anti-quarks, in a process where the success of the $1/N$ expansion is that it provides a proof that "at $N = \infty$ the quark sea is absent"! It would seem that in the Witten approach one should use two completely different models in the low momentum and in the DIS regimes. To emphasise the point we simply quote the result of Sonoda and Wise[15] that in the large-N limit, for Bjorken $x > 0.3$ the Skyrme model predicts $u(x) \sim \frac{4}{5} d(x)$. Experimentally $u(x)$ is greater than $2d(x)$ in this region!

To summarise, if our intention is to interpret data over a range of momentum transfers from low energy and intermediate energy physics, through to K-factories, SURA, Bonn, Saclay and SLAC within one consistent framework, it will be essential to use models (like the MIT bag or the CBM) which treat quark degrees of freedom explicitly.

THE SEARCH FOR QUARKS IN NUCLEI

As we have already remarked, most applications of quark models to phenomena other than elastic N-N scattering have relied on some kind of boundary condition model. That is, whenever two nucleons approach each other closer than some critical distance b (or r_o) one describes the system as a 6 q bag. Outside b one uses conventional nucleon wave functions.

By far the most popular system for such calculations has been the deuteron, where one typically finds a probability of order (5-6)% for the 6 q component of the wave function if $b \sim 1$ fm. This model has been used to describe deuteron electrodisintegration,[16] parity violation in $np \to d\vec{\gamma}$ [17] and pp elastic scattering,[18] as well as the asymptotic D/S ratio of the deuteron.[19] A crucial difference between this sort of calculation and the NRQM calculations is that in the former there is no suppression of the hidden colour components of the short distance wave function.[20] In the resonating group method the hidden colour components are suppressed because of their higher mass. At our present stage of knowledge we cannot unambiguously claim that either of these approaches is better than the other.

One fairly straightforward prediction of the quark model is the counting rule for elastic form-factors. Ideally the 6-q component of the deuteron wave function should dominate at high-Q^2 leading to a Q^{-10} behaviour. However, Gross [21] has shown that the best fit up to $Q^2 \sim 6$ GeV^2 is actually $Q^{-5.5}$. Thus even at 6 GeV^2 we must apply quark model ideas with some caution.

To actually establish the 6-q piece of the deuteron wave function unambiguously the obvious tool is deep inelastic scattering. Kobushkin suggested (9 years ago!) that the deuteron structure function in the region kinematically forbidden for a free nucleon (x>1) could be explained by a 6% 6-q component in the deuteron wave function.[22] Unfortunately the SLAC data which he used is not yet published. There are also tremendous uncertainties in the treatment of fermi motion in this region.

A much more recent suggestion by Mulders and Thomas [23] offers the promise of a reasonable determination of the deuteron's 6-q component. Experimentally it is known that the ratio d(x)/u(x) in the valence region drops well below the naive parton model prediction of 0.5. In fact an analysis of proton and deuteron electromagnetic data gives $\tilde{d}(x) = 0.57\ (1-x)\ \tilde{u}(x)$. The tildas are included to emphasise that this method will not work if there is a 6-q component of the deuteron wave function. Figure 1 shows the ratio of the true d/u ratio (from $\nu,\bar{\nu}$ scattering from hydrogen) compared with $\tilde{d}/\tilde{u}$. (In this ratio of ratios fermi motion is a negligible correction for x<0.8.) The five curves shown (calculated for P_{6q} = 5%) are probably an overestimate of the ambiguities in our knowledge of the valence distribution in a 6 q bag. Clearly the existing data do favour a 6-q component in the deuteron wave function. However no-one will be convinced without far better neutrino data on hydrogen, which should be a top priority in the Fermilab program.

