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Center vortices and the quark propagator in SU(2) gauge theory

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We study the behavior of the AsqTad quark propagator in Landau gauge on quenched SU(2) gauge configurations under the removal of center vortices. In contrast to recent results in SU(3), we clearly see the infrared enhancement of the mass function disappear if center vortices are removed, a sign of the intimate relation between center vortices and chiral symmetry breaking in SU(2) gauge-field theory. These results provide a benchmark with which to interpret the SU(3) results. In addition, we consider vortex-only configurations. On those, the quark dressing function behaves roughly as on the full configurations, and the mass function picks up an almost linear momentum dependence.

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I. INTRODUCTION

Dynamical breaking of chiral symmetry (D χ SB) is an essential nonperturbative property of quantum chromodynamics (QCD) which cannot be accounted for within perturbation theory at any order. Only nonperturbative approaches, such as those provided by lattice QCD simulations or studies of the Dyson-Schwinger equations, can be used to explore this phenomenon.

The other characteristic nonperturbative phenomenon of QCD is confinement: the fact that colored states are never observed. It is tempting to speculate that these two phenomena might be driven by the same basic mechanism, an idea supported by finite-temperature studies where the deconfinement and chiral restoration transitions are observed to occur at coincident temperatures [1]. Moreover, it was found that the low-lying modes of the quark operator not only bear witness on spontaneous chiral symmetry breaking but also on confinement [2].

One leading candidate for such a mechanism is the center vortex. Center vortices have been studied in lattice QCD simulations for more than a decade. The recovery of the string tension from “vortex-only” SU(2) gauge configurations [i.e., Z_2 projected from SU(2)] is well known, as is the recovery of the chiral condensate [3–6] (see also more recent Refs. [7–9]). In SU(3), however, the situation is far less compelling. For example, in Refs. [10,11] numerical evidence has been given that in SU(3) mass generation remains intact after removing center vortices, whereas the string tension vanishes as expected. This came as a surprise and immediately suggested a corresponding “benchmark” study for the case of SU(2) which is reported here.

We use the quark propagator as a probe of D χ SB. The Dirac scalar part of the propagator, related at large momenta to the perturbative running mass, is enhanced at low

momenta, even in the chiral limit [12]: a demonstration of D χ SB. We explicitly establish the relation between center vortices and D χ SB by investigating the quark propagator under the removal of center vortices. Specifically, we will provide numerical evidence that dynamical mass generation disappears if those vortices are removed, and surprisingly, much of it resides in the vortex-only part. Additionally, we present an improved method for generating the SU(2) maximal-center-gauge-projected configurations.

The paper is organized as follows. In an attempt to make it self-contained we briefly introduce the quark propagator and its realization on the lattices based on the AsqTad quark action. This is followed by a specification on how we gauge-fixed and identified center vortices in our gauge configurations. Then, results for the mass and quark dressing function are compared on full, vortex-removed and vortex-only configurations. A summary concludes the paper.

II. THE QUARK PROPAGATOR

The quark propagator is gauge dependent. In covariant gauges in Euclidean momentum space it can be parametrized in the general form

$$S(p^2) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}, \quad (1)$$

where M is the running mass and Z the quark dressing function. S can be calculated in regularized theories, as, for example, here in a lattice regularization where the lattice spacing a makes all expressions finite. At sufficiently small a , i.e., if scaling violations due to finite lattice spacings are negligible, the bare quark propagator, S , is related to the renormalized propagator via multiplicative renormalization:

$$S_R(p^2; \mu, g_R(\mu), m_R(\mu)) = Z_2 S(p^2; a, g_0(a), m_0(a)).$$

To ensure multiplicative renormalizability, all the dependence of S_R on the renormalization point μ is contained in the renormalized quark dressing function, Z_R . M does not depend on μ . A renormalization condition fixes Z_2 , the renormalization constant of the quark fields. Lattice calculations often use momentum space subtraction (MOM) schemes to fix renormalization constants. In MOM schemes, Z_2 is fixed by requiring S_R to be of the form of a free propagator at the renormalization point $p^2 = \mu^2$. This sets

$$Z_R(\mu^2, \mu^2) = 1 \quad \text{and} \quad M(\mu^2) = m_R(\mu), \quad (2)$$

where the latter denotes the renormalized mass at μ^2 .

