



# Optimal spatially fixed and moving virtual sensing algorithms for local active noise control

CORNELIS D. PETERSEN

School of Mechanical Engineering  
The University of Adelaide  
South Australia 5005  
Australia

*A thesis submitted in fulfillment of the requirements  
for the degree of Ph. D. in Mechanical Engineering  
on 15 June 2007. Qualified on 27 July 2007.*

**Ph. D. Thesis**

15 June 2007

Active Noise and Vibration Control Group  
School of Mechanical Engineering  
The University of Adelaide  
SA 5005 Australia

Typeset by the author with the  $\text{\LaTeX}$ .  
Printed in Australia.

Copyright © 2007, The University of Adelaide, South Australia.  
*All rights reserved. No part of this report may be used or reproduced in any form or by any means, or stored in a database or retrieval system without prior written permission of the university except in the case of brief quotations embodied in critical articles and reviews.*

# Abstract

Local active noise control systems aim to create zones of quiet at specific locations within a sound field. The created zones of quiet generally tend to be small, especially for higher frequencies, and are usually centred at the error sensors. For an observer to experience significant reductions in the noise, the error sensors therefore have to be placed relatively close to an observer's ears, which is not always feasible or convenient. Virtual sensing methods have been proposed to overcome these problems that have limited the scope of successful local active noise control applications. These methods require non-intrusive sensors that are placed remotely from the desired locations of maximum attenuation. These non-intrusive sensors are used to provide an estimate of the sound pressures at these locations, which can then be minimised by a local active noise control system. This effectively moves the zones of quiet away from the physical locations of the transducers to the desired locations of maximum attenuation, such as a person's ears.

A number of virtual sensing algorithms have been proposed previously. The difference between these algorithms is the structure that is assumed to compute an estimate of the virtual error signals. The question now arises as to whether there is an optimal structure that can be used to solve the virtual sensing problem, which amounts to a linear estimation problem. It is well-known that the Kalman filter provides an optimal structure for solving such problems. An optimal solution to the virtual sensing problem is therefore derived in this thesis using Kalman filtering theory. The proposed algorithm is implemented on an acoustic duct arrangement to demonstrate its effectiveness. The presented experimental results indicate that the zone of quiet was effectively moved away from the physical sensor towards the desired location of maximum attenuation.

The previously proposed virtual sensing algorithms have been developed with the aim to create zones of quiet at virtual locations that are assumed spatially fixed within the sound field. Because an observer is very likely to move their head, the desired locations of the zones of quiet are generally moving through the sound field rather than being spatially fixed. For effective control, a local active noise control system incorporating a virtual sensing method thus has to be able to create moving zones of quiet that track the observer's ears. A moving virtual sensing method is therefore developed in this thesis that can be used to estimate the error signals at virtual locations that are moving through the sound field. It is shown that an optimal solution to the moving virtual sensing problem can be derived using Kalman filtering theory. A practical implementation of the developed algorithm is combined with an adaptive feedforward control algorithm and implemented on an acoustic duct arrangement. The presented experimental results illustrate that a narrowband moving zone of quiet that tracks the desired location of maximum attenuation has successfully been created.

# Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, all the material that is presented in this thesis is my own original work and has not been published previously for the award of any other degree or diploma in any university or other tertiary institution. If accepted for the award of the degree of Doctor of Philosophy in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

Cornelis D. Petersen

# Acknowledgements

I would like to acknowledge all the people that have made a contribution to the work presented in this thesis, or that have supported me during my post-graduate studies. First of all, I would like to thank my supervisors Dr Anthony Zander, Dr Ben Cazzolato and Professor Colin Hansen for their input and for proof-reading this thesis during busy times. I would also like to acknowledge the input of Dr Rufus Fraanje. The discussions we had via e-mail were very insightful and have contributed significantly to the work presented here. I would like to thank the people from the electronics workshop, especially George Osborne, who helped me with my experimental work. I also gratefully acknowledge the University of Adelaide for providing an ASI scholarship, and the Australian Research Council for supporting this research.

My time as a post-graduate student would not have been so much fun would I have not shared an office with Will, Rohin and Zebb. Will, without your  $\LaTeX$  expertise and hours, probably even days, of help with the lay-out of this thesis, it would probably have looked less nice than it does today. Besides that, I have enjoyed sharing the office with you and using your i-Pod to annoy Rohin with pirate music and Sigur Ros. Rohin, thanks for the insightful discussions that helped me with the research presented in this thesis, and also for letting me annoy you with pirate music and Sigur Ros.

