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Appendix A

3RRR compliant mechanism analytical PRBM parametric constants

A.1 3RRR PRBM kinematic model ‘constant Jacobian’ parametric constants

These constants were derived using *Maple* software

$$\begin{aligned}
J_{11} = & (A_{1y} C_{3x} B_{1x} B_{3y} C_{2y} - A_{1y} C_{3x} B_{3y} C_{1x} C_{2y} + A_{1y} C_{3y} B_{3x} C_{1x} C_{2y} \\
& - A_{1y} C_{3y} B_{1x} B_{3x} C_{2y} + A_{1y} C_{3y} C_{1x} B_{2y} C_{2x} + A_{1y} C_{3y} B_{2x} B_{1x} C_{2y} \\
& - A_{1y} B_{2y} C_{1x} B_{3y} C_{2x} + A_{1y} B_{3y} C_{1x} B_{2x} C_{2y} - A_{1y} C_{3y} B_{2x} C_{1x} C_{2y} \\
& - A_{1y} B_{1x} B_{3y} B_{2x} C_{2y} + A_{1y} C_{3y} B_{1x} B_{2y} B_{3x} - A_{1y} C_{3y} B_{2y} C_{1x} B_{3x} \\
& + A_{1y} C_{3x} B_{2y} C_{1x} B_{3y} - A_{1y} C_{3x} B_{1x} B_{2y} B_{3y} + C_{1y} C_{3x} B_{1x} B_{2y} B_{3y} \\
& - C_{1y} B_{1x} B_{2y} B_{3y} C_{2x} + C_{1y} B_{1x} B_{3y} B_{2x} C_{2y} - C_{1y} C_{3x} B_{1x} B_{3y} C_{2y} \\
& + C_{1y} C_{3y} B_{1x} B_{3x} C_{2y} + C_{1y} C_{3y} B_{1x} B_{2y} C_{2x} - C_{1y} C_{3y} B_{2x} B_{1x} C_{2y} \\
& - C_{1y} B_{2y} C_{3x} B_{3y} A_{1x} + C_{1y} B_{2y} C_{3y} B_{3x} A_{1x} + C_{1y} B_{2y} A_{1x} B_{3y} C_{2x} \\
& + C_{1y} A_{1x} C_{3x} B_{3y} C_{2y} + C_{1y} A_{1x} C_{3y} B_{2x} C_{2y} - C_{1y} B_{2y} A_{1x} C_{3y} C_{2x} \\
& - C_{1y} A_{1x} B_{3y} B_{2x} C_{2y} - C_{1y} A_{1x} C_{3y} B_{3x} C_{2y} - C_{1y} C_{3y} B_{1x} B_{2y} B_{3x} \\
& + B_{1y} C_{3x} B_{3y} C_{1x} C_{2y} - B_{1y} C_{3y} B_{3x} C_{1x} C_{2y} - B_{1y} C_{3y} C_{1x} B_{2y} C_{2x} \\
& + B_{1y} B_{2y} C_{1x} B_{3y} C_{2x} + B_{1y} C_{3y} B_{2y} C_{1x} B_{3x} - B_{1y} C_{3x} B_{2y} C_{1x} B_{3y} \\
& - B_{1y} B_{3y} C_{1x} B_{2x} C_{2y} + B_{2y} C_{3x} B_{1y} B_{3y} A_{1x} - B_{1y} A_{1x} C_{3y} B_{2y} B_{3x} \\