Calculation of the quark propagator S proceeds like any correlation function in a lattice Monte Carlo (MC) calculation once the gauge has been fixed. For the gauge we used the ever popular Landau gauge. It is straightforward to implement on the lattice and allows for an easy comparison to other studies. The Landau-gauge quark propagator has been studied widely in SU(3) gauge theory using Wilson-clover, staggered-type, and overlap actions in quenched and unquenched simulations (see, e.g., Refs. [12–20]). It has been shown that the quark propagator obtained with the AsqTad-improved staggered quark action possesses good symmetry properties and is well behaved at large momenta (see, e.g., Ref. [21]). The AsqTad action was therefore a natural choice for this study.

III. DETAILS OF THE CALCULATION

Configurations were generated on a $16^3 \times 32$ lattice using a tadpole-improved Wilson gauge action with an inverse coupling constant $\beta = 1.35$. Around 120 configurations were used.

A. Identifying center vortices

The observation that the long-range static potential only depends on the center charge (also called “ N -ality”) of the quark representation led to the expectation that the center subgroup Z_N of SU(N) plays a crucial role for quark confinement. Center vortices emerge from the corresponding Z_N gauge theory: they form closed world sheets in four space-time dimensions carrying flux which takes values in the center of the SU(N) group. Early attempts to define this subgroup by the projection $SU(N) \rightarrow Z_N$ failed in the sense that the arising vortex matter did not have meaningful properties in the continuum limit of vanishing lattice spacing a . In the pioneering works [22,23] a two step process was proposed to define the links $Z_\mu(x)$ spanning the Z_N gauge theory:

$$(i) \quad \sum_{x,\mu} |\text{Tr} U_\mu^g(x)|^2 \xrightarrow{g} \max, \quad (3)$$

$$(ii) \quad \Re \text{Tr} (U_\mu^g(x) Z_\mu^\dagger(x)) \xrightarrow{g} \max. \quad (4)$$

Step (i) is the difficult part involving gauge fixing and the interference of Gribov ambiguities (see comments below). The projection step (ii) operates locally and can be implemented straightforwardly. Using a standard iteration over-relaxation procedure [23] for step (i) finally defines vortex matter with a sensible phenomenology in the continuum limit: for SU(2), it was observed that the so-called “vortex-only” configurations, defined by the center links $Z_N(x)$ reproduce a good deal of the string tension while “vortex-removed” configurations, spanned by the links

$$\tilde{U}_\mu(x) = Z_\mu^\dagger(x) U_\mu(x), \quad (5)$$

do not support a linear rising static potential at large distances [22–24]. It was put forward in [25] that this heuristically defined vortex texture has sensible properties in the continuum limit. It was subsequently discovered that these vortices provide an intriguing picture of the deconfinement phase transition at finite temperatures [26]. They even admit detailed insights in the critical phenomenon [27,28] in complete agreement with studies of the free energy of gauge-invariant center vortices over this transition [29].

We point out that presentday algorithms [30] are only capable of obtaining one of the many local maxima of the gauge-fixing condition (3) and that the vortex properties do depend on the set of maxima [31–33]. Localizing the global maximum of Eq. (3) would remove this ambiguity, but it is not clear whether the vortex matter arising from the global maximum would be of any phenomenological value. In fact, an anticorrelation was found [32]: the larger the gauge-fixing functional (3), the lesser the string tension obtained from vortex-only configurations. Although we have not yet found a concise mathematical description, the vortex matter best for phenomenological studies seems to arise from an average over Gribov copies.

Note also that the relation between vortices and quark confinement is far less striking for the gauge group SU(3): despite many attempts, the string tension arising from vortex-only configurations is systematically smaller than the full string tension [5,34,35]. The difference in quality between the SU(2) and SU(3) vortex picture indicates that we are still missing a point for SU(3) and partially motivated the present investigation of the SU(2) quark propagator.