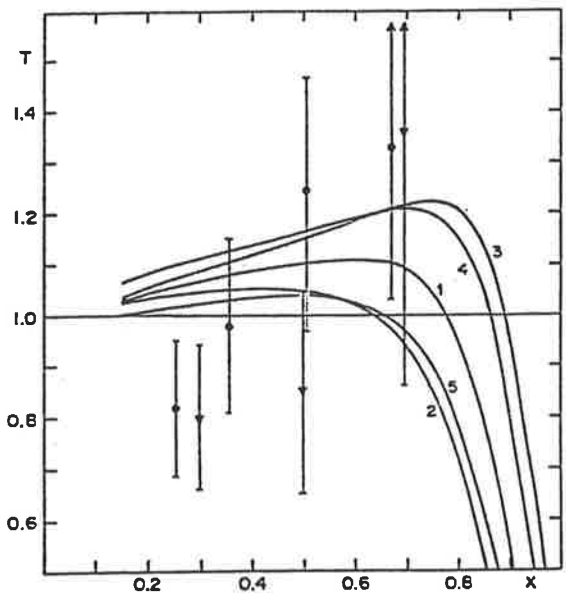

Figure 1:
The ratio [23] of the true to apparent d/u ratios for various values of (α, β) assuming a six quark valence distribution $n(x) \sim x^{\alpha-1}(1-x)^{\beta}$. (Curves 1-5 correspond to (α,β) = (0.5, 9) (0.5, 8), (0.5, 10), (0.4, 9), (0.6, 9) respectively. Curve 3 is theoretically favoured.)

When we move to systems with A>2 there has been very little work done - except recently in connection with the EMC effect. For ^3He Pirner and Vary [24)] showed in 1981 that the discrepancy between theory and experiment in the $^3He(e,e')$ reaction - again in the region kinematically forbidden for a free nucleon - could be understood in terms of 6-q and 9-q components in the ground-state wave function. (The best fit corresponded to $b \sim 0.9$ fm, which is close to the value needed in many calculations involving deuterium.) Unfortunately recent work by Laget [25)] has opened some doubt on this interpretation. In fact the greatest discrepancy occurred at energy transfers of order 200-300 MeV where scaling is simply not valid for quarks. Simple low energy physics like p-d final state interactions may explain much of the effect.

To summarise, there is so far no hard experimental evidence that quarks must be treated explicitly in low energy nuclear physics. We repeat our assertion that no single low energy measurement will ever provide unambiguous evidence of this kind. However, some experiments may be more useful than others in constraining hybrid models, and it is in this spirit that we turn to pion double charge exchange.

PION DOUBLE CHARGE EXCHANGE

Early in 1984 Gerry Miller [26)] reminded us rather dramatically of the sensitivity of pion DCX to short distance physics - see also ref. [(27)], and the talk of Bill Gibbs at this meeting.[28)] Whereas low energy single charge exchange (SCX) is strongly suppressed at small angles the first low energy DCX data, from the TRIUMF TPC, suggested a dramatic forward peak.[29)] It therefore seemed unlikely that the conventional mechanism of two sequential SCX's could reproduce the data. One the other hand, a relatively simple calculation involving pion absorption and re-emission from a 6-q bag did fit the data (which only existed for $\theta > 30^0$), and predicted a cross section as large as 10μb/sr on ^{14}C at 0°. This was really the state of play until very recently and we shall leave the later developments for the proceedings of this meeting. Instead we shall finish our introduction with some general remarks about the advantages of low energy pions in this sort of investigation.

At meson factories we tend to be blasé about the pion -

perhaps we get too many too cheaply! It should never be forgotten just how special the pion is in QCD. With small quark masses in the QCD Lagrangian one has an extra symmetry, namely chiral symmetry. In nature this is manifest in the fact that the pion mass is far less than any other meson mass. Indeed on the scale of m_ρ, m_{A_1} etc., m_π is essentially zero. This low mass is an extremely important theoretical tool which has not been adequately exploited up to now. In particular, the interaction of a low energy pion with not only nucleons, but also 6-q bags*, are governed by soft-pion theorems (plus small corrections). In this sense the pion is a more elementary probe of nuclear structure than any other hadron.[30)]

Nor should it be forgotten that the pion has a very long mean free path at low energy. The elementary pion-nucleon amplitude is small and provides a natural expansion parameter. Thus, there should ultimately be no excuse for not producing a successful microscopic description of pion-nucleus scattering below (say) 60 MeV.

