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AN ANALYTICAL SOLUTION FOR THE TRANSIENTS IN A PIPELINE WITH A VARIABLE BOUNDARY CONDITION: LEAK DETECTION IN PIPE NETWORKS USING CODED TRANSIENTS

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Abstract

A time-domain analytical solution for the transients in a pipeline with a small leak has been developed under variable boundary conditions. The solution shows that the presence of a small leak reduces the amplitudes of the resonant frequencies while its influence on the non-resonant frequencies is negligible. The amplitudes of the resonant components are a function of input signal, measurement locations, damping factor of the pipeline and leak size and location. As pipe section in a water distribution network can be considered as a single pipe with two variable boundary conditions at both ends. A new leak detection technique, which is able to detect and locate leaks in pipe networks, has been developed based on the analytical solution. The application of the new leak detection method is similar to the widely used acoustic leak detection techniques involving two monitoring locations across a leak. The new leak detection technique uses fully controlled and purposely generated fluid transient signals—coded transients. Compared to the acoustic based leak detection methods, the proposed method may be applied with a much greater measurement interval and is less influenced by the background noises.

Key words:

Leak detection, pipe network, coded transients, pipeline.

1. INTRODUCTION

Increasing water demand the costs associated with water treatment and supply have been forcing water authorities to reduce leakage in the water distribution systems (WDS). Many leak detection methods have been proposed. The leak detection methods based on fluid transients have shown some promise in terms of both quick response location (Liggett and Chen 1994, Liou and Tian 1995, Silva et al. 1996, Liou 1998, Vítkovský et al. 1999, Mpesha et al. 2001, Wang et al. 2002, Ferrante and Brunone 2003, Kapelan et al 2003, Covas et al. 2005, Lee et al. 2005). However, most of the fluid transient based leak detection techniques are based on either numerical simulations or experimental results for simple single pipelines. Theoretically, the inverse transient method (ITM) can be used to detect and locate the leaks in a WDS given sufficient measurement data using the inverse transient method (Liggett and Chen, 1994). However, a number of field tests have shown that there are still some difficulties in applying this transient leak detection technique in a WDS with a reasonable scale (Stephens et al. 2004). The modelled transients are not accurate enough compared to the measured transients for the purpose of leak detection.

In order to improve the performance of the leak detection in a WDS, the ITM was applied to a small section of a large WDS using the measured pressure transients at the boundaries of the selected section as shown in Fig 1. It was found that the modelled transient pressure heads using the measured transients as the boundary conditions are almost insensitive to the leak (Vítkovský 2002). As a result, it was suggested

that independent boundary conditions (e.g. valve conditions) be used as the boundary conditions in the ITM. This phenomenon has been further explored in this paper. A linear analytical solution is developed for the transients in a pipeline with a leak under a variable boundary condition. A sinusoidal input boundary perturbation has been applied and this enables an analytical solution expressed as a Fourier series solution to be obtained. For a general transient perturbation, the signal can be decomposed into a series of sinusoidal functions. Based on the analytical solution, a new technique to detect leaks in WDS using coded transients is proposed. The proposed method has been applied in a small network used in Liggett and Chen (1994).

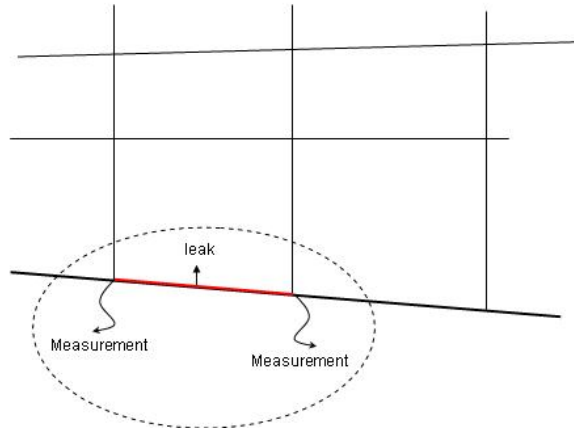


Figure 1 Leak detection using measured transient pressures as the boundary condition

