Compartmental Flow Modelling of Acute Care Hospital Bed Occupancy for Strategic Decision-Making

A thesis presented towards the degree of Doctorate of Philosophy

By

Mark Mackay BSc(Hons) BEc BComm

School of Psychology

University of Adelaide

Submitted July 2007
# Table of Contents

Table of Contents ......................................................................................... i

List of Figures ............................................................................................. vii

List of Tables ............................................................................................ xvii

Abstract .................................................................................................... xxi

Declaration .............................................................................................. xxiv

Acknowledgments ................................................................................... xxv

Glossary and Abbreviations ....................................................................... xxvii

Chapter 1 — Introduction ........................................................................ 1

1.1 The Australian Health System and the Emerging Problem ............. 2
1.2 The Need for Better Decision-Making ............................................. 9
1.3 Why Model Hospital Bed Occupancy? ........................................... 13
1.4 Decision-making and Hospital Beds .............................................. 14
1.5 Simple Models and the Average Length of Stay ............................ 14
1.6 The Introduction of Bed Occupancy Compartmental Flow Model... 15
1.7 Compartmental Flow Modelling and Health Services Research .... 19
1.8 The Need for More Research into Compartmental Flow Bed
Occupancy Modelling .......................................................................... 21
1.9 Personal Motivation for Undertaking the Research ....................... 22
1.10 The Research Project Outline ....................................................... 25
1.11 The Research Project Aims ............................................................ 25
1.12 The Research Project Questions ................................................... 26
1.13 About the Research Project Methodology .................................... 28
1.14 Contribution Towards Knowledge ............................................... 28
1.15 The Thesis Layout ......................................................................... 29
1.16 Author Publications .................................................................... 31
1.17 Conclusions ................................................................................... 32

Chapter 2 — Hospital Bed Modelling - A Chequered History ........... 33

2.1 Introduction .................................................................................... 34
2.2 Australian literature reviews ......................................................... 34
2.2.1 Review by Dwyer and Jackson ............................................... 35
2.2.2 Review by Anderson et al. ..................................................... 36
2.2.3 Review by Goddard and Mills ................................................. 37
2.2.4 Conclusions about Australian literature reviews ..................... 38
2.3 General hospital bed modelling techniques ................................. 38
2.3.1 Simple Methods .................................................................... 41
2.3.2 More Complex One-off Approaches ...................................... 44
2.3.3 Queueing Models .................................................................. 47
2.3.4  Simulation .................................................................53
2.3.5  Flow Models .............................................................58
2.3.6  Conclusion regarding the general research ....................64
2.4  The Harrison and Millard compartmental flow model .........67
2.4.1  The Subsequent Research Effort .................................67
2.4.2  A more in-depth look at the compartmental flow model ....78
2.4.3  Conclusion regarding the compartmental flow research ....87
2.5  Other sources of literature .............................................90
2.5.1  Bed management texts ..............................................90
2.5.2  Grey information ......................................................91
2.5.3  Conclusions regarding the other literature ...................92
2.6  Overall conclusion .......................................................93

Chapter 3 — Setting the Scene ........................................95
3.1  Introduction ..................................................................96
3.2  Flinders Medical Centre ................................................97
3.2.1  Background Information ..............................................97
3.2.2  Methodology ............................................................99
3.2.3  Results .................................................................100
3.3  Internal Medicine Department, HealthCare Otago ..........106
3.3.1  Background Information ..............................................106
3.3.2  Methodology ............................................................108
3.3.3  Results .................................................................110
3.4  Discussion .................................................................119
3.4.1  The Hospital Services ................................................119
3.4.2  Methods of Analysis ..................................................123
3.4.3  Problems with the ALOS .............................................127
3.4.4  Simple Alternatives to the ALOS ......................129
3.5  Conclusion .................................................................131

Chapter 4 — Modelling — Some Theoretical Background ........132
4.1  Introduction ..................................................................133
4.2  Modelling .................................................................135
4.3  The process of model building ......................................137
4.3.1  Testing the model validity and inputs .........................137
4.3.2  Testing the model outputs ...........................................138
4.4  Model choice .............................................................139
4.5  Model fit and complexity trade-off ...............................143
4.6  Model choice and performance ...................................145
4.6.1  Model performance — common approaches ...............145
4.6.2  Least Squares and Maximum likelihood ..................150
4.6.3  Bayesian Information Criterion (BIC) .........................153
4.6.4  Bayes factor ..........................................................155
4.6.5  Cross validation and Bayes factor ...............................157
4.7  Conclusion .................................................................159
# Chapter 5 — Choice of Models for the Analysis and Forecasting of Acute Care Hospital Beds

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Introduction</td>
<td>161</td>
</tr>
<tr>
<td>5.2 Methodology</td>
<td>162</td>
</tr>
<tr>
<td>5.2.1 The Harrison and Millard Approach</td>
<td>162</td>
</tr>
<tr>
<td>5.2.2 Methodology 1 – using all the data</td>
<td>166</td>
</tr>
<tr>
<td>5.3 Results</td>
<td>172</td>
</tr>
<tr>
<td>5.4 Discussion</td>
<td>184</td>
</tr>
<tr>
<td>5.4.1 Does the Model Succeed in Describing Acute Care Data?</td>
<td>184</td>
</tr>
<tr>
<td>5.4.2 Validity and Reliability</td>
<td>186</td>
</tr>
<tr>
<td>5.4.3 Should the Single Day Census Style of Model Continue?</td>
<td>187</td>
</tr>
<tr>
<td>5.4.4 The Fit and Complexity Trade-off</td>
<td>189</td>
</tr>
<tr>
<td>5.4.5 Other Issues</td>
<td>189</td>
</tr>
<tr>
<td>5.5 Conclusion</td>
<td>190</td>
</tr>
</tbody>
</table>

# Chapter 6 — Modelling New Zealand Acute Care Occupancy

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 Introduction</td>
<td>194</td>
</tr>
<tr>
<td>6.1.1 The Need to Establish Validity</td>
<td>194</td>
</tr>
<tr>
<td>6.1.2 Evaluation of Service Change</td>
<td>194</td>
</tr>
<tr>
<td>6.1.3 Application of Model Selection Techniques</td>
<td>195</td>
</tr>
<tr>
<td>6.2 Methodology 2 – capturing variation through the average</td>
<td>195</td>
</tr>
<tr>
<td>6.2.1 Data</td>
<td>195</td>
</tr>
<tr>
<td>6.2.2 Method</td>
<td>196</td>
</tr>
<tr>
<td>6.3 Results</td>
<td>200</td>
</tr>
<tr>
<td>6.4 Discussion</td>
<td>207</td>
</tr>
<tr>
<td>6.4.1 Model Fit</td>
<td>207</td>
</tr>
<tr>
<td>6.4.2 Explanatory value</td>
<td>209</td>
</tr>
<tr>
<td>6.4.3 Alignment with experience</td>
<td>210</td>
</tr>
<tr>
<td>6.4.4 Portability of health system measures and validation</td>
<td>211</td>
</tr>
<tr>
<td>6.4.5 Expert Judgment</td>
<td>213</td>
</tr>
<tr>
<td>6.4.6 Average models</td>
<td>214</td>
</tr>
<tr>
<td>6.4.7 Other issues</td>
<td>215</td>
</tr>
<tr>
<td>6.5 Conclusion</td>
<td>215</td>
</tr>
</tbody>
</table>

# Chapter 7 — Model choice and prediction: forecasting changes in bed occupancy profiles as a consequence of population change

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 Introduction</td>
<td>218</td>
</tr>
<tr>
<td>7.2 Methodology</td>
<td>220</td>
</tr>
<tr>
<td>7.2.1 Creation of the Compartmental Flow Model</td>
<td>220</td>
</tr>
<tr>
<td>7.2.2 Linkage to Population Change</td>
<td>222</td>
</tr>
<tr>
<td>7.3 Results</td>
<td>222</td>
</tr>
<tr>
<td>7.3.1 The Compartmental Flow Models</td>
<td>222</td>
</tr>
<tr>
<td>7.3.2 Parametric Parameter Forecasts Linked to Population Change</td>
<td>230</td>
</tr>
<tr>
<td>7.4 Discussion</td>
<td>237</td>
</tr>
<tr>
<td>7.4.1 Model Choice</td>
<td>238</td>
</tr>
<tr>
<td>7.4.2 Forecasts and policy</td>
<td>241</td>
</tr>
</tbody>
</table>
Chapter 8 — Incorporation of seasonal effects into the compartmental flow model: should model complexity increase or model design change?..........................256

8.1 Introduction..................................................................................257
8.2 Methodology ..............................................................................262
8.2.1 Data and the Bed Occupancy Compartmental Flow Model.......262
8.2.2 Weather and Regression...............................................................262
8.2.3 Linkage to Population Change .....................................................266
8.2.4 Analysis of vacancy rates.............................................................266
8.3 Results..........................................................................................267
8.3.1 Weather and regression...............................................................267
8.3.2 The amended forecast .................................................................276
8.3.3 Accounting for vacancy ..............................................................277
8.4 Discussion ....................................................................................282
8.4.1 Effect of weather on the model.....................................................282
8.4.2 Inclusion of the “vacancy” factor ..................................................289
8.4.3 The amended forecast .................................................................294
8.4.4 The role of modifiers .................................................................294
8.5 Conclusion....................................................................................297

Chapter 9 — The use of compartmental flow bed occupancy models for forecasting service change ..........................................................299

9.1 Introduction..................................................................................300
9.2 Methodology ..............................................................................301
9.3 Results..........................................................................................302
9.3.1 Post service change trends.........................................................302
9.3.2 Forecasting using the ALOS .......................................................304
9.3.3 Forecasting with compartmental flow models .........................306
9.3.4 Comparison of forecasts to actual outcomes............................307
9.3.5 Comparison of the expected stay and the average length of stay...312
9.4 Discussion ....................................................................................313
9.4.1 The ALOS Forecast .................................................................314
9.4.2 The Bed Occupancy Compartmental Flow Model Forecast ......316
9.4.3 Expected length of patient stay....................................................320
9.4.4 Model Choice ...........................................................................321
9.4.5 Predictive validity .....................................................................322
9.5 Conclusion....................................................................................322

Chapter 10 — Application of Bed Occupancy Compartmental Flow Modelling to Casemix............................................................324

10.1 Introduction..................................................................................325
Chapter 10 — The Australian Casemix-funding Model

10.1.1 The Australian Casemix-funding Model ........................................... 325
10.1.2 The Average Length of Stay and the Casemix-funding Model ....... 328
10.2 Methodology .................................................................................. 331
10.3 Results .......................................................................................... 332
10.3.1 DRG Profiles ............................................................................. 332
10.3.2 Overnight Stay Patient Results .................................................... 334
10.3.3 Elective Same-day Results ............................................................ 336
10.4 Discussion ...................................................................................... 338
10.4.1 Number of Data .......................................................................... 338
10.4.2 DRG 261 Model Fit ...................................................................... 339
10.4.3 Inpatient and Same-day Activity .................................................. 340
10.4.4 Business Rules – fix add distribution could change ...................... 341
10.4.5 Casemix and Benchmarking ......................................................... 342
10.4.6 Policy Implications ..................................................................... 343
10.4.7 Further Research ........................................................................ 344
10.5 Conclusion ...................................................................................... 345

Chapter 10 — Discussion ........................................................................... 380

10.1 Introduction .................................................................................... 381
10.2 The Research Questions .................................................................. 381
10.2.1 The Main Research Question ....................................................... 382
10.2.2 The Secondary Research Questions .............................................. 385
10.3 The Revised Methodology ............................................................... 388
10.4 The Preferred Methodology ............................................................ 392
12.4.1 The Strategic Level ................................................................. 392
12.4.2 Knowledge of the Business .................................................. 393
12.4.3 The Value of Information ....................................................... 393
12.5 Revisiting Bed Occupancy and the Role of Modelling .......... 394
12.5.1 A More General Consideration of Bed Occupancy .............. 394
12.5.2 Has the problem been over simplified? ............................... 398
12.5.3 Influence over the problem space ....................................... 402
12.6 Conclusion ............................................................................. 407

Chapter 13 — Future Research and Application of Bed Occupancy
Compartmental Flow Modelling ................................................. 408

13.1 Introduction ............................................................................. 409
13.2 Scope for Additional Research ............................................. 410
13.2.1 Research within the hospital ............................................ 410
13.2.2 Research beyond the hospital inpatient .......................... 412
13.2.3 Incorporating information from other modelling .............. 414
13.2.4 Beyond the health sector .................................................. 416
13.3 Potential for Application ......................................................... 416
13.3.1 Scope for Application ........................................................ 417
13.3.2 Drivers of Adoption .......................................................... 417
13.3.3 Barriers to Adoption ......................................................... 419
13.3.4 Possible Solutions ............................................................. 421
13.4 Conclusion ............................................................................. 424

Chapter 14 — Conclusions – The Beginning of a Longer Journey ....... 426

14.1 Introduction ............................................................................. 427
14.2 The problem is better understood ........................................ 427
14.3 Final comments and observations .......................................... 430

Appendices ..................................................................................... 431

Appendix I — Publications ............................................................. 432
Appendix II — BOMPS Formulae ..................................................... 437

References ...................................................................................... 449
List of Figures

Figure 1: The changing age structure of the Australian population is illustrated through the changing proportion of older people (aged 65 years or more). Historically the proportion of older people was low. While it has increased in recent times, a new period of rapid growth is forecast to commence. Source data: Australian Bureau of Statistics (2002 and 2006). ............................................................ 4

Figure 2: The hospital utilisation patterns for non-Indigenous Australians show a high use of services for the very young and increased utilisation as age increases. The difference between males and females aged between 15 and 40 relates to child bearing. Data source: AIHW (2006)............................ ............................................. 5

Figure 3: A schematic representation of the emerging demand and supply issues facing the health sector............................................................. 7

Figure 4: A diagrammatic representation of the flow of patients through compartments. The compartments may be virtual or real - the patients may not actually change location within the physical hospital (Mackay and Lee, 2005)...................................................... 17

Figure 5: Application of Powel and Baker’s creative problem solving process to the hospital bed problem. There is an apparent difference between my research and the approach undertaken by the hospital. ................................................................. 24

Figure 6: A diagrammatic representation to the research that is required to show that there is merit in adopting compartmental flow models for strategic decision-making purposes in the Australian acute care environment. ................................................................. 29

Figure 7: The cumulative growth in the number of publications per year since 1991 showed a period of marked increased from 1998. ........... 75

Figure 8: Journal publications have been the dominant form of disseminating written information about the modelling approach to others. ................................................................. 76

Figure 9: The research effort has been biased towards services relating to aged care. ............................................................................. 77

Figure 10: A diagrammatic representation of the flow of patients through compartments. The compartments may be virtual or real - the patients may not actually change location within the physical hospital (Mackay and Lee, 2005). ................................................................. 79
Figure 11: Scatterplot of estimated average inpatient occupancy and the proportion of same-day patients. The relationship between these two variables appears to be reasonably well described by a linear relationship, with approximately 65 per cent of the variance in occupancy explained by changes in the proportion of same-day patients. ................................................................. 102

Figure 12: A scatterplot of the average number of bed and the proportion of same-day patients. A linear relationship appears to exist between the two variables, with the changes in the proportion of same-day patients explaining approximately 75 per cent of the variation in the average number of available beds. ................................. 102

Figure 13: A scatterplot of the ALOS (inpatients) and estimated inpatient admissions. A linear relationship was found to explain the relationship, with a decrease in ALOS generally being associated with an increase in patient throughput. ............................................... 103

Figure 14: The length of stay distribution compared to a Normal distribution with the same mean and standard deviation. ................................. 105

Figure 15: Trend in total bed occupancy. A seasonal trend is evident, as is regular peaks and troughs. The regular peaks and trough relate to day of week trends that were found to occur. ............................ 106

Figure 16: The ALOS trend for the period 1990-2004. It is evident that the way patients flowed through the system changed after 1996 as the ALOS declined significantly. ................................................. 110

Figure 17: The average midnight bed census profile occupancy profile (ogive) for the Internal Medicine Department data. The shape of the distribution indicated that occupancy has a skewed distribution, which is expected, with most patients only being admitted for a relatively short period of time (for example, fewer than 10 days). ................................................................. 111

Figure 18: The total midnight bed occupancy trend for the Internal Medicine Department 1990 to 2003 is illustrated. The trend reveals weekly trends (illustrated by the many peaks and troughs occurring between, say December 1992 and June 1993); seasonal trends (for example, December 2001 and December 2002 are low points, while June 2002 represents a peak in occupancy); and change in service trends (the occupancy for period prior to 1997 is generally greater than the occupancy from 1997 onwards). These trends, however, can be highlighted better using moving averages. ................................................................. 112
Figure 19: A seven-day moving average of total midnight bed occupancy. The troughs represent low points of activity (such as weekends) and the peaks represent days of high admissions (typically a week day). However, when data covering many years is presented in this manner, as in this case, the weekly cycle becomes compressed. .......................................................... 113

Figure 20: A 90-day moving average of total midnight bed occupancy for the period 1990 to 2003. The trend highlights the seasonal variation associated with the functioning of this service and removes the noise arising from daily variations in service provision. .......................................................... 113

Figure 21: A 90-day moving average of short and long-stay patient midnight bed occupancy trends. It can be seen that the post 1996 growth in bed occupancy is due to short-stay patients. ........................................ 115

Figure 22: Profile of total bed midnight bed occupancy by patients aged 65 years or more. When viewed with the previous figures it can be seen that these patients occupy the majority of Internal Medicine Department beds. .......................................................... 116

Figure 23: Relative distribution of midnight bed occupancy for aged and non-aged patients. The non-aged patients account for less of the total bed occupancy post 1997. .......................................................... 117

Figure 24: Trends in aged and non-aged patient total midnight bed occupancy for patients admitted for 10 or fewer days. Increases in midnight bed occupancy for aged patients post 1997 have led to more beds being occupied than before 1997 for this group of patients. .......................................................... 118

Figure 25: Trends in aged and non-aged patient total midnight bed occupancy for patients admitted for more than 10 days. While post 1996 service delivery changes have been maintained, the occupancy for the aged patient group has increased. ..................... 119

Figure 26: According to Armstrong (1985) models built for forecasting purposes have two basic inputs or components, namely variables and relationships. The concepts of validity and reliability can be applied to these inputs. .......................................................... 137

Figure 27: According to Hastie, Tibshrani and Friedman (2001), model development should include the use of both training and test data sets. .......................................................... 139

Figure 28: The model space represents the total possible number of models that could be fitted to the data. The number of models actually tested represents a small proportion of the model space. .............. 140
Figure 29: Increasing model complexity achieves a gain in fit for both training and test data to a certain point, after which gains only occur in the fit obtained for the training data (based upon Hastie, Tibshirani and Friedman, 2001). .......................................................... 144

Figure 30: Harrison and Millard (1991) used a bed occupancy census from a single day, which is highlighted by the data enclosed in the red box. An alternative approach could have been to census a patient cohort, which is represented by the shaded numbers (along the diagonal). Ultimately, patients are discharged, which is not shown on the diagram. .......................................................... 163

Figure 31: All of the data is captured and used in this methodology as suggested by the box surrounding the data, whereas the Harrison and Millard (1991) approach is based upon a single day census approach. .......................................................... 168

Figure 32: In order to avoid early termination of the model optimisation process, seeding was used. Seeding provided a starting point for optimisation that resulted in the model fitting process being able to overcome the presence of local optima. ........................................... 170

Figure 33: Comparison of the absolute errors for census and the annual average bed occupancy compartmental flow models. The annual average model performance was better than any census model performance (in terms of absolute error). .............................................. 175

Figure 34: Comparative fit, in terms of maximising log-likelihood, of models against training and test data. .......................................................... 177

Figure 35: Comparison of available beds during the training and test years. ... 179

Figure 36: The “seasonal model” represents a less complex model compared to those illustrated in Figures 37 and 38. The more general nature of this model (that is, reduction in complexity) results in a better fit of the test data compared to the more complex models. .................. 181

Figure 37: The "monthly model with weeks" represents a more complex model than illustrated in Figure 36 and achieves a better fit to the training data. However, the model fit deteriorates when test data is used as seen by the emergence of greater variation between the model and test data. .......................................................... 182

Figure 38: The "weekly model with weekends" represents a highly complex model. The fit to the training data is superior compared to the models illustrated in Figures 36 and 37. However, the greater model complexity results in further deterioration of fit when compared to test data. .......................................................... 183
Figure 39: The average and standard deviation were calculated for each time period (using the reverse cumulative days since admitted profile) as opposed to using all the data points. .......................................... 197

Figure 40: One option was to create a model that described all the data as indicated by the blue line. The difficulty with this approach was that it covered two periods of different service provision arrangements ................................................. 200

Figure 41: The creation of a compartmental flow model of bed occupancy for each year was also possible. The blue lines indicate the points where the data was cut in order to achieve these models. This represented the most complex solution. .............................................. 201

Figure 42: The simplest splitting of the data that enabled the change in service provision arrangements to be captured was to model the pre and post service change periods (the blue line indicates the split of the data). ........................................................................... 201

Figure 43: The fit of the model to the data improved, generally, as the complexity of the model increased. While the usefulness of the model is improved by slightly increasing complexity, over-fitting occurred once the numbers of parameters exceed 20, as shown by increasing BIC values. ................................................................. 203

Figure 44: The model fitted the pre-service change period data for each year well, although some years were described better than others. ........ 204

Figure 45: The model fitted the post change period data for each year well. Variation between the model and data was expected, as the aim was to avoid over-fitting of the data. The maximum duration of patient stay was reduced compared to the pre-service change (hence the changed x-axis scale). .................................................. 204

Figure 46: Visual inspection of the models and data show that the fit was good. Furthermore, given the change in shape of the post-change model, it can be seen that there was a reduction in occupied beds and patients flowed through the system faster (the post change model line has a steeper slope). ..................................................... 206

Figure 47: Visual representation of the age groupings used for the analysis. The circled option was the preferred model. ..................... 223

Figure 48: The trade-off between model fit and generalisation with increasing complexity is shown. ................................................. 224

Figure 49: Fit of the data against the simplest model - the annual average model with no disaggregation of patients by age. The model did not fit the data well. ................................................................. 225
Figure 50: Fit of the data against the most complex model. All data groups are well-fitted (over-fitted) by the model, as evidenced by the difficulty in visually differentiating between the data and model. ... 226

Figure 51: Fit of the preferred model and data. The model does not perfectly fit the data and thereby avoided over-fitting (see Figure 50), but represented a significant improvement on the simplistic poor-fitting model (see Figure 49). ................................................. 226

Figure 52: The five age components of the preferred model exhibit a high level of fit to the data. ................................................................. 227

Figure 53: Trade-off between complexity and fit for the preferred model. Based upon the BIC value, a single compartment model was suggested. However, other factors can also be important for model selection. ............................................................. 228

Figure 54: Comparative population profile. It is predicted that there will be a large growth in the number of older people during this period. ... 231

Figure 55: The timing of the change of population with age groups matched to the preferred compartmental model. ..................................... 232

Figure 56: Forecast change in requirements for short-stay beds according to changes in the population size based upon age. ....................... 232

Figure 57: Forecast change in requirements for long-stay beds based upon population changes. The timing of changes is important for policy and planning reasons. ................................................. 233

Figure 58: Forecast total bed requirements. It is evident that those aged 65 years or more use the most beds. .............................................. 233

Figure 59: The forecast mix of bed usage based by age group over time. ....... 234

Figure 60: The assumption that the mix of short and long-stay beds was held constant is reflected in the forecast results. .......................... 235

Figure 61: Visual representation of effects of making changes to the system. The system becomes stable over time. ................................. 236

Figure 62: Representation of the different approaches to including seasonal weather factors in the bed occupancy compartmental flow model. In this scenario a switch (or model) is provided for each month, whereas based upon the findings reported in Chapter 6 fewer switches could be used. .................................................. 259

Figure 63: Total available beds = occupied beds + vacant beds. Vacant beds provide a buffer against variation in occupancy. ..................... 261
Figure 64:  Daily occupancy trends for total and age grouped bed occupancy. The moving average indicates the existence of a winter peak in total occupancy. ................................................................. 267

Figure 65:  Moving average occupied bed trends for the age-grouped data. Winter peaks in bed occupancy appear to exist for some aged-grouped data. ................................................................. 268

Figure 66:  Monthly movements in average monthly 9 a.m. air temperature and the relative change variable. Both variables show a strong winter trough. ................................................................. 270

Figure 67:  Scatterplot of bed occupancy for the model year and relative temperature change variables. The variation in occupancy was largely explained by the relative temperature change variable. ...... 272

Figure 68:  The weather adjusted compartmental flow model better reflects the actual occupancy data than the average compartmental flow model. ................................................................. 275

Figure 69:  Despite a better fit between the weather adjusted model and the daily occupancy data, the model is clearly suited, as intended, for exploring longer-term strategic issues as opposed to the operational issues associated with daily occupancy. .................. 276

Figure 70:  The weather adjusted forecast provided a visual indication of the seasonal occupancy fluctuations, unlike the forecast based upon the average compartmental flow model. ................................. 277

Figure 71:  Comparison of the original compartmental flow model and weather adjusted compartmental flow model. The weather-adjusted model required less vacancy to avoid patient turn-away. .. 278

Figure 72:  Comparison of the original, weather-adjusted and weather and vacancy-adjusted models with the data. ................................. 280

Figure 73:  Comparison of the original, weather-adjusted and no patient turn-away model forecasts. Inclusion of seasonality and vacancy is important for aiding planning decisions. ............................. 281

Figure 74:  The movement from the base model to the modified model resulted in a better fit between the model and data. ....................... 283

Figure 75:  Schematic representation of the change in the weather adjusted compartmental flow model of bed occupancy. The modification is adjusted to reflect business or policy implications. .................. 290

Figure 76:  The ALOS trend showed a period of continued decrease post service until about the year 2000 after which the ALOS rose.

xiii
The changes in ALOS were small, with an increase of approximately 2.5% over the period. ............................................. 303

Figure 77: The 90-day moving average trend shows an increase in total occupancy that commences from about the year 1999. ................. 304

Figure 78: The ALOS trend is fitted well by a polynomial model. However, the forecast is overstated. A linear model does not describe the post change period well. ......................................................... 305

Figure 79: Using fewer historic observations results in a better linear fit. The model appears to forecast the future ALOS well, although the ALOS during 2005 was greater than expected. .................... 306

Figure 80: Trends and forecasts for bed occupancy compartmental flow parameters over the post service change period. A constant model best described Parameters b, C and d, while a linear model best described parameter A. ........................................................... 307

Figure 81: The forecast of the model parameter A appeared reasonable. The implication of the forecast was that additional patients were admitted. ................................................................. 309

Figure 82: Comparison of short-stay group patient flow data based upon the original modelling period and the forecast period. There appeared to be an upward trend in the ALOS, which is contrary to the forecast for the short-stay flow parameter, b (see Figure 80). It is not possible to determine from the available data whether the increase in ALOS was solely flow related. .................. 310

Figure 83: The best forecast for parameter C was with a constant model (as shown in Figure 80). During the forecast period, there appeared to have been growth in the number of long-stay patient beds. ........ 311

Figure 84: Comparison of long-stay patient group flow data over the model and forecast period using the ALOS as a proxy measure for parameter D. The best forecast for parameter d was with a constant model (as shown in Figure 80). There appeared to be a downward trend in ALOS during the forecasted period, but it was not possible to determine from the available data whether this was solely related to patient flow. ................................. 312

Figure 85: The comparison of the compartmental flow expected length of patient stay and the average length of stay showed that the two measures were similar, though not identical. The partial data for 2003 and lack of data for 2004 for the compartmental flow model makes comparison of the measures more difficult. ...................... 313

Figure 86: Visual inspection reveals that the trimmed length of stay profile does not exhibit a Gaussian distribution. ............................... 330
Figure 87: The original data and the fitted model are illustrated. Visually the model appears to describe the data well. .............................. 335

Figure 88: The length of stay profile for AR-DRG 274 was different to that of AR-DRG 261. AR-DRG 274 appears to have a Gaussian distribution. ................................................................. 337

Figure 89: The length of stay profiles for AR-DRG 572 was different to that of AR-DRG 261. AR-DRG 572 appears to be uniformly distributed around a single value of stay. ............................. 337

Figure 90: The effect on patient admissions as a consequence of altering the number of available beds is illustrated. Over time the change stabilises. ............................................................... 359

Figure 91: Visualisation of the results presented in Table 41. ................ 360

Figure 92: The effect of altering the model parameters by +/- 10% on the first compartment release rate is illustrated. Changes to the second compartment parameters did have a very small effect, as opposed to no effect. ..................................................... 362

Figure 93: Daily admissions were most sensitive to changes in the first compartment parameters A and B. ........................................................... 363

Figure 94: The number of occupied beds was sensitive to changes in parameters A and C - the bed parameters. ........................................ 363

Figure 95: The overall expected length of patient stay was particularly sensitive to changes in parameter B - a flow rate parameter. It was also sensitive to changes in the other parameters, although to a much lesser extent. .......................................................... 364

Figure 96: The spread of total bed occupancy is illustrated after 10,000 runs... 365

Figure 97: Probability ranges for the number of total beds based upon 10,000 simulation runs using standard Monte Carlo methods. ...... 366

Figure 98: The distribution of total occupancy for the month of January arising from the Monte Carlo simulations. ............................ 367

Figure 99: The original model describing the number of beds required to avoid patient turn-away. The number of beds is represented deterministically, that is, there is no attempt to show the extent of uncertainty around the forecast. ..................................................... 368
Figure 101: Uncertainty was incorporated into the model designed to avoid patient turn-away by adding the range minimum and maximum lines. ................................................................. 369

Figure 102: Identification of secondary hypothesis and the chapter in which the question was answered. ................................................................. 387

Figure 103: Modification of the original (base) model has occurred to enable the incorporation of seasonal variation and variability of patient arrivals. The diagrams on the right-hand side are illustrative only and were presented in full detail in earlier chapters. ...................... 390

Figure 104: The use of the compartmental flow model of bed occupancy for planning and facilitating understanding has been extended. This research has shown the potential to apply this style of modelling in the areas of forecasting, evaluation and casemix. ....................... 391

Figure 105: The range of stakeholders in the decision-making process may be large. A solution that relies upon a single group may not yield the best solution. ................................................................. 403
List of Tables

Table 1: Recent reviews of state health systems have not always included bed modelling or access to modelling methodologies used. .............. 11

Table 2: Categorisation of research papers. Where papers could have been assigned to more than one category, the capitalised X indicates the primary category. .............................................................. 40

Table 3: Table of publications between 1991 and 2005 (inclusive) stemming from the original work by Harrison and Millard (1991). ................................................................. 70

Table 4: Summary statistics relating to the Flinders Medical Centre. While trends are evident, such as the increase in same-day admissions, these are often better visualised. ......................... 101

Table 5: Length of stay statistics revealed that the distribution is highly skewed. ................................................................. 104

Table 6: A search of “average length of stay” and “hospital” using the Google™ and Google Scholar™ internet search engines found many hits. The ratio of Google Scholar™ hits to Google™ hits was similar regardless of the country. ........................................... 134

Table 7: Types of models created and analysed. The number of model parameters reflects the complexity of the model, with the annual average being the least complex model. ...................... 172

Table 8: Comparative model fit, measured in terms of absolute errors. The complex models perform better than the single day census models. ................................................................. 178

Table 9: Correlation between models, training and test data. .................. 180

Table 10: Types of models created and analysed. The number of model parameters reflects the complexity of the model. .................... 202

Table 11: Model complexity and performance. The BIC value indicated that one of the more simple models was the preferred choice and the likelihood of this choice being incorrect was low given the Bayes Factor scores. ................................................................. 203

Table 12: Additional performance statistics supported the findings from visual inspection that although there was some variation between the model to the individual year data, the fit was very good. ........... 205

Table 13: Commonly used model performance statistics indicated that the model fitted the data well. ................................................................. 206
Table 14: Analysis of the model parameters shows that the change in bed numbers and flow rate was not uniformly implemented across short and long-stay patient groups, with the long-stay patient group being changed more than the short-stay group. .......................... 207

Table 15: Search results on “country name” and “hospital bed occupancy” using Google™ and Google Scholar™. ........................................... 211

Table 16: Scenarios tested to determine the best age grouping of the data for modelling bed occupancy. The preferred model is highlighted. ................................................................. 223

Table 17: Model performance statistics. The correlation between the individual components of the model and the data was very high and the absolute errors were all low. ................................................. 227

Table 18: Short-stay patients used disproportionately less bed-days than long-stay patients. Valuable information about the long-stay patients can be gained via a double compartment flow model. ...... 228

Table 19: Confidence (95 per cent) intervals for each parameter of the model. Such intervals are useful in showing that uncertainty around the exact model fit exists. ........................................... 229

Table 20: The comparison of the model to the actual data suggests that the model over-estimates the total number of beds actually occupied. ............................................................................. 230

Table 21: Summary of resource use (that is number of beds and admissions) by age group for 1998. ........................................................... 235

Table 22: Summary of resource use (that is the number of beds and admissions) by age group for 2019. The forecast increase since 1998 is also shown. ............................................................. 236

Table 23: Correlations between bed occupancy data and average climate variables. The mean 9 a.m. temperature was found to have the strongest correlation. ............................................................. 269

Table 24: Correlations between the four temperature variables and bed occupancy. The relative monthly average temperature change has strong correlations with occupancy and the over temperature variables. ........................................................................... 270

Table 25: Correlations between 1995-2000 average monthly bed occupancy and the average temperature variables. The correlations are weaker, but still significant and of reasonable strength. ................................................................. 271
Correlations between temperature variables and bed occupancy for individual years. The influence of the temperature on occupancy was not consistent; rather it was one of many factors that influenced occupancy. ............................................................ 273

The R-square value for the model was high, with 82 per cent of the variance in occupancy explained by the weather variable. ........ 274

The explained variation is significantly greater than the unexplained variation. Thus, from the data, it would appear that a linear relationship between the weather variable and occupancy existed. .......................................................................................... 274

Model coefficients. The model coefficients vary slightly to those in Figure 65, because a statistical package was used to run the regression analysis as opposed to the addition of a trend as part of the graphical capabilities of a spreadsheet. ........................................... 274

Vacancy rates for the original compartmental flow model and weather-adjusted model. ............................................................. 279

Absolute errors for each model. The weather-adjusted model achieved the lowest absolute error. .................................................. 280

Comparative statistics for the original and trimmed length of stay profiles. The trimmed profile is still skewed. .............................. 329

DRG profile for same-day patients during 1998. DRG 572 accounted for almost 65 per cent of the same-day activity. DRG 274 was the second most frequently assigned DRG for same-day patients. ................................. 333

There was little difference between the number of patient separations for the top two inpatient DRGs. Prior research with DRG 261 and its relative cost determined its selection for this research. .......................................................... 333

The patient stay and admission category profile for the three DRGs analysed. The majority of patients for each DRG related to the profiles analysed for the research. ........................................... 334

The average activity per DRG at three major teaching hospitals in South Australia. For many DRGs there is little regular activity. In smaller hospitals there tends be more DRGs where there is a low number of patient admissions per week or year. ...................... 334

Model performance statistics. The absolute error and correlation are indicative of reasonable fit. .................................................. 335
Table 38: Model parameters for a single and double compartment flow model. The flow rate of the double compartment model is not sufficiently different to justify the use of a double compartment model. ............................................................. 336

Table 39: Parameter variation ................................................................. 357

Table 40: BOMPS style bed occupancy measures for the original model for patients aged 65 to 79 years. ......................................................... 358

Table 41: The BOMPS style resource utilisation table for the model relating to patients aged 65-79 years. The majority of patients were short-stay patients. ................................................................. 359

Table 42: The effect on patient admissions when the number of beds was decreased by 10 per cent and short-stay patient flow was increased. ............................................................. 360

Table 43: The model parameters were altered by fixed amounts and the effects on certain aspects of patient discharge, bed numbers and length of stay were examined. ..................................................... 361

Table 44: Model parameter confidence intervals after 10,000 runs. ..... 365

Table 45: Occupancy statistics following a 10,000 simulation run. The simulated occupancy will be greater than the average monthly occupancy as the goal was to create a model that resulted in zero patient turn-away. ............................................................. 366

Table 46: Illustration of using the ALOS to calculate bed requirements. ....... 395

Table 47: Subdividing the data may not lead to a materially different determination of the number of beds required for a service. ............. 395
Abstract

The research presented in this thesis focuses on the investigation of whether the compartmental flow models of bed occupancy originally described by Harrison and Millard (1991) for decision-making around geriatric service care in the English National Health Service can be used to describe data from acute care hospitals in Australia and New Zealand.

Australia’s total health expenditure for 2004-05 was $87.3 billion. The use of health care services and expenditure pattern is well established and Australia follows the pattern found in most developed countries, with the greatest expenditure occurring on services for the elderly. Australia is experiencing a shift in population structure, with the proportion of older people forecast to increase. It is expected there will be a need for a greater level of expenditure on health care as the number of elderly people increase.

There is an emerging gap between the ability to supply health services and the demand for them. Furthermore, acute care hospital treatment is generally considered expensive and governments have been keen to control this expenditure.

It is imperative that governments are able to make decisions based upon robust policy advice. There are serious consequences in both economic resource allocation and patient (and population) health outcomes if decisions about future health service structures are incorrect. In particular, there is a need for better decision-making around bed management at the strategic level. Strategic decision-making relates to decisions that will occur in a longer time frame.
Decision-making can benefit from the use of modelling. Models represent a simplified version of reality that preserve the essential features of the situation being examined and can be used as a tool to investigate decision-making options, particularly in complex environments such as the health sector.

Historically decision-making relating to hospital beds has used either simple “back of the envelope” calculations or adherence to “rule of thumb” approaches. Most of the approaches have relied upon using the average length of stay metric. While the modelling of hospital bed numbers is not new, much of this work has relied upon the average length of stay, which is known to be a poor measure.

Harrison and Millard (1991) introduced the application of the compartmental flow model for modelling hospital bed occupancy and noted its potential to be used to influence policy decision-making. The flow model results are plausible and easily interpreted. However, relatively little work has focussed on the ability of these models to be generalized and be used for predictive purposes.

The research undertaken for this thesis consisted of a series of modelling experiments that can be grouped into two key stages: whether the models could be successfully applied to the acute care data; and whether the models could be used for novel purposes, such as forecasting, evaluation of service change, and benchmarking. This entailed the further development of the model, and a consideration of basic modelling issues such as the balance between data-fit and model complexity, in order to capture
better variation in the data and also to facilitate linkage to changes in population and seasonality.
Declaration Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give my consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

Signed…………………………………………………
(Mark Mackay)

Date…………………………………………………….
Acknowledgments

My research and this resulting thesis would not have been possible or as easily undertaken without the involvement of the following people or organisations:

- Emeritus Professor Peter Millard (London) who agreed to meet with me and discuss bed management problems during my visit to London during 1998 and then subsequently continued to help and encourage me to develop skills useful in the analysis of practical problems in the strategic management of hospital beds. Peter has responded to many emails over the years, welcomed my family and me into his home, visited Australia and provided contacts with other researchers in this area.

- Associate Professor Michael Lee (California) who has provided great support, assistance and helped me to develop further my skills in the area of modelling. His advice and guidance have been invaluable. His support to enable me to attend an international conference in London was also much appreciated.

- Dr Simon Dennis (Adelaide) who stepped in to provide supervision after Michael left the School of Psychology. Simon’s input has been much appreciated.

- The School of Psychology for enabling me to undertake my research with Michael Lee and providing opportunities for me to travel to conferences in Australia and England.

- The Department of Health for granting me study time.

- The library staff at the Department of Health for obtaining references for me and extending my loans on many occasions. Particular thanks go to Tricia Brooks, Jill Austin and Judy Gye.
To my modelling colleagues in Australia (particularly Professor Don Campbell, Professor Terry Mills, Dr Geoff McDonnell and Dr Chris Bain) and overseas (particularly Professor Gary Harrison, Professor Sally McClean, Professor Florin Gorunescu, Dr Brendon Rae, Dr Elia El-Darzi and Mrs Georgina Christodoulou) who have provided conversation, inspiration, assistance and answered many emails over the years.

To Don Mackay, my father, and Marisa Maio Mackay, my wife, for helping with the editing of my thesis.

My parents, Don and Jocie Mackay and Enzo and Tina Maio for their endless support.

Benjamin and Matthew, my sons, for being a source of inspiration to ensure that my work remained focussed and for all the smiles, hugs, games and good times.

Marisa, my wife, for keeping me company at night while working on my research; providing encouragement, advice and being a sounding board; keeping me focussed on finishing; believing in me; and her ongoing love and support.

Without this assistance, this research would not have been possible. I thank them all and I am indebted to them for their support.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>The first parameter of the first compartment in the flow model that relates to the total number of occupied beds or patients in the first or short-stay compartment.</td>
</tr>
<tr>
<td>ABS</td>
<td>Australian Bureau of Statistics</td>
</tr>
<tr>
<td>Acute care hospital</td>
<td>A hospital that provides short-term medical care especially for serious acute disease or trauma</td>
</tr>
<tr>
<td>ALOS</td>
<td>Average length of stay</td>
</tr>
<tr>
<td>ARDRG</td>
<td>Australian refined diagnostic related group</td>
</tr>
<tr>
<td>ARIMA</td>
<td>Auto-Regressive Inductive Moving Average</td>
</tr>
<tr>
<td>B or b</td>
<td>The second parameter of the first compartment in the flow model that relates to the flow of patients through the first or short-stay compartment.</td>
</tr>
<tr>
<td>BIC</td>
<td>Bayesian information criterion</td>
</tr>
<tr>
<td>BOMPS</td>
<td>Bed Occupancy Management and Planning System</td>
</tr>
</tbody>
</table>
C  The first parameter of the second compartment in the flow model that relates to the total number of occupied beds or patients in the second or long-stay compartment.

D or d  The second parameter of the second compartment in the flow model that relates to the flow of patients through the second or long-stay compartment.

DRG  Diagnostic related Groups (see also ARDRG)

Elective admission  A planned admission of a patient into a hospital bed, as opposed to emergency admission.

Emergency admission  An unplanned admission of a patient into a hospital bed.

Long-stay patients  Patients who stay for a longer period of time than short-stay patients. Arises from the fitting of a double compartmental flow model to acute care hospital data. Long-stay is a relative term and differs when applied to alternative types of care paradigms, for example, geriatric care services (see also short-stay patients).

LOS  Length of stay
Short-stay patients: Patients who stay for a short period of time in an acute care hospital. Arises from the fitting of a double compartmental flow model to acute care hospital data. Short-stay is a relative term and differs when applied to alternative types of care paradigms, for example, geriatric care services (see also long-stay patients).

WSSE: Weighted Sum Squared Error.
Chapter 1

Introduction

In this chapter the research topic of acute care hospital bed occupancy compartmental flow modelling bed is introduced. The issue of strategic decision-making in relation to hospital beds is identified as a real problem that warrants improvement and the gaps in current knowledge are identified. The research topic is shown to be of relevance to both researchers and policy makers in the health sector. The motivation for undertaking the research and the research questions are also presented. The chapter has the following structure:

1.1 The Australian Health System and the Emerging Problem........................... 2
1.2 The Need for Better Decision-Making.......................................................... 9
1.3 Why Model Hospital Bed Occupancy?.......................................................... 13
1.4 Decision-making and Hospital Beds............................................................ 14
1.5 Simple Models and the Average Length of Stay........................................... 14
1.6 The Introduction of Bed Occupancy Compartmental Flow Model............. 15
1.7 Compartmental Flow Modelling and Health Services Research............... 19
1.8 The Need for More Research Around Compartmental Flow Bed Occupancy Modelling.......................................................... 21
1.9 Personal Motivation for Undertaking the Research..................................... 22
1.10 The Research Project Outline.................................................................... 25
1.11 The Research Project Aims........................................................................ 25
1.12 The Research Project Questions................................................................ 26
1.13 About The Research Project Methodology................................................ 28
1.14 Contribution Towards Knowledge............................................................ 28
1.15 The Thesis Layout ..................................................................................... 29
1.16 Author Publications .................................................................................. 31
1.17 Conclusions ............................................................................................ 32
1.1 The Australian Health System and the Emerging Problem

There has been rarely a week, if not a day, go by in recent years when access to hospital beds or waiting times for elective surgery has not been in the media spotlight in Australia. In more recent times, the issue of sufficient clinical staff has also begun to receive significant media attention. Both are resource issues and both are intertwined – there is no point in opening hospital beds if there are insufficient staff to provide patient care.

So what is all the attention about? Australians are fortunate in having access to free medical care in public hospitals. Also, there is the option of accessing care through a considerable private sector. New medications and interventions provide cure or relief from many forms of disease that could not be treated previously.

Additionally, existing forms of treatment, such as dialysis, have been provided for more people, including older people, when previously a lack of access to equipment meant that such options were not always available. Thus, the expectation that access to high quality care that can treat more and more problems exists. Acute care hospital treatment, however, is generally considered expensive and governments have been keen to control expenditure (for example, see Generational Health Review, 2003).

In Australia, there are three levels of government: commonwealth, state and local governments. While the commonwealth government has the responsibility for primary health activity - that is it contributes to the cost of general practitioner and private specialist care in the doctor’s room and for diagnostic testing - the public acute care hospitals fall under the control of state governments.
Australia’s total health expenditure for 2004-05 was $87.3 billion or $4,319 per person and represented 9.8 per cent of the gross domestic product (AIHW, 2006). In terms of international comparison, Australia’s expenditure per person, a measure that overcomes the problems with using short-term measures of gross domestic product, was ninth highest when compared to 29 other OECD countries (AIHW, 2006). The level of expenditure has remained relatively stable over the last ten years, thus confirming that Australia applies considerable resources to the health sector.

The public health sector costs represent significant proportions of the budget in each state and territory. Almost 23 per cent of the Australian health expenditure for 2004-05 was met by state, territory and local governments (AIHW, 2006). While it is generally recognised that the demand for health care services is great, this must be balanced against the other services that state governments must also provide.

Almost all developed countries and many developing countries are experiencing a shift in population structure – the proportion of older people is increasing. Australia is no exception to this trend as shown in Figure 1.
The change in profile can be related to several events, including:

- A period of population growth post World War II (the baby boomers), and
- A period of contraction in the birth rate during the later part of the 20th century.

Although there may be some doubt about the ability of the Australian Bureau of Statistics to forecast accurately the population profile for the remainder of this century it is clear that the predicted rise in the proportion of the elderly will occur in the next few years.

The use of health care services and expenditure pattern is well established and Australia follows the pattern found in most developed countries, that is, with the exception of the first few years of life, the greatest expenditure occurs on services for
the elderly (OECD, 2003). This pattern is illustrated in Figure 2 using Australian patient separation data.

Figure 2: The hospital utilisation patterns for non-Indigenous Australians show a high use of services for the very young and increased utilisation as age increases. The difference between males and females aged between 15 and 40 relates to child bearing. Data source: AIHW (2006).

The Indigenous hospital utilisation pattern is similar to that of the non-Indigenous Australians except that utilisation rates are greater (that is, the distribution is shifted up) and the increased higher utilisation associated with ageing occurs at a younger age.

The ageing of the population presents a range of new challenges for communities and governments, including:

- A reduction in the size of the available workforce (based upon current patterns of working life)
• A smaller tax base (from individuals) upon which to seek revenue to fund services as the numbers in the workforce diminish

• A changing mix in service requirements – there may be demand to shift resources from the younger aged people, particularly as the number of children becomes less, to the older people (who also have voting power), and

• A period of greater expenditure on health and other social services as the volume of older people increases (see Figure 1).

This challenge is made even greater when combined with the shift in disease prevalence from acute infectious disease to one of chronic disease (Generational Health Review, 2003; Productivity Commission, 2005). Chronic diseases tend to affect people later in life and while we have been successful in achieving greater life expectancy we have also increased the burden of chronic disease (Duckett, 2005).

Health workers are not precluded from the processes of life – they too are ageing and the existing workforce is replete with baby boomers, many of who are, or will, reach the end of their working lives within the next 10 years (SA Government, 2005; Duckett, 2005). The expected large number of retirements comes at a time of forecast high demand for services, a reduction of available workers and following a period of insufficient succession planning. The health sector relies upon many highly skilled clinical professions whose training is often lengthy. In South Australia, estimations of workforce contraction vary, with contraction forecast to occur between 2011 and 2016 (Access Economics, 2001; Sphoehr, 2004).
The ability to rely upon other countries to provide a significant component of the South Australian health workforce no longer exists, because all developed countries (and other states within Australia) will be competing for the same resources. Consequently, South Australia will not only have to work harder to compete to attract clinical staff, but it will also face increased competition from other states or countries as they try to attract staff elsewhere. This competition has already begun with other states providing significant increases in remuneration to attract additional staff and prevent their own staff moving elsewhere. Thus, there is an emerging gap in the ability to supply services, both in terms of capital infrastructure and workforce, to meet demand as shown in Figure 3.

![Figure 3](image_url)

Figure 3: A schematic representation of the emerging demand and supply issues facing the health sector.

Hospital occupancy levels have also increased in recent years. The ability to provide services to the growing wave of baby boomers that are expected will not exist, *ceterus paribus*, unless capacity is increased (as shown in Figure 3). Additional capacity (capital infrastructure) can be created relatively quickly. There is, however, little point in doing so if there are insufficient staff available to provide services to patients.
Record breaking hospital inpatient activity levels reported in South Australia during 2006\(^1\) lends credence to such forecasts. The levels of demand on the South Australian health system forecast to occur by 2011 in the Generational Health Review report (Generational Health Review, 2003) were found to be occurring during 2006. Peak activity levels were such that one major teaching hospital, Flinders Medical Centre, implemented its disaster plan for the first time ever in order to deal with ordinary patient activity – not disaster activity. It entailed the early discharge of 120 people to take the pressure of the emergency department to allow for emergency admissions. Apart from Flinders Medical Centre, the Royal Adelaide Hospital (the State’s largest teaching hospital) and the ambulance service also report record activity levels. These records occurred during one of the mildest winters in recorded history for the State and in the absence of high levels of influenza. This activity resulted in the Minister for Health reporting that there was solid evidence that if the State continued to provide health care using current models of service delivery and care strategies, by 2043 the entire state budget will be required to meet the costs of providing health care (Hill, 2006).

Not everyone has supported the notion that the ageing of the population will pose problems for the health system. For example, Gray, Yeo and Duckett (2004) concluded that the ageing of the Australian population was not associated with an increase in the proportion of hospital beds used by older patients during the period 1993-2002. The authors identified that a significant increase in patient separations occurred and that much of this was attributable to the increase in same-day admissions. The authors, however, failed to take account of the relative mix of beds

\(^1\) At the time of writing, 2006 was not complete.
among the population groups, which is biased towards the elderly. Furthermore, the authors failed to note that despite an increase of almost 100 per cent in same-day patient activity over a ten-year period, the number of overnight patient bed days decreased by only one per cent and the overnight stay patient numbers increased. When these are considered a very different perception about recent activity can be reached as reported in the work by Mackay and Millard (2005a and 2005b).

Commentaries such as that provided by Gray, Yeo and Duckett (2004) are in the minority and most researchers, policy analysts and political or financial commentators are concerned that the level of health expenditure is set to rapidly increase as populations in developed countries age. For example, see Commission of the European Communities (1999), WHO (2002), OECD (2003), the Australian Productivity Commission (2005) and Gottret and Schieber (2006).

Consequently, governments are now exposed to having to respond to demand and supply problems that will have potentially long-term health consequences for their populations. Health is not the only industry that will be facing the challenges of ageing populations and thus the problems confronting governments are complex and widespread (Productivity Commission, 2005).

1.2 The Need for Better Decision-Making

Given the previously described set of circumstances it is imperative that governments are able to make decisions based upon policy advice that is robust and sound as possible given current understandings. As previously indicated, the health care
industry is a complex one and decision-making in such an environment can be difficult.

Models represent a simplified version of reality that preserve the essential features of the situation being examined (Denardo, 2002) and can be used as a tool to investigate decision-making options or possibilities, particularly in complex environments where many factors interplay (Jun, Jacobson and Swisher, 1999; Fone et al., 2003) or experimentation is not possible, both of which describe conditions that apply to the health sector. The interpretation of the model output, however, may be more of an art than an exact science at times (Powell and Baker, 2004).

Given the increasing demands being placed upon the health services and the likelihood of significant staff shortages, there are serious consequences in both economic resource allocation and patient (and population) health outcomes if decisions about future health service structures are incorrect. Given the recent advances in computing power and the need to improve decision-making, there has never been a more opportune time to apply modelling to facilitate improved decision-making in the health care sector. One aspect where modelling will become increasingly important in the health sector is in relation to modelling decisions around hospital beds.

Many states within Australia have reviewed their health systems in recent times, including South Australia, New South Wales, Victoria and Western Australia. In the review findings reference to hospital beds has been made, though any reference to modelling has not necessarily been accompanied by material that would enable
evaluation of any modelling undertaken as part of the review work as shown in Table 1.

Table 1: Recent reviews of state health systems have not always included bed modelling or access to modelling methodologies used.

<table>
<thead>
<tr>
<th>State</th>
<th>Modelling Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>South Australia</td>
<td>The South Australian Government sought a review of its health system during 2002. The findings of the review (Generational Health Review, 2003) found that without change a significant increase in hospital beds would be required. The methodology for arriving at the increase was not reported.</td>
</tr>
<tr>
<td>New South Wales</td>
<td>The report of the NSW Health Council (2000) found that NSW was facing increasing demand, but that a good health system is characterised by more than just the number of beds. No specific methodology for bed forecasting was stated in the report.</td>
</tr>
</tbody>
</table>
Western Australia

The Western Australian Government sought a review of its health system during 2003. The findings of the review (Department of Health, 2004) found that a major reprioritisation of the health system was required. Findings were in part based upon bed modelling. The methodology for the bed modelling was published (Strategic Planning Directorate, 2004). The modelling was based upon activity and utilisation data (bed days and separations), and linked to population projections.

Victoria

During November 2000, the Patient Management Task Force was set up to identify specific areas for improvement in in-hospital patient management processes and to advise on system factors that will encourage best practice in patient management (Patient Management Task Force, 2001). A patient flow-modelling project was commissioned by the Task Force to assess the feasibility of predicting the impact of changes to various elements of the health care system. The initial results led to the recommendation by the Task Force that Department of Human Services should commission the development of computer-based patient flow modelling tools to assist metropolitan health services in forward planning their elective caseloads. No details about the nature of the modelling were provided.
Thus, while there has been some move towards using modelling or recognition of the potential for using modelling to facilitate better decision-making around hospital beds this is not universal, the work is often simplistic (see section 1.5) and if often not transparent.

1.3 Why Model Hospital Bed Occupancy?

The ability to infer the underlying process that generated observed data is the goal of most behavioural research (Myung and Pitt, 1997) and the goal of modelling the use of acute care hospital beds is no different. Developing a formal and quantitative model allows:

- Interpretation, understanding, and insight into how and why the distribution changes
- The ability to generalise where data are not available (for example, other hospitals), and
- The ability to make predictions where data cannot be available (for example, into the future).

Ultimately, the use of modelling can be applied to policy and resource allocation issues, including the determination of the number of hospital beds required for service provision for a given hospital or community.
1.4 Decision-making and Hospital Beds

Historically decision-making relating to hospital beds has used either simple “back of the envelope” calculations or adherence to “rule of thumb” approaches. Most of these approaches have relied upon using the average length of stay metric.

The average length of stay measure is a ubiquitous measure in the health sector that is used for a range of purposes, from benchmarking, calculating bed numbers and more complex financial allocation methods, as can be verified by a simple search on the internet. For example, searching on Google™ (2005) using the criteria of a small number of country names and the term average length of stay yielded more than 15 million hits (see Table 5, Chapter 4 for more details).

The modelling of hospital beds and patient length of stay, which are intertwined, is not new. For example, work in this area has been undertaken Yates (1982), Pendergast and Vogel (1988), and Sorensen (1996). Some of this work, such as that by Sorensen, has the development of the models reliant upon the average length of stay (ALOS), which could be argued to be a slightly more advanced approach than what is often undertaken by health care managers and clinicians.

1.5 Simple Models and the Average Length of Stay

It has been recognised that the use of the ALOS for modelling hospital bed issues is flawed (for example, Farmer and Emami, 1990; Harrison and Millard, 1991; Mackay and Millard, 1999; Costa et al., 2003). There are several numerical and practical reasons that using the ALOS is inappropriate for use in the development of models. First, the length of stay profile typically has a highly skewed distribution and that is
not well summarised by its mean value. Second, the length of stay distribution is complex, often consisting of mixtures of patient types (that is, medical and surgical, planned and unplanned admissions, young and elderly) and mixtures of outcomes (that is, some patients die, some are discharged home, some to alternative care services such as nursing homes). While it might be argued that the introduction of casemix categories could reduce some of the complexity, recent work indicates that the problems associated with the average length of stay are still not overcome (Wang, Yau, and Lee, 2002). Furthermore, the ALOS does not take into account the time of day when a hospital is most busy. That is not to say, however, that some of the work previously undertaken has not yielded interesting findings, such as the focus on discharge destination by Sorensen (1996)\(^2\).

1.6 The Introduction of Bed Occupancy Compartmental Flow Model

In their seminal paper, Harrison and Millard (1991) introduced the application of the compartmental flow model. A compartmental model describes the flow of something, such as patients, through a system, where the system is comprised of a finite number of homogeneous subsystems known as compartments (Godfrey, 1983). According to Godfrey (1983), compartmental models have been widely applied as modelling solutions in the areas of biomedicine, pharmacokinetics and ecology.

---

\(^2\) See Chapter 2 for a more fuller appraisal of the literature on bed modelling.
The abstract of the paper describes the model developed by Harrison and Millard (1991) and its potential to be used to influence policy decision-making for bed allocation:

*The empirical distribution of length of stay of patients in departments of geriatric medicine is fit extremely well by a sum of two exponentials. Most of the patients in a geriatric department are rehabilitated and discharged or they die within a few weeks of admission, but the few who become long-stay patients remain for months or even years. A model is presented for the flow of patients through a geriatric department, which has analogies to models of drug flow in pharmacokinetics. The theoretical model explains why the empirical distribution, obtained from the midnight bed state report, can be used to study the effect of various policy decisions on both immediate and future admission rates for the department, and shows the benefits of policies which reduce long-stay patient numbers by improving long-stay rehabilitation.* (Harrison and Millard, 1991, pg 221).

Visually the hospital bed compartmental flow model can be represented as shown in the Figure 4.
Figure 4: A diagrammatic representation of the flow of patients through compartments. The compartments may be virtual or real - the patients may not actually change location within the physical hospital (Mackay and Lee, 2005).

Others joined Harrison and Millard in their work, notably McClean. Harrison, McClean and Millard continued publishing research that promulgated the notion of compartmental models of occupancy being used as a means of looking resource implications concerning hospital beds (for example, Harrison, 1994; McClean and Millard, 1994; 1995; 1998; Harrison, 2001; Harrison, Mackay and Schaeffer, 2005).

Work to date has focused on two or three compartment bed occupancy flow models to describe the patient stay profile within the hospital (Harrison, 1994, and Mackay, 2001), with additional compartments being added to incorporate the community (Taylor, McClean and Millard, 1996).
The modelling work of Harrison was incorporated into software known as the Bed Occupancy Management and Planning System (BOMPS) and attempts were made at commercialization. It was subsequently distributed as freeware. BOMPS provided two mechanisms for creating the bed occupancy profiles: a daily census, or an average census. Most of the work undertaken has focussed on the use of the daily census approach, although has not necessarily employed the BOMPS software (for example, Harrison and Millard, 1991; and McClean and Millard, 1993). The software, however, has not been updated to run on current computing environments.

The flow model results are plausible and easily interpreted. However, relatively little work has focussed on the ability of these models to be generalised and be used for predictive purposes. For example, if the census method of model creation is employed based upon sampling data from a Monday, it is reasonable to question whether the obtained LOS distribution will generalise to data from other days. It is also not clear whether generalisability is adequately addressed in the average day census approach.

Data relating to geriatrics patients from the South of England was used for the development of the model and also much of the subsequent research. The application of the model was extended to the acute care sector using data relating to the hospitalization of patients in acute care settings in Australia and New Zealand by Mackay with others (for example, Mackay and Millard, 1999; Millard, Mackay, Vasilakis and Christodoulou, 2000; Mackay, 2001; Mackay and Lee, 2004a, 2004b, 2005; Mackay, Lee, Millard and Rae, 2004; Harrison, Shafer and Schaefer, 2005; Mackay, 2006). This modelling also has the potential to influence decision-making in other areas, such as that of staffing. For example, the decision to increase bed numbers can only occur if sufficient nursing (and other) staff are available to provide
patient care services. Other potential applications include forecasting future bed
requirements, the ability to pre-test system changes through sensitivity and simulation
analysis, benchmarking, evaluation and the potential to influence resource allocation
funding models.

1.7 Compartmental Flow Modelling and Health Services Research

This type of modelling can be categorized as belonging to many particular academic
fields, including mathematical, statistical, economic or health services research. The
broadest and most inclusive category is that of health services research.

Health services research (HSR) has been defined by various individuals and
organisations, including Fraser (1997), Roper (1997), Aday (2001), Lohr and
Steinwachs (2002), Scott and Campbell (2002), and Academy Health, a US-based
organisation. It would appear that there is general agreement that HSR is a
multidisciplinary field, where researchers and others are interested in the application
of HSR are concerned about questions relating to the need, use, demand, supply and
outcome of health services (Last, 1988). The term “outcome” has a broad meaning
and may relate to the appropriateness, equity, effectiveness and efficiency of health
services (Pirkis et al., 2005). Research may be conducted across the range of care
situations, that is, from individual to populations; across the care types from
prevention through to palliative care; and across service provider types including
individuals through to organisations.

Some efforts are being made to expand this historic category to health services and
systems research. Bed occupancy modelling sits well in either health services research
or the wider health services and systems research category. HSR is a relatively newly recognised field of research in its own right in Australia, although as a research activity, it has a long history (Hall and Chinchen, 1999). Despite the large size of the health sector in terms of expenditure, Australia has not historically made large investments in such research (Hass, 2004; Pirkis et al., 2005). Consequently, the capacity to undertake and provide the necessary modelling in the more general sense is indeed limited. In terms of bed occupancy modelling, while there has been increased interest developing in the topic in recent years, there are few people in Australia actually involved in such research endeavours as I can attest to as one of the convenors of the International Health and Social Care Modelling and Applications Conference that was held during 2006.

Jun, Jacobson and Swisher (1999) observed that despite the obvious benefits of modelling – and they restricted their attention to discrete simulation health care modelling – they could not foresee modellers overcoming the difficulties in implementing models. Fone et al. (2003) have considered why health services research modelling may not have been more influential in the past. Their findings suggest that health care modelling has a somewhat potted history, with little known about the whether models were implemented, and even if they were, whether they were of value in guiding decisions. While this research was not specific to Australia and was restricted to particular types of modelling, particularly that relating to simulation, my experience suggests that their findings do resonate with experiences in Australia. From my experience other factors that have previously affected the use of modelling for decision-making purposes include the continual changes arising from political cycles, inflexible funding, a preference to adhere to existing service models
and a lack of understanding of modelling by decision-makers. Thus, notwithstanding
the challenges of the research, the challenges of gaining acceptance of the research
outcomes are considerable and cannot be ignored.

1.8 The Need for More Research into Compartmental Flow Bed Occupancy
    Modelling

Earlier in this chapter the need for good decision-making around the allocation of
health care resources was established. The remainder of this chapter will focus on the
need for better decision-making around bed management at the strategic level and
how research conducted for this thesis may usefully contribute to achieving that
outcome.

There is a continuum upon which decision-making occurs. Strategic decision-making
is concerned with decisions that will occur in a longer time frame, such as the
planning of future services. Conversely, operational or tactical decision-making
relates to decisions that come into effect immediately or perhaps in a short space of
time. For example, in a health care setting, this might be planning for the next shift, or
perhaps even the next hour, as queues begin to occur. Tactical and strategic decision-
making criteria may share some common inputs, but this will not always be the case.
Additionally, the weight placed upon the common factors may be different. Thus, it is
quite likely that the models that can assist decision-makers for tactical and strategic
decision-making purposes will be different.

Furthermore, the evidence for management solutions is rarely based upon the
interpretation of statistical tests, such as a double blind controlled experimental
environment, as this would not permit the flexibility or adaptability required by organisations, and could result in a loss of competitive advantage. Thus, the evidence about which method is or is not more suited to a particular management task is rarely concrete. The use of models, however, enables the study of systems (for example, organisations and integrated services) to aid in the design, understanding and construction without the need for real experimentation (McClean, 1994).

The personal motivation for undertaking the research and an outline of the remainder of the thesis will also be provided.

1.9 Personal Motivation for Undertaking the Research

The motivation for undertaking this research stems from involvement in a work project at the South Australian Health Commission (now Department of Health). A large teaching hospital had requested additional funding for extra beds that it believed would be required for the forthcoming financial year. The request came via a letter and included no business case or analysis to support the need for the additional funding. In the absence of any supporting analysis for provision of the funds a joint Health Commission-hospital project team was established to undertake the analysis to confirm whether the funding was required or not.

During this project the question of whether or not the hospital had any modelling or decision-making tools around hospital beds was asked. It was reported that the only measure required for this type of decision-making was the ALOS and that despite at least one attempt by the hospital, no modelling or decision-making tools relating to bed use had been found.
As a consequence of this feedback I undertook a literature search that resulted in the identification of numerous papers on decision-making and modelling hospital beds, including that of Harrison and Millard (1991). At that time Millard and his colleagues (Harrison and McClean) were the most prolific authors on the topic of bed modelling. A meeting with Millard in London convinced me that the models he had been developing with Harrison and McClean were suitable for application in the acute care sector in Australia, because the length of stay profiles in the geriatric and acute systems were of similar shape and it the difference between the two patient groups could be described as one of differing duration in hospital.

At the end of the project, the decision regarding the need for the additional funding was a political decision.

After discussions with Millard I prepared some findings for the hospital using the BOMPS software package. The feedback from that experience was that the clinicians saw some potential, but required further evidence that a “system developed for geriatric health services in the UK” could be applied to an acute care setting in Australia. This need for further evidence resulted in the identification of a gap in the academic literature relating to strategic management decision-making around acute care hospital beds. This eventually resulted in the research for this degree being undertaken.

The process for problem solving described by Powell and Baker (2004), which stems from the work of Couger (1995), is applied to this research problem and represented
in Figure 5. The first four stages of the process can be attributed to the initial work relating to the request for additional funding. The research work aligns with the fifth step of evaluating solutions.