I would like to thank my parents, Kees and Marijke, and my sister, Marlies. They have always supported me in everything that I do and gave me every opportunity to get where I am now.

Most of all, I would like to thank my beautiful partner Kerry for giving me the endless support and encouragement that I needed during the final stages of my post-graduate studies, and for everything else that we share.

# Contents

<b>Abstract</b>	<b>iii</b>
<b>Statement of originality</b>	<b>v</b>
<b>Acknowledgements</b>	<b>vii</b>
<b>List of Figures</b>	<b>xiii</b>
<b>Nomenclature</b>	<b>xix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Background . . . . .	1
1.2 Literature review . . . . .	5
1.2.1 Global active noise control . . . . .	5
1.2.2 Local active noise control . . . . .	7
1.2.3 Energy density sensing . . . . .	11
1.2.4 Virtual sensing . . . . .	13
1.3 Motivation and contributions . . . . .	25
1.3.1 Motivation and aim of research . . . . .	25
1.3.2 Contributions . . . . .	26
1.4 Outline of thesis . . . . .	28
1.5 Publications . . . . .	32
<b>I Algorithms for active noise control at virtual sensors</b>	<b>35</b>
<b>2 Active noise control algorithms</b>	<b>37</b>
2.1 Introduction . . . . .	37
2.2 Problem definition and assumptions . . . . .	38
2.2.1 Block diagram of active noise control system . . . . .	39
2.2.2 Standard state-space model . . . . .	45
2.2.3 Covariance properties . . . . .	46
2.3 Optimal narrowband control . . . . .	47

---

2.4	Optimal broadband control . . . . .	49
2.4.1	Optimal feedforward control . . . . .	50
2.4.2	Optimal feedback control . . . . .	59
2.4.3	General feedforward/feedback control . . . . .	61
2.5	Adaptive feedforward control . . . . .	63
2.5.1	Filtered-x LMS algorithm . . . . .	63
2.5.2	Normalised filtered-x LMS algorithm . . . . .	67
2.5.3	Filtered-x RLS algorithm . . . . .	67
2.5.4	Generating the virtual filtered-reference signals . . . . .	68
2.6	Conclusion . . . . .	69
<b>3</b>	<b>Spatially fixed virtual sensing algorithms</b>	<b>71</b>
3.1	Introduction . . . . .	71
3.2	Problem description . . . . .	74
3.3	Adaptive LMS virtual microphone technique . . . . .	75
3.3.1	Algorithm structure . . . . .	75
3.3.2	Optimal Wiener solution for physical sensor weights . . . . .	78
3.3.3	Extension to multiple spatially fixed virtual sensors . . . . .	83
3.3.4	Optimal solution from state-space model . . . . .	85
3.4	Remote microphone technique . . . . .	89
3.4.1	Algorithm structure . . . . .	90
3.4.2	Optimal estimation without measurement noise . . . . .	91
3.4.3	Optimal estimation with measurement noise . . . . .	95
3.5	Virtual microphone arrangement . . . . .	98
3.5.1	Algorithm structure . . . . .	98
3.5.2	Estimation performance . . . . .	99
3.6	Hybrid adaptive feedforward observer . . . . .	100
3.6.1	Algorithm structure . . . . .	100
3.6.2	Analysis of proposed observer method . . . . .	102
3.7	Kalman filter based spatially fixed virtual sensing algorithm . . . . .	104
3.7.1	Covariance and correlation properties . . . . .	105
3.7.2	Prediction and time/measurement update form . . . . .	108
3.7.3	Linear estimation given the innovations process . . . . .	111
3.7.4	Virtual sensing algorithm . . . . .	113
3.7.5	Practical implementation . . . . .	119
3.8	Conclusion . . . . .	122