Because of the above ambiguity, we are going to present the details of the gauge-fixing procedure which gives rise to an intriguing vortex phenomenology. For SU(2), the gauge-fixing matrix $g(x) \in SU(2)$ can be viewed as a 4-dimensional unit-vector in Euclidean color space:

$$g(x) = g_0(x) + i\vec{\tau} \vec{g}(x), \quad G(x) = \begin{pmatrix} g_0(x) \\ \vec{g}(x) \end{pmatrix},$$

where

$$g_0^2(x) + \bar{g}^2(x) = 1.$$

Maximizing the gauge-fixing functional (3) is an iterative procedure: choosing a particular site x_0 and setting $g(x \neq x_0) = 1$, the functional is locally maximized, and the links are updated accordingly: $U_\mu \rightarrow U_\mu^g$. Subsequently, all sites are visited and many sweeps through the lattice are performed until the gauge-fixing action does not change anymore within the required precision.

$$M(x) = \sum_{\mu=1}^4 \begin{pmatrix} (u_\mu^0(x))^2 + (u_\mu^0(x-\mu))^2 & -u_\mu^0(x)u_\mu^i(x) + u_\mu^0(x-\mu)u_\mu^i(x-\mu) \\ -u_\mu^0(x)u_\mu^i(x) + u_\mu^0(x-\mu)u_\mu^i(x-\mu) & u_\mu^i(x)u_\mu^k(x) + u_\mu^i(x-\mu)u_\mu^k(x-\mu) \end{pmatrix}. \quad (6)$$

We also introduce the eigenvectors and eigenvalues of the matrix M :

$$e_k, \lambda_k \quad \text{with } k = 1 \dots 4.$$

Choosing the largest eigenvector for the gauge transformation, i.e., $G(x_0) = e_{\max}$, the local increase of the gauge-fixing functional is maximized:

$$s_{\text{fix}} = \lambda_{\max}.$$

This choice gives rise to the standard iteration procedure which is usually employed for maximal center gauge (MCG) fixing [23].

Here, we depart from this standard procedure and introduce an aspect of simulated annealing: we choose $G(x_0) = e_k$ with a relative probability of $\exp\{\beta_f \lambda_k\}$, where β_f is an auxiliary parameter familiar from simulated annealing. For large β_f , the probability for picking the largest eigenvalue is high, and our method smoothly merges with the standard scheme. In practice, we started with $\beta_f = 0.02$ and performed 25 sweeps through the lattice until we increased β_f by 0.1. The procedure stopped when no further increase of the gauge-fixing functional was achieved.

It turns out that the vortex matter arising from this procedure has good phenomenological properties such as good scaling properties in the continuum limit. To test the latter aspect, we calculated the planar vortex area density ρ in units of the (measured) string tension for several values of the lattice spacing a using the standard Wilson action. Figure 1 illustrates how the vortex density becomes independent of the lattice regulator for sufficiently small values of the lattice spacing.

B. Fixing to Landau gauge

To fix configurations to Landau gauge an overrelaxation algorithm was used. This is an iterative algorithm that maximizes the Landau-gauge functional

$$F_U[g] = \frac{1}{4VN_c} \sum_{x,\mu} \Re \text{Tr}[g_x U_{x,\mu} g_{x+\hat{\mu}}^\dagger] \quad (7)$$

For the local update $g(x_0)$, let s_{fix} denote that part of the gauge-fixing functional (3) which is affected by a change of $g(x_0)$:

$$s_{\text{fix}} = G^T(x_0)MG(x_0) - \lambda(G^T(x_0)G(x_0) - 1),$$

where λ is a Lagrange multiplier and M is a real symmetric 4×4 matrix given in terms of the link fields [$U_\mu(x) = u_\mu^0(x) + i\vec{\tau}\vec{u}_\mu(x)$]:

by changing the gauge transformation fields g_x locally but keeping U fixed. The algorithm stopped if an accuracy of

$$\max_x \text{Tr}[\partial_\mu A_{x,\mu} \partial_\mu A_{x,\mu}^\dagger] < 10^{-13} \quad (8)$$

was reached [36]. Here the gauge potential is defined as

$$A_{x,\mu} := \frac{1}{2ia g_0} (U_{x,\mu} - U_{x,\mu}^\dagger).$$

For the vortex-only configurations, Fourier acceleration was also used [39]. We applied the standard technique where each configuration was gauge-fixed once.