2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

A dimensionless partial differential equation (PDE) for pressure transients in a pipeline including a small leak as shown in Figure 2 can be expressed as (Wang et al. 2002)

$$\frac{\partial^2 h^*}{\partial x^{*2}} = \frac{\partial^2 h^*}{\partial t^{*2}} + [2R + F_L \delta(x^* - x_L^*)] \frac{\partial h^*}{\partial t^*} \quad (1)$$

in which $h^* = (H - H_0)/H_1$ = the dimensionless head of the transient, H = transient head, H_0 = steady-state head, H_1 = a reference head at a tank, $x^* = x/L$ = dimensionless distance, $t^* = t/(L/a)$ = the dimensionless time, L = pipeline length, a = wave speed, $R = \frac{fLQ_0}{2DAa}$ = resistance term, Q_0 = steady-state flow rate, f = Darcy-Weisbach friction factor, D = pipe diameter, A = pipe cross sectional area, $F_L = \frac{C_d A_L a}{A \sqrt{2gH_{L0}}}$ = leak parameter, A_L = leak area, C_d = leak discharge coefficient, H_{L0} = steady-state head at the leak. $\delta(x^* - x_L^*)$ = Dirac delta function and $x_L^* = x_L/L$ = dimensionless leak location.

Assuming steady state conditions at $t^* = 0$ in the pipeline gives the initial conditions as

$$h^*(x^*, 0) = 0 \text{ and } \frac{\partial h^*(x^*, 0)}{\partial t^*} = 0 \quad (2)$$

Assuming a constant upstream reservoir and a sinusoidal perturbation at the downstream reservoir as shown in Figure 2, the boundary conditions for the problem are

$$h^*(0, t^*) = 0, \text{ and } h^*(1, t^*) = f(t^*) = E^* \sin(\omega t^*) \quad (t^* > 0) \quad (3)$$

in which $E^* = E/H_1 =$ dimensionless amplitude of the perturbation, $E =$ amplitude of the perturbation, and $\omega = 2\pi/(L/a) =$ angular frequency of the perturbation. The assumed sinusoidal reservoir perturbation in Eq. (3) is for the derivation of the analytical solution. A more practical transient generation method will be discussed in a later section.

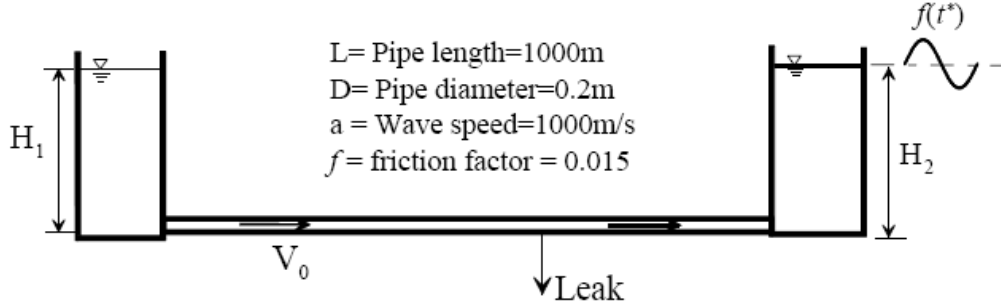


Figure 2 A pipeline connected to a constant reservoir and a downstream reservoir with a varying head

Eq. (1) assumes that pipe friction during a transient event is described by a constant steady-state Darcy-Weisbach friction factor. The main objective in considering the solution to this set of governing equations is to understand the influence of a leak on a transient event. The effects of the unsteady friction can be introduced into the solution once the governing equations have been developed based on the same approach as for the leaks by Wang et al. (2002). In addition, application of Dirac delta function has assumed that the leak size is small compared to the pipe size ($C_d A_L/A < 0.01$).

3. A FOURIER SERIES SOLUTION

The general solution for the problem expressed by Eqs. (1), (2) and (3) is (Wang 2002)

$$h^*(x^*, t^*) = \sum_{n=1}^{\infty} e^{-(R+R_{nL})t^*} [A_n \cos(n\pi x^*) + B_n \sin(n\pi x^*)] \sin(n\pi x^*) + x^* E^* \sin(\omega t^*) + \sum_{n=1}^{\infty} [A_{np} \cos(\omega t^*) + B_{np} \sin(\omega t^*)] \sin(n\pi x^*) \quad (4)$$