<table>
<thead>
<tr>
<th>Powell and Baker’s Framework</th>
<th>From My Perspective</th>
<th>The Hospital’s View</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Exploring the Mess</td>
<td>Initial Department &amp; Hospital Working Group Analysis</td>
<td></td>
</tr>
<tr>
<td>2. Searching for Information</td>
<td>Need for better planning around beds</td>
<td></td>
</tr>
<tr>
<td>3. Identifying a Problem</td>
<td>Literature review – including identification of Millard, Harrison, and McClean papers</td>
<td></td>
</tr>
<tr>
<td>4. Searching for Solutions</td>
<td>PhD research</td>
<td></td>
</tr>
<tr>
<td>5. Evaluating Solutions</td>
<td>Transfer of research into practice</td>
<td></td>
</tr>
<tr>
<td>6. Implementing a Solution</td>
<td>The Bed or Finance Problem</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5: Application of Powell and Baker's creative problem solving process to the hospital bed problem. There is an apparent difference between my research and the approach undertaken by the hospital.

The final step of the process relates to the transfer of research into practice. While some attempts have been made to publish and therefore alert others to the benefits of this research, that stage is yet to be completed.

The same process was applied to the approach adopted by the hospital. Without being privy to the entire decision-making within the hospital prior to the time of the request for additional funding, it would appear that the decision-making process did not follow that identified by Powell and Baker, but was truncated to the presentation of a
problem and a request for additional funding as a solution. The need for a joint project team to evaluate the initial funding request perhaps is evidence that the first two steps in the Powell and Baker process were bypassed. Additionally, the solution that the hospital sought was extra funding and no other solutions were apparently considered. Such a decision-making process, while not perhaps ideal, is perhaps reflective of the processes in place at the time and the political nature of any such decisions.

1.10 **The Research Project Outline**

This project will focus on the investigation of whether the compartmental flow models of bed occupancy originally described by Harrison and Millard (1991) for decision-making around geriatric service care in the English National Health Service can be used to describe data from acute care hospitals in Australia and New Zealand.

The research work will consist of a series of modelling experiments. The experiments can be broken into two key stages that addressing the following questions:

- Can the compartmental flow models be successfully applied to the acute care data?
- Can the models be used for novel purposes, such as forecasting, evaluation of service change, and the potentially altering funding policy?

1.11 **The Research Project Aims**

As previously indicated, the majority of the research conducted on bed occupancy compartmental flow models has relied upon an English geriatric data set. While this data has generated useful research about the application of flow models to patient occupancy issues, it can be argued that this research has little relevance to an acute care hospital setting or at least evidence is required to validate that the research using
geriatric data sets can be generalised to other parts of the health sector, including those from different counties.

Thus, the broad aims of this research project are to develop an understanding of:

- Whether the bed occupancy compartmental flow model will fit data from an acute care hospital
- Whether the bed occupancy compartmental flow model can be applied to novel research problems, such as population change
- Whether model choice theory can be introduced to improve model selection, and
- Whether the bed occupancy compartmental flow model can be further developed or modified to enable better fit of the data.

1.12 The Research Project Questions

The main question to be addressed by the research will be:

Can bed occupancy compartmental flow models be applied to acute care hospital data in order that better (compared to existing) or new information or understanding be developed and have the potential to result in improved strategic planning of service delivery (and thereby resource utilisation)?

Secondary research questions that will be tested during the research include:

1. How many data are required to create a bed occupancy compartmental flow model for an acute care hospital data set?
2. What level of model complexity is desirable in order that models can be used for generalisation and forecasting purposes?
3. Can bed occupancy compartmental flow models that incorporate the ageing of the population be used to forecast future bed (resource) usage in acute hospital care?

4. Can bed occupancy compartmental flow models be used to evaluate service change?

5. Can bed occupancy compartmental flow model parameters be used for forecasting purposes?

6. Can the bed occupancy compartmental flow model be adjusted to incorporate seasonal variation, where the term “seasonal” applies to weather seasons?

7. Can bed occupancy flow compartmental flow model parameters provide a substitute metric for the average length of stay in resource allocation models?

8. Can sensitivity and simulation techniques be used in conjunction with bed occupancy flow compartmental flow models to enable uncertainty to be incorporated into the modelling process?

These questions do not represent the full gamut of research that could be conducted in relation to strategic decision-making and bed occupancy compartmental flow models. Rather, it is intended to address the current gaps in the literature and also fundamental issues relating to the:

- Modelling technique (questions 1 and 2);
- Needs of the decision-maker and health policy worker (questions 3 to 7); and
- Need to address the notion of uncertainty (question 8).
1.13 About the Research Project Methodology

As previously indicated, the work of Harrison and Millard (1991) provides the foundation for this research. The research methodology will be fully described in Chapter 4. It is, however, useful to identify and reinforce that this research is of a multi-disciplinary nature and is likely to be of interest to researchers from a range of backgrounds, as well as those who might wish to apply the research findings in a health care setting. Consequently, and also to reflect the author’s own background, the research in this thesis relies on various other research that is often mathematical or statistical in nature, but is presented in a style that is deliberately devoid of extensive mathematical notation.

1.14 Contribution Towards Knowledge

The underlying purpose of conducting this research is to address the existing gaps in the literature, which includes queries initially raised about whether the work of Harrison and Millard (1991) can be applied to acute care hospital setting. It will also be shown that the research undertaken for this thesis covers new areas of findings, including:

a. Considering different means of modelling the data
b. Addressing the issue of model complexity and fit
c. Bed occupancy forecasting
d. Using the modelling for evaluation purposes
e. Modifying model to incorporate seasonal change, and
f. Considering whether there is merit in using the model output for funding purposes.
1.15 The Thesis Layout

This thesis is presented as a series of chapters, with those chapters relating to research findings being presented in the format of a lengthier academic journal publication. While each chapter can be read independently, it is recommended that the chapters be read in the order in which they are presented so that the development of the modelling can be more easily understood. The translation of the research questions into chapters in this thesis is presented diagrammatically in Figure 6.

Figure 6: A diagrammatic representation to the research that is required to show that there is merit in adopting compartmental flow models for strategic decision-making purposes in the Australian acute care environment.

In this chapter, I have introduced my topic and explained the importance of my research. In the following chapter I present a literature review of hospital bed modelling. In Chapter 3, contextual information is provided about the two hospitals
that provided the data for this research. Additionally, the bed occupancy data obtained from the hospitals is analysed using commonly used techniques in order to highlight the shortcomings of these techniques for strategic decision-making purposes. In Chapter 4 some of the theoretical background about modelling and model choice that underpins the research is discussed. Chapter 5 is based upon a published paper (Mackay and Lee, 2005) regarding model choice and considers how much data is required to model hospital bed occupancy in an Australian acute care hospital. Additionally, the issue of model complexity is examined through consideration of different models that enable the capture of seasonality. Chapter 6 presents the results of the evaluation of service change. These results provide further confirmation that the bed occupancy compartment flow model can be applied in an acute hospital setting using data from New Zealand and also introduce the novel application of the use of such models for evaluative purposes. In Chapter 7 the issue of complexity is again considered, but in relation to how many models are required to describe occupancy and patient age. These models are combined with population forecasts in a novel process to create forecasts of future occupancy. The presentation of an alternative method for modelling patient age and seasonality is presented in Chapter 8. In Chapter 9, the data from New Zealand is again considered, and the model parameters alone are used for forecasting future occupancy levels. In Chapter 10 the ability to substitute the ALOS with parameters from compartmental flow occupancy models as a means of improving the casemix funding methodology is considered. The issue of incorporation of uncertainty into the models through simulation and how this might be done using widely available and low cost software is discussed in Chapter

---

3 Seasonality in this sense refers to variation of weather patterns across the year, as opposed to the use of the term in operations research. This is discussed more fully in later chapters.
11. The overall discussion of the results, the need for further research and conclusions are presented in Chapters 12, 13 and 14.

1.16 Author Publications

I have already communicated the outcomes of much of the proposed research to the health sector and HSR sector as a consequence of:

a. Referred journal publications
b. Referred conference papers or abstracts
c. Published letters
d. Conference presentations (invited and other), and
e. Articles in newsletters.

The list of publications and presentations is detailed in Appendix I.

The communication of the research findings prior to submission of a completed thesis was a deliberate strategy designed to achieve two outcomes:

1. Engagement with the local health sector within Australia to raise awareness of the potential of the research and gauge interest in such research in a timely fashion that would not have been otherwise possible, and

2. Seek critical comment upon the nature of the research findings.

Both of these outcomes have been achieved and have assisted me in my research.
Additionally, the communication of the research findings has commenced the final stage of the problem-solving process suggested by Powell and Baker (2004), that of solution implementation.

1.17 Conclusions

In this chapter I have outlined the significant issues facing the health sector in Australia, and indeed, in most developed countries. The case for improved strategic decision-making around hospital beds has been presented.

The proposed research questions and aims should result in new and potentially useful decision-making tools for the acute care hospital sector. Clearly, much of this research has already been presented either in the form of published journal articles or as conference papers. The communication of the research findings to the health sector and broader academic community provides some measure of confidence in this research. The difficulties in translating this work into the health sector, however, have also been recognised.

The next chapter will consider the previous attempts that have been made to model hospital beds, why these past methods are problematic and identify the niche that can be filled by the bed occupancy compartmental flow model.
Chapter 2

Hospital Bed Modelling - A Chequered History

In this chapter I present examples from the published literature of various approaches to hospital bed planning and forecasting, together with the advantages and disadvantages of these methods. A simple classification system is introduced to group works of a similar nature. The work stemming from Harrison and Millard’s (1991) publication on the use of hospital compartmental flow models for modelling geriatric health service data is also presented. It is noted that Harrison and Millard’s (1991) has provided a general basis for modelling patient flows, but it now requires exploration in the acute care hospital sector. The chapter has the following structure:

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>34</td>
</tr>
<tr>
<td>2.2</td>
<td>Australian literature reviews</td>
<td>34</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Review by Dwyer and Jackson</td>
<td>35</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Review by Anderson et al.</td>
<td>36</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Review by Goddard and Mills</td>
<td>37</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Conclusions about Australian literature reviews</td>
<td>38</td>
</tr>
<tr>
<td>2.3</td>
<td>General hospital bed modelling techniques</td>
<td>38</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Simple Methods</td>
<td>41</td>
</tr>
<tr>
<td>2.3.2</td>
<td>More Complex One-off Approaches</td>
<td>44</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Queueing Models</td>
<td>47</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Simulation</td>
<td>53</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Flow Models</td>
<td>58</td>
</tr>
<tr>
<td>2.3.6</td>
<td>Conclusion regarding the general research</td>
<td>64</td>
</tr>
<tr>
<td>2.4</td>
<td>The Harrison and Millard compartmental flow model</td>
<td>67</td>
</tr>
<tr>
<td>2.4.1</td>
<td>The Subsequent Research Effort</td>
<td>67</td>
</tr>
<tr>
<td>2.4.2</td>
<td>A more in-depth look at the compartmental flow model</td>
<td>78</td>
</tr>
<tr>
<td>2.4.3</td>
<td>Conclusion regarding the compartmental flow research</td>
<td>87</td>
</tr>
<tr>
<td>2.5</td>
<td>Other sources of literature</td>
<td>90</td>
</tr>
<tr>
<td>2.5.1</td>
<td>Bed management texts</td>
<td>89</td>
</tr>
<tr>
<td>2.5.2</td>
<td>Grey information</td>
<td>91</td>
</tr>
<tr>
<td>2.5.3</td>
<td>Conclusions regarding the other literature</td>
<td>92</td>
</tr>
<tr>
<td>2.6</td>
<td>Overall conclusion</td>
<td>93</td>
</tr>
</tbody>
</table>
2.1 Introduction

The purpose of this chapter is to provide an illustration of the range of approaches that have been published as means of planning and forecasting hospital bed needs. In doing this, three sources of literature will be considered, namely: literature reviews, individual research papers and specifically written texts on the subject of bed management.

The aim of this chapter is not to present an exhaustive review of every publication that deals with hospital bed management and planning. Rather, the intent is to highlight that a myriad of approaches have been suggested in tackling bed management and planning over a period of more than 30 years and that little or no traction has been achieved in terms of ongoing or widespread application.

In order to provide context about developments in Australia, literature reviews on bed management emanating from Australia will be first presented. Illustrations of techniques originating from a variety of countries that have been published in the general academic journals will then be presented. This will be followed by a presentation of the work stemming from Harrison and Millard’s (1991) paper. Finally, the non-journal literature will be considered.

2.2 Australian literature reviews

In this section comment is provided on Australian literature reviews relating to hospital bed management (or patient flow). While literature reviews may not be frequently published, consideration of the literature reviews on the topic of bed management that have emanated from Australia is important in gaining an insight into
what Australian researchers and health departments consider important in relation to this topic. Literature reviews emanating from elsewhere are considered in other sections of this chapter.

In recent years, there have been three literature reviews relating to hospital bed management published by Australian authors, namely the work of Dwyer and Jackson (2001); Anderson, Bernath, Davies, Greene and Ludolf (2001); and Goddard and Mills (2003). Other reviews may exist, but were not discovered. It is likely that any such reviews would have been written for specific use within health care organisations (otherwise they would have been discovered) and were not intended to be made available to the general public, which is not an uncommon business practice.

2.2.1 Review by Dwyer and Jackson

The literature review by Dwyer and Jackson (2001) was commissioned by the Patient Management Task Force (Task Force) of the Victorian Department of Human Services. The Task Force was established in response to acknowledged difficulties in meeting the demand for acute inpatient care, particularly in winter. The Task Force was required to identify essential organisational and patient management practices that would maximise the ability of hospitals to respond to the demand for inpatient care. The review was commissioned to provide information about integrated bed and patient management to the Task Force. Given the terms of reference provided to the Task Force, the literature review reflected the published evidence relating to managing demand, improving throughput and balancing the system (that is, removing bottlenecks). Additionally, the authors also provided high-level policy advice that stemmed from their review to the Department of Human Services.
The review focussed on strategies for managing patients or work practices, rather than looking at mechanisms that would lead to better understanding of the existing problems. Consequently there was no reference to the benefits that could be derived from the application of modelling (or other analytical approaches) to the issue of bed management problems. Acknowledgment of the need for better data to facilitate management of emergency patients, however, was made.

This review encapsulates my experience of the health sector management approach, which is to identify a problem, look for a solution (often elsewhere) and implement it. Little emphasis was placed upon understanding the problem through beyond what would be considered simple analysis and the need for, or recognition of benefit of, undertaking more advance analysis or modelling did not occur.

### 2.2.2 Review by Anderson et al.

This review was published by the Centre for Clinical Effectiveness, Monash Institute of Public Health (Anderson et al., 2001), which is based in Melbourne (Australia). It occurred at a time when recognition of bed management problems had led to the creation of the Patient Management Task Force of the Victorian Department of Human Services, a group who were given responsibility for the identification of possible solutions to hospital management problems, particularly concerning access to hospitals. The stated aims of the review were to summarise and assess the literature relating to integrated bed and patient management, and identify pertinent issues for the Victorian health system.
The review was not particularly concerned with the application or use of modelling, although it was noted that some studies had confirmed that high occupancy levels contributed to bed crises, and that demand and supply issues could be modelled. Rather, the review concentrated on the identification of research relating to methods that may ameliorate hospital bed management problems, including demand management techniques, hospital in the home and appropriateness of admission.

As with the review by Dwyer and Jackson (2001), little emphasis was placed upon improving understanding of the problem through the use of modelling.

2.2.3 Review by Goddard and Mills

The literature review by Goddard and Mills (2003) arose as a consequence of a mathematician’s (Mills) involvement with a regional health service (Bendigo Health Care Group) and was published as an academic article. The stated aim of their paper was to alert mathematicians that they could usefully contribute to the task of better managing hospital beds. Thus, the intended audience and purpose were quite different from that of the review undertaken by Dwyer and Jackson (2001), and Anderson et al. (2001). The review commented upon the increasing congestion in Victorian public hospitals and provided illustration of how mathematics can be applied to gain an improved understanding of this congestion. Three approaches to investigating bed management issues were reviewed, namely the use of simulation, queueing and compartmental flow models. Goddard and Mills (2003) concluded that mathematics could contribute useful insights into bed management problems, but that the problems were complex and could not be solved by mathematicians alone.
This review did provide readers with an appreciation of how analysis and the application of modelling may lead to better understanding and solutions to health system management problems. However, it is unlikely to be read by many managers or analysts in the health care, because it was published in a highly specialised non-health related journal.

2.2.4 Conclusions about Australian literature reviews

The Australian literature reviews provide evidence that hospital bed management issues do indeed represent an issue about which there is current concern. Furthermore, given that some of these reviews were commissioned, it would appear that management is keen to base future decisions upon the published literature – at least to some extent. The reviews, however, indicate that much of the focus is based upon pre-existing solutions, such as demand management, and that the specialised field of modelling has received scant attention and continues to represent an area of interest to a niche group that are not necessarily main stream health sector players.

2.3 General hospital bed modelling techniques

Published research papers represented the primary source of information about modelling and associated methodologies concerning hospital bed management, planning and forecasting for this research. The investigation and reporting of hospital bed use is not a new subject, with the earliest paper I have noted being published by Benjamin and Perkins (1961) who reported upon the measurement of bed use and demand (cited by Barber and Johnson, 1973).
Apart from the fact that this field of research is very small in comparison to the field of health research in general, the difficulty in identifying relevant research papers was made more difficult due to the fact that publication occurs infrequently and in a diverse range of journals. In this section I present research findings that span more than 30 years and represent a range of approaches to modelling hospital bed issues. The approaches have been classified as belonging the following categories:

- Simple one-off approaches
- More complex one-off approaches
- Queueing models
- Simulation models, and
- Flow models.

These categories are not mutually exclusive and allocation was subjectively determined. Table 2 indicates the membership of papers to the various categories, with the principal category being highlighted.
Although the list of papers is not exhaustive, it is informative to note that prior to 1987 more complex studies were generally not performed. The expansion into more complex areas of analysis appears to coincide with an increase in the availability of the personal computer and an increase in computing power. It is also important to note that the publication of research that involved “simple” methods did not cease with the rise in publication of more complex methodologies, but has continued throughout the period of review. This is perhaps reflective of the divide between academic study of the health system and application of more simplistic methodologies in practice.

Table 2: Categorisation of research papers. Where papers could have been assigned to more than one category, the capitalised X indicates the primary category.

<table>
<thead>
<tr>
<th>Author/s</th>
<th>Year</th>
<th>Simple one-off approaches</th>
<th>More complex one-off approaches</th>
<th>Queueing Modelling</th>
<th>Simulation Modelling</th>
<th>Flow Modelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barber and Johnson</td>
<td>1973</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweeney and Ashley</td>
<td>1981</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bay and Nestman</td>
<td>1984</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sterk Shryock</td>
<td>1987</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worthington</td>
<td>1987</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pendergast and Vogel</td>
<td>1988</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Farmer and Emami</td>
<td>1990</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Worthington</td>
<td>1991</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Huang</td>
<td>1995</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Myeng-Ki</td>
<td>1995</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vissers</td>
<td>1995</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ahnel and Ulaj</td>
<td>1996</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sorensen</td>
<td>1996</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Milner</td>
<td>1997</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Huang</td>
<td>1998</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Law</td>
<td>1998</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bagast, Place and Posnett</td>
<td>1999</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fullerton and Crawford</td>
<td>1999</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coné and Stein</td>
<td>2000</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green and Nguyen</td>
<td>2001</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tousaint, Hereng, Gillis and Kohler</td>
<td>2001</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gorunescu, McClean and Millard</td>
<td>2002</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Isken and Bajuyogen</td>
<td>2002</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jones, Joy and Pearson</td>
<td>2002</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finaerelli and Johnson</td>
<td>2004</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latimer, Brailsford, Turnbull et al.</td>
<td>2004</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Myers and Green</td>
<td>2004</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nguyen, Six, Antoniouli et al.</td>
<td>2005</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Akcal, Coné and Lin</td>
<td>2006</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cochran and Bharti</td>
<td>2006</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fusco, Saito, Arcà and Penaci</td>
<td>2006</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.3.1 Simple Methods

Barber and Johnson (1973) identified that the range of statistics available for hospital activity that could form the basis of reporting to management was diverse. This diversity enabled alternative measures to be reported that were not consistent and could be used to suit the needs of the individual or unit, as opposed to those of the overall organisation. In relation to the management of hospital beds they proposed a graphical representation of the average patient length of stay (Y Axis) and patient turn-over interval (X Axis) that could be prepared at the unit or hospital level and tracked over time. Additionally, the effects of modifying patient length of stay or turn-over interval on the number of patient discharges or occupancy could be visually determined as lines of patient discharge numbers and bed occupancy levels were included on the graph. This approach, while combining a number of simple measures in a meaningful way has two drawbacks:

1. It relies upon the average length of stay, which is a flawed measure, and
2. The visual interpretation of the results is not straightforward (although training in the technique would overcome this drawback).

Given the era when this approach was advocated, it perhaps, represents a reasonable attempt to capture the effects of decision-making on hospital bed management. However, its use does not appear to have gained traction and the individual reporting of patient turnover and length of stay measures continues to occur.

A number of authors (Sweeney and Ashley, 1981; Bay and Nestman, 1984; Toussaint, Herengt, Gillois and Kohler, 2001) have reported on methods of looking at bed numbers that are based upon consideration of the population. Although these methods
incorporate consideration of population changes they rely upon simplistic measures of either bed occupancy, length of stay or a predefined level of beds per thousand head of population. As discussed in Chapter 1, the ALOS is highly skewed and therefore a poor measure (for example, Farmer and Emami, 1990; Harrison and Millard, 1991; Mackay and Millard, 1999; Costa et al., 2003). Thus, such simple approaches will not lead to improved decision-making. Setting a defined number of beds per thousand head of population relies upon a policy decision at some level within the system, for which no basis is obvious and thus cannot be said to rely upon a robust method.

Toussaint et al. (2001) also report on a method of setting a target level of occupancy. However, this methodology relies upon use of the ALOS and thus, while simple and trying to achieve an outcome that provides a mechanism to deal with daily variations in occupancy, is also flawed.

Sterk and Shryock (1987) suggested the use of a simple linear regression model of patient days to achieve improved financial outcomes in hospitals. The approach, while simple, does not facilitate planning based upon changes in the rate of patient flow or policy decision (for example, decisions to reduce the number of beds), and thus does not confer much advantage to those seeking to investigate bed requirements in a more thorough manner.

Finarelli and Johnson (2004) also consider linkage of simple bed occupancy statistics to population changes in the nine-step methodology they propose for determining bed requirements. However, they also include a supply-side component in their modelling. While the inclusion of a supply-side element is crucial (that is, there is little point planning for beds if the necessary staff are not available), the modelling is still based
upon use of the ALOS and thus is flawed. Myers and Green (2004) suggested a similar approach to that of Finarelli and Johnson (2004), although it was described as a two-step approach with the differentiation being modification of the forecast (for beds) by consideration of future changes (for example, changes in technology). Their approach is perhaps better described as a traditional simplistic methodology modified by inclusion of judgment about future events.

Nguyen, Six, Antonioli et al. (2005) suggested that achieving a minimum score on the means and standard deviations of ratios developed based on the number of unoccupied beds, patient transfers (due to full capacity) and capacity to deal with unplanned admissions enables determination of bed requirements. The authors claim that the method overcomes the flaws with the ALOS, but is only of use in assessing current, and not future, bed requirements and thus is of limited value for strategic planning purposes where changes in policy, population and other factors must be considered and analysed.

**Simple Methods - Summary**

It is interesting to find that the simple approaches to modelling hospital bed issues have continued to be developed and promulgated in spite of the availability of more powerful computers and better data collections that often underpin the development of more complex modelling.
2.3.2 More Complex One-off Approaches

Various researchers have undertaken the development of more complex models for forecasting hospital bed requirements. In many instances, it would appear that the work has been undertaken on a one-off basis as continued publication on the topic by the same author or authors has not been noted.

Farmer and Emami (1990), Myeng-Ki (1995), and Milner (1997) have all reported on research efforts involving auto-regressive inductive moving average (ARIMA) modelling. Farmer and Emami (1990) compared forecasts of ALOS made using simple approaches to that made using ARIMA. While they noted that the ALOS was a flawed measure they elected to use it for their work and found that forecasts could be improved by adopting ARIMA in preference to simpler methods, such as simple linear regression. Milner (1997) looked at forecasting patient attendances and found that ARIMA forecasts resulted in improved forecasts compared to simpler methods. Myeng-Ki (1995) used ARIMA to create short-term forecasts of bed occupancy. The work was development and although linked to the possibility of assisting management does not appear to have proceeded further. Jones, Joy and Pearson (2002) have undertaken similar, but have favoured in the generalised autoregressive conditional hetroskedasticity (GARCH) method in preference to the ARIMA model, because they contend it is better suited to modelling data with periods of high volatility which are followed by periods of less volatility. They also reported using the seasonal ARIMA (SARIMA) model. They found that they could predict bed occupancy, but not admissions, using SARIMA. The focus of their work was aimed at predicting crises.
and thus it had a more operational focus. Operational\textsuperscript{1} models are not suited for strategic decision-making purposes. An interesting conclusion reached by these researchers was that the success of the modelling should be gauged not by statistical measures, but rather by how it influenced patient care. Despite the improvements in forecasting gained through the application of ARIMA models, the ability to manipulate the model output to adjust for policy decisions (for example, changes in bed numbers) or changes in patient flow is limited (particularly when the ALOS is the basis of the modelling. Thus, alternative models are required.

Law (1998) reported on the application of neural networks for forecasting hotel room occupancy rates. It can be argued that a hospital is a hotel that has clinical services appended to it (for example, the provision of a bed, catering and cleaning are functions that occur in both institutions). Like hospital managers, hotel managers are also concerned about bed occupancy. However, the motivating factors are different. The aim of the hotel supplier is to ensure that investment in the provision of additional hotel beds only occurs if a minimum occupancy is reasonably certain. If demand exceeds supply this is not a problem for the hotelier, but rather a signal that investment in additional capital will be beneficial.

Law (1998) states that the superior pattern recognition capabilities of neural networking make it suitable for forecasting and that the methodology is superior to traditional statistical methods. Law found that the neural network model of room occupancy was more accurate than naïve extrapolation or multiple regression, two

\textsuperscript{1} The term “tactical” is often used in conjunction and relates to the development of the terminology in association with the military. However, the term “operational” is also appropriate and is perhaps more commonly used in health environments. For example, the local Department of Health (South Australia) has “strategic” and “operational” divisions and not a “tactical” division.
commonly used forecasting methods in the hotel industry, and concluded that the application of the approach would result in better decision-making. The potential for application in the health sector would appear to exist. However, neural networking can be viewed as a “black box” method and this can represent an impediment to implementation.

Fullerton and Crawford (1999) applied the method of Cosinor Rhythmometry to the analysis of hospital bed occupancy. The Cosinor Rhythmometry method enables the testing of the hypothesis that a seasonal sinusoidal curve exists. They analysed data based upon specialty grouping and found that General Medicine had a significant seasonal effect that explained much of the variation in bed occupancy. They concluded that the seasonal effect was predictable and that strategies could be adopted to ameliorate the effects of the winter peak in demand for beds. The approach, however, only confirms the presence of a seasonal effect and therefore is limited in its use in terms of strategic hospital bed decision-making.

Fusco, Saitto, Arcà and Peruci (2006) considered the effects of influenza outbreaks on hospital bed occupancy in Rome using Gaussian generalised additive models. It was reported that during the influenza season, influenza bed occupancy increased and the level of increase depended upon the specialty area. Daily and seasonal variation in occupancy was found. According to the Fusco et al. (2006) influenza bed crises were more a factor of an unresponsive system unable to exploit fully its resources rather than excessive demand. While informative in terms of identifying daily and seasonal trends in influenza related bed occupancy, the approach did not provide decision-makers with the tools to comprehensively plan, predict or test the system. At best, the
analysis might aid planning, but only in a general sense. The description of management as being “ineffective” (in terms of being able to allocate available beds) may result in rejection of the findings by hospital managers on the basis that the study did not identify the complexities that occur in a hospital environment (a commonly used strategy adopted when defending management of hospitals).

More Complex One-off Approaches - Summary

While the more complex approaches to studying hospital bed issues have yielded some interesting findings, these approaches, with the exception of the neural networking (which was not applied to hospital data), appear to offer decision-makers limited insights into hospital bed management and are therefore unlikely to be attractive to decision-makers.

2.3.3 Queueing Models

Hospitals and health services in general can be viewed in similar ways to other service provision industries, such as banks, shops or airlines, that is, there are customers (in this case called patients) who seek some kind of service and in order to receive the service, the customers must join queues. An admission to a hospital may see a patient join multiple queues, including a queue to get a bed, have diagnostic tests performed and analysed, and undergo surgery. Indeed, undergoing surgery may involve the patient joining multiple queues: waiting to see the anaesthetist to ensure that the patient is healthy enough to undergo the surgery, waiting to be prepared for theatre, waiting for someone to transport them to theatre, waiting to be moved into the theatre, waiting for biopsy results during surgery, waiting in recovery post theatre and waiting
Queueing theory has been applied to the analysis of resource problems when queues occur (Denardo, 2002). For example, how many bank tellers are required so that customers do not wait more than x minutes represents a typical problem that might be considered. A natural extension of the application of queueing theory extends to the analysis of health care queues. Often such modelling relates to the queues found in outpatients, emergency departments, but also hospital waiting lists. Queueing modelling is often combined with simulation and thus is not a mutually exclusive category of modelling. Examples of the application of queueing modelling to the analysis and development of solutions to hospital bed issues are now presented.

Worthington (1987 and 1991) applied queueing models to the analysis of hospital waiting-list problems. Application of the $M(\lambda_q)/G/S$ queueing model was undertaken in the work reported in 1987, where arrivals occur at a random rate $\lambda_q$, there are $q$ patients in the waiting list and there are $S$ servers. A server is the place where the customer receives the service – in this case it is the hospital bed, where the service time is the patient length of stay. Worthington (1987) described this type of queue as being a simpler version of the machine-interface problem. Using the models developed it was possible to illustrate the length of period patients would remain on waiting lists, the effect of altering the number of available beds and decreasing the patient length of stay. An important observation was that success in reducing a waiting list could be met with increased demand as a consequence of a recommendation by the College of Health that patients “shop” for the shortest queues. Positive feedback was considered likely to result in the original queue being re-
instated and no long-term benefit being attained from addressing the original waiting list problem.

Worthington’s second analysis (1991) reported work carried out with the Lancaster District Health Authority where queueing theory was applied to investigate waiting lists and included what-if analysis. The modelling incorporated the transition of patients through general practice to outpatients to inpatient and then back to general practice (although this may have included a transition through outpatients). Attempts were made to capture other features in the process, such as the classification of patients into different waiting list streams, such as routine or urgent. Worthington (1991) found, however, that the data were not uniformly collected making the modelling exercise difficult. The actual methodology was based upon system dynamics methodology, although the formulae were not stated. System dynamics software was not, however, used for the modelling exercise, but was performed using the more commonly available Microsoft Excel spreadsheet program. Projected waiting lists and waiting times were developed as the model output. Worthington (1991) reported that the use of the tool was very much textbook like and involved decision-makers in the development. Use of the tool achieved additional beds for one hospital consultant, but did not result in additional bed allocations for other consultants due to other constraints. Where consultants did not secure additional beds, the modelling was still deemed successful as it was used to highlight issues requiring action, such as improving the transition flow between inpatients and outpatients. Continued use of the spreadsheet-based model was not recommended due to the skills required to modify it for each scenario and a specific application was developed for generic use. While the achievement of additional bed allocations, identification of
related issues and development of a decision-support tool were beneficial outcomes from the research conducted by Worthington (1991), the lack of specificity around the mechanics of the modelling approach has masked much of the work, thus making independent replication or broader independent transfer difficult.

The application of queueing models was presented by Huang (1995) as an intermediate method that overcame the problems of using the ALOS and other simple measures in determining hospital bed requirements, and as being less onerous than using more complex methods such as simulation. The modelling extended previous work by Pike, Proctor and Wyllie (1963) to include a day of week effect and used the $M/G/\infty$ queueing model (unlimited beds) to investigate bed requirements. Consideration of other constraints – such as resource limitations – was not taken into account. While for analytical purposes, the approach appears reasonable, care in applying the approach when beds are limited, would be required. The results were found to be congruent with those obtained from a Monte Carlo simulation model. Huang (1995) concluded that once the number of beds was calculated using the queueing model, other performance measures could also be determined (for example, daily bed occupancy). Although the insights gained from the approach were useful, the need for appropriate training in implementation of the model and also the need to incorporate realistic constraints (such as a fixed number of beds) would likely deter use of these findings in a real decision-making environment.

Green and Nguyen (2001) investigated the potential impact of cost-cutting strategies on the delay for hospital beds using a queueing model. Additionally, the study was used to examine desirable occupancy levels, provide insights on reducing patient LOS
and consider the effects of demand variability. The majority of analysis performed by Green and Nguyen (2001) relied upon the application of M/M/s queueing models and this was justified on the basis of robustness, ease of use and wide application in industry. The analysis was conducted in the USA and used data from a surgical and obstetrics unit. The authors reported that for obstetric and surgical units of similar size and occupancy levels, the probability of delays in admissions were quite different. Similarly, combining small surgical units resulted in reductions in the overall number of beds, while there was little scope for bed reduction by combining large surgical units. Additionally they reported that efforts in attempting to reduce variation in LOS were not well rewarded in terms of the reduction in the number of required hospital beds when compared to the effects of reducing the ALOS. According to the authors the insights provided by the analysis were consistent with what was already known about queueing theory from application in other industries. The authors demonstrated that the application of queueing theory might provide useful insights for managers when considering how resources should be allocated (for example, combining units) or how particular efficiencies may be gained. The paper, however, focussed on how queueing theory may be applied. There was no reported intent to introduce the use of the modelling for decision-making purposes.

Goranescu, McClean and Millard (2002) applied queueing theory to describe the movement of patients through a hospital department and presented a means of optimising the number of beds required in order to meet specified delay. Additionally, they considered the need to balance the costs of empty beds against delays in admission. Previous research on compartmental flow modelling undertaken by McClean and Millard (and others – see Section 2.4 for more details) provided the
means of describing the patient stay. Compartmental flow models belong to a class of models known as phase-type distributions. A Poisson distribution was used to describe the arrival rate of patients and the number of beds was fixed, giving rise to a M/PH/c queueing model, where M denotes the Poisson arrivals, PH denotes the phase-type distribution of the patient length of stay and c denotes the number of beds (or servers in queueing language). The notion of patients being lost to the system due to insufficient beds was stated as not necessarily reflecting the observed practice, as patients will be admitted into other wards (overflow) or held in the emergency department and generally will not be turned away, as might occur in a cinema.

The base-stock policy, which according to the authors is used in inventory systems to determine base stock levels, was applied to determine the number of beds that would satisfy the need to balance unmet demand with the costs of unoccupied beds. The authors were able to illustrate that as the percentage bed occupancy increased, the number of patients turned-away (or forced to wait for a bed, or be transferred elsewhere) increased, which was consistent with known experience and other research (for example, Bagust, Place and Posnett, 1999). Additionally, they were able to illustrate that managers may be indifferent to decisions that suggest the provision of hospital bed numbers that will meet reasonable patient turn-away number (say up to three per cent of patients) outcomes and cost penalties. The provision of a sufficient number of hospital beds to avoid more than three per cent patient turn-away, however, was associated with an increased cost penalty. The cost modelling did not include the potential for different cost or patient outcomes should patients be admitted, but overflowed to a different part of the hospital, even though they recognised that overflow had the potential for such undesirable outcomes. As with some of the other queueing papers, while the authors suggested an application of queueing theory to aid
management decision-making, there was no evidence of the approach having being implemented. Furthermore, while the paper provides those managers keen to use the approach with sufficient methodological information to do so, it is likely that this is well beyond the capabilities of most people without either the development of a decision-support tool or the use of consultants to develop the requisite model.

**Queueing Theory - Summary**

While the application of queueing theory to investigate hospital bed management decision options appears to be able to offer useful insights to those seeking to improve decision-making, the actual uptake of the research appears poor. It is evident from the literature that there are numerous challenges to be overcome, including providing the appropriate tools for decision-makers to use this form of modelling and the need to address the reasons for variation in the modelling approaches illustrated. For example, each paper reviewed has applied different assumptions and queueing models, but with no reason as to why the given model should be chosen in preference to others already illustrated in the literature.

**2.3.4 Simulation**

According to Law and Kelton (1991) simulation is often used to model complex systems simply because the mathematical solution that could be developed is in itself complex and it is often difficult to develop an analytical solution.

A simple definition of simulation analysis is that it is undertaken by looking at the model performance results generated when the model is tested using a variety of input values (Law and Kelton, 1991). There are a variety of simulation approaches
including static and dynamic simulation. Given the complexity of health systems, it is not surprising that simulation analysis has been applied to the study of hospital bed occupancy problems.

Simulation modelling as a topic heading is not mutually exclusive. For example, queueing models and flow models have been combined with simulation analysis. The work of El-Darzi, Vasilakis, Chaussalet and Millard (1998) combines simulation, queueing and compartmental flow models and is referred to in the section on compartmental flow models (Section 2.4).

Altinel and Ulaş (1996) applied simulation analysis to study the bed requirements for the Emergency Surgical Department at the School of Medicine in Istanbul. The history of the Department was that it had experienced heavy demand and the simulation model was created to assist in the planning of the service to facilitate patient flow and determine bed numbers. The hospital system and processes were modelled using SLAM-II simulation software. The model was a dynamic simulation model. The model was used to test a number of plans for change at the hospital and found to be useful in guiding the configuration of the arrangement of the various clinics and bed numbers. However, creating a model that represented the hospital’s processes was found to be challenging and the lack of easily accessible data was found to be problematic.

Huang (1998) similarly applied dynamic simulation analysis as part of a re-engineering process to determine the allocation of beds in order to minimise patient over-flows at the Plymouth Hospital NHS Trust in the United Kingdom. The goals of
this project included achieving a reduction in general medical emergency admissions and a reduction in the ALOS by one day in order to fund the remaining goals of the project. Huang (1998) acknowledged that simulation was used in preference to attempting to solve the complex and difficult mathematics required to describe the hospital system. An interesting approach to achieving the required reduction in ALOS was to seek the reduction only by targeting those patients with long lengths of stays (defined as bed blockers). While a valid approach in terms of achieving LOS reductions, the cost savings profiles of bed blockers and the remainder of the patient population are not likely to be the same. The author did not state whether the required savings were successfully achieved. However, the model was used to test a variety of scenarios, particularly around patient over-flow. The author concluded that influencing decision-making to achieve better decisions was an appropriate goal for modellers.

The study by Bagust, Place and Posnett (1999) has received much attention as it validated a commonly held belief that an occupancy level of 85 per cent was appropriate, because higher levels of occupancy were associated with discernible levels of risk of bed crises. The study was based upon a discrete-event stochastic simulation model that reflected the relation between available bed capacity and demand. The statistics used to create the model were derived from the National Health Service (United Kingdom), but did not relate to an individual hospital. This represents the study’s strength and weakness – it was a hypothetical study designed to stimulate debate around hospital bed numbers, but it could not be validated against any particular hospital data. The lack of validation means that the particular occupancy
levels found to give rise to bed crises may not be relevant to an individual hospital. This can become important when such studies influence general policy.

Isken and Rajagopalan (2002) identified that simulation has a role to play in planning inpatient bed capacity, but that it is often difficult to determine the basis for creating patient groupings to use in a simulation model. Different patient types, for example cardiac patients and general medical patients, require different resource allocations (including bed occupancy). In order to develop a model that reflects reality it is necessary to create patient categories. However, creating a large number of patient categories can introduce too much complexity and make model validation difficult. Discrete-event stochastic modelling was used. They proposed data mining clustering techniques such as K-means as a way of determining appropriate patient groupings for use in simulation modelling. This paper demonstrated that methodological issues are still being considered in relation to hospital bed capacity simulation modelling.

Lattimer, Brailsford, Turnbull, et al. (2004) described the creation of a system dynamics model to describe an emergency and urgent care system within a health authority in the United Kingdom. System dynamic models represent a particular branch of modelling that stems from the work of J Forrester (1961) in the United States. The modelling is different from other forms of simulation modelling in so far as it emphasizes feedback behaviour. This study identifies operational issues, such as bottlenecks in patient flow as a consequence of high occupancy levels and also illustrates the application of a system dynamic model in a health setting. The model incorporated aspects beyond the hospital boundary, such as primary care, and thus is more indicative of a system. The authors did acknowledge that the provision of data,
particularly from primary care providers, limited the development of the model. They also acknowledged that the findings were unlikely to apply to other studies. The limitations highlight the downsides of simulation models: detailed simulation models are unlikely to be generalisable, and creating models of the wider health system (that is, greater than the hospital) will be difficult in Australia where much of primary care service is provided by small private businesses which increases the difficulty in capturing the necessary information required for such model development.

Cochran and Bharti (2006) applied discrete event simulation and queueing analysis to study the effects of increasing patient numbers on bed requirements in an obstetric hospital in the United States of America. The authors indicate that the modelling exercise was undertaken to help the owner of the hospital “organise the mess” or understand the complexity of the obstetric service. They found that using both queueing and simulation analysis was beneficial in order to attain the understanding that they required around bed utilisation. Data limitations were again identified as a problem in developing a simulation model. The study represents another typical application of simulation modelling.

**Simulation Modelling - Summary**

In general, the use of simulation modelling appears to enable decision-makers greater flexibility in gaining understanding of the system that is being examined and also the potential to test changes to the system when compared to the previously discussed approaches. However, the application of these models tends to be more operationally focussed, often requires significant additional data capture and can require significant development time. Additionally, the models are not generalisable. Thus, while there is
potential benefit to be gained from such models, as compared to the simple and more complex models, the development cost, data requirements and operational focus are likely to make such models unattractive for strategic bed management decision-making. The range of potential model types – for example, discrete-event, system dynamics, static – is also likely to hinder method selection by health care service managers, who are often not well trained in the possible approaches that can be applied to investigating hospital bed issues.

2.3.5 Flow Models

Flow models represent those models that aim to capture aspects of patient flow through some part of the hospital system. This class of model captures a range of work, although the word “flow” may have different meanings to different researchers. There is potential overlap with simulation and queueing modelling.

Pendergast and Vogel (1988) proposed a multistage model of hospital bed requirements. The model was designed at a time when excessively long hospital stays were suggested to be associated with hospital cost increases. Clinical judgment was used to determine the clinically meaningful phases of care. Basic probability theory was applied to determine the likelihood of transition from one to another. The authors claim that the approach relied upon standard operational research methodology and provided a sound basis for planning decisions around hospital beds. The model was applied to psychiatric patient data, where three phases of stay were identified, namely the acute, extended and long-term intensive stay. Ten possible paths through the various phases were identified. Patients could be discharged from any phase. Results were also presented in relation to medical and surgical patient data. There were only
two phases (acute and extended) for medical and surgical patients. According to the authors, the methodology represented a digression from other more complex approaches, such as simulation and queueing. The methodology was designed to identify patients in bed for non-hospital care, which is important when trying to quantify the level of hospital care provided to patients who should have received this care elsewhere, but could not (for example, in the case where discharge to another facility was delayed by that facility and has nothing to do with hospital decisions). The model also incorporated the population figures for the hospital catchment, which facilitated bed planning. While the method is relatively simple, it did rely upon the collection of specific data and also required the creation of phases based upon clinical judgment. Having the phases relate to clinical practice was considered to be the strength of the approach, as the authors believed the approach could be implemented widely with ease and reflect local clinical and organisational practice. The model, however, relied upon the average length of stay. The average length of stay is usually highly skewed and therefore a poor measure (for example, Farmer and Emami, 1990; Harrison and Millard, 1991; Mackay and Millard, 1999; Costa et al., 2003). Reliance upon the ALOS suggests that the value of the resultant model output will be considerably weakened.

Vissers (1995) considered the consequences of patient flow on the production capacity of Dutch hospitals. He found that the current practices of resource allocation resulted in peaks and troughs in the workload cycle of departments and in his opinion this led to a loss of capacity. Additionally, resource allocation practices resulted in departments competing for the same resource at times of peak activity, resulting in bottlenecks and thus leading to further inefficiencies. As part of Vissers’s (1995)
research a set of five computer models were developed to support resource allocation
decision-making. The models enabled the visualisation of patient flows and study of
consequences of different resource allocations decisions. The modelling relied upon
an industrial production control framework that was modified for the hospital setting.
The intent of the application of the model was to focus on the control of co-ordination
mechanisms between design, flow and clinical services to maximise output with the
available resources. The approach involved the development of a number of linked
models, with the patient flow component being treated as a low-level (operational)
component of the overall set of models. Vissers (1995) stated that simplistic models
were purposely developed to facilitate involvement of hospital staff and optimisation
of particular model outputs was not undertaken in order to achieve the desired level of
simplicity. Insufficient detail was provided to determine the methodology employed,
beyond noting the use of a production control framework. Thus, it is not clear how
patient flow was modelled and whether reliance was placed upon the ALOS or not.
According to Vissers (1995) the models were useful in acting as a catalyst for the
development of resource allocation solutions. While Vissers (1995) suggested that the
models could be used for both strategic and operational purposes, the results suggest
the main application was for operational purposes. For example, the models were used
to redesign surgeon timetables and improve use of x-ray facilities. While the reported
outcomes appear to support the use of the modelling, the lack of transparency
surrounding the exact model design was a significant oversight in the paper reducing
the likelihood of others being able to replicate the approach.

Sorenson (1996) continued to develop the approach advocated by Pendergast and
Vogel (1988). Sorenson stated that the specific purpose of the model was to assess the
effects of reducing the patient length of stay on the requirement and use of hospital beds. It was recommended that the approach be used in preference to more complex methods and only replaced by such alternative approaches when the model could not be used to analyse a particular bed issue. Unlike the Pendergast and Vogel (1988) approach, Sorenson (1996) subjectively determined the number of patient phases and the time spent in each phase. Sorenson (1996) created four patient phases of stay: same-day, short, extended and long-stay and applied these to medical, surgical and gynaecological patient data. For each phase, patients could be discharged home, to another institution or die. The focus on discharge destination is useful when considering issues relating to bed blockage that arise as a factor external to the hospital (for example, a lack of timely access to nursing home beds). While the model was easily created and could be implemented in a spreadsheet, a commonly available tool, it again relied upon the average length of stay as a key measure in the model, even though the length of stay profiles reported in the publication were clearly not Gaussian distributions. Thus, the model output will also be flawed.

Côté and Stein (2000) developed an Erlang-based stochastic model for describing patient flow. The authors described how patients could flow from one state, or phase, to another and that such flow represents a semi-Markov process. The authors developed theoretical justification for the adoption of the Erlang-based model, unlike many other authors who merely describe the application of a particular model to solve a given problem. Notably, the authors found that the skewed LOS distribution could be well described with exponential models. The models were used to study the flow of leukaemia and coronary patients. The authors spent considerable time justifying the approach. They noted several requirements that would preclude the use of the
approach, particularly when patients were found to relapse frequently and return to prior phases of treatment, and when the data was exceptionally different to the Erlang distribution. However, they stated that the patient data used in their work was representative of most patient flow and thus they could not foresee why the approach should not be used. They did concede that simulation was an alternative option, but the analytical model was superior, particularly in relation to what-if analysis. Given the journal the article appeared in, it is highly unlikely that intended audience were clinicians, managers or other professionals who may be charged with decision-making in the health sector. Thus, while the analysis presented by the authors was rigorous, it showed no sign of application and would more than likely be overlooked by applied decision-makers searching the literature for improved decision-making tools. This work represents an independent and parallel development of the flow model approaches fostered by Millard and his colleagues (see section 2.4 for more details), with the exception that the work relied upon acute patient data and not geriatric patient data, and lacked real world application. One point of divergence between the work of Millard and his colleagues and that of Côté and Stein (2000) is that the latter authors’ work included backwards patient flow (that is, flow to a previous state).

Akcali, Côté and Lin (2006) reported that despite the increasing number of services that could be accessed in an outpatient setting and a restriction on health care payment reimbursements, inpatient activity had grown in the United States. Prior to this activity growth, hospitals had undergone an extensive period of consolidation and contraction. The authors suggested that the increased inpatient growth and diminished resources has made hospital bed planning difficult. They examined how a network flow model could be applied to determine optimal hospital bed capacity. The model
developed was a large-scale non-linear integer optimisation model designed to
minimise cost while achieving targeted performance levels. The initial model
developed was modified to become a binary integer-programming problem in an
try to recognise that decisions about expansion or contraction of hospital beds
usually relates to a particular a quantum of beds (that is, a ward) as opposed to
opening one or two extra beds. The model was thus constrained to a single choice of
the same, more or less beds per period. In the study the quantum of beds that could be
opened or closed was 25 and the original number of beds opened was 300. A network
of options was formed by considering the consequences of options over a number of
periods. The use of flow is different from other “flow” models insofar as that the flow
is a balance of the number of beds in a previous period plus or minus changes to bed
capacity in the current period. The model was implemented using C++ programming
language. There is no evidence of this model being adopted in a practical sense and
the examples provided appear to be hypothetical. While the authors have identified
some key issues in relation to the allocation of beds, this model appears to be in the
early stages of development and further investigation is required before it could be
adopted in a real decision-making setting. Furthermore, it represents a flow of
decisions and does not relate to the flow of patients. This provides an illustration of
how the use of language in relation to bed modelling is not yet well developed.

Flow Models - Summary

Flow modelling represents a diverse range of work that shares some similarity insofar
as a notion of “flow” is being modelled. The meaning assigned to the term flow,
however, is not yet well defined and varies widely. In some instances there has been
reliance upon the average length of stay as a measure of patient flow, which is known
to be a poor measure, and in other instances the methodology is not clear. Both of these factors affect the credibility of this genre of modelling. Furthermore, there appears to be little evidence of uptake of this modelling.

The mathematically tractable flow model proposed by Harrison and Millard (1991) is discussed in section 2.4.

### 2.3.6 Conclusion regarding the general research

Research relating to hospital bed management and patient flow has been evolving and is increasing in volume and sophistication. The research has occurred in various countries, indicating the pervasiveness of hospital bed management problems.

Although more sophisticated methodologies are continuing to be developed it is noteworthy that the simple methods have not been abandoned. The continuation of the development and promulgation of simplistic methodologies reflects a desire to approach the issue of hospital bed management problems in a manner that is easily understood by everyone, even though such approaches are typically reliant upon a poor measure of patient stay, namely the ALOS. It may also reflect the investment in education around health service research topics within the health sector and the need to provide “simple political” solutions to hospital bed management problems. The continuation of the development of simplistic approaches is unlikely to result in improved decision-making, particularly where reliance upon the ALOS occurs. However, the use of simplistic models may represent the evolutionary path required in order for consideration of more complex approaches to occur.
Notably many research efforts resulted in outcomes that only partially addressed the needs of decision-makers. The work of Fullerton and Crawford (1999) is perhaps a good example, where the presence of seasonal variation was confirmed, but no management tool was developed to aid decision-makers. Furthermore, in some instances it is not clear that the intended purpose of the reported research was to develop a model that was intended for actual use or whether it was to illustrate the use of a particular methodology that might be applied.

The development of divergent approaches may lead to several conclusions, including:

- There are various views of the hospital bed management problem that require different methods of investigation
- Researchers from different backgrounds (for example, economics, mathematics and medicine) have developed different approaches to investigating what are essentially the same types of problems
- There has been a lack of sufficient details provided in some publications (for example, Vissers, 1995) to enable other researchers to test given types of models advocated in research journals
- There has been a failure of any one approach (or even a small number of approaches) to gain traction as a solution (or solutions) to the hospital bed management problem(s)
- The research field is not yet mature as a topic in its own right and thus results in a the application of a variety of research approaches, and
- The differences in the provision of care between different countries or regions and the dynamic nature of the health systems means that new ways of investigating hospital bed management issues will continue to evolve.
Clearly, the above conclusions are not mutually exclusive and the current state of hospital bed management research is most likely a mixture of the above.

Griffith and Wellman (1979) reviewed bed management recommendations stemming from six studies of hospitals conducted in Michigan (USA) during 1975 and found that the forecast bed requirements were inaccurate. Fone et al. (2003) have found that modelling has not been evaluated in the health sector. The absence of reliable forecasting and evaluation of modelling efforts also will foster an environment where divergent research methodologies occur.

The issue of poor forecasting also may relate to the problem of over-fitting data when models are developed. As an issue, the over-fitting of data and the impact on model generalisability and forecasting have been absent in the literature that has been reviewed. For example, there has been no representation that models are not generalisable when the data is over-fitted. Indeed the transferability of models on the basis of one-off studies is perhaps questionable practice anyway. This aspect of modelling is discussed in more detail in Chapter 4.

The flow modelling approach as originally proposed by Harrison and Millard (1991) offered a way forward in the approach to tackling hospital bed management issues at the strategic level. The resultant model provided a closer fit to the length of stay distribution, thereby improving upon models that were reliant upon the ALOS and also provided output that is meaningful in terms of management decision-making. The compartmental flow modelling research literature is now discussed.
2.4 The Harrison and Millard compartmental flow model

In Chapter 1 (see Section 1.6), the seminal work of Harrison and Millard (1991), which highlighted the potential for compartmental flow models of hospital bed occupancy for strategic decision-making, was introduced.

As previously mentioned, the compartmental model describes the flow of something, such as patients, through a system, where the system is comprised of a finite number of homogeneous subsystems known as compartments (Godfrey, 1983). The wide application of compartmental flow models (Godfrey, 1983) as modelling solutions in diverse areas such as biomedicine, pharmacokinetics and ecology indicates a degree of robustness as a solution for analogous problems in different fields. Indeed, Harrison and Millard (1991) drew an analogy between the flow patients through hospital beds to pharmacokinetics in their work.

2.4.1 The Subsequent Research Effort

Since the publication of the 1991 article by Harrison and Millard, various researchers have continued the investigation on various aspects of compartmental flow modelling. It is informative to consider this work as it shows the course of development over a period of time, something that is not achieved when methods are only published as one-off pieces of work.

As the key clinical proponent of using compartmental flow models for improving decision-making, Millard has played an ongoing role in the research. He has
maintained an electronic record of publications stemming from the original work by Harrison and himself. Millard provided me with an electronic copy of this collection of references, which I reviewed. As a consequence of my review, I omitted a number of references that were not directly related to the field of research and also included several papers that were absent from Millard’s list. The references covered the period 1991 to 2005 (inclusive). The published material included papers published in journals, letters, editorials and conference proceedings. It did not include book chapters, working papers, reports, conference or other presentations, or dissertations. Additionally, it did not include material published in “Nosokinetic News”, an informal newsletter established by Millard in 2004. It is possible that the reviewed collection is not exhaustive, but it is expected that most works have been captured in this process.

I developed a simple classification system in order that a number of facets pertaining to the research could be highlighted. Given that much of the work has stemmed from data relating to a geriatric service from England and the focus of my research related to the application of the modelling approach to the acute care hospital sector, I chose to classify the research efforts based upon data type. Four categories were used:

- Older persons, covering research using data from a geriatric health service and also data relating to aged care services
- Acute care, covering research using data from acute care hospitals
- Psychiatric care, covering research using data from a psychiatric hospital service, and
• Not related, covering research where the data type was not relevant and this mainly related to theoretical papers.

It is acknowledged that the classification is simplistic and that other classification systems could be developed. However, the approach was developed to enable the reporting of some simple statistics and more complex approaches were not required for this task.

The list of publications stemming from Harrison and Millard’s (1991) research is shown in Table 3.
Table 3: Table of publications between 1991 and 2005 (inclusive) stemming from the original work by Harrison and Millard (1991).

<table>
<thead>
<tr>
<th>Year</th>
<th>Author/s</th>
<th>Year</th>
<th>Pub type</th>
<th>Title</th>
<th>Jnl</th>
<th>Vol</th>
<th>Issue</th>
<th>Pgs</th>
<th>Data Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>Harrison G; Millard P</td>
<td>1991</td>
<td>Journal</td>
<td>Balancing acute and long-term care: the mathematics of throughput in departments of geriatric medicine</td>
<td>Methods of Information in Medicine</td>
<td>30</td>
<td>3</td>
<td>221-228</td>
<td>older persons</td>
</tr>
<tr>
<td>1993</td>
<td>McClean S; Millard P</td>
<td>1993</td>
<td>Journal</td>
<td>Modelling in-patient bed usage behaviour in a department of geriatric medicine</td>
<td>Methods of Information in Medicine</td>
<td>32</td>
<td>79-81</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td>1993</td>
<td>McClean S; Millard P</td>
<td>1993</td>
<td>Journal</td>
<td>Patterns of length of stay after admission in geriatric medicine</td>
<td>The Statistician</td>
<td>42</td>
<td>263-274</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td>1993</td>
<td>Millard P</td>
<td>1993</td>
<td>Journal</td>
<td>The seven principles of planning geriatric medical services</td>
<td>Health and Hygiene</td>
<td>14</td>
<td>95-98</td>
<td></td>
<td>not relevant</td>
</tr>
<tr>
<td>1994</td>
<td>Taylor G; McClean S; Millard P</td>
<td>1994</td>
<td>Journal</td>
<td>Go with the flow: Modelling bed occupancy and patient flow through a geriatric department</td>
<td>OR Insight</td>
<td>7</td>
<td>3</td>
<td>2-4</td>
<td>older persons</td>
</tr>
<tr>
<td>1997</td>
<td>Kearney D; Pearse R; Millard P</td>
<td>1997</td>
<td>Journal</td>
<td>The impact of the admission and discharge of stroke patients aged 65 and over on bed occupancy in English hospitals</td>
<td>Management in Health Care</td>
<td>1</td>
<td>2</td>
<td>151-157</td>
<td>mixed</td>
</tr>
</tbody>
</table>

Note: the table continues over the next four pages.
<table>
<thead>
<tr>
<th>Year</th>
<th>Author/s</th>
<th>Pub type</th>
<th>Title</th>
<th>Jnl</th>
<th>Vol</th>
<th>Issue</th>
<th>pgs</th>
<th>Data Group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Millard P; Lee C</td>
<td>Journal Article</td>
<td>The biochemistry of health care</td>
<td>CME Bulletin Geriatric Medicine</td>
<td>1</td>
<td>1</td>
<td>5-6</td>
<td>older persons</td>
</tr>
<tr>
<td>1998</td>
<td>Chaussalet T; Millard P; El Darzi E</td>
<td>Journal Article</td>
<td>Evaluating the costs of alternative options for dementia services</td>
<td>Health Care Management Science</td>
<td>1</td>
<td>1</td>
<td>125-131</td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>El-Darzi E; Vasilakis C; Chaussalet T; Millard P</td>
<td>Journal Article</td>
<td>A simulation modelling approach to evaluating length of stay, occupancy, emptiness and bed-blocking in a hospital geriatric department</td>
<td>Health Care Management Science</td>
<td>1</td>
<td>1</td>
<td>143-149</td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>Lee C; Vasilakis C; Kearney D; Pearse R; Millard P</td>
<td>Journal Article</td>
<td>An analysis of admission, discharge and bed occupancy of stroke patients aged 65 and over in English hospitals</td>
<td>Health Care Management Science</td>
<td>1</td>
<td>1</td>
<td>151-157</td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>McClean S; McAlea B; Millard P</td>
<td>Journal Article</td>
<td>Using a Markov reward model to estimate spend-down costs for a geriatric department</td>
<td>Journal of the Operational Research Society</td>
<td>49</td>
<td>1021-1025</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>McClean S; Millard P</td>
<td>Journal Article</td>
<td>A three compartmental model of the patient flows in a geriatric department</td>
<td>Health Care Management Science</td>
<td>1</td>
<td>1</td>
<td>159-163</td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>Millard P</td>
<td>Journal Article</td>
<td>The anatomy, physiology and biochemistry of health care for an ageing population</td>
<td>Archives of Gerontology and Geriatrics</td>
<td>6</td>
<td>325-334</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>Millard P; Chaussalet T</td>
<td>Journal Article</td>
<td>A modelling approach to the development of health and social services for dementia</td>
<td>CME Bulletin Geriatric Medicine</td>
<td>1</td>
<td>2</td>
<td>34-35</td>
<td>not relevant</td>
</tr>
<tr>
<td></td>
<td>Millard P; Lee C</td>
<td>Journal Article</td>
<td>The process of care</td>
<td>Reviews in Clinical Gerontology</td>
<td>8</td>
<td>345-352</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>Millard P; O’Connor M; McClean S</td>
<td>Journal Article</td>
<td>Measuring and modelling patient flows through rehabilitation and continuing care</td>
<td>Applied Stochastic Models and Data Analysis</td>
<td>14</td>
<td>165-174</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>Taylor G; McClean S; Millard P</td>
<td>Journal Article</td>
<td>Using a continuous-time Markov model with Poisson arrivals to describe the movement of geriatric patients</td>
<td>Applied Stochastic Models and Data Analysis</td>
<td>13</td>
<td>315-323</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td></td>
<td>Taylor G; McClean S; Millard P</td>
<td>Journal Article</td>
<td>Continuous-time Markov models for geriatric patient behaviour</td>
<td>Applied Stochastic Models and Data Analysis</td>
<td>13</td>
<td>315-323</td>
<td></td>
<td>older persons</td>
</tr>
<tr>
<td>Year</td>
<td>Author/s</td>
<td>Pub type</td>
<td>Title</td>
<td>Jnl</td>
<td>Vol</td>
<td>Issue</td>
<td>pgs</td>
<td>Data Group</td>
</tr>
<tr>
<td>------</td>
<td>----------</td>
<td>----------</td>
<td>-------</td>
<td>-----</td>
<td>-----</td>
<td>-------</td>
<td>-----</td>
<td>------------</td>
</tr>
<tr>
<td>1999</td>
<td>Faddy M; McClean S</td>
<td>Journal Article</td>
<td>Analysing data on lengths of stay of hospital patients using phase-type distributions</td>
<td>Applied Stochastic Models and Data Analysis Business and Industry</td>
<td>15</td>
<td>311-317</td>
<td>older persons</td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>Mackay M; Millard P</td>
<td>Journal Article</td>
<td>Application and comparison of two modelling techniques for hospital bed management</td>
<td>Australian Health Review</td>
<td>22</td>
<td>3</td>
<td>118-143</td>
<td>acute care</td>
</tr>
<tr>
<td>2000</td>
<td>Bennett M; Smith E; Victor C; Millard P</td>
<td>Journal Article</td>
<td>The right person? the right place? the right time? a pilot study of the appropriateness of nursing home placements.</td>
<td>Archives of Gerontology and Geriatrics</td>
<td>31</td>
<td>55-64</td>
<td>older persons</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Faddy M; McClean S</td>
<td>Journal Article</td>
<td>Analysing data on lengths of stay of hospital</td>
<td>Applied Stochastic Models and Data Analysis</td>
<td>15</td>
<td>311-317</td>
<td>older persons</td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Millard P; Christodoulou G; Kuhanjendran D; Millard P; Mackay M; Vasilakis C; Christodoulou G</td>
<td>Journal Article</td>
<td>Targeting the priorities of health and social care for an ageing population</td>
<td>CME Journal Geriatric Medicine</td>
<td>2</td>
<td>3</td>
<td>119-121</td>
<td>older persons</td>
</tr>
<tr>
<td>2000</td>
<td>Taylor G; McClean S; Millard P</td>
<td>Journal Article</td>
<td>Measuring and modelling surgical bed usage</td>
<td>Archives of the Royal College of Surgeons of England</td>
<td>82</td>
<td>2</td>
<td>75-82</td>
<td>acute care</td>
</tr>
<tr>
<td>2000</td>
<td>Christodoulou G; Taylor G</td>
<td>Journal Article</td>
<td>Using a continuous time hidden Markov process, with covariates, to model bed occupancy of people aged over 65 years. Analysis of bed usage and occupancy following the introduction of geriatric rehabilitation care in a hospital in Huesca, Spain</td>
<td>Health Care Management Science</td>
<td>4</td>
<td>1</td>
<td>21-24</td>
<td>older persons</td>
</tr>
<tr>
<td>2001</td>
<td>Harrison G</td>
<td>Journal Article</td>
<td>Implications of mixed exponential occupancy distributions and patient flow models for health care planning</td>
<td>Health Care Management Science</td>
<td>4</td>
<td>1</td>
<td>37-45</td>
<td>acute care</td>
</tr>
<tr>
<td>2001</td>
<td>Mackay M</td>
<td>Journal Article</td>
<td>Practical experience with bed occupancy management and planning systems: an Australian view</td>
<td>Health Care Management Science</td>
<td>4</td>
<td>1</td>
<td>47-56</td>
<td>acute care</td>
</tr>
<tr>
<td>Year</td>
<td>Author/s</td>
<td>Pub type</td>
<td>Title</td>
<td>Jnl</td>
<td>Vol</td>
<td>Issue</td>
<td>pgs</td>
<td>Data Group</td>
</tr>
<tr>
<td>--------</td>
<td>----------------------------------</td>
<td>--------------------</td>
<td>----------------------------------------------------------------------</td>
<td>------------------------------------------</td>
<td>-----</td>
<td>-------</td>
<td>---------</td>
<td>---------------------</td>
</tr>
<tr>
<td>2001 cont.</td>
<td>Marshall A; McClean S; Shapcott C; Hastie I; Millard P</td>
<td>Journal Article</td>
<td>Developing a Bayesian belief network for the management of geriatric hospital care</td>
<td>Health Care Management Science</td>
<td>4</td>
<td></td>
<td>23-30</td>
<td>older persons</td>
</tr>
<tr>
<td>2001</td>
<td>Marshall A; McClean S; Shapcott C; Millard P</td>
<td>Journal Article</td>
<td>Predicting patient survival using Bayesian belief networks</td>
<td>International Journal of Health Care Engineering</td>
<td>9</td>
<td></td>
<td>13-15</td>
<td>older persons</td>
</tr>
<tr>
<td>2001</td>
<td>Millard P</td>
<td>Journal Article</td>
<td>Nosokinetics</td>
<td>Craiova Medicala</td>
<td>3</td>
<td>2</td>
<td>91-96</td>
<td>not relevant</td>
</tr>
<tr>
<td>2001</td>
<td>Millard P; Christodoulou G; Jagger C; Harrison G; McClean S</td>
<td>Journal Article</td>
<td>Modelling hospital and social care bed occupancy and use by elderly people in an English health district</td>
<td>Health Care Management Science</td>
<td>4</td>
<td></td>
<td>57-62</td>
<td>older persons</td>
</tr>
<tr>
<td>2001</td>
<td>Vasilakis C; El-Darzi E</td>
<td>Journal Article</td>
<td>A simulation study of the winter bed crisis</td>
<td>Health Care Management Science</td>
<td>4</td>
<td></td>
<td>31-36</td>
<td>mixed</td>
</tr>
<tr>
<td>2001</td>
<td>Victor C; Hastie I; Christodoulou G; Millard P</td>
<td>Journal Article</td>
<td>The inappropriate placement of older people in nursing homes in England and Wales: a national audit</td>
<td>Quality in Ageing - Policy, Practice and Research</td>
<td>2</td>
<td>1</td>
<td>16-25</td>
<td>older persons</td>
</tr>
<tr>
<td>2001</td>
<td>Gorunescu F; Mackay M; Millard P; McClean S</td>
<td>Conference Proceedings</td>
<td>Queueing Models of the Dynamics of Bed Occupancy in Hospital Systems with Fixed or Limited Capacity.</td>
<td>Proceedings of the 10th International Symposium on Applied Stochastic Models and Data Analysis</td>
<td>1</td>
<td></td>
<td>475-480</td>
<td>acute care</td>
</tr>
<tr>
<td>2002</td>
<td>Gorunescu F; McClean S; Millard P</td>
<td>Journal Article</td>
<td>Using a queuing model to help plan bed allocation in a department of geriatric medicine</td>
<td>Health Care Management Science</td>
<td>5</td>
<td></td>
<td>307-312</td>
<td>older persons</td>
</tr>
<tr>
<td>2002</td>
<td>Gorunescu F; McClean S; Millard P</td>
<td>Journal Article</td>
<td>A queueing model for bed-occupancy management and planning of hospitals</td>
<td>Journal of the Operational Research Society</td>
<td>53</td>
<td></td>
<td>19-24</td>
<td>older persons</td>
</tr>
<tr>
<td>2002</td>
<td>Gorunescu M; Gorunescu F; Prodan A</td>
<td>Journal Article</td>
<td>Continuous-time Markov model for geriatric patients behavior: optimization of the bed occupancy and computer simulation</td>
<td>The Korean Journal of Computational &amp; Applied Mathematics</td>
<td>9</td>
<td>1</td>
<td>185 - 195</td>
<td>older persons</td>
</tr>
<tr>
<td>2002</td>
<td>Ivatts S; Millard P</td>
<td>Journal Article</td>
<td>Health care modelling: opening the &quot;black box&quot;</td>
<td>British Journal of Health Care Management</td>
<td>8</td>
<td>7</td>
<td>251-255</td>
<td>not relevant</td>
</tr>
<tr>
<td>2002</td>
<td>Ivatts S; Millard P</td>
<td>Journal Article</td>
<td>Health care modelling - why should we try?</td>
<td>British Journal of Health Care Management</td>
<td>8</td>
<td>6</td>
<td>212-216</td>
<td>not relevant</td>
</tr>
<tr>
<td>Year</td>
<td>Author/s</td>
<td>Year</td>
<td>Pub type</td>
<td>Title</td>
<td>Jnl</td>
<td>Vol</td>
<td>Issue</td>
<td>pgs</td>
</tr>
<tr>
<td>------------</td>
<td>---------------------------------</td>
<td>------</td>
<td>----------</td>
<td>----------------------------------------------------------------------</td>
<td>----------------------------</td>
<td>-----</td>
<td>-------</td>
<td>--------</td>
</tr>
<tr>
<td>2002</td>
<td>Marshall A; McClean S;</td>
<td>2002</td>
<td>Journal</td>
<td>Modelling patient duration of stay to facilitate resource management of geriatric patients</td>
<td>Health Care Management Science</td>
<td>5</td>
<td>4</td>
<td>313-319</td>
</tr>
<tr>
<td></td>
<td>Shapcott C; Millard P</td>
<td></td>
<td>Article</td>
<td>Modelling decisions of a multidisciplinary panel for admission to long-term care</td>
<td>Health Care Management Science</td>
<td>5</td>
<td>291-295</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Xie H; Chaussalet T;</td>
<td>2002</td>
<td>Journal</td>
<td>Models of congestion in hospitals</td>
<td>The Australian Mathematical Society Gazette</td>
<td>30</td>
<td>3</td>
<td>127-141</td>
</tr>
<tr>
<td></td>
<td>Thompson W; Millard P</td>
<td></td>
<td>Article</td>
<td>Optimization of Costs Policy in a Geriatric Queuing Model with Extra Beds Provision</td>
<td>Siberian Journal of Numerical Mathematics</td>
<td>6</td>
<td>2</td>
<td>139-147</td>
</tr>
<tr>
<td></td>
<td>Harrison G; Millard P;</td>
<td>2003</td>
<td>Journal</td>
<td>Mathematical modelling: how and why</td>
<td>British Journal of Health Care Management</td>
<td>9</td>
<td>4</td>
<td>144-150</td>
</tr>
<tr>
<td></td>
<td>Ivatts S</td>
<td></td>
<td>Article</td>
<td>Modelling demand - a rejoinder</td>
<td>British Journal of Health Care Management</td>
<td>9</td>
<td>2</td>
<td>67-71</td>
</tr>
<tr>
<td></td>
<td>Goddard P; Mills T</td>
<td>2003</td>
<td>Journal</td>
<td>Optimization of Costs Policy in a Geriatric Queuing Model with Extra Beds Provision</td>
<td>Health Care Management Science</td>
<td>5</td>
<td>3</td>
<td>470-473</td>
</tr>
<tr>
<td></td>
<td>Harrison G; Millard P;</td>
<td>2003</td>
<td>Journal</td>
<td>Mathematical modelling: how and why</td>
<td>Craiova Medicala</td>
<td>5</td>
<td>3</td>
<td>470-473</td>
</tr>
<tr>
<td></td>
<td>Ivatts S</td>
<td></td>
<td>Article</td>
<td>Modelling demand - a rejoinder</td>
<td>British Journal of Health Care Management</td>
<td>9</td>
<td>2</td>
<td>67-71</td>
</tr>
<tr>
<td></td>
<td>Harrison G; Millard P;</td>
<td>2003</td>
<td>Journal</td>
<td>Optimization of Costs Policy in a Geriatric Queuing Model with Extra Beds Provision</td>
<td>Health Care Management Science</td>
<td>5</td>
<td>3</td>
<td>470-473</td>
</tr>
<tr>
<td></td>
<td>Ivatts S</td>
<td></td>
<td>Article</td>
<td>Optimization of Costs Policy in a Geriatric Queuing Model with Extra Beds Provision</td>
<td>International Transactions in Operational Research</td>
<td>10</td>
<td>3</td>
<td>565-576</td>
</tr>
<tr>
<td></td>
<td>Gorunescu F; Gorunescu M</td>
<td>2003</td>
<td>Journal</td>
<td>Optimization of Costs Policy in a Geriatric Queuing Model with Extra Beds Provision</td>
<td>British Journal of Health Care Management</td>
<td>9</td>
<td>4</td>
<td>144-150</td>
</tr>
<tr>
<td></td>
<td>Millard P</td>
<td></td>
<td>Article</td>
<td>Modelling patient duration of stay to facilitate resource management of geriatric patients</td>
<td>British Journal of Health Care Management</td>
<td>10</td>
<td>12</td>
<td>368-372</td>
</tr>
<tr>
<td></td>
<td>McLean S; Papadopolou A;</td>
<td>2004</td>
<td>Journal</td>
<td>Continuous time Reward Models for homogeneous semi-Markov Systems</td>
<td>Health Care Management Science</td>
<td>8</td>
<td>3</td>
<td>221-230</td>
</tr>
<tr>
<td></td>
<td>Tsaklides G</td>
<td></td>
<td>Article</td>
<td>Local Authority fines: penny wise, pound foolish</td>
<td>Medical Journal of Australia</td>
<td>182</td>
<td>5</td>
<td>252-253</td>
</tr>
<tr>
<td></td>
<td>Harrison G; Millard P;</td>
<td>2003</td>
<td>Journal</td>
<td>Mathematical modelling: how and why</td>
<td>Health Care Management Science</td>
<td>5</td>
<td>3</td>
<td>470-473</td>
</tr>
<tr>
<td></td>
<td>Ivatts S</td>
<td></td>
<td>Article</td>
<td>Optimization of Costs Policy in a Geriatric Queuing Model with Extra Beds Provision</td>
<td>International Transactions in Operational Research</td>
<td>10</td>
<td>3</td>
<td>565-576</td>
</tr>
</tbody>
</table>
Figure 7 shows the number of publications achieved each year since 1991.

