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Measurement of radiation-pressure-induced optomechanical dynamics in a suspended Fabry-Perot cavity

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We report on experimental observation of radiation-pressure induced effects in a high-power optical cavity. These effects play an important role in next-generation gravitational wave detectors, as well as in quantum nondemolition interferometers. We measure the properties of an optical spring, created by coupling of an intense laser field to the pendulum mode of a suspended mirror, and also the parametric instability (PI) that arises from the coupling between acoustic modes of the cavity mirrors and the cavity optical mode. We measure an unprecedented optical rigidity of $K=(3.08\pm0.09)\times10^4$ N/m, corresponding to an optical rigidity that is 6000 times stiffer than the mechanical stiffness, and PI strength $R\approx3$. We measure the unstable nature of the optical spring resonance, and demonstrate that the PI can be stabilized by feedback to the frequency of the laser source.

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INTRODUCTION

Second-generation gravitational wave (GW) interferometers are anticipated to have circulating power in excess of 0.5 MW in the Fabry-Perot arm cavities [1]. High circulating power is needed to reduce the effect of photon shot noise, which limits the performance of these instruments at frequencies above 200 Hz. In this high-power regime, radiation pressure effects become important, and the dynamics of the suspended mirrors and the light field used to measure their motion cannot be treated separately. A rich variety of physical phenomena resulting from radiation pressure effects include parametric instability [2–5], optical tilt instability [6], *quantum* radiation pressure noise [1,8–11], and optomechanical rigidity [7–11]. In this paper, we report on the observation and characterization of two radiation pressure-induced phenomena: the optical spring (OS) effect and parametric instability (PI). Our measurements are carried out in a detuned Fabry-Perot cavity with mirrors suspended as pendulums. We explore regimes of mass, frequency, and optical rigidity that differ from previous measurements by orders of magnitude [12–15], and that are most relevant for GW interferometers [4,5,8] and quantum nondemolition (QND) devices [10,11].

For optomechanical coupling to occur in Fabry-Perot cavities via radiation pressure, some mechanism must linearly couple the phase fluctuations induced by mechanical motion to intensity fluctuations of the intracavity field. One such mechanism is to detune the laser frequency from the center of the cavity resonance, such that the stored power (and hence the radiation pressure force) has a linear dependence on the cavity length. Redshifting (or shortening the cavity) results in an antirestoring force, while blueshifting (or lengthening the cavity) results in a restoring force.

OPTICAL SPRING

The OS effect occurs when the optical restoring force on the cavity mirrors is comparable to, or greater than, the mechanical restoring force, and the resonant frequency of the optomechanical system is shifted. A previous demonstration of the OS effect [12] showed a 2% shift in the resonant frequency corresponding to an optical rigidity that was 25 times weaker than the mechanical rigidity. An important feature of the OS, which was not accessible to previous experiments due to the relative weakness of the OS, is that there is also a radiation pressure force proportional to the velocity of the cavity mirrors, which arises from the time delay in the cavity response. When the cavity detuning is blueshifted to create an OS, this time delay leads to a force in the same direction as the instantaneous mirror velocity, and the optical field pumps energy into the kinetic energy of the mirrors. If the mechanical viscous damping of the mirrors is not sufficient to remove this energy, then the OS will become unstable [7,9]. Another important feature of the OS is that at frequencies below the optomechanical resonance, the response of the system to external disturbances (e.g., driven by seismic or thermal forces) is suppressed by the optical rigidity. This effect makes the optical spring an important feature in QND interferometers [10,11], whose performance may otherwise be limited by thermal forces. In this paper, we report on an experiment that demonstrates an optical rigidity that is 6000 times stiffer than the mechanical rigidity. In the strong-coupling regime of our experiment, the unstable nature of the optical spring is exposed, and we show how it may be controlled.

