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Journal of Petroleum Science and Engineering xx (2005) xxx–xxx
Journal of Petroleum Science & Engineering
www.elsevier.com/locate/petrol

A splitting technique for analytical modelling of two-phase multicomponent flow in porous media

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Accepted 4 November 2005

Abstract

In this paper we discuss one-dimensional models for two-phase Enhanced Oil Recovery (EOR) floods (oil displacement by gases, polymers, carbonized water, hot water, etc.). The main result presented here is the splitting of the EOR mathematical model into thermodynamical and hydrodynamical parts. The introduction of a potential associated with one of the conservation laws and its use as a new independent coordinate reduces the number of equations by one. The $(n)\times(n)$ conservation law model for two-phase n-component EOR flows in new coordinates is transformed into a reduced $(n-1)\times(n-1)$ auxiliary system containing just thermodynamical variables (equilibrium fractions of components, sorption isotherms) and one lifting equation containing just hydrodynamical parameters (phase relative permeabilities and viscosities). The algorithm to solve analytically the problem includes solution of the reduced auxiliary problem, solution of one lifting hyperbolic equation and inversion of the coordinate transformation. The splitting allows proving the independence of phase transitions occurring during displacement of phase relative permeabilities and viscosities. For example, the minimum miscibility pressure (MMP) and transitional tie lines are independent of relative permeabilities and phases viscosities. Relative motion of polymer, surfactant and fresh water slugs depends on sorption isotherms only. Therefore, MMP for gasflood or minimum fresh water slug size providing isolation of polymer/surfactant from incompatible formation water for chemical flooding can be calculated from the reduced auxiliary system. Reduction of the number of equations allows the generation of new analytical models for EOR. The analytical model for displacement of oil by a polymer slug with water drive is presented.

Keywords: Reservoir engineering; Enhanced oil recovery; Analytical model; Gas injection; Polymer flooding; Hyperbolic systems; Conservation laws; Riemann problem

1. Introduction

Enhanced Oil Recovery (EOR) methods include injection of different fluids into reservoirs to improve oil displacement. Displacement of oil by any of these fluids involves complex physico-chemical interphase mass transfer, phase transitions and transport property changes. These processes can be divided into two main categories: that of thermodynamics and of hydrodynamics. They occur simultaneously during the displacement, and are coupled in the modern mathematical models of EOR.

The mathematical models for two-phase Enhanced Oil Recovery processes consist of mass conservation

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0920-4105/$ - see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.petrol.2005.11.009

for each component closed by thermodynamic relationships of phase equilibria. Thermal EOR models contain also the energy conservation law. The resulting systems of conservation laws (Gelfand, 1959; Dafermos, 2000) are hyperbolic (Logan, 1994). Solutions consist of continuous simple (rarefaction) waves and stable admissible shocks (Kulikovskii and Sveshnikova, 1995; Kulikovskii et al., 2001).

Continuous injection of EOR fluid corresponds to self-similar Riemann problem for the system of two-phase multi component flow equations. Injection of EOR fluid slugs with a water/gas drive results in non-self-similar problems of hyperbolic wave interactions.

Exact analytical solutions have been obtained for continuous chemical flooding by one component (Fayers, 1962; de Nevers, 1964; Claridge and Bondor, 1974; Helfferich, 1980), by two components (Braginskaya and Entov, 1980) and by any arbitrary number of components (Johansen and Winther, 1989; Johansen et al., 1989; Dahl et al., 1992). A graphical technique to solve the $(2)\times(2)$ system for two-phase three-component gas flooding was developed and several exact solutions for Riemann problems of continuous gas injection were obtained by Wachman (1964). Other solutions for different types of phase diagrams and boundary conditions related to injection of other fluids were found using the same technique (Hirasaki, 1981; Dumore et al., 1984; Lake, 1989).

Semi-analytical solutions for n-component gas flooding were obtained by numerical combination of shocks and rarefactions (Johns et al., 1993; Johns and Orr, 1996; Orr et al., 1995). The reduction of the continuous gas flood system dimension was developed through the lifting of the concentration waves from the system with lower dimension, and the exact solutions were obtained for the displacement of n-component ideal mixtures (Bedrikovetsky and Chumak, 1992a,b). These reduction technique and solutions were used for different initial-boundary data corresponding to different gas floods (Entov and Voskov, 2000; Entov et al., 2002). Non-self-similar analytical models for displacement of oil by chemical and gas/solvent slugs were derived explicitly by Bedrikovetsky (1993). The detailed study of these analytical EOR models can be find in monographs by Lake (1989), Barenblatt et al. (1991) and Bedrikovetsky (1993).

It was observed from semi-analytical and numerical experiments on the continuous displacement of oil by gases that several thermodynamic features (MMP, key tie lines, etc.) are independent of transport properties (Zick, 1986; Bedrikovetsky and Chumak, 1992a,b; Orr et al., 1995; Wang and Orr, 1997). The analytical modelling of multicomponent polymer/surfactant flood also allows observing that the concentration “path” of the solution is completely defined by adsorption isotherms and does not depend on relative permeability and phase viscosities (Johansen and Winther, 1989; Johansen et al., 1989; Bedrikovetsky, 1993). Nevertheless, the independence of thermodynamics and hydrodynamics for two-phase multi component flows in porous media has never been proved.

The model for one-dimensional displacement of oil by different EOR fluids is analysed in this paper. The main result is the splitting of thermodynamical and hydrodynamical parts in the EOR mathematical model. The introduction of a potential associated with one of the conservation laws and its use as an independent variable reduces the number of equations by one. The algorithm to solve the problem includes solution of the reduced auxiliary problem, solution of one lifting hyperbolic equation and inversion of the coordinate transformation.

The reduced auxiliary system contains just thermodynamical (equilibrium fractions of each phase, sorption isotherms) variables and the lifting equation contains just hydrodynamical (phases relative permeabilities and viscosities) parameters while the initial EOR model contains both thermodynamical and hydrodynamical functions. So, the problem of EOR displacement was divided into two independent problems: that of thermodynamics and that of hydrodynamics. The number of auxiliary equations is less than the number of equations in the compositional model by one. Explicit projection and lifting procedures are derived. The splitting is valid for either self-similar continuous injection problems or for non-self-similar slug injection problems.

Therefore, phase transitions occurring during displacement are determined by the auxiliary system, i.e. they are independent of hydrodynamic properties of fluids and rock. For example, the minimum miscibility pressure (MMP) and tie line sequences in displacement zones are independent of relative permeabilities and phases viscosities. Relative motion of polymer, surfactant and fresh water/brine slugs depends on sorption isotherms only. The splitting technique was used for the development of analytical model for non-self-similar displacement of oil by polymer slug with water drive.

Presently the development of 1D analytical models becomes particularly important in 3D streamline simulation. With respect to 3D flows, the splitting takes place only for the case of constant total mobility (where the stream line concept is valid). For the general case of the total mobility variation, mixing between fluids that enter different streamlines occurs, and splitting does not happen any more.

In Section 2 we present the splitting method for different two-phase multicomponent flows in porous media that correspond to various EOR methods: chemical flooding is given in 2.1, gasflooding is presented in 2.2, WAG injection is derived in 2.3, carbonised water flooding in 2.4 and non-isothermal waterflooding is presented in 2.5. The analytical model for 1D displacement of oil by a polymer slug with water drive as an illustration of the technique developed is presented in Section 3. Brief description of various applications in streamline simulation and laboratory EOR is shown in Section 4. Summary and conclusions are presented in Section 5. Proofs of splitting can be found in Appendixes.