& + B_{1y} A_{1x} B_{3y} B_{2x} C_{2y} - B_{1y} A_{1x} C_{3y} B_{2x} C_{2y} - B_{1y} A_{1x} C_{3x} B_{3y} C_{2y} \\
& + B_{1y} A_{1x} C_{3y} B_{3x} C_{2y} - B_{2y} C_{2x} B_{1y} B_{3y} A_{1x} + B_{2y} A_{1x} C_{3y} B_{1y} C_{2x} \\
& - B_{1x} A_{1y} C_{3y} B_{2y} C_{2x} + B_{1x} B_{2y} C_{2x} B_{3y} A_{1y} + B_{1y} C_{3y} B_{2x} C_{1x} C_{2y}) / (R_o (\\
& B_{1x} C_{1y} B_{3y} B_{2x} + C_{3x} C_{1y} B_{1x} B_{2y} + C_{3y} B_{1x} B_{2y} B_{3x} - C_{3y} B_{2y} C_{1x} B_{3x} \\
& - C_{3y} B_{2x} B_{1y} B_{3x} - C_{3x} B_{2y} C_{1x} B_{1y} + C_{3x} B_{2y} C_{1x} B_{3y} + C_{3x} B_{2x} B_{1y} B_{3y} \\
& - B_{3x} C_{1y} B_{1x} B_{2y} + B_{3x} B_{2y} C_{1x} B_{1y} - C_{3x} B_{1x} B_{2y} B_{3y} - B_{1y} C_{1x} B_{3y} B_{2x} \\
& + C_{3y} B_{2x} C_{1y} B_{3x} - C_{3y} B_{2x} C_{1y} B_{1x} + C_{3y} B_{2x} C_{1x} B_{1y} - C_{3x} B_{2x} C_{1y} B_{3y} \\
& + B_{3y} C_{1x} B_{2x} C_{2y} + B_{1x} B_{2y} B_{3y} C_{2x} - C_{3y} B_{2x} C_{1x} C_{2y} - C_{3x} B_{2x} B_{1y} C_{2y} \\
& + C_{3x} B_{2x} C_{1y} C_{2y} - B_{3x} B_{1y} B_{2y} C_{2x} + B_{3x} B_{1x} C_{1y} C_{2y} - B_{1x} B_{3y} B_{2x} C_{2y} \\
& + B_{3x} C_{1y} B_{2y} C_{2x} - B_{3x} B_{1y} C_{1x} C_{2y} + B_{3x} B_{2x} B_{1y} C_{2y} - B_{3x} B_{2x} C_{1y} C_{2y} \\
& - C_{3x} B_{1y} B_{3y} C_{2x} + C_{3x} B_{1x} B_{3y} C_{2y} - C_{3x} B_{3y} C_{1x} C_{2y} + C_{3x} B_{1y} C_{1x} C_{2y} \\
& + C_{3y} B_{3x} C_{1x} C_{2y} - C_{3y} B_{1x} B_{3x} C_{2y} + C_{3x} C_{1y} B_{3y} C_{2x} - C_{3x} B_{1x} C_{1y} C_{2y} \\
& - C_{3x} C_{1y} B_{2y} C_{2x} + C_{3x} B_{1y} B_{2y} C_{2x} + B_{1y} C_{1x} B_{3y} C_{2x} - B_{1x} C_{1y} B_{3y} C_{2x} \\
& + C_{3y} C_{1y} B_{1x} C_{2x} - C_{3y} C_{1y} B_{3x} C_{2x} + C_{3y} C_{1x} B_{2y} C_{2x} - C_{3y} C_{1x} B_{1y} C_{2x} \\
& - C_{3y} B_{1x} B_{2y} C_{2x} + C_{3y} B_{1y} B_{3x} C_{2x} + C_{3y} B_{2x} B_{1x} C_{2y} - B_{2y} C_{1x} B_{3y} C_{2x}))
\end{aligned}$$