where $R_{nL} = F_L \sin^2(n\pi x_L^*) =$ leak-induced damping coefficient, and the Fourier coefficients in (4) are defined as

$$A_{np} = \frac{C_a [(n\pi)^2 - \omega^2] - 2C_b (R + R_{nL})\omega}{[(n\pi)^2 - \omega^2]^2 + [2(R + R_{nL})\omega]^2}, \quad B_{np} = \frac{2C_a (R + R_{nL})\omega + C_b [(n\pi)^2 - \omega^2]}{[(n\pi)^2 - \omega^2]^2 + [2(R + R_{nL})\omega]^2}$$

$$A_n = -A_{np}, \quad B_n = \frac{2E^* \omega \cos(n\pi)}{(n\pi)^2} + \frac{(R + R_{nL})A_{np} - \omega B_{np}}{n\pi} \quad (5)$$

$$C_a = \frac{4RE^* \omega \cos(n\pi)}{n\pi} - 2F_L x_L^* \sin(n\pi x_L^*) E^* \omega, \quad C_b = \frac{-2E^* \omega^2 \cos(n\pi)}{n\pi}$$

Eq. (4) shows that a transient $h^*(x^*, t^*)$ induced by a sinusoidal boundary condition is composed of three parts in the right hand side of (4). The first part is independent of the boundary forcing function (noted by

parameters of E^* and ω), and is only related to the initial transients. Without the variable boundary condition, the second and the third parts at the right-hand side of (4) become zero and (4) is reduced to an exponential damping function, which was investigated in Wang et al. (2002). On the other hand, if the perturbation lasts long enough, the first part approaches zero, and Eq. (4) is simplified to a steady oscillation as

$$\begin{aligned}
h_0^*(x^*, t^*) &= x^* E^* \sin(\omega t^*) + \sum_{n=1}^{\infty} [A_{np} \cos(\omega t^*) + B_{np} \sin(\omega t^*)] \sin(n\pi x^*) \\
&= \sum_{n=1}^{\infty} -\frac{2E^* \cos(n\pi)}{n\pi} \sin(n\pi x^*) \sin(\omega t^*) + \sum_{n=1}^{\infty} [A_{np} \cos(\omega t^*) + B_{np} \sin(\omega t^*)] \sin(n\pi x^*) \\
&= \sum_{n=1}^{\infty} [A_{np} \cos(\omega t^*) + (B_{np} + C_{np}) \sin(\omega t^*)] \sin(n\pi x^*) \\
&= \sum_{n=1}^{\infty} [E_{np}^* \sin(\omega t^* + \delta)] \sin(n\pi x^*)
\end{aligned} \tag{6}$$

where

$$E_{np}^* = \sqrt{A_{np}^2 + (B_{np} + C_{np})^2} \tag{7}$$

which is the amplitude of a Fourier component, $\delta = \tan^{-1}(\frac{B_{np} + C_{np}}{A_{np}})$ = the phase angle,

and $C_{np} = -\frac{2E^* \cos(n\pi)}{n\pi}$. Note that h_0^* is used instead of h^* as this solution represents the long-term

steady oscillations. Therefore, after a sufficiently long time, the output corresponding to a sinusoidal input is a summation of a series of harmonic oscillations. The frequency of any of the harmonic oscillations is that of the input, and the amplitude of each of the harmonic component is defined by (7).

The value of the amplitude of each component expressed in (7) depends on the amplitude (E^*) and angular frequency of the input signal (ω^*) and the parameters of the pipeline and the leak, which may be characterised by parameters R , F_L and x_L^* . Because the amplitude and the frequency of the input signal are known, and the frictional parameter R can be calculated from the steady state conditions, the feasibility to detect the leaks is investigated in the following sections.

4. COMPARISON WITH NUMERICAL SOLUTION BASED ON MOC

The governing equation (1) is a linear equation (Wang et al. 2002). The analytical solution expressed in (4) is now compared with the nonlinear numerical results obtained from the method of characteristic (MOC). For the pipeline as shown in Figure 2 where $H_1 = 25$ m, $H_2 = 15 + E \sin(\omega t)$ in units of meters, and $E = 0.25$ m, two perturbation frequencies of $\omega = 1.0\pi$ and $\omega = 1.5\pi$ are considered. A leak of $C_d A_L / A = 0.001$ is located at 250 m ($x_L^* = 0.25$) downstream of the upstream reservoir. Based on the steady-state flow conditions, the friction damping parameter is calculated as $R = 0.0606$. A small transient is needed to keep the linearization error of the analytical solution at a low level.