![Cumulative Growth in Publications Relating to Compartmental Flow Modelling of Bed Occupancy](image)

Figure 7: The cumulative growth in the number of publications per year since 1991 showed a period of marked increased from 1998.

While publication has continued each year since the publication by Harrison and Millard (1991), it is not until 1998 that the growth in publications becomes more marked. The most publications achieved in any one year were 13 during 2001, highlighting that this represents a niche area of research.

Not surprisingly, journal publications were found to have dominated the mix of publication types, as shown in Figure 8.
The introduction of Nosokinetic News, an informal newsletter created by Millard during 2004, may have helped to increase the frequency of information delivery about patient flow modelling to people.

It is evident that much of the research to date has been based upon data relating to older persons as shown in Figure 9.

Figure 8: Journal publications have been the dominant form of disseminating written information about the modelling approach to others.
The bias in publications relating to older people is likely to relate to access to data. Millard, a geriatrician, has been pivotal in this research and has provided much of the data. While this has facilitated research efforts, it is necessary to reduce the bias if this research is going to be taken up in other sectors of the health care system.

Although publication was spread over 37 journals, multiple publication was achieved in only 10 journals, with 25 per cent of the publications occurring in the Health Care Management Science (a journal). While the Health Care Management Science is an appropriate source for dissemination of this research, it is also evident that publication has occurred in some journals that would not be considered mainstream.
The 73 papers involved 23 different principal authors, although five principal authors have contributed 55 per cent of the publications. It is evident that Millard and McClean have driven much of the publication effort. More than one quarter of principal authors have only published once as principal or contributing author, thus highlighting the fact the topic area still does not have ongoing buy-in by even those sufficiently interested to have published in the area. This is also evident by the fact that almost half of the 43 authors involved in the 73 publications have only contributed once.

2.4.2 A more in-depth look at the compartmental flow model

The compartmental flow model put forward by Harrison and Millard (1991) was introduced in section 1.6 of Chapter 1. In this section, the Harrison and Millard model, and subsequent developments will be examined more closely.

As previously illustrated in Chapter 1 (see Figure 4) the hospital bed compartmental flow model can be represented diagrammatically. It is useful to repeat the diagram here as shown in the Figure 10.
In creating their model, Harrison and Millard (1991) relied upon a number of assumptions, which were:

1. That the discharge and transfer rates (measured in patients/day) were proportional to the number of patients present in the compartment and these rates do not change with time. It should be noted that this also necessitates the assumption that the patient casemix also remains constant in order to achieve this explicitly stated first assumption. Depending on the period of analysis (for example, a year) this latter assumption may be approximately true.

2. That discharge and transfer rates are independent of the length of stay, thus given this assumption and assumption 1, the discharge rate of short-stay patients is constant, the discharge rate of long-stay patients is constant and the conversion rate from short to long-stay patient is also constant. It is not expected that this assumption is necessarily totally valid, but it does result in a simplified model.
3. That the system has reached steady state with respect to total occupancy. This appears to be a reasonable assumption given that hospitals tend to be run at high levels of occupancy. Clearly, to achieve this state, a hospital would have to be operated for some period of time and not have just commenced operations. The period of “warm-up” will depend upon the nature of the hospital, with an acute care hospital having a much shorter warm-up period prior to stability than a long-stay geriatric hospital.

Using midnight patient census data (that is, counting the number of patients who were in bed and also recording their length of stay since admission) and relying on these assumptions, Harrison and Millard (1991) found that a mixed exponential model fitted the data well, which can be represented as:

\[ Y = Ae^{-bx} + Ce^{-dx}, \]

where

- \( A \) = the number of beds in the short-stay compartment
- \( b \) = the flow rate through the short-stay compartment
- \( C \) = the number of beds in the long-stay compartment, and
- \( d \) = the flow rate through the long-stay compartment.

The parameters, \( A, b, C \) and \( d \), were equated to the theoretical equation used to describe the short and long-stay compartments by Harrison and Millard (1991), such that:

\[
A = \frac{(1-k)A_0}{v+r} \\
e^{-b} = (1 - v - r)
\]
\[ C = \frac{kA_0}{d} \]

\[ e^{-vd} = (1 - d) \]

where

\[ A_0 = \text{admission rate per day} \]

\[ k = \text{fraction of patients who would be long-stay if identified on admission} \]

\[ v = \text{conversion rate between compartments} \]

\[ r = \text{release rate per bed for short-stay patients} \]

The importance of doing this becomes clear when seeking to generate information that will be relevant to decision-making. For example, the half-life and expected length of stay of short-stay patients is given by (respectively):

\[ \text{Half life}_{\text{short-stay}} = \frac{\ln(1/2)}{\ln(1 - v - r)} \]

\[ \text{Expected length of stay}_{\text{short-stay}} = \frac{1}{v + r} \]

(The parameters were defined for the previous equations.)

This representation of the data can be used to generate a range of useful information about bed occupancy (see Appendix II for the performance statistics formulae). The derivation of the formula is more fully explored in the original paper by Harrison and Millard (1991), and is also summarised by Goddard and Mills (2003). The mathematics of compartmental models is not a new subject and for those seeking a fuller description of the mathematics and application of these models I refer you to Godfrey (1983), Manton, Singer, and Suzman (1993), or Matis and Kiffe (2000).
Goddard and Mills (2003) found it interesting that the resultant fit was excellent despite the fact that the model relied upon simplistic assumptions. Certainly, the issue of whether the assumptions are reasonable is a valid one. For example, the assumption that the system has reached a steady state with respect to total occupancy could be argued to be valid on the basis that the tendency to operate hospitals at high levels of occupancy and that the data was obtained from hospitals that had been in operation for many years. A counter argument can be made, however, as evidence exists that hospital systems are generally unstable and are challenged by differing workload demands at different times of the week or year (St George, 1988; Mackay and Gorunescu, 2001; MacStravic, 2001). Millard and his colleagues also acknowledged system instability in later work (for example, Taylor, McClean and Millard, 1996; Harrison, 2001; Harrison, Shafer and Mackay, 2005).

The issue of stability has implications for the methodology used to achieve a model. From a methodological point of view, the reliance upon a census of patient occupancy may be problematic. If the assumption that a hospital is in a steady state were to hold true, then basing the model on a single census should result in essentially the same model as if all the data were used. As previously stated, it is unlikely that the system will be stable, or at least stable enough to rely upon the use of a single census, as the data source for model development. Research relating to amount of data needed for model development is presented in Chapter 5.

Millard and Harrison (1991) illustrated the application of the model in relation to a number of hypothetical policy decisions, including the implications of increasing the bed stock to the geriatric service. While in general it is evident that the model can be
used to support strategic decision-making, not all the conclusions drawn by Millard and Harrison would appear to be reasonable. In relation to the increase in bed stock it was noted that a 10 per cent increase in beds would necessitate a 10 per cent increase in staff. Generally staffing, particularly that of nursing, is based upon some kind of agreed formula that stipulates the number of patients per nurse. This typically results in a step function and thus while an increase in beds may occur, it may or may not lead to a change in the number of nurses (it will depend upon the number of beds and the staffing formula).

Others joined Harrison and Millard in their work, notably McClean, as shown in Table 3. Harrison, McClean and Millard continued publishing research that promulgated the notion of compartmental models of occupancy being used as a means of looking at resource implications concerning hospital beds.

Marshall, Vasilakis and El-Darzi (2005) provided a review of the literature relating to what they describe as being length-of-stay-based patient flow models. These authors have been involved with Millard and McClean (and also myself). The review concentrated on the efforts of research stemming from the original Harrison and Millard (1991) paper. Apart from commenting upon the compartmental flow model, Marshall, Vasilakis and El-Darzi (2005) noted that there had been three other main areas of development relating to modelling patient flow, namely the introduction of stochastic models; the introduction of phase-type models; and the use of queueing models. While brief examples of each of these types of models will be presented it is not the intention to replicate the work of Marshall, Vasilakis and El-Darzi (2005) who provide a more thorough review of these developments.
The work by Harrison and Millard (1991) was based upon a deterministic approach. Deterministic models make no account of variation. Stochastic models, however, do take account of variation. The Markov-chain models represent a class of models that capture variation. Harrison and Millard (1991) noted that the assumption concerning the independence of the discharge and transfer rates from the length of stay was also an assumption that could be applied for Markov chain model development. They noted that the focus of the Markov model, however, would relate to patient transition probabilities and not the length of stay.

Irvine, McClean and Millard (1994) developed stochastic models to describe the movement of patients through a geriatric hospital using a two-stage continuous-time Markov model. As with the Harrison and Millard (1991) model, patients move through compartments or stages, until they are discharged or die. Admissions were modelled in two ways, namely as replacements for discharged patients (including death) or as a Poisson stream. Irvine, McClean and Millard (1994) stated that this method had the advantage of taking into account different types of patients and introducing variability, thus making it calculate not only the means of numbers of patients requiring hospital care, but also the variances. Other work on Markov models has also been undertaken, for example, using a Markov reward model to estimate spend-down costs for a geriatric department (McClean, McAlea and Millard, 1998) and using continuous-time Markov models to describe geriatric patient behaviour (Taylor, McClean and Millard, 1998).
Phase-type models are a generalisation of the Erlang distribution (Marshall, Vasilikas and El-Darzi, 2005), both of which are Markovian models. The phase type model enables movement between the phases (or states, or the compartments) and the absorption state (that is, in the case of bed modelling, death or discharge) at any point, whereas movement between phases or states is sequential for the Erlang models. While seemingly useful, the phase-type model is difficult to solve and the Coxian phase-type model was introduced (Neuts, 1981). In the Coxian phase-type model, the transition between phases or states is ordered, although progression to the absorption state can still occur at any time. Faddy and McClean (1999) used the Coxian phase-type model to analyse geriatric patient length of stay data. According to Marshall, Vasilikas and El-Darzi (2005) such models are mathematically sound, account for the long tail of the length of stay profile and still lead to the notion of compartments, which can be understood by clinicians.

The Coxian phase-type model has been augmented with the inclusion of Bayesian belief networks (Marshall, McClean, Shapcott, Hastie and Millard, 2001; Marshall, McClean, Shapcott and Millard, 2002; Marshall and McClean, 2003; Marshall and McClean, 2004). The Bayesian belief networks were used to condition the length of stay models by taking into account various patient factors, including age and gender, thus, giving rise to the potential to forecast patient length of stay in advance, which may be useful for managing costs (Marshall, McClean, Shapcott and Millard, 2002). Such an approach, according to Marshall, Vasilikas and El-Darzi (2005) has the potential to become a useful explanatory tool as it can provide insights into the interactions of variables (including social variables) that can affect length of stay, but
due to the additional complexity of the approach, additional development and testing is required compared to simpler methods.

Marshall, Vasilakis and El-Darzi (2005) suggested compartmental flow model output could be augmented with queueing model output. The output from queueing models can be used to analyse different aspects of bed problems, such as methods to overcome bottlenecks in the system. According to Marshall, Vasilakis and El-Darzi (2005) discrete event simulation (DES) is used to implement such models, because it offers flexibility.

El-Darzi, Vasilakis, Chaussalet and Millard (1998) developed a simulation modelling approach to evaluate length of stay, occupancy, emptiness and bed blocking that incorporated three compartments. One of the findings from this work was the observation that a long warm-up time was required to achieve stabilisation or steady state. The implication of this finding was that any change to the system (that is, changes in patient length of stay or increasing beds) would require a long period before a new steady state was achieved. This finding, however, was not generalisable to the acute care sector, as the research was based upon the study of a geriatric service with very long-stay patients.

Vasilakis and El-Darzi (2001) reported on the use of a queueing system and a discrete event simulation model to analyse the winter bed crises, which appears in British hospitals every year, two or three weeks after Christmas. As previously described in the section 2.3.3, Gorunescu, McClean and Millard (2002) have also applied queueing
theory to describe the movement of patients through a hospital department and presented a means of optimising the number of beds required in order to meet specified delay.

Marshall, Vasilakis and El-Darzi (2005) suggest that the incorporation of queueing methodologies into the study of patient length of stay confers some advantages over using compartmental flow models alone, including the ability to incorporate variation and to analyse the effects of bed constraints and blockages. Although technical solutions to simplify the generation of output have been achieved (Marshall, Vasilakis and El-Darzi (2005), it would appear that the approach is unlikely to be adopted as a routine management tool in its current form.

2.4.3 Conclusion regarding the compartmental flow research

Marshall, Vasilakis and El-Darzi (2005) suggest that building on the success of current models and currently evolving hybrid approaches is where the future of modelling patient flow and hospital bed type issues lay, and that interdisciplinary collaboration will be required to successfully work in this problem arena. The weakness with their forecast, as identified by Harrison (2001), is that to date there has been no apparent uptake of these modelling tools. According to Goddard and Mills (2003) the real value of the work of Millard and his colleagues has been to provide a general approach to modelling patient flows in hospitals. Furthermore, given that the models fit the data well and appear to align with intuition, Goddard and Mills (2003) suggest that there is potential to improve understanding among decision-makers of how compartments within a hospital interact, thereby providing a mechanism to drive change. Based upon my experience in the health sector, it would seem that the
necessary developments required for these tools to be implemented regularly on a wide scale are: expansion of model development to the acute care sector, which is a gap in the literature (and which is in part achieved by this thesis); education of management about how modelling can help improve decision-making (in appropriate forums such as under-graduate and post-graduate training courses); education of management that a variety of modelling approaches will be required to answer different questions relating to the health system (in appropriate forums); and the adoption of the modelling by a champion (ideally a head of a significant part of the health sector, such as a division within a large hospital) who will help “sell” the solution to others.

Given the need to address the use of this modelling in the acute care sector, and adopting Goddard and Mills’ (2003) notion that the original Harrison and Millard (1991) work provides a general platform for the development of flow modelling, the need to explore phase type and queueing models as part of this work cannot be justified. Rather it is appropriate the issues of whether the compartmental flow model can be used in the acute care sector be examined, together with consideration of issues pertaining to the number of data and model selection. The issue of variation is important and is also addressed in this research.
2.5 Other sources of literature

In this section comment is provided on the number of specific books relating to hospital bed management (or patient flow) and the grey literature base.

2.5.1 Bed management texts

There is little value in commenting upon the material in the books, suffice to say that much of the material could be obtained by combing the journal publications. Books, however, can be an economical way in which to access a range of ideas for which there is usually a range of published evidence, or at least an interpretation of the material presented in journals.

The numbers of books written that specifically address the topic of hospital bed management or patient flow and address modelling in any depth is limited. However, books on this topic have been published since at least 1982 when Yates wrote *Hospital beds: a problem for diagnosis and management*. Most recently, Hall (2006) edited the tome *Patient Flow: Reducing Delay in Healthcare Delivery*, which presents a collection of papers that cover managing demand, workforce issues, quality and safety issues, forecasting, logistics in relation to a full range of health service sectors (that is, primary care through to the acute care hospital sector). In between this period, other texts have also been produced, including those by Millard and McClean (1994 and 1996), and Vissers (2005). Of these five texts, only Millard and McClean (1994), and Yates (1982) can be considered as dealing exclusively with modelling and length of stay or occupancy. The other three texts incorporate topics that affect patient flow, such as personnel, waiting lists, outpatients and logistics (Hall, 2006; Vissers, 2005), and surgical audit (Millard and McClean, 1996). The inclusion of wider topic
material in these texts is perhaps indicative of the connectedness of the wider health system and the effects on patient flow.

The motivation for publishing these texts appears to be consistent. Millard and McClean (1994 and 1996) highlighted that their books stemmed from a passion for seeing the introduction of a scientific basis for the planning of health services, particularly as the population ages. In the book by Vissers and Beech (2005), the need for increased application of health operations management is identified, while in the book edited by Hall (2006) it is stated that the intent of the book is to illustrate the mechanisms for improving patient flow that are necessary so that medical practice can keep pace with medical science.

Other forms of texts do exist, but usually fall into one of two categories, namely belonging to the health services planning genre (for example, Schulz and Johnson, 1990; Mohan, 2002), or belonging to the more general operational research technique genre (for example, Ozcan, 2005). The health services planning genre, in my experience, tend to focus on economic policy, workforce issues, capital issues and strategic planning. Modelling is usually not a significant feature of such texts, and when present, simple modelling techniques are presented or discussion is limited to the acknowledgment that modelling exists, but details about its use and potential to aid decision-making are omitted. Conversely, the operational research texts, such as Ozcan’s book (2005), provide significant detail about various methodologies that can be applied to address a range of problems from scheduling through to capacity planning. While informative, and extensive discussion about modelling may be present, such texts do not focus on any one problem or technique and issues
pertaining to flaws with the LOS are not usually mentioned, and discussion about techniques such as compartmental flow models are rare.

The publishing of texts, however, does not guarantee the widespread adoption of methodologies, as evidenced by the work of Harrison and Millard (1991) and their colleagues (as discussed earlier), and also that of Yates (1982).

Given the investment in hospitals, and other forms of care in which accommodation is also provided (for example, aged care organisations), it is, however, surprising that few texts exist that deal with hospital bed management and modelling have been written. When compared to many other topics where significant investment has been and will continue to be made, and where problems in managing the system occur, it might be expected that many more texts on the topic would be found.

Although these texts present interesting material, reliance upon journal articles has been preferred for this research, because of the greater number of methods that can be discovered and the better timeliness of the material published in journals. Reference to these books has been included as a matter of completeness.

2.5.2 Grey information

Apart from the sources of literature already discussed, grey or miscellaneous sources of information regarding modelling and bed management or patient flow do exist. These can be broken down into two categories: research dissertations and reports.
Research dissertations do contain reviews of the literature. However, discovery and access to such work is problematic. During my research I have seen two theses that relate to hospital bed modelling – one by Millard (1989) and the other by Vasilakis (2003). These theses, however, have not been used to inform this research, due to various factors, such as issues of access, but most importantly the differing focus of the research.

Health departments (or the equivalent establishments) and hospitals frequently commission reports and reviews on a range of topics, including bed management, as evidenced by the work of Dwyer and Jackson (2001). This work may be undertaken within the organisation or be undertaken by external consultants (for example, Dwyer and Jackson, 2001). Often, while the outcome of such work may be visible, the actual decision-making, modelling and other aspects behind the report or review may not be available to the public (for example, the determination of the increase in beds referred to in the Generational Health Review (2003) is not supported by published methodology). The reasons for this are many and varied, but often include the need for political sensitivity.

Thus, the miscellaneous sources of literature have not played a significant role in providing methodological related information for this research.

2.5.3 Conclusions regarding the other literature

Non-journal based publications have not been a significant source of information for this research.
2.6 Overall conclusion

Publications in scholarly journals have been the primary source of information regarding modelling approaches used in relation to hospital bed management decision-making for this research.

It is evident that numerous methods have been proposed for investigating and improving decision-making pertaining to hospital bed management. Some of these methodologies relate to operational decision-making, while others relate to strategic decision-making. It is apparent that no single approach has gained sufficient traction with end users such that novel research regarding hospital bed modelling has ceased.

While it should not be expected that a single methodology would be able to provide management with answers to all of their questions, it may be expected that the development of a number of modelling approaches spanning the operational and strategic decision-making continuum will be required. What may be surprising to some is that methods reliant upon flawed measures and overly simplistic models continue to be promulgated. This suggests that the phase of development in hospital bed modelling has not yet reached maturity.

From a strategic decision-making perspective, the compartmental flow models appear to confer a number of advantages over the other categories of models:

- It is not too simplistic or too complex
- A body of evidence has been developing that supports the approach, albeit the evidence has largely related to a geriatric service based in the United Kingdom
• The approach has been shown to have the ability to provide meaningful output in a timely fashion, and
• It is not overly data or other resource hungry.

While the body of research relating to compartmental flow models used in relation to hospital bed management is continuing to grow, there is still significant scope for research activity. For example, the issues of model complexity (technical), variation across the year (technical) and the use of compartmental flow models in the acute hospital sector (scope of application) are yet to be fully explored. The research presented in this thesis contributes to the growing of the body of knowledge regarding compartmental flow models and hospital bed management.

In the next chapter examples of how high-level information, such as the ALOS, has been used is explored. Such information has historically been used to influence or base future decisions about hospital service, particularly in relation to costs and hospital bed use. In doing so, the flaws in using such information can be demonstrated.
Chapter 3

Setting the Scene

In this chapter I provide contextual detail about the two hospitals from which the data for this research has been provided. In doing so, recent trends in hospital activity and bed occupancy are examined using commonly available measures, such as the average length of stay, and simple methods, such as trend analysis, that have historically been used to influence or base future decisions about hospital service, particularly in relation to costs and hospital bed use. Comments highlighting the limitations of these measures and approaches are also provided. The chapter provides further support for the use of the Harrison and Millard (1991) compartmental flow model. The chapter has the following structure:

3.1 Introduction ................................................................................................. 96
3.2 Flinders Medical Centre ............................................................................. 97
3.2.1 Background Information ........................................................................ 97
3.2.2 Methodology .......................................................................................... 99
3.2.3 Results ................................................................................................... 100
3.3 Internal Medicine Department, HealthCare Otago ..................................... 106
3.3.1 Background Information ....................................................................... 106
3.3.2 Methodology .......................................................................................... 108
3.3.3 Results ................................................................................................... 110
3.4 Discussion .................................................................................................. 119
3.4.1 The Hospital Services ........................................................................... 119
3.4.2 Methods of Analysis ............................................................................. 123
3.4.3 Problems with the ALOS ..................................................................... 127
3.4.4 Simple Alternatives to the ALOS ......................................................... 129
3.5 Conclusion .................................................................................................. 131
3.1 Introduction

The purpose of this chapter is to provide some contextual information, including information relating to the average length of stay and hospital bed occupancy trends, relating to the two hospitals from which the data for this research has been provided. Reliance is placed upon the simple measures and methods that have historically been used to influence or base future strategic decisions about hospital service, particularly in relation to costs and hospital bed use. The limitations of using these simple measures and methods in relation to strategic decision-making are also documented.

The data for this research has been drawn from two primary sources:

- The Flinders Medical Centre, a tertiary acute care teaching hospital based in South Australia, and
- The Internal Medicine Department at HealthCare Otago, New Zealand.

The background information about each hospital, the methods of analysis used and the results are presented separately. A single discussion and conclusion about the various methodologies and findings is then presented.

Some of the information relating to the Internal Medicine Department at HealthCare Otago was presented at the Fourth IMA International Conference on Quantitative Modelling in the Management of Healthcare (Mackay, Lee, Rae and Millard, 2004).
3.2 Flinders Medical Centre

3.2.1 Background Information

All three tiers of the Australian government – commonwealth, state and local governments – have some responsibility for the delivery of public health care services (Duckett, 2004). The federal or commonwealth government operates the Medical Benefits Schedule (MBS) and Pharmaceutical Benefits Schedule (PBS) which part fund (co-pay) services provided by private medical practitioners and subsidize therapeutic drug prescriptions, respectively. These are significant schemes. The federal government also provides incentive for individuals to become members of private insurance funds to reduce the cost of insurance to individuals in order that greater use of the private health sector occurs. The funding of the veterans’ health service is another significant contribution to the provision of health services.

The state governments are responsible for the operation of public hospitals, as well as a range of other functions, such as community health services. The federal government contributes approximately 42 per cent of the funds required by the states for the operation of public hospitals (AIHW, 2006).

Local governments have various roles in the health sector including the provision of health inspectors and the provision of vaccination clinics.

As previously indicated, a private health sector exists. The private health sector provides a range of services, including hospital-based services that are funded by individuals, health insurance funds or both.
The Flinders Medical Centre is a tertiary level public teaching hospital and was established in 1976 (Flinders Medical Centre, 2006). It is located in the metropolitan area of South Australia. The hospital was the first medical school and teaching hospital in Australia to be planned and built as one institution. It services a local catchment area in the southern metropolitan area, as well as providing services for people across the state and also beyond the state boundaries (for example, Darwin in the Northern Territory).

The Flinders Medical Centre currently operates with approximately 430 beds (Flinders Medical Centre, 2006), although when opened the number of beds was greater. The reductions have occurred for a variety of reasons, including the expansion of same-day admission services. When this research was commenced the hospital was operating with approximately 430 available beds, although this subsequently decreased to 410 during 1999-00. A private 130-bed hospital (Flinders Private Hospital) is co-located on the campus and staff from the public hospital are able to provide services in the private hospital.

The hospital provides a full range of services, including paediatric, obstetric, mental health and surgical and medical services. It is also a major trauma centre and provides an around the clock emergency retrieval service. It is the only major public teaching hospital in South Australia to provide services for patients of all ages.

The hospital has a staff of approximately 3,300 and is supported by the largest volunteer service in a public hospital in South Australia (Flinders Medical Centre, 2006).
Approximately 80 per cent of inpatients are emergency patients at the hospital. The high proportion of emergency inpatients reduces the hospital’s flexibility in relation to bed management options. Hospitals with a lower proportion of emergency inpatients can use more elective inpatient beds (relative to Flinders) to cope with periods of high emergency patient demand. Flinders Medical Centre also provides many services (particularly elective services) on a same-day basis.

The hospital had identified a need for additional funding for more beds to alleviate difficulties in dealing with activity levels. The additional funding would have also potentially ameliorated the financial pressures being experienced by the hospital, which were in part attributed to activity levels. Prior to commencing this research I was involved in a project that considered the request for additional beds. As indicated in the first chapter, this work resulted in the identification for the need for improved strategic decision-making tools for hospital bed planning.

3.2.2 Methodology

Analysis of data relating to patient numbers and bed occupancy was conducted at two levels, namely:

- Examination of trends based upon high level summary statistics about the hospital that were publicly reported (Flinders Medical Centre, 1995, 1996, 1997, 1998, 1999, 2000, 2001 and 2002), such as the ALOS and available bed numbers, and
- Bed occupancy data relating specifically to the medical division was examined. The data was provided by the Hospital for this research.
The high-level summary data was analysed using scatterplots, which are the most popular method for examining bivariate relationships (Hair, Anderson, Tatham and Black, 1995). This method, originally developed by Galton, can suggest the nature of a relationship between variables, which is often linear, and provides a reasonably good account of many relationships found in health research (Kirk, 1990).

The daily bed occupancy data was analysed over time. This is a simple trend analysis method designed to highlight the variation in occupancy that occurs over a period of time (for example, a year) and provides a means to visualise trends (Kohler, 1984).

### 3.2.3 Results

Summary statistics from the annual reports were used to compile trends about bed occupancy for the period of the 1994-95 financial year to 2001-02 financial year (Flinders Medical Centre, 1995, 1996, 1997, 1998, 1999, 2000, 2001 and 2002). These summary statistics are detailed in Table 4.
Table 4: Summary statistics relating to the Flinders Medical Centre. While trends are evident, such as the increase in same-day admissions, these are often better visualised.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Admissions</td>
<td>40,942</td>
<td>39,580</td>
<td>40,832</td>
<td>43,938</td>
<td>46,645</td>
<td>45,603</td>
<td>44,780</td>
<td>44,976</td>
</tr>
<tr>
<td>Percentage same-day</td>
<td>38%</td>
<td>43%</td>
<td>45%</td>
<td>45%</td>
<td>46%</td>
<td>48%</td>
<td>51%</td>
<td>50%</td>
</tr>
<tr>
<td>Inpatient admissions (approximate)</td>
<td>25,220</td>
<td>22,640</td>
<td>22,417</td>
<td>24,166</td>
<td>25,048</td>
<td>23,622</td>
<td>21,942</td>
<td>22,578</td>
</tr>
<tr>
<td>Emergency (patient discharges)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>46%</td>
<td>44%</td>
<td>42%</td>
<td>42%</td>
<td>43%</td>
</tr>
<tr>
<td>Medical (patient discharges)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>75%</td>
<td>74%</td>
<td>65%</td>
<td>65%</td>
<td>66%</td>
</tr>
<tr>
<td>Average length of stay (excluding same day)</td>
<td>5.56</td>
<td>5.74</td>
<td>5.77</td>
<td>5.74</td>
<td>5.51</td>
<td>5.51</td>
<td>6.05</td>
<td>6.01</td>
</tr>
<tr>
<td>Average daily estimated occupied bed days (excluding same-day)</td>
<td>384</td>
<td>355</td>
<td>354</td>
<td>380</td>
<td>378</td>
<td>356</td>
<td>364</td>
<td>372</td>
</tr>
<tr>
<td>Estimated percentage occupancy of average daily available beds (excluding same-day patients)</td>
<td>83%</td>
<td>84%</td>
<td>83%</td>
<td>87%</td>
<td>87%</td>
<td>87%</td>
<td>88%</td>
<td>89%</td>
</tr>
<tr>
<td>Average Daily Available Beds</td>
<td>464.6</td>
<td>424.7</td>
<td>428.5</td>
<td>436.4</td>
<td>432.5</td>
<td>410.3</td>
<td>414.9</td>
<td>418.6</td>
</tr>
</tbody>
</table>

It is evident from Table 4 that during the period 1994-95 to 2001-02, the following events occurred:

- The number of inpatient and same-day patient admissions increased. Admissions, however, peaked during 1998-99.
- The proportion of patients that were same-day patients increased.
- The ALOS for inpatients increased, and
- The estimated average daily occupancy increased.

Additionally, emergency patients represented more than 40 per cent of the patient discharges and medical patients represented the majority of patients.

The increase in estimated average occupancy increased at the same time as the proportion of same-day patients increased as shown in Figure 11.
Figure 11: Scatterplot of estimated average inpatient occupancy and the proportion of same-day patients. The relationship between these two variables appears to be reasonably well described by a linear relationship, with approximately 65 per cent of the variance in occupancy explained by changes in the proportion of same-day patients.

A scatterplot of the average number of daily available beds and the percentage of same-day admissions is shown in Figure 12.

Figure 12: A scatterplot of the average number of bed and the proportion of same-day patients. A linear relationship appears to exist between the two variables, with the changes in the proportion of same-day patients explaining approximately 75 per cent of the variation in the average number of available beds.
Given the relationship between the average occupancy and the proportion of same-day patients it was not surprising that a relationship between the average number of available beds and the proportion of same-day patients also existed, as bed availability determines the resultant level of occupancy.

The relationship between the ALOS (excluding same-day patients) and inpatient admissions was examined and this is illustrated in Figure 13.

Figure 13: A scatterplot of the ALOS (inpatients) and estimated inpatient admissions. A linear relationship was found to explain the relationship, with a decrease in ALOS generally being associated with an increase in patient throughput.

The remainder of the analysis is restricted to the data that will be used for the modelling undertaken as part of this research. The data relate to the Medical Division of the Hospital and excludes elective same-day patients, as the business processes associated with the management of these patients is known to be different and thus
should not be included the modelling. Separate modelling of the elective same-day patients could occur, but was not undertaken as part of this research.

The length of stay statistics for patients discharged during 1998 and included in the modelling is detailed in Table 5.

Table 5: Length of stay statistics revealed that the distribution is highly skewed.

<table>
<thead>
<tr>
<th>Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients</td>
<td>9558</td>
</tr>
<tr>
<td><strong>Length of stay</strong></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>5.8</td>
</tr>
<tr>
<td>Median</td>
<td>4</td>
</tr>
<tr>
<td>Mode</td>
<td>1</td>
</tr>
<tr>
<td>Std. Deviation</td>
<td>7.2</td>
</tr>
<tr>
<td>Skewness</td>
<td>4.3</td>
</tr>
<tr>
<td>Std. Error of Skewness</td>
<td>0.0</td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
</tr>
<tr>
<td>Maximum</td>
<td>148</td>
</tr>
</tbody>
</table>

The fact that the distribution is highly skewed is not obvious from the annual high-level statistics. While the reporting of average occupancy or length of stay without any other measures of spread is common practice in the health sector, it is not a useful measure. Comparison between periods or across different hospitals, or parts of hospitals, is not easily undertaken without the degree of spread being understood and also understanding whether the distribution is skewed or not. The ramifications of a skewed distribution are highlighted in Figure 14.
Maximum length of stay was 148. The tail of the actual data distribution extends out here, but is difficult to see.

Figure 14: The length of stay distribution compared to a Normal distribution with the same mean and standard deviation. Note – the variation in the hypothetical distribution can be attributed to the variation in random numbers used to generate the data.

The visual comparison of the length of stay distribution to a Normal distribution with the same mean and standard deviation highlights the differences in distribution shape. If the Normal distribution were applicable, there would be more patients with a length of stay of between 10 and 22 days (the positive part of the Normal curve that is greater than the observed data for length of stay). The Normal distribution assumptions also give rise to (or density to) negative length of stay values, which cannot be observed. The ramifications of using the ALOS when the distribution is skewed are discussed later in this chapter (see the discussion).

The high level statistics also do not enable the variation in daily occupancy to be understood. The trend in variation in total occupancy across a single year is illustrated in Figure 15.
Figure 15: Trend in total bed occupancy. A seasonal trend is evident, as is regular peaks and troughs. The regular peaks and trough relate to day of week trends that were found to occur.

3.3 Internal Medicine Department, HealthCare Otago

3.3.1 Background Information

The New Zealand health care system is different to the Australian health care system. This section draws on the material presented by Mackay, Lee, Rae and Millard (2004). Although a central government responsibility exists, devolution of planning and administration functions has occurred. Responsibility for the administration of health care services lies with 21 District Health Boards. While global budgets are determined by the central administration, each health board has responsibility for the administration of its budget. Unlike Australia, the health care system is not split between a state and federal government, but rather a single system exists. The implications of the single system are various, but one significant difference to Australia is that the Region controls the budget for both acute care and primary health care services.
The Otago Province is located on the South Island of New Zealand. The only tertiary hospital for the Province is the Dunedin Hospital. Secondary care is shared with three small rural hospitals located in the Province. The Dunedin Hospital has a drainage population of greater than 180,000 people for tertiary services and 120,000 people for secondary services.

The Hospital’s Respiratory and Cardiology services share responsibility for the admission of acute medical patients with the Internal Medicine Department. The Internal Medicine Department is responsible for approximately 61 per cent of all acute medical admissions. The majority of these patients (approximately 73 per cent) are aged 65 years or older.

Changed work processes following the introduction of a stroke pathway produced reduced bed occupancy for the Internal Medicine Department (Rae, Busby and Millard, 2007). The major driver of the reduction in occupancy was getting patients home sooner and without increasing the level of illness on discharge than under the previous policy.

A further change to the practices of the Department occurred as a consequence of the review of Geriatric Medicine work process, which occurred during 1996-97. The review resulted in approximately 30 per cent of elderly patients being transferred to the Assessment Treatment and Rehabilitation, which is part of the Geriatric Medicine service, prior to discharge. This represented a significant increase in patient transfers.
to this service. This change in business practice altered the occupancy profile and this is evident in 1997 (see later results, for example, Figure 18).

While there was no growth in the total patient numbers for the Dunedin Hospital, there was an annual growth in patient numbers being admitted into the Internal Medicine Department. This growth was driven by a growth in the admission of elderly patients. During 1997 growth in admissions into the Internal Medicine Department unexpectedly increased. This growth was not driven by an increase in elderly patient admissions, but arose primarily due to financially motivated changes of practice in other hospital services, with the consequence of patients being deflected to the Internal Medicine Department.

3.3.2 Methodology

Public reporting for the activities of Otago HealthCare incorporates the activities of the various units within the region, including the Dunedin Hospital. Unlike the information that is available for Flinders Medical Centre, recent public reporting focuses on the region as a whole as opposed to reporting at unit level. Also, the focus of the research was different – it only related to the activities of the Internal Medicine Department, as opposed to the opportunity to consider the wider hospital. Consequently, the summary statistics available in relation to Dunedin Hospital were different to those available from Flinders Medical Centre.

The summary statistics considered in this analysis were derived from two primary sources, namely:
• ALOS data supplied by Dr Brendon Rae, Director of Internal Medicine (Rae, 2004), and

• Occupancy profile data relating to the Internal Medicine Department (also supplied by Dr Rae).

An ogive, which is the term for cumulative frequency distributions (Kohler, 1984), was used to describe the occupancy profile.

Trend information was created from the data in relation to bed occupancy profile and average length of stay. The trends covered the period 1990 to 2004\(^1\). Seven day and 90 day moving averages were used to smooth the bed occupancy trends (Kohler, 1984) in order to remove the noise associated with daily and weekly variation in the data. The trends were presented graphically.

Additional contextual information (beyond that provided in section 3.1.2) was detailed about the Dunedin Hospital to provide an enhanced understanding of any meaning that could be derived about the trends.

\(^{1}\) The data for the 1990 year were incomplete – only July to December. The data for the 2004 year were also incomplete – January to 19 April 2004. All other years were based upon complete data.
3.3.3 Results

The ALOS trend for the period 1990 to 2004\(^2\) is detailed in Figure 16.

![Internal Medicine - Average Length of Stay Trend](image)

Figure 16: The ALOS trend for the period 1990-2004. It is evident that the way patients flowed through the system changed after 1996 as the ALOS declined significantly.

The trend reveals two distinct periods of differing patient rates of flow as measured by the ALOS, namely:

- 1990 to 1996 – a steadily declining ALOS was evident, and
- 1997 to 2004 – a much larger decline in ALOS was initially achieved followed by a period a steadily increasing ALOS, although the ALOS was well below that of 1996 for the entire period.

Although the ALOS is often inappropriately used as illustrated in Chapter 2 (for example, see Farmer and Emami, 1990), there are circumstances where it can be used, but even then it reveals little about the total bed usage. The profile of the average

---

\(^2\) The data for the 1990 year were incomplete – only July to December. The data for the 2004 year were also incomplete – January to 19 April 2004. All other years were based upon complete data.
midnight bed census profile (that is, the time in days since a patient was admitted) for the entire data set is detailed in Figure 17.

Figure 17: The average midnight bed census profile occupancy profile (ogive) for the Internal Medicine Department data. The shape of the distribution indicated that occupancy has a skewed distribution, which is expected, with most patients only being admitted for a relatively short period of time (for example, fewer than 10 days).

The trend in the daily total midnight bed occupancy is useful revealing patterns that can be informative for management purposes. The information that can be gained from such analysis, however, is not complete. Figure 18 illustrates such a trend relating to the Internal Medicine Department.
The total midnight bed occupancy trend for the Internal Medicine Department 1990 to 2003 is illustrated. The trend reveals weekly trends (illustrated by the many peaks and troughs occurring between, say December 1992 and June 1993); seasonal trends (for example, December 2001 and December 2002 are low points, while June 2002 represents a peak in occupancy); and change in service trends (the occupancy for period prior to 1997 is generally greater than the occupancy from 1997 onwards). These trends, however, can be highlighted better using moving averages.

A seven-day moving average trend of midnight total occupancy is more useful to highlight weekly patterns in bed occupancy. A 90-day moving average trend of midnight total occupancy is useful to highlight the seasonal variations\(^3\) in bed occupancy. Such trends are shown in Figure 19 and Figure 20, respectively.

---

\(^3\) In this context the term “seasonal variation” is used to describe variation associated with changes in the time of the year (for example, summer and winter) and is not used in the operational research context where it relates to cyclical variation in general.
Figure 19: A seven-day moving average of total midnight bed occupancy. The troughs represent low points of activity (such as weekends) and the peaks represent days of high admissions (typically a week day). However, when data covering many years is presented in this manner, as in this case, the weekly cycle becomes compressed.

Figure 20: A 90-day moving average of total midnight bed occupancy for the period 1990 to 2003. The trend highlights the seasonal variation associated with the functioning of this service and removes the noise arising from daily variations in service provision.

Both of the moving average trends highlight the sudden decline in total occupancy that occurred at the end of 1996. The 90-day moving average trend, however, does not
reveal the extreme variations in short-term occupancy like the seven-day moving average trend.

It is evident in all three figures of midnight bed occupancy that while the service underwent apparent change at the end of 1996, bed occupancy also gradually increased over the subsequent years. When analysis of the midnight bed occupancy trend is combined with the analysis of the trend in average length of stay (refer Figure 16 and Figure 18), it would appear that part of the explanation for the increase is due to an increase in the ALOS.

A small increase in long-stay patients often explains an increase in the overall ALOS. To determine whether the increased occupancy was due to a change in long-stay patients, the data was partitioned into short and long-stay sets. Partitioning was based upon my expert understanding of health care systems, insofar as that a stay of longer than 10 days in acute care hospital service (and as supported by Figure 17) is less common. The resultant 90-day moving average trend for the partitioned data is illustrated in Figure 21.
Figure 21: A 90-day moving average of short and long-stay patient midnight bed occupancy trends. It can be seen that the post 1996 growth in bed occupancy is due to short-stay patients. Although Figure 21 suggests that the post 1997 increase in bed occupancy arose from a growth in the number of short-stay patients, it is also consistent that part of the increase was attributable to the increase in ALOS. This form of analysis, however, does not lead to a definitive conclusion.

Additionally, it can be seen in Figure 21 that the post 1997 amplitude in seasonal variation is increasing, suggesting that the winter peaks experienced at the Hospital are leading to additional pressure on beds and other associated resources.

The Internal Medicine Department contributes to the care of the elderly and the midnight total bed occupancy profile for this subset of patient is illustrated in Figure 22.
Figure 22: Profile of total bed midnight bed occupancy by patients aged 65 years or more. When viewed with the previous figures it can be seen that these patients occupy the majority of Internal Medicine Department beds.

While Figure 22 indicates a growth in total midnight bed occupancy that is attributable to patients aged 65 years or more, it is not evident if the growth in occupancy also occurs for the other patients. The distribution of occupancy for aged (that is, 65 years and older) and non-aged patients is shown in Figure 23.
Figure 23: Relative distribution of midnight bed occupancy for aged and non-aged patients. The non-aged patients account for less of the total bed occupancy post 1997.

It would appear that the post 1996 service change had little effect on the mix of aged and non-aged patients initially. The growth in aged patients has, however, led to an alteration in the mix of the patients (on the basis of age) over time.

In terms of the short-stay patient trend, the change in service delivery that occurred at the end of 1996 resulted in a decline in the non-aged group bed occupancy as illustrated in Figure 24. This initial decline was sustained to the extent that bed occupancy was still less than prior to the change.
Figure 24: Trends in aged and non-aged patient total midnight bed occupancy for patients admitted for 10 or fewer days. Increases in midnight bed occupancy for aged patients post 1997 have led to more beds being occupied than before 1997 for this group of patients.

The trend in total midnight bed occupancy for the short-stay aged group reveals an increase in bed occupancy post 1997 to the extent that occupancy now exceeds the occupancy prior to the 1996.

The long-stay trend is similar except that the levels of occupancy have not returned to the original pre-shock or service change state. However, there has been growth in the number of long-stay aged patients in recent years as shown in Figure 25.
3.4 Discussion

The results have provided some contextual information about the hospitals that have provided data for this research. Another purpose of this chapter has been to illustrate measures, such as the average length of stay, and methods, such as trends analysis, that are commonly used for strategic decision-making purposes in the health sector. The methods of analysis lead to three areas of discussion, namely: information that the hospitals may gain from the analysis, the methods of analysis and the problems associated with the ALOS.

3.4.1 The Hospital Services

The analysis of the high level measures and daily occupancy totals served to confirm the contextual information about both services statistics as detailed in sections 3.2.1

Figure 25: Trends in aged and non-aged patient total midnight bed occupancy for patients admitted for more than 10 days. While post 1996 service delivery changes have been maintained, the occupancy for the aged patient group has increased.
and 3.3.1. The following sections discuss the results in light of the contextual information.

**Flinders Medical Centre**

Table 4 substantiated the Hospital’s claim that activity was increasing, although at the time of this research the number of admissions declined. Same-day patient activity grew during the period 1994-95 to 2001-02.

The Hospital has highlighted its high level of emergency inpatient workload. The summary statistics, however, fail to enable this to be determined, because inpatient and same-day patient are not separated. From a strategic decision-making perspective, this reduces the quality of this information.

Increasing bed occupancy was reported. A reduction in vacancy combined with a large proportion of admitted patient emergency workload would have resulted in a reduced buffer of vacant beds being available to cope with the variation in daily activity. Bagust, Place and Posnett (1999) have reported, based upon a simulation study, that bed occupancy of greater than 85 per cent can result in bed management crises, thus supporting the Hospital’s position.

The relationships shown in the scatterplots in Figures 11 and 12 confirmed that increases in same-day patient admissions were associated with increased inpatient bed occupancy and a reduction in the number of available inpatient beds. Thus, it is apparent that same-day patient activity was not only increasing, but that a substitution of inpatient beds was also occurring. From personal knowledge of the hospital, at
least some same-day capacity was created through conversion of inpatient areas into same-day patient areas. The management of the creation of same-day beds thus may have contributed to the perceived need for additional inpatient beds.

Figure 13 illustrated the relationship between patient length of stay and the number of patients admitted. It would seem apparent that greater patient admissions were associated with a reduction in average patient stay. It would be easy to mount an argument that a busy hospital (in terms of patient numbers) is a stimulus for faster patient throughput. The information available from this graph, however, does not enable a user to determine if patients were being turned away, or were being turned away more frequently, when the average patient stay increased. Patient turn-away may be measured in various ways, including ambulance bypass and waiting lists. Nor is it possible to determine if changes in patient length of stay was associated with changing acuity levels in the patients, which may have resulted in fewer patients being admitted, but for longer periods of time. Thus, while the information in Figure 13 was interesting, it was not conclusive and further information about patient flow, numbers and whether there was a change in proportions of short and long-stay patients is required to gain a more complete understanding about the system and for strategic decision-making purposes. This is clearly an area where compartmental flow models are able to provide a range of information that more comprehensively addresses the issues raised by the analysis presented in Figure 13. This will be illustrated in subsequent chapters.

Table 5 and Figure 14 highlight the skewed nature of the patient data that was available for the modelling exercise. It is worth noting that often the only measure of patient stay and rate of flow available to managers is the average length of stay. While
educationally useful, insofar as that the information could be used to educate staff about the length of stay profile, there is little value to the decision-maker in having the additional information presented in Table 5 and Figure 14.

The weekly and overall seasonal fluctuations in bed occupancy were illustrated in Figure 15. Such information is useful to the extent that it confirms the presence of trends in the data.

While this type of information supports the notion that the hospital was experiencing some kind of additional stress with greater levels of activity, or reduced inpatient bed vacancy rates, or a combination of both, it is not possible to explore the causes of this without more in-depth analysis.

**Internal Medicine Department**

The analysis of the average length of stay (see Figure 16) and occupancy data highlighted the change in the service that occurred at the end of 1996 (see Figure 18, Figure 19 and Figure 20). The post 1996 service change was also associated with a reduction in number of beds occupied by long-stay patients (see Figure 21).

A post 1996 increase in average length of stay was found to have occurred. While the analysis of the ALOS was able to highlight this fact, the cause of the increase it is not evident from the analysis. For example, it is possible that the increase in ALOS is due to an increase in stay of the short-stay patients or long-stay patients. This style of analysis cannot tackle the drivers of ALOS and thus there is a need for a more definitive approach. These methods, however, are still useful in identifying trends.
Figure 22 confirmed that the Internal Medicine Department devoted a significant amount of resource to the care of the elderly, both pre and post the service change. The change to the service at the end of 1996 had little or no effect on the proportion of beds used by the aged and non-aged as shown in Figure 23. The proportion of beds being used by the aged, however, seemed to be growing in recent times and the total occupancy trend analysis confirms the increase in the number of beds occupied by aged patients as shown in Figure 24. This suggests that either there was a growing ageing population or a change in external services for the aged. In this instance, external services could be within the hospital. If the overall length of stay for the aged was increasing this might suggest a change in external services outside the hospital. A combination of service change and an ageing population may also explain the trend. The high use of resources by the aged, as measured by bed occupancy, is not unexpected and is consistent with trends in other OECD countries (for example, OECD, 2003). The style of analysis alone cannot be used to indicate whether a particular patient group is receiving the appropriate level of service.

3.4.2 Methods of Analysis

The use of high-level measures for determining the performance of hospitals is not new. The ALOS and level of occupancy are two key measures that are used to inform managers at the health service level, and also the bureaucracy, about issues concerning patient flow and bed use. For example, the Scottish National Health Service (NHSScotland, 2004) published the performance measures it used to assess health unit performance and this included the average length of stay and percentage of bed occupancy. Additionally, the average length of stay and various high level
statistics are reported in annual reports, such as the Flinders Medical Centre Annual Reports (for example, Flinders Medical Centre, 2003).

To ensure accountability of the use of scarce resources it is appropriate to measure the performance of aspects of the system. There are a number of uses of such measures, including:

- Comparison of performance between units (hospital or division)
- Tracking the performance of a measure for a single unit over time, and
- In the case of bed related measures, using these to determine required capacity.

The analysis undertaken of the performance measures relating to bed use for this research was related to tracking the performance of a single unit (hospital or division of a hospital) over time.

While the methods of analysis presented in this chapter are commonly used in the health care sector, there are issues with the methods, which are detailed in the remainder of this section.

**Scatterplots and Regression**

The sample size is an important consideration when undertaken regression. Samples with fewer than 20 observations are generally considered to be small (Hair et al., 1995). Simple regression analysis (that is, one independent variable) may be appropriate in instances where there is a small sample size provided that there is a strong relationship between the variables (Hair et al., 1995).
The analysis of the high level data was based upon only eight observations and thus is representative of a small sample size analysis. While Figure 10 appears, on the basis of a naked eye appraisal, to have a good linear fit, the sample size may not be appropriate to provide a good level of certainty. Similarly, based upon a visual fit, the relationship depicted in Figure 13 would benefit from inclusion of more observations.

Strategic management decisions, however, cannot necessarily rely upon the existence of long-term data collections that would improve the appropriateness of the regression analysis. Even if the data was available, data collections are often subject to changes in data definitions. Also, management practices (both clinical and other) change over time and thus analysis of data over a long period may not be useful.

Thus, while scatterplots and regression analysis can be useful in alerting management to the existence of indicative relationships that affect patient flow and bed occupancy, it is likely that the usefulness of the relationships will be limited in terms of the strategic management of hospital beds and patient flow. Furthermore, while the relationships that were identified are useful when considering issues about bed management and patient flow, there is little ability to manipulate the drivers affecting patient flow. Other forms of analysis are necessary for this type of analysis.
Moving Averages

Moving averages, while useful, are not without disadvantages. The Slutsky-Yule effect (Kohler, 1984) occurs when a non-existence cyclic trend is observed as a consequence of applying moving averages to a set of data. As the weekly and seasonal trends are evident in the non-smoothed data relating to the Internal Medicine Department results (for example, see Figures 19 to 22), there can be certainty about the existence of the trends and thus the Slutsky-Yule effect is not of concern in this instance.

Other weaknesses associated with the use of moving averages identified by Kohler (1984), such as sensitivity to extreme values and prolongment of trends, can affect bed occupancy trends that are based upon moving averages. The prolongment of a trend increases as the amount of data used to create the average increases. Thus, there is little prolongment of trends in a seven day moving average, but trends are likely to be prolonged when a 90-day moving average is used.

Exponential smoothing may reduce sensitivity to extreme values (Kohler, 1984). Extreme values, and in particular periods of high bed occupancy, are important in the analysis of bed occupancy and patient flow issues, as the management of beds involves consideration of peaks and troughs to ensure that beds are available to avoid the likelihood of unnecessarily long delays in patient admission. Thus, the need for exponential smoothing is unlikely.

Moving averages merely help clarify the existence of either weekly or seasonal trends through improved ability to visualise the trends. Providing that the moving average is
used in conjunction with presentation of the underlying raw data, the disadvantages stated are unlikely to cause significant issues.

As a tool for helping managers with strategic management decisions relating to hospital beds, however, they offer little value beyond the identification of generally appreciated trends. This stems from the fact that moving averages do not enable managers to understand and manipulate the drivers of the occupancy profile, namely patient numbers and length of stay.

**Ogives (Cumulative Frequency Distribution)**

Cumulative frequency distributions, such as Figure 17, are useful in illustrating the shape of the census or occupancy profile. The addition of confidence intervals provides some degree of indication of the variability around the profile. There is little surprise in the shape of the profile, however, as the length of stay distribution statistics (see Table 5) indicate patient stay is skewed and patient stay and occupancy are linked.

Without further analysis, however, such as the fitting of equations to describe the profile, little use can be made of the distribution for management purposes. The fitting of equations to this distribution is the subject for subsequent chapters.

**3.4.3 Problems with the ALOS**

The implicit assumption made in using the ALOS is that it is Normally distributed. Yet data from both hospitals clearly indicates that this is not the case (see Table 5 and in particular, Figure 14, which highlights the difference between the Normal and
observed distributions in relation to Flinders Medical Centre; and Figure 17 in relation
to the Internal Medicine Department). This is of no surprise given that other research
relating to maternity, geriatric, acute surgical services and flow modelling in general
has also recognised that the ALOS is skewed and is a poor measure for modelling and
resource allocation decision purposes (for example, Harrison and Millard, 1991;
Mackay and Millard, 1999; Millard, Mackay, Vasilikas and Christodoulou, 2000;
Harrison, 2001; Harpler and Shahani, 2002; Wang, Yau and Lee, 2002; Costa, Ridley,
Shahani, Harper, De Senna and Nielsen, 2003; Nguyen, Six, Antonioli, Lombrail and
What perhaps is of surprise is the lack of understanding about the ALOS and the
ramifications of the fact that its distribution is generally skewed.

There are theoretical and practical reasons that using the ALOS is inappropriate for
use in the development of models. First, the length of stay profile typically has a
highly skewed distribution and that is not well summarised by its mean value. Second,
the length of stay distribution is complex, often consisting of mixtures of patient types
(that is, medical and surgical, planned and unplanned admissions, young and elderly)
and mixtures of outcomes (that is, some patients die, some are discharged home, some
to alternative care services such as nursing homes). While it might be argued that the
introduction of casemix categories could reduce some of the complexity, recent work
indicates that the problems associated with the ALOS are still not overcome even with
the introduction of such approaches (Wang, Yau and Lee, 2002). It is also my
experience from having reviewed regional hospitals in South Australia that casemix
approaches also tends to focus attention on disease related groups as opposed to the
strategic decision level (Beltchev and Mackay, 2000).
The ALOS does not take into account the time of day when a hospital is most busy. The admission and discharge practices of a given hospital or service is a significant factor in determining the number of beds required throughout a day. If new patient admissions occur prior to existing patient discharges then additional bed capacity will be required (Mackay and Gorunescu, 2001). The use of the ALOS in determining bed requirements does not take into account the time of day effect. Other temporal effects may also exist, including variation across the week, and the ALOS does not take these effects into account. From a strategic point of view, these other effects may not be as significant as the time of day effect, because planning may occur for an average day.

Models for bed occupancy have been developed that do use the ALOS (for example, Sorensen, 1996). While the ALOS is flawed, some of the work previously undertaken has yielded interesting findings, such as the focus on discharge destination by Sorensen (1996). Consequently, the information gained from such flawed approaches should not be disregarded, but rather treated as providing some of the building blocks for future model development.

3.4.4 Simple Alternatives to the ALOS

The average is usually the preferred measure of central tendency, because among other things, it can be used in mathematical and algebraic manipulations, that is, it is mathematically tractable (Kirk, 1990). However, when the distribution is skewed this does not hold.
The median is another measure of central tendency. This measure is less sensitive to extreme values and thus is a more representative measure of central tendency when a distribution is skewed (Kirk, 1990). It could be argued that the median length of stay is a more appropriate measure to report given that the ALOS is skewed. While this is the case, the median is less mathematically tractable (Kirk, 1990) and thus, does not offer significant potential as a useful alternative to the ALOS. The ongoing use of the ALOS within the health sector highlights the apparent desire to summarize the patient length of stay distribution with a single number. However, the ability to use this single measure for the purposes to which it has often been applied has been demonstrated in this chapter to be not only flawed (given it is not Normally distributed), but also insufficient to address many of the issues that are considered as part of strategic bed management issues.

Rae and Millard (2004) have suggested that using a colour coded chart that highlights whether length of stay percentile values change might be a useful alternative to the ALOS. The system is simple and does overcome the problem of using a single measure of length of stay (that is, the average) to monitor a complex system. The approach, however, does not facilitate better future decision-making as it is not linked to a model that can be adapted for future change. It is a system of monitoring past performance.

In the absence of an alternative suitable single measure or simple approach, such as the use of percentiles, to replace the ALOS, the need to consider alternative approaches for describing, modelling and making predictions about bed occupancy exists. The compartmental flow model as described by Harrison and Millard (1991)
provides a suitable alternative – it enables the length of stay profile to be summarized
economically using a small number of parameters (two, four or six, depending upon
the number of compartments used), and these parameters can be used to generate
output which is easily interpretable and can be used to assist decision-making in
relation to many aspects of strategic bed management.

3.5 Conclusion

In this chapter I have presented information that can be, and often is, readily made
available to hospital and health care managers. The information was derived from
three sources, namely the publicly available information relating to the hospital, the
data used for this research and via personal contact with hospital staff.

While this information is of use in setting the context under which these hospitals
have operated, it has little or limited value for strategic management decision-making
purposes. It is proposed that the creation of models, and in particular the bed
occupancy compartmental flow model as originally promulgated by Harrison and
Millard (1991) for the geriatric health care sector, can fill some of this information
void and also provide information for strategic decision-making purposes in the acute
care sector.

In the next chapter the theoretical background that underpins the basis of modelling
used in this research is presented.
Chapter 4

Modelling – Some Theoretical Background

The essential underlying issue being addressed by the research presented in this thesis is whether the model originally proposed by Harrison and Millard (1991) for modelling geriatric hospital patient bed occupancy can be applied to the acute care sector for strategic decision-making purposes. In order to address this question and to consider other pertinent issues relating to the development of models of hospital bed occupancy that can be used to answer questions of a strategic nature, such as the issue of model selection, it is necessary to provide some background about the theories that underpin this research. The purpose of this chapter is thus to provide some of the theoretical background about modelling and model selection that is relevant to my research. The chapter has the following structure:

4.1 Introduction ................................................................. 133
4.2 Modelling ........................................................................ 135
4.3 The process of model building ........................................ 137
4.3.1 Testing the model validity and inputs.......................... 137
4.3.2 Testing the model outputs ........................................... 138
4.4 Model choice ................................................................. 139
4.5 Model fit and complexity trade-off ................................. 143
4.6 Model choice and performance ....................................... 145
4.6.1 Model performance – common approaches ................ 145
4.6.2 Least Squares and Maximum likelihood...................... 150
4.6.3 Bayesian Information Criterion (BIC) ......................... 153
4.6.4 Bayes factor ............................................................. 155
4.6.5 Cross validation and Bayes factor ............................... 157
4.7 Conclusion ................................................................. 159
4.1 Introduction
The ability to collect and analyse data relating to bed occupancy has existed for some
time, as illustrated in Chapter 3. Improvements in computing systems have meant that
the ability to extract such data is now easier. Consequently, the ability to investigate
issues requiring data extraction, analysis and modelling has become more
communplace. This was confirmed in a recent study on the use of simulation
modelling where it was reported that the number of papers reporting work on
modelling population health and health care delivery had increased, particularly since
the 1990s (Fone et al., 2003). The downside to this situation is the increased number
of avenues that can be explored without necessarily yielding something of value.