2. Mathematical models of enhanced oil recovery processes

In this part, several systems of equations that arise in enhanced oil recovery processes are presented, and the splitting technique applied.

2.1. Chemical flooding

We consider the linear displacement of oil by an aqueous solution of n-components (polymer, salts) in a reservoir of constant permeability and porosity. The reservoir is initially saturated with oil and water. The fluid system contains two incompressible phases (oil and water). There are also n low concentration components dissolved in the aqueous phase, so the change of concentrations does not affect the aqueous phase density. The components can be adsorbed by the porous rock. The following conditions are assumed:

- Neglected capillary pressure and diffusion;
- Instantaneous thermodynamics equilibrium;
- Constant pressure and temperature.

Under the conditions of thermodynamic equilibrium, the concentrations of the components adsorbed (a_i) and dissolved in water (c_i) are governed by adsorption isotherms:

$$\vec{a} = \vec{a}(\vec{c}), \quad \vec{a} = (a_1, a_2, \ldots, a_n), \\ \vec{c} = (c_1, c_2, \ldots, c_n) \tag{1}$$

The closed system of governing equations includes the conservation laws for the aqueous phase volume and for the mass of each component under equilibrium sorption conditions. The unknowns in the $(n+1)\times(n+1)$ system are the scalar water saturation function $s(x_D, t_D)$ and the vector-valued function $\vec{c}(x_D, t_D)$:

$$\begin{aligned} &\frac{\partial s}{\partial t_D} + \frac{\partial f(s, \vec{c})}{\partial x_D} = 0 \\ &\frac{\partial(\vec{c}s + \vec{a}(\vec{c}))}{\partial t_D} + \frac{\partial \vec{c} f(s, \vec{c})}{\partial x_D} = 0 \end{aligned} \tag{2}$$

where the following dimensionless coordinates are used:

$$x_D = \frac{x}{l}, \quad t_D = \frac{ut}{\Phi l} \tag{3}$$

where Φ is porosity.

The fractional flow function is defined as:

$$f = f(s, \vec{c}) = \left(1 + \frac{k_{ro}(s, \vec{c})\mu_w}{\mu_o k_{rw}(s, \vec{c})}\right)^{-1} \tag{4}$$

Initial and boundary conditions for continuous polymer injection correspond to:

$$\begin{cases} s(x_D, 0) = s^I \\ \vec{c}(x_D, 0) = 0 \end{cases} \quad \begin{cases} s(0, t_D) = s^J \\ \vec{c}(0, t_D) = \vec{c}^J \end{cases} \tag{5}$$

The boundary conditions for the displacement of oil by a polymer slug with water drive are:

$$\vec{c}(0, t_D) : \begin{cases} \vec{c}^J, & t_D < 1 \\ 0, & t_D > 1 \end{cases} \tag{6}$$

The conservation law for the aqueous phase allows the introduction of the following potential:

$$s = -\frac{\partial \varphi}{\partial x_D}, \quad f = \frac{\partial \varphi}{\partial t_D} \tag{7}$$

Consider any trajectory $x_D = x_D(t_D)$ that starts at $x_D = 0$ at the moment t_D. The potential $\varphi(x_D, t_D)$ is the water volume flowing through the trajectory during the period t_D:

$$\varphi(x_D, t_D) = \int_{0,0}^{x_D, t_D} f\,dt_D - s\,dx_D \tag{8}$$

and the integral (8) is a function of x_D and t_D, which is independent of the trajectory.

After the following transformation of independent variables:

$$\Theta : (x_{\mathrm{D}}, t_{\mathrm{D}}) \rightarrow (x_{\mathrm{D}}, \varphi) \tag{9}$$

system (2) becomes

$$\frac{\partial}{\partial \varphi}\left(\frac{s}{f}\right) - \frac{\partial}{\partial x_{\mathrm{D}}}\left(\frac{1}{f}\right) = 0 \tag{10}$$

$$\frac{\partial \vec{a}(\vec{c})}{\partial \varphi} + \frac{\partial \vec{c}}{\partial x_{\mathrm{D}}} = 0 \tag{11}$$

Derivation of system (11) is presented in Appendix A. The most important feature of the system (10), (11) is the independence of the n equations (11) from the first Eq. (10). The unknowns in the system (11) are c_i, $i=1, 2, 3, \ldots, n$. The hyperbolic Eq. (10) contains the unknown $s(x_{\mathrm{D}}, \varphi)$ and the known vector function $\vec{c}(x_{\mathrm{D}}, \varphi)$, which is the solution of (11).

The system (11) is called the auxiliary system of the large system (2). It is important to mention that the system (2) contains thermodynamic functions and transport properties, while the auxiliary system contains only thermodynamic functions.

The initial and boundary conditions (5) and (6) allow the calculation of the potential φ where these conditions are set. Integration of the potential Eq. (8), accounting for (5), determines the initial and boundary conditions for continuous chemical injection in plane $(x_{\mathrm{D}}, \varphi)$:

$$\begin{aligned} t_{\mathrm{D}} = 0 &: \varphi = -s^{\mathrm{I}} x_{\mathrm{D}} \\ x_{\mathrm{D}} = 0 &: \varphi = f^{\mathrm{J}} t_{\mathrm{D}} \end{aligned} \tag{12}$$

Then, the initial-boundary conditions (5) become

$$\begin{aligned} \varphi = -s^{\mathrm{I}} x_{\mathrm{D}} &\begin{cases} s = s^{\mathrm{I}} \\ \vec{c} = 0 \end{cases} \\ \varphi = -f^{\mathrm{J}} t_{\mathrm{D}} &\begin{cases} s = s^{\mathrm{J}} \\ \vec{c} = \vec{c}^{\,\mathrm{J}} \end{cases} \end{aligned} \tag{13}$$

Finally, the boundary conditions (6), for the displacement of oil by a polymer slug with water drive take the form:

$$\varphi = f^{\mathrm{J}} t_{\mathrm{D}} \begin{cases} s = s^{\mathrm{J}}, \forall t_{\mathrm{D}} \\ \vec{c} = \vec{c}^{\,\mathrm{J}}, t_{\mathrm{D}} < 1 \\ \vec{c} = 0, t_{\mathrm{D}} > 1 \end{cases} \tag{14}$$

It is possible to prove that any Cauchy or initial-boundary value problem for the model (2) can be projected onto the corresponding Cauchy or initial-boundary value problem for the auxiliary system.