$$\begin{aligned}
J_{21} = & - (A_{1x} C_{3x} B_{1y} B_{2y} C_{2x} - C_{1x} C_{3x} B_{1y} B_{2y} C_{2x} - C_{1x} C_{3y} B_{1y} B_{3x} C_{2x} \\
& + C_{1x} B_{2y} C_{2x} C_{3x} A_{1y} - C_{1x} B_{2y} C_{2x} B_{3x} A_{1y} - C_{1x} C_{2x} C_{3x} B_{3y} A_{1y} \\
& + C_{1x} C_{2x} C_{3y} B_{3x} A_{1y} + C_{1x} B_{2x} A_{1y} C_{3x} B_{3y} - C_{1x} B_{2x} A_{1y} C_{3y} B_{3x} \\
& + C_{1x} C_{3y} B_{2x} B_{1y} B_{3x} - C_{1x} C_{3x} B_{2x} B_{1y} B_{3y} + C_{1x} C_{3x} B_{2x} B_{1y} C_{2y} \\
& + C_{1x} B_{3x} B_{1y} B_{2y} C_{2x} - C_{1x} B_{3x} B_{2x} B_{1y} C_{2y} + C_{1x} C_{3x} B_{1y} B_{3y} C_{2x}
\end{aligned}$$

$$\begin{aligned}
& -A1x C3y B2x B1y B3x + A1x C3x B2x B1y B3y + A1x C3y B2x C1y B3x \\
& -A1x C3x B2x C1y B3y - A1x C3x B2x B1y C2y - A1x B3x B1y B2y C2x \\
& + A1x B3x C1y B2y C2x + A1x B3x B2x B1y C2y - A1x C3x B1y B3y C2x \\
& + A1x C3x C1y B3y C2x - A1x C3x C1y B2y C2x - A1x C3y C1y B3x C2x \\
& + A1x C3y B1y B3x C2x + B2x C3y B1x B3x A1y - B2x C3y C1y B1x B3x \\
& - C2x C3y B1x B3x A1y - B2x B3x C2y B1x A1y - C1x B2x C2y C3x A1y \\
& + C1x B2x C2y B3x A1y + B1x B3x B2x C1y C2y + B1x C3y C1y B3x C2x \\
& + B2y C2x B1x B3x A1y + B1x C1y C2x C3x B2y - B1x C1y C2x B3x B2y \\
& - B2x C2y C1y B3x A1x + B1x C2x C3x B3y A1y - B1x C2x C3x C1y B3y \\
& + B2x C1y C3x B1x B3y + B2x C2y C3x B1x A1y - B1x B2x C2y C3x C1y \\
& - B1x B2y C2x C3x A1y + B2x C2y C3x C1y A1x - B2x B1x A1y C3x B3y)/(Ro (\\
& B1x C1y B3y B2x + C3x C1y B1x B2y + C3y B1x B2y B3x - C3y B2y C1x B3x \\
& - C3y B2x B1y B3x - C3x B2y C1x B1y + C3x B2y C1x B3y + C3x B2x B1y B3y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - C3x B1x B2y B3y - B1y C1x B3y B2x \\
& + C3y B2x C1y B3x - C3y B2x C1y B1x + C3y B2x C1x B1y - C3x B2x C1y B3y \\
& + B3y C1x B2x C2y + B1x B2y B3y C2x - C3y B2x C1x C2y - C3x B2x B1y C2y \\
& + C3x B2x C1y C2y - B3x B1y B2y C2x + B3x B1x C1y C2y - B1x B3y B2x C2y \\
& + B3x C1y B2y C2x - B3x B1y C1x C2y + B3x B2x B1y C2y - B3x B2x C1y C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y C1x C2y + C3x B1y C1x C2y \\
& + C3y B3x C1x C2y - C3y B1x B3x C2y + C3x C1y B3y C2x - C3x B1x C1y C2y \\
& - C3x C1y B2y C2x + C3x B1y B2y C2x + B1y C1x B3y C2x - B1x C1y B3y C2x \\
& + C3y C1y B1x C2x - C3y C1y B3x C2x + C3y C1x B2y C2x - C3y C1x B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y C1x B3y C2x))
\end{aligned}$$

$$\begin{aligned}
J_{31} = & \\
& - (C1x C2x C3y A1y + B1x C1y B3y B2x + C3x C1y B1x B2y - C3x B2y C1x B1y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - B1y C1x B3y B2x - C3y B2x C1y B1x \\
& + C3y B2x C1x B1y + C1x B3x C2y A1y - C1x C2y C3x A1y - A1x C3y C1y C2x \\
& - B1x B3y B2x A1y + B3y C1x B2x A1y - C3y B2x C1x A1y + B3x C1y B2y A1x \\
& - B3x B2y C1x A1y + B3x B1x B2y A1y - B3x B1y B2y A1x + C3x B1y B2y A1x \\
& - C3x B1x B2y A1y + C3x B2y C1x A1y - C3x C1y B2y A1x + C3y B2x C1y A1x \\
& + B1y B2x B3y A1x - B2x C1y B3y A1x + B3x B1x C1y C2y - C3y B2x B1y A1x \\
& + C3y B2x B1x A1y - B3x B1y C1x C2y + C3x B1y C1x C2y - C3x B1x C1y C2y \\
& + B1y C1x B3y C2x - B1x C1y B3y C2x + C3y C1y B1x C2x - C3y C1x B1y C2x \\
& - B3x C2y C1y A1x + C1y A1x B3y C2x - C1x A1y B3y C2x - C2x C3y B1x A1y \\
& - B1y A1x B3y C2x - C2y C3x B1y A1x + A1x C3y B1y C2x - B3x C2y B1x A1y \\
& + C2y C3x B1x A1y + B1x A1y B3y C2x + B3x C2y B1y A1x + C2y C3x C1y A1x)/(\\
& Ro (B1x C1y B3y B2x + C3x C1y B1x B2y + C3y B1x B2y B3x - C3y B2y C1x B3x \\
& - C3y B2x B1y B3x - C3x B2y C1x B1y + C3x B2y C1x B3y + C3x B2x B1y B3y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - C3x B1x B2y B3y - B1y C1x B3y B2x
\end{aligned}$$