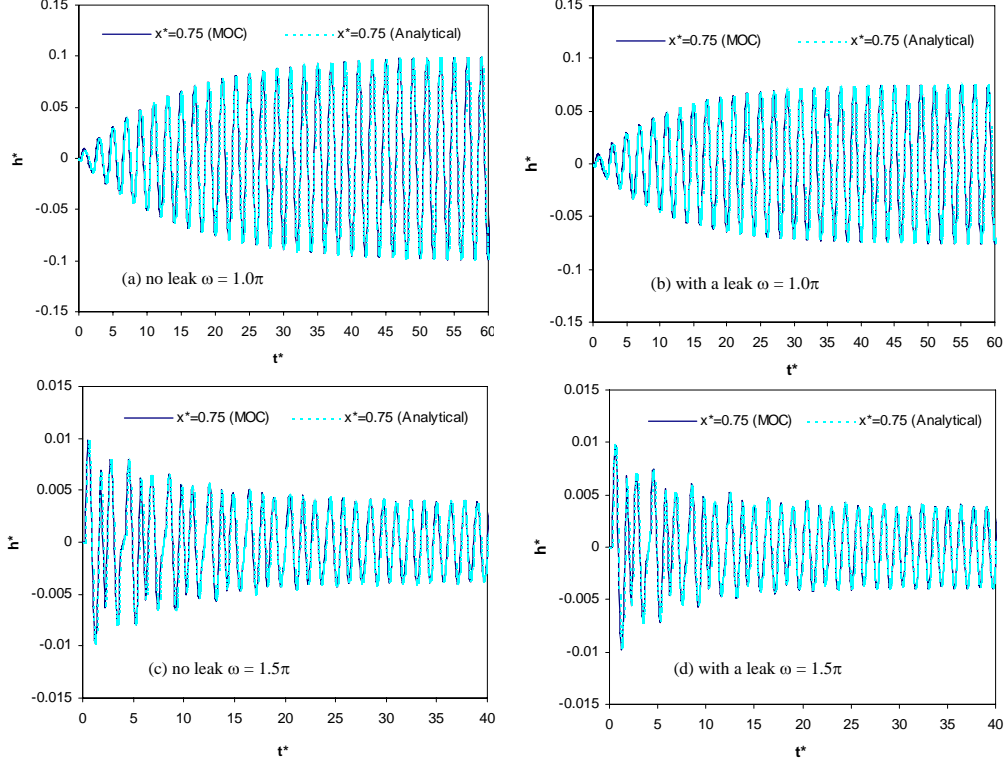


Figure 3 Comparison of the analytical solution and the numerical results based on the MOC

The transients measured 750 m downstream from the upstream reservoir ($x^* = 0.75$) based on the analytical solution and the MOC are presented in Figure 3. For all cases, with and without a leak and two input frequencies, the transients calculated based on the analytical solution of (4) agree very well with the numerical results from the MOC. In the analytical solution, components of $n \leq 20$ are considered. When ω is close to 1.0π , the frequency of the boundary perturbation approaches the natural frequency of the pipeline, a resonance condition is created with a significantly larger amplitude than the input signal. For the case without a leak, the transient stops growing and a steady oscillation appears after about 25 periods ($t^* > 50.0$). When a leak is present, the steady oscillation establishes after about 20 periods ($t^* > 40.0$). A steady oscillation is defined here in such a way that the increase of transient amplitudes over 10 periods is less than 0.1%. The presence of the leak reduces the amplitude of the steady oscillation by about 40%, but it has no influence on the frequency of the transient. When $\omega = 1.5\pi$, a transient with a fundamental period of $T^* = 1.33$, is observed for both cases of with and without a leak. However, the presence of the leak has little influence on the amplitude of the steady oscillation for non-resonance inputs.