Consider the trajectory $x_{\mathrm{D}} = x_{\mathrm{D}}(t_{\mathrm{D}})$ and its image $\varphi = \varphi(t_{\mathrm{D}})$ by the mapping (9):

$$\varphi(t_{\mathrm{D}}) = \varphi(x_{\mathrm{D}}(t_{\mathrm{D}}), t_{\mathrm{D}}) \tag{15}$$

Define the trajectory speeds

$$\begin{aligned} D &= \frac{dx_{\mathrm{D}}}{dt_{\mathrm{D}}} \\ V &= \frac{dx_{\mathrm{D}}}{d\varphi} \end{aligned} \tag{16}$$

Using x_{D} as a parameter for both curves $x_{\mathrm{D}} = x(t_{\mathrm{D}})$ and $\varphi = \varphi(t_{\mathrm{D}})$ it is possible to obtain

$$\frac{1}{V} = \frac{f}{D} - s \tag{17}$$

from which follows the relationship between elementary wave speeds in planes (x_{D}, t_{D}) and (x_{D}, φ):

$$D = \frac{f}{s + 1/V} \tag{18}$$

For example, the eigenvalues of the large and auxiliary systems for c waves are related by:

$$\Lambda_{i+1}(s, \vec{c}) = \frac{f}{s + 1/\lambda_i}, i = 1, \ldots, n. \tag{19}$$

2.2. Gas flooding

Consider 1D two-phase multicomponent gas flooding under the following assumptions:

- Neglected capillary pressure and diffusion;
- Instantaneous thermodynamic equilibrium;
- Constant pressure and temperature;
- Equal component individual densities in both phases.

Thermodynamic equilibrium implies $n-2$ independent phase fractions. We choose components $i=2, 3, \ldots, n-1$ in gas phase for the vector of independent phase fractions:

$$\vec{g} = \left(c_{2g}, c_{3g}, \ldots, c_{(n-1)g}\right) \tag{20}$$

Under the above mentioned conditions, the total two-phase flux is conserved, and n mass balances for n-components are replaced by $n-1$ volume conservation laws for $n-1$ components:

$$\frac{\partial C_i}{\partial t_D}+\frac{\partial F_i}{\partial x_D}=0 \qquad x_D=\frac{x}{l},\, t_D=\frac{ut}{\Phi l} \tag{21}$$

where the overall i-th component fraction and flux are

$$C_i - c_{il}S + c_{ig}(1-S) \tag{22}$$

$$F_i = c_{il}f + c_{ig}(1-f) \tag{23}$$

Here f is the fractional flow of liquid:

$$f(S,\vec{g})=\frac{k_{rl}(S,\vec{g})/\mu_l(\vec{g})}{k_{rl}(S,\vec{g})/\mu_l(\vec{g})+k_{rg}(S,\vec{g})/\mu_g(\vec{g})} \tag{24}$$

Initial and boundary conditions for continuous gas injection correspond to given compositions of injected gas and displaced oil:

$$C_i(x_D,0)=C_i^I \qquad C_i(0,t_D)=C_i^J \tag{25}$$

The boundary conditions for the displacement of oil by solvent slug with lean gas drive are:

$$C_i(0,t_D):\begin{cases} C_i^J, & t_D<1\\ C_i^D, & t_D>1\end{cases} \tag{26}$$

where C_i^D is the composition of gas driving the solvent slug.

At this point we introduce new variables:

$$\alpha_i(\vec{g})=\frac{c_{il}-c_{ig}}{c_{nl}-c_{ng}},\, i=2,3,\ldots n-1 \tag{27}$$

$$\beta_i(\vec{g})=c_{ig}-\alpha_i c_{ng},\, i=2,3,\ldots n-1 \tag{28}$$

Fig. 1 shows the geometrical meaning of α_i and β_i. Vertices 1, 2, …, n correspond to pure components in phase diagram. Tie line GL connects equilibrium phase compositions, G_iL_i is the tie line projection on the plane (C_i, C_n). The slope of the straight line G_iL_i is equal to α_i, the intersection of G_iL_i with the axes C_i is equal to β_i.

System (21) takes the form:

$$\frac{\partial C}{\partial t_D}+\frac{\partial F\left(C,\vec{\beta}\right)}{\partial x_D}=0 \qquad \frac{\partial\left(\vec{\alpha}\left(\vec{\beta}\right)C+\vec{\beta}\right)}{\partial t_D}+\frac{\partial\left(\vec{\alpha}\left(\vec{\beta}\right)F+\vec{\beta}\right)}{\partial x_D}=0 \tag{29}$$

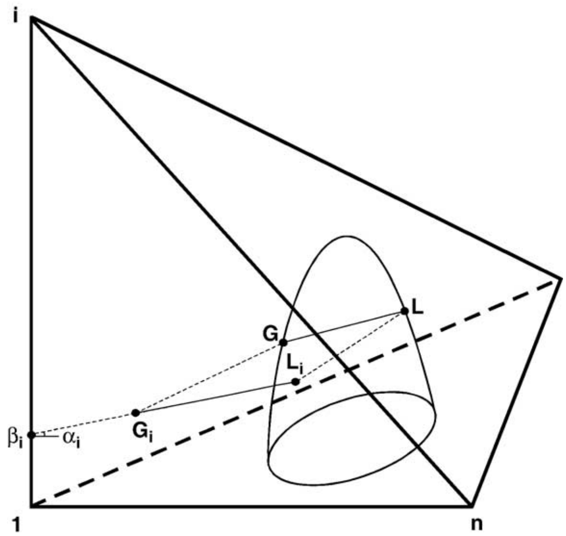

Fig. 1. Phase diagram for n-component fluids.

In system (29), C is equal to C_n, the overall volumetric fraction of n-th component, and F is equal to F_n, the overall volumetric fractional flow of n-th component.

The unknowns in system (29) of $n-1$ equations are C and β_i, i=2, 3, …, $n-1$.

After the introduction of variables (27) and (28), the initial and boundary conditions (25) for continuous gas injection become

$$C(x_D,0)=C_n^I \qquad \beta_i(x_D,0)=\beta_i(\vec{g}^I) \tag{30}$$

$$C(0,t_D)=C_n^J \qquad \beta_i(0,t_D)=\beta_i(\vec{g}^J) \tag{31}$$

For displacement of oil by a rich gas slug with lean gas drive, the boundary conditions (26) take the form

$$C(0,t_D):\begin{cases} C_n^J, & t_D<1\\ C_n^D, & t_D>1\end{cases} \qquad \beta_i(0,t_D):\begin{cases} \beta_i(\vec{g}^J), & t_D<1\\ \beta_i(\vec{g}^D), & t_D>1\end{cases} \tag{32}$$

The conservation law form of the first Eq. (29) allows the introduction of the following potential:

$$c=-\frac{\partial\varphi}{\partial x_D},\, F=\frac{\partial\varphi}{\partial t_D} \tag{33}$$

The potential $\varphi(x_D,t_D)$ is equal to the n-th component volume flowing via a trajectory connecting points (0, 0) and (x_D, t_D):

$$\varphi(x_D,t_D)=\int_{0,0}^{x_D,t_D} F\,dt_D - C\,dx_D \tag{34}$$

and the integral (34) is a function of x_D and t_D, and is independent of the trajectory.

Let us introduce the variable

$$\psi = x_D - t_D \tag{35}$$

From the incompressibility of the total flux follows that $\psi(x_D, t_D)$ is equal to the overall mixture volume flowing via a trajectory connecting points (0, 0) and (x_D, t_D).