---

<b>4</b>	<b>Active noise control at spatially fixed virtual sensors</b>	<b>127</b>
4.1	Introduction . . . . .	127
4.2	Optimal narrowband control at virtual sensors . . . . .	129
4.2.1	Quadratic optimisation . . . . .	129
4.2.2	Fully determined and under-determined cases . . . . .	130
4.3	Adaptive feedforward control at virtual sensors . . . . .	135
4.3.1	Block diagram of modified implementation . . . . .	136
4.3.2	Adaptive LMS virtual microphone technique . . . . .	139
4.3.3	Remote microphone technique . . . . .	141
4.3.4	Virtual microphone arrangement . . . . .	145
4.3.5	Virtual secondary transfer path equalisation . . . . .	147
4.4	Optimal feedback control at virtual sensors . . . . .	149
4.4.1	Causal Wiener solution . . . . .	150
4.4.2	State-space solution . . . . .	153
4.5	Conclusion . . . . .	155
<b>5</b>	<b>Active noise control at moving physical and virtual sensors</b>	<b>157</b>
5.1	Introduction . . . . .	157
5.2	Problem description and assumptions . . . . .	159
5.2.1	Desired moving locations of zones of quiet . . . . .	159
5.2.2	Plant model and covariance properties . . . . .	160
5.2.3	Non-stationary virtual primary disturbances . . . . .	163
5.3	Active noise control at moving physical sensors . . . . .	165
5.3.1	Adaptive feedforward control at moving physical sensors . . . . .	167
5.3.2	Practical implementation . . . . .	168
5.3.3	Tracking of non-stationarities . . . . .	175
5.4	Moving virtual sensing algorithms . . . . .	175
5.4.1	Problem definition and assumptions . . . . .	177
5.4.2	Adaptive LMS moving virtual microphone technique . . . . .	178
5.4.3	Remote moving microphone technique . . . . .	184
5.4.4	Kalman filter based moving virtual sensing algorithm . . . . .	187
5.5	Conclusion . . . . .	192
<b>II</b>	<b>Experimental validation of algorithms</b>	<b>195</b>
<b>6</b>	<b>Acoustic duct arrangement</b>	<b>197</b>
6.1	Introduction . . . . .	197
6.2	Acoustic duct arrangement . . . . .	197
6.3	Numerical modelling . . . . .	200
6.3.1	Travelling wave model . . . . .	201
6.3.2	Modal model . . . . .	204

---

6.3.3	Comparison of models . . . . .	205
6.4	System identification . . . . .	206
6.4.1	Subspace model identification . . . . .	207
6.4.2	Identification example . . . . .	208
6.5	Conclusion . . . . .	212
<b>7</b>	<b>Adaptive LMS virtual microphone technique</b>	<b>213</b>
7.1	Introduction . . . . .	213
7.2	Narrowband estimation . . . . .	215
7.2.1	Analytical solution from travelling wave model . . . . .	216
7.2.2	Travelling wave decomposition . . . . .	217
7.3	Broadband estimation performance . . . . .	219
7.3.1	Numerical broadband estimation performance . . . . .	220
7.3.2	Experimental broadband estimation performance . . . . .	226
7.4	Narrowband adaptive feedforward control . . . . .	232
7.4.1	Narrowband control performance versus frequency . . . . .	234
7.4.2	Narrowband control performance versus virtual distance . . . . .	237
7.5	Broadband adaptive feedforward control . . . . .	242
7.6	Forward difference prediction techniques . . . . .	246
7.7	Conclusion . . . . .	258
<b>8</b>	<b>Kalman filter based spatially fixed virtual sensing algorithm</b>	<b>261</b>
8.1	Introduction . . . . .	261
8.2	Acoustic duct arrangement . . . . .	262
8.3	Broadband estimation performance . . . . .	265
8.4	Narrowband adaptive feedforward control . . . . .	268
8.5	Broadband adaptive feedforward control . . . . .	274
8.6	Conclusion . . . . .	279
<b>9</b>	<b>Narrowband active noise control at a moving physical sensor</b>	<b>281</b>
9.1	Introduction . . . . .	281
9.2	Acoustic duct arrangement . . . . .	282
9.2.1	Adaptive feedforward control implementation . . . . .	283
9.2.2	Speed of physical sensor and spatial characteristics of sound field . . . . .	287
9.2.3	Preliminary identification stage . . . . .	290
9.3	Filtered-x LMS algorithm . . . . .	290
9.4	Normalised filtered-x LMS algorithm . . . . .	293
9.5	Filtered-x RLS algorithm . . . . .	296
9.6	Tracking performance comparison . . . . .	298
9.6.1	Slow moving physical sensor with period $T_v = 10$ s . . . . .	299
9.6.2	Medium pace moving physical sensor with period $T_v = 5$ s . . . . .	301
9.6.3	Fast moving physical sensor with period $T_v = 2.5$ s . . . . .	303