To summarise, a priori low energy pion double charge exchange has many attractions. The pion itself is a special probe, we can hope for a microscopic understanding of the pion nucleus interaction, and finally the process itself probes short distances in the N-N relative co-ordinate. It is therefore a prime candidate to yield constraints on those models of nuclear structure which explicitly include quarks.

Of course, we must not be totally blinded by the delights of low energy pion physics. The lack of momentum transfer means that at best we will extract global information about short distance phenomena. It may appear from our discussions over the next few days that the resonance region is unexpectedly useful in this regard - perhaps through sensitivity to the short distance "N-Δ force". Finally, keeping in mind possible future facilities we should not forget that the pion-nucleus mean free path again drops at high energy and that region may be most valuable in the long term.

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*We have hopes of explaining some of the S-wave pion-nucleus repulsion at low energy in terms of the interaction with 6-q bags.

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CONSTRAINTS ON THE COLOUR DIELECTRIC MODEL OF THE NUCLEUS

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We examine the consequences of an up–down quark mass splitting in the colour dielectric model of the nucleus. In particular it is seen that this introduces a binding energy difference for protons and neutrons in the nucleus. We show that if this binding energy difference is to be acceptably small then the colour dielectric model of the nucleus in its present form is insufficient to explain the EMC effect. However it does provide a possible explanation of the Nolen–Schiffer anomaly.

The announcement by the European Muon Collaboration (EMC) [1] of a significant change in the structure function of a "nucleon" in iron compared with one in the deuteron has led to considerable theoretical speculation on the microscopic structure of atomic nuclei. At least in respect of changes in the valence quark distribution most of these explanations can be summarised by the statement that there is a change in the confinement scale proportional to the nuclear density [2]. Many mechanisms for this change of scale have been proposed – including exotic configurations of 6, 12 and 168 quarks [3–9], swelling of individual nucleons [10], and so on. The proposal of a colour dielectric model of the nucleus by Chanfray, Nachtmann and Pirner [11,6] is of particular interest since in this model the confinement scale increases with nuclear density and with increasing resolution (increasing Q^2). The effective lagrangian for this model was obtained by Nielsen and Patkos [12] from studies of QCD. Seen in this light the EMC effect may provide much needed insight into how confinement is realised in Nature. Our aim is to test, in its simplest form, the colour dielectric explanation of the EMC effect against our established knowledge of nuclear physics.

The effective lagrangian density obtained by Nielsen and Patkos can be written as [11,12]

$$\mathcal{L} = \mathrm{i}\chi\bar{\psi}\gamma^{\mu}(\partial_{\mu} - \mathrm{i}B_{\mu}/\chi)\psi - m\bar{\psi}\psi + \tfrac{1}{2}\sigma_{\mathrm{v}}^{2}(\partial_{\mu}\chi)^{2} - U(\chi) - (1/4g^{2})\chi^{4}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \qquad (1)$$

with

$$F_{\mu\nu} = (\partial_{\mu} - \mathrm{i}B_{\mu}/\chi)B_{\nu}/\chi - (\partial_{\nu} - \mathrm{i}B_{\nu}/\chi)B_{\mu}/\chi\,. \qquad (2)$$

χ is an effective scalar field, B^{μ} is an $(N_{\mathrm{c}} \times N_{\mathrm{c}})$ matrix of effective gauge fields (N_{c} is the number of colours), m is the quark mass (later to be identified with the (running) current quark mass), $U(\chi)$ is some self-interaction of the scalar field and σ_{v} and g are unknown real constants. It is assumed that $U(\chi)$ has an absolute minimum at $\chi = 0$ and a local minimum at $\chi = 1$. In particular the bag constant B is defined by $B \equiv U(\chi = 1)$. Expanding about $\chi = 0$ up to second order gives for small χ

$$U(\chi) \simeq \tfrac{1}{2}\partial^{2}U/\partial\chi^{2}|_{\chi=0}\,\chi^{2} \equiv \tfrac{1}{2}m_{\mathrm{GB}}^{2}\sigma_{\mathrm{v}}^{2}\chi^{2}\,, \qquad (3)$$

where m_{GB} can be thought of as the glueball mass. The colour dielectric picture of confinement [13,12] has a space–time dependent dielectric constant $\epsilon(\chi)$ which is zero in the vacuum and non-zero only in small localized regions. Quark and gluon confinement follows automatically. It can be seen that χ is related to ϵ by $\epsilon = \chi^{4}$ [11,12].