After the following transformation of independent variables

$$\Theta : (x_D, t_D) \rightarrow (\psi, \varphi) \tag{36}$$

system (29) becomes

$$\frac{\partial}{\partial \varphi}\left(\frac{C}{F-C}\right) - \frac{\partial}{\partial \psi}\left(\frac{1}{F-C}\right) = 0 \tag{37}$$

$$\frac{\partial \vec{\beta}}{\partial \varphi} + \frac{\partial \vec{\alpha}\left(\vec{\beta}\right)}{\partial \psi} = 0 \tag{38}$$

Derivation of system (38) is presented in Appendix B. The most important feature of the system (37), (38) is the independence of the $n-2$ Eq. (38) from the first Eq. (37). The unknowns in the system (38) are β_i, $i=2$, 3, ..., $n-1$. The hyperbolic Eq. (37) contains the unknown $C(\psi, \varphi)$ and the known vector function $\beta_i(\psi, \varphi)$, which is the solution of (38).

The system (38) is called the auxiliary system of the large system (29). It is important to mention that the system (29) contains thermodynamic functions and transport properties, while the auxiliary system contains only thermodynamic functions.

The initial and boundary conditions (30), (31) and (32) allow the calculation of both potentials along the axes x_D and t_D where the conditions are set.

Performing the integration (34) in x_D accounting for (30) we obtain the potential φ along the axes x_D:

$$t_D = 0 : \varphi = 0 - C_n^I \psi \qquad \psi = x_D \tag{39}$$

So, the initial conditions (30) in coordinates (ψ, φ) become

$$\varphi = -C^I \psi : C = C^I \tag{40}$$

$$\varphi = -C^I \psi : \vec{\beta} = \vec{\beta}^I \tag{41}$$

Integrating (34) in t_D accounting for boundary condition (31) allows calculation of the potential φ along the axes t_D:

$$x_D = 0 : \varphi = 0 - F_n^J \psi \qquad \psi = -t_D \tag{42}$$

The boundary conditions (31) take the form:

$$\varphi = -F^J \psi : C = C^J \tag{43}$$

$$\varphi = -F^J \psi : \vec{\beta} = \vec{\beta}^J \tag{44}$$

The boundary condition (32) for slug injection gives the following value of potential φ:

$$x_D = 0 : \varphi \begin{cases} -F_n^J \psi, & -1<\psi<0 \\ F_n^J - F_n^D(\psi+1), & -\infty<\psi<-1 \end{cases} \tag{45}$$

So, the boundary conditions (32) become:

$$C = \begin{cases} C^J, \varphi = -F^J \psi, & -1<\psi<0 \\ C^D, \varphi = -F^J - F^D(\psi-1), & -\infty<\psi<-1 \end{cases} \tag{46}$$

$$\vec{\beta} = \begin{cases} \vec{\beta}^J, \varphi = -F^J \psi, & -1<\psi<0 \\ \vec{\beta}^D, \varphi = -F^J - F^D(\psi-1), & -\infty<\psi<-1 \end{cases} \tag{47}$$

Therefore, the transformation (36) separates the initial and boundary conditions for the large system (29) into initial-boundary value problem for auxiliary system (38) and the initial-boundary value problem for the lifting Eq. (37).

It is worth mentioning that the elementary wave speeds of the auxiliary system are linked with the wave speeds of the large system by

$$D = \frac{F+V}{C+V} \tag{48}$$

The eigenvalues of the large and auxiliary systems for β waves are related by:

$$\Lambda_k\left(C, \vec{\beta}\right) = \frac{F + 1/\lambda_k\left(\vec{\beta}\right)}{C + 1/\lambda_k\left(\vec{\beta}\right)}, \quad k = 2, 3, \ldots, n-1. \tag{49}$$

The phase transitions occurring during gas-based EOR displacements throughout the 1D reservoir are determined just by thermodynamics of the oil–gas system and are independent of transport properties.

The solution of the large system $\beta_i(x_D, t_D)$ realizes the mapping from the plane (x_D, t_D) to the set of tie lines in n-vertices simplex of n-component phase diagram. The image of the domain of the plane (x_D, t_D); $x_D>0$, $t_D>0$, defines 2D surfaces in the simplex. The auxiliary solution $\beta_i(\psi, \varphi)$ also maps the domain of the plane (ψ, φ), where the initial-boundary value problem is defined, into 2D surface in the simplex. From the splitting of the compositional model (29) into auxiliary (38) and lifting (37) problems follow that these surfaces coincide.

The auxiliary solution depends on thermodynamic functions α_i and β_i and on the composition fractions of the initial and boundary conditions. So, the 2D solution image in the simplex is independent of transport properties, i.e. fractional flow curves, relative phase permeability and phase viscosities.

2.3. Wag injection

During miscible WAG (water-alternate-gas) flooding, aqueous phase contains just water component, and oleic phase is an n-component mixture of the virgin oil with hydrocarbon components of the gaseous solvent:

$$\begin{aligned}&\frac{\partial s}{\partial t_D}+\frac{\partial f(s,\vec{c})}{\partial x_D}=0\\&\frac{\partial(\vec{c}s)}{\partial t_D}+\frac{\partial \vec{c}f(s,\vec{c})}{\partial x_D}=0\end{aligned}\qquad(50)$$

Here $\vec{c}$ is an n-vector of hydrocarbon components in the oleic phase and s is saturation of oleic phase. When gas composition in all slugs is the same, the problem (50) is equivalent to the case of binary oil–gas mixture. System (50) is mathematically equivalent to the system of multi component polymer flooding with no adsorption, $\vec{\alpha}(\vec{c})=0$. So, the introduction of potential (8) transforms the system (50) into the form

$$\frac{\partial \vec{c}(x_D,\varphi)}{\partial x_D}=0\qquad(51)$$

The proposed splitting technique significantly simplifies exact solution for miscible WAG if compared with that derived in Bedrikovetsky (1993).

2.4. Carbonised waterflooding

Displacement of oil by carbonised water is described by $(n+1)\times(n+1)$ hyperbolic system

$$\begin{aligned}&\frac{\partial s}{\partial t_D}+\frac{\partial f(s,\vec{c})}{\partial x_D}=0\\&\frac{\partial\left(\vec{c}s+\vec{b}(\vec{c})(1-s)\right)}{\partial t_D}+\frac{\partial\left(\vec{c}f(s,\vec{c})+\vec{b}(\vec{c})(1-f)\right)}{\partial x_D}=0\end{aligned}\qquad(52)$$

Here low concentration of gases in injected water $\vec{c}$ and low equilibrium concentration of gases in oil $\vec{b}(\vec{c})$ do not change overall volume balance of water and oil phases if compared with immiscible waterflooding.

The introduction of coordinates φ and ψ, (8) and (35), results in the following $(n)\times(n)$ auxiliary system

$$\frac{\partial \vec{b}(\vec{c})}{\partial\varphi}+\frac{\partial\left(\vec{c}-\vec{b}\right)}{\partial\Psi}=0.\qquad(53)$$

2.5. Hot waterflood with heat losses for surround formations

Displacement of oil by hot/cold water is described by a $(2)\times(2)$ hyperbolic system of quasi-linear equations of water volume balance and of heat balance for water–oil–rock system

$$\begin{aligned}&\frac{\partial s}{\partial t_D}+\frac{\partial f(s,T)}{\partial x_D}=0\\&\frac{\partial(T(s+b))}{\partial t_D}+\frac{\partial(T(f+h))}{\partial x_D}=\alpha(T-1)\end{aligned}\qquad(54)$$

where T is the temperature. A quasi steady state heat flux from the reservoir into surround formations (Newton's law) is assumed, and α is a heat transfer coefficient.