9.7	Conclusion . . . . .	305
<b>10</b>	<b>Narrowband active noise control at a moving virtual sensor</b>	<b>307</b>
10.1	Introduction . . . . .	307
10.2	Acoustic duct arrangement . . . . .	308
10.2.1	Adaptive feedforward control implementation . . . . .	309
10.2.2	Moving virtual sensing implementation . . . . .	311
10.2.3	Speed of virtual sensor and spatial characteristics of sound field	312
10.3	Kalman filter based moving virtual sensing algorithm . . . . .	312
10.3.1	Preliminary identification stage . . . . .	313
10.3.2	Real-time experimental results . . . . .	315
10.4	Adaptive LMS moving virtual microphone technique . . . . .	324
10.4.1	Preliminary identification stage . . . . .	324
10.4.2	Real-time experimental results . . . . .	325
10.5	Comparison . . . . .	332
10.6	Conclusion . . . . .	333
<b>11</b>	<b>Conclusions &amp; Future research</b>	<b>335</b>
11.1	Conclusions . . . . .	335
11.2	Future research . . . . .	340
<b>A</b>	<b>Proof of causal Wiener solutions</b>	<b>343</b>
A.1	Proof of causal Wiener Theorem 2.1 . . . . .	343
A.2	Proof of Theorem 3.2 . . . . .	347
<b>B</b>	<b>State-space solutions of virtual sensing algorithms</b>	<b>351</b>
B.1	Calculus with state-space realisations . . . . .	351
B.2	State-space solution of the remote microphone technique . . . . .	358
B.3	Comparison to Kalman filter based algorithm . . . . .	362
B.4	State-space solution of the virtual microphone arrangement . . . . .	364
	<b>References</b>	<b>365</b>

# List of Figures

1.1	Schematic representations of the zone of quiet created by a local active noise control system using a <i>traditional</i> sensing method or a <i>virtual</i> sensing method.	4
1.2	Electronic sound absorber used in an airplane or automobile to reduce the noise in the vicinity of the occupant's head.	8
1.3	Local active noise control system assembled on a headrest of a passenger's seat.	10
2.1	Block diagram of an active noise control system incorporating a virtual sensing algorithm.	40
2.2	Block diagram of the multi-channel quadratic optimisation problem.	47
2.3	Block diagram of the <i>optimal feedforward control problem</i> .	50
2.4	Block diagram of the <i>optimal feedback control problem</i> .	59
2.5	Block diagram of the internal model control parameterisation for optimal feedback control.	60
2.6	Block diagram of the <i>general feedforward/feedback control problem</i> .	62
2.7	Block diagram of the multi-channel filtered-x LMS algorithm.	64
3.1	Block diagram of the original implementation of the adaptive LMS virtual microphone technique.	75
3.2	Block diagram of the LMS algorithm as applied in the adaptive LMS virtual microphone technique.	76
3.3	Block diagram of the modified implementation of the adaptive LMS virtual microphone technique.	78
3.4	Block diagram of the filter problem that is solved to derive the optimal Wiener solution for the physical sensor weights.	79
3.5	Vector diagram illustrating the tonal estimation problem.	83
3.6	Block diagram of the remote microphone technique without measurement noise.	90
3.7	Block diagram for determining a causal Wiener solution for the filter $\mathbf{H}$ .	92
3.8	Block diagram of the remote microphone technique including measurement noise.	96
3.9	Block diagram of the virtual microphone arrangement.	99