PARAMETRIC INSTABILITY

For cases in which the optical rigidity is much weaker than the mechanical rigidity, it would seem that optomechanical effects should be negligible. However, though the optomechanical resonant frequency will be only slightly shifted from the mechanical resonant frequency, the viscous optical force may still have a strong effect, if it is of the same order as the mechanical viscous damping. The behavior of the system at resonance is dominated by the damping, and may show parametric instability if the optical damping is stronger than the mechanical damping. The optical damping may either viscously damp (blueshifted, cold damping) or

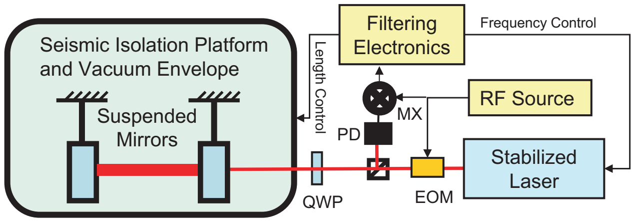

FIG. 1. (Color online) Schematic representation of the experiment, showing the 1-m-long Fabry-Perot cavity suspended in vacuum. 3 W of the 1064 nm Nd:YAG laser light is incident on the suspended cavity. The input mirror of the cavity has a transmission of 0.63%, giving a linewidth of 75 kHz. EOM refers to electro-optic modulator, MX to mixer, PD to photodetector, and QWP to quarter-wave plate.

antidamp (redshifted, PI) the motion. This effect is predominantly important for the acoustic modes of mirrors, which are extremely stiff and have extremely small mechanical damping. PIs have been observed in resonant bar detectors with microwave resonator readouts [12] and in optical microcavities [13]. Kippenberg *et al.* observed PIs in ultrahigh-Q toroidal optical microcavities, at frequencies of 4.4–49.8 MHz and modal masses of 10^{-8}–10^{-9} kg. Enhancement or reduction of the damping of a microlever at a similar mass scale has also been demonstrated [15]. PIs are also predicted to occur in advanced gravitational-wave detectors [2–5], although the optomechanical coupling mechanism is different. In these detectors, the linewidths of the cavities are on the order of 100 Hz, while the acoustic mirror modes have frequencies of 10 kHz, well outside the linewidth of the cavities. For PIs to occur in this regime, it is necessary to excite higher-order eigenmodes of the optical beam, which overlap both spectrally and spatially with the mirror acoustic mode. Additionally, the lower and upper (Stokes and anti-Stokes) optical modes must have different optical gains to provide phase to intensity coupling [2,5]. The experiment presented in this paper differs from previous experiments in that it demonstrates PI for a 28.188 kHz acoustic mode with an effective (modal) mass of 0.125 kg in a suspended cavity apparatus. The mass and frequency regime of this experiment are of particular interest to GW detectors and ponderomotive squeezing experiments [11]. Moreover, we demonstrate that the PI may be stabilized by locking the laser frequency to the cavity mode, an important result for any experiment under threat of PI, such as Advanced LIGO [1].

UNIFIED MODEL

We present a model that describes both the OS and PI, valid for the PI when it occurs within the fundamental (TEM_{00}) optical mode. The experimental setup is shown in Fig. 1. The cavity mirrors, suspended by single loops of wire, are located in a vacuum chamber where they are mounted on a vibrationally isolated platform. The motion of the mirrors can be controlled by forces applied through current-carrying coils placed near magnets that are glued to the back surface of each mirror. The Pound-Drever-Hall (PDH) technique is used to control the length of the cavity.

Defining Ω as the angular frequency of measurement, the equation of motion for the cavity mirrors in the frequency domain is

$$-\Omega^2 x = -\left(\Omega_0^2 + \frac{i\Omega\Omega_0}{Q_m} + \frac{K(\Omega)}{M_{\rm eff}}\right)x + \frac{F}{M_{\rm eff}}, \quad (1)$$

where x is the cavity length change due to mirror displacement, Q_m and Ω_0 are the quality factor and resonant frequency of the mechanical mode, respectively, F is the difference of the forces applied to the mirrors, $M_{\rm eff}$ is the effective (modal) mass of the mechanical mode being considered, and $K(\Omega)$ is the optical rigidity, given by [10,11,16]

$$K(\Omega) = K_0 \frac{1 + (\delta/\gamma)^2}{(1 + i\Omega\gamma)^2 + (\delta/\gamma)^2}, \quad (2)$$

where

$$K_0 = \frac{2}{c}\frac{dP}{dx} = \frac{128\pi I_0(\delta/\gamma)}{\mathcal{T}_I^2 c\lambda_0}\left[\frac{1}{1 + (\delta/\gamma)^2}\right]^2. \quad (3)$$