5. EFFECTS OF A LEAK ON STEADY OSCILLATION

The above example indicated that a leak has an influence on the amplitude of a steady oscillation and that the amplitude is a function of the frequency of the input signals. The amplitude values of the components of $n < 11$ defined in (7) are plotted in Figure 4 for the cases given in Figure 3. When $\omega = 1.0\pi$, in cases with and without a leak, the component of $n = 1$ is dominant, as the amplitudes of other components are negligible. When a linear system is excited at one of the resonant frequencies, the output contains only that frequency with other frequencies being zero. As a result, the summation sign in the response function defined in (6) for a sinusoidal input signal with a resonant frequency can be dropped out and (6) is reduced to one component as

$$h_0^*(x^*, t^*) = \left[E_{np}^* \sin(\omega t^* + \delta) \right] \sin(n\pi x^*) \quad (n\pi = \omega) \quad (8)$$

Figure 4 shows that when $\omega = 1.0\pi$ the presence of a leak decreases the amplitude of the dominant component $n = 1$ significantly (from $E_{np} = 0.17$ to $E_{np} = 0.12$). As a result, the amplitude of the whole transient is decreased by the presence of the leak as shown in Figure 3(b) compared to Figure 3(a). On the contrary, for the input signal whose frequency is not resonant, the presence of the leak has no obvious influence on the amplitude of the components as shown in Figure 3(c) and Figure 3(d). In addition, when compared to the case of resonance, there is no obvious dominant component in the response signal if the dimensionless frequency of the input signal is not resonant.

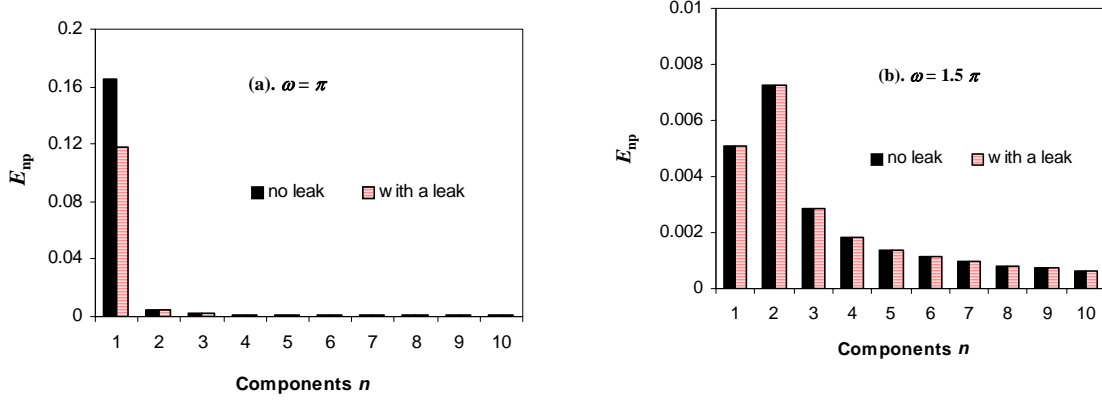


Figure 4 Magnitude of the Fourier components

Substituting $\omega = n\pi$ into (8) gives the resonance amplitude as

$$E_{np} = \frac{E^*}{R + R_{nL}} \sqrt{1 - \left[\frac{2R_{nL} \cos(n\pi) + n\pi F_L x_L^* \sin(n\pi x_L^*)}{n\pi} \right]^2} \approx \frac{E^*}{R + R_{nL}} \quad (9)$$

Since the value of leak parameter R_{nL} is positive, Eq. (9) indicates that the presence of a leak in a pipeline decreases the amplitude of the component whose frequency is that of the input signal. Substituting (9) into (8) gives

$$h_0^*(x^*, t^*) = E_p \sin(\omega t^* + \delta) \quad (n\pi = \omega) \quad (10)$$

where $E_p = \frac{E^*}{R + R_{nL}} \sin(n\pi x^*)$ = amplitude of the steady oscillation. Therefore, the transient in a pipeline

under a resonant sinusoidal input signal is a sinusoidal function, and the amplitude of the transient is a function of the amplitude of the input signal E^* , friction damping parameter of the pipeline R , leak damping parameter R_{nL} and measurement location x^* .