3.10	Block diagram of the hybrid adaptive feedforward observer. . . . .	101
3.11	Block diagram of the Kalman filter based spatially fixed virtual sensing algorithm. . . . .	118
4.1	Block diagram of the remote microphone technique for the case of tonal disturbances. . . . .	132
4.2	Contour plot of expected attenuation. . . . .	135
4.3	Block diagram of the multi-channel filtered-x LMS algorithm, using a virtual sensing algorithm to compute an estimate of the virtual error signals. . . .	136
4.4	Block diagram of the <i>equivalent optimal feedforward control problem</i> that is solved to determine the optimal control performance that can be obtained at the virtual locations when adaptively minimising an estimate of the virtual error signals. . . . .	138
4.5	Block diagram for determining the optimal broadband feedforward control performance that can be obtained at the virtual locations when using the <i>adaptive LMS virtual microphone technique</i> . . . . .	140
4.6	Block diagram for determining the optimal broadband feedforward control performance that can be obtained at the virtual sensors when using the <i>remote microphone technique</i> . . . . .	142
4.7	Bode diagrams that illustrate Example 4.1. . . . .	145
4.8	Block diagram for determining the optimal broadband feedforward control performance that can be obtained at the virtual sensors when using the <i>virtual microphone arrangement</i> . . . . .	146
4.9	Block diagram of the virtual sensing algorithm based on virtual secondary transfer path equalisation. . . . .	147
4.10	Block diagram for determining a causal Wiener solution for the compensation filter $\mathbf{H}_u$ . . . . .	148
4.11	Block diagram of the <i>optimal feedback control at virtual sensors problem</i> . . . . .	150
4.12	Block diagram of the <i>optimal feedback control at virtual sensors problem</i> reformulated as an equivalent feedforward control problem. . . . .	151
4.13	Generalised plant formulation for the <i>optimal feedback control at virtual sensors problem</i> . . . . .	153
5.1	Illustration of the active control of engine-induced noise inside a mining vehicle cabin using two moving virtual sensors that track the driver's ears. . . . .	160
5.2	Block diagram of the multi-channel filtered-x LMS algorithm for the case of moving physical sensors that directly measure the virtual error signals at the moving virtual locations during real-time control. . . . .	167
5.3	Block diagram of the practical implementation of the multi-channel filtered-x LMS algorithm for the case of moving physical sensors that directly measure the virtual error signals at the moving virtual locations during real-time control. . . . .	170

---

5.4	Illustration of the active control of engine-induced noise inside a mining vehicle cabin using two moving virtual sensors that track the driver's ears.	173
5.5	Block diagram of the multi-channel filtered-x LMS algorithm including a moving virtual sensing algorithm.	176
5.6	Block diagram of the <i>adaptive LMS moving virtual microphone technique</i> .	179
5.7	Block diagram of the practical implementation of the <i>adaptive LMS moving virtual microphone technique</i> .	183
5.8	Block diagram of the <i>remote moving microphone technique</i> .	185
5.9	Block diagram for determining a time-variant optimal solution for the filter $\mathbf{H}(n)$ .	185
5.10	Block diagram of the practical implementation of the <i>remote moving microphone technique</i> .	187
5.11	Block diagram of the <i>Kalman filter based moving virtual sensing algorithm</i> .	188
5.12	Block diagram of the practical implementation of the <i>Kalman filter based moving virtual sensing algorithm</i> .	191
6.1	Schematic diagram of the experimental acoustic duct arrangement.	198
6.2	Schematic diagram of a rectangular acoustic duct.	201
6.3	Bode diagram of the estimated <i>deterministic</i> part of the plant.	210
6.4	Power spectral density plots of (a) the measured physical primary disturbance, its predicted estimate, and the associated innovation signal; and (b) the measured virtual primary disturbance, its predicted estimate $\hat{d}_v(n n-1)$ , and the associated innovation signal.	211
7.1	Schematic diagram of an acoustic duct of length with arbitrary termination conditions.	215
7.2	Travelling wave decomposition of the sound field inside an acoustic duct.	218
7.3	Schematic diagram of the acoustic duct implementation for the <i>adaptive LMS virtual microphone technique</i> .	219
7.4	Numerical optimal physical sensor weights plotted against virtual distance for the two, three, and five physical sensor configurations.	222
7.5	Numerical normalised mean-square virtual output error plotted against virtual distance for primary and secondary sound fields.	223
7.6	Numerical power spectral density plots of the virtual primary and secondary disturbances, and the virtual output error for the two, three, and five physical sensor configurations.	225
7.7	Numerical magnitude and phase plots of the transfer impedance between the virtual <i>primary</i> disturbance and its estimate, and between the virtual <i>secondary</i> disturbance and its estimate, for the two, three, and five physical sensor configurations.	226