If, following ref. [11], the effective gauge fields B^{μ} are neglected, the fermion fields can be rescaled to yield the lagrangian density

$$\mathcal{L} = \bar{\psi}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m/\chi)\psi + \tfrac{1}{2}\sigma_{\mathrm{v}}^{2}(\partial_{\mu}\chi)^{2} - U(\chi)\,. \qquad (4)$$

This effective lagrangian density is expected to have soliton-bag type solutions [14] with $\chi \simeq 1$ inside and $\chi \to 0$ outside. If σ_{v} is sufficiently small, and if $U(\chi)$

has its local minimum at $\chi = 1$ and its absolute minimum at $\chi = 0$ separated by a sufficiently high barrier, then it is expected that the transition between $\chi = 1$ and $\chi = 0$ would be well approximated by a step function. The familiar MIT bag model results, with the energy

$$E(R) = \alpha \sum_{i=1}^{3} \omega_i(R) + \tfrac{4}{3}\pi R^3 B \,, \tag{5}$$

where the $\omega_i(R)$ are the quark energies, B is the bag constant and α (with $0 < \alpha < 1$) is a correction due to centre-of-mass and zero-point energies etc. The radius of the bag is that R at which E is a minimum (i.e. $\partial E/\partial R|_R = 0$). For light quarks $\omega(R) \simeq x_0/R$ where $x_0 = 2.04$. Minimizing eq. (5) then eliminates the bag constant and gives

$$E \simeq 4\alpha x_0/R \,. \tag{6}$$

Nucleons contain only the light up and down quarks and so we have for the nucleon mass $m_N \simeq 4\alpha x_0/R$. Typical MIT fits [15] give $R \simeq 1$ fm and $\alpha \simeq 70\%$ for $m_N = 938$ MeV. Since α is unknown it will be chosen for a given R such that eq. (6) gives the correct nucleon mass.

It will be assumed that the minimum-energy configuration has stationary nucleons evenly distributed throughout the nucleus. Fig. 1 is a sketch of how the χ-field might be expected to vary along a line joining the centres of two neighbouring nucleons. Also shown is a step-function approximation to this. Making this step-function approximation we have $\chi = 1$ in the nucleons, $\chi = \chi_N$ between the nucleons and $\chi = 0$ outside the nucleus. Using the Wigner–Seitz approximation each nucleon is placed in a spherical cell of radius r_0, the volume of which is $1/\rho \equiv \frac{4}{3}\pi r_0^3$ (where ρ is the average nuclear density). For light quarks and small χ_N (i.e. $m_i \ll x_0/R \ll m_i/\chi_N$) we have, to first order in χ_N, the quark energies [11]

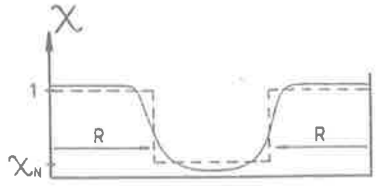

Fig. 1. A sketch of how the χ-field might vary along a line joining the centres of two nucleons in the nucleus (solid line) and a step function approximation to it (dashed line).

$$\omega_i \simeq (x_0/R)(1 - \chi_N/2m_i R) \,, \tag{7}$$

where the m_i are the appropriate quark masses. The energy associated with each nucleon of radius R in its Wigner–Seitz cell is approximately given by

$$E(\chi_N, R) = \frac{\alpha x_0}{R} \sum_{i=1}^{3} (1 - \chi_N/2m_i R) + \tfrac{4}{3}\pi R^3 B + \tfrac{4}{3}\pi(r_0^3 - R^3)\tfrac{1}{2} m_{GB}^2 \sigma_v^2 \chi_N^2 \,. \tag{8}$$