C. The AsqTad quark propagator

On those gauge-fixed configurations we calculate the quark propagator $S(x, y)$ in coordinate space by inverting the AsqTad fermion matrix. After Fourier-transforming

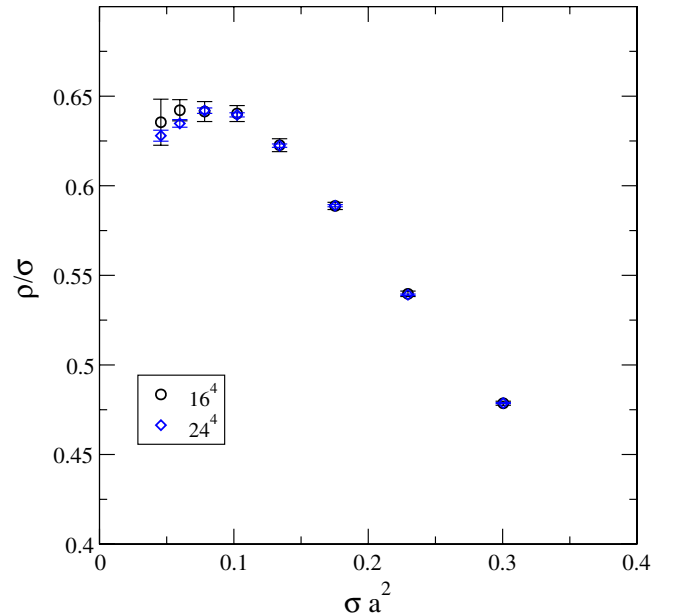


FIG. 1 (color online). Scaling of the planar vortex area density ρ in units of the string tension σ as function of the lattice spacing a .

$S(x, 0)$, the mass and dressing functions, M and Z , are extracted from S by suitable projections in Dirac space (see below). At this step it is important to know how the discrete lattice momenta

$$p_\mu = \pi k_\mu / L_\mu \quad \text{with } k_\mu \in (-L_\mu/2, L_\mu/2]$$

are related to the physical momenta $a^2 q^2(p)$ in lattice units. From experience we know it is always good practice to look at the tree-level form of the lattice propagators, and to define momenta such that the continuum tree-level expression is retrieved. This, known as tree-level correction [40], accounts for the lowest order discretization effects. With the AsqTad action the tree-level form of the quark propagator in Landau gauge reads

$$S_L^{-1}(a^2 p^2) = i \sum_{\mu=1}^4 \tilde{\gamma}_\mu a q_\mu (p_\mu) + ma,$$

where p_μ is as above and

$$a q_\mu (p_\mu) = \sin(p_\mu) [1 + \frac{1}{6} \sin^2(p_\mu)] \quad (9)$$

is the ‘‘kinematic momentum’’ (see, e.g., Ref. [15]). The

four matrices $\tilde{\gamma}_\mu$ form a staggered Dirac algebra (Eqs. (A.6) and (A.7) of Ref. [41]). Consequently, we use Eq. (9) to define momenta.

The functions M and Z are then extracted from $S_L(p)$ by taking the two traces in Dirac space

$$A_L(q, a) = \frac{i}{4N_c q^2 a^2} \text{tr}(\tilde{\gamma}_\mu q_\mu S_L(q, a)), \quad (10)$$

$$B_L(q, a) = \frac{1}{4N_c} \text{tr} S_L(q, a), \quad (11)$$

and combining them to (here $N_c = 2$)

$$Z_L(q, a) = \frac{q^2 a^2 A_L^2(q, a) + B_L^2(q)}{A_L(q, a)} \quad (12)$$

and

$$M_L(q, a) = \frac{B_L(q, a)}{A_L(q, a)}. \quad (13)$$

In the region of asymptotic scaling, $M_L(q, a)$ becomes independent of a and equals the running mass. Z needs to be renormalized as explained above. As we are only interested in qualitative changes of the momentum dependence