While raw length of stay data can be extracted, the volume of data is such that visual
inspection of the data is highly unlikely to yield sufficient information to be of value
to health system managers and clinicians. Nor will it be timely.

The ability to summarise such data using basic statistics, such as the average, has been
used for many years. It is also widely used as shown in Table 6.
Indeed, the high number of Google™ hits may provide support that the average length of stay is an entrenched and ubiquitous measure in the health systems across the world, regardless of the status of development of the country. As previously discussed, however, the average length of stay is of limited value, particularly in relation to using it to assist with informed and defensible decision-making in the health sector.

The creation of a bed occupancy compartmental flow model provides decision-makers with the opportunity to replace the average length of stay with a more useful and defensible decision-making tool in relation to strategic decisions concerning bed occupancy. The creation of such a model is not necessarily straightforward – at least in terms of the development required to support justification of the use of such a model. For example, should a bed occupancy compartmental flow model of an acute

<table>
<thead>
<tr>
<th>Country name used</th>
<th>Number of search hits</th>
<th>Ratio of Scholar™ to Google™</th>
</tr>
</thead>
<tbody>
<tr>
<td>none specified</td>
<td>48,800</td>
<td>3,480,000</td>
</tr>
<tr>
<td>Australia</td>
<td>10,100</td>
<td>728,000</td>
</tr>
<tr>
<td>New Zealand</td>
<td>5,260</td>
<td>432,000</td>
</tr>
<tr>
<td>USA or America</td>
<td>24,100</td>
<td>1,580,000</td>
</tr>
<tr>
<td>UK or Britain</td>
<td>15,300</td>
<td>1,260,000</td>
</tr>
<tr>
<td>England</td>
<td>11,800</td>
<td>1,230,000</td>
</tr>
<tr>
<td>France</td>
<td>10,400</td>
<td>764,000</td>
</tr>
<tr>
<td>Japan</td>
<td>7,880</td>
<td>1,130,000</td>
</tr>
<tr>
<td>China</td>
<td>6,870</td>
<td>1,100,000</td>
</tr>
<tr>
<td>India</td>
<td>6,080</td>
<td>546,000</td>
</tr>
<tr>
<td>Russia</td>
<td>3,840</td>
<td>597,000</td>
</tr>
<tr>
<td>Canada</td>
<td>13,000</td>
<td>1,730,000</td>
</tr>
<tr>
<td>Finland</td>
<td>3,510</td>
<td>191,000</td>
</tr>
<tr>
<td>Italy</td>
<td>7,850</td>
<td>502,000</td>
</tr>
<tr>
<td>Romania</td>
<td>922</td>
<td>121,000</td>
</tr>
<tr>
<td>Indonesia</td>
<td>1,460</td>
<td>195,000</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1,280</td>
<td>138,000</td>
</tr>
</tbody>
</table>
care hospital service be constructed with one, two or three compartments? The remainder of this chapter will provide information on modelling, model performance and model selection – all topics of importance when developing a model. Such information provides the theoretical foundations for the research conducted for this thesis.

4.2 Modelling

In general, modelling provides a means of summarizing a relationship in a concise manner. Models may be verbally specified when first developed and this can be sufficient to lead to discussion about the system under examination (Myung, Pitt and Kim, 2005). Creation of mathematical and statistical models, however, can be useful as such models can facilitate better understanding of the system under investigation, enable prediction of future events and can facilitate wider understanding through generalisation (Myung, Pitt and Kim, 2005).

Mathematics can be used to describe statistical ideas with precision and efficiency (Davison, 2003). Mathematical solutions tend to be evaluated on the basis of generality of the solution and its elegance (Davison, 2003). Statistical solutions, however, tend to arise from applied questions linked to data and often lack the elegance and generality of the mathematical solution. Despite the lack of a single over-arching theory, Davison (2003) posits that shared threads between statistical solutions exist and have lead to the notion of the statistical model.

A statistical model is one where variability is represented through the use of some form of probability distribution and that the probability distribution forms the basis of the model (Davison, 2003). There are two types of variation that must be captured
with such models: haphazard variation in the data and systemic variation. The haphazard variation within the data should be captured through the probability distribution, while the systemic variation should be captured through the model structure (Davison, 2003). It may be argued, contrary to Davison, that such modelling is as elegant as mathematical modelling, but differs from mathematical modelling insofar as that such modelling must contend with an additional source of variability. Such variation in opinion, however, does not have bearing on this thesis.

The problem for which a solution is being sought should guide the development of the model, particularly its complexity. Thus, it is possible that the same data may be used to generate different models to answer different questions (Davison, 2003). In the case of hospital bed occupancy models, the range of possible models can be best described as existing on a continuum where models designed to address operational issues lie at the more complex end and models designed to address the strategic issues lie at the less complex end.

The formulation of models involves experience, judgement, and trial and error (Davison, 2003). This also applies to the development of models that describe hospital bed occupancy. Useful models arise when the model is consistent with knowledge of the system under study and that the explanations derived from the model match experience with the system (Davison, 2003). Also, the model can be used for generalisation to other similar situations. Furthermore, the model should have reasonable mathematical and statistical properties (Davison, 2003). For example, if a given data set exhibits an exponential-like distribution, as is often the case with patient length of stay, then it is reasonable to adopt a model that relies upon the
exponential distribution as opposed to one that relies upon the Gaussian (or Normal) distribution.

4.3 The Process of Model Building
The creation of a model is a process that has two inputs, namely variables and relationships (Armstrong, 1985), and can be represented as shown in Figure 26.

![Model Building Process Diagram](image)

Figure 26: According to Armstrong (1985) models built for forecasting purposes have two basic inputs or components, namely variables and relationships. The concepts of validity and reliability can be applied to these inputs.

4.3.1 Testing the model validity and inputs
Model input validity and reliability are important issues in model building, particularly given that the creation of models involves subjective decisions (Armstrong, 1985). It is possible to test model reliability and credibility and in doing so it achieves a greater assurance that the model is of use.

The test of reliability is whether another researcher can follow the same approach to model building and arrive at the same outcome. In terms of this research, it will be shown that it is possible to generate census type compartmental flow models of patient occupancy (see Chapter 5).
There are three tests of model validity, namely: face validity, predictive validity and construct validity (Armstrong, 1985). Face validity is the weakest of the validity tests. It involves determining whether the model appears reasonable by people who should be in a position to judge such an outcome (Armstrong, 2001). In relationship to this research, it will be shown that there is a high level of face validity (see later sections of this chapter and also Chapter 5).

Predictive validity relates to determining whether the model inputs are valid in terms of being used to create the intended model output (Armstrong, 1985), or whether the model is useful for making forecasts (Armstrong, 2001). Given that the research interest relates to modelling hospital bed occupancy, it will be demonstrated that the variables used to generate the model output, both in terms of explanatory and forecasting purpose, are appropriate (see Chapters 5 and 9).

Constructive validity aims to determine if a measurement is measuring what is intended to be measured (Armstrong, 1985). In terms of the patient flow occupancy models this will be clearly demonstrated (see Chapter 5).

### 4.3.2 Testing the model outputs

Increased assurance that a model is useful is also gained from testing the model outputs (Armstrong, 1985). Such testing can lead to further model development and refinement. Hastie, Tibshirani and Friedman (2001) suggest that models be created using training data, evaluated using test data (cross validation) and then, if successful, used for forecasting purposes as illustrated in Figure 27.
Figure 27: According to Hastie, Tibshrani and Friedman (2001), model development should include the use of both training and test data sets.

In terms of the research undertaken for this thesis, the benefit of using training and test data sets is demonstrated in Chapter 5.

Differences in approaches to testing model outputs are discussed later in this chapter.

4.4 Model choice

As previously indicated, more than one approach may be adopted for modelling data (Davison, 2003). Although the number of potential models that could be created is large (known as the model space), only a subset of the potential models are evaluated as shown in Figure 28.
Clearly, limiting the number of potential models that are tested is a reasonable course of action in order to achieve:

- A timely solution – often modelling solutions take time to develop and the testing of a large number of models will prolong model development with the potential for little real gain, and
- A cost effective solution – often models are created using limited resources, thus preventing the testing of a large number of alternative solutions.

Additionally, modelling may be undertaken on a contractual basis in a commercial setting, where those undertaking the work have successfully won the right to develop a particular solution. Under such circumstances it could be assumed that those selecting the model have considered the consequences of not testing a greater range of models beyond the mere consideration of initial development costs and timing considerations. From personal experience, this assumption is unlikely to hold true.

Figure 28: The model space represents the total possible number of models that could be fitted to the data. The number of models actually tested represents a small proportion of the model space.
Fone et al. (2003) reported that the value of the discrete event simulation models created for health sector problems remains unclear. It is likely that this finding extends to all modelling undertaken in the health sector. Furthermore, in the review undertaken by Fone et al. (2003), it was reported that the quality of models has not been routinely evaluated, and nor has there been investigation to establish the extent to which findings have been translated into policy. Given the political sensitivity of many health care issues, it is perhaps not surprising that the evaluation of modelling and its use does not occur.

The value of modelling is that it facilitates understanding and enables predictions to be made about the future (Hastie, Tibshirani and Friedman, 2001). However, for this to occur, there is a requirement that the choice of the model being used is appropriate. In terms of the research carried out for this thesis, the number of different types of models tested could be argued to be limited. The decision to limit the work to compartmental flow models was made on the following bases:

- The understanding of the problems associated with using the ALOS as the basis for modelling, that is, the extent to which the model addressed a skewed LOS data profile
- There was (and will be) an ability to use the modelling for strategic decision-making purposes (as opposed to operational decision-making purposes)
- The extent to which research had already been carried out on the modelling approach (many approaches are only reported once in the literature, which suggests insufficient research or interest in the modelling approach)
- That previous work has shown that the compartmental flow models describe the acute care situation well although without appropriate validation (Mackay and
• At the strategic level, there was an absence of alternative models, and
• The pragmatic need to limit the research undertaken to meet the resource and time requirements associated with the research degree being undertaken.

As previously indicated, the work of Millard and his colleagues ((Harrison and Millard, 1991; McClean and Millard, 1993; Harrison, 1994; McClean and Millard, 1994; McClean and Millard, 1995; McClean and Millard, 1998) provided a strong foundation for this research. They have shown that the compartmental model can be used to describe patient occupancy profiles, and so provide a useful basic model.

While the focus of this research has been on compartmental flow models this has not prevented the issue of model choice from being explored. There are two approaches that can be used to fit a model to the data – trialling different types of model classes and improving a given class. Within the class of compartmental flow models it is possible to create a range of differing models that describe bed occupancy and can be assessed to determine if model performance is improved. In terms of model types, my intent was not to consider alternative model classes, but to focus on different applications of the compartmental flow model originally proposed by Harrison and Millard (1991). There were good reasons for this, namely that previous work has shown that compartmental flow models describe the acute care situation well although without appropriate validation (Mackay and Millard, 1999; Millard, Mackay, Vasilakis and Christodoulou, 2000; Mackay, 2001) and that at the strategic level, there is an absence of alternative models.
Modelling, however, must strike the right balance between fit and complexity (Hastie, Tibshirani and Friedman, 2001).

4.5 Model Fit and Complexity Trade-Off
It is possible to develop models that fit the data very well. Often the ability to gain improvement in model fit is achieved by increasing the level of model complexity. There is, however, a trade-off in achieving an improved level of model fit as the model may become less useful for generalising similar scenarios where there is no data (for example, another hospital that lacks the appropriate data) or forecasting future events (for example, at the same hospital for a different time period), that is the model is over-fitted (Myung and Pitt, 1997; Roberts and Pashler, 2000; Lee, 2004; Hastie, Tibshirani and Friedman, 2001). The problem of increasing model fit and the ability to generalise is illustrated in Figure 29.
According to Hastie, Tibshirani and Friedman (2001), the generalisation performance of models relates to the predictive ability of the model using independent test data. Consequently, model development requires at least two stages: model development and model testing. Each stage requires the use of separate data sets, namely, training data and test data, respectively. Analysis of the errors arising from the training data is insufficient to determine the predictive ability of the model. The absolute error and squared error are typical measures used to analyse model performance (Hastie, Tibshirani and Friedman, 2001).

While the compartmental flow model for describing bed occupancy has been well documented (for example, Harrison and Millard, 1991), at least in relation to an English geriatric patient system, discussion about appropriateness of the level of model complexity, the degree of fit and the ability to use the model for generalisation
and forecasting has been lacking. For example, BOMPS, the resulting software package from Harrison and Millard’s 1991 work, provided statistics concerning the fit of the model to the data, but had no mechanism that enabled the user to create a testing set of data to examine the predictive ability of the model.

4.6 Model Choice and Performance

The task of model choice is twofold (Hastie, Tibshirani and Friedman, 2001). First, model selection must occur, that is, the best model from those being scrutinised must be chosen. Second, the predictive performance of the chosen model must be evaluated, that is, model assessment.

4.6.1 Model performance – common approaches

There are various approaches to determining whether a model describes the data. The manual accompanying the BOMPS software package suggests using visual inspection of fit, and correlations and least squares statistics as a means of gauging model fit (for example, BOMPS manual, 1992; Harrison and Millard, 1991). These methods can provide those interpreting the model output with some guide as to whether the modelling has described the data well or not.

The Harrison and Millard model achieved a good fit to the underlying data. It is recommended in the BOMPS manual (1992) that models with a correlation of less than 0.9 be discarded and that a low least squared error can help discriminate between two or more models. However, reliance upon only these statistics may result in selection of an over-fitted model. As previously described, there is a trade-off in terms
of the usefulness of a model for generalisation and forecasting purposes if it is over-
fitted.

Various authors (for example, Hastie, Tibshirani and Friedman, 2001; Berenson, Levine and Krehbiel, 2002) have suggested that the absolute error and squared error are typical measures used to analyse model performance. Other measures, such as the log likelihood have also been recognised as measures of fit (Hastie, Tibshirani and Friedman, 2001).

The various model performance statistics used in this research are now discussed.

**Visual inspection**
It is stated in the BOMPS manual that visual inspection of the fit of the model to the data should occur (BOMPS manual, 1992). Visual inspection can reveal whether a model fits the data or not. Motulsky and Ransnas (1987) concur with this view and state that many potential problems are best identified from a visual inspection of the model superimposed on the data. However, visual inspection alone is insufficient to enable selection between models that fit the data well. Consequently, other measures of model fit are also required (Motulsky and Ransnas, 1987).

**Correlation**
The sample coefficient of correlation, r, measures the strength of association (Berenson, Levine and Krehbiel, 2002) between two variables (X and Y) and is calculated according to:
where

\[ r = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^{n} (Y_i - \bar{Y})^2}} \]

\( X_i \) and \( Y_i \) represent the \( i \)th observations of variable \( X \) and \( Y \), respectively, and \( \bar{X} \) and \( \bar{Y} \) represent the mean of variable \( X \) and \( Y \), respectively.

The relationship between the model and the data can be measured using correlation. Compartmental flow models of bed occupancy have achieved high correlations, with \( r \) often exceeding 0.95 (for example, Mackay and Lee, 2005). As previously indicated, it is indicated in the BOMPS manual that models should achieve a correlation value (\( r \)) greater than 0.9 (BOMPS manual, 1992).

It is possible for the strength of association between the model and the data to be high and the model still not represent a good fit. An example of this is where the model lies parallel to the data, but is shifted up or down. In such a situation, a high correlation will be reported, yet the model may not be a good fit of the data. Thus, as a method of model assessment, the correlation must be used in conjunction with other measures such as the absolute error.

**Absolute and Squared Errors**

The difference between a model (the predicted value, \( \hat{Y}_i \)) and the observed data (\( Y_i \)) is known as the residual or estimated error (\( e_i \)) (Berenson, Levine and Krehbiel, 2002) and can be expressed as:
\[ e_i = Y_i - \hat{Y}_i \]

One means of assessing model fit is to plot the residuals (Berenson, Levine and Krehbiel, 2002). In the case of the bed occupancy compartmental flow models, the literature and experience (for example, Harrison and Millard, 1991; Mackay and Millard, 1999) has shown that there is a high level of fit between the model and data. Thus, there is little to be gained from the inspection of the residuals. (Good practice, however, would suggest that either visual inspection of the model fit to the data or inspection of the residuals should occur.)

This research has not involved the comparison of models of different types, but rather models within the same class. According to Berenson, Levine and Krehbiel (2002), if assessment of a residual analysis is not sufficient to distinguish between two types of models, measurement of the magnitude of the residual error is appropriate.

Summation of the estimated error is not useful as errors may lay either side of a fitted model. Thus, the positive and negative values may cancel each other out and not yield useful information about the nature of the model fit. Two approaches overcome this limitation – the use of the absolute error and least squared error. Both of these approaches remove the negative value that may arise with the estimated error, one though involves taking the absolute value of any difference (absolute error), while the other involves squaring any error (squared error). Summation of only positive values can provide useful information about the fit of the model to the data.
According to Hastie, Tibshirani and Friedman (2001), but keeping the same notation as Berenson, Levine and Krehbiel (2002), the absolute and squared errors are found by:

\[
\text{Absolute error} = |Y - \hat{Y}|
\]

\[
\text{Squared error} = (Y - \hat{Y})^2
\]

Summing the errors is therefore of use. The difference between the two errors is the weighting placed upon larger errors. This is perhaps best illustrated with an example.

<table>
<thead>
<tr>
<th>Error size</th>
<th>Absolute Error</th>
<th>Squared Error</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Error is the same.</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>16</td>
<td>With a larger error, the weighting in the squared error is revealed.</td>
</tr>
</tbody>
</table>

The squared error places greater emphasis on large errors.

It is also common to use the mean squared error (MSE) (Myung, Pitt and Kim, 2005) or mean absolute error (MAD) (Berenson, Levine and Krehbiel, 2002). Using the mean does not overcome the weighting placed upon large errors when calculating the squared error.

Berenson, Levine and Krehbiel (2002) state that statisticians have not reached consensus as to which measure of error is preferred. As the absolute error is perceived
as being more easily understood, primarily as a consequence of the lack of weighting of the error, it was used as a measure of fit for this research.

### 4.6.2 Least Squares and Maximum likelihood

The original work by Harrison and Millard (1991) relied upon minimising the squared error for model fitting purposes, which is commonly known as the least squares method (Berenson, Levine and Krehbiel, 2002). The sum of squared errors (SSE) is given by (Myung, Pitt and Kim, 2005):

\[
SSE(w) = \sum_{i=1}^{m} (y_i - y_{i,\text{prd}}(w))^2
\]

where \(y_{i,\text{prd}}(w)\) is the model’s prediction for observation \(y_i\) and \(w\) are the parameters of the model (that is, for \(A\), \(b\), \(C\) and \(d\) for the bed occupancy compartmental flow model).

It is recommended in the BOMPS manual that the selection of the bed occupancy model with the lowest least squares value occur (BOMPS manual, 1992). The use of least squares is a common approach (Hastie, Tibshirani, and Friedman, 2001; Berenson, Levine and Krehbiel, 2002; Davison, 2003).

The residual sum of squares or least squares continues to decrease as the complexity of the model increases, that is, the number of model parameters increase. Consequently, model choice based upon least squares will always result in the most complex model being preferred (assuming that the models being tested all fit the data.
well). This, however, is not consistent Ockham’s Razor (Sivia, 1996; Davison, 2003) and the notion of parsimony (Motulsky and Ransnas, 1987; Davison, 2003) where the least complex solution that achieves the description of the data is preferred. In order to achieve good model fit without undue complexity it was necessary to use an alternative approach for this research.

The other generally accepted approach for parameter estimation is the maximum likelihood estimation (MLE) (Myung, Pitt and Kim, 2005). The maximum likelihood value was used to determine the fit of model parameters given the data. Maximum likelihood was used in preference to least squares for the following reasons:

- It is mathematically tractable (Berger, 1985)
- Under common-variance Gaussian conditions, least squares and maximum likelihood are the same (Murshudov, Vagin and Dodson, 1997)
- The use of the maximum likelihood was consistent with Murshudov, Vagin and Dodson (1997), who found that refinement of models was better when maximum likelihood methods were used in preference to least squares, and
- Enables the calculation of other values, such as the Bayesian Information Criterion, that can be used in model selection (see the next section for more detail).

According to Myung and Pitt (2002) maximising either the log likelihood or the likelihood will lead to the same outcome, as both are monotonically related to each other. The log-likelihood, however, is computationally easier and therefore preferred. The log-likelihood (loglik) function is given as (Myung, Pitt and Kim, 2005):
In terms of parameter estimation, by maximising the likelihood function, the parameter values are those that are most probable given the data (Sivia, 1996; Myung, Pitt and Kim, 2005). The best-fit likelihood ratio will be the dominant factor in model choice where the data is of good quality (Sivia, 1996). However, where the performance of competing models is similar, in terms of fit, then preference will be given to the model that enables more values to be selected for model parameters, that is, the data range for each parameter is greater (Sivia, 1996). In essence, this will mean that the more complex model is less preferred than the simpler model. This is in keeping with the previously mentioned Ockham’s Razor (Sivia, 1996; Davison, 2003) and the notion of parsimony (Motulsky and Ransnas, 1987; Davison, 2003). While this may seem counter intuitive, at least initially, the selection of a model that enables more data to be fitted, facilitates model generalisation as well as description of the underlying data. For example, if hospital occupancy models were generated using 2, 4 and 6 parameters (for example, as done with compartmental flow models) and have maximized log likelihood values of 0, 10 and 11, respectively, then the four parameter (for example, double compartment) model is preferred over the two parameter (for example, single compartment), but it is not clear whether the four or six parameter (for example, double or triple compartment) model is preferred (Davison, 2003).
The use of the maximum likelihood also enables different model selection statistics to be used. This leads to discussion about likelihood values, Bayes Factor and Bayesian Information Criterion (BIC).

4.6.3 Bayesian Information Criterion (BIC)

A range of statistical performance indicators for model fit has been discussed. Some of these statistics come from developments in classical probability theory. Classical probability theory relies upon observation of data and those subscribing to this theory are sometimes referred to as frequentists. Frequentists rely upon hypothesis testing and accept or reject a given hypothesis on the basis of a probability value \(p\).

However, the Bayesian school of thought relies on conditional probability, that is, prior information is taken into account when finding the probability of another event (Berenson, Levine and Krehbiel, 2002; Davison, 2003). While Bayes developed the initial work that supports this school of thought in the 18th century, the work of Laplace, and then later Jeffreys, significantly contributed to its advancement (Davison, 2003).

The performance statistics discussed thus far (and others) do not penalise selection of complex models or do not provide sufficient penalty (for example, the use of the chi-squared statistic). The BIC, which is a Bayesian development, does impose a penalty for complexity and is therefore better suited for model selection purposes (Tashman and Hoover, 2001). BIC is given by:
\[ \text{BIC} = -2LL + k \ln n \]

where

- \( LL \) is the log likelihood,
- \( k \) is the number of parameters, and
- \( n \) is the number of observations (Myung, Pitt and Kim, 2005).

Another similar model selection statistic is the Akaike Information Criterion (AIC) (Tashman and Hoover, 2001). The AIC formula does not include the number of observations. Armstrong (2001) suggests that there is some preference by practitioners for using BIC over AIC. Hastie, Tibshirani and Friedman (2001), however, state that there is no clear choice between the BIC and AIC. For very large \( n \), the AIC will lead to selection of a model that is too complex (Hastie, Tibshirani and Friedman, 2001). BIC on the other hand will give consistent model selection for large \( n \), and as \( n \to \infty \) the probability of BIC selecting the correct model, assuming that the true and correct model has been fitted to the data, approaches 1. For finite samples, however, the heavier penalty placed upon complexity by BIC may lead to the selection of less complex models with the consequence of under fitting or a too parsimonious model (Hastie, Tibshirani and Friedman, 2001; Davison, 2003). Contrary to the views of Hastie, Tibshirani and Friedman (2001), Kass and Raftery (1995) have indicated that in the more usual situation BIC will tend to favour simpler models (when compared to the AIC), is easy to use and does not require evaluation of prior distributions. They conclude that it is well suited for summarizing results in scientific communication. Given the inclusion of the number of data in the BIC
calculation, and taking into consideration the views of Kass and Raftery (1995), the BIC statistics was used in this research.

One potential criticism of the BIC (and also of the AIC) is that it does not take into account the functional form of models (Pitt and Myung, 2002). The functional form of a model is the way in which model parameters are combined. Thus, it is possible to have two models with the same number of parameters, but have the model parameters used in different ways (for example, additive versus multiplicative), and assuming the model fit was similar the BIC would not be of use in selecting between the two models. Model selection methods, such as the Bayesian model selection and minimum description length, do take into account the functional form of models (Pitt and Myung, 2002). Ideally functional form would have been taken into account in this research, but it is computationally difficult. Additionally, it was considered that the BIC and Bayes Factor approximations would be sufficiently accurate given conservative interpretation of results, and the availability of large and informative data sets when comparing models with similar functional forms.

The calculation of the BIC enables the comparison of models through the calculation of the Bayes factor, which is now discussed.

4.6.4 Bayes factor

According to Goodman (1999), the Bayes factor provides a comparison of how well the data is predicted by two hypotheses. Furthermore, it is objective and can be used as a measure of evidential strength in lieu of the frequentists’ $p$ value. The Bayes factor ($B_{ij}$) for model selection is given by:
\[ B_{ij} = \frac{p(D|M_i)}{p(D|M_j)} \]

where \( D \) is the training data and \( M \) is the model (Hastie, Tibshirani and Friedman, 2001).

In this case model \( M_i \) is preferred over \( M_j \) if the odds are greater than one (Hastie, Tibshirani and Friedman, 2001). Although model \( M_i \) may be preferred over \( M_j \) the strength of the evidence in making this choice needs to be considered (Goodman, 1999). Various ranges of the Bayes factor have been reported in relation to the strength of the evidence provided for accepting (in this case) one model over another (for example, Jeffreys, 1967; Kass and Raftery, 1995; Goodman, 1999). A Bayes factor that is low provides weak evidence for accepting one model over another, while a Bayes factor that is large (for example, greater than 150) provides very strong evidence.

The Bayes factor \( (B_{ij}) \) is difficult to calculate (Kass and Raftery, 1995), but may be approximated by:

\[ -2 \ln B_{ij} = BIC_i - BIC_j \]

Rearrangement of the above equation yields the Bayes factor, \( B_{ij} \). The above can be used and the difference between the two BIC values is referred to as “log odds”. These are posterior odds and according to Berger (1985) is something that many more people prefer instead of using probabilities.
While the BIC provides a statistic that can be used for model selection purposes, it does not take into account the qualitative need of increasing complexity, that is, the value of the extra information gained from the inclusion of additional model complexity. Thus, the BIC, and therefore also the Bayes factor, must be used in conjunction with judgment to facilitate selection of the most appropriate model.

This research was concerned, in part, with model selection and thus use was made of the BIC and Bayes factor.

4.6.5 Cross validation and Bayes factor

As previously indicated (see section 4.3.2), Armstrong (1985) and Hastie, Tibshirani and Friedman (2001) support the notion of model testing using test data. This is known as cross validation and aids in the selection of models that generalise better (that is, cross validation should highlight when models fit the test data poorly as a consequence of increased complexity or other factors) Kass and Raftery (1995) indicate that such an approach can be adopted no matter whether an individual holds Bayesianist or frequentist views. Cross validation provides a practical solution to model selection.

Cross validation, however, relies upon the existence of two sets of data – the training and test data. Ideally the training and test data should be independent and representative. For example, in the case of bed occupancy data the variation that occurs across a year should be captured. Thus, splitting the data into two six month periods would lead to independent training and test data, but the variation attributable
to seasonality (for example, summer and winter) would not be represented similarly in both data sets. An alternative method of generating the test data set is to withhold some of the training data (Hastie, Tibshirani and Friedman, 2001). In the case of an annual bed occupancy model this would require random selection of records relating to approximately 180 days across the year for the training and test data sets. The downside to such an approach is that the data available for model training is reduced which is not preferred (see Chapter 5 for results relating to the number of data). While repeated sampling of the data to create multiple training and test data sets, known as bootstrapping methods (Hastie, Tibshirani and Friedman, 2001), may increase the confidence about the resultant model, it necessitates additional model development and testing time. Furthermore, in some instances it may be difficult to split the available data into two parts, thus leading to the situation where no test data is available.

In the absence of test data, the Bayes factor, as approximated by the Bayesian information criterion enables model selection decisions to occur and thereby reducing the possibility of selection of overly complex models. While there is theoretical support for using the Bayes factor for model selection purposes (see section 4.6.4), it perhaps provides a less practical approach to model selection compared to cross validation. Furthermore, the adoption of this approach for model selection purposes can lead to model development using all the available data (that is, there is no requirement for data splitting to create a test data set).

Kass and Raftery (1995) indicate that the degree to which the forecasts from a model predict the future ultimately determines how well a modelling task has succeeded.
The aim of model selection using either cross validation or the Bayesian information criterion is to select a model that generalise well and that can be used for forecasting. Thus, while it may be expected that the same models may not be chosen using these techniques, it would be hoped that the models would be sufficiently close to achieve forecasts that are reasonably similar.

4.7 Conclusion

The purpose of this chapter was to provide the necessary background and theoretical foundations relating to the development of models of hospital bed occupancy and model selection techniques that relate to the research presented in the remainder of this thesis. This has been done.

The subsequent chapters present the methodologies and results of the modelling approaches used in my research. Chapter 5 reports on whether the model originally proposed by Harrison and Millard (1991) for modelling geriatric hospital patient bed occupancy can be applied to an Australian acute care sector hospital. Additionally, the questions of how much data should be used to model acute hospital bed occupancy and the level of model complexity required for compartmental flow models are explored.
Chapter 5

Choice of Models for the Analysis and Forecasting of Acute Care Hospital Beds

In this chapter I investigate whether the model originally proposed by Harrison and Millard (1991) for modelling geriatric hospital patient bed occupancy can be applied to an Australian acute care sector hospital. Additionally, I address the questions of how much data should be used to model acute hospital bed occupancy and the level of model complexity required for compartmental flow models. The chapter has the following structure:

5.1 Introduction ............................................................................................... 161
5.2 Methodology ............................................................................................ 162
5.2.1 The Harrison and Millard Approach ...................................................... 162
5.2.2 Methodology 1 – using all the data......................................................... 162
5.3 Results ....................................................................................................... 172
5.4 Discussion .................................................................................................. 184
5.4.1 Does the Model Succeed in Describing Acute Care Data? ...................... 184
5.4.2 Validity and Reliability ........................................................................... 186
5.4.3 Should the Single Day Census Style of Model Continue? ....................... 187
5.4.4 The Fit and Complexity Trade-off ......................................................... 189
5.4.5 Other Issues ........................................................................................... 189
5.5 Conclusion ................................................................................................ 190
5.1 Introduction

The background regarding the need for this research has been covered in previous chapters and will not, in the main, be reiterated here. It is useful, however, to highlight the two areas in which the issue of data and model application can be explored, namely:

1. Increasing the number of data (that is, census type models versus models that are based upon consideration of more data), and

2. Creating more models to describe smaller parts of the period being modelled that were previously described with fewer models (that is, nesting models).

Consequently, research was undertaken to:

- Further validate that the compartmental flow model could be used to describe acute care hospital data.
- Explore the ramifications of increasing the complexity of the bed occupancy compartmental modelling approach promulgated by Harrison and Millard (1991) and others (Harrison, 1994; McClean and Millard, 1994, 1995 and 1998), and
- Demonstrate the use of measures that can be applied to such work to help identify the trade-off between the level of model fit, complexity and generalisation.

The results of the work investigating model complexity have been presented at a conference (Mackay and Lee, 2004a) and also have been published in a paper (Mackay and Lee, 2005).
5.2 Methodology

As previously indicated (see section 4.3) my intent is not to consider alternative model classes, but to focus on different applications of the compartmental flow model originally proposed by Harrison and Millard (1991). There are three areas in which the issue of data and model application can be explored, namely:

1. Increasing the number of data (that is, census type models versus models that are based upon consideration of more data)
2. Creating more models to describe smaller parts of the period being modelled that were previously described with fewer models (that is, nesting models), and
3. Creating models that adjust the compartmental flow models for other factors, such as seasonal variation, without the need to create multiple compartmental flow models.

The methodology dealing with the first two areas of exploration specified above is now presented. The methodology dealing with the last area of interest is covered in later chapters (see Chapters 6 and 8).

5.2.1 The Harrison and Millard Approach

It is useful to revisit the approach adopted by Harrison and Millard (1991) in their seminal work in order that the differences in methodologies used for this research can be more easily understood.

The approach adopted by Harrison and Millard (1991) relied upon taking snapshots or censuses of patient data. This is illustrated in Figure 30.
There are two possible types of occupancy data censuses that could be undertaken, namely one based on a date and the other based upon a patient cohort. Harrison and Millard (1991) chose to census on the basis of date.

Harrison and Millard (1991) found that a mixed exponential model fitted the data well. This can be represented as:

\[ Y = Ae^{-bx} + Ce^{-dx}, \]

where

- \( A \) = the number of occupied beds in the short-stay compartment
- \( b \) = the flow rate through the short-stay compartment

**Figure 30:** Harrison and Millard (1991) used a bed occupancy census from a single day, which is highlighted by the data enclosed in the red box. An alternative approach could have been to census a patient cohort, which is represented by the shaded numbers (along the diagonal). Ultimately, patients are discharged, which is not shown on the diagram.
C = the number of occupied beds in the long-stay compartment, and
d = the flow rate through the long-stay compartment.

The model parameters can be used to generate a range of useful information about bed occupancy (see Appendix II for the performance statistics formulae) and can facilitate what-if scenario testing. As previously stated, this modelling was incorporated in the BOMPS software package, a decision support system (McClean and Millard, 1993).

The stated assumption (Harrison and Millard, 1991) underlying the ability to census using a single date is that the occupancy profile will be relatively stable and thus a patient occupancy census taken on day $x$ will be very similar to that taken on days $y$ or $z$ or any other day. Another assumption is that patient mix is homogenous (Harrison and Millard, 1991; McClean and Millard, 1993). Using a single day census is very economical and providing the assumption holds true is therefore not unreasonable.

These assumptions would also apply to a census based upon a patient cohort. An occupancy profile based upon a patient cohort census will be made up of the differing LOS of patients admitted on the same date. Taking a census based upon a given date implies an interest in the activities of the hospital service on any given day as opposed to an interest in a particular group of patients. Consideration of occupancy profiles based upon patient cohorts has received attention more recently (Harrison, Mackay and Schaefer, 2005).

The census methodology, however, is not free from potential criticism. St George (1988) and MacStravic (2001) have reported that an annual average model of acute
hospital services will be insufficient to enable bed planning, as the variation within
the data will not be detected. Variation in the data stems from day of week and
seasonal differences in workloads as well as other factors. The exception to this will
be in the case of a stable system, where the term “system” refers to the factors that
generate the observed patient LOS profile. While Millard and his colleagues have
stated that the system used for the development of the model was stable (for example,
Harrison and Millard, 1991; McClean and Millard, 1993), evidence exists that
hospital systems are generally unstable and are challenged by differing workload
demands at different times of the week or year (St George, 1988; Mackay and
Gorunescu, 2001; MacStravic, 2001). Millard and his colleagues also acknowledged
system instability in later work (for example, Taylor, McClean and Millard, 1996;
Harrison, 2001).

As previously stated (see Chapter 4), useful models arise when the model is consistent
with knowledge of the system under study and that the explanations derived from the
model match experience with the system (Davison, 2003). From the work I have
undertaken in the Australian acute care hospital setting, it is known that the system is
not stable and that variation occurs on a weekly basis (Mackay and Gorunescu, 2001)
and across the year (Beltchev and Mackay, 2000). Furthermore, the research on
Australian data initially undertaken using the BOMPS software package made use of
the “average” data option as opposed to the “census” option and the resultant models
were found to fit the data well (Mackay and Millard, 1999; Millard, Mackay,
Vasilikas and Christodoulou, 2000). Consequently, the use of the census approach
does not underpin the modelling used for this research. Comparison to census models,
however, is made.
As previously stated (see Chapter 2), the work of Harrison and Millard (1991) provided a deterministic model of bed occupancy. The work of McClean and Millard (for example see McClean and Millard, 1994) introduced uncertainty and involved the development of a stochastic model of bed occupancy.

The reliance upon the census methodology by Harrison and Millard (1991) and the lack of consideration about model complexity and data needs in early research around compartmental flow models and acute care hospitals (Mackay and Millard, 1999; Millard, Mackay, Vasilakis and Christodoulou, 2000; Mackay, 2001) suggests new approaches to research. The methodology used to address the questions of how much data should be used to model acute hospital bed occupancy and the level of model complexity required for compartmental flow models is now described.

5.2.2 Methodology 1 – using all the data

De-identified data were used as the basis for this research. The data related to patients treated within the Medical Division and therefore excluded the majority of patients who had been admitted for surgical procedures. The contextual details about the data were described in Chapter 3 (see sections 3.2 and 3.3).

In keeping with Davison’s (2003) observations that useful models arise when the model is consistent with knowledge of the system under study and that the explanations derived from the model match experience with the system, the elective same-day patients were excluded from this analysis. While it is likely that same-day elective bed occupancy could also be modelled using compartmental flow models, it
is recognised that elective same-day patients are managed quite separately to other admitted patients. For example, different patients use some same-day elective beds several times during the same day. The same-day service may only operate on certain days of the week and for certain hours each day (that is, the beds are generally not available for patients admitted overnight or for emergency care). Thus, the business process of patient management is different. The ability to use compartmental flow models to model elective same-day patient activity is considered briefly in Chapter 10.

The data included the date and time of patient admission and discharge. A subset of the data was used to create a profile of the busiest time of day at the hospital based upon bed occupancy at various times of the day using the admission and discharge data. As a consequence of this analysis, instead of using midnight census data for the remainder of the analysis, which was the approach used by Harrison and Millard (1991), a midday day bed census profile was created for each day of 1998 and 1999 calendar years. The 1998 calendar year data was used for model training, while the 1999 data was retained for model testing.

The profiles provided a count of how many patients were in bed at midday for a given date and how many days patients had been in bed (that is, days since admission). The “days since admission” profiles did not indicate the total number of beds occupied on a given day for all patients admitted on that day or before, which is how the data would be represented in BOMPS software. To put the data in the format of that used in the BOMPS software, a reverse cumulative distribution was created (a type of ogive). This profile showed how many patients had been in bed for at least x days.
Thus, at 0 days, all patients currently admitted at the midday census would have been in bed for at least 0 days. The difference between the profile creation used in this approach and that adopted by Harrison and Millard (1991) is shown in Figure 31.

The advantage of using all of the data was that the resultant model would be based not upon a single day’s occupancy profile, but that of a year, which included various sources of variation (for example, daily and monthly variation). Thus, the model should better reflect the period and therefore be more useful for decision-making purposes.

The BOMPS software uses the least squares method to determine the model parameters that best-fit the cumulative pattern of bed occupancy. This assumes that

```
<table>
<thead>
<tr>
<th>Date</th>
<th>Reverse cumulative profile of days since admitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Sep-94</td>
<td>45 40 36 31 27 27 25 21 19 18 17</td>
</tr>
<tr>
<td>2-Sep-94</td>
<td>42 39 35 31 28 24 24 23 20 18 17</td>
</tr>
<tr>
<td>3-Sep-94</td>
<td>37 36 34 31 27 25 21 21 20 18 17</td>
</tr>
<tr>
<td>4-Sep-94</td>
<td>40 37 36 34 31 27 25 21 21 20 18 17 continues</td>
</tr>
<tr>
<td>5-Sep-94</td>
<td>36 32 29 28 26 24 21 19 16 16 16</td>
</tr>
<tr>
<td>6-Sep-94</td>
<td>31 28 24 22 22 20 18 15 13 11 11</td>
</tr>
<tr>
<td>7-Sep-94</td>
<td>32 25 22 20 18 18 16 15 13 11 9</td>
</tr>
<tr>
<td>8-Sep-94</td>
<td>36 30 23 20 18 16 16 15 14 12 11</td>
</tr>
<tr>
<td>9-Sep-94</td>
<td>39 30 25 19 17 15 14 14 13 12 11</td>
</tr>
<tr>
<td>10-Sep-94</td>
<td>35 30 25 21 15 14 12 11 11 11 10</td>
</tr>
<tr>
<td>11-Sep-94</td>
<td>39 33 28 23 19 14 13 11 10 10 10</td>
</tr>
<tr>
<td>12-Sep-94</td>
<td>46 36 30 26 22 18 14 13 11 10 10</td>
</tr>
<tr>
<td>13-Sep-94</td>
<td>50 44 35 29 26 22 18 14 13 11 10</td>
</tr>
<tr>
<td>14-Sep-94</td>
<td>48 38 33 27 21 20 17 13 11 11 9</td>
</tr>
</tbody>
</table>
```

Figure 31: All of the data is captured and used in this methodology as suggested by the box surrounding the data, whereas the Harrison and Millard (1991) approach is based upon a single day census approach.
each count is drawn from a Gaussian distribution with common variance. A more
common statistical assumption in modeling count data is that the counts follow a
Poisson distribution (for example, see Kohler, 1985). Previous research involving the
counts that define length of stay distributions in hospitals has used this assumption
successfully (for example, Wang, Yau and Lee 2002; Xiao, Lee and Vemuri 1999).

Although BOMPS relied upon least squares for optimization of the curve fitting,
maximum likelihood was chosen in preference. A two-compartment model for a range
of scenarios was fitted to the training data using the method of maximum likelihood.
Technically, this was done by minimising the negative log likelihood.

The model fitting process may be terminated too soon if the optimised parameter,
which in this case was the maximum likelihood, reaches a local optimum. This
situation is analogous to the presence of local peaks being present on a larger
mountain – it is possible to reach a local peak without actually reaching the top of the
mountain as illustrated in Figure 32.
Consequently, to avoid early termination of the model fitting process, the established
technique of seeding of parameters was employed (Motulsky and Ransnas, 1987;
Powell and Baker, 2004). Seeding involves the setting of the starting point for the
fitting of the model parameters. Seeding did not always result in improved model fit
and nor was it used for the fitting of the most complex models tested, as the
computing time was too great.

With the exception of two models, all models were based upon commonly used
periods that describe periods of time. For example, an annual model, a seasonal
model, a weekly model and models that separated the weekend periods from the
weekdays. Two models were based upon the analysis of the deviation of the
underlying data from the annual average model.

Figure 32: In order to avoid early termination of the model optimisation process, seeding was used.
Seeding provided a starting point for optimisation that resulted in the model fitting process being able to
overcome the presence of local optima.
Single day census profiles were also created for purposes of comparison. The profiles were generated on the basis of random selection of dates during the training data period, as well as choosing the days of minimum and maximum bed occupancy.

The goodness-of-fit achieved by optimisation was measured for models against the training data and against the test data. The test data were based upon the 1999 calendar year, although the period was amended to ensure that there was alignment of days of the week between the training and test data sets. Although a variety of measures of goodness-of-fit are possible (Hastie, Tibshirani and Friedman, 2001), the absolute error was used as well as the likelihood measure.

In order to test whether any benefit was derived from using a single day census type model as opposed to one that relied upon being trained with more data, single day census models were also generated on the basis of random selection of dates during the training data period, as well as choosing the days of minimum and maximum bed occupancy. These models were fitted in the same manner described for the models based upon consideration of more data.

A range of software was used to conduct this research. SPSS® for Windows (various versions) was used to generate the occupancy profile data. The data was consolidated using Microsoft® Excel. Microsoft® Excel was used for general analysis of the data sets and model results. Matlab® (version 6.1.0450, Release 12.1) was used to create the bed occupancy models.
5.3 Results

Table 7 details the types of models created for modelling acute care hospital bed occupancy training data from the Flinders Medical Centre. The training period data represented one calendar year.

Table 7: Types of models created and analysed. The number of model parameters reflects the complexity of the model, with the annual average being the least complex model.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Number of parameters</th>
<th>Increasing complexity (compared to annual average model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual average</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>annual with weekends</td>
<td>8</td>
<td>2 times</td>
</tr>
<tr>
<td>seasonal</td>
<td>8</td>
<td>2 times</td>
</tr>
<tr>
<td>seasonal with weekends</td>
<td>16</td>
<td>4 times</td>
</tr>
<tr>
<td>based upon annual model residuals</td>
<td>36</td>
<td>9 times</td>
</tr>
<tr>
<td>based upon annual model residuals - more detail</td>
<td>40</td>
<td>10 times</td>
</tr>
<tr>
<td>monthly</td>
<td>48</td>
<td>12 times</td>
</tr>
<tr>
<td>monthly with weeks</td>
<td>96</td>
<td>24 times</td>
</tr>
<tr>
<td>fortnightly</td>
<td>108</td>
<td>27 times</td>
</tr>
<tr>
<td>weekly</td>
<td>212</td>
<td>53 times</td>
</tr>
<tr>
<td>fortnightly with weekends</td>
<td>216</td>
<td>54 times</td>
</tr>
<tr>
<td>weekly with weekends</td>
<td>424</td>
<td>106 times</td>
</tr>
</tbody>
</table>

Most models were based upon disaggregation of the training data into periods that coincided with calendar events or seasonal events, where the term “seasonal” related to the common weather patterns of summer, autumn, spring and winter, as opposed to the specialised use of the term in operational research. Increased model complexity was achieved in two ways: increasing the complexity of the same model, which resulted in nested models; and model redesign, which resulted in a new model. An example of a nested model is where the less complex model is further disaggregated into smaller parts, such as the monthly model becoming the weekly model.
Two models were based upon disaggregation of the training data into periods that reflected similar patterns in the residuals arising from comparison of the total actual bed occupancy with the total occupancy arising from the annual average model. While these models were still nested in terms of sitting under the annual average model, they represented an alternative model design (that is, they were not based upon calendar or seasonal events, but residual error patterns).

The annual average model represents the least complex model used. An alternative manner of representing the increase in model complexity is expressing the number of parameters relative to simplest model. Thus, based on this approach the weekly model is 53 times as complex as the annual model (see Table 7).

In keeping with the original methodology used by Harrison and Millard (1991), census models were also created for single days from the training period. Census dates were randomly generated until dates for each day of the week were achieved.

The training models were optimised on the basis of maximum log-likelihoods. As previously indicated, this was done by minimising the negative log-likelihood. The smaller these negative log-likelihood values, the stronger the likelihood and better the model fitted the training data. Cross validation using the test data was performed to determine which model generalised best.

The absolute error for the census models and the annual average models was calculated for the training and test period. The absolute error for each census model
was also calculated using the training data from the corresponding census day. The errors are compared in Figure 33.
Figure 33: Comparison of the absolute errors for census and the annual average bed occupancy compartmental flow models. The annual average model performance was better than any census model performance (in terms of absolute error).
Figure 34 graphically represents the results of the model fit using the maximum likelihood as the performance measure. Model fit improved with increasing complexity as expected. The fit of the models to the test data, however, did not show an improvement with overly complex models. Rather the test showed initial improvement with a slight increase in complexity until the seasonal model was reached, after which model performance declined slightly, but remained stable.
Figure 34: Comparative fit, in terms of maximising log-likelihood, of models against training and test data.
The fit of the single day census models are also shown in Figure 35, which confirms
the findings shown in Figure 34 that the annual average model performance was
better than that of any census model evaluated. The absolute errors for all models are
reported in Table 8.

Table 8: Comparative model fit, measured in terms of absolute errors. The complex models perform
better than the single day census models.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>No. of parameters</th>
<th>Absolute Errors Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Training data - day</td>
</tr>
<tr>
<td>Single Day Census Models</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tuesday - random selection</td>
<td>4</td>
<td>102</td>
</tr>
<tr>
<td>Saturday - random selection</td>
<td>4</td>
<td>84</td>
</tr>
<tr>
<td>Thursday - random selection</td>
<td>4</td>
<td>65</td>
</tr>
<tr>
<td>Monday - random selection</td>
<td>4</td>
<td>63</td>
</tr>
<tr>
<td>Wednesday - random selection</td>
<td>4</td>
<td>73</td>
</tr>
<tr>
<td>Sunday - random selection</td>
<td>4</td>
<td>71</td>
</tr>
<tr>
<td>Friday - random selection</td>
<td>4</td>
<td>101</td>
</tr>
<tr>
<td>Monday - maximum occupancy</td>
<td>4</td>
<td>96</td>
</tr>
<tr>
<td>Friday - minimum occupancy</td>
<td>4</td>
<td>49</td>
</tr>
<tr>
<td>Complex Models</td>
<td></td>
<td></td>
</tr>
<tr>
<td>annual average</td>
<td>4</td>
<td>226</td>
</tr>
<tr>
<td>annual with weekends</td>
<td>8</td>
<td>225</td>
</tr>
<tr>
<td>seasonal</td>
<td>8</td>
<td>188</td>
</tr>
<tr>
<td>seasonal with weekends</td>
<td>16</td>
<td>181</td>
</tr>
<tr>
<td>based upon annual model residuals</td>
<td>36</td>
<td>149</td>
</tr>
<tr>
<td>based upon annual model residuals - more detail</td>
<td>40</td>
<td>143</td>
</tr>
<tr>
<td>monthly</td>
<td>48</td>
<td>136</td>
</tr>
<tr>
<td>monthly with weeks</td>
<td>96</td>
<td>131</td>
</tr>
<tr>
<td>fortnightly</td>
<td>108</td>
<td>125</td>
</tr>
<tr>
<td>weekly</td>
<td>212</td>
<td>114</td>
</tr>
<tr>
<td>fortnightly with weekends</td>
<td>216</td>
<td>117</td>
</tr>
<tr>
<td>weekly with weekends</td>
<td>424</td>
<td>108</td>
</tr>
</tbody>
</table>

The test data was checked twice. The first analysis involved fitting each model to the
test year data. The second analysis involved adjusting each model in a consistent
manner to allow for policy changes that affected the number of beds available. Figure
35 illustrates the differences between the two years in terms of the number of beds
made available. The second analysis was required as the health care system under examination was not static and consideration of model performance without such adjustment may not provide a correct reflection of model performance.

Figure 35: Comparison of available beds during the training and test years.

While the log-likelihood values were of use in terms of model optimisation, other measures, such as the correlation and absolute error, may be of value in interpretation of the fit between models and data. The correlation between the model performance with and without adjustment for policy change and the various models were similar as shown in Table 9.
Table 9: Correlation between models, training and test data.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Correlation Model v Training Data</th>
<th>Correlation Model v Test Data</th>
<th>Correlation Model v Test Data adjusted for policy change</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual average</td>
<td>0.975</td>
<td>0.979</td>
<td>0.979</td>
</tr>
<tr>
<td>annual with weekends</td>
<td>0.976</td>
<td>0.979</td>
<td>0.979</td>
</tr>
<tr>
<td>seasonal</td>
<td>0.984</td>
<td>0.983</td>
<td>0.982</td>
</tr>
<tr>
<td>seasonal with weekends</td>
<td>0.985</td>
<td>0.984</td>
<td>0.983</td>
</tr>
<tr>
<td>based upon annual model residuals</td>
<td>0.990</td>
<td>0.979</td>
<td>0.978</td>
</tr>
<tr>
<td>based upon annual model residuals - more detail</td>
<td>0.991</td>
<td>0.980</td>
<td>0.979</td>
</tr>
<tr>
<td>monthly</td>
<td>0.992</td>
<td>0.981</td>
<td>0.979</td>
</tr>
<tr>
<td>monthly with weeks</td>
<td>0.993</td>
<td>0.981</td>
<td>0.979</td>
</tr>
<tr>
<td>fortnightly</td>
<td>0.993</td>
<td>0.980</td>
<td>0.977</td>
</tr>
<tr>
<td>weekly</td>
<td>0.994</td>
<td>0.979</td>
<td>0.976</td>
</tr>
<tr>
<td>fortnightly with weekends</td>
<td>0.994</td>
<td>0.981</td>
<td>0.978</td>
</tr>
<tr>
<td>weekly with weekends</td>
<td>0.995</td>
<td>0.979</td>
<td>0.976</td>
</tr>
</tbody>
</table>

It can be seen that the correlations between the models and the training and test data were all high, indicating that the fit between the model and the data was good. The correlations also exhibit similar trends as the log-likelihood data and absolute error data, that is correlations improved with complexity in relation to the training data, but in terms of generalisation model performance, was best for the seasonal models.

Figures 36 to 38 illustrate the performance of the models in relation to the total bed occupancy. Total bed occupancy represents only a single data point of the data fitted to the models on each day.
Figure 36: The “seasonal model” represents a less complex model compared to those illustrated in Figures 37 and 38. The more general nature of this model (that is, reduction in complexity) results in a better fit of the test data compared to the more complex models.
Figure 37: The “monthly model with weeks” represents a more complex model than illustrated in Figure 36 and achieves a better fit to the training data. However, the model fit deteriorates when test data is used as seen by the emergence of greater variation between the model and test data.
Figure 38: The "weekly model with weekends" represents a highly complex model. The fit to the training data is superior compared to the models illustrated in Figures 36 and 37. However, the greater model complexity results in further deterioration of fit when compared to test data.
It can be seen that the fit of the training data total occupancy profile achieved by the model improved as model complexity increased. Increased model complexity, however, was not associated with improved model fit to test data total occupancy. The adjustment made to the models to reflect the changes in the total available bed numbers did result in a closer fit between the test data total occupancy and model total occupancy profiles.

### 5.4 Discussion

#### 5.4.1 Does the Model Succeed in Describing Acute Care Data?

Previous work using BOMPS suggested that acute care hospital data could be described well by a double compartmental flow model (see Mackay and Millard, 1999; Millard, Mackay, Vasilakis and Christodoulou, 2000; Mackay, 2001). Other work also supports the use of the compartmental flow model for modelling acute care activity (Riddington and Kearney, 1994; and Scully and Kearney, 1994). The results obtained from the methodology used in this work have confirmed these findings.

The data from an acute care medical service have been described well by the compartmental flow models (see Figures 33 to 38 and Tables 7 and 8). Based upon visual observation of total occupancy and a range of statistics, the compartmental models fit the data well in both instances.

There are limitations to the extent that I have validated the use of compartmental flow models being of use in describing acute hospital data, including:

- The modelling has excluded same-day elective patient data as the management of these patients often relies upon a different model of practice (notably the use
of one bed for many patients in a single day). This is discussed in more detail in Chapter 10.

- The focus on medical service provision. The way services are clustered varies from hospital to hospital. However, it is evident that surgical, obstetric, mental health and paediatric services have not been included in this analysis.
- The visualisation of the fit has been limited to consideration of total occupancy (see Figures 36 to 38) as plotting the data and model requires a three dimensional graph that is not easily interpreted (that is, there are occupancy profiles (data) for each day of the year and numerous models).

Millard, Mackay, Vasilakis and Christodoulou (2000) found that compartmental models described surgical data from an Australian acute care hospital. There is no reason to suggest that other acute care services will not be able to be similarly described using compartmental models. The exceptions will occur if the business model of how services are provided gives rise to fixed time service provision and this may be the case in relation to some aspects of obstetric activities due to the push to reduce length of stay to a fixed period of service for particular types of deliveries and also in relation to the provision of elective same-day patient services (see Chapter 10).

The issue regarding visualisation of the model fit arises from the difficulty of creating a graph that can be interpreted. For a model developed upon a census approach there is little difficulty in creating an appropriate graph – there are only two lines to represent, namely the data and the resultant model. Whereas for a model based upon more data, the ability to represent the data and the resultant model becomes more complex. For example, it is difficult to create a three dimensional graph that
represents data from 365 days and the resulting models and can still be easily interpreted. The use of the performance statistics overcomes this limitation to some extent. Given the resultant statistics there is no reason not to be confident of the model fit. Furthermore, the development of the second methodology provides a complete remedy (see Chapter 6).

5.4.2 Validity and Reliability

As discussed in Chapter 4, there are three tests of model validity, namely: face validity, predictive validity and construct validity (Armstrong, 1985). Face validity is the weakest of the validity tests and relies upon the judgment of experts in the field in which the modelling is occurring. Bed occupancy compartmental flow models for geriatric services already have established face validity through the involvement of Peter Millard, who was, at the time the original research was undertaken, a clinician in charge of a large geriatric health service in the England. Furthermore, face validity can be claimed on the basis of the involvement of mathematicians and statisticians such as Gary Harrison and Sally McClean, co-authors on bed occupancy flow models with Peter Millard, who can attest to the reasonableness of the modelling approach. Face validity of the model in relation to the acute care sector is achieved through reliance upon the previous work with the geriatric data, but also my views as an expert in the analysis of acute care health services.

Predictive validity relates to determining whether the model inputs are valid in terms of being used to create the intended model output (Armstrong, 1985), or whether the model is useful for making forecasts (Armstrong, 2001). The ability to generate a model that, subject to the incorporation of extraneous factors such as policy change,
provides a good fit of the test data confirms the predictive validity of the approach. In terms of explanatory power, Chapter 11 provides an illustration of how the explanatory power of the model can be harnessed (see section 11.3.1).

Constructive validity is achieved if a measurement is measuring what is intended to be measured (Armstrong, 1985). In terms of the patient flow occupancy models this is clearly the case, because the patient occupancy profile is the data on which the compartmental flow model of bed occupancy is based. Three separate pieces of research support this: the creation of a compartmental flow model for a chest pain service (Mackay and Millard, 1999) using BOMPS; the modelling of the surgical service using BOMPS (Millard, Mackay, Vasilakis and Christodoulou, 2000); and this research.

In terms of meeting the reliability criterion, it has been shown that the creation of bed occupancy compartmental flow models can be created using the same approach as that of Harrison and Millard (1991). For the acute care compartmental flow modelling reliance can be placed upon this research and also prior research using BOMPS (Mackay and Millard, 1999; Millard, Mackay, Vasilakis and Christodoulou, 2000).

5.4.3 Should the Single Day Census Style of Model Continue?

There are various approaches to determining whether a model describes the underlying data. The employment of simple statistics, such as least squares or correlations has been used, as has the visual inspection of graphed results. Such an approach is recommended in the manual that accompanies the BOMPS software package (BOMPS, 1992). This approach is not inconsistent with methods suggested
by Hastie, Tibshirani and Friedman (2001), in which least squares and the squared error are listed as typical examples of measures used to analyse the performance of models. These methods may provide those interpreting the model output with some guide as to whether the modelling has described the underlying data well or not.

The data of choice for the majority of work undertaken by Millard and his colleagues has been the single day census. The limitations of such an approach are described in section 5.2.1.

Model selection on the basis of model performance in relation to training data only is likely to lead to an over-fitted model that describes the training data well, but generalises poorly (Hastie, Tibshirani and Friedman, 2001). In order to determine if the model fits the data well it is necessary to analyse the model performance against both training and test data. Performance against both the training and test data was undertaken in this research and is shown in Figures 33 and 34, and Table 8 for both census models and models based upon more data.

The findings support the obvious point that more data are always better, providing appropriate model selection methods are used. Models learned from more data perform better than models constructed on the basis of a single census day when applied to an extended period of time, such as a year using data relating to medical patients from an acute care hospital. In many ways this is to be expected, as such models are more able to capture the variation across the year in the acute care setting.
5.4.4 The Fit and Complexity Trade-off.

As previously described in Chapter 4, model fit can be measured in a number of ways. In this research the maximum likelihood was used as a means of gauging model fit, along with measures such as correlation and absolute error.

Given the same level of data, increasing model complexity led to a better fit of the underlying data as might be expected (see Table 8 – absolute errors, Table 9 – correlations, and Figure 34 – maximum likelihood). While increased model complexity may result in a model that fits the training data better than a simpler model, the same results cannot be guaranteed when applying the models for predictive purposes as illustrated by the results for all model performance statistics used. The analysis has shown that increasing model complexity did result in over-fitting and parsimony was achieved with a relatively simple model. This finding is illustrated in Figures 36 to 38, which show the outcome on total bed occupancy when the preferred model and other models are used. It can be seen that while improved fit was achieved to the training data with the more complex models, this was not the case when the test data was evaluated.

5.4.5 Other Issues

The Influence of the Weather

The fact that the complexity of the seasonal model was found to be preferred to that over the simpler annual average model suggests that the weather is important in determining hospital bed requirements. While this finding may not be of great surprise to those working in the hospital sector, the influence of the weather on
The annual average bed occupancy compartmental flow model provides a better fit of the data than the census model. As with the census models, there are only four model parameters required for this model and it is easy to use this model for scenario testing purposes.

As stated earlier (section 5.2.1), St George (1988) and MacStravic (2001) have reported that an annual average model of acute hospital services will be insufficient to enable bed planning, as the variation within the data will not be detected. Thus, the preferred model as based upon the model selection techniques used in this research overcomes the difficulties identified by St George and MacStravic. However, the increased model complexity does increase the difficulty of undertaking scenario testing. Another approach that captures the variation in the data, while facilitating easier scenario testing, is perhaps necessary if such models are to be adopted for real world strategic decision-making purposes. Such an approach is developed and discussed in Chapter 8.

**5.5 Conclusion**

The issues of how much data is required for modelling and the ability to select models of differing complexity are important and have not previously been considered in relation to bed occupancy models. The research findings presented in this Chapter have:
- Provided further validation that the compartmental flow model can be used to describe acute care hospital data.
- Shown that creating models of bed occupancy based upon more data, as opposed to basing models upon census data, results in a bed occupancy model that better captures the variation in an acute care hospital setting.
- Shown that model complexity can be increased to better describe occupancy data, but that there is a trade-off in ability to generalise, particularly in relation to forecasting future periods, and
- Demonstrated that various approaches, including the use of the maximum likelihood statistic, can be applied to help identify the trade-off between the level of model fit, complexity and generalisation.

The validation and complexity issues are important as they enable the results of hospital occupancy modelling that relies upon compartmental flow models to be used for forecasting or generalisation purposes as opposed to gaining an understanding of specific historic events.

While this work has shown that there is a pay-off for increasing the level of model complexity, many avenues of model development have yet to be explored. There is growing concern in various countries that the methods of providing health services are, if not already, approaching a level that cannot be sustained by the population (Commission of the European Communities, 1999; World Health Organisation, 2002; and OECD, 2003) and this provides support for the need to link population changes to bed models. Clearly, the need to understand the relationship between resource use and the underlying population structure requires that further development of more complex models that capture age and resource use. The issues of linking
compartmental flow models to population change and model development will be explored in a subsequent chapter (see Chapter 7). The need to consider alternative methods of incorporating seasonality, where seasonality relates defined periods of the year, is also considered later (see Chapter 8).

In the next chapter I consider whether bed occupancy compartmental flow models can be used for the purpose of evaluation of change and I also introduce the use of the BIC statistic in relation to model choice.
Chapter 6

Modelling New Zealand Acute Care Occupancy

In this chapter the investigation into whether the model originally proposed by Harrison and Millard (1991) for modelling geriatric hospital patient bed occupancy can be applied to the acute care sector is continued using data from a different hospital and country. The application of new model selection techniques to the bed occupancy compartmental flow modelling approach is also continued. The ability to use the model for evaluation of past events is also illustrated. The chapter has the following structure:

6.1 Introduction ............................................................................................ 194
6.1.1 The Need to Establish Validity ............................................................. 194
6.1.2 Evaluation of Service Change .............................................................. 194
6.1.3 Application of Model Selection Techniques ........................................ 195
6.2 Methodology 2 – capturing variation through the average ....................... 195
6.2.1 Data ..................................................................................................... 195
6.2.2 Method ................................................................................................ 196
6.3 Results .................................................................................................... 200
6.4 Discussion .............................................................................................. 207
6.4.1 Model Fit ............................................................................................. 207
6.4.2 Explanatory value ................................................................................ 209
6.4.3 Alignment with experience ................................................................. 210
6.4.4 Portability of health system measures and validation ......................... 211
6.4.5 Expert Judgment .................................................................................. 213
6.4.6 Average models ................................................................................... 214
6.4.7 Other issues ........................................................................................ 215
6.5 Conclusion .............................................................................................. 215
6.1 Introduction

6.1.1 The Need to Establish Validity

The results from the previous chapter validated the use of the compartmental flow model for modelling acute care hospital data. It may be argued, however, that the validation is limited, because it was a single validation study. The results from the analysis of surgical data by Millard, Mackay, Vasilikas and Christodoulou (2000) provide further support that the compartmental model can be fitted to acute care hospital data. In this study, the patient type was different, that is, the data related to surgical patients. The data, however, were sourced from the same hospital as the data used in Chapter 5. It could be argued that the practices in the hospital around patient length of stay are not necessarily dissimilar between the medical and surgical patients (for example, in terms of the application of hospital wide policies) and thus, the additional validation is of limited value. Validation using data sourced from a different hospital is therefore of use and represents the basis of the investigation reported in this chapter. The investigation relies upon data sourced from an acute care hospital in New Zealand.

6.1.2 Evaluation of Service Change

As previously stated (see Chapter 3), the Internal Medicine Department at HealthCare Otago, New Zealand, underwent a period of significant service change. Apart from the issue of establishing validity of whether compartmental flow models can be used to model acute care, the aim of this investigation was also to determine if the compartmental flow models could be used to evaluate service change.
The results of the research undertaken for this research were presented at a conference (Mackay, Lee, Rae and Millard, 2004; Mackay, Lee and Walton, 2004) and form the basis of this chapter.

6.1.3 Application of Model Selection Techniques

In the previous two chapters the issue of model selection has been considered. The trade-off between increasing model complexity and model fit when generalising to the test data was obvious when the maximum likelihood statistic was plotted (see Chapter 5). The other model performance statistics supported this finding and there was no need to introduce the BIC or Bayes factor.

The BIC statistic and Bayes factor were discussed in Chapter 4 and are introduced in this chapter for model selection purposes.

6.2 Methodology 2 – capturing variation through the average

6.2.1 Data

De-identified data were used as the basis for this research. The data related to patients treated within the Internal Medicine Department of the Dunedin Hospital, which is not a surgical service. Same-day elective patient data were not excluded from the analysis. As the census was undertaken at midnight, however, there would be no same-day patient data included in the profile. The Dunedin Hospital is part of organisation HealthCare Otago. The contextual details about the data were described in Chapter 3 (see sections 3.2.2 and 3.3.2).
6.2.2 Method

The results obtained from the modelling using the methodology described in Chapter 5 (see Methodology 1) guided the development of the methodology described in this chapter. This methodology was used to examine the HealthCare Otago data and re-examine different aspects of the Flinders Medical Centre data (see later chapters).

The first methodology (see Chapter 5) relied upon every data point being examined in order for the model fitting to occur. When analysing data over the period of one year this generates a matrix in the order of 365 (days) by 140 (maximum time since admission) or 51,100 data points. Fortunately, not every day generated the maximum time since admission and the actual number of data was somewhat fewer than the maximum possible.