We will assume equal numbers of protons and neutrons in the nucleus for the purposes of defining χ_N. Let $\bar{E}(\chi_N, R)$ be the average of $E(\chi_N, R)$ for protons and neutrons. χ_N and R are then determined by minimizing $\bar{E}$ with respect to χ_N and R. As shown by Chanfray, Nachtmann and Pirner [11] the change in R due to $\chi_N \neq 0$ is a reduction of a few percent only. It is therefore a good approximation to assume that the radius R of the free nucleon bag does not change when it is bound in the nucleus. It is also a good approximation to neglect the small difference in R for the proton and neutron induced by the quark mass splitting [16]. Minimizing $\bar{E}$ with respect to χ_N and using $m_N = 4\alpha x_0/R$ then gives

$$\chi_N = \tfrac{3}{4} m_N \tfrac{1}{2}(1/m_u + 1/m_d) \times [R\tfrac{8}{3}\pi(r_0^3 - R^3) m_{GB}^2 \sigma_v^2]^{-1} \,, \tag{9}$$

where m_u and m_d are the up and down quark masses.

Using eq. (8) with χ_N determined by eq. (9) the energy associated with the nucleons is

$$E = m_N + W \,, \tag{10}$$

where W, the binding energy for the nucleons, is defined by

$$W \equiv -\tfrac{1}{2}\alpha x_0 \chi_N R^{-2} \sum_{i=1}^{3} \frac{1}{m_i} + \tfrac{4}{3}\pi(r_0^3 - R^3)\tfrac{1}{2} m_{GB}^2 \sigma_v^2 \chi_N^2 \,. \tag{11}$$

The neutron–proton binding energy difference ΔW is given by

$$\Delta W = W_n - W_p = \tfrac{1}{2}\alpha x_0 \chi_N R^{-2}(1/m_u - 1/m_d) \,. \tag{12}$$

Note that since $m_d > m_u$ the proton is bound more strongly than the neutron. The potential keeping the quarks in the nucleon is m_i/χ_N, which means that the heavier the quark the lower the probability of it being outside the nucleon. Furthermore, since eq. (9) implies that χ_N will increase with increasing nuclear density, the rms radius of the quark distributions (R_{rms}) will increase. That is, the confinement scale increases with increasing nuclear density.

Defining $\hat{m} \equiv \frac{1}{2}(m_u + m_d)$ we see from eq. (9) that to a good approximation $\chi_N \propto 1/\hat{m}$. Hence the average potential $m_0 \equiv \hat{m}/\chi_N$ which binds quarks in nucleons approximately satisfies $m_0 \propto \hat{m}^2$. The strong dependence of R_{rms} on the quark mass is turned into a Q^2-dependence by identifying the quarks in this model as the current quarks with running masses $m_i \equiv m_i(Q^2)$ for $Q^2 \gtrsim 1$ GeV^2. For $Q^2 \lesssim 1$ GeV^2 the quark masses are assumed constant. So $m_0(Q^2)$ begins to decrease as Q^2 increases beyond about 1 GeV^2. For example, increasing Q^2 from 1 to 100 GeV^2 approximately halves m_0 [11]. Thus the confinement scale increases with increasing Q^2 for $Q^2 \gtrsim 1$ GeV^2, in agreement with the ideas of colour conductivity postulated by Nachtmann and Pirner [17].