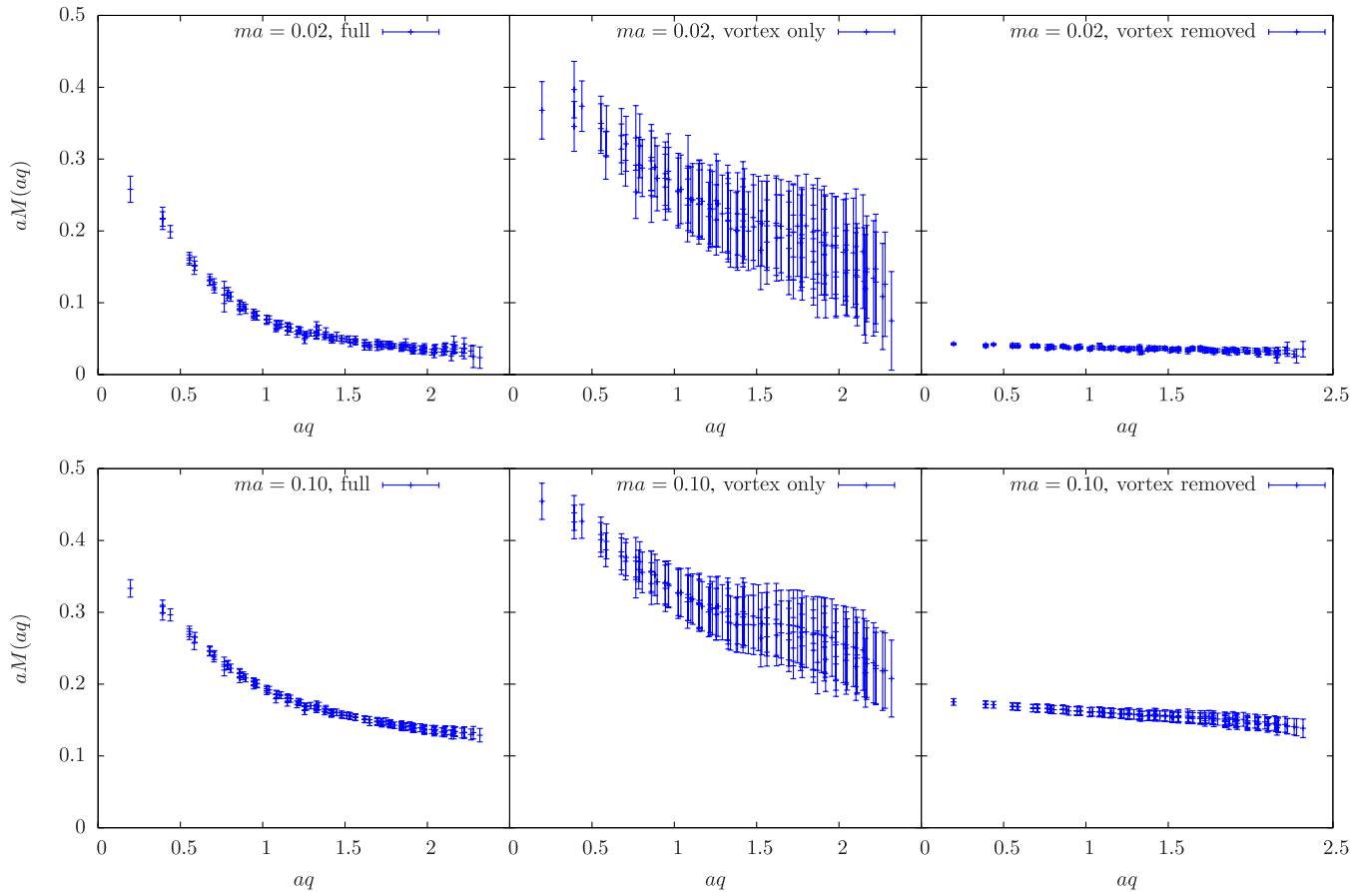


FIG. 2 (color online). The mass function $aM(q)$ versus kinematic momentum for two bare quark masses $ma = 0.02$ (top) and $ma = 0.10$ (bottom). We show data on full, vortex-only and vortex-removed configurations from left to right. Data has been cylinder cut. The nonperturbative enhancement of the mass function at low momenta is associated with the presence of center vortices.

of M and Z under the removal of center vortices, we prefer to work directly with unrenormalized quantities. Therefore, the presentation of the data is simplified by considering only bare lattice functions at a fixed lattice spacing. Consequently, lattice artefacts could not be directly assessed; however, for this value of the coupling, discretization errors are not expected to be significant.

A cylinder cut [40] is applied to all the data to reduce the effects of rotational symmetry violation.

IV. RESULTS

A. The mass function

In Fig. 2 we compare the mass function on our sets of full, vortex-only and vortex-removed configurations. Data was obtained for a range of bare quark masses from $ma = 0.020$ to $ma = 0.100$, and the two extremes are shown here as functions of momentum.

Our results on the full SU(2) configurations (see Fig. 2, left column) show a large enhancement near zero momentum, while data drops rapidly to its expected asymptotic

behavior at large momentum. Also, as expected, the infrared enhancement is stronger for the smaller bare quark mass. That is, our data for $M(q^2)$ on the full configurations clearly reproduce the well-known characteristics expected for the mass function in both the nonperturbative and perturbative regime.