Introduction of potential φ (8) and $\psi=bx_D-ht_D$ results in the linear auxiliary equation

$$\frac{\partial T}{\partial\varphi}+\frac{\partial T}{\partial\Psi}=-\alpha(T-1)\qquad(55)$$

and the solution of the auxiliary problem (55) decreases along the characteristic lines $\varphi-\psi=$constant with decrement α. It allows derivation of the exact solution for alternate injection of hot and cold water in oil reservoir accounting for heat losses.

3. An analytical model for oil displacement by polymer slug with water drive

In this section the splitting technique is applied to the analytical modelling of oil displacement by a polymer slug with water drive. The same procedure may be applied to the solution of the problem of gas slug injection with lean gas drive.

We assume a linear sorption isotherm $a(c)=\Gamma c$. Typical fractional flow functions are shown in Fig. 2.

The chemical flooding problem with only one chemical component in solution is a $(2)\times(2)$ hyperbolic system. For the linear adsorption isotherm con-

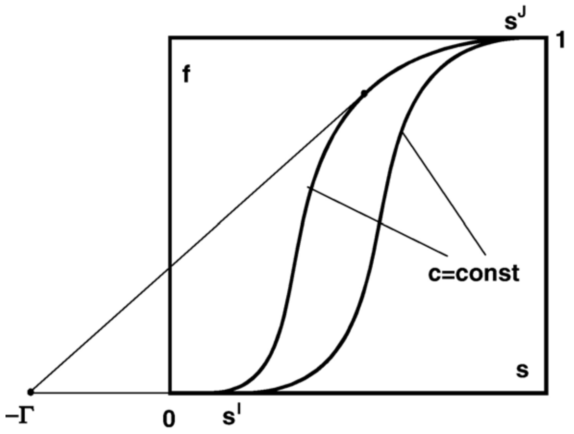

Fig. 2. Typical forms of fractional flow curves.

sidered here, the auxiliary system is a linear hyperbolic equation:

$$\Gamma\frac{\partial c}{\partial \varphi}+\frac{\partial c}{\partial x}=0 \tag{56}$$

subject to the initial and boundary conditions

$$\begin{aligned}&\varphi=-s^{\mathrm{I}}x_{\mathrm{D}}: c=0\\ &x_{\mathrm{D}}=0: c(0,\varphi)=\begin{cases}1, 0<\varphi<1\\ 0, 1<\varphi<+\infty\end{cases}\end{aligned} \tag{57}$$

The solution of the auxiliary problem is given by:

$$c(x_{\mathrm{D}},\varphi)=\begin{cases}0, -s^{\mathrm{I}}x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}}\\ 1, \Gamma x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}}+1\\ 0, \Gamma x_{\mathrm{D}}+1<\varphi<+\infty\end{cases} \tag{58}$$

For the sake of simplicity, we define two new dependent variables for the lifting Eq. (10):

$$U=\frac{1}{f}, F(U,\overrightarrow{c})=-\frac{s}{f} \tag{59}$$

that becomes

$$\frac{\partial U}{\partial x_{\mathrm{D}}}+\frac{\partial F(U,\overrightarrow{c})}{\partial \varphi}=0 \tag{60}$$

The lifting problem for these new variables corresponds to the following boundary conditions:

$$\begin{aligned}&x_{\mathrm{D}}=0; U=1\\ &\varphi=-s^{\mathrm{I}}x_{\mathrm{D}}: U=+\infty\end{aligned} \tag{61}$$

There are two discontinuities in the boundary conditions of this problem: at the points (0, 0) and (0, 1). The evolution of the discontinuity at the point (0, 0) is given by the path $s^{\mathrm{I}}\rightarrow 3\rightarrow 2\text{–}(-s^{\mathrm{J}})$. Fig. 3 shows the solution path through two fractional flow functions $f(s,c)$ in new variables U and F. The speed of the shock $s^{\mathrm{I}}\rightarrow 3$ is equal to $(-s^{\mathrm{I}})^{-1}$. The speed of the shock $3\rightarrow 2$ is $1/\Gamma$, and point 2 is a tangent point of the curve $F=F(U, c=1)$ and the straight line 2–3.

$$\frac{U_2-U_3}{F_2-F_3}=\frac{1}{F_U^{/}(U_2,1)}=\Gamma \tag{62}$$

The area between the fronts $\varphi=\Gamma x_{\mathrm{D}}$ and $\varphi=\Gamma x_{\mathrm{D}}+1$ is filled by the s-wave $2\text{–}(-s^{\mathrm{J}})$. The values U^{+} ahead of the front $\varphi=\Gamma x_{\mathrm{D}}+1$ are determined by the s-wave

$$U=U^{0}\left(\frac{\varphi'}{x'_{\mathrm{D}}}\right), F_U^{/}\left(U^{0}, c=1\right)=\frac{\varphi'}{x'_{\mathrm{D}}} \tag{63}$$

The points ahead of and behind the shock, U^{+} and U^{-}, are linked by the Hugoniot–Rankine conditions:

$$\Gamma=\frac{F(U^{+},1)-F(U^{-},0)}{U^{+}-U^{-}} \tag{64}$$

In the domain behind the shock $\varphi=\Gamma x_{\mathrm{D}}+1$, the values of U are constant along the s-characteristics:

$$\begin{aligned}&U(x_{\mathrm{D}},\varphi)=U^{-}(x'_{\mathrm{D}},\varphi')\\ &\frac{\varphi-\varphi'}{x_{\mathrm{D}}-x'_{\mathrm{D}}}=F_U^{/}(U^{-},0)\end{aligned} \tag{65}$$

Now we consider the s-characteristic passing through a point $(x_{\mathrm{D}}, \varphi)$ from the area behind the shock $\varphi=\Gamma x_{\mathrm{D}}+1$. This characteristic crosses the front $\varphi=\Gamma x_{\mathrm{D}}+1$ at the point $(x'_{\mathrm{D}}, \varphi')$ (Fig. 4). So, the system of four transcendental equations (63) (64) and (65)

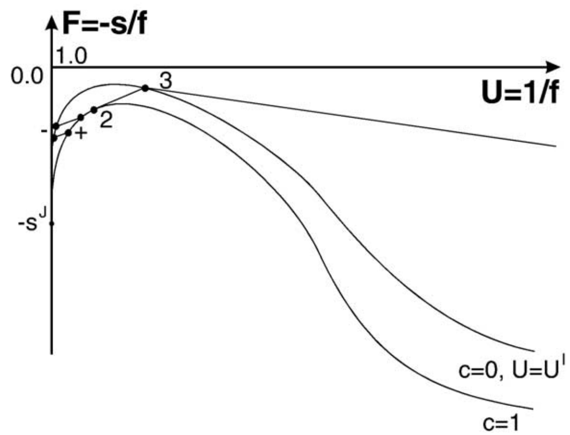

Fig. 3. The lifting problem in plane (U, F).