7.8	Experimental optimal physical sensor weights and normalised mean-square virtual output error plotted against virtual distance, for the two, three, and five physical sensor configurations. . . . .	228
7.9	Experimental normalised mean-square virtual output error plotted against virtual distance for primary and secondary sound fields. . . . .	229
7.10	Experimental power spectral density plots of the virtual primary and secondary disturbances, and the virtual output error for the two, three, and five physical sensor configurations. . . . .	231
7.11	Experimental magnitude and phase plots of the frequency response function between the virtual <i>primary</i> disturbance and its estimate, and between the virtual <i>secondary</i> disturbance and its estimate, for the two, three, and five physical sensor configurations. . . . .	232
7.12	Schematic diagram of the acoustic duct implementation for the <i>adaptive LMS virtual microphone technique</i> . . . . .	233
7.13	Numerical and expected experimental narrowband attenuation plotted against frequency for the two, three, and five physical sensor configurations. . . . .	235
7.14	Magnitude and phase of the <i>ratio</i> of the complex transfer impedances $T_d$ and $T_y$ plotted against frequency for the two, three, and five physical sensor configurations. . . . .	237
7.15	Numerical and experimental sound pressure distribution plotted against virtual distance for excitation frequencies considered. . . . .	239
7.16	Numerical and experimental narrowband attenuation plotted against virtual distance for the three excitation frequencies considered. . . . .	240
7.17	Surface plot of the numerical narrowband attenuation plotted against frequency and virtual distance. . . . .	241
7.18	Numerical and real-time broadband feedforward control performance at virtual location after convergence of adaptive algorithm. . . . .	243
7.19	Experimental broadband feedforward control performance obtained while minimising the physical error signal. . . . .	244
7.20	Numerical and experimental primary and controlled sound pressure distributions plotted against virtual distance. . . . .	245
7.21	Numerical and experimental overall attenuation obtained at the virtual distance. . . . .	246
7.22	Linear forward difference prediction. . . . .	247
7.23	Analytical optimal weights and linear forward difference prediction weights plotted against frequency and virtual distance. . . . .	248
7.24	Over-constrained linear forward difference prediction. . . . .	249
7.25	Analytical optimal weights and over-constrained linear forward difference prediction weights plotted against frequency and virtual distance. . . . .	250
7.26	Quadratic forward difference prediction. . . . .	251

---

7.27	Analytical optimal weights and quadratic forward difference prediction weights plotted against frequency and virtual distance. . . . .	252
7.28	Numerical narrowband attenuation that can in theory be obtained with forward difference prediction techniques plotted against frequency and virtual distance. . . . .	253
7.29	Analytical optimal weights, linear forward difference prediction weights, and optimal broadband weights for the two physical sensor configuration plotted against frequency and virtual distance. . . . .	256
7.30	Numerical narrowband attenuation obtained using linear forward difference prediction weights or optimal broadband weights for the two physical sensor configuration, plotted against frequency and virtual distance. . . . .	257
8.1	Schematic diagram of the acoustic duct implementation for the <i>remote microphone technique</i> . . . . .	262
8.2	Numerical and experimental estimation performance of the Kalman filter based spatially fixed virtual sensing algorithm. . . . .	266
8.3	Numerical and experimental normalised mean-square virtual output error plotted against virtual distance. . . . .	268
8.4	Numerical and experimental sound pressure distributions plotted against virtual distance for the excitation frequencies considered. . . . .	271
8.5	Numerical and experimental narrowband attenuation plotted against virtual distance for the three excitation frequencies considered. . . . .	272
8.6	Numerical and real-time experimental broadband feedforward control performance at the virtual location after convergence of adaptive algorithm. . . . .	275
8.7	Experimental broadband feedforward control performance obtained while minimising the physical error signal. . . . .	276
8.8	Numerical and experimental primary and controlled sound pressure distributions plotted against virtual distance. . . . .	277
8.9	Numerical and experimental overall attenuation obtained at the virtual distance. . . . .	278
9.1	Schematic diagram of the acoustic duct implementation for the moving physical sensor case. . . . .	283
9.2	Measured spatial rate of change of primary and secondary sound fields over the target zone inside the acoustic duct. . . . .	289
9.3	Influence of the convergence coefficient $\mu$ on the tracking performance of the filtered-x LMS algorithm for narrowband control at a moving physical sensor. . . . .	292
9.4	Influence of the convergence coefficient $\alpha$ on the tracking performance of the <i>normalised filtered-x LMS algorithm</i> for narrowband control at a moving physical sensor. . . . .	295