While such an approach does enable the variation in the data to be captured it is inefficient in terms of time. Additionally, while consideration of the entire data set is always sufficient (that is, encapsulates all variation), it is not always desirable to consider the entire data set (Schervish, 1995) and it is often possible to use a statistic to reduce the data and still attain sufficiency.
The second method employed relied upon the fact that for each period since admission, the data appeared to be Normally distributed. Thus, variation could be captured using the average occupancy profile for the year with the standard deviation. The number of data points was reduced to a matrix of 2 (average and standard deviation) by approximately 100 (maximum time since admission) or 200 data points. This is illustrated in Figure 39.

The reduction in computing time makes the methodology more appealing as a potential decision-making tool. A tool that takes several days to generate a bed occupancy model was considered unlikely to be tolerated well other than in academic exercises.

<table>
<thead>
<tr>
<th>Date</th>
<th>Reverse cumulative profile of days since admitted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>1-Sep-94</td>
<td>45</td>
</tr>
<tr>
<td>2-Sep-94</td>
<td>42</td>
</tr>
<tr>
<td>3-Sep-94</td>
<td>37</td>
</tr>
<tr>
<td>4-Sep-94</td>
<td>40</td>
</tr>
<tr>
<td>5-Sep-94</td>
<td>36</td>
</tr>
<tr>
<td>6-Sep-94</td>
<td>31</td>
</tr>
<tr>
<td>7-Sep-94</td>
<td>32</td>
</tr>
<tr>
<td>8-Sep-94</td>
<td>36</td>
</tr>
<tr>
<td>9-Sep-94</td>
<td>39</td>
</tr>
<tr>
<td>10-Sep-94</td>
<td>35</td>
</tr>
<tr>
<td>11-Sep-94</td>
<td>39</td>
</tr>
<tr>
<td>12-Sep-94</td>
<td>46</td>
</tr>
<tr>
<td>13-Sep-94</td>
<td>50</td>
</tr>
<tr>
<td>14-Sep-94</td>
<td>48</td>
</tr>
</tbody>
</table>

average: 39.7 | 34.1 | 29.6 | 25.9 | 22.6 | 20.3 | 18.1 | 16.1 | 14.6 | 13.5 | 12.6
standard deviation: 5.8 | 5.3 | 5.2 | 5.0 | 4.9 | 4.6 | 4.4 | 4.0 | 3.8 | 3.6 | 3.5

Figure 39: The average and standard deviation were calculated for each time period (using the reverse cumulative days since admitted profile) as opposed to using all the data points.
The HealthCare Otago data were provided in census format. Unlike the work relating to the data from the other hospital, no analysis of the busiest time of day was undertaken and the census was based at midnight. The data covered the period from June 1990 to September 2003. The data provided a count of how many patients were in bed at midday for a given date and how many days the patients had been in bed (that is, days since admission).

The “days since admission” profiles did not indicate the total number of beds occupied on a given day for all patients admitted on that day or before, which is how the data are represented in BOMPS software. To put the data in the format of that used in the BOMPS software, a reverse cumulative distribution was created (a type of ogive). This profile showed how many patients had been in bed for at least x days. Thus, at 0 days, all patients currently admitted at the midday census would have been in bed for at least 0 days.

Compartmental models were fitted to the data with the number of compartments varying between one and seven. The method of maximum likelihood was used to optimize the fit. Technically, this was done by minimizing the negative log likelihood. Given that it was assumed that the distribution was Gaussian (or Normal) the weighted summed squared error (WSSE) was used for fitting the data, as this is the log likelihood in this instance. The formula for the squared error was given in section 4.6.2. Maris (1993) contends that a more appropriate statistic is one where the squared errors ($SE_n$) are weighted by the reciprocal of the estimated variances ($\hat{\sigma}_n^2$) and this gives rise to the weighted squared error (WSE) as denoted by:
Summing the weighted square errors over \( n \) gives the weighted sum squared error (WSSE).

\[
WSE_n = \frac{SE_n}{\sigma_n^2}
\]

As with the first method, seeding of model parameters was also undertaken.

The goodness-of-fit achieved by optimization was measured for the model against the training data. Although a variety of measures of goodness-of-fit are possible (Hastie, Tibshirani and Friedman, 2001), the absolute error and Bayesian Information Criterion (BIC) were used. The BIC value provided information about the level of fit and complexity in the absence of using a training and test set of data. The absolute error only provided information about model fit to the data.

A range of software was used to conduct this research. Microsoft® Excel was used to generate the occupancy profiles relating to the HealthCare Otago data and also used for general analysis of data sets and model results. Matlab® (version 6.1.0450, Release 12.1) was used to create the bed occupancy models.

The methodology detailed in this chapter is relied upon in subsequent chapters. Consequently, the methodology presented in subsequent chapters is truncated and only aspects of the methodology that were not detailed in this chapter are reported.
The results of the research based upon this method are detailed in this chapter and also Chapters 7 to 11.

6.3 Results

The shape of the occupancy profile was established in Chapter 3 (Figure 17) and was consistent with being well described by a mixed exponential equation as used in the bed occupancy compartmental flow model.

Given the volume of data the smallest period to be modelled was a year, which is appropriate at the strategic level. The issue of how to best model the data is illustrated in Figures 40 to 42 where three of the obvious data groupings are presented.

Figure 40: One option was to create a model that described all the data as indicated by the blue line. The difficulty with this approach was that it covered two periods of different service provision arrangements.
Figure 42: The simplest splitting of the data that enabled the change in service provision arrangements to be captured was to model the pre and post service change periods (the blue line indicates the split of the data).

Figure 41: The creation of a compartmental flow model of bed occupancy for each year was also possible. The blue lines indicate the points where the data was cut in order to achieve these models. This represented the most complex solution.
Nine configurations of models that captured different time periods in the data were investigated and these are detailed in Table 10. The configurations were created on the basis of expert opinion and represent a small subset of the total possible configurations that could have been investigated. The creation of all possible model types, however, was not pragmatic and would lead to the creation of many models that would be dismissed as not being useful by those attempting to use the model results for strategic planning or evaluation purposes.

Table 10: Types of models created and analysed. The number of model parameters reflects the complexity of the model.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Number of Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>average 1990-2003</td>
<td>4</td>
</tr>
<tr>
<td>grouped years: 1990-96 and 1997-03</td>
<td>8</td>
</tr>
<tr>
<td>grouped years: 1990-96 and individual years: 1997, ..., 2003</td>
<td>32</td>
</tr>
<tr>
<td>individual years 1990, ..., 2003</td>
<td>56</td>
</tr>
</tbody>
</table>

Some models were created on the basis of ensuring that the pre and post service change periods were modelled separately. Other model configurations occurred on the basis of reducing the 14 years of data into groups of two or three year periods.

Four of the models created had the same number of parameters, that is, the same level of complexity. Three of these models were discarded for the remainder of the analysis, because the purpose of the analysis was to consider the effect of complexity
on model choice and the complexity of these four models was the same. The retained model was one that included disaggregation of the data on the basis of service change.

As expected, the model fit, as measured by the weighted sum squared error (WSSE), improved with increased complexity as reported in Table 11.

Table 11: Model complexity and performance. The BIC value indicated that one of the more simple models was the preferred choice and the likelihood of this choice being incorrect was low given the Bayes Factor scores.

<table>
<thead>
<tr>
<th>Model Description</th>
<th>Number of Model Parameters</th>
<th>WSSE</th>
<th>BIC</th>
<th>Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>average 1990-2003</td>
<td>4</td>
<td>1,237.9</td>
<td>1,265.8</td>
<td>9.9839E+240</td>
</tr>
<tr>
<td>grouped years: 1990-96 and 1997-03</td>
<td>8</td>
<td>100.2</td>
<td>155.9</td>
<td>1</td>
</tr>
<tr>
<td>grouped years: 1990-91, 1992-93, 1994-95, 1996, 1997-03</td>
<td>20</td>
<td>47.1</td>
<td>186.4</td>
<td>4213692.923</td>
</tr>
<tr>
<td>grouped years: 1990-96 and individual years: 1997,…, 2003</td>
<td>32</td>
<td>79.4</td>
<td>302.3</td>
<td>6.02286E+31</td>
</tr>
<tr>
<td>individual years 1990, …, 2003</td>
<td>56</td>
<td>7.3</td>
<td>397.4</td>
<td>2.65375E+52</td>
</tr>
</tbody>
</table>

The relationship between complexity, fit (error) and the BIC is illustrated in Figure 43.

Figure 43: The fit of the model to the data improved, generally, as the complexity of the model increased. While the usefulness of the model is improved by slightly increasing complexity, overfitting occurred once the numbers of parameters exceed 20, as shown by increasing BIC values.

The relationships between the data for each year and the resultant models are illustrated in Figures 44 and 45.
Figure 44: The model fitted the pre-service change period data for each year well, although some years were described better than others.

Figure 45: The model fitted the post change period data for each year well. Variation between the model and data was expected, as the aim was to avoid over-fitting of the data. The maximum duration of patient stay was reduced compared to the pre-service change (hence the changed x-axis scale).
The relationship between the data from each year and the resultant models were examined using correlations and absolute errors, which are performance measures similar to those recommended in the BOMPS manual (1992), as reported in Table 12.

Table 12: Additional performance statistics supported the findings from visual inspection that although there was some variation between the model to the individual year data, the fit was very good.

<table>
<thead>
<tr>
<th>Year</th>
<th>Pearson Correlation</th>
<th>Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990</td>
<td>0.9972</td>
<td>103.6</td>
</tr>
<tr>
<td>1991</td>
<td>0.9988</td>
<td>58.6</td>
</tr>
<tr>
<td>1992</td>
<td>0.9999</td>
<td>26.2</td>
</tr>
<tr>
<td>1993</td>
<td>0.9993</td>
<td>41.2</td>
</tr>
<tr>
<td>1994</td>
<td>0.9987</td>
<td>68.6</td>
</tr>
<tr>
<td>1995</td>
<td>0.9991</td>
<td>25.0</td>
</tr>
<tr>
<td>1996</td>
<td>0.9989</td>
<td>35.5</td>
</tr>
<tr>
<td>1997</td>
<td>0.9987</td>
<td>10.9</td>
</tr>
<tr>
<td>1998</td>
<td>0.9999</td>
<td>22.4</td>
</tr>
<tr>
<td>1999</td>
<td>0.9993</td>
<td>14.1</td>
</tr>
<tr>
<td>2000</td>
<td>0.9998</td>
<td>5.8</td>
</tr>
<tr>
<td>2001</td>
<td>0.9999</td>
<td>15.0</td>
</tr>
<tr>
<td>2002</td>
<td>0.9997</td>
<td>46.5</td>
</tr>
<tr>
<td>2003</td>
<td>0.9992</td>
<td>28.7</td>
</tr>
</tbody>
</table>

The fit of the model to the data, where the data has been averaged over the two periods described by the preferred model, is illustrated in Figure 46.
Figure 46: Visual inspection of the models and data show that the fit was good. Furthermore, given the change in shape of the post-change model, it can be seen that there was a reduction in occupied beds and patients flowed through the system faster (the post change model line has a steeper slope).

In terms of other model performance statistics, the correlations between the models and data were all very high, and the absolute error values are all very low indicating that the models fit the data well as shown in Table 13.

Table 13: Commonly used model performance statistics indicated that the model fitted the data well.

<table>
<thead>
<tr>
<th>Period</th>
<th>Correlations</th>
<th>Absolute Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-1996</td>
<td>0.9999</td>
<td>9.1</td>
</tr>
<tr>
<td>1997-2003</td>
<td>0.9998</td>
<td>8.0</td>
</tr>
</tbody>
</table>

While choosing the minimum BIC enables the preferred model to be selected, the Bayes factor (see Table 10) enabled comparison of the relative performance of the models tested. The odds of one of the other models performing better than the
preferred model are very small (almost negligible). Consequently, there was no reason to choose one of the other models.

The double compartment model facilitated analysis of the service through examination of the change in the model parameters. This information can be obtained from analysis of the model parameters as shown in Table 14.

Table 14: Analysis of the model parameters shows that the change in bed numbers and flow rate was not uniformly implemented across short and long-stay patient groups, with the long-stay patient group being changed more than the short-stay group.

<table>
<thead>
<tr>
<th>Service Period and % Change</th>
<th>Bed number</th>
<th>Flow rate</th>
<th>Service Period</th>
<th>Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre change</td>
<td>30.61</td>
<td>14.15</td>
<td>0.189</td>
<td>0.064</td>
</tr>
<tr>
<td>Post change</td>
<td>25.24</td>
<td>4.13</td>
<td>0.325</td>
<td>0.146</td>
</tr>
<tr>
<td>% change</td>
<td>-18%</td>
<td>-71%</td>
<td>72%</td>
<td>127%</td>
</tr>
</tbody>
</table>

6.4 Discussion

The purpose of this study has been threefold, namely:

- To provide a measure of validity regarding the use of compartmental flow models of bed occupancy for modelling acute care data
- To demonstrate that compartmental flow models of bed occupancy can be used as a means of evaluating the effect of service change, and
- To highlight the need to use model choice selection methodologies.

These issues will now be discussed.

6.4.1 Model Fit

It is evident from the results that the compartmental flow model originally proposed to model the behaviour of geriatric bed occupancy by Harrison and Millard (1991)
describes the bed occupancy data from an acute care setting in New Zealand well. Evidence is provided in terms of various statistical measures, such as the correlation and absolute errors (see Table 12 and Table 13) and also in terms of visual inspection (see Figures 42 to 45).

Fit and complexity trade-off

As previously stated, the value of modelling is that it facilitates understanding and enables predictions to be made about the future (Hastie, Tibshirani and Friedman, 2001). Fulfillment of two requisite conditions is necessary to achieve successful modeling, namely:

- The choice of the model being used is appropriate, and
- The right balance between fit and complexity is struck.

The use of the BIC value and the Bayes Factor provides evidence that the model selection has resulted in the choice of a model that adequately describes the data, particularly in the absence of test data, while not being overly complex and leading to a loss in ability to use the model to generalise or forecast. Such evidence cannot be gained from commonly used performance statistics, such as the correlation or absolute error (Hastie, Tibshirani and Friedman, 2001). Indeed Mayer (1975 and 1980) reported that when only training (or sample) data was used, reliance upon model selection methods, such as the R-squared statistic, resulted in models that predicted future outcomes poorly.

Figure 42 illustrates that model fit generally improves as complexity increases, but that the BIC value is minimised for a relatively simple model.
The calculation of the Bayes Factor (see Table 11) indicates that the odds of one of the other models being preferred to the chosen model (data split into two periods) are very small. This provides increased confidence in the use of the model for its intended purpose.

It may be argued that for the purposes of service change evaluation there is little value in the application of model choice methodologies given that no generalisation or forecasting was linked to the development of these models. However, the application of model choice may help to reduce to the tendency to create models that are designed to represent the finest detail (that is, they are unnecessarily complex) purely because of the fact that it can be done, which I have frequently seen occur in the health sector. This may stem from the fact that people can imagine how a large number of variables might impact on a process, without appreciating that their measures of the process are not accurate enough to support such a complicated model. Furthermore, model choice provides a defensible position when reporting the results of the evaluation (that is, a simple model is appropriate for evaluating the service change), and it can facilitate subsequent evaluation or analysis, such as extension of the results for other purposes.

6.4.2 Explanatory value

While the BIC value can be used to determine model choice, the value itself does not provide an indication of additional explanatory value gained through increasing model complexity.
The models were created using double compartments as it was understood that the service was used predominantly by older people, who tend to have longer lengths of stay, and that the service redesign affected longer staying patients more so than shorter staying patients.

Table 14 showed that the service redesign affected the flow rates and bed numbers of both groups, but that the long-stay patients experienced the greatest change in both relative and absolute terms (a greater reduction in absolute and relative bed numbers, and a greater increase in flow rate). The value in using a double compartmental flow model was supported by the additional information gained in understanding the change in service provision that occurred, namely that:

- Both short and long-stay patient bed numbers were reduced and the amount of reduction was different for each group, and
- The flow rate of both short and long-stay patients was increased and the rate of change was different for each group.

Thus, while a single compartment model may have been indicated, the loss of explanatory value derived from the model would negate its use. This issue becomes particularly important in a situation where the evaluation results in benchmarks that can be used to track ongoing service performance.

6.4.3 Alignment with experience

The pre and post-service change model, which was the preferred model, aligns with the actual experience of the service change. While the creation of a model based upon the timing of the service change may seem an intuitive choice, the fact that it was also
the preferred model from those tested enables the results to be communicated more easily. This is important, because communication of the results, particularly as this analysis concerned a significant historic service redesign, adds credibility to the approach.

While defensible and sound modelling is important, if such modelling is going to progress from an academic pursuit to a widely applied tool, it is equally important to recognise that communication of results to end-users must be a key goal.

6.4.4 Portability of health system measures and validation

In Chapter 4 (see section 4.1) the ALOS was shown to be a widely used measure across health systems around the world. Hospital bed occupancy is also widely used, though not as much as the average length of stay as shown in Table 15 (search conducted 29 December 2005).

<table>
<thead>
<tr>
<th>Country name used</th>
<th>Number of search hits</th>
<th>Google Scholar</th>
<th>Google</th>
</tr>
</thead>
<tbody>
<tr>
<td>none specified</td>
<td>6,810</td>
<td>818,000</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>1,090</td>
<td>109,000</td>
<td></td>
</tr>
<tr>
<td>America</td>
<td>1,980</td>
<td>428,000</td>
<td></td>
</tr>
<tr>
<td>England</td>
<td>1,540</td>
<td>206,000</td>
<td></td>
</tr>
<tr>
<td>New Zealand</td>
<td>573</td>
<td>59,000</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>1,410</td>
<td>172,000</td>
<td></td>
</tr>
</tbody>
</table>

The portability of the average length of stay and bed occupancy suggests that measures of patient stay are of importance and are reported irrespective of local differences in practices and policy. Indeed, one simple strategy often employed to
reduce hospital costs is to cut the number of beds that can be occupied (for example, Bannerman, 1995; Taheri, Butz and Greenfield, 2000).

Given that perspective on patient stay metrics, international variation between the styles of health service provision should have minimal impact on the ability to apply the compartmental flow model as a tool for looking at bed occupancy issues. Furthermore, the data used in this research related to medical (as opposed to surgical) type services and the elderly dominate the use of these services. While there may be differences in the way services are provided, England, Australia and New Zealand are developed countries and therefore the disease profiles will be similar (that is, one of a developed country as opposed to one of a developing country). Consequently, it would be unexpected that the casemix of a particular hospital from any of these countries would prevent the use of compartment flow models for evaluating changes to patient bed occupancy.

The research presented in Chapter 5 indicated that the modelling approach developed by Harrison and Millard (1991) using geriatric data could be used for modelling acute care data. The results from the analysis of the New Zealand data confirm that the compartmental model can be used to model acute care data and thus validate the findings of the results obtained in Chapter 5.

While the research presented in this and the previous chapters has focussed on the modelling of medical patient data, modelling has also been undertaken on surgical patient data (Millard, Mackay, Vasilikas and Christodoulou, 2000). This suggests, that subject to understanding the business processes involved in the provision of care
within an acute care hospital, there is good reason to believe that the compartmental model can be fitted to acute care hospital inpatient data in general.

Despite the likelihood that there will be little inter-health system variation that would prevent the use of compartmental flow models of bed occupancy, the successful use of data from various countries and from various service types (geriatrics, surgical and medical) to fit such models should provide some measure of comfort that the results have not arisen as a consequence of peculiarities around bed occupancy for one service, but rather that there is now evidence that the application of the model in relation to acute hospital data in general is validated and there is no reason that more widespread adoption of the compartmental flow modelling technique should not occur to aid improved decision-making in the health care environment.

6.4.5 Expert Judgment

The fit of the model with 28 parameters to the data deteriorated despite an increase in complexity when compared to slightly less complex models. When dealing with the same class of models, a decline in model fit can only happen if the simpler model is not nested in the more complicated one, and this occurred in relation to the 28 parameter model – the eight and 20 parameter models were not nested in this more complex model. The improvement in fit from the four parameter model to the 28 parameter model, where nesting occurred, is also consistent with this notion.

This perhaps is evidence to suggest that the “expert” judgment does not always involve the selection of models that lead to better fit despite increasing model
complexity, that is, structural mis-specification occurs. The observed relationships were otherwise consistent with expectations, that is, fit improved with complexity.

6.4.6 Average models

As previously stated, St George (1988) and MacStravic (2001) have reported that an annual average model of acute hospital services will be insufficient to enable bed planning, as the variation within the data will not be detected.

The annual average model fits the data well as shown in these results. Additionally, it is possible to train the model to using a methodology that captures variation. The notion that there is no “right” answer may be a more appropriate response to the shortcoming identified by St George (1988) and MacStravic (2001) and in fact, it is better to be confident that the answer lies within some bounds. Indeed, according to Dasgupta (1998), Wildon Carr, a noted British philosopher, is attributed to having suggested the notion that it is “better to be vaguely right than precisely wrong”, which is another way of expressing this notion.

Given the primary purpose of the modelling in this instance, which was to enable evaluation of the service redesign as opposed to plan for future bed allocations, the views of St George (1988) and MacStravic (2001) perhaps have less weight. This is not to say that incorporation of additional complexity, such as seasonality, may not be useful, but rather it is not relevant given this particular task and that use of the model should determine the level of complexity that should be sought.
6.4.7 Other issues

The methodology used for the modelling exercise undertaken in this chapter was
different to that used in the previous chapter. A reliance was placed upon using the
average to capture the variation in the data to be captured compared to incorporating
every data point into the calculations performed. This enabled a significant reduction
in the volume of data used in the modelling to occur and thereby gain significant
increases in model run time. The method used in Chapter 5 was only applied to data
relating to a single year and complex models required significant computing time (that
is, many hours). In fact, given the volume of data, it would not have been practical to
run the models using the first method as in most cases the data spanned several years
and this would have increased computing time. The reduction in computing time
makes the methodology more appealing as a potential tool in the real world.

6.5 Conclusion

The bed occupancy compartmental flow model described the data from the Internal
Medicine Department at HealthCare Otago, New Zealand, well. The successful
development of compartmental flow models using this data validates the application
of the modelling approach.

Ensuring that the model suits the purpose of the analysis is important. Model selection
methodologies provide a defensible means of selecting the appropriate level of model
complexity. Consideration of the value of the information gained from the inclusion
of additional model complexity, such as the use of a second compartment,
nevertheless requires judgment.
There are limitations to the extent that I have validated the use of compartmental flow models being of use in describing acute hospital data, including:

- The modelling has excluded same-day elective patient data as the management of these patients often relies upon a different model of practice (notably the use of one bed for many patients in a single day). This is discussed in more detail in Chapter 10, and
- The focus on medical service provision - it is evident that surgical, obstetric, mental health and paediatric services have not been included in this analysis.

The validation and complexity issues are important as they enable the results of hospital occupancy modelling that relies upon compartmental flow models to be used for forecasting or generalisation purposes as opposed to gaining an understanding of specific historic events.

Given this research and other work not detailed here (Millard et al., 2000), it is reasonable to suggest that other acute care services will be able to be similarly described using compartmental models. The exceptions will occur if the business model gives rise to fixed time service provision and this may be the case in relation to some aspects of obstetric activities due to the push to reduce length of stay to a fixed period of service for particular types of deliveries (this issue is examined more generally in Chapter 10).

The issues of linking compartmental flow models to population change and model development is explored in the next chapter and the examination of the New Zealand data is continued in Chapter 8.
Chapter 7

Model choice and prediction: forecasting changes in bed occupancy profiles as a consequence of population change

In this chapter I investigate whether the model selection methods used in prior chapters can be applied to determine a suitable level of complexity for a bed occupancy compartmental flow model that incorporates patient age. The ability to link the aged based bed occupancy model to forecast population change is also investigated. The chapter has the following structure:

7.1 Introduction ............................................................................................ 218
7.2 Methodology ............................................................................................ 220
  7.2.1 Creation of the Compartmental Flow Model ........................................ 220
  7.2.2 Linkage to Population Change ............................................................. 222
7.3 Results .................................................................................................... 222
  7.3.1 The Compartmental Flow Models ........................................................ 222
  7.3.2 Parametric Parameter Forecasts Linked to Population Change ............. 230
7.4 Discussion .............................................................................................. 237
  7.4.1 Model Choice ...................................................................................... 238
  7.4.2 Forecasts and policy ............................................................................. 241
  7.4.3 Population profiles and assumptions .................................................... 243
  7.4.4 Resource use and scenario testing ........................................................ 246
  7.4.5 Technical Issues ................................................................................... 249
7.5 Conclusion .............................................................................................. 254
7.1 Introduction

The Generational Health Review (2003) reported that the current South Australian health system was not sustainable (see Chapter 1 for more details). Some of the issues affecting sustainability were:

- The costs of services (the acute care sector costs are generally greater than primary care costs on a per service basis, and at a higher level, the health care budget represents a large part of the State budget)
- Older people use more health services (OECD, 2003), and
- A change in the population profile is occurring that will result in a greater number of older people.

The authors of the Generational Health Review (2003) suggested that if current practices were continued then it would be expected that:

- Admissions would increase by ten per cent
- There would be a requirement for an additional 420 beds (same-day and inpatient) – a 16 per cent increase, and
- Costs would increase by nine per cent or $88 million per annum (based upon 2001 prices).

As the modelling for this work was not made public it is difficult to comment upon the credibility of the findings published in the report. In particular, it is not clear whether or not the model selection issues discussed in Chapter 4, and explored in Chapters 5 and 6, were considered as part of that model development.
Irrespective of the technical issues surrounding the findings of some aspects of the Generational Health Review, there is widespread support for the notion that health care costs are likely to increase as populations age (see earlier discussion on this in Chapters 1 and 4). Consequently, the possibility of forecasting future resource use associated with the ageing of the population is an important topic that warrants examination.

In this chapter I create a linkage between the strategic bed occupancy models and population change. This linkage enables the resource use differences for different age groups to be measured in terms of both number of beds and also the rate of flow through the system. The work on model choice in Chapter 5 is extended to investigate whether there is benefit in disaggregating the data into age groups that can be used to assist decision-making around the expensive resource of hospital beds. This work is then used as the basis of forecasting future bed occupancy. Explicitly, research was undertaken to:

- Explore the application of model choice methodology when selecting compartmental flow models of bed occupancy disaggregated on the basis of patient age, and
- Explore the implications of forecasting future bed occupancy based upon linking compartmental flow model parameters with population forecasts.

As previously stated, the results of the work investigating model complexity and population changes were presented at a conference (Mackay and Lee, 2004a).
7.2 Methodology

7.2.1 Creation of the Compartmental Flow Model

The data considered here only relate to that drawn from the Flinders Medical Centre. The contextual details about the data were described in Chapter 3 (see sections 3.2.1 and 3.3.1).

The data included the patient age at the time of discharge and this age was used to create bed occupancy data sets that reflected various patient age groupings. This was necessary to enable a matching of the population age profile to the bed census profiles. The population profiles group age in five-year intervals, and consequently, the bed census profiles were created to match the age ranges, with the first age range commencing at 20 years and the last age range representing all those aged 85 years or more. The data contained few occurrences of patients aged less than 20 years. This was due in part to the fact that the diseases treated in the medical ward tend to be more prevalent in the elderly. Additionally, patients aged between 15-19 years may not necessarily receive treatment in adult wards. Consequently, data relating to patients aged less than 20 years was omitted from the study. Thus, it was necessary to create 14 bed census profiles.

The methodology used to create the occupancy models for the age related occupancy data was fully described in Chapter 6 (see section 6.2.2).

The number of possible models that could be constructed on the basis of combining the various age groups was very large. The simplest model was a single model that
covered all ages and this equated to just using the “annual average” model for all patients. The most complex was one that involved the computation of a compartmental flow model for each of the age groups. While the ideal model was to be found within the bounds of the simplest and most complex model groupings, it was not pragmatic to construct every possible model that could be tested. My expert knowledge of the health system was used to guide the development of a range of models that could be tested. This lead to the development of 19 possible configurations of age grouped compartmental flow models being constructed. Of the 19 configurations, 13 configurations had differing numbers of parameters.

The goodness-of-fit achieved by optimization was measured for each model against the training data. Although a variety of measures of goodness-of-fit were possible (Hastie, Tibshirani and Friedman, 2001), the Bayesian Information Criterion (BIC) was used. The BIC value provided information about the level of fit in the absence of using a training and test set of data.

The number of compartments was varied for the preferred model to test whether increasing the number of compartments improved the model.

Confidence intervals for model parameters were calculated using standard Monte Carlo methods (Hillier and Lieberman, 2001; Powell and Baker, 2004))
7.2.2 Linkage to Population Change

The projected population profile for South Australia was obtained from the Australian Bureau of Statistics (1999 and 2000). The projections were matched to the catchment area of the hospital.

The rates of change for age groups matching those developed for the preferred compartmental flow model were applied to the model parameters relating to the numbers of beds. For the purpose of this exercise, the parameters relating to the rates of patient flow were held constant.

The timing of predicted changes in the population profile was compared to the changes in the number of beds.

7.3 Results

7.3.1 The Compartmental Flow Models

The groupings used for the ages, together with the number of model parameters and fit and complexity measures are shown in Table 16.
Table 16: Scenarios tested to determine the best age grouping of the data for modelling bed occupancy. The preferred model is highlighted.

<table>
<thead>
<tr>
<th>Parameter and Model Type</th>
<th>No. of Parameters</th>
<th>WSSE</th>
<th>BIC</th>
<th>Log Odds</th>
<th>Bayes Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 Single Model</td>
<td>4</td>
<td>883.9</td>
<td>925.5</td>
<td>662.6</td>
<td>8.E+143</td>
</tr>
<tr>
<td>8 20-64, 65+</td>
<td>8</td>
<td>240.9</td>
<td>296.1</td>
<td>37.2</td>
<td>1.E+08</td>
</tr>
<tr>
<td>12 20-44, 45-69, 70+</td>
<td>12</td>
<td>189.5</td>
<td>272.3</td>
<td>13.4</td>
<td>8.E+02</td>
</tr>
<tr>
<td>12 20-64 same, 65-84 same, 85+</td>
<td>12</td>
<td>239.5</td>
<td>322.3</td>
<td>63.4</td>
<td>6.E+13</td>
</tr>
<tr>
<td>16 20-39, 40-59, 60-79, 80+</td>
<td>16</td>
<td>173.1</td>
<td>283.5</td>
<td>24.6</td>
<td>2.E+09</td>
</tr>
<tr>
<td>16 20-64 same, 65-74 same, 75-84 same, 85+</td>
<td>16</td>
<td>187.7</td>
<td>298.0</td>
<td>39.2</td>
<td>3.E+08</td>
</tr>
<tr>
<td>20 20-34, 35-49, 50-64, 65-79, 80+</td>
<td>20</td>
<td>120.9</td>
<td>258.9</td>
<td>0.0</td>
<td>1</td>
</tr>
<tr>
<td>20 20-34, 35-44, 45-54, 55-64, 65+</td>
<td>20</td>
<td>123.6</td>
<td>261.6</td>
<td>2.7</td>
<td>4</td>
</tr>
<tr>
<td>24 20-34, 35-44, 45-54, 55-64, 60-84, 85+</td>
<td>24</td>
<td>122.2</td>
<td>287.8</td>
<td>28.9</td>
<td>2.E+06</td>
</tr>
<tr>
<td>24 20-64 same, other years unique</td>
<td>24</td>
<td>155.9</td>
<td>321.5</td>
<td>62.6</td>
<td>4.E+13</td>
</tr>
<tr>
<td>28 20-34, 2 adjoining age groups joined, 65-74, 75-84, 85+</td>
<td>28</td>
<td>70.7</td>
<td>263.9</td>
<td>5.0</td>
<td>12</td>
</tr>
<tr>
<td>28 every 2 groups joined</td>
<td>28</td>
<td>90.7</td>
<td>283.9</td>
<td>25.0</td>
<td>3.E+05</td>
</tr>
<tr>
<td>28 decades joined, 80+</td>
<td>28</td>
<td>90.7</td>
<td>283.9</td>
<td>25.0</td>
<td>3.E+05</td>
</tr>
<tr>
<td>32 decades joined, 80-84, 85+</td>
<td>32</td>
<td>82.9</td>
<td>303.7</td>
<td>44.8</td>
<td>5.E+09</td>
</tr>
<tr>
<td>40 decades joined, 60-64, 65-69, 70-74, 75-79, 80-84, 85+</td>
<td>40</td>
<td>31.4</td>
<td>307.3</td>
<td>48.5</td>
<td>3.E+10</td>
</tr>
<tr>
<td>44 decades joined, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, 80-84, 85+</td>
<td>44</td>
<td>31.4</td>
<td>307.3</td>
<td>48.4</td>
<td>3.E+10</td>
</tr>
<tr>
<td>48 decades joined, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70-74, 75-79, 80-84, 85+</td>
<td>48</td>
<td>25.8</td>
<td>356.9</td>
<td>98.1</td>
<td>2.E+21</td>
</tr>
<tr>
<td>52 20-29, rest separate</td>
<td>52</td>
<td>17.3</td>
<td>376.0</td>
<td>117.1</td>
<td>3.E+25</td>
</tr>
<tr>
<td>56 All age groups uniquely modelled</td>
<td>56</td>
<td>5.4</td>
<td>391.8</td>
<td>132.9</td>
<td>7.E+28</td>
</tr>
</tbody>
</table>

The groupings with the preferred model choice are shown in a visual representation in Figure 47 to facilitate interpretation of Table 16.

Figure 47: Visual representation of the age groupings used for the analysis. The circled option was the preferred model.

Where the number of parameters was the same, the BIC values will not discriminate between the models well, as the BIC is determined by the fit, the number of
parameters and the number of data. The latter two are constant in models with the same number of parameters and in general terms the compartmental flow models all fit the data well. A small difference in BIC values does not provide a good basis for the selection of one model over another. This outcome is shown in Table 16 where the models with the same number of parameters have similar BIC values. Consequently, only results for 13 unique configurations are reported in the remainder of the results.

Figure 48 shows the increasing fit that was achieved with model complexity (that is, the weighted sum squared error decreased with increasing complexity), but that the BIC value was minimized for a less complex model.

![Graph showing Fit, Complexity and BIC](image)

Figure 48: The trade-off between model fit and generalisation with increasing complexity is shown.

This result is also shown in Table 16 where the Bayes factor and log odds are reported. The preferred model was compared to the other models and it can be seen that the log odds value suggests a strong preference for the chosen model in relation
to most of the modelled alternatives. There were several models where the log odds value was not large, indicating that the preference between the models was not great.

In these instances the alternative models could have been used without much loss of generalisability. However, there was no good reason to adopt the alternative models, as the preferred model related well to the accepted understanding of resource use and age, and it achieved parsimony.

The fit of the most simple and complex models, together with the fit of the preferred models is visually shown in the following three figures.

Figure 49: Fit of the data against the simplest model - the annual average model with no disaggregation of patients by age. The model did not fit the data well.
Figure 50: Fit of the data against the most complex model. All data groups are well-fitted (over-fitted) by the model, as evidenced by the difficulty in visually differentiating between the data and model.

Figure 51: Fit of the preferred model and data. The model does not perfectly fit the data and thereby avoided over-fitting (see Figure 50), but represented a significant improvement on the simplistic poor-fitting model (see Figure 49).
When the data is grouped into the preferred model age ranges it can be seen that the model fits the data as shown in Figure 52.

Aside from visual inspection, performance statistics confirm that the degree of model fit to the data was high as reported in Table 17.

Table 17: Model performance statistics. The correlation between the individual components of the model and the data was very high and the absolute errors were all low.

<table>
<thead>
<tr>
<th>Age group</th>
<th>20-24</th>
<th>25-49</th>
<th>50-64</th>
<th>65-79</th>
<th>80+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute Error</td>
<td>3.45</td>
<td>2.97</td>
<td>3.86</td>
<td>25.46</td>
<td>6.48</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.992</td>
<td>0.998</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Ten per cent of patients (long-stay) used a much greater proportion of bed days (37 per cent) as shown in Table 18.
Given the importance of the long-stay patient group in influencing bed use, as illustrated in Table 18, consideration was given to the question of whether there was merit in creating a model with additional compartments to improve the explanatory power of the model. Single, double, triple and quadruple compartment models based upon the preferred age groups established earlier were created. Based upon the BIC value, the number of model compartments used should be one, that is, a less complex model is preferred, as shown in Figure 53.

Table 18: Short-stay patients used disproportionately less bed-days than long-stay patients. Valuable information about the long-stay patients can be gained via a double compartment flow model.

<table>
<thead>
<tr>
<th>Length of stay</th>
<th>Bed days</th>
<th>Percentage of total bed days</th>
<th>Patients</th>
<th>Percentage of total patients</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-14 days</td>
<td>35,093</td>
<td>63%</td>
<td>8,628</td>
<td>90%</td>
</tr>
<tr>
<td>15 or more days</td>
<td>20,739</td>
<td>37%</td>
<td>930</td>
<td>10%</td>
</tr>
<tr>
<td>Total</td>
<td>55,832</td>
<td>100%</td>
<td>9,558</td>
<td>100%</td>
</tr>
</tbody>
</table>

Figure 53: Trade-off between complexity and fit for the preferred model. Based upon the BIC value, a single compartment model was suggested. However, other factors can also be important for model selection.
Given that the perceived additional value in understanding the behaviour of long-stay patients and that increase in BIC value was not substantial, which gives rise to a low log odds value (47.5) and Bayes factor (2.1E+10), combining judgement with the BIC value would suggested that a double compartment model is still useful and does not lead to a significant gain in unnecessary complexity. The need for additional complexity through the creation of a third compartment was not so easily justified, particularly given the increased log odds value (115.7).

It was possible to generate confidence intervals around each of the model parameters using standard Monte Carlo methods, which is useful for demonstrating that the actual model is not deterministic, as shown in Table 19. The model formula has been previously given (see Chapter 5, section 5.2.1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Statistic</th>
<th>20-34</th>
<th>35-49</th>
<th>50-64</th>
<th>65-79</th>
<th>80+</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>lower 95% CI</td>
<td>6.6417</td>
<td>12.7284</td>
<td>22.6711</td>
<td>63.6663</td>
<td>40.7524</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>6.6505</td>
<td>12.7372</td>
<td>22.6799</td>
<td>63.6751</td>
<td>40.7612</td>
</tr>
<tr>
<td></td>
<td>upper 95% CI</td>
<td>6.6592</td>
<td>12.7459</td>
<td>22.6886</td>
<td>63.6838</td>
<td>40.7699</td>
</tr>
<tr>
<td>B</td>
<td>lower 95% CI</td>
<td>0.3210</td>
<td>0.2501</td>
<td>0.2010</td>
<td>0.1362</td>
<td>0.1284</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>0.3298</td>
<td>0.2589</td>
<td>0.2098</td>
<td>0.1450</td>
<td>0.1372</td>
</tr>
<tr>
<td></td>
<td>upper 95% CI</td>
<td>0.3386</td>
<td>0.2677</td>
<td>0.2186</td>
<td>0.1538</td>
<td>0.1460</td>
</tr>
<tr>
<td>C</td>
<td>lower 95% CI</td>
<td>0.9008</td>
<td>1.6952</td>
<td>4.6090</td>
<td>4.8869</td>
<td>3.8520</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>0.9095</td>
<td>1.7040</td>
<td>4.6178</td>
<td>4.8957</td>
<td>3.8608</td>
</tr>
<tr>
<td></td>
<td>upper 95% CI</td>
<td>0.9183</td>
<td>1.7127</td>
<td>4.6265</td>
<td>4.9044</td>
<td>3.8695</td>
</tr>
<tr>
<td>D</td>
<td>lower 95% CI</td>
<td>0.0510</td>
<td>0.0487</td>
<td>0.0523</td>
<td>0.0311</td>
<td>0.0416</td>
</tr>
<tr>
<td></td>
<td>mean</td>
<td>0.0597</td>
<td>0.0574</td>
<td>0.0611</td>
<td>0.0399</td>
<td>0.0504</td>
</tr>
<tr>
<td></td>
<td>upper 95% CI</td>
<td>0.0685</td>
<td>0.0662</td>
<td>0.0698</td>
<td>0.0486</td>
<td>0.0591</td>
</tr>
</tbody>
</table>

Table 19: Confidence (95 per cent) intervals for each parameter of the model. Such intervals are useful in showing that uncertainty around the exact model fit exists.
Model performance can be measured in several ways. The degree of fit has been previously established (see Figure 52 and Table 17). One aspect of fit in this application relates to the total number of occupied beds that is suggested from the model as opposed to the actual number of beds occupied. Table 20 reports the model fit with respect to total bed occupancy.

Table 20: The comparison of the model to the actual data suggests that the model over-estimates the total number of beds actually occupied.

<table>
<thead>
<tr>
<th>Age Group</th>
<th>Bed Number Model Parameters (A+C)</th>
<th>Total Ave Occupancy</th>
<th>% over estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-34</td>
<td>7.6</td>
<td>7.1</td>
<td>7.0%</td>
</tr>
<tr>
<td>35-49</td>
<td>14.4</td>
<td>13.8</td>
<td>4.7%</td>
</tr>
<tr>
<td>50-64</td>
<td>27.3</td>
<td>25.6</td>
<td>6.7%</td>
</tr>
<tr>
<td>65-79</td>
<td>68.6</td>
<td>67.0</td>
<td>2.3%</td>
</tr>
<tr>
<td>80+</td>
<td>44.6</td>
<td>41.5</td>
<td>7.6%</td>
</tr>
<tr>
<td>Total</td>
<td>162.5</td>
<td>154.9</td>
<td>4.9%</td>
</tr>
</tbody>
</table>

7.3.2 Parametric Parameter Forecasts Linked to Population Change

The Australian Bureau of Statistic’s (ABS) forecast population profiles for the years of 1999 and 2019 are shown in Figure 54.
It is forecast that the growth in the number people aged 55 years or more will be 60 per cent during this period, while the number of people aged 20-54 years will decrease by 10 per cent. For the entire population aged 20 years or more there is an increase in the size of the population by approximately 13 per cent during the same period.

The timing of the changes in population size is not uniform as shown in Figure 55.
Parametric forecasting of the bed number parameters using the change in population as the basis for the forecast enables the number of short and long-stay beds to be predicted. The forecast for short-stay beds is shown in Figure 56.

Figure 55: The timing of the change of population with age groups matched to the preferred compartmental model.

Figure 56: Forecast change in requirements for short-stay beds according to changes in the population size based upon age.
Similarly, changes in requirements for long-stay beds were also forecast as shown in Figure 57.

Figure 57: Forecast change in requirements for long-stay beds based upon population changes. The timing of changes is important for policy and planning reasons.

The forecast total bed requirement – that is both short and long-stay beds – is shown in Figure 58.

Figure 58: Forecast total bed requirements. It is evident that those aged 65 years or more use the most beds.
While Figure 58 shows the forecast bed requirements, the mix of bed use is better represented by showing the changes over time as a proportion of 100 per cent capacity. This is done in Figure 59.

Figure 59: The forecast mix of bed usage based by age group over time.

The assumption that the mix of services, and therefore bed usage, was held constant is confirmed in Figure 60.
The change in mix of resources can also be tabulated. The situation at 1998 and the forecast 2019 bed forecast are shown in the following two tables (see Tables 21 and 22).

Table 21: Summary of resource use (that is number of beds and admissions) by age group for 1998.

<table>
<thead>
<tr>
<th>Model</th>
<th>Admissions (day)</th>
<th>Admissions (year)</th>
<th>Average stay (days)</th>
<th>Number of admissions discharged (%)</th>
<th>Average stay (days)</th>
<th>Number of beds used (%)</th>
<th>Number of admissions discharged (%)</th>
<th>Average stay (days)</th>
<th>Number of beds used (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 to 34 years age model</td>
<td>1.9</td>
<td>701.2</td>
<td>3.9</td>
<td>685.9 98%</td>
<td>3.6</td>
<td>6.8 90%</td>
<td>15.3 2%</td>
<td>17.2 10%</td>
<td>98% 90%</td>
</tr>
<tr>
<td>35 to 49 years age model</td>
<td>3.0</td>
<td>1095.2</td>
<td>4.8</td>
<td>1060 94%</td>
<td>4.4</td>
<td>13.2 91%</td>
<td>26.2 2%</td>
<td>17.9 13%</td>
<td>98% 91%</td>
</tr>
<tr>
<td>50 to 64 years age model</td>
<td>4.6</td>
<td>1666.5</td>
<td>6.0</td>
<td>1597 96%</td>
<td>5.3</td>
<td>24.1 80%</td>
<td>46.8 4%</td>
<td>16.9 32%</td>
<td>96% 88%</td>
</tr>
<tr>
<td>65 to 79 years model</td>
<td>8.8</td>
<td>3206.5</td>
<td>7.8</td>
<td>3156 94%</td>
<td>7.4</td>
<td>65.1 89%</td>
<td>49.6 2%</td>
<td>25.6 3.5</td>
<td>98% 95%</td>
</tr>
<tr>
<td>80 or more years model</td>
<td>5.4</td>
<td>1976.3</td>
<td>8.2</td>
<td>1933 98%</td>
<td>7.8</td>
<td>42.2 99%</td>
<td>42.7 2%</td>
<td>20.3 2.4</td>
<td>98% 95%</td>
</tr>
<tr>
<td>Overall</td>
<td>23.7</td>
<td>8645.7</td>
<td>30.8</td>
<td>8447 94%</td>
<td>28.4</td>
<td>155.1</td>
<td>202.5 2%</td>
<td>98.0 11.4</td>
<td>98% 95%</td>
</tr>
</tbody>
</table>
Table 22: Summary of resource use (that is the number of beds and admissions) by age group for 2019. The forecast increase since 1998 is also shown.

<table>
<thead>
<tr>
<th>Model</th>
<th>Admissions (day)</th>
<th>Admissions (year)</th>
<th>Average stay (days)</th>
<th>Number of admissions discharged (%)</th>
<th>Average stay (days)</th>
<th>Number of beds used (%)</th>
<th>Number of admissions discharged (%)</th>
<th>Average stay (days)</th>
<th>Number of beds used (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 to 34 years age</td>
<td>1.8</td>
<td>646.2</td>
<td>3.9</td>
<td>632.1</td>
<td>98%</td>
<td>3.6</td>
<td>6.3</td>
<td>14.3</td>
<td>17.2</td>
</tr>
<tr>
<td></td>
<td>35 to 49 years age</td>
<td>2.6</td>
<td>930.8</td>
<td>4.8</td>
<td>908.5</td>
<td>98%</td>
<td>4.4</td>
<td>11.2</td>
<td>22.3</td>
</tr>
<tr>
<td></td>
<td>50 to 64 years age</td>
<td>6.1</td>
<td>2216.9</td>
<td>6.0</td>
<td>2125.6</td>
<td>90%</td>
<td>5.3</td>
<td>32.1</td>
<td>91.3</td>
</tr>
<tr>
<td></td>
<td>65 to 79 years age</td>
<td>12.5</td>
<td>4569.2</td>
<td>7.8</td>
<td>4408.5</td>
<td>90%</td>
<td>7.4</td>
<td>92.8</td>
<td>70.7</td>
</tr>
<tr>
<td></td>
<td>80 or more years</td>
<td>8.2</td>
<td>11338.6</td>
<td>8.2</td>
<td>2911.3</td>
<td>90%</td>
<td>7.8</td>
<td>63.6</td>
<td>64.3</td>
</tr>
<tr>
<td>Overall</td>
<td>31.1</td>
<td>2692.9</td>
<td>0.0</td>
<td>2.633</td>
<td>98%</td>
<td>0.0</td>
<td>209.6</td>
<td>262.7</td>
<td>60.2</td>
</tr>
<tr>
<td>Change</td>
<td>7.4</td>
<td>13738.6</td>
<td>0.0</td>
<td>2.633</td>
<td>98%</td>
<td>0.0</td>
<td>209.6</td>
<td>262.7</td>
<td>60.2</td>
</tr>
</tbody>
</table>

The ability to undertake what-if analysis exists. Various hypothetical examples are shown in Figure 61.

Figure 61: Visual representation of effects of making changes to the system. The system becomes stable over time.
7.4 Discussion

The need to influence future decision-making about the provision of health services is clear, as shown by the forecasts made in the Generational Health Review (2003). The ability to make such forecasts, however, has not been well explored at least in the literature. Myers and Green (2004) suggest that focussing efforts on managing the length of stay is an alternative way of managing bed capacity compared to increasing bed capacity. Some countries or studies have used a particular rate of beds per head of population as the means of determining the number of beds required now and in the future (Griffith and Wellman, 1979; Bay and Nestman, 1984; Toussaint, Herengt, Gillois and Kohler, 2002). The basis formula for this approach is given in by Farmer and Emami (1990):

\[
\text{Beds} = \frac{\text{population served} \times \text{admission rate} \times \text{ALOS} \times \text{efficiency factor}}{365}
\]

Others, such as Sweeney and Ashley (1981), have considered the morbidity pattern in the population as a driver in the determination of beds. Farmer and Emami (1990) consider that time series forecasting (ARIMA) methods may be of more use than simple population models. Pendergast and Vogel (1988) and Sorensen (1996) have advocated a disaggregation of the ALOS on the basis of clinical care, time spent in hospital and discharge destination before the calculation of bed numbers. Commercial and other interests may prevail over the need to publish the detail of how forecasts, if made, are established. The Generational Health Review (2003) provides a useful example to highlight this situation, as the methodology surrounding projected future needs was not published, although no reason for this was provided.
This is the first time population change has been linked to bed occupancy compartmental flow modelling and also where model choice theory has been used to determine the complexity of the age groupings used to generate the compartmental model. This approach provides this modelling with a broader horizon of application, therefore making it a more powerful and potentially useful approach. I have subsequently also been involved with others in pursuing this the use of forecasting, though with a slightly different approach (see Harrison, Mackay and Shafer, 2005).

Various aspects of the modelling approach employed, including model choice, population forecasts and assumptions will be discussed in more detail in the following sections.

7.4.1 Model Choice

The data were disaggregated on the basis of age only. The previous work on model choice considered the effects of the how disaggregation of the data for one year should occur on the basis of time (for example, not at all, seasons or weeks - see Chapter 5). This work found that a seasonal model was the preferred choice of the possible models tested. While creating a model on the basis of age and seasonality was possible, the additional level of complexity was not desired for two reasons:

- The desire to focus only on issues pertaining to patient and population age in order that understanding of the analysis of this particular component of analysis was maximised, and
- It was planned that subsequent work would introduce alternative mechanisms to consider this more complex scenario (see Chapter 8).
As demonstrated and discussed in Chapter 4, the level of model fit increases with model complexity. This again has been demonstrated in this analysis as shown in Figure 48. Visual inspection was used to highlight the consequences of choosing models that were too simple or complex, as well as showing the fit of the preferred model (see Figures 49 to 51).

The preferred model was found to fit the data well, as shown in Table 17 with a high correlation between the data and model being found, together with low absolute errors. These findings are consistent with the recommendations made in the BOMPS manual (1992), namely that models with low errors and high correlations should be retained or were found to exist when working with geriatric data. The inclusion of model complexity and fit trade-off decisions has not detracted from a well fitting model, but has served to improve the usefulness of the resultant model, insofar as that the preferred model can be used better for generalisation and forecasting than an over-fitted model.

The recommended use of a double compartment flow model is made on the basis of the resultant BIC values obtained when testing the number of compartments to use (see Figure 53) and expert judgment about the importance of long-staying patients (see Table 18). Long-stay patients are often credited with the blame for blocking the acute care health system (for example, see Mackay, 2001; Cameron and Campbell, 2003). Given that a single long-stay patient occupies a bed for a period that could have accommodated multiple short-stay patients this explanation appears credible to some extent. The perceived problem with long-stay patients, is however, not that they
block beds, but rather whether the service has been planned to cope with the number of long-stay patients that exist.

The length of stay and number of long-stay patients can be influenced by factors outside the control of the hospital, for example, the provision of community-based services that would enable a patient to be discharged or availability of nursing home beds. When outside factors increase the number of long-stay patients or increase their length of stay, then long-stay patients do contribute to bed blockage.

Internal hospital factors can also influence the length of stay and number of long-stay patients, and if not addressed, can also contribute to bed block problems. However, at Flinders Medical Centre work on the reasonableness of admissions (for example, see Baggoley, Phillips and Aplin, 1994; Finucane et al., 2000) and more recently, the application of lean thinking approaches to ensure the patient journey does not involve wasted time has been undertaken (for example, see King, Ben-Tovim and Bassham, 2006). Thus, there is evidence that hospital management was cognisant of the implications of not addressing internal factors that may contribute to bed blockage.

While the BIC value was useful in helping to determine whether over-fitting of the data occurred, it did not take into account the value of other aspects of the model, such as the ability to demonstrate behaviour of a particularly important group of patients, that is, the long-stay patients. Had the difference in BIC values between a single and double compartment model been far greater, then the value of the additional information gained through increased complexity would have been questionable.
7.4.2 Forecasts and policy

It is evident from Figure 54 that the ageing of the local population is occurring. It can be seen that the relative changes in population growth are substantial (for example, a growth of more than 40 per cent of people aged 80 years of more is expected). Not all population groups, however, are expected to experience growth during this period, with a decline in the population aged less than 49 years expected. The timing of these changes is not uniform with some age groups experiencing growth (both positive and negative) sooner than others as shown in Figure 55.

Given the linkage between health care resource use and ageing it is important to understand the ramifications of the timing and sized of population change. The growth in demand for short-stay beds is illustrated in Figure 56. Patients aged 65-79 years were the highest users of beds at the base year for this analysis. The need for additional beds for patients aged 65-79 years, however, does not increase until midway through the forecast period. The need for additional beds for those aged 50-64 years and 80 years or more is expected to beginning to stabilize midway through the forecast period.

While the shape of the forecasted trends for long-stay patients illustrated in Figure 57 are similar to those forecast for short-stay patients (see Figure 56), it is evident that there are differences. Although the patient group aged 65-79 years occupied the greatest number of long-stay beds (on average) at the beginning of the forecast period, other age groups, namely those aged 50-64 years and then 80 or more years, exceed the numbers of beds used by this group until almost the end of the forecast period.
This has several important policy implications that would not have been identified if the analysis had not been undertaken using age-based disaggregated data and a double compartment flow model.

It is forecast that the total number of beds required to service the medical patients will increase by approximately 37 per cent over the period as shown in Figure 58. The potential policy decisions arising from the forecasts illustrated in Figures 56, 57 and 58 include:

- The need to fund an increase in hospital activity
- The need to fund capital works required to provide the physical infrastructure required to meet the forecast additional activity
- The need to attract and retain the necessary workforce in order to be able to deliver the additional services
- The need to plan for and implement alternative care models if additional capacity is not to be provided
- The need to recognise those aged 65-79 years will continue to be the dominant age group for short-stay patients and that growth of this group will not increase until post 2008
- That patients aged 50-64 years will become the dominate long-stay patient group until approximately 2016 when the patient group aged 65-79 years will dominate
- That the timing of the implementation of new alternative intervention strategies (demand management or other services) should consider patient age, as some groups will experience growth prior to others and the quantum of growth varies for each group, and
• The need to fund additional non-hospital activities that already occur, but will experience increased demand as a result of the ageing of the population.

The opportunity to influence operational or tactical activity through strategic decision-making occurs prior to the time that increased demand eventuates. This is not to say that strategic decisions cannot be made once increased demand has occurred. However, the situation may be less ideal, as some decisions have significant lead times (for example, building extra capacity often takes years).

Although the purpose of the forecast is designed to influence strategic decision-making, Bay and Nestman (1984) note that ultimately political priorities of governments determine what health care services are provided. Consequently, one such additional use of the forecast information is to influence the decision-making processes of elected governments.

7.4.3 Population profiles and assumptions

Population forecast
The population forecasts used in this analysis were created by the ABS (1999 and 2000). The ABS provides three forecasts around future population change: low, medium and high forecasts. The forecasts are based upon a variety of factors and take into account various factors including health and birth rates. The need for multiple views of the future world reflects that these forecasts are not certain, but rather indicative and rely heavily upon the underlying assumptions used.
The analysis conducted has relied upon the middle range forecast of the population, which is deemed reasonable from a number of perspectives, including:

- It is the forecast recommended for use within the Department of Health
- It is the forecast recommended for use by the ABS if the middle forecast is sought and the more extreme views are not required (Australian Bureau of Statistics, 1999), and
- On the basis of being a demonstration of technique for this research.

The ABS population forecasts are known to generate debate on occasion. For example, from my role within the Department of Health I know that there has been debate as to whether the ABS or the Planning SA population forecasts should be used. This debate has since been resolved in favour of using the ABS population forecast.

For the purpose of this research, the technical issues associated with forecasting the future population are not being considered as this is outside the bounds of this research. Rather, the population forecasts are accepted as reliable on the basis that ABS has a significant responsibility in trying to produce reliable statistical information for the benefit of the Australian public. Reliance is thus placed upon the ABS’s credibility. Acknowledgment of the limitations of such forecasts is, however, given.

**Clinical activity and practice**

The resultant bed occupancy forecasts are not only reliant upon the population forecast, but also of the activity occurring within the hospital during the year the activity was measured. For the purpose of forecasting future bed occupancy, it was assumed that clinical activity on a population basis remained constant. Figure 60
showed that the mix of short and long-stay beds forecasted remained constant. This is a proxy measure for confirming that clinical activity used in the forecast remained constant.

In measuring bed occupancy, the activity of the hospital is not questioned, but assumed to be appropriate. There are a number of aspects that could be questioned, including whether:

- The admission of patients was appropriate
- The services offered are those required by the community
- The mix of services and the related volumes of services appropriate
- Variation in clinical practice within is significant and affects bed use
- Variation in clinical practice across hospitals exists and if it does, whether the hospital performs well or not,
- There is significant levels of unmet demand, and
- Funding or other decisions outside the hospital affect the level of services provided, and therefore also the number of beds that are occupied.

All of these factors are important in determining whether the forecast number of beds is reasonable. Flinders Medical Centre hospital has previously commissioned studies on whether admission to hospital is appropriate and concluded that almost all admissions were appropriate (Baggoley, Phillips and Aplin, 1994; Finucane et al., 2000).

The introduction of casemix funding has enabled comparison of activity between hospitals. The role of benchmarking in casemix funding would be greatly diminished
if variation between hospitals did not exist and the argument that casemix funding
would lead to greater efficiencies would be lessened.

Clearly, as identified by Bay and Nestman (1984), the political process strongly
influences the quantum and type of health care services provided.

Thus, this reinforces the view that the forecast should be viewed as a potential that
might arise if current clinical activity is continued, rather than that it is immutable.
The use of such forecasts would benefit from concomitant activities that strove to
achieve increased efficiencies in service delivery and also quantify or better
understand the effects of variation, unmet demand, the potential for new technology
(including medicines) and other factors that could significantly affect bed occupancy.

7.4.4 Resource use and scenario testing

Resource use summary information
The bed occupancy model parameters can enable quantification of resource use, that
is, the average number of occupied beds, and information about the likely number of
patient admissions as shown in Tables 21 and 22.

The information reported in Tables 21 and 22 was calculated using spreadsheets that
were developed for this purpose for Millard. The information provides perspective on
bed occupancy statistics at the commencement and conclusion of the forecasting
period. The presentation of the summary information contained in the tables is

---

1 Ms Georgina Christodoulou developed the spreadsheets Microsoft Excel in her capacity as a research
assistant for Peter Millard. I liaised with Ms Christodoulou during the development of these
spreadsheets and provided some limited input into the development. I was given access to these
through my collaboration with Peter Millard and his research colleagues.
valuable, as it quantifies summary information about the change in bed occupancy over the forecast period. Senior decision-makers frequently do not have the time to delve deeply into how information has been gained, and must often rely upon summary information. Without the aid of such summary tables the ability to influence senior decision-makers would be limited, particularly when the catch cry of many time poor senior decision-makers is “just give me the number(s)”. There is a risk, however, in that the use of the resource summary information may be misinterpreted unless the end user is apprised of the methodology used to derive the information and the issues and assumptions that are not conveyed in the summary table, but affect the interpretation of the results. However, this is a risk that exists with all such similar information and should not deter attempts to improve the basis for decisions around bed occupancy.

The presentation of the tables alone also hides some of the policy implications revealed in Figures 56 to 59. Thus, augmentation of the information contained in the two tables with at least two of the Figures is suggested as being necessary to reduce the likelihood of misguided decision-making.

**Scenario testing**

The ability to alter the model to reflect possible or desired changes in the rate of admission or patient flow gives rise to the ability to undertake scenario testing before implementation of real change. This is a powerful feature of the compartmental flow model and was a feature in the original BOMPS package (BOMPS, 1992). The ability to independently alter the flow and bed number parameters for each compartment confers considerable advantage over using a single ALOS measure (that is flawed
anyway) compared to many other methods (see the review of the literature, Chapter 2).

Figure 61 provides graphical output on the effects of testing a range of changes to the number of beds, rate of patient flow and simultaneous change to both rate of flow and beds. What becomes evident from analysis of Figure 61 is that the implementation of change requires time for the system to re-adjust and stabilize. The time required to reach system stability is relatively short in the scenarios examined and is less than three months. This can be contrasted to a geriatric service where the time required to reach stability was much longer, being around five and a half years in one system (El-Darzi, Vasilakis, Chaussalet and Millard, 1998). Despite the much shorter time to system stability, the dynamic nature of acute care hospitals may mean that this position is never reached before the next change is implemented (As an example, see Figure 35 in Chapter 5 where there are four changes shown in the number of available beds during a single year).

While the ability to implement endless configurations of scenario changes exists, the ability to actually implement the change at the operational level needs to be considered. For example, a goal of saving a certain level of funds may be achieved by reducing patient length of stay by some percentage, this may not translate to real savings for a variety of reasons including:

- The achieved reduction in length of stay (for example) does not result in a reduction of staffing (for example, only one bed will close, but nursing numbers require that eight beds must close for the staffing to be reduced)
• The planned reduction in length of stay is marginal (perhaps as little as a few hours) and does not translate into any bed closures

• Reduction in length of stay may have already reached a plateau that cannot be changed further without new technologies, and

• Demand is such that despite the planned reductions being achieved, the ability to close the bed is not achieved due to the need to provide additional services.

Consequently, the use of the scenario testing ability provides the mechanism to initiate discussions about system change between strategic decision-makers and operational workers as opposed to guaranteeing that planned changes designed by strategic decision-makers will eventuate. The issue of scenario testing and simulation is examined further in Chapter 11.

7.4.5 Technical Issues

Model selection and test data
Cross validation using test data represents an alternative approach to model selection (Hastie, Tibshirani and Friedman, 2001), but is only viable when test data is available. When forecasting future occupancy levels, the availability of test data may not exist. For example, the results presented in Chapter 5, which used the same data, relied upon the 1999 data as test data, but in this work, the 1999 period was a forecast period and thus the 1999 could not be used as test data. From my experience in the health sector, this is not unusual.

The need to take account of variability across the year and also split the data into small age groups meant that withholding some data from the training year for the
purpose of providing test data was also not a reasonable option. Had this option been pursued, it may have resulted in some data groupings having little data and thus potentially adversely affected the resulting model.

In the absence of model selection methodologies, sole reliance upon performance measures, such as error measures or correlations, are likely to result in selection of over-fitted models. The ability to use the Bayesian information criterion and the Bayes factor (or log odds) provides an important model selection tool in this instance.

**Bayes factor and log odds**
The Bayes factor values are generally very large and provide the same information as the log odds values. However, in terms of ease of interpretability, easier interpretation occurs when using the log odds values and for implementation purposes, the log odd values would be preferred.

**Catchment population**
Although every endeavour was made to match the ABS population projection data to the hospital’s catchment area, it is unlikely that this has been achieved perfectly. However, there were few differences in the rates of changes of the population for the overall State when compared to subgroups created on the basis of geography. Thus, the inability to create a perfect match between the population forecasts and the hospital catchment should not impact the results.

**Period of forecast**
The issue of how far out should strategic bed occupancy models forecast has not been considered. The research presented here only projects data for 20 years. The
projections have used the available population projection data as the limitation factor for the forecast. While it may be argued that it is useful to forecast out using all the available data, this is probably questionable given that services are affected by a myriad of factors (see earlier section on clinical activity and practice), most of which are volatile (for example, political influence and technology). Thus, while it is perhaps unrealistic to some extent to project forward bed occupancy too far into the future, it is nevertheless useful from a strategic perspective, because it opens up discussion about what the future might be and how it might be altered. A pragmatic view might be that forecasts might be viewed as more likely to represent a likely outcome if not more than ten years into the future, with longer forecasts of bed occupancy representing a general indication of possible outcomes. The usefulness of the bed occupancy forecast should be considered in terms of whether the methodology is reliable, and tested in terms of measuring the bed occupancy and whether the population forecasts have been sourced from a respected and credible source.

**Bed numbers**
The fit of one part of the model to one data point, namely the total bed occupancy, is reported in Table 20. The model consistently over estimates the data at this single point. Given the intended purpose of the model output, it is preferable that over-estimation occurs rather than under-estimation of the total number of occupied beds. In both instances, it is imperative that the modeller and users of the model output are aware of such variation.

In terms of model fit, particularly at a single point, it must also be recognised that the model does not provide a deterministic answer, but rather aims to provide an answer
that lies within some degree of variation. The use of confidence intervals, as shown in Table 19 is one means of reporting that uncertainty exists.

There is also one possible point of variation that may lead to a difference in model fit at the point of total occupancy when comparing the HealthCare Otago and FMC model outputs, and this is associated with the census time. The HealthCare Otago model was based upon midnight census data, whereas the FMC model was based upon midday census data. Midday census data will capture same-day patient and possible admitted and discharged patient overlap if admission occurs before patient discharge. This may have affected the fit of the FMC data slightly at the point of total occupancy, but can be easily remedied by the use of data where the census time used is midnight. The need to accommodate inefficiencies arising from the overlap of patient admission and discharge can be factored into the model separately, as can the need for beds required to accommodate same-day emergency patients (which were few in number on a patient per day basis). Such modifications represent trivial changes to the modelling process.

Profile construction
The data profiles used to construct the models were manually constructed. While this is not a particularly difficult task, it was nevertheless a time consuming task. This became amplified as multiple profiles had to be created on the basis of patient age.

For application in applied situations the use of automated data profile creation is recommended as has happened with the HealthCare Otago data. Failure to adopt such an approach may limit the adoption of such techniques in applied settings.
Efficiency factors
Farmer and Emami (1990) have suggested that an efficiency factor be incorporated into the calculation of bed numbers. There is no need to incorporate such a factor into the compartmental flow model in most circumstances as the ability to undertake scenario testing provides for this (see earlier section on Scenario testing).

