The Modelling and Optimal Design of a Three Degree-of-Freedom XYθ_Z Micro-Motion Stage

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Abstract

This thesis presents an investigation of the modelling and optimal design of a particular 3-degree-of-freedom (DOF) $XY\theta_Z$ micro-motion stage. This stage provides micron-scale motion in X and Y directions and a rotation about the Z-axis. Such a stage can be used for applications where positioning of components with micrometre, or even nanometre positioning accuracy is required. Some applications are; the positioning of samples in a scanning-electron-microscope; the positioning of masks in lithography; aligning fibre-optics and lasers; and manipulation of micro-scale objects in micro-biology or micro-systems assembly.

The $XY\theta_Z$ micro-motion stage investigated in this study uses a particular topology of monolithic compliant mechanism and three stack piezoelectric actuators. The compliant mechanism used is a 3RRR (three revolute-revolute-revolute) parallel compliant mechanism using flexure hinges. This parallel mechanism uses three RRR linkages. Each of the three RRR linkages uses three circular profile flexure hinges. Each flexure hinge provides predominantly rotational motion about one axis. This topology of mechanism has a symmetrical structure and provides numerous advantages that make it appropriate for use in a micro-motion stage. However, as yet this topology of compliant mechanism has only been investigated by a handful of researchers and it has not been used in any commercially developed systems. The

design methodology of a stage using the 3RRR compliant mechanism has not been investigated in detail.

In this thesis a study is presented that investigates different approaches to model the 3RRR compliant mechanism and also considers the piezo-actuator modelling, to give the complete $XY\theta_Z$ micro-motion stage. Three models are presented and compared; the Pseudo-Rigid-Body Model (PRBM); a two-dimensional Finite-Element-Model (2-D FEM); and a third model is developed that is similar to the PRBM, but uses analytical equations to model the multiple degree-of-freedom compliance of the flexure hinges. The models developed are then used in parametric study so that the relationship between design parameters and output behaviour can be understood. An optimal design approach is then presented to develop an $XY\theta_Z$ micro-motion stage for a particular application in a Scanning-Electron-Microscope (SEM). Finally experimental validation of the models is presented.

The results of this study indicate which modelling approaches are accurate enough to prove useful for design, while also considering which models are computationally simple enough to be efficient and easy to use. The kinematic and dynamic behaviour of the 3RRR compliant mechanism and $XY\theta_Z$ micro-motion stage is discussed in detail. This includes; a comprehensive description of the stage workspace, defining reachable and constant-rotation workspace areas; a discussion of actuator coupling; and in depth investigation of the modes of vibration. The results of the parametric study provide useful insight to aid the design of the $XY\theta_Z$ micro-motion stage and help simplify optimal design. The parametric study also highlights the difference in trends predicted by different modelling methods, which demonstrates the importance of using an appropriate model in design. The experimental validation demonstrates the accuracy of some modelling approaches while highlighting the limited accuracy of others.

Statement of originality

To the best of my knowledge, except where otherwise referenced and cited, everything that is presented in this thesis is my own original work and has not been presented previously for the award of any other degree or diploma in any University. If accepted for the award of the degree of Ph.D. in Mechanical Engineering, I consent that this thesis be made available for loan and photocopying.

Daniel Handley

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