On the vortex-removed configurations (see Fig. 2, right column), the mass function is more or less flat taking values slightly above ma . The momentum dependence of M is almost linear with a bigger slope for $ma = 0.1$. Thus, the dynamical contribution to the mass function, which we clearly see on the full configurations, disappears when center vortices are removed.

Interestingly, the mass function on the vortex-only configurations (see Fig. 2, middle column) depends quite strongly on momentum. Even though the signal is quite noisy, $M(q^2)$ grows almost linearly upon decreasing a^2q^2 . That is, much of the infrared enhancement of M is contained in the vortex-only part which clearly underlines the importance of center vortices as IR degrees of freedom.

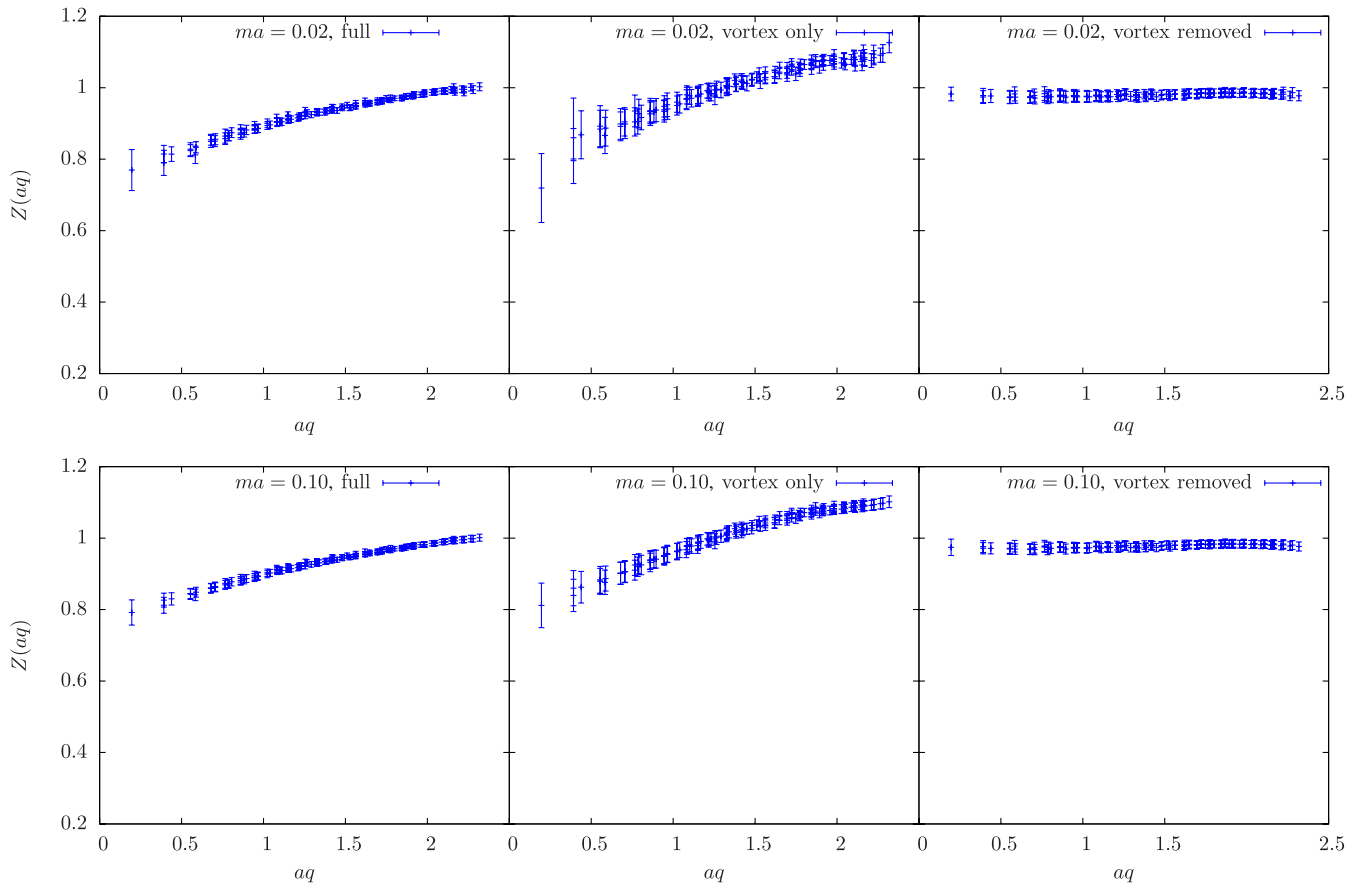


FIG. 3 (color online). The bare quark dressing function, Z , as a function of momentum for two bare quark masses $ma = 0.02$ (top) and $ma = 0.10$ (bottom). The left column displays data on full configurations, whereas in the middle and right column the data on vortex-only and vortex-removed configurations are shown, respectively. Data has been cylinder cut. The nonperturbative suppression at low momenta is associated with the presence of center vortices.

B. The quark dressing function

The same comparison for the bare quark dressing function, Z , is shown in Fig. 3. Again, we show data at $ma = 0.02$ and $ma = 0.10$ on full SU(2) configuration in the left column, while the middle and right column displays data on vortex-only and vortex-removed configurations, respectively. As expected, on the full configurations, the quark dressing function takes values around one at large momenta and becomes suppressed towards lower momenta. The smaller