Annual average models and the need for other model adjustment
In Chapter 5 it was noted that St George (1988) and MacStravic (2001) have reported that an annual average model of acute hospital services will be insufficient to enable bed planning, as the variation within the data will not be detected. Clearly, it is possible to capture variation within the bed occupancy model as shown by the reporting of confidence intervals for the model parameters in Table 19.

The usefulness of reporting an annual average level of occupancy is, however, acknowledged as being deficient in some instances as such a figure relies upon the user understanding that acute care hospital services do exhibit seasonal trends. The results from Chapter 5 support the position put forward by St George (1988) and MacStravic (2001) insofar as that the preferred model was not the annual average, but the “seasonal” model. Thus, strategic planning for the annual average may be appropriate for some activities, while for other activities it may be more relevant to plan for a peak season level of bed occupancy.

The introduction of a method to modify the annual average model that has been disaggregated on the basis of age groups so that it can reflect seasonal variation is discussed further in Chapter 8.
7.5 Conclusion

In this chapter, the exploration of model choice methodology when selecting compartmental flow models of bed occupancy disaggregated on the basis of patient age, and the implications of forecasting future bed occupancy based upon linking compartmental flow model parameters with population forecasts has been undertaken. This has established a linkage between the strategic bed occupancy models and population change.

The age grouped annual average occupancy model created indicates that should the assumptions underlying the forecasts hold true, a 37 per cent increase in the number of occupied beds will eventuate. Such an outcome has significant implications for the health system as it would imply that significant levels of additional resources will be required to meet new capital works (to provide additional physical capacity) and additional operational costs.

The ability to use the Bayesian information criterion and the Bayes factor as a guide to model selection in the absence of test data has also been demonstrated in this chapter. Adoption of such measures should improve model selection and result in less over-fitted models being chosen.

While the average occupancy model can be easily linked to forecast changes in the population, it does not reflect the seasonal variations that are widely reported to occur. Chapter 8 extends some of the work presented in this chapter to include analysis of a mechanism that provides linkage between population change and seasonality issues.
The issue of how best to incorporate additional model complexity arising from seasonality is also considered.
Chapter 8

Incorporation of seasonal effects into the compartmental flow model: should model complexity increase or model design change?

In the previous chapter, the ability to select a preferred compartmental flow model of average bed occupancy using data that had been disaggregated on the basis of patient age was demonstrated. It was then demonstrated that such models could be linked to population forecasts in order to generate information that would help strategic planners better understand the demand for hospital beds as the population age profile changed. In this chapter, exploration of forecasting is continued with the population linked bed occupancy compartmental flow model being adjusted for seasonality. The chapter has the following structure:

8.1 Introduction....................................................................................................257
8.2 Methodology..................................................................................................262
  8.2.1 Data and the Bed Occupancy Compartmental Flow Model .......................262
  8.2.2 Weather and Regression .............................................................................262
  8.2.3 Linkage to Population Change.................................................................266
  8.2.4 Analysis of vacancy rates............................................................................266
8.3 Results............................................................................................................267
  8.3.1 Weather and regression...............................................................................267
  8.3.2 The amended forecast .................................................................................267
  8.3.3 Accounting for vacancy.............................................................................277
8.4 Discussion......................................................................................................282
  8.4.1 Effect of weather on the model.................................................................282
  8.4.2 Inclusion of the “vacancy” factor .............................................................289
  8.4.3 The amended forecast .................................................................................294
  8.4.4 The role of modifiers ..................................................................................294
8.5 Conclusion .....................................................................................................297
8.1 Introduction

At a strategic level, the ability to link population age profiles to bed occupancy compartmental flow models for forecasting purposes has value, because it links to the differences in resource use associated with different stages of life. However, it is also clear that bed occupancy varies at different times of the year. From the literature (for example, St George, 1988; Fullerton and Crawford, 1999; Heyworth, Anderson and Belstead (2000); MacStravic, 2001; Jones, Joy and Pearson, 2002; Menec VH, Roos NP and MacWilliam, 2002; and Matter-Walstra, Widmer and Busato, 2006) and also from my experience in working in the health system it is evident that some of this variation is seasonal. For example, the existence of winter bed peaks or crises is well accepted in the research literature (Vasilakis and El-Darzi, 2001) and also the media (for example, Clarke and Crouch, 2002; Patty, 2000; and Pollard, 2004). The term “seasonal” is used in this context as to relating to particular periods of the year, such as summer and winter, as opposed to the meaning given in operational research texts (for example, Ozcan, 2005), which define the term “seasonal” to mean a short-term relatively frequent variation. The more general definition may encompass variation that can be attributed to the weather seasons, but it also can relate to many other types of variations. The findings presented in Chapter 5 also support the notion of seasonal influence on the data analysed. Thus, it is posited that the inclusion of a seasonal adjustment factor would add value to bed occupancy compartmental flow models.

Given the scenario that forecasting bed occupancy on the basis of age grouping is useful for planning purposes, the question of how to incorporate seasonality into this modelling arises. There are at least two options that could be considered, namely:
The creation of seasonal and age group disaggregated data sets to create compartmental flow models, and

The retention of the approach developed in Chapter 7 with the addition of a seasonality modifier that does not rely upon the development of more compartmental flow models.

If the first scenario was to be adopted then the number of sub-models would expand from five age related models to five age related models multiplied by four seasonal models (assuming that the findings from Chapter 6 are adopted), that is, twenty compartmental flow models would be required to model the FMC dataset. Clearly, a move to such an approach would involve increasing the overall model complexity. Alternatively, new data groupings would be required to reduce the number of model sub-groups. Such an approach would require significant investment in resources and in terms of application in real world scenarios may possibly work against uptake of the research.

The addition of a modifier to the approach developed in Chapter 7 may provide the opportunity to retain the existing age based compartmental flow models without an undue increase in model complexity. A useful analogy may be to consider the first option as one of a system controlled by push buttons or switches, with a switch for each compartmental flow sub-group, while the second option is more akin to a system that is controlled with far fewer sliding scale controls as illustrated in Figure 62.
The sliding scale model is still a parameterised model, but the parameters can now be thought of in qualitatively different ways.

Consequently, research was undertaken to consider whether a “sliding scale” modified compartmental flow could be developed to add a seasonal or weather factor to the age related compartmental flow models developed in Chapter 4. The weather provides measures, such as air temperature, that change across the seasons and is also known to be related to the prevalence of some diseases (Jones, Joy and Pearson, 2002). The research was limited to demonstrating the approach for one age group and
was not fully implemented, as the intended purpose of the research was to investigate the potential of this approach and not to develop it fully.

The research was, however, extended to include consideration of the effect of vacancy rates on the resultant sliding scale model. Bed vacancy occurs when a patient does not occupy a given bed and occupancy rates are frequently measured as an indicator of hospital performance (for example, Ozcan, 2005). There is a tension between the number of occupied beds and the number of vacant beds: vacant beds represent an expensive resource, particularly if they have been staffed; yet, vacancy enables hospitals to cope with daily variation in patient arrivals, particularly emergency patient arrivals. This is illustrated in Figure 63.
Bagust, Place and Posnett (1999) reported that exceeding 90 per cent bed occupancy would, according to the results of their simulation modelling, result in frequent bed crises. Separate research undertaken in relation to the use of inpatient beds in Australia undertaken by myself and Millard (2005a and 2005b), suggests that a reduction in supply of over-night stay inpatient beds combined with a marginally altered demand for multi-stay beds has led to the occurrence of increasing numbers of bed crises in Australia. Consequently, the inclusion of bed vacancy in this research is important.

Figure 63: Total available beds = occupied beds + vacant beds. Vacant beds provide a buffer against variation in occupancy.
Some of the findings from this analysis were presented at a conference (Mackay and Lee, 2004a).

8.2 Methodology

8.2.1 Data and the Bed Occupancy Compartmental Flow Model

The data considered here only relate to that drawn from the Flinders Medical Centre. The contextual details about the data were described in Chapter 3 (see sections 3.2.1 and 3.3.1).

The methodology used to create the occupancy models for this work was fully described in Chapter 6 (see section 6.2.2).

This analysis only used the sub-model relating to the patients aged 65-79 years created and reported in Chapter 7. The basis for only using this group of patients was twofold:

- Of the five age groups, those aged 65-79 years occupied the greatest number of beds and therefore from a resource perspective represented a key group, and
- There was an obvious “winter” peak in the annual bed occupancy trend, which suggested a strong likelihood of seasonal influence.

8.2.2 Weather and Regression

Weather data was obtained from the Bureau of Meteorology (2004). The data were collected at the Adelaide Airport (Latitude (deg S): -34.9524; Longitude (deg E):
The data obtained were climate averages. The data provided monthly average figures for various climate measures, such as the temperature, rainfall and humidity. The data represented the period 1955 to 2003. Average climate data were used as this accounted for variation (for example, warmer and colder winters), which was considered a benefit given the intended use in forecasting.

Correlations between the monthly average bed occupancy and all the climatic averages were determined. The climatic average with the highest correlation, the mean 9 a.m. temperature measured in degrees Celsius, was selected as the target variable to explore for further analysis. A high degree of collinearity between the weather variables meant that there was little value in exploring other variables for the purpose of this work.

The mean 9 a.m. temperature had been collected for 48.5 years and there were no missing data. Additional variables were created from the mean 9 a.m. temperature variable: a lagged temperature variable, a lead temperature variable and a relative average monthly temperature change. This latter variable was calculated according to the formula given by:

$$\text{Relative average monthly temperature change} = \frac{(\bar{x}_{AT} - \bar{x}_{MT})}{\bar{x}_{MT}}$$, where

$$\bar{x}_{AT} = \frac{1}{n} \sum_{j=1}^{n} x_j$$

= annual monthly average temperature

$$\bar{x}_{MT}$$

= mean 9 a.m. temperature
where

\[ x_j \] is monthly average 9 a.m. temperature for month \( j \)

\( n \) is the number of months in the year

and

\[ \bar{x}_{MT} = \text{monthly average temperature for a given month.} \]

\[ = \frac{1}{n} \sum_{j=1}^{n} x_j \]

where

\( x_j \) is temperature recorded at 9 a.m. on day \( j \)

\( n \) is the number of days in the month

The greatest correlation of each of these variables with monthly average occupancy was achieved with the relative average monthly temperature change. This variable was used to create a simple regression model. The regression model was given by:

\[ Y = mx_r + c \]

where

\( m \) was the regression equation co-efficient

\( x_r \) was the relative difference between the 9 a.m. mean monthly temperature and the annual average temperature weighted for the given month

\( c \) was the regression equation constant

The original model compartmental flow model formula was given by:
\[ Y = A e^{-bx} + C e^{-dx} \]

where

- \( A \) is the total number of short-stay occupied beds
- \( b \) is the flow rate of the patients in the short-stay compartment
- \( C \) is the total number of long-stay occupied beds, and
- \( d \) is the flow rate of the patients in the long-stay compartment.

This was adjusted to incorporate the influence of the weather giving:

\[ Y = \left( (m x_r + c) \times \frac{A}{A+C} \right) e^{-bx} + \left( (m x_r + c) \times \frac{C}{A+C} \right) e^{-dx} \]

where

- \( m \) was the regression equation co-efficient
- \( x_r \) was the relative difference between the 9 a.m. mean monthly temperature and the annual average temperature weighted for the given month
- \( c \) was the regression equation constant, and
- \( A, b, C \) and \( d \) were as per the Harrison and Millard (1991) original equation.

The regression relationship was established between the weather variable and total bed occupancy. Consequently, the compartmental flow model was adjusted to take account of the weather. The adjustment was only applied to the bed number parameters and flow was assumed to remain constant.
8.2.3 Linkage to Population Change

The methodology for the creation of the linked bed occupancy compartmental flow model and population forecast was described in Chapter 7. The forecast was amended to take account of the revised model of bed occupancy.

8.2.4 Analysis of vacancy rates

The actual bed occupancy was compared to the weather adjusted compartmental flow total occupancy. From this it was possible to calculate the number of beds required to achieve zero days where there would be insufficient beds each month and also the number of beds required to achieve not more than five per cent of days where patients would be turned away.
8.3 Results

8.3.1 Weather and regression

Bed occupancy trends

Figure 64 shows the daily bed occupancy for the base year. The total bed occupancy trend suggests the presence of a winter bed occupancy peak, though such peaks for the age-grouped data are difficult to ascertain.

Figure 65 shows the moving averages, a more useful mechanism for seeing trends over time, for the age-grouped data.

Figure 64: Daily occupancy trends for total and age grouped bed occupancy. The moving average indicates the existence of a winter peak in total occupancy.
Visual inspection of the trends presented in Figure 65 show that bed occupancy for the 50-64, 65-79 and 80 or more year age groups exhibited some degree of seasonal variation.

In terms of bed occupancy for this data set, the 65-79 year age group was the most important as they occupied the most beds. Consequently, given the limited intended scope of this piece of research, all further analysis in this chapter relates to the data pertaining to this age group.

**Correlations**
Correlations between the average climate data obtained from the Bureau of Meteorology and the 65-79 year age-grouped bed occupancy data are reported in Table 23.
Given the strength of the correlation between the mean 9 a.m. temperature and bed occupancy, the mean 9 a.m. temperature was further investigated for use in modifying the bed occupancy compartmental flow model. Figure 66 shows a trend that has similar timing to the bed occupancy trend, but is the inverse, that is a winter trough occurs instead of a winter peak.

Table 23: Correlations between bed occupancy data and average climate variables. The mean 9 a.m. temperature was found to have the strongest correlation.
Table 24 shows the correlations between the four 9 a.m. mean air temperature variables and monthly average bed occupancy.

Table 24: Correlations between the four temperature variables and bed occupancy. The relative monthly average temperature change has strong correlations with occupancy and the over temperature variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>average monthly air temperature</th>
<th>lagged average monthly air temperature</th>
<th>lead average monthly air temperature</th>
<th>relative monthly average temperature change</th>
</tr>
</thead>
<tbody>
<tr>
<td>monthly average occupancy</td>
<td>-0.906*</td>
<td>-0.858*</td>
<td>-0.711*</td>
<td>-0.915*</td>
</tr>
<tr>
<td>average monthly air temperature</td>
<td>0.859*</td>
<td>0.859*</td>
<td>0.982*</td>
<td></td>
</tr>
<tr>
<td>lagged average monthly air temperature</td>
<td></td>
<td>0.483</td>
<td></td>
<td>0.841*</td>
</tr>
<tr>
<td>lead average monthly air temperature</td>
<td></td>
<td></td>
<td></td>
<td>0.837*</td>
</tr>
</tbody>
</table>

Number of observations = 12. *Correlation is significant at the 0.01 level (two-tailed).
Table 25 indicates that the correlations between the weather variables and the occupancy data existed even when more occupancy data was included in the analysis. This was important in identifying that the influence of the weather was not a one-off incident.

Given the strength of the relationship between the relative change temperature variable and bed occupancy this relationship was explored further. Figure 67 shows a scatterplot and regression equation of this relationship.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>average monthly air</td>
</tr>
<tr>
<td></td>
<td>temperature</td>
</tr>
<tr>
<td>monthly average</td>
<td>-0.471*</td>
</tr>
<tr>
<td>occupancy</td>
<td>lagged average monthly</td>
</tr>
<tr>
<td></td>
<td>air temperature</td>
</tr>
<tr>
<td></td>
<td>-0.338*</td>
</tr>
<tr>
<td></td>
<td>lead average monthly</td>
</tr>
<tr>
<td>lead average</td>
<td>air temperature</td>
</tr>
<tr>
<td></td>
<td>0.859*</td>
</tr>
<tr>
<td>monthly average</td>
<td>relative monthly</td>
</tr>
<tr>
<td>temperature</td>
<td>average temperature</td>
</tr>
<tr>
<td></td>
<td>0.483*</td>
</tr>
<tr>
<td></td>
<td>change</td>
</tr>
<tr>
<td></td>
<td>0.837*</td>
</tr>
</tbody>
</table>

| Number of observations = 72. *Correlation is significant at the 0.01 level (two-tailed). |

Table 25: Correlations between 1995-2000 average monthly bed occupancy and the average temperature variables. The correlations are weaker, but still significant and of reasonable strength.
Table 26 reports the correlations between the temperature variables and the average monthly bed occupancy for each year of data held. It is evident that the weather accounted for much variation in some, but not all years.

Figure 67: Scatterplot of bed occupancy for the model year and relative temperature change variables. The variation in occupancy was largely explained by the relative temperature change variable.

\[ y = -26.572x + 69.909 \]
\[ R^2 = 0.8347 \]
The temperature did not appear to be uniformly correlated for each year, suggesting the influence of other factors. Overall, average monthly occupancy was correlated with the relative temperature change variable ($r = -0.486, p<0.01$).

<table>
<thead>
<tr>
<th>Variables</th>
<th>average monthly air temperature</th>
<th>lagged average monthly air temperature</th>
<th>lead average monthly air temperature</th>
<th>relative monthly average temperature change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995 monthly average occupancy</td>
<td>-0.357</td>
<td>-0.070</td>
<td>-0.572</td>
<td>-0.396</td>
</tr>
<tr>
<td>1996 monthly average occupancy</td>
<td>-.835*</td>
<td>-0.560</td>
<td>-.826*</td>
<td>-.840*</td>
</tr>
<tr>
<td>1997 monthly average occupancy</td>
<td>-0.391</td>
<td>-0.008</td>
<td>-.730*</td>
<td>-.328</td>
</tr>
<tr>
<td>1998 monthly average occupancy</td>
<td>-.906*</td>
<td>-.858*</td>
<td>-.711*</td>
<td>-.915*</td>
</tr>
<tr>
<td>1999 monthly average occupancy</td>
<td>-0.574</td>
<td>-0.571</td>
<td>-0.427</td>
<td>-.650*</td>
</tr>
<tr>
<td>2000 monthly average occupancy</td>
<td>-0.333</td>
<td>-0.442</td>
<td>-0.163</td>
<td>-0.389</td>
</tr>
<tr>
<td>average monthly air temperature</td>
<td></td>
<td></td>
<td>0.859*</td>
<td>0.982*</td>
</tr>
<tr>
<td>lagged average monthly air temperature</td>
<td></td>
<td></td>
<td>0.483*</td>
<td>0.841*</td>
</tr>
<tr>
<td>lead average monthly air temperature</td>
<td></td>
<td></td>
<td></td>
<td>0.837*</td>
</tr>
</tbody>
</table>

Number of observations = 12. *Correlation is significant at the 0.01 level (two-tailed).

Table 26: Correlations between temperature variables and bed occupancy for individual years. The influence of the temperature on occupancy was not consistent; rather it was one of many factors that influenced occupancy.
Regression and modified compartmental flow model
The regression model output is detailed in Tables 27 to 29.

Table 27: The R-square value for the model was high, with 82 per cent of the variance in occupancy explained by the weather variable.

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>R square</th>
<th>Adjust R Square</th>
<th>Standard Error of the Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.914</td>
<td>0.836</td>
<td>0.820</td>
<td>3.64</td>
</tr>
</tbody>
</table>

Note: the predictors were a constant and the relative temperature change

Table 28: The explained variation is significantly greater than the unexplained variation. Thus, from the data, it would appear that a linear relationship between the weather variable and occupancy existed.

<table>
<thead>
<tr>
<th>Model</th>
<th>unstandardized coefficients</th>
<th>standard coefficients</th>
<th>t</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>standard error</td>
<td>Beta</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>69.802</td>
<td>1.079</td>
<td>-0.914</td>
<td></td>
</tr>
<tr>
<td>Relative temperature change</td>
<td>-26.554</td>
<td>3.717</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>64.704</td>
<td>-7.144</td>
<td>p &lt;0.001</td>
<td></td>
</tr>
</tbody>
</table>

Note: the dependent variable was occupancy.

Table 29: Model coefficients. The model coefficients vary slightly to those in Figure 65, because a statistical package was used to run the regression analysis as opposed to the addition of a trend as part of the graphical capabilities of a spreadsheet.

Anova

<table>
<thead>
<tr>
<th></th>
<th>sum of squares</th>
<th>degrees of freedom</th>
<th>mean square</th>
<th>F</th>
<th>significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>674.75</td>
<td>1</td>
<td>674.75</td>
<td>51.04</td>
<td>p &lt;0.001</td>
</tr>
<tr>
<td>Residual</td>
<td>132.20</td>
<td>10</td>
<td>132.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>806.95</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: the predictors were a constant and the relative temperature change. The dependent variable was occupancy.

The regression model was then combined with the compartmental flow model previously obtained (see Chapter 7). The amended model, original average compartmental flow model and the actual data are shown in Figure 68.
The adjusted model provided a better fit to the data at the monthly occupancy level compared to the average occupancy compartmental flow model. The improved fit was also achieved with the daily occupancy data as shown in Figure 69.

Figure 68: The weather adjusted compartmental flow model better reflects the actual occupancy data than the average compartmental flow model.
8.3.2 The amended forecast

The forecast bed occupancy based upon population change for patients aged 65-79 years was recalculated using the weather adjusted compartmental flow model. The comparison of the original forecast and the weather adjusted forecast is shown in Figure 70.

Figure 69: Despite a better fit between the weather adjusted model and the daily occupancy data, the model is clearly suited, as intended, for exploring longer-term strategic issues as opposed to the operational issues associated with daily occupancy.
As shown in Figures 68, 69 and 70, the weather adjusted compartmental flow model enables visualisation of the effect of seasonality on total average bed occupancy, but does not reflect daily bed requirements well. While this is to be expected, as the model was not constructed to reflect the occupancy for each day, there is some value in understanding the level of vacancy required so that all occupancy levels can be fulfilled.

Figure 71 illustrates the improvement in performance of the model, as measured by the percentage of the year when bed shortages would occur, by adding the weather adjustment to the model and also considering the effect of allowing for a vacancy rate.

8.3.3 Accounting for vacancy

As shown in Figures 68, 69 and 70, the weather adjusted compartmental flow model enables visualisation of the effect of seasonality on total average bed occupancy, but does not reflect daily bed requirements well. While this is to be expected, as the model was not constructed to reflect the occupancy for each day, there is some value in understanding the level of vacancy required so that all occupancy levels can be fulfilled.
The level of vacancy required to avoid overload or patient turn-away was on average 12 and 11 per cent with standard deviations of 9 and 6 per cent for the original and weather-adjusted model, respectively as shown in Table 30.

Figure 71: Comparison of the original compartmental flow model and weather adjusted compartmental flow model. The weather-adjusted model required less vacancy to avoid patient turn-away.
It can be seen that the variation in the vacancy rate required to avoid patient turn-away was reduced for the weather-adjusted model (lower standard deviation), as is the maximum vacancy rate. Figure 71 also illustrated that for any particular level of vacancy, the percentage of days that would result in patient turn-away was lower.

Table 30: Vacancy rates for the original compartmental flow model and weather-adjusted model.

<table>
<thead>
<tr>
<th>Month</th>
<th>Vacancy rate - Base Model</th>
<th>Vacancy rate - Weather adjusted Model</th>
<th>Difference Between Models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>6%</td>
<td>16%</td>
<td>-10%</td>
</tr>
<tr>
<td>Feb</td>
<td>5%</td>
<td>14%</td>
<td>-9%</td>
</tr>
<tr>
<td>Mar</td>
<td>12%</td>
<td>19%</td>
<td>-7%</td>
</tr>
<tr>
<td>Apr</td>
<td>10%</td>
<td>12%</td>
<td>-2%</td>
</tr>
<tr>
<td>May</td>
<td>13%</td>
<td>8%</td>
<td>5%</td>
</tr>
<tr>
<td>Jun</td>
<td>20%</td>
<td>5%</td>
<td>15%</td>
</tr>
<tr>
<td>Jul</td>
<td>27%</td>
<td>11%</td>
<td>16%</td>
</tr>
<tr>
<td>Aug</td>
<td>22%</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>Sep</td>
<td>21%</td>
<td>17%</td>
<td>4%</td>
</tr>
<tr>
<td>Oct</td>
<td>9%</td>
<td>9%</td>
<td>0%</td>
</tr>
<tr>
<td>Nov</td>
<td>1%</td>
<td>5%</td>
<td>-4%</td>
</tr>
<tr>
<td>Dec</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>St Dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12%</td>
<td>9%</td>
<td>0%</td>
<td>27%</td>
</tr>
<tr>
<td></td>
<td>11%</td>
<td>6%</td>
<td>0%</td>
<td>19%</td>
</tr>
</tbody>
</table>

Figure 72 illustrates the effect of including a vacancy rate in the weather-adjusted model. Two vacancy rates were used: one to achieve no more than five per cent of days with patient turn-away, and the other to achieve no days of patient turn-away.
Visual inspection indicates that the weather-adjusted model fitted the monthly average occupancy data well. While the shape of the weather and vacancy adjusted models were similar to that of the weather-adjusted model, there were some differences. In terms of fit, examination of the absolute error, as shown in Table 31, was lowest for the weather-adjusted model.

![Figure 72: Comparison of the original, weather-adjusted and weather and vacancy-adjusted models with the data.](image)

Similar results were found for the absolute errors even when calculated using the daily occupancy. The absolute errors for each model were as follows: the average model had an absolute error of 3112.4; the weather-adjusted model had an absolute error of 2220.2; the bed shortages on five per cent of days model had an absolute error

<table>
<thead>
<tr>
<th>Model Type</th>
<th>Month</th>
<th>Total Absolute Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>average model</td>
<td>Jan 9.2</td>
<td>Feb 11.7</td>
</tr>
<tr>
<td>weather adjusted average model</td>
<td>2.6</td>
<td>5.3</td>
</tr>
<tr>
<td>bed shortages 5% of days</td>
<td>11.0</td>
<td>13.8</td>
</tr>
<tr>
<td>no bed shortages</td>
<td>14.4</td>
<td>15.5</td>
</tr>
</tbody>
</table>
of 3661.4; and the no patient turn-away model had an absolute error of 4298.0. These may be more easily interpreted as an average absolute error per day, giving values of 8.5, 6.1, 10.0 and 11.8, respectively. It was not unexpected that the vacancy-adjusted models performed less well in terms of absolute error, because they were adjusted to increase the number of beds to reduce or remove the effect of days of high occupancy (or extreme variation). In terms of performance against patient turn-away indicators, however, these models perform better than the non-vacancy adjusted models.

Incorporation of the vacancy adjustments to the forecast for future bed requirements is shown in Figure 73.

![Weather and Overload Adjusted Model Bed Forecast](image)

**Figure 73:** Comparison of the original, weather-adjusted and no patient turn-away model forecasts. Inclusion of seasonality and vacancy is important for aiding planning decisions.
It can be seen that the capacity required to cope with future activity levels (based upon anticipated population change) could be under-estimated unless the effects of seasonality and vacancy rates are understood.

8.4 Discussion

I am not aware of factors such as the weather being incorporated previously as part of compartmental flow models of bed occupancy. Consequently, this research, while being limited to one of demonstration, has shown that the bed occupancy compartmental flow model can be adjusted to incorporate the effect of weather, at least for medical patients aged between 65-79 years treated in an acute care hospital environment. Such modification should make the model more useful to planners and decision-makers, and therefore more likely to be adopted as a planning tool.

The creation of a modified compartmental flow model raises various issues and these are discussed in the remainder of the discussion.

8.4.1 Effect of weather on the model

It is evident from the results presented in this chapter that the weather effected bed occupancy. While this should be of little or no surprise given recent publications in the literature regarding the existence of “winter bed crises” (Vasilakis and El-Darzi, 2001) and other literature concerning the influence of weather on the patterns of disease (Jones, Joy and Pearson, 2002), the inclusion of such a modifier is novel in regards to the work on compartmental flow models of bed occupancy.
The modified model
As stated in the methodology (see section 6.2.2), the original model put forward by Harrison and Millard (1991) was modified to facilitate consideration of more data and incorporation of a simple regression model of weather and bed occupancy.

Schematically this can be represented as moving from the “base” model to a “modified” model as illustrated in Figure 74.

Figure 74: The movement from the base model to the modified model resulted in a better fit between the model and data.

Figure 74 consolidates the results presented in Figures 68 and 69, and Table 31 into a single illustration – modification of the base model results in a better fit of the data.

The method has enabled the modification of the original base model in a relatively simple manner. Rather than increasing the number of compartmental flow models
required to reflect seasonal changes, as suggested in Chapter 5, there was no increase in the number of compartmental flow models. Rather the existing compartmental flow model has been retained. There are several benefits from such an approach. First, the complexity of the model does not increase as much as it would have if additional compartmental flow models had been used, which is important in terms of model generalisation. The number of model parameters would have been 48 using individual monthly compartmental models (see Figure 62), or 16 based upon the results from Chapter 5. The adopted approach resulted in a model that had six parameters. Using the number of model parameters as a measure of complexity, the adopted approach is eight times less complex than the individual monthly compartmental model approach and 2.7 times less complex than the approach based upon the results in Chapter 5.

The ability to more easily undertake what-if scenario analysis is also related to model complexity. The number of parameters available for modification in a what-if scenario analysis does not increase greatly using the sliding scale model (an increase in one modifiable parameter as opposed to an additional 12 parameters using the multiple compartmental flow models as suggested from the results of Chapter 5). This is also a benefit, as it reduces the effort required to undertake what-if analysis.

An additional benefit from the modification of the model is that the weather parameter is modifiable at an increased level of detail (monthly as opposed to seasonal). Furthermore, the weather parameter relates to the weather as opposed to the seasonal compartmental flow models, which have no actual weather variable, thus making what-if scenario analysis relating to weather change possible (see Chapter 11 for additional analysis).
Investigation of whether the relationship with the weather was different for short and long-stay patient bed occupancy compartments was not undertaken due to the demonstration nature of this work. There is a possibility that variation in the relationship with the weather may exist and this represents an avenue for further research.

**Measuring complexity**

The modification of the model affects the ability to measure complexity using the Bayesian information criterion (BIC). As identified in Chapter 4 (see section 4.6.3), a potential criticism of the BIC (and also of the AIC) is that it does not take into account the functional form of models (Pitt and Myung, 2002). The modification of the model has resulted in a changed functional form and thus the ability to measure and draw conclusions about two different models using the BIC in this situation is difficult and was not attempted. However, appreciating the implications of the complexity and over fitting trade off, as discussed in Chapter 4, it is not unreasonable to conclude that achievement of improved model fit with a less complex model is the preferred outcome.

Model selection methods, such as the Bayesian model selection and minimum description length, do take into account the functional form of models (Pitt and Myung, 2002). These methods are computationally difficult and were not explored for this thesis (see Chapter 4).
The ability to compare to the alternative option
The alternative option of capturing the effect of seasonality was mentioned in the introduction in this chapter, namely, the creation of a seasonal and age compartmental flow model for bed occupancy (that is, a combination of the approaches presented in Chapters 5 and 7). It has already been stated that this would lead to a greater complexity than the approach investigated.

The output from Chapter 5 is not comparable with the results presented in this chapter as the seasonal model created in Chapter 4 was based upon all the data and not a subset based upon patient age. In order for comparison to be made, a subset of the data would need to be created that was based upon age and also “seasons”. While technically this was possible, there is little pragmatic reason for pursuing this line of research for the reasons already stated, particularly relating to increased complexity, generalisation and the ability to actually modify the factor under investigation (that is, the weather).

The results from the analysis presented in this chapter are supportive of this stance insofar as:

- The age grouping was obtained from the work in Chapter 7 and the best model tested was used, and
- The absolute error of the modified model improves with the weather adjustment (see Figures 68, 69 and 72 for visual inspection and Table 31 for the absolute errors).
The benefit of the modification
The use of the compartmental flow model to reflect bed occupancy overcomes the inherent problems with the ALOS. In terms of forecasting, however, the use of a model based upon average occupancy has limitations as suggested by St George (1988) and MacStravic (2001). The primary limitation is that bed occupancy is affected by seasonal variation. Thus, any planning based without consideration of the variation will be flawed. While there is no reason why an “average” bed occupancy model could be used providing that the ramifications of the variation are understood, it is difficult for planners and strategists to undertake scenario testing.

The amended forecast illustrated in Figure 70 shows the seasonal variation well. The visualisation of the influence of the weather on the forecast not only enables better what-if scenario testing to occur, but should improve the acceptance of the model by showing that it captures the winter peaks and summer troughs in bed occupancy. Clearly, decisions about bed occupancy and related resource issues (such as capital planning, workforce and avoidance strategies) would be flawed if they were only made using the average occupancy.

The role of other factors that affect bed occupancy
It is evident from Figures 64 and 65 that seasonal variation in bed occupancy occurred at least at the overall level and also for some subsets of data based upon patient age. Tables 25 and 26 confirm that the chosen weather variable correlated with bed occupancy and that this relationship was variable (for example, $r = -0.389, p=0.211$ in the year 2000; $r = -0.915, p<0.01$ in the year 1998; and for the six year period $r = -0.486, p<0.01$).
Weather is only one of many possible influences on occupancy. It is posited that in some years, the influence of weather is stronger as other influences are less dominate (such as policy changes, actions arising from financial considerations, etc) and this results in the observed different correlation strengths between the weather variable and the average monthly bed occupancy variable. While this has not been investigated, experience in the health system would suggest that it is a reasonable hypothesis.

The data used for the modelling exercise showed a strong correlation year with the weather variable. While correlation is not causation, it is not unreasonable to expect a relationship between medical patient activity and the weather (Jones, Joy and Pearson, 2002). The forecast that represents a situation where weather variable accounts for a high proportion of the variation in monthly average bed occupancy ($R^2 = 0.83$). This is not an unreasonable approach insofar as that the forecast makes various assumptions about the base year, such as the mix of the type of activity for a given age group will remain constant. Similarly, it is reasonable to include an assumption that the influence of the weather on average monthly occupancy will also remain constant.

For strategic decision-making purposes it may be useful to also provide a scenario where weather accounts for less variation in the monthly average bed occupancy. Basing the regression on a more extended period of average monthly bed occupancy would achieve this ($R^2 = 0.24$). The problem with this approach is that the assumptions about the base year data become mixed, with some relating to the base year only, while those relating to weather are based upon bed occupancy from other
years. Nevertheless, providing that such issues are identified, it may provide an alternative scenario that stimulates discussion around future resource decisions that involve bed occupancy. The better alternative is to attempt to identify the actual cause(s) of non-weather variation in years where the correlation is low and quantify the effect of this on occupancy in order to see if the variation around seasonality would have otherwise been relatively constant. This, however, is likely to be a difficult task at a strategic level, though at a lower level (such as if modelling the activity relating to an operational unit or related to a disease) may be achievable.

8.4.2 Inclusion of the “vacancy” factor

In recent years there has been much written about hospital bed crises and the need for more beds or the operation of hospitals at occupancy levels that avoid crises (for example, see Bagust, Place and Posnett, 1999; and Utley, Gallivan, Treasure and Valencia, 2003). The rule of thumb that hospitals should operate at 85 per cent occupancy has been supported by research such as that of Bagust, Place and Posnett, 1999) that found, based upon simulation studies of theoretical hospitals, when levels of occupancy exceeded 90 per cent bed crises occurred. Furthermore, the inclusion of an occupancy factor becomes more important when it is realised that there has been little change in the levels of inpatient bed day use in recent years, yet the number of available beds has declined as consequence of the increase in provision of same-day patient services (Mackay and Millard, 2005a; Mackay and Millard, 2005b).

The compartmental flow models of bed occupancy, however, have not included an occupancy component. Clearly, not only is there a need to understand bed occupancy, but there is also a need to understand the level of vacancy that is necessary to avoid
The compartmental flow model was further modified to take account of vacancy requirements.

The modified model
Schematically, the modification of the compartmental flow model to incorporate the vacancy adjustment is shown in Figure 75.

Unlike the modification made to reflect the effect of the weather on bed occupancy, the adjustment made for the required level of vacancy to avoid patient turn-away relates to a business or policy decision. The resultant adjustment means that the occupancy level has been shifted upwards away from the average in order that the extreme or rare events in terms of occupancy levels are covered. This modification takes into account changes at the daily level as opposed to the more general
incorporation of the weather modification. Furthermore, it is possible to set the level of occupancy such that a given percentage of days in any year will experience patient turn-away, as shown in Figure 72 (see the section on *The ability to attach costs* later in this chapter for more details).

**The ideal occupancy level**

Table 30 supported the notion put forward by Bagust, Place and Posnett (1999) that percentage bed occupancy should be on average less than 90 per cent if patient turn-away is to be avoided. The result was also consistent with that found for a geriatric service using queueing theory (Gorunescu, McClean and Millard, 2002). The vacancy rate, however, is affected by the given mix of emergency and elective patient workload and the type of patients included in the analysis (that is, medical patients in this instance). It was interesting to note that the required vacancy rate is not uniform across the year, but reflected the influence of the seasonal variation for the base model. The weather adjusted model vacancy was also not uniform, as shown in Table 30, but as the influence of seasonality was largely captured by the inclusion of the weather adjustment, the pattern of vacancy appears to be more random.

In reality, the ability to identify what is operationally acceptable in terms of patient turn-away is not as simple for a variety of factors. For example, in reality patients may not be turned away, but admission to a different part of the hospital where there is capacity to take the patient may occur; elective surgery may be cancelled to provide additional emergency bed capacity; people may wait longer in the emergency department for admission to a bed; ambulance bypass may be enacted so that patients are admitted at other hospitals, etc. While overflow and demand management strategies exist, they may not be optimal in terms of patient outcomes (for example,
anecdotally it is suggested that admission to the wrong area in the hospital leads to different care and longer length of stay), but may facilitate the attainment of other goals, such as the management of financial resources.

Figure 71 illustrated the trade-off between decreasing occupancy and decreasing the amount of the year when patient turn-away occurs. Such a diagram is useful for informing decisions about bed crises. Notably, the weather-adjusted model performed better as it captures the seasonal variation.

**The ability to attach costs**
The inclusion of a vacancy adjustment reflects a policy decision – in this case a policy decision about the number of days that patient turn-away may occur during the year. It can be seen from Figure 72 and Table 31 that the model is shifted up – to meet days when occupancy is high and consequently, the modified model fits the data less well (in terms of visual inspection – it is further away from the data - and absolute error, which is greater than the weather adjusted model). However, it must be recognised that the data does not relate to the maximum occupancy for each month, but rather average occupancy levels. This highlights one of the many issues regarding model fit, namely that the original model was created to fit the average occupancy profile, but the adjusted model relates to maximum occupancy levels and thus the performance of the vacancy model may be better judged against maximum monthly occupancy data. The point of the introduction of the vacancy factor, however, was to improve understanding about capacity required to cope with occupancy levels that exceed the average.
From a resource management perspective, it must be recognised that the introduction of a vacancy factor involves various costs. Reducing the cost of running hospitals is often the focus of hospital management. Consequently, reducing the number of staffed, but unoccupied beds is often one way to achieve reduced running costs. There is, however, some tension in such an aim insofar as that political decision-making often appears to be concerned with avoiding “front page” headlines about bed crises.

Figure 71 is useful in that it shows the turn-away rates for given levels of percentage occupancy. Given that staffing vacant beds often involves additional cost (for example, additional nursing staff), achieving zero patient turn-away comes at a higher cost than having some level of patient turn-away. The attachment of costs of opening beds and the cost of patient turn-away enables the quantification of the trade-off to occur. Certainly the attachment of costs of staffing beds can occur. The cost of patient turn-away, however, is a more difficult task insofar as some of the outcomes are either difficult to cost (for example, is there a financial cost to the hospital for the achievement or failure to achieve political outcomes), or may not be considered as part of the decision-making process (for example, if patient outcome changes slightly does the cost, if any, associated with this matter to those making financial decisions about occupancy levels?).

**The benefit of the modification**

The inclusion of the vacancy modification leads to the following benefits:

- Visually it provides additional information to the user in that the maximum levels of occupancy can be appreciated, given the accepted level of patient turn-away, and
- It facilitates a better understanding of the difference between planning for the average occupancy (even if modified for the weather) and the maximum that may result.

- It helps overcome the limitations of forecasting using a model based upon average occupancy, which St George (1988) and MacStravic (2001) suggest has limitations.

8.4.3 The amended forecast

The weather and vacancy modified forecast of future bed occupancy levels is illustrated in Figure 73. In terms of improving strategic planning decisions about bed occupancy and other related resources (staffing levels, future training requirements, capital works, etc), the inclusion of the modifications provides additional useful information.

8.4.4 The role of modifiers

A general need

Two forms of bed occupancy compartmental flow model modifications have been examined. The weather modification provides an example of how factors that affect bed occupancy through the impact on disease and other factors can be incorporated into the compartmental flow model. The vacancy modification provides an example of how business or policy decisions can be incorporated into the compartmental flow model. Presumably other potential modifications could also be considered.

While a general need to include modifiers has been demonstrated, the inclusion of other modifications should be based upon the impact on the purpose of the model.
(that is, in this case strategic decision-making). The goals of inclusion should thus be threefold:

- Provision of better understanding of bed occupancy behaviour
- Inclusion without undue complexity, and
- Relevancy to better decision-making.

The outcome of implementation of these goals is that modification of the base compartmental flow model should result in additional information that will be of benefit to decision-makers. The benefits of the weather and vacancy modifications have been previously discussed.

**The importance of business rules**
The modification of the model to facilitate consideration of the influence of business or policy rules is important. It highlights that bed occupancy is not just a factor of patient demand, but rather service provision is a function of patient demand and business or policy rules. The ability to incorporate such factors should make the model a more useful tool for those undertaking strategic decision-making.

**The implications for what-if analysis**
The inclusion of modifiers is important insofar as that it provides additional capacity to examine alternative scenarios using what-if analysis. In the case of weather variables, the modifiers can be altered to examine the influence of more varied weather patterns that may arise as a consequence of global warming, a much discussed topic in the general media. The means of undertaking the what-if modelling, however, may need to be altered and this is now discussed.
The original what-if calculations relied upon an unmodified compartmental flow model. In this model, the compartmental flow model is assumed to be full, that is, there are no unoccupied beds.

The assumptions adopted when introducing the vacancy and weather modifications to the compartmental flow model have meant that the flow rates of the model (that is, parameters “b” and “d”) were not altered. The assumptions did, however, result in the alteration of parameters relating to the number of beds in each compartment (that is, parameters “A” and “C”). In the case of the vacancy modification, the point to the modification was to enable the effect of “vacancy” to be considered. The consequence of the increased bed requirement was not that an additional number of patients would be seen, but rather that patients would not be turned away due to a lack of beds arising from the daily variation in patient numbers.

For the intended forecasting purposes, the modification of the model parameters had value insofar as the only aspect being forecast was the total number of beds, and the modification resulted in improved visualization of the trends across a year and also an improved understanding of the actual bed capacity required in future years at different times of the year and in order to avoid patient turn-away.

The ability to conduct what-if analysis was a feature of the original BOMPS package and this feature has been transferred to Microsoft Excel (as mentioned in previous chapters). While what-if analysis can still be conducted, care must be adopted when doing so in order to avoid confusion about the capabilities of the original what-if analysis methodology – it was designed to work with the original compartmental flow
model and not the modified version where the compartments are not full (that is, vacancy exists). A simple work around exists – what-if scenario testing can be undertaken using the original compartmental flow model parameters and the effects of the vacancy modifier can be included subsequently (assuming that the what-if scenario testing merely involves the compartmental flow model – see Chapter 11).

Changes to the weather can also be examined (for example, the effects of a warmer winter could be examined), but can be conducted using the original what-if analysis with the “bed” parameters (that is, A and C) being substituted with the weather adjusted bed parameters, as increased throughput would occur. Alternatively, scenarios considering changes to bed number or patient flow would be made using the base model and subsequently modified to incorporate the influence of the weather.

8.5 Conclusion

Research has been undertaken to investigate whether a “sliding scale” modified compartmental flow could be developed to add a seasonal or weather factor to the age related compartmental flow models developed in Chapter 7. The research was limited to demonstrating the approach for the most important subset of the data based upon age grouping.

It has been demonstrated that the use of a bed occupancy compartmental flow model modifier, such as the weather variable used in this work, can lead to a better fit with the data while not increasing the model complexity greatly. The benefits that arise from this approach extend to the potential for improved representation of the data and
better forecasting, which should result in more credibility for the approach by users
and better decision-making.

The extension of the modification to include consideration of the effect of vacancy
rates on the resultant sliding scale model highlighted the ability to incorporate
business or policy rules into the resulting model. This is also important as it should
contribute to improved credibility of the model by users and lead to better decision-
making.

The scope to undertake further research exists. For example, the ability to explore the
relationship between the weather and bed occupancy across other patient age groups
exists. Investigation of other potential model modifiers may also be warranted.

The bed occupancy compartmental flow model can be used for other purposes, such
as evaluation of service change and forecasting future outcomes. The ability to
undertake short-term forecasting using the model parameters is the subject of Chapter
9.
Chapter 9

The use of compartmental flow bed occupancy models for forecasting service change

In this chapter I investigate whether the bed occupancy compartmental flow can be used to forecast short-term changes in the acute care hospital setting using the New Zealand hospital data. The chapter has the following structure:

9.1 Introduction ............................................................................................ 300
9.2 Methodology ........................................................................................... 301
9.3 Results .................................................................................................... 302
9.3.1 Post service change trends .................................................................... 302
9.3.2 Forecasting using the ALOS ................................................................. 304
9.3.3 Forecasting with compartmental flow models ....................................... 306
9.3.4 Comparison of forecasts to actual outcomes ......................................... 307
9.3.5 Comparison of the expected stay and the average length of stay .......... 312
9.4 Discussion .............................................................................................. 313
9.4.1 The ALOS Forecast ............................................................................. 314
9.4.2 The Bed Occupancy Compartmental Flow Model Forecast ................ 316
9.4.3 Expected length of patient stay ............................................................ 320
9.4.4 Model Choice ...................................................................................... 321
9.4.5 Predictive validity ................................................................................ 322
9.5 Conclusion .............................................................................................. 322
9.1 Introduction

In Chapter 6 compartmental flow models were used to evaluate the change in service in Internal Medicine at HealthCare Otago. Evaluation of change is an activity that is valuable as it helps determine whether the aims of a given change process were achieved. Evaluation, however, relates to past events. While decision-makers in the health sector are interested in past events, consideration of the future events is probably more important, as it provides information that will indicate whether an existing service, or even a newly created service, will be able to meet demand with its given physical resources (for example, beds), financial resources and workforce.

In Chapter 7 forecasting future bed occupancy in light of a changing population profile and using bed occupancy compartmental flow models was illustrated. The time horizon for the forecasting period was 20 years, although it was recognised that forecasting too far out into the future did not necessarily lead to useful forecasts when many underlying factors may alter (for example, technology).

While longer-term forecasts are useful, many decision-makers are more concerned about the shorter-term (that is, the next few years, as opposed to the next 20 years). Thus, in this chapter I explore the ability to use the bed occupancy compartmental flow model to create short-term forecasts about bed occupancy without incorporation of population change. Given the ubiquitous nature of the ALOS, such forecasting is contrasted with short-term forecasts of the ALOS and reinforces the deficiencies of ALOS for short-term forecasting. The modelling uses the data from the Internal Medicine Department at HealthCare Otago.
The results of the research undertaken for this research were presented at a conference (Mackay, Lee, Rae and Millard, 2004) and form the basis of this chapter.

9.2 Methodology

The contextual details about the Internal Medicine data were described in Chapter 3 (see Chapter 3 sections 3.2.2 and 3.3.2). The data relating to the period following the reduction in bed numbers that occurred as part of the service change, that is from 1997 to 2003, were used for this research.

The method used for the creation of the compartmental flow models reported in this chapter was described in Chapter 6 (see section 6.2.2).

Compartmental flow models were created for each year for the period 1997 to 2003. Confidence intervals for model parameters were calculated using standard Monte Carlo methods (Hillier and Lieberman, 2001; Powell and Baker, 2004). The model parameters were analysed for the presence of any trend. A constant or simple linear regression model was selected and used to create forecasts of movement in parameters for the next two years (that is, 2004 and 2005).

The ALOS was also determined for each year for the same period. Linear and non-linear regression models were fitted to the ALOS trend.

Additional data were obtained relating to the forecast years 2004 and 2005. This data enabled the calculation of average patient numbers (or the average number of occupied beds) and the ALOS, but it was not suitable for the calculation of
compartmental flow parameters. This necessitated the use of approximate measures of short and long-stay bed numbers and patient flow as comparisons to the compartmental flow model forecasts.

The estimated length of patient stay was calculated using the compartmental flow model parameters\(^1\) and was compared to the ALOS measure.

9.3 Results

9.3.1 Post service change trends

Although a general analysis of the trends in ALOS and bed occupancy was presented in Chapter 3 (see section 3.3.2) it is useful to revisit these trends. In revisiting these trends the focus of the analysis was shifted from the entire data set (that is, pre and post service change) to only that of the post service change period. The reason for reducing the period of analysis was that it was post service change period that would be used as the basis for forecasting future patient flow rate and bed occupancy.

---

\(^1\) Ms Georgina Christodoulou developed the spreadsheets in Microsoft Excel in her capacity as a research assistant for Peter Millard. I liaised with Ms Christodoulou during the development of these spreadsheets and provided some limited input into the development. I was given access to these through my collaboration with Peter Millard and his research colleagues.
Figure 76 illustrates the change in the ALOS over the post service change period.

Figure 76: The ALOS trend showed a period of continued decrease post service until about the year 2000 after which the ALOS rose. The changes in ALOS were small, with an increase of approximately 2.5% over the period.

Figure 77 shows the 90-day moving average trend in bed occupancy for the post service change period.
Neither of these measures adequately explained the changes in either patient flow or patient numbers, that is, was occupancy increasing because of more patients or slower patient flow or both, and was the change in ALOS due to a general decline in patient flow, an increase in long-stay patients or a decline in the flow rate of short-stay patients?

### 9.3.2 Forecasting using the ALOS

The trend in the ALOS post changed service consisted of two phases: an initial decrease, followed by a period of increase. The ALOS for 2003 had increased by 2.5 per cent when compared to 1997. Figure 78 illustrates the trend and also shows a linear and polynomial model fitted to the data.

Figure 77: The 90-day moving average trend shows an increase in total occupancy that commences from about the year 1999.
If only the period where the ALOS increased was included in a forecast (that is, from the year 2000), which would be consistent with using data that represented current trends and also frequent practice in the health sector, a linear model described the data well as shown in Figure 79.

Figure 78: The ALOS trend is fitted well by a polynomial model. However, the forecast is overstated. A linear model does not describe the post change period well.
9.3.3 Forecasting with compartmental flow models

The compartmental flow models fitted the data well as previously reported in Chapter 6. The models created using data relating to individual years had a very low squared error indicating a good fit (for example, see Table 10). The trend in the model parameters (that is, A, b, C and d) is illustrated in Figure 80.
The trends showed that movement in parameters was not necessarily uniform. For example, parameters A, C and d all increased in 2001 (compared to the prior year), while parameter b declined.

Models were fitted to the parameter trends as shown in Figure 80, with model selection being determined by the fit across both the time period and also the range of each parameter (as shown by the error bars in Figure 80).

**Figure 80**: Trends and forecasts for bed occupancy compartmental flow parameters over the post service change period. A constant model best described Parameters b, C and d, while a linear model best described parameter A.

### 9.3.4 Comparison of forecasts to actual outcomes

The additional data provided for analysis of the parameter forecasts were different to the data used to generate the compartmental flow models. Only bed occupancy
measures and patient stay measures (that is, the ALOS) could be derived from the data.

It was possible to split the data into approximate short and long-stay groups in order to provide measures that could be used for comparison to the compartmental flow model output. This was done on the basis of the half-life of the short-stay patient group, as seven times (7x) the half-life\(^2\) (in days) of the short-stay patient group equated to clearance of more than 99 per cent of the short-stay patients from the short-stay compartment. The half-life value was obtained using the model parameters from 2001 to 2003 and using additional features of the resource tables\(^3\) reported in Chapter 7. The lowest half-life value was used.

The average daily occupancy for the derived short-stay patient group was a reasonable estimate of parameter A. The forecast of parameter A was best described with a linear fit and the additional data confirmed that this forecast was reasonable as illustrated in Figure 81.

\(^2\) The half-life formula was incorporated as part of the BOMPS software and the formula is shown, with other BOMPS formulae, in Appendix II.

\(^3\) Ms Georgina Christodoulou developed the spreadsheets in Microsoft Excel in her capacity as a research assistant for Peter Millard. I liaised with Ms Christodoulou during the development of these spreadsheets and provided some limited input into the development. I was given access to these through my collaboration with Peter Millard and his research colleagues.
The ALOS for the derived short-stay patient group was used to provide an estimate of the patient flow parameter B. The difficulty of using the ALOS – even if for only part of the patient population (that is, the short-stay patient group) – was that it is a complex measure. The short-stay patient group ALOS trend is illustrated in Figure 82.
Figure 82: Comparison of short-stay group patient flow data based upon the original modelling period and the forecast period. There appeared to be an upward trend in the ALOS, which is contrary to the forecast for the short-stay flow parameter, b (see Figure 80). It is not possible to determine from the available data whether the increase in ALOS was solely flow related.

The average daily occupancy for the derived long-stay patient group was a reasonable estimate of parameter C. The forecast of parameter C was best described with a constant model. The additional data suggested that there was a period of growth in the number of long-stay patient beds as illustrated in Figure 83.
The ALOS for the derived long-stay patient group was used to provide an estimate of the patient flow parameter $d$. The long-stay patient group ALOS trend is illustrated in Figure 84.

Figure 83: The best forecast for parameter $C$ was with a constant model (as shown in Figure 80). During the forecast period, there appeared to have been growth in the number of long-stay patient beds.
Reducing the training period for the development of the compartmental flow model parameter forecasts (that is, only using the parameters for the 2000 to 2003 compartmental flow models) was not found to generate improved forecasts.

9.3.5 Comparison of the expected stay and the average length of stay

The comparison of the expected length of patient stay calculated from the compartmental flow model parameters to the ALOS is illustrated in Figure 85.

Figure 84: Comparison of long-stay patient group flow data over the model and forecast period using the ALOS as a proxy measure for parameter D. There appears to be a downward trend in ALOS during the forecasted period, but it is not possible to determine from the available data whether this was solely related to patient flow.
The trends of the expected and average lengths of stay were found to be similar. The trends appear to diverge during the 2003 year, but this may be artefact due to the compartmental flow model being generated with data for only part of the year. Additionally, the differences between the expected length of stay and ALOS did not exceed more than four per cent for any one year, and could thus be considered small.

9.4 Discussion

Bed occupancy compartmental flow models were shown to be of use in evaluating past service change in the Internal Medicine Department at HealthCare Otago (see Chapter 6). While there is much that can be learned from history, it is my experience that decision-makers in the health care sector are pre-occupied with the present and then with the short-term future.
The ability to use bed occupancy models in a simple manner to create short-term forecasts was demonstrated and to some extent evaluated. The ability to create a short-term forecast of the ALOS over the same period – a competitive measure to the compartmental flow model parameters- was also undertaken. The results are now discussed.

9.4.1 The ALOS Forecast

The ALOS is a measure of how long patients stay (on average) in the hospital. It is a measure of time spent admitted in a hospital bed and is sometimes used as a proxy measure of flow (for example, see Sorensen, 1996). For example, from my experience in the health sector a reduction in ALOS is often assumed to mean that most patients are flowing faster through the system, whereas the decrease in ALOS may be explained by a reduction in the length of stay of a small number of long-stay patients. It is a complex measure. The complexity with this measure arises from the fact that it is a composite measure of patient stay and patient numbers. Thus, while it is possible to create a forecast from an ALOS trend and achieve an apparent good fit to the resulting data in the forecast periods, as illustrated in Figure 79, it is not clear how this trend should be interpreted in terms of management decision-making. For example, it is not clear whether the forecast in increase in the ALOS would translate into a need for additional beds, as this outcome depends upon whether patient numbers will be lessened or maintained, assuming that existing levels of occupancy are to remain unaltered. Consideration of trends in patient numbers is a separate analytical exercise. Consequently, there is no single simple measure that can be used for trend analysis and forecasting around acute care hospital bed occupancy.
The creation of the ALOS forecast did highlight several of the many issues that affect modelling, namely the choice of data and the number of data. Figure 78 illustrated several forecasts created using the ALOS data for the period of 1997 to 2003 – the post service change period data. The resultant simple linear regression model did not fit the data well and only had a r-square score of 0.13, which indicated that the model did not explain much of the variance and thus was considered a poor alternative compared with the polynomial model that fitted the data well and explained most of the variance (r-square score of 0.91). The polynomial model, despite fitting the data well, would have been a poor choice for forecasting the future ALOS, because it would have overstated the likely ALOS. This outcome would have occurred because of the application of a model without judgment. It is evident that the ALOS declined during the period 1997 to 2000 and then increased from the year 2000 (see Figure 78). Thus, a model of ALOS could be established over the entire period (for example, the polynomial model) or judgment would suggest that only the data relating to the most recent movement in ALOS should be used (that is, from the year 2000 onwards). Powell and Baker (2004) have identified that modelling is not just the application of statistical methods, but rather it involves both the application of statistical methods together with judgment and is an art as opposed to a pure science. Armstrong and Collopy (1998) also identified that the integration of judgment with statistical methods can lead to significant better forecasts, particularly where the judgment is based upon expert opinion and is unbiased. The use of judgment suggested that only the data from the year 2000 should be considered and a simple linear regression model fitted the data well as shown in Figure 79.
While judgment may suggest the use of only the most recent data to establish the forecasting model, this gives rise to the problem of whether there were a sufficient number of observations upon which to base the forecast. In terms of management decision-making, it is not always practical or possible to wait for additional data and thus judgment must be used when interpreting the suitability of any model for the forecasting task to which it is being applied. In the case of the ALOS, the r-square score was high and the visual fit appeared to be good, thus it would appear reasonable to use the regression model established using the data from the year 2000 onwards for forecasting changes to the ALOS in the near term future. Consideration of the meaning and usefulness of the forecast as discussed earlier, however, supports the notion of Powell and Baker (2004) that modelling is an art and not a science, or requires a mixture of judgment and science (Armstrong and Collopy, 1998) as a good statistical outcome does not necessarily translate to a useful forecasting model. For example, the forecast does not provide management with information about why the ALOS is increasing (is the change due to patient flow, bed occupancy or both) and thus, it is more difficult to assess how to act to change the service if the forecast future is not desired.

9.4.2 The Bed Occupancy Compartmental Flow Model Forecast

The bed occupancy compartmental flow model overcomes the problem of complexity with the ALOS, because it captures the number of occupied beds and the rate of patient flow in separate model parameters. At its most simple level, the compartmental flow model captures flow and occupancy with two measures. Previous justification has been provided to show that in an acute care hospital setting, a double compartmental flow model is justified in order to capture the valuable information
about the behaviour of long-stay patients. Thus, the model has two flow parameters and two occupancy parameters. These parameters can be used to create a variety of measures that are useful for decision-making purposes (for example, see the resource tables in Chapter 6). Furthermore, the ability to undertake meaningful what-if or scenario testing type analysis provides a substantial benefit over using the ALOS for decision-making purposes (see Chapter 11 for a discussion on what-if analysis).

The separation of the rate of flow and number of occupied beds provided additional benefit over the ALOS in that additional data could be used for model building. The separate model parameters did not over-ride the need for judgment in determining the period from which the data were drawn for the model building purposes. However, it was evident (see Figure 81) that all of the observations could reasonably contribute to the development of the linear model of parameter A. Similarly, there was no reason to exclude data from the modelling of parameters b, C and d.

Given the post modelling data, the ability to compare the model forecast with the actual outcome was possible for parameters A and C – the parameters relating to the number of occupied beds. The forecast that the number of beds required for short-stay patients would continue to increase was found to hold true as shown in Figure 82. A constant model was found to describe parameter C – the number of beds required for long-stay patients. It would appear that during the forecast period there was a linear growth in the number of beds used for long-stay patients and a constant model did not forecast this outcome (see Figure 83). This outcome highlighted two factors, one concerning the evaluation technique and one about the nature of forecasting changes in acute care hospital environments. The evaluation technique relied upon the
approximate identification of the number of occupied beds that would have been
determined had compartmental flow models been developed. The data for this was not
available and thus these estimates represented a reasonable alternative. It is unlikely,
but nevertheless possible, that a different trend may have been reported for the
forecast period had compartmental flow models been developed. Had a different trend
been reported this would have affected the interpretation of the results.

This research has highlighted that forecasting the future bed occupancy based upon
the compartmental flow model parameters alone in an acute care hospital – even for a
short period of time – can be difficult. The problem of forecasting with unstable
predictive variables relates to the use of non-stationary data. Armstrong (2001)
describes stationary data as time series data that have means and variances that are
unaffected over time. Clearly, the compartmental flow model parameters (and for that
matter the ALOS) were non-stationary for the system under examination. In Chapter 5
it was identified that hospital systems are generally unstable (St George, 1988;
Mackay and Gorunescu, 2001; MacStravic, 2001). While the issue of general
instability was identified as being important for decision-making relating to the period
within a year, I contend that the multiple factors affecting the health system (including
political decision-making, policy change, resource allocation decision-making,
population change and changes in other services) means this is also generally the case.

The practical solution to this issue of non-stationary data is to forecast with fewer data
that relate to a period of approximate stationarity and only to forecast into the future a
little way forward, as was done here. The outcome of such an approach is that the
forecast will ideally be useful. This is not an easy thing to achieve, as indicated by
these findings. While short-term stationarity appeared to hold true for the forecast of parameter A (short-stay occupied bed numbers), the issue of rapid change was highlighted in relation to parameter C (long-stay occupied bed numbers), as shown in Figure 83.

The increase in the number of occupied long-stay patient beds suggests that either there was a drift back towards the pre-service change model of care where the number of long-stay patients was greater or the service was affected by changes in a downstream service (that is, it was required to keep more long-stay patients). It is believed that there was no deliberate change in resource allocation policy during this period and thus this is ruled out as a possible explanation for the change in pattern of parameter C. Indeed, Rae, Busby and Millard (2007) acknowledge a 56 per cent increase in admissions to the service between 1998 and 2002, but cannot offer adequate explanation as to the causes that account for this. While judgment can be important in improving the accuracy of forecasts (Armstrong and Collopy, 1998), judgment may not always capture the likely changes in internal (that is, within another part of the hospital) or external (that is, outside of the hospital) factors that affect acute care hospitals for many reasons. For example, the likelihood of a change in policy in another service that affects the service of interest is not widely known and it is only discovered once the effect is observed and explanation is sought as to why the service of interest has changed. The fact that Rae, Busby and Millard (2007) could not account for the increase in patient admissions corroborates the difficulty in identification of such factors, even after the event.
Given the post modelling data and the inability to determine the flow parameters from this data, the ability to compare the model forecast with the actual outcome was
difficult for parameters b and d – the parameters relating to the patient flow. The
ALOS was used as a proxy evaluation guide, but as already stated, the ALOS is a
composite measure that captures changes in both flow and patient numbers. Thus,
despite Figure 82 showing an increase in ALOS for the short-stay patients and Figure
84 showing a decrease in ALOS for the long-stay patients, it is not clear whether the
observed trends reflect changes in the number of occupied beds or rate of patient flow
and no meaningful conclusion about the illustrated trends and patient flow can be
reached.

The growth in the number of beds occupied by long-stay patients may have explained
the observed increase in the ALOS (as illustrated in Figure 79). For the reasons
previously detailed it is not clear if there was also a change in the rate of patient flow
and thus, the possible contribution of a changed rate of patient rate of flow to the
observed ALOS could not be ruled out.

9.4.3 Expected length of patient stay

The comparison of the expected length of patient stay generated from the
compartmental flow model parameters to the ALOS (see Figure 85) highlights the
power of the compartmental flow model - it provides the means to create basic
parameters that collectively have far more utility than the ALOS alone. For example,
using the compartmental flow model parameters it is possible to generate a length of
stay measure from the model, as well as gain information about short and long-stay
patient bed occupancy and flow, and undertake what-if type analysis about service
change. The reverse, that is, using the ALOS to generate compartmental flow parameters is, however, not possible.

Given the earlier observations about the widespread use of the ALOS, the ability to generate an expected length of stay that is similar to the ALOS may be an important feature of the model that can be used to assist with the adoption of this research in the field (that is, users will feel more comfortable knowing that a length of stay measure exists and may therefore use the modelling).

9.4.4 Model Choice

In Chapter 6, the post service change model based upon modelling each year separately was abandoned in preference for a simpler model. The choice of models was made in relation to the task being undertaken. In this instance, and using the same data, models were constructed in relation to each year of service (post the service change). However, the purpose of modelling was different. Thus, despite using the same data, it is important to recognise that different models are required to answer different questions.

It is also acknowledged that only two types of model were fitted to the compartmental flow model parameters for the research presented in this chapter, namely a constant and linear model. While other models could have been fitted, these models seemed to offer reasonable fit and also could be explained. It is unlikely that meaningful explanations could have been attached to more complex models and again highlights the need for judgment when constructing models.
9.4.4 Predictive validity

As previously stated, predictive validity relates to determining whether the model inputs are valid in terms of being used to create the intended model output (Armstrong, 1985). The bed occupancy compartmental flow model was developed using patient occupancy data (see the Chapter 4, section 4.6.3 for more details). This data reflected both the number of beds that are occupied on any given day and also the length of stay. The model parameters captured the rate of patient flow and total occupancy. These parameters were used for forecasting both future occupancy and flow. Predictive validity was achieved as total occupancy and patient flow parameters were used to forecast total occupancy and patient flow, respectively.

9.5 Conclusion

In this chapter the ability to use the bed occupancy compartmental flow model parameters for forecasting has been examined. The output was compared to forecasts of the ALOS.

The ALOS is a composite measure that is based upon both patient numbers and patient flow. Forecasting using the ALOS is of limited value, as a change in ALOS does not provide useful information for decision-makers without additional analysis. It also has poor what-if scenario analysis use.

The bed occupancy compartmental flow models, however, provide parameters that exclusively capture patient flow or the number of occupied beds. These parameters can be used for forecasting. Without incorporation of other information (such as population change) parametric forecasting should be restricted to a small number of
future periods, because bed occupancy is sensitive to many factors. It was also shown that even the short-term forecasting of occupied bed numbers was difficult due to issues of data stationarity.

In terms of this particular service, it is evident that despite the planned change in service that saw a reduction in patient numbers, there has been a growth of short-stay patients. It would appear that a growth in long-stay patients is now also being experienced. Unless this growth was planned, it would appear that there is some risk of a return to pre-service change activity levels.