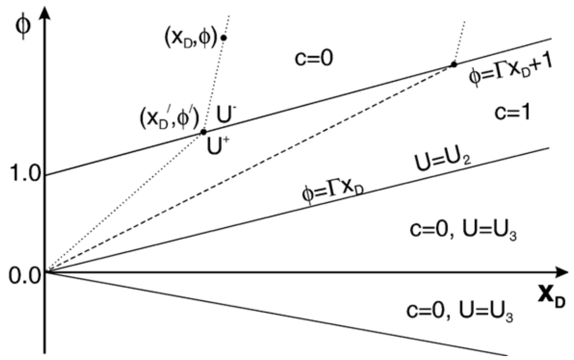

Fig. 4. Solution of the auxiliary and lifting problem.

determines the unknowns x'_{D}, φ', U^{-} and U^{+} for given x_{D} and φ.

The solution of the lifting problem is given by the formula:

$$U(x_{\mathrm{D}},\varphi)=\begin{cases} U_3 & -s^{\mathrm{I}}x<\varphi<\Gamma x_{\mathrm{D}} \\ U^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right) & \Gamma x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}}+1 \\ U^{-}(x_{\mathrm{D}},\varphi) & \Gamma x_{\mathrm{D}}+1<\varphi<+\infty \end{cases} \tag{66}$$

The expression of unknown s via U is obtained from (59):

$$s=-UF(U,c) \tag{67}$$

Finally, the solution $s(x_{\mathrm{D}},\varphi)$ is:

$$s(x_{\mathrm{D}},\varphi)=\begin{cases} s_3 & -s^{\mathrm{I}}x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}} \\ s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right) & \Gamma x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}}+1 \\ s^{-}(x_{\mathrm{D}},\varphi) & \Gamma x_{\mathrm{D}}+1<\varphi<+\infty \end{cases} \tag{68}$$

In order to invert the mapping (9), we calculate the variable $t_{\mathrm{D}}(x_{\mathrm{D}},\varphi)$ from (8). In the area ahead of the front $\varphi=\Gamma x_{\mathrm{D}}$, the dependent variables s and f are constant:

$$t_{\mathrm{D}}=\frac{1}{f_3}\int_0^{\varphi}d\varphi'+\frac{s_3}{f_3}\int_0^{x_{\mathrm{D}}}dx' \tag{69}$$

Repeating the integration in the area between fronts $\varphi=\Gamma x_{\mathrm{D}}$ and $\varphi=\Gamma x_{\mathrm{D}}+1$, where s and f are constant along each characteristic line, we get:

$$t_{\mathrm{D}}=\frac{\varphi}{f\left(s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right),1\right)}+\frac{s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right)}{f\left(s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right),1\right)}x_{\mathrm{D}} \tag{70}$$

Next we determine time t along the front $\varphi=\Gamma x_{\mathrm{D}}+1$. The expressions linking x_{D} and φ with the variable s ahead of the front are:

$$\varphi=\Gamma x_0(\varphi)+1$$
$$\frac{\varphi}{x_0(\varphi)}=\frac{f(s^+,1)-s^+f_s'(s^+,1)}{f_s'(s^+,1)} \tag{71}$$

From (71) follows the expression for $x_0(\varphi)$ in a parametric form:

$$x_0(s^+)=\frac{f'(s^+,1)}{f(s^+,1)-f'(s^+,1)(\Gamma+s^+)}$$
$$\varphi(s^+)=\frac{f'(s^+,1)-s^+f'(s^+,1)}{f(s^+,1)-f'(s^+,1)(\Gamma+s^+)} \tag{72}$$

then, along the front

$$t_{\mathrm{D}}=\frac{1}{f(s^+,1)-f'(s^+,1)(\Gamma+s^+)}$$
$$x_0(t_{\mathrm{D}})=\frac{f'(s^+,1)}{f(s^+,1)-f'(s^+,1)(\Gamma+s^+)} \tag{73}$$

Fig. 4 shows s-characteristics of the lifting equation ahead of and behind the rear front $x_0(\varphi)$.

The final expression for $t_{\mathrm{D}}(x_{\mathrm{D}},\varphi)$ is:

$$t_{\mathrm{D}}(x_{\mathrm{D}},\varphi)=\begin{cases} \dfrac{\varphi}{f_3}+\dfrac{s_3}{f_3}x_{\mathrm{D}} & -s^{\mathrm{I}}x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}} \\ \dfrac{\varphi}{f\left(s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right),1\right)}+\dfrac{s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right)}{f\left(s^0\left(\dfrac{\varphi}{x_{\mathrm{D}}}\right),1\right)}x_{\mathrm{D}} & \Gamma x_{\mathrm{D}}<\varphi<\Gamma x_{\mathrm{D}}+1 \\ \dfrac{\varphi}{f(s^{-}(x_{\mathrm{D}},\varphi),0)}+\dfrac{s^{-}(x_{\mathrm{D}},\varphi)}{f(s^{-}(x_{\mathrm{D}},\varphi),0)}x_{\mathrm{D}} & \Gamma x_{\mathrm{D}}+1<\varphi<+\infty \end{cases} \tag{74}$$

Finally, the solution for $c(x_{\mathrm{D}},t_{\mathrm{D}})$ and $s(x_{\mathrm{D}},t_{\mathrm{D}})$ is given by the following expressions:

$$c(x_{\mathrm{D}},t_{\mathrm{D}})=\begin{cases} 0, & \dfrac{(s_3-s^{\mathrm{I}})}{f_3}x_{\mathrm{D}}<t_{\mathrm{D}}<\dfrac{(s_3+\Gamma)}{f_3}x_{\mathrm{D}} \\ 1, & \dfrac{\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)+\Gamma\right)}{f\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)\right)}x_{\mathrm{D}}<t_{\mathrm{D}}<\dfrac{\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)+\Gamma\right)x_{\mathrm{D}}+f\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)\right)}{f\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)\right)} \\ 0, & \dfrac{(s^{-}(x_{\mathrm{D}},t_{\mathrm{D}})+\Gamma)x_{\mathrm{D}}+f(s^{-}(x_{\mathrm{D}},t_{\mathrm{D}}))}{f(s^{-}(x_{\mathrm{D}},t_{\mathrm{D}}))}<t_{\mathrm{D}}<+\infty \end{cases} \tag{75}$$

$$s(x_{\mathrm{D}},t_{\mathrm{D}})=\begin{cases} s_3 & \dfrac{(s_3-s^{\mathrm{I}})}{f_3}x_{\mathrm{D}}<t_{\mathrm{D}}<\dfrac{(s_3+\Gamma)}{f_3}x_{\mathrm{D}} \\ s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right) & \dfrac{\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)+\Gamma\right)}{f\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)\right)}x_{\mathrm{D}}<t_{\mathrm{D}}<\dfrac{\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)+\Gamma\right)x_{\mathrm{D}}+f\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)\right)}{f\left(s^0\left(\dfrac{x_{\mathrm{D}}}{t_{\mathrm{D}}}\right)\right)} \\ s^{-}(x_{\mathrm{D}},t_{\mathrm{D}}) & \dfrac{(s^{-}(x_{\mathrm{D}},t_{\mathrm{D}})+\Gamma)x_{\mathrm{D}}+f(s^{-}(x_{\mathrm{D}}t_{\mathrm{D}}))}{f(s^{-}(x_{\mathrm{D}},t_{\mathrm{D}}))}<t_{\mathrm{D}}<+\infty \end{cases} \tag{76}$$

From now we use the following dimensionless space and time:

$$x_D = \frac{\Phi x}{\Delta}, t_D = \frac{ut}{\Delta} \tag{77}$$

where Δ is the slug volume.