For the example as shown in Figure 2, the transient responses under different input frequencies ($0.0 \sim 10\pi$) are plotted in Figure 5. As discussed in the above analysis, Figure 5 shows that under a sinusoidal perturbation, the presence of a leak reduces the amplitude of the transient if the input signal is that of the resonant frequency of the pipeline. For an input signal that has a non-resonant frequency, the presence of a leak has little influence on the transient. In addition, the transients measured at different locations along the pipeline ($x^* = 0.5$, and $x^* = 0.75$ in Figure 5), the transient responses are different for both cases of with and without a leak.

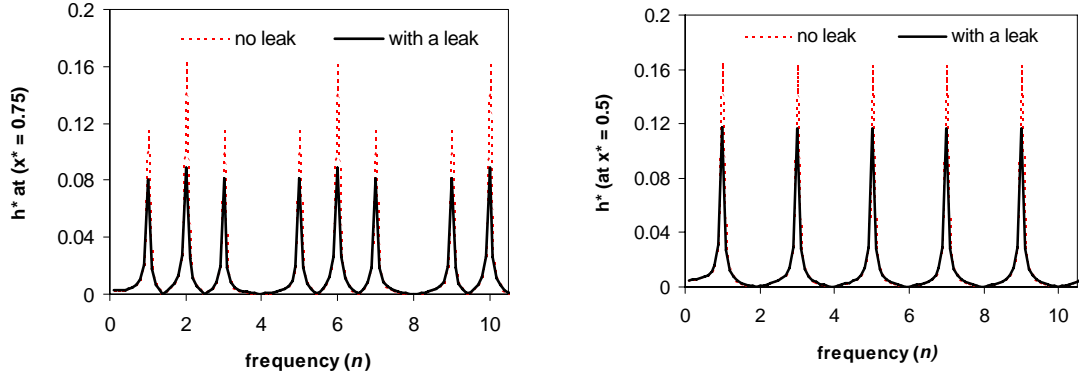


Figure 5 Transient response (measured at $x^* = 0.75$ and $x^* = 0.5$) of a leak under different input frequencies

Previous studies (Vítkovský 2001) have found that the transient pressure heads measured from a pipeline cannot be used as boundary conditions in the inverse transient leak detection analysis because under such a boundary condition, the transients measured from the pipeline are very insensitive to the leak. The analytical solution in (6) gives the reason for this observed phenomenon. If a measured transient is used as a boundary condition in a transient simulation modelling, the measured transient is a variable boundary condition. The example given in the previous section has shown that presence of a leak has little influence on the transients in a pipeline if the variable boundary condition is not resonant. In another word, transients initiated by a non-resonant boundary condition don't have enough response with leaks. Under a normal condition, a transient initiated by the common methods (e.g. a valve disturbance) are not likely a resonant transient. As a result, if such a non-resonant transient event is applied into a transient simulation model as a boundary condition, different leak sizes (including no leak) will produce similar transient responses, which means the transients fails to tell the information about the leak. Therefore, normally generated transients can not be used as boundary conditions in the inverse transient leak detection analysis. However, because the presence of a leak has significant influence on the resonant frequencies, the resonant components will be a good media for leak detection.

6. LEAK DETECTION USING RESONANT FLUID TRANSIENTS IN PIPE NETWORKS

As indicated in the previous section, the amplitude of the resonant steady oscillation is a function of magnitude of the input signal E^* , friction damping parameter of the pipeline R , leak damping parameter R_{nL} and measurement location x^* . Since the values of E^* , R , and x^* are known or can be calculated from the flow conditions in the pipeline, the value of a leak damping parameter R_{nL} can be obtained from the amplitude of a steady oscillation defined in (16). The leak location (x_L^*) and leak size ($C_d A_L$) can be obtained from the ratio of two leak damping coefficients (Wang et al. 2002).

$$\frac{R_{n_2 L}}{R_{n_1 L}} = \frac{\sin^2(n_2 \pi x_L^*)}{\sin^2(n_1 \pi x_L^*)} \quad (n_1 = 1, 2, 3, \dots, n_2 = 1, 2, 3, \dots, n_1 \neq n_2) \quad (11)$$

$$C_d A_L = \frac{R_{nL} A (2gH_{L0})^{0.5}}{a \sin^2(n \pi x_L^*)} \quad (n = 1, 2, 3, \dots) \quad (12)$$