In Chapter 10 the case for considering the inclusion of the bed occupancy compartmental flow model parameters in funding allocation models is presented.
Chapter 10

Application of Bed Occupancy Compartmental Flow Modelling to Casemix

In the previous chapters, the emphasis of the research has been on considering whether or not bed occupancy compartmental flow models as originally suggested by Harrison and Millard (1991) could be used to model acute care hospital data and how the resultant models could be modified, and be used for forecasting and evaluation. In this chapter I present preliminary research findings about how the bed occupancy flow model parameters could be incorporated into existing financial allocation models such as the casemix-funding model. Funding models can be key tools for the implementation of financial control of health expenditure (Duckett, 2004) and thus this is an important area of application. The chapter has the following structure:

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.1</td>
<td>Introduction</td>
<td>325</td>
</tr>
<tr>
<td>10.1.1</td>
<td>The Australian Casemix-funding Model</td>
<td>325</td>
</tr>
<tr>
<td>10.1.2</td>
<td>The Average Length of Stay and the Casemix-funding Model</td>
<td>328</td>
</tr>
<tr>
<td>10.2</td>
<td>Methodology</td>
<td>331</td>
</tr>
<tr>
<td>10.3</td>
<td>Results</td>
<td>332</td>
</tr>
<tr>
<td>10.3.1</td>
<td>DRG Profiles</td>
<td>332</td>
</tr>
<tr>
<td>10.3.2</td>
<td>Overnight Stay Patient Results</td>
<td>334</td>
</tr>
<tr>
<td>10.3.3</td>
<td>Elective Same-day Results</td>
<td>336</td>
</tr>
<tr>
<td>10.4</td>
<td>Discussion</td>
<td>338</td>
</tr>
<tr>
<td>10.4.1</td>
<td>Number of Data</td>
<td>338</td>
</tr>
<tr>
<td>10.4.2</td>
<td>DRG 261 Model Fit</td>
<td>339</td>
</tr>
<tr>
<td>10.4.3</td>
<td>Inpatient and Same-day Activity</td>
<td>340</td>
</tr>
<tr>
<td>10.4.4</td>
<td>Business Rules – fix add distribution could change</td>
<td>341</td>
</tr>
<tr>
<td>10.4.5</td>
<td>Casemix and Benchmarking</td>
<td>342</td>
</tr>
<tr>
<td>10.4.6</td>
<td>Policy Implications</td>
<td>343</td>
</tr>
<tr>
<td>10.4.7</td>
<td>Further Research</td>
<td>344</td>
</tr>
<tr>
<td>10.5</td>
<td>Conclusion</td>
<td>345</td>
</tr>
</tbody>
</table>
10.1 Introduction

10.1.1 The Australian Casemix-funding Model

According to Duckett (2004), the emphasis on fiscal control of health expenditure during the Fraser Government’s reign between 1975 and 1983 in Australia resulted in the introduction of policies such as the casemix funding mechanisms in the states of Victoria and South Australia. The implementation of the casemix funding policy took many years to develop. As an example, casemix funding was introduced into South Australia during 1994-95 (Duckett, 2004; Department of Health, 2004), although work on the funding mechanism commenced during the early to mid 1980s. Casemix funding is now used widely in Australia, although it is applied in varying ways in different parts of Australia (e.g. funding based upon casemix or adjusted by casemix).

Duckett (2004) indicates that Australian public hospital services have been funded in a variety of ways, including the historical budget approach, the negotiated budget approach, per deim approach (based on days of stay), prescribed minimum payment (which only applies to private patients), and casemix. Casemix funding is based upon the notion that hospitals should be funded on the basis of the number and types of patients treated (that is, the mix of cases). Diagnostic related groups (DRGs) were developed to capture the admitted patient casemix of hospitals. The wide variety of hospital inpatient services are described by relatively few DRGs – about 660 in total. DRGs are meant to describe patient services that are homogenous in terms of resource use. There have been various versions of DRGs used throughout the casemix-funding era and casemix has been extended to outpatient and community based services.
One of the consequences of the introduction of casemix has been the increased attention paid by management to patient length of stay and this is of interest for a number of reasons. First, DRGs are meant to exhibit homogeneity of resource use and patient length of stay is one measure of how resources are used (that is, a bed is occupied and services are delivered to the patient). Second, patient length of stay is considered one aspect of patient treatment that can be manipulated. Third, it is the average length of stay that is used in the casemix-funding model. Thus, in order to “do well” under the casemix system, attempts have been made to manipulate the average length of patient stay.

Casemix was originally developed as a benchmarking tool and not a funding tool. Thus, its use as a funding tool can be questioned. Duckett (2004) suggests that the application of expenditure control as a goal, and therefore the development of funding mechanisms such as casemix, as being based upon flawed perceptions by policy makers that the level of health care expenditure is somehow wrong. A more appropriate policy decision, according to Duckett (2004), may have been to consider what benefit, in terms of health outcomes, additional expenditure on health conferred to the Australian public. The measurement of health outcomes, however, is still a developing area and thus perhaps it is easy to see why financial control was targeted.

Given that the average length of stay is a key driver in casemix funding, it is relatively easy to understand that gaming strategies were developed that involved the patient length of stay. Casemix was designed to reward activity providing that the costs of the activity were less than the cost of providing the service. The large number of DRGs and the variation in a range of factors, including patient severity (that is some patients
are sicker than others), service provider decisions and the use of the average length of stay, meant that it was expected that hospitals would be “winners” and “losers” on some DRGs. With sufficient information, hospitals could try to become more efficient and one available mechanism was to alter patient length of stay, such that it was less than the funding model average length of stay, which is used to determine the funding weight.

Based upon personal experience in helping to develop aspects of the casemix system and other work in the health sector, I have come across some aspects of the gaming strategies that occurred in order to secure better funding outcomes for individual hospitals. These include:

- Increasing patient throughput for specific DRGs. In this case, the attempt was unsuccessful, as the casemix reimbursement was less than the cost of the activity, but a lack of information systems prevented this outcome from being determined before the outcome occurred.

- Focussing on certain DRGs known to generate profits for the hospital. This can only occur when management know that consistent performance is achieved that results in a large surplus per patient being generated. The downside to this strategy is that hospitals may focus on profitable services as opposed to meeting the health needs of their local communities. Also, this activity can only occur in hospitals that can limit the type of services provided (generally smaller hospitals).

- Undertaking research to show why certain hospitals should be excluded from the casemix system or looking at aspects of casemix that require refinement (for specific gain of the hospital). While this has resulted in an evolving funding
mechanism, the cost of undertaking this work means that other areas of work may not receive the appropriate attention from management.

- Realising that the length of stay distribution is skewed (as a consequence of the publication by Millard, Mackay, Vasilakis and Christodoulou G (2000)) and that attention to only a few patients with longer lengths of stay will alter the average and result in a casemix “win”. While giving increased attention to patients with longer stays may be appropriate, the “win” can be achieved without altering service delivery to the majority of patients and while a good financial strategy, may have some unintended ethical and equity issues.

10.1.2 The Average Length of Stay and the Casemix-funding Model

I have previously shown that the length of patient stay is skewed and therefore the use of the average length of stay in health management decision-making is often flawed, because the assumption underlying the decision-making process requires that the average be distributed Normally.

The technical bulletins developed by the Department of Health (2005a and 2005b) require that the average length of stay be modified for use in the casemix-funding model. Modification is required in order that payments for patients who stay in hospital for either a very short time (short-stay outliers) or a very long time (long-stay outliers) are made on a different basis so that the hospitals do not either profit or lose as a consequence of treating such patients. Additionally, the funding for same-day patients is treated separately.
The trim points used to determine the short and long-stay outliers are based upon simple modification of the length of stay calculation (Department of Health, 2005a), which are described by the following steps:

- Same-day patients are excluded from the calculation
- If the average length of stay is greater than four days, the low trim point is calculated as one third of the average length of stay
- The high trim point is calculated as three times the average length of stay.

Although the trim points would be calculated separately for each DRG, it is possible to apply the trim points to the data previously used to show that the average length of stay was skewed. The comparisons are shown in Table 32\(^1\) and Figure 86.

<table>
<thead>
<tr>
<th>Comparative Statistics</th>
<th>All LOS</th>
<th>Inlier LOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients</td>
<td>9,060</td>
<td>6,856</td>
</tr>
<tr>
<td>Total bed days</td>
<td>55,832</td>
<td>40,772</td>
</tr>
<tr>
<td>Average length of stay</td>
<td>6.2</td>
<td>5.9</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>&gt;7</td>
<td>3.9</td>
</tr>
<tr>
<td>Skewness</td>
<td>&gt;4</td>
<td>1.2</td>
</tr>
<tr>
<td>Minimum</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Maximum</td>
<td>148</td>
<td>18</td>
</tr>
</tbody>
</table>

\(^1\) Note: as the table was for illustrative purposes only approximate standard deviation and skewness figures were reported, as the number of patients varied slightly from the original profile.
In constructing this analysis more than three quarters of the patient records (76 per cent) were found to lie within the inlier region. While it could be argued that this is an artificial construct and therefore has little application to casemix, the example length of stay profile reported for Australian Refined (AR) DRG 169B (version 5.0), which relates to bone disease with severe complications, used in the Technical Bulletin 94:10 (Department of Health, 2005a), exhibits a similar profile to that shown in Figure 86. Thus, the consideration of whether the compartmental flow measure flow parameters could be used to replace the flawed average length of stay measure in the casemix funding mechanism is therefore important. Successful application could alter current funding allocations and also reduce the ability for hospital managers to game the system. Consequently, in this chapter I investigate whether the bed occupancy compartmental flow model can be used at the DRG level for overnight stay patients. Additionally, given that the funding model is applied to elective same-day patients,
the implications of using the compartmental funding model for elective same-day patients at the DRG is also considered.

The findings presented in this chapter formed part of the content for the conference presentation *Benchmarking Using Flow Modelling* (Mackay and Lee, 2004). Additionally, previous research using the BOMPS package was undertaken on one of the DRGs (DRG 261) used in this research (Mackay and Millard, 1999).

### 10.2 Methodology

The data used for this research related to the Flinders Medical Division data that has been previously described. The general contextual details about the data were described in Chapter 3 (see sections 3.2.1 and 3.3.1). Extracts from these data were taken based upon the AR-DRG categories. Profiles were created for three AR-DRGs (V3.0):

- 261 – chest pain
- 572 – renal dialysis, and
- 274 – circulatory disease without acute myocardial infarction with invasive cardiology investigative procedures without complicating diseases and without major complications.

The chest pain patients were generally admitted overnight and were not elective patients. The same-day patients were omitted for this analysis. The renal dialysis and circulatory disease patients were the elective same-day patients and any inpatients
were omitted for this analysis. These DRGs were chosen because of the large number of patients or cost (or both).

At the DRG level, the patient numbers per day for most DRGs were found to be very low. Thus, it is not particularly useful to construct an average profile of patient occupancy across the year. Rather, it was considered more appropriate to artificially construct an annual profile. This was done by creating a matrix where:

- each row related to an individual patient admission
- each column related to the number of days of admission
- for each day of stay, a patient was given a score of 1 (in the case of elective same-day patients, the time profile was in hours)
- the column totals were summed and used as the occupancy profile.

For AR-DRG 261 a compartmental flow model was constructed using the second methodology described in Chapter 6 (see section 6.2.2). Performance statistics were generated. For AR-DRGs 572 and 274 the occupancy profiles were constructed only.

10.3 Results

10.3.1 DRG Profiles
The top ten same-day and inpatient DRG profiles for the Medical Division for 1998 are reported in Tables 33 and 34 to substantiate the choice of DRGs chosen for this research, that is, high volume (and also high cost or both).
Table 33: DRG profile for same-day patients during 1998. DRG 572 accounted for almost 65 per cent of the same-day activity. DRG 274 was the second most frequently assigned DRG for same-day patients.

<table>
<thead>
<tr>
<th>DRG</th>
<th>Admission Category</th>
<th>Total</th>
<th>% of Total</th>
<th>Cum. %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elective</td>
<td>Emergency</td>
<td>Elective - Booking List</td>
<td></td>
</tr>
<tr>
<td>572</td>
<td>4,700</td>
<td>1</td>
<td>4,701</td>
<td>64%</td>
</tr>
<tr>
<td>274</td>
<td>629</td>
<td>5</td>
<td>634</td>
<td>9%</td>
</tr>
<tr>
<td>509</td>
<td>358</td>
<td>1</td>
<td>359</td>
<td>5%</td>
</tr>
<tr>
<td>514</td>
<td>235</td>
<td>1</td>
<td>237</td>
<td>3%</td>
</tr>
<tr>
<td>780</td>
<td>148</td>
<td></td>
<td>148</td>
<td>2%</td>
</tr>
<tr>
<td>484</td>
<td>146</td>
<td></td>
<td>146</td>
<td>2%</td>
</tr>
<tr>
<td>261</td>
<td>14</td>
<td>121</td>
<td>136</td>
<td>2%</td>
</tr>
<tr>
<td>758</td>
<td>91</td>
<td>1</td>
<td>92</td>
<td>1%</td>
</tr>
<tr>
<td>889</td>
<td>62</td>
<td></td>
<td>62</td>
<td>1%</td>
</tr>
<tr>
<td>280</td>
<td>40</td>
<td>6</td>
<td>47</td>
<td>1%</td>
</tr>
<tr>
<td></td>
<td>6,361</td>
<td>191</td>
<td>6,562</td>
<td>89%</td>
</tr>
</tbody>
</table>

Table 34: There was little difference between the number of patient separations for the top two inpatient DRGs. Prior research with DRG 261 and its relative cost determined its selection for this research.

<table>
<thead>
<tr>
<th>DRG</th>
<th>Admission Category</th>
<th>Total</th>
<th>% of Total</th>
<th>Cum. %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Elective</td>
<td>Emergency</td>
<td>Elective - Booking List</td>
<td></td>
</tr>
<tr>
<td>177</td>
<td>11</td>
<td>519</td>
<td>530</td>
<td>6%</td>
</tr>
<tr>
<td>261</td>
<td>5</td>
<td>519</td>
<td>524</td>
<td>6%</td>
</tr>
<tr>
<td>252</td>
<td>6</td>
<td>403</td>
<td>410</td>
<td>4%</td>
</tr>
<tr>
<td>297</td>
<td>194</td>
<td>197</td>
<td>397</td>
<td>4%</td>
</tr>
<tr>
<td>274</td>
<td>140</td>
<td>176</td>
<td>317</td>
<td>3%</td>
</tr>
<tr>
<td>170</td>
<td>3</td>
<td>274</td>
<td>277</td>
<td>3%</td>
</tr>
<tr>
<td>269</td>
<td>1</td>
<td>244</td>
<td>245</td>
<td>3%</td>
</tr>
<tr>
<td>270</td>
<td>1</td>
<td>225</td>
<td>226</td>
<td>2%</td>
</tr>
<tr>
<td>273</td>
<td>38</td>
<td>167</td>
<td>205</td>
<td>2%</td>
</tr>
<tr>
<td>280</td>
<td>6</td>
<td>178</td>
<td>184</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>405</td>
<td>2,902</td>
<td>3,315</td>
<td>36%</td>
</tr>
</tbody>
</table>
While the analysis was restricted to particular patient types, based upon admission category and patient stay status, for DRGs 261 and 274 there were other patient types as shown in Table 35. The other patient types, however, represented the minority of patients.

Table 35: The patient stay and admission category profile for the three DRGs analysed. The majority of patients for each DRG related to the profiles analysed for the research.

<table>
<thead>
<tr>
<th>DRG &amp; Brief Description</th>
<th>Patient Stay Status</th>
<th>Admission Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Emergency</td>
</tr>
<tr>
<td>261 Chest pain</td>
<td>same-day</td>
<td>18%</td>
</tr>
<tr>
<td></td>
<td>inpatient</td>
<td>79%</td>
</tr>
<tr>
<td>274 Circulatory disease</td>
<td>same-day</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>inpatient</td>
<td>19%</td>
</tr>
<tr>
<td>572 Renal dialysis</td>
<td>same-day</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>inpatient</td>
<td>0%</td>
</tr>
</tbody>
</table>

In terms of the total DRG profile and for comparative purposes the number of patient separations per DRG was analysed. It was found that the number of patient separations per DRG for large teaching hospitals was not uniform as shown in Table 36.

Table 36: The average activity per DRG at three major teaching hospitals in South Australia. For many DRGs there is little regular activity. In smaller hospitals there tends be more DRGs where there is a low number of patient admissions per week or year.

<table>
<thead>
<tr>
<th>DRGs with more than:</th>
<th>Hospital 1</th>
<th>Hospital 2</th>
<th>Hospital 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>% of total DRGs</td>
<td>N</td>
</tr>
<tr>
<td>&gt;=1 separation per day</td>
<td>23</td>
<td>3%</td>
<td>20</td>
</tr>
<tr>
<td>&lt; 1 sep per day, &gt; 1 sep per week</td>
<td>212</td>
<td>32%</td>
<td>183</td>
</tr>
<tr>
<td>&lt;= 1 sep per week, 0 seps</td>
<td>370</td>
<td>56%</td>
<td>422</td>
</tr>
<tr>
<td>no separations recorded</td>
<td>53</td>
<td>8%</td>
<td>33</td>
</tr>
<tr>
<td>Total</td>
<td>658</td>
<td>100%</td>
<td>658</td>
</tr>
</tbody>
</table>

10.3.2 Overnight Stay Patient Results

The data for DRG 261 were found to be well described by the compartment flow model as shown in Figure 87 and Table 37.
While the fit to the data was good, the model did overshoot the data when the time since admission was zero days. The implication of this was that the model overstated the number of occupied beds required. While this may not be significant for many modelling situations, for bed management purposes it would be preferable if there had been closer fit between the model and data at this point.

Despite using seeding in the model fitting process, the data were well described by a single compartment model as shown in Table 38.
In terms of using the model parameter A to determine average daily bed requirements, the value would have to be averaged across the year due to the use of the constructed annual occupancy profile.

### 10.3.3 Elective Same-day Results

The most frequently recorded DRGs accounted for 90 per cent of the Medical Division’s activity and included the two elective DRGs examined in this research. The length of stay profiles for the two DRGs are shown in Figures 88 and 89.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Number of Compartments</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>One</td>
</tr>
<tr>
<td>A</td>
<td>694.8</td>
</tr>
<tr>
<td>B</td>
<td>0.57457</td>
</tr>
<tr>
<td>C</td>
<td>377.1</td>
</tr>
<tr>
<td>D</td>
<td>0.57457</td>
</tr>
</tbody>
</table>

Table 38: Model parameters for a single and double compartment flow model. The flow rate of the double compartment model is not sufficiently different to justify the use of a double compartment model.
Figure 88: The length of stay profile for AR-DRG 274 was different to that of AR-DRG 261. AR-DRG 274 appears to have a Gaussian distribution.

Figure 89: The length of stay profiles for AR-DRG 572 was different to that of AR-DRG 261. AR-DRG 572 appears to be uniformly distributed around a single value of stay.
10.4 Discussion

The research conducted has shown that a compartmental flow model can be used to describe the length of stay distribution of an inpatient DRG. As previously identified in earlier chapters, compartmental flow models are easily constructed and overcome the flaws associated with the ALOS, and the model parameters can also be used for what-if scenario analysis. Based upon the examination of two DRGs that relate to same-day activity it is evident that there will be some DRGs where there will be little value in using exponential compartmental flow models to generate flow parameters. The ramifications of these preliminary findings for policy, along with the technical issues arising from the research, are now discussed.

10.4.1 Number of Data

The methodology for this particular piece of research required that the training data profile for the creation of the compartmental flow model be artificially constructed, because of low patient numbers per day. This was a realistic option given that for most hospitals there are few DRGs where there are at least one patient per day admitted and for the majority of DRGs (more than 60 per cent) there are, on average, fewer than 1 patient admitted per week as shown in Table 36. The biased DRG profile was expected and is a consequence of a number of factors, including the type of services that can be provided at a particular hospital (for example, complex surgery is rarely performed at a remote country hospital, but is a routine occurrence in a major teaching hospital), and the underlying burden of disease (or needs) of the population.

The same-day DRG profile for the Medical Division was biased by the inclusion of the dialysis activity, with the two most frequently recorded DRGs (one of which was
dialysis) accounting for more than 70 per cent of the Division’s activity in terms of same-day patient numbers. Renal dialysis was the most frequently recorded DRG for the Division. It is the only DRG where there would have been sufficient data to avoid the construction of an artificial data profile for a compartmental flow model.

While construction of an artificial occupancy profile still enabled the modelling of the data, obtaining a greater number of data per DRG would be more helpful for a number of reasons:

- The bed number parameters (A, C and E – if C and E used) would not need averaging across the year to determine average daily occupancy, and
- There may be implications for model fit (see next section).

As this research was conducted in order to determine whether compartment flow modelling could be applied to casemix, there was no imperative to seek more data. It may have been possible to obtain additional data in relation to the whole State and this would have improved the number of data for each of these DRGs. Since this research was conducted, The Health Roundtable Pty Ltd announced that data from many hospitals around Australia and also New Zealand will be made available to researchers and thus would be an appropriate data source for further research.

10.4.2 DRG 261 Model Fit

Tables 37 and 38, together with Figure 87, indicate that the model described the data well. This is, perhaps, almost a trivial result insofar as it has been previously shown

---

2 This announcement was made at The International Health and Social Care Modelling and Applications Conference held in Adelaide during April 2006. I was the Australian convenor for this conference.
that acute care inpatient data is well described by the compartmental flow model and
that DRG 261 represents a subset of this inpatient data and also shares the same
general length of stay profile. Additionally, together with Millard, I have previously
shown that DRG 261 could be modelled with the BOMPS software package (Mackay
and Millard, 1999). Also, the work of Wang, Yau and Lee (2002) has previously
identified that maternity patient length of stay in Western Australia could be better
described using a mixture model in place of the ALOS.

It is evident, however, that the point when the number of patients have been in
hospital for zero days, that is, \( x = 0 \) or total occupancy, is overstated by the model.
While this would be of concern for a bed allocation decision-model, it is less of a
concern in this instance for two reasons. First, the intention is to use the flow
parameters (that is, the B parameter in a single compartment flow model) to replace
the ALOS in the casemix funding model. Second, the number of data were relatively
few and with more data the variation between the data and the model may be further
reduced.

10.4.3 Inpatient and Same-day Activity
This research has focussed on modelling and profiling the occupancy patterns of
inpatient or same-day activity. All the DRGs included in this analysis had both
inpatient and same-day activity, as shown in Table 35, although for DRG 572 the
inpatient activity was negligible (less than 0.1 per cent) and possibly questionable in
terms of data quality.
The focus on inpatient activity in relation to DRG 261 was appropriate as this represented approximately 80 per cent of the activity, as shown in Table 35. Additionally, the same-day activity would be funded differently according to casemix funding rules (Department of Health, 2005a).

The same-day activity for DRG 274 represented almost 70 per cent of the patient separations, as shown in Table 35. While the inpatient occupancy profile of the remaining 30 per cent of patient separations was not examined, this is perhaps a moot point, as the funding rules differ between the inpatient and same-day components (Department of Health, 2005a). Rather, the requirement for at least a dual funding mix, that is, an inpatient and same-day component, for any given DRG highlights the acceptance that the ALOS is not a perfect measure for resource allocation. This research has, however, identified that while compartment flow model parameters may be suitable for inpatient DRGs, for at least some same-day DRGs they will not represent a useful means of obtaining an alternative parameter, even when the occupancy profile is measured in hours rather than days.

10.4.4 Business Rules – fix add distribution could change

The previous section identified that compartment flow models based upon mixed exponential distributions may not generate parameters that can be used to replace average length of stay parameters for same-day DRGs, such as DRG 572. This fact does not mean that compartment flow models are flawed per se, but rather that the modelling needs to take into account the business rules, that is, the accepted practices that exist. Clearly, for DRG 572 (renal dialysis), the business practice is well known and there is minimal variation in patient stay.
Casemix is a benchmarking tool that has been adapted for funding. The incorporation of the ALOS in the funding mechanism ignores to a large extent, the existence of business rules. It could be argued, however, that the separation of same-day activity from inpatient activity is an acceptance, albeit not an ideal solution, to the existence of business rules.

The use of compartment flow models parameters would, however, represent an improvement in the casemix funding environment, as any changes in patient length of stay profile should be derived from changes to business practices, as opposed to gaming the system to try and achieve financial advantage through manipulation of the flawed ALOS metric.

The primary issue that remains to be investigated is whether the compartmental flow model parameters could be used to replace the ALOS for all the inpatient component of activity in the funding model. It is possible that for some inpatient activity there may be business practices that result in the compartment flow model parameters being less than ideal as indicators of patient stay.

10.4.5 Casemix and Benchmarking

As previously stated, the casemix-funding model was derived from a benchmarking application. The implications are clear – benchmarking of DRGs across different hospitals within a state or across different states in Australia should be possible.
Notwithstanding the previous comments regarding the implications of business practices, the use of the ALOS as a benchmarking parameter is poor. For example, it is possible that two hospitals will treat the majority of their patients within a similar time frame, yet, through manipulation of the length of stay profile of the longer staying patients, which may yield better clinical outcomes for that small group of patients, one hospital will have a lesser ALOS than another hospital (this is one of the gaming options under casemix).

In Chapter 9 the potential to use the compartmental flow model parameters for benchmarking purposes was highlighted. Clearly, the flow rate parameters, if adopted for use in a casemix-funding mechanism, could also result in the improved comparison of patient flow between hospitals at the DRG level as these parameters overcome the weaknesses previously identified with the ALOS.

10.4.6 Policy Implications

Leaving aside the need for additional research to provide further evidence that the compartmental flow parameters should replace the ALOS in the casemix-funding model based upon technical merit, there are potential policy implications from this research. Benefits of using the compartmental flow parameters for forecasting and evaluation have already been discussed in previous chapters.

As already identified early in this thesis, the ALOS is a ubiquitous measure and is routinely used in funding models, performance measures and for many other purposes. Furthermore, the limitations of the ALOS are either poorly understood by many or ignored. Measures, such as the median length of stay, would not readily be
able to be substituted into the current range, albeit often incorrectly, uses of the ALOS. Thus, it is expected that the well-cemented position of the ALOS will be difficult to shift in the short-term. As a consequence, even with the continued use of the trimmed data as required in line with the funding model policy (Department of Health, 2005a), the length of stay distribution remains skewed as shown in Figure 86, and hence, gaming around the modified ALOS is still possible.

The research findings of Wang, Yau and Lee (2002) have shown that maternity patient length of stay in Western Australia could be better described using a mixture model. These authors have suggested that the ALOS should be replaced by the parameters from a mixture model. However, to date, these findings have not been persuasive in altering the use of the ALOS in the casemix model.

10.4.7 Further Research

This research represents an initial foray into the application of compartmental flow model parameter use in resource allocation. The research of Wang, Yau and Lee (2002) supports the need to consider alternative drivers other than the ALOS in funding models.

While there are approximately 660 DRGs, many hospitals do not treat patients across the entire DRG spectrum. Consequently there is a paucity of data for many DRGs at the hospital level. Thus, it would seem appropriate that any additional research occur either at the whole of state or commonwealth level to ensure the availability of sufficient data.
Given the results for the two same-day DRGs, it is considered likely that other DRGs will be identified where a mixed exponential compartmental flow model will not be the best method for describing the occupancy profile. Thus, the primary issue that requires investigation is whether the compartmental flow model parameters could be used to replace the ALOS for all the inpatient component of activity in the casemix-funding model. Additional research will be required to determine what other models will be required to describe the occupancy parameters and whether such parameters can be integrated into a single funding model without undue complication.

The research described in Chapters 6 and 9 has highlighted the potential for using compartmental flow models for benchmarking and forecasting. The casemix funding mechanism was derived from a benchmarking application. While resource allocation is an important area of work, it represents only one area where benchmarking may be usefully applied. There is a potential to use flow models, and in particular the flow rate parameters, in other benchmarking situations that may also incorporate the use of DRGs. For example, there is the potential for research to:

- Compare alternative care options
- Compare the flow of patients based upon different demographic aspects (for example, sex, ethnicity), and
- Consider the effect of discharge destination on patient stay and the implications for funding.

10.5 Conclusion
As previously identified in earlier chapters, compartmental flow models are easily constructed and overcome flaws of ALOS. The research conducted in relation to this chapter has revealed that in some, but not all, instances exponential compartmental
flow models will yield parameters that overcome the flaws associated with using the average length of stay in relation to measuring patient occupancy at the DRG level. The fact that compartmental flow models work at the DRG level comes as no surprise given knowledge of the length of stay profiles of many DRGs and prior research of Wang, Yau and Lee (2002).

The identification that other business rules may be relevant in determining patient length of stay profiles that are unlikely to be usefully modelled with a mixed exponential compartmental flow model was also identified. This is an important revelation in itself, as it provides further justification as to why the ALOS should not be used as a measure for comparative or resource allocation purposes.

Given that the introduction of casemix funding was delivered from a model initially established to facilitate benchmarking, it would seem appropriate to adopt a measure of occupancy that enables the best possible comparison across services at the DRG level that can be achieved. I, along with Wang, Yau and Lee (2002) believe that there is merit in replacing the ALOS as a key driver in the casemix funding allocation model with other kinds of flow rate parameters in order to overcome the deficiencies of the ALOS.

Replacement of the ALOS is an issue of policy and politics. Until the examination of other DRGs has occurred in relation to this issue, it is unlikely that consideration of policy changes will be considered. Furthermore, given that a change may lead to some reallocation of resources, it is likely that resistance to such change would occur.
Consequently, in order to facilitate change, the benefits of any policy change will have to be sold well.

The application of compartmental flow models at the DRG level, therefore should be viewed as highlighting the potential to improve resource allocation mechanism, but requiring further research to justify the necessary policy change required for implementation.

The next chapter considers the need for sensitivity and simulation analysis as a component of bed occupancy research. Examples of sensitivity and simulation analysis using results from earlier chapters are provided.
Chapter 11

The use of sensitivity and simulation analysis in conjunction with compartmental flow bed occupancy models

In this chapter I investigate the need and value of incorporating sensitivity and simulation analysis with the bed occupancy compartmental flow modelling. Sensitivity and simulation analysis are means of considering the consequence of variation on model output. The chapter has the following structure:

11.1 Introduction .............................................................................................349
11.1.1 Sensitivity Analysis ..............................................................................351
11.1.2 Simulation Analysis .............................................................................353
11.2 Method ....................................................................................................354
11.2.1 Data and Base Model ...........................................................................354
11.2.2 Research Tools ....................................................................................354
11.2.3 Sensitivity Analysis ..............................................................................355
11.2.4 Simulation ............................................................................................356
11.3 Results .....................................................................................................358
11.3.1 Sensitivity Analysis – BOMPS Style .....................................................358
11.3.2 Tornado Sensitivity Analysis .................................................................361
11.3.3 Parameter Simulation ...........................................................................364
11.3.4 Patient Turn-away Model Simulation ....................................................366
11.4 Discussion ...............................................................................................369
11.4.1 Sensitivity Analysis – BOMPS Style .....................................................371
11.4.2 Tornado Sensitivity Analysis .................................................................373
11.4.3 Parameter and Patient Turn-away Model Simulation.............................374
11.4.4 Technical Issues ....................................................................................376
11.4.5 Migration of Research into the Applied Setting .....................................377
11.5 Conclusion ...............................................................................................378
11.1 Introduction

Recent experience in the Australian public health sector has shown that the planning for future health care services does involve forecasting future bed numbers (Generational Health Review, 2003; Strategic Planning Directorate, 2004). For example, the Generational Health Review (2003) reported that by 2011 the increase in total admissions would be ten per cent, a 16 per cent increase in bed stock would be required (overnight and same-day) and total cost per annum would increase by nine per cent. These forecasts appear to be deterministic in nature, because the forecasts were reported without any sense of variation and the accompanying text did not indicate that variation had been explicitly considered.

The methodology for the determination of future bed requirements for Western Australia presented by the Strategic Planning Directorate (2004) provided transparent methodology that could be examined. While the methodology enabled sensitivity analysis to be undertaken, the inclusion of the notion of variability, or uncertainty, was absent.

The absence of reporting any uncertainty in the model output is perhaps not surprising as many of the methods previously discussed do not incorporate the inclusion of uncertainty in the reported methodology (for example, see Sorensen, 1996). It may be surmised that the authors have not ignored the concept, but by choosing to report “average” occupancy or bed requirements (where this has occurred), they are implicitly reporting that variation either side of the average should be expected. This interpretation, however, may be generous.
My personal experience in the health sector has led me to the conclusion that most “business” forecasts around resource issues are deterministic in nature, with many users and even forecasters having limited knowledge of forecasting methodologies beyond stating some basic assumptions that have been incorporated into the forecasts. These comments do not apply to forecasts created for other purposes, such as academic research. Some attempts to raise this issue have, however, been made in Australia in relation to workforce planning, with Joyce, McNeil and Stoelwinder (2004) recommending that forecasts need to move beyond deterministic models and providing an example of such a model (Joyce, McNeil and Stoelwinder, 2006).

The previous chapters have shown how compartmental flow models can be applied to the acute care health sector. Generally, the results have involved the use of some kind of average model, whether it was for forecasting future bed requirements (see Chapters 5, 7 and 9) or the evaluation of service change (see Chapter 6). Variation around parameter values using standard Monte Carlo methods (Hillier and Lieberman, 2001; Powell and Baker, 2004)) was referred to in Chapters 7 and 9. The notion of altering an average model to incorporate variation across the year to overcome the limitations of using average models of bed occupancy for forecasting and planning as reported by St George (1988) and MacStravic (2001) was, however, introduced in Chapter 8. Although the acknowledgment of a “what-if” or scenario testing feature in the original BOMPS package has been mentioned, the discussion of how the model output may vary, however, has been limited to consideration of differences between current and future occupancy based upon population change. Consequently, the purpose of this chapter is to: further discuss sensitivity analysis; further develop the notion of variation around the model results using simulation (in particular, using
standard Monte Carlo methods (Hillier and Lieberman, 2001; Powell and Baker, 2004)); and to discuss the ramifications for decision-making. It should be noted that it is not the purpose of this chapter to explore the extensive literature around sensitivity and simulation analysis from other areas of endeavour, but to limit consideration to that affecting hospital bed occupancy.

Hass (2004) has reported that health service research is poorly funded within Australia. Given the low level of investment in health service research, translating research from academia to the health sector must not be expensive. Thus, one attribute that may facilitate translation and therefore adoption of research is that it uses already commonly available software (where software is required). Consequently, a further aim of the research conducted for this chapter was to demonstrate this work could be undertaken using software that is readily available to all decision-makers within the Australian health care sector.

### 11.1.1 Sensitivity Analysis

According to Powell and Baker (2004), sensitivity analysis has three functions:

1. To assess the sensitivity of the base or original model to variations in the model parameters, given that model parameters are subject to errors or other changes,
2. To examine if changing model parameters leads to a better outcome, and
3. To examine differences in model structure (for example, examination of the effect of using non-linear relationships in place of linear relations).

The terms “what-if analysis” and “sensitivity analysis” are often used interchangeably (Powell and Baker, 2004). In some situations, such as model optimisation, the term
“sensitivity analysis” has quite a specific meaning. However, here the terms will be used interchangeably. Furthermore, the use of sensitivity analysis to examine different types of model structure is not explored.

The potential for examining changes in bed occupancy compartmental flow model parameters created by Harrison and Millard (1991) was recognised and incorporated as a feature of BOMPS (BOMPS, 1992; McClean and Millard, 1995). According to McClean and Millard (1995) and the BOMPS manual (1992), the value of what-if testing was that it enabled the user to pre-test the impact of clinical decision-making and resource allocation and compare the merits of different solutions to planning problems. Typical what-if questions that could be answered were stated in the BOMPS manual and also by McClean and Millard (1995) and these included:

- How many less patients will we treat in the coming year if x beds were closed?
- What rate of discharge is required to counterbalance the impact of bed closures?
- What would be the immediate and long-term effects on admissions of opening five new beds?

While some of the questions posed by McClean and Millard (1995) and listed in the BOMPS manual (1992) were related to a geriatric care service, many questions relating to an acute care service can be posed and examined. While the ability to test the sensitivity of the model to changes in the parameters was also possible with the what-if analysis, this was not explicitly stated as something that could, or should, be
done in the BOMPS manual (1992), or by McClean and Millard (1995) and thus it raises a new area of potential investigation.

11.1.2 Simulation Analysis
Simulation is a tool that enables the modelling of events that involve uncertainty (Hillier and Lieberman, 2001; Denardo, 2002; Powell and Baker, 2004; Ozcan, 2005). Computer simulation involves the generation of many observations about a particular event, such as bed occupancy. Due to the speed of the computer, many thousands of observations can be created quickly leading to a probability distribution that can be interpreted in order to provide insights about the system under investigation.

Motulsky and Ransnas (1987) suggest that simulation is also of value in checking the fit of models to data. Indicators for use include systems that will continue to operate for a long time, uncertainty and complexity (Hillier and Lieberman, 2001; Powell and Baker, 2004). Hospital bed problems can meet these criteria and Bagust, Place and Posnett (1999), and Vasilakis and El-Darzi (2001) provide examples of published research in this area.

Bagust, Place and Posnett (1999) examined the daily bed requirements arising from the flow of emergency admissions to an acute hospital in order to identify the implications of fluctuating and unpredictable demands for emergency admission for the management of hospital bed capacity, and to quantify the daily risk of insufficient capacity for patients requiring immediate admission. Their research involved the use of a discrete-event stochastic simulation model that reflected the relationship between demand and available bed capacity. On the basis of their simulation modelling, they found that when average bed occupancy rates exceeded about 85% an acute hospital can expect regular bed shortages and periodic bed crises if average bed occupancy
rises to 90% or more. Consequently they supported the retention of spare bed capacity in order for the effective management of emergency admissions.

Vasilakis and El-Darzi (2001) evaluated the British winter hospital bed crisis using simulation. As a consequence of their modelling, they suggested that the withdrawal of social services during the Christmas period offered a good explanation for the bed blockages.

Simulation was not a feature of the BOMPS software. However, given the long-term horizon for strategic hospital bed management decision-making and the uncertainty associated with bed occupancy, the incorporation of simulation represents a new area of investigation that may yield beneficial outcomes.

11.2 Method

11.2.1 Data and Base Model

In Chapter 8 the development of a bed occupancy compartmental flow model that incorporated patient age, variation in seasonal weather and a vacancy factor was reported. That model and data provides the foundation for the sensitivity and simulation analyses conducted for this research. Details regarding the data and the methodology were reported in Chapter 8.

11.2.2 Research Tools

Microsoft® Excel, is the most commonly available spreadsheet program (Denardo, 2002) and is widely available to health sector staff in Australia, unlike more sophisticated tools, such as Matlab®. Given the desire to see that the use of compartmental flow models of bed occupancy become a more commonly adopted
strategic planning tool, the work in this chapter was undertaken using Microsoft® Excel which is typically available for hospital and health administrators, clinicians and bureaucrats use.

Crystal Ball 2000.2 Student Edition by Decisioneering, Inc. is a Microsoft® Excel add-in that was used for the simulating aspects relating to patient turn-away.

### 11.2.3 Sensitivity Analysis

Two approaches to sensitivity analysis were trialled. The first was based upon the output generated in the original BOMPS software package and the second was based upon general decision-science methodologies.

**Bomps style what-if analysis**

The effect of two hypothetical policy decisions was examined. The first hypothetical decision involved examining the alteration of bed numbers. Bed numbers were increased and decreased through a range of −10 to +10 per cent.

The second hypothetical decision involved examining what reduction in the short-stay patient group length of stay (or increased flow) was required to offset a 10 per cent reduction in beds.

The output for the analysis was calculated using spreadsheets that were developed for this purpose for Millard\(^1\).

---

\(^1\) Ms Georgina Christodoulou developed the spreadsheets in Microsoft Excel in her capacity as a research assistant for Peter Millard. I liaised with Ms Christodoulou during the development of these spreadsheets and provided some limited input into the development. I was given access to these through my collaboration with Peter Millard and his research colleagues.
**Tornado sensitivity analysis**

Sensitivity analysis of the compartmental flow model parameters was undertaken. Each parameter was varied by plus and minus 10 per cent. Parameters were varied separately to enable the effect on various aspects of the model that may be the subject of real decision-making value to be gauged. The visual reporting of such information can be done using Tornado charts (Powell and Baker, 2004). The aspects of the model that were measured after parameter variation were:

- Release rate (patient/day) - first compartment
- Release rate (patient/day) - second compartment
- Overall admissions per day
- Overall total occupied beds, and
- Overall expected length of stay.

The output was generated using the what-if spreadsheet developed for Millard. The Tornado charts were created using Microsoft® Excel based on those described by Powell and Baker (2004).

**11.2.4 Simulation**

Two approaches were taken in terms of simulation, both being forms of Monte Carlo simulation.

**Variation Around Compartmental Flow Model Parameters**

Variation around model parameters was generated using standard Monte Carlo methods (Hillier and Lieberman, 2001; Denardo, 2002; Powell and Baker, 2004; Ozcan, 2005). This was generated in both Matlab® and Microsoft® Excel.
**Patient Turn-Away Model**  
The second simulation was applied to aspects of the model created to look at patient turn-away and again involved using standard Monte Carlo methods (Hillier and Lieberman, 2001; Denardo, 2002; Powell and Baker, 2004; Ozcan, 2005).

Crystal Ball 2000.2 Student Edition by Decisioneering, Inc. is a Microsoft® Excel add-in that was used to run the simulations in order to save time, although the simulations could have been set up without this add-in.

The simulation was based upon incorporation of variation for a limited number of parameters using differing distributions as shown in Table 39.

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Distribution used for simulation</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compartmental flow model parameters A and C</td>
<td>Normal</td>
<td>n/a</td>
</tr>
<tr>
<td>Relative Average Air Temperature Change</td>
<td>Uniform</td>
<td>+/- 10%</td>
</tr>
<tr>
<td>Vacancy rate for 0 days with shortages</td>
<td>Uniform</td>
<td>+/- 10%</td>
</tr>
</tbody>
</table>

The target output for the simulation was the total bed occupancy required to achieve 0 days of shortages or patient turn-away.
11.3 Results

11.3.1 Sensitivity Analysis – BOMPS Style

The ability to generate a range of bed occupancy performance measures from the bed occupancy compartment flow model is illustrated in Table 40.

Table 40: BOMPS style bed occupancy measures for the original model for patients aged 65 to 79 years.

<table>
<thead>
<tr>
<th></th>
<th>FIRST COMPARTMENT</th>
<th>SECOND COMPARTMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of patients</td>
<td>65.0</td>
<td>3.5</td>
</tr>
<tr>
<td>Release Rate</td>
<td>0.14</td>
<td>0.04</td>
</tr>
<tr>
<td>Release Rate (patients/day)</td>
<td>8.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Expected Length of Stay (days)</td>
<td>7.3</td>
<td>26.0</td>
</tr>
<tr>
<td>Percentage of Beds Occupied</td>
<td>95</td>
<td>5.1</td>
</tr>
<tr>
<td>Half-Life (days)</td>
<td>4.7</td>
<td>17.6</td>
</tr>
<tr>
<td>Rehabilitation Benefit</td>
<td>1.0</td>
<td>3.4</td>
</tr>
<tr>
<td>Percentage of Patients Treated</td>
<td>98.5</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Conversion rate to the 2nd COMPARTMENT</th>
<th>Conversion rate to the 3rd COMPARTMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0021</td>
<td>n/a</td>
</tr>
</tbody>
</table>

|                        | Conversion rate to the 2nd COMPARTMENT (patient/day) = 0.14 |
|                        | Conversion rate to the 3rd COMPARTMENT (patient/day) = n/a |

<table>
<thead>
<tr>
<th></th>
<th>OVERALL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admissions (per day)</td>
<td>8.9</td>
</tr>
<tr>
<td>Derived Total</td>
<td>68.6</td>
</tr>
<tr>
<td>Expected Length of Stay (days)</td>
<td>7.7</td>
</tr>
</tbody>
</table>

The performance measures relate to both the overall patient group and also the individual short and long-stay patient groups.

Table 41 provides a high level summary about patient admissions and the short and long-stay compartments. Both Table 40 and 41 contain similar information, with Table 41 being of more use to the strategic planner, while Table 40 is probably of more interest to the clinicians, managers and other decision-makers working with this patient group.
The effect of altering the number of available beds is shown in Figure 90. In each example of changed bed numbers the effect on admissions is initially greater than the change made to the number of beds. Overtime the system stabilises and eventually the percentage change in admissions equals the percentage change in beds numbers.

The effect of changing both bed numbers (reduction) and the short-stay patient group flow rate is reported in Table 42 and Figure 91.

<table>
<thead>
<tr>
<th>Model</th>
<th>Admissions (day)</th>
<th>Admissions (year)</th>
<th>Average stay (days)</th>
<th>Number of admissions discharged (%)</th>
<th>Number of beds used (%)</th>
<th>Average stay (days)</th>
<th>Number of beds used (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65 to 79 years age model</td>
<td>8.9</td>
<td>3,257</td>
<td>8</td>
<td>3.208</td>
<td>98.5%</td>
<td>7</td>
<td>65</td>
</tr>
</tbody>
</table>

Table 41: The BOMPS style resource utilisation table for the model relating to patients aged 65-79 years. The majority of patients were short-stay patients.

The majority of patients were short-stay patients.

Figure 90: The effect on patient admissions as a consequence of altering the number of available beds is illustrated. Over time the change stabilises.

In each example of changed bed numbers the effect on admissions is initially greater than the change made to the number of beds. Overtime the system stabilises and eventually the percentage change in admissions equals the percentage change in beds numbers.

The effect of changing both bed numbers (reduction) and the short-stay patient group flow rate is reported in Table 42 and Figure 91.
Table 42: The effect on patient admissions when the number of beds was decreased by 10 per cent and short-stay patient flow was increased.

<table>
<thead>
<tr>
<th>Percentage of Short Length of Stay Reduction</th>
<th>Percentage Reduction in Beds</th>
<th>Admissions (at stability)</th>
<th>% change in admissions (initial)</th>
<th>% change in admissions (at stability)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0%</td>
<td>8.92</td>
<td>-10.4%</td>
<td>-10.0%</td>
</tr>
<tr>
<td>0%</td>
<td>-10%</td>
<td>8.03</td>
<td>-9.5%</td>
<td>-9.1%</td>
</tr>
<tr>
<td>-1%</td>
<td>-10%</td>
<td>8.11</td>
<td>-8.6%</td>
<td>-8.3%</td>
</tr>
<tr>
<td>-2%</td>
<td>-10%</td>
<td>8.19</td>
<td>-7.7%</td>
<td>-7.4%</td>
</tr>
<tr>
<td>-3%</td>
<td>-10%</td>
<td>8.27</td>
<td>-6.7%</td>
<td>-6.5%</td>
</tr>
<tr>
<td>-4%</td>
<td>-10%</td>
<td>8.35</td>
<td>-5.7%</td>
<td>-5.5%</td>
</tr>
<tr>
<td>-5%</td>
<td>-10%</td>
<td>8.43</td>
<td>-4.8%</td>
<td>-4.6%</td>
</tr>
<tr>
<td>-6%</td>
<td>-10%</td>
<td>8.52</td>
<td>-3.7%</td>
<td>-3.6%</td>
</tr>
<tr>
<td>-7%</td>
<td>-10%</td>
<td>8.60</td>
<td>-2.7%</td>
<td>-2.6%</td>
</tr>
<tr>
<td>-8%</td>
<td>-10%</td>
<td>8.69</td>
<td>-1.7%</td>
<td>-1.6%</td>
</tr>
<tr>
<td>-9%</td>
<td>-10%</td>
<td>8.78</td>
<td>-0.6%</td>
<td>-0.6%</td>
</tr>
<tr>
<td>-10%</td>
<td>-10%</td>
<td>8.87</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

Figure 91: Visualisation of the results presented in Table 42.

It can be seen that the initial reduction in the percentage change in admissions is greater than the final percentage change in admissions. Stability takes approximately 160 days to achieve.
Achieving policy neutrality around the reduction in beds through making concomitant changes to the short-stay patient flow rate or length of stay was possible, but required a greater percentage change in patient flow rate compared to the reduction in beds numbers.

11.3.2 Tornado Sensitivity Analysis

The effect of altering each of the model parameters by +/- 10% separately on a range of bed management measures was examined using Tornado analysis and is reported in Table 43 and Figures 92 to 96.

Table 43: The model parameters were altered by fixed amounts and the effects on certain aspects of patient discharge, bed numbers and length of stay were examined.

<table>
<thead>
<tr>
<th>Feature of Interest</th>
<th>Variation</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>Release rate (patient/day) - first compartment</td>
<td>-10%</td>
<td>7.9</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>9.7</td>
</tr>
<tr>
<td>Release rate (patient/day) - second compartment</td>
<td>-10%</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>0.1</td>
</tr>
<tr>
<td>Overall admissions per day</td>
<td>-10%</td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>9.8</td>
</tr>
<tr>
<td>Overall total occupied beds</td>
<td>-10%</td>
<td>62.2</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>74.9</td>
</tr>
<tr>
<td>Overall expected length of stay</td>
<td>-10%</td>
<td>7.7</td>
</tr>
<tr>
<td></td>
<td>+10%</td>
<td>7.6</td>
</tr>
</tbody>
</table>
Figure 92: The effect of altering the model parameters by +/- 10% on the first compartment release rate is illustrated. Changes to the second compartment parameters did have a very small effect, as opposed to no effect.

Figure 93: The second compartment release rate was affected by changed to parameters B, C and D, but not by equal amounts.
Figure 94: Daily admissions were most sensitive to changes in the first compartment parameters A and B.

Figure 95: The number of occupied beds was sensitive to changes in parameters A and C - the bed parameters.
The various measures showed different sensitivities to the effects of uniform changes to the model parameters. Usually the change in measure had a uniform direction of change across all model parameters. However, it is evident in Figure 96 that the uniform change in parameter C resulted in a change in the expected length of stay that had a different direction to that of the changes arising from modifying the other model parameters.

### 11.3.3 Parameter Simulation

Following 10,000 simulation runs using standard Monte Carlo methods (Hillier and Lieberman, 2001; Denardo, 2002; Powell and Baker, 2004; Ozcan, 2005) confidence intervals for the model parameters were estimated and are reported in Table 44.
The implications of uncertainty around the model parameters is perhaps best illustrated in terms of the effect this has on representing the number of total beds as shown in Figure 97.

The total number of beds is no longer a unique number, but can be described in terms of probabilities. Thus, the total number of beds can be reported as lying within a confidence interval. Confidence intervals can be expressed numerically, or they can be represented graphically, as shown in Figure 98.

Table 44: Model parameter confidence intervals after 10,000 runs.

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>Mean</td>
<td>63.67326</td>
</tr>
<tr>
<td>Std Deviation</td>
<td>0.99936</td>
</tr>
<tr>
<td>N</td>
<td>10,000</td>
</tr>
<tr>
<td>Lower 95% confidence interval</td>
<td>63.65367</td>
</tr>
<tr>
<td>Upper 95% confidence interval</td>
<td>63.69285</td>
</tr>
</tbody>
</table>

Figure 97: The spread of total bed occupancy is illustrated after 10,000 runs.
11.3.4 Patient Turn-away Model Simulation

Standard Monte Carlo simulation methods (Hillier and Lieberman, 2001; Denardo, 2002; Powell and Baker, 2004; Ozcan, 2005) were again employed to generate occupancy statistics for the patient turn-away model created in Chapter 8. The resulting statistics after 10,000 simulation runs are reported in Table 45.

Table 45: Occupancy statistics following a 10,000 simulation run. The simulated occupancy will be greater than the average monthly occupancy as the goal was to create a model that resulted in zero patient turn-away.

![Trend Chart - Probability Ranges for Total Occupied Beds (10,000 runs)](image)

Figure 98: Probability ranges for the number of total beds based upon 10,000 simulation runs using standard Monte Carlo methods.
For each month, the simulation resulted in a distribution of total bed occupancy that could be illustrated. The distribution for the month of January is shown in Figure 99.

Figure 99: The distribution of total occupancy for the month of January arising from the Monte Carlo simulations.

The original model to avoid patient turn-away was reported as shown in Figure 100 (see also Chapter 8).
The outputs from the Monte Carlo simulation can be used to create a visual representation that better illustrates the existence of uncertainty as shown in Figure 101.
The mean total bed numbers arising from the simulation follow the original model closely. In the month of December the mean and the model deviate a little, which may be a reflection of the Monte Carlo process.

11.4 Discussion

The underpinning reason for the need for better modelling of bed occupancy was that the ALOS was a flawed measure. While creating a model that describes the occupancy data well is a goal in itself, it is of little value unless the model can be used to either gain a better understanding about the current situation or for forecasting purposes.

Harrison and Millard’s (1991) original model described a geriatric health service data set from England and achieved the goal of finding a better means of describing bed occupancy.
occupancy than the ALOS. The subsequent development of BOMPS as a decision support system for decision-makers and planners highlighted the thinking of those involved in the early research. Not only did they aim to explain and gain understanding about a specific data set (that is, parts of the English geriatric health service) in order to aid management decisions for Millard, but also to provide a tool that could be more widely used by others when seeking to explore data from the health services in which they worked. This intent was clearly promulgated by McClean and Millard (1995).

The ability to address questions of policy, particularly in relation to service change, was a key feature of BOMPS and this was incorporated in the package’s what-if module. While the what-if module – a sensitivity tool – enabled the user to examine various scenarios involving changes to particular aspects of the system, such as the number of beds or the length of stay of patients in a particular phase of the model, the output was deterministic and fixed in what could be analysed.

There is no doubt that much of the resource utilisation information, as reported in Tables 39 and 40, and the ability to undertake what-if analysis provided in the original BOMPS package achieved the second goal of seeking a replacement for the ALOS, namely, to replace it for decision-making purposes in relation to strategic planning and decision-making activities relating to beds. In fact, the adoption of the compartmental flow model exceeds the ability of the ALOS to be used to address many issues that were not previously able to be addressed, such as the notion of different patient groups (short and long-stay) and differing flow rates.
Although the what-if features of BOMPS have been replicated in Microsoft® Excel there was a need to address the limited what-if or sensitivity analysis capabilities originally developed for BOMPS and the deterministic nature of the modelling. While some aspects of this have been achieved (for example, Irvine, McClean and Millard, 1994), there has been no further work on converting the modelling into output that is useable by decision-makers. Unless decision-makers and planners perceive the value of compartmental flow modelling this methodology is likely to remain one of largely academic interest. The expansion of the sensitivity analysis and also incorporation of simulation analysis in a manner that provides output that can be understood by non-academics is therefore of paramount importance. It is not, however, without risk and this will be further discussed later (see section 11.4.3).

11.4.1 Sensitivity Analysis – BOMPS Style

Simulation and sensitivity analysis provide mechanisms for exploring the acute care hospital bed system without recourse to undertaking real experiments, which is of considerable benefit, as it avoids wasting resources and the fact that the management of hospital facilities cannot be subject to a considerable range of changes for the purpose of experimentation (Denardo, 2002; Ozcan, 2005).

Sensitivity analysis can help a user better understand the model and also address questions of policy. However, even before undertaking sensitivity analysis, the model output as shown in the resource tables (see Tables 39 and 40) provide information that can help decision-makers and planners address questions of policy. For example, while long-stay patient use proportionally more beds compared to the number of patients admitted, there are only a few of these patients admitted at any time. Thus,
the rate of discharge is low. This has implications for programs that might attempt to
target the number of long-staying patients, that is, such a small number may make it
difficult to achieve desired goals. Consequently, while sensitivity analysis or
simulation could be undertaken, perusal of the resource tables can help determine
where the best outcomes may be gained when looking at policy change issues.

The analysis of changes to bed numbers and the impact on patient admissions was
performed as an example of the type of what-if analysis that could have been
undertaken using the BOMPS package. The results are illustrated in Figure 90. These
results are important in that they showed changes to bed numbers:

1. Disturb the system for a period of time, and
2. The percentage change to admissions initially planned was initially exceeded.

The system was found to re-stabilise in approximately 160 days. However, acute care
health systems may experience changes on a more rapid basis (for example, summer
bed closures, additional beds opened in winter), thus suggesting that the likelihood of
system stability being achieved for a prolonged period of time, if ever, is small. This
fact may not be well understood by decision-makers and planners.

The initial greater percentage change in admission (that is, if there was a 10 per cent
increase in beds, there would be an increase in admissions of more than 10 per cent) is
probably trivial in the sense that the difference between when the change occurs and
stability is only small. This effect was also shown in Table 42.

The other policy question that was illustrated using the BOMPS style what-if analysis
was whether changing the patient length of stay could offset a reduction in the number
of beds and these results were reported in Table 42 and Figure 91. While intuitively this question appears simple (that is, of course this can be achieved), the result was perhaps not so. In this instance, the proportion change in patient length of stay exceeded the proportion decrease in beds in order that there was no change in patient admissions. The power of such analysis is significant and the ability to translate such analysis into meaningful service change may be great. For example, it is relatively easy to open or close beds (subject to the required resources being available), but achieving an 11 per cent reduction in patient length of stay (for the short-stay patients) may be hard to achieve, when the length of stay is already short and the reduction amounts to hours and not whole days.

11.4.2 Tornado Sensitivity Analysis

Parameter sensitivity was examined and the results were illustrated in Figures 92 to 96. Examination of parameter sensitivity is useful in that it:

1. Provides the individual with insights as to how the model behaves when parameters are changed, and
2. Can be used to show the differences in effect on given outputs arising from the modification of particular parameters.

Assuming the amount of change applied to a given parameter is within the bounds of normal behaviour, the model output can be anticipated and thus the credibility of the model assessed. Alternatively, the novice can better understand the behaviour of the system through such sensitivity analysis. This activity provides limited redress to the criticism of Fone et al. (2003) regarding the lack of evaluation of models.
The modification of the model parameters in relation to expected patient length of stay (see Figure 96) highlights the effects of parameter sensitivity on model outputs. In this instance, modifying the different parameters not only has a different size of effect on expected length of stay, but also a different direction of change. For example, a reduction in parameter C achieved a reduction in patient length of stay, while for other parameters, a reduction in their values achieved an increased patient length of stay. The additional benefits of such sensitivity analysis are considerable and warrant inclusion as part of the normal model bed occupancy compartmental flow model building and analysis process.

11.4.3 Parameter and Patient Turn-away Model Simulation

Simulation analysis is advantageous when exploring bed management issues, as there are many factors that are uncertain, and mathematical solutions to such problems are too complex to easily develop (Ozcan, 2005). Mathematical analysis is, however, more powerful (Denardo, 2002) and simulation does not provide an indication of how the system should behave, or the settings of the parameters required to achieve the system’s best performance (Denardo, 2002; Ozcan, 2005).

The simulation of parameter values using Monte Carlo methods was both trivial and yet important. It was trivial in the sense that it could be argued that other forms of analysis such as the tornado sensitivity analysis could have provided insights around parameter variability. While this was the case to some extent, the actual benefit of the tornado style sensitivity analysis comes from the insight gained about the effect of changes of individual parameters on various parts of the bed system (for example, the number of admissions). Simulation analysis conveys a different perspective on variability.
The value of the information to end users presented in Table 44, which detailed the mean parameter value and a confidence interval around the parameter mean, was undoubtedly low. Rather, decision-makers would most likely seek the conversion of the parameter uncertainty into something that is more concrete in terms of their needs. The distribution of total bed occupancy, as shown in Figure 97, is one means of achieving such an outcome. An alternative outcome of such simulation is also the ability to express particular values, such as total bed occupancy, as mean values with confidence intervals and this can be presented numerically or graphically (see Figure 98). While such an approach helps convey that uncertainty exists, this, as previously mentioned, is not without risk. From my personal experience in the health sector, and also observation of published reports regarding future bed requirements such as those already mentioned (Generational Health Review, 2003; Strategic Planning Directorate, 2004), there is generally a lack of acknowledgment of variation around model outputs. This, in my opinion, stems from two factors:

1. That health decision-makers have operated in an environment that tries to convey certainty as opposed to uncertainty, and

2. A single “number” is easier to explain than an average that is reported with a standard deviation or a confidence interval.

Thus, while there is much benefit in considering the range of a forecast, as illustrated using the patient turn-away model (see Table 45 and Figures 99 and 101), as it can be used to show the range of activity that may have to be planned for, there is a risk that it will be more difficult to convince end users of the value of such information. This impediment may be overcome, but requires the education of the end user of the
modelling about the need and value of such information. This has implications for the actions required in order for the migration of this strategic decision-making tool from an academic exercise to being adopted more widely as part of normal business practice in the health sector.

In relation to the patient turn-away model simulation, the simulation could have been extended to include the population forecast. For purposes of illustration of the simulation technique, however, this was not necessary. The comparison of the original model output, as shown in Figure 100, to the simulated model output shown in Figure 101 highlights the fact that variation around the forecast can be visually communicated better using the simulated model output.

Simulation of the entire model parameters provides a mechanism of partial redress to the criticism by Fone et al. (2003) that models are not evaluated. Simulation, as with sensitivity analysis, enables end users and also the modellers to better understand the behaviour of the system described by the model and therefore judge whether the model is appropriate or not. Simulation has the added benefit that it is easier to vary many parameters at the same time and repeatedly compared to sensitivity analysis.

**11.4.4 Technical Issues**

There are two technical issues that merit discussion. First, alternative distributions, such as the normal or triangular distributions, could have been chosen to represent the variation around air temperature and vacancy rates. The variation around vacancy rates may have been better described using the triangular distribution, with the mean representing a more likely outcome and the extremes less likely outcomes. As the
standard deviation for the variation around the weather variables was not included in the data a uniform distribution was appropriate without recourse to purchasing additional data. The choice of parameter variation was, however, unlikely to have affected the outcome of the results greatly. That is, the intent was to illustrate the variation around the model output. In an applied setting the purchase of additional data or experimentation with different distributions to describe parameter variation (or both) would occur.

Second, the simulation could have been undertaken without the use of the Crystal Ball add-in to Microsoft® Excel. The add-in enabled the research to be undertaken more easily and there is evidence of use of similar add-ins in the Australian health sector (Joyce, McNeil and Stoelwinder, 2006) to facilitate simulation modelling.

11.4.5 Migration of Research into the Applied Setting
BOMPS provided a decision support system that was capable of creating the bed occupancy model and also enabling users to undertake what-if analysis. The BOMPS is no longer supported and an upgraded version has not been created.

It could be argued that the ability to fit models to the data is a task that should reside with persons with the appropriate expertise (for example, modelling, statistical or mathematical expertise) and thus, such a task could be performed using a variety of software packages. It may also be argued that the incorporation of the compartmental flow model into other models, such as that illustrated in Chapter 8, also requires the skills of appropriate trained individuals.
The main benefit of this work, however, will occur when decision-makers and planners can undertake what-if analysis and also simulation analysis. Consequently, the ability to undertake such analysis using readily available software, such as Microsoft® Excel, is important. The provision of a range of spreadsheet templates or add-in tools (developed using visual basic facilities) may facilitate the uptake of this work.

11.5 Conclusion
The use of sensitivity and simulation analysis in relation to bed occupancy compartmental flow models has been demonstrated. Previously developed sensitivity analysis approaches to bed occupancy compartmental flow models were presented along with the parameter sensitivity analysis. Monte Carlo simulation was applied to previously developed models of bed occupancy and enabled the simultaneous variation of bed occupancy, weather and occupancy parameters to be demonstrated.

In terms of management application, and therefore the ability to translate this research from the academic to the applied setting, sensitivity and simulation analysis are likely to represent the most appropriate vehicle for the communication of the benefits of bed occupancy compartmental flow modelling. The ability to do such analysis using readily available software will be important in this task.

This chapter concludes the presentation of the results of the research undertaken for this thesis. In the next chapters I draw together the discussions arising from the separate pieces of research presented in this and previous chapters, present ideas for
future research and present an overall conclusion regarding the application of bed occupancy compartmental flow models for strategic decision-making in the acute hospital sector.
Chapter 12

Discussion

In this chapter I draw together the discussion arising from the results presented in previous chapters. The discussion about the research is at the collective level, as opposed to the individual experiment or chapter level. In doing so, the original research questions are examined, as are the implications arising from the research for the bed occupancy compartmental flow model and the general issue of strategic decision-making around hospital beds. The chapter has the following structure:

12.1 Introduction ............................................................................................ 381
12.2 The Research Questions ........................................................................ 381
12.2.1 The Main Research Question ............................................................... 382
12.2.2 The Secondary Research Questions ...................................................... 385
12.3 The Revised Methodology and Model .................................................. 388
12.4 The Preferred Methodology .................................................................. 392
12.4.1 The Strategic Level .............................................................................. 392
12.4.2 Knowledge of the Business .................................................................. 393
12.4.3 The Value of Information ..................................................................... 393
12.5 Revisiting Bed Occupancy and the Role of Modelling ......................... 394
12.5.1 A More General Consideration of Bed Occupancy ................................ 394
12.5.2 Has the problem been over simplified? ................................................. 398
12.5.3 Influence over the problem space ......................................................... 402
12.6 Conclusion .............................................................................................. 407
12.1 Introduction

The research conducted for this thesis has been presented as a series of different experiments, with each experiment being reported in a separate chapter. The research work, however, does not represent a series of completely disjointed and unconnected experiments. Rather, the research represents a journey along a path of inquiry seeking to determine whether compartmental flow models of bed occupancy can be usefully applied in the acute care hospital sector, with the “experiments” linking to each other either through the use of methodology or the expansion of a particular idea.

The purpose of this chapter, is therefore, to present a discussion about the research work that is at the level of the whole body of research as opposed to the details presented in the preceding chapters. In doing so, the original research questions will be examined, as will the implications arising from the research for the bed occupancy compartmental flow model and the more general issue of ownership of strategic decision-making around hospital beds.

12.2 The Research Questions

The research findings validates the belief that the compartmental flow model of occupancy, as first described by Harrison and Millard (1991) for a geriatric service operating in the health sector in London, can be applied to medical patient data from acute care hospitals in Australia and New Zealand.

The creation of double compartmental flow models of bed occupancy for the acute hospital data used in this research were based upon understanding the trade-off between model complexity and fit, and also the need to secure information of value
(that is, the need to distinguish between short and long-staying patients). While the BIC could help determine the model of choice in terms of trade-off between model fit and complexity, expert opinion was required to determine the value of additional information gained through increasing model complexity.

The primary and secondary research questions are now examined separately.

12.2.1 The Main Research Question
In the first chapter, I stated that the main research question to be addressed by the research was:

Can bed occupancy compartmental flow models be applied to acute care hospital data in order that better (compared to existing) or new information or understanding be developed and have the potential to result in improved strategic planning of service delivery (and thereby resource utilisation)?

Throughout this thesis it has been demonstrated that compartmental flow models of bed occupancy can be applied to acute care hospital data. These models have been generated in a variety of ways:

- Using the now outdated BOMPS package (for example, Mackay and Millard, 1999)
- Using the method where all the data were incorporated into the modelling process (see Chapter 5), and
- Using the method where the data were first summarised and then incorporated into the modelling process (See Chapters 6, 7, 8, 9 and 10).
Furthermore, the data used for this research have come from hospitals existing in separate health systems thereby increasing the validation of the approach and results. It could be argued that the successful application of the approach using only two data sets is not sufficient evidence to validate the approach. However, reliance can be placed upon additional sources of evidence, including:

- The successful modelling of acute care surgical data (Millard, Mackay, Vasilikas and Christodoulou, 2000)
- The representation of a length of stay profile as being highly skewed in the casemix manual Technical Bulletin 94:10 (Department of Health, 2005a), thereby implying this profile is the normal length of patient stay profile.
- The work undertaken by Wang, Yau and Lee (2002) that found the maternity patient stay was better modelled by a hierarchical Poisson mixture regression model than the ALOS.