The graphical solution of the problem (5), (6) is presented in Fig. 5. Fig. 6 shows movements of concentration and saturation fronts in plane (x_D, t_D). The shock speeds D_2 and D_3 are given by:

$$\begin{aligned} D_2 &= \frac{f_2}{s_2 + \Gamma} = \frac{f_3}{s_3 \Gamma} \\ D_3 &= \frac{f_3}{s_3 - s^I} \end{aligned} \tag{78}$$

and are obtained graphically in plane (s, f). Here D_2 is the velocity of the oil bank, D_3 is the slug front velocity.

Trajectory of the rear slug front is given by the parametric formulae (73)). The explicit dependency $x_0(t_D)$ can be found geometrically. Draw the tangent to the fractional flow curve $c=1$ at point $s^+(x_0)$ to meet axis f at point A and axis s at point B. Then

$$A_0 = \frac{1}{t_D}, B_0 = \frac{1}{x_0(t_D)} \tag{79}$$

Let us fix time t_D and calculate A_0. From (79) it follows that if the segment A_0 is marked up and the tangent to the curve $c=1$ is drawn from the point A, then it meets the curve 1 at the point $s^+(x_0)$, and the intersection with the axes s at point B defines the coordinate $x_0(t_D)$. The straight line $(-\Gamma)$–$s^+(x_0)$ is

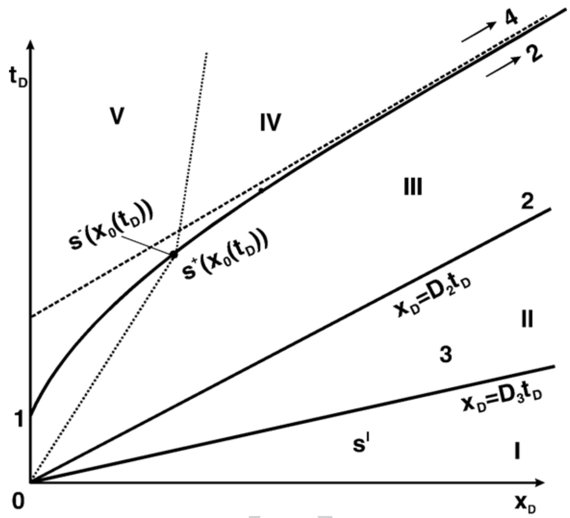

Fig. 6. Trajectories of fronts in plane (x_D, t_D) for the slug problem.

then produced to meet the curve $c=0$ at point $s^-(x_0)$.

The structure of the displacement zone during polymer slug injection (Fig. 6) is:

I. Zone of displaced oil, $c=0$, $s=s^I$;
II. Water–oil bank formed ahead of the slug, $c=0$, $s=s_3$, velocity of the leading front of the bank is D_3;
III. Polymer slug, $c=1$, saturation decreases from $s^+(x_0)$ ahead of the rear front of the slug up to s_2 on the leading front of the slug; the leading slug front velocity is equal to D_2;
IV. Water drive zone with mobile oil; $c=0$, saturation decreases from s^J at the stagnant front up to $s^-(x_0)$ behind the rear slug front; the position of the stagnant front is determined by equality $s^-(x_0)=s^J$;
V. Water drive zone with immobile oil, $s=s^J$.

Sizes of the first, second and fourth zones grow unlimitedly. The slug size grows with time and stabilizes at $t_D \gg 1$. Saturation in slug tends to s_3; it allows calculating the limit of the slug size from the polymer mass conservation $-1/(s_3+\Gamma)$. The thickness of the water drive zone with immobile oil becomes constant after the slug rear front passes this zone.

Stabilization of the slug volume with time results in different outcomes, depending on the flow geometry. In case of linear flow (rows of injectors and producers), since the slug volume is proportional to the distance between the leading and rear slug fronts, the slug thickness stabilizes. For radial flow with injection in a single well $x=r^2/2$, and slug

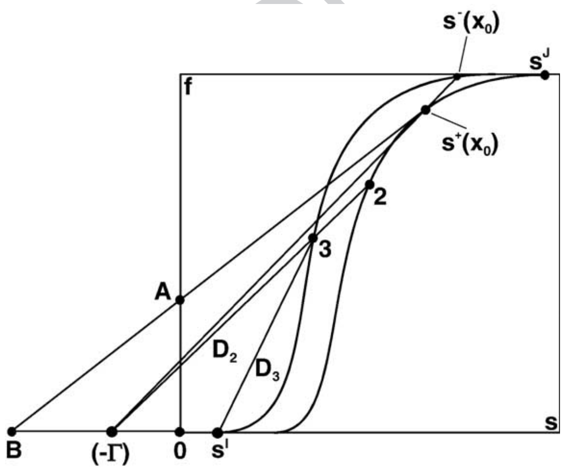

Fig. 5. Solution of the slug problem in the phase plane (s, f).

thickness tends to zero with order $(t_D)^{1/2}$. This fact should be considered when designing the slug size preventing the slug destruction by more mobile driving water.

Compared with waterflooding, the use of a polymer slug increases the period of water free production, reduces the water cut at initial water drive period, and enhances the ultimate displacement at a stage after breakthrough. Water drive does not disturb the flow ahead of the oil bank and in the front part of the slug.

For low sorption (small Γ), the slug injection results in prolongation of water free production while for high sorption slug injection does not change water free period, if compared with waterflooding.

4. Applications

The obtained analytical models for 1D gas injection and polymer slug flood can be used in streamline modelling. The structure of the displacement zone, as obtained from the exact solution, can be used for the interpretation of laboratory and field data.

For an n-component polymer flooding test, the auxiliary system (11) can be used to determine sorption isotherms of each component through relationships linking the sorption isotherms with breakthrough component concentrations $c_i(1, t_D)$ measured during the continuous chemical injection. Integrating the left hand side of the auxiliary system over the closed triangle with vortexes in points (0, 0), (1, 0) and (1, φ) with Green's formula

$$\int_\Delta \int \left(\frac{\partial \vec{a}(\vec{c})}{\partial \varphi} + \frac{\partial \vec{a}}{\partial x_D}\right) dx_D d\varphi = \int_{\partial \Delta} \vec{c} d\varphi - \vec{a}(\vec{c}) dx_D \qquad (80)$$

The right hand side integral over the side (0, 0)–(1, 0) is equal zero due to initial conditions, where all concentrations are zero. The integral over the side (0, 1)–(1, φ) is equal to the mass of the i-th component during the production of the volume φ of water. The solution of the auxiliary system for continuous polymer injection is self-similar, so $\vec{c}$ is constant along (0, 0)–(1, φ). The right hand side of the integral (80) is equal zero:

$$\int_0^\varphi \vec{c}(1,y)dy - \vec{c}(1,\varphi)\varphi + \vec{a}(\vec{c})(1,\varphi) = 0 \qquad (81)$$

The expression above allows calculating $\vec{a}(\vec{c})$ for each value of breakthrough concentrations $\vec{c}(1,\varphi)$. The function $\vec{a}(\vec{c})$ is calculated by (81) only along the trajectory $\vec{c}(1,\varphi)$, i.e. sorption isotherms can be determined only for measured concentrations during the test.