The technique for detecting and quantifying a leak based on leak-induced damping coefficients has been discussed in Wang et al. (2002) in which the damping coefficients were obtained by measuring the decayed transient components using an exponential fitting function. In this paper, the focus is on how to obtain the leak-induced damping coefficients (R_{nL}) from the amplitude of the steady oscillation for the case of a pipe network. A small network as shown in Figure 6, which is similar to that in Liggett and

Chen (1994) has been used as an example to illustrate the detailed procedures. The major parameters of the small network are:

Node number:	46
Pipe number:	50
Pipe diameter:	200 mm
Single pipe length (m):	50 m
Wave speed:	1000 m/s
Pipe friction factor:	0.02
Leak location:	node 8
Relative leak size ($C_d A_L/A$):	0.1%

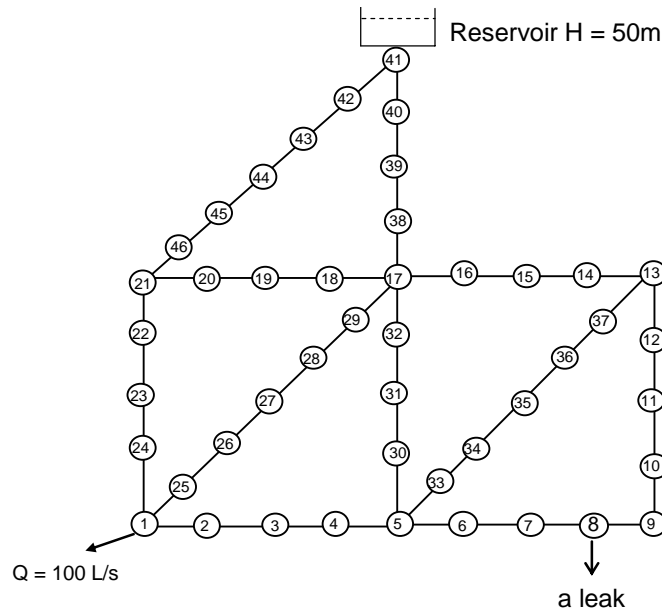


Figure 6 A small network with a leak (flow injection at node 6 and node 10)

For the purpose of leak detection, two flow injections are applied at node 6 and node 10 with the magnitudes as shown in Figure 7. Given the distance of two boundary perturbations, the character length of the pipe (from node 6 to node 10) is 200m. The magnitude of the perturbation pressure is approximately 0.05m ($E^* = 0.001$). The steady flow velocity in the pipe section from node 6 to node 10 is 0.22m/s and the friction damping factor based on the steady flow conditions is approximately $R = 0.0022$. The transient head variation calculated using the MOC are plotted in Figure 8. A state of steady oscillation forms for all the cases when $t^* > 200$, so only transients in the period of $230 < t^* < 240$ are shown in Figure 8, and are used for the leak detection analysis. The decrease of the amplitude of the steady oscillation caused by the leak is obvious as shown in Figure 8. The amplitude of the steady oscillation is $E_p = 0.00748$ for no leak case and $E_p = 0.00441$ for the case of with a leak (E_p was defined in Eq. 10). Substituting these values into (10) and considering $E^* = 0.001$, $n = 1$ and $x^* = 0.5$ gives $R_{1L} = 0.0118$. Applying the ratio of $R_{2L}/R_{1L} = 0.00$ into (11) gives the leak location of $x_L^* = 0.5$. Applying $R_{1L} = 0.0118$ and $x_L^* = 0.5$ gives the size of the leak of $C_d A_L/A = 0.00101$. Both leak location and leak size are almost the same as the real leak location $x_L^* = 0.5$ and size of $C_d A_L/A = 0.001$.