9.5	Influence of the forgetting factor $\lambda$ on the tracking performance of the <i>filtered-x RLS algorithm</i> for narrowband control at a moving physical sensor.	297
9.6	Tracking performance of the <i>filtered-x LMS algorithm</i> , the <i>normalised filtered-x LMS algorithm</i> , and the <i>filtered-x RLS algorithm</i> for narrowband control at a moving physical sensor, with period $T_v = 10$ s.	300
9.7	Tracking performance of the <i>filtered-x LMS algorithm</i> , the <i>normalised filtered-x LMS algorithm</i> , and the <i>filtered-x RLS algorithm</i> for narrowband control at a moving physical sensor, with period $T_v = 5$ s.	302
9.8	Tracking performance of the <i>filtered-x LMS algorithm</i> , the <i>normalised filtered-x LMS algorithm</i> , and the <i>filtered-x RLS algorithm</i> for narrowband control at a moving physical sensor, with period $T_v = 2.5$ s.	304
10.1	Schematic diagram of the acoustic duct implementation for the moving virtual sensor case.	309
10.2	Narrowband control performance at the moving virtual location when using either a spatially fixed virtual sensor, a moving virtual sensor, or a moving physical sensor, with period $T_v = 10$ s.	317
10.3	Narrowband control performance at the moving virtual location when using either a spatially fixed virtual sensor, a moving virtual sensor, or a moving physical sensor, with period $T_v = 5$ s.	320
10.4	Narrowband control performance at the moving virtual location when using either a spatially fixed virtual sensor, a moving virtual sensor, or a moving physical sensor, with period $T_v = 2.5$ s.	322
10.5	Narrowband control performance at the moving virtual location when using either a spatially fixed virtual sensor, a moving virtual sensor, or a moving physical sensor, with period $T_v = 10$ s.	326
10.6	Narrowband control performance at the moving virtual location when using either a spatially fixed virtual sensor, a moving virtual sensor, or a moving physical sensor, with period $T_v = 5$ s.	328
10.7	Narrowband control performance at the moving virtual location when using either a spatially fixed virtual sensor, a moving virtual sensor, or a moving physical sensor, with period $T_v = 2.5$ s.	330

# Nomenclature

## Signals

$d_p$	physical primary disturbance
$d_v$	virtual primary disturbance
$e_p$	physical error signal
$e_v$	virtual error signal
$r_p$	physical filtered-reference signal
$r_v$	virtual filtered-reference signal
$s$	disturbance source signal
$u$	control signal
$x$	feedforward reference signal
$x_{fb}$	intrinsic feedback signal
$y_p$	physical secondary disturbance
$y_v$	virtual secondary disturbance
$z$	state
$\varepsilon_p$	physical innovation, physical output error
$\varepsilon_v$	virtual innovation, virtual output error
$\rho$	state estimation error

## Transfer paths and impedances, controllers and filters

$C$	feedback controller
$G$	arbitrary plant or system
$G_{ci}$	co-inner factor of $G$ , $G_{ci}G_{ci}^* = I$
$G_{co}$	co-outer factor of $G$ , $G_{co}G_{co}^* = GG^*$
$G_i$	inner factor of $G$ , $G_i^*G_i = I$
$G_o$	outer factor of $G$ , $G_o^*G_o = G^*G$
$G_{ci}^\perp$	co-inner factor perpendicular to $G_{ci}$ , $G_{ci}^\perp G_{ci}^{\perp*} = I$ and $G_{ci}G_{ci}^{\perp*} = 0$
$G_i^\perp$	inner factor perpendicular to $G_i$ , $G_i^{\perp*}G_i^\perp = I$ and $G_i^{\perp*}G_i = 0$

## NOMENCLATURE

---

$G_{ps}$	physical primary transfer path
$G_{pu}$	physical secondary transfer path
$G_{xs}$	detector transfer path
$G_{xu}$	intrinsic feedback path
$G_{vs}$	virtual primary transfer path
$G_{vu}$	virtual secondary transfer path
$H$	filter
$W$	feedforward controller
$Z_{ps}$	physical primary transfer impedance
$Z_{pu}$	physical secondary transfer impedance
$Z_{xs}$	detector transfer impedance
$Z_{xu}$	intrinsic feedback impedance
$Z_{vs}$	virtual primary transfer impedance
$Z_{vu}$	virtual secondary transfer impedance

