Exact Solution to the Stochastic Spread of Social Contagion - Using Rumours

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Abstract

This Thesis expands on the current developments of the theory of stochastic diffusion processes of rumours. This is done by advancing the current mathematical characterisation of the solution to the Daley-Kendall model of the simple S-I-R rumour to a physical solution of the sub-population distribution over time of the generalised simple stochastic spreading process in social situations. After discussing stochastic spreading processes in social situations such as the simple epidemic, the simple rumour, the spread of innovations and ad hoc communications networks, it uses the three sub-population simple rumour to develop the theory for the identification of the exact sub-population distribution over time. This is done by identifying the generalised form of the Laplace Transform Characterisation of the solution to the three sub-population single rumour process and the inverse Laplace Transform of this characterisation. In this discussion the concept of the Inter-Changeability Principle is introduced. The general theory is validated for the three population Daley-Kendall Rumour Model and results for the three, five and seven population Daley-Kendall Rumour Models are presented and discussed. The $\alpha - \rho$ model results for pseudo-Maki-Thompson Models are presented and discussed. In subsequent discussion it presents for the first time a statement of the Threshold Problem for Stochastic Spreading Processes in Social settings as well as stating the associated Threshold Theorem. It also investigates limiting conditions.

Aspects of future research resulting from the extension of the three subpopulation model to more than three subpopulations are discussed at the end of the thesis. The computational demands of applying the theory to more than three subpopulations are restrictive; the size of the total population that can be considered at one time is considerably reduced. To retain the ability to compute a large population size, with an increase in the number of possible subpopulations, a possible method of repeated application of the three population solution is identified. This is done through the medium of two competing mutually exclusive rumours. The final discussion occurs on future investigation into the existence of limit values, zero states, cyclic states and absorbing states for the $M$ subpopulation case.

The generalisation and inversion of the Laplace Transform as well as the consequential statement of the threshold theorem, derivation of the transition probabilities and discussion of the limiting conditions are significant advances in the theory of rumours and similar social phenomena.
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Declaration

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being made available in all forms of media, now or hereafter known.

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