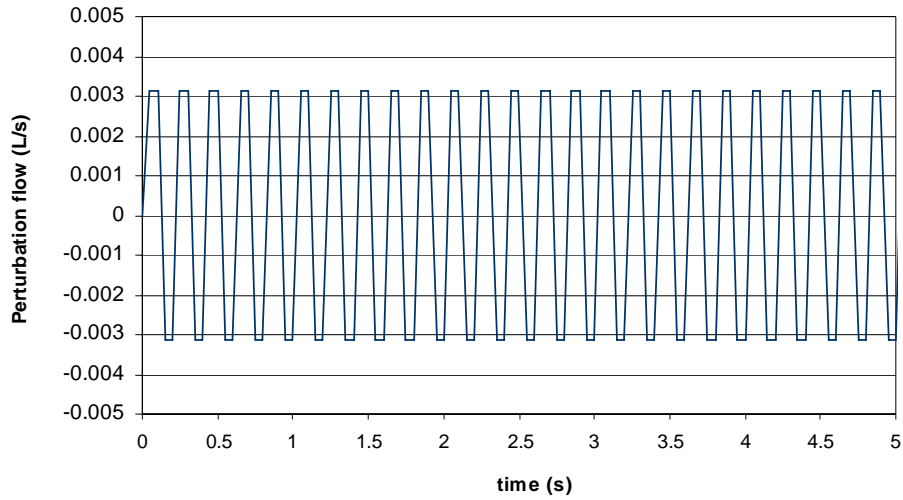


Figure 7 Flow injections at node 6 and node 10

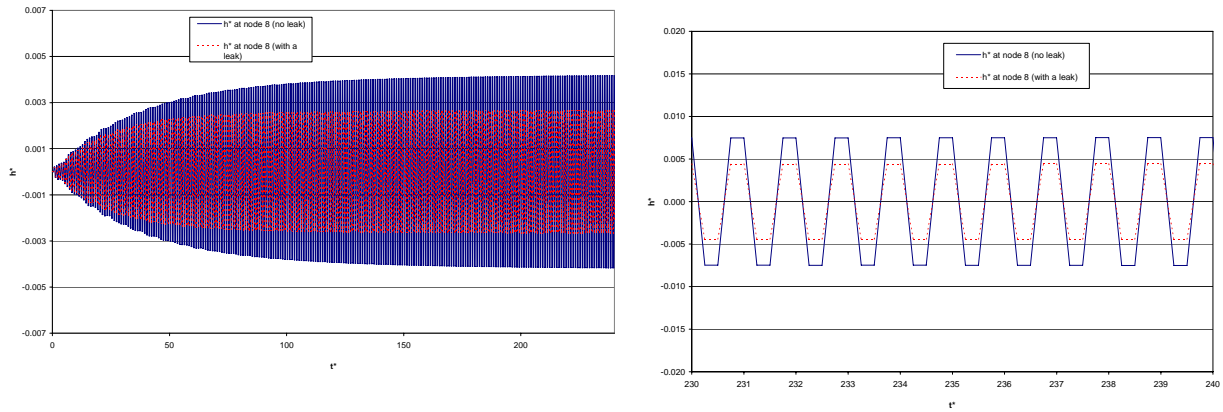


Figure 8 Transients at node 8

7. CONCLUSIONS

Transients in a pipeline under variable boundary conditions have been studied analytically, and an analytical solution expressed as a Fourier series has been developed under a sinusoidal boundary condition. By comparison to the numerical solution based on the method of characteristics (MOC) in which the non-linear effects are included, the analytical solution based on a linearised governing equation shows a high degree of accuracy for both cases of with and without a leak in a pipeline.

The analytical solution shows that a steady oscillation forms in a pipeline under a continuously varying boundary perturbation, and the formation time and the amplitude of the steady oscillation depends on the pipe friction, the leaks in the pipeline and input signals. The analytical solution also shows that the presence of a leak in a pipeline only influences the amplitude of a resonant transient, and almost has no influence on the amplitude of a non-resonant transient. An analytical relationship between the leak and the resonant amplitude has been obtained. Because the influence of the leak on the resonant amplitude is frequency dependent, and depends on the leak location and size, a new leak detection technique that are able to detect the presence, location and magnitude, have been developed by examining the magnitudes of resonant amplitudes of different frequencies.

Considering that a pipe section in a pipe network is a single pipe with two variable boundary conditions, the new leak detection technique proposed for single pipeline can be applied into a pipe network as tested in a numerical example.

The analytical solution and the leak detection method proposed in this paper has assumed that the leak is small ($C_d A_L/A < 0.01$). A larger leak will not only reduce the resonant transient amplitudes, but also change the system frequency. In addition, the effect of the unsteady friction, which is important for the fast transient event, has not been included in the current study and should be considered in the experimental verifications.

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