Here γ is the cavity linewidth ($2\pi\times 75$ kHz), δ is the detuning of the cavity from resonance, I_0 is the power incident on the cavity (3 W), $\mathcal{T}_I$ is the transmission of the cavity input mirror (0.63%), and λ_0 is the wavelength of the laser (1064 nm). K_0 is the rigidity at $\Omega=0$ (dc). An imaginary spring constant corresponds to viscous damping.

Both the PI and the OS that occur in our experiment are derived by considering Eq. (1), but in different regimes. The OS effect occurs at low frequency, where the optical rigidity is larger than the stiffness of the mechanical pendulum. The PI, on the other hand, occurs at much higher frequency, namely at a flexural normal mode of the mirror, whose stiffness is much higher than both the pendular mode and the optical rigidity.

OPTICAL SPRING MODEL

First we consider the pendular mode with oscillation frequency, $\Omega_0=2\pi\times 1$ Hz, and with $M_{\rm eff}\equiv\mu=0.125$ kg, the reduced mass of the two mirror system. For sufficient detuning δ, the optical restoring force will be much larger than the mechanical restoring force, i.e., $K \gg \mu\Omega_0^2$. In this regime $\Omega \ll \gamma$, allowing us to make the approximation

$$K \approx K_0\left[1 - \frac{2i\Omega}{\gamma}\frac{1}{1 + (\delta/\gamma)^2}\right], \quad (4)$$

which leads to a modified equation of motion

$$-\Omega^2 x \approx -\left(\Omega_0'^2 + \frac{i\Omega_0'}{Q'}\Omega\right)x + \frac{F}{\mu}, \quad (5)$$

where

$$\Omega_0'^2 \equiv \frac{K_0/\mu}{1 + (\delta/\gamma)^2}, \quad (6)$$

$$Q' \equiv -[1 + (\delta/\gamma)^2]\frac{\gamma}{2\Omega_0'} \quad (7)$$

are the modified resonant frequency and quality factor of the *optomechanically coupled* system. Note that the new quality factor Q' is negative, indicating that this resonance is unstable. In this case, both the resonant frequency and the quality factor of the mode are changed, important features of the optical spring.

PARAMETRIC INSTABILITY MODEL

For the PI we consider a regime where the dominant mechanical motion is the drumhead mode of the mirrors. For this *acoustic* mode we have resonant frequency $\Omega_0 \to \Omega_0^* = 2\pi \times 28.188$ kHz, and the quality factor Q_m^* and effective (modal) mass $M_{\rm eff} \equiv M^*$, respectively, are determined experimentally [17]. Other acoustic modes of the mirror are not excited due to their poor spatial overlap with the cavity optical mode. The mechanical restoring force is much larger than the optical restoring force, i.e., $K \ll M^* \Omega_0^{*2}$, and the frequency of the mechanical mode will hardly changed. The response of the system at the resonant frequency, however, may be modified if the optical damping is comparable to the mechanical damping. The gain of the parametric loop, R, is estimated by the product of two factors: (i) the response of the mirror surface to forces at the resonant frequency (displacement/force), given by $Q_m^*/(M^*\Omega_0^{*2})$; and (ii) the viscous radiation pressure force exerted by the intracavity field due to the motion of the mirror surface (force/displacement), given by $\mathrm{Im}[K(\Omega_0^*)]$. Then

$$R \approx \frac{\mathrm{Im}[K(\Omega_0^*)]Q_m^*}{M^*\Omega_0^{*2}} \tag{8}$$

quantifies the strength of the PI. $R > 1$ results in PI, while $R < 1$ results in cold damping, depending on the side to which the cavity is detuned. Our model gives similar results to that of Braginsky *et al.* [4], who first defined the dimensionless PI susceptibility, R. We note that while this is an illustrative estimation for R, an exact numerical solution for Eq. (1) was used for the PI results below.