$$\begin{aligned}
& + C3y B2x Cl1 B3x - C3y B2x Cl1 B1x + C3y B2x Cl1x B1y - C3x B2x Cl1 B3y \\
& + B3y Cl1 B2x C2y + B1x B2y B3y C2x - C3y B2x Cl1x C2y - C3x B2x B1y C2y \\
& + C3x B2x Cl1 C2y - B3x B1y B2y C2x + B3x B1x Cl1 C2y - B1x B3y B2x C2y \\
& + B3x Cl1 B2y C2x - B3x B1y Cl1 C2y + B3x B2x B1y C2y - B3x B2x Cl1 C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y Cl1 C2y + C3x B1y Cl1 C2y \\
& + C3y B3x Cl1 C2y - C3y B1x B3x C2y + C3x Cl1 B3y C2x - C3x B1x Cl1 C2y \\
& - C3x Cl1 B2y C2x + C3x B1y B2y C2x + B1y Cl1x B3y C2x - B1x Cl1 B3y C2x \\
& + C3y Cl1 B1x C2x - C3y Cl1 B3x C2x + C3y Cl1 B2y C2x - C3y Cl1 B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y Cl1 B3y C2x)
\end{aligned}$$

$$\begin{aligned}
J_{12} = & (B2y C2x C3x B3y B1y + Cl1 B1x B2y B3y C2x - Cl1 B1x B3y B2x C2y \\
& - Cl1 C3y B1x B2y C2x + Cl1 C3y B2x B1x C2y + B1y C3y Cl1 B2y C2x \\
& - B1y B2y Cl1 B3y C2x + B1y B3y Cl1 B2x C2y - B2y C2x C3y B3x B1y \\
& + B1y C3y B2x Cl1 A2y + Cl1 B2x C2y C3x B3y + Cl1 C2x C3x B3y A2y \\
& + Cl1 B1x B3y B2x A2y - Cl1 C2x C3y B3x A2y - Cl1 B2x C2y C3y B3x \\
& - Cl1 A2x C3x B3y C2y - Cl1 B1x A2y B3y C2x + Cl1 C2y B1x B3y A2x \\
& + Cl1 B2y C3x B3y A2x - Cl1 B2y C2x C3x B3y + Cl1 B2y C2x C3y B3x \\
& + Cl1 A2x C3y B3x C2y + Cl1 C3y B1x B2y A2x + Cl1 C2x C3y B1x A2y \\
& - Cl1 A2x C3y B1x C2y - Cl1 C3y B2x B1x A2y - Cl1 B2y C3y B3x A2x \\
& + Cl1 B2x A2y C3y B3x - Cl1 B2x A2y C3x B3y - Cl1 B1x B2y B3y A2x \\
& - B1y C3y Cl1 B2y A2x - B1y B2x C2y C3x B3y - B1y C2x C3x B3y A2y \\
& + B1y C2x C3y B3x A2y + B1y B2x C2y C3y B3x + B1y A2x C3x B3y C2y \\
& - B1y B2y C3x B3y A2x - B1y A2x C3y B3x C2y + B1y B2y C3y B3x A2x \\
& - B1y B2x A2y C3y B3x + B1y B2x A2y C3x B3y - B1y Cl1 C2x C3y A2y \\
& + B1y Cl1 A2y B3y C2x - B1y Cl1 C2y B3y A2x - B1y B3y Cl1 B2x A2y \\
& + B1y Cl1 A2x C3y C2y + B1y B2y Cl1 B3y A2x - B1y C3y B2x Cl1 C2y)/(Ro (\\
& B1x Cl1 B3y B2x + C3x Cl1 B1x B2y + C3y B1x B2y B3x - C3y B2y Cl1 B3x \\
& - C3y B2x B1y B3x - C3x B2y Cl1 B1y + C3x B2y Cl1 B3y + C3x B2x B1y B3y \\
& - B3x Cl1 B1x B2y + B3x B2y Cl1 B1y - C3x B1x B2y B3y - B1y Cl1 B3y B2x \\
& + C3y B2x Cl1 B3x - C3y B2x Cl1 B1x + C3y B2x Cl1 B1y - C3x B2x Cl1 B3y \\
& + B3y Cl1 B2x C2y + B1x B2y B3y C2x - C3y B2x Cl1 C2y - C3x B2x B1y C2y \\
& + C3x B2x Cl1 C2y - B3x B1y B2y C2x + B3x B1x Cl1 C2y - B1x B3y B2x C2y \\
& + B3x Cl1 B2y C2x - B3x B1y Cl1 C2y + B3x B2x B1y C2y - B3x B2x Cl1 C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y Cl1 C2y + C3x B1y Cl1 C2y \\
& + C3y B3x Cl1 C2y - C3y B1x B3x C2y + C3x Cl1 B3y C2x - C3x B1x Cl1 C2y \\
& - C3x Cl1 B2y C2x + C3x B1y B2y C2x + B1y Cl1 B3y C2x - B1x Cl1 B3y C2x \\
& + C3y Cl1 B1x C2x - C3y Cl1 B3x C2x + C3y Cl1 B2y C2x - C3y Cl1 B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y Cl1 B3y C2x))
\end{aligned}$$