Thus, while further research may be undertaken in relation to compartmental flow models of bed occupancy, the question of whether such models can be used in the acute care sector has been validated using primary (my research for this thesis) and secondary sources (my research prior to commencing this thesis and other sources) of evidence.

The question, however, also included the condition that better (compared to existing), or new information, or understanding result as a consequence of the application of the compartmental flow models of occupancy. As previously stated, the ALOS is a simple measure of patient stay that is widely reported. It is a single measure. The application of a double compartmental flow model of occupancy yields four parameters: two
parameters relating to the number of occupied beds or patients, and two parameters relating to rate of flow of patients through the compartments. Thus, *prima facie* evidence shows that new information is gained as a consequence of applying these models. Further information can be gleaned about patient flow and numbers (or numbers of beds) through the use of sensitivity and simulation analysis.

It can only be surmised that the additional information gained through the application of the compartmental flow model will result in better understanding about patient flow and as a consequence lead to improved strategic decision-making, because the opinions of potential end users regarding this research were not sought. However, as someone who has previously had to rely upon the ALOS, I can attest to the improved understanding gained about patient flow as a consequence of the generation of new information from compartmental flow models, as compared to what is possible to learn from analysis of the ALOS alone. Feedback from colleagues as a consequence of my research also indicates that the compartmental flow model has resulted in improved understanding about patient flow for them (Chris Bain and Don Campbell, personal communication).

The translation of new or better information into improved decision-making was not tested. While it can be surmised that this should be an expected outcome should the modelling approach be adopted, it may not necessarily follow. For example, while better understanding around a particular strategic patient flow or bed management issue may result from the application of compartment flow modelling, political or financial imperatives may prevent the decision from changing. Under such circumstances, however, the additional understanding should enable more informed discussion about the consequence or consequences of the decision.
12.2.2 The Secondary Research Questions

The secondary research questions that were examined during the research were:

1. How many data are required to create a bed occupancy compartmental flow model for an acute care hospital data set?

2. What level of model complexity is desirable in order that models can be used for generalisation and forecasting purposes?

3. Can bed occupancy compartmental flow models that incorporate the ageing of the population be used to forecast future bed (resource) usage in acute hospital care?

4. Can bed occupancy compartmental flow models be used to evaluate service change?

5. Can bed occupancy compartmental flow model parameters be used for forecasting purposes?

6. Can the bed occupancy compartmental flow model be adjusted to incorporate seasonal variation, where the term “seasonal” applies to weather seasons?

7. Can bed occupancy flow compartmental flow model parameters provide a substitute metric for the average length of stay in resource allocation models?

8. Can sensitivity and simulation techniques be used in conjunction with bed occupancy flow compartmental flow models to enable uncertainty to be incorporated into the modelling process?
It was possible to explicitly answer each of these secondary research questions. Figure 102 details the research question, the chapters in which the research question was resolved and the linkages between the chapters.
Figure 102: Identification of secondary research questions and the chapter in which the question was answered.
The successful resolution of these secondary research questions has resulted in a contribution to the body of knowledge around strategic decision-making in relation to acute care hospital beds. Some of this knowledge has already been communicated to the wider community through journal articles and conference papers. More importantly, the research has led to consideration of technical issues, such as model choice, and the modification of the original model upon which this research was predicated.

12.3 The Revised Methodology and Model

The original work of Harrison and Millard (1991) relied upon the patient census as a means of obtaining data for the model. The research presented in this thesis, however, found that consideration of more data resulted in improved fitting of the model to the data. Although this is not a surprising outcome, it did lead to a justification to use data based upon a year as opposed to a single census for the creation of the bed occupancy compartmental flow model.

Consideration of every data point across the year, however, was found to be impractical and would certainly not provide a tool for managers working in applied settings (for example, hospitals or health departments). Consequently, further modification of the approach was adopted, resulting in the creation of a second base methodology (see Chapter 6) that still captured variation in the data, but resulted in a much improved model creation time.

Prior research undertaken by Millard and others has been based upon a single geriatric patient data set. The desire to investigate more complex models for a variety of
reasons (see Chapters 5 and 6) represented a new area of research. While model complexity could be increased easily, there was a risk that the resultant model would over-fit the data and therefore be of limited use for generalisation and forecasting. The introduction of a variety of methods to determine model choice was therefore required and was incorporated into the methodology. While such methods are not free of the need for judgment, such as when deciding whether a single compartment or double compartment model is most suited to the modelling task, they do provide clear choices between competing models and are to be valued, particularly in the data rich health sector, where I have observed that the practice has often been to create models at the finest level of detail possible.

The original model was developed assuming that the system under examination was stable (Harrison and Millard, 1991). The issue of stability has been discussed in earlier chapters and it was noted that the acute care sector and even the geriatric care hospital sector were not stable and exhibited variation that could be attributed to a variety of sources, including seasonality and policy changes. Some of the work presented in this thesis has attempted to address the issue of instability, particularly attributed to seasonal variation, through the development of more complex models (see Chapter 5), which were later abandoned in favour of a less complex approach (see Chapter 8). Variability associated with daily variation in patient numbers – or vacancy – was also incorporated into the model (see Chapter 8). Uncertainty surrounding various aspects of the model was also incorporated through the use of Monte Carlo simulation (see Chapter 9). This progression of development of the model is illustrated in Figure 103.
The other aspect of variation is acknowledgment that policy decisions may affect a given system, but that the model can be varied to accommodate such changes.

The development of the model co-incided with the exploration of how the model may be applied in different ways. For example, applications relating to forecasting (see Figure 103: Modification of the original (base) model has occurred to enable the incorporation of seasonal variation and variability of patient arrivals. The diagrams on the right-hand side are illustrative only and were presented in full detail in earlier chapters.)
Chapters 7 and 9), evaluation (see Chapter 6) and casemix (see Chapter 10) were considered. This is illustrated in Figure 104.

While the potential to apply the model to a range of health sector issues has always existed, early work, including the development of BOMPS, focussed on facilitating greater understanding of bed occupancy and providing tools that would result in better planning decisions. For example, BOMPS provided a means of pre-testing service change, but did not incorporate a mechanism to link it to population change. This is not a criticism of the early work, but rather is indicative of a period of evolving research and development.

New approaches for conveying information from the modelling approach to users were also explored. For example, the tornado charts (see Chapter 11) enable users to
see what model parameters have the greatest impact upon a particular aspect of bed occupancy.

12.4 The Preferred Methodology

The discussion in this section focuses on the use, technical issues and limitations of the modelling approach that has evolved during the course of this research. These issues and limitations are of a generalised nature and more specific comments relating to particular aspects of the research can be found within individual chapters (see Chapters 5 to 11).

12.4.1 The Strategic Level

This research has not altered the focus of the use of the compartmental flow model of bed occupancy. The inclusion of uncertainty does enable communication of the fact that there will not be an exact answer, particularly in relation to forecasting, but rather future bed requirements will fall within a given range, assuming the assumptions hold true.

The model is a tool that can assist decision-makers to make better decisions at the strategic level. It is not a tool for guiding operational decisions that occur on a day-to-day basis, although the analytical methods associated with this work, as well as a working understanding of the model output, can be used to highlight the impact of short-term decisions. For example, temporary short-term closures of wards, such as those that are often implemented around Christmas time, should be accompanied by an appropriate reduction in admissions well before the closure date in order to provide sufficient time for clearance of the system. Without such clearance, as soon as or
shortly after implementation of the reduced bed numbers, it is inevitable that bed blockages will occur and crises occur.

12.4.2 Knowledge of the Business

The research has focussed on modelling inpatient bed occupancy and not elective same-day patient bed occupancy. While the ability to model data exists using various software packages (BOMPS, Microsoft Excel, Matlab, etc) and achieve a good fit to the data, the ability to create a bed occupancy compartmental flow model is not the key to the success of this modelling approach. Rather, it is imperative that the modeller must understand the business processes. The elective same-day bed occupancy profile is perhaps illustrative of the importance of having knowledge of the business. Aside from the fact that elective same-day patients are usually managed separately to inpatients, the potential to apply the compartment flow model to these patients exists. It was shown, however, that at least some elective same-day patient occupancy profiles are very different to inpatient profiles and may be better described using alternative modelling approaches (see Chapter 10).

12.4.3 The Value of Information

The implementation of a means to choose among models of varying complexity was considered important, as this approach resulted in the selection of models that were not over-fitted and thus more useful for forecasting purposes.

However, model selection methodology cannot be divorced from judgment, because current approaches (for example, the use of the BIC value) do not incorporate the value of additional information into the measure. The example from this research that
illustrates this point is the decision to use double compartmental flow models as opposed to single compartment flow models. The double compartment model enables generation of information about a small group of patients that use a significant proportion of resources and can result in bed blockages.

This is perhaps, closely related to a need for a strong understanding of the business environment and signals the fact that naïve use of the modelling approach is fraught with problems.

12.5 Revisiting Bed Occupancy and the Role of Modelling

In this section I reflect upon the notion of why modelling bed occupancy is of value and consider where the role of modelling, and in particular bed occupancy compartmental flow modelling, fits in solving strategic bed management issues. This reflection occurs after having spent time undertaking the research presented in this thesis, reading the works of others and co-convening an international conference on health and social care modelling.

This section of work draws upon an article I wrote for Nosokinetics News that published during June 2006.

12.5.1 A More General Consideration of Bed Occupancy

It is clear that the strategic decision-making task relating to acute care hospital beds, or for that matter geriatric service beds, is affected by many factors. The ALOS is a ubiquitous measure and is simple – it is a single measure, that when combined with
one other measure can yield the number of beds required for a given hospital service.

A hypothetical example is illustrated in Table 46.

Table 46: Illustration of using the ALOS to calculate bed requirements.

<table>
<thead>
<tr>
<th>ALOS (days)</th>
<th>5.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admissions per year</td>
<td>9500</td>
</tr>
<tr>
<td>Days per year</td>
<td>365</td>
</tr>
<tr>
<td>Number of beds required per day (nearest whole number)</td>
<td>141</td>
</tr>
</tbody>
</table>

In determining the bed requirements, many factors affecting the resultant patient length of stay are assumed, either explicitly or implicitly, to be constant. The 9,500 patients admitted will not be the same, unless of course analysis of a single disease is occurring (for example, across a state or country). Admissions will occur typically for a variety of reasons, and involve people from a variety of backgrounds and ages. To breakdown the ALOS by any of the many possible groupings takes effort and may not lead to a materially different outcome as shown in the hypothetical example in Table 47.

Table 47: Subdividing the data may not lead to a materially different determination of the number of beds required for a service.

<table>
<thead>
<tr>
<th>ALOS</th>
<th>Group A</th>
<th>Group B</th>
<th>Group C</th>
<th>Group D</th>
<th>Group E</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admissions per year</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
<td>1500</td>
<td>9500</td>
</tr>
<tr>
<td>Number of beds required per day (nearest whole number)</td>
<td>37</td>
<td>16</td>
<td>27</td>
<td>28</td>
<td>30</td>
<td>141</td>
</tr>
<tr>
<td>Beds required on basis of individual groups</td>
<td>138</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Difference</td>
<td>-2%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Marmot (1999) highlights the fact that there are socio-economic determinants of health. That is, factors such as childhood environment, education, unemployment and social relationships help determine health outcomes. Consequently bed occupancy is a
result of many factors and perhaps might be represented according to the following equation:

\[ \text{Bed occupancy } \alpha(f(pm_c, c_m, \text{age}, \text{sex}, \text{ttt}, c_d, f_s, a_{toc}, s_{ed}, r_{a}, b_{dm}, p_{dm}, \text{etc})} \]

Where

- \( pm_c \) is primary medical condition
- \( c_m \) is co-morbidities
- \( t_{tt} \) is time to treatment
- \( cd \) is clinical decisions
- \( f_{m} \) is family support
- \( a_{toc} \) is access to other care
- \( s_{ed} \) is socio-economic determinants
- \( r_{a} \) is resource allocation constraints
- \( b_{dm} \) is bureaucratic decision-making
- \( p_{dm} \) is political decision-making

Administrative data, which is usually the source for the calculation of the ALOS, and in the case of this research the occupancy profiles, does not capture all the factors that result in an individual patient’s length of stay.

As a consequence, the planning and forecasting of changes in the number of hospital beds is based upon the manipulation of what is measured. This is not uncommon – that is, what is measured is managed. In fact, Berwick has made the following observation:
Every system is perfectly designed to achieve the results it achieves


The immediate question arising from the adoption of this view is whether such an outcome is reasonable. I contend that it is reasonable for managers to exert control only over issues for which they have control. While the length of hospital stay is a consequence of many factors, most of these factors lie outside the control of decision makers within the hospital setting and many lie outside the control of health departments. For example, while poor education and employment status may be associated with poor health outcomes (at the population level), a hospital clinician is usually dealing with a patient who has presented with an illness that, in our society is more likely to be related to non-infectious disease, and thus may have developed over a period of time and be associated with a variety of risk factors (for example, a common chronic illness is diabetes and this can be associated with admission to hospital). The hospital clinician cannot remedy the problems of the past that have led to the admission, but merely attend to the current (and future) needs of the patient. Additionally, while the clinician and associated hospital staff (for example, epidemiologists) may identify patterns of disease and be able to advocate changes that will affect future admission patterns (for example, the use of seat belts, promotion of sun-safe behaviour and reduction in smoking rates) they do not control the decisions that will alter the admission patterns. Furthermore, it can be argued that even at a health department level many of the socio-economic determinants of health lay outside the jurisdiction of the health sector.
Clearly, the use of the ALOS is not advocated as the basis for the planning or forecasting of hospital beds at the strategic level due to the previously identified flaws with this measure. The research presented for this thesis provides an approach that overcomes the flaws and can incorporate other factors, such as patient age, gender and reason for admission (for example at the broad level, this is captured through casemix categories). Linkages to other factors that may be deemed to be within the control of decision-makers, such as patient outcomes could occur. However, such data is not routinely collected and is still in many ways in its infancy.

Given that this approach appears to be reasonable, or at least defensible, a further two questions arise, namely:

1. Has the problem been over simplified, and
2. Who influences the problem space?

### 12.5.2 Has the problem been over simplified?

Plsek and Greenhalgh (2001) suggest that the simple reductionist approaches to analysing health sector problems, be they relating to clinical practice or organisational leadership, is no longer appropriate. Reductionist thinking suggests that work and organisations can be thoroughly planned and optimised.

According to Plsek and Greenhalgh (2001), the reductionist or machine metaphor, as described by Morgan (1986), is deficient insofar as that it performs poorly when no part of the equation is constant, independent or predictable – features often used to describe health systems or parts thereof. In many ways this is consistent with the writing of Rosenhead (1978) and colleagues Best and Parston (Best, Parston and
Rosenhead, 1986) who contended that the application of operational research
techniques to health service planning problems is fraught with difficulties for the
following reasons:

1. Problems are often treated as though they are static in nature and attempts are
   made to remove uncertainty when providing an answer.

2. The goal of many proffered solutions is to maximise a single goal, as opposed
to recognising that solutions to many goals must be concurrently achieved.

3. There is reliance upon quantification and the exclusion of factors that cannot
   be quantified can result in models that poorly reflect the behaviour observed in
   the real world.

4. Solutions do not allow for multiple decision-makers or environments where
   rational decisions may give way to the need to implement political solutions.

To counter these problems, Rosenhead (1978) and his colleagues (Best, Parston and
Rosenhead, 1986) suggested that approaches that met the following criteria should be
favoured:

1. Incorporates or recognises uncertainty

2. Adopts co-ordinated solutions in preference to optimised solutions

3. Relies upon fewer data

4. Facilitates participation, as opposed to solely relying upon hierarchial
deduction, and

5. Recognises that technocratic solutions will not always meet political needs.

The use of the compartmental flow model, as described in this research, meets the
first and third criteria. The use of the compartmental flow model output can be used to
achieve participation in decision-making (see the next section). The recognition of political agenda is not a criterion that can be addressed through the creation of a model, but rather it should be recognised that the output of a model can be used to inform decisions at all levels, including the political level, but may not necessarily give rise to the outcome suggested by the model.

Alternatively, Plsek and Greenhalgh (2001) would argue that the science of complex adaptive systems might provide better support for decision-makers who are trying to find a path through the complexity of the health system. A complex adaptive system is defined by the ability of a collection of individual agents being able to act in ways that are not always totally predictable and where an individual agent is altered (or their behaviour is altered) as a consequence of the action of another agent that is connected to them (Plsek and Greenhalgh, 2001). There are many examples of such systems, and one common example is families.

So where does this leave the bed occupancy compartmental flow model?

Such views as those espoused by Plsek and Greenhalgh (2001) are representative of those promulgating their own particular solution and while they have merit, the grounds for abandoning the compartmental flow model in favour of a complex adaptive system model, or for that matter other models, are not yet sufficient for the following reasons:

- The research confirms the ability of the model to describe the occupancy data well
- The compartmental flow model fulfils a range of uses, including providing users with a better understanding of patient flow; the ability to plan and pre-test
decisions relating to the use of beds; the ability to forecast future bed requirements; and the ability to evaluate past changes and benchmark against other organisations.

- From the strategic perspective many issues involving complexity are held constant through the assumptions that underlie the modelling approach.

- Plsek and Greenhalgh (2001) acknowledge that not all problems lie in a complex modelling space. There is certainty about the ageing of the population, existing occupancy profiles and disease patterns, and for the purpose of establishing forecasts around future bed requirements, it may be argued that there is little value in adopting the complex adaptive system model.

- It is possible to introduce uncertainty into the compartmental flow modelling approach.

- Given the findings of Fone et al (2003) that few models are evaluated, the fact that the compartmental flow model of bed occupancy has been well researched provides a reasonable basis for its adoption and use.

- While it is recognised that the health system comprises of more than just hospitals, the data required to create models of a larger part of the system is not routinely collected, and

- Given the current reliance upon simpler modelling methods, that are relatively cheap, relatively quick to create and apparently easy to understand (though clearly there is a lack of understanding around the flaws associated with the use of the ALOS), it is unlikely that most decision-makers are ready to adopt more complex decision-making tools which are often expensive to implement, require long development times and require an extensive understanding of a range of topics, including modelling methodologies and complexity.
The multiple methods approach has been proposed in the management and information systems literature as a means of better understanding complex situations. This represents a means of capturing more than one source of information to guide the decision-making process and does not require the abandonment of the compartmental flow model of bed occupancy. This is discussed in the context of who has influence over the problem space in the next section.

**12.5.3 Influence over the problem space**

Given the conclusion in the previous section that the compartmental flow model is a reasonable choice as a tool to improve strategic bed management decision-making, it is worthwhile to consider whether, as recommended by Best, Parston and Rosenhead (1986), the model can be used to facilitate participation in the decision-making process.

The health system is comprised of numerous groups, including doctors and nurses, as well as many other groups. Decision-makers, or those that can assist with decision-making work within the health sector, may also be brought in to the health sector from other sectors for specific assistance, for example as consultants or researchers. The range of stakeholders in the decision-making process is illustrated in Figure 105.
It is important to recognise that the solutions for problems relating to strategic issues, such as decision-making around beds, cannot be owned by a single group of stakeholders, but rather collaboration among stakeholders is required to ensure their successful development and adoption. Thus, while it is possible to develop a model, such as the compartmental flow model of bed occupancy, which can lead to better decisions, the usefulness of such a model is reliant upon the acceptance of and inclusion of others in developing the solution to the question that is being addressed. Engaging stakeholders during the model development phase and also in discussion around the model output is one means of achieving participation in the decision-making process. It will not, however, guarantee that organisational politics that are frequently a feature of the health sector environment will be reduced.

From the modeller’s perspective, a number of key factors can contribute to successful collaboration, including:

Figure 105: The range of stakeholders in the decision-making process may be large. A solution that relies upon a single group may not yield the best solution.
• Ensuring that the problem is being solved as opposed to implementing a particular tool

• Providing education about the modelling approach to those involved in the collaboration, as often the appreciation and understanding of the benefits of modelling varies greatly among the professions working in the health sector,

• Provide regular information to the collaborators, and

• Create some output early in the life of the project, even if it is basic analysis, so that those involved with the work have sufficient time to absorb the information that is generated and interest is maintained.

The multiple methods approach was previously suggested as a mechanism that facilitates the capture of more than one source of information to guide the decision-making process and is particularly relevant to forecasting decision-making. A multiple methods framework not only means more information is collected, but also a better understanding of stakeholder’s worldviews should be gained (Winkler 1989) and thus supports the need for participation and collaboration in the decision-making process. There is also a consensus among many forecast researchers (for example, Makridakis and Winkler, 1983; Armstrong, 1986; Clemen, 1989) and practitioners that combining methods improves forecast accuracy and forecast relevance. Clemen (1989) has stated that the notion that combining forecasts may be beneficial is not new and noted that Laplace identified that combining methods will lead to a result that has a lower probability of error during the early 19th century.

The act of combining and giving weights to the different forecasts can be problematic (Armstrong, 1985), but empirical evidence does demonstrate improved accuracy. A
multiple methods approach is also likely to improve the integration of forecasting information with strategic planning, and helps to ensure that unquantifiable catastrophic factors are accounted for (Morrison and Metcalfe, 1996). The adoption of this approach should therefore enable the inclusion of factors that are not necessarily captured in the routine administrative data (which is the basis for the data used in this research) to be incorporated into the compartmental flow model and further promote collaborative efforts. To some extent, the creation of the sliding scale model methodology used to incorporate seasonality, represents an initial foray into the application of multiple methods, as it involved the use of temperature data.

There are various approaches to the implementation of a multiple methods approach and it is not my intention to provide an account of these approaches. Rather, I will use *multiple perspective* approach defined by Mitroff and Linstone (1993) to help illustrate why seeking multiple sources of information may be beneficial in the health sector, particularly in relation to strategic decision-making relating to hospital beds. The multiple perspective approach encourages decision-makers to consider a given problem from particular perspective, namely the technical (T), organisational (O) and personal (P) perspectives. The three perspectives contribute to the discovery of a more complete understanding of the problem and may result in improved decision-making, as opposed to relying upon a single analytical view of the problem (Lindstone, 1999), and are particularly useful when addressing complex social problems (Mitroff and Linstone, 1993). Clearly, decision-making in relation to hospital beds is a complex social problem – it involves different organisations, subgroups within organisations (both professional and organisational), patients and politicians. According to Linstone:
in the political arena highly technical information is usually and properly, discounted in favour of social interests and considerations of values involved – and these can never be encompassed by the T perspective (1999, p38).

The multiple perspectives approach is a means of reducing the disparity between a model and reality (Linstone, 1981). Consequently, the multiple perspective approach can be justified on the basis that:

- Each perspective (T, O and P) provides mutually exclusive insights, and
- To bridge the gap between imperfect models (the technical view) and the real world, it is essential that the organisational and personal perspectives be sought.

I also contend that the approach can be used to justify the development of improved technical perspectives, so that highly political decisions, such as those often relating to the planning of hospital beds, are based upon sound technical input into the decision-making process. Thus, not only is it important to ensure collaboration so that other perspectives are brought to bear upon the decision-making process, but also that the appropriate technical solution is sought. In the case of strategic decision-making relating to hospital beds, the development of the compartmental flow model of bed occupancy represents achievement in improving the technical view of the strategic bed management problem. Collaboration with others is required to achieve incorporation of the organisational and political perspectives.
12.6 Conclusion

The goal of seeking to determine whether the work of Harrison and Millard (1991) in applying compartmental flow models of bed occupancy to a geriatric health service in London could be applied to the acute care hospital sector has been achieved. In achieving this outcome, the original methodology has been modified in a variety of ways. The modelling approach has shifted from a deterministic approach to a stochastic one. Consequently, the communication that there is uncertainty in the output is now easy to communicate.

Better modelling of matters affecting the decision-making relating to hospital beds alone, however, will not necessarily result in better decision-making outcomes. The health sector is a complex system and it is recognised that collaboration by many stakeholders will be required to enable better decisions to be made. The modelling work and also the model output may be used to facilitate collaboration, and ultimately achieve better decision-making in relation to the planning, forecasting and evaluation of acute care hospital beds.

In the next chapter I discuss the potential future directions for this research and whether the ability to transfer this research from an academic pursuit to one of improving decision-making in the health sector exists.
Chapter 13

Future Research and Application of Bed Occupancy Compartmental Flow Modelling

In this chapter I comment upon the opportunities for further research in relation to bed occupancy compartmental flow models and the acute care health sector, and also, the ability to move the research into other areas. The scope for potential application in the health sector business setting (as opposed to the academic environment) is also discussed. The chapter has the following structure:

13.1 Introduction ......................................................................................... 409
13.2 Scope for Additional Research ............................................................. 410
13.2.1 Research within the hospital .............................................................. 410
13.2.2 Research beyond the hospital inpatient.............................................. 412
13.2.3 Incorporating information from other modelling .............................. 414
13.2.4 Beyond the health sector ................................................................... 416
13.3 Potential for Application ...................................................................... 416
13.3.1 Scope for Application ....................................................................... 417
13.3.2 Drivers of Adoption .......................................................................... 417
13.3.3 Barriers to Adoption ........................................................................ 419
13.3.4 Possible Solutions ............................................................................. 421
13.4 Conclusion ........................................................................................... 424
13.1 Introduction

In this thesis I have presented research that supports the adoption of the bed occupancy compartmental flow model as a method of overcoming the shortcomings of using the ALOS, a widely used performance measure.

While compartmental flow models represent only one aspect of modelling, and are particularly useful for strategic decision-making, it would seem that during the course of my research, others are reaching similar conclusions regarding the need for better management decision-making in relation to acute care hospitals. For example Jones, Joy and Pearson (2002) reached the following conclusion in relation to the need for forecasting demand:

*in 44BC, Cicero wrote that after carefully examining the pros and cons of being able to foresee the future, he felt that it was better to remain ignorant. Nevertheless, many writers on management argue that for an organization to be able to plan effectively, it is vital that it can in some way anticipate the future. Perhaps nowhere is this more pressing in the day to day management of an acute hospital.* (Jones, Joy and Pearson, 2002, pg 297)

I believe that the research that I have conducted has made a strong case for the adoption of the bed occupancy compartmental flow model in the acute care sector and when considered with the research undertaken in relation to the geriatric care service in England (for example, Harrison and Millard, 1991; McClean and Millard, 1993; Harrison, 1994; Taylor, McClean, and Millard, 1996; McClean and Millard, 1998),
there is a convincing case for its adoption more widely. However, the need for ongoing research does not abate.

13.2 Scope for Additional Research

The topics of short-term and long-term forecasting, benchmarking, evaluation, resource allocation and model choice were discussed in earlier chapters in relation to the bed occupancy compartmental flow model and their use in the acute care hospital setting. Although the research presented for this thesis has focussed on medical patient data, there is no reason such analysis will not be applicable to other patient groups providing that the business model results in a similar distribution of bed occupancy. For example, work has previously been reported in relation to the use of compartmental flow models and surgical inpatients (Millard, Mackay, Vasilikas, and Christodoulou, 2000).

13.2.1 Research within the hospital

The findings of the research presented in this thesis the potential to use bed occupancy compartmental flow models to investigate a wide range of issues exists within the acute care hospital environment, including:

- Comparison across similar services
- Comparison of alternative care options or system redesign
- Comparison in differences in patient flow rate, particularly relating to patient sex, ethnicity, patient type (elective or emergency), source of the patient (outpatient clinic, emergency department, etc) and patient discharge destination.
- Patient flow in relation to chronic disease
- Patient flow in relation to different times of the year, and
- Patient flow in relation to funding or resource allocation data (DRGs and
  service related groups (SRGs)).

Investigations into some of these areas of research have already commenced. For example, Harrison, Shafer and Mackay (2005) have reported on the creation of a compartmental flow model that is based upon patient cohorts created on the date of patient admission. This approach is perhaps too complex for adoption in a routine decision-making environment, because the methodology relied upon models for each day of the week (that is, the number of model parameters was relatively large making manipulation in a business setting a difficult task). Nevertheless, the approach demonstrated the ability to successfully examine patient occupancy on the basis of date of admission, as opposed to examining occupancy on the basis of date, and there may be potential to look at the rate of patient flow for similar patient groups across the year using this methodology. Such analysis may provide insight into whether similar patients are treated differently at different times of the year. For example, it could be used to answer the question of whether the rate of patient flow is faster during winter when the demand for beds is greater compared to other times in the year.

Bayesian belief networks were implemented by Marshall, McClean and Millard (2004) as a means of predicting differences in occupancy based upon various factors, such as age and sex, prior to admission. Although this work was instigated independently of my research, my research nevertheless, supports this as an area of further research. Indeed, there is significant potential to undertake further research in this area, particularly as the population ages and chronic diseases drives up the cost of care.
In association with others from the Department of Health I have also commenced a preliminary study of how compartmental flow models may be used in the evaluation of service increases associated with chronic diseases, such as diabetes, and population change.

The ability to link forecasting and sensitivity and simulation analysis to such work should increase researcher interest in this modelling approach. Furthermore, the necessary data for such analyses is now readily available within most health care agencies and the cost of analysis is not great. For example, in Australia there are now organisations that are prepared to provide researchers with access to such data from a wide variety of hospitals for free.

13.2.2 Research beyond the hospital inpatient
A further source of research endeavour centres on the incorporation of the compartmental flow modelling approach as a part of a bigger overall model. A bigger model may be beneficial in representing other parts of the hospital setting, such as the emergency department (or accident and emergency) and outpatient services, and also in incorporating aspects of the health and social service sector that operate outside of the hospital setting. The combination of an emergency department model and inpatient bed occupancy flow model is likely to be of significant interest, as the impact of bed blockages is often acutely felt in the emergency department resulting in long waits for admission to an inpatient bed. Whether or not a compartmental flow model could be used to model the emergency department section of a larger model requires investigation. Modelling the combined services, however, is not reliant upon
the emergency department being able to be described with a compartmental flow model. In fact, in the more general sense, where the bed occupancy compartmental flow model is contra-indicated due to the business model used to run a particular aspect of a health or social system, it will be appropriate to use other approaches in conjunction with the bed occupancy compartmental flow model.

The acute care hospital, or indeed any hospital, does not operate separately from the rest of society. The ability to discharge patients from an acute care hospital can be affected by factors outside of the hospital. For example, patients who cannot care for themselves following discharge may rely upon access to community services, such as home care services, or access to a nursing home bed. The ability to secure such services can delay the discharge of patients. Consequently, while modelling may facilitate the discovery of strategies that will improve the operation of the hospital, the successful implementation of the strategies may be contingent upon the continuation, or even change, of external services. Thus, there is every reason to consider the creation of a model that can describe a larger part of the system, assuming that such a model is of value in answering the research questions being investigated. Taylor, McClean and Millard (1996) have already reported on incorporation of the community as part of the geriatric patient flow model. Also, given the work by Millard and his colleagues (for example, Harrison and Millard, 1991) on modelling the geriatric service data, where long-stay patients were admitted for many years, an obvious extension of the research into the compartmental flow model in the Australian setting would be the modelling of the aged care sector, where people may be residents for very long periods of time.
The notion that the measure of time varies in different parts of the system should also be more generally recognised. Thus, the potential to consider extending the compartmental flow model research into areas such as the Emergency Department or Intensive Care Unit, where the duration of patient stay is better measured in hours, also represents a worthwhile area of potential investigation. Rehabilitation services, where patient duration of stay may be considerable, may also represent a worthwhile area of potential investigation in terms of the use of compartmental flow model.

Access to data from different organisations (as opposed to hospital data) and the need to link patient data may make such modelling exercises more difficult to undertake. Linked patient data does exist in the state of Western Australia and subject to particular conditions may be used for such research by researchers from other states of Australia. Even with this data set, however, it is known that much of the data necessary to create some models that would be of interest when modelling a larger part of the health system is not yet collected.

13.2.3 Incorporating information from other modelling

The potential to explore the use of forecasts based upon the use of the multiple method approach, as suggested by Linstone (for example, 1981 and 1999) also exists. Alternative approaches to investigating the effect of decision-making based upon the use of multiple methods may also result in interesting findings.

From an organisational perspective, determining the effect of compartmental flow modelling, or perhaps the interpretation of the output from such modelling, on collaboration between different professional groups or decision-makers may also
generate useful information regarding health sector culture, collaboration and
decision-making.

The potential for further research is not limited to only developing greater insights about how compartmental flow models may be used to facilitate better strategic decisions in relation to the planning and forecasting of hospital beds. Some examples of areas where research may be linked with bed occupancy modelling are now provided. Workforce planning issues are also topical as the population ages. Holmes (2004) has noted that there is little point in planning for additional hospital beds if there are insufficient nurses to staff them. Thus, the linkage between bed modelling and workforce represents an important area of potential research.

From a clinical perspective, patient safety is also of current interest in the health sector. Borg (2003) has reported that the level of bed occupancy is a determinant factor in the incidence of methicillin-resistant Staphylococcus aureus (MRSA) infections within general ward settings. Thus, there may be potential to link bed occupancy modelling to quality and safety issues, such as infection levels. However, before investing in such research it will be important to understand whether occupancy is merely an indicator of risk of an adverse patient outcome (that is, other factors, such as staffing levels, cause the adverse event) or whether it is a controllable factor that directly contributes to adverse patient outcomes.

The notion that reducing the patient length of stay is a good mechanism for controlling cost is a widely held belief in the health sector. Taheri, Butz and Greenfield (2000) suggest that reducing patient length of stay has minimal impact on
the cost of hospital admission. While their study identified that the later days in a patient stay are less expensive than the early days of a patient stay, there was little to suggest that analysis of different patient groups had been undertaken. Thus, being able to link costs to the rate of flow of patients (particularly when discriminating between short and long-stay patients) and other factors, such as age, is important, because policy decisions usually involve cost considerations. Marshall, McClean and Millard (2004) have also suggested that the linkage of costs to patient flow modelling is important.

13.2.4 Beyond the health sector

Exploration of the use of the compartmental flow model of occupancy in other industries may also be worthwhile. For example, the “occupancy profiles” of people provided with supported accommodation in Australia (that is, temporary accommodation for homeless people) or those incarcerated in the prison system may be well described by compartmental flow models.

13.3 Potential for Application

Given the political nature of hospital bed decisions, the ability to improve strategic planning and forecast decision-making and also evaluate past service change would seem desirable, if not a political necessity. Churchill, the former British Prime Minister, stated:

*The most essential qualification for a politician is the ability to foretell what will happen tomorrow, next month and next year, and to explain afterwards why it did not happen.* Winston Churchill (via Cetron and Ralph, 1983) as cited by Armstrong 1985, Pg 322.
I believe that there is real potential for application of the compartmental flow model of occupancy in the health sector and that such application could contribute to improved decision-making.

The following discussion sets out the necessary conditions that I believe are required for the transfer of this research to application to occur.

13.3.1 Scope for Application

The bed occupancy compartmental flow model is not limited to a single use, which enhances its attraction in relation to application. Previous chapters have detailed the range of applications for which bed occupancy compartmental flow models can be applied. Applications include generating data that leads to better understanding about existing systems, forecasting future bed requirement information, the ability to pre-test system change through sensitivity and simulation analysis, benchmarking, evaluation and the potential to influence resource allocation funding models.

13.3.2 Drivers of Adoption

Millard has already found through experience in England that the creation of a “modelling tool” that can be sold is not sufficient to result in the widespread adoption of the compartmental flow model in the health care setting (personal communication).

The necessary factors that I believe to be critical in the widespread adoption of the compartmental flow model are:
• Research to support the validity of the use of the modelling approach in the acute care sector and also in other settings such as the geriatric care service in England (for example, Harrison and Millard, 1991).

• The potential to use the modelling approach to investigate a wide range of strategic issues.

• Communication of the research results through journal publications, academic and most importantly conference presentations and papers.

• A business need – particularly in relation to short-term strategic decision-making around acute care hospital bed use.

• A lack of confidence in, or at least a questioning of, the failure of methods that rely upon the ALOS.

• Creation of a business case that details what is necessary to enable the creation of bed occupancy compartmental flow models so that decision-makers understand that the data demands are not necessarily onerous, and

• Successful adoption of the modelling approach by one or more health care organisations (for example, a health department or large hospital).

Some of these factors have already been or are currently being addressed. For example, research to validate the modelling approach has occurred (as presented in this thesis) and work towards communicating the results has already commenced (for example, see Appendix I). Some, however, may argue that additional research is required. While there is merit in undertaking further research, and potential areas of further research have been identified (see earlier in this chapter), it is my opinion that transfer to the applied setting should occur concurrently.
Recognition of the need to be able to forecast future bed requirements is also evident. For example, the findings in the reports of the NSW Health Council (2000), Generational Health Review (Generational Health Review, 2003) and Western Australian Government (Department of Health, 2004) all forecast increases in future bed requirements.

The lack of questioning of the use of the ALOS by decision-makers and the early adoption of the modelling by an organisation represent the greatest challenges in achieving the widespread adoption of the modelling approach.

13.3.3 Barriers to Adoption

It is useful to consider Michael Crichton’s observations about human decision-making in complex environments:

*The total system we call the biosphere is so complicated that we cannot know in advance the consequences of anything we do. … This uncertainty is characteristic of all complex systems, including man-made systems.*

*…That is why even our most enlightened past efforts have had undesirable outcomes – either because we did not understand enough, or because the ever-changing world responded to our actions in unexpected ways. … The fact that the biosphere responds unpredictably to our actions is not an argument for inaction. It is, however, a powerful argument for caution, and for adopting a tentative attitude toward all we believe, and all we do. …We think that we know what we are doing. We have always thought so.*

*(Crichton, 2002, pg vii-ix).*
These observations hold true in relation to strategic decision-making relating to the health care system and is particularly true in relation to the use of the ALOS. The lack of questioning of the ALOS is perhaps an outcome that has arisen because of two factors: a failure to educate decision-makers about the use of the average in general, and the inability to provide a reasonable alternative. The latter is now addressed as the flow model parameters provide good alternative measures that overcome the flaws of the ALOS.

The need to communicate the flaws of the ALOS, however, will be a task that will require a significant investment in time. In the short-term, communication of the problems associated with using the ALOS may be best presented via conference and journal papers. In the longer-term, it may be desirable to seek that better or additional information about basic statistics is included in courses commonly undertaken by health care-managers.

The need for education of decision-makers about the shortcomings of the ALOS may be reduced if an early adopter of the modelling approach can be found. Successful adoption of the modelling approach by a large organisation may be sufficient to establish interest by other organisations. This assumes that the early adopter reports favourably on the use of the approach or indeed even champions its use. However, given that I have previously identified that Hass (2004) reported that there is significant under-investment in health services research in Australia finding an earlier adopter of the technology may be a difficult task as there may not be people who are sufficiently skilled to advocate its uptake.
There are a number of strategies that may further assist the uptake of the methodology in the business setting and these are now discussed.

13.3.4 Possible Solutions

Not Overselling the Approach

I have found that health care managers tend to be cynical about the potential of modelling approaches to help them address their problems. One source of frustration has arisen from the fact that advocates of modelling tend to sell their solutions, as opposed to attempting to address the problem. This perhaps is borne out by Fone et al. (2003) who have reported that there is a lack of evaluation of modelling and understanding of whether models are implemented.

In terms of the bed occupancy compartmental flow model it is important that it is used in relation to strategic decision-making and not short-term operational decision-making. Consequently, this modelling approach is not suitable for forecasting the number of beds required for the next hour, day, week or month.

Acknowledgment of Complexity

In the discussion in Chapter 12, it was acknowledged that the modelling approach could not exist in isolation and that the decision-making occurred in a complex environment. Rather, the purpose of the modelling tool was to assist decision-makers improve their strategic decision-making in relation to hospital bed planning and use. The information derived from such modelling, however, can provide the mechanism to bring those involved in such decision-making together. Acknowledgment of the
complexity of the decision-making environment and role of the modelling approach will be critical in establishing the credibility of this modelling approach.

**Joint Research**

While health related research funding (for example, National Health and Medical Research Council funding) may be difficult to obtain, as traditionally health services research has not been well funded in Australia (for example, Hass, 2004), other funding sources may be of use in facilitating adoption of the modelling tool in the applied setting. The Australian Research Council provides grants that enable industry and academics to undertake research projects that will provide a benefit to industry. Successful application for such a research grant with either a public or private hospital partner may encourage adoption of the research tool, because it would reduce the financial risk and provide confidence about the usefulness of the approach using the hospitals own data.

Another mechanism less reliant upon grant funding may be the offer of free use (that is, a trial) of the modelling tool for a limited period in exchange for commendation to other potential users.

**Need for Software Tools**

While Millard found that the development of a modelling tool – BOMPS – did not assist in the uptake of the compartmental flow model, it is nevertheless a factor that will facilitate transfer of the research into the applied setting.
While academic researchers may prefer to conduct their research with more specialised software, unless the compartmental flow model can be created easily using readily available software, the likelihood of widespread uptake of compartmental flow modelling is unlikely to be great. The work presented in Chapter 11 showed that sensitivity and simulation analysis could be undertaken using Microsoft® Excel. Attempts have also been made by Rae to create the compartmental flow model using Microsoft® Excel (personal communication).

Whether the modelling tool should be created using Microsoft® Excel or some other approach is perhaps a moot point. In terms of seeking a commercial reward from the application of the modelling approach this is unlikely to come from the development of the software, but rather the provision of expert analysis and transfer of skills to end users. Indeed, the tension about a readily available tool and the need for skilled users is significant and is perhaps a key to the successful transfer of this research to the applied setting.

**The Need for Documentation**

A critical requirement for the successful transfer of the bed occupancy compartmental flow modelling approach to the business setting will be the provision of documentation. This requirement will exist irrespective of the transfer approach chosen. There are at least two documents that will be required to be developed, namely a business plan and an information prospectus. It is not the purpose of this thesis to fully outline the material that would be contained in these documents, but rather to identify the need for their existence.
The information prospectus is a document that is designed to provide information about the modelling approach to potential users of the application (and others). This documentation will assist interested parties to see whether the application may be of benefit to them. It is not expected that the documentation alone will be sufficient to encourage people to adopt the modelling approach, but rather will assist other communication methods, such as presentations.

The business plan is a document that sets out the path or paths for transferring the application to the business setting. The development of such a plan is particularly important in identifying aspects of additional development, financial considerations, possible business structures and risks.

13.4 Conclusion

The potential to expand the research base regarding the use of compartmental flow models of bed occupancy in hospitals, and even in wider social care setting contexts, exists. Much of the research can use existing data and thus can be undertaken at relatively low cost. Given the history of funding health service research in Australia (Hass, 2004), this may be beneficial. More complex modelling, particularly where modelling attempts to represent activity beyond the hospital, will involve challenges in gaining access to the appropriate data.

The potential to transfer the research findings into practice resulting in improved strategic decision-making is, perhaps, a more significant reason to further the evidence base regarding the use of this modelling approach. Transferring the modelling methodology on a commercial basis, however, will not be an easy task, as
evidenced by the failure of the BOMPS package to become established as a commercial software package. While an appropriate transfer plan is yet to be developed, critical features of such a plan were identified.

In the next chapter I present an overall conclusion regarding the research undertaken for this thesis and comment upon the contribution made to knowledge by this work.
Chapter 14

Conclusions – The Beginning of a Longer Journey

In this chapter I reflect upon the journey from the initial problem with hospital bed management to the conclusion of this research. I also highlight the contribution made towards knowledge as a consequence of the research undertaken for this thesis. The chapter has the following structure:

14.1 Introduction ..........................................................................................427
14.2 The problem is better understood ..........................................................427
14.3 Final comments and observations..........................................................430
14.1 Introduction

In the first chapter of this thesis I provided an account of the personal motivation for undertaking this research (see section 1.9). My involvement in this research stemmed from the identification of the gap in strategic management tools that were available to facilitate better decision-making in relation to hospital bed management.

In this final chapter I will reflect upon the original purpose of my research, the outcomes and present some concluding comments.

14.2 The problem is better understood

As I have indicated many times during this thesis, the work of Harrison and Millard (1991) formed the basis for this research. The research was structured to address a number of aims (see section 1.11) and this has been achieved (see earlier chapters, particularly the Chapter 12, the Discussion).

It is my opinion that had this work been undertaken in almost any other large industry, it is likely that:

- Significant investment would have occurred to better understand the problem
- Investment in operational research methodologies would have played a role in such work, and
- That this work would have occurred earlier – perhaps during the early 1990s.

This is not to say that investment in the management of bed problems has not occurred. There are now “patient flow managers” and funds have been allocated to various projects to better manage demand or avoid demand. These efforts are,
however, usually at the operational end of the decision-making spectrum, and are subject to the vagaries of the funding cycles and often lack the rigorous evaluation of pilot or demonstration projects, making it difficult to determine if the outcomes achieved were as good as suggested prior to implementation and that the return on the investment is worthwhile. However, little has been done to improve the strategic decision-making abilities of the health system in relation to hospital bed management. Without appropriate strategic decision-making tools, I suggest that the implementation of bed management options at the operational end of the hospital bed management problem spectrum will continue to be difficult and not lead to the outcomes sought by politicians and desired by communities.

Despite my lament regarding the under investment in health services research in Australia, which is supported by evidence (Hass, 2004), much has been achieved given the level of funding for this work, including:

- The modification of the Harrison and Millard (1991) model
- Research concerning the number of data to use when generating a compartmental flow model
- Incorporation of model selection techniques
- Incorporation of seasonal variation
- Linkage to population change
- Consideration of vacancy levels
- Application to casemix and benchmarking
- Consideration of how variation in the data may be captured
- Incorporation of additional sensitivity analysis
• Incorporation of simulation analysis as a means of highlighting the uncertainty associated with model results
• Generation of research papers
• Invited presentations at industry seminars and conferences, and
• Presentation of papers at academic conferences both locally and internationally.

The above highlights the contribution towards knowledge generated as a consequence of the research activities undertaken for this thesis. Additionally, I have sought to increase the knowledge of others through:

• The convening of the several local seminars and workshops that were attended by Australian researchers and Millard, and
• The convening of the inaugural international conference on health and social care modelling with Millard that was held in Adelaide.

The successful convening of the inaugural international conference on health and social care modelling perhaps represents a sign that this field of work is gaining momentum and recognition. Of course, the case for not using the ALOS for decision-making has also been promulgated.

Thus, while there is still much more research that can be done (as suggested in the previous chapter) and regular implementation of the bed occupancy compartmental modelling methodologies has not yet been achieved, my understanding of the initial problem that led to this research is now much greater. Furthermore, the understanding of bed modelling and strategic decision-making by those who have
been following the published works of myself and Millard and his colleagues should
also have increased, thereby making a contribution to the world’s knowledge base.

14.3 Final comments and observations

In recent communication with Millard (February 2007), he noted that:

Together we are changing the world - a bit like a stalagmite - ever so

slowly growing.

The completion of my research marks the end of the studies towards my doctorate.
The need for such research is unlikely to abate in the next few years, particularly as
we enter a period of declining workforce numbers and increased demand for hospital
beds.

My involvement in continuing to contribute to the development of improved strategic
decision-making approaches in relation to hospital bed management will continue,
subject to opportunity, and in many ways this marks the beginning of the next part of
the journey.
References

Academy Health. What is health services research? http://www.academyhealth.org/about/whatisshr.htm [accessed 4 October 2006]


http://www.abs.gov.au/AUSTATS/abs@nsf/Previousproducts/A2CD8D204F84E882CA2470EC001117a1?opendocument accessed on March 2006.


Bellin E and Kalkut G (2004). Is Time-Slice Analysis Superior to Total Hospital Length of Stay in Demonstrating the Effectiveness of a Month-long Intensive Effort on a Medicine Service? *Quality management in Health Care*, 13:(2) 143-149.


Flinders Medical Centre (2002). *Flinders Medical Centre Annual Report 2001-02*, Adelaide (South Australia).

Flinders Medical Centre (2006). Flinders Medical Centre Corporate Brochure. 


Strategic Planning Directorate (2004). *Projecting Demand For WA Hospital Inpatient Activity – Methodology*. Department of Health, Western Australia.


APPENDICES

Appendix I: List of academic publications and activities relating to hospital bed and health care modelling

Appendix II: BOMPS Formulae
Appendix I

List of academic publications and activities relating to hospital bed and health care modelling

The purpose of this appendix is to demonstrate my involvement in the generation of research articles and involvement in related research activities. This appendix has the following structure:

Refereed Journal Articles ............................................................................................ 433
Letters ........................................................................................................................ . 433
Conference Papers (Abstracts Refereed)...................................................................... 434
Invited Presentations ................................................................................................... 434
Nosokinetic News – A Health Care Modelling News Letter......................................... 435
Collaborations ............................................................................................................. 435
Convenor of Seminars and Conferences ................................................................. 435
Attracted International Guests ..................................................................................... 436
International Sponsorship ............................................................................................ 436
Editorials


Refereed Journal Articles


Letters


Refereed Conferences Papers (published in proceedings)


Conference Papers (Abstracts Refereed)


Invited Presentations

Royal Melbourne Hospital 2000

Department of Human Services (Vic) 2001

Melbourne University 2005

LaTrobe University 2005


Nosokinetic News – A Health Care Modelling News Letter


Collaborations

Founding member of the Nosokinetics Group – an international collaboration of health care modellers focussing on compartmental models relating to patient flow. This includes published research with:

- Professor Peter Millard (UK)
- Professor Gary Harrison (USA)
- Professor Florin Gorunescu (Romania)

Additionally, research relating to health services in Dunedin, New Zealand, has been undertaken in collaboration with Dr Brendon Rae.

Collaboration with Prof Terry Mills and colleagues from LaTrobe University, Victoria, regarding patient flow modelling and health services research.

Collaboration with Prof Mills, Prof Don Campbell (Monash), Prof Gary Harrison (USA), Prof Sally McClean (Northern Ireland), Dr Geoff McDonnell (UNSW), Dr M Faddy (QUT), Assoc Prof Peter Sprivulis (Dept of Health and UWA), Assoc Prof Drew Richardson (ANU Medical School), Dr Dan Navarro (Adelaide), Dr A van Deth (Flinders) seeking NHMRC funding for bed modelling and related service investigation funding.

Convenor of Seminars and Conferences

Australian Convenor of the International Health and Social Care Modelling Conference being held in Adelaide 2006.

April 2002 - Workshop held at University of Adelaide regarding health care modelling. Included interstate and local participants and one overseas guest.
October 2002 - Workshop held at University of Adelaide regarding health care modelling. Included interstate and local participants and one overseas guest.

**Attracted International Guests**

Professor Peter Millard – April and November 2002 to address the Department of Human Services and be involved in workshops

Professor Sally McClean to speak at Adelaide University and Flinders Medical Centre

**International Sponsorship**

The Novartis Foundation offered funding to me so that I could attend a conference relating to bed modelling that was held in England during 1999.
Appendix II

BOMPS Formulae

The purpose of this appendix is to detail the formulae used in BOMPS. The material was put together by Mrs Georgina Christodoulou when she worked with Professor Peter Millard. The appendix has the following structure:

The exponential equations ................................................................. 438
  One compartment exponential equation ......................................... 438
  Two compartment exponential equation ....................................... 438
  Three compartment exponential equation ..................................... 438
Actual number ............................................................................... 439
Admissions per day (p/d) ............................................................... 439
Conversion rate (c/p/d) ................................................................. 439
Release rate (r/p/d) ........................................................................ 440
Number in each group ................................................................. 441
Release rate (p/d) .......................................................................... 442
Conversion rate (c/d) ................................................................. 442
Derived total ................................................................................ 443
Admission rate per day (a/p/d) ....................................................... 443
Fraction of beds occupied .......................................................... 443
Expected length of stay ............................................................... 444
Half-life ..................................................................................... 445
Rehabilitation benefit ................................................................... 446
Rehabilitation percentage ............................................................ 447
Percentage of admissions leaving ............................................... 447
**THE EXPONENTIAL EQUATIONS**

Where:

\[ y = \text{the number of patients that have been in the system for } x \text{ days} \]

\[ e = \text{exponential constant} = 2.7182818 \ldots \]

\[ x = \text{the length of stay in the system} \]

\[ A = \text{the calculated number of inpatients in the first sector of the system} \]

\[ B = \text{the rate of change in the first sector of the system} \]

\[ C = \text{the calculated number of inpatients in the second sector of the system} \]

\[ D = \text{the rate of change in the second sector of the system} \]

\[ E = \text{the calculated number of inpatients in the third sector of the system} \]

\[ F = \text{the rate of change in the third sector of the system} \]

\[ G = \text{the constant that is added to the equations below. It represents the data that does not follow the calculated rates: these are described as bed-blockers} \]

**One Compartment Exponential Equation**

\[ y = Ae^{-Bx} \]  
\[(i)\]

**Two Compartment Exponential Equation**

\[ y = Ae^{-Bx} + Ce^{-Dx} \]  
\[(ii)\]

**Three Compartment Exponential Equation**

\[ y = Ae^{-Bx} + Ce^{-Dx} + Fe^{-Fx} \]  
\[(iii)\]

The co-efficients of these models are calculated when the curve is fitted to the data set. It is these co-efficients (A, B, C, D, E, F & G if a constant is used) that will be used to derive the following information that is presently in the results table created by the BOMPS program.
**Actual Number**

This is the actual number of residents/patients that are in the data set. This must be a count function within the BOMPS program.

**Admissions Per Day (P/D)**

This will be denoted as $A_0$:

$$A_0 = \left(1 - e^{-B}\right)^* A$$  

The co-efficients $A$ and $B$ are obtained from equation (i).

$$A_0 = \left(\left(1 - e^{-B}\right)^* A\right) + \left(\left(1 - e^{-D}\right)^* B\right)$$  

The co-efficients $A$, $B$, $C$ and $D$ are obtained from equation (ii).

$$A_0 = \left(\left(1 - e^{-B}\right)^* A\right) + \left(\left(1 - e^{-D}\right)^* C\right) + \left(\left(1 - e^{-F}\right)^* E\right)$$  

The co-efficients $A$, $B$, $C$, $D$, $E$ and $F$ are obtained from equation (iii).

Note: If the release rate ($r_i$) and the ‘number in the group’ ($N_i$) are known then the following equation can be used:

$$A_0 = \sum_{i=1}^{1,2,3} r_i N_i$$  

**Conversion Rate (C/P/D)**

Denoted as $v_i$:

$$v_i = \left(\frac{1 - e^{-B}}{A_0}\right) \left(\left(1 - e^{-B}\right) - \left(1 - e^{-D}\right)\right)$$  

The co-efficients $B$, $C$ and $D$ are obtained from equation (ii) and the parameter $A_0$ from (1.2).
\[
v_1 = \left(\frac{1-e^{-D}}{A_0}\right)C - \left(\frac{E(1-e^{-F})}{(1-e^{-B})-(1-e^{-D})}\right)\left(\frac{(1-e^{-B})-(1-e^{-D})}{(1-e^{-B})-(1-e^{-D})}\right)\left(\frac{(1-e^{-B})-(1-e^{-D})}{(1-e^{-B})-(1-e^{-D})}\right) \text{three compartment model (2.1.2)}
\]

\[
v_2 = \frac{E(1-e^{-F})}{A_0}(1-e^{-D})-(1-e^{-F})\left(\frac{(1-e^{-B})-(1-e^{-D})}{(1-e^{-B})-(1-e^{-D})}\right) \text{three compartment model (2.2)}
\]

The co-efficients B, C, D, E and F are obtained from equation (iii) and the parameter \( A_0 \) from (1.3).

**RELEASE RATE \((r_i/P/D)\)**

Denoted as \( r_i \):

\[
r_i = (1-e^{-B}) - v_1 \quad (3.1)
\]

\[
r_2 = (1-e^{-D}) - v_2 \quad (3.2)
\]

\[
r_3 = (1-e^{-F}) \quad (3.3)
\]

The co-efficients of these equations depends on the type of model being used. For example, if a triple compartment model were being used then the co-efficients B, D and F would be obtained from equation (iii), the parameter \( v_1 \) from equation (2.1.2) and the parameter \( v_2 \) from equation (2.2).
**NUMBER IN EACH GROUP**

Denoted as $N_i$:

$$N_1 = \frac{A_0}{r_1^i} \quad \text{one compartment model (4.1.1)}$$

Parameter $A_0$ obtained from equation (1.1) and parameter $r_1$ obtained from equation (3.1).

$$N_1 = \frac{A_0}{(r_1 + v_1)} \quad \text{2&3 compartment model (4.1.2)}$$

Parameter $A_0$ obtained from either equation (1.2) or (1.3) depending on the type of model being used. Parameter $v_1$ obtained from either equation (2.1.1) or (2.1.2) and parameter $r_1$ obtained from equation (3.1).

$$N_2 = \frac{A_0 v_1}{(r_1 + v_1)(r_2 + v_2)} \quad \text{two compartment model (4.2.1)}$$

Parameter $A_0$ is obtained from equation (1.2). Parameter $v_1$ is obtained from equation (2.1.1), parameter $r_1$ is obtained from equation (3.1) and parameter $r_2$ is obtained from equation (3.2).

$$N_3 = \frac{A_0 v_1}{(r_1 + v_1)(r_2 + v_2)} \quad \text{three compartment model (4.2.2)}$$

Parameter $A_0$ is obtained from equation (1.3). Parameter $v_1$ is obtained from equation (2.1.2), parameters $r_1$ and $r_2$ are obtained from equations (3.1) and (3.2), respectively.

$$N_3 = \frac{A_0 v_1}{(r_1 + v_1)(r_2 + v_2)} \quad \text{three compartment model (4.3)}$$
Parameter $A_0$ is obtained from equation (1.3). Parameters $v_1$ and $v_2$ are obtained from equations (2.1.2) and (2.2), respectively. Parameters $r_1$, $r_2$ and $r_3$ are obtained from equations (3.1), (3.2) and (3.3), respectively.

**RELEASE RATE (P/D)**

Denoted as $R_i$:

$$R_i = N_i r_i, \quad \text{where } i = 1, 2, 3 \quad (5.1)$$

Parameter $N_1$ is obtained from equation (4.1.1) or (4.1.2) depending on which model is being used and parameter $r_1$ from equation (3.1).

Parameter $N_2$ is obtained from equation (4.2.1) or (4.2.2) depending on which model is being used and parameter $r_2$ from equation (3.2).

Parameter $N_3$ is obtained from equation (4.3) and parameter $r_3$ from equation (3.3).

**CONVERSION RATE (C/D)**

Denoted as $V_i$:

$$V_i = N_i v_i, \quad \text{where } i = 1, 2 \quad (6.1)$$

Parameter $N_1$ is obtained from equation (4.1.1) or (4.1.2) depending on which model is being used and parameter $v_1$ from equation (2.1.1) or (2.1.2) depending on which model is being used.

Parameter $N_2$ is obtained from equation (4.2.1) or (4.2.2) depending on which model is being used and parameter $v_2$ from equation (2.2).
**DERIVED TOTAL**

Denoted as T:

\[ T = \sum_{i=1}^{3} N_i \]  

Parameter \( N_1 \) obtained either from the equation (4.1.1) or (4.1.2).

Parameter \( N_2 \) obtained either from the equation (4.2.1) or (4.2.2).

Parameter \( N_3 \) obtained from the equation (4.3).

**ADMISSION RATE PER DAY (A/P/D)**

Denoted as \( a_0 \):

\[ a_0 = \frac{A_0}{T} \]  

(8.1)

\( A_0 \) is obtained from the equation (1.1), (1.2) or (1.3) depending on the compartment model being used.

**OR**

For the one compartment model the Admissions per day is equal to the release rate per day because the system is assumed to be in equilibrium.

\[ a_0 = r_1 (r_1 + v_r)/(r_2 + v_r) \]  

two compartment model  

(8.2)

**FRACTION OF BEDS OCCUPIED**

Denoted by \( b_i \):

Note: for the single compartment model this is set to 100 per cent.

\[ b_i = r_2/(r_2 + v_r) \times 100 \]  

2 & 3 compartment model  

(9.1.1)

**OR**
\[ b_1 = \frac{N_1}{T} \quad \text{2 & 3 compartment model (9.1.2)} \]

\( r_2 \) is obtained from equation (3.2) and \( v_1 \) obtained either from equations (2.1.1) or (2.1.2) depending on the type of model being used.

\( N_1 \) is obtained either from equation (4.1.1) or (4.1.2) and \( T \) is obtained from equation (7.1).

\[ b_2 = \frac{N_2}{T} \quad \text{2 & 3 compartment model (9.2.2)} \]

\( r_2 \) is obtained from equation (3.2) and \( v_1 \) is obtained either from equations (2.1.1) or (2.1.2) depending on the type of model being used.

\( N_2 \) obtained either from equation (4.2.1) or (4.2.2) and \( T \) obtained from equation (7.1).

\[ b_3 = \frac{N_3}{T} \quad \text{2 & 3 compartment model (9.1.2)} \]

\( N_3 \) is obtained from equation (4.3) and \( T \) is obtained from equation (7.1).

**EXPECTED LENGTH OF STAY**

Denoted as \( L_i \):

\[ L_i = \frac{1}{r_1} \quad \text{one compartment model (10.1.1)} \]

\[ L_i = \frac{1}{(r_1 + v_1)} \quad \text{2 & 3 compartment model (10.1.2)} \]

\( r_1 \) is obtained from equation (3.1) and \( v_1 \) is obtained either from equation (2.1.1) or (2.1.2) depending on the model being used. \( B \) is obtained either from equation (i), (ii) or (iii).
\[ L_2 = \frac{1}{r_2} \quad \text{OR} \quad L_i = \frac{1}{D} \quad \text{two compartment model (10.2.1)} \]

\[ L_2 = \frac{1}{(r_2 + v_2)} \quad \text{three compartment model (10.2.2)} \]

\( r_2 \) is obtained from equation (3.2) and \( v_2 \) is obtained from equation (2.2). \( D \) is obtained either from equation (ii) or (iii).

\[ L_3 = \frac{1}{r_3} \quad \text{OR} \quad L_i = \frac{1}{F} \quad \text{three compartment model (10.3)} \]

\( r_3 \) is obtained from equation (3.3) and \( v_2 \) is obtained from equation (2.2). \( F \) is obtained from equation (iii).

The total length of stay, for all patients is denoted by \( TL \):

\[
TL = \left( \frac{1}{(r_1 + v_1)} \right) + \left( \frac{v_1}{(r_1 + v_1)(r_2 + v_2)} \right) + \left( \frac{1}{r_2} \right) \quad \text{two compartment model (10.4.1)}
\]

\( r_1 \) is obtained from equation (3.1), \( r_2 \) is obtained from equation (3.2) and \( v_1 \) is obtained from equation (2.1.1).

\[
TL = \left( \frac{1}{(r_1 + v_1)} \right) + \left( \frac{v_1}{(r_1 + v_1)(r_2 + v_2)} \right) + \left( \frac{v_1 v_2}{(r_1 + v_1)(r_2 + v_2)r_3} \right) \quad \text{three compartment model (10.4.2)}
\]

\( r_1 \) is obtained from equation (3.1), \( r_2 \) is obtained from equation (3.2), \( r_3 \) is obtained from equation (3.3), \( v_1 \) is obtained from equation (2.1.2) and \( v_2 \) is obtained from equation (2.2).

**HALF-LIFE**

Denoted by \( h_i \):

\[
\]
\[ h_1 = \frac{\ln(\frac{1}{2})}{\ln(\frac{1}{r_1})} \quad \text{one compartment model } \quad (11.1.1) \]

\[ h_2 = \frac{\ln(\frac{1}{2})}{\ln(\frac{1}{r_2})} \quad \text{two compartment model } \quad (11.2.1) \]

\[ h_3 = \frac{\ln(\frac{1}{2})}{\ln(\frac{1}{r_3})} \quad \text{three compartment model } \quad (11.3) \]

\[ r_1 \text{ is obtained from equation (3.1) and } v_1 \text{ is obtained either from equation (2.1.1) or (2.1.2).} \]

\[ r_2 \text{ is obtained from equation (3.2) and } v_2 \text{ is obtained from equation (2.2).} \]

\[ r_3 \text{ is obtained from equation (3.3).} \]

**Rehabilitation Benefit**

Denoted by \( R_B \):

Note: The rehabilitation benefit for Group 1 in all the compartment models is set to 1.

\[ R_B = \frac{r_1 + v_1}{r_2 + v_1} \quad \text{Group 2: two compartment model } \quad (12.1.1) \]

\[ R_B = \frac{1}{(1 - e^{-\sigma}) + (1 - e^{-\sigma})r_3} \quad \text{Group 2: three compartment model } \quad (12.1.2) \]

\[ r_3 \text{ is obtained from equation (3.3), } D \text{ is obtained from equation (iii) and } TL \text{ is obtained from equation (10.4.2).} \]
Group3: three compartment model (12.2)

\[ r_3 \text{ is obtained from equation (3.3) and TL is obtained from equation (10.4.2).} \]

**Rehabilitation Percentage**

Denoted by RP₁:

Note: Group1 in a single compartment model, Group2 in a double compartment model and Group3 in a triple compartment model is set to 100 per cent, respectively.

\[ RP_1 = \left( \frac{r_1}{r_1 + v_1} \right) \times 100 \quad \text{Group1: 2&3 compartment model (13.1)} \]

\[ \text{r}_1 \text{ obtained from equation (3.1) and v}_1 \text{ is obtained either from equation (2.1.1) or (2.1.2).} \]

\[ RP_2 = \left( \frac{r_2}{r_2 + v_2} \right) \times 100 \quad \text{Group2: 3 compartment model (13.2)} \]

\[ \text{r}_2 \text{ is obtained from equation (3.2) and v}_2 \text{ is obtained either from equation (2.2).} \]

**Percentage of Admissions Leaving**

Denoted by APᵢ:

\[ AP_1 = RP_1 \quad \text{Group1: All compartment model (14.1)} \]

\[ AP_1 = 100 - AP_1 \quad \text{Group2: 2 compartment model (14.2.1)} \]

\[ AP_1 \text{ obtained from equation (14.1).} \]

\[ AP_2 = \left( \frac{100 - RP_1}{100} \right) RP_2 \quad \text{Group2: 3 compartment model (14.2.2)} \]

\[ RP_1 \text{ is obtained from equation (13.1) and RP}_2 \text{ is obtained from equation (13.2).} \]
\[ AP_3 = \frac{(100 - RP_2) \cdot AP_2}{100} \quad \text{(14.3)} \]

Group 3: 3 compartment model

RP₂ obtained from equation (13.2) and AP₂ obtained from equation (14.2.2).