the quark mass the more pronounced the dip at lower momenta. If center vortices are removed, the infrared suppression disappears and Z is a flat function of momentum (see the right column). There, Z roughly stays at its tree-level value. Surprisingly, on the vortex-only configurations, Z has a similar momentum dependence to the full configurations. Again, the results are noisier, but the infrared suppression is unambiguous.

V. CONCLUSIONS

We have studied the Landau-gauge quark propagator in quenched SU(2) gauge theory under the removal of center vortices. Our implementation of this propagator is based on the AsqTad-improved staggered quark action modified to SU(2). The full propagator is found to strongly resemble that of the SU(3) theory.

Our results for the mass and quark dressing functions unambiguously show the disappearance of $D\chi_{SB}$ when center vortices are removed. This is in contrast to the situation in SU(3) [10,11]. There, even after center-vortex removal dynamical mass generation survives while the string tension is flat.

Additionally, we have studied the quark propagator on vortex-only configurations. Even though the signal is quite noisy, both parts of the propagator reveal a form qualitatively similar to the full, untouched configurations.

In summary, our SU(2) results clearly represent a strong relationship between the vortex picture and spontaneous breaking of chiral symmetry. We do not expect this to be merely a feature of staggered fermions, however it might be valuable to repeat this study with another type of fermion, in particular, after the new results presented in Ref. [9]. More importantly, whether it is possible to observe similar behavior in SU(3) [or ultimately SU(N)] is still an open question.