$$\begin{aligned}
J_{22} = & - (C2x C3x B1x Cl1y A2y + Cl1x C3x B1y B2y C2x - Cl1x C3x B2x B1y C2y \\
& - Cl1x B3x B1y B2y C2x + Cl1x B3x B2x B1y C2y - B1x B3x B2x Cl1y C2y \\
& - B1x Cl1y C2x C3x B2y + B1x Cl1y C2x B3x B2y + B2x C3x B3y Cl1x C2y \\
& + B1x B2x C2y C3x Cl1y - B2x C3y B3x Cl1x C2y + B1x C2y Cl1y B3x A2x \\
& - B1x A2y Cl1y B3x C2x - B1x B2y Cl1y B3x A2x + B1x A2y B2x Cl1y B3x \\
& - B2y A2x C3x B1x B3y - C2x C3y B1x B2y B3x - A2x C3y B1x B3x C2y \\
& + C2x C3x B1x B2y B3y + B2x C3y B1x B3x C2y - B2x C3y B1x B3x A2y \\
& + Cl1y B1x A2x C3x B2y - B2x C2y C3x B1x B3y + B2y A2x C3y B1x B3x \\
& + C2x C3y B1x B3x A2y + A2x C3x B1x B3y C2y - A2x C3x B1x Cl1y C2y \\
& - B1x C2x C3x B3y A2y - B2x A2y C3x Cl1y B1x + B2x B1x A2y C3x B3y \\
& + Cl1x C2x B3x B1y A2y - Cl1x C2x C3x B1y A2y + Cl1x C2x C3x B3y A2y \\
& - Cl1x C2x C3y B3x A2y - Cl1x A2x C3x B3y C2y + Cl1x B2y C3x B3y A2x \\
& - Cl1x B2y C2x C3x B3y + Cl1x B2y C2x C3y B3x + Cl1x A2x C3x B1y C2y \\
& + Cl1x A2x C3y B3x C2y - Cl1x C3x B1y B2y A2x + Cl1x B3x B1y B2y A2x \\
& - Cl1x B3x B2x B1y A2y + Cl1x C3x B2x B1y A2y - Cl1x B2y C3y B3x A2x \\
& + Cl1x B2x A2y C3y B3x - Cl1x B2x A2y C3x B3y - Cl1x A2x B3x B1y C2y) / (Ro (\\
& B1x Cl1y B3y B2x + C3x Cl1y B1x B2y + C3y B1x B2y B3x - C3y B2y Cl1x B3x \\
& - C3y B2x B1y B3x - C3x B2y Cl1x B1y + C3x B2y Cl1x B3y + C3x B2x B1y B3y \\
& - B3x Cl1y B1x B2y + B3x B2y Cl1x B1y - C3x B1x B2y B3y - B1y Cl1x B3y B2x \\
& + C3y B2x Cl1y B3x - C3y B2x Cl1y B1x + C3y B2x Cl1x B1y - C3x B2x Cl1y B3y \\
& + B3y Cl1x B2x C2y + B1x B2y B3y C2x - C3y B2x Cl1x C2y - C3x B2x B1y C2y \\
& + C3x B2x Cl1y C2y - B3x B1y B2y C2x + B3x B1x Cl1y C2y - B1x B3y B2x C2y \\
& + B3x Cl1y B2y C2x - B3x B1y Cl1x C2y + B3x B2x B1y C2y - B3x B2x Cl1y C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y Cl1x C2y + C3x B1y Cl1x C2y \\
& + C3y B3x Cl1x C2y - C3y B1x B3x C2y + C3x Cl1y B3y C2x - C3x B1x Cl1y C2y \\
& - C3x Cl1y B2y C2x + C3x B1y B2y C2x + B1y Cl1x B3y C2x - B1x Cl1y B3y C2x \\
& + C3y Cl1y B1x C2x - C3y Cl1y B3x C2x + C3y Cl1x B2y C2x - C3y Cl1x B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y Cl1x B3y C2x))
\end{aligned}$$

$$\begin{aligned}
J_{32} = & - (-B3y Cl1x B2x A2y + Cl1x A2x C3y C2y + B2y Cl1x B3y A2x - C2y C3x Cl1y A2x \\
& + A2y C3x Cl1y C2x + C3y B2x Cl1x A2y + B3x C2y Cl1y A2x - B3x A2y Cl1y C2x \\
& - Cl1x C2x C3y A2y + Cl1x A2y B3y C2x - Cl1x C2y B3y A2x + B3y Cl1x B2x C2y \\
& + B1x B2y B3y C2x - C3y B2x Cl1x C2y - C3x B2x B1y C2y + C3x B2x Cl1y C2y \\
& - B3x B1y B2y C2x - B1x B3y B2x C2y + B3x Cl1y B2y C2x + B3x B2x B1y C2y
\end{aligned}$$