### Symbols

$f$	frequency (Hz)
$h$	physical sensor weight
$n$	time index
$x_p$	spatial location of physical sensor/primary source
$x_s$	spatial location of secondary source
$x_v$	spatial location of virtual sensor
$w$	control filter coefficient
$z$	complex variable in the z-transform, unit shift forward operator
$I$	number of control filter coefficients
$J$	cost function
$K$	number of feedforward reference sensors
$L$	number of control sources
$M_p$	number of physical sensors
$M_v$	number of virtual sensors
$N$	order of state-space system
$P$	covariance of state estimation error, inverse correlation matrix
$Q$	covariance of process noise
$R$	covariance of measurement noise
$S$	cross-covariance between process and measurement noise
$\alpha$	convergence coefficient (normalised filtered-x LMS algorithm)
$\delta$	regularisation parameter (filtered-x RLS algorithm)
$\epsilon$	regularisation parameter (normalised filtered-x LMS algorithm)

---

$\eta$	attenuation (dB)
$\lambda$	forgetting factor (filtered-x RLS algorithm)
$\mu$	convergence coefficient (filtered-x LMS algorithm)
$\omega$	angular frequency (rad/s)
$\sigma$	variance
$\Pi$	covariance of states
$*$	discrete-time convolution
$\otimes$	Kronecker matrix product
$\mathbb{R}$	field of real numbers
$\mathbb{C}$	field of complex numbers
$\in$	belong to
$\perp$	perpendicular to
$\forall$	for all
$\square$	end of proof
$\sim$	defined by
$\triangleq$	defined as
$\approx$	approximately
$\simeq$	similar or equivalent
$\mathcal{RH}^{m \times n}$	set of all all rational $m \times n$ transfer function matrices with real coefficients and excluding singularities on the unit circle
$\mathcal{RH}_{\infty}^{m \times n}$	set of all asymptotically stable rational $m \times n$ transfer functions matrices with real coefficients
$\mathcal{RH}_p^{m \times n}$	set of all all rational proper $m \times n$ transfer function matrices with real coefficients
${}^{10}\log[\cdot]$	logarithm operator with base 10
$\text{var}[\cdot]$	variance operator
$E[\cdot]$	expectation operator
$[\cdot]^*$	adjoint operator
$[\cdot]_+$	causality operator
$[\cdot]_-$	non-causality operator
$\ \cdot\ _2$	$H_2$ -norm operator
$ \cdot $	magnitude operator
$\angle$	phase operator

**Matrix conventions**

$a$	scalar
$\mathbf{a}$	column vector
$\mathbf{a}^T$	transpose of $\mathbf{a}$
$\mathbf{a}^H$	complex conjugate transpose of $\mathbf{a}$
$\mathbf{A}$	matrix
$\mathbf{A}^H$	complex conjugate transpose of $\mathbf{A}$
$\mathbf{A}^T$	transpose of $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of $\mathbf{A}$
$\mathbf{A}^{-H}$	inverse of $\mathbf{A}^H$
$\mathbf{A}^{-T}$	inverse of $\mathbf{A}^T$
$\mathbf{A}^\dagger$	generalised or pseudo-inverse of $\mathbf{A}$
$\mathbf{I}$	identity matrix
$\text{tr}(\mathbf{A})$	trace of $\mathbf{A}$
$\kappa(\mathbf{A})$	condition number of $\mathbf{A}$

**Abbreviations**

AD	analogue to digital
ANC	active noise control
BPF	blade passage frequency
DA	digital to analogue
DARE	discrete-time algebraic Ricatti equation
FIR	finite impulse response
IIR	infinite impulse response
IMC	internal modal control
LMS	least mean-squares
LQG	linear quadratic gaussian
MIMO	multiple input multiple output
NMSV	normalised mean-square virtual output error
PEM	prediction error method
PO-MOESP	past output multi-variable output error state space
RLS	recursive least squares
SISO	single input single output
SMI	subspace model identification
SNR	signal to noise ratio
VAF	variance accounted for