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- [1] E. Laermann and O. Philipsen, *Annu. Rev. Nucl. Part. Sci.* **53**, 163 (2003).
 - [2] F. Synatschke, A. Wipf, and K. Langfeld, *Phys. Rev. D* **77**, 114018 (2008).
 - [3] P. de Forcrand and M. D'Elia, *Phys. Rev. Lett.* **82**, 4582 (1999).
 - [4] C. Alexandrou, M. D'Elia, and P. de Forcrand, *Nucl. Phys. B, Proc. Suppl.* **83**, 437 (2000).
 - [5] K. Langfeld, *Phys. Rev. D* **69**, 014503 (2004).
 - [6] J. Gattnar *et al.*, *Nucl. Phys.* **B716**, 105 (2005).
 - [7] P. Y. Boyko *et al.*, *Nucl. Phys.* **B756**, 71 (2006).
 - [8] V. G. Bornyakov *et al.*, *Phys. Rev. D* **77**, 074507 (2008).
 - [9] R. Höllwieser, M. Faber, J. Greensite, U. M. Heller, and S. Olejnik, arXiv:0805.1846.
 - [10] D. B. Leinweber *et al.*, *Nucl. Phys. B, Proc. Suppl.* **161**, 130 (2006).
 - [11] P. O. Bowman *et al.* (unpublished).
 - [12] P. O. Bowman, U. M. Heller, D. B. Leinweber, A. G. Williams, and J. B. Zhang, *Lect. Notes Phys.* **663**, 17 (2005).
 - [13] J. I. Skullerud and A. G. Williams, *Phys. Rev. D* **63**, 054508 (2001).
 - [14] J. Skullerud, D. B. Leinweber, and A. G. Williams, *Phys. Rev. D* **64**, 074508 (2001).
 - [15] P. O. Bowman, U. M. Heller, and A. G. Williams, *Phys. Rev. D* **66**, 014505 (2002).
 - [16] P. O. Bowman, U. M. Heller, D. B. Leinweber, and A. G. Williams, *Nucl. Phys. B, Proc. Suppl.* **119**, 323 (2003).
 - [17] F. D. R. Bonnet, P. O. Bowman, D. B. Leinweber, A. G. Williams, and J.-b. Zhang (CSSM Lattice), *Phys. Rev. D* **65**, 114503 (2002).
 - [18] J. B. Zhang, P. O. Bowman, D. B. Leinweber, A. G. Williams, and F. D. R. Bonnet (CSSM Lattice), *Phys. Rev. D* **70**, 034505 (2004).
 - [19] M. B. Parappilly *et al.*, *Phys. Rev. D* **73**, 054504 (2006).
 - [20] W. Kamleh, P. O. Bowman, D. B. Leinweber, A. G. Williams, and J. Zhang, *Phys. Rev. D* **76**, 094501 (2007).
 - [21] P. O. Bowman, U. M. Heller, D. B. Leinweber, A. G. Williams, and J.-b. Zhang, *Nucl. Phys. B, Proc. Suppl.* **128**, 23 (2004).
 - [22] L. Del Debbio, M. Faber, J. Greensite, and S. Olejnik, *Phys. Rev. D* **55**, 2298 (1997).
 - [23] L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and S. Olejnik, *Phys. Rev. D* **58**, 094501 (1998).
 - [24] J. Greensite, *Prog. Part. Nucl. Phys.* **51**, 1 (2003).
 - [25] K. Langfeld, H. Reinhardt, and O. Tennert, *Phys. Lett. B* **419**, 317 (1998).
 - [26] M. Engelhardt, K. Langfeld, H. Reinhardt, and O. Tennert, *Phys. Rev. D* **61**, 054504 (2000).
 - [27] K. Langfeld, *Phys. Rev. D* **67**, 111501 (2003).
 - [28] K. Langfeld, G. Schulze, and H. Reinhardt, *Phys. Rev. Lett.* **95**, 221601 (2005).

- [29] P. de Forcrand and L. von Smekal, Phys. Rev. D **66**, 011504 (2002).
- [30] L. Giusti, M. L. Paciello, C. Parrinello, S. Petrarca, and B. Taglienti, Int. J. Mod. Phys. A **16**, 3487 (2001).
- [31] T. G. Kovacs and E. T. Tomboulis, Phys. Lett. B **463**, 104 (1999).
- [32] V. G. Bornyakov, D. A. Komarov, and M. I. Polikarpov, Phys. Lett. B **497**, 151 (2001).
- [33] M. Faber, J. Greensite, and S. Olejnik, Phys. Rev. D **64**, 034511 (2001).
- [34] A. O. Cais *et al.*, Proc. Sci., LAT2007 (2007) 321 [arXiv:0710.2958].
- [35] A. O. Cais *et al.*, arXiv:0807.0264.
- [36] Here a note is in order. Maximizing $F_U[g]$ is not unique as there are different g 's all satisfying relation (8). This ambiguity, known as the Gribov-copy problem, has been shown to systematically affect data, e.g., of the Landau-gauge ghost propagator, while for others, e.g., for the gluon propagator in the same gauge, the impact stays within statistical errors (see, e.g., Ref. [37]). Neither does maximization sample all Gribov copies. In that, lattice Landau gauge crucially differs from the continuum Landau gauge which is based on a BRST average over *all* copies. The modified lattice Landau gauge of [38], however, provides a framework for lattice BRST without Neuberger 0/0 problem which prevented that for 20 odd years.
- [37] A. Sternbeck, E.-M. Ilgenfritz, M. Müller-Preussker, and A. Schiller, Phys. Rev. D **72**, 014507 (2005).
- [38] L. von Smekal, D. Mehta, A. Sternbeck, and A. G. Williams, Proc. Sci., LAT2007 (2007) 382 [arXiv:0710.2410].
- [39] C. T. H. Davies *et al.*, Phys. Rev. D **37**, 1581 (1988).
- [40] D. B. Leinweber, J. I. Skullerud, A. G. Williams, and C. Parrinello (UKQCD Collaboration), Phys. Rev. D **60**, 094507 (1999).
- [41] P. O. Bowman *et al.*, Phys. Rev. D **71**, 054507 (2005).