$$\begin{aligned}
& - B3x B2x C1y C2y - C3x C1y B2y C2x + C3x B1y B2y C2x + C3y C1x B2y C2x \\
& - C3y B1x B2y C2x + C3y B2x B1x C2y - B2y C1x B3y C2x - C3x B2x C1y A2y \\
& - B3x C1y B2y A2x + B3x B2x C1y A2y + C3x C1y B2y A2x - C3y C1x B2y A2x \\
& + C2x B3x B1y A2y - C2x C3x B1y A2y + B1x B3y B2x A2y - B1x A2y B3y C2x \\
& + C2y B1x B3y A2x + A2x C3x B1y C2y - C3x B1y B2y A2x + C3y B1x B2y A2x \\
& + C2x C3y B1x A2y - A2x C3y B1x C2y - C3y B2x B1x A2y + B3x B1y B2y A2x \\
& - B3x B2x B1y A2y + C3x B2x B1y A2y - B1x B2y B3y A2x - A2x B3x B1y C2y)/(\\
& Ro (B1x C1y B3y B2x + C3x C1y B1x B2y + C3y B1x B2y B3x - C3y B2y C1x B3x \\
& - C3y B2x B1y B3x - C3x B2y C1x B1y + C3x B2y C1x B3y + C3x B2x B1y B3y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - C3x B1x B2y B3y - B1y C1x B3y B2x \\
& + C3y B2x C1y B3x - C3y B2x C1y B1x + C3y B2x C1x B1y - C3x B2x C1y B3y \\
& + B3y C1x B2x C2y + B1x B2y B3y C2x - C3y B2x C1x C2y - C3x B2x B1y C2y \\
& + C3x B2x C1y C2y - B3x B1y B2y C2x + B3x B1x C1y C2y - B1x B3y B2x C2y \\
& + B3x C1y B2y C2x - B3x B1y C1x C2y + B3x B2x B1y C2y - B3x B2x C1y C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y C1x C2y + C3x B1y C1x C2y \\
& + C3y B3x C1x C2y - C3y B1x B3x C2y + C3x C1y B3y C2x - C3x B1x C1y C2y \\
& - C3x C1y B2y C2x + C3x B1y B2y C2x + B1y C1x B3y C2x - B1x C1y B3y C2x \\
& + C3y C1y B1x C2x - C3y C1y B3x C2x + C3y C1x B2y C2x - C3y C1x B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y C1x B3y C2x))
\end{aligned}$$

$$J_{13} =$$

$$\begin{aligned}
& - (B2y C2x C3x B3y B1y + B1y B3y B2y C1x A3x - B1y C1x C2y B3y A3x \\
& + C1y C3x B1x B2y B3y - C1y C3x B1x B3y C2y + C1y C3y B1x B3x C2y \\
& - C1y C3y B1x B2y B3x + B1y C3x B3y C1x C2y - B1y C3y B3x C1x C2y \\
& + B1y C3y B2y C1x B3x - B1y C3x B2y C1x B3y - B2y C2x C3y B3x B1y \\
& + C1y B2x C2y C3x B3y - C1y B2x C2y C3y B3x - C1y B2y C2x C3x B3y \\
& + C1y B2y C2x C3y B3x - B1y B2x C2y C3x B3y + B1y B2x C2y C3y B3x \\
& + B1y C1x C2y C3y A3x - B1y B3x B2y C1x A3y + B1y C3x B2y C1x A3y \\
& - B1y C3y B2y C1x A3x + B1y C1x B3x C2y A3y - B1y B2x C2y B3x A3y \\
& + B1y B2x C2y C3x A3y - B1y B2x C2y C3y A3x - B1y B2y C2x C3x A3y \\
& + B1y B2x C2y B3y A3x + B1y B2y C2x B3x A3y + B1y B2y C2x C3y A3x \\
& - B1y B2y C2x B3y A3x + C1y B2x C2y B3x A3y - C1y B2x C2y C3x A3y \\
& - C1y B3x C2y B1x A3y + C1y B2x C2y C3y A3x + C1y C2y C3x B1x A3y \\
& + C1y C2y B3y B1x A3x + C1y B2y C2x C3x A3y - C1y B2x C2y B3y A3x \\
& - C1y B2y C2x B3x A3y - C1y B2y C2x C3y A3x - C1y B1x C2y C3y A3x \\
& + C1y B2y C2x B3y A3x - C1y B3y B1x B2y A3x + C1y B3x B1x B2y A3y)
\end{aligned}$$