OPTICAL SPRING MEASUREMENT

The OS effect is characterized at the cavity detuning that maximizes the optical rigidity, $\delta \approx 40$ kHz (to avoid the PI at this operating point, it is actively stabilized, as described later). We measure the swept-sine transfer function from a force applied on the mirror to the mirror position, using the PDH error signal and feedback loop response. The transfer function (Fig. 2) shows that with the OS, the eigenfrequency has moved from 1 to 79 Hz, and that the new eigenmode shows a negative damping constant (the phase *increases* by 180° at the resonance), as predicted by the model. The smearing out of the sharp predicted peak in the data is likely caused by fluctuations of the intracavity power. The fit of the transfer function data to the theoretical prediction yields an OS resonance frequency of (79±1) Hz. Since the reduced mass is (0.125±0.003) kg, we determine $K = (3.08 \pm 0.09) \times 10^4$ N/m.

PARAMETRIC INSTABILITY MEASUREMENT

To study the PI experimentally, we measure the value of R for the mode as a function of detuning and power, by measuring ringup and ringdown times of the drumhead mode for various detunings. The time constants are related to R by

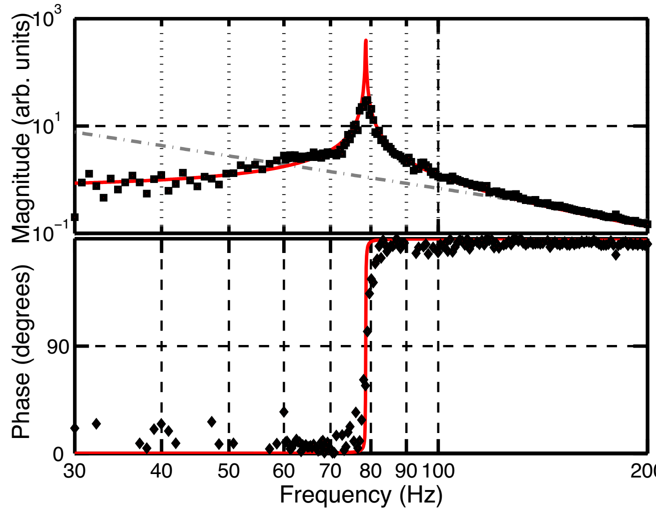

FIG. 2. (Color online) The transfer function of applied force to displacement of the end mirror of the cavity, showing the optical spring resonance. The solid (red) curve is the theoretical prediction, based on measurement of the intracavity power, and the quadrangles are the measured data. An important feature is the *increase* in phase from 0° to 180° at the resonance, showing that the resonance is unstable. The dashed (gray) curve is the expected response of the mirror in the absence of the OS, illustrating modest noise suppression below the OS resonance.

$$\tau' = \tau/(1 - R), \tag{9}$$

where τ' is the modified ringing time, and τ is the natural ringing time, both on the order of seconds for the 28.188 kHz drumhead mode. The length of the cavity is locked to the frequency of the laser with a ~1 kHz bandwidth feedback loop that is strongly filtered to ensure that there is negligible gain at the 28.188 kHz drumhead mode. For the cases in which $R > 1$, the mode is unstable, so we allow the mode to begin oscillating. We capture the ringup of the mode via measurement of the error signal time series (which is proportional to changes in cavity length) with a lock-in amplifier, and fit a sinusoidal function with exponential growth to find the ringup time, the oscillation frequency, and the value of R. To measure values at $R < 1$, we first detune the cavity to a point where $R > 1$, thus making the unstable mode ring up, then quickly switch the detuning to the desired $R < 1$ point and capture the ringdown. In this case, we fit an exponentially decaying sinusoid to find the value of R. The measurement of R at various detunings, for a fixed input power, is shown in Fig. 3. Also shown are the time series for the exponentially growing and decaying oscillations, for $R > 1$ and $R < 1$, respectively, and the theoretical prediction, with no free parameters. The errors for R are from 95% confidence for the fit of the time series, and from a 20% error in the value of Q_m^*, which changed on that order from measurement to measurement of the ring down. The error in detuning comes from ±2% accuracy in measuring the cavity transmitted power, which is used to calibrate the