Consider the parameters of refs. [11,6], i.e. the low-Q^2 ($Q^2 \lesssim 1$ GeV^2) quark masses $\hat{m} = m_u = m_d$ = 20 MeV, R = 0.83 fm, $m_{GB}^2 \sigma_v^2 = 0.4$ GeV^4 and ρ = nuclear matter density = 0.17 fm^{-3} (i.e. r_0 = 1.12 fm). Using m_N = 938 MeV gives for this R, $\alpha \simeq m_N R/4x_0$ = 0.48. From eqs. (9) and (11) we obtain $\chi_N = 0.024$, $m_0 \simeq 0.8$ GeV and $W \simeq -50$ MeV. Now consider a typical mass-splitting of $\Delta m \equiv m_d - m_u = 5$ MeV with say m_d = 22.5 MeV and m_u = 17.5 MeV. Then χ_N = 0.025 and is virtually unchanged. However ΔW is no longer zero and we obtain from eq. (12) ΔW = 8.9 MeV. The difference in binding energies for protons and neutrons can be determined from experiment. After subtracting Coulomb corrections the remaining discrepancy, referred to as the Nolen–Schiffer anomaly, is $\lesssim$1 MeV [18]. Clearly then, since ΔW = 8.9 MeV $\gg$ 1 MeV the above parameters are unacceptable.

From eq. (12) we see that ΔW decreases with increasing R, increasing $\hat{m}$, decreasing Δm and decreasing χ_N. The largest nucleon radius which is consistent with the model presented, in particular with respect to the initial neglect of multi-quark bag formation etc., is $R \lesssim 1$ fm. Ref. [19] gives $\Delta m/\hat{m} = (56 \pm 6)\%$ and $\hat{m}(Q^2 = 1 \text{ GeV}^2) = 7 \pm 2$ MeV for the running light quark masses. We therefore can take $\Delta m/\hat{m} = 33\%$ and $\hat{m}$(low Q^2) = 15 MeV as reliable lower and upper limits, respectively. We require $\Delta W \leqslant 1$ MeV in the low Q^2 region ($Q^2 \lesssim 1$ GeV^2) where the quark masses are constant. Then using m_u = 12.5 MeV, m_d = 17.5 MeV and R = 1 fm (implies α = 57%) an upper bound can be found for χ_N from eq. (12)

$$\chi_N(\text{low } Q^2) \lesssim 0.0019 . \tag{13}$$

A lower bound then follows for the average potential barrier keeping the quarks inside nucleons ($m_0 \equiv \hat{m}/\chi_N$)

$$m_0(\text{low } Q^2) \gtrsim 8.0 \text{ GeV} , \tag{14}$$

(c.f. 0.8 GeV). Note that this lower bound does not depend on which nucleus is being considered. Furthermore, since the upper bound on χ_N for fixed $\Delta m/\hat{m}$ goes approximately as $\hat{m}$, the lower bound on m_0 is essentially independent of the choice of $\hat{m}$. Thus a larger choice of $\hat{m}$(low Q^2) will not reduce the lower bound on m_0(low Q^2).

From table 2 of ref. [6] we see that nucleons in the deuteron have less than a 1% increase in R_{rms} compared to that of a free nucleon and that this corresponds to m_0 = 7.4 GeV. The average quark distribution radius R_{rms} will be determined by m_0 and R only. Hence from eq. (14) ($m_0 \geqslant 8$ GeV) the maximum allowed increase in R_{rms} is $\lesssim$1% for low Q^2 in any nucleus. Even at Q^2 = 100 GeV^2 where m_0 will be approximately halved, the increase is still $\lesssim$2%, (see the ^{3}He case in abovementioned table 2). This is much less than the increase of approximately 15% in confinement scale in iron needed to explain the EMC effect [2–4]. Thus it has been shown that if the neutron–proton binding energy difference at low Q^2 ($Q^2 \lesssim 1$ GeV^2) is to be acceptably small, then the colour dielectric model of the nucleus in its present form cannot explain the EMC effect.

It is interesting that through the appearance of χ_N in eq. (12), ΔW will increase with nuclear density. Since similar behaviour occurs with the Nolen–Schiffer anomaly [18], it may be possible to achieve a reasonable fit of ΔW to the anomaly. This model would then predict an increase in the Nolen–Schiffer anomaly (ΔW) as Q^2 increases beyond about 1 GeV^2.

The colour dielectric model of the nucleus is an elegant and simple way to model partial deconfinement of quarks and gluons in the nucleus. In this sense it probably contains an element of whatever is the complete picture of the nucleus. However, it is insufficient to explain the EMC effect without the addition of new features such as multi-quark bags, sea quark enhancement and so on.