$$\begin{aligned}
& - C1y C3x B1x B2y A3y + C1y C3y B1x B2y A3x - B1y C1x C2y C3x A3y) / (Ro (\\
& B1x C1y B3y B2x + C3x C1y B1x B2y + C3y B1x B2y B3x - C3y B2y C1x B3x \\
& - C3y B2x B1y B3x - C3x B2y C1x B1y + C3x B2y C1x B3y + C3x B2x B1y B3y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - C3x B1x B2y B3y - B1y C1x B3y B2x \\
& + C3y B2x C1y B3x - C3y B2x C1y B1x + C3y B2x C1x B1y - C3x B2x C1y B3y \\
& + B3y C1x B2x C2y + B1x B2y B3y C2x - C3y B2x C1x C2y - C3x B2x B1y C2y \\
& + C3x B2x C1y C2y - B3x B1y B2y C2x + B3x B1x C1y C2y - B1x B3y B2x C2y \\
& + B3x C1y B2y C2x - B3x B1y C1x C2y + B3x B2x B1y C2y - B3x B2x C1y C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y C1x C2y + C3x B1y C1x C2y \\
& + C3y B3x C1x C2y - C3y B1x B3x C2y + C3x C1y B3y C2x - C3x B1x C1y C2y \\
& - C3x C1y B2y C2x + C3x B1y B2y C2x + B1y C1x B3y C2x - B1x C1y B3y C2x \\
& + C3y C1y B1x C2x - C3y C1y B3x C2x + C3y C1x B2y C2x - C3y C1x B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y C1x B3y C2x))
\end{aligned}$$

$$J_{23} =$$

$$\begin{aligned}
& - (C1x C3y B1y B3x C2x - C1x C3y B2x B1y B3x + C1x C3x B2x B1y B3y \\
& - C1x C3x B1y B3y C2x + B2x C3y C1y B1x B3x - B1x C3y C1y B3x C2x \\
& - B2x C3x B3y C1x C2y + B1x C2x C3x C1y B3y - B2x C1y C3x B1x B3y \\
& + B2x C3y B3x C1x C2y + C2x C3y B1x B2y B3x - C2x C3x B1x B2y B3y \\
& - B2x C3y B1x B3x C2y + C1x B2x C2y B3y A3x + B2x C2y C3x B1x B3y \\
& - B1x C1y C2x C3x A3y + B1x C3x B2x C1y A3y - B2x C2y C3x B1x A3y \\
& - B2x C3y B1x C1y A3x - C2x C3y B1x B2y A3x + C2x C3y B1x C1y A3x \\
& - C2x B3x B1x B2y A3y + C2x B3x B1x C1y A3y + C2x C3x B1x B2y A3y \\
& + B2x B3x C2y B1x A3y + B2x C1y A3x B1x B3y - C2x B3y B1x C1y A3x \\
& + C2x B3y B1x B2y A3x - B3x B1x B2x C1y A3y + C1x B2x C2y C3x A3y \\
& - C1x B2x C2y B3x A3y - C1x B2x C2y C3y A3x + B2x B1x C2y C3y A3x \\
& - B2x C2y B3y B1x A3x + C1x B2y C2x C3x B3y - C1x B2y C2x C3y B3x \\
& - C1x C2x B3x B1y A3y + C1x B1y C2x C3x A3y - C1x C2x C3y B1y A3x \\
& - C1x B2y C2x C3x A3y - C1x C3x B2x B1y A3y + C1x B2y C2x B3x A3y \\
& + C1x B2y C2x C3y A3x + C1x B1y C2x B3y A3x - C1x B2y C2x B3y A3x \\
& + C1x C3y B2x B1y A3x - C1x B3y B2x B1y A3x + C1x B3x B2x B1y A3y) / (Ro (\\
& B1x C1y B3y B2x + C3x C1y B1x B2y + C3y B1x B2y B3x - C3y B2y C1x B3x \\
& - C3y B2x B1y B3x - C3x B2y C1x B1y + C3x B2y C1x B3y + C3x B2x B1y B3y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - C3x B1x B2y B3y - B1y C1x B3y B2x \\
& + C3y B2x C1y B3x - C3y B2x C1y B1x + C3y B2x C1x B1y - C3x B2x C1y B3y
\end{aligned}$$