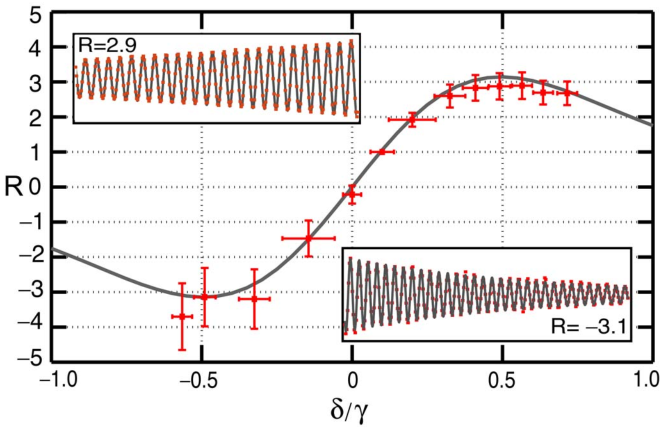

FIG. 3. (Color online) The instability measure R is plotted as a function of detuning of the cavity from resonance, for fixed input power. The solid (gray) curve is the theoretical prediction, given by a numerical solution to Eq. (1) with no free parameters. The squares show the measured data; the error bars are explained in the text. The inset time series show 3 s of data along with fits for the ringup and ringdown.

detuning. M^* is obtained by measuring the small frequency shift of the 28.188 kHz mode between the case of $\delta_1=0$ and $\delta_2=2\pi\times 40$ kHz, the same detuning at which the OS measurement is performed. Since K is known from the OS measurement, the value of M^* can be obtained from $M^*=K/[\Omega_0^{*2}(\delta_2)-\Omega_0^{*2}(\delta_1)]$. The measured frequency shift is 0.11 ± 0.01 Hz, giving $M^*=0.12\pm 0.04$ kg [18]. The theoretical prediction of R as a function of the detuning δ, shown in Fig. 3, has no free parameters.

The PI was also stabilized by a feedback loop that makes the frequency of the laser follow the length of the cavity at frequencies from ~300 Hz to 50 kHz, which suppresses the formation of the 28.188 kHz sidebands, thereby reducing the parametric gain and hence stabilizing the system.

CONCLUDING REMARKS

In summary, we have measured and controlled two important radiation pressure-induced effects, the OS and PI, in a high-power Fabry-Perot cavity with mirrors suspended as pendulums. We find a maximum optical rigidity of $K=(3.08\pm 0.09)\times 10^4$ N/m and a PI with $R\approx 3$. We show that the OS resonance is unstable, and that it can be stabilized with a control system. The demonstration of an OS that significantly dominates the mechanical restoring forces is a result relevant to QND experiments that rely on radiation pressure to create nonclassical states of light [11], and to experiments that rely on the optical spring to reach and exceed the standard quantum limit. The stabilization of the PI by feedback to the laser frequency is an important confirmation that this is a feasible method for controlling PIs in Advanced LIGO [1], or any optical system with high circulating power.

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[17] M^* is a material property of the mirror, and has no relation to the reduced mass for the optical spring.
[18] An estimate based on finite element analysis gives $M^*=0.09$ kg. The higher value in our measurement is consistent with the optical beams not being laterally well centered on the mirrors. We note that the similar value of M^* and the reduced mass μ is coincidental.