Splitting of compositional model into thermodynamics and hydrodynamics equations can be used for testing numerical 1D models. For example, in order to test a polymer simulator, we model two cases that differ from each other by oil viscosity. The time-dependencies of accumulated water production $\varphi(1, t_D)$ and of outlet concentrations $c_i(1, t_D)$ must be different for the two cases, but the outlet concentrations versus accumulated water production $c_i(1,\varphi)$ must be the same. The concentrations $c_i(1,\varphi)$ must be the same for different oil and water viscosities, relative permeabilities and resistance factors that could vary in wide intervals during the model testing. The concentration equality allows validation of the numerical simulator.

The problem of the compatibility of polymer with formation water can be overcome by the injection of a compatible water slug before the polymer slug injection. In order to avoid contact between the polymer and the formation water, the polymer front should not bypass the compatible waterfront before they both reach the production row $x_D = 1$, which could be achieved by the injection of a sufficient volume of compatible water. Determination of the minimum water slug size can be achieved by the solution of the auxiliary system only—if the polymer and compatible water fronts do not meet for $x_D < 1$ in the solution of the auxiliary system, they also do not meet in the solution of the general system.

Design of injection gas composition and minimum miscibility pressure calculations may be performed using the auxiliary system only and does not involve transport properties of rock and fluids.

5. Summary and conclusions

The $(n+1) \times (n+1)$ system of conservation laws for two-phase n-component chemical flooding in porous media with adsorption can be splitted into an $(n) \times (n)$ auxiliary system and one independent lifting equation. The splitting is obtained from the change of independent variables (x_D, t_D) to (x_D, φ). This change of coordinates also transforms the water conservation law into the lifting equation. In the case of gas/solvent injection, the $(n-1) \times (n-1)$ system of conservation laws is splitted into an $(n-2)\times(n-2)$ auxiliary system and one independent lifting equation through the change of independent variables (x_D, t_D) to flow potentials (ψ, φ). This change of coordinates transforms the conservation law for the n-th component into the lifting equation.

The lifting procedure for the solution of the large system consists of:

- Solution of the auxiliary system;
- Solution of the lifting equation;
- Inverse transformation of independent variables.

The auxiliary system contains only equilibrium thermodynamic variables, while the large system contains both hydrodynamic (phases relative permeabilities and viscosities) functions and equilibrium thermodynamic variables. Therefore, phase transitions occurring during displacement are determined by the auxiliary system, i.e. they are independent of hydrodynamic properties of fluids and rock. For example, the minimum miscibility pressure (MMP) is independent of relative permeabilities and phase viscosities.

Nomenclature

a_i	Concentration of i-th adsorbed component
b_i	Equilibrium concentration
c_i	Chemical concentration in water, volumetric fraction
C	Overall volumetric fraction of n-th component
C_i	Overall volumetric fraction of i-th component
D	Shock speed for the large system
f	Liquid fractional flow
F	Overall volumetric fractional flow of n-th component
F_i	Overall volumetric fractional flow of i-th component
$\vec{g}$	Vector of independent fractions of gas phase
G	Gas phase composition
k_r	Relative permeability
l	Reservoir size
L	Liquid phase composition
n	Number of components
s	Saturation
S	Volumetric liquid fraction
t	Time
T	Temperature
t_D	Dimensionless time
u	Total flux
V	Shock speed for the auxiliary system
x	Distance
x_0	Position of rear slug front
x_D	Dimensionless distance

Greek letters

α	Geometric parameter of thermodynamic equilibrium
β	Geometric parameter of thermodynamic equilibrium
Δ	Polymer slug volume, solvent slug volume
Φ	Porosity
Γ	Proportionality coefficient
φ	Potential
λ	Eigenvalue of auxiliary system
Λ	Eigenvalue of large system
μ	Viscosity
Θ	Transformation of independent variables
Ω	Closed domain
ψ	Flow potential of overall flux

Subscripts

g	Gas phase
i	Component index
k	Wave index
l	Liquid phase
o	Oil phase
w	Water phase

Superscripts

+	Value ahead of the shock
−	Value behind the shock
D	Drive condition
I	Initial condition
J	Injection condition
L	Behind the slug
R	Inside the slug

Acknowledgements

Authors are grateful to Dr. A. Mailibaev (Institute of Mechanics, Moscow Lomonosov State University) for clearing the physical meaning of the potential and auxiliary system, to Prof. Y. Yortsos (Southern California University) and Prof. D. Marchesin (Institute of Pure and Applied Mathematics, Brazil) for fruitful discussions.

A. P. Pires also thanks the Brazilian Petroleum National Agency (ANP) for partial sponsorship of his Ph.D. research.

Authors thank Eneida Pires, Themis Carageorgos and Nina Gade for support and encouragement.

Appendix A. Proof of splitting for chemical flooding

If $s(x_D, t_D)$, $c_i(x_D, t_D)$, $i = 1, 2, \ldots, n$ is a solution of system (2), and $\varphi(x_D, t_D)$ is the potential function (8), then the function $c_i(x_D, \varphi)$ obeys the following conservation law:

$$\oint_{\partial\Omega} c_i d\varphi - a_i dx_D = 0 \tag{A-1}$$

where Ω is a closed domain $\Omega \subset R^2$.

System (2) can be derived from the following conservation laws in the integral form:

$$\oint_{\partial\Omega} (c_i f) dt_D - (c_i s + a_i) dx_D = 0 \tag{A-2}$$

Applying the definition of the potential (8) in (A-2):

$$\oint_{\partial\Omega} c_i (f dt_D - s dx_D) - a_i dx_D = \oint_{\partial\Omega} c_i d\varphi - a_i dx_D = 0 \tag{A-3}$$

In domains Ω where the solution is a smooth function, from the integral conservation laws (A-3) follows the system of partial differential equations (11). In narrow domains around shock trajectories, from (A-3) follows the Hugoniot–Rankine conditions.

Appendix B. Proof of splitting for gas flooding

If $C(x_D, t_D)$, $\beta_i(x_D, t_D)$, $i = 2, 3, \ldots, n-1$ is a solution of system (26), and $\varphi(x_D, t_D)$ and $\psi(x_D, t_D)$ are the potential functions (34) and (35), then the function $\beta_i(\psi, \varphi)$ obeys the following conservation law:

$$\oint_{v\partial\Omega} \alpha_i d\varphi - \beta_i d\psi = 0 \tag{B-1}$$

where Ω is a closed domain $\Omega \subset R^2$.

The system (29) was derived from the conservation law of i-th component volume balance in the integral form:

$$\oint_{\partial\Omega} (\alpha_i F + \beta_i) dt_D - (\alpha_i C + \beta_i) dx_D = 0 \tag{B-2}$$

From (B-2), and using the definition of potentials (34) and (35), we obtain:

$$\oint_{\partial\Omega} \alpha_i (F dt_D - C dx_D) - \beta_i (dx_D - dt_D) = \oint_{\partial\Omega} \alpha_i d\varphi - \beta_i d\psi = 0 \tag{B-3}$$

In domains Ω where the solution is a smooth function, from the integral conservation law (B-3) follows the system of partial differential equations (38). In narrow domains around shock trajectories, from (B-3) follows the Hugoniot–Rankine conditions.

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