$$\begin{aligned}
& + B3y C1x B2x C2y + B1x B2y B3y C2x - C3y B2x C1x C2y - C3x B2x B1y C2y \\
& + C3x B2x C1y C2y - B3x B1y B2y C2x + B3x B1x C1y C2y - B1x B3y B2x C2y \\
& + B3x C1y B2y C2x - B3x B1y C1x C2y + B3x B2x B1y C2y - B3x B2x C1y C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y C1x C2y + C3x B1y C1x C2y \\
& + C3y B3x C1x C2y - C3y B1x B3x C2y + C3x C1y B3y C2x - C3x B1x C1y C2y \\
& - C3x C1y B2y C2x + C3x B1y B2y C2x + B1y C1x B3y C2x - B1x C1y B3y C2x \\
& + C3y C1y B1x C2x - C3y C1y B3x C2x + C3y C1x B2y C2x - C3y C1x B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y C1x B3y C2x)
\end{aligned}$$

$$J_{33} =$$

$$\begin{aligned}
& (-B3y B2x C1y A3x - C1x C2y C3x A3y + C1x C2y C3y A3x - C2x C3y C1y A3x \\
& - C3x B2x C1y A3y - B3x B2y C1x A3y + B3x B2x C1y A3y + C3x B2y C1x A3y \\
& - C3y B2y C1x A3x + C3y B2x C1y A3x - B3x C1y C2x A3y - C1x C2y B3y A3x \\
& + C1x B3x C2y A3y + C1y C2x C3x A3y + C1y C2x B3y A3x + B3y B2y C1x A3x \\
& - C3y B1x B2y B3x + C3y B2y C1x B3x + C3y B2x B1y B3x - C3x B2y C1x B3y \\
& - C3x B2x B1y B3y + C3x B1x B2y B3y - C3y B2x C1y B3x + C3x B2x C1y B3y \\
& + C3x B1y B3y C2x - C3x B1x B3y C2y + C3x B3y C1x C2y - C3y B3x C1x C2y \\
& + C3y B1x B3x C2y - C3x C1y B3y C2x + C3y C1y B3x C2x - C3y B1y B3x C2x \\
& + C2x B3x B1y A3y - B1y C2x C3x A3y - B3x C2y B1x A3y + C2x C3y B1y A3x \\
& + C2y C3x B1x A3y + C2y B3y B1x A3x + C3x B2x B1y A3y - B1y C2x B3y A3x \\
& - B1x C2y C3y A3x - C3y B2x B1y A3x + B3y B2x B1y A3x - B3y B1x B2y A3x \\
& + B3x B1x B2y A3y - B3x B2x B1y A3y - C3x B1x B2y A3y + C3y B1x B2y A3x)/(\\
& Ro (B1x C1y B3y B2x + C3x C1y B1x B2y + C3y B1x B2y B3x - C3y B2y C1x B3x \\
& - C3y B2x B1y B3x - C3x B2y C1x B1y + C3x B2y C1x B3y + C3x B2x B1y B3y \\
& - B3x C1y B1x B2y + B3x B2y C1x B1y - C3x B1x B2y B3y - B1y C1x B3y B2x \\
& + C3y B2x C1y B3x - C3y B2x C1y B1x + C3y B2x C1x B1y - C3x B2x C1y B3y \\
& + B3y C1x B2x C2y + B1x B2y B3y C2x - C3y B2x C1x C2y - C3x B2x B1y C2y \\
& + C3x B2x C1y C2y - B3x B1y B2y C2x + B3x B1x C1y C2y - B1x B3y B2x C2y \\
& + B3x C1y B2y C2x - B3x B1y C1x C2y + B3x B2x B1y C2y - B3x B2x C1y C2y \\
& - C3x B1y B3y C2x + C3x B1x B3y C2y - C3x B3y C1x C2y + C3x B1y C1x C2y \\
& + C3y B3x C1x C2y - C3y B1x B3x C2y + C3x C1y B3y C2x - C3x B1x C1y C2y \\
& - C3x C1y B2y C2x + C3x B1y B2y C2x + B1y C1x B3y C2x - B1x C1y B3y C2x \\
& + C3y C1y B1x C2x - C3y C1y B3x C2x + C3y C1x B2y C2x - C3y C1x B1y C2x \\
& - C3y B1x B2y C2x + C3y B1y B3x C2x + C3y B2x B1x C2y - B2y C1x B3y C2x))
\end{aligned}$$

A.2 3RRR PRBM dynamic model parametric constants

The parametric form of the inertia and stiffness matrix constants, ψ , σ , α , and β are similar to the Jacobian constants given in Appendix A.1, but include more parameters and are far longer. Each matrix constant is given by a parametric equation that is over 10 pages long! Therefore, these parametric equations are not particularly useful. The ‘linear-Cartesian’ method is best solved by using real parameter values in the four kinematic constraint equations, (3.42) to (3.45), which are then solved simultaneously. The subsequent equations, to give the inertia and stiffness matrix constants, can then be easily solved using either Maple or Matlab.